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Law and Decisions in Corporations

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by

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Contents

List of Figures

List of Tables

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Dedicated to Philip H. Dybvig.

ABSTRACT OF THE DISSERTATION

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This dissertation studies the impact of corporate laws and bankruptcy laws on decisions in corporations from a theoretical perspective. Chapter 1 studies the impact of liability rules on firms' choices of care (affecting the frequency of tort damages) and scale (level of output) at the extensive and intensive margins. Chapter 2 focuses on gambling using derivatives, made more available by recent changes in the bankruptcy law granting repos and other derivatives "superpriority," which is exemption from the automatic stay and clawback in bankruptcy.

Limited liability is a birth right given by law to corporations, LLCs, and to differing extents special forms of partnerships (LPs, LLPs, and LLLPs). Chapter 1 starts with a question: does limited liability on damages improve social efficiency? The results show that when the outside stakeholders (consumers, employees, suppliers, communities, governments, etc.) obtain benefits from the firm, the tradeoff between damages to the tort claimants and benefits to the outside stakeholders determines the efficiency of liability rules. Full liability induces efficient care but also increases marginal and average costs, inducing less-than-efficient scale. Limited liability externalizes some

damages, resulting in less than efficient care, but the higher profitability encourages larger intensive and extensive margins for scale than full liability. Compared to full liability, limited liability tends to be more efficient if the benefits from larger scale covers the loss from lower care, i.e., if the outside stakeholders have a larger potential value. As potential value to the outside stakeholders falls, the equilibrium with full liability converges to the first best, whereas the equilibrium with limited liability deviates from the first best, encouraging low care level and large scale. Therefore, limited liability is not a one-size-fits-all policy to achieve the optimum for different firms. This opens up possibilities of other rules, for example requiring insurance for some activities, to adjust for cross-firm differences.

Chapter 2 is a joint work with Philip Dybvig. Myers (1977) described how firms can gamble using asset substitution, which is switching to a less efficient and more volatile project. Gambling using derivatives is a sharper instrument, allowing the firm to gamble just to what is needed, and with negligible efficiency loss. In our model, “gambling for redemption” operates at small scale and is socially beneficial, while “gambling for ripoff” operates at large scale and is socially inefficient but benefits stockholders at the expense of bondholders. Superpriority laws reduce firm value by making it harder for firms to borrow due to the anticipation of gambling for ripoff.

Chapter 1

Limited Liability and Scale

1.1 Introduction

For decades, Manville Corporation (formerly Johns-Manville) hid the health damages of the asbestos in its products. As a result, Manville was subject to over 16,500 tort[1] lawsuits pending against it by the year 1982, a result of "an average filing rate of 3 cases per hour, every hour of every business day."[2] Manville filed for Chapter 11 bankruptcy in that year and set up a pool of funds in excess of $2 billion using its future profits and contributions from insurers. As in any other mass tort litigation of public corporations, Manville's shareholders were not held personally liable for the damage caused by the firm.[3] Following standard practice, I write "piercing the corporate

[1]A tort is a civil wrong other than a breach of contract that causes a claimant to suffer loss or harm, resulting in legal liability for the person who commits the tortious act. The persons sued for a tort and lose usually have to pay "damages" – that is, a sum of money – to the person who they wronged. This paper focus on the "pure" tort cases in which no contracts are involved.

[2]Delaney (1992) has a chapter of detailed discussion of the case and the firm's bankruptcy resolution. It had been observed since 1906 that Asbestos, a fibrous mineral material used for insulation, can lead to lung cancer. However, the asbestos manufacturer claimed not to know about the damage before 1964.

[3]Consider, e.g., *A.H. Robins and the Dalkon Shield, Union Carbide and Bhopal, Johns-Manville and asbestos, Exxon and the Valdez oil spill*, and *Dow-Corning and silicone breast implants*. Roe (1986) provides rationales and evidence that public firms subject to mass tort have all filed for Chapter 11 and in all cases, the valuable assets in the firm were untouched. Though "piercing the corporate veil" provides an exception of limited liability, it is more related to closely held corporations and the veil piercing cases are generally rare, involving directors or managers who have decision making power in the firm. Specifically, Ramberg (2011) notes empirical evidence that no public corporation has ever been pierced the corporate veil.

vail" to refer to the situation in which shareholders or directors are held personally liable for the corporation's actions or debts. Besides corporations, limited liability also applies to other business entities including limited liability companies (LLCs) and even partnership-like firms (LPs, LLPs and LLLPs).[4] The question is not new: should the investors in these enterprises that potentially cause damages also have the "privilege" of limited liability?

To answer the question, I analyze a theoretical model that features 1) liability coming from torts on a third party, 2) outside stakeholders obtaining potential value from the firm, 3) the firm making a two-dimensional choice of care (or safety) and scale (or quantity) under limited and unlimited liability rules, and 4) only the firm but not the "victims" can affect the probability of accidents. Feature 4) assumes unilateral care in which only one side affects the probability, and it is an approximation if the other side is unaware of the damages or has very small influence on the losses from accidents. A firm bears more liability has stronger care incentives to reduce the liability cost. The increase in care from the increase in liability has a side effect of reducing the incentives for producing at scale. On the extensive margin, the increased cost of care and the private payment to the tort claimants may make the firm unprofitable, decreasing the scale to zero. On the intensive margin, the cost of care may increase the marginal cost of production, decreasing the scale incentives. The choices of care and scale affect the benefits to the tort claimants and other outside stakeholders. A higher care level - for example, by monitoring activities and developing safer technology - reduces the frequency (and probably also severity) of damages to the tort claimants. A larger scale increases benefits to the outside stakeholders. For example, communities would have lower crime rates and more job creation, governments could collect more taxes, and consumers would have larger fraction of consumer surplus[5]. The results show that when there is potential

[4]Limited liability partnerships (LLPs) and limited liability limited partnerships (LLLPs) have limited liability by nature, as a result of expansion of limited liability in the recent decades. Limited partnerships (LPs) usually set corporations as general partners as a shield against liability.

[5]Consumers have larger consumer surplus is under the assumption that the demand for each firm is not perfectly elastic and the firm does not price discriminate

value received by the outside stakeholders, full liability results in efficient care but lower scale on the intensive margin compared to the first best. Adding fixed cost may also affect the extensive margin of production, because the scale can decrease to zero if investing is unprofitable. However, limited liability mitigates this inefficiency in scale by externalizing some damages. When the potential benefits received by the outside stakeholders is big, limited liability is more efficient since the increased social benefit from scale is greater than the decreased social loss from care. If the potential value to the outside stakeholders vanishes, full liability converges to the first best because the firm aligns more of its interests with society by internalizing all the damages. Limited liability moves the firm away from the first best and encourages lower care and higher scale than what is optimal. These results suggest that applying limited liability to all firms is a blunt instrument that can impose too much liability for some firms but not enough for the others compared to what would be socially optimal.

Limited liability is at the firm level, but we can effectively have the same impact with unlimited liability if the investor is judgment-proof, which means that the investor has zero available assets to seize.[6] Full liability in the model setting requires both unlimited liability and what is referred to as "deep pockets," namely that the owner has sufficient assets that can be seized in satisfaction of the obligations. If an owner instead has a "small pocket, " the obligations are only partially satisfied.[7] Limited liability is a basic feature of laws for these entities. A *de facto* full liability can hardly be achieved directly given the existing laws for corporations, LLCs and special forms of partnerships. However, minimum capital requirements, requiring insurance, and setting up funds represent ways

[6]The limited liability rule means that investors are not liable for any amount exceeding what they invest (Easterbrook and Fischel (1985)). Unlimited liability says that unmet obligations of the firm are obligations of the owner, and is always bounded by the debtor's wealth unless one can take a penalty of a pound of flesh as in *The Merchant of Venice* or in a form of indentured servitude or even one's life. All are not accepted in today's civilized world. Even if possible, unlimited liability is still limited more or less. I also analyze the case with zero liability which society would likely not make as a rule, but the relevant scale level is the same with costless care and is a useful comparison.

[7]"Judgment-proof" is a special case of small pocket. For instance, a family have all their wealth in a family firm is "judgment-proof."

of increasing liability. Minimum capital requirements often refer to standardized regulations for banks and other depository institutions to include a minimum amount of liquid assets against their risky assets.[8] In the United States, capital requirements on partnerships, corporations and LLCs are only imposed by contracting between the firms and lenders or other third parties rather than by law. In many other countries, corporate laws require some organizations to hold minimum assets, and the primary purpose was "to protect creditors and nurture confidence in financial markets."[9] For example, all public firms in European Union have to hold capital of value at least €25,000,[10] and in the United Kingdom (England and Wales) the amount required is £50,000.[11] Requiring insurance is relatively easy when bonded with other regulation. For example, in the US most of the states require proof of insurance with car registration and only 7 states have exceptions, and contractors are often required to be insured and bonded.[12] Setting up funds are common for entities involved in environmental contamination.[13] The financing of the funds varies. For example, The Oil Spill Liability Trust Fund (OSLTF) in the US mainly collects barrel tax on the petroleum industry, and the Nuclear Decommissioning Trusts (NDTs) can be a combination of prepayments, sinking funds,[14] and insurances or even guarantees from parent companies. It is relatively simple to change the amount of insurance and funds required, and consequently firms can bear liability.

The first feature - the liability comes from torts - suggests that the wrongful acts studied in this paper do not include breach of contract or criminal wrongs that would potentially result in jail time. Unlike murders or assaults, of which we impose punishment to forbid the occurrences, I

[8] Haubrich (2020)

[9] Bank (2013)

[10] `https://www.dlapiperintelligence.com/goingglobal/corporate/index.html?t=03-minimum-capital-requirement`

[11] `https://www.dlapiperrealworld.com/law/index.html?t=corporate-vehicles&c=GB-ENG-WLS&s=setting-up-a-corporate-vehicle&q=minimum-capital`

[12] A liability insurance covers claims or losses the contractors may be responsible for, and surety bonds reduces the damages if the contractors walk off the construction work halfway.

[13] Bayon et al. (1999).

[14] A prepayment is a deposit by the licensee at the start of operation, similar to capital installation. A sinking fund is the account to accumulate funds set aside by the firm over time.

consider in this paper a "better" liability rule to induce higher social value, which is an aggregate of all the benefits received by all the parties minus the total damages minus the costs to reduce the damages.[15] The focus of the pure tort case also suggests that liability rules do not affect market demand: either a third party bears damages (for example, an Amazon truck delivering packages hit pedestrians) or the consumers are not aware of the damages (for example, in *Union Carbide and Bhopal* and *Johns-Manville and asbestos* cases). This is distinct from the situation that social efficiency can be achieved when the damages are priced out between the tortfeasors and victims. Particularly, product warranties studied in the previous literature such as Dybvig and Lutz (1993) and Yang (2010) can affect behaviors of the manufacturers and consumers.[16]

The second feature that the firm's other stakeholders obtain potential value from operation distinguishes this paper from most previous theoretical literature on liability. The positive externalities are affected by the firm's investment, for example, when consumers and/or workers obtain surplus from each individual firm, when beneficiaries spend more from higher taxes, or when communities benefit from job creation and lowered crime rate, etc. These outside stakeholders would always prefer larger scale. The mechanism is different for different stakeholders. When the firm has market power, the demand curve for each individual firm is downward sloping. Consequently, a larger scale results in lower equilibrium price and a greater consumer surplus over total benefits received by the firm. The same argument applies to the parallel labor market. When the firms' market power decreases, scale (as the total output of all the firms) in equilibrium increases. In particular, the scale under full liability increases to the first-best scale with perfect competition, and limited liability that externalizes damages would result in over-invetsment in scale. On the care level, full

[15]This welfare approach may not fully align with some notions of "corrective justice" concerning punishment and/or compensation. The notion of fairness concerning punishment says that the injurers should pay for any harm that is associated with the act, whereas one concerning compensation says that victims should be made whole. Kaplow and Shavell (2001) has a very detailed discussion of fairness and welfare.

[16]Dybvig and Lutz (1993) has a feature of two-sided moral hazard similar to the bilateral precaution tort case. A complete warranty worsens consumer's maintenance incentives, and similarly, full liability would also decrease consumer's precaution incentives.

liability always results in efficient care and limited liability results in less than efficient care. Thus, without market power limited liability should be prevented by society as a shield to externalize risk and costs.[17]

The analysis is also related to the financing of particular new technologies by venture capital investors. Lerner and Nanda (2020) shows from 1985 to 2019 a decreased proportion of funding for materials, energy and computer hardware, whereas the proportion for software and consumer and business products and services has increased substantially. Of course, both the disproportional investment in different industries and the changing trends of financing can be due to different gross returns, particularly in the short run, but the impact of evolution of the liability laws to investment may also play a role. For example, Bellon (2021) studies the superfund laws regarding lenders liability and Akey and Appel (2021) studies liability on parent firms. While computer hardware such as semi-conductors and software has less concern of the liability rule, since these products might not be involved in tort litigation, materials, energy and biopharmaceuticals are more prone to environmental and/or bodily damages. Hence, imposing certain liability rules may affect financing of some certain industries more while having little impact on others. Moreover, Lerner and Nanda (2020) observes a concentration of capital of a few deep-pocket venture capital investors and the small-pocket investors mainly focus on the early stage of start-ups. In this paper, I also study investors who endogenously choose to be deep-pockets or small-pockets. Interestingly, unlike common belief that small-pocket investors do worse than deep-pockets in providing care, the two-dimensional tradeoff can result in higher care level of small-pockets and even higher social outcomes, compared to the deep-pocket investors.

