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Essays in Macroeconomics, Development, and Entrepreneurship

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Essays in Macroeconomics, Development, and Entrepreneurship

by
Alexandros Loukas

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Table of Contents

List of Figures

List of Tables

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Alexandros Loukas

Washington University in St. Louis

May 2024

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οὔτε λέγει

οὔτε κρύπτει

ἀλλὰ σημαίνει

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ABSTRACT OF THE DISSERTATION

Essays in Macroeconomics, Development, and Entrepreneurship

by

Alexandros Loukas

Doctor of Philosophy in Economics

Washington University in St. Louis, 2024

Professor Costas Azariadis, Chair

Professor Ping Wang, Co-Chair

This dissertation consists of three chapters that contribute to the fields of macroeconomics, economic development, and entrepreneurship.

In the first chapter, *"Entrepreneurship Selection and Performance in the U.S. and Across Countries: The Role of Human Capital,"* I seek to establish a set of stylized facts related to entrepreneurship and human capital, the latter being proxied by years of formal education. Analyzing individual-level survey data from nearly 100 countries unveils new empirical facts: there is a strong positive link between the mean-adjusted rate of entrepreneurship for higher educated individuals and output per worker or estimated total factor productivity (TFP). Further focus on the U.S. economy, again at the micro level, reveals a non-linear and time-varying relationship between the rate of entrepreneurship and educational attainment exhibiting an asymmetric U-shape with its left branch declining over time. At the same time, higher education is strongly positively associated—not U-shaped—with numerous measures of business outcomes among active firm owners/managers. Conditioning on a rich set of socioeconomic and demographic observables, the robustness of these results is confirmed under proper repeated imputation inference.

The second chapter, *"Entrepreneurship, Human Capital, and the Allocation of Talent,"* raises new points of inquiry and attempts to enrich the discussion in the relevant literature. Is the allocation of human capital between entrepreneurs and workers a key determinant of aggregate productivity and income? How pervasive are its implications for macro-development? To organize the discourse on addressing these questions, I propose a versatile heterogeneous-agent model with occupational and educational choices, which is able to rationalize the empirical findings of Chapter 1 while remaining broadly consistent with aggregate and survey data. Under the hypothesis that entrepreneurial human capital may enhance productive capacities via costly technology adoption, the entrepreneurship-education nexus has first-order aggregate and distributional consequences. As new generations build skills through schooling and form expectations about their future labor market prospects, this mechanism also affects the accumulation of human capital economy-wide. Calibrating the model to the U.S. economy is successful in replicating a wide spectrum of targeted and non-targeted moments, thereby capturing salient features of micro and macro data. Quantitative explorations suggest sizeable and persistent losses in output and total factor productivity (TFP) across nations due to inadequate complementarity between idiosyncratic talent and human capital. This novel channel can often account for a major share of cross-country income differences vis-à-vis the U.S., as it drastically affects both factor accumulation and endogenous TFP formation.

In the third chapter, *"Sometimes Less is More: Growth, Risk Aversion, and the Suboptimality of Entrepreneurial Insurance"* (joint work with Neville N. Jiang, Ping Wang, and Haibin Wu), we aim to address two major research questions. Is promoting entrepreneurship always conducive to long-run economic growth? To what extent should policymakers strive to insure entrepreneurial risk away? We study these questions by developing a tractable endogenous growth model with occupational choice, where individuals are heterogeneous in their risk attitude and entrepreneurial ability. Less risk-averse and sufficiently productive

agents become entrepreneurs and contribute to growth by expanding product variety. More risk-averse and less productive agents become workers and foster growth by enhancing human and physical capital formation. As occupational choice induces an inverse association between risk tolerance and entrepreneurial talent at the margin, encouraging firm creation may hinder aggregate productivity. The interplay of these forces leads to a non-monotone relationship between the rates of entrepreneurship and balanced growth. A competitive equilibrium entails suboptimal allocations with either too few or too many active entrepreneurs, even in the absence of distortions or financial frictions. Insuring some entrepreneurial risk away is almost always growth-enhancing, but it is almost surely never optimal to provide full insurance. Calibrating the model to U.S. data reveals substantial output-side misallocation, with most of income growth and aggregate TFP losses stemming from the intensive margin due to the presence of risk aversion.

Chapter 1

Entrepreneurship Selection and Performance in the U.S. and Across Countries: The Role of Human Capital

1.1 Introduction

The prominent role of entrepreneurship in driving economic growth and development has long been recognized by economists and policymakers. While the literature has emphasized various aspects of individual heterogeneity as determinants of occupational choices and entrepreneurial outcomes, such as entrepreneurial ability, wealth and access to credit, or non-pecuniary benefits, less attention has been paid to the heterogeneity in human capital as proxied by educational attainment.

This study aims to fill this gap by shedding light on the importance of human capital in shaping occupational choices and entrepreneurial outcomes, both across countries and within the United States. By doing so, I seek to address two key research questions: First, is there a systematic relationship between the allocation of human capital across entrepreneurs and workers and aggregate economic performance? Second, how does educational attainment influence individuals' selection into entrepreneurship and their subsequent business outcomes?

The paper has two main empirical objectives. At the outset, I seek to establish whether the allocation of human capital between entrepreneurs and workers is empirically linked to economic development across nations. The search for such evidence is non-trivial as it requires large samples of high-quality harmonized survey data at the individual level. I address this precondition by constructing an extensive micro-level dataset covering nearly 100 countries, with repeated cross-sectional samples drawn from annual Global Entrepreneurship Monitor (GEM) adult population surveys. To begin tackling the question in quantitative terms, I introduce a simple statistic called the ERGON (Entrepreneurship Rate of Graduates Over National rate) index: the normalized/mean-adjusted rate of entrepreneurship for higher educated individuals. Merging the GEM dataset with national-level data reveals a new empirical fact: the ERGON index varies strongly positively and significantly with output per worker, as well as with estimated Hicks-neutral TFP. In other words, higher educated people in richer and more productive countries become entrepreneurs at rates significantly higher than those in poorer countries. These findings are robust after controlling for variables that are known to be firmly connected with development, such as countries' overall rate of entrepreneurship and average human capital in the labor force.

Focusing further on the U.S. economy, I utilize 11 waves of nationally representative samples drawn from the Survey of Consumer Finances (SCF), spanning a period of 30 years. I document large differences in various economic and demographic characteristics across entrepreneurs and workers in the U.S. labor force, as well as within entrepreneurs. Special attention is given to variations stemming from formal education. I find the relationship between the rate of entrepreneurship and educational attainment to be non-linear and time-varying, exhibiting an asymmetric U-shape with its left branch declining over time. Moreover, U.S. entrepreneurs are more educated than workers on average, and this difference is somewhat increasing over time. Among active firm owners/managers, higher education is strongly positively associated—not U-shaped—with numerous measures of business outcomes, such as pre-tax profits (both hourly and total), sole proprietorship income, and firm employment size. Conditioning on a rich set of observables—experience, hours worked, health condition, willingness to take financial risk, past and future inheritances, marital status, sex, race, and others—the robustness of the descriptive results is confirmed through reduced-form regressions under proper repeated imputation inference (RII).

The above stylized facts cannot be jointly accommodated by existing theories without imposing strong distributional assumptions or implausible restrictions on primitives, and thus warrant closer investigation. Therefore, these findings serve as the empirical motivation for the theoretical framework developed in the subsequent chapter, which seeks to rationalize the observed patterns and explore their implications for macroeconomic development and cross-country income differences.

Related Literature. This paper relates and contributes to two strands of literature. In terms of cross-country empirics, this is the first study documenting the fact that differences in entrepreneurship rates by educational attainment associate with cross-country output and estimated (Hicks-neutral) TFP differences.

Moreover, it adds to an expanding branch of literature studying the determinants of selection into entrepreneurship and entrepreneurial performance. For the important case of the U.S., some prominent examples are Evans and Leighton (1989), Hamilton (2000), Moskowitz and Vissing-Jørgensen (2002), Hurst and Lusardi (2004), Hipple (2010), and Levine and Rubinstein (2017). For the cases of the UK and OECD countries, two noteworthy examples are Blanchflower and Oswald (1998) and Blanchflower (2000); see also the references therein.

Most relevant studies do not examine potential non-linearities in the relationship between education and entrepreneurship, and those few that do find suggestive evidence, do not elaborate on the subject. In fact, more often than not, the estimated relationships are found to be particularly weak and not at all decisive. This is indeed a primary finding in Van der Sluis, Praag, and Vijverberg (2008), who survey a considerable portion of the empirical literature on educational attainment and entrepreneurship in industrial countries, and report that the impact of formal schooling on entry is largely statistically insignificant. Such a finding is not necessarily surprising, since an underlying U-shaped association could easily lead to insignificant estimates of linear coefficients.[1]

One noteworthy exception is the work of Poschke (2013), which brings to the forefront new evidence about the prevalence of the U-shaped relationship between the likelihood of individuals engaging in entrepreneurship and their level of education or other measures of ability. This study complements and expands on the aforementioned for the case of the U.S. using a different dataset, time periods under consideration, and research design.

The rest of the paper is outlined as follows. Section II presents new motivating facts coming from individual-level data across countries. Section III contains the contributions of the

[1] Upon entertaining linear specifications and controlling for a wide range of covariates along with RII bootstrapped standard errors, my findings also suggest strongly positive and significant coefficients on education of the entrepreneur, which arise because the non-monotonic relationship is asymmetric.

paper relating to several descriptive and reduced-form results for the case of the U.S. Section *IV* briefly summarizes the study and offers suggestions for future work.

1.2 Cross-Country Evidence

Not all entrepreneurs are created equal. Heterogeneity matters in various ways, and this paper argues that human capital is a principal factor. In the absence of readily available motivating facts, one needs to uncover whether there is observational evidence in support of the inquiries the study poses. The underlying assumption throughout is that formal educational attainment proxies a primary measure of human capital fairly well.

As an initial pass, the first subsection seeks to address the following empirical questions: Is there a strong cross-country correlation between the mean-adjusted rate of entrepreneurship for higher educated individuals and output per worker? Does the same apply to estimated TFP? Are these relationships robust upon conditioning on some key variables related to macro-development? By analysing an international micro-level dataset consisting of survey data from nearly 100 countries, in conjunction with national-level data, the answer to all three inquiries is a resounding *yes*.

1.2.1 International Individual-Level Data

I construct an extensive dataset with repeated adult population surveys over the period 2009−2019, collected annually by the Global Entrepreneurship Monitor (GEM).[2] The GEM

[2] The GEM consortium was founded in 1999 as a joint research project between Babson College and London Business School, and since then has expanded its presence through collaborations with numerous academic institutions, national organizations, market research firms, and government agencies in more than 100 countries. The quality of GEM data and the consistency of corresponding findings with those observed using alternative sources has been documented by several studies; see Poschke (2018) and the references therein. For more information on methodology, see https://www.gemconsortium.org.

data collection process constitutes perhaps the largest global study on entrepreneurship, and above all, it employs a standardized methodology across participating countries in order to produce comparable and nationally representative samples. This approach is particularly suitable for the purposes of our analysis and it provides a wealth of individual-level information. Importantly, all surveys report individuals' level of educational attainment according to the International Standard Classification of Education (ISCED), a harmonized framework administered by UNESCO for comparing educational qualifications across countries.

National-level data. The pooled GEM dataset is merged with aggregate data coming from the latest version of the Penn World Table (PWT 10.01) by Feenstra, Inklaar, and Timmer (2015). In what follows, I consider the geometric mean of variables relative to the U.S. over the time period dictated by GEM data availability.[3]

I obtain Hicks-neutral TFP estimates for each country by carrying out a development accounting exercise in levels, using an extension of the Hall and Jones (1999) methodology.[4] Instead of positing a Cobb-Douglas aggregate production function, however, I incorporate heterogeneous entrepreneurs/producers and workers along with decreasing returns to scale in the spirit of Lucas (1978) in order to sustain a non-degenerate firm distribution. In this way I employ a framework that is consistent with entrepreneurship and occupational choice, and simultaneously reflects the model structure that will be presented in chapter 2. Please refer to section A.1 for further details.

[3] Specifically, "output per worker" refers to output-side real GDP at chained PPPs divided by numbers of people engaged *(rgdpo/emp)*, relative to the U.S. Apart from dealing with issues of multiplicative relationships and serial correlation, an additional advantage of the geometric mean is its invariance to whether we consider average of ratios or ratios of averages.

[4] The Hall-Jones decomposition exemplifies the so-called calibration approach and is often used a natural benchmark. For similarly influential contributions, see Klenow and Rodríguez-Clare (1997) and Caselli (2005), as well as the references therein for previous important work.

Identification of entrepreneurs in the GEM dataset. A nontrivial issue is how to identify entrepreneurs in survey data. I refine a common and quite uncontroversial approach, e.g., Hamilton (2000), Moskowitz and Vissing-Jørgensen (2002), Cagetti and De Nardi (2006), which is also consistent with the identification scheme I employ for the U.S. in the next section. I classify as entrepreneurs all survey respondents that **(i)** report being self-employed as their primary occupation; **(ii)** are currently the owner and manager of a business; **(iii)** personally own all or part of the business; **(iv)** have received wages, profits, or payments in kind from the business. I shall refer to *self-employed business owners/managers (SEBO)* simply as *entrepreneurs*. Likewise, I shall refer to everyone else as *workers/wage-earners*. I consider only respondents who participate in the labor force, which excludes individuals who are retired, homemakers, students, and those who haven't reported their work status.

1.2.2 The ERGON index

The intent is to construct a simple statistic which is informative about the allocation of human capital between entrepreneurs and workers, and which does not neglect cross-country differences in average levels of self-employment and educational attainment. I propose a measure that I call the *ERGON index* (Entrepreneurship Rate of Graduates Over National rate), defined for each country i as

$$\frac{ER_i(\text{educ} \geq \overline{S})}{ER_i(\text{total})} = \frac{\mu(\mathcal{E}_i \,|\, \text{educ} \geq \overline{S})/\mu(\Omega_i \,|\, \text{educ} \geq \overline{S})}{\mu(\mathcal{E}_i)/\mu(\Omega_i)} \tag{1.1}$$

where $\mu(\mathcal{E}) :=$ measure of entrepreneurs and $\mu(\Omega) :=$ measure of the labor force. The index essentially quantifies the mean-adjusted rate of entrepreneurship for higher educated individuals. For example, a value of 1.10 means that among the echelons of people with higher education, we observe a 10% higher entrepreneurship rate compared to the national average.

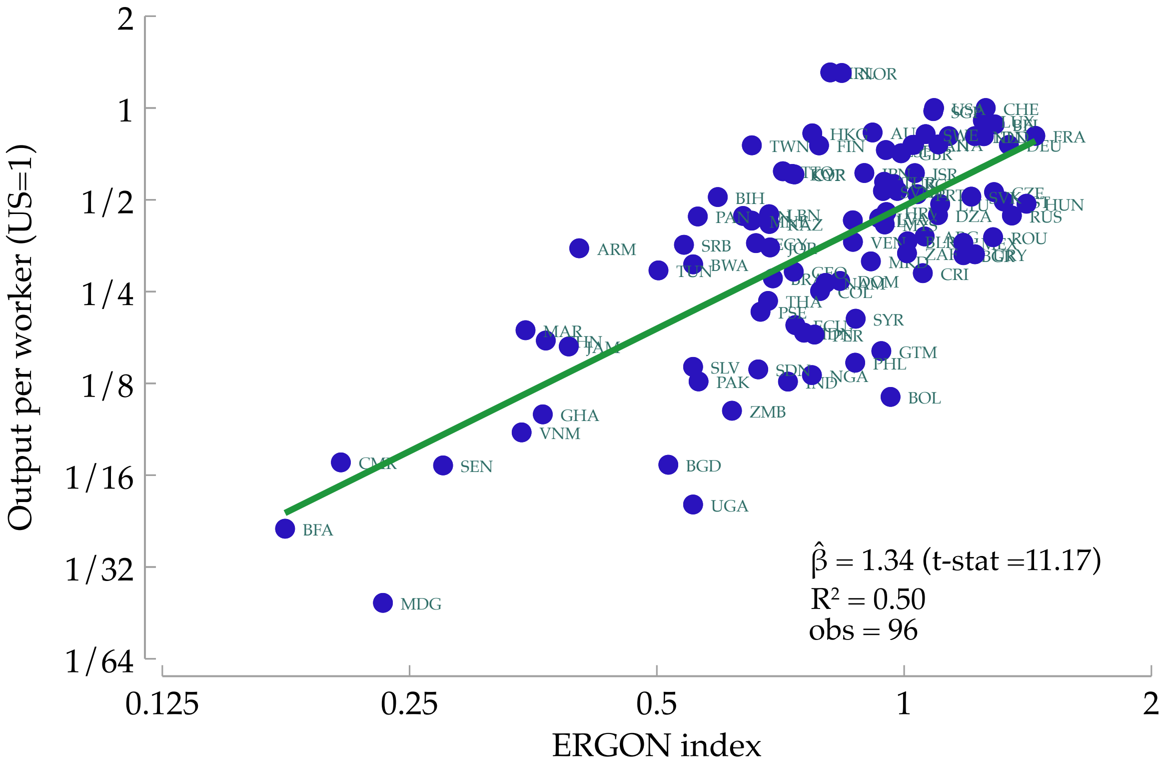
Accordingly, lower values indicate that—in relative terms—more educated people become entrepreneurs at lower rates.

To complete the construction of the ERGON index, one needs to determine a threshold above which individuals are considered higher educated, or "graduates" as in the acronym. Given that the vast majority of people with tertiary education in less developed countries do not pursue a four-year Bachelor's degree, I argue that an appropriate threshold is $\overline{S}$ = ISCED level 5, as reported in the harmonized GEM datasets. Individuals with ISCED level 5 education are those with *at least* short-cycle tertiary education, which typically corresponds to at least 14 years of schooling.

Figure 1.1 and Figure 1.2 plot the ERGON index against output per worker relative to the U.S. as well as against estimated Hicks-neutral TFP, all in the logarithmic scale. OLS fitted values and their corresponding t-statistics and R^2 are also included. In both cases the observed relationships are striking and remarkably strong, highlighting an underlying (weak) power-law function. The ERGON index is undoubtedly positively associated with development patterns among nations. These variations can also account for about half of the variance in relative output per worker and TFP, a goodness-of-fit that is notably high.

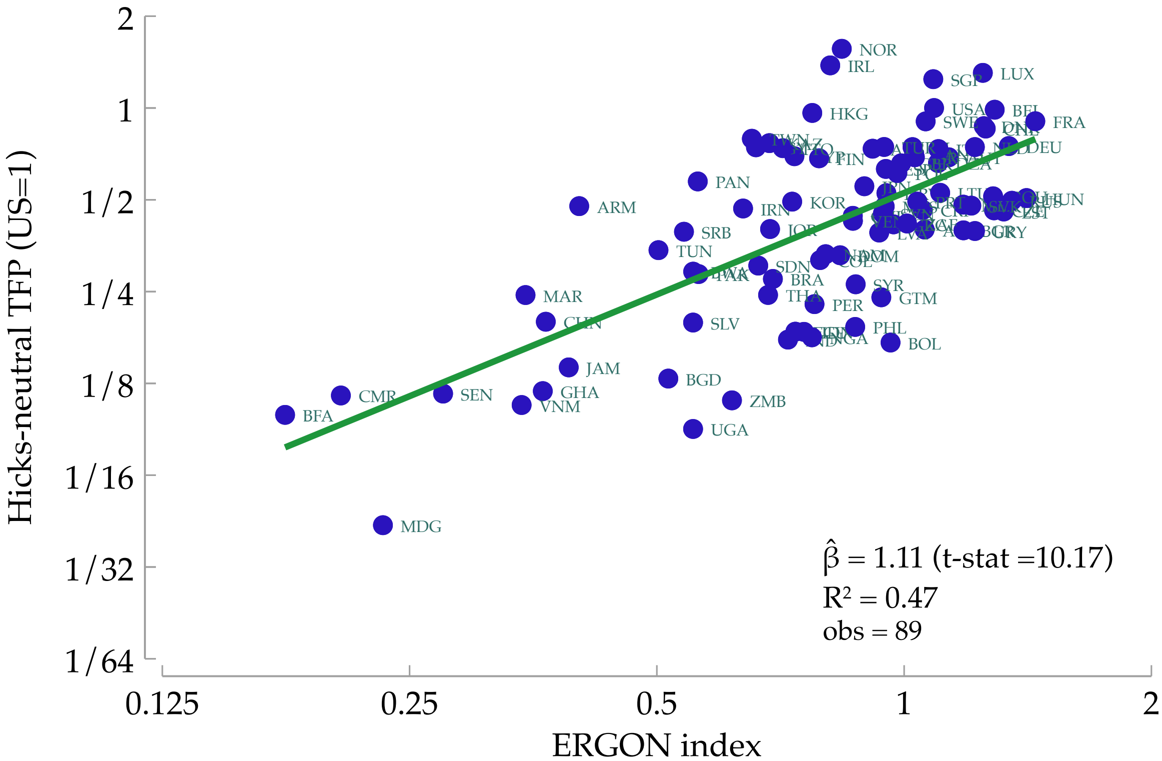
Moreover, one would like to examine whether the above results are sufficiently robust to the inclusion of variables that can naturally influence the numerator and denominator of the ERGON index, and which are ubiquitously relevant for macro-development. As evidenced by Table 1.1, their significance holds firmly even after controlling for the overall entrepreneurship rate calculated using GEM data, as well as average human capital proxied by the PWT human capital index.

Figure 1.1: ERGON Index vs Output per Worker



SOURCES: Global Entrepreneurship Monitor, Penn World Table, and author's calculations.

Figure 1.2: ERGON Index vs Hicks-Neutral TFP



SOURCES: Global Entrepreneurship Monitor, Penn World Table, and author's calculations.

Although these observational findings cannot be taken as direct evidence for causality, it is hard to overlook their sheer magnitude and robustness or to easily reconcile them with existing theories. Why do higher educated people in richer and more productive countries become entrepreneurs at rates significantly higher than those in poorer countries? The model economy presented in the second chapter of the dissertation provides a quantitative theory of how the above findings can arise endogenously in general equilibrium and assesses their implications for macro-development. But before that, let's delve into the U.S. economy and take a closer look at how differences in educational attainment influence individuals' selection into entrepreneurship together with a variety of business outcomes. These are indeed salient features that a pertinent theory should be also able to explain.

Table 1.1: OLS Regressions; Robustness of Observed Relationships

	Dependent variable: relative output per worker (log)		
Covariates	(1)	(2)	(3)
ERGON index (log)	1.336***	1.033***	0.523***
	(0.120)	(0.134)	(0.135)
Entrepreneurship rate (×100)		−0.034***	−0.023***
		(0.006)	(0.005)
PWT human capital index			1.040***
			(0.125)
Observations	96	96	89
Adjusted R^2	0.503	0.594	0.763
	Dependent variable: relative Hicks-neutral TFP (log)		
Covariates	(1)	(2)	(3)
ERGON index (log)	1.107***	0.779***	0.474***
	(0.109)	(0.118)	(0.135)
Entrepreneurship rate (×100)		−0.038***	−0.029***
		(0.006)	(0.005)
PWT human capital index			0.621***
			(0.130)
Observations	89	89	89
Adjusted R^2	0.474	0.613	0.683

Notes: Robust standard errors in parentheses. All regressions include intercepts but are omitted for brevity. *** $p < 0.01$.

1.3 Evidence from the U.S.

Focusing further on the U.S. economy, this section explores in more detail certain determinants of active entrepreneurship, with emphasis on human capital as proxied by educational attainment. We begin by describing the micro-level dataset employed and explain how the study addresses several practical and methodological issues.

1.3.1 U.S. Household-Level Data

I consolidate a comprehensive dataset with nationally representative samples, collected triennially by the Survey of Consumer Finances (SCF) over the period 1989 – 2019. I do not consider the two waves prior to 1989 (1983 and 1986) as those surveys are known to be of lower overall quality, do not include multiply imputed missing data, and do not ask a number of questions related to entrepreneurship.

There are at least three key advantages to utilizing this dataset. First, it is crucial to have many nationally representative samples that adequately capture demographic and occupational characteristics, educational attainment, as well as the full distribution of income and wealth. Second, the survey design is beneficial to the study's goal since the over-sampling of the wealthy results in more observations of entrepreneurs. Third, and perhaps chiefly, SCF interviewers ask a variety of important questions on each participant's business activities, which enables more precise identification of households associated with entrepreneurship and their business outcomes. The main disadvantage is the lack of a panel structure; however, this is not vital to the purpose of the study.

There is a number of methodological aspects that need to be taken into account when working with SCF data, especially those related to multiple imputation. The theory on proper statistical inference in such settings, called repeated imputation inference (RII), is well-understood; see the original contribution of Rubin (1987) as well as Van Buuren (2018) for a good example of more recent advances. Details can be found in section A.1.

Identification of U.S. entrepreneurs in the SCF dataset. The approach I follow is inspired by Cagetti and De Nardi (2006) among others, but instead of relying solely upon the answers of the arbitrarily defined "household head", I take into account both the respondent (R) and the spouse/partner (S/P).[5] I classify as entrepreneurs the households in which either R or S/P meets the following four criteria: **(i)** engages in some form of self-employment as their primary occupation,[6] **(ii)** owns or shares ownership in at least one privately-held business, **(iii)** has an active management role in at least one business, and **(iv)** the net value of actively managed businesses in greater than zero. As in the previous section, I refer to *self-employed business owners (SEBO)* simply as *entrepreneurs*; the rest of the labor force is referred to as *workers* or wage-earners.

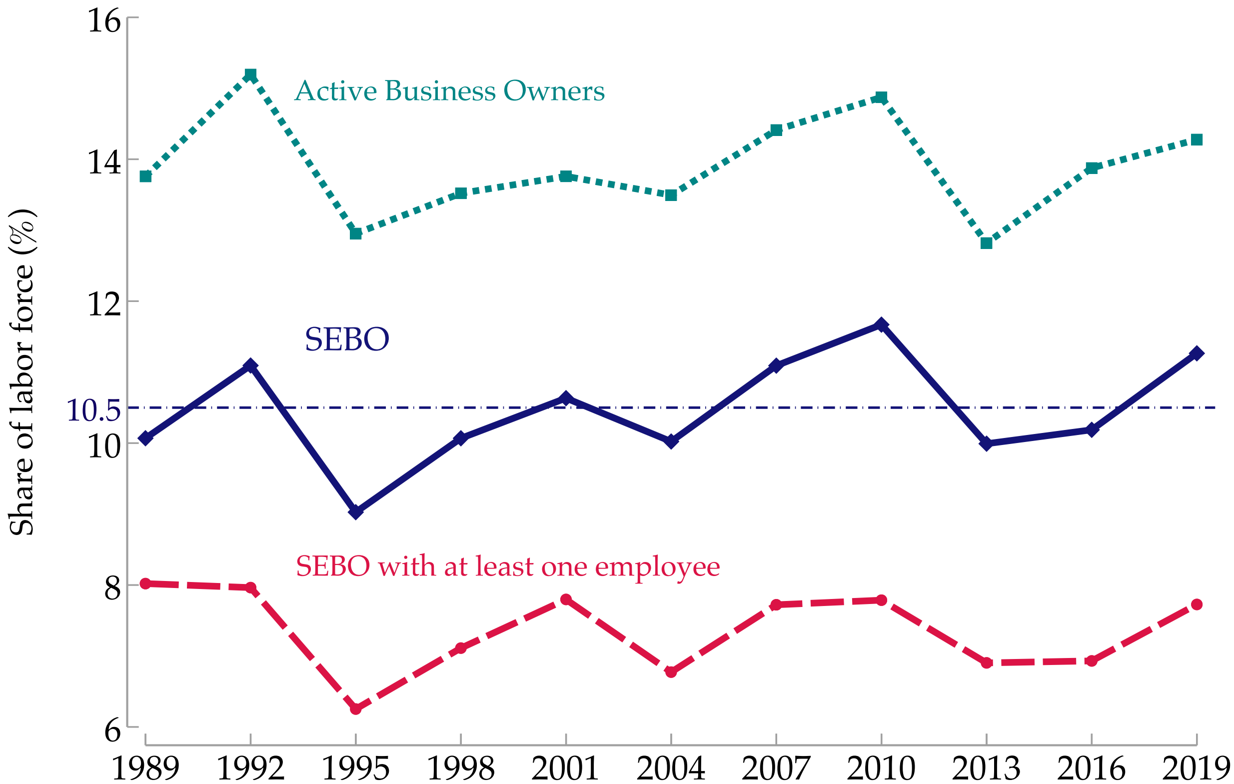
The above classification is fairly noncontroversial since the mapping of related models to data calls for entrepreneurs to have an investment stake in their business and to be working as managers with some span of control over hired capital and labor. The requirement of self-employment eliminates people who are predominantly employed by someone else and only help in a business as mere pastime. The requirement of owning at least part of the business helps not counting as entrepreneurs people who are temporarily self-employed or

[5] Although most empirical analyses using the SCF have considered the personal characteristics of the "head of the PEU", it is important to realize that the respondent and the head in couple households are not necessarily interchangeable. R is identified by the SCF staff in the initial screening interview as the more financially knowledgeable person. For mixed-sex couple households, the SCF always assigns the title to the male partner, and for same-sex couple households the older person in the partnership is assigned that title. There is no reason why to simply assume that the head's demographic and personal characteristics (e.g., sex, race, age, experience) correspond to those of the SEBO in the household.

[6] To be more precise, the SCF provides the following answers relating to self-employment status: i) Self-employed; other closely held business owned by PEU; ii) Partnership; law firm; medical/dental partnership; other non-publicly traded business in which R/S/P has an interest; iii) Consultant/contractor.

switching between jobs. The requirement of a managerial role in at least one firm phases out potentially wealthy individuals who may participate in a business only as passive investors. Also note that this definition does not discriminate against the legal status of business. More details can be found in Appendix A.

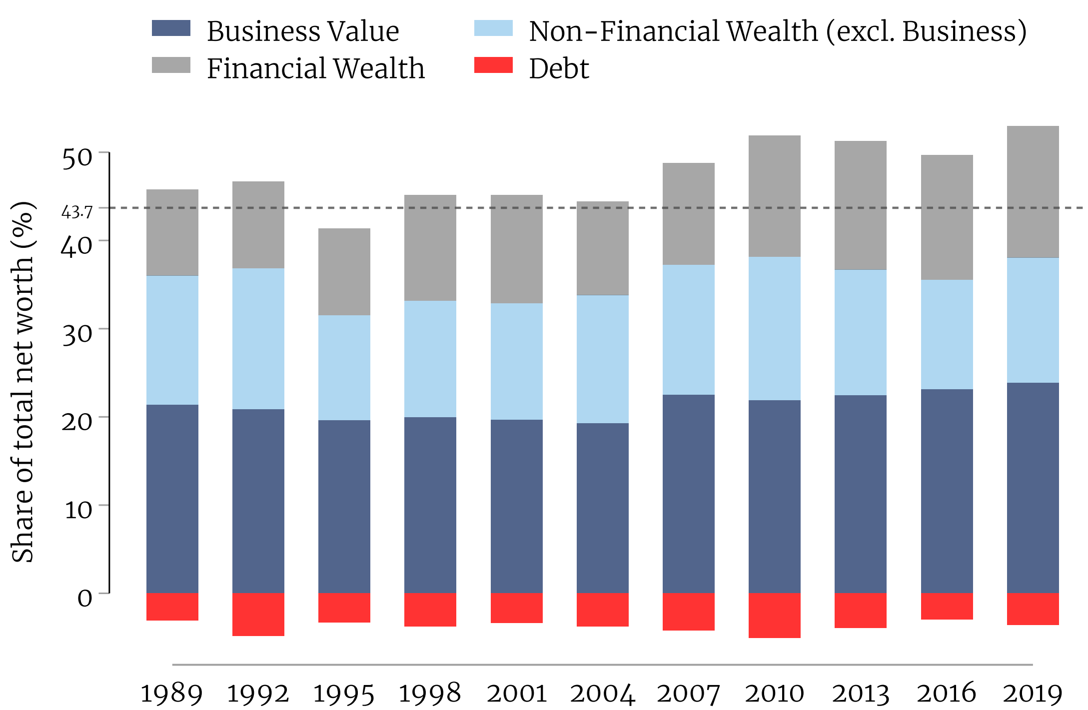
Figure 1.3: Rate of Entrepreneurship in the U.S. Labor Force



SOURCES: Survey of Consumer Finances and author's calculations.

Figure 1.3 shows the rate of entrepreneurship in the U.S. labor force according to three different definitions, with SEBO being the adopted measure throughout this study. The fraction of entrepreneurs has remained relatively stable over the thirty years, hovering around 10.5 percent of the labor force. This average value is markedly close to the arithmetic mean of 10.4 percent for the U.S. entrepreneurship rate over 1990 − 2009, as inferred from Current Population Survey (CPS) data reported by Hipple (2010).[7] The apparent consistency with U.S. Census/BLS data lends further credence to the procedures I employ and to the argument for truly nationally representative empirics.

Figure 1.4: Share of U.S. Labor Force Wealth Held by Entrepreneurs

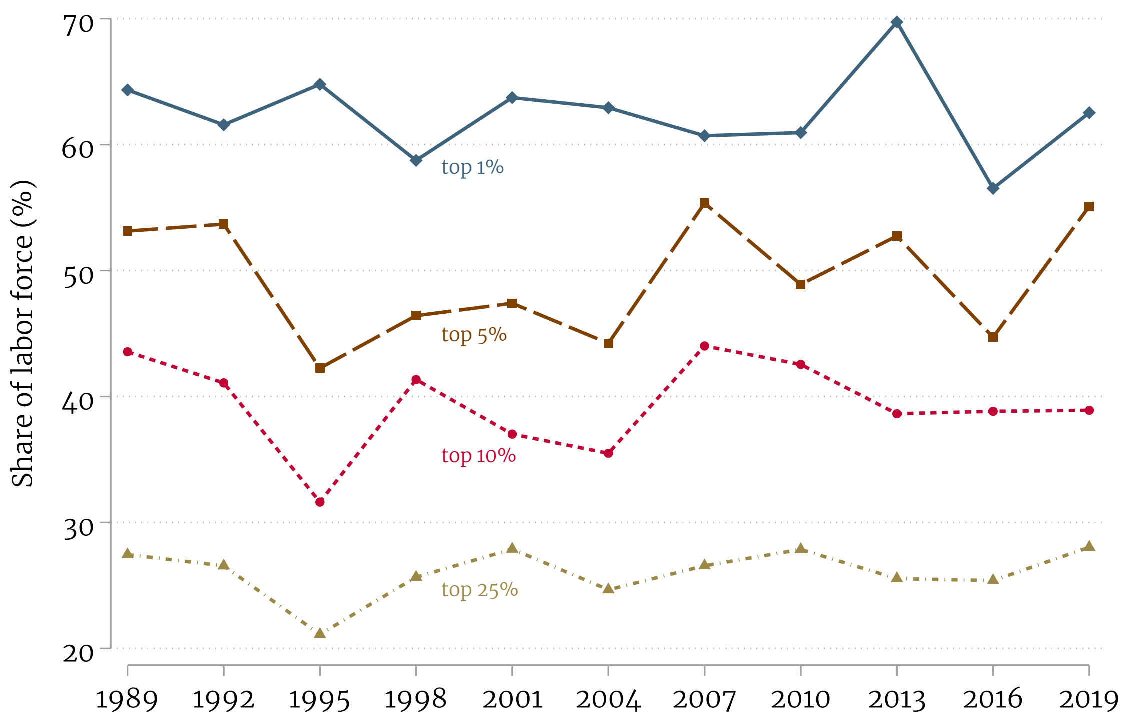


Sources: Survey of Consumer Finances and author's calculations.

Entrepreneurs form a relatively small group compared to the working population, but their economic significance is disproportionately large. Some of the most conspicuous examples

[7] This number corresponds to the sum of the incorporated plus unincorporated self-employed divided by total employment in all non-agricultural industries; see Table 1 and Table 2 in Hipple (2010). Results are identical to the first decimal point whether calculating the average of ratios or the ratio of averages.

Figure 1.5: Fraction of Entrepreneurs in Selected Wealth Percentiles

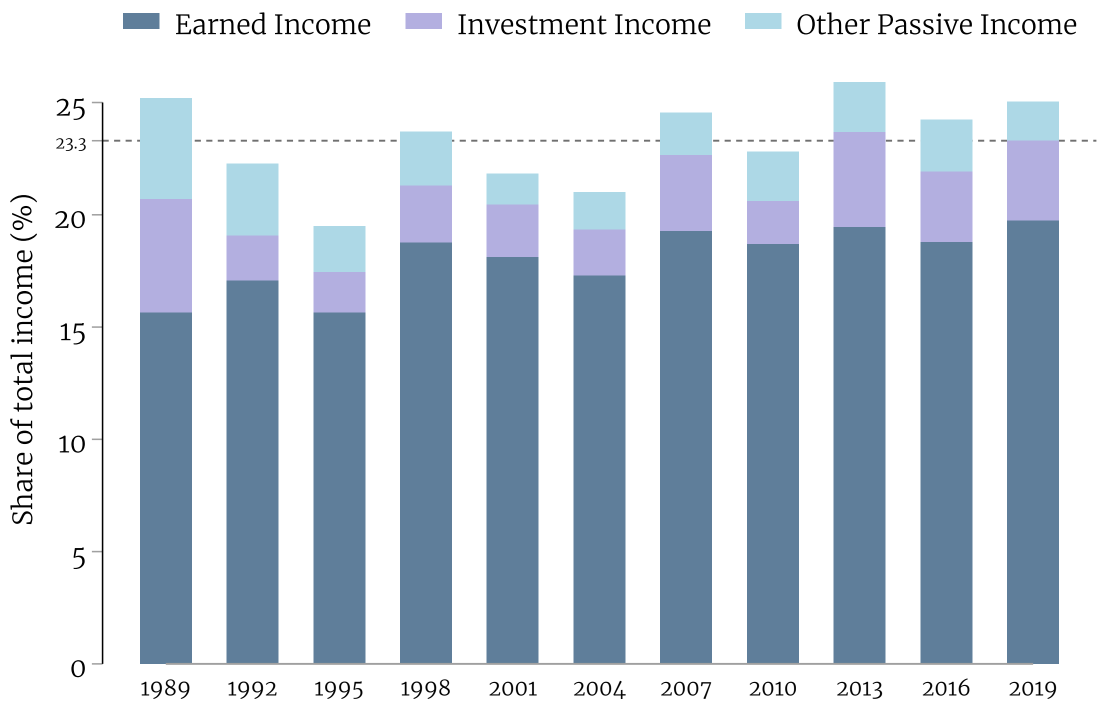


SOURCES: Survey of Consumer Finances and author's calculations.

are depicted in the following figures. Despite being on average only 10.5 percent of the labor force, U.S. entrepreneurs they hold 44 percent of total net worth (Figure 1.4). The systematic concentration of wealth in the hands of SEBO becomes even more apparent when we look at higher percentiles of the distribution (Figure 1.5). About 4 in 10 working adults with net worth in the top decile are entrepreneurs; around 1 in 2 in the top 5 percent; and this number climbs to almost two thirds when considering the top 1 percent of wealth holders.

