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Essays in Macroeconomics and Financial Economics

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Essays in Macroeconomics and Financial Economics

by

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ABSTRACT OF THE DISSERTATION

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This dissertation consists of three independent articles in the fields of Macreconomics and Financial Economics. Chapter one investigates the determinants of the demand for bonds of different maturities and the relationship with differences in idiosyncratic risk in an heterogeneous agent framework. Chapter two studies the effect of policy instability on the risk-return trade-off of different financial assets. Chapter three studies macroeconomic risk in an incomplete market economy.

In the first chapter ”Heterogenous Liquidity Demand and the Term Structure of Interest Rates” I study what determines differences in the demand for bonds of different maturities. I focus on the effect of differences in idiosyncratic risk. I find evidence that relates the demand for bonds of different maturities with earnings risk. To provide a rationale for these findings, I build a continuous-time, general equilibrium model with heterogeneous agents, two assets and incomplete markets. The model succesfully reproduces the fact that high idiosyncratic risk is associated with high demand for short-term assets, while low idiosyncratic risk is related to high demand for long-term assets.

The second chapter ”Policy Instability and the Risk-Return Trade-Off”, coauthored with Rodolfo Manuelli, we study what is the impact of large swings in economic policy on the risk-return trade-off faced by investors. We use data from Argentina—a country that has experienced frequent and very large regime changes—and find that the risk-return for individual assets and minimum variance portfolios are quite different across regimes. We then develop a dynamic model to understand

optimal portfolios when investors are cognizant that regimes can change. We find that when portfolios are unrestricted, it is optimal for investors to take a large amount of risk. On the other hand, when portfolios are restricted to include only long positions, a real asset (real estate) dominates financial assets.

The third chapter ”Incomplete Markets and Macroeconomic Risk” analyze the equilibrium dynamics of asset prices, investment and risk premia in an incomplete financial market economy that is subject to aggregate risk shocks. It also addresses the question on how economic conditions endogenously affect risk in the economy. To this end, I use a continuous time macroeconomic model with financial frictions. The main findings of this article are that an exogenous increase in aggregate risk causes an increase in asset price volatility, an increase in risky asset returns through an increase in risk-premia, a decline in asset prices, investment and risk free interest rates. Moreover, the model presented here is able to reproduce counter-cyclical endogenous risk. This results also depend on how constrained are financial intermediaries. If financial intermediaries are constrained some of the results are amplified.

Chapter 1: Heterogeneous Liquidity Demand and the Term Structure of Interest Rates

What determines the demand for bonds across maturities? Canonical macroeconomic models of the term structure of interest rates rely on the assumption of exogenous financial market segmentation. This paper provides a theory of the demand for assets across maturities that is both consistent with the empirical facts and the behavior of the yield curve. First, I find micro-evidence that relates the demand for different maturities with earnings risk. Second, I present a continuous time macroeconomic model with incomplete markets, heterogeneous agents and two assets, where agents have heterogeneous demand for liquidity, driven by differences in idiosyncratic risk. The model provides micro-foundations for differences in the demand for bonds of different maturities and shows that, consistent with the empirical findings, agents with high idiosyncratic risk tend to allocate their assets in short-term horizons, while agents with low idiosyncratic risk results in higher demand for long-term bonds.

Keywords: Term Structure of Interest Rates, Heterogeneous Agents, Incomplete Markets, Continuous-Time Macroeconomics

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1.1 Introduction

What determines the demand for bonds across maturities? Time-series on the yields of safe assets (*e.g.* US Treasuries) show on average a positive difference between long- and short-term interest rates. Hence, long-term bonds like US Treasuries command a positive term premium. Standard macroeconomic models have difficulties accounting for this term premium (see Mehra and Prescott, 1985). This has given rise to many theories that try to explain asset prices.[1] One of the most popular theories of the term structure is the preferred habitat theory, which successfully reproduces term premia across maturities. However, it relies on the fundamental assumption of exogenous market segmentation, which means that investors have specific preferences across maturities (a *preferred habitat*) and these are exogenously given. Similar types of exogenous market segmentation are also present in some models that analyze the effects of policies on the long end of the yield curve, like Quantitative Easing or Tightening, described in Gertler and Karadi (2011) and Carlstrom, Fuerst and Paustian (2017). Indeed, what these models show is that introducing market segmentation is an important feature in explaining the term structure of interest rates or the effect of some policies on the yield curve.

The objective of this paper is to study empirically and theoretically what determines the demand for bonds of different maturities. To that end, I focus on the effect of differences in idiosyncratic risk across agents on the demand for different maturities. In particular, differences in idiosyncratic risk translate into differences in the demand for liquidity, which ultimately translate into differences in the demand for bonds of different maturities. In the empirical part of the paper I merge the Panel Study of Income Dynamics (PSID) with the Survey of Consumer Finances (SCF) by matching on observables. Using shared demographic factors in both surveys, I estimate income risk using the PSID. After obtaining these estimates, I predict earnings risk using the SCF in order to match asset holdings with a measure of income dynamics. I provide two main empirical findings: First, I find micro-evidence that negatively relates the demand for long-term assets with earnings risk. This

[1] See Cochrane (2017) for a review of the main theories in the asset pricing literature and Gürkaynak and Wright (2012) for the specific literature on the term structure of interest rates.

means that conditional on observables, higher income volatility translates into a higher fraction of the portfolio allocated in short-term bonds and vice-versa. Second, I find that households with low income hold a larger fraction of their portfolios in long-term assets.

Motivated by these findings, I develop a continuous-time general equilibrium model with two assets –short- and long-term bonds–, incomplete markets and heterogeneous agents. I introduce two sources of heterogeneity: (*i*) *ex-ante* heterogeneity, which results from the introduction of different types or groups of agents that face differences in idiosyncratic risk and (*ii*) *ex-post* heterogeneity, which results from the realization of idiosyncratic income shocks, as in standard Bewley-Aiyagari-Huggett models. These features create heterogeneity beyond the standard models of wealth distribution, they generate a heterogeneous demand for assets across types, mainly driven by differences in earnings risk. In particular, this environment creates heterogeneity in the demand for liquidity, without assuming differences in preferences or in the degree of access to financial markets.

The model is able to reproduce differences in the demand for maturities across agents, endogenously generating partial market segmentation, where in line with the empirical findings, households with stable income tend to hold a larger fraction of long-term assets, while households with riskier income, due to a higher demand for liquidity, hold a larger fraction of shorter maturities. Households with lower income hold a higher fraction of their wealth in long-term assets.

I perform welfare analyses for multiple scenarios using comparative statics. First, a higher supply of short-term bonds relative to long-term bonds, maintaining public debt constant, which can have a similar interpretation of an *Operation Twist*, yields to lower long-term interest rates and higher short-term rates, resulting in a flattening of the yield curve. The provision of liquidity is welfare improving. Second, scenarios of market distress, modeled as lower market liquidity, result in a steepening of the yield curve and a decline in welfare. Finally, changes in financial conditions, modeled as changes in the borrowing constraints, have an inverted U-shaped effect on welfare, meaning that too lose or too tight financial conditions decrease welfare in the economy.

Finally, I introduce transition dynamics to the model. The yield curve inverts after an unanticipated negative and transitory shock to supply, modeled as a productivity shock. The main mechanism behind this result is that, following a negative aggregate income shock, individuals respond with fire sales, mainly in short-term bonds, which generate a spike in the short-term rate, leading to a yield curve inversion.

The reminder of the paper is organized as follows. The rest of Section 1.1 discusses related literature. In Section 2, I present the empirical evidence on the demand for different financial assets. Section 3 describes the model used to rationalize the empirical findings. Section 4 describes the solution of the model and the results. Section 5 concludes.

Related literature

An overview of the literature linking macroeconomic factors to the term structure of interest rates can be found in Gürkaynak and Wright (2012). This paper relates to three main branches of the literature on macroeconomics and the term structure of interest rates. First, it is related to the literature of bond market segmentation, like the preferred habitat theory described in Vayanos and Vila (2021) and Ray (2019), or models for Quantitative Easing like Gertler and Karadi (2011) or Carlstrom, Fuerst and Paustian (2017) that also assume some form of segmentation in bond markets. As mentioned before, the difference is that I present a model that generates endogenous preferences for maturities, where agents are allowed to invest both in short- and long-term bonds. However, agents lean towards longer or shorter maturities depending on their idiosyncratic risk, leading to a different degree of exposure to different interest rates.

Second, the paper is related to the literature on heterogeneous agents and the term structure of interest rates.[2] Challe, Le Grand and Ragot (2013) analyze the term structure of interest rates in a model with incomplete markets, aggregate shocks and positive net supply of bonds. All these

[2] The use of heterogeneity to explain asset prices and risk premia dates from Heaton and Lucas (1992, 1996). Constantinides and Duffie (1996) and Krusell, Mukoyama and Smith (2011) explore asset prices under uninsurable idiosyncratic risk, reproducing asset pricing features, including the term premium on long-term bonds. However, these models adopt a maximally strict borrowing constraint, resulting in an autarkic –no trade– equilibrium.

features generate a liquidation risk, which arises from the risk of selling bonds at a low price, which with risk-averse investors, dominates the risk of selling bonds at higher prices. Thus, long-term bonds command a positive term premium. This paper diverges from existing literature in heterogeneous agents and the term structure of interest rates in the sense that these models do not have any differences in idiosyncratic risk. An implication of this is that in Challe, Le Grand and Ragot (2013), the same idiosyncratic state across agents will be reflected in the same demand for the same assets. In this paper, the same idiosyncratic state for different types of agents leads to a different demand for assets.

Third, this paper is also related to the literature that associates liquidity and the term structure of interest rates as in Mishkin (2015), Geromichalos, Herrenbrueck and Salyer (2016), Williamson (2016), Kozlowski (2021), and Wang (2023). The latter deserves more attention due to its similarities with the paper presented here. Wang (2023) studies how monetary policy influences the term premium through the liquidity channel. In the model, agents face a cash-in-advance constraint, as well as idiosyncratic liquidity demand shocks and asset market participation shocks, where the degree of asset market participation is what mainly determines the term premium. The model achieves endogenous market segmentation after introducing heterogeneous households, which differ in two dimensions: (i) the intertemporal discount rate, and (ii) the degree of asset market participation. This paper differs from Wang (2023) since households in my model have exactly the same preferences and full access to financial markets. The heterogeneity across types comes from the income process which, as shown in Section 3, all types have an income process with the same stationary distribution and unconditional moments, they only differ in the conditional distribution, given by the frequency at which idiosyncratic shocks arrive. Therefore, differences in the demand for liquidity arise from a very subtle difference in the income process. Moreover, the model I present in this paper is motivated by the empirical findings in Section 2.

Finally, the paper also contributes to recent literature on heterogeneity in long-term portfolios and the wealth distribution (see Greenwald et al.; 2021). In particular, it provides an explanation for heterogeneity in portfolio duration across households.

1.2 Empirical Facts

This section presents the empirical findings of the paper. I show new evidence that other models of the term structure ignore. I focus on earnings risk, measured by income volatility over time, and its impact on the allocation of financial assets across households. I merge the Survey of Consumer Finances with the Panel Study of Income Dynamics by matching on observable characteristics. I show evidence that supports that higher income volatility is associated with a higher demand or short-term liquid assets, while low earnings risk is associated with higher demand for long-term assets. Second, I show that households with low income hold a larger fraction of their portfolio in long-term assets.

1.2.1 Evidence on the demand for liquidity across households

This section contains evidence on the demand for different types of assets across households. The objective of the empirical analysis is to find evidence that help us understand the demand for liquidity and maturity in order to explain the degree of participation in different bond markets. I present a new fact that other papers leave aside. In particular, I focus on the relationship between income stability and asset allocations. The empirical analysis aims to show that households with more stable income tend to allocate their savings into long-term financial assets, while households with higher income volatility tend to hold a larger fraction of short-term financial assets.

The study utilizes data from the Survey of Consumer Finances (SCF) for the years 2013, 2016, and 2019. Notably, one main problem is that the SCF is a repeated cross-sectional survey and lacks individual income dynamics data. To address this limitation and link income dynamics with asset holdings, I use the Panel Study of Income Dynamics (PSID) for the years 2015, 2017 and 2019, which contains data from 2013 to 2018. There are some reasons to pick this sample and not longer samples. First, I drop all households from the PSID that have missing values, this ensures having a balanced panel, hence, using a larger sample period would imply to drop more observations resulting in a smaller sample. Second, due to methodological changes, I prefer not to use versions of the SCF that are made prior to 2013. Third, income exhibits large variations across the life-cycle, which could make the analysis of income volatility more complex, so I restrain the analysis to a short period of time.

To establish a connection between the two surveys and match income dynamics with asset holdings, I use the PSID to estimate relevant variables related to income dynamics based on shared demographic factors. These demographic variables are employed to estimate income variability across households. By using the PSID's estimated model, income volatility is predicted using the same demographic variables from the SCF, thereby enabling an observation of household portfolios based on estimated income dynamics. For specific details on data and estimation, see Appendix B.

It has been well-documented that income varies significantly over the life-cycle. To avoid factors inherent to the life-cycle affect measures of income stability over time, I use the the volatility of the real rate of growth of labor income as a proxy. With this measure I remove income trend over the life cycle. It is important to note that labor income refers to income earned by all activities that required households to work, whether they are working for someone else or self-employees.

Table 1.1: Descriptive statistics

	Log of Income Growth Volatility
Mean	-1.811862
S.D.	0.9164099
Skewness	0.0438141
Kurtosis	3.309285

Figure 1.1 shows income growth rate volatility in log scale across households for the sample period, which is close to normal as the skewness is close to zero and the kurtosis is close to 3. Table 1.1 shows descriptive statistics. The average variance of income growth across households is close to 0.18, which will be later used to calibrate the average income volatility in the model presented in Section 1.3.

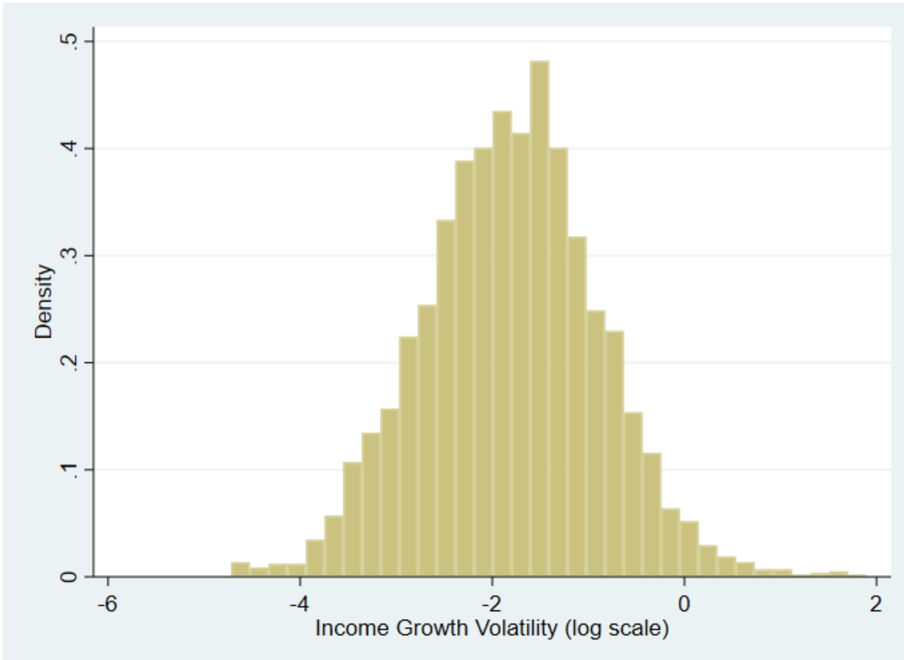

Figure 1.1: Distribution of the variance of income growth rate in logs

Income growth volatility in logarithmic scale is estimated through Ordinary Least Squares (OLS) with robust standard errors. Details on this intermediate estimation are in Appendix B. These estimations are subsequently applied to the SCF data to predict income volatility based on shared demographic variables. Since we want to estimate the demand for liquidity in order to find an explanation for market segmentation across bond maturities, we need to specify the dependent variables. Since the SCF does not report bond holdings for different maturities, I use proxies for these variables. I use cash and deposits as a proxy for short-term bonds and fixed income securuties as a proxy for long-term bonds. Since we want to remove possible effects from wealth, I use cash and deposits as a fraction of financial wealth and fixed income as a fraction of financial wealth as dependent variables. This removes possible biases due to differences in wealth. Below is a list of assets that are cathaloged as cash and deposits, as well as fixed income.

- Cash and deposits as short-term investments:
  - Checking accounts
  - Savings accounts

  - Money Market accounts
  - Call accounts
  - Certificates of deposit
  - Prepaid cards
- Fixed income as long-term investments
  - Retirement accounts
  - Cash value of life-insurance
  - Annuities and trusts
  - Mutual Funds excluding stocks
  - Savings bonds
  - Directly held bonds
  - Other miscelaneous financial assets

I describe the data from the SCF used in the analysis below. First, the main independent variable is:

- Conditional Income Growth Volatility (log scale; $\log(CondIncVol)$): This variable is obtained by employing the estimates from the PSID to demographic factors in the SCF. Since it is obtained by conditioning on these demographic variables, it is a conditional variance for the real rate of growth of income between 2013 and 2018. This is the main independent variable.
- Income (in log scale; $\log(Inc)$): Refers to labor income, as in the PSID, coming from actibvities that requires households to work, wether they are employees or self-employees.

The control variables are:

- Financial Wealth (in log scale; $\log(Fin)$): This consists on all financial assets held by households, minus liabilities. I use it as a control.

- Attitude towards risk ($Risk$): Refers to the response of households on how willing are them to take financial risks. The lowest value is 0, meaning that the household reports the highest degree of risk tolerance, and is bounded above by 4, which is the highest degree of risk aversion. I use it as a control.
- Age (Age): I use it as a control.

I estimate the portfolio shares on fixed income first and then for cash and deposits through ordinary least squares (OLS) with robust standard errors. Note that since financial wealth also includes stocks, the sum of cash the two dependent variables do not add to 1. Hence, I present results on both regressions. Later, I show an alternative measure of wealth that excludes stocks.

The econometric specification for long-term bonds is given by equation 1.1. For short-term bonds, the specification is similar except for the dependent variable. Results are presented in Tables 2 and 3.

$$FI/FW = \beta_0 + \beta_1 \log(\mathit{CondIncVol}) + \beta_2 \log(\mathit{Inc}) + \underline{\beta}\underline{X} + \varepsilon \tag{1.1}$$

where $\underline{X}$ is a vector of controls and ε is normally distributed with zero-mean.

Table 1.2: Fixed Income/Financial Wealth (%)

	(1)	(2)	(3)
$\log(CondIncVol)$	-4.638***	-4.306***	-4.268***
	(-8.01)	(-7.49)	(-7.41)
$\log(FW)$	8.361***	9.229***	9.300***
	(64.73)	(55.09)	(53.33)
Age	0.0178	0.0322	0.0246
	(0.62)	(1.13)	(0.85)
$RiskAversion$	0.712		0.675
	(1.62)		(1.53)
$\log(Inc)$		-3.927***	-3.926***
		(-7.53)	(-7.53)
$Constant$	-45.04***	-8.707*	-11.06**
	(-20.45)	(-1.78)	(-2.16)
R^2	0.32	0.32	0.32

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Results are highly encouraging. In Table 1.2, where the dependent variable is fixed income as a fraction of financial wealth, the coefficient associated with income volatility is negative and statistically significant for all specifications. Similarly, in Table 1.2.1, where the independent variable is cash and deposits as a fraction of financial wealth, the coefficient associated with income volatility is positive and statistically significant in all estimations. Higher income volatility is associated with larger fraction of the portfolio in short-term bonds. On the other hand, higher income is associated with a higher fraction of short-term assets, as a results of precautionary savings.

Table 1.3: Cash & Deposits/Financial Wealth (%)

	(1)	(2)	(3)
log($CondIncVol$)	3.823***	3.494***	3.551***
	(7.64)	(7.02)	(7.13)
log(FW)	-10.15***	-10.98***	-10.87***
	(-84.01)	(-66.11)	(-62.91)
Age	-0.0524*	-0.0436	-0.0551**
	(-1.93)	(-1.62)	(-2.02)
$Risk$	0.962**		1.014**
	(2.32)		(2.44)
log(Inc)		3.045***	3.048***
		(6.19)	(6.20)
$Constant$	151.8***	128.8***	125.3***
	(72.80)	(28.55)	(26.51)
R^2	0.46	0.46	0.46

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

I control for income and financial wealth (in log scale), age and attitude towards risk. All regressors are statistically significant. Both higher income and financial wealth have a positive impact on the portfolio share of fixed-income and a negative correlation with cash and deposits. And as expected, the higher is the degree of risk aversion, the lower is the portfolio share in fixed income and higher is the share on cash and deposits.

An alternative measure of financial wealth

Here I present results from using an alternative measure of wealth, which excludes stocks from it, so the fraction of cash and deposits and that of fixed income add to 1. This means that one dependent variable is perfectly negatively correlated with the other one, allowing us to show only one table, since results are the same, except they need to be multiplied by -1. Table shows the results of this estimation.

Table 1.4: $FI/(FI + C\&D)$ (%)

	(1)	(2)	(3)
log($CondIncVol$)	-4.230***	-3.909***	-3.923***
	(-7.74)	(-7.19)	(-7.21)
log(FW)	9.640***	10.41***	10.39***
	(77.73)	(61.75)	(59.22)
Age	0.0514*	0.0544**	0.0571**
	(1.85)	(1.96)	(2.03)
$Risk$	-0.207		-0.238
	(-0.48)		(-0.55)
log(Inc)		-3.127***	-3.127***
		(-6.16)	(-6.16)
$Constant$	-52.19***	-25.92***	-25.10***
	(-24.19)	(-5.54)	(-5.11)
R^2	0.41	0.41	0.41

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

1.2.2 The Yield Curve and the Business Cycle

This section presents the main facts of the term structure related to business cycles. I use data for the United States from the Federal Reserve Economic Data (FRED), that has measures for real interest rates, which are computed by the Federal Reserve Bank of Cleveland by subtracting inflation expectations to nominal interest rates. I use the 1-year and the 10-year real interest rates and I compute the real yield spread by subtracting the 10Y real rate minus the 1Y real rate. The data is quarterly and the sample goes from 1982Q1 until 2021Q4.

