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## A criterion for the solvability of a $\mu$ -synthesis problem

### Abstract

We give a solvability criterion for the following  $\mu$ -synthesis problem. Let  $\mu$  be the structured singular value for the diagonal matrices with entries in  $\mathbb{C}$ .

**Problem.** Given distinct points  $\lambda_1, \dots, \lambda_n$  in the open unit disc  $\mathbb{D}$  and target  $2 \times 2$  complex matrices  $W_1, \dots, W_n$  such that  $\mu(W_j) \leq 1$  for all  $j = 1, \dots, n$ , find a holomorphic  $2 \times 2$  matrix function  $F$  on  $\mathbb{D}$  such that  $F(\lambda_j) = W_j$  for each  $j$ , and  $\mu(F(\lambda)) \leq 1$  for all  $\lambda \in \mathbb{D}$ .

By [1, Theorem 9.2], this problem is equivalent to the following interpolation problem: does there exist a holomorphic function  $x$  from the disc to the tetrablock  $\overline{\mathbb{E}}$  such that  $x(\lambda_j) = (w_{11}^j, w_{22}^j, \det W_j)$  for each  $j$ ? The tetrablock is the domain in  $\mathbb{C}^3$  defined by

$$\overline{\mathbb{E}} := \{(x_1, x_2, x_3) \in \mathbb{C}^3 : 1 - x_1 z - x_2 w + x_3 z w \neq 0 \text{ for all } z, w \in \mathbb{D}\}.$$

In this talk we show such an  $x$  exists if and only if, for distinct  $z_1, z_2, z_3 \in \mathbb{D}$ , there are positive  $3n$ -square matrices  $[N_{il,jk}]$ , of rank 1, and  $[M_{il,jk}]$  such that

$$\left[ 1 - \frac{\overline{z_l} x_{3i} - x_{1i} \overline{z_k} x_{3j} - x_{1j}}{x_{2i} z_l - 1} \frac{z_k x_{3j} - x_{1j}}{x_{2j} z_k - 1} \right] \geq [(1 - \overline{z_l} z_k) N_{il,jk}] + [(1 - \overline{\lambda_i} \lambda_j) M_{il,jk}],$$

where  $(x_{1j}, x_{2j}, x_{3j}) = (w_{11}^j, w_{22}^j, \det W_j)$  for each  $j$ .

The talk is based on a joint work with Z. A. Lykova and N. J. Young. [1] A. A. Abouhajar, M. C. White and N. J. Young, A Schwarz lemma for a domain related to  $\mu$ -synthesis, *J. Geom. Anal.* 17, (2007), pp. 717-750

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