Furthermore, entrepreneurs receive on average between one fifth and one fourth of total U.S. labor force income (Figure 1.6). They are also substantially overrepresented in the top percentiles of the income distribution (Figure 1.7), for example by a factor of 3 and 5 when looking at the top decile and top 1 percent, respectively. However, it is worth

Figure 1.6: Share of U.S. Labor Force Income Held by Entrepreneurs



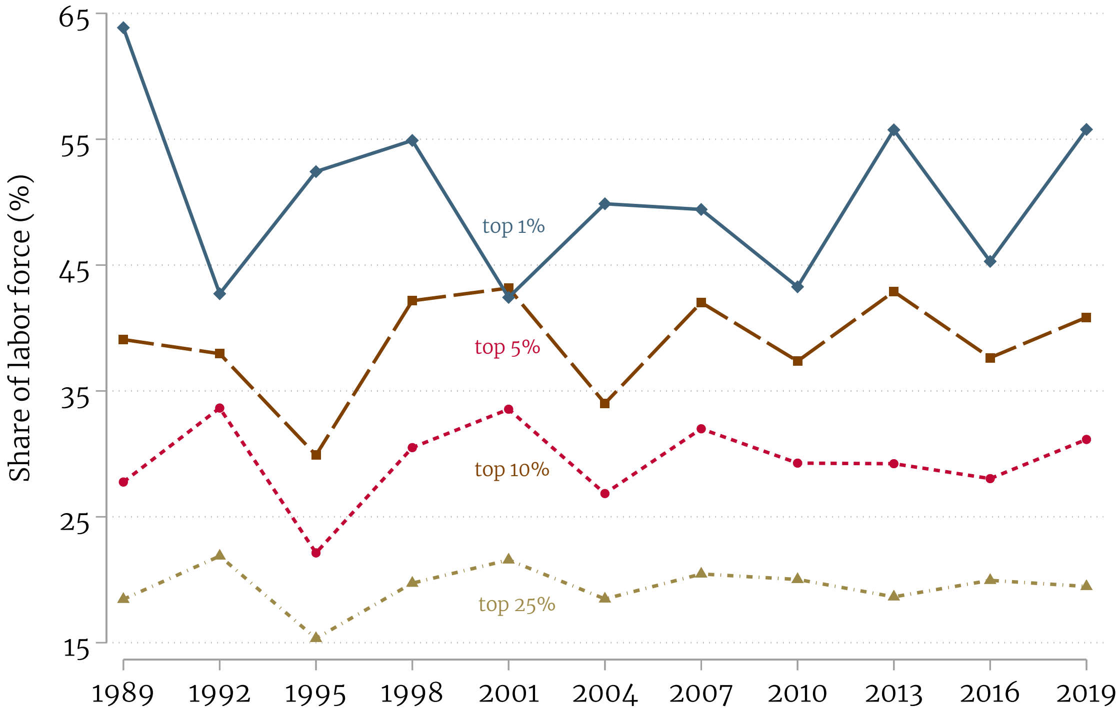
Sources: Survey of Consumer Finances and author's calculations.

stating that these numbers pertaining to entrepreneurial income should be taken more as lower bounds rather than definitive estimates, as it has been known in the literature (e.g. Hamilton (2000)) that legally disclosed net business profit may be deceptively low because of various tax incentives to underreport it to the IRS.

Finally, Table 1.2 documents ample differences in various socioeconomic and demographic characteristics across U.S. entrepreneurs and workers.

In a nutshell: entrepreneurs have more years of potential experience in the labor market; they work longer hours on a yearly basis; they score better when it comes to self-assessing their general health condition; and they report higher willingness to take financial risk. There seem to be no systematic differences in their rates of retirement or disability. Additionally, they are

Figure 1.7: Fraction of Entrepreneurs in Selected Income Percentiles



SOURCES: Survey of Consumer Finances and author's calculations.

predominantly married (or in a civil union), male, and White/Caucasian, although this has been steadily changing over the 21st century in line with demographic shifts. For example, Black/African-American and Hispanic/Latino populations that have been underrepresented in the entrepreneurial group are catching up in the second part of the subsample, whereas the Other Race category, consisting mostly of Asian people, has been equally represented in both groups. There is no doubt that many of the above disparities deserve a deeper look, and some of them have already been studied in the cited literature.

Table 1.2: Descriptive Statistics: U.S. Entrepreneurs vs Workers

	Pooled 1989–2004 Sample			Pooled 2007–2019 Sample		
Variable	non-SEBO (N_1=13,932)	SEBO (N_2=5,187)	p-value of diff.	non-SEBO (N_1=16,601)	SEBO (N_2=5,502)	p-value of diff.
Education	13.6	14.4	$<.01$	14.0	14.9	$<.01$
Potential experience	22.3	26.5	$<.01$	24.2	30.7	$<.01$
Worker experience	21.2	15.7	$<.01$	23.2	17.4	$<.01$
Self-employed experience	1.1	10.8	$<.01$	1.0	13.3	$<.01$
Annual labor supply	$2,142$	$2,418$	$<.01$	$2,109$	$2,230$	$<.01$
Health $(1-4)$	3.19	3.31	$<.01$	3.08	3.20	$<.01$
Risk willingness $(1-4)$	1.87	2.14	$<.01$	1.88	2.17	$<.01$
Employment variety	3.15	2.98	$<.05$	3.34	3.08	$<.05$
Unemployed partner (0/1)	0.25	0.22	$<.01$	0.26	0.23	$<.01$
Retired (0/1)	0.007	0.010	$<.10$	0.02	0.03	$<.10$
Disabled (0/1)	0.002	0.003	.65	0.010	0.008	.61
Male	0.68	0.77	$<.01$	0.65	0.73	$<.01$
Married	0.65	0.82	$<.01$	0.63	0.82	$<.01$
White/Caucasian	0.74	0.89	$<.01$	0.64	0.79	$<.01$
Black/African-American	0.12	0.03	$<.01$	0.14	0.05	$<.01$
Hispanic/Latino	0.09	0.03	$<.01$	0.12	0.06	$<.01$
Other race category	0.05	0.05	.96	0.10	0.10	.79

Notes: All statistics refer to weighted arithmetic means. Sample means and standard errors are calculated under RII using all 5 SCF implicates for every observation and all 999 SCF bootstrap replicate draws and weights. The equality of means/proportions is assessed with a weighted, two-sided, unequal-variance hypothesis test.

1.3.2 U.S. Entrepreneurs and Educational Attainment

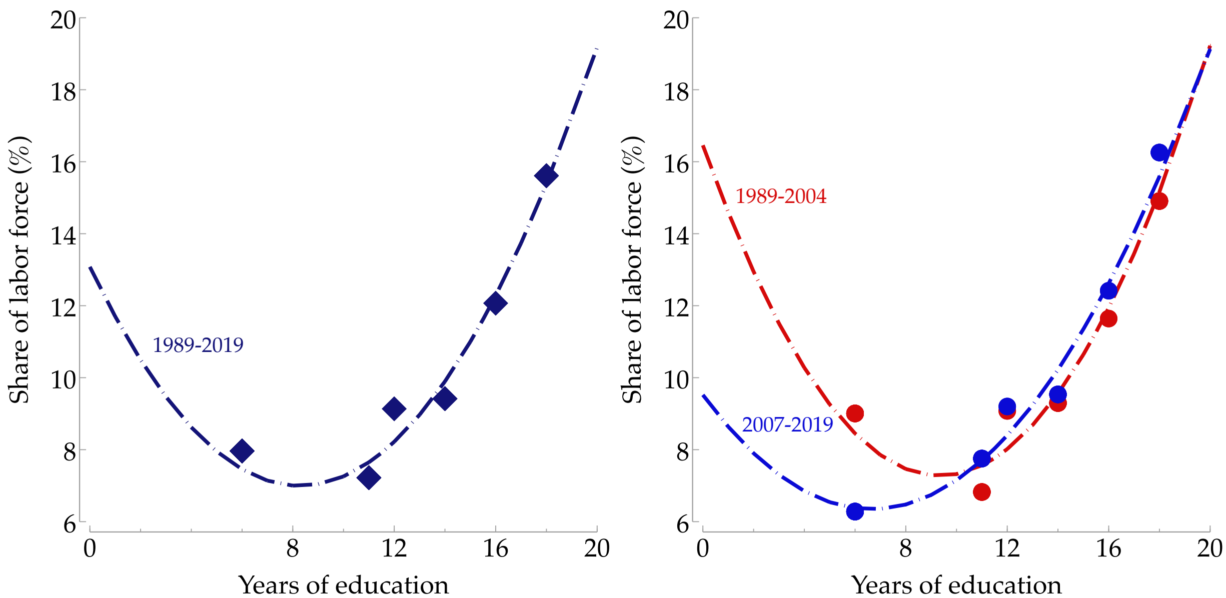
This section throws some light on the importance of heterogeneity in educational attainment across entrepreneurs and wage earners, as well as within entrepreneurs. I document that differences in years of schooling are critical in explaining both selection into entrepreneurship and systematic variations in business outcomes. The results are robust upon conditioning on a wide-ranging set of characteristics in subsequent reduced-form regressions.

Fact 1: *The rate of entrepreneurship by educational attainment follows an approximate U-shape in the full sample; closer to a* J*-shape in more recent years.*

Figure 1.8 documents the relationship between the share of the U.S. labor force engaging in entrepreneurship (SEBO) and completed years of formal education. The left panel contains estimates using the pooled sample for $1989 - 2019$; the right panel splits the sample roughly in half and shows estimates for the periods $1989 - 2004$ and $2007 - 2019$. Note that the bin scatter points correspond to raw data for six educational attainment categories, and the dashed lines represent (weighted RII) quadratic regression fitted values of entrepreneurship rates against educational attainment over the full set of points.

In relative terms, active entrepreneurship is characterized mostly by individuals from the extremes of the education distribution. Such an occupational choice pattern results in an approximately U-shaped relationship in the full sample, but this masks an important additional fact. When we look at the two consecutive subsamples, the shape of the relationship changes from a clear U-shape to something closer to a J-shape. In other words, among people with few years of schooling, less and less of them select into entrepreneurship in more recent years. Meanwhile, entrepreneurship rates are consistently high among higher educated people in the U.S. labor force.

Figure 1.8: U.S. Entrepreneurship Rate by Educational Attainment



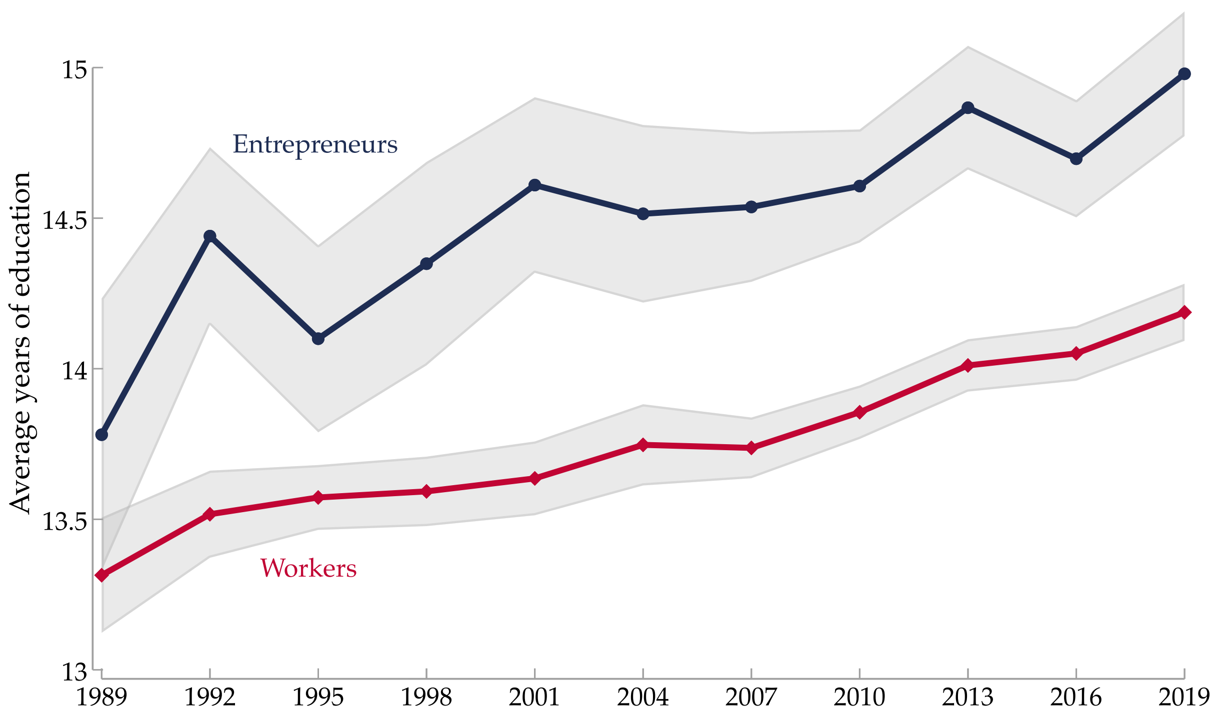
SOURCES: Survey of Consumer Finances and author's calculations.
NOTES: Dashed lines represent weighted quadratic regression fitted values using entrepreneurship rates over the full set of educational attainment; a bin scatter of raw data for six educational attainment categories (indicated by x-axis labels) is used to reduce visual clutter. Full pooled sample (left panel) and two consecutive subsamples (right panel).

Fact 2: *Entrepreneurs are more educated than workers on average.*

The evolution of average years of formal schooling for U.S. entrepreneurs and workers is shown in Figure 1.9. Mean differences in educational attainment across occupational groups are extensive and somewhat widening over time. The plotted 95% RII confidence intervals for the mean act as visual t-tests and suggest very high statistical significance. A discrepancy of $0.8 - 0.9$ years of schooling on average should be considered abundant, not only within a country's labor force but also across different countries.[8]

[8] One can consult the September 2021 version of the Barro and Lee (2013) educational attainment dataset for population aged 25-64. Over the period $1995 - 2015$, if we compare average years of schooling in some of the world's richest advanced economies such as Ireland, Denmark, Sweden, Germany, Australia, and

Figure 1.9: Average Years of Education: U.S. Entrepreneurs vs Workers



Sources: Survey of Consumer Finances and author's calculations.
Notes: Connected markers refer to weighted arithmetic means by occupational group. Sample means and standard errors are calculated under RII using all 5 SCF implicates for every observation and all 999 SCF bootstrap replicate draws and weights. Shaded areas represent 95% confidence intervals for the mean.

Fact 3: *Average entrepreneurial outcomes are strictly increasing—not U-shaped—in the educational level of the firm owner/manager.*

Japan, to some less advanced and/or upper-middle income countries such as Hungary, Poland, Lithuania, Latvia, Kazakhstan, and Ukraine, the differences are less or equal to 0.9 years of schooling. A difference of 0.5 years or less can be obtained if we look at some of the relatively successful African countries such as Tunisia, Egypt, and Namibia, and compare them to much poorer countries like the Republic of the Congo, Syria, and Cameroon.

Table 1.3 presents weighted RII averages for various business outcomes along six educational attainment categories to facilitate the comparison. In terms of annual business income reported to the IRS, I consider total business income, which may include earnings from incorporated or unincorporated businesses, as well as sole proprietorship income. These are also reported in per-hour terms after dividing income quantities by the annual labor supply of the firm/owner manager (number of working weeks times number of working hours per week) and then take averages. Estimates for average firm employment sizes are provided; these include the number of employed workers for firms with at least one paid employee, as well as for all firms. To capture the extent of tangible and intangible capital accumulation, I calculate the average firm net value per employee in each educational category.[9] The table also displays average differences in potential experience (age - years of education - 6) and its components, in addition to some personal and demographic characteristics of the firm owner, ranging from self-reported willingness to take financial risk and health status (indices in ascending order) to sex, race, and marital status. These factors together with numerous others will be included in the reduced-forms regressions below.

A clear message stands out. Entrepreneurs with higher educational attainment—more human capital in the conventional sense—fare better on average across a range of business outcomes. This is yet another stylized fact that a relevant theory should be able to explain in a robust manner. Notably, there is no *prima facie* evidence that business outcomes are strictly increasing in educational attainment because more educated firm owners own a larger share of their main business, but the opposite pattern prevails. They are also not typically associated with either more years of self-employed experience and they have not worked more as wage-earners in the past.

[9] Firm net value for businesses where the household has an active interest is defined by the SCF as net equity if the business were sold today, minus loans, plus value of personal assets used as collateral.

Table 1.3: Within-Entrepreneur Heterogeneity by Educational Attainment

Variable	Years of Education (pooled 1989–2019 Sample)					
	$[0, 8]$	$(8, 12)$	12	$(12, 16)$	16	$(16, 21]$
Total business income (> 0)	53,302	64,625	82,502	93,122	188,941	260,210
Total business income	38,831	45,332	54,886	64,102	129,745	195,047
Sole proprietorship income	26,014	33,849	38,948	42,098	59,181	100,865
Total business income per hour (> 0)	23.4	35.1	54.9	64.5	114.5	229.0
Total business income per hour	17.1	24.5	37.0	42.0	78.6	170.5
Sole proprietorship income per hour	12.5	19.9	28.1	33.3	34.2	60.2
Firm employment size (> 0)	5.4	6.3	8.2	16.8	49.1	80.9
Firm employment size	3.4	4.1	5.7	10.9	35.5	62.1
Firm net value per employee	85,725	90,667	137,249	140,997	209,560	233,104
Main business ownership share	0.92	0.91	0.91	0.89	0.82	0.77
Potential years of experience	35.1	33.4	30.7	27.8	26.9	27.5
Self-employed years of experience	14.6	13.6	12.7	11.0	11.1	12.8
Prior worker years of experience	20.5	19.8	18.1	16.8	15.7	14.8
Risk willingness (1 − 4)	1.59	1.66	1.94	2.18	2.36	2.37
Health (1 − 4)	3.32	3.28	3.35	3.37	3.50	3.52
Ever received inheritance (0/1)	0.18	0.17	0.26	0.30	0.34	0.36
Expect to receive inheritance (0/1)	0.10	0.12	0.16	0.22	0.26	0.25
Male (0/1)	0.92	0.82	0.76	0.68	0.72	0.77
Married (0/1)	0.89	0.77	0.85	0.79	0.83	0.82
White/Caucasian (0/1)	0.66	0.75	0.84	0.83	0.85	0.86

Notes: All statistics refer to weighted arithmetic means, calculated under RII using all 5 SCF implicates for every observation and all 999 SCF bootstrap replicate draws and weights. All monetary variables (measures of income and net firm value) are inflation-adjusted to 2019 U.S. dollars by the SCF staff.

1.3.3 Further Reduced-Form Evidence

The descriptive results show dramatic differences along the dimension of educational attainment, both between and within occupational groups. It is nonetheless possible that these differences stem from sources aside from education. It could be the case, for example, that highly educated individuals actually work longer hours and have more job market experience, or that educated entrepreneurs are on average healthier and more risk-loving. In such cases the apparent impact of education could be biased upward, so one would want to control for additional confounding variables.

In order to further sort out the effect of formal education on the rate of entrepreneurship and entrepreneurial outcomes, this section presents reduced-form results (Probit and OLS) for pertinent outcome variables. I consider regressions of the form[10]

$$y_i = \beta_0 + \beta_1 \cdot educ_i + \beta_2 \cdot educ_i^2 + X_i \boldsymbol{\gamma} + \Gamma_s + \Gamma_o + \Gamma_s \times \Gamma_o + \Gamma_t + \varepsilon_i$$

where X_i is a rich set of controls such as potential experience, annual hours worked, self-reported health condition, willingness to take risk, past and future inheritances, marital status, sex, race, and others. To mitigate potential concerns that results might be driven by occupational and sectoral differences, I also control for sector fixed effects (Γ_s), occupation fixed effects (Γ_o), their interactions ($\Gamma_s \times \Gamma_o$), and year fixed effects (Γ_t).

The coefficients β_1 and β_2 are of primary interest. Results are reported in Table A.2 and confirm the robustness of Fact 1 presented above. In Probit regressions with the binary

[10] Regressions are based on the entire pooled sample and are weighted by the appropriate SCF sampling weights; estimated parameters are based on repeated imputation inference using all 5 SCF implicates for every observation; bootstrapped standard errors are calculated using 999 SCF bootstrap replicate draws and their respective replicate weights. Results using a Logit specification for selection into entrepreneurship yield nearly identical results. As before, please refer to Appendix A for more details.

outcome $SEBO = \{0, 1\}$ as the dependent variable, both hypotheses $\widehat{\beta}_1 = 0$ and $\widehat{\beta}_2 = 0$ are strongly rejected so that a non-linear relationship emerges even after controlling for a wide range of factors. The exact combination of signs and magnitudes of $(\widehat{\beta}_1, \widehat{\beta}_2)$ determine the vertex and consequently the shape and asymmetry of the parabola within the support of *educ*. Results based on Logit regressions yield nearly identical conclusions.

The OLS regressions in Table A.3 in the appendix confirm the robustness of Fact 3. Considering hourly total business income, the coefficient $\widehat{\beta}_1$ is strongly positive in linear specifications—but becomes insignificant in all quadratic specifications—and at the same time the hypothesis $\widehat{\beta}_2 \neq 0$ cannot be rejected. Hourly entrepreneurial income does not appear to be U-shaped but instead strictly increasing and convex in educational attainment. Highly similar results are obtained upon considering other business income measures as the dependent variable.

All in all, the evidence suggests that very successful entrepreneurs without higher formal education are the exception rather than the rule, which can be useful in addressing a famous myth surrounding business success. Reflecting on the link between education and entrepreneurial achievements, one may be tempted to conclude that they are unrelated or even inversely related. Famous examples of extraordinary entrepreneurs who happen to be college dropouts come to mind: Paul Allen, Michael Dell, Larry Ellison, Bill Gates, Steve Jobs, Steve Wozniak, Mark Zuckerberg. One could even consider famous high school dropouts such as Richard Branson, Amancio Ortega, and Francois Pinault. But such examples are so memorable exactly because they are so rare. They form a subset made by tail events, with a relatively small measure compared to the set of entrepreneurs, and this is yet another reason why their cases are so exceptional.

1.4 Summary and Concluding Remarks

In order to highlight the importance of conventional human capital as a key source of heterogeneity in shaping occupational choices and entrepreneurial outcomes, this paper has sought to establish a set of stylized facts related to active entrepreneurship and human capital, the latter being proxied by educational attainment.

There are two main messages emerging from this study. First, robust empirical evidence from international survey data suggests that higher educated people in richer and more productive countries become entrepreneurs at rates significantly higher than those in poorer countries. Second, further focus on the U.S. economy—again at the micro-level—shows that the relationship between schooling and the rate of entrepreneurship exhibits an asymmetric U-shaped form, while average business outcomes are strictly increasing for more educated firm owners/managers. These stylized facts cannot be easily or adequately explained by existing theories, and thus deserve more thorough investigation.

Moving forward, I would suggest two avenues for future research. There is undoubtedly more work to be done in exploring the link between entrepreneurship and educational attainment, along with establishing further stylized facts. Analyses mirroring this study's for the U.S. can be carried out for other countries to uncover whether similar results apply. In particular, one would want to examine if selection into entrepreneurship is indeed non-linear and/or U-shaped in more countries Second, it would be useful to examine how these relationships evolve over time by exploiting detailed panel survey data. Regarding the expanding literature on the role of entrepreneurship in macroeconomics, I would advocate for putting more emphasis on the dimension of education/human capital as a source of individual heterogeneity, the allocation of which. This is one of the issues I will be exploring in the second chapter of this dissertation.

Chapter 2

Entrepreneurship, Human Capital, and the Allocation of Talent

2.1 Introduction

Entrepreneurship is renowned as a major driver of economic development for more than a century, dating back to the pivotal works of Schumpeter (1911) and Knight (1921). Since then an expansive array of research has emerged offering a fundamental insight: *who* becomes an entrepreneur is paramount. Occupational choices are not only determined by but also determine macroeconomic outcomes, as the allocation of resources between and within occupations governs total factor productivity (TFP) and output.

A substantial body of literature studies economies in which heterogeneous agents may become entrepreneurs who create firms and shape productive capacities. In such settings, the decision to own and run a firm usually hinges on differences in ability/productivity; risk aversion; wealth and access to credit; taste for entrepreneurship; or a combination of the above. The sources of heterogeneity have a crucial bearing on occupational sorting, hence on the macroeconomy and firm size distributions.[11]

However, there is scant research on the nature and impact of occupational choices due to heterogeneity in conventional human capital, as proxied by years of formal education. I advocate for greater emphasis on how differences in entrepreneurship rates by educational attainment can arise, and how they affect long-run economic outcomes. In the sphere of macroeconomics, even less endeavor has been devoted to understanding the aggregate and distributional consequences of the entrepreneurship-human capital nexus. I argue that these are critical research gaps that need to be further explored.

Is the allocation of human capital between entrepreneurs and workers a key determinant of aggregate productivity and income? If so, how pervasive are its implications for macro-development? This study employs theory and quantitative assessment aiming to address these inquires. To the best of my knowledge, this is the first paper that attempts to enrich the discussion in the relevant literature by raising these questions.

The study has both theoretical and quantitative objectives, and is motivated by the empirical results presented in the first chapter of the dissertation. In particular, these stylized facts cannot be jointly accommodated by existing theories without imposing strong distributional assumptions or implausible restrictions on primitives, and thus warrant closer investigation.

[11]A cornerstone of this line of research can be traced back to the seminal article by Lucas (1978), along with subsequent work by Calvo and Wellisz (1980) and Rosen (1982), where occupational decisions are guided by differences in (latent) managerial ability. Parallel to those efforts, Kihlstrom and Laffont (1979) and Kanbur (1979) emphasized the relevance of risk aversion in choosing between occupations and analyzed the consequent general equilibrium effects. Later contributions by Evans and Jovanovic (1989), Banerjee and Newman (1993), and Aghion and Bolton (1997), to name a few, paved the road to the modern literature by stressing the importance of wealth heterogeneity and financial frictions. Some attention has also been drawn to the role of differences in non-pecuniary benefits/tastes related to business ownership, as in Hurst and Pugsley (2015) and Poschke (2018). More generally, the idea that the allocation of entrepreneurial talent is instrumental for economic growth has been examined by Baumol (1990) in a historical context, and further explored by Murphy, Shleifer, and Vishny (1991), among others.

I develop a versatile dynamic general equilibrium model with the aim of rationalizing the empirical findings established in the previous chapter, and at the same time, operate within a framework that has the capacity to be broadly consistent with aggregate and survey data. The overarching goal is to understand the importance of the allocation of human capital between entrepreneurs and workers in determining cross-country differences in output and *endogenous TFP*.[12] Simultaneously, an empirically relevant theory should be able to generate realistic predictions across important micro and macro aspects, ranging from the firm size distribution to cross-sectional income and wealth inequality.

To organize the discourse, I propose a micro-founded heterogeneous-agent model that features occupational and educational choices, along with incomplete markets and the option of some financial frictions. Upon entering the labor force, individuals differ in three dimensions (state variables): entrepreneurial ability, liquid assets, and human capital. The latter is accumulated endogenously prior to joining the labor force, with agents choosing the amount of time to invest in formal schooling. For the reasons explained below, I argue that incorporating my elaborations to the workhorse model would be beneficial to the growing literature on entrepreneurship and macroeconomics, both in terms of theoretical predictions and quantitative performance. Conditional on the new hypothesis I introduce, the proposed theory remains valid and flexible without the need for any atypical assumptions on preferences and technologies, and without presupposing any dependence between entrepreneurial talent and human capital at the population level. In principle, the main results hold under any sequence of joint distributions.

[12] The emphasis follows from a well-known consensus in the literature. Cross-country variations in output per worker are not primarily driven by variations in physical or human capital, but are mostly determined by some form of unexplained (Solow-type) residual, often called total factor productivity (TFP); see Klenow and Rodríguez-Clare (1997); Prescott (1998); Hall and Jones (1999); Hendricks (2002); and Caselli (2005). The contribution of TFP seems to be all-important—but in the absence of an endogenous theory of TFP, it simply becomes a "measure of our ignorance".

At the crux of the analysis lies the following hypothesis: there exists a disembodied technology that coalesces entrepreneurial ability and human capital to form the fundamental "effective productivity" that is ultimately used in production. The output of this process is assumed to entail positive complementarity between idiosyncratic ability and education. To reap the underlying benefits at each point in time, active firm managers face a costly intratemporal technology choice. Technology adoption enables them to complement their talent with the skills and competences that education entails. In general equilibrium, among equally talented entrepreneurs those with higher education will be at least as productive. Put simply, entrepreneurial human capital may serve as an additional factor of production under some constraints.[13]

Competitive equilibria encompass threshold levels of education beyond which agents choose to adopt the technology. Aggregate total factor productivity is a pivotal endogenous quantity determined by occupational and technological choices, with entrepreneurial human capital being a key component. Concurrently, new generations make their schooling decisions before entering the labor force based on expectations about future income streams and factor prices. The latter are largely affected by prevailing occupational patterns and technology adoption choices; therefore, the outlook on entrepreneurial success will directly influence the accumulation of human capital for all individuals. These two mechanisms, together with their ensuing general equilibrium effects, form the main channels through which the entrepreneurship-education nexus has first-order consequences for macro-development.

The degree of complementarity between factors of production in the disembodied technology is of primary interest. *Ceteris paribus*, it is the central parameter governing the shape

[13] As discussed later in more detail, one can view the potential input of entrepreneurial human capital as a source of firm-specific intangible capital. This leads to interpreting the effects of technology adoption as contributions to firms' *organization capital*, e.g., Prescott and Visscher (1980); Atkeson and Kehoe (2005); or more specifically to their *managerial capital*, e.g., Bruhn, Karlan, and Schoar (2010).

and form of the entrepreneurship-human capital relationship, which has a major impact on multiple facets of the macroeconomy—the structure of endogenous TFP, the number of workers and entrepreneurs, the future accumulation of human capital. The complementarity parameter is not presumed to be constant across economies or over time. In fact, a main message is that cross-country differences in aggregate TFP and per capita output are driven by differences in the capacity of nations to foster synergy between entrepreneurial ability and human capital.

The quantitative analysis focuses on stationary competitive equilibria, the recursive formulation of which corresponds to a (continuous-time) Mean Field Game without common noise. The model is fully solved numerically using an implicit upwind finite difference scheme based on the efficient methods by Achdou et al. (2022).

The model economy is calibrated to U.S. data and is able to closely match key moments across varied dimensions. This is achieved without relying on unconventional specifications or functional forms, and without producing unconventional parameter values compared to the pertinent literature. The calibration exercise is also successful in replicating a range of non-targeted moments capturing salient features of aggregate and survey data. The results feature, for instance, large dispersion in business outcomes that approximate firm size distributions; widespread wealth inequality with the appropriate concentration of assets in the hands of entrepreneurs; substantial income inequality without any shocks to human capital or labor earnings; empirically plausible cross-sectional variation in educational attainment; and sensible firm dynamics. Notably, the relationship between the rate of entrepreneurship and educational attainment arises as of an asymmetric U-shaped form like the one found in the data, under a modest degree of complementarity upon technology adoption.

Further model-based assessment illustrates the quantitative importance of the two mechanisms put forward by the theory. The goal is to obtain a clearer picture about the magnitude and decomposition of implied long-run output differences with respect to the U.S. (baseline calibration). The first experiment involves varying only the complementarity parameter—in essence emulating a series of economies with ower ERGON index, but otherwise similar in terms of preferences, processes, technologies, and parameters. *Ceteris paribus* shifts in the entrepreneurship-education nexus generate up to -45% output vis-à-vis the U.S. These sizeable and persistent differences are mostly driven by endogenous TFP, accompanied by considerable variation in educational attainment leading to lower human capital of workers.

In a similar vein, the second experiment involves varying only a prior distribution-related parameter—in essence emulating a series of economies with uniformly lower educational attainment, but otherwise similar. The ramifications are far-reaching when considering *ceteris paribus* variations in schooling, which can alone account for up to about -70% output per person engaged. Such abundant differences arise not only due to economies with lower aggregate/average human capital having less productive workers, but also because a less educated labor force will eventually result in less productive entrepreneurs and firms. This is a new perspective on how prevalent the role of human capital can be for macro-development, and complements recent work offering explanations such as quality-adjusted education and varying returns to experience.[14]

To get even more precise about the extent of international income differences the model can explain, I vary key parameters as above to target related moments for a selected group of countries. All in all, findings suggest that economies differ markedly in their ability to complement entrepreneurial talent with the benefits that human capital can offer. This leads to a direct misallocation channel that has drastic implications for both factor accumulation

[14] See for example Schoellman (2012), Manuelli and Seshadri (2014), and Lagakos et al. (2018).

and aggregate TFP formation, and sheds light on a novel proximate cause of cross-country output differences over the long run.

Related Literature. This paper relates and contributes to several strands of literature. In terms of non-linearities in the entrepreneurship-education nexus emerging from a general equilibrium framework, not much have been done in the macroeconomics and macro-labor literature. One noteworthy exception is the work of Poschke (2013), which explicitly considers how self-selection into entrepreneurship can arise predominantly from the bottom and top of the ability distribution, using a labor market search structure embedded in Roy-type model of occupational choice with two-dimensional heterogeneity (general productive ability and firm-level productivity). The study focuses primarily on explaining the aforementioned together with additional facts akin to firm entry/exit snd entrepreneurial returns; however, it abstains from exploring any implications for macro-development and cross-country output/TFP differences, which is the central focus of the present paper. In addition, the author relies on a modeling framework that is quite different to the approach I employ; does not consider endogenous human capital formation and its interactions with occupational choices; and many of his main results depend crucially on the assumption that people with higher ability are endowed with productivity draws from first-order stochastic dominant distributions, whereas a similar equilibrium result about effective productivities arises endogenously in my model through technology choices and without presupposing any distributional dependence at the population level. See also Poschke (2018) for some related results.

In regard to the dynamic general equilibrium framework I contribute to, the model expands on a number of antecedents. The approach I follow is inspired by the more recent dynamic literature on entrepreneurship and macroeconomics, spearheaded by the important contributions of Cagetti and De Nardi (2006), Banerjee and Moll (2010), Buera, Kaboski, and

Shin (2011), Buera and Shin (2013), Midrigan and Xu (2014), and Moll (2014), among others.[15] Relative to these papers, which are most often concerned with the impact of financial frictions on the macroeconomy and the wealth distribution, my elaborations feature a more general environment set in continuous time with three-dimensional heterogeneity, endogenous schooling choices/human capital formation, an intratemporal technology choice for entrepreneurs, as well as the new hypotheses discussed above concerning the role of entrepreneurial human capital.

There is a rather limited amount of research exploring the link between entrepreneurship and conventional human capital (in the Becker-Mincer view), and it is particularly distinct in terms of core research questions and methodologies. Below I will briefly draw attention to some notable studies that are partially related to this paper in certain aspects. The idea that entrepreneurial/managerial human capital—measured using formal education—may enter as a separate factor of production that raises firm-level productivity has been put forward by Gennaioli et al. (2013), integrated in their famous "Lucas-Lucas" spatial model, and shown to be an important determinant of regional development. Mestieri, Schauer, and Townsend (2017) build and quantify a detailed dynastic life-cycle heterogeneous-agent model to study how households' schooling investments and occupational choices impact each other inter-generationally and over time, with the analysis heavily involved in the persistent effects of market incompleteness and credit constraints.

Furthermore, Gomes and Kuehn (2017) and more recently Allub, Gomes, and Kuehn (2023) stress the aggregate consequences of occupational choices and skilled-unskilled labor under capital-skill complementarity. The authors demonstrate how enhancing the human capital of the labor force can lead to higher average firm size and total factor productivity, and how

[15] As this literature is too extensive to cover, please see the excellent surveys of Quadrini (2009) and Buera, Kaboski, and Shin (2015).

these underlying forces interact non-trivially with financial frictions. By complementing and extending this set of views, the present study argues that effective firm productivity is the combination of entrepreneurial ability and accumulated human capital coming together through costly technology adoption, and shifts the focus on endogenous aggregate TFP formation and on the importance of the entrepreneurship rate by educational attainment for macro-development.

The rest of the paper is outlined as follows. Section II provides a detailed exposition of the model economy along with the recursive formulation of its stationary competitive equilibrium. Section III presents a number of theoretical results and offers insights into how occupational and educational choices unfold. Upon obtaining numerical solutions, Section IV parametrizes and calibrates the model to the U.S. economy. Section V carries out a further quantitative exploration with emphasis on the implications of the entrepreneurship-education nexus for macro-development. Section VI briefly summarizes the study and offers concluding remarks and suggestions for future work.

2.2 An Economy with Occupational and Educational Choices

This section presents a model economy where heterogeneous agents make consumption, schooling, and occupational choices to maximize the expected present value of lifetime utility. Entrepreneurs/firms maximize profits and face an intratemporal technology choice that determines the effective productivity of an entrepreneur, thus output and profit functions. It starts with a brief overview of the economy's structure, followed by a more comprehensive exposition of its components in the next subsections.

2.2.1 Setup, Demographics, Information

The model describes a heterogeneous-agent economy with a simple overlapping- generations structure, set in continuous time under the usual conditions: a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ equipped with a right-continuous filtration $\{\mathcal{F}_t\}$ under common knowledge. The sample space is the fixed set $\Omega = (\mathcal{A} \times \mathcal{Z} \times \mathcal{H}) \subset \mathbb{R}^3_+$, generated by the joint support of the three state variables discussed below. Admissible controls are chosen from the space of square-integrable $\mathcal{F}_t$-adapted processes $\mathcal{L}^2(\Omega, \mathcal{F}, \mathbb{P})$.

Heterogeneity. Agents in the continuum are heterogeneous in terms of *assets*, $a \in \mathcal{A}$, *entrepreneurial ability*, $z \in \mathcal{Z}$, and *human capital*, $h \in \mathcal{H}$, upon entering the labor force, thereby facing a series of occupational choices between *worker* and *entrepreneur* roles.[16] Prior to that, they differ only in terms of *learning affinity*, $\beta \in \mathcal{B}$, and form their human capital by choosing the amount of time to invest in formal schooling, $S(\beta) \in \mathcal{S}$.[17]

Demographics. I adopt a subtle variation of the Blanchard-Yaari "perpetual youth" framework as in Blanchard (1985). Individuals face a constant probability of death throughout their life, $\eta > 0$, hence lifetime is a finite *a.s.* exponential random variable with expectation $1/\eta$. Assume that a measure η of both learner and worker/producer populations perishes

[16] We can interpret entrepreneurial talent (z) with a wide perspective: the efficiency in combining factors of production, the quality of business ideas and management, or the ability to market the consumption good. Human capital (h) in this context is to be viewed more narrowly as part of the Becker-Mincer view: a unidimensional measure of the benefits of formal education embodied in a person—abstracting from health, on-the-job training, etc. In other words, a set of (inalienable) acquired productive skills that are valued in the labor market and can be used to generate earnings.

[17] Heterogeneity in learning affinity (β) encapsulates differences in flow utility from education: the sustained task of going to school, studying, and learning may be burdensome for some, while less arduous and enjoyable for others. The idea that this 'consumption" component of schooling is a notable element of the value of education dates at least back to Schultz (1963), who argues: "Schooling can contribute satisfactions either in the present (for example, immediate enjoyment of association with one's college fellows), or in the future (increased capacity to enjoy good books)." See also Bils and Klenow (2000).

per unit of time . At every instant a new cohort of the same measure is born, so that the size of the labor force is normalized to $\int_{-\infty}^{t} \eta e^{-\eta(t-\tau)} d\tau = 1$.[18]

Information structure and timing of decisions. Each member of a cohort born at time v is endowed with a random draw of learning affinity (β) from a non-singular invariant distribution, $G_\beta(\beta)$, and proceeds by choosing length of schooling, S. In what follows I retain two simplifying assumptions. First, all learners consume a constant amount $\bar{c} > 0$ of a single good during the time spent in school (normalize $u(\bar{c}) = 0$), so that no intertemporal decisions are made $\forall t \leq v+S$. Second, the initial state (a, z) is not observed until $t = v+S$, when each labor force participant starts with a random draw from the (endogenous) conditional distribution $G_t(a, z|h)$. Educational decisions are effectively made behind a "veil of ignorance" about one's starting point after leaving school, hence agents form rational expectations about the evolution of $G_t(a, z|h)$ and prevailing factor prices. Alongside dimensionality reduction and spareness, these conditions highlight the relevance of the model without presupposing any dependence between entrepreneurial talent and human capital *ex-ante at the population level*; see below for more details.