[17] See Bainbridge and Henderson (2016). Similar claims are made in some literature. For example, Hansmann and Kraakman (1990) claims that "there may be no persuasive reasons to prefer limited liability over a regime of unlimited pro rata shareholder liability for corporate torts." LoPucki (1994) also proposes the abolition of limited liability to achieve "the goal of minimizing the externalization of tort liability."

In Section 2, I start the analysis with the example of the product market in which a single investor chooses care and scale. The monopoly firm sets price and the consumers decide the amount to consume depending on the reservation value. In this benchmark, the firm uses rental capital so that limited liability allows only the proceeds in the firm to be seized when damages occur, and the claimants can grab assets outside of the firm under unlimited liability and without any cost.

With monopoly power, a firm cannot internalize all the benefits of the project and consumers obtain consumer surplus. If a deep-pocket investor has unlimited liability, the result is correct incentives for care but under-provision of scale: the more inelastic the demand is, the greater the externalization, and the more inefficient in scale. Section 3 extends the model to multiple firms with Cournot competition. I assume that the homogeneous firms invest in the same technology and each has an investor with the same wealth. With less market power, I show that full liability moves to the first-bestand limited liability would result in over-investment in scale and under-investment in care. If the investors instead have small pockets, even with unlimited liability the equilibrium would be very similar to limited liability.

Section 4 compares purchased capital versus rental capital or capital financed by non-recourse debt. With rental capital (or capital financed by non-recourse debt), The owners (or the secured creditors) have higher priority than the tort claimants who can only be paid with the firm's proceeds. However, purchased capital is part of the firm's assets and can be used as payments to the tort claimants. When market is more competitive, firms under limited liability would overinvest in scale and underinvest in care, but buying capital can mitigate the inefficiency in both care and scale. Particularly, purchased capital internalizes more damages and hence induces more care, and higher care level also increases the marginal and average costs of production, driving scale down. However, purchased capital does not have such benefits under unlimited liability but creates an entry barrier for the firms.

The model can also be applied to other markets as discussed in Section 5. An analogous market is the labor market where employees are the stakeholders. When a firm is the only buyer of labor in the market, the firm has monopsony power. The marginal cost of employing workers would be greater than the social cost which is also workers' reservation utility. Because of this, the firm does not capture full benefits from the employees working in the firm. Because of this, the firm does not capture full benefits from the employees working in the firm. A classic example would be a coal mining company in a geographically remote areas such as the West Virginia where finding substitutions of a job is costly. Parallel to the product market, more liability on torts results in higher care. On the extensive margin, higher care level as well as fixed costs may reduce scale to zero. On the intensive margin, higher care level increases the marginal cost of production and decreases scale. Limited liability on accidents would induce higher labor employed by the firm compared to full liability and is efficient if the welfare gain from the increased employment and the operation of business covers the loss from lower investment in care. However, having a lot of competitors is where limited liability is inefficient, and similarly, the inefficiency can be lowered by requiring some capital installed in the firm as a cushion for tort claimants. In this section, governments and communities as stakeholders are also discussed.

1.1.1 Some literature

The focus of this paper is the tradeoff between the externalities to the outside stakeholders. In the literature, the factors justifying limited liability usually fall into four categories: the investors' preferences considerations (risk-aversion and reputation), care incentives of the victims under bilateral precautions, the transaction cost argument, and strategic evasion of liabilities. Investors may decline investing in the potentially tortious firms because they over-react in negative returns, or their morality distastes investments that can damage environment. With bilateral care, firms and tort victims can both take actions to decrease the frequency of damages. In such a framework like Landes and Posner (1985), it is usually assumed that joint care is most efficient. Therefore, limited liability is better than full liability because it encourages precaution of the potential tort claimants by externalizing some damages to them. Bilateral care can also be between two potential tortfeasors who jointly cause damages. For example, Hay and Spier (2005) studies the optimal shared liability between the suppliers and manufactures when both parties are liable. There are no positive externalities in these models[18].

The majority of the law literature follows the premise that investors should internalize tort risk.[19] Following this premise, it is argued that limited liability is appropriate for public corporations because forcing to internalize tort risk generates social costs outweighing social benefits. In particular, one big cost for the joint and several unlimited liability regime is to know the available wealth of other shareholders since the shareholders who have more assets bear more tort liability. This could result in large investors heavily monitoring the firm, or no investing, or even highly leveraged firm with a very large amount of secured debt that has priority over tort liability.[20] A famous paper Hansmann and Kraakman (1990) proposes pro rata unlimited liability over joint and several rule to solve the information and monitoring problems. Bainbridge and Henderson (2016) disagrees on the rule made by Hansmann and Kraakman (1990) to identify responsible persons, suggesting that it is not practical given the fast changes of the firm's shareholder pool especially with today's financial market. Leebron (1991) also argues that the collection costs would be too high considering possibly hundreds of thousands of shareholders among whom some are out-of-state, some are off-shore, some own too few shares to worth the effort, and some have small pockets. In addition,

[18] In Hay and Spier (2005), since the consumers and the manufacturers jointly bear all the liabilities, we can view them as a whole.

[19] This premise can be justified on the grounds of certain notions of fairness which has a logic of corrective justice: "If A wrongfully injures B, A must pay B for the loss B suffers as a consequence of A's act." See Coleman (1995). Internalizing all the damages can also be justified by a welfare consideration if the firm does not have positive externalities, because the firm would align the interests with society – only a special case in this paper.

[20] Grundfest (1992) argues that there would be "more exotic debt-equity hybrid."

investors could always take advantage of other law to evade personal liability, e.g., investing in some real estate or employing independent contractors, etc.[21] The investors are judgment-proof if they can effectively evade all the personal liabilities. In papers study judgment-proof investors, Shavell (1986) shows that too much risk and too little care would be taken, and Che and Spier (2008) claims that injurers strategically using senior bonds to judgment proof themselves will result in less precaution. Hansmann and Kraakman (1990) has claimed to solve these problems, for example, by proposing mandatory insurance. For closely held firms, unlimited liability is usually claimed appropriate because forcing to internalize tort risk has relatively small social costs.[22] This paper gives an argument that limited liability might be better even for closely held entities.

1.2 The Benchmark Model

For now, I consider consumers as the only other stakeholders of the monopoly firm selling a potentially dangerous product, and I look at the implications of allowing the investor to organize the risky activity with different liability rules. Almost the same arguments could be made for other stakeholders who benefit from the existence and the scale of the firm. These arguments would not of course hold if there are only negative externalities to other stakeholders, in which case full liability would dominate limited liability. In this section, the single firm uses rental capital which cannot be grabbed by the tort claimants. The firm can only use revenue in compensation of the tort claimants. In the following sections, I increase competition by increasing the number of the firms and I also discuss the impact of rental capital versus purchased capital on the investors' choices of care (how much the firm invests in ensuring that the product is safe) and scale (the quantity

[21] In California, money home mortgages are non-recourse. In agency law, if the tort is committed by an independent contractor, the principal would have limited liability. See Bainbridge and Henderson (2016)

[22] Easterbrook and Fischel (1985).

produced). I will discuss other stakeholders such as employees and government who also capture some of the rents from operation through various institutions.

I focus on a single investor with wealth W who can choose either to invest in a firm or to invest in a safe technology with a payoff $R_f > 0$ (one plus the rate of return) per unit invested.[23] The firm's product potentially causes damages. The investor decides the allocation of investment into a firm with input Q, which produces Q units of goods. The care level or safety of the product is $s \in [\underline{s}, 1), (\underline{s} > 0,)$ which is the probability of each unit of good being safe. The unit cost of care $C(s)$ satisfies $C'(s) > 0$, $C''(s) > 0$ for $s \in (\underline{s}, 1)$, $C(\underline{s}) = C'(\underline{s}) = 0$ and $\lim_{s \uparrow 1} C'(s) = \infty$. The convexity of cost function implies that the marginal cost of care level is increasing in care. Whatever wealth is not invested in the risky technology is invested in the safe technology with the payoff $R_f \geq 1$. With probability s each product is safe, and with probability $1-s$ it causes damage $d > 0$ to tort claimants. With strict liability, tort is identified with probability 1 and effective liabilities (or compensations) are the actual damages subject to limited liability or small pockets. We denote Λ as the total effective compensation paid to the tort claimants. By saying "effective," here I mean the actual payment instead of the liability that the firm owes. For example, if the firm is protected by limited liability, the effective liability is the revenue in the firm at most. The damage d is a constant in the model. In reality, the higher care level may also reduce the per unit damage. For example, requiring fastening safety belt while driving reduces both frequency and severity of bodily damages. I only assume that higher care level only reduces the frequency for simplicity. This assumption does not qualitatively change our conclusion.

The investor's firm is a monopolist in the product market and sets price p for each unit of good. There is an identical continuum of consumers with measure one, each has an inverse demand function $m(x)$ (satisfying $m'(x) < 0$) which is also the marginal utility of the x^{th} good consumed,

[23] Wealth W is large enough to avoid discussing financial constraint and trade-offs of investments in operation and safety at the boundary, but it is not unlimited so that the investor is possibly small-pocket.

excluding any damages not compensated. In this paper, I proceed with the pure tort case in which the consumers are not subject to damages, or they act as if they are not subject to damages.[24] We can simply think that the damages are fully borne by a third party who is unknown until the damages occur. Given price p and the inverse demand function $m(x)$, a representative consumer chooses a scale (quantity) $Q \in \mathbb{R}_{\geq 0}$ to maximize consumer surplus

$$V_c = \int_{x=0}^{Q} \Big[m(x) - p\Big] dx \tag{1.1}$$

Since all the consumers are identical, V_c is also the total consumer surplus. In equilibrium, $p = m(Q)$, namely, the price should be equal to the marginal benefit of consuming Q goods. In equilibrium, the total consumer surplus is

$$V_c = \int_{x=0}^{Q} \Big[m(x) - m(Q)\Big] dx. \tag{1.2}$$

Under the limited liability rule, liability is restricted to firm's available assets which is the total revenue from selling the product, i.e. $\Lambda(Q, p, w) = (Qp) \wedge (Qd)$, where $a \wedge b \equiv \min\{a, b\}$ denotes the minimum of a and b, and $a \vee b \equiv \max\{a, b\}$ denotes the maximum of a and b. The effective liability of a deep-pocket investor under unlimited liability rule is full liability Qd, but that of a small-pocket also depends on the appropriable wealth outside of the firm and is $w \equiv [R_f(W - (1 + C(s))Q)]$. For an investor with limited wealth, the larger the investment in the firm, the more likely the investor is to be small-pocket.[25]

[24]For example, a customer purchasing goods from Amazon cares about the prices and not the probability that they would get hit by the Amazon trucks, or the consideration of accidents is negligible.

[25]I don't explicitly model the uncertainty in the litigation process, but that can be implicit embedded in s. If the strict liability rules apply, the firm is responsible whenever an accident occurs; if under the negligence laws, there can be proof of burden and there is uncertainty whether the liability is going into litigation (add reference) especially when the tortious technology is new and not well understood even by the experts. Firms could then perform strategically to affect the litigation. It is an interesting topic but here I assume that the expected liability is foreseeable.

Given the investor's choices of care and scale, the social value of the project is

$$S(s,Q) = \int_{x=0}^{Q} \Big[m(x) - (1-s)d\Big] dx - R_f(1+C(s))Q. \tag{1.3}$$

$$\text{s.t.} \quad Q \geq 0 \tag{1.4}$$

Because we are looking at total surplus, liability is just a transfer from firms to tort claimants and is not in the social value function. With first-best care level s_s^* and scale Q_s^*, the first-best social value is equal to the consumer surplus[26]

$$\begin{aligned} S(s_s^*, Q_s^*) &= \Big(\int_{x=0}^{Q_s^*} \Big[m(x) - (1-s_s^*)d\Big] dx - R_f(1+C(s_s^*))Q_s^* \Big)^+ \\ &= \Big(\int_{x=0}^{Q_s^*} m(x)dx - m(Q_s^*)Q_s^* \Big)^+. \end{aligned} \tag{1.7}$$

With smaller reservation utility, firms tend to produce more[27] and create larger total damages, so that there are more incentives to choose a higher care level. Similarly, Equation (1.6) indicates that the risk-neutral investor either will not invest at all when the potential damage is too high or the risk-free investment has a high return, or will keep investing until the marginal return of investment in the firm is exactly equal to the risk-free payoff.

[26] The first-best s_s^* and Q_s^* satisfy

$$C'(s_s^*) = \frac{d}{R_f}, \tag{1.5}$$

$$m(Q_s^*) - (1-s_s^*)d = R_f(1+C(s_s^*)). \tag{1.6}$$

Equation (1.7) is derived when replacing the total cost of consumption with $m(Q_s^*)$ (from equation (1.6)), and the social value equal to the consumer surplus is always non-negative.