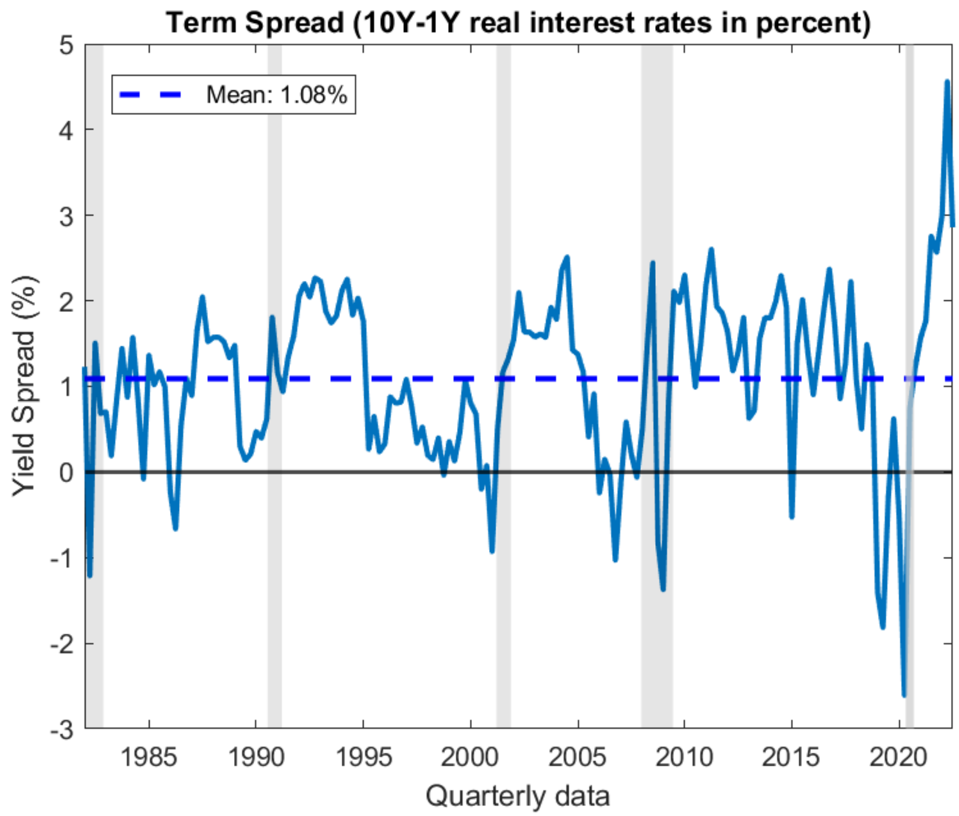

Figure 1.2: Real yield spread over time

Figure 1.2 shows the time series for the yield spread between 10Y and 1Y Treasuries. On average, the slope of the yield curve is positive and fluctuates around 1.08 percent. About the cyclical properties of the slope of the yield curve, it tends to be negative prior to recessions and exhibits a spike as recessions hit the economy. After recessions, during the recovery, it remains high and decreases as the economy enters the expansionary phase of the business cycle.

1.3 The Model

This section presents a macroeconomic model motivated by the empirical findings shown in the previous section. The purpose is to develop a dynamic model to provide a rationale for the demand for bonds across maturities, consistent with the data. I use a continuous-time general equilibrium model with incomplete financial markets, two assets and heterogeneous agents. There is no aggregate uncertainty, but agents face idiosyncratic risk. There are two types of households: stable income households, which face idiosyncratic labor income shocks at low frequencies, and risky households, which face idiosyncratic income shocks at high frequencies. Households are allowed to trade long-term bonds before maturity, subject to a portfolio adjusting cost. This implies that (i) long-term bonds are less liquid than short-term bonds and (ii) purchases or sales of long-term bonds are staggered.

For simplicity, I model long-term bonds as perpetual bonds, or console bonds, which pay a fixed coupon every period.[3] Finally, there is a representative firm that hires labor to produce a consumption good and a government that issues long- and short-term debt.

1.3.1 Households

The economy is populated by a continuum of households that face idiosyncratic income shocks. Households are divided in two groups. First, stable income households that are a fraction μ of the population, that face idiosyncratic labor income shocks at a low frequency rate. Second, a continuum of households with measure 1 μ that face idiosyncratic income shocks at a high frequency rate, hereafter referred to as risky income households. Both types of households can buy short-term and long-term bonds, and are allowed to issue debt in both markets subject to a borrowing constraint. Long term bonds can be traded at a cost, reflecting the lack of liquidity when bonds are *off-the-run*, while short-term bonds are traded at no cost.

The income process

This section describes the details of the income process. Both types of households face idiosyncratic labor income shocks given by a two-state Poisson process that jumps from l_1 to l_2 ($l_2 > l_1$

[3]It slightly deviates from Woodford (2001) where long-term bonds are perpetual bonds with decaying coupons. In fact, the constant coupon is a particular case of it. The difference is that with a decaying coupon structure, the state variable is the cummulated coupons earned by the long-term portfolio, while in the constant coupon case, the state variable is the stock of long-term bonds in the portfolio. So assuming constant coupons allow us to look at the portfolio. Since we are trying to explain differences in long-term portfolios, the constant coupon is more appropriate for our case.

without loss of generality) and vice versa. The only difference between the two groups of households is the stability of income or the intensity rate of idiosyncratic income shocks. In particular, denote λ_S the intensity rate for the stable income process and λ_R for the risky process. Given that stable households face shocks at a lower frequency rate than risky households, it must be the case that

$$\lambda_S < \lambda_R$$

and given that it is a Poisson process, the expected duration of the state (level) of income is the inverse of the intensity rate. Therefore, the expected duration of a given state or level of income is larger for the stable process than for the risky process.

What are the statistical properties of the income processes? First, the stable and the risky processes are symmetric. This means that, given that the arrival rates are different across processes, for each one –stable or risky– the arrival rate for switching from low to high income is the same arrival rate to switch from high to low income state, which implies that in the stationary distribution both income states have probability 1/2, from which follows that both processes have the same unconditional mean and the same unconditional variance. See Appendix C for the derivation of the stationary distribution of income.

Given that both income processes have the same stationary distribution and the same unconditional moments then, how do they differ between them? Since the intensity rates are different for stable or risky households, there must be some way to differentiate them. Specifically, the aim is to capture the findings from Section 1.2. This difference comes from the conditional distribution.

Consider the discrete-time analog of a two-state Poisson process, which is a two-state Markov chain. Instead of an intensity matrix, the process evolves according to a 2x2 transition matrix. If we condition on the risky household, and on income, the probability of switching to the different state is higher relative to the conditional –on type and income state– probability of a stable household for switching to a different income level, because the stable household is expected to stay in the same level of income for a longer amount of time. This means that there would be more dispersion in the risky process than in the stable process, hence higher conditional variance. Finally, since both processes are symmetric, the intensity matrices are symmetric, which in the discrete-time analog means that the transition matrices are symmetric. Therefore, the conditional variance is the same for each level of income, if we also condition on the type of agent. All this can be summarized in the following condition:

$$Var(l|l_1, R) = Var(l|l_2, R) > Var(l|l_1, S) = Var(l|l_2, S)$$

This is what our measure of income volatility in Section 1.2 is capturing, since we are conditioning on demographic factors and in income. For analogy purposes, these demographic differences that lead to higher variance are captured by S and R, denoting the stable and risky type, respectively.

The rest of the model aims to show that, consistent with the evidence in Section 1.2, households with higher conditional variance allocate their assets in shorter horizons and households with lower volatility allocate their savings in long-term assets.

The consumer problem

Both types of households maximize their utility, given by

$$E\left[\int_0^\infty e^{-\rho t} u(c_t) dt\right]$$

subject to the budget constraint and the borrowing constraints:

$$db_t = \left((1-\nu)w_t l_t + r_t(b)b_t - c_t - q_t(x_t - b_t^L) - \frac{\varphi}{2}\frac{(x_t - b_t^L)^2}{b_t^L - \underline{b}^L} - T_t\right) dt \quad (1.2)$$

$$q_t db_t^L = \left(\nu w_t l_t + \kappa b_t^L + q_t(x_t - b_t^L)\right) dt \quad (1.3)$$

$$b_t \geq \underline{b} \quad (1.4)$$

$$b_t^L \geq \underline{b}^L \quad (1.5)$$

where c_t is consumption, w_t is the wage, $r_t(b)$ is the instantaneous interest rate, which as in Kaplan, Moll & Violante (2018), has a piece-wise structure according to

$$r_t(b_t) = \begin{cases} r_t & \text{if } b_t \geq 0, \\ \\ r_t + \zeta & \text{if } b_t < 0. \end{cases} \tag{1.6}$$

where b_t denotes short term bond holdings, b_t^L denotes long-term bonds holdings at the beginning of period t, which is one state variable, x_t is a control variable and denotes long-term bond holdings after net purchases in period t, so the term $x_t - b_t^L$ is the net demand for long-term bonds at time t, q_t denotes the price at time t of long term bonds, $\underline{b} < 0$ and $\underline{b}^L \leq 0$ are the borrowing constraints faced by households. Finally, l_t is labor, which fluctuates according to the Poisson process described above.

The first two equations jointly describe the budget constraint and the evolution of wealth. The first one represents the evolution of short-term bond holdings, while the second equation describes the evolution of long-term bond holdings. Notice that there is a quadratic term in the budget constraint referring to portfolio adjustment costs for long-term bonds, which has several implications. First, it makes long-term bonds less liquid than short-term bonds. Second, it generates staggered purchases/sales of long-term bonds, consistent with the trading of these assets. Third, it serves as an entry cost to the long-term bond market. Finally, the parameter ν could be interpreted as the fraction of labor income allocated in social security or retirement savings. The purpose of that is to allow for a non-degenerate stationary distribution. Below, I calibrate this parameter as low as possible (1%), so that it does not play a significant role in the dynamics of the model. The last two inequalities are the borrowing constraints. This constitutes a consumer problem with two assets, similar to Kaplan, Moll & Violante (2018).

The Hamilton-Jacobi-Bellman equation is given by

$$\rho V(b, b^L, l_t^j) = \max_{\{c_t, x_t\}} \quad \{u(c_t) \quad + V_b(b, b^L, l_t^j)[(1-\nu)w_t l_t^j + r_t(b)b_t - c_t$$

$$-q_t(x_t - b_t^L) - \frac{\varphi}{2}\frac{(x_t - b_t^L)^2}{b_t^L \ \underline{b}^L} - T_t]$$

$$+V_{b^L}(b, b^L, l_t^j)\frac{1}{q_t}[\nu w_t l_t^j + \kappa b_t^L + q_t(x_t - b_t^L)]$$

$$+\lambda_j[V(\cdot, l_t^{-j}) - V(\cdot, l_t^j)]\}$$

where the index j refers to high or low labor income and $i \in \{S, R\}$ to risky or stable income household. The last term represents the regime switching process for the state of income level. The first order conditions are given by

$$c_t: \qquad u'(c_t) = V_b(b, b^L, l_t^j) \tag{1.7}$$

$$x_t: \quad V_b(b, b^L, l_t^j)\left(q_t + \varphi\frac{(x_t - b_t^L)}{b_t^L}\right) = V_{b^L}(b, b^L, l_t^j) \tag{1.8}$$

The LHS of the first equation is the marginal cost of saving in short-term bonds, while the RHS is the marginal benefit of saving in long-term bonds. The same idea is behind the second equation, where the marginal cost is given by the opportunity cost of saving in long-term bonds, times the marginal cost of buying a long-term bonds.

1.3.2 Production

There is a representative firm that hires labor and invest in physical capital. Therefore, the firm solves

$$\max \quad Y_t - w_t L_t$$

$$s.t. \quad Y_t = A_t L_t \tag{1.9}$$

where Y is total output, L is aggregate labor and A_t is the Total Factor Productivity, which, in a stationary equilibrium is constant. Since the production functions has constant returns to scale, there are no profits.

The first order conditions of the firm imply that the marginal productivity of labor should be equal to the wage, which for this case implies that

$$w_t = A \tag{1.10}$$

1.3.3 Government

Government issues short and long-term debt. In particular, long-term government debt has the form of perpetual bonds that pay coupons to bondholders according to the description above. The budget constraint of the government is given by

$$dB_t + q_t dB_t^L = (G_t - T_t + r_t B_t + \kappa B_t^L)dt \tag{1.11}$$

where B_t denotes short-term debt, B_t^L is long-term debt, G_t is government expenditure and T_t are lump-sum taxes paid by households in order to satisfy the government's budget constraint.

1.3.4 Market Clearing

Market clearing for the goods market is given by

$$
\begin{aligned}
Y_t &= \mu \left(\int_{\underline{b}^L}^{\infty} \int_{\underline{b}}^{\infty} c_t(b, b^L) g_H^S(b, b^L) db db^L + \int_{\underline{b}^L}^{\infty} \int_{\underline{b}}^{\infty} c_t(b, b^L) g_L^S(b, b^L) db db^L \right) \quad (1.12) \\
&+ (1-\mu) \left(\int_{\underline{b}^L}^{\infty} \int_{\underline{b}}^{\infty} c_t(b, b^L) g_H^R(b, b^L) db db^L + \int_{\underline{b}^L}^{\infty} \int_{\underline{b}}^{\infty} c_t(b, b^L) g_L^R(b, b^L) db db^L \right) \\
&+ \Theta_t + G_t
\end{aligned}
$$

for short term bonds, implies

$$
\begin{aligned}
B &= \mu \left(\int_{\underline{b}^L}^{\infty} \int_{\underline{b}}^{\infty} b_t(b, b^L) g_H^S(b, b^L) db db^L + \int_{\underline{b}^L}^{\infty} \int_{\underline{b}}^{\infty} b_t(b, b^L) g_L^S(b, b^L) db db^L \right) \quad (1.13) \\
&+ (1-\mu) \left(\int_{\underline{b}^L}^{\infty} \int_{\underline{b}}^{\infty} b_t(b, b^L) g_H^R(b, b^L) db db^L + \int_{\underline{b}^L}^{\infty} \int_{\underline{b}}^{\infty} b_t(b, b^L) g_L^R(b, b^L) db db^L \right)
\end{aligned}
$$

while the market for long term bonds clears if

$$
\begin{aligned}
B^L &= \mu \left(\int_{\underline{b}^L}^{\infty} \int_{\underline{b}}^{\infty} b_t^L(b, b^L) g_H^S(b, b^L) db db^L + \int_{\underline{b}^L}^{\infty} \int_{\underline{b}}^{\infty} b_t^L(b, b^L) g_L^S(b, b^L) db db^L \right) \quad (1.14) \\
&+ (1-\mu) \left(\int_{\underline{b}^L}^{\infty} \int_{\underline{b}}^{\infty} b_t^L(b, b^L) g_H^R(b, b^L) db db^L + \int_{\underline{b}^L}^{\infty} \int_{\underline{b}}^{\infty} b_t^L(b, b^L) g_L^R(b, b^L) db db^L \right)
\end{aligned}
$$

where g_j^S and g_j^R are the stationary distributions for stable and risky households, respectively, and $j \in \{H, L\}$ denotes if the household has high or low income, and Θ_t are all transaction costs and short-term interest rate spreads that can be considered as consumption of financial intermediation.

1.4 Solution

The model is solved using the finite difference method presented in Achdou et al. (2022) with an extension for two assets, similar to Kaplan, Moll & Violante (2018), except that instead of kinked portfolio adjustment costs, I use simple quadratic costs.[4]

1.4.1 Calibration

This section describes the calibration of the model. The risk aversion coefficient, γ, is calibrated according to the standard literature on macro-finance models, which is a value of 2. The coupon κ is set to 2.4%, so that the equilibrium price of the long-term bonds, q_t, is one. Note that if κ takes a different value, the price of the bond would be different, such that the long-term interest rate is in equilibrium.

For the gap between high labor and low labor, I follow Table 1.1 where the average variance of labor income growth is 0.18, which in terms of the model translates into the following:

$$\begin{aligned} Var(wl) &= w^2 Var(l) \\ &= w^2(E[l^2] - E^2[l]) \\ &= w^2 \left(\frac{l_2 - l_1}{2}\right)^2 \end{aligned}$$

which means that the gap is given by

$$l_2 - l_1 = 2\sqrt{\frac{Var(wl)}{w^2}}$$

which gives a value of 0.38 and, to normalize fluctuations in l around 1, then $l_2 = 1.19$ and l_1 satisfying the gap.

For the parameter in the adjustment cost function, φ, I use estimates coming from Roll (1984) model explained in Appendix A. Roll (1984) produces estimates of the transaction costs given by 2c –where, as shown in the Appendix A, c is the cost per trade faced by buyers or sellers–, which the analog of the model comes from the marginal cost of adjusting the long-term portfolio which is given by

$$\varphi \left(\frac{x_t - b_t^L}{b_t^L - \underline{b}^L}\right) = 2c$$

[4] I follow the algorithm available in Ben Moll's website.

after aggregating across all households in the economy, gives a value of 0.52 for φ.

For the Poisson intensity rates for the two income processes it is important to mention some points. First, individual data on income dynamics usually has, at most, annual frequency. Second, almost the entire sample of households report income variations on a yearly basis. This poses a problem on the identification of the arrival rates of idiosyncratic income shocks.[5] Given that the arrival rates have the property of being the inverse of the average duration of the income state, one alternative path is to look at the average job duration. In the PSID and SCF households report how many years they have with their current employer –or business for those who are self-employed–, the advantage of the PSID is that it keeps track of individual households over time, so it is easy to see if at some point in time, the job tenure is short because they moved to a different job, but they tend to have stability in their jobs. I also take into account only people between 30 and 50 since younger people tend to have low job duration and elderly people tend to have large job duration. I compute the average job duration of each head of household. This gives us a proxy of income stability over time. I split the sample into two groups, households with average job duration above the median and households with an average job duration below the median. I find that for the first group the average job duration is 2.7 years and for the second group is 13 years, which means that the intensity rate for risky households is going to be 1/2.7 and for stable households is 1/13.

Government expenditure is set to 22% of GDP close to 19% for the United States government. The supply of bonds targets two moments. First, to set the composition of long-term debt relative to total public debt. I use the total debt in Treasury Bills, whose maturities are shorter than 2 years, and Treasury Notes, whose maturities are between 2 and 10 years. I leave aside Treasury Bonds for two main reasons: (*i*) these instruments have maturities of 20 and 30 years, while the standard measure of the slope of the yield curve takes into account the 10Y interest rate; (*ii*) Treasury Bonds represent less than 20% of government's debt. Hence, Treasury Bills represent 27% of our measure of public debt, while Treasury Notes represent 73 percent. The model is calibrated such that the short-term debt is 41%, while long-term debt is 59 percent. Second, I target the average debt

[5]For different ways to identify this parameter see Kaplan et al. (2018) and Parra-Alvarez, Posch and Wang (2017).

over GDP from the end of the Great Recession until the first quarter of 2020, prior to the Covid shock, which averages 99.9%, so the model is set to target 99.9 percent of debt over GDP. The fraction of mandatory savings, since it is used only for computational reasons, I leave it as small as possible to prevent it from playing an important role in the dynamics of the model, so I set it to 1 percent. Values below 1% create difficulties for the convergence of the solution. The full external calibration of the model is described in Table 3.1.

Table 1.5: External Calibration (Annual frequency)

Parameter	Notation	Value	Target	Model	Data
Risk aversion coefficient	γ	2	Standard	-	-
Coupon (Perpetual bond)	κ	2.4%	Normalize $q_t = 1$	-	-
Income volatility	$l_2 - l_1$	0.38	Avg. income variance (PSID)	0.18	0.18
Adjustment cost parameter	φ	0.6	Roll (1984) model	0.6	0.52
Intensity rate for stable households	λ^S	1/13	Avg. job duration in years	13	13
Intensity rate for risky households	λ^R	1/2.7	Avg. job duration in years	2.7	2.7
Government Expenditure	G/Y	0.22	Expenditure as % of GDP	22%	19%
Government short-term debt	$B/(B+B^L)$	0.41	Share of short-term debt	41%	27%
Government long-term debt	$B^L/(B+B^L)$	0.59	Debt over GDP	99.9%	99.9%
Fraction of low risk households	μ	0.5	Same weight for each group	-	-
Mandatory savings	ν	1%	-	-	-

Finally, the intertemporal discount rate, ρ, the spread between borrowers and savers, ζ, and the borrowing constraints, $\underline{b}$ and $\underline{b}^L$, are internally calibrated to match the short-term real interest rate, the fraction of agents with short-term debt and the coefficient of variation–standard deviation normalized by the mean– of the marginal distributions of long-term assets and short-term assets in the data. Table 1.6 reports the value of parameters that were internally calibrated and Table 1.7 reports the targeted moments for the internal calibration.

Table 1.6: Internal Calibration (Annual frequency)

Parameter	Notation	Value
Intertemporal discount rate	ρ	0.029
Spread between borrowers and savers	ζ	2.7%
Borrowing constraint (Short-term)	$\underline{b}$	-2
Borrowing constraint (Long-term)	$\underline{b}^L$	-0.7

Table 1.7: Targeted Moments

Moment	Model	Data
Short-term interest rate	1.39%	1.39%
Fraction of agents with short-term debt ($b < 0$)	16.8%	15%
Coefficient of variation long-term bond distribution	4.15	4.15
Coefficient of variation short-term bond distribution	2.07	5.87

1.4.2 Stationary Equilibrium

This section contains the stationary equilibrium of the economy. To solve the model in general equilibrium, one has to iterate the short term interest rate and the price of the bond, until the excess demand functions for the two types of bonds are zero.

The equilibrium prices are such that they clear both markets and by Walras law, the goods market clears. Table 1.8 summarizes the equilibrium prices and the yield spread in the stationary equilibrium.

Table 1.8: Equilibrium Term spread

Variable	Notation	Model	Data
Short-Term real interest rate	r_t	1.39%	1.39%
Real term spread	$r_t^L - r_t$	1.01%	1.08%

Stationary Distribution

In this section I present the stationary wealth distribution for households depicted in Figures 1.3 and 1.4. To find the stationary distribution, we need to solve the Kolmogorov-Forward Equation, given by

$$0 = -\frac{\partial}{\partial b} s_b(b, b^L, l) g_j(b, b^L, l) - \frac{\partial}{\partial b^L} s_{b^L}(b, b^L, l) g_j(b, b^L, l) - \lambda_i g(b, b^L, l^i) + \lambda_{-i} g(b, b^L, l^{-i})$$

where $s_b(b.b^L, l)$ and $s_{b^L}(b, b^L, l)$ are the drifts for the evolution of short-term and long-term bond holdings in the budget constraint of the household, respectively, $j \in \{S, R\}$ denotes stable or risky household and $i \in \{1, 2\}$ denotes high or low income according to the definition above.

Here is the main result of the paper. Households with stable income (having the level of income constant) save more in long-term bonds than households with an unstable income process, reflecting a lower need for liquidity. This result follows from the fact that households with riskier income have higher needs for liquidity, thus are less willing to hold long-term bonds, since they are trading more than stable households in order to smooth consumption. On the other hand, stable income households are more willing to hold long-term bonds, because they do not expect changes in labor income in the near future, so their demand for liquidity is low.