At every $t \geq v + S$ individuals are in the labor force, supply their time inelastically to the market, and face a dichotomous occupational choice. They decide between becoming workers/employees who earn labor income contingent on their skills (wh), or entrepreneurs/producers who run firms and earn profits ($\pi(h, a, z)$). All work/production occurs within each time interval, at the end of which consumption decisions are made.

[18] Normalizing the size of the working instead of the total population comes with technical advantages, as we don't need to seek alternative normalizing constants.

2.2.2 State Variables, Controls, Preferences

Endogenous states. The only relevant state during the time spent in school is (constant) individual learning affinity. Post-schooling, each agent observes two endogenous state variables: her level of wealth (a), which is optimally determined by forward-looking behavior; and her (constant) level of human capital, which is determined by

$$h = e^{\phi(S(\beta))},\ \forall t \geq v + S \tag{2.1}$$

In the vein of Hall and Jones (1999) and Bils and Klenow (2000), the function $\phi(S)$ embodies the relative efficiency of quality-adjusted labor between S and zero years of schooling, with $\phi'(S) > 0$ capturing Mincerian returns to education.

Exogenous state. Entrepreneurial ability is subject to exogenous idiosyncratic shocks and evolves stochastically according to some Itô process

$$dz_t = \widetilde{\mu}_z(z,t)\,dt + \widetilde{\sigma}_z(z,t)\,dW_t,\ \forall t \geq v + S \tag{2.2}$$

where the drift and diffusion functions are globally Lipschitz-continuous. I consider processes that admit non-singular invariant measures. In addition, due to technological reasons, every sample path of z_t is reflected both above and below.[19] Note the lack of dependence between z_t and $h(S)$ *at the population level*, even for $t \geq v + S$, as the increments of the Wiener process (W_t) are independent across time and states.[20]

[19] A necessary and sufficient condition for the existence of an invariant probability measure μ_x is $\int_{\mathbb{R}} \mathcal{A}_x f(x)\mu_x(dx) = 0$, for all càdlàg $f(.)$ in the domain of the infinitesimal generator $\mathcal{A}_x$. The natural assumption of reflecting barriers gives rise to two Neumann boundary conditions (NBC); see (2.12) below.

[20] This does not imply independence or uncorrelatedness *within* each occupational group in equilibrium. Even with frictionless capital markets, ability (z) cutoffs still depend on human capital, thus joint distributions for entrepreneurs (workers) will turn out to have some dependency structure.

Controls. The control variables are instantaneous consumption and occupational choice. Regarding the latter, I avoid unnecessary complexity by abstracting from costs of switching between occupations, as well as from start-up or adjustment costs for the operation of production technologies. When it comes to entry (exit) into (from) entrepreneurship, the only cost is foregone wage income (profits). In conjunction with the model structure, this implies that maximization between worker/entrepreneur value functions is equivalent to maximization between labor income/entrepreneurial profits.[21]

Preferences. I assume common CRRA preferences over utility flows from enjoying a homogeneous consumption good, $u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$, discounted at the constant rate $\rho > 0$.

In sum, agents botn in time v seek to maximize the expected present value of their lifetime utility over admissible controls $S(\beta), c_t(h,a,z) \in \mathcal{L}^2(\Omega, \mathcal{F}, \mathbb{P})$

$$\max_{S,c_t} \int_v^{v+S} e^{-(\rho+\eta)(t-v)}\, \psi(\beta)\, dt + \mathbb{E}_v \left[\int_{v+S}^{\infty} e^{-(\rho+\eta)(t-v)}\, u(c_t)\, dt \; |\mathcal{F}_v \right] \tag{2.3}$$

$$\text{s.t.} \quad da_t = (\mathcal{Y}_t + (r_t + \eta)a_t - c_t)dt, \; a_t \geq 0, \; \forall t \geq v + S \tag{2.4}$$

$$\mathcal{Y}_t(h,a,z) = \max\Big\{ \widetilde{\pi}_t(h,a,z), w_t h \Big\}, \forall t \geq v + S, \text{ together with } (2.1), (2.2) \tag{2.5}$$

where (common) expectations are formed conditional on the information set at time v. The function $\psi(.)$ parametrizes flow utility from attending school and for simplicity it depends only on learning affinity (β). As explained above, occupational choice is fully embedded in (2.5), i.e., $\max\{V_{\mathcal{W}}(h,a,z), V_{\mathcal{E}}(h,a,z)\} \iff \max\{\widetilde{\pi}_t(h,a,z), w_t h\}, \forall t$.

[21] This result can be easily proved as a "separation-type theorem" and renders the solution tractable, as occupational choice is only a control and not a state variable in the next period.

2.2.3 Factor Markets

Financial market. Physical capital is the only productive asset, depreciating at the rate δ. There exists a large number of competitive financial intermediaries that receive deposits from savers and create capital loans for firms. There are no state-contingent securities, so the market is fully liquid but incomplete. Agents have access solely to safe deposits, accruing real interest at the rate r_t. Credit transactions are settled within each period and everyone faces a common *borrowing constraint*: $a_t \geq 0, \forall t$.[22] The equilibrium interest rate (r_t) is determined endogenously through the supply of assets from savers and the demand for capital by entrepreneurs. Free entry and the zero-profit condition for intermediaries implies a rental rate equal to the user cost of capital $(r_t + \delta)$.

Insurance market. Individuals in the model operate under no intergenerational altruism, so unforeseen bequests may occur because of random death. To circumvent such issues I follow Blanchard (1985) and assume the existence of a large number of competitive life insurance companies. Under actuarially fair pricing, each agent buys an annuity contract that pays a flow ηa_t throughout their lifetime, with the insurance company assuming ownership of any remaining assets upon the agent's passing.

Collateral constraint. Producers can finance their capital expenditures (k_t) using either internal funds (a_t), or external financing from intermediaries subject to frictions. Due to limited contract enforceability entrepreneurs face a common *collateral constraint*, restricting their maximum debt position (d_t) to a fraction of their hired capital. Specifically, $d_t \leq \vartheta k_t$, and since $d_t = \max\{k_t - a_t, 0\}$, the constraint becomes

$$k_t \leq \frac{1}{1-\vartheta} a_t, \quad \vartheta \in [0, 1] \tag{2.6}$$

[22] The borrowing constraint gives rise to a state constraint boundary condition (SCBC); see (2.11) below.

The parameter ϑ captures the degree of financial frictions in the economy, where $\vartheta = 0$ corresponds to financial autarky, and $\vartheta \to 1$ results in a frictionless capital market.[23]

Labor market. The market for human capital is perfectly competitive. Workers supply labor to firms inelastically, denominated in units of human capital. Under perfectly substitutability of inputs, employees receive labor income equal to $w_t h_t$. The equilibrium effective wage rate, w_t, is determined endogenously through the supply of human capital by workers and the demand for labor by entrepreneurs. This setup captures the common assumption of efficiency units of labor, i.e., there is no assortative matching.

2.2.4 Entrepreneurs and Technology

Entrepreneurs behave competitively in product and factor markets, hiring k units of physical capital and ℓ units of human capital to maximize per-period profits. Revenue is generated via an individual-specific technology that turns capital and labor into the homogeneous consumption good (numéraire). Production takes place intraperiod, during which individual states are known and fixed, so there can be no default.

Given a wage rate per unit of human capital (w) and a rental rate of capital (r), an entrepreneur's indirect profit function is

$$\widetilde{\pi}(h_e, a, z|w, r) = \max_{k,\ell \geq 0} \; y(h_e, a, z) - w\ell - (r + \delta)k, \text{ s.t. (2.6)} \tag{2.7}$$

[23] This form of static collateral constraint is quite standard in the relevant literature; e.g., Evans and Jovanovic (1989), Banerjee and Moll (2010), Buera and Shin (2013), and Midrigan and Xu (2014). It can be derived from a simple limited enforcement problem. Upon entering a contract, entrepreneurs can renege on their obligations and embezzle their full debt position; in retaliation, lenders can seize a fraction ϑ of the firm's hired capital. In a zero-profit equilibrium, intermediaries will lend only up to the amount they can recover, hence $d_t \leq \vartheta k_t$. As Moll (2014) has showed, the collateral constraint can take more general forms without necessarily affecting the core results, as long as the constraint is linear in wealth.

I expand upon Lucas (1978) and consider the production technology

$$y(h_e, a, z) = \zeta(h_e, z)\left(k^{\alpha}\ell^{1-\alpha}\right)^{1-\nu} \tag{2.8}$$

where $h_e \coloneqq h_i \cdot \mathbb{1}_{\mathcal{E}}$ denotes human capital of entrepreneur i. ν is the span-of-control parameter inducing diminishing returns to scale as a firm grows, since it becomes increasingly difficult for the manager to exert control over production plans. The input of entrepreneurship is rival, excludable, and necessary for production.[24]

Effective productivity and technology adoption. The nature of the function $\zeta(z, h_e)$ is consequential. I hypothesize the existence of a *disembodied technology* that coalesces an entrepreneur's talent (z) and human capital (h_e) to form the fundamental "effective entrepreneurial productivity." This is of ultimate use in the production process. Moreover, it is assumed that $\zeta(z, h_e)$ exhibits positive complementarity between z and h_e, i.e., the theorized link function is log-supermodular.

Active entrepreneurs face an intratemporal *technology adoption choice* in the beginning of every period. Reaping the underlying benefits incurs some costs due to the complexity conjoined with adopting the technology. To keep things uncomplicated while retaining interesting results, I propose the following specification:

$$\zeta(h_e, z) \coloneqq \begin{cases} z & \text{at no cost} \\ z h_e^{\omega} & \text{at cost } \kappa \cdot y(h_e, a, z),\ \kappa \in (0,1) \end{cases} \tag{2.9}$$

[24] Several papers posit the additional assumption of *indivisibility*, but there is no reason for such imposition. In settings where occupational choices come down to comparing earned income, if we allow individuals to work both as workers and entrepreneurs the optimal choice will always be a corner solution, i.e., they would devote their whole time endowment to one occupation.

Upon adoption, the form of $\zeta(z, h_e)$ is quite familiar to economists as a baseline assumption: an isoelastic function that is uniformly continuous on any convex set in $\mathbb{R}^2_+$.

In principle, one can allow the cost function to take more general forms. I would, however, argue against a fixed adoption cost in this setting. The technological expansion of a firm is naturally associated with a variety of expenses, ranging from higher infrastructure, legal, accounting, and incorporation costs, to the need for enhancing management practices and keeping up with increased competition. I also embrace the assumption of a proportionally constant tradeoff denominated in common units of output for at least two reasons. First, it leads to more transparent analytical solutions; second, it results in better identification of the parameter κ because of a global minimum property of the entrepreneurship rate by human capital; see Proposition 1.

The complementarity parameter ω (elasticity) is of primary interest, and importantly, it is not presumed to be constant across economies or over time. In fact, one of the main messages of the paper is that differences in TFP and output per worker across countries are driven by differences in the complementarity between entrepreneurial talent and human capital, which also determines variations in the ERGON index.

Intuition and related concepts. A few remarks are in order regarding the hypothesis I introduce and how it relates to certain established concepts in the literature. I postulate that entrepreneurial human capital may serve as an additional factor of production under some constraints. This standpoint is an enrichment of the classic papers by Lucas (1978) and Rosen (1982), and most of their epigones. The central assumption in these studies is that some entrepreneurs/firms are more efficient than others at all levels of output, due to some unobserved aspect such as "talent for managing". I argue that this aspect is also related to the accumulated (general-purpose) human capital of the entrepreneur, which can become a source of competitive advantage for the firm.

To gain further intuition, one can interpret the disembodied technology and the effect of technology adoption as another perspective on the theory of *organization capital*, a form of firm-specific intangible capital. The inceptive paper of Prescott and Visscher (1980) draws attention to the importance of firms expanding their stock of information in order to broaden their production possibilities. Atkeson and Kehoe (2005) concentrate on manufacturing plants and regard organization capital as a function of plant-specific productivity and age, the accumulation of which yields substantial rents to the firm owner. This paper advocates the idea that more educated individuals may choose to complement their entrepreneurial talent with the skills and competences that human capital entails, thereby enhancing the organization capital of the firm they own and run. This view can also be linked to even more specific subcomponents of intangible capital such as *managerial capital*; see Bruhn, Karlan, and Schoar (2010).

2.2.5 Recursive Formulation and Stationary Competitive Equilibrium

Let $V(h, a, z) = \mathbb{E}\left[\int_{v+S}^{\infty} e^{-(\rho+\eta)(t-v)} u(c_t)\, dt\right]$ be the value function of an agent in the labor force with state vector (h, a, z). Henceforth I restrict my attention to stationary competitive equilibria, i.e., $\partial V/\partial t = 0$. The optimal control problem admits a recursive representation, with solutions characterized by a Hamilton-Jacobi-Bellman equation together with a set of boundary conditions over the state space:

$$
\begin{aligned}
(\rho+\eta)V(h,a,z) &= \max_{c\in\mathcal{C}} u(c) + V_a(h,a,z)\left(\mathcal{Y}(h,a,z)) + (r+\eta)a - c\right) \\
&\qquad + V_z(h,a,z)\widetilde{\mu}_z(z) + \frac{1}{2}V_{zz}\,\widetilde{\sigma}_z^2(z) \qquad (2.10)\\
\rho V(h,a,z) &= \max_{c\in\mathcal{C}} u(c) + \mathcal{A}\,V(h,a,z) \qquad (\overline{11})\\
V_a(h,\underline{a},z) &\geq u'\left(\mathcal{Y}(h,\underline{a},z)\right) \quad \forall z,h \quad (\text{SCBC for } \underline{a}=0) \qquad (2.11)\\
V_z(h,a,\underline{z}) &= V_z(h,a,\bar{z}) = 0 \quad \forall a,h \quad (\text{NBC for } \underline{z} \text{ and } \bar{z}) \qquad (2.12)\\
\lim_{t\to\infty} e^{-\rho t} a_t\, V_a(h,a,z) &= 0 \quad \forall h,a,z \quad (\text{TVC}) \qquad (2.13)
\end{aligned}
$$

The aggregate state of the economy is fully captured by the non-singular endogenous joint distribution $G_t(h,a,z)$[25], with corresponding density $g(h,a,z)$, which satisfies the stationary Kolmogorov Forward (or Fokker-Planck) equation:

$$
\begin{aligned}
0 &= -\frac{\partial}{\partial a}\left[\widetilde{d}(h,a,z)\,g(h,a,z)\right] - \frac{\partial}{\partial z}\left[\widetilde{\mu}_z(z)\,g(h,a,z)\right] + \frac{1}{2}\frac{\partial^2}{\partial z^2}\left[\widetilde{\sigma}_z^2(z)\,g(h,a,z)\right] \\
&\quad - \eta g(h,a,z) + \eta \iint_{\mathcal{A}\,\mathcal{Z}} g_h(h)\,g(a,z|h)\,da dz \qquad (2.14)\\
&= \mathcal{B}g(h,a,z) + \eta g_h(h) \qquad (\overline{15})\\
g_h(h) &= g_\beta\big(\upsilon^{-1}(h)\big)\left|\frac{d\upsilon^{-1}(h)}{dh}\right|,\ h := \upsilon(\beta) = e^{\phi(S(\beta))} \qquad (2.15)
\end{aligned}
$$

[25] By virtue of structure and the absence of aggregate shocks, a weak law of large numbers applies and the dynamics of the state distribution are deterministic. To avoid potential measurability problems one can appeal to the WLLN theorems of Uhlig (1996). Frequency and probability distributions coincide on subsets of positive measure, thus multiple integration over Ω yields aggregate quantities. The σ-finiteness of spaces and the measurability of all functions in the present setting allows to switch from multiple to repeated integration by the Fubini-Tonelli theorem.

together with the boundary condition,

$$0 = -\widetilde{d}\, g\Big|_{\mathcal{A}} - \left(\widetilde{\mu}_z - \frac{1}{2}\frac{\partial}{\partial z}\widetilde{\sigma}_z^2\right) g\,\Big|_{\mathcal{Z}} \tag{2.16}$$

The marginal distribution of human capital, $g_h(h)$, is obtained via a composite Jacobian transformation using the single-valued inverse for h. The term $\widetilde{d}(h, a, z) := (\mathcal{Y}(h, a, z) + (r + \eta)a - \widetilde{c}(h, a, z))$ denotes optimal savings (optimal control drift). Note that the additional boundary condition (2.16) ensures the adjointness of the KFE differential operator $\mathcal{B}$ to the HJB infinitesimal generator $\mathcal{A}$; see Appendix B. That is, $\mathcal{B} = \mathcal{A}^*$.

A **stationary recursive competitive equilibrium** consists of a value function $V(h, a, z) : \Omega \mapsto \mathbb{R}$; $\mathcal{F}_t$-adapted policy functions $\widetilde{c}(h, a, z)$, $\widetilde{d}(h, a, z)$, $\widetilde{\ell}(h, a, z)$, $\widetilde{k}(h, a, z)$, $\mathcal{Y}(h, a, z) : \Omega \mapsto \mathbb{R}_+$ and $S(\beta) : \mathcal{B} \mapsto \mathbb{R}_+$; factor prices (w, r); and a non-singular joint distribution function $G(h, a, z)$: $(\Omega, \mathcal{F}) \mapsto [0, 1]$, such that,

1. Given prices, the value function and policy functions solve the optimal control problem (2.10) with boundary conditions (2.11)–(2.13);
2. Given prices, entrepreneurs/firms maximize profits with factor demand functions given by (2.19) and (2.20);

3. All markets clear;

$$\iiint\limits_{\mathcal{H}\,\mathcal{A}\,\mathcal{Z}} a\; dG(h,a,z) = \iiint\limits_{\mathcal{H}_{\mathcal{E}}\mathcal{A}_{\mathcal{E}}\mathcal{Z}_{\mathcal{E}}} \widetilde{k}\; dG(h,a,z) \qquad \textit{(physical capital/assets)}$$

$$\iiint\limits_{\mathcal{H}_{\mathcal{W}}\mathcal{A}_{\mathcal{W}}\mathcal{Z}_{\mathcal{W}}} h\; dG(h,a,z) = \iiint\limits_{\mathcal{H}_{\mathcal{E}}\mathcal{A}_{\mathcal{E}}\mathcal{Z}_{\mathcal{E}}} \widetilde{\ell}\; dG(h,a,z) \qquad \textit{(human capital/labor)}$$

$$\iiint\limits_{\mathcal{H}_{\mathcal{E}}\mathcal{A}_{\mathcal{E}}\mathcal{Z}_{\mathcal{E}}} \widetilde{y}\; dG(h,a,z) = \iiint\limits_{\mathcal{H}\,\mathcal{A}\,\mathcal{Z}} \widetilde{c}\; dG(h,a,z) + \delta K(r,w) \qquad \textit{(net goods)}$$

4. The joint density of human capital, wealth, and entrepreneurial ability, $g(h,a,z)$, satisfies the Kolmogorov-Forward equation (2.14) with boundary condition (2.16).

where $\mathcal{X}_{\mathcal{E}} := \{x_i \in \mathcal{X} : i \text{ is an entrepreneur}\} \subset \mathcal{X};\; \mathcal{X}_{\mathcal{W}} := \Omega \backslash \mathcal{X}_{\mathcal{E}}, \forall \mathcal{X} = \{\mathcal{H}, \mathcal{A}, \mathcal{Z}\}$.

Obtaining an equilibrium amounts to solving a pair of coupled, nonlinear, second-order partial differential equations (HJB and KFE). Classical PDE solutions are not guaranteed to hold in this setting, as the non-convexities induced by occupational and technology choices give rise to convex kinks. The appropriate solution method requires the powerful theory of constrained *viscosity solutions*; see Crandall and Lions (1983).

The assumption of a continuum of non-atomic agents that face idiosyncratic—but not aggregate shocks—and interact strategically only through the incentives actuated by a common set of prices (w,r) is a typical case of what Lasry and Lions (2007) call a *Mean Field Game* without common noise.

2.3 Theoretical Findings

This section delineates some theoretical predictions of our framework and offers insights into how the entrepreneurship-human capital nexus shapes aspects of macro-development. We begin with some useful results on entrepreneurial factor demands and technology adoption.

LEMMA 1: *Net firm output can be expressed as*:

$$\widetilde{y} = z\tilde{h}\left(\widetilde{k}^{\alpha}\,\widetilde{\ell}^{\,1-\alpha}\right)^{1-\nu} \tag{2.17}$$

$$\tilde{h} := \underset{h\leq\hbar}{\mathbb{1}} + \underset{h>\hbar}{\mathbb{1}}\left(\frac{h_e}{\hbar}\right)^{\omega} \tag{2.18}$$

where, given any joint distribution, $\exists\,\hbar(\kappa,\omega) = \left(\frac{1}{1-\kappa}\right)^{1/\omega}$ *such that entrepreneurs choose to adopt the disembodied technology if and only if* $h \geq \hbar$.

Optimal factor demands depend on individual state vectors (h_e, a, z) *and are given by:*

$$\widetilde{\ell}(h_e,a,z) = (z\widetilde{h})^{\frac{1}{\nu}}\left(\frac{\alpha(1-\nu)}{\widehat{r}_t+\delta}\right)^{\frac{\alpha(1-\nu)}{\nu}}\left(\frac{(1-\alpha)(1-\nu)}{w_t}\right)^{\frac{1-\alpha(1-\nu)}{\nu}} \tag{2.19}$$

$$\widetilde{k}(h_e,a,z) = (z\widetilde{h})^{\frac{1}{\nu}}\left(\frac{\alpha(1-\nu)}{\widehat{r}_t+\delta}\right)^{\frac{1-(1-\alpha)(1-\nu)}{\nu}}\left(\frac{(1-\alpha)(1-\nu)}{w_t}\right)^{\frac{(1-\alpha)(1-\nu)}{\nu}} \tag{2.20}$$

The entrepreneur-specific shadow interest rate is defined as $\widetilde{r}_t(h_e,a,z) = r_t + \lambda(h_e,a,z)$ *where* $\lambda(.) \geq 0$ *is the Lagrange multiplier on the collateral constraint, and corresponds to:*

$$\widetilde{r}_t(h_e,a,z) = \begin{cases} r_t \;\text{ if }\; a > (1-\vartheta)\,(z\widetilde{h})^{\frac{1}{\nu}}\left(\dfrac{\alpha(1-\nu)}{r_t+\delta}\right)^{\frac{1-(1-\alpha)(1-\nu)}{\nu}}\left(\dfrac{(1-\alpha)(1-\nu)}{w_t}\right)^{\frac{(1-\alpha)(1-\nu)}{\nu}} \\ \left(\dfrac{(z\widetilde{h})^{\frac{1}{\nu}}}{\frac{1}{1-\vartheta}a}\right)^{\frac{\nu}{1-(1-\alpha)(1-\nu)}}\left(\dfrac{(1-\alpha)(1-\nu)}{w_t}\right)^{\frac{(1-\alpha)(1-\nu)}{1-(1-\alpha)(1-\nu)}}\alpha(1-\nu)-\delta \quad \text{otherwise} \end{cases} \tag{2.21}$$

The ensuing technology choice has an intuitive interpretation: for low levels of human capital, individuals choose to fully rely on their entrepreneurial ability in the production process, whereas more educated individuals will choose to complement their idiosyncratic talent with the benefits stemming from their accumulated human capital. *Ceteris paribus*, "more productive entrepreneur = more talented or more educated". In equilibrium, $\hbar$ is the threshold level above which human capital enhances the agent's productivity—a *local threshold externality* inducing a non-convexity.

In the presence of a certain wealth heterogeneity in the economy, the second part of Lemma 1 establishes that collateral constraints will alter the production scale and profits of some entrepreneurs. This shows up in the *shadow cost of funds*[26] each capital-constrained producer is facing, i.e., through the size of the multiplier $\lambda(h_e, a, z)$. As the demand for capital and labor is strictly increasing in z and $\widetilde{h}$, agents on the higher end of both distributions are being disproportionately affected. The dispersion of marginal products thus creates a persistent case of capital misallocation on the intensive margin.

2.3.1 Occupational Choices and the Entrepreneurship-Human Capital Nexus

As in many studies in the literature, an object of interest is the productivity cutoff above which individuals decide to become entrepreneurs. In contrast to most studies, this threshold is neither unique for all agents, nor does it hinge on wealth alone, but also depends on the level of human capital in a non-linear way. The unique cutoff for each realization of the joint vector is determined by a three-dimensional fictitious marginal agent that is indifferent between becoming an entrepreneur or a wage worker. A first finding is summarized below.

[26] This is the terminology that Midrigan and Xu (2014) use and their work contains similar results.

LEMMA 2: *(a) The entrepreneurial ability threshold for unconstrained agents ($\widehat{r} = r$) is independent of assets and given by*

$$\underline{z}(h_e) = \begin{cases} h_e^{\nu} \left(\frac{w}{\nu}\right)^{\nu} \left[(1-\nu)\left(\frac{\alpha}{r+\delta}\right)^{\alpha}\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\right]^{\nu-1} & \forall h_e \leq \hbar \\ h_e^{\nu-\omega}\,\hbar^{\omega} \left(\frac{w}{\nu}\right)^{\nu} \left[(1-\nu)\left(\frac{\alpha}{r+\delta}\right)^{\alpha}\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\right]^{\nu-1} & \forall h_e > \hbar \end{cases} \tag{2.22}$$

Therefore, $\underline{z}(h_e)$ is strictly increasing in human capital $\forall\, h_e \leq \hbar$, and under the condition $\omega > \nu$, it is strictly decreasing $\forall h_e > \hbar$.

(b) The entrepreneurial ability threshold for each constrained agent i depends on her assets, $a_i < (1-\vartheta)k(a_i, z, h_e)$, and is given by

$$\underline{z}^c(h_e, a_i) = \begin{cases} h_e^{1-\upsilon} \left(\frac{w}{1-\upsilon}\right)^{\frac{1}{1-\upsilon}} \left[\left(\frac{a_i}{1-\vartheta}\right)^{\widehat{\alpha}}\left(\frac{\upsilon}{w}\right)^{\upsilon} - \frac{a_i(r+\delta)}{1-\vartheta}\right]^{\upsilon-1} & \forall h_e < \hbar \\ \frac{h_e^{1-\upsilon-\omega}}{\hbar^{-\omega}} \left(\frac{w}{1-\upsilon}\right)^{\frac{1}{1-\upsilon}} \left[\left(\frac{a_i}{1-\vartheta}\right)^{\widehat{\alpha}}\left(\frac{\upsilon}{w}\right)^{\upsilon} - \frac{a_i(r+\delta)}{1-\vartheta}\right]^{\upsilon-1} & \forall h_e \geq \hbar \end{cases} \tag{2.23}$$

$\upsilon := (1-\alpha)(1-\nu), \widehat{\alpha} := \alpha(1-\nu)$. *Therefore, $\underline{z}^c(h_e, a)$ is strictly increasing in human capital $\forall\, h_e \leq \hbar$, and under the condition $\omega > \alpha + \nu - \alpha\nu$, it is strictly decreasing $\forall h_e > \hbar$.*

Lemma 2 demonstrates that occupational choices depend on human capital non-monotonically, under a necessary and sufficient condition. Considering individuals with human capital less than $\hbar$, breaking even requires draws from a progressively higher part of the entrepreneurial talent distribution. In contrast, as long as the complementarity parameter ω is not too low, higher human capital when $h > \hbar$ makes entry into entrepreneurship progressively easier (in terms of z-cutoffs) since profits grow faster than labor income in that direction of h.

The results also show that the tightness of the collateral constraint will impact occupational choices and the allocation of talent for individuals who would be constrained should they decided to become entrepreneurs. For a given wealth level, tighter borrowing conditions make it more difficult for profits to match foregone labor income, inducing a higher cutoff for z. Combining the above leads to a central theoretical result of the paper, namely, the behavior of the entrepreneurship rate by educational attainment.

PROPOSITION 1: *For any sequence of equilibrium prices* $\{w_t, r_t\}_{t\in\mathbb{R}_+}$*, joint distributions* $\{G_t(h,a,z)\}_{t\in\mathbb{R}_+}$*, and collateral constraint* $\vartheta\in[0,1]$*, there exists a non-increasing measurable function* $\chi(\vartheta):[0,1]\mapsto[\nu,\alpha+\nu-\alpha\nu]$*, such that the entrepreneurship rate by human capital is:*

- *strictly decreasing* $\forall h\in\mathcal{H}$ *almost surely, if and only if* $\omega<\chi(\vartheta)$*;*
- *strictly decreasing* $\forall h<\hbar(\kappa,\omega)$ *and strictly increasing* $\forall h\geq\hbar(\kappa,\omega)$ *almost surely, if and only if* $\omega>\chi(\vartheta)$*;*
- *strictly decreasing* $\forall h<\hbar(\kappa,\omega)$ *almost surely, and anything goes* $\forall h\geq\hbar(\kappa,\omega)$*, if and only if* $\nu<\omega\leq\chi(\vartheta)$*.*

where $\hbar=\left(\frac{1}{1-\kappa}\right)^{1/\omega}$*.*

The conclusions of Proposition 1 are stark, flexible, and in line with the empirical findings under minimal restrictive assumptions. The pivotal condition bears on the complementarity parameter in tandem with financial frictions; it requires ω to be larger than an endogenous cutoff value that is non-decreasing in ϑ, meaning that tighter financial markets dictate a larger ω to achieve entrepreneurship rates that are increasing somewhere in the support of human capital. These results hold under *any* joint distribution for (z, h) that is non-flat almost everywhere in its support. The steepness and curvature of this relationship will obviously depend on the magnitude of parameters, but the main consequences are nonetheless clear-cut. For example, the probability of being an active entrepreneur in this economy will arise as asymmetric U-shaped in the second case of Proposition 1, with the lowest probability occurring right on the threshold $\hbar(\kappa, \omega)$.

This global minimum property offers not only a sharp theoretical prediction but also an empirical advantage. Given a mapping between years of schooling and human capital, $\hbar$ is an identifiable parameter determined by observables—either directly from descriptive data or through a reduced-form model—that can be calibrated or estimated using standard structural methods. This makes the model particularly attractive to use with micro-level datasets as well as for cross-country analysis.

PROPOSITION 2: *For any sequence of equilibrium prices* $\{w_t, r_t\}_{t \in \mathbb{R}_+}$*, joint distributions* $\{G_t(h, a, z)\}_{t \in \mathbb{R}_+}$*, and collateral constraint* $\vartheta \in [0, 1]$*, the expected value of any entrepreneurial outcome* X *(production plans, profits, capital and labor demands) is strictly increasing in human capital with probability one. That is,* $\frac{\partial \mathbb{E}[X|\mathcal{E}, h_e]}{\partial h_e} > 0 \; \forall h \in \mathcal{H}$ *a.s..*

Proposition 2 establishes a prevalent result in the empirics of Section II (Fact 3): the central tendency of business outcomes is monotonically increasing in the educational attainment of the firm owner/entrepreneur. The result is guaranteed to hold irregardless of the value of ω, and the level of the cost κ is extraneous to the core prediction.

This occurs for two reasons. For $h < \hbar$, there is a self-selection effect: choosing entrepreneurship to wage work implies higher draws from the z-distribution as h increases. For $h \geq \hbar$, technological choices lead to a rightward shift of the effective productivity distribution; i.e., for the same level of z, the "more educated" distribution dominates the "less educated" one in the first-order stochastic dominance sense.

2.3.2 Educational Choices

To derive optimal schooling choices, it is straightforward to maximize (2.3) subject to the intertemporal (lifetime) budget constraint

$$\int_{v+S}^{\infty} e^{-R(S,t)} c_t \, dt \leq a_{v+S} + \int_{v+S}^{\infty} e^{-R(S,t)} \, \mathcal{Y}_t \, dt, \;\; R(S,t) := \int_{v+S}^{t} (r_\tau + \eta) d\tau \tag{2.24}$$

which holds almost surely, so it will also hold in expectation. The necessary first-order condition can be rearranged to yield

$$\frac{\psi(\beta)}{\mathbb{E}_v \left[u'(c_{v+S})\right]} + \mathbb{E}_v \left[\int_{v+S}^{\infty} e^{-R(S,t)} \frac{\partial}{\partial S} \mathcal{Y}(v+S) \, dt \right] = \frac{\mathbb{E}_v \left[u(c_{v+S})\right]}{\mathbb{E}_v \left[u'(c_{v+S})\right]} + \mathbb{E}_v[\widetilde{d}_{v+S}] \tag{2.25}$$

where rational expectations are taken at time v regarding the evolution of the joint distribution and prices. The left-hand side represents the expected marginal benefit of the S^{th} period of schooling (expressed in time $v+S$ units)—the sum of flow utility from attending school and the present value of future income gains. The right-hand side is the expected marginal cost of the S^{th} period of schooling (also in time $v+S$ units)—the sum of foregone utility from not joining the labor force and the opportunity cost of time spent in school, $\widetilde{d}_S = \mathcal{Y}_S + (r_S + \eta) a_S - c_S$. When expected marginal cost rises faster than marginal benefit, or vice versa, the above first-order condition is also sufficient. In a stationary equilibrium where prices are constant, the market discount factor becomes simply $R(S,t) = (r+\eta)(t-S)$ and heterogeneity in schooling choices reflect solely differences in learning aptitude (β).

The most interesting aspect is how prevailing macroeconomic conditions affect educational choices through prices and expected quantities. These are largely determined by occupational choices and the allocation of human capital between entrepreneurs and workers, we should thus expect to have rich interactions. For example one would like to assess the importance of complementarity between entrepreneurial talent and human capital, which has a direct impact on expected future income gains and consumption/savings. The quantitative analysis below reveals that the influence of ω is indeed central to promoting more investment in schooling.

2.3.3 Aggregate Output and Total Factor Productivity

The model admits an endogenous aggregate production function in the sense of a stable relationship among per-person-engaged net total output, aggregate human capital of workers, aggregate physical capital, and the rate of entrepreneurship. Total factor productivity appears in the form of a refined Solow-type residual, which depends directly on occupational, educational, and technology adoption choices.

PROPOSITION 3: *In equilibrium, the production side of the economy aggregates and net total output per person engaged can be expressed as*

$$Y = \underbrace{\frac{\left(\mathbb{E}\left[\left(z\widetilde{h}\right)^{\frac{1}{\nu}}\left(\widehat{r}(h,a,z)+\delta\right)^{-\frac{\widehat{\alpha}}{\nu}}\Big|\mathcal{E}\right]\right)^{\widehat{\alpha}+\nu}}{\left(\mathbb{E}\left[\left(z\widetilde{h}\right)^{\frac{1}{\nu}}\left(\widehat{r}(h,a,z)+\delta\right)^{-\frac{(\widehat{\alpha}+\nu)}{\nu}}\Big|\mathcal{E}\right]\right)^{\widehat{\alpha}}}}_{\text{Total Factor Productivity}}\ \mu(\mathcal{E})^{\nu}\left(K^{\alpha}H^{1-\alpha}\right)^{1-\nu} \tag{2.26}$$

where expectations are taken with respect to the conditional distribution $G(h, a, z|\mathcal{E})$, K is aggregate physical capital, H is aggregate human capital of workers, $\mu(\mathcal{E})$ is the measure of entrepreneurs in the labor force, and $\widehat{\alpha} := \alpha(1-\nu)$.

In the absence of financial frictions ($\vartheta \to 1$), net total output per person employed becomes

$$Y_* = \underbrace{\left(\mathbb{E}\left[\left(z\widetilde{h}\right)^{\frac{1}{\nu}} \middle| \mathcal{E}_*\right]\right)^{\nu}}_{\text{Total Factor Productivity}} \mu(\mathcal{E}_*)^{\nu}\left(K_*^{\alpha} H_*^{1-\alpha}\right)^{1-\nu} \tag{2.27}$$

Note that the adopted definition of TFP is closer to "true TFP" as opposed to conventionally measured counterparts. It is important to highlight this demarcation as, more often than not, studies include some function of the entrepreneurship rate in their definition of TFP. The measure $\mu(\mathcal{E})$ reflects number of people, which is a measurable factor of production in and of itself. This perspective is also shared by Hopenhayn (2014).

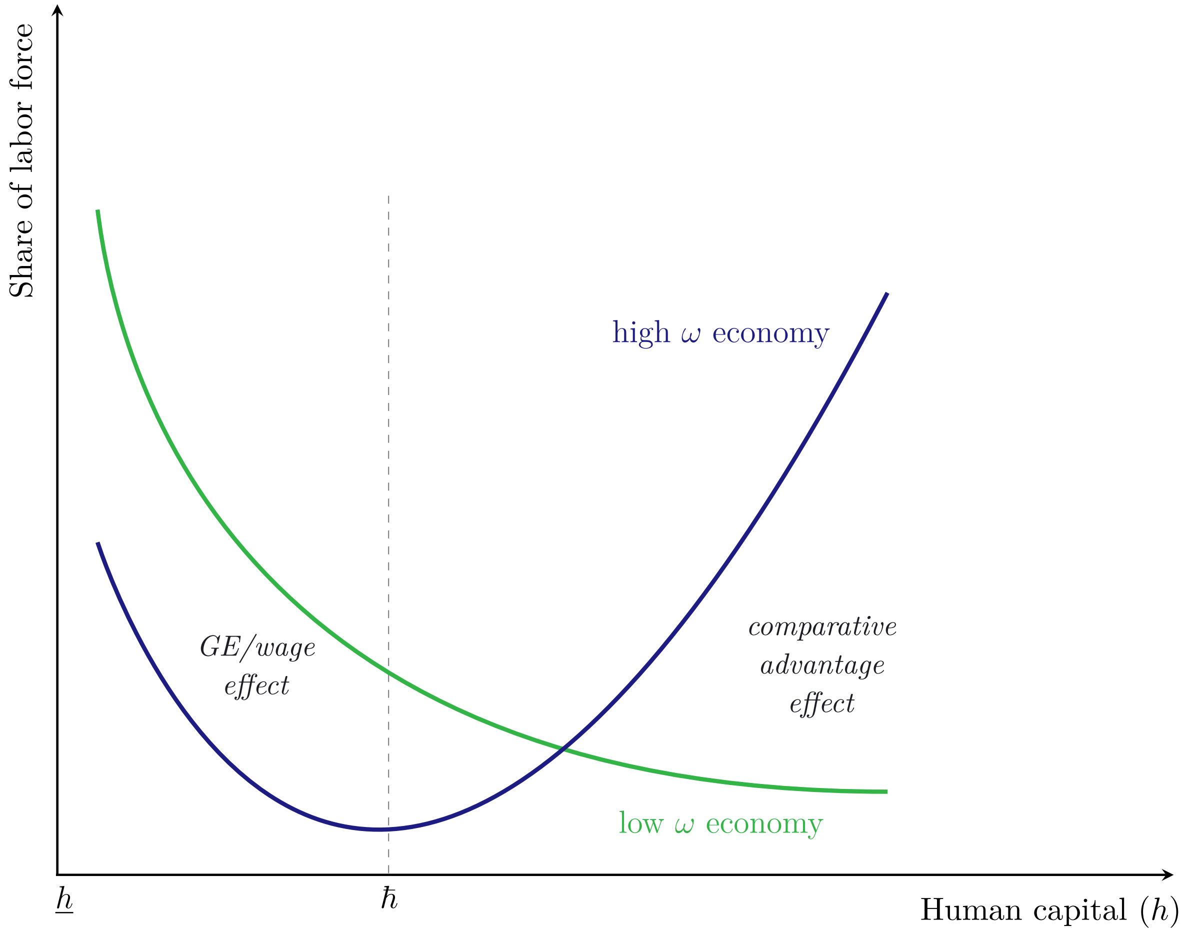
What are the implications for macro-development? Proposition 3 establishes that the contribution of entrepreneurial human capital ($\widetilde{h}$), and thus the allocation of human capital between entrepreneurs and workers, are paramount for long-run output and TFP differences. The key is the formation of individual effective productivities through technology adoption choices, which depends critically on the degree complementarity.