[27] $dQ_s^*/dR_f = \frac{1+C(s_s^*)+R_f}{m'(Q_s^*)} < 0.$

I assume interior solutions so that $w \equiv R_f(W-(1+C(s))Q)$ is strictly positive. The investor's problem is to choose care s and scale Q, followed by the consumer's consumption choice. Formally,

$$\max_{s,Q}\left(Qp-(1-s)\Lambda(Q,p,w)+w\right) \tag{1.8}$$

$$\text{s.t.} \quad p=m(Q), 0\le Q \tag{1.9}$$

$$w\equiv R_f(W-(1+C(s))Q)>0 \tag{1.10}$$

$$\text{and} \quad \underline{s}\le s<1, \tag{1.11}$$

where

$$\Lambda(Q,p,w)=\begin{cases} 0, & \text{no liability} \\ (Qp)\wedge(Qd), & \text{limited liability} \\ \Big(Qp+w]\Big)\wedge(Qd) & \text{unlimited liability with small pockets} \\ Qd, & \text{unlimited liability with deep pockets (like } w=\infty) \end{cases}$$

We can see that the optimal choices of care and scale by firm are affected by the externalization of benefit (through consumer surplus) and the internalization of damage (through liability rule). To ensure the existence and nice features of solution, I make the following assumptions:

Assumpt. *A: $m(Q)$ is continuous and smooth with $m(Q)>0, m'(Q)<0$ and $\lim_{s\to\infty} m(Q)\le 0$.*

Assumpt. *B: $m(0)>R_f, m'(0)>-\infty$.*

Assumpt. *C: $d^2[m(Q)Q]/(dQ)^2 = 2m'(Q)+m''(Q)Q<0$, or equivalently*

$$2+\frac{d\log m'(Q)}{d\log Q}>0. \tag{1.12}$$

Assumption A says that the demand curve is downward-sloping, and Assumption B is necessary for the project to be socially valuable because consuming the very first unit should generate more than the reservation payoff. Assumption C ensures unique solution in most cases and will be used in the next section. The following Table 1.1 compares the investor's choices with different liability rules to the first-best. For more detailed computation see Table 1.2.

Rules	s^*	Q^*	Q_M^*
FL	$=$	$<$	$<$
LL	$<$	$\lesseqgtr$	$<$
UL-SP	$\lesseqgtr$	$\lesseqgtr$	$<$
NL	$<$	$\lesseqgtr$	$=$

Table 1.1: ***(Comparison of different liability rules to first-best and pure monopoly)*** FL: full liability (unlimited liability + deep-pocket investors); LL: limited liability; UL-SP: unlimited liability + small pockets; NL: no liability; (s^*, Q^*): first-best safety and quantity; Q_M^*: equilibrium quantity when safety has zero cost.

PROPOSITION 1. *With full liability, investment in care achieves first-best but there is always under-investment in scale because the investor does not internalize all the benefits. Limited liability under-provides care but increases investment in scale compared to full liability. This can be socially harmful when demand is very elastic and potential tort liability is large.*

Proof. With full liability, the first order conditions are

$$\begin{aligned}
(s) \quad & C'(s_u^*) = \frac{d}{R_f} \Rightarrow q_u^* = q_s^* \\
(Q) \quad & \frac{m(Q_u^*) + m'(Q_u^*)Q_u^* - (1 - s_u^*)d}{1 + C(s_u^*)} = R_f
\end{aligned}$$

Compare the second equation to Equation (1.6),

$$m(Q_s^*) = m(Q_u^*) + m'(Q_u^*)Q_u^*$$

(1)	(2)	(3)	(4)	(5)	(6)	(7)
rule	$\Lambda(Q,p,w)$	FOC:(s)	FOC:(Q)	Firm started ($Q^* > 0$)	$s^* < s^*_s$	$Q^* > Q^*_s$
FB	-	$C'(s^*_s) = \frac{d}{R_f}$	$m(Q^*_s) \leq R_f\big(1+C(s^*_s)\big) +(1-s^*_s)d$	$m(0) > R_f\big(1+C(s^*_s)\big) +(1-s^*_s)d$	(always $=$)	(always $=$)
FL	Qd	$C'(s^*_u) = \frac{d}{R_f}$	$m(Q^*_u)+m'(Q^*_u)Q^*_u \leq R_f\big(1+C(s^*_u)\big)+(1-s^*_u)d$	$m(0) > R_f\big(1+C(s^*_u)\big)$	(always $=$)	(always $<$)
LL	$Qp(< Qd)$	$C'(s^*_l) = \frac{m(Q^*_l)}{R_f}$	$m(Q^*_l)+m'(Q^*_l)Q^*_l \leq \frac{R_f\big(1+C(s^*_l)\big)}{s^*_l}$	$m(0) > \frac{R_f\big(1+C(s^*_l)\big)}{s^*_l}$	(always $<$)	$e_p(Q^*_l) > \big[1-\frac{R_f\big(1+C(s^*_l)\big)}{s^*_l m(Q^*_s)}\big]^{-1}$
UL-SP	$Qp+w$ $(< Qd)$	$C'(s^*_j) = \frac{m(Q^*_j)+R_f(W/Q-1-C(s^*_j))}{s^*_j R_f}$	$m(Q^*_j)+m'(Q^*_j)Q^*_j \leq R_f\big(1+C(s^*_j)\big)$	$m(0) > R_f\big(1+C(s^*_j)\big)$	(can be $\lesseqgtr$)	$e_p(Q^*_j) > \big[1-\frac{R_f\big(1+C(s^*_j)\big)}{m(Q^*_s)}\big]^{-1}$
NL	0	$C'(s^*_0) = 0$	$m(Q^*_0)+m'(Q^*_0)Q^*_0 \leq R_f$	$m(0) > R_f$	(always $<$)	$e_p(Q^*_0) > \big[1-\frac{R_f}{m(Q^*_s)}\big]^{-1}$

$e_p \equiv -\frac{dI/I}{dp/p} = -\frac{m(Q)}{m'(Q)Q}$

Table 1.2: ***(Liability rules and solutions)*** When the firm has market power and demand is not perfectly elastic, full liability (FL) or unlimited liability with deep-pocket investors (UL) always results in first-best care but lower scale; the firm with limited liability has less incentives for care but more incentives in scale compared to FL. However, when the firm has unlimited liability, with small pockets (UL-SP) there can be under- and over-investment in both care and scale depending on the parameter values. With no liability (NL), the firm invests minimum in care and there can be under- and over-investment in scale depending on the demand elasticity.

Since $m'(Q_u^*)Q_u^* < 0$, it must be true that $m(Q_s^*) < m(Q_u^*)$, so that $Q_s^* > Q_u^*$. Similarly, we can prove the proposition with limited liability by comparing the first order conditions. See Appendix A.1 for details. □

Unlimited liability with deep-pocket investors can undermine the incentives for scale, and the inefficiency can be huge when demand is less elastic and the consumer surplus is large. Limited liability may generate slightly less incentives for safety, but the inefficiency can be compensated by higher scale and result in higher social value. Limited liability can be regarded as a subsidy from the tort claimants to the investor and higher quantity compared to full liability. The incentives for safety is dampened due to the externalization of tort damages, which also mitigates the externalization of benefits and encourages bigger scale. This can be good when too much benefits are externalized due to inelastic demand. However, limited liability is not a one-size-fits-all rule, firstly because there can be over-compensation when the tort claims are large and when demand is very elastic, and secondly, when the investor is small-pocket, the liability is capped even with unlimited liability.

PROPOSITION 2. ***(Unlimited liability with small pockets)*** *If $\Lambda(Q, p, w) = Qp + w < Qd$,*[28]

(1) Compared to first-best, there can be under-provision or over-provision of both safety and quantity.

(2) Compared to limited liability, if $Q_j^* < Q_l^*$, then $s_j^* > s_l^*$.

(3) The investor is more likely to have small pockets with larger damage d.

[28]The investor being small-pocket or deep-pocket is endogenous. The original firm's problem is not smooth at $\Lambda(Q_f, p, w)/Q_f = d$, since the gradients are different when approaching $\Lambda(Q_f, p, w)/Q_f = d$ on different sides. To solve such a problem I obtain the optimal solution on each side (i.e., conditional on whether $\Lambda(Q_f, p, w)/Q_f$ is greater than d or not) and compare them to get the global optimum. If conditional on $\Lambda(Q_f, p, w)/Q_f < d$ but instead the interior solution satisfies that $\Lambda(Q_f^*, p, w)/Q_f* \geq d$, the solution should be at the corner on this side and the global optimum is on the other side where $\Lambda(Q_f, p, w)/Q_f \geq d$. Then the investor is not small-pocket. This proposition considers the results conditional on the investor optimally chooses to have a small pocket.

(4) $\frac{dQ_j^*}{dW}\frac{ds_j^*}{dW} < 0$. *That is, safety and quantity always move to different directions when wealth changes.*

Proof. See Appendix A.2. The investor's care level can be higher than the socially optimum. When the investor is small-pocket, all the wealth is wiped out if damages occur (with probability $1-s$). The investor only cares about the cost and revenue when the firm stays (with probability s). For the same level of care s, the investor's effective marginal cost (the marginal cost "discounted" by the probability s) is smaller than the the social planner's marginal cost. We also know that the marginal revenue for the investor (the effective unit liability Λ/Q) is also smaller than the social planner's MR (unit damage d) by definition of "small-pockets." Therefore, for the socially optimum care level, MC(social)=MR(social), but MC(investor)<MR(investor) can happen, and in this case the firm's choice of care is greater than the socially optimum. □

The tort claimants bear some damages if the investor has a small pocket. When the firm increases care s, the tort claimants would bear more damages when tort is realized. This gives an extra incentive for the firm to invest more in care. Compared to the first best, the firm's choice of care can be higher. Below is a numeric example.

An example of overinvestment in care

Suppose $m(Q) = 7 - .2Q, C(s) = \frac{3}{1-s} - 12s, s \in [.5, 1).d = 6, w = 20$, and $R_f = 1.01$. Social planner's problem is:

$$
\begin{aligned}
&\max_{s,Q} \Big\{ \int_0^Q [m(Q) - (1-s)d]dQ - R_f(1 + C(s))Q \Big\} \\
=&\max_{s,Q} \Big\{ \int_0^Q [7 - .2Q - (1-s) * 6]dQ - 1.01(1 + \frac{3}{1-s} - 12s)Q \Big\} \\
=&\max_{s,Q} \Big\{ -.1Q^2 - .01Q - \frac{3.03Q}{1-s} + 18.12sQ \Big\}
\end{aligned}
$$

First best solution $s^* = .59, Q^* = 16.45$.

Investor's problem:

$$
\begin{aligned}
&\max_{s,Q} \Big\{ (1-s)Q\Big(m(Q) + R_f[w/Q - (1 + C(s))] - d\Big)^+ + sQ\Big(m(Q) + R_f[w/Q - (1 + C(s))]\Big) \Big\} \\
=&\max_{s,Q} \Big\{ (1-s)Q\Big(7 - .2Q + 1.01[20/Q - (1 + \frac{3}{1-s} - 12s)] - 6\Big)^+ \\
&\qquad\qquad\qquad + sQ\Big(7 - .2Q + 1.01[20/Q - (1 + \frac{3}{1-s} - 12s)]\Big) \Big\}
\end{aligned}
$$

The solution is $s_f^* = .60, Q_f^* = 14.17$ which satisfies $m(Q_f^*) + R_f[w/Q_f^* - (1 + C(s_f^*))] = 4.26 < d(= 6)$. That is, the investor has a small pocket. Notice that, with greater value of wealth, for example, when $w = 40$ the choices become $s_f^* = .59, Q_f^* = 8.23$. In this case, $m(Q_f^*) + R_f[w/Q_f^* - (1 + C(s_f^*))] = 9.01 > d$ and the investor has a deep pocket.

Contrast to what people may believe that small pockets result in larger social inefficiency compared to deep-pocket investors - since they don't internalize damages - the equilibrium of a small-pocket can instead improve social efficiency and may even be close to the social optimum.

PROPOSITION 3. ***(No liability)*** *If $\Lambda(Q, p, w) = 0$, compared to other liability cases, the firm invests least in care (zero) and most in scale, which can be greater or less than the first-best quantity depending on the demand elasticity.*

Proof. the first order conditions are

$$(s) \quad C'(s_0^*)R_f = 0, \text{ and}$$
$$(Q) \quad m(Q_0^*) + m'(Q_0^*)Q_0^* = R_f(1 + C(s_0^*))$$

$C(s_0^*) = 0$ is immediate, thus $m(Q_0^*) + m'(Q_0^*)Q_0^* = R_f$.

To compared to other liability rules, here we take limited liability as an example. Under limited liability, the first order condition for scale Q is $m(Q_l^*) + m'(Q_l^*)Q_l^* = R_f\big(1 + C(s_l^*)/s_l^*$. The inequality $R_f\big(1 + C(s_l^*)\big)/s_l^* > R_f$ indicates $m(Q_l^*) + m'(Q_l^*)Q_l^* > m(Q_0^*) + m'(Q_0^*)Q_0^*$, and by Assumpt. C we have $Q_l^* < Q_0^*$. The proofs are similar for other liability cases.

Compared to first-best, the condition for scale greater than the first-best is

$$e_p(Q_0^*) > \big[1 - R_f/m(Q_s^*)\big]^{-1}.$$

□

Having no liability for damages seems unacceptable, but can happen. If the law system is not well established, powerful firms can evade liabilities easily. More commonly, tort may involve technologies that are not well understood or the diseases show their symptoms only chronically. The original investors may grab the revenue early and the original firms may shut down before the problem reveals. Tracing the parties who should be responsible after decades can be very costly, too. Even if the firm survives, should the tort claimants go after the current firms and investors? What if the firm has already advanced the technology and what if the management has changed? Going after the wrong group does not push incentives to the right direction and can damage the current business.