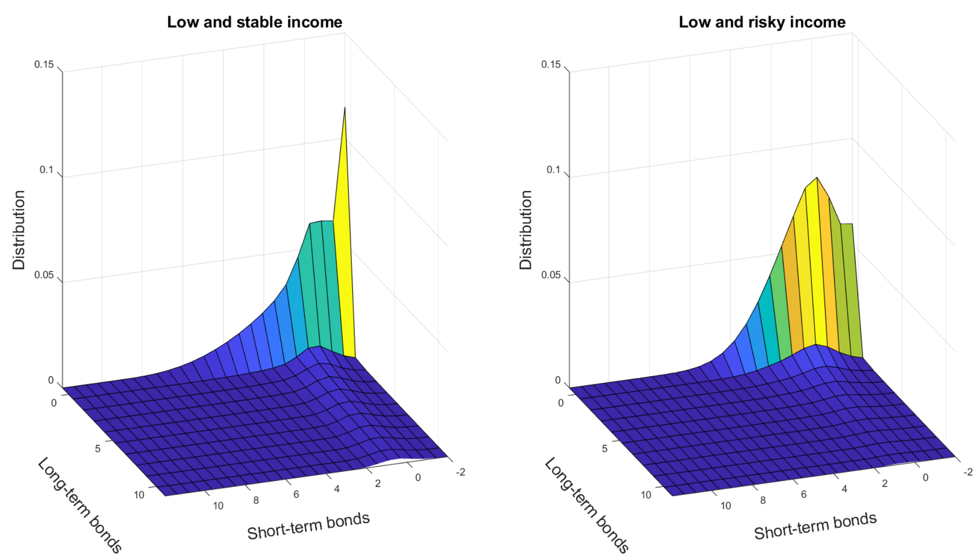

Figure 1.3: Stationary Distribution (Low income households)

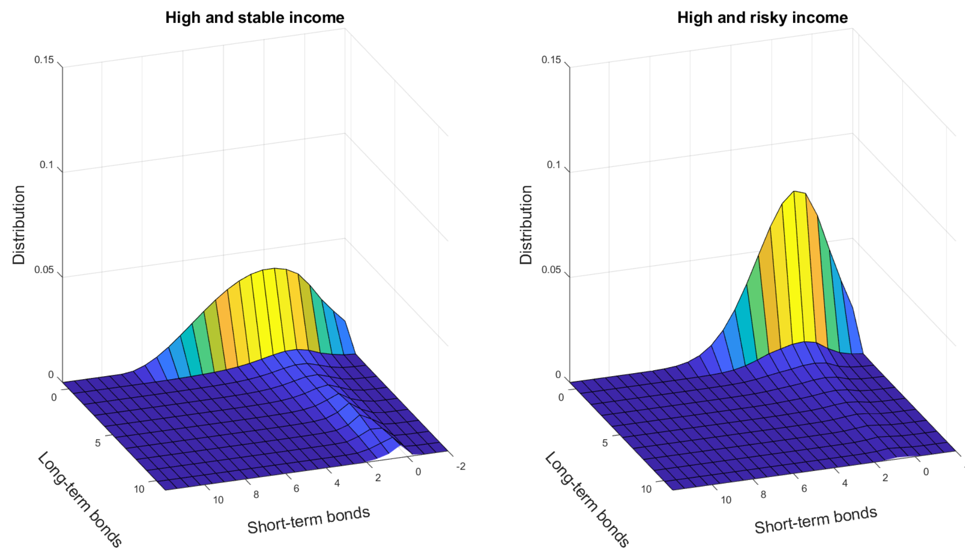

Figure 1.4: Stationary Distribution (High income households)

Tables 1.9 and 1.10 present the distribution of long-term bonds across households as a percentage of total bonds.

Table 1.9: Long-term bond market share by type of agent

Percentage	Stable household	Risky household
High income	59.96%	0.76%
Low income	39.65%	-0.37%

Table 1.10: Short-term bond market share by type of agent

Percentage	Stable household	Risky household
High income	51.31%	33.24%
Low income	4.03%	11.42%

As mentioned above, stable households tend to hold more long-term bonds than risky households, despite the level of income. Here, we can see that, after aggregation within types, stable households hold practically 100% percent of long-term bonds issued by the government, while risky households hold, on average, 0% approximately, close to full segmentation in the long-term bond market. As expected, high income stable households hold more long-term bonds than low income stable households. The same applies to risky households. The explanation is simple, given that stable households receive shocks at a lower rate, it is easier for them to smooth consumption, since by the Poisson process statistical properties, lower intensity rate implies a lower conditional variance of income, this allows them to hold a higher share of their wealth in long-term bonds since they do not need a high level of liquidity. On the other hand, risky households experience shocks at a high frequency, implying higher conditional variance of their income process. This means that they need to trade assets with a higher frequency in order to smooth consumption, therefore, their demand for liquidity is higher, resulting in lower long-term bond holdings. Therefore, stable households lean towards longer maturities, while risky households lean towards shorter maturities, generating an endogenous market segmentation.

Tables 1.9 and 1.10 only report market shares, but not portfolio shares. Table 1.11 reports long-term bond holdings as a fraction of wealth. As mentioned before, stable households allocate a higher fraction of their wealth in long-term bonds, compared to risky households. This is consistent with the findings in tables 1.2, 1.2.1 and 1.4. Additionally, Table 1.11 says that households with high income, whether they are risky or stable, hold a lower fraction of wealth in long-term bonds, which is also consistent with the results reported in Tables 1.2, 1.2.1 and 1.4. This means that, when agents face negative shocks, they sell their liquid assets before selling illiquid assets, resulting in a higher share of long-term assets in the portfolio. For the case of high income and risky households, on average, they are shorting long-term bonds in order to have enough liquidity to face idiosyncratic risk. In other words, they are willing to pay a price for liquidity.

Table 1.11: Long-term bond holdings as % of portfolio

Percentage	Stable household	Risky household
High income	33.78%	-23.52%
Low income	65.58%	13.94%

1.4.3 Welfare Analysis

In this section I present welfare analysis using comparative statics. I present different experiments, varying some parameters, which are intended to measure the effect of different policies and scenarios. First, I show how differences in the composition of government debt affect the term structure and welfare in the economy. Second, I present comparative statics for differences in the degree of liquidity in the economy by varying the adjustment cost parameter. Third, I show the effect of different regulations by varying the borrowing constraints.

To do welfare analysis, I look at consumption. In particular, I look at the life-time utility and I compare the effects of different parameters on the life-time utilities and then I find the equivalent consumption. Note that I am comparing the long-run effects of the policies. In other words, I am comparing different steady states of the model. Let U denote the life-time utility of a household, then

$$U = E_0 \left[\int_0^{\infty} e^{-\rho t} u(c_t) dt \right] \tag{1.15}$$

and the life-time utility of a consumer after a change in the relevant parameter is given by

$$\tilde{U} = E_0 \left[\int_0^{\infty} e^{-\rho t} u(\tilde{c}_t) dt \right] \tag{1.16}$$

to find the welfare effects of a change in the parameters of the model, we need to find $\phi > 0$, such that

$$\tilde{U} = E_0 \left[\int_0^{\infty} e^{-\rho t} u(\phi c_t) dt \right] \tag{1.17}$$

for CRRA utility, we get that

$$\tilde{U} = \phi^{1-\gamma} U \tag{1.18}$$

which yields to

$$\phi = \left(\frac{\tilde{U}}{U} \right)^{\frac{1}{1-\gamma}} \tag{1.19}$$

I classify households according to their type of income. I group stable households regardless of their level of income and risky households on the other side.

Changes in the composition of government debt

In this exercise I present the effect of a reduction in long-term government debt as a fraction of total public debt, maintaining the level of debt constant. In some sense, this exercise mimics the *"Operation Twist"*, where central banks buy long-term bonds and sell short-term bonds, maintaining the size of their balance sheet constant, aiming to provide more liquidity to financial markets and reduce long-term interest rates.

Table 1.12: Operation Twist

$B/(B+BL)$	r	r^L	$r^L - r$	ϕ Risky	ϕ Stable
36%	1.322%	2.416%	1.095%	0.9983	0.9983
38%	1.350%	2.414%	1.064%	0.9990	0.9990
41%*	1.390%	2.410%	1.020%	1	1
43%	1.428%	2.406%	0.978%	1.0009	1.0009

*Note: The value at 41% is is from the benchmark calibration.

Table 1.12 shows the effects of a change in the composition of government debt. First, the short-term interest rate increases as the supply of short-term bonds increases, while the long-term rates decreases as a response to a lower supply of long-term debt. These effects result in a flatter yield curve. In terms of welfare, it increases as the relative supply of long-term bonds is lower. Holding a lower amount of long-term bonds in the balance sheet, due to a reduction in supply results in lower trading on long-term bonds. Since trading decreases, the welfare costs of trading these illiquid assets, decreases as well, increasing welfare.

Changes in the degree of market liquidity

In this section I present comparative statics for different value of the transaction cost parameter, φ, simulating differences in market liquidity.

As shown in Table 1.13, as the portfolio adjustment costs on long-term bonds increase, the short-term rate decreases, while the long-term rate increases, reflecting lower market liquidity. As liquidity in the long-term bond decreases, the cost of holding them is higher, resulting in higher

Table 1.13: Financial distress

φ	r	r^L	$r^L - r$	ϕ Risky	ϕ Stable
0.55	1.394%	2.409%	1.015%	1.0001	1.0001
0.6*	1.390%	2.410%	1.020%	1	1
0.65	1.386%	2.411%	1.025%	0.9999	0.9999
0.7	1.382%	2.412%	1.030%	0.9998	0.9998

*Note: The value at 0.6 corresponds to the benchmark calibration

demand for short-term bonds and lower-demand for long-term bonds. This results in a steepening of the yield curve. As the transaction cost increases, welfare is lower for all groups of households, since the deadweight loss from trading long-term assets is higher.

Differences in long-term borrowing constraints

In this section, I present the effects of varying the long-term borrowing constraint shown in Table 1.14.

Table 1.14: Financial Regulation

$\underline{b}^L$	r	r^L	$r^L - r$	ϕ Risky	ϕ Stable
-0.7*	1.390%	2.410%	1.020%	1	1
-0.4	1.417%	2.280%	0.862%	1.0010	1.0010
-0.2	1.414%	2.205%	0.791%	1.0006	1.0006
0	1.399%	2.126%	0.727%	0.9996	0.9996

*Note: The value at -0.7 corresponds to the benchmark calibration

This case is particularly interesting due to an inverted U-shaped effect on welfare and the short-term interest rate. First, recall two facts from the model: (*i*) there is a spread, ζ, between borrowers and savers in the short-term bond market, and (*ii*) the closer households are to the long-term credit constraint, the higher is the cost of trading long-term assets, due to the term in the denominator of the adjustment cost function.

In this case it is easier to start the analysis at $\underline{b}^L = 0$, households can only issue debt in the short-term bond market at a rate $r_t + \zeta$, which is higher than the long-term interest rate. As the credit limit eases, the supply of long-term bonds increases, which results in higher long-term interest rates. Despite higher long-term rates, some households with short-term debt might prefer to switch to long-term debt, in order to reduce the cost of debt, since the long-term rate is still lower than the short-term rate on loans. This has a positive wealth effect on debtors, but also on savers, since they are earning $r_t^L > r_t$ in the long-term market. As the credit limit keeps rising, long-term interest rates are higher. However, less households are willing to borrow at the borrowing constraint. The reason is because unwind short positions in the long-term bond close to the borrowing constraint are quite costly, which means that some households would prefer to borrow at the short-term rate than in the long-term rate, which results in lower short-term interest rates. Finally, since households are borrowing at a higher price, the effect on welfare is negative, creating the inverted U-shaped effect.

1.4.4 Aggregate Dynamics: Transition

This section analyzes the response of the economy to an MIT Shock to productivity, were agents are initially at the stationary equilibrium, where aggregate variables are constant, and unexpectedly the economy faces an aggregate shock, followed by a deterministic transition to the steady state.

Given that aggregate variables now are time-varying, the consumer problem has to be written as

$$\max_{x_t, c_t} \qquad E\left[\int_0^\infty e^{-\rho t} u(c_t) dt\right]$$

$$\begin{aligned}
s.t. \quad db_t &= (1-\nu) w_t l_t dt + r_t(b) b_t dt - c_t dt - q_t(x_t - b_t^L) dt - \frac{\varphi}{2} \frac{(x_t - b_t^L)^2}{b_t^L - \underline{b}^L} dt - T_t dt \\
b_t dq_t + q_t db_t &= \nu w_t l_t dt + \kappa b_t^L dt + q_t(x_t - b_t^L) dt \\
b_t &\geq \underline{b} \\
b_t^L &\geq \underline{b}^L
\end{aligned}$$

For computational reasons, the law of motion for long-term bonds can be written as

$$db_t^L = \frac{1}{q_t}[\nu w_t l_t dt + \kappa b_t^L dt + q_t(x_t - b_t^L) dt - b_t^L dq_t] \tag{1.20}$$

Then, the time-varying HJB equation is

$$\begin{aligned}
\rho V(b_t, b_t^L, l_t, t) &= \max_{x_t, c_t} \{ u(c_t) \\
&+ V_b(b_t, b_t^L, l_t, t)((1-\nu) w_t l_t + r_t(b) b_t - c_t - q_t(x_t - b_t) - \frac{\varphi}{2} \frac{(x_t - b_t)^2}{b_t - \underline{b}^L} - T_t) \\
&+ V_{b^L}(b_t, b_t^L, l_t, t) \frac{1}{q_t}[\nu w_t l_t + \kappa b_t^L + q_t(x_t - b_t^L) - b_t^L dq_t] \\
&+ \lambda_j (V(b_t, b_t^L, l_t^{-j}, t) - V(b_t, b_t^L, l_t^j, t)) \\
&+ \frac{\partial V(b_t, b_t^L, l_t, t)}{\partial t} \}
\end{aligned}$$

The F.O.C. are given by

$$c: \quad u'(c_t) = V_b(b_t, b_t^L, l_t)$$

$$x: \quad V_b(b_t, b_t^L, l_t)\left(q_t + \varphi\frac{(x_t - b_t^L)}{b_t^L}\right) = V_{b^L}(b_t, b_t^L, l_t)$$

Similarly, the time-varying KFE is

$$\frac{\partial g_j(b, b^L, l, t)}{\partial t} = -\frac{\partial s_b g_j(b, b^L, l, t)}{\partial b} - \frac{\partial s_{b^L} g_j(b, b^L, l, t)}{\partial b^L} - \lambda_j g(b, b^L, l^j, t) + \lambda_{-j} g(b, b^L, l^{-j}, t)$$

Finally, the long-term interest rate is given by

$$r_t^L dt = \frac{\kappa}{q_t} dt + \frac{dq_t}{q_t} \tag{1.21}$$

Negative Supply Shock

Here I present the transition dynamics of the economy after an unanticipated negative supply shock. First, assume that productivity, from its long-run value declines 1% and mean reverts according to

$$dA_t = \theta(\bar{A} - A_t)dt \tag{1.22}$$

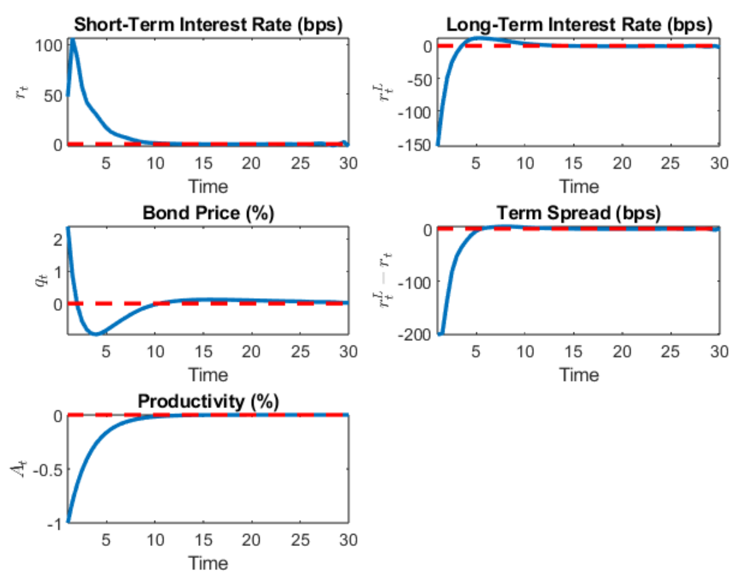

Figure 1.5: MIT Shock to productivity

Figure 1.5 shows the response to a negative unanticipated shock to productivity. The short-term interest rate increases by around 100 bps due to expected consumption recovery, but long-term interest rates decline in a similar magnitude and return to the steady state. This causes, on impact, a yield curve inversion, measured by the term spread and a decline in consumption. The interpretation is the following: due to a negative income shock, households try to smooth the shock by decreasing their asset holdings. Since risky households mainly hold short-term assets, they are more willing to sale these assets, on average, relative to stable households. This causes an increase in short-term interest rates and, since someone has to hold these assets, they enter into the balance sheet of stable households. For the wealthiest households, buying short-term assets at a higher interest rate is causing a net positive wealth effect, which leads to an increase in the demand for long-term bonds. The higher demand is supplied by stable households with low liquidity and by risky households with relative –to their type– high long-term asset holdings, leading to an increase in the price of the long-term bond, with the consequential decrease in long-term interest rates. The opposite effects on interest rates is what is causing the yield curve inversion.

1.5 Conclusions

This paper studies empirically and theoretically, differences in the demand for bonds of different maturities based on differences in idiosyncratic risk across agents. First, I find household level evidence on the determinants for the demand for different maturities which is related to earnings risk. Micro data suggests that households with stable income over time tend to have low needs for liquidity, allocating their assets in long-term investments, while households with more volatile income over time, tend to demand more liquidity, allocating their assets in short-term investments. In order to rationalize these findings, I present a continuous-time general equilibrium model with incomplete markets and heterogeneous agents that is able to generate two main features: First, households demand liquidity depending on their income process. Stable income households hold a larger fraction of the portfolio in long-term bonds, while unstable or risky households, have higher needs for liquidity, demanding more short-term bonds. Second, the model is able to generate a large enough yield spread with a very reasonable calibration.

This paper also provides an explanation on the wealth distribution and portfolio duration. In particular, the paper explains, bot empirically and theoretically, why households' wealth exhibit high differences in portfolio duration. Income stability is associated with a higher duration of the portfolio, while income instability is reflected in shorter investment horizons.

1.6 Appendix A: Roll (1984) model

This section describes Roll (1984) model to estimate transaction costs when bid-ask speads are not available. First, assume agents buy and sell assets through a broker. Let a be the ask price, b the bid price, q the efficient price of the asset and c the cost per trade. Assume that

$$q_t = q_{t-1} + u_t$$

where u_t is $i.i.d.$ with mean zero and variance σ_u^2, the bid and ask prices are given by, respectively by

$$\begin{aligned} b_t &= q_t - c \\ a_t &= q_t + c \end{aligned}$$

then the bid-ask spread is

$$a_t - b_t = 2c$$

then the trade price is

$$p_t = q_t + dc$$

where $d = 1$ if the agent buys or $d = -1$ if sells.

It can be shown (see Roll (1984)) that

$$c = \sqrt{|cov(\Delta p_t, \Delta p_{t-1})|}$$

which is the squared root of the absolute value of the first order autocovariance of price changes.

1.7 Appendix B: Data

This section describes the data used in the paper and the estimations. I explore micro-level evidence to test the main result of the model, which is that households with stable income over time tend to have a preference for savings or investments with longer horizons, while households with high earnings risk tend to prefer short-term investments.

1.7.1 Microevidence

The empirical analysis aims to show that households with more stable income tend to allocate their savings into less liquid long-term financial assets, while households with greater income volatility tend to hold more liquid short-term financial assets. The study utilizes data from the Survey of Consumer Finances (SCF) for the years 2013, 2016, and 2019. Notably, one main problem is that the SCF is a repeated cross-sectional survey and lacks individual income dynamics data. To address this limitation and link income dynamics with asset holdings, I use the Panel Study of Income Dynamics (PSID) for 2015, 2017 and 2019, which contains data from 2013 to 2018.

To establish a connection between the two surveys and match income dynamics with asset holdings, I employ the Panel Study of Income Dynamics (PSID) to estimate relevant variables related to income dynamics based on shared demographic factors. These demographic variables are subsequently employed to estimate income volatility across households. By using the PSID's estimated model, income volatility is predicted using demographic factors from the SCF, thereby enabling an observation of household portfolios based on estimated income dynamics.

Panel Study of Income Dynamics

The first step in the estimation process involves employing pertinent demographic factors to estimate income volatility. To mitigate potential effects related to household life cycles, income stability is better assessed by measuring the volatility of income growth rather than income volatility itself.

A notable consideration is that while the PSID is structured as a panel survey, the SCF is a cross-sectional survey. The analysis focuses on households that reported being in the labor force for at least 24 weeks per year during the sample period, and those reporting labor income less than $20,000$ per year or exceeding 1 million per year are excluded. Income variability is estimated through ordinary least squares (OLS) regression with robust standard errors. The variables employed in the regression are outlined in the table below:

Several key observations are pertinent. First, labor income considers income generated from labor activities, including working for others, farming or working on own business. Second, given that the PSID is a biannual survey, all observations are biannual except labor income which is reported twice in each survey (*e.g.* the 2019 survey reports labor income for 2018 and 2017, but only reports wages and salaries for 2018 and income from farming and businesses for 2018). Third, occupation and industry indicators consistent with the SCF are available only in the 2017 and 2019 surveys, so I take the one reported in 2019. All income measures encompass the sum of the head of the household and the spouse's incomes, while qualitative variables pertain solely to the head of the household, consistent with the summary data from the SCF.

These estimations are subsequently applied to the SCF data to generate income volatility estimates based on demographic variables.

Survey of Consumer Finances

This section outlines the results obtained from the SCF after incorporating the estimations derived from the PSID. The study utilizes data from the 2013, 2016, and 2019 surveys, each containing different sets of individuals, due to a repeated cross-section structure. The analysis begins by excluding households that reported being out of the labor force and those with zero or negative financial wealth, as the focus is on asset holdings as a fraction of financial wealth. Income volatility estimates, obtained from the PSID, serve as the primary explanatory variable in assessing whether households with greater income variability allocate more to short-term bonds and those with income stability invest more in longer-horizon assets as a fraction of their financial wealth.