First, an economy with sufficiently high ω is able to permanently boost aggregate demand for physical and human capital beyond what its capabilities would dictate in the absence of technology adoption (low ω environment). Second, equilibrium aggregation leads to an endogenous TFP term that is a positive function of $z\widetilde{h}$: a Hölder mean with exponent $1/\nu$—also known as generalized mean—of entrepreneurs' effective productivities weighted by the appropriate conditional distribution. Such a high-ω economy is also characterized by higher entrepreneurship rates for higher levels of education, i.e., higher ERGON index. Net output is thus determined both by how many entrepreneurs are active in the economy (extensive margin/$\mu(\mathcal{E})$), and by what type of entrepreneurs they actually are (intensive-margin/TFP).

To gain more perspective on how the entrepreneurship-education nexus in linked to macro-development, consider two economies that have the same endowments, parameters, and are identical in every way apart from the prevailing degree of complementarity ω. Some illustrative guidance is provided by Figure 2.1. In the low-ω economy ($\omega < \chi(\theta)$) the entrepreneurship rate by educational attainment is (endogenously) strictly decreasing, and TFP is low due to the lack of technology adoption by firms. In the relatively high-ω economy ($\omega > \chi(\theta)$), e.g. the U.S., some active entrepreneurs adopt the disembodied technology, reinforce their effective productivities, and promote aggregate factor and TFP formation. I call this channel the *comparative advantage effect*. Furthermore, increased demand for workers pushes the equilibrium effective wage rate higher, which sorts out less productive firms and results in even more enhanced TFP. I call this channel the *wage/general equilibrium effect*. In theory, the corresponding fall in $\mu(\mathcal{E})$ could be large enough to counteract the positive impact of the wage effect, but in practice it is not so under realistic parameter restrictions.

Lastly, the following indication about existing frameworks is worth mentioning. Consider similar macro models with occupational choice that involve two distinct dimensions of productivity—entrepreneurial (z) and worker (h) ability—but do not allow for complementarities and do not impose any *ex-ante* statistical dependence. In such cases, the entrepreneurship rate by human capital will necessarily be strictly decreasing: higher labor income (wh) leads to a higher opportunity cost at each point in time, which leads to higher z-cutoffs that occur with lower probability.

Figure 2.1: Entrepreneurship Rate along h: High vs Low Complementarity



Notes: This illustrative diagram depicts the fraction of the labor force opting into entrepreneurship for each level of human capital h. The curve for the "high ω economy" is drawn for the case of $\omega > \boldsymbol{\omega}(\vartheta) \in [\nu, \alpha + \nu - \alpha\nu]$. The curve for the "low ω economy" is drawn for the case of $\omega < \boldsymbol{\omega}(\vartheta)$. The latter case also describes macro models that do not allow for complementarities human capital/labor ability and productivity/entrepreneurial outcomes, or do not impose any functional relationship between the distributions of z and h.

2.4 Calibrating the Model to U.S. Data

Solving the full model requires a PDE numerical scheme that yields unique viscosity solutions to the HJB and KF equations. To that end, I employ the implicit upwind finite difference method of Achdou et al. (2022), which is monotone, consistent, and stable in the Barles-Souganidis sense. All computational details are relegated to Appendix B.[27]

The model economy is calibrated to U.S. data for the period 1989 – 2019. The length of a period is taken to be one year to allow for internal consistency with aggregate, survey, and firm-level data. The calibration strategy is on the side of parsimony; I reduce the number of degrees of freedom first by preassigning values to five conventional and well-accepted parameters in the literature, second by estimating one parameter that can be inferred from the SCF data used in the empirical section. The model is then disciplined via the joint calibration of eight parameters that are less standard, thus of principal interest. Before doing so, the section proceeds by specifying some functional forms.

2.4.1 Specification of Functional Forms

Itô process for z_t. Entrepreneurial ability is assumed to obey the diffusion process

$$d \log z_t = \varphi_z(\mu_z - \log z_t)dt + \sigma_z dW_t \tag{2.28}$$

This is an Ornstein-Uhlenbeck (O-U) process in natural logs—the continuous-time analogue of a log AR(1) process—with drift μ_z, speed of reversion (persistence) governed by φ_z, and innovation dispersion σ_z. By Itô's lemma, the process in levels becomes

$$dz_t = \left[\varphi_z(\mu_z - \log z_t) + \frac{1}{2}\sigma_z^2\right] z_t \, dt + \sigma_z z_t \, dW_t \tag{2.29}$$

Due to mean-reversion and the properties of W_t, the marginal stationary distribution is log-normal: $\log z_t \sim \mathcal{N}(\mu_z, \frac{\sigma_z^2}{2\varphi_z})$. Since the distribution of shocks to entrepreneurial ability

[27] Apart from the thorough numerical appendix of Achdou et al. (2022), many useful details can be also found in Nuño and Moll (2018).

predominantly shapes the firm size distribution in the model, this result is rather empirically relevant; see for example Kondo, Lewis, and Stella (2021).[28]

Human capital formation. The functional form connecting individual schooling choices (S_i) to stocks of human capital (h_i) needs to be specified. I draw upon a standard practice in the literature, e.g., Hall and Jones (1999), Caselli (2005), and consider the following simple bijective map:

$$h_i = e^{\phi(S_i)},\ \phi(S) = \begin{cases} 0.117{\cdot}S & \text{if } S \leq 5 \\ 0.117{\cdot}5 + 0.097(S-5) & \text{if } 5 < S \leq 10 \\ 0.117{\cdot}5 + 0.097{\cdot}5 + 0.075{\cdot}(S-10) & \text{if } 10 < S \end{cases} \tag{2.30}$$

The assumption of a piecewise-linear function is made to reconcile the log-linearity of wages and schooling at the country level with the observed concavity of this relationship across countries, e.g., Psacharopoulos (1994), with coefficient values drawn from conventional Mincerian returns-to-schooling estimates. This is an updated version of the Hall-Jones/Caselli approach in two minor ways. FIrst, I use more recent average Mincerian estimates reported by Psacharopoulos and Patrinos (2004) for sub-Saharan Africa, the world as a whole, and the OECD, respectively. Second, I assume that the function changes slope after the 5$^{\text{th}}$ and 10$^{\text{th}}$ year of schooling, as opposed to the 4$^{\text{th}}$ and 8$^{\text{th}}$. This is to account for the prevalent increases in primary and secondary educational attainment across the globe in the decades since these studies were written.

[28] Using an extensive confidential Census Bureau panel dataset, one of the main stylized facts the authors establish is that "a lognormal fits both firm and establishment size employment distributions better than the commonly used Pareto, even far in the truncated upper tail." This is an important empirical finding that casts doubt on the conventional wisdom that the U.S. firm size distributions are adequately approximated by a Pareto distribution with a shape parameter close to one (Zipf's law).

Utility during schooling and distribution of learning affinity. The functional form of utility from attending school in (2.3) needs to be specified. To keep things transparent and uncomplicated, I assume that $\psi(.)$ is a function that depends only on individual learning affinity and of the same power form as utility from consumption

$$\psi(\beta) = \frac{\beta^{1-\gamma}}{1-\gamma} \tag{2.31}$$

Finally, regarding individuals' endowment of learning affinity upon birth, random draws from a lognormal distribution is a rather conventional choice: $\log \beta \sim \mathcal{N}(B, \sigma_\beta^2)$.

2.4.2 Parametrization and Calibration

To sum up, the model requires 14 parameter values to be pinned down. On the household side: ρ, γ, and η. Three technological parameters: α, δ, and ν. Three parameters characterizing the O-U process for entrepreneurial ability: μ_z, σ_z, ϕ_z. Two parameters determining the distribution of learning affinity and thus of human capital: B, σ_β. The extent of financial frictions captured by ϑ. The level of the complementarity parameter ω, and the technology adoption cost κ. A more thorough discourse is provided below.

Assigned parameters. The set of assigned parameters is $\{\alpha, \gamma, \delta, \eta, \mu_z\}$. Values for the first three are fairly canonical in the macro-development literature. I fix the coefficient of relative risk aversion to $\gamma = 1.5$; the annual capital depreciation rate is set to $\delta = 0.06$; the elasticity of output with respect to capital is chosen to be $\alpha = 0.36$, which, in the presence of moderate financial frictions, results in an aggregate capital share of income slightly above 0.34 for the U.S. The Poisson death rate is set to $\eta = 0.01667$ implying an average "lifetime" of about 60 years (ages 6 to 66). As normalization, the drift of entrepreneurial ability is $\mu_z = 0$, which simply translates to a zero-mean process in logs.

Estimated parameter. I make use of the theoretical results in Section IV together with the Probit reduced-form results for the U.S. in Section II to identify and infer the technology adoption cost (κ). Proposition 1 established that, for sufficiently high ω, the entrepreneurship rate by human capital attains its global minimum at $\hbar = \left(\frac{1}{1-\kappa}\right)^{1/\omega}$. Given equation (2.30) we can determine the level of the resulting threshold externality ($\hbar$) in the data. The average estimate from Probit regressions suggests that the minimum of the U-shaped relationship occurs at 9.25 years of schooling. Having obtained the numerical grid for human capital $h_i, i = 0, 1, \ldots, 20$, I set $\kappa = 1 - 1/(e^{\phi(9.25)})^{\omega}$.

Discussion of calibrated parameters. There remain eight parameters to be jointly calibrated for the model to best fit eight relevant moments in the data. The vector under consideration is $\{\nu, \vartheta, \sigma_z, \sigma_\beta, \phi_z, \rho, B, \omega\}$. Most calibrated parameter values are easily comparable and close to alternative estimates in the relevant literature.

$\{\nu \models$ *entrepreneurship rate in the labor force*$\}$. The span-of-control parameter governs the shape and scale of production possibilities and profit functions, hence is paramount when in comes to occupational decisions. This conclusion can be also reached by observing the dominant role of ν in Lemma 2. In SCF samples from Section II, the fraction of entrepreneurs has remained relatively stable over the thirty years, hovering around 10.5 percent of the labor force. This average value is markedly close to the arithmetic mean of 10.4 percent for the U.S. entrepreneurship rate over $1990 - 2009$, as inferred from Current Population Survey (CPS) data reported by Hipple (2010).[29]

The calibrated value $\nu = 0.232$ is somewhat higher compared to economies without financial frictions and/or with a corporate sector (around $0.15-0.20$), but it is not so in comparison to

[29] This number corresponds to the sum of the incorporated plus unincorporated self-employed divided by total employment in all non-agricultural industries; see Table 1 and Table 2 in Hipple (2010). Results are identical to the first decimal point whether calculating the average of ratios or the ratio of averages.

the germane class of models. For instance, the calibration of Buera, Kaboski, and Shin (2011) to the U.S. economy results in $\nu = 0.21$ ($\alpha + \theta = 0.79$ in their model), which in turn generates an entrepreneurship rate of only 5 percent.

$\{\vartheta \models$ *External finance-to-GDP ratio*$\}$ To calibrate the extent of financial market imperfections via the collateral constraint, a relevant data counterpart is the external finance-to-GDP ratio. It is defined as domestic credit provided to the private sector as a share of GDP with data coming from World Bank's Global Financial Development Database (GFDD).[30] This moment has been widely used in a multitude of studies; see for example Midrigan and Xu (2014), Moll (2014) and the references therein.

The calibrated value of $\vartheta = 0.864$ is not too far from the perfect-credit benchmark, but still implies moderate financial frictions. It is worth noting that the calibration takes a more conservative stance by not considering the U.S. a frictionless economy.

$\{(\sigma_z, \sigma_\beta) \models$ (*dispersion of log TFP*, *total income Gini coefficient*)$\}$ Distributional scale parameters—the standard deviation of entrepreneurial ability shocks and standard deviation of learning affinity—are the dominant sources of variation among production plans and total income. It is thus palpable to match the cross-sectional dispersion of U.S. firms' TFP together with the Gini coefficient for total (earned plus passive) income.

Regarding the former, a benchmark choice is given by the summary statistics on firms' log productivity dispersion in Foster, Haltiwanger, and Syverson (2008), specifically their measure of "traditional TFP" in Table 1 (0.21). This is also in line with Haltiwanger (2011), who states that the bulk of empirical evidence suggests "…estimates of the standard deviation

[30] I use the September 2022 version of the GFDD database, which is publicly available at https://www.worldbank.org/en/publication/gfdr/data/global-financial-development-database.

of innovations to productivity shocks of about 0.20 (in terms of log total factor productivity)." I choose the average of the two values (0.205).

Regarding the latter, I consult data from the U.S. Bureau of Economic Analysis (BEA) on the distribution of personal income. The average of available estimates for $2000-2019$ is 0.446; I set the target value to 0.430 to correct for the fact that income inequality has been somewhat lower in the period $1989-2000$.[31]

$\{\phi_z \models$ *firm entry/exit rate*$\}$ The persistence of entrepreneurial ability is by and large the main driver of firm dynamics in this setting. As such, I choose to target the average U.S. entry rate for $1989-2019(0.11)$, based on annual observations from the latest Business Dynamics Statistics (BDS) dataset.[32]

The calibrated value $e^{-\varphi_z}=0.901$ is reasonable and falls right between the estimates of Asker, Collard-Wexler, and De Loecker (2014) for the U.S. using an AR(1) specification for (the log of) TFPR, and the corresponding values from the replication of Hsieh and Klenow (2009) using their dataset.[33]

$\{\rho \models$ *real rate of return on capital*$\}$ The household discount rate, ρ, is set to match an average real interest rate of 4 percent per annum, a value commonly used in the literature. The calibrated value of $\rho = 0.091$ implies an economy with fairly impatient agents, a result that is quite standard in the sphere of incomplete-market models with entrepreneurship. For instance, the baseline calibration of Cagetti and De Nardi (2006) requires a (discrete time) discount factor of $\beta = 0.865$ to match a 6.5 percent equilibrium interest rate; the model of

[31] I use the December 2022 version of the BEA release, which is publicly available at https://www.bea.gov/data/special-topics/distribution-of-personal-income.

[32] I use the economy-wide dataset of the 2021 BDS release, which is publicly available at https://www.census.gov/data/datasets/time-series/econ/bds/bds-datasets.html.

[33] See Table 1 at http://www.johnasker.com/ACWDLcomment.pdf.

Buera and Shin (2013) targets a 4.5 percent interest rate and requires a discount factor of $\beta = 0.904$.

$\{B \models$ *mean years of schooling*$\}$ The distribution of learning affinity, assumed to be $\log \beta \sim \mathcal{N}(B, \sigma_\beta^2)$, is the primary source of variation in educational choices among individuals. The location parameter B governs the central tendency of the distribution and is calibrated so that the model matches average years of schooling in the data. The full SCF dataset indicates an average of 13.83 among labor force participants, and Barro and Lee (2013) data for population aged $25 - 64$ suggest 13.30 for the period $1990 - 2015$. I choose to target 13.50 years of schooling and the resulting value is $B = 0.106$.

$\{\omega \models$ *ERGON index*$\}$ Much of the analysis in Section IV is about the allocation of human capital between occupational groups induced by technology adoption choices. These depend crucially on the degree of complementarity between entrepreneurial ability and human capital. Therefore a natural target is the ERGON index, defined as $\frac{ER_i(\text{educ}\geq 14\text{years})}{ER_i(\text{total})}$ in line with the motivating facts presented in Section II. The calibrated value is $\omega = 0.515$. Although there is no reference paper we can compare this value to, it results in the isoelastic function $\zeta(h_e, z)$ exhibiting moderate diminishing returns.

2.4.3 Baseline Model Output

Table 3.1 reports the output of the calibration exercise and summarizes the parametrization. The model is able to match the targeted moments very closely. As emphasized above, this is achieved prudently by targeting only as many moments as parameters, through typical functional specifications, and without producing unconventional parameter values. Although strong local first-order identification is rather difficult in this class of non-linear general equilibrium models, good performance in varied dimensions shows that the selected moments are sufficiently informative about the calibrated parameters so that the objective function is not locally flat.

In terms of non-targeted moments, the model is able to generate quite reasonable results across a wide range of aspects, as evidenced in Table 2.2. To begin with, the predicted rate of entrepreneurship by educational attainment follows the data fairly well. Replicating such a highly disaggregated and non-linear relationship is intricate; it requires getting both endogenous occupational and educational choices right (the law of total probability has to hold). This is indeed one of the model's accomplishments, especially given its parsimony and unconditional independence between z and β.

Another feature of the U.S. economy the model can come close to is the firm size distribution by employment size. Although not perfect, the match should be considered adequate as it is non-targeted and often notoriously hard to attain. The main discrepancy is found at the right tail of the distribution, which is expected since I am not assuming a Pareto or extreme-value distribution for z.

Finally, it seems fair to say that the model is also doing reasonably well in generating additional non-targeted cross-sectional moments. For instance, it predicts substantial total income (earned plus capital income) inequality, with realistic concentrations across different quintiles; sizeable wealth inequality, with the appropriate large share of wealth in the hands of entrepreneurs together with realistic occupational representations at the top wealth decile; as well as accurate within-entrepreneurs heterogeneity with respect to completed years of schooling.

Table 2.1: Model Calibration Summary; Targeted Moments and Parametrization

Targeted Moments	Data	Model	Data Source
Entrepreneurship rate in the labor force	0.105	0.105	SCF & CPS
External finance-to-output ratio	1.650	1.650	GFDD/World Bank
Dispersion of (log) TFP	0.205	0.205	Foster et al. (2008) & Haltiwanger (2011)
Total income Gini coefficient	0.430	0.425	BEA/CPS ASEC
Firm entry/exit rate	0.110	0.110	BDS
Real rate of return on capital	0.040	0.040	Standard
Mean years of schooling	13.50	13.50	Barro and Lee (2013) & SCF
ERGON index	1.161	1.210	SCF
Calibrated Parameters	**Value**		**Comment**
Span-of-control parameter (ν)	0.232		See text
Collateral constraint (ϑ)	0.864		See text
Dispersion of entrep. ability shock (σ_z)	0.204		See text
Scale parameter of learning affinity (σ_β)	0.167		See text
Autocorrelation of entrep. ability z ($e^{-\phi_z}$)	0.901		See text
Subjective discount rate (ρ)	0.091		See text
Location parameter of learning affinity (B)	0.106		See text
Complementarity between z and h_e (ω)	0.515		See text
Assigned & Estimated Parameters	**Value**		**Comment**
Elasticity of output w.r.t. capital (α)	0.360		Standard
Capital depreciation rate (δ)	0.060		Standard
Coefficient of relative risk aversion (γ)	1.500		Standard
Mean (log) entrepreneurial productivity (μ_z)	0.000		Normalization
Poisson death rate (η)	0.017		60 years until retirement
Technology adoption cost (κ)	0.402		Estimated from Probit-RII regressions

Table 2.2: Model Calibration Output; Non-Targeted Moments

Non-Targeted Moments	Data	Model	Source
Entrepreneurship rate by educational attainment			
Years of education: $[0-8]$	0.080	0.080	SCF
Years of education: $(8-12)$	0.072	0.070	SCF
Years of education: 12	0.091	0.088	SCF
Years of education: $(12-16)$	0.094	0.097	SCF
Years of education: 16	0.121	0.121	SCF
Years of education: $(16, 20]$	0.156	0.149	SCF
Firm size distribution			
Employment size: $1-9$	0.769	0.780	BDS
Employment size: $10-19$	0.115	0.126	BDS
Employment size: $20-99$	0.096	0.090	BDS
Employment size: 100+	0.020	0.004	BDS
Total Income distribution statistics			
Share received by top 20%	0.505	0.514	BEA/ CPS ASEC
Share received by middle 20%	0.141	0.132	BEA/ CPS ASEC
Share received by bottom 20%	0.052	0.077	BEA/ CPS ASEC
Mean-to-median ratio	1.465	1.494	BEA/ CPS ASEC
Wealth distribution statistics			
Share owned by entrepreneurs	0.437	0.434	SCF
Share of entrepreneurs in top 10%	0.394	0.400	SCF
Ratio of mean assets; entrepreneurs to workers	6.876	6.463	SCF
Miscellaneous statistics			
Job destruction rate: deaths	0.046	0.041	BDS
Fraction of entrepreneurs with ≤ 12 years education	0.329	0.329	SCF
Fraction of entrepreneurs with ≥ 16 years of education	0.438	0.455	SCF

2.5 Quantitative Exploration

This section features some model-based evidence illustrating the quantitative importance of the central mechanisms put forward by the theory. The main exercises consist of varying key parameters of interest in order to gauge the magnitude and decomposition of implied long-run output differences relative to the U.S. (baseline calibration). These counterfactuals highlight the *ceteris paribus* nature of the misallocation argument: countries with otherwise identical structure and endowments attain strictly worse outcomes because they are unable to put their resources to their most efficient use.

The model gives rise to a "development accounting" framework in levels by equation (3.20), as real output per person employed in country i can be equivalently expressed as[34]

$$Y = \underbrace{A(\mathcal{E}|\omega, B, \vartheta)_i}_{\text{TFP contribution}} \; \mu(\mathcal{E})_i^{\frac{\nu}{1-\hat{\alpha}}} \left(\frac{K_i}{Y_i}\right)^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} H_i^{1-\frac{\nu}{1-\hat{\alpha}}} \qquad \widehat{\alpha} := \alpha(1-\nu) \tag{2.32}$$

with the contribution of TFP being the endogenous term $A_i := Z_i^{\frac{1}{1-\hat{\alpha}}}$, and where Z_i is the Hölder-type weighted mean term of effective productivities among active producers as in equation (3.20). The terms "TFP" and "worker human capital" used in upcoming figures refer to the first and last term of (2.32), respectively.

[34] See Klenow and Rodríguez-Clare (1997), Hall and Jones (1999), and the references therein for reasons why it is preferred to express the development accounting equation in terms of capital-output ratios.

2.5.1 The Impact of Complementarity on Macro-Development

The first counterfactual experiment involves varying the degree of complementarity (ω) between entrepreneurial ability and human capital—essentially the elasticity of the disembodied technology upon adoption—all else being equal. By doing so one can effectively simulate a series of economies that differ only with respect to a crucial underlying driver of the entrepreneurship-human capital nexus, in order to compare certain aspects of the resulting stationary equilibria. Specifically, I consider economies characterized by strictly lower values of ω compared to the U.S. in the baseline calibration, thus exhibiting strictly lower ERGON indexes.[35]

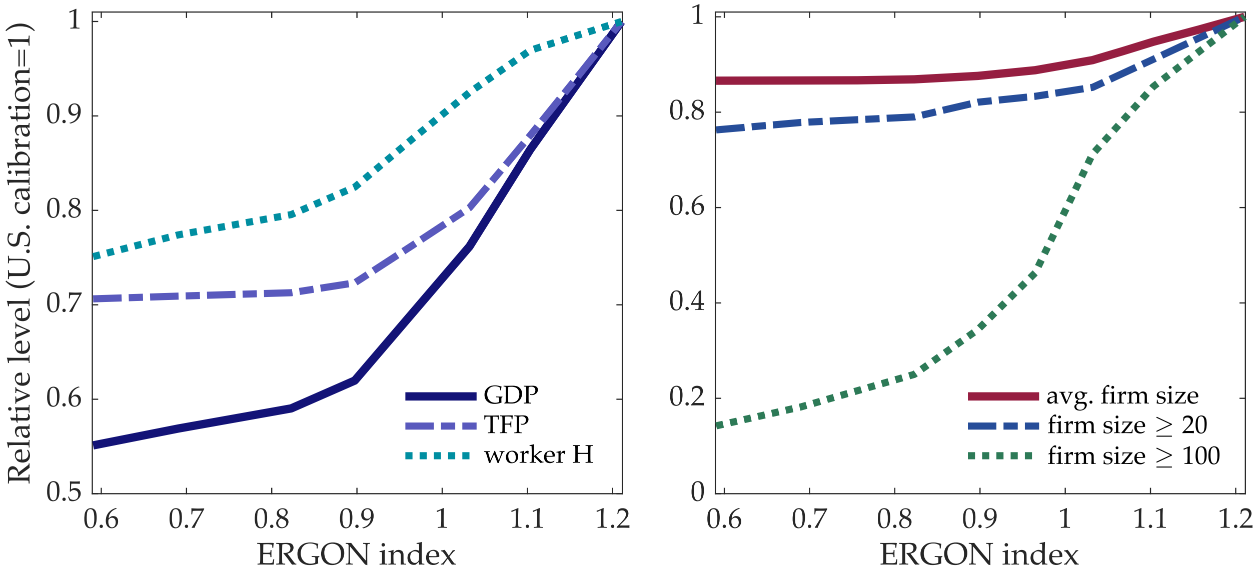
The results of the first exercise are summarized in Figure 2.2; the outcomes of the U.S. calibration are normalized to one (top right corner). Starting with the left panel comparing output per person employed ("GDP") together with two components accounting for almost all of its variation, namely, total factor productivity ("TFP") and human capital of workers ("worker H"), against the attained ERGON index.

To begin with, moderate *ceteris paribus* shifts in the entrepreneurship-education nexus can generate sizeable and persistent misallocation losses up to 45% less vis-á-vis the U.S. Moreover, most of these differences are due to endogenous TFP formation, with the role of accumulated worker human capital being second but markedly important.

Why does lower complementarity entail such considerable long-term losses in output per laborer? There are three major forces at play. First, a lower ω increases the cutoff value $\hbar = (1/(1-\kappa))^{1/\omega}$ above which entrepreneurs choose to adopt the disembodied technology. Such a drastic impact on equilibrium technology adoption restrains productive capacities among

[35] To remain consistent with the motivating facts of Section II and the calibration of section V, the ERGON index is once again defined as $\frac{ER_i(\text{educ}\geq 14\text{years})}{ER_i(\text{total})}$.

Figure 2.2: The Impact of Complementarity (ω) on Macro-Development

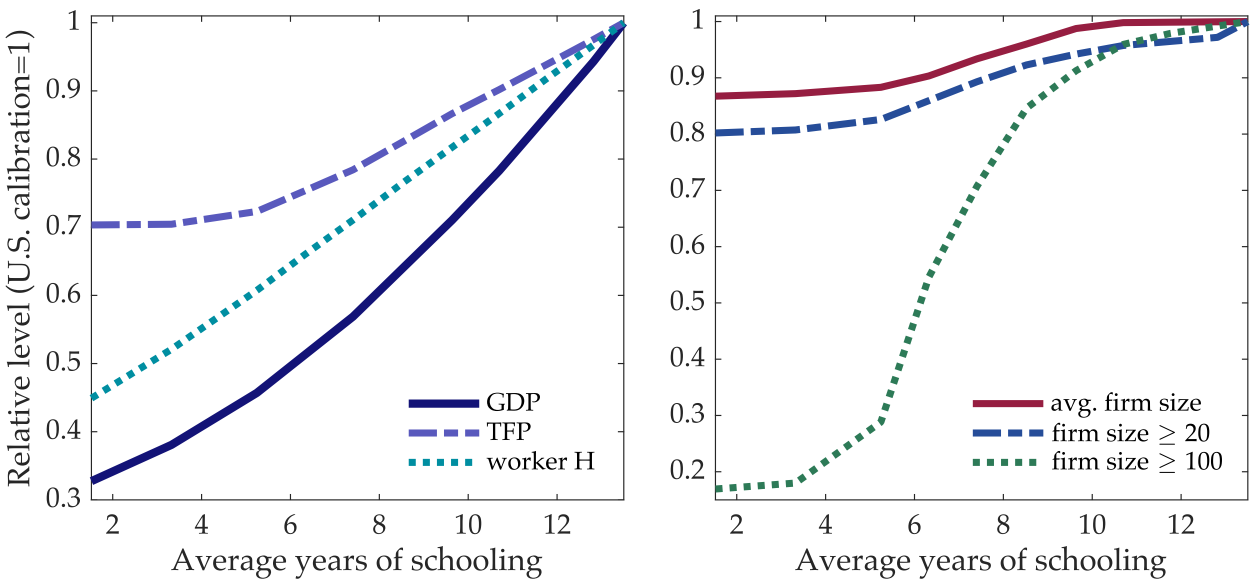


Notes: Stationary equilibria of the model obtained by varying the complementarity parameter ω from 0.515 (U.S. calibration) to 0.282, *ceteris paribus*.

firms with talented managers, and in turn maps into subdued aggregate TFP formation. Second, lower complementarity is an impediment to the accumulation of human capital by future generations. Observe that both sides of equation (2.25) depend positively on ω, which affects both the present value of expected future income gains and the expected foregone utility from not joining the labor force, thus leading to lower schooling choices for any level of learning affinity.

Third, there are important general equilibrium effects associated with changes in entrepreneurship rates by education—as discussed in the context of Figure 2.1—that have additional implications for the firm size distribution. Specifically, a lower ω exerts simultaneous downward pressure on the aggregate demand for labor by entrepreneurs and on the aggregate supply of human capital by the rest of the labor force. Th equilibrium effective wage rate (w) drops substantially, which successively alters the patterns of occupational choice. Lower labor income dampens the opportunity cost of running a firm and induces plenty of less talented individuals to become entrepreneurs. Such a critical mass of small firms leads to a reduced average firm size, and in conjunction with weakened technology adoption by more able entrepreneurs, the reduction is reflected even more strongly among larger firms. In other words, lower degrees of complementarity lead to higher overall rates of entrepreneurship together with firm size distributions that are more concentrated towards smaller and less productive firms.[36]

Figure 2.3: The Impact of Aggregate Human Capital on Macro-Development



Notes: Stationary equilibria of the model obtained by varying the location parameter B of learning affinity, from $B = 0.106$ (U.S. calibration) to -0.760, *ceteris paribus*.

2.5.2 The Impact of Aggregate Human Capital on Macro-Development

The second exercise involves the simulation of a series of economies with less educated labor forces, all else being equal. The setup of the model does allow one to carry out such an experiment in a simple and transparent manner, as explained in Section V. Recall that

[36] As a reminder, in this setting average firm size is simply pinned down by $(1 - \mu(\mathcal{E}))/\mu(\mathcal{E})$.

the distribution of learning affinity is $\log \beta \sim \mathcal{N}(B, \sigma_\beta^2)$; since heterogeneity in random draws of β governs individual-level variations in optimal schooling choices, the location parameter B determines the central tendency of educational attainment. By varying only the distributional parameter B the model will endogenously generate different total/average years of schooling. The results of this exercise are summarized in Figure 2.3, with the outcomes of the U.S. calibration being normalized to one.

Responding to realistic variations in B, lower total/average years of schooling engender long-run differences of up to about -70% output per laborer. It is certainly no surprise that lower educational attainment has a considerable influence on macroeconomic activity through the conventional channel of accumulated worker human capital. What is indeed notable is how startling these differences come to be due to the additional impact of endogenous TFP formation, which accounts for a large share in the decline of output. This sizeable effect is primarily a result of the intensive-margin nature of technology adoption decisions: A lower stock of human capital leads to less available entrepreneurial human capital that can contribute to firms' productive capacities, even when there is enough scope for complementarity between education and idiosyncratic ability (high ω) as in this exercise.[37]

[37] Recall that a conventional heterogeneous-agent model in which human capital accumulation alters only worker's productivity would not be able to generate similar aggregate TFP dynamics.

Table 2.3: Accounting For Cross-Country Output Differences Vis-à-Vis the U.S.

COUNTRY	SHARE OF OUTPUT P.W. DIFFERENCE EXPLAINED BY		
	varying only ω	varying (ω, B)	varying (ω, B, ϑ)
Italy — median in 5^{th} quintile	89.0%	96.2%	100.1%
Poland — median in 4^{th} quintile	71.7%	72.9%	92.3%
Malaysia — median in 3^{rd} quintile	59.8%	60.5%	71.2%
Peru — median in 2^{nd} quintile	31.6%	32.8%	47.5%
Bangladesh — median in 1^{st} quintile	22.1%	32.0%	40.3%
ADDITIONAL COUNTRIES			
Sweden — 5^{th} income quintile	99.2%	99.4%	99.9%
Canada — 5^{th} income quintile	99.1%	99.1%	99.8%
Japan — 4^{th} income quintile	98.9%	99.3%	99.7%
Greece — 4^{th} income quintile	79.8%	83.2%	97.4%
Egypt — 3^{rd} income quintile	56.2%	66.7%	94.8%
Argentina — 3^{rd} income quintile	50.1%	52.6%	81.3%
Brazil — 2^{nd} income quintile	44.2%	50.3%	63.5%
Colombia — 2^{nd} income quintile	42.9%	49.7%	61.5%
China — 1^{st} income quintile	34.0%	36.2%	38.9%
India — 1^{st} income quintile	27.3%	39.0%	49.5%

NOTES: Each quantitative experiment for each county follows the strategy of the U.S. calibration: varying only ω targets the ERGON index (GEM); varying (ω, B) targets the ERGON index and average years of schooling (Barro-Lee); and varying (ω, B, ϑ) targets the ERGON index, average years of schooling, and the external finance-to-output ratio (GFDD).

Moreover, parallel to the previous quantitative exercise there are significant general equilibrium forces affecting the firm size distribution. A novel implication of the model is that uniformly lower educational attainment depresses not only the aggregate supply of human capital, but also the aggregate demand for labor. Since the entrepreneurship-education nexus is similar to the U.S. in this exercise—meaning that a significant fraction of entrepreneurs adopts the disembodied technology—the decline in aggregate labor demand dominates and the market-clearing effective wage rate moves permanently lower. Lower labor income at each level of human capital decreases the opportunity cost of entrepreneurship and leads to a larger number of less-able producers running smaller and less productive firms. In this sense, and in relation to the first counterfactual, it essentially reflects the other side of the same coin. [38]

2.5.3 Accounting For Cross-Country Output Differences

In light of the pervasive implications brought out by the counterfactuals, it would be instructive to get a clearer picture of how important the underlying mechanisms are in explaining actual cross-country differences in output per worker. The idea is to examine the model's ability to account for income gaps vis-à-vis the U.S. by varying *only* a limited subset of components. First I vary the degree of complementarity (ω) to target the ERGON index of country i (GEM data); second both (ω, B) to target the ERGON index and average years of schooling (Barro-Lee data); third (ω, B, ϑ) to target the previous two moments together with the external finance-to-GDP ratio (GFDD data).

[38] To all of the above counterfactuals, it should be mentioned that the capital-output ratios, interest rates, investment rates, and related moments are virtually the same as in the U.S. calibration.

To calculate the share of the income gap between county i and the U.S. accounted for by the model in each quantitative exercise j, $\mathcal{S}_{ij}$, I proceed as follows. Let m_{ij} be the model-generated output per worker in country i (relative to the U.S.) under exercise j, and let d_i be relative output per worker in the data. Then, $\mathcal{S}_{ij} = \frac{\log(1/x_{ij})}{\log(1/x_{ij})+\log(x_{ij}/d_i)}$.[39]

A key observation is that varying *only* ω can alone account for an ample share of cross-country differences. Indeed, for each country above the second income quintile it can explain more than half of the income gap. When it comes to poorer countries like Bangladesh or India the gap explained is much smaller, suggesting the need for more radical modifications of the baseline model. Apart from generating large drops in aggregate TFP, these variations in ω are also able to close much of the gap in educational attainment, as evidenced by the limited explanatory power from adjusting B. Accounting for differences in financial frictions is undoubtedly crucial in line with numerous studies, but at least for the countries considered and without taking into account aggregate transitions, the marginal gains from varying ϑ are not as extensive as those coming from ω. All in all, the results further substantiate the central prediction of the theory: the mechanism governing the effective use and allocation of entrepreneurial human capital plays a crucial role in the determination of cross-country income differences.

2.6 Summary and Concluding Remarks

Entrepreneurship and human capital are widely recognized drivers of economic performance. At the same time, the allocation of resources between and within occupations shapes total factor productivity and output. This paper raises new points of inquiry and employs theory and quantitative assessment in order to address them: Is the allocation of human capital

[39] As appropriate, $\mathcal{S}_{ij}$ is defined in terms of log-factors of income gaps and adheres to the underlying ratio scale; it is strictly decreasing in x_{ij} with $\mathcal{S}_{ij} = 1$ at $x_{ij} = d_i$ and $\mathcal{S}_{ij} = 0$ at $x_{ij} = 1$. Surprisingly, it is not too uncommon to encounter erroneous calculations using gross factors or an inconsistent scale.

between entrepreneurs and workers a key determinant of aggregate productivity and income? How pervasive are its implications for macro-development?

There are two main messages emerging from this study. First, a versatile heterogeneous-agent model with occupational and educational choices is able to rationalize the empirical findings of the first chapter, while remaining broadly consistent with aggregate and survey data. A central hypothesis is that the costly adoption of a disembodied technology enables individuals to enhance their idiosyncratic ability with the competences of their human capital. In general equilibrium, the extent of technology adoption determines the contribution of entrepreneurial human capital in firms' productivities, which is a key component of endogenous TFP. As new generations build skills through schooling and form expectations about their future prospects as workers/entrepreneurs, the above mechanism also affects the accumulation of human capital economy-wide. An important insight is that the entrepreneurship-education nexus has first-order aggregate and distributional consequences.

Second, the calibrated model does well in replicating a wide spectrum of targeted and non-targeted U.S. moments, thereby capturing salient features of micro and macro data. Quantitative explorations illustrate how different allocations of human capital between occupational groups can lead to sizeable and persistent losses in TFP and output, with additional results accentuating the role of human capital in the process of development. The analysis indicates that if economies differ in their capacities to complement entrepreneurial talent with the benefits that human capital can offer, there are drastic implications for both factor accumulation and TFP formation that shed light on a novel proximate cause of long-run cross-country output and productivity differences.

Moving forward, I would suggest two avenues for future research. Regarding the expanding literature on the role of entrepreneurship in macroeconomics, I would advocate for putting more emphasis on the dimension of education/human capital and technology adoption. Pertinent elaborations can benefit the quantitative performance of workhorse heterogeneous-agent models without high computational costs, and enable them to account for a range of facts that current practices mostly ignore. Moreover, much remains to be learned about the nature of the hypothesized disembodied technology and its ensuing impact on productivity. It would be constructive to formulate new theories on the origin of the complementarity process at a more granular level, which could be related to sector-specific effects or possibly to deeper cultural considerations. Altogether, the entrepreneurship-human capital nexus seems crucial in understanding various aspects of macro-development. In my opinion, this is an issue worth pursuing.

Chapter 3

Sometimes Less is More: Growth, Risk Aversion, and the Suboptimality of Entrepreneurial Insurance

Joint work with Neville N. Jiang, Ping Wang, and Haibin Wu

3.1 Introduction

For over a century, at least since the pioneering work of Schumpeter (1911) and Knight (1921), the role of active entrepreneurship in fostering long-run economic growth has been emphasized by economists and taken into serious consideration by policymakers. Either implicitly or explicitly, conventional wisdom dictates that encouraging business formation and dynamism has a decidedly positive impact on economic growth; that is, "more is more" when it comes to entrepreneurship and the process of development. Is this a theoretically robust prediction that public policies should *always* aim to accommodate for more entrepreneurial activities?[40]

Entrepreneurs form a special occupational group in the sense that, despite its small size relative to an economy's labor force, it holds a disproportionately large share of total wealth and income.[41] Individual heterogeneity plays a crucial role in shaping occupational decisions. In addition to entrepreneurial ability emphasized by Schumpeter and risk tolerance highlighted by Knight, Kihlstrom and Laffont (1979) also advocate wealth and access to capital markets as an important aspect of entrepreneurship. There is indeed a large array of literature focusing on the Schumpeterian view starting from the seminal articles by Lucas (1978) and Rosen (1981), and an even larger body of work focusing on capital-market access factor stemming from the original contributions of Banerjee and Newman (1993), Aghion and Bolton (1997), and Lloyd-Ellis and Bernhardt (2000).[42] The Knightian factor of risk attitude, however, remains largely unexplored despite its plausibility and empirical relevance.[43]

We develop a dynamic general equilibrium model of occupational choice, taking into account individuals' heterogeneity not only in entrepreneurial ability but also in risk attitude. By designing such a framework we aim to address two major research questions: (i) *is promoting*

[40] A plethora of classic papers such as Baumol (1990) and Murphy, Shleifer, and Vishny (1991) suggest that policies aimed at supporting productive (as opposed to rent-seeking) entrepreneurship will spur innovation and growth. At the same time, virtually all major international and intergovernmental institutions such as the World Bank, IMF, OECD, and UNCTAD are systematically advocating for policies and programs that encourage more inclusive entrepreneurship and promote small and medium-sized enterprises.