Considering such situation in which the firm is not responsible for any liability, tort is a pure externality and the investor has no incentives for care. It thus becomes a price-quantity tradeoff determined by demand elasticity as in the basic choice theory. It is more socially harmful when individual damage is large and demand is elastic, because compared to the social optimal choices, a more elastic demand would "internalize" more positive part while larger damage externalizes the negative part.[29] When the demand is not elastic, too large fraction of benefit goes to consumer surplus, and the firm would want to decrease the investment in scale. If the firm does not internalize enough damages, scale level can be too high instead.

1.2.1 The monopoly quantity: a special case with costless safety

It may be useful to compare the model to the intermediate microeconomics monopoly firm's problem, which is a special case when the firm is always safe. Then we have the following conclusion:

PROPOSITION 4. ***(The monopoly quantity)*** *If $C(s) \equiv 0$, the firm invests in full safety and the same quantity compared to the no liability quantity, and higher quantity compared to the quantities under any other liabilities.*

Proof. Suppose Q_M^* is the equilibrium quantity when the firm is always safe, then the first order condition for quantity is $m(Q_M^*) + m'(Q_M^*)Q_M^* = R_f$. This is the same when there is no liability

[29] With perfect price discrimination, the firm internalizing all the benefit would inevitably invest more compared to the "social optimum" because there is only negative externality.

(i.e., $\Lambda(Q,p,w)=0$) in which case the firm chooses minimal safety and the cost of safety is also zero. We can get the rest part of conclusion from Proposition 3. $\square$

The choice of quantity is the highest without any consideration of safety, and is the same level when there is no liability at all. We can think of different liability rules as adding different caps to the liability. With lower caps the investor externalizes more benefits. The result is lower safety incentives and higher quantity incentives. Then there is some level of cap such that it is most socially beneficial along the line – the "second best" choice since we can never reach the first-best because of the market power. The choice of liability should be related to how close they are to this "second best."

The remaining part of this section provides a numerical example of the model and shows equilibria of different liability cases with different parameter values. I also add fixed cost to the model as a comparison. Some observations besides the above propositions are discussed.

1.2.2 An example of linear demand

In this example I assume linear demand. The cost function is assumed so that corner solutions are avoided.

$$\begin{aligned} m(Q) &= a - bQ \\ C(s) &= \frac{c}{1-s} + \frac{2\underline{s}c - (1+s)c}{(1-\underline{s})^2}, \quad s \in [\underline{s}, 1) \end{aligned}$$

For limited liability, we have the first order conditions

$$
\begin{aligned}
&\text{when } \left(\frac{a-d}{b}\right)^{+} \leq Q < \frac{a}{b} \\
&\qquad (s) \quad Q(s) = \frac{a}{b} - \frac{R_f}{b} C'(s) \\
&\qquad\qquad\qquad = \frac{a}{b} - \frac{cR_f}{b}\left[\frac{1}{(1-s)^2} - \frac{1}{(1-\underline{s})^2}\right] \\
&\qquad (Q) \quad Q(s) = \frac{a}{2b} - \frac{R_f}{2bs}(1+C(s)) \\
&\qquad\qquad\qquad = \frac{a}{2b} - \frac{cR_f}{2bs}\left[\frac{1}{c} + \frac{1}{1-s} + \frac{2\underline{s}-(1+s)}{(1-\underline{s})^2}\right] \\
&\text{when } 0 < Q \leq \left(\frac{a-d}{b}\right)^{-} \\
&\qquad (s) \quad s = 1 - \left[\frac{d}{cR_f} + \frac{1}{(1-\underline{s})^2}\right]^{-\frac{1}{2}} \\
&\qquad (Q) \quad Q(s) = \frac{a}{2b} - \frac{R_f}{2b}(1+C(s)) - \frac{(1-s)d}{2b}
\end{aligned}
$$

For unlimited liability, we have

$$
\begin{aligned}
&\text{when } \left(\left(\frac{a-d}{2b} - \frac{R_f(1+C(s))}{2b}\right) + \sqrt{\left(\frac{a-d}{2b} - \frac{R_f(1+C(s))}{2b}\right)^2 + \frac{R_fW}{b}}\right)^{+} \leq Q < \frac{a}{b} \\
&(s) \quad Q(s) = \left(\frac{a-sR_fC'(s)}{2b} - \frac{R_f(1+C(s))}{2b}\right) + \sqrt{\left(\frac{a-sR_fC'(s)}{2b} - \frac{R_f(1+C(s))}{2b}\right)^2 + \frac{R_fW}{b}} \\
&(Q) \quad Q(s) = \frac{a}{2b} - \frac{R_f}{2b}(1+C(s)) \\
&\text{when } 0 < Q \leq \left(\left(\frac{a-d}{2b} - \frac{R_f(1+C(s))}{2b}\right) + \sqrt{\left(\frac{a-d}{2b} - \frac{R_f(1+C(s))}{2b}\right)^2 + \frac{R_fW}{b}}\right)^{-} \\
&(s) \quad s = 1 - \left[\frac{d}{cR_f} + \frac{1}{(1-\underline{s})^2}\right]^{-\frac{1}{2}} \\
&(Q) \quad Q(s) = \frac{a}{2b} - \frac{R_f}{2b}(1+C(s)) - \frac{(1-s)d}{2b}
\end{aligned}
$$

The following figures assume some parameter values. Besides the propositions discussed above, there are several observations:

Observation 1. *In Figure 1.1, when unit damage d is relatively small, full liability (endogenously chosen) has a worse outcome than limited liability because of inefficiency in scale. Limited liability results in less care but this inefficiency of care can be compensated by the improved efficiency in scale, making the choice more efficient overall. However, as the damages become bigger, society prefers higher care level and lower scale. In this case, limited liability generates too little incentives for care and too much incentives for scale, and full liability tends to be more efficient.*

Observation 2. *Figure 1.2 shows an example that an investor with less wealth may end up with an equilibrium closer to the first best under unlimited liability rule. In the first graph, the investor with higher wealth level is deep-pocket and chooses an efficient care and inefficiently low scale under unlimited liability rule. In the second graph, however, the equilibrium is even more efficient than limited liability when the investor has less wealth and chooses to have a small pocket.*

Observation 3. *Figure 1.3 provides an observation of the sensitivity of firm's choices regarding changes of demand elasticity. With larger wealth* $W(=40)$*, demand elasticity has no impact on care but affects firm's choices of scale only proportionally. When demand is more elastic (which means a flatter* $m(Q)$*), society and investor both prefer higher level of scale and the choices of care and scale have the same sensibility to the change of elasticity. With limited wealth, however, the increment of scale as a result of higher elasticity of demand can be disproportionate with a sharper increase in scale under unlimited liability rule. This is suggesting that equilibrium under unlimited liability rule can be more sensitive subject to changes in demand, particularly when the investor has limited wealth.*

Observation 4. *Figure 1.4 compares different costs of care. Not surprisingly, increasing the prevention costs of damages lowers incentives for care for both the investor and society, and with* *unaffected scale level. This result may not be taken literally, since it does not necessarily hold true for every case. When the firm is small-pocket, the choices of the two dimensions can both change.*

1.2.3 With fixed cost

When the firm has fixed cost, average cost is decreasing. In this case the firm would not operate if they have to internalize all the damages, which would result in low quantity ex post. Ex ante, low quantity makes average cost of investment higher than the marginal benefit, and the firm is less likely to start as a consequence. This can be good or bad depending on how big the damages are, shown by the two cases in Figure 1.5.

Observation 5. *Figure 1.5 shows that with full liability, investment in care reaches first-best but there is always underinvestment in scale (the first figure). The firm will even not start when there is fixed costs (the second figure). Limited liability under-provides care but increases scale and even makes possible some profitable projects to be undertaken.*

So far I have analyzed a single firm who has market power and can set price for its product. The firm cannot fully internalize all the benefit from operation and the consumers obtain the "triangle" of the demand function. As a consequence, full liability can undermine the incentives to produce and can even halt a beneficial project especially when the demand elasticity is relatively low and the "triangle" is substantial. With fixed cost, the firm is likely not started at all. Limited liability can improve social welfare by increasing scale a large amount, and the social gain from it possibly offsets the social loss from increased probability of damage. The investor and the consumers benefit from limited liability, which hurts the potential tort claimants because their probability of

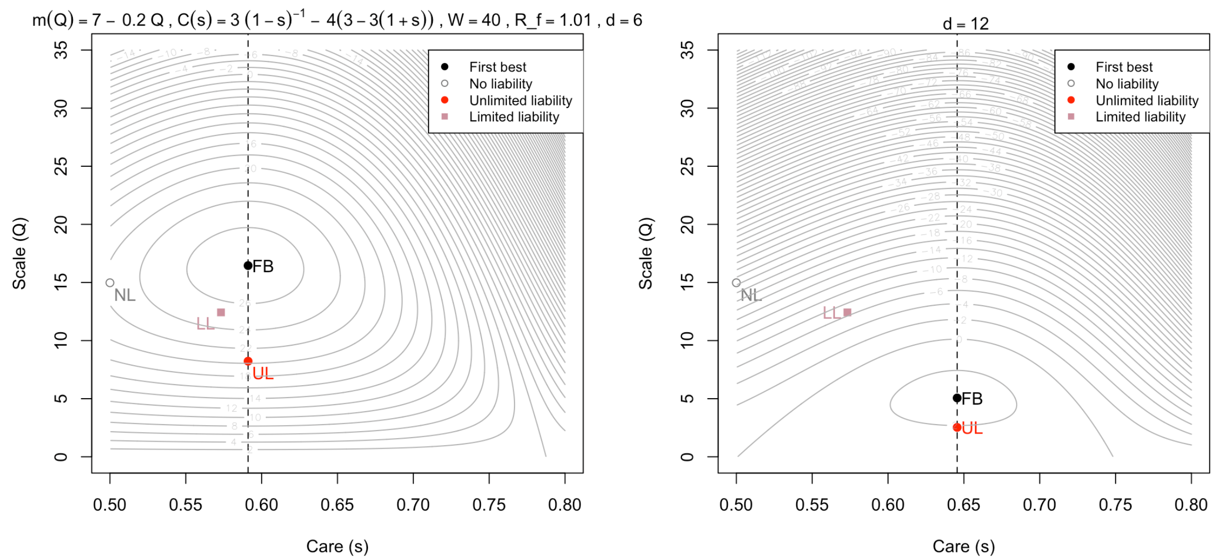


Figure 1.1: ***(Unit damage** d)* The x axis is care, and y axis is scale. The dots of different shapes and colors represent the equilibrium of different liability rules. The contour plot represents indifference curves of social value of the project. In these two graphs, the first graph shows the investor's choice when damage *d* is relatively small (=6), and the second graph shows a larger unit damage(=12). In these two examples, the investor is deep-pocket because wealth is big enough, and full liability is worse than limited liability in terms of social welfare because of inefficiency in under-investment in scale. However, as damages increase and other things equal, society prefers higher care level and lower scale. In this case, full liability is closer to then first best than limited liability.

suffering a loss is greater and they are not to be fully compensated. In the next section, I analyze how competition changes the equilibria and the implications of different liability rules.

1.3 Cournot Competition

I show in this section that with more competition, full liability tends to be more socially efficient than limited liability. For better discussion, I only focus on full liability and limited liability. It might be interesting to also talk about the investor with small pockets, but the comparisons between limited full liability is more intuitive. The formal investor's problems also include the possibility

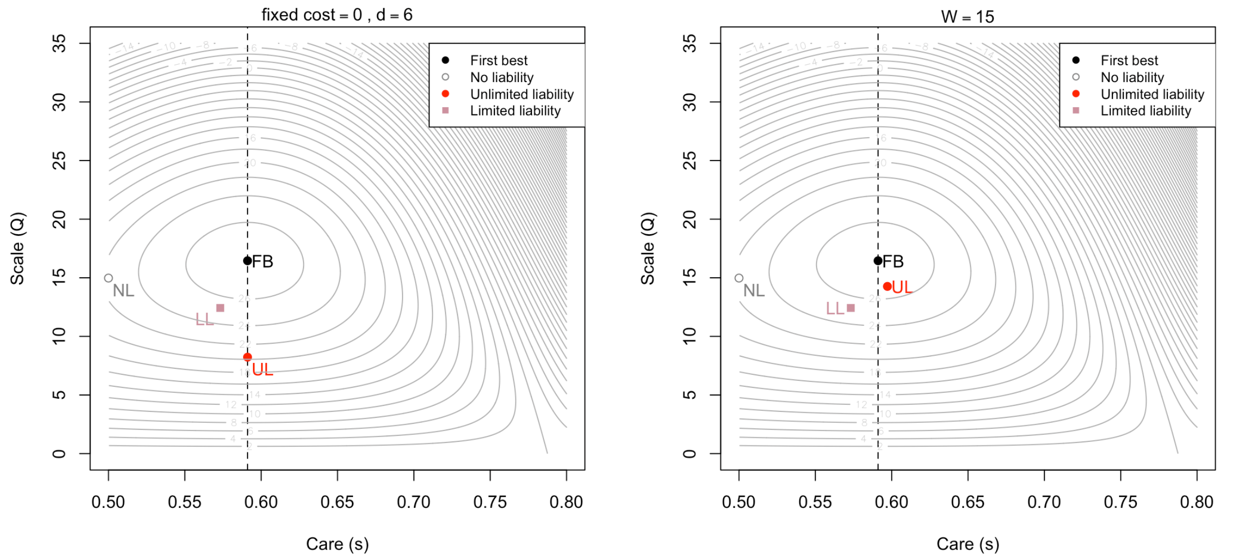
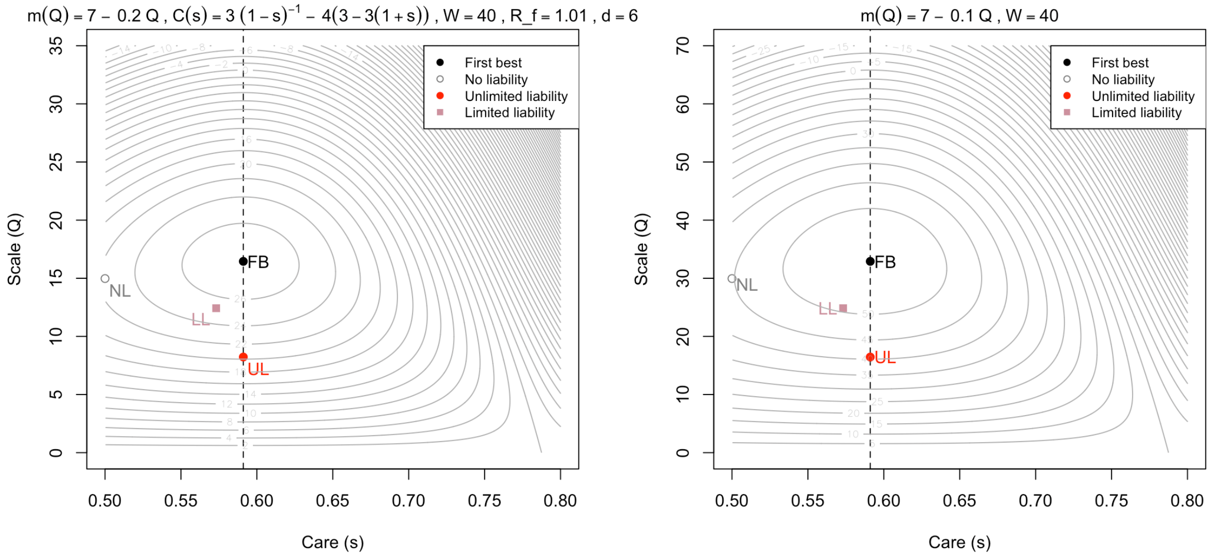


