Orr Shalit

Technion

Dilations, inclusions of matrix convex sets, and completely positive maps

Abstract

In this talk I will present a part of a recent joint work with Davidson, Dor-On, and Solel (a complementary talk will be given by Adam Dor-On in the *Multivariable Operator Theory* special session).

If $A = (A_1, \ldots, A_d)$ is a tuple of operators on H and $B = (B_1, \ldots, B_d)$ is a tuple of operators on K, then B is said to be a *dilation* of A, denoted $A \prec B$, if $A_i = P_H B_i\big|_H$ for all i. For a long time it seemed that the name of the game was: given a commuting tuple of operators A, find a commuting tuple of *normal* operators B such that $A \prec B$ (usually with additional conditions on the joint spectrum $\sigma(B)$, and requiring the dilation to hold for powers as well). Quite recently, Helton, Klep, McCullough and Schweighofer changed the rules, and started dilating tuples of *noncommuting* operators to commuting tuples of normal operators. They showed that there is a universal constant ϑ_n, such that given a tuple of $n \times n$ selfadjoint contractions A, there exists a tuple of commuting selfadjoints B, such that $\sigma(B) \subseteq [-1,1]^d$ and $\frac{1}{\vartheta_n} A \prec B$. This result had deep implications to spectrahedral inclusion problems.

The constant ϑ_n behaves roughly like $\sqrt{n}$, and was shown to be the best constant possible. We were led to ask whether it is possible to obtain such a dilation result with a constant that does not depend on $n = \operatorname{rank} A$ (necessarily fixing d). Moreover, we sought a normal dilation B with more precise control on the joint spectrum $\sigma(B)$. As a representative of our results, I will present the following theorem, as well as some applications.

Theorem. *Let K be a convex set in $\mathbb{R}^d$ satsfying some reasonable conditions. Then for every d-tuple A of selfadjoint operators with a joint numerical range contained in K, there is a d-tuple of commuting selfadjoint operators B with joint spectrum $\sigma(B) \subseteq K$, such that*

$$\frac{1}{d} A \prec B.$$

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