[41] As documented by Cagetti and De Nardi (2006), entrepreneurs defined as self-employed business owners account for only 7.6% of the U.S. population, but hold about one third of total net worth. Furthermore, according to Quadrini (2000) entrepreneurial households receive more than 20% of total U.S. income.

[42] For a broader coverage of the literature on entrepreneurship and macroeconomics, the reader is referred to the comprehensive surveys of Quadrini (2009) and Parker (2018).

[43] For example, see the results of Caliendo, Fossen, and Kritikos (2009) using an experimentally validated survey.

entrepreneurship always conducive to long-run growth and (ii) *to what extent should policy-makers strive to ensure entrepreneurial risk away?* In exploring these questions, we also examine whether a decentralized equilibrium is suboptimal in the absence of typical output distortions or financial frictions. Thus, we intentionally leave out the capital-market access factor that may dilute a chief purpose of this study.

We study an economic environment with heterogeneous abilities and endogenous occupational decisions in the spirit of Lucas (1978) as well as more recent studies by Ghatak, Morelli, and Sjöström (2007), Jiang, Wang, and Wu (2010), and Inci (2013). In contrast to this strand of literature, however, we explicitly consider the influence of risk attitude to fill the knowledge gap in that regard. We begin by examining how the presence of risk aversion affects entry into entrepreneurship and the formation of aggregate total factor productivity (TFP) in dynamic general equilibrium. We then inquire whether the decentralized outcome is efficient. We further study the scope for insurance policy against entrepreneurial risk for enhancing long-run economic growth by comparing the outcome of full (actuarially fair) insurance of entrepreneurial risk with the (uninsured) decentralized equilibrium.

Specifically, we develop an overlapping-generations endogenous growth model with occupational choices, in which agents who are heterogeneous in their risk attitude and idiosyncratic productivity choose whether to become workers or entrepreneurs. In the absence of any capital market imperfections, young workers are by construction net loanable fund suppliers, while entrepreneurs are net borrowers. A young entrepreneur endowed with an inalienable business idea can transform loanable funds into productive capital under a risk of failure. If succeeded, she can then combine capital and labor to generate an intermediate good of a distinct variety akin to her business idea. The final consumption good is produced by a representative competitive firm that aggregates the available basket of intermediate goods.

The central insights of the paper can be summarized in the following points. First, the relationship between the rate of entrepreneurship and long-run growth need not be positive or even monotone. Balanced growth depends non-monotonically on the number of active entrepreneurs in the economy, as well as on an *endogenous* aggregative measure of their productive capacities. There are three opposing forces at play in dynamic general equilibrium. On the one hand, a higher number of people opting into entrepreneurship expands the variety of intermediate products, leading to increased production of the final good and subsequently higher growth. On the other hand, the ensuing reduction in the number of workers decreases both the aggregate supply of labor and of loanable funds, which in turn depresses capital formation and lowers economic growth. Apart from how many individuals become entrepreneurs, it is of first-order importance what type of individuals do so. We show that occupational choices induce an inverse association between risk tolerance and entrepreneurial talent at the margin; thus promoting entrepreneurship in a decentralized economy will hinder aggregate TFP formation as firm entry is accommodated by the lower parts of the ability distribution.

The ambiguous relationships (of both sign and magnitude) predicted by our theory can help rationalize an important empirical fact that may contradict conventional wisdom: increases in the rate of entrepreneurship need not be positively associated with the rate of economic growth, e.g. Blanchflower (2000) and papers cited in Jiang, Wang, and Wu (2010). Importantly, our findings suggest that public and/or industrial policies aimed at promoting entrepreneurship *unconditionally* need not be growth-enhancing. In the presence of insufficient loanable fund supply, reducing the number of workers/savers can also be harmful for growth even in the absence of distortions or financial frictions.

Second, the allocations in the decentralized equilibrium are almost surely suboptimal and result in misallocation both on the extensive and intensive margins. Specifically, a competitive market economy features either over- or under-entrepreneurship (depending on parameter values and distributions) compared to the growth-maximizing solution of a constrained social planner. This occurs due to three reasons, none of which stems from frictions or distortions. Individuals will consistently undervalue the marginal benefit of becoming entrepreneurs via the variety effect, and they also undervalue the marginal cost of becoming entrepreneurs incurred through the loanable fund supply effect. At the same time, heterogeneity in risk aversion distorts the efficient pattern of occupational sorting and leads to lower aggregate TFP formation. Notably, this inefficiency is a result of an intergenerational externality arising from endogenous occupational choice and an intersectoral externality stemming from differentiable intermediate goods over an endogenous operative range that is again dependent on occupational choice.[44]

In the empirically plausible case where the capital income share is lower than the labor income share, an additional policy implication arises. Introducing an actuarially fair insurance market that eliminates entrepreneurial risk will indeed align private and social marginal benefits, but it will still fail to correct for the undervaluation of private marginal costs, thereby leading to excessive entrepreneurship rates. This finding provides a reasonable theoretical explanation for the empirical evidence documented by Astebro (2003), namely that potential entrepreneurs can be overly optimistic to invest in less lucrative projects.

This study has an additional theoretical ramification that is perhaps worth mentioning. Ever since the seminal work of Akerlof (1970), it is well understood that asymmetric information—with or without limited liability and/or financial frictions— can lead to inefficient outcomes in the market for entrepreneurial talent, which in turn deters aggregate investment and

[44]These aspects of externality are less obvious but can indeed cause suboptimality or complex dynamics, as pointed out by Mino, Shimomura, and Wang (2005) and Varvarigos and Gil-Moltó (2016).

productivity, e.g., Stiglitz and Weiss (1981) and De Meza and Webb (1987). Our analysis provides an alternative explanation: in economies with risk-averse individuals and/or insufficient loanable fund supply, promoting entrepreneurship can be harmful to long-run growth and endogenous TFP formation even without any informational or credit market imperfections.

To further explore the quantitative implications of our framework, the model economy is calibrated to aggregate and establishment-level data for the U.S. and it is able to closely match all targeted moments without producing unconventional parameter values. In addition, it fits the full establishment size distribution together with the full employment distribution by size quite well, even though we are targeting only a single moment from each distribution.

We then compute the decentralized equilibrium under full insurance against entrepreneurial risk as well as the planner's solution. Removing misallocation related to occupational choices leads to sizeable balanced growth gains of about 0.6% on an annualized basis under full risk insurance, and up to 0.7% per annum under the efficient allocation. The results also indicate that the U.S. entrepreneurship rate (as per our measurement) is lower than the optimal one, and in the case of full insurance would lead to far too many entrepreneurs. We also find that about 97% of income growth losses vis-à-vis the planner's solution is due to misallocation on the intensive margin caused by the presence of risk aversion: *who* becomes an entrepreneur is far more important for long-run growth than *how many* people do so. A crucial policy insight is that encouraging a small number of highly skilled individuals to start up and operate firms would be more beneficial than incentivizing a larger number of less capable entrepreneurs to do so.

In sum, our answer to the first question on the nexus between entrepreneurship and growth is: more entrepreneurship may yield less growth if more risk tolerant but less talented entrepreneurs are attracted to firm creation. Regarding the second question on whether to insure some entrepreneurial risk away, our answer is: while such insurance is almost always growth-enhancing, full insurance is almost surely suboptimal. More can be less, and the presence of risk aversion may induce misallocation and cause substantial aggregate TFP and income growth losses.

The rest of the paper is structured as follows. Section 2 provides a detailed exposition of the structure and components heterogeneous-agent economy, its . Section 3 contains the main results pertaining to decentralized competitive equilibria, along with sharp characterizations of endogenous quantities and factor prices. Section 4 presents a number of related results for centralized economies (planner's solutions) and offers insights into how the introduction of an actuarially fair market for entrepreneurial risk shapes occupational choices and the macroeconomy. Section 5 carries out a further characterization of balanced growth equilibria with respect to changes in key parameters. Section 6 calibrates the model economy to U.S. data and presents quantitative evidence in favor of the potentially large misallocation losses predicted by our theory. Section 7 briefly summarizes our main results and offers some concluding remarks.

3.2 An Endogenous Growth Model with Occupational Choices

In this section we delineate the environment of the model economy: a overlapping-generations endogenous growth model populated by heterogeneous agents making occupational choices and carrying out production plans. In Figure 3.1 we outline the basic structure of the economy, in which the timing of events is numerically ordered from 1 to 5.

3.2.1 Environment, Endowments, Preferences

There is a continuum of unit measure of two-period lived agents. After an initial old generation at time $t = -1$, the economy consists of an infinite sequence of two-period lived overlapping generations without any population growth. Individuals are *ex-ante* heterogeneous in their *risk attitude*, $\rho \in \mathcal{P}$, and *entrepreneurial ability/productivity*, $z \in \mathcal{Z}$. All young agents are endowed with a draw from a (non-singular) stationary joint distribution, $G(\rho, z)$, along with one unit of labor, and an idea i of designing a particular intermediate good. The sample space is the set $\Omega = (\mathcal{P} \times \mathcal{Z}) \subset \mathbb{R}^2_+$ generated by the joint support of the two state variables. We do not impose any restrictions on $G(\rho, z)$ apart from the natural assumption that it is continuous a.e.

An agent born in period t chooses to become a worker or an entrepreneur when young, and supplies one unit of labor inelastically to market activities. Individuals value consumption only when old and have no positive bequest motive. Their preferences are represented by,

$$U(c_{t+1}; \rho) = (c_{t+1})^{1/(1+\rho)}$$

where c_{t+1} is consumption of the final good in old age, and $\rho \geq 0$ is an index of risk aversion measuring agents' attitude towards intertemporal risk. Notice that this simple power utility form implies strictly increasing and concave cardinal utility for any positive $\rho < \infty$, and nests the case of risk-neutral preferences when $\rho = 0$.

A young worker of generation t supplies her entire labor endowment augmented by human capital h to an old entrepreneur of generation $t-1$. Subsequently, she saves the entirety of her income for consumption in the second period of her lifetime.

A young entrepreneur of generation t borrows from a bank to transform the loan into capital subject to some uncertainty; each business project is bound to succeed with (constant) probability $q < 1$. The success rate q is the source of risk in the economy. If failed, she cannot produce and need not repay the loan under limited liability. If succeeded, she can then combine her capital with young workers to implement her idea and produce an intermediate good of a specific variety when old. As long as varieties are imperfect substitutes in the aggregation process, firm owners gain some pricing power due to the downward-sloping demand for each variety. With perfect capital markets, she will fully repay her debt and consume her remaining profit.

3.2.2 Production and Financial Markets

The economy consists of three sectors: a perfectly competitive final good sector, a monopolistically competitive intermediate goods sector, and a frictionless banking sector.

Intermediate goods sector. Intermediate producers (entrepreneurs) operate in a monopolistically competitive market and treat all prices but their own as given. Varieties are assumed to be imperfect substitutes in the final-good aggregation process thereby allowing producers to charge a fixed markup over their marginal cost, which in turn depends on their entrepreneurial ability/productivity (z). A young entrepreneur of generation t with a unique business idea and productivity z borrows $x_t(z)$ from a bank with a view to transforming the loan into productive capital, subject to an exogenous success rate $q < 1$,

$$k_{t+1}(z) = \begin{cases} x_t(z) & \text{with probability } q \\ 0 & \text{with probability } 1-q. \end{cases}$$

If succeeded, she hires young labor at $t+1$ to produce the intermediate good i according to the individual-specific technology,

$$y_{t+1}(z) = z\, k_{t+1}(z)^{\alpha}\left(\ell_{t+1}(z)h_{t+1}\right)^{1-\alpha} \tag{3.1}$$

The input of entrepreneurship is essential for production and higher entrepreneurial ability serves as Hicks-neutral technical progress, in the sense that the firm owner/manager is more efficient in combining the variable factors of production.

Final good sector. The final good sector is assumed to be perfectly competitive resulting in zero economic profit for the representative firm, so there is no need to specify its ownership structure. All intermediate goods provided by active entrepreneurs are aggregated into the production of a homogeneous consumption good—the numéraire, which is consumed by everyone in the economy—according to a standard CES technology,

$$Y_{t+1} = A\left(\iint\limits_{\mathcal{P}\times\mathcal{Z}} y_{t+1}(z)^{\theta}\, dG(\rho, z, \mathcal{E})\right)^{\frac{1}{\theta}} \tag{3.2}$$

The solution to the cost minimization problem of the representative firm yields

$$p_{t+1}(z) = A^{\theta}\left[\frac{y_{t+1}(z)}{Y_{t+1}}\right]^{-(1-\theta)} \tag{3.3}$$

The price of intermediate good i produced with ability z, $p_{t+1}(z)$, is inversely related to the relative demand $\frac{y(z)}{Y}$, subject to a constant elasticity of demand equal to $-\frac{1}{1-\theta}$. We show below that the maximized level of output is strictly increasing in z, so more productive firms have lower marginal costs and can compete by charging a lower price for their product.

Financial market. The banking sector consists of financial intermediaries that receive deposits from workers and provide loans to potential entrepreneurs, without any operational costs. Under perfect capital markets and limited liability, the zero profit condition determines the (gross) loan rate (δ) to simply be a markup of the (gross) deposit rate (r_t), depending on the entrepreneurial success rate: $\delta_t = \frac{r_t}{q}$.

3.2.3 Entrepreneurs, Workers, and Occupational Choice

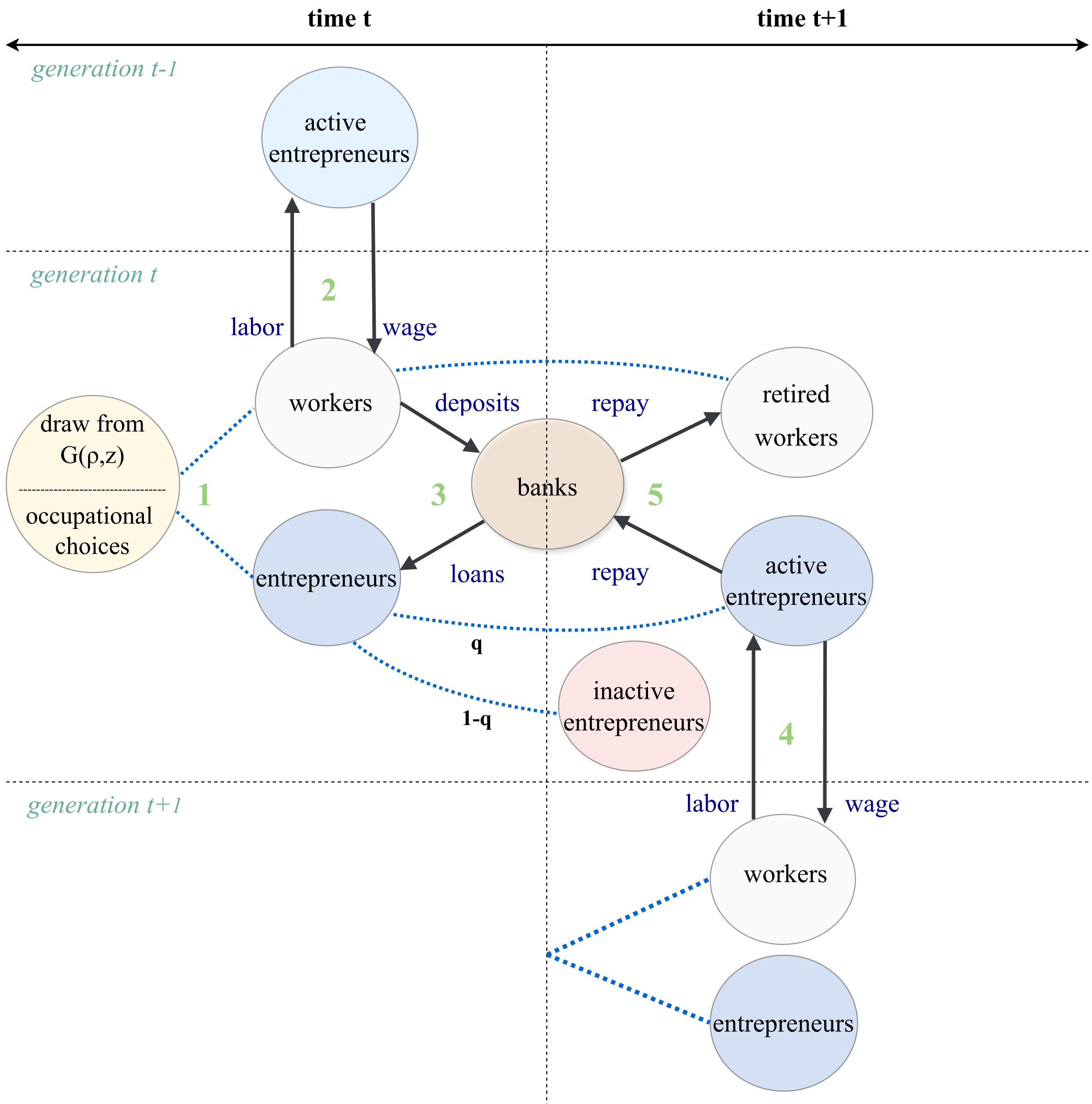
Entrepreneurs. The decision by an entrepreneur i of generation t at period $t+1$ is specified as follows. Given the effective wage rate w_{t+1}, the market loan rate δ_t, as well as quantities $k_{t+1}(z)$, Y_{t+1} and h_t, she determines her demand for labor by solving

$$\max_{\ell_{t+1}(z)} \pi_{t+1}(z) = p_{t+1}(z)y_{t+1}(z) - w_{t+1}\ell_{t+1}(z)h_{t+1} - \delta_t k_{t+1}(z)$$
$$\text{subject to (3.3)}$$

The necessary and sufficient first-order condition implies

$$\ell_{t+1}(z)h_{t+1} = \left[z^\theta \left(\theta A^\theta Y_{t+1}^{1-\theta}\right)\left(\frac{1-\alpha}{w_{t+1}}\right) k_{t+1}(z)^{\alpha\theta}\right]^{\frac{1}{1-(1-\alpha)\theta}} \tag{3.4}$$

Figure 3.1: The Main Structure of the Model Economy



Employed capital has a positive effect on labor demand due to factor complementarity (in the Pareto sense). The above can be substituted into (3.1) and the profit function to derive

$$y_{t+1}(z) = \left[z \left(\theta A^{\theta} Y_{t+1}^{1-\theta} \right)^{1-\alpha} \left(\frac{1-\alpha}{w_{t+1}} \right)^{1-\alpha} k_{t+1}(z)^{\alpha} \right]^{\frac{1}{1-(1-\alpha)\theta}} \tag{3.5}$$

$$\pi_{t+1}(z) = \Big[1-(1-\alpha)\theta \Big] \left[z^{\theta} \left(\theta A^{\theta} Y_{t+1}^{1-\theta} \right) \left(\frac{1-\alpha}{w_{t+1}} \right)^{(1-\alpha)\theta} k_{t+1}(z)^{\alpha\theta} \right]^{\frac{1}{1-(1-\alpha)\theta}} - \delta_t k_{t+1}(z) \tag{3.6}$$

We now go back one step to determine the entrepreneur's loan demand when young. Consider an entrepreneur with an inalienable business idea i and with risk attitude ρ whose optimization problem at this stage is

$$\max_{k_{t+1}(z)} \mathbb{E}\left[U(c_{t+1}) \right] = q \cdot (\pi_{t+1}(z))^{1/(1+\rho)} + (1-q) \cdot 0 \tag{3.7}$$

We rearrange the first-order condition to obtain the loan demand function

$$k_{t+1}(z) = \left[z^{\theta} \left(\theta A^{\theta} Y_{t+1}^{1-\theta} \right) \left(\frac{1-\alpha}{w_{t+1}} \right)^{(1-\alpha)\theta} \left(\frac{\alpha}{\delta_t} \right)^{1-(1-\alpha)\theta} \right]^{\frac{1}{1-\theta}} \tag{3.8}$$

and accordingly the labor demand function

$$\ell_{t+1}(z) h_{t+1} = \left[z^{\theta} \left(\theta A^{\theta} Y_{t+1}^{1-\theta} \right) \left(\frac{1-\alpha}{w_{t+1}} \right)^{1-\alpha\theta} \left(\frac{\alpha}{\delta_t} \right)^{\alpha\theta} \right]^{\frac{1}{1-\theta}} \tag{3.9}$$

A higher aggregate demand for the final good or a lower labor cost raises profitability and thus loan demand, whereas an increase in the loan rate reduces it. Notice that $k_{t+1}(z)$ is independent of ρ, which means all entrepreneurs of the same generation with identical ability will borrow the same amount from banks. From (3.5), (3.6), and (3.8), the amount of intermediate good i produced and the corresponding profit become

$$y_{t+1}(z) = \left[z\left(\theta A^{\theta} Y_{t+1}^{1-\theta}\right) \left(\frac{1-\alpha}{w_{t+1}}\right)^{1-\alpha} \left(\frac{\alpha}{\delta_t}\right)^{\alpha} \right]^{\frac{1}{1-\theta}} \tag{3.10}$$

$$\pi_{t+1}(z) = \frac{1-\theta}{\theta} \left(\frac{\delta_t}{\alpha}\right) k_{t+1}(z) \tag{3.11}$$

The expected utility of an entrepreneur of type (ρ, z) corresponds to

$$\mathbb{E}[U^E(i, z, \rho)] = q \ (\pi(z)_{t+1})^{1/(1+\rho)} \tag{3.12}$$

which is clearly strictly decreasing in ρ, and strictly increasing in z since $\theta \in (0, 1)$.

Workers. Upon becoming a worker, an agent's decision is trivial: she works full time and deposits her entire wage income into a bank for consumption in old age. Her expected utility corresponds to

$$\mathbb{E}\left[U^W(i)\right] = (r_t w_t h_t)^{1/(1+\rho)} = (q\delta_t w_t h_t)^{1/(1+\rho)} \tag{3.13}$$

where the deposit rate is $r_t = q\delta_t$ under the zero profit condition of the banking sector.

Occupational choice. The occupational decision between workers and entrepreneurs comes down to comparing the indirect utilities given by (3.12) and (3.13).

For a fixed level of risk aversion ρ, the ratio $\frac{\mathbb{E}[U^e]}{\mathbb{E}[U^w]}$ is continuous and strictly increasing in z from zero to infinity. Accordingly, for a fixed productivity level $z > \underline{z}$ where $\underline{z}$ ensures that the LHS is larger than the RHS for $\rho = 0$, the ratio $\frac{\mathbb{E}[U^e]}{\mathbb{E}[U^w]}$ is continuous and strictly decreasing in ρ, ranging from a constant greater than one to $q < 1$.[45] It follows that there exists a unique pair of critical values $\left(\rho^D(z), z^D(\rho)\right), \forall(\rho, z) \in \mathcal{P} \times \mathcal{Z}$, such that

$$q(\pi_{t+1}(i, z^D))^{1/(1+\rho^D)} = (q\delta_t w_t h_t)^{1/(1+\rho^d)} \tag{3.14}$$

Individuals of joint type $(\rho < \rho(z)^d, z > z(\rho)^d)$ choose to be entrepreneurs each period. Apart from agents who are sufficiently productive and have a large absolute advantage in starting a firm, the economy will also feature a (potentially sizeable) mass of only moderately productive and less risk-averse agents. This is a source of misallocation in the economy, both on the intensive and extensive margin. A further characterization together with the determination of the measure of active entrepreneurs, N_{t+1}^d, is presented below.[46]

3.2.4 Human Capital Accumulation

To close the model we need to specify the human capital accumulation process. As the focus of our paper is on risk attitude and entrepreneurship, there is no need for further complication on that front. We simply assume that the human capital stock evolves according to

$$h_{t+1} = Y_t^{\beta} h_t^{1-\beta} \tag{3.15}$$

That is, total output of the final product is used as a proxy for aggregate current activity, which in turn contributes to the accumulation of human capital. Since real GDP is taken as given by entrepreneurs, the analysis is substantially simplified.

[45] The case $z < \underline{z}$ means that no agent with such z will choose entrepreneurship, as the cutoff for ρ is negative.

[46] Throughout the paper we use the superscript "d" to denote solutions in the decentralized economy. Accordingly, the superscript "c" will denote planner solutions in the centralized economy.

3.3 Competitive Equilibrium

We are now ready to study the equilibrium in the decentralized economy. We start by obtaining the loan and labor market clearing conditions. There is no need to consider the goods market separately as it automatically clears by virtue of Walras's law.

3.3.1 Market Clearing Conditions

The total demand for loans is simply the integral across all entrepreneurial loan demands, which can be also expressed as the measure of entrepreneurs (in the labor force) times the average amount of loan demanded within entrepreneurs. The total supply of loans comes from the wage income of workers. The loan market clearing condition is specified as

$$\iint_{\mathcal{P}\times\mathcal{Z}} k_{t+1}(z)\, dG(\rho, z, \mathcal{E}) = \iint_{\mathcal{P}\times\mathcal{Z}} w_t h_t\, dG(\rho, z, \mathcal{W})$$

$$N^D_{t+1}\bar{k}_{t+1} = (1 - N^D_{t+1}) w_t h_t, \forall t \geq 0 \tag{3.16}$$

where $\bar{k}_{t+1} = \iint k_{t+1}(z)\, dG(\rho, z|\mathcal{E})$ is the *average* firm capital.[47] Similarly for the demand for labor, with the difference that only those entrepreneurs who succeeded in transforming loans into capital goods (qN^D_t) can hire labor to undertake production. The labor market clearing condition thus becomes

$$q \iint_{\mathcal{P}\times\mathcal{Z}} \ell_t(z)\, h_t\, dG(\rho, z, \mathcal{E}) = \iint_{\mathcal{P}\times\mathcal{Z}} h_t\, dG(\rho, z, \mathcal{W})$$

$$qN^D_t \bar{\ell}_t = 1 - N^D_{t+1}, \forall t \geq 0. \tag{3.17}$$

[47] Average quantities are obtained by integrating with respect to the conditional distribution after occupational choices have been made, i.e., $\mathcal{E} := \{(\rho_i, z_j) \in \mathcal{P}\times\mathcal{Z} : \big(\rho_i < \rho^d(z_j)\big) \wedge \big(z_j > z^d(\rho_i)\big)\}$.

where $\bar{\ell}_t$ is the *average* firm size by employment. Note that the time subscript on the RHS is $t+1$ because the number of entrepreneurs in the next period is determined by occupational choice in the current period. We proceed by defining the dynamic competitive equilibrium.

Definition. *Given a non-singular joint distribution of risk attitude and entrepreneurial productivity,* $G(\rho, z)$, initial stock of human capital, h_0, *the measure of initial successful entrepreneurs* qN_0, *and the initial average amount of capital they hire,* $(\bar{k}_0)$, *a* ***dynamic competitive equilibrium*** *is a collection of quantity sequences* $\{Y_t, \bar{\ell}_t, \bar{k}_{t+1}, x_t, h_{t+1}, N^D_{t+1}\}_{t=0}^{\infty}$, *and a collection of price sequences* $\{w_t, r_t, p_t\}_{t=0}^{\infty}$, *such that:*

1. *given prices and endowments, every agent maximizes her expected utility for all* $t \geq 0$;
2. *an agent of type* (ρ, z) *born in period* t *chooses to become an entrepreneur if and only if* $\rho \leq \rho^d_t(z)$ and $z \geq z^d_t(\rho)$, where ρ^d_t and z^d_t *are determined by* (3.14)
3. *the measure of entrepreneurs is* $N^D_{t+1} = \int_0^{\rho^d_t} \int_{z^d_t}^{\infty} dG(\rho, z)$, *and a fraction* q *of them succeeds;*
4. *human capital evolves according to (3.15) for all* $t \geq 0$;
5. *the labor, capital, and goods markets clear at all* $t \geq 0$

We thereby focus on perfect-foresight ***balanced growth equilibria*** in which the real variables, $Y, \bar{k}$, and h, all grow at constant rates, and N^d, $\bar{\ell}$, p, w,, r, $\widetilde{\rho}^D$, and $\widetilde{z}^D$, are all constant.

3.3.2 Occupational Choice

We are now ready to obtain the cutoff level for each pair of risk aversion/productivity that completely determines each agent's occupational choice. As shown below, $\rho^D_t(z)$ is unique and time invariant for each agent of type $(\rho, z) \in \mathcal{P} \times \mathcal{Z}$. By utilizing $\pi_{t+1}(z)$ in (3.11), the occupational choice condition (3.14) reads

$$q^{\rho^d} \frac{1-\theta}{\alpha\theta} k_{t+1}(z) = w_t h_t, \; \forall t \tag{3.18}$$

which can be further combined with (3.8) and (3.16) to yield

$$q^{\rho^d} \frac{1-\theta}{\alpha\theta} z^{\frac{\theta}{1-\theta}} = \frac{N_{t+1}^D}{1-N_{t+1}^D} \mathbb{E}_t \left[z^{\frac{\theta}{1-\theta}} | \mathcal{E} \right], \; \forall t \tag{3.19}$$

where the expectation is taken with respect to the joint distribution conditional on agents being entrepreneurs, $G(\rho, z|\mathcal{E})$. Since the LHS of (3.19) is strictly decreasing from $\frac{1-\theta}{\alpha\theta}$ to 0 and the RHS is strictly increasing in ρ from 0 to infinity, $\rho_t^d(z)$ is uniquely determined $\forall z \in \mathcal{Z}$ and is time-invariant. Thus, the number of entrepreneurs does not change over time.

3.3.3 Dynamics and the Balanced Growth Path

Since the number of people who choose to become entrepreneurs in each generation is the same, it turns out that the dynamics of this economy hinge solely on the average physical-to-human capital ratio. To express total output in terms of $(\bar{k}_t/h_t)$ for any $t \geq 0$, start by integrating across all individual production plans in the final good production function

$$Y_t = A \left(qN^d\right)^{\frac{1}{\theta}} \left(\iint\limits_{\mathcal{T}_\mathcal{E} \times \mathcal{Z}_\mathcal{E}} z^{\frac{\theta}{1-\theta}} \, dG(\rho, z|\mathcal{E}) \right)^{\frac{1}{\theta}} \left[\left(\theta A^\theta Y_{t+1}^{1-\theta}\right) \left(\frac{1-\alpha}{w_{t+1}}\right)^{1-\alpha} \left(\frac{\alpha}{\delta_t}\right)^{\alpha} \right]^{\frac{1}{1-\theta}}$$

By manipulating the above equation and using the labor market clearing condition, we can express total output per unit of human capital as[48]

$$
\begin{aligned}
Y_t &= A \left(\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E} \right] \right)^{\frac{1-\theta}{\theta}} \left(qN^d\right)^{\frac{1}{\theta}} \left(\bar{\ell}_t \, h_t\right)^{1-\alpha} \left(\bar{k}_t\right)^{\alpha} \\
\frac{Y_t}{h_t} &= A \, \mathcal{M}_{\vartheta} \left(qN^d\right)^{\frac{1}{\theta}} \left(\frac{1-N^d}{qN^d} \right)^{1-\alpha} \left(\frac{\bar{k}_t}{h_t} \right)^{\alpha}
\end{aligned}
\tag{3.20}
$$

Apart from the exogenous level term A, the Solow residual is no longer a measure of our ignorance in this economy. Specifically, it consists of two endogenous quantities: a composite extensive-margin term shaped by the measure of entrepreneurs and workers; and an intensive-margin/TFP term, $\mathcal{M}_{\vartheta} := \left(\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E} \right] \right)^{\frac{1-\theta}{\theta}}$, which is in fact a Hölder mean (also known as generalized mean) with exponent $\vartheta := \frac{\theta}{1-\theta}$ of active entrepreneurs' productivities, weighted by the conditional joint distribution of (ρ, z) on the support $\mathcal{T}_{\mathcal{E}} \times \mathcal{Z}_{\mathcal{E}}$. These are indeed key quantities, as they highlight the distributional consequences of occupational choice on aggregate productivity. Total income is determined both by how many entrepreneurs are active in the economy, as well as by what type of entrepreneurs they actually are.

The next step is to show how the market-clearing effective wage rate is related to the ratio of the two state variables, k and h. Equation (3.20) can be combined with (3.4) and (3.17) to derive

$$
\begin{aligned}
w_t &= A^{\theta} \theta \left(1-\alpha\right) \left(\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E} \right] \right)^{1-\theta} \left(\frac{1-N^d}{qN^d} \right)^{-1+(1-\alpha)\theta} \left(\frac{Y_t}{h_t} \right)^{1-\theta} \left(\frac{\bar{k}_t}{h_t} \right)^{\alpha\theta} \\
&= A \, \theta \left(1-\alpha\right) \mathcal{M}_{\vartheta} \left(qN^d\right)^{\frac{1-\theta}{\theta}} \left(\frac{1-N^d}{qN^d} \right)^{-\alpha} \left(\frac{\bar{k}_t}{h_t} \right)^{\alpha}
\end{aligned}
\tag{3.21}
$$

[48] Throughout the derivations, it is clear that we can switch from multiple to repeated integration by the Fubini-Tonelli theorem, given the σ-finiteness of probability spaces and measurability of functions.

Following a similar procedure we can derive the market-clearing deposit/loan rate

$$\delta_t = r_t/q = A\,\theta\,\alpha\,\mathcal{M}_\vartheta \left(qN^d\right)^{\frac{1-\theta}{\theta}} \left(\frac{1-N^d}{qN^d}\right)^{1-\alpha} \left(\frac{\bar{k}_t}{h_t}\right)^{\alpha-1} \tag{3.22}$$

Turning to the dynamics of the economy, the task is to analyze the path of the ratio of the two states. Total wage earnings at period t will be loaned to generation-t entrepreneurs. From the loan market clearing condition we have that

$$\frac{\bar{k}_{t+1}}{h_{t+1}} = \frac{1-N^d}{N^d}\frac{h_t}{h_{t+1}}\,w_t \tag{3.23}$$

After substituting in the human capital evolution equation (3.15) together with (3.20) and (3.21), the dynamic ratio of average physical to human capital becomes

$$\begin{aligned}\frac{\bar{k}_{t+1}}{h_{t+1}} &= \frac{1-N^d}{N^d}\left(\frac{Y_t}{h_t}\right)^{-\beta} w_t \\ &= \left[A^{1-\beta}\,\theta\,(1-\alpha)\,\mathcal{M}_\vartheta^{1-\beta}\left(qN^d\right)^{\frac{1-\theta-\beta}{\theta}}\left(\frac{1-N^d}{N^d}\right)\left(\frac{1-N^d}{qN^d}\right)^{-\alpha-\beta+\alpha\beta}\right]\left(\frac{\bar{k}_t}{h_t}\right)^{\alpha(1-\beta)}\end{aligned}$$

It follows from the above equation that, for any given initial conditions, $\frac{\bar{k}_t}{h_t}$ will converge in finite time to its balanced growth value

$$\left(\frac{\bar{k}}{h}\right)^{BGP} = \left[A^{1-\beta}\theta\,(1-\alpha)\,\mathcal{M}_\vartheta^{1-\beta}\,(qN^d)^{\frac{1-\theta-\beta}{\theta}}\left(\frac{1-N^d}{N^d}\right)\left(\frac{1-N^d}{qN^d}\right)^{-\alpha-\beta+\alpha\beta}\right]^{1/(1-\alpha+\alpha\beta)} \tag{3.24}$$

Once $(\frac{\bar{k}}{h})^{BGP}$ is reached, the system is on the balanced growth path where Y_t, h_t and k_t all grow at the same constant rate g^d:

$$1 + g^d = \frac{h_{t+1}}{h_t} = \left(\frac{Y_t}{h_t}\right)^{\beta}$$
$$1 + g^d = \left[A\left(q\theta(1-\alpha)\right)^{\alpha}\mathcal{M}_{\vartheta}\left(qN^d\right)^{\frac{1-\theta}{\theta}}(1-N^d)^{1-\alpha}\right]^{\beta/(1-\alpha+\alpha\beta)} \tag{3.25}$$

We can thus conclude that any competitive balanced-growth equilibrium features a non-monotone relationship between economic growth and the rate of entrepreneurship. In addition, the attained growth rate is suboptimal with probability one. We summarize these findings below.

Proposition 1. *In a decentralized equilibrium, encouraging entrepreneurship may or may not promote balanced growth. Moreover, if the joint distribution $G(\rho, z)$ is strictly monotone on all measurable sets of Ω, the attained balanced growth rate is suboptimal almost surely. Specifically,*

$$\frac{dg^d}{dN^d} \propto \underbrace{\frac{1-\theta}{\theta}\frac{1}{N^d}}_{\text{variety effect}} - \underbrace{\frac{1-\alpha}{1-N^d}}_{\substack{\text{loanable fund}\\ \text{supply effect}}} + \underbrace{\frac{1-\theta}{\theta}\frac{\frac{\partial}{\partial N^d}\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^d\right]}{\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^d\right]}}_{\text{TFP effect}} \gtrless 0 \quad a.s. \tag{3.26}$$

In the heart of Proposition 1 lies the fact that increasing the number of entrepreneurs gives rise to three opposing effects, as suggested by (3.26). On the one hand, more entrepreneurs means that greater intermediate product variety can be achieved, whose importance is gauged by $\frac{1-\theta}{\theta}$. We call this the *variety effect*. On the other hand, more entrepreneurs means less workers, and since workers are net savers, capital formation is being reduced. The labor income share $(1-\alpha)$ measures the importance of the *loanable fund supply effect*. Furthermore, the heterogeneity in (ρ, z) shapes the average productivity of active entrepreneurs in the economy increasing N^d induces a negative *TFP effect*.

We have already shown that occupational choices result in a positive relationship between risk aversion and entrepreneurial ability, hence the decentralized allocation will necessarily involve a potentially large mass of less risk-averse agents with lower productivity. Put differently, due to risk aversion, $\mathcal{M}_\vartheta$ will not be (conditionally) maximal almost surely, even if the variety and loanable fund supply effects exactly offset each other. thereby deviating from the (constrained) optimal balanced growth rate.

3.4 Centralized Economy and Insurance Markets

In this section we analyze the differences between the number of entrepreneurs, endogenous TFP, and the balanced growth rate attained in the decentralized equilibrium and those obtained in a centralized economy. In addition, we examine the long-run implications of an actuarially fair insurance market for entrepreneurial risk.

3.4.1 A Constrained Central Planning Problem

Consider a central planner who wishes to maximize the long-run growth rate of the economy. The choices consist of a sequence of allocations $\{k_t^C(z), \ell_t^C(z)\}_{t=0}^{\infty}$, together a time-invariant measure N^c and the set of entrepreneurs $\mathcal{E}^c$, under the constraint that the saving rate is equal to the worker income share, $\theta(1-\alpha)$, as in the decentralized economy.[49] The latter condition constitutes a realistic perspective. If there is a government/institutional policy that allows the decentralized economy to achieve (or bring it closer to) the constrained

[49] Note that although the labor share of aggregate income is $(1-\alpha)$, this accounting identity attributes a "labor"' income component to entrepreneurial profits, which is a fraction $(1-\theta)$ of total income. Therefore, the worker share of income is $\theta(1-\alpha)$.

planner's allocation, then it is the preferred policy choice. In such a case, what N^c and $\mathcal{E}^c$ would the planner pick, and how do they relate to N^d and $\mathcal{E}^d$?

