On singular integral operators with linear-fractional involutions

Abstract

We denote the Cauchy singular integral operator along a contour $\mathcal{L}$ by $(S_\mathcal{L}\varphi)(t) = \frac{1}{\pi i} \int_{\mathcal{L}} \frac{\varphi(\tau)}{\tau - t} \, d\tau$ and the identity operator by $(I_\mathcal{L}\varphi)(t) = \varphi(t)$.

In the paper [1,2] we constructed a similarity transformation $F^{-1} AF = D$, between the singular integral operators $A$ with the rotation operator $W_T$ through the angle $2\pi/m$ on the unit circle $T$, acting on the space $L_2^2(T)$, and a certain matrix characteristic singular integral operator without shifts acting on the space $L_2^2(T)$. For $m = 2$, we have $(W_T\varphi)(t) = \varphi(-t),

A = a_0 I_T + b_0 S_T + a_1 W_T + b_1 S_T W_T, \quad A \in [L_2(T)], \quad D = u I_T + v S_T, \quad D \in [L_2^2(T)].$

the right hand and left-hand side we reduced

$$B_R = a I_R + b Q_R + c S_R + d Q_R S_R, \quad B_R \in [L_2(R)], \quad R = (+\infty, -\infty),$$

where involution $(Q_R\varphi)(x) = \sqrt{\delta^2 + \beta x - \delta^2\varphi(x)}$, $\alpha(x) = \frac{\delta^2 + \beta}{x - \delta}$, $\delta^2 + \beta > 0$,

to a matrix characteristic singular integral operator without shift:

$$\mathcal{H}BF = D_{R_+}, \quad D_{R_+} = u I_{R_+} + v S_{R_+},$$

acting on the space $L_2^2(R_+, x^{-\frac{1}{2}})$, $R_+ = (0, +\infty)$. We will refer to the formulas as operator equalities. Different applications of operator equalities to singular integral operators and to boundary value problems are considered.
