Horn inequalities for singular values of products of operators

Abstract

Consider Hermitian matrices $A, B, C$ such that $A + B = C$. Let $\{\lambda_j(A)\}$, $\{\lambda_j(B)\}$, and $\{\lambda_j(C)\}$ be sequences of eigenvalues of $A, B,$ and $C$ counting multiplicity, arranged in decreasing order. In 1962, A. Horn conjectured that the relations of $\{\lambda_j(A)\}$, $\{\lambda_j(B)\}$, and $\{\lambda_j(C)\}$ can be characterized by a set of inequalities defined inductively. This conjecture was proved true by Klyachko and Knutson-Tao in the late 1990s. A related question, the multiplicative Horn problem, asks to describe the possible singular values of $AB$ when the singular values of $A$ and $B$ are given. This problem is fully solved in the case where $A$ and $B$ are invertible matrices as Klyachko showed that it is equivalent to the additive problem after taking logarithms. In this talk, we will discuss the case when $A$ and $B$ are not necessarily invertible and its generalization to the von Neumann algebra setting. This is joint work with H. Bercovici, B. Collins, and K. Dykema.

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