The problem is solved in two stages. First, the planner chooses the level of capital $k_t^C(z)$ and labor $\ell_t^C(z)$ to be hired by each firm operated by z-type agents, keeping N^c and $\mathcal{E}^c$ fixed; second, he effectively picks N^c and $\mathcal{E}^c$, to satisfy the optimality condition for the balanced growth rate. Since the planning problem involves the maximization of a value *functional* over a suitable Banach space, it is accordingly formulated as a variational calculus program:

$$\max_{\substack{k_{t+1}(z)\ell_{t+1}(z),\\ N^c(\rho,z),\mathcal{E}^c(\rho,z)}} A^{\beta}\left(\iint\limits_{\mathcal{P}\times\mathcal{Z}} z^{\theta}\left(k_{t+1}(z)/h_{t+1}\right)^{\alpha\theta}\ell_{t+1}(z)^{(1-\alpha)\theta}\,dG(\rho,z,\mathcal{E}^c)\right)^{\frac{\beta}{\theta}} \tag{3.27}$$

$$\text{subject to } \iint\limits_{\mathcal{P}\times\mathcal{Z}} k_{t+1}(z)\,dG(\rho,z,\mathcal{E}^c) = \theta(1-\alpha)Y_t \tag{3.28}$$

$$q\iint\limits_{\mathcal{P}\times\mathcal{Z}} \ell_{t+1}(z)\,dG(\rho,z,\mathcal{E}^c) = 1-N^c \tag{3.29}$$

Proposition 2. *In the centralized economy, the optimal balanced growth rate corresponds to*

$$1+g^c = \left[A\left(q\theta(1-\alpha)\right)^{\alpha}\mathcal{M}_{\vartheta}^c\left(qN^c\right)^{\frac{1-\theta}{\theta}}\left(1-N^c\right)^{1-\alpha}\right]^{\beta/(1-\alpha+\alpha\beta)} \tag{3.30}$$

where the optimal number of entrepreneurs satisfies

$$\underbrace{\frac{1-\theta}{\theta}\frac{1}{N^c}}_{\text{variety effect}} - \underbrace{\frac{1-\alpha}{1-N^c}}_{\substack{\text{loanable fund}\\ \text{supply effect}}} + \underbrace{\frac{1-\theta}{\theta}\frac{\frac{\partial}{\partial N^c}\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^c\right]}{\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^c\right]}}_{\substack{\text{TFP effect}\\ (<0)}} = 0 \quad a.s. \tag{3.31}$$

$$N^c = \frac{\Phi^c(1-\theta)}{(1-\alpha)\theta+\Phi^c(1-\theta)},\ \ \Phi^c := \left(\frac{\overset{*}{z}}{\mathcal{M}_{\vartheta}^c}\right)^{\frac{\theta}{1-\theta}} \tag{3.32}$$

The central decision maker must choose N^c so that the sum of the three effects exactly offset each other. Specifically, he will set N^c such that the variety effect is appropriately higher than the loanable fund supply effect in order to compensate for the necessarily negative TFP effect. In addition, the planner will choose the most able entrepreneurs (highest z) conditional on their total number being N^c, thereby eliminating misallocation on the intensive margin.

In the decentralized equilibrium, why does N^d fail to satisfy the optimality condition? To ease the comparison we can rewrite (3.19) and rearrange (see ??) to get

$$\frac{q^{\rho^d}(1-\theta)}{\theta}\frac{1}{N^d} = \frac{\alpha}{1-N^d} \tag{3.33}$$

$$N^d = \frac{q^{\hat{\rho}}\Phi_j^d(1-\theta)}{\alpha\theta + \Phi_j^d(1-\theta)}, \quad \Phi_j^d := \left(\frac{\hat{z}_j}{\mathcal{M}_\vartheta}\right)^{\frac{\theta}{1-\theta}} \tag{3.34}$$

We see that N^d differs from N^c in three distinct ways. First, individual decision-makers discount the importance of the variety effect due to the presence of risk aversion. The impact of this channel leads to lower-than-optimal entrepreneurship in the decentralized equilibrium, as N^d tends to be smaller than N^c ($q^{\hat{\rho}} < 1$). Second, exactly because agents' risk aversion impacts their occupational decisions, it is clear that $\Phi_j^d \neq \Phi^c$ capturing misallocation on the intensive margin. Third, note the term $\alpha\theta$ in the denominator of N^d instead of $(1-\alpha)\theta$, which suggests that there will be misallocation on the extensive margin as long as $\alpha \neq 1/2$, even if there is an actuarially fair market for entrepreneurial risk. This is the issue which we now turn to.

3.4.2 Actuarially Fair Insurance Market

Suppose there exists an *actuarially fair* insurance market for entrepreneurial risk in the decentralized economy: a large number of competitive risk-neutral insurance companies are willing to insure potential producers against the risk of failing to transform their loans into productive capital. In equilibrium, the zero profit condition leads to an insurance price equal to $p^{FI} = 1 - q$, and all entrepreneurs will choose to be fully insured.

The occupational choice condition in that case can be expressed as if agents were risk-neutral,

$$\pi_{t+1}(z) = \delta_t w_t h_t \tag{3.35}$$

Then the number of entrepreneurs in the decentralized economy with full insurance satisfies,

$$\frac{1-\theta}{\alpha\theta}\,\bar{z}^{\frac{\theta}{1-\theta}} = \frac{N^{FI}}{1-N^{FI}}\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^{FI}\right] \tag{3.36}$$

Since the above is independent of ρ, the cutoff $\bar{z}$ is unique and applies to every agent in the economy. We are now ready to compare the rate of entrepreneurship rate in the decentralized equilibrium without insurance to that of the decentralized economy with full insurance, as well as to the constrained-first-best allocation in the centralized economy.

Proposition 3. *The equilibrium allocations in a decentralized economy with actuarially fair insurance markets, and thus full insurance against entrepreneurial risk, results in more entrepreneurs than one without such a market:* $N^{FI} > N^d$ *a.s.*

By eliminating entrepreneurial risk, full insurance always encourages entrepreneurship. However, observe that the importance of the loanable fund effect from the individuals' viewpoint amounts to α, not $(1-\alpha)$. The reason for this distortion is an intergenerational externality: becoming a worker contributes to current total output, which in turn raises the human capital of the next generation. The current generation internalizes only part of this externality since a higher level of human capital complements physical capital owned by current-period workers, thus individual decision-makers value the importance of the loanable fund supply effect by α instead of $1-\alpha$. Interestingly, the prevailing level of the capital income share is necessary and sufficient in determining whether N^{FI} will be higher or lower than N^c

Proposition 4. *The equilibrium allocations in a decentralized economy with actuarially fair insurance markets, i.e. full insurance against entrepreneurial risk, results in more (less) entrepreneurs than the centralized economy, if and only if α is less (greater) than $1/2$:* $N^{FI} > N^c \iff \alpha < 1/2$ *a.s.;* $N^{FI} < N^c \iff \alpha > 1/2$ *a.s.*

An interesting case is when $N^d < N^c < N^{FI}$. In that case it is straightforward to combine the above Propositions to show that providing *some* insurance to entrepreneurial risk in a decentralized economy without an insurance market can be growth-enhancing, whereas providing *too much* insurance can be growth-retarding.

Proposition 5. *It is never optimal to provide full insurance. Moreover, if $\alpha < 1/2$,*

(i) *when the decentralized economy features less entrepreneurs than the centralized economy, i.e., $N^d < N^c$, providing some insurance to entrepreneurial risk in a decentralized economy without an insurance market is growth-enhancing;*

(ii) *when the decentralized economy features more entrepreneurs than the centralized economy, i.e., $N^d > N^c$, any provision of insurance to entrepreneurial risk in a decentralized economy without an insurance market is growth-retarding.*

Astebro (2003) provides empirical evidence that potential entrepreneurs can be overly optimistic to invest in less-desirable projects. In our paper, full insurance that removes entrepreneurial risk can align private and social marginal benefits but fail to correct the undervaluation of the private marginal costs. When the capital income share is less than the labor income share, the decentralized equilibrium under full insurance features too much entrepreneurship. Thus, too much insurance results in too much optimism. Our finding therefore offers a plausible theoretical explanation for the empirical evidence specified above.

3.5 Further Characterization of the Balanced Growth Equilibrium

The results up to now allow us to investigate how the cutoff degree of risk-aversion ρ^d and the equilibrium number of entrepreneurs N^d respond to a change in the probability of success q or in the marginal distribution of risk attitude, $G(\rho)$.

3.5.1 Changes in the Probability of Success

Consider an increase in the probability of entrepreneurial success (q). A greater q will raise the RHS of (3.19) without affecting the LHS. The result is obvious: a greater q leads to a higher cutoff degree of risk-aversion and more entrepreneurs. This result is also intuitive: a greater chance of success makes entrepreneurs a more attractive occupation.

Will a higher q raise the balanced growth rate? Intuitively, raising q should be growth-enhancing for two reasons. First, a higher fraction of the savings can be transformed into productive physical capital. Second, a greater variety of the intermediate goods can be produced under a higher q and the variety effect is growth-enhancing. We can see these two direct effects from the terms (q^α) and $(qN^d)^{\frac{1-\theta}{\theta}}$ in (3.30), respectively. There, however, is an indirect effect through changing the number of entrepreneurs. As shown in Proposition 1, there is no definite relationship between number of entrepreneurs and growth. If there are already too many entrepreneurs in the decentralized economy, more entrepreneurs resulting from a higher q will further lower the growth rate. Taking both the direct and the indirect effects into account, a higher q may or may not lead to a higher growth rate. In summary, we have:

Proposition 6. *An increase in the entrepreneurial probability of success q leads to higher threshold degrees of risk aversion ρ^d, and thus more entrepreneurs N^d. Its effect on the balanced growth rate is, however, ambiguous.*

3.5.2 Changes in the Risk-Attitude Distribution

We are particularly interested in changes in the risk-attitude distribution in two specific ways: the new distribution is a mean-preserving spread of $G(\rho)$, and the new distribution first-order stochastically dominates $G(\rho)$. It should be noted that, unlike changing q, modifying G does not have direct effects on the balanced growth rate; it does have, however, a potentially sizeable indirect impact through occupational choices and general equilibrium effects.

Exercise 1: First-Order Stochastic Dominance ($G \to G^{FOSD}$)

Under distribution F^{FOSD} which first-order stochastically dominates G, the LHS of (3.19) will be lowered for any given ρ. Once again, our intuition is confirmed: If agents in an economy become more risk averse, there will be fewer agents choosing to become entrepreneurs and the the cutoff degree of risk-aversion will become higher.

Exercise 2: Mean-Preserving Spread ($G \to G^{MPS}$)

Since G^{MPS} is a mean-preserving spread of G, $G^{MPS}(\rho)$ is greater (smaller) than $G(\rho)$ all for ρ smaller (greater) than μ_ρ, the statistical mean. Consequently, graphing the function $\Psi(N) \equiv \frac{N}{1-N}$ under G^{MPS} one can see that it crosses the one under G from above at ρ^*. The effect of a mean-preserving spread on the equilibrium number of entrepreneurs depends on whether the cutoff degree is greater or smaller than the mean. Think a mean-preserving spread as changing the mass of people who are around-the-mean-risk-averse to either less or more risk-averse. In the case of $\rho^d > \mu_\rho$, the change that makes people less risk-averse will not create many new entrepreneurs because most of them would have become entrepreneurs anyway. Some people who are made more risk-averse, however, may change their occupational choice from entrepreneurs to wage workers. The net effect in this case should be less entrepreneurs If $\rho^d < \mu_\rho$, this argument is reversed and the number of entrepreneurs will increase. The next proposition summarizes the findings obtained in the two exercises above.

Proposition 7. *(Changing the underlying distribution of risk attitude)*

(i) *If the new distribution first-order stochastically dominates the old one, there will be more entrepreneurs but the effect on the balanced growth rate is ambiguous.*

(ii) *If the new distribution is a (non-trivial) mean-preserving spread of the old one, the number of entrepreneurs will decrease if $\rho^d > \mu_\rho$ and will increase if $\rho^d < \mu_\rho$. In either case, the effect of the mean-preserving spread on the balanced growth rate is ambiguous.*

3.6 Calibrating the Model to U.S. Data and Quantifying Misallocation

This section describes our calibration strategy and presents quantitative evidence in favor of the potentially large misallocation losses predicted by our theory. The decentralized model economy is calibrated to post-war U.S. time series and cross-sectional establishment-level data coming from the U.S. Census Business Dynamics Statistics (BDS) for the period $1978 - 2019$. The length of one model period/generation is taken to be 25 years.[50]

3.6.1 Parametrization, Calibration, and Baseline Model Output

Marginal and joint densities. Each agent is characterized by a realization of the random vector (ρ, z), for which we need to determine an invariant joint distribution. A simple and transparent way to do is by first specifying each marginal density.

In line with a plethora of studies on entrepreneurship and firm dynamics, the distribution of entrepreneurial ability (z) is assumed to follow a Pareto distribution with scale parameter equal to one and shape parameter $\eta > 0$; its corresponding probability density is $g_z(z) = \eta z^{-(\eta+1)}$. In our numerical analysis we consider the finite support $[1, z_{max}]$, discretized on 600 equispaced grid points, with the upper bound set such that $G_z(z_{max}) = 0.99991$.

Regarding the distribution of risk attitude (ρ), one would naturally want to consider functions with non-negative support that result in modes/medians being towards relatively low values of ρ. A reasonable assumption is the Lognormal distribution, i.e, $\log \rho \sim \mathcal{N}(\mu_\rho, \sigma_\rho^2)$. In our numerical analysis we consider the support $[\rho_{min}, t_{max}]$, discretized on 600 equispaced grid points, with the bounds set such that $G_\rho(\rho_{min}) = 0.00001$. and $G_\rho(\rho_{max}) = 0.95$.

To pin down the joint distribution in the baseline calibration, we simply assume that the two random variables are independent at the population level, such that the joint density of entrepreneurial ability and risk aversion becomes $g_{\rho,z}(\rho, z) = g_\rho(\rho)\, g_z(z)$. In a following subsection we examine cases where the random vector does exhibit *a priori* statistical dependence by coupling the marginals into the joint via the use of parametric copulas.

[50] Accordingly, annualized variables, e.g., real GDP growth and interest rates, are calculated as $x^{1/25} - 1$.

Assigned parameter. The parameter θ governing the elasticity of substitution among intermediate goods produced by entrepreneurs, given by $\sigma = \frac{1}{(1-\theta)}$, is externally calibrated. We set θ to 2/3, or equivalently, $\sigma = 3$, which is close to the median values of σ estimated by Broda and Weinstein (2006) across 4-digit industries, as well as across different levels of disaggregation.[51]

Calibrated parameters. There remain seven parameters to be jointly calibrated for the model to best fit seven relevant moments in the data. In particular, the vector under consideration is $\{A, \alpha, \beta, q, \eta, \mu_\rho, \sigma_\rho\}$. Although strong local first-order identification is rather difficult in this class of non-linear general equilibrium models, the selected moments are sufficiently informative about the calibrated parameters so that the objective function is not locally flat along any direction. Below we discuss the determination and measurement of the targeted moments.

We start by describing the moment conditions used to calibrate the model's technological parameters: the productivity scaling parameter (A), the physical capital elasticity of output (α), and the output elasticity in the production of human capital (β). By observing equations (3.22), (3.24), and (3.25) it is clear that the parameters $\{A, \alpha, \beta\}$ are paramount in determining the balanced growth rate of real output per capita, the equilibrium loan rate, and the employed physical capital-to-output ratio. Following standard practice in the literature, we target an annualized growth rate of real GDP per capita of 2% and a (net) loan rate of 3.5% per annum; that is, $(g^d)^{1/25} = 1.020$ and $\delta^{1/25} = 1.035$. The target for the U.S. physical capital-to-output ratio is set to its long-run average of 2.9, as measured by the current-cost net stock of fixed assets in the BEA fixed assets tables divided by GDP.

[51] The implied price markup, $\mu = \frac{\sigma}{\sigma-1} = \frac{1}{\theta} = 1.5$, is consistent with the estimates of De Loecker, Eeckhout, and Unger (2020) for the median markup of manufacturing firms in the U.S. Censuses.

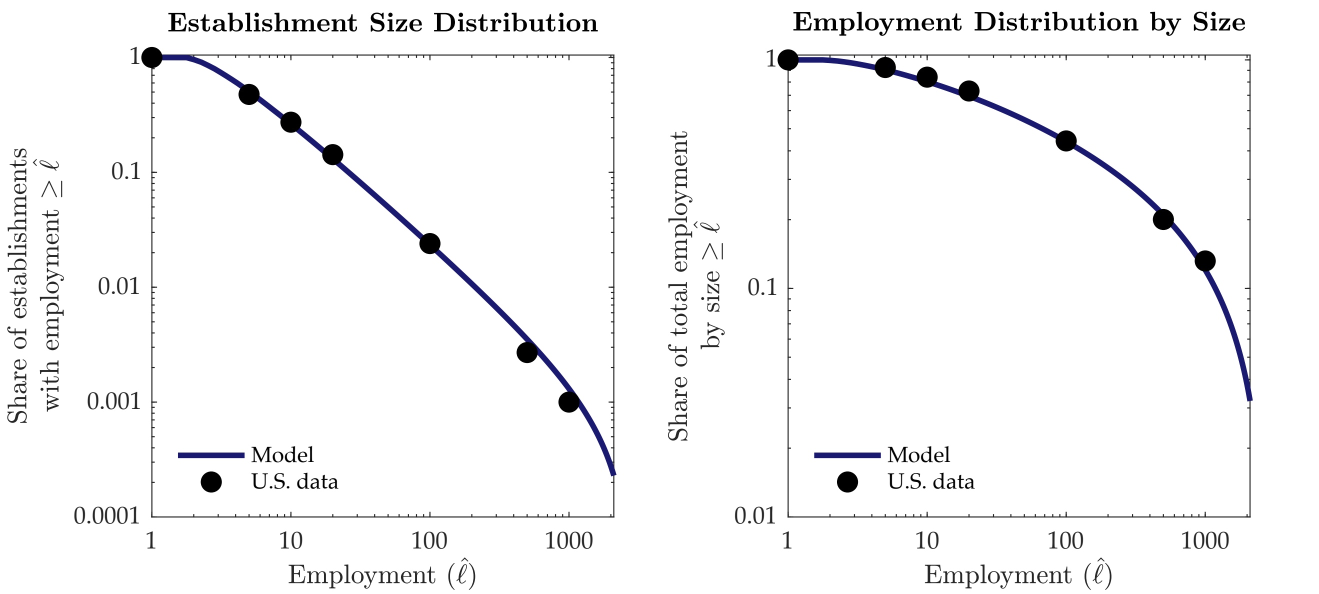
Table 3.1: Model Calibration to U.S. Data; Moments and Parameters

Targeted Moments	Model	Data
Annualized growth rate of real GDP per capita (g^d)	0.020	0.020
Measure of active entrepreneurs (qN^d)	0.052	0.052
Annualized real loan rate (δ)	0.035	0.035
Annualized real deposit rate (r)	0.005	0.005
Physical capital-to-output ratio	2.900	2.900
Share of establishments with $\ell \geq 100$	0.024	0.024
Employment share of establishments with $\ell \geq 100$	0.442	0.442
Calibrated Parameters	**Value**	
Productivity scaling parameter (A)	3.389	
Physical capital elasticity of output (α)	0.410	
Output elasticity of human capital (β)	0.896	
Probability of entrepreneurial success (q)	0.479	
Pareto tail parameter (η)	2.286	
Mean of $\log \rho$ (μ_ρ)	0.942	
Standard deviation of $\log \rho$ (σ_ρ)	1.911	
Elasticity of substitution $\left(\frac{1}{1-\theta}\right)$ [assigned]	3.000	

Next we explain how to infer the entrepreneurial success rate. Due to the simple setting of our model, q is uniquely determined by the proportional difference between the real deposit and lending rates: $q = r_t/\delta_t$. We target a spread of 3% per annum, i.e., $r^{1/25} = 1.005$, based on the estimates of lending spreads in the novel dataset of Zimmermann (2019). Therefore, $q = 0.4793$.

The three distributional parameters—the Pareto tail parameter (η) for the distribution of z, and the mean (μ_ρ) and standard deviation (σ_ρ) for the Lognormal distribution of ρ—are primary determinants of occupational choice patterns and the distribution of factor demands. The first evident target is the measure of active entrepreneurs/producers in the

Figure 3.2: Establishment Size and Employment Distributions: Model and Data



Notes: The source of U.S. data on establishment size and employment is the Business Dynamics Statistics (1978 – 2019); data points correspond to sample averages. Quantities are displayed on a log scale.

labor force, which can be deduced from the labor market clearing condition: $qN^d\bar{\ell} = 1 - N^d$ or $qN^d = q/(1 + q\bar{\ell})$, where $\bar{\ell}$ corresponds to average number of employees. Using this expression along with estimates for the total number of employees and establishments in the BDS data, the sample average for 1978 – 2019 is 0.052.[52] Next, relevant targets that are capable of providing further discipline come from the establishment-size and employment distributions. Two particularly instructive moments are the share of establishments with $\ell \geq \hat{\ell}$ and the employment share of establishments with $\ell \geq \hat{\ell}$, for some $\hat{\ell} > 0$ number of hired employees. Given the large concentration of entrepreneurship in small firms together with the disproportionate importance of large firms in terms of hiring, an evenhanded option is $\hat{\ell} = 100$ employees. The BDS sample averages correspond to 0.024 and 0.442, respectively. Finally, it is worth noting that varying α has little quantitative impact on occupational

[52] Since annual average employment in the BDS data is mostly between 16 and 18 employees, the value of qN^d is quite insensitive to q for reasonable values of the success rate.

choices and size/employment distributions in general equilibrium, while different values of $\{A, \beta\}$ do not affect at all any of the above moments.

Table 3.1 reports the output of the calibration exercise and summarizes our parametrization. The model replicates the targeted moments very closely; this is achieved prudently by targeting only as many moments as parameters and through typical distributional specifications. As evidenced in Figure 3.2, the model is also successful in matching the full extent of U.S. establishment size and employment distributions, despite having targeted only one data point from each distribution.

Table 3.2: Balanced Growth Equilibria vs Centralized Economy; Model Output

Model Output (U.S. Calibration)	Decentralized	Full Insurance	Planner
Annualized growth rate of real GDP per capita	0.020	0.026	0.027
Measure of active entrepreneurs/producers	0.052	0.097	0.070
Annualized real loan rate	0.035	0.042	—
Annualized real deposit rate	0.005	0.011	—
Physical capital-to-output ratio	2.900	2.467	2.450
Share of establishments ($\ell \geq 100$)	0.024	0.009	0.014
Employment share of establishments ($\ell \geq 100$)	0.442	0.290	0.326

3.6.2 Balanced Growth Equilibria vs Centralized Economy and Misallocation Losses

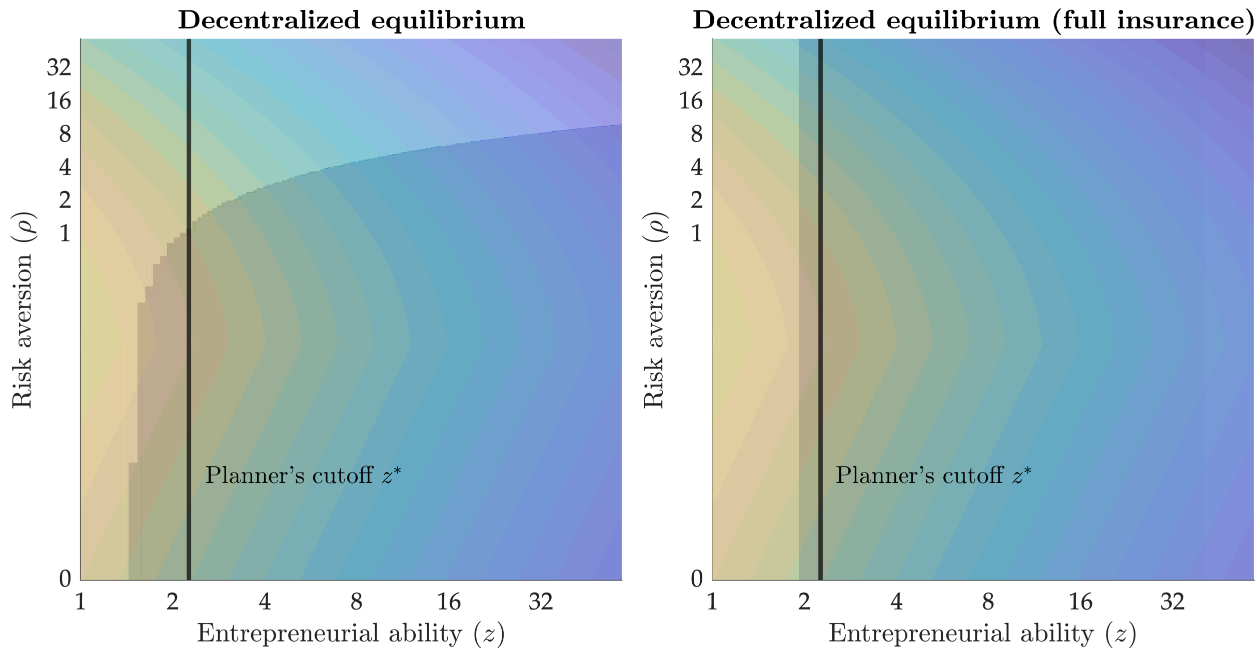
The next step is to further employ the calibrated model with a view to answering the following questions: How far from efficient are the allocations in the U.S. economy? How much of the associated losses is due to misallocation on the intensive/extensive margin? How would the competitive equilibrium change if we introduced full insurance against entrepreneurial risk?

We compute the planner's solution as well as the competitive equilibrium with actuarially fair insurance markets and compare them to the baseline U.S. calibration. We remind the reader that while the planner is able to eradicate both types of misallocation, full insurance in the decentralized economy eliminates only misallocation on the intensive margin (risk aversion becomes irrelevant), but features misallocation on the extensive margin ($N^{FI} > N^c$ iff $\alpha < 1/2$). Key statistics for these exercises are reported in Table 3.2.

The most salient points to observe are the striking gains in terms of balanced growth rates: about 0.6% on an annualized basis under full risk insurance and up to 0.7% per annum under the efficient allocations. In other words, upon removing misallocation related to occupational choices, it would take around 10 years less for real income per capita to double. The results also indicate that the U.S. entrepreneurship rate—as per our measurement—is lower than the optimal one (7%), and introducing full insurance would lead to far too many entrepreneurs (9.7%).

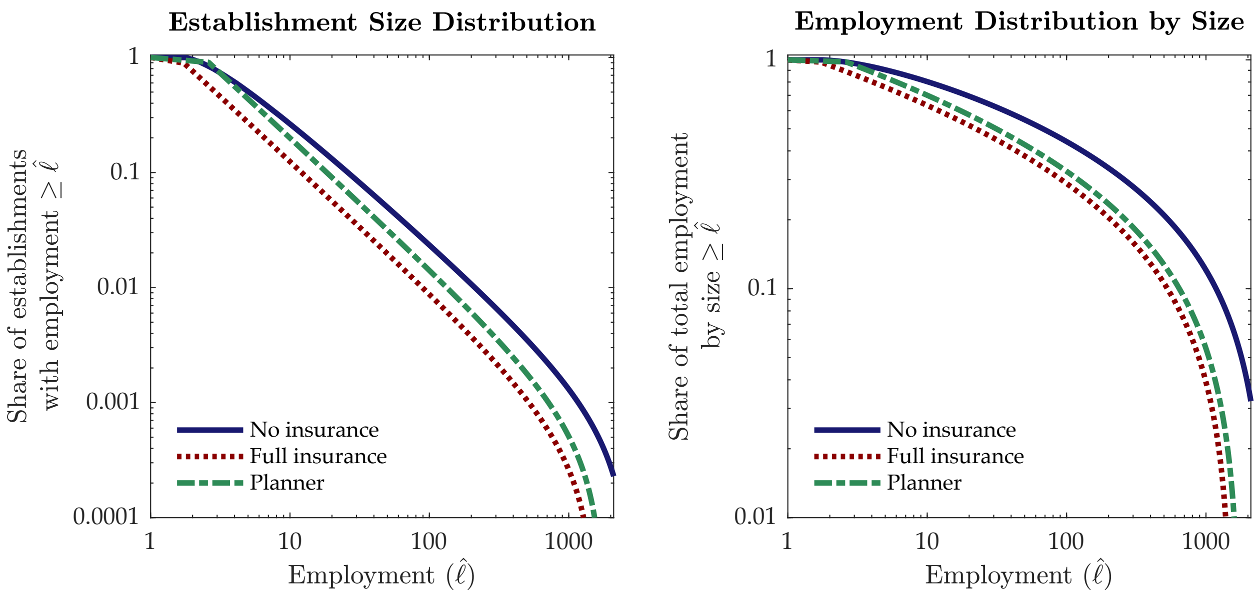
The occupational choice maps in Figure 3.3 offer a closer look into the anatomy of misallocation stemming from occupational sorting. Unshaded areas to the right of the planner's unique z-cutoff value represent the sizeable number of high-ability agents who do not become entrepreneurs due to high risk aversion, and the shaded areas to the left of the threshold represent a significant measure of excess entrepreneurship with lower-ability individuals. We also compute the efficient solution under the constraint that $N^c = N^d$ and find that about 97% of income growth losses (vis-à-vis the first-best) is due to misallocation on the intensive margin. That is, in the presence of misallocation due to risk aversion, *who* becomes an entrepreneur is far more important for long-run growth than *how many* people do so. A crucial policy insight is that encouraging a small number of highly skilled individuals to operate firms would be more beneficial than incentivizing a larger number of less capable entrepreneurs to do so.

Figure 3.3: Occupational Choice Maps: Decentralized Equilibria vs Planner



NOTES: Shaded areas represent selection into entrepreneurship. The background is a filled contour plot of the joint pdf $g(\rho, z)$, with cooler colors denoting lower densities. Quantities are displayed on a log scale.

Figure 3.4: Comparing Establishment Size and Employment Distributions



NOTES: Model solutions under the baseline parametrization. Quantities are displayed on a log scale.

3.7 Concluding Remarks

When it comes to promoting economic growth, is it always beneficial to encourage more entrepreneurship? Is it always desirable to have entrepreneurial risk insured away? This paper has explored the role of risk aversion and entrepreneurial ability in shaping occupational choices and balanced growth within a highly tractable endogenous growth model with heterogeneous agents. Several key insights emerge from the analysis, such as the finding that entrepreneurship and insurance provision against entrepreneurial risk may be harmful for long-run growth.

First, the relationship between the rate of entrepreneurship and balanced growth is non-monotone in general equilibrium. Increasing the number of entrepreneurs has three distinct effects on growth: a positive variety effect from expanding the range of intermediate goods, a negative loanable fund supply effect from reducing the number of workers/savers, and a TFP effect from lowering/increasing the average productivity of active firms due to occupational choices. The interplay of these forces leads to an ambiguous link between the rate of entrepreneurship and balanced growth, contrary to the conventional wisdom that "more is more." Indeed, sometimes less is more.

Second, the decentralized equilibrium allocations are suboptimal—even without any firm-level distortions or financial frictions—and feature misallocation on both the extensive and intensive margins. Due to the presence of risk aversion, the competitive market consistently undervalues the marginal social benefits and costs of entrepreneurship, and the inverse association between risk tolerance and entrepreneurial ability induced though occupational choices leads to lower aggregate TFP.

Third, introducing actuarially fair insurance markets to eliminate entrepreneurial risk in the decentralized economy does not restore the first-best allocations. While full insurance aligns private and social marginal benefits, it still fails to correct the undervaluation of marginal costs, resulting in excessive entry when the capital share is less than the labor share. Some insurance is almost always growth-enhancing, but full insurance is never optimal.

Calibrating the model to U.S. data reveals substantial output-side misallocation, with most of income growth and aggregate TFP losses stemming from the intensive margin due to risk aversion. Moreover, the U.S. entrepreneurship rate (inferred using BDS data) is lower than socially optimal; providing full insurance would result in too many, less productive entrepreneurs, but is still able to induce substantial growth gains since it eliminates intensive-margin misallocation. This suggests policies aimed at encouraging a small mass of highly talented individuals to start firms may be more effective than broad-based incentives for entrepreneurship.

All in all, even in cases where entrepreneurial entry should be encouraged to some degree, optimizing the number of entrepreneurs is not equivalent to maximizing growth. Ultimately, *what type* of individuals will choose to start firms and shape the productive capacity of an economy matters substantially more than *how many* will do so.

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Appendix A

Appendix to Chapter 1

A.1 Development Accounting, Measurement, and Additional Tables

Below I describe a variant of the standard development accounting exercise, e.g., Hall and Jones (1999), Caselli (2005), used for Hicks-neutral TFP calculations in Figure 1.2. The difference is that I incorporate heterogeneous entrepreneurs/producers and workers, along with decreasing returns to scale in the spirit of Lucas (1978) in order to sustain a non-degenerate firm distribution. The intent is to have a framework that is consistent with entrepreneurship and occupational choice, while reflecting the model structure employed in this paper. Net output aggregation in country i corresponds to the model presented in Section IV without financial frictions (when $\vartheta \to 1$). Specifically,

$$Y_i = Z_i \left(K_i^{\alpha} H_i^{1-\alpha}\right)^{1-\nu} \mu_i(\mathcal{E})^{\nu} \tag{A.1}$$

where K_i is the stock of physical capital; H_i is the stock of worker human capital; $\mu(\mathcal{E})$ is the rate of entrepreneurship; and Z is the ν-Hölder mean of effective productivities among active producers. Following Hall and Jones (1999), the accounting equation is given by expanding the expressing the above in terms of capital-output ratios,

$$Y_i = A_i \left(\frac{K_i}{Y_i}\right)^{\frac{\widehat{\alpha}}{1-\widehat{\alpha}}} h_i^{1-\frac{\nu}{1-\widehat{\alpha}}} \mu_i(\mathcal{E})^{\frac{\nu}{1-\widehat{\alpha}}} (1-\mu_i(\mathcal{E}))^{1-\frac{\nu}{1-\widehat{\alpha}}} \tag{A.2}$$

where h_i is average human capital per worker in country i and $\widehat{\alpha} = \alpha(1-\nu)$. I carry out a development accounting exercise in levels using (A.2) under conventional parameter values, $\alpha = 0.36$ and $\nu = 0.20$. After accounting for the observable factors, I obtain the (endogenous) residual $A_i := Z_i^{\frac{1}{1-\widehat{\alpha}}}$ that is interpreted as TFP.

Measurement. Observations coming from PWT 10.01 data are geometric means of their levels relative to the U.S. as dictated by the availability of GEM data for $2009-2019$. "Output per worker" refers to output-side real GDP at chained PPPs divided by numbers of people engaged *(rgdpo/emp)*. The capital-output ratio corresponds to *(cn/cgdpo)*. Average worker human capital is measured by the PWT human capital index *(hc)*. The entrepreneurship rate is calculated as the fraction of self-employed business owners-managers in the labor force using the pooled sample of GEM data for each country.

All countries with available GEM–PWT data are included apart from three small sets of exceptions. First, as it is common in the literature, I omit countries with very low numbers of labor force participants ($< 150,000$ persons). The two countries excluded are Belize and Barbados. Second, to mitigate the risk of analyzing samples of potentially lower quality, I do not consider exceedingly poor countries, defined as those with relative output per worker $< 1/64$ of the U.S. The two countries excluded are Ethiopia and Malawi. Third, I leave out exceedingly resource-rich countries, defined as those with total natural resources rents

$\geq$ 25% of their GDP (World Development Indicators). The five countries excluded are Angola, Oman, Qatar, Saudi Arabia, and the UAE.

Table A.1: GEM Adult Population Survey Data Details

Country code | name	Survey years	Obs	Country code | name	Survey years	Obs
ARG | Argentina	2009 − 2018	14,505	JOR | Jordan	2009, 2016, 2019	2,956
ARM | Armenia	2019	1,330	JPN | Japan	2009 − 2014, 2017 − 2019	13,707
AUS | Australia	2010, 2011, 2014 − 2019	9,639	KAZ | Kazakhstan	2014 − 2017	5,854
AUT | Austria	2012, 2014, 2016, 2018	13,931	KOR | South Korea	2009 − 2013, 2015 − 2019	13,070
BEL | Belgium	2009 − 2015	12,326	LBN | Lebanon	2009, 2015 − 2018	9,045
BFA | Burkina Faso	2014 − 2016	5,275	LTU | Lithuania	2011 − 2014	5,506
BGD | Bangladesh	2011	541	LUX | Luxembourg	2013 − 2019	9.994
BGR | Bulgaria	2015 − 2018	6,093	LVA | Latvia	2009 − 2013, 2015 − 2019	14,025
BIH | Bosnia & Herzegovina	2009 − 2014, 2017	6,784	MAR | Morocco	2009, 2015 − 2019	8,465
BLR | Belarus	2019	1,594	MDG | Madagascar	2017 − 2019	5,487
BOL | Bolivia	2010, 2014	2,363	MEX | Mexico	2010 − 2017, 2019	20,481
BRA | Brazil	2009 − 2019	31,440	MKD | North Macedonia	2010, 2012 − 2016, 2019	6,052
BWA | Botswana	2012 − 2015	4,486	MNE | Montenegro	2010	1,129
CAN | Canada	2013 − 2019	17,168	MYS | Malaysia	2009 − 2017	12,454
CHE | Switzerland	2009 − 2019	17,628	NAM | Namibia	2013	1,187
CHL | Chile	2009 − 2019	55,251	NGA | Nigeria	2011 − 2013	4,846
CHN | China	2009 − 2019	32,521	NLD | Netherlands	2009 − 2019	21,627
CMR | Cameroon	2014 − 2016	4,867	NOR | Norway	2009 − 2015, 2019	14,194
COL | Colombia	2009 − 2019	34,583	PAK | Pakistan	2010 − 2012, 2019	3,257
CRI | Costa Rica	2010, 2012, 2014	3,099	PAN | Panama	2009, 2011 − 2019	14,295
CYP | Cyprus	2016 − 2019	5,538	PER | Peru	2009 − 2018	13,130
CZE | Czech Republic	2011, 2013	5,496	PHL | Philippines	2013 − 2015	3,650
DEU | Germany	2009 − 2019	40,677	POL | Poland	2011 − 2019	24,281
DNK | Denmark	2009 − 2012, 2014	9,064	PRT | Portugal	2010 − 2016, 2019	11,052
DOM | Dominican Republic	2009	2,016	PSE | Palestine	2009 − 2012	4,239
DZA | Algeria	2009, 2011 − 2013	6,214	ROU | Romania	2009 − 2015	8,417
ECU | Ecuador	2009, 2010, 2012 − 2019	13,278	RUS | Russia	2009 − 2014, 2016 − 2019	17,694
EGY | Egypt	2010, 2012, 2015 − 2019	10,291	SDN | Sudan	2018	1,120
ESP | Spain	2009 − 2019	169,512	SEN | Senegal	2015	1,707
EST | Estonia	2012 − 2017	9,683	SGP | Singapore	2011 − 2014	5,503
FIN | Finland	2009 − 2016	13,457	SLV | El Salvador	2012, 2014, 2016	3,004
FRA | France	2009 − 2014, 2016 − 2018	11,947	SRB | Serbia	2009	1,626
GBR | United Kingdom	2009 − 2019	71,087	SVK | Slovakia	2011 − 2019	13,053
GEO | Georgia	2014, 2016	1,255	SVN | Slovenia	2009 − 2019	16,819
GHA | Ghana	2010, 2012, 2013	5,132	SWE | Sweden	2010 − 2019	24,310
GRC | Greece	2009 − 2019	14,047	SYR | Syria	2009	2,148
GTM | Guatemala	2009 − 2011, 2013 − 2019	15,892	THA | Thailand	2011 − 2018	13,998
HKG | Hong Kong	2009, 2016	3,766	TTO | Trinidad & Tobago	2010 − 2014	6,004
HRV | Croatia	2009 − 2019	13,542	TUN | Tunisia	2009, 2010, 2012, 2015	5,163
HUN | Hungary	2009 − 2016	12,065	TUR | Turkey	2010 − 2013, 2016, 2018	19,937
IDN | Indonesia	2013 − 2018	17,994	TWN | Taiwan	2010 − 2019	15,110
IND | India	2012 − 2019	13,928	UGA | Uganda	2009, 2010, 2012 − 2014	9,339
IRL | Ireland	2010 − 2019	14,043	URY | Uruguay	2009 − 2018	12,390
IRN | Iran	2009 − 2019	18,871	USA | United States	2009 − 2019	28,990
ISL | Iceland	2009, 2010	4,360	VEN | Venezuela	2009, 2011	2,932
ISR | Israel	2009, 2010, 2012 − 2019	14,961	VNM | Vietnam	2013 − 2015, 2017	6,654
ITA | Italy	2009, 2010, 2012 − 2019	14,216	ZAF | South Africa	2009 − 2017, 2019	12,707
JAM | Jamaica	2009 − 2014, 2016	10,619	ZMB | Zambia	2010 − 2013	3,678

NOTES: These are the countries considered in the empirical section of the paper, for which there exist available data in both the PWT 10.01 and GEM datasets for the period 2009−2019. "Obs" denotes the total number of (unweighted) GEM adult population survey observations in each country that were used for calculations, which include persons belonging to the labor force with non-missing educational attainment responses. A person belongs to the labor force if they work full-time/part-time for someone else or if they are self-employed according to the GEM harmonized work status (GEMWORK); the categories excluded from the labor force are "retired, disabled," "homemaker," "student," "not working," and "other".