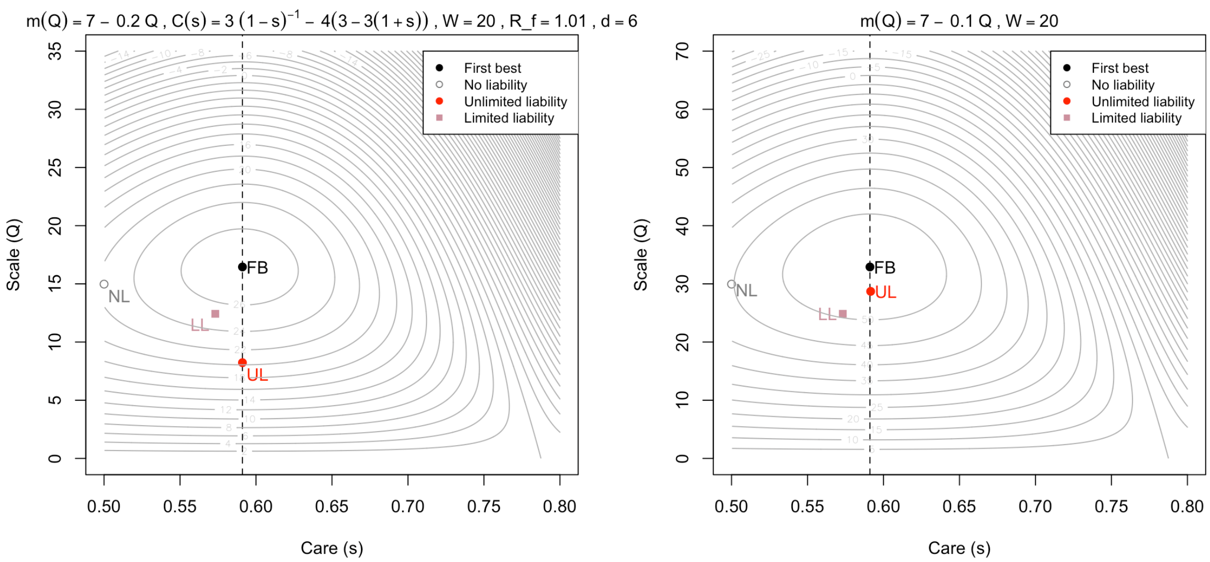
Figure 1.2: ***(Choice of deep-pocket or small-pocket)*** The notations follows the previous graphs. In these two graphs, the left graph shows the investor's choice when wealth W is relatively big (=40), and the right graphs shows a smaller wealth(=15). In the first graph, the investor with higher wealth level is deep-pocket and chooses an efficient care and inefficiently low scale under unlimited liability rule. In the second graph, however, the equilibrium is even closer to the first best when the investor has less wealth and chooses to have small pocket.

of small pockets for the readers who are interested in this case. The results can also be applied to the labor market and is discussed in more detail in a later section. If the market is very competitive instead, full liability is close to the first best because scale is also efficient. Competition intensity is measured by the number of firms. Suppose there are N homogeneous firms investing in the same technology, and a representative firm i chooses care s_i and scale Q_i. In the simplest case the firm's choice of safety is independent of other firms' choices.[30] The social welfare function is the same

[30] In some cases, though, a tort litigation would trigger a series of litigation on similar products which are produced by other firms.



(a) ***Change demand elasticity with larger wealth.*** *With larger wealth $W(=40)$, demand elasticity has no impact on care but affects firm's choices of scale only proportionally. When demand is more elastic (flatter $m(Q)$), society and investor both prefer higher level of scale.*



(b) ***Change demand elasticity with smaller wealth.*** *When demand become more elastic, scale increases for every equilibrium, but can be disproportionate with a sharper increase of scale under unlimited liability rule. This is suggesting that equilibrium under unlimited liability can be more sensitive to changes in demand, particularly when the investor has limited wealth.*

Figure 1.3: ***(Change demand elasticity with high/low level of wealth)***

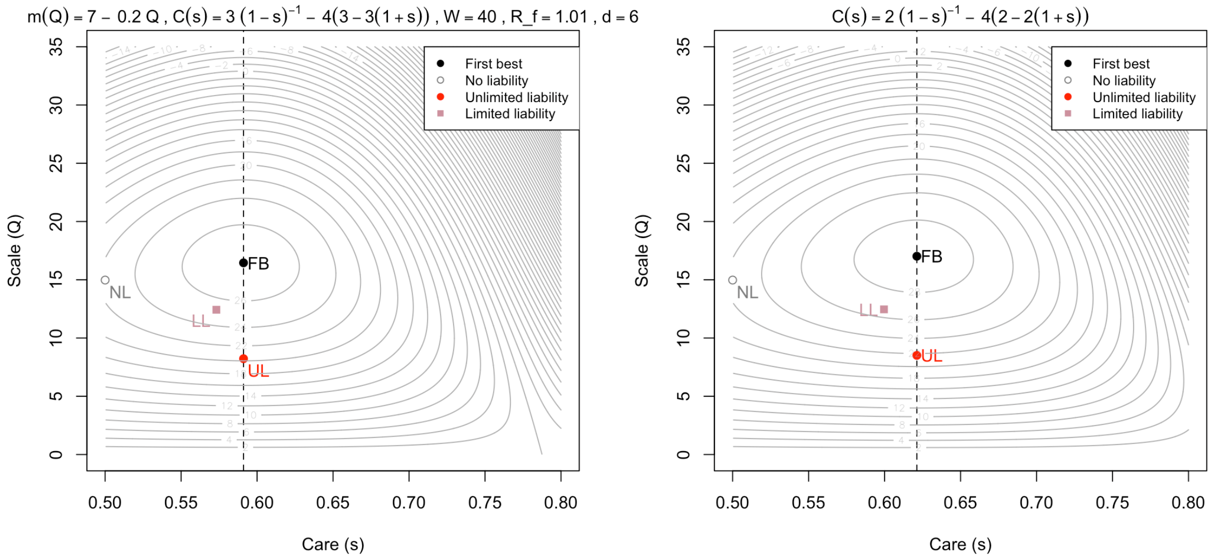


Figure 1.4: ***(Cost of care)*** The notations follows the previous graphs. When it is more expensive to prevent damages, incentives for care decrease for everyone and incentives for scale do not change.

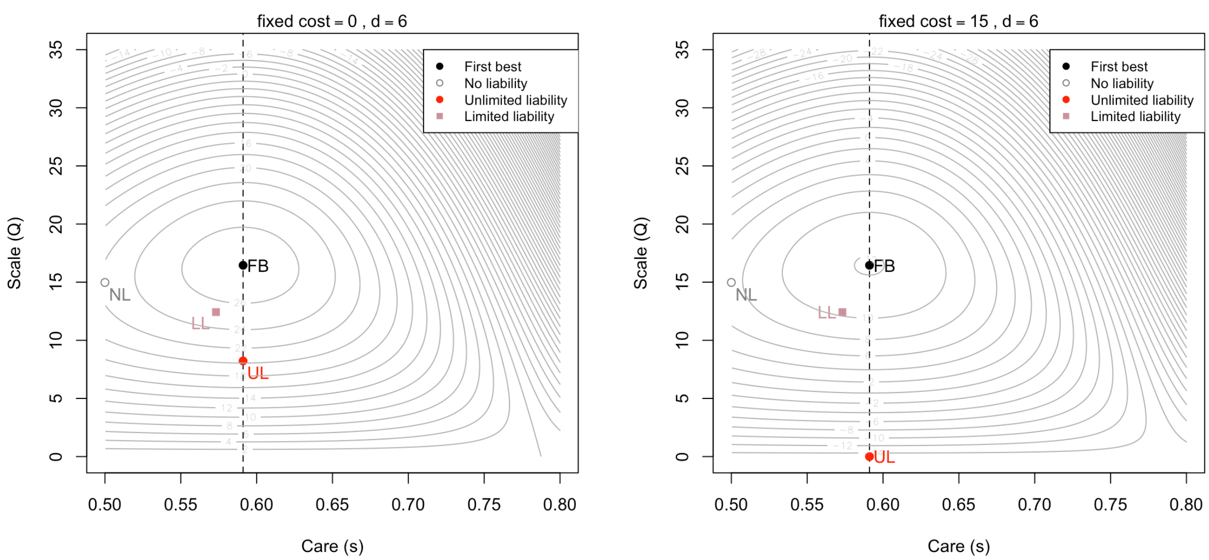


Figure 1.5: ***(Change fixed cost)*** The notations follows the previous graphs. With full liability, investment in safety reaches first-best but there is always underinvestment in quantity (figure on the left). The firm will even not start when there is fixed costs (figure on the right). Limited liability under-provides safety but increases quantity compared to full liability.

as (1.3), and firm i's problem is

$$\max_{s_i,Q_i}\left(Q_i p-(1-s_i)\Lambda(Q_i,p,w_i)+w_i\right) \tag{1.13}$$

$$\text{s.t.}\quad p=m(Q_{-i}+Q_i), 0\leq Q_i \tag{1.14}$$

$$w_i\equiv R_f[\frac{W}{N}-Q_i(1+C(s_i))]>0 \tag{1.15}$$

$$\text{and}\quad \underline{s}\leq s_i<1 \tag{1.16}$$

Where

$$\Lambda(Q_i,p,w_i)=\begin{cases}0, & \text{if no liability}\\ (Q_i p)\wedge(Q_i d), & \text{if limited liability}\\ \left(Q_i p+w_i\right)\wedge(Q_i d), & \text{unlimited liability with small pockets}\\ Q_i d, & \text{unlimited liability with deep pockets}\end{cases}$$

In equilibrium, all the firms choose the same safety and productivity levels, i.e., $s_i=q, Q_i=I/N$. See Table 1.3 for detailed computation of the first order conditions. We then have the following proposition:

PROPOSITION 5. ***(Cournot competition)*** *When the number of firms increases to infinity, equilibrium of full liability converges to first-best, whereas limited liability diverges from the first best and results in overinvestment in scale and underinvestment in care and is socially inefficient.*

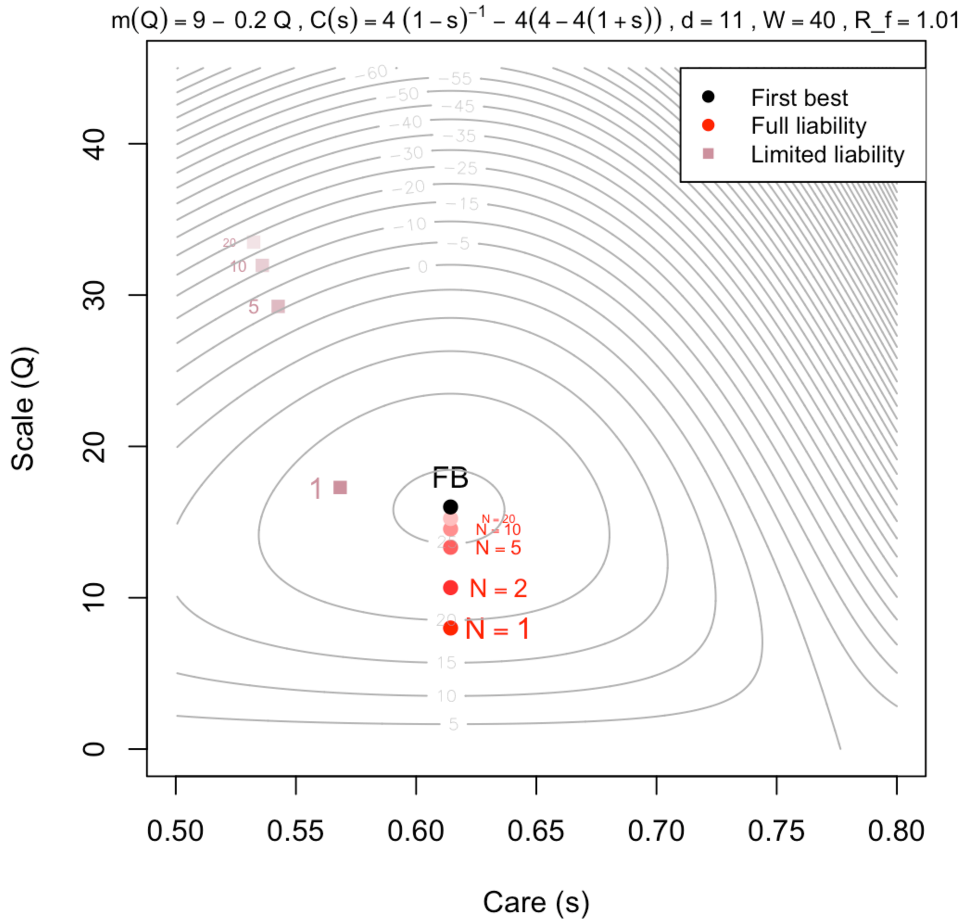


Figure 1.6: ***(Cournot competition)*** The notations follow the previous figures. When the number of firms increases (N =1,2,5,10,20 shown in the graph with the same shapes and colors but reduced saturation), equilibrium of full liability converges to first best (red dots), whereas with limited liability the equilibrium deviates from first best and result in over-provision of scale and under-provision of care.