Table 1.15: Dependent Variable: Logarithm of Income Variability

Variable	Description	Coefficient
Sex	1 if male	0.293***
		(6.13)
Age	Age reported in the 2019 survey	-0.00646***
		(-3.85)
Education	Years of education reported in 2019	-0.0205
		(-1.63)
Married	1 if married 2015-2019	-0.217***
		(-5.59)
Kids	Average number of kids during the sample period	-0.0320**
		(-2.22)
Race	1 if white	-0.0663*
		(-1.96)
Mean Labor Income	Mean of income from main occupation 2013-2018	0.00000231
		(0.76)
Wage Income	Mean of wages excluding business/farm income 2013-2018	-0.00000201
		(-0.69)
Business and Farm Income	Mean from 2013-2018	0.0000300***
		(5.07)
Worker/Self-employed	1 if majority of observations reported worker	-0.550***
		(-9.54)
Tenure	Average number of years in current job 2013-2018	-0.0197***
		(-9.41)
Belong to Union	1 if reported being in union the majority of observations	-0.0480
		(-1.00)
Occupation2	1 if code from SCF=2 in 2019	-0.00407
		(-0.02)
Occupation3	1 if code from SCF=3 in 2019	0.164***
		(2.81)
Occupation4	1 if code from SCF=4 in 2019	0.0797
		(1.45)
Occupation5	1 if code from SCF=5 in 2019	0.103**
		(2.44)
Occupation6	1 if code from SCF=6 in 2019	0.0925
		(0.53)
Industry2	1 if code from SCF=2 in 2019	-0.0931
		(-0.75)
Industry3	1 if code from SCF=3 in 2019	-0.0707
		(-0.60)
Industry4	1 if code from SCF=4 in 2019	-0.111
		(-0.94)
Industry5	1 if code from SCF=5 in 2019	-0.0293
		(-0.24)
Industry6	1 if code from SCF=6 in 2019	-0.135
		(-1.18)
Industry7	1 if code from SCF=7 in 2019	-0.325***
		(-2.69)
Constant	Constant	-0.669***
		(-3.44)
N	2976	

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

One additional problem of the SCF is that it does not report the maturity of bonds. To address this problem I make one assumption. Cash and Deposits serve as a proxy for short-term bonds, while Fixed Income securities represent long-term bonds. A few reasons of doing this are that first, following Greenwald et al. (2021) the cashflow duration of Cash and Deposits is 0.25 years, while fixed income is 4 years. Also, since these measures contain direct and indirect held assets, cash and deposits include funds allocated in Money Market Funds or other financial institutions that tend to invest in short-term assets, while the measure of fixed income includes assets in pension funds and insurance companies which typically invest in longer horizons. I use Cash and Deposits and Fixed Income as a fraction of total financial wealth as dependent variables.

The independent variables are the estimated variance of labor income, logarithm of financial wealth, logarithm of income, age, and risk attitude. I estimate through ordinary least squares (OLS) with robust standard errors. The results of this estimation are in the main text.

1.8 Appendix C: Stationary Distribution of Income

This section describes the stationary equilibrium of labor income. For each type of agent (stable or risky), the balance equations need to be satisfied. Notice that the two-state Poisson process is symmetric, which means the arrival rate of a positive shock (going from l_1 to l_2) is the same as the arrival rate of having a negative shock (going from l_2 to l_1). Define p_1 as the stationary probability of being in the low income state and p_2 the stationary probability of high income state, the the balance equations are

$$p_1\lambda_i = p_2\lambda_i$$

where $i \in \{S, R\}$, which means that $p_1 = p_2 = 1/2$, consistent with the symmetry of the process of each type of household. The expected value of labor income is

$$\begin{aligned} E(wl) &= wE(l) \\ &= w(0.5l_1 + 0.5l_2) \end{aligned}$$

Now, to compute the variance of labor income, compute

$$\begin{aligned} Var(wl) &= w^2 Var(l) \\ &= w^2(E[l^2] - E^2[l]) \\ &= \frac{w^2}{2}(l_1^2 + l_2^2) - \frac{w^2}{4}(l_1 + l_2)^2 \end{aligned}$$

This means that the distribution of labor income does not depend on the intensity rate, due to symmetry. This is important, because it means that both types of households have the same expected value for income and the same variance of income. What changes is the conditional variance.

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Chapter 2: Policy Instability and the Risk-Return Trade-Off

Co-authored with Rodolfo E. Manuelli.

What is the impact of large swings in economic policy on the risk-return trade-off faced by investors? What is the impact of changes in policy regimes on investment strategies? In this paper we study the impact on returns of switches between periods of market-friendly economic policies and periods of populist policies. To quantify the impact of policy instability, we use data from Argentina—a country that has experienced frequent and very large regime changes—and find that the risk-return for individual assets and minimum variance portfolios are quite different across regimes. We then develop a dynamic model to understand optimal portfolios when investors are cognizant that regimes can change. We find that when portfolios are unrestricted, it is optimal for investors to take a large amount of risk. On the other hand, when portfolios are restricted to include only long positions, a real asset (real estate) dominates financial assets.

Keywords: Portfolio Allocation, Risk-Return Trade-Off, Policy Instability

JEL Codes: E44, G11, G12

2.1 Introduction

Over the last few years, many countries have adopted economic policies that can be broadly defined as populist. Typically, these policies include different forms of interventions that disrupt market mechanisms.[1] The impact of a given policy is determined by, not only its features, but also its stability. Policy regimes that change very frequently create uncertainty and negatively affect investment decisions.[2] The historical records of many Latin American economies show that many have experienced frequent switches between (relatively) market-friendly and populist regimes, and some view these changes as imposing significant costs.[3]

A country's economic performance depends crucially on its ability to direct savings to the most productive uses. Economic policies have a large impact on how investors choose to allocate their savings. In this paper we document how the risk-return trade-off faced by an investor changes with the policy regime and we illustrate how portfolios that perform well in one regime can generate large losses when the regime changes. We then develop a model of dynamic portfolio selection to study how a rational investor should choose his portfolio, accounting for the possibility of regime changes and the costs—both in terms of time and resources—of adjusting the portfolio.

To illustrate the forces at work, we study the impact of policy instability in Argentina, a country characterized by frequent and dramatic swings in economic policies. We use monthly data on the *real* returns on a collection of assets that include time deposits (both fixed and adjustable rate), real estate, and foreign exchange (U.S. dollar) at both the official exchange rate—which is typically controlled by the government during populist periods—and the black market rate that is easily

[1]For a discussion, see Edwards (2019).

[2]The negative effects of high uncertainty on economic performance have been studied by Bloom (2009, 2014) and Bloom et al. (2018), among others. In particular, the effects of policy uncertainty have been documented by Boutchkova et al. (2012), Fernandez-Villaverde et al. (2015), and Baker, Bloom, and Davis (2016). While these studies focus mainly on the U.S. economy, others focus on small open economies. For example, Fernandez-Villaverde et al. (2011) document that interest rate volatility at which small open economies borrow can trigger a contraction in output, consumption, and investment.

[3]In the Latin American context, see Dornbusch and Edwards (1991). Edwards (2019) draws lessons from Latin America for the rest of the world. Hopkin and Blyth (2019) discuss the impact of populism in Eastern Europe. Rodrik (2018) discusses the interplay between globalization and populism. In a series of papers edited by Kehoe and Nicolini (2022), several authors study the structural changes and economic performances of many Latin American countries in the last six decades.

accessible to individual investors. The sample period is from 1981 to 2019 and includes four populist periods and three market periods.

We find that the risk-return trade-off using the full sample—which corresponds to the appropriate approach if one ignores regime changes—is very misleading of the actual options available to investors. If we allow for unrestricted portfolios—that is, portfolios in which some assets can be shorted—the minimum variance frontier during market periods uniformly dominates that of populist periods. This means that for a given riskiness of the portfolio, expected returns are higher during the market regime.

This finding, somewhat surprisingly, depends crucially on the assumption that the investor can go short in some assets. In the case of Argentina, the returns on investing in foreign exchange are negative during market periods and positive during populist periods. Thus, a policy of contracting debts in U.S. dollars during market periods is behind the large returns of the optimal portfolio. This result is roughly consistent with the observation that Argentina has, in the past, significantly increased borrowing in foreign currency during market periods. It also shows that high returns are associated with leverage and the regime-dependent returns encourage even risk-averse investors to take significant risk by highly leveraging their portfolios.

To capture the trade-offs faced by investors that cannot short any asset, we compute the minimum variance frontier, imposing the restriction that no asset can be used to borrow to finance long positions. The results are radically different. Two extreme observations give a good sense of the differences. First, the safest (lowest variance) portfolio that can be constructed using returns during the populist period has a level of risk—as measured by the standard deviation of the returns—that is about 50 percent higher than the riskiest portfolio during the market period. Second, the highest expected return that is possible to attain in the market regime falls short of 9 percent, while the portfolio with the highest expected return in the populist period earns over 60 percent per year.

To better understand optimal investment decisions, we develop a dynamic portfolio choice model. We consider a long-lived investor who understands that regime changes are stochastic and that it is costly—both in terms of time and resources—to adjust a portfolio. We consider several scenarios and find that the composition of the optimal portfolio depends, crucially, on whether assets can be shorted or not. In the case that the investor can borrow, they take advantage of this possibility by creating high return-high risk portfolios during market regimes by borrowing in foreign exchange and investing in domestic real estate. The negative positions are undone during populist regimes to reduce the riskiness of the portfolio, but investments in real estate are still a major component.

These large differences in the composition of the optimal portfolios are a reflection of the large differences in returns across policy regimes. These differences imply that a fixed portfolio, apart from one invested in real estate, that performs well in one regime can earn poor returns upon a regime change.

A more general, although somewhat speculative, message from our exercise is that policy instability that is associated with increased uncertainty will generally induce large changes in positions and hence in the price of different assets. Even though Argentina is an extreme example of poor and unstable policy, it is a perfect laboratory to study the potential costs of instability as they appear to be large.

The rest of the article is as follows. Section 2.2 briefly describes the major features associated with populist and market-friendly policies. Section 2.3 describes the risk-return trade-offs across policy regimes. Section 2.4 develops a dynamic model of optimal portfolio choice and illustrates—using data from Argentina—the impact of regime changes on the allocation of wealth across assets. Section 2.5 concludes.

2.2 Argentina: Populist and Market-Friendly Regimes

Simon Kuznets is said to have remarked that there were four types of countries: developed, developing, Japan, and Argentina. If Kuznets were writing today, he would probably subtract Japan from that list as its economic performance can be readily understood using standard models. However, Argentina, a country characterized by an above-average endowment of natural resources and a relatively high endowment of human capital, remains a puzzle (and interesting case study) due to its frequent and large policy changes and poor performance.

It is impossible to summarize the economic history of Argentina since 1980 in a few paragraphs.[4] At a general level, the economic policies implemented in the last 100 years alternate between a version of *populism*[5] and more market-friendly policies. It is misleading to believe that, within a regime, policies are stable. Typically, the first few months of the pro-market regime are devoted to undoing the regulations and fixing the distortions inherited from the populist regime. Similarly, a populist regime spends the first few months creating the institutional framework to implement its preferred policies.

There is no agreement on what constitutes a populist economic policy. Edwards (2019) distinguishes between *classical* and *new* populism. He views most populist experiments in Latin America before 1990 as being of the classical variety that relies on heterodox macroeconomic policies. New populism emphasizes "blanket regulations, deep protectionist policies, large expansions of the public sector, and mandated minimum wage increases"(Edwards 2019). However, given that our interest is in the relationship between policies and portfolio choices, it is useful to describe some features of both policy regimes that directly influence asset returns and, consequently, optimal portfolios.

Populist economic policies typically include (especially in the latter stages) the following:

- Exchange rate and capital mobility controls
- Significant regulation of financial intermediaries, including caps on yields and quantitative restrictions

[4]Recent short summaries of the economic history of Argentina include Buera and Nicolini (2019) and the various papers that appeared in the December 2018 issue of the *Latin American Review*. A good summary of the economic outcomes can be found in the introduction of Glaeser et al. (2018). See also Cavallo and Cavallo Runde (2018) and Della Paolera and Taylor (2003). De Pablo (2019) (in Spanish) discusses the difficulties of designing economic policy in Argentina.

[5]See Dornbush and Edwards (1991) for traditional populism and Edwards (2019) for the new populism as applied to Latin America.

- Use of extreme adjustment (or unorthodox taxation) mechanisms (confiscation of assets either through mispriced mandatory exchanges or inflation)
- High taxes on the formal sector (which promote informality)
- Price controls, including rents

From the perspective of an investor choosing his portfolio, there are two important features. First, price (or rate of return) controls and regulations that require some economic agents to invest in those assets as part of their economic activity distorts portfolio choices and rates of return. Some assets might display a "convenience yield" if they provide a way of bypassing costly regulations. The returns of other assets might reflect the existence of, for want of a better word, a "convenience tax," which is the case when holding these assets exposes the investor to some form of penalty. This includes assets that have low liquidity (e.g., real estate) as well as assets that expose investors to risk (black market operations in foreign exchange).

Second, during periods of populist policies, governments have resorted to a variety of actions that are tantamount to expropriation. Examples of this type of policy include exchanging at par bank deposits for government bonds whose market price was about 30 percent of their par value, episodes of hyperinflation that amount to a tax on nominal assets, and "unilaterally rewriting contracts in U.S. dollars in depreciated pesos, imposing huge losses to investors and international firms"(Edwards 2019, 95).

Some of the main features of market-friendly policies are the following:

- Elimination of many regulations and controls
- Minimal restrictions on capital mobility and restrictions on portfolios (e.g., allowing portfolios to include assets denominated in foreign currency)
- Low probability of expropriation

In a market-friendly regime, the standard approach to asset pricing should yield a better fit conditional on the regime. However, since regime changes are rightfully viewed as random events, the pricing equation has to take that into account.

To make progress on understanding asset valuation, we use the sample in Mosquera and Sturzenegger (2020), which contains data on returns on a variety of assets for the period 1981-2019, and split it into two subsamples according to the policy regime. As mentioned above, there is no uncontroversial procedure for determining whether a particular policy is populist or market friendly. We use the following criteria:

- Market regime: This includes the period during which Argentina followed a traditional monetary policy with a constant exchange rate from April 1991 to November 2001; December 2002 to March 2011, during which there were few restrictions on asset transactions; and January 2016 to August 2019, when the Macri government liberalized the economy and did not impose exchange controls.
- Populist regime: This consists of the rest of the sample.

It is clear that there is a fair amount of policy heterogeneity within each of these phases. However, to preserve degrees of freedom, we ignore the within-regime differences.[6]

In this article we present results on the real returns of a collection of assets, and we discuss the evidence from the perspective of a standard asset pricing model. The set of assets that we consider include the following:

- Time deposits (CD): These are regular time deposits (the minimum term varies greatly over time, but they could be as short as 7 days and as long as a year). We use the 30-day CD rate. The interest is set in nominal terms, but the returns are deflated by a measure of inflation using the consumer price index (CPI).

[6]This classification is arbitrary. We have experimented including the 2002-2011 period as part of the populist regime, and the results are virtually identical. Ocampos (2018) developed an index that includes the gap between official and black market exchange rates, fiscal deficit, and differences between import and export exchange rates, among other variables. His sample includes the years 1982–2013. According to his Index 1, the relevant value for the years that we consider market friendly is 3.90, while the corresponding value for the populist years is 6.42. In the appendix we report the results of the exercise where portfolios are chosen optimally for alternative definitions of the two regimes. The results are similar.

- Adjustable bank deposits (UVA): Interest paid is adjusted using a formula that, effectively, is a distributed lag of the inflation rate during the previous two months. The resulting nominal rate is deflated using the CPI.
- Real estate (RE): The return is an index of the change in house prices and an allocation for the monthly value of a lease.
- U$S dollar (U$S): This is the real return in pesos of holding non-interest earning dollars valued at the "official" (legal) exchange rate. Thus if the peso-dollar exchange rate is denoted as S_t (pesos per dollar), then the return is computed as

$$e^{r_{Dt}} = \frac{S_t/S_{t-1}}{P_t/P_{t-1}},$$

  where P_t is a measure of the aggregate price index.
- U$S dollar "blue" (B): It is also the return from holding U$S dollars except we use the black market exchange rate instead of the official exchange rate. Even though there are some costs associated with exchanging dollars at this rate, it is relatively easy for middle-class Argentinians (but not necessarily for low-income households) to access this informal market.
- The data are monthly and have not been seasonally adjusted.

From the perspective of the U.S., it might be surprising that we exclude investments in some form of security that tracks the overall value of the stock market. However, the reason for this exclusion is the lack of a consistent index that covers the period under study.[7]

How different are these two regimes? Table 2.1 presents data for the whole sample and each subsample separately for the growth rate of the real wage and inflation. The differences across regimes are stark. Real wages (a proxy for consumption) grow faster and are more stable during market-friendly periods. At an annual level, they exhibit zero growth during populist periods

[7]It is possible to use official statistics corresponding to the MERVAL index for the period 2004–20, but we could not find data covering the whole period. The stock market capitalization relative to GDP in Argentina is very small. According to the World Bank, it was less than 9 percent in 2019, while the average for Latin American countries exceeds 50 percent, and it reaches 190 percent in the U.S.: https://databank.worldbank.org/reports.aspx?source=2series=CM.MKT.LCAP.GD.ZScountry=

Table 2.1: Monthly Growth Rates (%)

	Real wage		Prices	
Regime	Mean	St. dev.	Mean	St. dev.
Full sample	0.121	3.55	5.37	13.92
Populist	0.010	5.30	13.63	20.12
Market	0.200	1.40	1.03	1.34

and about 0.2 percent (per month) during periods in which the prevailing macro policy is market friendly. Relative to more developed economies, Argentina shows a very large variability of the growth rate of our proxy for consumption. The ratio of the mean growth rate to its standard deviation for the whole period is about 29, while in the U.S. the ratio is about 1.88, measured by the Real Personal Consumption Expenditure, from 1981 to 2019.

The differences in inflation across regimes are even larger. The monthly inflation rate is about 11 times higher, on average, in populist periods. The standard deviation is also higher. At these levels of inflation, it is reasonable to assume that minimally informed investors can distinguish between nominal and real returns. Thus, our choice of only focusing on real returns appears to be justified.

Overall, we find that these two indicators convey the basic message: economic outcomes under the two policy regimes are starkly different. Ignoring the possibility of regime switching is likely to result in mistakes in understanding the performance of individual portfolios.

2.3 Risk and Return

In this section we present the basic features of the risk-return trade-off for a variety of assets.

2.3.1 Individual Assets

What is the return-risk trade-off for individual assets? Table 2.2 shows average monthly (real) returns, $\bar{r}_i^S$, (in percentage terms) as well as their standard deviations, σ_i^S, (also in percentages)

Table 2.2: Monthly Asset Returns (%)

Asset	Mean ($\bar{r}_i^S$)	St. deviation (σ_i^S)
CD	-0.77	4.65
UVA	0.42	8.27
RE	0.79	7.92
U$S	0.61	12.17
B	2.02	29.14

corresponding to the full sample.[8] The differences are large. Investing in blue (B) dollars earns the highest return but also has the highest standard deviation. Investing in real estate (RE) earns the second-highest return. From the perspective of a mean-variance investor, these two assets dominate the returns of investing in UVAs and U$S dollars.

Even though we have ignored the possibility of default (at least conditional on the regime), it is important to emphasize that some "safe" investments from the perspective of an American investor (e.g., bank CDs) are risky in Argentina due to the large (and many times hard to predict) swings in the inflation rate. Thus, the riskiness of some assets is associated to the large change in their value in terms of goods associated with unanticipated changes in inflation.

These results hide large differences in the first and second moments of asset returns depending on the policy regimes. Table 2.3 presents the same statistics but distinguishes the policy regime.

The differences are shocking: First, investments in dollars (both official (U$S) and blue (B)) earn high returns during populist periods and negative returns during periods in which financial markets operate more freely. Second, the standard deviations of the returns are also much smaller during periods in which the policy is more market friendly, which reflects the overall stability of the economy during those periods. The third interesting feature is that the asset that displays the smallest difference between regimes is real estate: the expected returns are similar across periods, and the standard deviation during market periods is about one-third of the value in the populist periods. This is a much smaller relative decrease than the corresponding changes for other asset

[8] All the data were kindly provided by Federico Sturzenegger and Santiago Mosquera and were used in Mosquera and Sturzenegger (2020). The appendix contains a brief description of the data.

Table 2.3: Monthly Asset Returns (%)

	Populist sample		Market sample	
Asset	Mean	St. Dev.	Mean	St. Dev.
CD	-1.92	7.03	0.03	0.92
UVA	0.78	12.86	0.17	1.18
RE	0.86	11.78	0.74	3.21
U$S	2.08	18.65	-0.41	2.80
B	5.68	45.17	-0.52	3.46

Table 2.4: Correlation Coefficient (Market)

	CD	Dollar	Real estate	UVA	Blue
CD	1.000	0.237	0.296	0.259	0.070
Dollar	0.237	1.000	0.708	-0.102	0.781
Real estate	0.296	0.708	1.000	-0.087	0.507
UVA	0.259	-0.102	-0.087	1.000	-0.090
Blue	0.070	0.781	0.507	-0.090	1.000

Table 2.5: Correlation Coefficient (Populist)

	CD	Dollar	Real estate	UVA	Blue
CD	1.000	0.111	-0.017	0.087	0.164
Dollar	0.111	1.000	0.813	-0.083	0.890
Real estate	-0.017	0.813	1.000	0.107	0.740
UVA	0.087	-0.083	0.107	1.000	-0.011
Blue	0.164	0.890	0.740	-0.011	1.000

classes, and in part it reflects the preference of Argentinean middle-class investors for saving in the form of "bricks," as investments in real estate are popularly known. Tables 2.4 and 2.5 display the correlation matrices for the two regimes. In general, except for real estate and the two measures of the returns to foreign exchange, the correlations are rather small. A low return asset (UVA) is the only one that displays a negative correlation with real estate, official dollar, and dollar blue.

Another tool to describe the risk-return trade-off is the minimum variance frontier.[9] This fron-

[9]The original pioneering works are Markowitz (1952), Sharpe (1964), and Lintner (1965). For a good summary of the Capital Asset Pricing Model, see Perold (2004).

tier displays the highest possible return from combining all assets for a given measure of the portfolio's risk (its variance). To highlight how different regimes result in different risk-returns trade-offs, we compute the minimum variance frontier for the whole sample and for each subsample. When we allow investors to form unrestricted portfolios (which allow shorting), we find that for any given level of risk—as measured by the standard deviation—the expected return in a market regime is uniformly higher.

Figure 2.1 shows the minimum variance frontier for the whole sample and for each of the two subsamples. It shows that not only are expected returns higher (for a given standard deviation) during market periods, but the lowest risk portfolio in a market regime is also several times safer than the minimum variance portfolio in the populist regime, consistent with the differences in the covariances between assets in the two regimes.