Table A.2: Weighted Probit Regressions; Probability of Being an Active Entrepreneur

	Dependent variable: SEBO = 1						
COVARIATES	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Education	0.047***	0.028***	0.017***	−0.080 ***	−0.036***	−0.037***	−0.039***
	(0.0034)	(0.0035)	(0.0036)	(0.0122)	(0.0128)	(0.0128)	(0.0146)
Education^2 (×100)				0.347***	0.246***	0.248***	0.191***
				(0.0419)	(0.0436)	(0.0435)	(0.0501)
Potential experience					0.020***	0.033***	
					(0.0008)	(0.0026)	
Potential experience^2 (×100)						−0.0224***	
						(0.0044)	
Self-Employed experience							0.073***
							(0.0016)
Worker experience							−0.002**
							(0.0009)
ADDITIONAL CONTROLS	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Annual labor supply (log)			0.018	0.018	0.086***	0.076***	0.125***
			(0.0151)	(0.0151)	(0.0170)	(0.0171)	(0.0198)
Ever received inheritance (0/1)			0.245***	0.244***	0.156***	0.156***	0.125***
			(0.0199)	(0.0199)	(0.0200)	(0.0202)	(0.0230)
Expects to receive inheritance (0/1)			−0.010	−0.009	0.080***	0.078***	0.073***
			(0.0231)	(0.0229)	(0.0238)	(0.0240)	(0.0239)
Risk willingness (1−4)			0.133***	0.136***	0.168***	0.166***	0.174***
			(0.0094)	(0.0095)	(0.0096)	(0.0097)	(0.0101)
Health (1−4)		0.130***	0.119***	0.116***	0.153***	0.155***	0.132***
		(0.0177)	(0.0178)	(0.0178)	(0.0185)	(0.0185)	(0.0196)
Married (0/1)		0.499***	0.491***	0.490***	0.474***	0.457***	0.511***
		(0.0231)	(0.0236)	(0.0237)	(0.0239)	(0.0243)	(0.0258)
Male (0/1)		0.106***	0.088***	0.078***	0.083***	0.093***	−0.100***
		(0.0220)	(0.0219)	(0.0220)	(0.0228)	(0.0232)	(0.0247)
Black (0/1)		−0.433***	−0.393***	−0.391***	−0.346***	−0.353***	−0.302***
		(0.0323)	(0.0329)	(0.0329)	(0.0336)	(0.0337)	(0.0368)
Hispanic (0/1)		−0.417***	−0.366 ***	−0.414***	−0.291***	−0.302***	−0.254***
		(0.0368)	(0.0358)	(0.0367)	(0.0384)	(0.0384)	(0.0429)
Other non-white (0/1)		−0.083***	−0.047	−0.056*	0.011	0.010	0.010
		(0.0310)	(0.0314)	(0.0313)	(0.0312)	(0.0314)	(0.0324)
Sector Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Occupation Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector × Occupation Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
McFadden's adjusted R^2	0.117	0.152	0.161	0.163	0.186	0.187	0.331
Observations	40, 413	40, 413	40, 413	40, 413	40, 413	40, 413	40, 413

NOTES: All regressions are weighted by the appropriate SCF sampling weights. Estimated parameters are based on repeated imputation inference using all 5 SCF implicates. Bootstrapped standard errors (in parentheses) are calculated using all 999 SCF replicate draws and weights, and account for both imputation and sampling variability. All specifications include additional controls for *retirement*; *disability*; *employment variety*; *whether S/P is unemployed*; and a *constant term*, not reported here due to space limitations. *** p <0.01; ** p <0.05; * p <0.1. Results using a Logit specification for selection into entrepreneurship yield nearly identical results.

Table A.3: Weighted Least Squares Regressions; SEBO Hourly Business Income

	Dependent Variable: log hourly business income (> 0)						
COVARIATES	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Education	0.079***	0.076***	0.075***	−0.047	−0.040	−0.033	−0.047
	(0.0078)	(0.0078)	(0.0080)	(0.0332)	(0.0357)	(0.0319)	(0.0315)
Education^2(×100)				0.417***	0.393***	0.346***	0.368***
				(0.1126)	(0.120)	(0.1081)	(0.1076)
Worker experience	0.007***	0.008***	0.009***	0.009***	0.006	0.003	0.002
	(0.0024)	(0.0024)	(0.0023)	(0.0023)	(0.0083)	(0.0075)	(0.0075)
Worker experience^2(×100)					0.007	0.014	0.016
					(0.0222)	(0.020)	(0.020)
Self-empl. experience	0.018***	0.017***	0.019***	0.018***	0.046***	0.039***	0.028***
	(0.0019)	(0.0019)	(0.0019)	(0.0019)	(0.0057)	(0.0055)	(0.0055)
Self-empl. experience^2(×100)					−0.070***	−0.058***	−0.043***
					(0.0141)	(0.0138)	(0.0139)
Business employment size (log)						0.283***	0.176***
						(0.0185)	(0.0209)
Net business value (log)							0.167***
							(0.0123)
ADDITIONAL CONTROLS	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Ever received inheritance (0/1)			−0.081*	−0.080*	−0.089**	−0.072*	−0.045
			(0.0435)	(0.0434)	(0.0438)	(0.0432)	(0.0413)
Expects to receive inheritance (0/1)			−0.035	−0.032	−0.041	−0.008	0.014
			(0.0464)	(0.0470)	(0.0471)	(0.0501)	(0.0483)
Inherited any business (0/1)			0.221**	0.235**	0.229**	0.181**	0.051
			(0.0983)	(0.0989)	(0.0961)	(0.0868)	(0.0874)
Risk willingness (1−4)			0.073***	0.079***	0.080***	0.062***	0.027
			(0.0247)	(0.0247)	(0.0246)	(0.0237)	(0.0232)
Health (1−4)		0.140***	0.137***	0.135***	0.122***	0.107***	0.097**
		(0.0376)	(0.0377)	(0.0375)	(0.0381)	(0.0386)	(0.03888)
Male (0/1)		0.106**	0.104**	0.089**	0.088*	0.014	−0.065
		(0.0457)	(0.0458)	(0.0458)	(0.0465)	(0.0467)	(0.0473)
Black (0/1)		−0.240***	−0.250**	−0.244***	−0.239***	−0.276***	−0.216**
		(0.0886)	(0.0885)	(0.0885)	(0.0887)	(0.0898)	(0.090)
Hispanic (0/1)		−0.003	−0.007	−0.044	−0.055	−0.114	−0.122
		(0.0796)	(0.0795)	(0.0814)	(0.0809)	(0.0757)	(0.0786)
Other non-white (0/1)		0.078	0.070	0.057	0.047	0.047	0.046
		(0.0685)	(0.0687)	(0.0686)	(0.0683)	(0.0651)	(0.0650)
Sector Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Occupation Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector × Occupation Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Legal Entity Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	7,810	7,810	7,810	7,810	7,810	7,810	7,810
Adjusted R^2	0.149	0.154	0.157	0.159	0.165	0.215	0.249
RMSE	1.272	1.268	1.266	1.264	1.260	1.22	1.195

NOTES: All regressions are weighted by the appropriate SCF sampling weights. Estimated parameters are based on repeated imputation inference using all 5 SCF implicates. Bootstrapped standard errors (in parentheses) are calculated using all 999 SCF replicate draws and weights, and account for both imputation and sampling variability. All specifications include additional controls for *firm ownership shares*; *number of firms owned and managed*; *marital status*; *retirement*; *disability*; *employment variety*; *whether S/P is unemployed*; and a *constant term*, not reported here due to space limitations. *** $p<0.01$; ** $p<0.05$; * $p<0.1$. OLS regressions for different definitions of business income as dependent variables yield similar conclusions.

Table C5: Description of Variables Used in the Survey of Consumer Finances

Variable	Measure of	SCF Code and Description
Age	Individual's Age	X14 (for R) and X19 (for S/P) in the Full Public Dataset.
Education	Years of Formal Education	X5901, X5905, X5931 (for R) and X6101, X6105, X6111 (for S/P) in the Full Public Dataset. Variables for "grade completed" and "highest degree earned" are appropriately combined and translated into years of education, ranging from 0 to 20.
Potential Experience	Years of Potential Experience	Age - Education - 6, both defined as above.
Self-Employed Experience	Years of Experience Working Self-Employed	X4115, X4515, X4518, X4519, X4535, X4538, X4539 (for R) and X4715, X5115, X5115, X5118, X5119, X5135, X5138, X5139 (for S/P) in the Full Public Dataset. Variables are combined to calculate the years of past self-employed experience for non-SEBO and current self-employed experience for SEBO.
Worker Experience	Years of Experience Working as an Employee (not Self-Employed)	Potential Experience - Self-employed Experience, both defined as above.

Labor Income	Annual Labor Income	X5702 in the Full Public Dataset. *Q:* "In total, what was your annual income from wages and salaries, before deductions for taxes and anything else?"
Business Income	Annual Entrepreneurial Profit	X5704 + X5714 in the Full Public Dataset. *Q:* "In total, what was your net annual income from a sole proprietorship or a farm, before deductions for taxes and anything else?". *Q:* "In total, what was your annual income from other businesses or investments, net rent, trusts, or royalties, before deductions for taxes and anything else?"
Earned Income	Annual Earned Income	Labor Income + Business Income, both defined as above.
Active Business Value	Hired Capital by Entrepreneur	ACTBUS in the Summary Public Dataset. Net equity if (share of) businesses were sold today, plus loans from the household to business, minus loans from business to HH, plus value of personal assets used as collateral for business loans.
Number of Employees	Hired Labor by Entrepreneur	X3111 + X3211 in the Full Public Dataset. All paid workers in the businesses, both full-time and part-time, including the entrepreneur, members of his/her family, and anyone who is working without pay.
Net Worth	Total Wealth	NETWORTH in the Summary Public Dataset. The sum of total reported financial and non-financial wealth, minus total reported debt.

Net Worth excl. Business	Entrepreneurial Wealth Net of Business Interests	NETWORTH minus ACTBUS in the Summary Public Dataset. Net Worth minus the corresponding Active Business Value, both defined as above.
Business Share	Share of Firm owned by the household	X3128 and X3228 in the Full Public Dataset. *Q:* "What percentage of the business do you own?"
Labor Supply	Total hours of work in a typical year (hours per week × weeks per year)	X4110, X4111 (for R) and X4710, X4711 (for S/p) in the Full Public Dataset. *Q:* "How many hours do you work on your main job in a normal week?" *Q:* "Counting paid vacations as weeks of work, how many weeks do you work on your main job in a normal year?"
Employment Variety	Number of Different Past Employers	X4513 (for R) and X5113 (for S/p) in the Full Public Dataset. *Q:* "Including any self-employment and your current job, for how many different employers have you worked in full-time jobs lasting one year or more?"
Inherited Business (0/1)	Whether the entrepreneur has inherited any of his/her businesses or not	X3108=3 or X3108=4 or X3208=3 or X3208=4. *Q:* "How did you first acquire this business; was it bought or invested in, started by you, inherited, given to you, or some other way?"
Past Inheritance (0/1)	Whether the household has received any substantial inheritance in the past	X5801 in the Full Public Dataset. *Q:* "Have you ever received an inheritance, or been given substantial assets in a trust or in some other form? Please do not include inheritances from a deceased spouse."

Future Inheritance (0/1)	Whether the household expects to receive any substantial inheritance in the future	X5819 in the Full Public Dataset. *Q:* "Do you expect to receive a substantial inheritance or transfer of assets in the future?"
Risk Willingness (1–4)	Willingness to take Financial Risk	X3014 (jointly for R and S/P) in the Full Public Dataset, recoded in inverse order. *Q:* "4) Take substantial financial risks expecting to earn substantial returns, 3) Take above average financial risks expecting to earn above average return, 2) Take average financial risks expecting to earn average returns, 1) Not willing to take any financial risks"
Health (1–4)	Health Condition	X6030 (for R) and X6124 (for S/P) in the Full Public Dataset, recoded in inverse order. *Q:* "Would you say your health in general is 4) excellent, 3) good, 2) fair, or 1) poor?"
Retired (0/1)	Whether the individual has reported being retired from a previous job.	X4100=13 or X4100=50 if year $\leq$ 1992 and X6670=7 or X6671=7 ...or X6677=7 if year>1992 (for R); X4700=13 or X4700=50 if year $\leq$ 1992 and X6678=7 or X6679=7 ... or X6685=7 if year >1992 (for S/P) in the Full Public Dataset.

Disabled (0/1)	Whether the individual has reported to have some disability	X4100=12 or X4100=52 if SCF year $\leq$ 1992 and X6670=6 or X6671=6 ...or X6677=6 if SCF year > 1992 (for R); X4700=12 or X4700=52 if SCF year $\leq$ 1992 and X6678=6 or X6679 =6 ... or X6685=6 if SCF year > 1992 (for S/P) in the Full Public Dataset.
Male (0/1)	Sex	X8021 (for R) and X103 (for S/P) in the Full Public Dataset. Individual has reported to be male or female.
Married (0/1)	Individual is either married or living with a partner	X8023 (for R) and X105 (for S/P) in the Full Public Dataset. *Q:* "Are you currently married or living with a partner, separated, divorced, widowed, or never been married?"
Unemployed Partner (0/1)	Individual is married/living with a partner who does not work	Married=1 and Labor Supply (of the other) = 0, both defined as above. *Q:* "Are you currently married or living with a partner, separated, divorced, widowed, or never been married?"
Race (1–4)	Individual's race	RACECL4 in the Summary Public Dataset. 1) White or Caucasian (includes Middle-Easter and Arab), 2) Black or African-American, 3) Hispanic or Latino, 4) Any other race.

A.1.1 Some Methodological Considerations When Using the SCF

The SCF is a detailed triennial cross-sectional survey of U.S. households sponsored by the Board of Governors of the Federal Reserve System in cooperation with the Statistics of Income Division of the IRS. Samples are drawn based on a dual-frame sampling scheme, which includes both a carefully designed area-probability and a list component. The area-probability sample is selected in three stages in order to provide robust national coverage of a broad range of behaviors and characteristics, and the list sample is selected using information from administrative data in order to disproportionately sample more wealthy households.

As expected in such detailed surveys the data will involve some missing responses, either because the interviewee(s) did not provide some answers, or for confidentiality/non-disclosure reasons. The Federal Reserve imputes missing values using a statistical procedure called *multiple imputation*.[53] This procedure replaces each missing or deficient value with more acceptable values representing a realistic distribution of possibilities, with individual imputations generated by drawing repeatedly from an estimate of the conditional distribution of the data. Imputations are stored as five successive *implicates* of each data record. Thus, the total number of observations in each SCF wave is five times the actual number of responses. However, it is critical not to treat all five implicates as independent observations when conducting statistical analysis.

The SCF is based on a complicated structure and research design, which if ignored, it is guaranteed to yield errors when computing parameters and standard errors, and will seriously affect any hypothesis tests. The theory on proper inference in a multiple imputation setting is well-understood; see the original work of Rubin (1987) and Van Buuren (2018) for more

[53] The SCF research staff has developed the FRITZ (Federal Reserve Imputation Technique Zeta) model of multiple imputation, incorporatating insights from the Gibbs sampling algorithm together with statistical techniques developed in the image processing literature. See Kennickell (2017) for details.

recent advancements. Below I summarize some of these issues and I explain how the current study addresses them.

1) When calculating sample statistics, estimating regressions, or carrying out any form of analysis, the appropriate SCF survey weights are used; i.e., all reported statistics correspond to *weighted* sample estimates. This is essential in order to obtain accurate estimates while working with a truly nationally-representative sample.

2) The SCF contains five implicates of every observation; each implicate may or many not have different values for any variable. To obtain valid point estimates one must compute the average point estimate across the five implicates. For instance, the correct sample mean/median/variance of a variable should equal the average of the means/medians/variances across all five implicates, or the correct $\hat{\beta}$ coefficient vector of an OLS regression should be computed as the average of the estimated $\hat{\beta}$ vectors after running the model five times using each of the implicates.

For the multivariate case, let $\widehat{\mathbf{Q}}$ be a $k \times 1$ vector of (properly weighted) estimates of the parameter vector $\mathbf{Q}$ that would have been obtained if no data were missing. Each imputed data set $m = 1, \ldots, M$ admits an estimate of $\widehat{\mathbf{Q}}$, denoted $\widehat{\mathbf{Q}}_m$, along with a standard error $\sqrt{\mathbf{W}}_m$. The RII estimate of $\mathbf{Q}$ based on M imputed datasets is given by,

$$\overline{\mathbf{Q}} = \frac{1}{M} \sum_{m=1}^{M} \widehat{\mathbf{Q}}_m \tag{A.3}$$

3) Standard error calculations in the SCF call for special techniques. If we naively treated each of the five implicates as an independent observation, the resulting standard errors would be substantially smaller and the corresponding hypothesis tests would overstate the significance of the estimates. The reason is the following. When using all M complete-data versions of the potentially incomplete dataset and combine them into one pooled result, the uncertainty due to the missing data has to be taken into account. That is, we need to account for both *within-imputation* and *between-imputation* variability. Within variability is the sampling variance within each implicate; between variability is the variance contributed across implicates.

Specifically, within variability is given by the mean of the squared standard errors within the imputed data sets

$$\overline{\mathbf{W}} = \frac{1}{M-1} \sum_{m=1}^{M} \widehat{\mathbf{W}}_m \tag{A.4}$$

Between variability is given by the sample variance of the five parameters computed within each implicate

$$\mathbf{B} = \frac{1}{M-1} \sum_{m=1}^{M} \left(\widehat{\mathbf{Q}}_m - \overline{\mathbf{Q}}\right) \left(\widehat{\mathbf{Q}}_m - \overline{\mathbf{Q}}\right)' \tag{A.5}$$

Finally, the estimate of the total covariance matrix of the parameter vector is given by

$$\mathbf{V} = \overline{\mathbf{W}} + \left(1 + \frac{1}{M}\right) \mathbf{B} \tag{A.6}$$

4) All datasets come with a total of 999 replicates for each variable to simulate the complex sampling scheme of the SCF. The vector of replicate variables contains the number of times an observation was drawn for each replicate, and the *bootstrap* serves as a useful analog to the replicate process. For the first replicate, keep only the first implicate, and create a dataset with as many copies of each observation as are contained in the replicate variable. Then compute and store the desired parameter, using the newly expanded dataset. Repeat with another replicate variable.

A.1.2 More Details on Identifying Entrepreneurs in the SCF

An essential difference between my analysis and Cagetti and De Nardi (2006) is that they consider entrepreneurs in juxtaposition to the U.S. population, but instead I focus on households in the *labor force*. The need to compare economic outcomes between SEBO and non-SEBO requires excluding minors and people who are economically inactive, e.g., fully disabled persons, fully retired persons, volunteers, etc. I therefore designate a household to be part of the labor force if either R or S/P satisfies the following three requirements: **(i)** reports being economically active in some way, excluding volunteering; **(ii)** provides positive annual labor supply (number of weeks worked times hours worked per week); and **(iii)** declares non-zero income. The analysis considers only households participating in the labor force, i.e., the sum of SEBO and non-SEBO.

Finally, I use the following procedure to decide whether the respondent or the spouse/ partner should be considered the primary entrepreneur or worker. *1)* Assign SEBO status to either R or S/P according to which person has declared to engage in some form of self-employment. *2)* Assign SEBO status to S/P if they participate in the operation of the business and R is marked as a non-participant. Moreover, to wipe out the possibility of R working at the business as a hobby, I assign SEBO status to S/P is R has documented no weeks of active work on their primary job in a normal year, whereas S/P has a non-zero labor supply on their primary job as self-employed. *3)* If both persons are self-employed in some way, then S/P is assigned as SEBO if the person fulfills all of the above criteria and supplies more annual working hours in the current business than R. Otherwise, R is given the assignment since it is more plausible for the more financially knowledgeable person to be a SEBO. Once this procedure is completed, the demographic and personal characteristics to be analyzed are assigned to be those of R or S/P.

Appendix B

Appendix to Chapter 2

B.1 Proofs and Additional Derivations

Proof of Lemmas 1 **and** 2**:** We start by deriving entrepreneurial factor demands under the general productivity function $\zeta(h_e, z)$, without specifying the impact of technology adoption yet. Optimal demands for capital and labor correspond to

$$\widetilde{k}(h_e, a, z) = \frac{\alpha\,(1-\nu)}{\widetilde{r}+\delta}\,\widetilde{y}(h_e, a, z) \tag{B.1}$$

$$\widetilde{\ell}(h_e, a, z) = \frac{(1-\alpha)(1-\nu)}{w}\,\widetilde{y}(h_e, a, z) \tag{B.2}$$

The entrepreneur-specific shadow interest rate is $\widetilde{r}_t(h_e, a, z) = r_t + \lambda(h_e, a, z)$, where $\lambda(.) \geq 0$ is the Lagrange multiplier on the collateral constraint. In the unconstrained case when an individual possess assets $a > (1-\vartheta)\widetilde{k}(h_e, a, z)$, we have $\lambda = 0$ and thus $\widetilde{r} = r$. Manipulating through yields the optimal level of production

$$\widetilde{y}(h_e, a, z) = \zeta^{\frac{1}{\nu}} \left[\left(\frac{\alpha(1-\nu)}{\widetilde{r}+\delta} \right)^{\alpha} \left(\frac{(1-\alpha)(1-\nu)}{w} \right)^{1-\alpha} \right]^{\frac{1-\nu}{\nu}} \tag{B.3}$$

By plugging B.3 into B.1 and B.2 we can recover unconstrained factor demands

$$\widetilde{k}(h_e, z) = \zeta^{\frac{1}{\nu}} \left(\frac{\alpha(1-\nu)}{\widetilde{r}+\delta} \right)^{\frac{1-(1-\alpha)(1-\nu)}{\nu}} \left(\frac{(1-\alpha)(1-\nu)}{w} \right)^{\frac{(1-\alpha)(1-\nu)}{\nu}} \tag{B.4}$$

$$\widetilde{\ell}(h_e, z) = \zeta^{\frac{1}{\nu}} \left(\frac{\alpha(1-\nu)}{\widetilde{r}+\delta} \right)^{\frac{\alpha(1-\nu)}{\nu}} \left(\frac{(1-\alpha)(1-\nu)}{w} \right)^{\frac{1-\alpha(1-\nu)}{\nu}} \tag{B.5}$$

Solving for the multiplier $\lambda(h_e, a, z)$ allows to express the shadow interest rate as

$$\widetilde{r}_t(h_e, a, z) = \begin{cases} r_t \ \text{ if } \ a > (1-\vartheta)\, \zeta^{\frac{1}{\nu}} \left(\frac{\alpha(1-\nu)}{r_t+\delta} \right)^{\frac{1-(1-\alpha)(1-\nu)}{\nu}} \left(\frac{(1-\alpha)(1-\nu)}{w_t} \right)^{\frac{(1-\alpha)(1-\nu)}{\nu}} \\ \left(\frac{\zeta^{\frac{1}{\nu}}}{\frac{1}{1-\vartheta} a} \right)^{\frac{\nu}{1-(1-\alpha)(1-\nu)}} \left(\frac{(1-\alpha)(1-\nu)}{w_t} \right)^{\frac{(1-\alpha)(1-\nu)}{1-(1-\alpha)(1-\nu)}} \alpha(1-\nu) - \delta \quad \text{otherwise} \end{cases}$$

The indirect profit function for unconstrained entrepreneurs is given by

$$\pi^{uc}(h_e, z) = \zeta^{\frac{1}{\nu}}\ \nu \left[(1-\nu) \left(\frac{\alpha}{r+\delta} \right)^{\alpha} \left(\frac{1-\alpha}{w} \right)^{1-\alpha} \right]^{\frac{1-\nu}{\nu}} \tag{B.6}$$

The indirect profit function for constrained entrepreneurs depends on their wealth level and for $a < (1-\vartheta)k(h_e, a, z)$ is given by

$$\pi^{c}(h_e, a, z) = (1-\upsilon) \left[\zeta \left(\frac{a}{1-\vartheta} \right)^{\alpha(1-\nu)} \left(\frac{\upsilon}{w} \right)^{\upsilon} \right]^{\frac{1}{1-\upsilon}} - \frac{a(r+\delta)}{1-\vartheta} \tag{B.7}$$

where $\upsilon := (1-\alpha)(1-\nu)$.

Using the above results, it is now straightforward to use the indirect profit functions and set up the marginal conditions for occupational choice. Setting up the equations $\pi^{uc}(h_e, z \mid r, w) = wh_e$ and $\pi^c(h_e, a, z \mid r, w) = wh_e$, using B.6 and B.7, and manipulating through to solve for z, yields the desired results. ■

A quick note on the proofs to follow. It is clear that the set of entrepreneurs can be partitioned as $\mathcal{E} = \mathcal{E}^{uc} \cup \mathcal{E}^c$, $\mathcal{E}^{uc} \cap \mathcal{E}^c = \emptyset$. The superscripts stand for capital "unconstrained" and "constrained." In addition, conditioning on the set $\mathcal{E}^{uc}$ or $\mathcal{E}^c$ is shorthand for the σ-algebra generated by the truncated random variables given by Lemma 3. To reduce notational in the next proof, I will also use shorthand notation based on discretized probability distributions, with the understanding that the calculations involve proper integrals over specific domains, e.g., $\mathcal{P}[h = k]$ refers to $\int_{k_1}^{k_2} g(h)\, dh$.

Proof of Proposition 1: The object of interest is the fraction of the labor force with human capital $h = j \in h_{min} \ldots h_{max}$ that opts into entrepreneurship. First, we can break this fraction down to the sum of unconstrained ($\mathcal{E}^{uc}$) and capital-constrained ($\mathcal{E}^c$) entrepreneurs, as the two sets are disjoint:

$$\frac{\mathcal{P}[\mathcal{E} \cap h = k]}{\mathcal{P}[h = k]} = \frac{\mathcal{P}[(\mathcal{E}^{uc} \cup \mathcal{E}^c) \cap h = k]}{\mathcal{P}[h = k]} = \frac{\mathcal{P}[(\mathcal{E}^{uc} \cap h = k]}{\mathcal{P}[h = k]} + \frac{\mathcal{P}[(\mathcal{E}^c \cap h = k]}{\mathcal{P}[h = k]} \tag{B.8}$$

Using the rules of conditional probability and the conditions for occupational choice:

$$\begin{aligned}\frac{\mathcal{P}[\mathcal{E}\cap h=k]}{\mathcal{P}[h=k]} &= \frac{\mathcal{P}[(\mathcal{E}^{uc}|h=k]\,\mathcal{P}[h=k]}{\mathcal{P}[h=k]} + \frac{\mathcal{P}[(\mathcal{E}^{c}\cap h=k]\,\mathcal{P}[h=k]}{\mathcal{P}[h=k]} \\ &= \mathcal{P}[(\mathcal{E}^{uc}\,|\,h=k] + \mathcal{P}[(\mathcal{E}^{c}\,|\,h=k] \\ &= \mathcal{P}\left[z > \underline{z}^{uc}(h_k)\,|\,h=k\right]\;\mathcal{P}[a_i > a^*(z,h_k)\,|\,z > \underline{z}^{uc}(h_k), h=j] \\ &+ \mathcal{P}\left[z > \underline{z}^{c}(a_i,h_k)\,|\,h=k\right]\;\mathcal{P}[a_i \leq a^*(z,h_k)\,|\,z > \underline{z}^{c}(a_i,h_k), h=k] \\ &= (\mathbf{1}-\boldsymbol{\Theta_k})'(\mathbf{1}-\boldsymbol{G_z}[\underline{\boldsymbol{z}}^{\boldsymbol{uc}}(\boldsymbol{h_k})]) + \boldsymbol{\Theta_k}'(\mathbf{1}-\boldsymbol{G_z}[\underline{\boldsymbol{z}}^{\boldsymbol{c}}(\boldsymbol{a_i},\boldsymbol{h_k})])\end{aligned} \tag{B.9}$$

where the vector of $\boldsymbol{\Theta}$'s and complementary cdf's are defined accordingly to express the previous equality as a non-negative linear combination ($0 \leq \Theta_i \leq 1$) of non-negative functions ($0 \leq (1 - G_z(z_j)) \leq 1$) for every pair of (a_i, z_j).

The cutoffs for the tail distributions are determined by the conditions given in Lemma 3. It is now simple to derive necessary and sufficient conditions for B.9 to be strictly decreasing/increasing in the domain of human capital. Note that we can safely deduce strict (instead of only weak) monotonicity of B.9 because of strict monotonicity of $G_z(z)$: a non-singular cdf is strictly increasing on its support if the support is a finite interval, and in our case z has indeed compact support because of reflecting barriers.

[$\Theta = 0$ ($\vartheta \to 1$)] This is the case when equation (2.22) characterizes the solution. The cutoff $\underline{z}^{uc}(h)$ is weakly increasing in human capital for $h \leq \hbar$, and for $\omega > \nu$, it is weakly decreasing for $h > \hbar$. If the marginal CDF of z is strictly increasing (no flat intervals), then the object of interest is strictly decreasing for $h \leq \hbar$ and strictly increasing for $h > \hbar$. Therefore, the necessary and sufficient condition is simply $\omega > \nu$.

[$\Theta = 1$ ($\vartheta = 0$)]] This is the case when equation (2.23) applies and is analogous to the previous one with the only exception that the parameter restriction is different. The necessary and sufficient condition in this case is simply $\omega > \nu + \alpha - \alpha\nu$.

[$0 \leq \Theta \leq 1$] The general case is analyzed using the continuity of B.9. All functions are decreasing for $h \leq \hbar$, so we only need to analyze the region above the threshold. For a given level of financial frictions ϑ and thus a vector $\boldsymbol{\Theta_k}$ for every h, there exists some $\widehat{\omega} \in [\nu, \nu + \alpha - \alpha\nu]$ such that both components of B.9 are strictly increasing in h for all $h > \hbar$. Since any non-negative linear combination of non-negative decreasing (increasing) functions is itself decreasing (increasing), and since both components have the same minimum in $\mathcal{H}$, the result follows. ■

Proof of Proposition 2**:** For every entrepreneurial outcome $X(h, a, z)$ – net production, capital demand, labor demand, profits – the focus is on the evolution of the conditional (truncated) expectation along the dimension of human capital: $\partial\,\mathbb{E}[X|\mathcal{E}, h_e]/\partial h_e$. To save space I will illustrate the proof for the case of indirect profit functions, with the procedure for every other $X(h, a, z)$ being analogous.

The key observation is that every outcome $X(h, a, z)$ is homogeneous of the same degree, as well as strictly increasing and convex in z by the Envelope theorem. It is helpful to consider the case $\mathcal{E}^{uc}$ and $\mathcal{E}^{c}$ separately, as well as the cases $h_e \leq \hbar$ and $h_e > \hbar$.

[$\mathcal{E}^{uc}$ and $h_e \leq \hbar$] For unconstrained entrepreneurs without technology adoption, the conditional expectation involves only a function of the random variable $z > \underline{z}^{uc}(h_e)$. From the first part of Lemma 3, we know that $\underline{z}^{uc}(h_e)$ is strictly increasing for all $h_e \leq \hbar$, and $\pi^{uc}(z)$ is strictly increasing and convex in z, hence $\partial \underline{z}^{uc}(h_e)/\partial h_e > 0$ implies $\partial\,\mathbb{E}[\pi^{uc}(z)|\mathcal{E}, h_e]/\partial h_e > 0$.

To see this more clearly, use the definition of the truncated expectation together with the Leibniz integral rule to derive

$$\frac{\partial\,\mathbb{E}[\pi^{uc}(z)|z > \underline{z}^{uc}]}{\partial h_e} = \frac{\partial\,\mathbb{E}[\pi^{uc}(z)|z > \underline{z}^{uc}]}{\partial \underline{z}^{uc}}$$
$$= \frac{g_z(\underline{z}^{uc})}{1 - G_z(\underline{z}^{uc})}\{\mathbb{E}[\pi^{uc}(z)|z > \underline{z}^{uc}] - \pi^{uc}(\underline{z}^{uc})\} > 0 \tag{B.10}$$

where the term in curly brackets is always positive due to Jensen's inequality together with truncation from below. It is clear that this holds for all non $\mathbb{P}$-null sets.

[$\mathcal{E}^{uc}$ and $h_e > \hbar$] For unconstrained entrepreneurs with technology adoption, using a similar argument as above together with the homogeneity property of indirect profits, it suffices to consider how the lowest effective productivity $\zeta(\underline{z}^{uc}, h_e)$ evolves along h_e. That is, $\partial\zeta(\underline{z}^{uc}(h_e))/\partial h_e > 0 \Longrightarrow \partial\,\mathbb{E}[\pi^{uc}(z)|\mathcal{E}, h_e]/\partial h_e > 0$. By Lemmas 2 and 3, $\zeta(\underline{z}^{uc}(h_e)) \propto h_e^{1-\nu}$, which is strictly increasing in h_e.

[$\mathcal{E}^{c}$ and $h_e \leq \hbar$] For constrained entrepreneurs without technology adoption, the same arguments as above apply, *mutatis mutandis.* We employ the second part of Lemma 3 and we work along the dimension of assets by conditioning on each a_i. If we analyze the conditional (truncated) expectation point-by-point, then we can also understand the total expectation $E\left[\pi^c(h_e, a, z)\right] = \sum_i \mathbb{E}\left[\pi^c(h_e, z|a_i)\right]\mathcal{P}(a_i)$. As above:

$$\frac{\partial\,\mathbb{E}[\pi^{c}(z)|z > \underline{z}^{c}, a_i]}{\partial h_e} = \frac{\partial\,\mathbb{E}[\pi^{c}(z)|z > \underline{z}^{c}, a_i]}{\partial \underline{z}^{c}}$$
$$= \frac{g_{a,z}(\underline{z}^{c})}{1 - G_{a,z}(\underline{z}^{c})}\{\mathbb{E}[\pi^{c}(z)|z > \underline{z}^{c}, a_i] - \pi^{c}(\underline{z}^{c})\} > 0 \tag{B.11}$$

where the term in curly brackets is always positive due to the same reasons as before.

[$\mathcal{E}^c$ and $h_e > \hbar$] For constrained entrepreneurs with technology adoption, as in the unconstrained case, it suffices to consider how the lowest effective productivity evolves along h_e. By Lemmas 2 and 3, it is clear that $\zeta(\underline{z}^{uc}, h_e|a_i)$ is strictly increasing in h_e.

By integrability and the partition property of $\mathcal{E}$, we can directly apply the law of total expectation to obtain

$$\begin{aligned}\mathbb{E}\left[X|\mathcal{E}, h_e\right] = \mathbb{E}\left[X|\mathcal{E}^{uc}, h_e \leq \hbar\right] \mathcal{P}(\mathcal{E}^{uc}, h_e \leq \hbar) + \mathbb{E}\left[X|\mathcal{E}^{uc}, h_e > \hbar\right] \mathcal{P}(\mathcal{E}^{uc}, h_e > \hbar) \\ + \mathbb{E}\left[X|\mathcal{E}^{c}, h_e \leq \hbar\right] \mathcal{P}(\mathcal{E}^{c}, h_e \leq \hbar) + \mathbb{E}\left[X|\mathcal{E}^{c}, h_e > \hbar\right] \mathcal{P}(\mathcal{E}^{c}, h_e > \hbar)\end{aligned}$$

Since we have shown that all conditional expectations on the rhs are strictly increasing in h_e almost surely, so is any positive linear combination of them. ■

Proof of Proposition 3**:** Aggregate physical capital (K) and aggregate human capital (H) can be derived using the expressions in Lemma 1 as follows:

$$\begin{aligned}K &= \iiint_{\mathcal{H}\times\mathcal{A}\times\mathcal{Z}} \widehat{k}(h_e, a, z)\, dG(h, a, z) \\ &= \iiint_{\mathcal{H}\times\mathcal{A}\times\mathcal{Z}} (z\widetilde{h}_e)^{\frac{1}{\nu}} \left(\widetilde{r}(h, a, z) + \delta\right)^{-\frac{\widehat{\alpha}+\nu}{\nu}} dG(h, a, z)\, \widehat{\alpha}^{\frac{\widehat{\alpha}+\nu}{\nu}} \Psi^{\frac{1-(\widehat{\alpha}+\nu)}{\nu}} \qquad \text{(B.12)}\end{aligned}$$

$$\begin{aligned}H &= \iiint_{\mathcal{H}\times\mathcal{A}\times\mathcal{Z}} \widehat{\ell}(h_e, a, z)\, dG(h, a, z) \\ &= \iiint_{\mathcal{H}\times\mathcal{A}\times\mathcal{Z}} (z\widetilde{h}_e)^{\frac{1}{\nu}} \left(\widetilde{r}(h, a, z) + \delta\right)^{-\frac{\widehat{\alpha}}{\nu}} dG(h, a, z)\, \widehat{\alpha}^{\frac{\widehat{\alpha}}{\nu}} \Psi^{\frac{1-\widehat{\alpha}}{\nu}} \qquad \text{(B.13)}\end{aligned}$$

where $\widehat{\alpha} \equiv \alpha(1-\nu)$ and $\Psi \equiv \frac{(1-\alpha)(1-\nu)}{w}$. It is also useful to derive the factor-only part of aggregate net output:

$$\left(K^{\alpha} H^{1-\alpha}\right)^{1-\nu} = \left(\int\limits_{\Omega} (z\widetilde{h}_e)^{\frac{1}{\nu}} \, (\widetilde{r}+\delta)^{-\frac{\widehat{\alpha}+\nu}{\nu}} \, dG \right)^{\widehat{\alpha}} \left(\int\limits_{\Omega} (z\widetilde{h}_e)^{\frac{1}{\nu}} \, (\widetilde{r}+\delta)^{-\frac{\widehat{\alpha}}{\nu}} \, dG \right)^{1-(\widehat{\alpha}+\nu)} \widehat{\alpha}^{\frac{\widehat{\alpha}}{\nu}} \Psi^{\frac{1-(\widehat{\alpha}+\nu)}{\nu}} \tag{B.14}$$

where for notational simplicity I use the shorthand $\int\limits_{\Omega}$ for the triple integral.