Proof. The first order conditions for full liability are

$$(s_i) \quad s_i^* = s_s^*$$

$$(Q_i) \quad m(Q_u^*) + m'(Q_u^*)Q_u^*/N - (1 - s_i^*)d - R_f(1 + C(s_i^*)) = 0.$$

We can compute how Q_u^* changes when the number of firms increases:

$$\frac{dQ}{dN} = \frac{Q/N}{N+1+\frac{m''(Q)Q}{m'(Q)}}.$$

Equation (1.12) indicates that $\frac{dQ}{dN} > 0$, so the equilibrium quantity is increasing when the number of firms increases. As $N \to \infty$,

$$m(Q_u^*) + m'(Q_u^*)Q_u^*/N - (1-s_i^*)d - R_f(1+C(s_i^*)) \to m(Q_u^*) - (1-s_i^*)d - R_f(1+C(s_i^*))$$

since Q cannot be infinite. When the number of firm increases, while safety is always optimal, the first order condition for quantity is closer to first-best. We can do the same calculation for other liability rules. See Appendix A.3. $\square$

At extreme, the market becomes perfect competitive. Demand elasticity is perfect. The efficiency of full liability comes immediately from the fact that the firm internalizing all the benefits and costs fully aligns its interest with that of society. However, limited liability which does not fully internalize damages is more inefficient with increased competition and would result in inefficiency of under-investment in care and overinvestment in scale. In order for full liability under intense competition to work, one also has to make sure that the investors have enough assets to cover all the liability, that is, the investor has to be deep pocket, otherwise the evasion of liability would make investor's choices less socially efficient. The equilibrium would deviate from the first best with more competition and is similar to the case under limited liability.

(1) rule	(2) $\Lambda(Q,p,w)$	(3) FOC:(s)	(4) FOC:(Q)
FB	-	$C'(s_s^*) = \frac{d}{R_f}$	$m(Q_s^*) \leq R_f\left(1+C(s_s^*)\right) +(1-s_s^*)d$
FL/UL	Qd	$C'(s_u^*) = \frac{d}{R_f}$	$m(Q_u^*)+m'(Q_u^*)Q_u^*/N \leq R_f\left(1+C(s_u^*)+k\right)+(1-s_u^*)d$
LL	$Qp+Qk(<Qd)$	$C'(s_l^*) = \frac{m(Q_l^*)+k}{R_f}$	$m(Q_l^*)+m'(Q_l^*)Q_l^*/N+k \leq \frac{R_f\left(1+C(s_l^*)+k\right)}{s_l^*}$
UL-SP	$Qp+R_f(W/N-Q(1+C(s))-Qk)(<Qd)$	$C'(s_j^*) = \frac{m(Q_j^*)+R_f(W/Q-1-C(s_j^*)-k)+k}{s_j^*R_f}$	$m(Q_j^*)+m'(Q_j^*)Q_j^*/N \leq R_f\left(1+C(s_j^*)+k\right)$

red: adding number of firms.
blue: adding capital installation.

Table 1.3: ***(Comparisons of variables: increase competition and add capital)*** When the number of firms N increases to infinity, equilibrium of full liability converges to first-best, whereas limiting liability (either because of the limited liability rule or the investors having small pockets) results in too high investment in scale and too low investment in care and is socially inefficient.
Requiring purchased capital as an example of general capital requirement may push the equilibria back to first-best for limited liability, but either do not change incentives for small pockets under unlimited liability or tend to be inefficient when the investor has deep-pocket.

1.4 Lease versus Buy

In the previous section, the only assets in the firm available to pay tort claimants are the proceeds from selling the product. This is true if the firm rents capital or uses debt to finance its capital and can pledge the capital to ensure that the repayment to the lender has higher priority in resolution. In tort litigation, the firm has to return the rental capital or transfer the capital to the lender so that it is out of reach of the tort claimants. This section discusses the situation in which the firm can only buy capital to produce. This may happen when the operating capital is very specialized (especially for new technology) and the firm may not be able to rent existing capital or find lenders to provide funds. It may also come from capital requirement for regulatory purpose or requirement by other contractual creditors as a cushion. In this section I assume proportional capital requirement, namely, each unit of production requires k units of capital. Capital does not depreciate when tort occurs as a simplification in the model, but if it does (probably because capital is firm specific and should be liquidated with loss), the loss would make rental price higher if priced out. With firm purchasing capital, this depreciation is fully absorbed by the tort claimants in the limited liability case, and can be partially or fully borne by the investor under unlimited liability. I focus on the case in which there is no depreciation and the firm can sell the capital at the original value kQ_i.

The social welfare function is the same as (1.3). Firm Q's problem is

$$\max_{s_i, Q_i} \left(Q_i p - (1 - s_i)\Lambda(Q_i, p, w_i) + w_i \right)$$

$$\text{s.t.} \quad p = m(Q_{-i} + Q_i), 0 \leq Q_i$$

$$w_i \equiv R_f[W/N - (1 + k + C(s_i))Q_i] > 0$$

$$\text{and} \quad \underline{s} \leq s_i < 1$$

Where

$$\Lambda(Q_i, p, w_i) = \begin{cases} 0, & \text{if no liability} \\ [Q_i(p+k)] \wedge (Q_i d), & \text{if limited liability} \\ \Big(Q_i p + Q_i k + w_i\Big) \wedge (Q_i d), & \text{unlimited liability with small pockets} \\ Q_i d, & \text{unlimited liability with deep pockets} \end{cases}$$

PROPOSITION 6. ***(Buying capital)*** *If capital is purchased, more damages are internalized under limited liability so that equilibrium is pushed towards first-best with sufficient competition. Capital installation does not change the equilibria of unlimited liability with small pockets, but pushes the equilibria of deep pockets away from the first-best and may even turn down the investment.*

Proof. Limited Liability requires $m(Q_l^*) < d - k$ and the first order conditions

$$(s_i) \quad m(Q_l^*) = -k + R_f C'(s_l^*)$$

$$(Q_i) \quad m(Q_l^*) + m'(Q_l^*)Q_l^*/N = -k + R_f(1 + k + C(s_l^*))/s_l^*.$$

Detailed computation see Table 1.3. It is easy to prove that $\frac{dQ_l^*}{dk} > 0$ for the first equation and $\frac{dQ_l^*}{dk} < 0$ for the second, hence the function $Q_l^*(s_l^*)$ shifts upward for the first equation, and shifts downward for the second as k increases, resulting in lower s_l^* and higher Q_l^*. With full liability, we have

$$(s_i) \quad s_i^* = s_s^*$$

$$(Q_i) \quad m(Q_u^*) + m'(Q_u^*)Q_u^*/N - (1 - s_i^*)d - R_f(1 + k + C(s_i^*)).$$

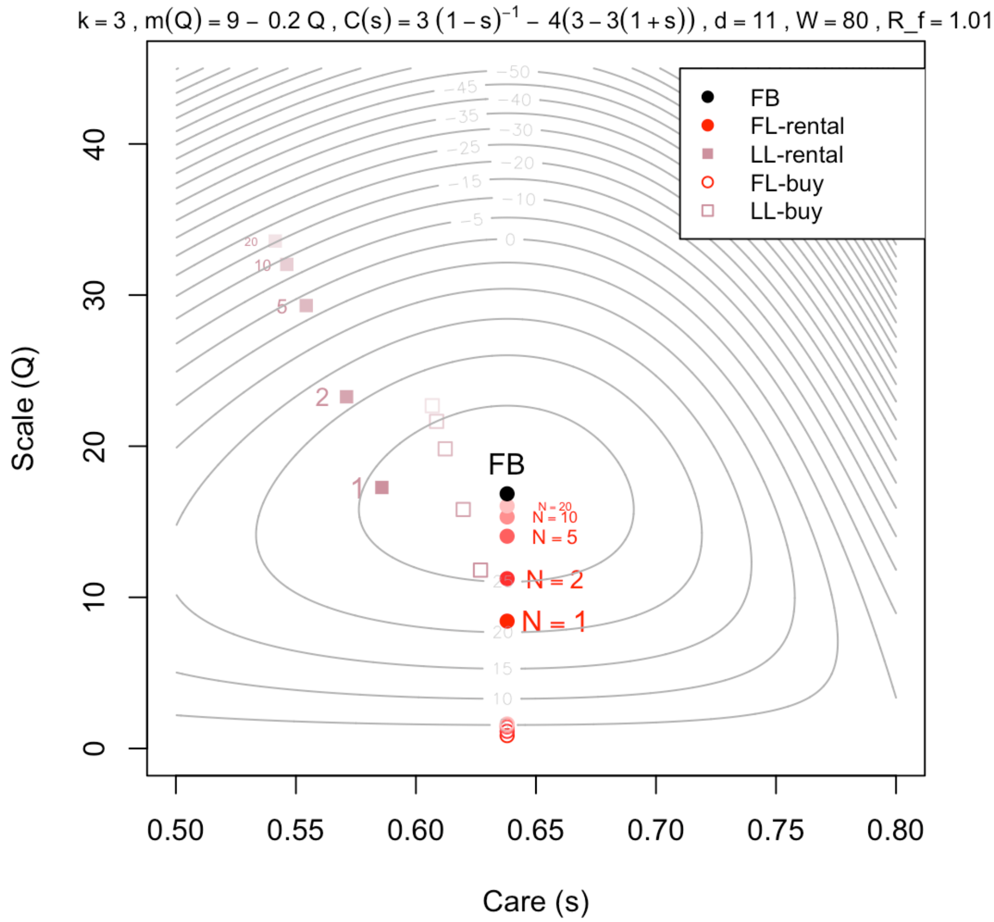


Figure 1.7: ***(Lease versus buy)*** The solid shapes represent the same as in Figure 1.6. The hollow shapes represent the relevant liability rules with purchased capital instead of rental capital. The plot shows that requiring purchased capital may push the equilibrium to first-best for limited liability shown by the hollow pink dots, but tends to be inefficient when the investors are deep-pocket (depicted by the red hollow dots).

Since $\frac{dQ_l^*}{dk} < 0$ for the second equation, the equilibrium care level does not change but the scale drops as k increases. For unlimited liability:

$$\text{if } m(Q_u^*) + R_f(W/Q_u^* - 1 - k - C(s_u^*)) < d - k$$

$$(s_i) \quad m(Q_u^*) + k + R_f(W/Q_u^* - 1 - k - C(s_u^*)) - s_u^* R_f C'(s_u^*)$$

$$(Q_i) \quad m(Q_u^*) + m'(Q_u^*)Q_u^*/N + k - R_f(1 + k + C(s_u^*))$$

When R_f is close to 1, the problem is close to the problem with rental capital. It is true that capital in the firm does not earn risk-free interest outside of the firm, but if the difference is negligible, capital requirement would only act as a fixed cost and shut down the firm if sufficient. $\square$

With unlimited liability, requiring (purchased) capital in effect adds overhead, either a fixed number or in this setting a proportional cost, to the investor. As shown in Figure 1.7, the deep pockets still invest in first-best care but underinvestment in scale. It would not change the equilibrium for small pockets because the capital inside or outside of the firm can always be seized under unlimited liability rule. However, requiring capital internalizes damages for limited liability, increasing incentives for care and at the same time decreasing incentives for scale and improves social welfare particularly when there is more competition.

Notice that, minimum capital requirement also moves the budget down, and if sufficient enough, makes some equilibria above the budget line infeasible. In this case, investments in care and scale for limited liability may move along the budget line. Along the line, there is also a substitution effect of care and scale.