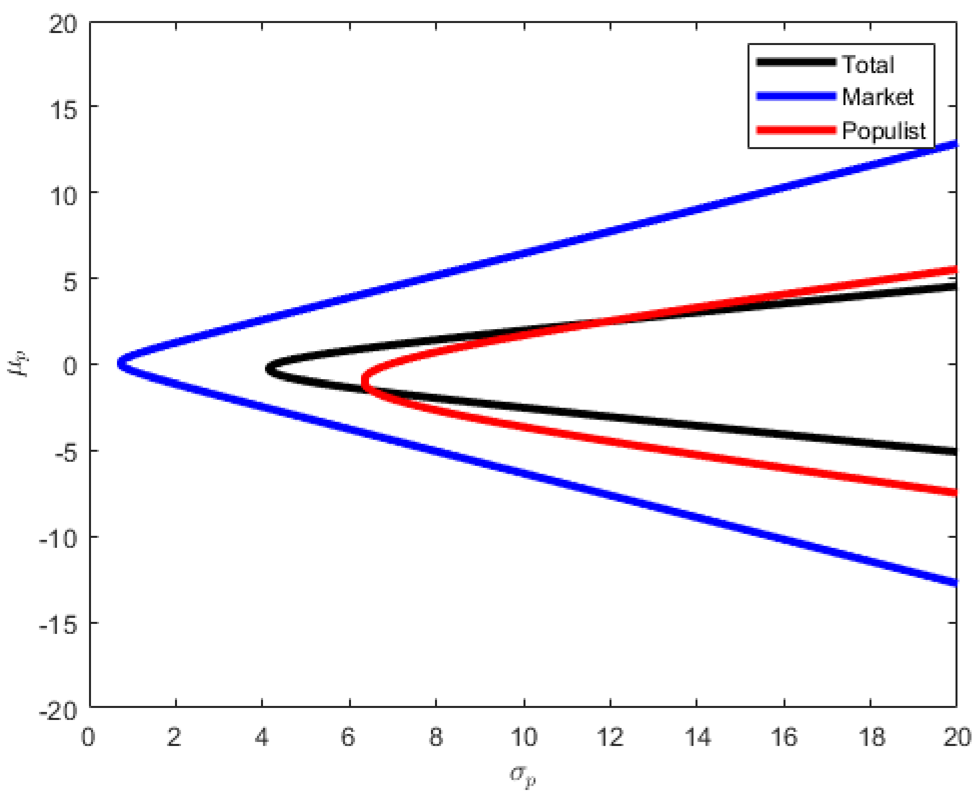

Figure 2.1: Minimum Variance Frontier (Unconstrained)

The previous result allows investors to short every asset. If we restrict portfolios to contain only long positions, the differences across regimes are starker. Figure 2.2 displays the risk-return trade-offs in the no-shorting case. The differences across regimes are very large: the safest portfolio in the populist regime has a risk that is almost twice the standard deviation of the riskiest portfolio in the market regime. On the other hand, the highest expected returns during populist periods greatly exceed those of the market periods.

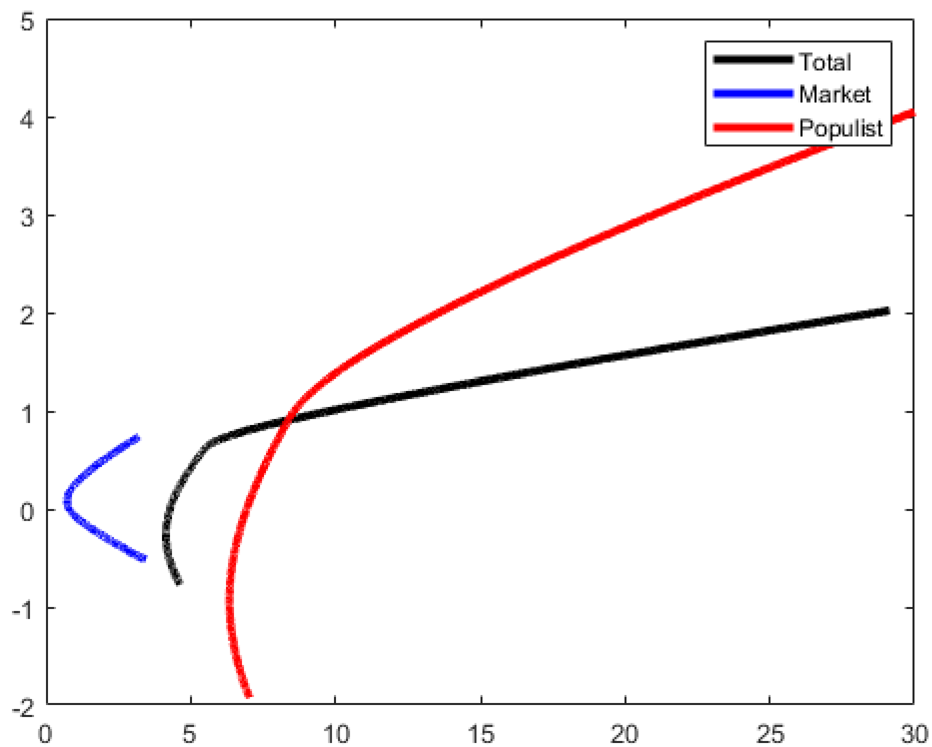

Figure 2.2: Minimum Variance Frontier (Constrained)

Overall, this first look at the risk-return trade-offs across regimes shows there are large differences in returns (and in the standard deviation) and that even the minimum variance portfolios vary across regimes. In Appendix D we report changes in the sample to allow financial crisis in the market periods, and we find no significant differences with the benchmark case.

2.4 Portfolio Returns and Regime Change

A natural next step is to go beyond the simple measures of risk and return and to determine what is the *optimal* portfolio for an investor who understands that returns vary across regimes and regimes are not permanent; that is, there is a nonzero probability of a regime change at any given time.[10]

The previous exercise shows that, depending on the desired rate of return, a portfolio's composition changes dramatically across policy regimes. In this section we make progress on understanding the optimal portfolio for risk-averse investors who account for regime changes and know it is not costless to change their portfolio's composition.[11]

In our setting a portfolio is a set of weights, $\alpha = (\alpha_{CD}, \alpha_{U\$S}, \alpha_{RE}, \alpha_{UVA}, \alpha_B)$, that add up to one. A restricted portfolio requires that, in addition, the α_k cannot be negative. We assume that the expected return of a portfolio in state $j \in \{M, P\}$ is given by

$$\mu_j(\alpha) = \sum_{k \in \Upsilon} \alpha_k r_{k,j}$$

and the variance of the portfolio is

$$\sigma_j^2(\alpha) = \sum_{k \in \Upsilon} \left(\alpha_k r_{k,j} - \mu_j(\alpha)\right)^2,$$

where the set Υ is $\{CD, U\$S, RE, UVA, B\}$.

Preferences are then given by

$$U_j(\alpha) = \mu_j(\alpha) - \frac{\theta}{2}\sigma_j^2(\alpha).$$

We assume that the investors care about expected returns and dislike uncertainty.

[10]Note that our specification of regimes does not coincide with political mandates. In other words, the same administration can choose populist and market-friendly policies.

[11]There is extensive literature on portfolio adjustment costs. There are two types of costs, transaction costs and observation costs. The former generate state-dependent portfolio rebalancing, while the latter generate time-dependent portfolio rebalancing. Our model falls into the first category; some examples of these can be found in Bonaparte and Cooper (2010); Bonaparte, Cooper, and Zhu (2012); and Muhle-Karbe, Reppen, and Soner (2017). For our case, fixed adjustment costs generate an inaction region, so investors tend to make infrequent adjustments of their portfolios. In particular, Rieger (2015) documents that these costs tend to lower the volume of trading but increase the volatility of asset prices. On the other hand, for models with observational costs, the adjustment is time dependent. Examples of these can be found in Abel, Eberly, and Pangeas (2007); Alvarez, Guiso, and Lippi (2012); and Huang and Liu (2007).

We consider the problem of an investor over a long horizon who understands there will be regime changes and that it is (potentially costly) to change a portfolio. Formally, the investor solves

$$\max_{\alpha \in \Upsilon} E \int_0^\infty e^{-\rho t} U_{j(t)}(\alpha_t) dt - \sum_{n=0}^{\infty} e^{-\rho n} c_{j(n)},$$

where the expectation is taken over the stochastic process of regime change and individual states that capture frictions in adjusting the optimal portfolios. Here $j(s)$ indicates the state (either M or P) at time s, while $c_{j(n)}$ is the fixed cost of changing a portfolio when the state is $j(n)$ time n. We use n to denote the jump times when the economy switches from one regime to the other.

Since regime changes are often periods in which many activities are disrupted, it is not obvious that investors can adjust their portfolio instantaneously. We capture this delay by creating a state after a regime switch in which portfolios are unchanged. We view the switch from state M to P as driven by a Poisson process with parameter π_M, implying that the expected duration of a market period is $1/\pi_M$. The switch from P to M is captured by a Poisson process with parameter π_P.

Suppose the economy is in state M and it switches to P. An individual i cannot immediately change his portfolio (at any cost) for a random period of time with expected duration $1/\eta^i_M$. There is a similar waiting period when the switch is from P to M. In this case the relevant expected time is $1/\eta^i_P$.

It is convenient to describe the value of a portfolio using a recursive formulation. Let $V_j(\alpha)$ be the value of holding portfolio α in state $j \in \{M, P\}$. Then, the appropriate valuation formula is

$$\rho V^i_M(\alpha) = U^i_M(\alpha) + \pi_M \left[V^i_{MP}(\alpha) - V^i_M(\alpha) \right],$$

where $V^i_{MP}(\alpha)$ is the value of the (fixed) portfolio α in state P *before* the investor has had a chance to make any adjustments. It follows that $V^i_{MP}(\alpha)$ is the solution to

$$\rho V^i_{MP}(\alpha) = U_P(\alpha) + \eta^i_P \left[\max\left(\max_{\alpha'} V^i_P(\alpha') - c^i_P, V^i_P(\alpha) \right) - V^i_{MP}(\alpha) \right].$$

Note that when the individual can change the portfolio, the optimal decision depends on both the cost of switching and the value of the “old” portfolio in the new regime.

The value of switching (net of costs) is simply $\max_{\alpha'} V_P^i(\alpha') - c_P^i$. If this exceeds the value of the old portfolio in state P, $V_P^i(\alpha)$, then it is optimal to pay the cost and switch. In this case the new payoff is $\max_{\alpha'} V_P^i(\alpha') - c_P^i$. If the cost of switching is high, then the investor does not adjust the portfolio and the value is $V_P^i(\alpha)$.

The Hamilton-Jacobi-Bellman equations that describe the value in state P are similar.

2.4.1 Small Switching Costs

If monetary switching costs are small, that is, if the c_j^i are small, then the investor will choose the best portfolio after a regime switch as soon as this is possible. In this section we let the monetary switching costs be small but keep the time switching costs unchanged. Formally, we assume that $\max_{\alpha'} V_P^i(\alpha')\ \ c_P^i > V_P(\alpha)$. In this case, the optimal portfolio in state M is given by

$$\alpha_M^* = \operatorname{argmax}_\alpha H_M(\alpha) \equiv U_M(\alpha) + \frac{\pi_M}{\rho + \eta_P} U_P(\alpha).$$

The optimal portfolio maximizes a weighted average of the payoffs in each of the two states. The magnitude of the factor $\pi_M / (\rho + \eta_P)$ determines how much weight an investor who is choosing his portfolio during a market period will assign to the performance of the portfolio in the populist regime. This factor increases as the likelihood of a switch to the populist regime becomes higher (the higher is π_M) and as the waiting period until the portfolio can be adjusted grows longer (the lower is η_P).

The optimal portfolio in the populist regime solves an analogous equation, and it is given by

$$\alpha_P^* = \operatorname{argmax}_\alpha H_P(\alpha) \equiv U_P(\alpha) + \frac{\pi_P}{\rho + \eta_M} U_M(\alpha).$$

Let

$$V_j^+ \equiv \max_{\alpha'} V_j(\alpha'), \text{ for } j \in \{M, P\}.$$

The appendix describes the expressions for V_j^+, which gives the value of an investor in state j of following the optimal strategy, taking into account switching regimes and costs.

Let $\hat{\alpha}_M$ be the optimal portfolio for an investor in state M if the economy were to stay in that state forever. Thus, the value that this investor would obtain is

$$\hat{V}_M = \frac{U_M(\hat{\alpha}_M)}{\rho} = \frac{\mu_M(\hat{\alpha}_M) - \frac{\theta}{2}\sigma_M^2(\hat{\alpha}_M)}{\rho}.$$

Then we can define an "equivalent expected return," $\tilde{\mu}_M$, as the expected return that an investor who faces risk $\sigma_M^2(\hat{\alpha}_M)$ would demand to be indifferent between this portfolio and the value $\bar{V}_M$. Thus,

$$\frac{\tilde{\mu}_M - \frac{\theta}{2}\sigma_M^2(\hat{\alpha}_M)}{\rho} = V_M^+.$$

It follows that

$$\tilde{\mu}_M = \mu_M(\hat{\alpha}_M) - \rho\left(\hat{V}_M - V_M^+\right).$$

The term $\rho\left(\hat{V}_M - V_M^+\right)$ measures the loss of utility—in expected returns—associated with the instability of the Argentine economy relative to the ideal alternative in which the economy is always in the M regime.

2.4.2 Large Switching Costs

In this section we examine the optimal decision of individuals that face large switching costs. Effectively, this assumption implies that $\max_{\alpha'} V_j^i(\alpha') - c_j^i < V_j(\alpha)$ for $j \in \{M, P\}$. This, in turn, implies that in the market regime, the value of a portfolio α is given by

$$\rho V_M^i(\alpha) = U_M^i(\alpha) + \pi_M\left[V_{MP}^i(\alpha) - V_M^i(\alpha)\right],$$

and $V_{MP}^i(\alpha)$ is

$$\rho V_{MP}^i(\alpha) = U_P(\alpha) + \eta_P^i\left[V_P^i(\alpha) - V_{MP}^i(\alpha)\right].$$

Solving for $V_M^i(\alpha)$ (see the appendix), it follows that its value is proportional to

$$G_M(\alpha) = H_M(\alpha) + \frac{\pi_M}{\rho + \eta_P}\left(Z_M - 1\right)U_P(\alpha)$$

and $Z_M > 1$ if and only if

$$(\rho + \eta_P)(\rho + \eta_M) > \pi_P \pi_M,$$

which is satisfied when the expected time required to adjust the portfolio is relatively short (this corresponds to a high η_j) relative to the duration of a regime. These restrictions are clearly satisfied in the data. Consequently, the implications are as follows. Since the optimal portfolio with small costs maximizes $H_M(\alpha)$, the optimal portfolio for agents with large switching costs puts more weight on the return of the portfolio after a switch: during the market period, these investors choose a portfolio (relative to the small-cost investors) that puts more weight on the payoff in the populist period. A similar expression holds for the investor who enters the market in the populist regime.[12]

2.4.3 Taking Stock

For an investor who can be characterized as a "small switching cost" investor based on the amount of time he has to wait until he can adjust his portfolio (as captured by η_M and η_P) and the actual costs he faces when changing the composition of his portfolio (as captured by (c_P, c_M)), the model implies that he continuously readjusts his portfolio every time a regime changes. At the other end, a "large switching cost" investor chooses his optimal portfolio—which depends on the regime when he first entered the market—and never changes.

The truth for a given investor is probably a mixture of the two extremes: an individual sometimes faces small costs and sometimes large costs. In what follows we will explore—under a variety of possible parameterizations— the differences in the portfolios across types of investors (high and low switching costs) and regimes (market and populist).

2.4.4 Calibration

To quantify the impact of regime changes, we must estimate the parameters of the model. In this section we describe the strategy that we use to select reasonable parameter values. One key parameter is the degree of risk aversion θ. To estimate risk aversion, we consider the expected value of an investor in the U.S. who chooses between a risky portfolio and safe portfolio using the

[12] The appendix contains the expressions for the value of the dynamic problem in all cases.

same risk-variance preferences. Standard calculations show that the share of the risky portfolio is given by

$$\alpha = \frac{E\left(r^{s}\right) - r^{f}}{\theta \sigma_{s}^{2}},$$

where $E\left(r^{s}\right)$ is the expected return to the risky asset and σ_s^2 its variance, and r^f is the risk-free rate. In the U.S. the equity premium is somewhere between 4 and 8 percent, and the standard deviation of a broad index of the stock market is about 16 percent. There is some controversy regarding the share of the U.S. portfolio that is composed of safe assets (which correspond to $1-\alpha$ in this calculation). Gorton, Lewellen, and Metrick (2012) estimate the safe share to be somewhere between 31 and 33 percent. Martin (2018), using a more conservative definition, estimates it at 25 percent. We take 30 percent as a compromise, and hence the risky asset share is 70 percent. This implies that, depending on the assumption about the equity premium, $\theta \in \{2, 4\}$.

The value of $1/\pi_j$ measures the expected duration of regime j in months. In our sample we find that, on average, market regimes last 91 months and populist regimes last 62. Thus, we estimate π_M=0.011 and π_P=0.016.

There are no estimates that we are aware of for time (delay) costs and rate of return costs. If there is no change of regime, the value of a portfolio is

$$V = \frac{\mu - \frac{\theta}{2}\sigma^{2}}{\rho}.$$

If we measure cost in terms of expected return, we have that

$$V\,c = \frac{\mu - \frac{\theta}{2}\sigma^{2}}{\rho} - \frac{\bar{c}}{\rho}.$$

Then if $\bar{c} = x$, then $c = x/\rho$. A small switching cost is 0.01 percent on a monthly basis (which is about 12 basis points on an annual level).

If the time delay is about one month, then $\eta = 1$; if it is two weeks, then $\eta = 2$; and if it is two months, then $\eta = 1/2$. We experiment with those values. Our benchmark scenario assumes the delay is one month for both regimes, but we do a sensitivity analysis for different values of θ and η. The full calibration of our benchmark scenario is described in Table 2.6.

Table 2.6: Calibration

Parameter	Notation	Value
Discount rate	ρ	0.0025
Market regime intensity	π_M	0.011
Populist regime intensity	π_P	0.016
Market delay	η_M	1
Populist delay	η_P	1
Market adjustment cost	c_M	0.0001
Populist adjustment cost	c_P	0.0001

2.4.5 Findings

In this section we describe the results of our benchmark scenario. Table 2.7 shows the optimal portfolios for the low- and high-cost case.[13]

Table 2.7: Optimal Portfolios

	Low costs				High costs			
Asset	α_M^U	α_M^C	α_P^U	α_P^C	α_M^U	α_M^C	α_P^U	α_P^C
CD	1.2774	0.0001	-0.3551	0.0000	1.2770	0.0000	-0.4198	0.0000
Dollar	-6.1307	0.0000	0.1786	0.1184	-6.1303	0.0000	-0.0484	0.0000
Real estate	5.5760	0.9994	0.5623	0.3945	5.5760	1.0000	0.8291	0.5531
UVA	-0.3486	0.0004	0.6252	0.4714	-0.3484	0.0000	0.6246	0.4302
Blue	0.6259	0.0000	-0.0109	0.0156	0.6257	0.0000	0.0145	0.0167
Mean	6.26%	0.74%	1.97%	1.04%	6.26%	0.74%	1.99%	0.91%
Variance	10.30%	3.21%	10.66%	8.70%	10.30%	3.21%	11.03%	8.46%

Our previous analysis showed that the minimum variance frontiers vary significantly depending on whether we assume (as in standard portfolio composition analysis) that the investor can hold negative positions in some asset (borrow) or not. To highlight how this distinction is critical, we separately analyze the two cases.

[13] In the Appendix, we report the results using an alternative definition of regimes that includes, in the market regime, some crises. We do not find significant differences in the results.

Unrestricted Portfolios The optimal portfolios for this case (when switching costs are relatively low) are in the columns labeled $\alpha^U_{M,L}$ for the market regime and $\alpha^U_{P,L}$ for the populist regime (Table 2.7). The corresponding portfolios for the high-switching cost case are $\alpha^U_{M,H}$ and $\alpha^U_{P,H}$.

There are several remarkable results. First, the existence of the two regimes encourage investors to take a large amount of risk. For example, in the market regime, $\alpha^U_{M,L}$, it is optimal to borrow a large amount (six times the value of the capital) in foreign currency to finance investments in real estate and time deposits. When the regime changes (the optimal portfolio is in the column labeled $\alpha^U_{P,L}$), the positions are undone: the only significant short positions is in domestic currencies at a fixed rate, while the most significant long positions are in real estate and adjustable deposits.

To illustrate how it is optimal to leverage a position, consider the return of the portfolio in the market regime. The expected return is a staggering 6.26 percent per month, compared with Table 2.3, where the highest return in the market regime is 0.74 percent. The standard deviation of the portfolio is over 10 percent. Interestingly, the riskiness of the portfolio is similar in both regimes, although the expected return is much lower (1.97 percent) in the populist regime.

We next estimate the costs of switching. Using a conservative estimate, the cost of a regime change from market to populism (relative to the alternative of a permanent market regime) is equivalent to a decrease in expected returns (controlling for the variability) of about 3 percent. This is a significant difference.

One way to summarize these results is that the market encourages investors to take risky positions with high leverage borrowing in foreign currency during market periods and a more conservative stance in populist periods. Note that the riskiness of the portfolio is about the same in both regimes but (see Table 2.3) the volatility of returns of individual assets is much higher in the populist regime.

The results for the large cost of switching (which is close to myopic investors) are very similar. The reason is simple: Given the (relatively small) instantaneous probability of change, it is optimal to invest for the present, paying little attention to the costs of switching.

Restricted Portfolios The results when investors cannot borrow are also surprising: In the market regime, basically 100 percent of the investment is allocated to real estate. During the populist regime, a little over 50 percent is invested in real estate and the rest in adjustable rate deposits. The returns are much lower than in the unrestricted case and so is the riskiness of the portfolio.

Sensitivity Analysis Since there is some uncertainty about some of the parameters section, Appendix shows the results of changing some of the parameters. We find that decreases in risk aversion increase, as expected, the riskiness of the portfolio. Changes in the expected time of adjusting the portfolio have a small impact on the results. We also experiment with changes in the expected duration of a policy regime. When we make—contrary to the evidence—the market regime more transitory, the difference in the portfolios across regimes is very small, and the expected returns and the riskiness of the portfolio are much lower. When the market regime is transient, the optimal portfolio is close to the optimal portfolio in the base case in the populist regime. When only the populist period is transient, we get the opposite result. In addition, when the expected duration of the regimes changes, the differences between high- and low-cost switching become larger.

2.5 Conclusion

It is not surprising that in a country like Argentina—characterized by large and dramatic changes in economic policy, including changes that at times have amounted to confiscation of assets—the risk-return menus available to investors change with the policy regime. Using data from Argentina to better understand the consequences of populist economic policies relative to market policies, we find that relatively safe portfolios that perform well during market periods display a large negative return and very high risk during populist periods. In general, a robust finding is that if investors are constrained in terms of leverage (no shorting), then it is inevitable that a switch to a populist regime results in higher risk.