Then, aggregate output (real GDP) per person engaged can be expressed as

$$\begin{aligned}
Y &= \iiint\limits_{\mathcal{H}\times\mathcal{A}\times\mathcal{Z}} z\widetilde{h} \left(\widetilde{k}(h_e,a,z)^{\alpha} \, \widetilde{\ell}(h_e,a,z)^{1-\alpha} \right)^{1-\nu} dG(h,a,z) \\
&= \left(\int\limits_{\Omega} (z\widetilde{h}_e)^{\frac{1}{\nu}} \, (\widetilde{r}+\delta)^{-\frac{\widehat{\alpha}}{\nu}} \, dG \right) \widehat{\alpha}^{\frac{\widehat{\alpha}}{\nu}} \Psi^{\frac{1-(\widehat{\alpha}+\nu)}{\nu}} \\
&= \frac{\left(\int\limits_{\Omega} (z\widetilde{h}_e)^{\frac{1}{\nu}} \, (\widetilde{r}+\delta)^{-\frac{\widehat{\alpha}}{\nu}} \, g(h,a,z|\mathcal{E}) dh \, da \, dz \right)^{\widehat{\alpha}+\nu}}{\left(\int\limits_{\Omega} (z\widetilde{h}_e)^{\frac{1}{\nu}} \, (\widetilde{r}+\delta)^{-\frac{(\widehat{\alpha}+\nu)}{\nu}} \, g(h,a,z|\mathcal{E}) dh \, da \, dz \right)^{\widehat{\alpha}}} \Big(1 - G(h,a,z) \Big)^{\nu} \left(K^{\alpha} H^{1-\alpha}\right)^{1-\nu} \\
&= \frac{\left(\mathbb{E}\left[\left(z\widetilde{h}\right)^{\frac{1}{\nu}} \, \left(\widehat{r}(h,a,z)+\delta\right)^{-\frac{\widehat{\alpha}}{\nu}} \middle| \mathcal{E} \right] \right)^{\widehat{\alpha}+\nu}}{\left(\mathbb{E}\left[\left(z\widetilde{h}\right)^{\frac{1}{\nu}} \, \left(\widehat{r}(h,a,z)+\delta\right)^{-\frac{(\widehat{\alpha}+\nu)}{\nu}} \middle| \mathcal{E} \right] \right)^{\widehat{\alpha}}} \, \mu(\mathcal{E})^{\nu} \left(K^{\alpha} H^{1-\alpha}\right)^{1-\nu}
\end{aligned} \tag{B.15}$$

By following the same procedure and using the fact that $\widetilde{r}(h,a,z) = r$, we can derive aggregate output in the frictionless case ($\vartheta \to 1$):

$$Y_* = \left(\mathbb{E}\left[(z\widetilde{h})^{\frac{1}{\nu}} | \mathcal{E}_* \right] \right)^{\nu} \mu(\mathcal{E}_*)^{\nu} \left(K_*^{\alpha} H_*^{1-\alpha} \right)^{1-\nu} \tag{B.16}$$

$$\text{where} \quad K_* = \iint\limits_{\mathcal{H}_{\mathcal{E}} \times \mathcal{Z}_{\mathcal{E}}} (z\widetilde{h}_e)^{\frac{1}{\nu}} dG(h,z) \left(\frac{\widehat{\alpha}}{r+\delta} \right)^{\frac{\widehat{\alpha}+\nu}{\nu}} \Psi^{\frac{1-(\widehat{\alpha}+\nu)}{\nu}} \tag{B.17}$$

$$H_* = \iint\limits_{\mathcal{H}_{\mathcal{E}} \times \mathcal{Z}_{\mathcal{E}}} (z\widetilde{h}_e)^{\frac{1}{\nu}} dG(h,z) \left(\frac{\widehat{\alpha}}{r+\delta} \right)^{\frac{\widehat{\alpha}}{\nu}} \Psi^{\frac{1-\widehat{\alpha}}{\nu}} \tag{B.18}$$

■

Existence and uniqueness of a non-singular joint density. It is very common in the literature to simply posit the existence and uniqueness of a joint density $g(.)$, but there is no need for such imposition. It is straightforward to prove it in a fairly general setting with multiple Itô processes, $X_i, i = 1, \ldots, n$. The only assumptions one needs to retain are the standard Lipschitz continuity conditions for the existence and uniqueness of SDE solutions for each univariate process. Then, it is known that each induced measure μ_i is absolutely continuous w.r.t. Lebesgue measure λ. By absolute continuity and σ-finiteness, it follows that the product measure $\mu^n := \mu_1 \times \cdots \times \mu_n$ is absolutely continuous w.r.t. to n-dimensional Lebesgue measure λ^n. As $\mu^n << \lambda^n$, the Radon-Nikodym theorem ensures the existence and uniqueness λ^n-almost everywhere of a joint density $g(x_1, \ldots, x_n)$, which is simply the Radon-Nikodym derivative of the induced product measure w.r.t. λ^n.

Necessary KFE boundary condition. The purpose of this short proof is to establish an important theoretical result in this setting, and to highlight the necessity of boundary conditions so that the desired result holds. The procedure is instructive and straightforward as in Gabaix et al. (2016). The domain of both infinitesimal generators $\mathcal{A}$ and $\mathcal{B}$ is the Hilbert space $\mathcal{L}^2(\Omega)$: the space of square-integrable continuous functions v equipped with the appropriate inner product. In our case, the inner product between two real continuous trivariate functions $v(\mathbf{x})$ and $w(\mathbf{x})$ is defined as $\langle v, w\rangle = \iiint v(\mathbf{x})\, w(\mathbf{x})\, d^3\mathbf{x}$. To prove the adjointness of $\mathcal{A}$ and $\mathcal{B}$ we need to show that $\langle v, \mathcal{B}g\rangle = \langle \mathcal{A}v, g\rangle$, for all $v \in \mathcal{L}^2(\Omega)$, which implies that $\mathcal{B} = \mathcal{A}^*$. As before, the use of the Fubini-Tonelli theorem is crucial.

$$
\begin{aligned}
\langle v, \mathcal{B}g\rangle &= \iiint\limits_{\Omega} v\left[-\frac{\partial}{\partial a}(\widetilde{d}\, g) - \frac{\partial}{\partial z}(\widetilde{\mu}_z\, g) + \frac{1}{2}\frac{\partial^2}{\partial z^2}(\widetilde{\sigma}_z^2\, g) - \eta g\right] d^3\mathbf{x} \\
&= \underbrace{-\int\limits_{\mathcal{H}}\int\limits_{\mathcal{Z}} v(\widetilde{d}g)\Big|_{\mathcal{A}} dz\, dh}_{\Delta 1=0} + \iiint\limits_{\Omega}\left(v_a\widetilde{d} - \eta\right) g\; d^3\mathbf{x} \\
&\quad + \underbrace{\int\limits_{\mathcal{H}}\int\limits_{\mathcal{A}} v\left(-\widetilde{\mu}_z\, g + \frac{1}{2}\frac{\partial}{\partial z}(\widetilde{\sigma}_z^2\, g)\right)\Big|_{\mathcal{Z}} da\, dh}_{\Delta 2=0} + \iiint\limits_{\Omega} v_z\left(\widetilde{\mu}_z\; g - \frac{1}{2}\frac{\partial}{\partial z}(\widetilde{\sigma}_z^2\, g)\right) d^3\mathbf{x} \\
&= \iiint\limits_{\Omega}\left(v_a\widetilde{d} + v_z\widetilde{\mu}_z - \eta\right) g\; d^3\mathbf{x} - \underbrace{\int\limits_{\mathcal{H}}\int\limits_{\mathcal{A}} \frac{1}{2}\,\widetilde{\sigma}_z^2\, g\, v_z\Big|_{\mathcal{Z}} da\, dh}_{\Delta 3=0} + \iiint\limits_{\Omega}\frac{1}{2} v_{zz}\widetilde{\sigma}_z^2\, g\, d^3\mathbf{x} \\
&= \iiint\limits_{\Omega}\left(v_a\widetilde{d} + v_z\,\widetilde{\mu}_z + \frac{1}{2}v_{zz}\widetilde{\sigma}_z^2 - \eta\right) g\; d^3\mathbf{x} \\
&= \langle \mathcal{A}v, g\rangle
\end{aligned}
$$

I use integration by parts to get the second equality, and once again to obtain the third equality. We need the term $\Delta 3 = 0$, which requires the boundary conditions $v_z(h, a, \underline{z}) = v_z(h, a, \bar{z}) = 0$. When $v(.)$ is the value function $V(h, a, z)$, these boundary conditions are automatically satisfied because the reflecting barriers in $[\underline{z}, \bar{z}]$ imply the corresponding NBC (2.12) . Finally, the boundary condition (2.16) is due to the fact that $(\Delta 1 + \Delta 2) = 0$ needs to be satisfied.

Ornstein-Uhlenbeck Process. Although this is definitely not new information, it would be helpful to illustrate *(i)* the exact way in which an Ornstein-Uhlenbeck process is the continuous-time counterpart of an AR(1) process (both in natural logs), and *(ii)* determine its distributional properties. In discrete time, a general log AR(1) process takes the form

$$\log z_{t+1} = (1-\phi)\theta + \phi \ln z_t + \tilde{\sigma}\,\varepsilon_t, \qquad \varepsilon \overset{i.i.d.}{\sim} N(0,1)$$

A general Ornstein-Uhlenbeck process takes the form

$$dx = \phi(\mu - x)ds + \sigma dW_s$$

Given that the drift and diffusion terms are constant and thus fulfill the sufficient growth and Lipschitz conditions, we can employ the method of multiplying by the integrating factor $e^{\phi s}$ and integrate over the interval $[t, t+\Delta]$ to get

$$e^{\phi(t+\Delta)}x(t+\Delta) - e^{\phi t}x(t) = \mu(e^{\phi(t+\Delta)} - e^{\phi t}) + \sigma \int_t^{t+\Delta} e^{\phi s}\, dW_s$$

Rearranging we obtain the *strong* SDE solution,

$$x(t+\Delta) = \mu(1-e^{-\phi\Delta}) + e^{-\phi\Delta}x(t) + \underbrace{\sigma e^{-\phi\Delta}\int_t^{t+\Delta} e^{\phi(s-t)}\, dW_s}_{\mathcal{I}(t,\Delta)} \tag{B.19}$$

Using the martingale property of Itô integrals along with Itô's isometry we obtain

$$\begin{aligned}\mathbb{E}\left[\mathcal{I}(t,\Delta)\right] &= 0,\\ Var\left[\mathcal{I}(t,\Delta)\right] &= \sigma^2 e^{-2\phi\Delta}\int_t^{t+\Delta} e^{2\phi(s-t)}\, ds = \frac{\sigma^2(1-e^{-2\phi\Delta})}{2\phi},\end{aligned}$$

Therefore we can express the unique Markov solution as

$$x(t+\Delta) = \mu(1-e^{-\phi\Delta}) + e^{-\phi\Delta}x(t) + \sigma\sqrt{\frac{1-e^{-2\phi\Delta}}{2\phi}}\,\varepsilon, \qquad \varepsilon \overset{i.i.d.}{\sim} N(0,1) \tag{B.20}$$

In addition, the distributional properties of this process are as follows. For $\phi > 0, \sigma > 0$, $\mu \in \mathbb{R}$ and $X_0 = x_0$, recall that the O-U process for $X_t := \log Y_t$ admits the strong SDE solution

$$X_t = \mu + (x_0 - \mu)e^{-\phi t} + \sigma \int_0^t e^{-\phi(t-s)} dW_s \tag{B.21}$$

This is a sum of deterministic terms and an integral of a deterministic function with respect to a Wiener process with normally distributed increments, so the distribution of X_t has to be Normal. The conditional expectation is

$$\begin{aligned}
\mathbb{E}[X_t|X_0 = x_0] &= \mathbb{E}\left[\mu + (x_0-\mu)e^{-\phi t} + \sigma\int_0^t e^{-\phi(t-s)}dW_s|X_0 = x_0\right] \\
&= \mu + (x_0-\mu)e^{-\phi t}
\end{aligned}$$

The conditional variance, calculated using Itō's isometry is

$$\begin{aligned}
Var[X_t|X_0 = x_0] &= \mathbb{E}\left[\left(\sigma\int_0^t e^{-\phi(t-s)}dW_s|X_0 = x_0\right)^2\right] \\
&= \sigma^2\,\mathbb{E}\left[\int_0^t e^{-2\phi(t-s)}ds\right] \\
&= \frac{\sigma^2}{2\phi}\left(1-e^{-2\phi t}\right)
\end{aligned}$$

The conditional distribution of X_t is therefore

$$(X_t|X_0 = x_0) \sim \mathcal{N}\left(\mu + (x_0-\mu)e^{-\phi t}, \frac{\sigma^2}{2\phi}\left(1-e^{-2\phi t}\right)\right)$$

which converges asymptotically to its stationary and ergodic distribution

$$X_t \xrightarrow{d} X \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{2\phi}\right)$$

Making the direct association with the AR(1) process in the beginning, we deduce that

$$\begin{aligned} x(t+\Delta) &\iff \log z_{t+1} \\ x(t) &\iff \log z_t, \\ e^{-\phi\Delta} &\iff \phi \\ \mu(1-e^{-\phi\Delta}) &\iff \theta(1-\phi) \\ \sigma\sqrt{\frac{1-e^{-2\phi\Delta}}{2\phi}} &\iff \tilde{\sigma} \end{aligned}$$

As in a discrete setting the interval between sampled observations is equal to one unit of time, by setting $\Delta = 1$ and using the dimensionless time variable $t = \widehat{t}/\Delta$, we arrive at the exact continuous-time counterpart.

B.2 Numerical Methods

To solve the model numerically I use an *implicit upwind finite difference* scheme similar to Achdou et al. (2022). In this part of the appendix I summarize the most important features of the employed numerical methods.[54]

B.2.1 Construction of Grids

The value functions are approximated on the state space $(\mathcal{A}\times\mathcal{Z}\times\mathcal{H})$, discretized at $(I\times J\times K)$ grid points. I use a standard uniform grid for the entrepreneurial ability (z) process on the interval $[0.5797, 4.1658]$, at 30 equidistant grid points. The reflecting barriers are chosen such that $G_z(\underline{z}) = 0.1111$ and $G_z(\bar{z}) = 0.9993$, with $G_z(.)$ being the stationary lognormal distribution under the calibrated mean and variance.

The approximation for h is necessarily made on a non-uniform grid in view of the nonlinear mapping between schooling and human capital (2.30). The grid consists of 21 points, one for every full year of education: $S = 0, 1, 2, \ldots, 20$. The bounds are determined by the mapping itself: $\underline{h} = 1$ and $\bar{h} = 6.1719$.

I work with a non-equispaced grid of 200 points for the dimension of assets (a). As it is well-known in our field or in computational physics and finance, it is often useful to adapt the grid in order to drastically improve the accuracy of finite difference calculations. Moreover, it is an established result that in Bewley-Aiyagari model economies the value function displays significant curvature in low-wealth regions (near the borrowing constraint), whereas it becomes approximately linear for larger values of wealth. To concentrate the

[54] A good list of additional references can be found in the numerical appendix of Achdou et al. (2022). My computer programs is also based on the very useful set of codes and notes of Benjamin Moll and his collaborators, which can be found at https://benjaminmoll.com/codes/.

mesh in areas where the value function is more sensitive, I use an affine transformation to construct a grid that is around 20 times more dense (compared to a uniform grid) in its first third, around 10 times more dense towards the middle, and progressively more sparse for larger asset holdings. Finally, there are two actions I take to determine an appropriate upper bound for assets. *i)* The resulting grid covers the full support of the stationary wealth distribution, so that the last grid point results in $g_a(a_{\max}) \approx 10^{-22}$. *ii)* The upper bound does not affect the saving decisions of the highest-type agents with non-zero measure. One of the lowest upper bounds that satisfies the above is $\bar{a} = 11,000$.

B.2.2 Hamilton–Jacobi–Bellman Equations

Denote by Δx^+ (Δx^-) the forward (backward) inter-grid distances for each variable $x = a, z$. To make notation easier to read I use the shorthand $V_{i,j,k} := V(a_i, z_j, h_k)$. At each point on the grid the first-order partial derivatives of $V_{i,j,k}$ are computed with either a *forward* or a *backward* finite difference approximation:

$$\begin{aligned} \partial_a^F V_{i,j,k} &:= \frac{V_{i+1,j,k} - V_{i,j,k}}{\Delta a^+} \qquad & \partial_a^B V_{i,j,k} &:= \frac{V_{i,j,k} - V_{i-1,j,k}}{\Delta a^-} \\ \partial_z^F V_{i,j,k} &:= \frac{V_{i,j+1,k} - V_{i,j,k}}{\Delta z^+} \qquad & \partial_z^B V_{i,j,k} &:= \frac{V_{i,j,k} - V_{i,j-1,k}}{\Delta z^-} \end{aligned} \tag{B.22}$$

and the second-order partial derivatives using a *central* difference approximation:

$$\partial_{zz} V_{i,j,k} := \frac{V_{i,j+1,k} - 2V_{i,j,k} + V_{i,j-1,k}}{(\widetilde{\Delta z})^2} \tag{B.23}$$

where $(\widetilde{\Delta z})^2 := \frac{1}{2}(\Delta z^+ + \Delta z^-) \cdot (\Delta z^+ \cdot \Delta z^-)$. Note that in the simplest case of equispaced grids, $(\widetilde{\Delta z})^2$ reduces to $(\Delta z)^2$. The backward and forward difference approximations for savings—the optimal control drift—are defined as:

$$\begin{aligned} s^F_{i,j,k} &:= \mathcal{Y}_{i,j,k} + ra_i - (u')^{-1}(\partial^F_a V_{i,j,k}) \\ s^B_{i,j,k} &:= \mathcal{Y}_{i,j,k} + ra_i - (u')^{-1}(\partial^B_a V_{i,j,k}) \end{aligned} \tag{B.24}$$

Analogously, define the discretized forward and backward *Hamiltonians*:

$$\begin{aligned} H^F_{i,j,k} &:= u(c^F_{i,j,k}) + \partial^F_a V_{i,j,k}\ s^F_{i,j,k} \\ H^B_{i,j,k} &:= u(c^B_{i,j,k}) + \partial^B_a V_{i,j,k}\ s^B_{i,j,k} \end{aligned} \tag{B.25}$$

Non-convexities may result in value functions that are not strictly concave in the endogenous state variable. In fact, it gives rise to a *convex kink*, so problems may come up in the approximation of the optimal drift when we have both $s^F_{i,j,k} > 0$ and $s^B_{i,j,k} < 0$. A fast and reliable solution comes from the field of computational physics: a so-called *upwind scheme*. The main idea is to use a forward difference whenever the drift of each state variable is positive, and a backward difference whenever the drift is negative. In our case, the appropriate upwind scheme is the following approximation:

$$\begin{aligned} v'_{i,j} = v'_{i,j,F} &\left(\mathbb{1}_{\{s_{i,j,F}>0\}}\mathbb{1}^{unique}_i + \mathbb{1}_{\{H_{i,j,F}\geq H_{i,j,B}\}}\mathbb{1}^{both}_i\right) \\ &+ v'_{i,j,B}\left(\mathbb{1}_{\{s_{i,j,B}<0\}}\mathbb{1}^{unique}_i + \mathbb{1}_{\{H_{i,j,F}\leq H_{i,j,B}\}}\mathbb{1}^{both}_i\right) \\ &+ \bar{v}'_{i,j}\ \mathbb{1}_{\{s_{i,j,F}\leq 0\leq s_{i,j,B}\}} \end{aligned} \tag{B.26}$$

where the indicator $\mathbb{1}^{both}_i$ is defined for the "problematic case" in which both $s_{i,j,F} > 0$ and $s_{i,j,B} < 0$, and the indicator $\mathbb{1}^{unique}_i$ is defined for the "unproblematic" cases $s_{i,j,F} < 0$ and $s_{i,j,B} > 0$, as well as $s_{i,j,F} < 0$ and $s_{i,j,B} < 0$.

Putting it all together, the implicit upwind finite difference approximation reads

$$\frac{V_{i,j,k}^{n+1} - V_{i,j,k}^{n}}{\Delta} + (\rho+\eta) V_{i,j,k}^{n+1} = u\left(c_{i,j,k}^{n}\right) + \partial_a^F V_{i,j,k}^{n+1}\left(s_{i,j,k}^{F_n}\right)^{+} + \partial_a^B V_{i,j,k}^{n+1}\left(s_{i,j,k}^{B_n}\right)^{-} + \partial_z^F V_{i,j,k}^{n+1}\left(\tilde{\mu}_{z_j}\right)^{+} + \partial_z^B V_{i,j,k}^{n+1}\left(\tilde{\mu}_{z_j}\right)^{-} + \frac{1}{2}\tilde{\sigma}_{z_j}^2 \partial_{zz} V_{i,j,k}^{n+1} \tag{B.27}$$

Superscripts denote the iteration counter; the positive and negative part of the drift is $f(x)^+ = \max\{f(x), 0\}$ and $f(x)^- = -\min\{f(x), 0\}$, respectively. Notice both the n superscript on the LHS and the $n+1$ superscripts on the RHS to see why the scheme is (semi)-implicit, keeping in mind that the HJB equation is solved backwards in time.[55]

The result is an $(I \times J \times K)$-dimensional system of equations that can be expressed in matrix form and solved very efficiently using sparse matrix procedures. Substitute for the partial derivative approximations and rearrange to get

$$\begin{aligned}\frac{V_{i,j,k}^{n+1} - V_{i,j,k}^{n}}{\Delta} + \rho V_{i,j,k}^{n+1} = {} & u\left(c_{i,j,k}^{n}\right) \\ & + V_{i-1,j,k}^{n+1}\,\alpha_{i,j,k}^{n} + V_{i,j,k}^{n+1}\left(\beta_{i,j,k}^{n} + \chi_j^z + \chi_k^h\right) + V_{i+1,j,k}^{n+1}\,\gamma_{i,j,k}^{n} \\ & + V_{i,j-1,k}^{n+1}\,\phi_j^z + V_{i,j+1,k}^{n+1}\,\psi_j^z\end{aligned} \tag{B.28}$$

The auxiliary variables $(\alpha^n, \beta^n, \gamma^n)$ are encoding information about agents' optimal savings, i.e., the drift of the *endogenous* state (in the absence of a diffusion term), at each iteration and each sets of grid points (i, j, k):

$$\alpha_{i,j,k}^{n} := -\frac{\left(s_{i,j,k}^{B_n}\right)^{-}}{\Delta a^{-}},\ \beta_{i,j,k}^{n} := -\frac{\left(s_{i,j,k}^{F_n}\right)^{+}}{\Delta a^{+}} + \frac{\left(s_{i,j,k}^{B_n}\right)^{-}}{\Delta a^{-}} - \eta,\ \gamma_{i,j,k}^{n} := \frac{\left(s_{i,j,k}^{F_n}\right)^{+}}{\Delta a^{+}} \tag{B.29}$$

[55] In contrast to explicit schemes being computationally slow because of the Courant-Friedrichs-Lewy (CFL) condition, this semi-implicit scheme achieves fast convergence because it allows for arbitrarily an large time step. Under value function iteration, the term $\frac{V_{i,j,k}^{n+1} - V_{i,j,k}^{n}}{\Delta} \to 0$ as $V_{i,j,k}^{n+1} \to V_{i,j,k}^{n}$.

and the auxiliary variables $(\phi^z, \phi^h, \chi^z, \chi^h, \psi^z, \psi^h)$ are encoding information about the drift and diffusion of the exogenous state at each grid point:

$$\phi_j^z = -\frac{(\tilde{\mu}_j^z)^-}{\Delta z^-} + \frac{\tilde{\sigma}_{z_j}}{2(\Delta z)^2}, \chi_j^z = \frac{(\tilde{\mu}_j^z)^-}{\Delta z^-} - \frac{(\tilde{\mu}_j^z)^+}{\Delta z^+} - \frac{\tilde{\sigma}_{z_j}}{(\Delta z)^2}, \psi_j^z = \frac{(\tilde{\mu}_j^z)^+}{\Delta z^+} + \frac{\tilde{\sigma}_{z_j}}{2(\Delta z)^2} \tag{B.30}$$

Using the above definitions, the discretized system can be represented in matrix form

$$\frac{1}{\Delta}\left(\mathbf{v}^{n+1} - \mathbf{v}^n\right) + \rho \mathbf{v}^{n+1} = \mathbf{u}^n + \mathbf{A}^n \, \mathbf{v}^{n+1} \tag{B.31}$$

where $\mathbf{v}^{n+1} = \mathrm{vec}(\mathbf{V}^{n+1})$, $\mathbf{v}^n = \mathrm{vec}(\mathbf{V}^n)$, and $u^n = \mathrm{vec}(\mathbf{u}^n)$. The (very sparse) matrix $\mathbf{A}^n = \left(\widetilde{\mathbf{A}}^n + \widetilde{\mathbf{Z}}\right)$ is of size $(I \times J \times K) \times (I \times J \times K)$. It is understood that the "HJB operator," $\mathcal{A}$, is the infinitesimal generator of the joint stochastic process (a, z, h). Such a differential operator can be thought of as the infinite-dimensional analogue of a continuous time transition matrix. The matrix $\mathbf{A}^n$ is the discretized version of $\mathcal{A}$. Hence, the elements of $\mathbf{A}^n$ satisfy the three properties of a proper *Markov transition rate matrix* (or *intensity matrix*): *i)* $0 \leq -q_{ii} \leq \infty$; *ii)* $0 \leq q_{ij}, \ \forall\, i \neq j$; *iii)* $\sum_i q_{ij} = 0, \ \forall\, i$.

Finally, re-express the system as a sparse system of linear equations of the general form $\mathbf{Qx} = \mathbf{b}$, so that the system can be solved for $\mathbf{v}^{n+1}$ in one step:

$$\mathbf{Q}^n \mathbf{v}^{n+1} = \mathbf{b}^n, \text{where } \mathbf{Q}^n := \left(\frac{1}{\Delta} + \rho\right)\mathbf{I} - \mathbf{A}^n, \ \mathbf{b}^n := \mathbf{u}^n + \frac{1}{\Delta}\mathbf{v}^n \tag{B.32}$$

B.2.3 Computational Algorithm for the Stationary Equilibrium

I solve for the stationary equilibrium of the model using an extension of the iterative methods described in Achdou et al. (2022). The main differences are that the model *(i)* involves guessing and updating expectations with respect to measurable functions of random variables; *(ii)* it requires iteration on two prices (w, r) until all markets clear.

1. Guess the equilibrium interest rate r^l for $l = 0, 1, 2, \ldots$ on a reasonable interval $[r_{min}, r_{max}]$ with initial guess $r_{min} \leq r^0 \leq r_{max}$. Guess the equilibrium wage rate w^l for $l, = 0, 1, 2, \ldots$ on a reasonable interval $[w_{min}, w_{max}]$ with initial guess $w_{min} \leq w^0 \leq w_{max}$. Guess initial values for the conditional expectations $\mathbb{E}\left[f(a, z)|h; r^l, w^l\right]$, for all relevant measurable functions $f(.)$.

2. Given guesses for factor prices, solve the Hamilton-Jacobi-Bellman equation for agents in the labor force using the implicit FD upwind scheme explained in the previous section.

3. Given guesses for factor prices and expectations, as well as solutions to the HJB equations, solve the optimal schooling problems for newborn agents and obtain the marginal density of human capital $g_h(h)$.

4. Solve the stationary Kolmogorov-Forward equation in line with the implicit FD upwind scheme and obtain the joint density $g(h, a, z)$.

5. Compute the resulting expectations with respect to the joint density $g(h, a, z)$ and check if they coincide with the guesses. If not, update them accordingly for the next iteration.

6. Compute the excess demand for physical capital/assets and the excess demand for labor/human capital, and check whether both markets clear. If not, update r^l and w^l accordingly for the next iteration.

7. Repeat steps 2 through 6 until markets clear and expectations converge.

Appendix C

Appendix to Chapter 3

C.1 Proofs and Additional Derivations

Proof of Proposition 2: The constrained planner solves a variational calculus problem in two steps in order to maximize the balanced growth rate, $1 + g^C = h_{t+1}/h_t = (Y_t/h_t)^\beta$, subject to the resource constraints. This is done by choosing feasible allocations over the set of admissible controls on the Sobolev space $\mathcal{H}^1(\Omega)$:

$$\max_{\substack{k_{t+1}(z)\ell_{t+1}(z),\\ N^c(\rho,z),\mathcal{E}^c(\rho,z)}} A^\beta \left(\iint\limits_{\mathcal{P}\times\mathcal{Z}} z^\theta \left(k_{t+1}(z)/h_{t+1}\right)^{\alpha\theta} \ell_{t+1}(z)^{(1-\alpha)\theta} \, dG(\rho, z, \mathcal{E}^c) \right)^{\frac{\beta}{\theta}} \tag{C.1}$$

$$\text{subject to } \iint\limits_{\mathcal{P}\times\mathcal{Z}} k_{t+1}(z) \, dG(\rho, z, \mathcal{E}^c) = \theta(1-\alpha)Y_t \tag{C.2}$$

$$q \iint\limits_{\mathcal{P}\times\mathcal{Z}} \ell_{t+1}(z) \, dG(\rho, z, \mathcal{E}^c) = 1 - N^c \tag{C.3}$$

The first step of the optimization problem involves the optimal choice of $k_{t+1}(z), \ell_{t+1}(z)$, while keeping $N^c(\rho, z), \mathcal{E}^c(\rho, z)$ fixed. Let J denote the value functional (C.1) and H_k, H_ℓ the constraints (C.2), (C.3) in standard form. Define the Lagrangian functional $\mathcal{L}[k(z), \ell(z)] = J - \lambda_k H_k - \lambda_\ell H_\ell$. By strict concavity and differentiability, the first-order conditions for a global maximum are necessary and sufficient,

$$\delta_\xi \mathcal{L}(k; \ell, \xi) = \frac{\partial}{\partial \varepsilon} \mathcal{L}(k + \varepsilon \xi; \ell)\Big|_{\varepsilon=0} = 0 \tag{C.4}$$

$$\delta_\xi \mathcal{L}(\ell; k, \xi) = \frac{\partial}{\partial \varepsilon} \mathcal{L}(\ell + \varepsilon \xi; k)\Big|_{\varepsilon=0} = 0 \tag{C.5}$$

where δ_ξ denotes the Gateaux derivative in the direction of ξ, for all compactly supported smooth functions ξ vanishing at $\partial\Omega$. Rearranging the FOCs and applying the fundamental lemma of the calculus of variations yields the multipliers,

$$\lambda_k = \frac{\alpha\beta(Y_t/h_t)^{\beta-1}}{\theta(1-\alpha)} \tag{C.6}$$

$$\lambda_\ell = \frac{(1-\alpha)\beta(Y_t/h_t)^{\beta}}{1-N^c} \tag{C.7}$$

By substituting the multipliers back into the FOCs and resource constraints, one can solve the resulting system of equations to uncover the optimal entrepreneurial allocations,

$$\frac{k_{t+1}(z)}{h_{t+1}} = \left[z^\theta A^\theta (q\theta(1-\alpha))^{1-(1-\alpha)\theta} (1-N^c)^{\theta(1-\alpha)} \left(\frac{Y_t}{h_t}\right)^{(1-\beta)(1-(1-\alpha)\theta)-\theta} \right]^{\frac{1}{1-\theta}} \tag{C.8}$$

$$\ell_{t+1}(z) = \left[z^\theta A^\theta (q\theta(1-\alpha))^{\alpha\theta} (1-N^c)^{1-\alpha\theta} \left(\frac{Y_t}{h_t}\right)^{\theta(\alpha-\alpha\beta-1)} \right]^{\frac{1}{1-\theta}} \tag{C.9}$$

$$\frac{y_{t+1}(z)}{h_{t+1}} = \left[z A^\theta (q\theta(1-\alpha))^{\alpha} (1-N^c)^{1-\alpha} \left(\frac{Y_t}{h_t}\right)^{\alpha-\alpha\beta-\theta} \right]^{\frac{1}{1-\theta}} \tag{C.10}$$

Finally, substituting the production plan into the objective functional and manipulating through yields the social planner's long-run growth rate,

$$1 + g^C = \left[A \left(q\theta(1-\alpha) \right)^{\alpha} \mathcal{M}_{\vartheta}^{C} \left(qN^c \right)^{\frac{1-\theta}{\theta}} \left(1 - N^c \right)^{1-\alpha} \right]^{\beta/(1-\alpha+\alpha\beta)} \tag{C.11}$$

where $\mathcal{M}_{\vartheta}^{C} := \left(\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E}^C \right] \right)^{\frac{1-\theta}{\theta}}$ and the planner determines the set $\mathcal{E}^C$ so that there is no misallocation on the intensive margin from occupational sorting stemming from risk aversion.

The second step involves choosing the optimal rate of entrepreneurship, N^c, and the set $\mathcal{E}^C$. By totally differentiating $1 + g^C$ the optimality condition reads,

$$\frac{1-\theta}{\theta} \frac{1}{N^c} - \frac{1-\alpha}{1-N^c} = -\frac{1-\theta}{\theta} \frac{\frac{\partial}{\partial N^c} \mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E}^c \right]}{\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E}^c \right]}$$

Next we derive an expression for the numerator on the RHS by making use of the following key observations. In the absence of misallocation, the above expression is independent of $G_{\rho}(\rho)$, hence the change in $\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E} \right]$ by increasing N^c is exactly equal to the change induced by decreasing the threshold $\overset{*}{z}$ adjusted by the derivative of the cdf (the pdf) at the cutoff point $\overset{*}{z}$. Also, $\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E} \right]$ is a positive and monotonic function, hence its partial derivative has the same magnitude whether we are increasing or decreasing the function w.r.t. N^c. Therefore, using the definition of truncated conditional expectation together with the Leibniz integral rule we get,

$$\frac{\partial}{\partial N^c} \mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E}^c \right] = -\frac{g(\overset{*}{z}) \left(\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E}^c \right] - \overset{*}{z}^{\frac{\theta}{1-\theta}} \right)}{g(\overset{*}{z})(1 - G_z(\overset{*}{z}))} = -\frac{1}{N^c} \left(\mathbb{E}\left[z^{\frac{\theta}{1-\theta}} | \mathcal{E}^c \right] - \overset{*}{z}^{\frac{\theta}{1-\theta}} \right) \tag{C.12}$$

Diving through by $\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^c\right]$ and substituting back into the optimality condition,

$$\frac{1-\theta}{\theta}\frac{\Phi^c}{N^c} = \frac{1-\alpha}{1-N^c}$$

which yields the optimal number of entrepreneurs,

$$N^c = \frac{\Phi^c(1-\theta)}{(1-\alpha)\theta + \Phi^c(1-\theta)}, \quad \Phi^c := \left(\frac{\overset{*}{z}}{\mathcal{M}^c_\vartheta}\right)^{\frac{\theta}{1-\theta}} \tag{C.13}$$

where optimal TFP is defined as $\mathcal{M}^c_\vartheta := \mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^c\right]^{\frac{1-\theta}{\theta}}$. ■

Proof of Proposition 3: We start by deriving an expression for N^{FI}. Recall that in the case of actuarially fair markets against entrepreneurial risk (full insurance), the occupational choice condition reads

$$\frac{1-\theta}{\alpha\theta}\bar{z}^{\frac{\theta}{1-\theta}} = \frac{N^{FI}}{1-N^{FI}}\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^{FI}\right]$$

Since the above is independent of ρ, the cutoff $\bar{z}$ is unique and applies to every agent in the economy. We can thus easily solve for the number of entrepreneurs under full insurance:

$$N^{FI} = \frac{\Phi^{FI}(1-\theta)}{\alpha\theta + \Phi^{FI}(1-\theta)}, \quad \Phi^{FI} := \left(\frac{\bar{z}}{\mathcal{M}^{FI}_\vartheta}\right)^{\frac{\theta}{1-\theta}} \tag{C.14}$$

with corresponding TFP defined as $\mathcal{M}^{FI}_\vartheta := \mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}^{FI}\right]$. We continue by inferring N^d. Recall that in the decentralized economy there exist uncountably many occupational choice conditions:

$$q^{\hat{\rho}}\frac{1-\theta}{\alpha\theta}\hat{z}^{\frac{\theta}{1-\theta}} = \frac{N^d}{1-N^d}\mathbb{E}\left[z^{\frac{\theta}{1-\theta}}|\mathcal{E}\right]$$

This equation holds for all joint realizations $(\rho_i < \hat{\rho}(z_j), z_j > \hat{z}(\rho_i)) \in \mathcal{P} \times \mathcal{Z}$, thus N^d can be pinned down from an infinite number of pairs. Define the set $\mathcal{F} := \{\Phi_j^d := \hat{z}_j^{\frac{\theta}{1-\theta}} / \mathbb{E}[z^{\frac{\theta}{1-\theta}} | \mathcal{E}] : \rho_i < \hat{\rho}(z_j), z_j > \hat{z}(\rho_i)) \in \mathcal{P} \times \mathcal{Z}\}$. In general, the number of entrepreneurs satisfies:

$$N^d = \frac{q^{\hat{\rho}} \Phi_j^d (1-\theta)}{\alpha\theta + \Phi_j^d(1-\theta)}, \quad \Phi_j^d := \left(\frac{\hat{z}}{\mathcal{M}_\vartheta}\right)^{\frac{\theta}{1-\theta}} \tag{C.15}$$

We now seek appropriate values for Φ_j^d to ease the comparison with N^{FI} and examine the two cases that can arise depending on distributional assumptions.

Case 1: $\exists \Phi_j^d = \Phi^{FI} \in \mathcal{F}$. In this case it is straightforward to conclude that $N^{FI} > N^d$ (*a.s.*), since $q^{\hat{\rho}} < 1$ given that $\tilde{\rho} > 0$.

Case 2: $\nexists \Phi_j^d = \Phi^{FI} \notin \mathcal{F}$. This case may apply if and only if $\min(\hat{z}_j) \geq \bar{z}$. Instead of examining Φ_j^d, it is easier to consider the simple fact that under full insurance occupational choice is independent of risk aversion. Hence, there exists at least one measurable set such that $\mathcal{S} = \{(z = \min(\hat{z}_j)) \wedge (\rho \in \mathcal{P})\} \subset \mathcal{E}^{FI}$ and $\mathcal{S} \cap \mathcal{E} = \varnothing$. The set has positive measure and $\mathcal{P}(\mathcal{E}^{FI}) \geq \mathcal{P}(\mathcal{E} \cup \mathcal{S})$, which implies that $N^{FI} > N^d$ *a.s.*. ■.

Intermediate Lemma: The proof is immediate once we identify a crucial insight. Although the number of entrepreneurs under full insurance markets may differ from the centralized economy, in both scenarios there is no misallocation on the intensive margin, as occupational choices are independent of ρ and the z-cutoff is unique (but not necessarily the same). One could think of the resulting occupational choice sets as forming two *similar rectangles*, defined on the measurable space $(\Omega, \mathcal{B}(\Omega))$ equipped with the push-forward probability measure $\mathbb{P}_{T,Z}$.

For this reason the endogenous TFP term – a function of a truncated conditional expectation – will be (conditionally) maximal in both cases, and since the lower truncation point is respectively determined by a single z-cutoff, this translates to:

$$\frac{\bar{z}}{\mathcal{M}_{\vartheta}^{FI}} = \frac{\overset{*}{z}}{\mathcal{M}_{\vartheta}^{c}}$$

$$\left(\frac{\bar{z}}{\mathcal{M}_{\vartheta}^{FI}}\right)^{\frac{\theta}{1-\theta}} =: \Phi^{FI} = \Phi^{c} := \left(\frac{\overset{*}{z}}{\mathcal{M}_{\vartheta}^{c}}\right)^{\frac{\theta}{1-\theta}} \tag{C.16}$$

■

Proof of Proposition 4: The proof is straightforward given the result of the intermediate lemma above. Rearrange (C.14) and (C.13) to get,

$$\alpha = \frac{\Phi^{FI}(1-\theta)(1-N^{FI})}{\theta N^{FI}}$$

$$1-\alpha = \frac{\Phi^{c}(1-\theta)(1-N^{c})}{\theta N^{c}}$$

Since $\Phi^{FI} = \Phi^{c} = \Phi \in \mathbb{R}_{+}$, this proves that $N^{FI} = N^{c}$ if and only if $\alpha = \frac{1}{2}$. As both equations are strictly decreasing in N, we can conclude directly that $N^{FI} > N^{c} \iff \alpha < \frac{1}{2}$ and $N^{FI} < N^{c} \iff \alpha > \frac{1}{2}$. ■