1.5 Other Stakeholders

Previously, consumers are the only stakeholders of the firm. The inefficiency of underinvestment in quantity is a result of firm's market power in the product market that externalizes benefits, and as discussed before, limited liability has a flavor of Ramsey pricing and may enhance social welfare by "subsidizing" the firm through reducing the liability from damages. This is also true if the firm has other stakeholders. For example, large corporations usually have big impact on communities where the firms located in. They create jobs, provide investment opportunities, safety, unique community identity, economic health and development, etc. Much of the characteristics are valuable but not captured by the profits. Another aspect is the discrepancy of management interest and shareholder interest, which are not necessarily align because of separation of ownership and control. Management nowadays usually do not take full liability or even exempt from liabilities on the consequences of decision making, except for fraudulent conveyance and breach of duty. Part of the reason is that they do not capture all the benefits in the firm and therefore bearing full liability would probably result in too conservative investment strategy and may let go of profitable investment opportunities. In this section, I discuss two other firm's stakeholders: governments and employees.

1.5.1 Government as a stakeholder: taxation

Governments are considered as a major stakeholder of a corporation because they collect corporate income taxes from the firm, payroll taxes from the employees, as well as other taxes (sales taxes, etc.). In some states, certain corporations also pay franchise taxes for the right to be chartered. With higher taxes, the firm externalizes larger proportion of benefits. A lump-sum tax such as a franchise tax would be similar to a fixed cost to a firm. It may not distort incentives once the firm is established, but it makes it less attractive to start the firm in the first place. I assume unit tax with tax rate $\tau \in [0,1)$. The investor's problem is

$$\max_{s_i,Q_i}\left(\left[Q_i p-(1-s_i)\Lambda(Q_i,p,w_i)\right]+w_i-\text{Tax}\right) \tag{1.17}$$

$$\text{s.t.}\quad \text{Tax}=\left[(1-s_i)(Q_i p-\Lambda(Q_i,p,w_i))^{+}+s_i Q_i p\right]\tau \tag{1.18}$$

$$p=m(Q_{-i}+Q_i),\quad 0\leq Q_i \tag{1.19}$$

$$w_i\equiv R_f[W/N-(1+C(s_i))Q_i]>0, \tag{1.20}$$

$$\text{and}\quad \underline{s}\leq s_i<1 \tag{1.21}$$

Where

$$\Lambda(Q,p,w_i)=\begin{cases} 0, & \text{if no liability} \\ Q_i p\wedge Q_i d & \text{if limited liability} \\ \left(Q_i p+w_i\right)\wedge Q_i d & \text{unlimited liability (small pocket)} \\ Q_i d, & \text{unlimited liability (deep pocket)} \end{cases}$$

Figure 1.8 shows an example when per unit taxes are 0, 0.25 and 0.5, respectively.

PROPOSITION 7. ***(Taxation)*** *Full liability results in underinvestment in quantity when there is taxation. With limited liability, increasing unit taxes undermines both quantity and quality incentives.*

Proof. See Appendix A.4. □

Higher tax rate would externalize more benefits and results in lower quantity for all cases. When liability is capped, taxation results in less assets in the firm to compensate tort claimants. This also

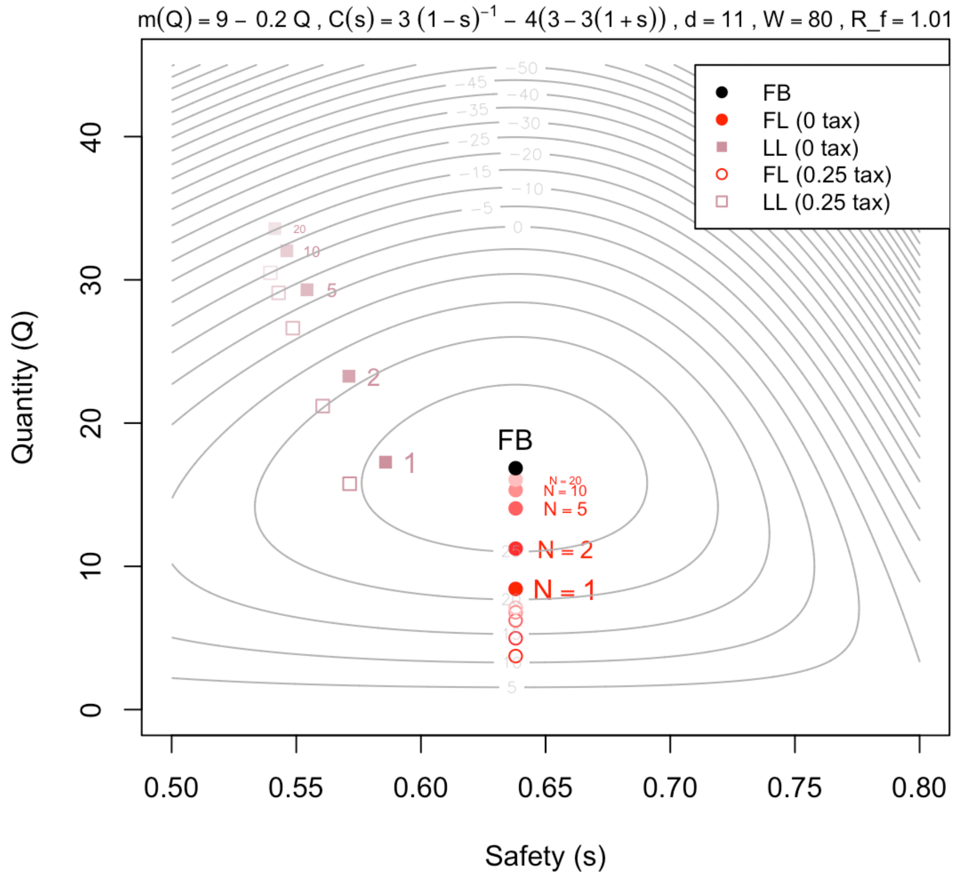


Figure 1.8: ***(Taxation)*** The solid shapes represent the same as in Figure 1.6 in which there is no tax. The hollow shapes represent the relevant liability rules with unit tax .25. Full liability results in underinvestment in quantity when there is taxation. When liability is capped, increasing unit taxes undermines both quantity and quality incentives.

discourages safety incentives. As shown in Figure 1.8, taxation is bad socially with deep pockets because the firm may not start. Yet it is not necessarily bad socially when liability is capped, especially when there is more competition. With intense competition, quantity can be way too high.

1.5.2 Employees as stakeholders

In the model I focus on the product market, but we can have similar analysis on the labor market. If the firm has monopsony power in the labor market, then the firm does not capture the full benefits of people working in the firm and will tend to operate in a smaller scale. Monopsony is not unusual in the U.S. labor market. A typical example is a mining town in the mountains, where it is remote and has only few mining employers. If the firm is the only employee, the marginal cost is bigger than the workers' reservation utility, because to hire one more worker the wage has to increase for every worker that the firm hires. This probably partly explains small scales of firms in small places where firms have monopsony/monopoly power in the labor market as well as other factor markets. Similar to the conclusion before, if there is also fixed cost, the firm probably would not start in the first place. Beneficiaries have long advocated for unionization and increased wages to a level comparable to a competitive outcome to achieve a more "equitable economy," and the thought can be traced back to as early as Robinson (1933), but this would externalize more benefits and consequently reduces demand for labor and social welfare.

Our model suggests an analysis parallel to the product market analysis: limited liability would mitigate the inefficiency of not internalizing all the benefits, and social efficiency in quantity may be improved. Increasing competition only improves social welfare when the firm bears full liability, which requires unlimited liability and investors have deep pockets. Increasing competition will result in too low safety and too high quantity when liability is capped. With proper capital requirement, the equilibrium under limited liability may be drawn towards the first-best, but that does not work for unlimited liability.

1.6 Conclusion

This paper provides a theoretical framework to study choice of care (safety) and scale (quantity) under limited and unlimited liability rules. When the firm's other stakeholders obtain large benefits from the firm, full liability results in under provision of quantity. I mainly focus on the product market in which such inefficiency is a result of big consumer surplus when the firm has monopoly power and faces a less elastic demand. We can also extend the discussion when the firm has other stakeholders, such as communities, governments, and when the firm has market power in other markets such as the labor market. Limited liability mitigates the inefficiency caused by externalization of benefits, because it reduces the damages taken by the investors as a means of "subsidy." An actual subsidy to a firm may not be possible in the real world for social and political reasons. Alleviating liability for damages can be an easier way to increase incentives for scale.

With intensified competition, firms capture higher fraction of benefits would have equilibrium converge to the first best under full liability, but that also requires the investors to be deep-pocket. If the investors have limited liability, the equilibrium would overinvest in scale and underinvest in care, which is also true when the investor has a small pocket under unlimited liability. However, one advantage of limited liability is that it is flexible to include other policies to adjust for cross firm differences. For example, minimum capital requirements, requiring insurance, and setting up funds as a buffer increases liability paid by the firm.

So far I have only studied the single investor's problem. Even when competition is discussed, each firm only has one investor because the intention is to focus on the effect of reduced market power. Future research can study multiple investors in the firm, namely, shareholders and bondholders, and to answer questions such as who should be in control and how they are sharing liabilities.

In that setting, priority order of different claimants and monitoring also matter for the investors' decisions and social efficiency.

Chapter 2

Gambling for Redemption or Ripoff, and the Impact of Superpriority

(joint with Philip Dybvig)

2.1 Introduction

In the early days of Federal Express, the company's cash once dwindled to \$5,000, too little to cover the \$24,000 jet fuel bill due the following Monday. With the firm hanging on the edge, the founder Frederick Smith flew to Las Vegas over the weekend and played blackjack to convert the \$5,000 into \$32,000, enough to keep the company afloat for another week.[31] This gambling was obviously beneficial to the firm's owners since it provided a positive probability to avoid bankruptcy, and it was probably also beneficial to the other claimants including the fuel company, who were unlikely to receive much in bankruptcy. Gambling by a firm can also benefit owners at the expense of creditors as in the asset substitution studied in Myers (1977) because owners

[31] Frock (2006)

receive most of the upside of large gambles but most of the downside is borne by the creditors. In this paper, we study pure gambling by the firm. We can understand the impact of this gambling through two polar cases. *Gambling for redemption*, which means gambling just enough to stay in business as in the Federal Express example, is good for the owners, the creditors, and for overall efficiency. *Gambling for ripoff*, which is at a larger scale, benefits the owners at the expense of the creditors and overall economic efficiency. We study gambling using derivatives, which allows more control over the payoff distribution and negligible efficiency loss compared to asset substitution. Our results show that when gambling at large scale is possible, the anticipation of gambling for ripoff makes it hard for firms to borrow in the first place and reduces the equilibrium value of equity.

Gambling for ripoff is of special current interest because of legislation before the financial crisis that exempts repos and other derivative securities from important provisions of bankruptcy, including the automatic stay and clawbacks, causing some people to call them superpriority claims.[32][33] One important aspect of superpriority is that it enables the firm to gamble away assets, which may also be possible due to poor specification or enforcement of property rights and bankruptcy law in under-developed countries. In the United States, it has traditionally been difficult to redeploy assets for gambling. While common law allows for asset seizure in satisfaction of debts, seizure or sales in violation of bond covenants can be clawed back in bankruptcy. Traditionally, bonds contain covenants to prevent asset sales, typically placing the firm in default on the loan if the covenant is violated. Together with cross-default clauses in bonds saying that a default on one bond places the company in default on all its bonds, this would normally result in the firm entering bankruptcy. According to the original bankruptcy law, an asset sale in satisfaction of a particular

[32] Roe (2010) describes the law. Roe argues that these laws accelerated the financial crisis.

[33] Superpriority protects the contractual right of derivatives counterparties to "terminate, liquidate, or accelerate" a derivatives contract before the commencement of the case. See 11 U.S. Code §362(b)(6), §546(e). Stockholders who are members of Securities Investor Protection Corporation (SIPC) are liquidated under SIPA with similar rules, instead of the Code, see 15 U.S. Code §78eee(b)(2)(C).

claim within 90 days (or in some cases longer[34]) before bankruptcy is considered preferential if the firm cannot satisfy all claimants and can be clawed back (reversed by the court), which tends to make asset seizure or sale pointless. Consequently, any promise by the firm to transfer assets to pay off on a failed gamble would not be credible unless the gambling counterparties are sure that the firm will not be pushed into bankruptcy. However, the new exemption from bankruptcy law for "superpriority" claims sidesteps these laws. The gambling counterparties are now granted immunity from the clawback and can redeem their gains without being stayed in the firm's estate in bankruptcy. Thus, it makes possible for a financially shaky firm on a path to bankruptcy to gamble assets, and derivative securities makes it easier to shape the exact distribution of the gamble a firm chooses.

Following Myers (1977), we know that limited gambling, in the form of "asset substitution" to an inefficient but noisier production technology, can benefit firm owners at the expense of bondholders and overall efficiency. Gambling with derivatives is a sharper tool for gambling just what is needed, and permits gambling with negligible efficiency loss. For example, a firm can buy a digital option paying off exactly the maturing debt due this period. This is different from the previous literature focusing on choices of investment risk, oftentimes the choices of variance of a normal distribution, such as in Ericsson (1997), Ross et al. (1998), Leland (1998), Gong (2004) and Della Seta et al. (2020).[35] Under pre-assumed continuous distributions, selections of risk are constrained by the shapes. For example, increasing risk with normal distribution thickens both tails. In our framework, gambling is fairly priced and is more flexible and precise - the firm can concentrate the probability in a targeted payoff, e.g., the amount needed to repay the debt. If the face value of maturing debt is less than the equity's continuation value, the owners want the firm

[34] The clawback extends back one year for a preferential transfer to an insider, or up to two years for a fraudulent conveyance.

