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A band formula for a Toeplitz commutant lifting problem

Abstract

The band method plays a fundamental role in solving a Toeplitz and Nehari interpolation problem; see [2]. The solution to the Nehari problem involves the inverses of \( I - HH^* \) and \( I - H^*H \) where \( H \) is the corresponding Hankel matrix. Here we will derive a similar result for a certain commutant lifting problem.

Let \( \Theta \) be an inner function in \( H^\infty(\mathcal{E}, \mathcal{Y}) \) and \( \mathcal{H}(\Theta) \) the subspace of \( \ell^2_+ \) defined by

\[
\mathcal{H}(\Theta) = \ell^2_+ (\mathcal{Y}) \ominus T^* \Theta \ell^2_+ (\mathcal{E})
\]

where \( T_\Theta \) is the Toeplitz operator determined by \( \Theta \). Clearly, \( \mathcal{H}(\Theta) \) is an invariant subspace for the backward shift \( S^*_Y \). Consider the data set \( \{ A, T', S_Y \} \) where \( A \) is a strict contraction mapping \( \ell^2_+ (\mathcal{U}) \) into \( \mathcal{H}(\Theta) \), the operator \( T' \) on \( \mathcal{H}(\Theta) \) is the compression of \( S_Y \) to \( \mathcal{H}(\Theta) \), that is,

\[
T' = \Pi_{\mathcal{H}(\Theta)} S_Y |_{\mathcal{H}(\Theta)}
\]

where \( \Pi_{\mathcal{H}(\Theta)} \) is the orthogonal projection from \( \ell^2_+ (\mathcal{Y}) \) onto \( \mathcal{H}(\Theta) \). Moreover, \( A \) intertwines \( S_\mathcal{U} \) with \( T' \), that is, \( T'A = AS_\mathcal{U} \). Given this data set the commutant lifting problem is to find all contractive Toeplitz operators \( T_\Psi \) such that

\[
\Pi_{\mathcal{H}(\Theta)} T_\Psi = A.
\]

(1)

This lifting problem includes the Nevanlinna-Pick and Leech interpolation problems. Using two different methods we will show that the set of all solutions are given by

\[
\Psi = (\Upsilon_{12} + \Upsilon_{11}g) (\Upsilon_{22} + \Upsilon_{21}g)^{-1}.
\]

Here \( g \) is a contractive analytic function acting between the appropriate spaces. Analogous to the band formulas in the Nehari interpolation problem, \( \Upsilon_{jk} \) are determined by the inverses of \( I - AA^* \) and \( I - A^*A. \) The proofs rely on different techniques. Finally, this is joint work with S. ter Horst and M.A. Kaashoek.

References


Talk time: 07/18/2016 3:00PM—07/18/2016 3:20PM
Talk location: Cupples I Room 113

Special Session: State space methods in operator and function theory. Organized by J. Ball and S. ter Horst.