Elementary operators and their lengths

Abstract

Elementary operators on an algebra, which are finite sums of operators $x \mapsto axb$, provide a way to study properties of the algebra. In particular, for $C^*$-algebras we consider results that are related to the length $\ell$ of the operator, defined as the minimal number of summands required. We will review some results concerning complete positivity or complete boundedness. Although all elementary operators on a $C^*$-algebra $A$ are completely bounded, that is induce uniformly bounded operators on the algebras $M_n(A)$, the supremum is always attained for $n = \ell$, or for smaller $n$ in case $A$ has special structure. For positivity, there are also results couched in analagous terms, but with different bounds.

In recent work with I. Gogić, we have shown that for prime $C^*$-algebras $A$ the elementary operators of length (at most) 1 are norm closed, but that for the rather tractable class of homogeneous $C^*$-algebras more subtle considerations are required for closure. For instance $A = C_0(X, M_n)$ fails to have this closure property if $X$ is an open set in $\mathbb{R}^d$ with $d \geq 3$, $n \geq 2$ ($X \neq \emptyset$).