Regular and positive noncommutative rational functions

Abstract

Call a noncommutative rational function \( r \) regular if it has no singularities, i.e., \( r(X) \) is defined for all tuples of self-adjoint matrices \( X \). In this talk regular noncommutative rational functions \( r \) will be characterized via the properties of their (minimal size) linear systems realizations \( r = c^*L^{-1}b \). Our main result states that \( r \) is regular if and only if \( L = A_0 + \sum_j A_j x_j \) is privileged. Roughly speaking, a linear pencil \( L \) is privileged if, after a finite sequence of basis changes and restrictions, the real part of \( A_0 \) is positive definite and the other \( A_j \) are skew-adjoint. Afterwards I will speak about a solution to a noncommutative version of Hilbert’s 17th problem: a positive regular noncommutative rational function is a sum of squares.

The talk is based on the joint work with I. Klep and J. E. Pascoe.

Talk time: 07/21/2016 3:30PM— 07/21/2016 3:50PM
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