We also find that an investor who understands that regimes change randomly and that it is costly to adjust his portfolio will pick a portfolio that both reflects the current regime and accounts for the returns of that portfolio when the regime changes. In addition, we find that when investors (individuals as well as firms) are free to have short positions, a clear pattern emerges: It is optimal during market periods to borrow heavily in foreign exchange to invest (mostly) in domestic real estate. When the policy regime changes (to populism), the short position in U$S dollar is turned into a long position, and investments in real estate and adjustable deposits in domestic currency make up most of the portfolio. Investors are thus willing to take on a significant amount of risk.

This instability is a reflection of the costs of regime switches (and the poor economic performance of populism), but it is totally justified from the perspective of an individual: A fixed portfolio that performs relatively well in one regime can perform poorly when it changes. Portfolio adjustment —with the consequent disruption and changes in relative prices—is a necessity in turbulent economies.

We also find that when investors are not allowed to borrow to finance their portfolios—a friction that captures rigidities in the financial sector—the optimal portfolios include almost exclusively real estate in the market regime and a mix of real estate and time deposits in the populist regime. Overall, our results show that regime switches between populist and market regimes result in portfolio compositions that are quite different from what is observed in a more stable environment such as the U.S.

2.6 Appendix D

2.6.1 Data

The data were shared by Santiago Mosquera and Federico Sturzenegger from the University de San Andres in Argentina and were used in Mosquera and Sturzenegger (2020).

Time deposits (CD): These correspond to 30 certificates of deposits in the formal banking system in nominal terms. The real returns were deflated using a version of the CPI modified for the periods in which the economic authorities reported incorrect values.

Adjustable bank deposits (UVA): These are deposits in the formal banking system, and the nominal return is adjusted depending on a weighted average of the inflation over the previous two months.

Real estate: This is an index in real terms with an imputation for the market value of leases.

U$S dollar and U$S dollar blue: See the text for a description.

2.6.2 Chronology of Economic Policies

1975–1991

- Real per capita income decreases by 20 percent
- Annual inflation exceeds 300 percent
- External debt increases
- The real exchange rate is overvalued
- Capital flight occurs

1991–2001

- Free market reforms and increases in foreign investment
- Privatization of state-owned enterprises
- External shocks (Long-Term Capital Management collapse and Russian debt crises)

2001–2002

- Large restrictions on withdrawals of bank deposits

2002–2011

- Increases in regulation
- Renationalization of some formerly state-owned enterprises
- Little interference with asset markets

2011–2015

- Large sovereign debt crises
- Major devaluation
- Increases in regulation
- Exchange controls and, in the latter part of the period, capital controls
- Renationalization of many firms

2016–2019

- Pro-market reforms
- Large deficits that were lowered gradually
- Flexible exchange rates
- No capital controls

2.6.3 Minimum Variance Frontier

The first step is to build the Minimum Variance Frontier, or the efficient frontier, by choosing optimal asset allocations to minimize the variance of the portfolio given a specific return. Mathematically, this problem can be expressed for our case as

$$
\begin{aligned}
\min_{\{\omega_i\}_{i\in\Theta}} \quad & \sigma_p^2 = \sum_{i\in\Theta} \omega_i^2 \sigma_i^2 + \sum_{\{(i,j)\in\Theta \,:\, i\neq j\}} 2\omega_i\omega_j Cov(i,j) \\
s.t. \quad & \mu_p = \sum_{i\in\Theta} \omega_i \mu_i \\
& 1 = \sum_{i\in\Theta} \omega_i \\
& \underline{\omega} \leq \omega_i \leq \bar{\omega},
\end{aligned}
$$

where $\Theta = \{CD, UVA, RE, U\$S, B\}$ and CD denote CDs described above, UVA denotes UVA, RE denotes real estate, $U\$S$ denotes dollars, and B denotes blue dollars. Finally, μ_i and σ_i^2 denote the mean and the variance of each asset, respectively; μ_p is the mean of the whole portfolio; and σ_p^2 denotes its variance. To find the Minimum Variance Frontier, we solve the portfolio problem for different values of μ_p, which we describe below.

2.6.4 Portfolios

Unrestricted

Table 2.8: Unconstrained Portfolios: Total Sample

μ_p	σ_p	ω_{CD}	$\omega_{U\$S}$	ω_{RE}	ω_{UVA}	ω_B
0.1%	4.4014%	0.3747	-0.0271	0.3693	0.2838	-0.0007
0.3%	4.739%	0.2374	-0.0412	0.4594	0.3414	0.003
0.5%	5.1831%	0.1001	-0.0553	0.5495	0.399	0.0067
0.7%	5.7089%	-0.0372	-0.0695	0.6396	0.4565	0.0105

Table 2.9: Unconstrained Portfolios: Market Sample

μ_p	σ_p	ω_{CD}	$\omega_{U\$S}$	ω_{RE}	ω_{UVA}	ω_B
0.1%	0.7175%	0.5798	0.1461	0.0325	0.3558	-0.1142
0.3%	0.8072%	0.6538	-0.0234	0.2043	0.3017	-0.1364
0.5%	0.9915%	0.7231	-0.188	0.3756	0.2516	-0.1622
0.7%	1.2286%	0.793	-0.35	0.5469	0.2004	-0.1903

Table 2.10: Unconstrained Portfolios: Populist Sample

μ_p	σ_p	ω_{CD}	$\omega_{U\$S}$	ω_{RE}	ω_{UVA}	ω_B
0.1%	7.0662%	0.2706	0.0566	0.3473	0.3344	-0.0089
0.3%	7.3371%	0.2052	0.0697	0.3694	0.3648	-0.0092
0.5%	7.6575%	0.1363	0.0835	0.3927	0.3969	-0.0094
0.7%	7.9915%	0.0708	0.0966	0.4148	0.4274	-0.0097

Constrained

Table 2.11: Constrained Portfolios: Total Sample

μ_p	σ_p	ω_{CD}	$\omega_{U\$S}$	ω_{RE}	ω_{UVA}	ω_B
0.1%	4.4081%	0.372	0.0007	0.3395	0.2874	0.0003
0.3%	4.7473%	0.2299	0.0002	0.4236	0.3462	0.0001
0.5%	5.1948%	0.0881	0.0006	0.5056	0.4052	0.0005
0.7%	5.9384%	0	0.0001	0.5734	0.3818	0.0446

Table 2.12: Constrained Portfolios: Market Sample

μ_p	σ_p	ω_{CD}	$\omega_{U\$S}$	ω_{RE}	ω_{UVA}	ω_B
0.1%	0.7415%	0.5783	0.0167	0.0431	0.3569	0.005
0.3%	1.1603%	0.2328	0.0001	0.2879	0.4791	0.0001
0.5%	1.9889%	0.0038	0.0002	0.584	0.4117	0.0002
0.7%	3.0138%	0.0011	0.0001	0.9349	0.0637	0.0001

Table 2.13: Constrained Portfolios: Populist Sample

μ_p	σ_p	ω_{CD}	$\omega_{U\$S}$	ω_{RE}	ω_{UVA}	ω_B
0.1%	7.0628%	0.28	0.0377	0.3491	0.3331	0.0001
0.3%	7.3402%	0.213	0.0507	0.3719	0.3644	0
0.5%	7.6522%	0.1464	0.0633	0.3945	0.3956	0.0003
0.7%	7.9943%	0.0795	0.0762	0.4172	0.4268	0.0003

Returns of Fixed Portfolio across Regimes

Table 2.14: Returns of a Fixed Portfolio across Regimes (%)

	Unrestricted		Constrained	
	Market	Populist	Market	Populist
Mean	0.1	-1.155	0.1	-0.7342
SD	0.7175	7.9707	0.7415	7.0104
Mean	0.5	-2.1846	0.5	0.8203
SD	0.9915	10.925	1.9889	8.3279
Mean	0.7	-2.7079	0.7	0.856
SD	1.2286	13.2963	3.0138	10.9845

Table 2.15: Returns of a Fixed Portfolio across Regimes (%)

	Unrestricted		Constrained	
	Populist	Market	Populist	Market
Mean	0.1	0.3025	0.1	0.3069
SD	7.0662	1.347	7.0628	1.3355
Mean	0.5	0.3318	0.5	0.3364
SD	7.6575	1.5511	7.6522	1.5374
Mean	0.7	0.346	0.7	0.3511
SD	7.9915	1.6553	7.9943	1.6435

2.6.5 Small Switching Costs

The relevant value functions are

$$
\begin{aligned}
V_M^+ &= \frac{(\rho+\pi_P)\, H_M(\alpha_M^*) + \frac{\pi_M\eta_P}{\rho+\eta_P} H_P(\alpha_P^*)}{\Delta} \\
&\quad - \left(\frac{\pi_M\eta_P}{\rho+\eta_P}\right)\left(\frac{(\rho+\pi_P)\, c_P + \frac{\pi_P\eta_M}{\rho+\eta_M} c_M}{\Delta}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
V_P^+ &= \frac{(\rho+\pi_M)\, H_P(\alpha_P^*) + \frac{\pi_P\eta_M}{\rho+\eta_M} H_M(\alpha_M^*)}{\Delta} \\
&\quad - \left(\frac{\pi_P\eta_M}{\rho+\eta_M}\right)\left(\frac{(\rho+\pi_M)\, c_M + \frac{\pi_M\eta_P}{\rho+\eta_P} c_P}{\Delta}\right),
\end{aligned}
$$

where

$$\Delta = (\rho + \pi_M)\,(\rho + \pi_P)\,(1 - \varkappa(\pi_M)\varkappa(\pi_P)\varkappa(\eta_M)\varkappa(\eta_P)) > 0,$$

and for any $x \geq 0$,

$$\varkappa(x) \equiv \frac{x}{\rho + x} \in [0, 1).$$

In these formulations, α_j^* is the maximizer of $H_J(\alpha)$.

2.6.6 Large Switching Costs

In this case the value functions are

$$\tilde{V}_M(\alpha) = \frac{(\rho + \pi_P)\, H_M(\alpha) + \frac{\pi_M \eta_P}{\rho + \eta_P} H_P(\alpha)}{\Delta}$$

and

$$\tilde{V}_P(\alpha) = \frac{(\rho + \pi_M)\, H_P(\alpha) + \frac{\pi_P \eta_M}{\rho + \eta_M} H_M(\alpha)}{\Delta}.$$

The highest possible value for an investor who enters the market in regime M is

$$\tilde{V}_M^+ = \max_\alpha \frac{(\rho + \pi_P)\, H_M(\alpha) + \frac{\pi_M \eta_P}{\rho + \eta_P} H_P(\alpha)}{\Delta}.$$

Let the maximizer be denoted as $\tilde{\alpha}_M$. The corresponding value for an investor who joins the market in the P regime is

$$\tilde{V}_P^+(\alpha) = \max_\alpha \frac{(\rho + \pi_M)\, H_P(\alpha) + \frac{\pi_P \eta_M}{\rho + \eta_M} H_M(\alpha)}{\Delta}.$$

As above, the maximizer is denoted as $\tilde{\alpha}_P$.

2.6.7 Large and Small Switching Costs: A Comparison

When will an investor choose to pay the switching costs? In the M regime, an investor will choose to pay the switching costs if $V_M^+ > \tilde{V}_M^+$. It follows that

$$\begin{aligned} V_M^+ - \tilde{V}_M^+ \;\; = \;\; & \frac{(\rho + \pi_P)\,(H_M(\alpha_M^*) - H_M(\tilde{\alpha}_M)) + \frac{\pi_M \eta_P}{\rho + \eta_P}\,(H_P(\alpha_P^*) - H_P(\tilde{\alpha}_P))}{\Delta} \\ & - \left(\frac{\pi_M \eta_P}{\rho + \eta_P}\right) \left(\frac{(\rho + \pi_P)\, c_P + \frac{\pi_P \eta_M}{\rho + \eta_M} c_M}{\Delta}\right). \end{aligned}$$

It is clear that the first term is positive since the investor who pays the cost can tailor his portfolio to the regime, while an investor who does not pay the cost has to suffer a potentially lower value of his (fixed) portfolio when the regime switches. The second term is negative and converges to zero as the vector (c_P, c_M) becomes arbitrarily small.

To the extent that waiting times to change the portfolio (as captured by η_M and η_P) and actual return costs (as captured by (c_P, c_M)) vary across investors, the model is consistent with a fair amount of heterogeneity in optimal portfolios even though all investors share the same preferences for risk and return.

2.6.8 Sensitivity Analysis

Table 2.16: Optimal Portfolios (%)

	Low Costs				High Costs			
Asset	α_M^U	α_M^C	α_P^U	α_P^C	α_M^U	α_M^C	α_P^U	α_P^C
CD	189.22	0.01	-134.42	0	189.23	0.00	-148.58	0.00
Dollar	-1232.38	0	37.33	14.59	-1232.41	0.00	-8.41	0.00
Real Estate	1110.82	99.97	90.01	26.53	1110.86	100.00	145.06	53.74
UVA	-95.14	0.01	108.5	50.3	-95.15	0.00	108.31	39.84
Blue	127.48	0	-1.42	8.58	127.48	0.00	3.62	6.42

Case $\theta = 2$

Table 2.17: Optimal Portfolios (%)

	Low Costs				High Costs			
Asset	α_M^U	α_M^C	α_P^U	α_P^C	α_M^U	α_M^C	α_P^U	α_P^C
CD	-91.13	0	-31.49	0	-91.12	0.00	-43.12	0.00
Dollar	0	0	0	0	0.00	0.00	0.00	0.00
Real Estate	255.56	99.99	66.98	48.18	255.57	100.00	80.04	55.31
UVA	53.59	0.01	61.35	47.54	53.59	0.00	62.79	43.02
Blue	-118.02	0	3.16	4.28	-118.03	0.00	0.29	1.67

Case $\alpha_{U\$S} = 0$

Table 2.18: Optimal Portfolios (%)

	Low Costs				High Costs			
Asset	α_M^U	α_M^C	α_P^U	α_P^C	α_M^U	α_M^C	α_P^U	α_P^C
CD	45.74	0	-35.62	0	45.76	0	-41.97	0
Dollar	-447.97	0	17.54	11.5	-448.04	0	-4.8	0
Real Estate	438.91	99.96	56.62	39.81	438.95	100	82.86	55.3
UVA	9.36	0.04	62.52	47.09	9.35	0	62.46	43.02
Blue	53.97	0	-1.05	1.6	53.98	0	1.45	1.68

Case $\eta_M = \eta_P = 1/2$

Table 2.19: Optimal Portfolios (%)

	Low Costs				High Costs			
Asset	α_M^U	α_M^C	α_P^U	α_P^C	α_M^U	α_M^C	α_P^U	α_P^C
CD	214.62	0.01	-35.46	0	214.62	0.00	-41.98	0.00
Dollar	-742.6	0	18.02	12.01	-742.60	0.00	-4.86	0.00
Real Estate	663.23	99.95	56.03	39.28	663.23	100.00	82.93	55.32
UVA	-86.76	0.03	62.52	47.17	-86.76	0.00	62.46	43.01
Blue	51.51	0	-1.11	1.54	51.51	0.00	1.45	1.67

Case $\eta_M = \eta_P = 2$

Table 2.20: Optimal Portfolios (%)

	Low Costs				High Costs			
Asset	α_M^U	α_M^C	α_P^U	α_P^C	α_M^U	α_M^C	α_P^U	α_P^C
CD	127.74	0.01	-35.46	0	127.70	0.00	-41.95	0.00
Dollar	-613.07	0	18.02	12.01	-613.03	0.00	-4.72	0.00
Real Estate	557.6	99.94	56.03	39.28	557.59	100.00	82.77	55.28
UVA	-34.86	0.04	62.52	47.17	-34.83	0.00	62.46	43.04
Blue	62.59	0	-1.11	1.54	62.57	0.00	1.44	1.68

Case $\eta_M = 2$ **and** $\eta_P = 1$

Table 2.21: Optimal Portfolios (%)

	Low Costs				High Costs			
Asset	α_M^U	α_M^C	α_P^U	α_P^C	α_M^U	α_M^C	α_P^U	α_P^C
CD	214.62	0.01	-35.51	0	214.62	0.00	-42.01	0.00
Dollar	-742.6	0	17.86	11.84	-742.60	0.00	-4.98	0.00
Real Estate	663.23	99.95	56.23	39.45	663.23	100.00	83.07	55.36
UVA	-86.76	0.03	62.52	47.14	-86.76	0.00	62.46	42.99
Blue	51.51	0	-1.09	1.56	51.50	0.00	1.47	1.65

Case $\eta_M = 1$ **and** $\eta_P = 2$

2.6.9 Different Sample

This section reports the results after changing the samples to avoid including/excluding key episodes in each sample, such as the 2001 crisis and the 2019 crisis. The new sample is constructed as follows:

- Populist periods
  - May 1981 to March 1991
  - January 2002 to November 2002
  - April 2011 to November 2015
- Market periods
  - April 1991 to December 2001
  - December 2002 to March 2011
  - December 2015 to December 2019

Table 2.22 reports the mean and standard deviation of each asset for both samples. After including the two crisis periods in the market sample, the results are not significantly different from the benchmark sample.

Table 2.22: Monthly Asset Return (%)

	Populist Sample		Market Sample	
Asset	Mean	St. Dev.	Mean	St. Dev.
CD	-1.98	7.09	0.02	0.92
Dollar	2.16	18.84	-0.42	2.78
Real Estate	0.91	11.90	0.70	3.19
UVA	0.81	12.99	0.16	1.18
Blue	5.74	45.63	-0.45	3.61

Unconstrained Case

As shown in the graph below, the results of changing the sample to include crisis episodes in a different regime are almost identical to the benchmark case shown in Section 2.3. During market periods, given a specific level of risk, the return in the market regime is always higher than in the populist regime. Moreover, as shown in Section 2.3, the minimum possible risk that can be achieved during the populist period is many times higher than the minimum possible risk in the market period.

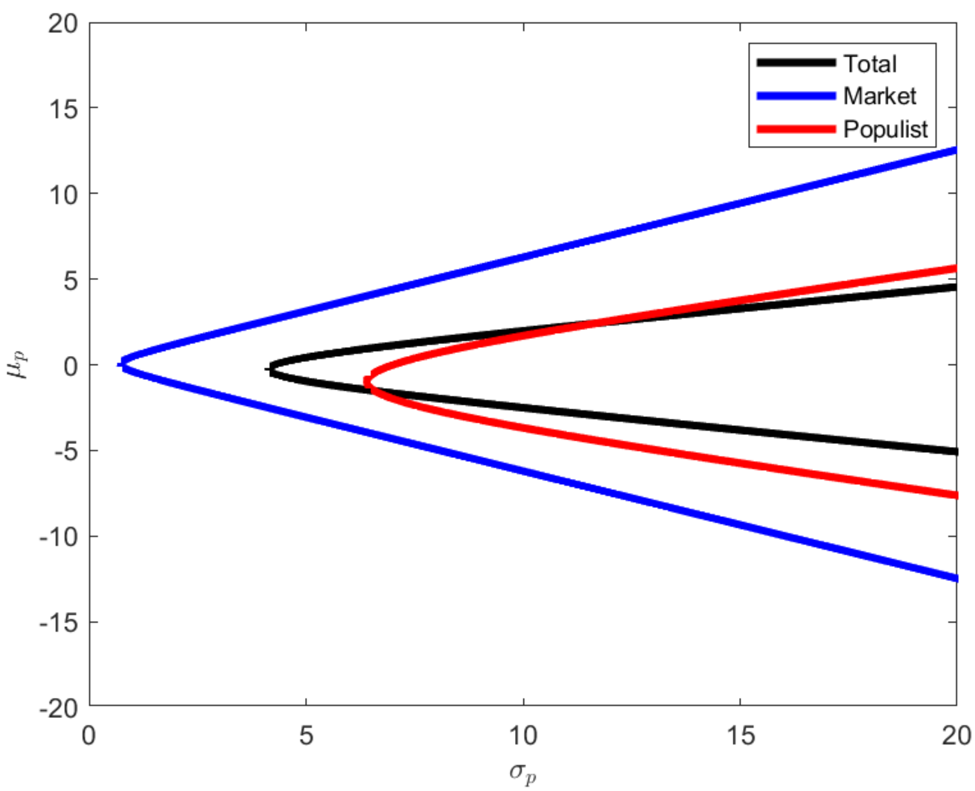

Figure 2.3: Minimum Variance Frontier (Unconstrained)

Constrained Case

The graph below shows the Minimum Variance Frontier for the case where agents cannot short any asset. In line with the previous results, the findings do not change significantly.

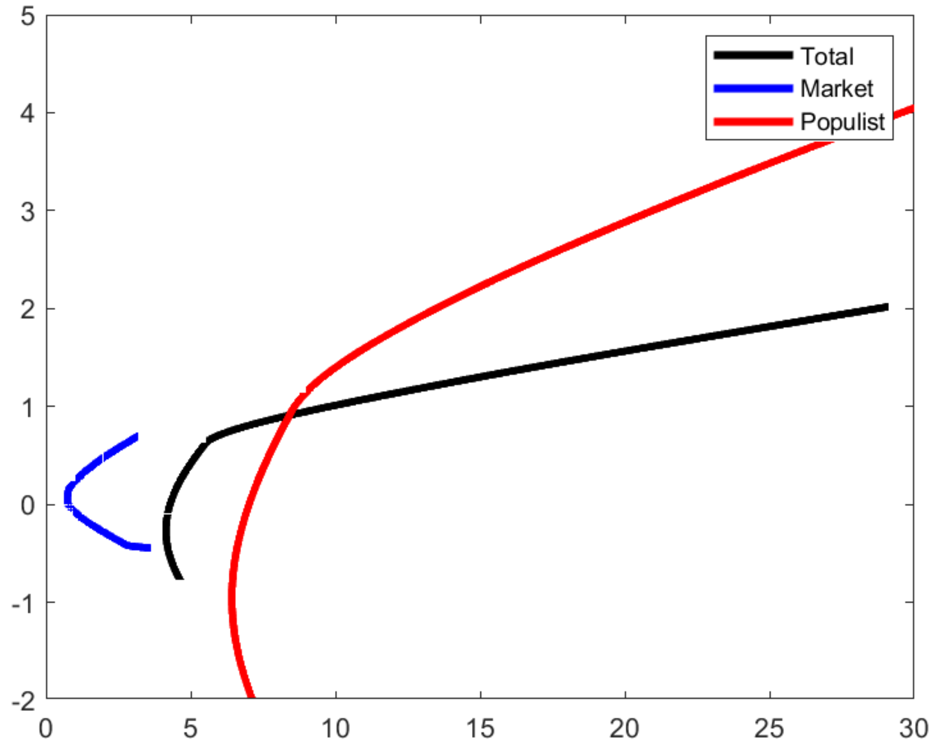

Figure 2.4: Minimum Variance Frontier (Constrained)

Optimal Portfolios

Table 2.23 presents optimal portfolio allocations for the new sample. Compared with our benchmark case, the differences are small enough to confirm that our results are robust for the sample selection.

Table 2.23: Optimal Portfolios (%)

	Low Costs				High Costs			
Asset	α_M^U	α_M^C	α_P^U	α_P^C	α_M^U	α_M^C	α_P^U	α_P^C
CD	130.71	0.00	-36.86	0.00	130.72	0.00	-42.31	0.00
Dollar	-627.09	0.00	18.80	12.51	-627.10	0.00	-3.53	0.00
Real Estate	548.94	99.98	56.86	39.47	548.95	100.00	81.97	55.09
UVA	-30.46	0.01	62.77	46.83	-30.46	0.00	62.66	43.16
Blue	77.89	0.00	-1.57	1.19	77.89	0.00	1.21	1.75
Mean	6.10	0.70	2.07	1.08	6.10	0.70	2.08	0.95
Variance	10.13	3.20	10.85	8.78	10.13	3.20	11.17	8.58

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Chapter 3: Incomplete Markets and Macroeconomic Risk

The objective of this paper is to study the equilibrium dynamics of an economy with incomplete financial markets that is subject to aggregate risk shocks. In particular, it studies how volatility shocks affect returns and asset prices, as well as consumption and investment. To this purpose, we use a continuous time macroeconomic model in which agents can trade financial assets, but we assume incomplete markets. The main findings are that an exogenous increase in volatility leads to an increase in risky returns and risk premium, and a decline in asset prices, interest rate and investment. Some of these effects are amplified when financial constraints are binding, suggesting that incomplete markets play an important role in the propagation of risk shocks.

Keywords: Incomplete Markets, Uncertainty Shocks, Asset Pricing

JEL Codes: D52, E44, G10

3.1 Introduction

Macroeconomic time-series tend to exhibit time-varying risk premia (see Fernandez-Villaverde and Rubio-Ramirez (2013)). The negative effects of high on uncertainty on economic activity have been well documented by many authors. Bloom (2009) argues that the main mechanism behind uncertainty shocks is a reduction of investment. Christiano, Motto and Rostagno found that variations in uncertainty are an important driver of business cycles, Di Tella (2017) explains why uncertainty shocks concentrate aggregate risk, leading to balance sheet recessions. Fernandez-Villaverde et al. (2011), discuss the effects or volatility of interest rates at which small open economies borrow, which triggers a decline in output, consumption and investment. Other papers argue that uncertainty shocks negatively impact economic activity through a decline in investment, which is amplified by financial frictions (see Bloom (2014), Arellano, Bai and Kehoe (2019)). However, some argue that uncertainty responds to economic conditions (endogenous risk) rather than being an exogenous source of aggregate fluctuations, instead it works as an amplifier. Moreover, it is argued that this endogenous uncertainty is counter-cyclical (see Bachman and Moscarini (2011), Fernandez-Villaverde and Guerron-Quintana (2020)).

The purpose of this paper is to analyze the equilibrium dynamics of asset prices, investment and risk premia in an incomplete financial market economy that is subject to aggregate risk shocks. It also addresses the question on how economic conditions endogenously affect risk in the economy. To this end, I use a continuous time macroeconomic model similar to Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012) and Di Tella (2017). The model has two agents, experts and households, where only experts can operate the production technology and households finance investment projects investing either in risk free assets or in risky assets. In the model, experts have limited participation.

The main findings of this paper are that an exogenous increase in aggregate risk causes an increase in asset price volatility, an increase in risky asset returns through an increase in risk-premia, a decline in asset prices, investment and risk free interest rates. Moreover, the model presented here is able to reproduce counter-cyclical endogenous risk. This results also depend on how constrained are financial intermediaries. If financial intermediaries are constrained some of the results are amplified.

This paper is related to the literature of Basak & Cuoco (1998) and Brunnermeier & Sannikov (2014, 2016) in the sense that it introduces financial frictions in continuous time macroeconomic models that arise from incomplete financial markets. This causes the financial sector to play an important role in determining the equilibrium of the economy. It differs from Brunnermeier and Sannikov (2014) since financial intermediaries trade capital with less productive households, while in this paper financial intermediaries trade equity, but only they operate capital. Also it differs in the sense that, given that they have linear preferences, the interest rate is constant, while in our paper interest rates play an important role.

The paper is also related to He and Krishnamurthy (2012, 2013) in the sense that financial intermediaries are constrained in the amount of equity that they can issue to households, which means that financial intermediaries have to hold, at least, a minimum amount of risk due to a moral hazard problem. To this end, the model introduces an equity capital constraint from derived from a moral hazard problem. The main difference with this paper is that the model that I present here is able to reproduce counter-cyclical endogenous risk. It also introduces a production economy. As in He and Krishnamurthy (2012), the model allows for closed form solutions.

The main purpose of the paper is to analyze how equilibrium changes after a shock to volatility, so it is related to Christiano, Motto and Rostagno (2014) and, in particular, to Di Tella (2017). These papers assume heterogenous agents and idiosyncratic risk. Christiano, Motto and Rostagno (2014) find that the main force driving business cycles are risk shocks, while Di Tella (2017) studies how uncertainty shocks can drive balance sheet recessions. Even though the model is remarkably similar to Di Tella (2017), there are many differences. The most important is that, while in his model financial intermediaries are constraint in the amount of idiosyncratic risk they can share, they can freely trade aggregate risk, while in our model financial intermediaries are constrained in the amount of aggregate risk they can trade. This, as mentioned in Di Tella (2017), implies that in our model the balance sheet channel of Brownian TFP shocks plays a role in determining the equilibrium dynamics of the model, while in his model, this difference implies that the balance sheet channel of TFP shocks disappears. However, this fact is the one that allows for countercyclical endogenous risk, absent in Di Tella (2017).

Related literature can be found in Basu and Bundick (2018) which find that uncertainty shocks cause contractions in output, investment, consumption and hours worked. Also, the effects of volatility shocks have been widely studied in finance. An example is the Heston model and all its variations. On the other hand, most of the work studying volatility shocks in macroeconomics focuses mainly in real aggregate variables. For example, Seoane (2017) studies the role of endogenous markups in the transmission of volatility shocks in real models. Seoane (2019) studies how volatility changes affect sovereign spreads in strategic default models. Fernández-Villaverde et al. (2011) analyze how changes in the volatility of interest rates at which small open economies borrow affects real variables. Fernández-Villaverde et al. (2015) studies the effects of changes in uncertainty about future fiscal policy on aggregate economic activity.

3.2 Model

The model is related to Basak and Cuoco (1998), Brunnermeier and Sannikov (2014, 2016), He and Krishnamurthy (2012) and Di Tella (2017). The main differences are that (i) we introduce TFP shocks which will make the price of capital vary with productivity, unlike Brunnermeier and Sannikov (2014) that shock the quality of capital, which, in my model does not have any impact on asset prices, (ii) we allow agents to trade risky assets, unlike Basak and Cuoco (1998), where only experts can hold risky assets, and (iii) we introduce an equity capital constraint, derived from a moral hazard problem, as in He and Krishnamurthy (2012); however, this paper is built on a pure exchange economy, while our paper has endogenous production, that allows us to capture counter-cyclical endogenous risk. This combination better reflects how financial markets affect real economic activity. Finally, we add aggregate risk shocks as a two-state regime switching Poisson process for volatility.

The economy has two types of agents, experts and households. Only experts can operate the technology to produce goods. The economy has two assets, one risky asset (capital), with positive net supply and a risk free asset (bonds) of zero net supply. Both agents can trade bonds and stocks (share of capital of the firm). However, there is a constraint on the amount of equity that households can hold. This forces experts to have some "*skin in the game*" in order to avoid a moral hazard problem.

3.2.1 Production

The production function of this economy is given by a continuum of firms of mass 1, that produce according to.

$$y_t = a_t^j k_t \tag{3.1}$$

and dividends are given by

$$(a_t - \iota_t)k_t \tag{3.2}$$

where ι_t is the investment per unit of capital, k_t is the capital of the firm and a_t is the output per unit of capital or the productivity of the firm, which follows a Cox-Ingersoll-Ross process

$$da_t = \theta(\bar{a} - a_t)dt + \sigma_t a_t^j dZ_t, \tag{3.3}$$

and σ_t is a two-state Poisson process, that takes values σ_1 and σ_2, where $\sigma_1 < \sigma_2$, with intensity parameters λ_1 and λ_2 and denote low volatility and high volatility, respectively. Notice that, since σ_t follows a Poisson process, the intensity parameters are the inverse of the average duration of each state. Following Brunnermeier and Sannikov (2014), we can refer to σ_t as the exogenous risk in the economy, which in our case is not constant. Finally, θ is the rate at which productivity mean-reverts.

The law of motion of capital is given by

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta)dt \tag{3.4}$$

where the function Φ satisfies $\Phi(0) = 0$, $\Phi'(\cdot) > 0$ and $\Phi''(\cdot) < 0$ and can be interpreted as the investment function, which reflects adjustment costs to the capital stock.

3.2.2 Asset prices and returns

Let q_t be the price of capital, then the total wealth in the economy is $q_t k_t$. For the price of capital, let us postulate

$$\frac{dq_t}{q_t} = \mu_t^q dt + \sigma_t^q dZ_t \tag{3.5}$$

Again, following Brunnermeier and Sannikov (2014), we might refer to σ_t^q as the endogenous risk of this economy.

The total return of the risky asset is given by

$$\begin{aligned} dr_t^k &= \frac{a_t - \iota_t}{q_t} dt + \frac{d(q_t k_t)}{q_t k_t} - X_t s_t \\ &= \left(\frac{a_t - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q - s_t X_t \right) dt + \sigma_t^q dZ_t \end{aligned}$$

where $s_t X_t$ is the efficiency loss associated to the moral hazard problem explained in the following section. Define $\bar{r}_t^k(a_t)$ as the return of the risky asset after removing risk, which is given by

$$\bar{r}_t^k(a_t) = \frac{a_t - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q - s_t X_t \tag{3.6}$$

3.2.3 Experts

The technology can be only operated by a continuum of identical experts, equivalent to a representative expert. Experts play the role of a financial intermediary. They borrow from households to invest in risky assets, they can also issue equity to households to offload risk from their balance sheets; however, they face an equity capital constraint, so they can only issue a limited amount of stocks. For this purpose, following He and Krishnamurthy (2012) I introduce a moral hazard problem that leads to a capital constraint. The moral hazard arises from the possibility of the expert to work ($s_t = 0$) or shirk ($s_t = 1$). If the expert shirks, the firm loses $X_t dt$, but the expert earns $B_t dt$, where $0 < B_t < X_t$. Also, the expert charges the household with a fee F_t for managing risky assets owned by households, which will be determined in equilibrium and it will be helpful to derive the capital constraint. This fee will play an important role equilibrating the financial intermediation market.

Let $\beta_t \in [0, 1]$ be the share of the firms owned by the experts, so equity held by the experts is given by $E^e_t = \beta_t E_t$, where $E_t = q_t k_t$ is total equity of the firm. Experts consolidated wealth evolves according to

$$\begin{aligned} dN_t &= \beta_t q_t k_t dr^k_t + (N_t - \beta_t q_t k_t) r_t dt - c_t dt + F_t dt + B_t s_t dt - T_t N_t dt \\ &= E^e_t dr^k_t + (N_t - E^e_t) r_t dt - c_t dt + F_t dt + B_t s_t dt - T_t N_t dt \end{aligned}$$

where $F_t = p_t N_t$ is the fee which, from the experts' view is linear in their wealth, T_t are taxes, and E^e_t is the equity held by the experts. The last term of the equation describes how default affects the wealth evolution of experts. In other words, the last term is the fraction of experts that default and become households. Suppose for now that the expert does not shirk, so that $s_t = 0$ (later we will derive conditions under which $s_t = 0$). We assume that all agents have log utility. Let α_t be the share of expert's portfolio in risky assets, therefore the expert's problem is given by

$$\max_{c_t, \alpha_t} \quad E\left[\int_0^\infty e^{-\rho t} \ln(c_t) dt\right]$$

$$s.t. \quad dN_t = [(r_t + (\bar{r}^k_t(a_t) - r_t)\alpha_t) N_t - c_t + F_t - T_t N_t] dt + \alpha_t N_t \sigma^q_t dZ_t$$

Since volatility follows a two-state Poisson process, we need a Hamilton-Jacobi-Bellman equation for each state $i \in \{1,2\}$ as defined above. Then, the HJB equation can be written as

$$\begin{aligned}
\rho V_{i,t}(N_t,a_t) &= \max_{c_t,\alpha_t}\{\ln(c_t) \\
&+ \frac{\partial V_{i,t}(N_t;a_t)}{\partial N_t}\left(\alpha_t N_t\left(\frac{a_t-\iota_t}{q_t}+\Phi(\iota_t)-\delta+\mu_t^q\right)+(1-\alpha_t)N_t r_t - c_t + F_{i,t} - T_t N_t\right) \\
&+ \frac{1}{2}\frac{\partial^2 V_{i,t}(N_t,a_t)}{\partial {N_t}^2}\alpha_t^2 N_t^2(\sigma_t^q)^2 \\
&+ \frac{\partial V_{i,t}(N_t,a_t)}{\partial a_t}\theta(\bar{a}-a_t)dt + \frac{1}{2}\frac{\partial^2 V_{i,t}(N_t,a_t)}{\partial {a_t}^2}a_t^2\sigma_t^2 dZ_t \\
&+ \lambda_i(V_{-i,t}(N_t)-V_{i,t}(N_t))\}
\end{aligned}$$

The HJB equation is solved by guess and verify method, where the guess for the value function is given by

$$V_{i,t}^0(N_t,a_t) = A_t + \frac{\ln(N_t)}{\rho+\lambda_i} \tag{3.7}$$

where A_t contains all the irrelevant terms of the consumer problem and

$$\frac{\partial V_{i,t}^0(N_t,a_t)}{\partial N_t} = \frac{1}{(\rho+\lambda_i)N_t}$$

Therefore, the solution of the maximization problem is

$$c_{i,t} = (\rho+\lambda_i)N_t \tag{3.8}$$

$$\alpha_{i,t} = \frac{\bar{r}_{i,t}^k(a_t)-r_{i,t}}{(\sigma_{i,t}^q)^2} \tag{3.9}$$

3.2.4 Households

Household's wealth evolves according to

$$\begin{aligned} dH_t &= (1-\beta_t)q_t k_t dr_t^k + (H_t - (1-\beta_t)q_t k_t)r_t dt - c_t^h dt - F_t dt + T_t N_t dt \\ &= E_t^h dr_t^k + (H_t - E_t^h)r_t dt - F_t dt - c_t^h dt + T_t N_t dt \\ &= E_t^h dr_{i,t}^k + (H_t - E_t^h)r_t dt - \tau_t E_t^h dt - c_t^h dt + T_t N_t dt \end{aligned}$$

where E_t^h is equity held by the household. Also, from the households perspective, $F_{i,t} = \tau_{i,t} E_t^h$, where $\tau_{i,t}$ can be interpreted as the unit price of risk exposure. Later, an equilibrium relationship between $\tau_{i,t}$ and $p_{i,t}$ will be derived. Finally, T_t are lump-sum transfers that will help to generate a stationary distribution.

Similar to the experts' problem, households maximize their utility function subject to the budget constraint and taking into account the regime switch from the Poisson process. With log utility, the problem for the household yields

$$c_{i,t}^h = (\rho + \lambda_i) H_t \tag{3.10}$$

$$E_t^h = \frac{(\bar{r}_t^k - r_t) - \tau_t}{\sigma_t^{q2}} H_t \tag{3.11}$$

3.2.5 Moral Hazard Problem

In this section, the capital constraint is derived according to He and Krishnamurthy (2012). The only difference is that the capital constraint here depends on σ_t, the exogenous risk of the economy. This implies that the model is going to have a risky steady state, so risk shocks affect the steady state of the model. This is one of the main differences with Di Tella (2017). This will make the economy fluctuate between to what we will call later the constrained equilibrium and the unconstrained equilibrium.

Proposition 1

Let $\frac{B_t}{X_t} \equiv \frac{\sigma_t}{1+m}$, then

$$s_t = 0 \Leftrightarrow \beta_t \geq \frac{\sigma_t}{1+m}$$

Proof. To induce the expert to work ($s_t = 0$), we need that the cost of shirking is greater than the benefit of it. We know that the benefit of shirking is B_t and the cost is $\beta_t X_t$. Therefore, $s_t = 0$ if and only if

$$B_t \leq \beta_t X_t$$

if and only if

$$\begin{aligned} \beta_t &\geq \frac{B_t}{X_t} \\ &= \frac{\sigma_t}{1+m} \end{aligned}$$

Then

$$\beta_t \geq \frac{\sigma_t}{1+m} \tag{3.12}$$

■

The inequality described above can be interpreted as the incentive compatibility constraint, to avoid a moral hazard problem. In other words, experts must have enough skin in the game to avoid shirking. The fact that this constraint depends on the exogenous risk, σ_t, means that it is harder for experts to offload risk when uncertainty is higher. From the incentive compatibility constraint, we can derive the equity capital constraint. Recall that equity held by the expert is

$$\begin{aligned} E_t^e &= \beta_t E_t \\ &= \beta_t q_t K_t \end{aligned}$$

which combining with the equity held by the household yields to

$$\begin{aligned} E_t^h &= (1-\beta_t)E_t \\ &= (1-\beta_t)\frac{E_t^e}{\beta_t} \\ &\leq \frac{1-\frac{\sigma_t}{1+m}}{\frac{\sigma_t}{1+m}}E_t^e \\ &= \tilde{m}(\sigma)E_t^e \end{aligned}$$

where $\tilde{m}(\sigma) \equiv \sigma_t^{-1}(1+m-\sigma_t)$. Then, the constraint on equity held by households is given by

$$E_t^h \leq \tilde{m}(\sigma)E_t^e \tag{3.13}$$

The following proposition summarizes some results in He and Krishnamurthy (2012) that are needed in order to compute equilibrium results for shareholdings between households and experts.

Proposition 2

There are two regions in which equilibrium occurs:

1. Unconstrained region, where the following conditions hold

$$\tau_t = 0 \Leftrightarrow E_t^h < \tilde{m}(\sigma)E_t^e \Leftrightarrow \beta_t > \frac{\sigma_t}{1+m}$$

2. Constrained region

$$\tau_t > 0 \Leftrightarrow E_t^h = \tilde{m}(\sigma)E_t^e \Leftrightarrow \beta_t = \frac{1}{1+m}$$

Moreover, given the risk exposure $\tau_{i,t}$, define

$$p_t = \begin{cases} 0 & if \quad \tau_t = 0 \\ \\ \tilde{m}(\sigma)\frac{\bar{r}_t^k - r_t}{(\sigma_t^q)^2}\tau_t & if \quad \tau_t > 0 \end{cases}$$

then, the fee is linear in the expert's wealth

$$F_t = p_t N_t$$

Since this model has the same assumptions than He and Krishnamurthy (2012), we take this results as given. However, it can be proved that the above conditions hold. For the formal proof see He and Krishnamurthy (2012).

3.2.6 Market Clearing

Since both agents have logarithmic preferences, the market clearing condition for goods can be written as

$$(\rho + \lambda_i)q_t k_t = (a_t - \iota_t(q_t))k_t \tag{3.14}$$

where the LHS is aggregate consumption and the RHS is production minus investment.

On the other hand, as in Brunnermeier and Sannikov (2016), assume that the investment function $\Phi(\cdot)$ is given by

$$\Phi(\iota_t) = \frac{\log(\kappa\iota_t + 1)}{\kappa} \tag{3.15}$$

and after maximizing expected returns of the risky asset dr_t^k, the first order conditions give

$$\Phi'(\iota_t) = \frac{1}{q_t} \tag{3.16}$$

which yields to

$$\iota(q_t) = \frac{q_t - 1}{\kappa} \tag{3.17}$$

Plugging into the market clearing condition and solving for q_t we get

$$q_t = \frac{\kappa a_t + 1}{\kappa(\rho + \lambda_i) + 1} \tag{3.18}$$

Recall that the process for productivity is given by

$$\frac{da_t}{a_t} = \theta(\bar{a} - a_t)dt + \sqrt{a_t}\sigma_t dZ_t \tag{3.19}$$

applying Ito's lemma for $q_t(a_t)$, we get

$$\frac{dq_t}{q_t} = \frac{\kappa\theta(\bar{a} - a_t)}{\kappa a_t + 1}dt + \frac{\kappa\sigma_t\sqrt{a_t}}{\kappa a_t + 1}dZ_t \tag{3.20}$$

where

$$\mu_t^q = \frac{\kappa\theta(\bar{a} - a_t)}{\kappa a_t + 1}$$

and the endogenous risk is given by

$$\sigma_t^q = \frac{\kappa\sigma_t\sqrt{a_t}}{\kappa a_t + 1} \tag{3.21}$$

Here are two important results. First, endogenous risk of the economy is countercyclical if and only if $a_t > \kappa^{-1}$. Since $\bar{a}$ will be calibrated later at 3 and κ takes the value of 2, we know that around the steady state, endogenous risk is countercyclical. What happens if $a_t < \kappa^{-1}$? First, that scenario happens with low probability. Second, we know from the data (*e.g.* VIX) that volatility peaks in the first stage of a crisis. Therefore, starting from the steady state, a sharp decline in productivity will cause an increase in the endogenous risk of the economy, but if productivity decreases further than κ^{-1}, endogenous risk starts to decrease. One interpretation is that, after a sharp decline in economic activity, agents expect a recovery, despite that productivity stills falling. Low asset prices imply good investment opportunities, therefore, agents improve their expectations about the future, reducing endogenous risk in the economy.

3.2.7 Interest rate and the evolution of relative wealth

This section describes the behavior of the relative wealth of experts, which can be interpreted as the balance sheet of experts. High values of η reflect that experts are well capitalized or have strong balance sheets, and low values of η means that their capitalization is low, reflecting weak balance sheets. As we will see below, this is the main state variable of the model.

First, define the relative wealth of the expert by

$$\eta_t \equiv \frac{N_t}{q_{i,t}k_t} \tag{3.22}$$

Define $\theta_t \equiv u'(c_t) = \frac{1}{c_t}$, then let's postulate

$$d\theta_t = \mu_t^\theta \theta_t dt + \sigma_t^\theta \theta_t dZ_t \tag{3.23}$$

After applying Ito's lemma to $\frac{\partial V_t^0(N_t, a_t)}{\partial N_t}$ we obtain that

$$-\sigma_t^\theta = \frac{\bar{r}_t^k - r_t}{\sigma_t^q} \tag{3.24}$$

Now, rewrite the expert's wealth as

$$\begin{aligned} dN_t &= \beta_t q_t k_t dr_t^k + (N_t - \beta_t q_t k_t) r_t dt - c_t dt + F_t dt - T_t N_t dt \\ &= (r_t N_t + \beta_t q_t k_t (\bar{r}_t^k - r_t) - c_t + F_t - T_t) dt + \beta_t q_t k_t \sigma_t^q dZ_t \end{aligned}$$

which implies that

$$\frac{dN_t}{N_t} = \left(r_t + \beta_t \frac{(\bar{r}_t^k - r_t)}{\eta_t} - (\rho + \lambda_i) + p_t - T_t \right) dt + \beta_t \frac{\sigma_t^q}{\eta_t} dZ_t \tag{3.25}$$

Now, recall that

$$\theta_t \equiv u'(c_t) = \frac{1}{c_t} = \frac{1}{(\rho + \lambda_i) N_t} \tag{3.26}$$

applying Ito's lemma to $\theta_t(N_t)$ and after some manipulations

$$\frac{d\theta_t}{\theta_t} = \left[r_t + \beta_t \frac{\bar{r}_t^k - r_t}{\eta_t} - (\rho + \lambda_i) + p_t - T_t + \beta_i^2 \frac{(\sigma_t^q)^2}{\eta_t^2} \right] dt - \beta_t \frac{\sigma_t^q}{\eta_t} dZ_t \tag{3.27}$$

from which follows that

$$\mu^{\theta}_{i,t} = r_t + \beta_t \frac{\bar{r}^k_t - r_t}{\eta_t} - (\rho + \lambda_i) + p_t - T_t + \beta_t^2 \frac{(\sigma^q_t)^2}{\eta_t^2} \tag{3.28}$$

$$\sigma^{\theta}_t = -\beta_t \frac{\sigma^q_t}{\eta_t} \tag{3.29}$$

Now, combining our previous results we have that

$$\frac{\bar{r}^k_t - r_t}{\sigma^q_t} = -\sigma^{\theta}_t = \beta_t \frac{\sigma^q_t}{\eta_t} \tag{3.30}$$

multiplying both sides by $\frac{\sigma^q_t}{\eta_t}$ yields to

$$\frac{\bar{r}^k_t - r_t}{\eta_t} = \beta_t \frac{(\sigma^q_t)^2}{\eta_t^2} \tag{3.31}$$

substituting in the budget constraint of the expert yields

$$\frac{dN_t}{N_t} = \left(r_t + \beta_t^2 \frac{(\sigma^q_t)^2}{\eta_t^2} - (\rho + \lambda_i) + p_t - T_t \right) dt + \beta_t \frac{\sigma^q_t}{\eta_t} dZ_t \tag{3.32}$$

Now, consider the process that describes the evolution of total wealth in the economy

$$\frac{d(q_t K_t)}{q_t K_t} = (\Phi(\iota_t) - \delta + \mu^q_t) dt + \sigma^q_t dZ_t \tag{3.33}$$

and recall that

$$\begin{aligned} \beta_t \frac{\sigma^q_t}{\eta_t} &= \frac{\bar{r}^k_t - r_t}{\sigma^q_t} \\ &= \frac{(\rho + \lambda_i) + \Phi(\iota_t) - \delta + \mu^q_t - r_t}{\sigma^q_t} \end{aligned}$$

from which follows that

$$\Phi(\iota_t) - \delta + \mu^q_t = r_t + \beta_t \frac{(\sigma^q_t)^2}{\eta_t} - (\rho + \lambda_i) \tag{3.34}$$

hence

$$\frac{d(q_t K_t)}{q_t K_t} = \left(r_t + \beta_t \frac{(\sigma^q_t)^2}{\eta_t} - (\rho + \lambda_i) \right) dt + \sigma^q_t dZ_t \tag{3.35}$$

and the interest rate is given by

$$r_t = (\rho + \lambda_i) + \Phi(\iota_t) - \delta + \mu_t^q - \beta_t \frac{(\sigma_t^q)^2}{\eta_t}. \tag{3.36}$$

After substituting the expressions obtained for μ_t^q and σ_t^q from the market clearing condition we have that, in equilibrium, the interest rate is given by

$$r_t = (\rho + \lambda_i) + \Phi(\iota_t) - \delta + \frac{\kappa\theta(\bar{a} - a_t)}{\kappa a_t + 1} - \left(\frac{\kappa\sqrt{a_t}}{\kappa a_t + 1}\right)^2 \beta_t \frac{\sigma_t^2}{\eta_t} \tag{3.37}$$

Define the risk premium as

$$\bar{r}_t^k(a_t) - r_t \tag{3.38}$$

Until now, we have expressed consumption and risk exposure as functions of wealth, which can be rewritten in terms of relative wealth. Also, the interest rate and, therefore, the risk premium is in terms of relative wealth η_t, hence we need and expression for the evolution of the state variable.

Proposition 2

$$\frac{d\eta}{\eta} = \left(\sigma_t^{q2} \frac{\beta_t^2 - \eta_t^2}{\eta_t^2} + p_t - T_t\right) dt + \sigma_t^q \left(\frac{\beta_t - \eta_t}{\eta_t}\right) dZ_t \tag{3.39}$$

Proof. Recall that

$$\frac{dN_t}{N_t} = \left(r_t + \beta_t^2 \frac{(\sigma_t^q)^2}{\eta_t^2} - (\rho + \lambda_i) + p_t - T_t\right) dt + \beta_t \frac{\sigma_t^q}{\eta_t} dZ_t$$

and

$$\frac{d(q_t K_t)}{q_t K_t} = \left(r_t + \beta_t \frac{\sigma_t^{q2}}{\eta_t} - (\rho + \lambda_i)\right) dt + \sigma_t^q dZ_t$$

Using Ito's lemma for a quotient as it is shown below

$$d\left(\frac{X}{Y}\right) = \frac{X}{Y}\left(\frac{dX}{X} - \frac{dY}{Y} - \frac{dX}{X}\frac{dY}{Y} + \left(\frac{dY}{Y}\right)^2\right)$$

$$
\begin{aligned}
\frac{d\eta_t}{\eta_t} &= \left(r_t + \beta_t^2 \frac{\sigma_t^{q2}}{\eta_t^2} - (\rho + \lambda_i) + p_t - T_t\right) dt + \beta_t \frac{\sigma_t^q}{\eta_t} dZ_t \\
&- \left(r_t + \beta_t \frac{\sigma_t^{q2}}{\eta_t} - (\rho + \lambda_i)\right) dt - \sigma_t^q dZ_t \\
&- \left[\left(r_t + \beta_t^2 \frac{(\sigma_t^q)^2}{\eta_t^2} - (\rho + \lambda_i) + p_t - T_t\right) dt + \beta_t \frac{\sigma_t^q}{\eta_t} dZ_t\right] \left[\left(r_t + \beta_t \frac{(\sigma_t^q)^2}{\eta_t} - (\rho + \lambda_i)\right) dt + \sigma_t^q dZ_t\right] \\
&+ \left[\left(r_t + \beta_t \frac{\sigma_t^{q2}}{\eta_t} - (\rho + \lambda_i)\right) dt + \sigma_t^q dZ_t\right]^2 \\
&= \left((\sigma_t^q)^2 \frac{\beta_t^2 - \eta_t^2}{\eta_t^2} + p_t - T_t\right) dt + \sigma_t^q \left(\frac{\beta_t - \eta_t}{\eta_t}\right) dZ_t
\end{aligned}
$$

where the last equality follows from the fact that $(dt)^2 = dtdZ_t = 0$ and $(dZ_t)^2 = dt$. ■

3.3 Constrained and Unconstrained Equilibrium

Proposition 1 states that equilibrium occurs in two regions. In the unconstrained region, the unconstrained equilibrium occurs and the capital constraint is binding. On the other hand, in the constrained region the constrained equilibrium occurs and the capital constraint is slack. This section discusses how equilibrium changes across regions.

First, in order to compute the equilibrium, it is important to obtain the equilibrium value of the intermediation fee τ_t. Proposition 1 states that the equilibrium fee is positive in the constrained region, and equals zero in the constrained region.

Proposition 3

In equilibrium, the intermediation fee is given by

$$\tau_t = \begin{cases} 0 & in\ the\ unconstrained\ equilibrium \\ \\ (\bar{r}_t^k - r_t)\left(\frac{1-(1+\tilde{m}(\sigma))\eta_t}{1-\eta_t}\right) & in\ the\ constrained\ equilibrium \end{cases}$$

Proof. Recall that household's equity holdings are given by

$$E_t^h = \frac{(\bar{r}_t^k - r_t) - \tau_t}{(\sigma_t^q)^2} H_t \tag{3.40}$$

solving for τ_t yields

$$\tau_t = (\bar{r}_t^k - r_t) - \frac{E_t^h (\sigma_t^q)^2}{H_t} \tag{3.41}$$

using the capital constraint $E_t^h \leq \tilde{m}(\sigma) E_t^e$ gives

$$\tau_t = (\bar{r}_t^k - r_t) - \frac{\tilde{m}(\sigma) E_t^e \sigma_t^{q2}}{H_t} \tag{3.42}$$

plugging $E_t^e = \frac{\bar{r}_t - r_t}{(\sigma_t^q)^2} N_t$ from the expert's maximization problem and $q_t k_t = N_t + H_t$

$$\tau_t = (\bar{r}_t^k - r_t)\left(\frac{q_t k_t - (1+\tilde{m}(\sigma))N_t}{q_t k_t - N_t}\right) \tag{3.43}$$

multiplying and dividing the RHS by $q_t k_t$ and rearranging yields

$$\tau_t = (\bar{r}_t^k - r_t)\left(\frac{1-(1+\tilde{m}(\sigma))\eta_t}{1-\eta_t}\right) \tag{3.44}$$

which is the equilibrium fee for the constrained equilibrium.

From Proposition 1, we know that $\tau_t = 0$ in the unconstrained equilibrium. ■

It is clear that equilibrium changes across regions. However, it has not been defined which are the constrained and unconstrained region. Proposition 4 establish under which conditions the economy is in the constrained region or the unconstrained region.

Proposition 4

In the unconstrained region, where the unconstrained equilibrium occurs, the state variable η satisfies

$$\eta_t \leq \frac{1}{1 + \tilde{m}(\sigma)} \tag{3.45}$$

and for the constrained region, the state is such that

$$\eta_t > \frac{1}{1 + \tilde{m}(\sigma)} \tag{3.46}$$

Proof. From Proposition 1, we know that in the constrained region $\tau_t > 0$, while in the unconstrained region $\tau_t = 0$.

Therefore, $\tau_t > 0$ if and only if

$$(\bar{r}_t^k - r_t)\left(\frac{1 - (1 + \tilde{m}(\sigma))\eta_t}{1 - \eta_t}\right) > 0 \tag{3.47}$$

which holds if and only if

$$\eta_t < \frac{\sigma_t}{1 + m} \tag{3.48}$$

■

Now that the constrained and unconstrained regions have been defined, we are still missing how households and experts allocate their resources. In particular, it is important to find the share of the firm that each one owns. In other words, we need to find the equilibrium in the stock market, which is determined by the amount of stocks each agent has.

Proposition 5

In equilibrium the share of the firm owned by the specialist is given by

$$\beta_t = \begin{cases} \eta_t & in\ \ the\ \ unconstrained\ \ equilibrium \\ \\ \frac{\sigma_t}{1+m} & in\ \ the\ \ constrained\ \ equilibrium \end{cases}$$

Proof. The stock market clearing condition is given by

$$E_t^e + E_t^h = E_t \tag{3.49}$$

In the unconstrained region

$$\alpha_t^e = \alpha_t^h = \frac{\bar{r}_t^k - r_t}{(\sigma_t^q)^2}$$

Since $E_t^e = \alpha_t N_t$ and $E_t^h = \alpha_t H_t$ and, $q_t k_t = E_t$ then

$$\alpha_t (N_t + H_t) = q_t k_t$$

which implies that $\alpha = 1$.

Now, recall that $\alpha_t = \frac{\beta_t E_t}{N_t} = \frac{(1-\beta_t)E_t}{H_t}$, therefore

$$\beta_t = \frac{N_t}{E_t}$$

$$= \frac{N_t}{q_t k_t}$$

$$= \eta_t$$

For the case of the constrained equilibrium ($\eta_t < \frac{\sigma_t}{1+m}$), recall that β_t is bounded below by $(1 + \tilde{m}(\sigma))^{-1}$, which is the incentive compatibility constraint of the moral hazard problem, therefore

$$\beta_t = \frac{\sigma_t}{1+m} \tag{3.50}$$

■

Having established the equilibrium values for β_t, we can find the equilibrium value for the portfolio share on risky asset for the constrained region.

Proposition 6

In the constrained region, $\alpha_t^e = \frac{\sigma_t}{(1+m)\eta_t}$. *Moreover,* $\alpha_t^e > 1$.

Proof. Since in the constrained region $\beta = (1 + \tilde{m}(\sigma))^{-1}$, then

$$\alpha_t^e = \beta_t \frac{q_t k_t}{N_t} = \frac{\sigma_t}{(1+m)\eta_t}$$

Since in the constrained region $\eta_t < \frac{\sigma_t}{1+m}$, then

$$\frac{\sigma_t}{(1+m)\eta_t} > 1$$

Therefore, $\alpha_t^e > 1$.

We already showed in the previous Proposition that in the unconstrained region $\alpha_t^e = 1$. ■

The previous two propositions are particularly relevant. If the relative wealth of the expert falls below $(1 + \tilde{m}(\sigma))^{-1}$ then, since the capital constraint is binding, the expert cannot offload risk. However, since its relative wealth is decreasing, it is necessary to borrow from households in order to hold enough equity so the capital constraint holds. This leads to an excessive risk exposure. In the next section, we will analyze the equilibrium response to this situation.

3.4 Results

This section contains the numerical results of the model. Here it is important to notice that high volatility (riskier) episodes are associated with a low intensity rate of the state, while low volatility episodes are associated with a high intensity rate. This means that, on average, low volatility episodes last longer that high volatility episodes.

The model is calibrated according to Table 3.1.

Table 3.1: Calibration

Parameter	Value
ρ	0.03
m	1
δ	0.025
κ	1.2
σ_1	0.5
σ_2	1
λ_1	36[1]
λ_2	6[1]

To calibrate the Poisson process, λ_1 and λ_2 need to be calibrated. Recall that the intensity parameter is the inverse of the average duration of the state. Therefore, λ_1 is the intensity parameter for the low volatility state (σ_1), while λ_2 for the high volatility state (σ_2). The calibration above implies that average duration of low volatility (normal times) is 36 months. On the other hand, high volatility episodes have an average duration of 6 months. Once the model is calibrated, the policy functions are computed.

3.4.1 Policy Functions and Comparative Statics

This subsection analyzes the equilibrium dynamics of the model for the different states of exogenous risk, σ_t. We compare the policy functions of the model across all possible values for the endogenous state, the relative wealth of the expert, η, for the low risk state and the high risk state.

Table 3.2 shows how some variables change within the states of volatility across the state variable η and for a fixed level of productivity. First, we can see that risky asset prices are higher in the low volatility state, and lower in the high volatility state, consistent with the hypothesis. This reflects that in riskier episodes, agents demand safe assets and sale risky assets leading to a decrease in their price and, as a result of that, an increase in the return of the risky asset, as shown in the table. Given how asset prices vary across states, investment, which is positively related to the price of the risky asset, is higher in the low volatility state and low during high volatility episodes. Also, given the lower investment in the risky state, the investment function is also lower. Finally, the endogenous risk of the economy, σ_t^q, increases as a result of higher exogenous risk.

Table 3.2: Equilibrium values

Variable	σ_1	σ_2
q_t	5.3785	4.3062
ι_t	2.1892	1.6531
$\Phi(\iota_t)$	0.96694	1.0178
$\bar{r}_t^k$	0.8412	0.73003
σ_t^q	0.1304	0.2609

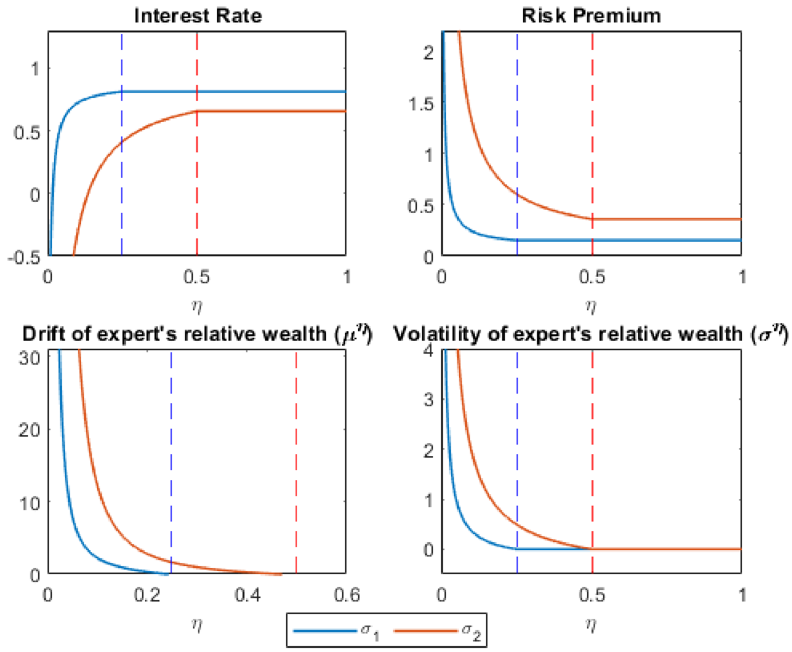

Figure 3.1: Interest rate, risk premium and relative wealth evolution

Figure 3.1 shows how interest rates, risk premia and balance sheets vary across states. First, as the balance sheet of the expert (η) weakens, interest rates go down, while the risk premium increases reflecting the high demand for safe assets, given that experts become constrained on how much equity they can issue. The drift and variance of the evolution of relative wealth are decreasing on the balance sheet of experts. An important thing to notice is that in the low risk state, the vertical line that separates the constrained and unconstrained region is to the left of the horizontal line in the riskier state, reflecting that households are less willing to bear risk in high uncertainty state, making the economy more financially constrained. This means that the steady state of the model ($\mu_t^\eta = 0$) changes if the exogenous risk changes similar to Brunnermeier & Sannikov (2014), but different from Di Tella (2017) and He & Krishnamurthy (2012).

Figure 3.1 also shows that interest rates decrease in the riskier state as a result of a higher demand for safe assets. Risk premium is also higher. Finally, the drift of relative wealth in the riskier state is higher than the one of the low risk state as $\eta_t \to 0$, then lower, meaning that in that region the recovery of the balance sheet is slower during riskier episodes, and goes to zeros as $\eta_t \to \eta_{ss}$. In the unconstrained region, the drift is negative due to lump sum transfers from experts to households; hence, the model does not have an absorbing state. Also, notice that the volatility of experts' relative wealth goes to zero as the economy enters the unconstrained region and goes to infinity as $\eta_t \to 0$. This means that once the economy enters the unconstrained region, the system deterministically pushes the economy back to the unconstrained equilibrium.

Figure 3.2 shows equilibrium consumption and fraction of the portfolio in risky assets, α_t, both for experts and households, to which we refer to as the risk exposure. Consumption of both agents is increasing in their own relative wealth, and in the riskier state consumption is higher as a result of a decrease in investment with constant output. On the other hand, risk exposure (α^j) is 1 in the unconstrained region, meaning that both agents allocate all their assets in stocks, but in the constrained region, since the capital constraint is binding, experts cannot offload risk from their balance sheets and as long as their relative wealth decreases, in order to hold enough equity to satisfy the capital constraint, experts issue bonds to households, increasing leverage, leading to an excessive risk exposure.

3.5 Conclusions

This paper studies macroeconomic risk in an economy with incomplete financial markets, where the main distortion arises from a moral hazard problem which implies that the experts cannot offload all risk. The model is able to reproduce counter-cyclical endogenous risk, as opposed to existing literature that assumes that changes in macroeconomic uncertainty are exogenous. The model is also able to reproduce the main facts about high uncertainty, where exogenous increases in risk, cause a decline in investment and asset prices. Hence, the counter-cyclical endogenous risk amplifies negative TFP shocks, but dampens positive shocks. We use a continuous time macroeconomic model based on Brunnermeier and Sannikov (2014, 2016), He and Krishnamurthy (2012) and Di Tella (2017). The equilibrium occurs in two regions, the constrained region, where experts,

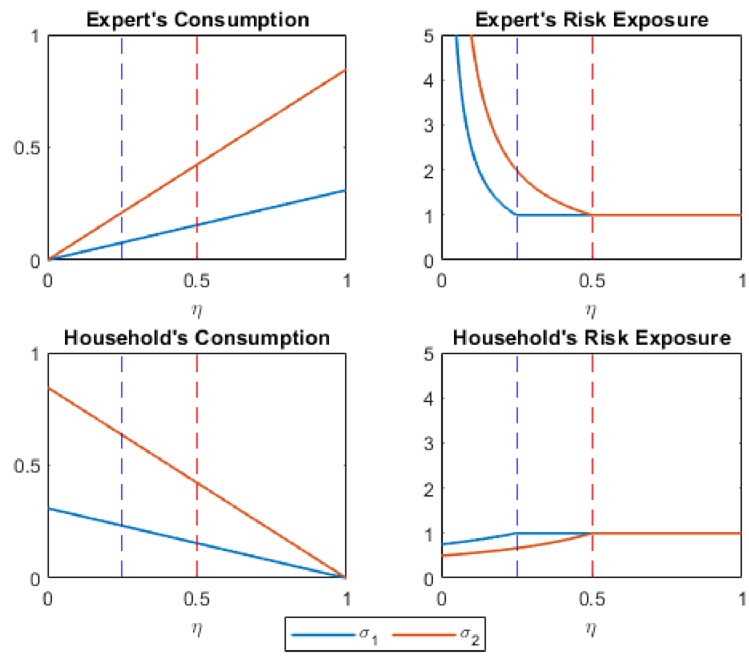

Figure 3.2: Consumption and Risk Exposure

that play the role of financial intermediaries, are constrained, and the unconstrained region where agents can trade both risky and safe assets. Volatility shocks are introduced as a two-state Poisson process which makes the computation of the equilibrium easier. The simplicity of the model allows for closed-form solutions.

The effects on interest rates, risk premium or the balance sheets are amplified in the constrained region when the capital constraint is binding, suggesting that incomplete markets play an important role in the propagation of risk shocks. As the capital constraint starts to bind, experts have to borrow from households, which leads to an excessive risk exposure of experts. Then, risk premium is higher and interest rates are lower, since the expected return on risky assets does not depend either on η or m.

The model also captures the idea of a risky steady state. This means that the steady state of the model changes with risk. This allows the economy to switch between regions and makes the economy more financially constrained during riskier episodes.

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