[35] Ericsson (1997) studies firm's one-time choice between two levels of risk, Gong (2004), Ross et al. (1998), Leland (1998) and Della Seta et al. (2020) extend the choice of variance to an interval.

to survive and will gamble up to the face value of debt (if possible) but not further, to maximize the probability of survival. This is consistent with Federal Express's gambling, but gambling using superpriority can also operate at a much larger scale and in the presence of accounting controls. When the face value of maturing debt is more than the equity's continuation value, the owners have an incentive to gamble to as large a level as possible to capture the whole value while paying off the bondholders as infrequently as possible.

We start our analysis in a single-period model. The required payment on debt coming due may or may not be covered by the incoming cash flow. Managers' incentives are assumed to be aligned with the owners' to maximize equity value.[36] Bankruptcy has two costs: the loss of the continuation value, and an administrative cost paid out of the surviving assets. In the model, the loss of continuation value is borne primarily by the owners and the administrative cost is borne primarily by the bondholders. Before paying off the debt or going into bankruptcy, the owners can choose to undertake a fair gamble subject to having enough cash flow to handle the downside. If the firm gambles for redemption, just to the level needed to repay the debt, the gambling benefits both the owners and the bondholders by minimizing both costs. Absent the administrative cost paid out of the surviving assets, the bondholders would be just indifferent about gambling for redemption since we stay on the linear part of their payoff (receiving 50 cents on the dollar 100% of the time and receiving 100 cents on the dollar 50% of the time have the same expected value). With the administrative cost, gambling for redemption is actually better because it allows the bondholders to avoid the cost with some probability. However, whenever the face value of debt is larger than the continuation value lost in bankruptcy, the firm owners prefer for the firm not to continue since the lost continuation value is trumped by not having to pay off the debt. The firm owners may

[36]Our firms are more like proprietorships than corporations in order to focus on the role of gambling. Sometimes we use "the firm" to refer to the firm owners, and whenever we have "firm owners" in the model we simply mean the single entity who makes decisions as a whole. Similarly, "bondholders" are also considered as a whole. Future work could study gambling more generally, for example by a rogue trader or a CEO who does not maximize on behalf of shareholders.

simply “take the money and run,” and gambling provides a legal way of doing it. With gambling for ripoff, the bondholders are worse off since they may only receive full repayment .1% of the time and zero 99.9% of the time. If it is possible to gamble some of the assets as well (for example, due to superpriority), gambling for ripoff becomes more attractive because it also transfers part or all of the asset value (which would be subject to a stay or a clawback absent superpriority) to the owners and making bankruptcy more appealing to them.

In the single-period model, the face value of maturing debt is exogenous. This might be a good assumption at the time of the superpriority legislation, if the legislation is a surprise to the bondholders with debt in place. It is perhaps more interesting to think about the impact of the law once it is understood by bondholders and is priced out in the lending decision. For this, we have a multi-period model that endogenizes the level of borrowing and equity’s continuation value. In the multi-period model, the firm chooses gambling and new financing after the realization of shock in each period. After the shock, if the continuation value is greater than debt coming due but current cash flow plus potential borrowing and liquidation does not cover the debt, the firm will gamble for redemption. However, if the shock leaves the continuation value small enough, the owners prefer to lose the continuation value rather than pay the debt. Gambling for ripoff avoids paying off the debt most of the time while still capturing the expected cash flow by concentrating the gamble’s payoff in a small set of states. In general, whether gambling is beneficial depending on how often there is gambling for redemption and ripoff. Our main result of the multi-period model shows that if there is significant liquidation value (for example due to superpriority), being able to gamble against assets reduces the bond value and hence the maximum amount the firm can borrow, and also reduces the market value of equity. This suggests that superpriority may not be appealing for the firm owners at first place, but the adoption of the law makes large gambles ex post optimal for the firm owners and results in damage of value ex ante.

How much can be gambled in a firm is crucial for our analysis and depends on legal considerations. As a consequence of superpriority law, the bondholders cannot rely on protections in bankruptcy through negative pledge covenants which preclude asset sales, but security interests if perfected are still honored under UCC Article 9. In a parallel provision, the liquidation of stockbrokers who are members of Securities Investor Protection Corporation (SIPC) is governed by SIPA[37] rather than the Bankruptcy Code. SIPA generally provides similar superpriority protections to qualified financial contracts, except that foreclosure of related securities collateral may still subject to a stay.[38]

The "superpriority" claims we are talking about obtained their exemption from bankruptcy in a series of laws passed between 1978 and 2006. See Schwarcz and Sharon (2014) for a detailed history of the law. The game changer seems to be the 2005 amendment to bankruptcy code (BAPCA), which extends the exemption, which started with some commodity futures and previously extended to repos and swaps, to all derivative securities. Taken together, these laws exempt qualified contracts (including securities contracts, commondity contracts, forward contracts, repos, swaps, and contracts, etc.) from the automatic stay and clawbacks of bankruptcy.[39] BAPCA and the subsequent 2006 Act also added and extended protections for "master netting agreements," an arrangement between counterparties to net or set off any qualified contracts described above. If a counterparty and the firm owed each other one dollar, without netting, when the firm is in bankruptcy the counterparty has to repay the one dollar and may receive only 50 cents out of the dollar from the firm. With a netting agreement, the counterparty can set off beforehand and be paid 100 cents out of a dollar. This treatment makes gambling even easier.

[37]The Securities Investor Protection Act in 1970.

[38]15 U.S. Code §78eee.

[39]Supepriority also favors derivatives by exempting clawback of constructive (but not actual) fraudulent transfers. See Vasser (2005). However, the exemption from avoidance of fraudulent transfer may not apply in the context of gambling, since the transfer is in satisfaction of an existing claim and reflects a fair value.

The superpriority treatment has drawn a lot of attention since the 2008 financial crisis. Roe (2010) observes a soaring volume of interest rate derivatives from $13 trillion in 1994 to $430 trillion in 2009, nearly 40 times of increase. During this period, the private business debt only tripled from $11 trillion to $34 trillion.[40] Baily et al. (2008) also shows an exponential increase in value of CDS outstanding since 2001. Roe suggests that this is because superpriority provides a cheaper way of financing, facilitating more liquidity that otherwise would not occur. This shifts the firms away from using traditional financing and lower the incentives of derivatives counterparties to monitor the firm. As a consequence of the expansion of market, the "too big to fail" problem is worsened if the superpriority claims are heavily used by the systemically important firms. Besides the costs, Duffie and Skeel (2012) provides "benefits" of the safe harbor exemption on QFCs, such as increasing reliance of firms using critical hedges and reducing self-fulfilling security runs. Previous economics literature also focus on the repo market fire sales, which dilute the collateral value for the secured creditors.[41] Our paper suggests another angle to understand the impact of the law by incorporating firms' gambling decisions and provides meaningful economic insights to understand gambling.

In normal times (when it is beneficial for the firm owners to continue the firm), gambling would not be a problem because a firm with sufficient cash to pay debt would not gamble. Even if the cash cannot cover the maturing debt, the firm will gamble for redemption to maximize the probability of survival. In either case, bankruptcy costs are minimized. Gambling becomes a problem when the firm's continuation value is small compared to debt, probably because of a decrease in asset value. When the decrease is large enough, gamble for ripoff maximizes owners' benefits by looting the value that should have been collected by the bondholders. Interestingly, if the owners benefit from bankruptcy, they favor such extreme gambling regardless having enough cash to cover debt or not.

[40] Roe (2010) Figure 1.

[41] See Infante (2013), Oehmke (2014), Antinolfi et al. (2015) and Auh et al. (2018).

When cash is not enough, gambling for ripoff transfers value from the bondholders to the owners; but with enough cash, gambling for ripoff also dissipates equity's continuation value.

When the economy goes south, superpriority law makes gambling for ripoff more appealing for the owners. Providing liquidity in a downturn may help to keep the firm temporarily, but it may not be useful enough to change the risk taking by the firm with poor going concern surplus. Rather, policies that increase firms' going concern surplus or block superpriority gambling may be more socially efficient.

If the firm owners are potentially worseoff because of the laws, as claimed in our multi-period model, the owners would have incentives to use more defensive measures (secured debt, short-term debt, and even repos) to protect against the laws. This is supported by some empirical evidence. For example, Benmelech et al. (2020) documents an increase of secured debt over total debt since 1995 and an upward jump in 2005.[42] Baily et al. (2008) shows that the issuance of total value of short term (with 1-4 days maturity) asset-backed commercial paper has increased significantly from 2005 to mid 2007, whereas the commercial paper with longer terms (with 21-40 days and >40 days maturities) stayed steady during the period.[43]. There was also a surge in the growth in the market for repurchase agreements, a much higher growth rate compared to the total debt in the financial sector, particularly after 1999 (Roe (2010)). In another empirical study, Lewis (2020) provides causal evidence of expansion of repo collateral rehypothecation as a result of the law and estimates a money multiplier of private-label mortgage collateral to be 4.5 times that of Treasuries. The overuse of collateral can be inefficient. Donaldson et al. (2019, 2020) suggest that it damages the flexibility of assets redeployment, possibly resulting in underinvestment in good projects in the future.

[42] See Benmelech et al. (2020) Figure 8a.
[43] Baily et al. (2008) Figure 6

The analysis of gambling can further extend to other scenarios. For instance, it may encourage gambling for ripoff after Congress enacted Chapter 11 section 1114 to give retiree medical benefits special priority in 1988, because an increase of debt obligation will reduce the going concern surplus of the firm. Consequently, gambling for ripoff wipes out most of the assets and the assets dilution effect may be larger than purely having additional debt. This highly risky behavior may make it harder for the firm to raise funds, and bondholders may only be willing to lend if the firm promises to file for Chapter 7 liquidation to evade the legislation with underfunded retiree insurance benefits.[44]

The paper is organized as follows. Section 2 focuses on the two polar cases of *"gambling for redemption"* and *"gambling for ripoff"* by examining a stripped-down single-period model. Section 3 presents a multi-period model using the building block in Section 2 and with endogenous decision making to study the ex ante effect of gambling with and without superpriority. Section 4 characterizes equilibrium properties of the model and provides numerical examples to illustrate the results, and Section 5 concludes.

2.2 Optimal gambling: the single-period model

We start with a stripped-down model in which we show there can be two different forms of gambling: *"gambling for redemption"* and *"gambling for ripoff."* *Gambling for redemption* occurs when the firm cannot pay the debt immediately, and the owners would suffer a net loss from bankruptcy. In this case, bankruptcy is bad for firm owners, so they will minimize the probability of bankruptcy by gambling just to what is needed to pay the debt, and the firm owners and the bondholders are all better off. By contrast, *gambling for ripoff* occurs when the owners would

[44] For further reading of this issue, see Keating (1990, 1991).

gather a net gain from bankruptcy. In this case, bankruptcy is good for the owners, so they will maximize the probability of bankruptcy to evade debt obligations but still collect the firm's value by gambling to a large payoff, benefiting the owners at the expense of the bondholders. Superpriority claims reduce the net loss to owners because they allow them to collect part of the firm's asset value directly without paying the bondholders, which makes gambling for ripoff more appealing to the firm owners.

To illustrate these main ideas, this section assumes that everything except for the firm's gambling decision is exogenous. Absent gambling, if the current cash flow π exceeds the face value of the debt F, the debt is paid off and the value to owners is $\pi - F + C$, where C is the continuation value. However, if $\pi < F$ and there is no gambling, then the firm fails, the bondholders collects $(1-c)(\pi+L)$ and the owners get nothing, where $L \in [0,C)$ is the liquidation value of the firm's capital, the liquidation value of the entire firm excluding the cash flow π. $(1-c) \in [0,1]$ is the fraction of remaining value that the bondholders can collect in bankruptcy. Therefore, the total cost of bankruptcy to all financial claimants is $C + (\pi+L)c$.

With gambling, the owner can gamble π plus the part of the liquidation value that is available for gambling. Superpriority increases the proportion of L available for gambling, other things equal, and therefore allows the owners to make large gambles. In this section, for simplicity that superpriority increases the amount available for gambling from 0 to the entire liquidation value L. Gambling is not unlimited. We require it to be "fair" and "feasible," meaning that the firm can purchase mean-zero market claims with any payoff distribution subject to the limitation of what is available to pay. If $\mathbf{p}$ is a stochastic gambling payoff, the two requirements are

$$\begin{aligned} &\text{gambling being fair:} && \mathrm{E}[\mathbf{p}] = \pi, \quad \text{and} \\ &\text{gambling being feasible:} && -\sigma L \le \mathbf{p} \le \bar{\pi}, \end{aligned}$$

where $\bar{\pi}$ is the upper bound of gambling and can be infinite, and $\sigma \in [0,1]$ is the fraction the liquidation value of capital that can be grabbed. The fraction σ is impacted by superpriority laws, bond covenants, and the amount of perfected collateral. Without superpriority, bond covenants and perfected liens can preclude seizure of capital. Superpriority laws increase the amount of capital that can be grabbed unless the capital is all pledged as perfected collateral. Without loss of generality, we focus on the extreme case by assuming that the firm does not have perfected liens. Without superpriority, firm's capital is protected by bond covenants and $\sigma = 0$; with superpriority, the owner can gamble away all the liquidation value of the assets and hence $\sigma = 1$. In conclusion,

$$\sigma \equiv \begin{cases} 1, & \text{with superpriority} \\ 0, & \text{absent superpriority.} \end{cases}$$

"Fair gambling" assumes the efficiency of gambling, because using derivatives largely reduces the cost of gambling (for example, using a digital option which pays x with probability $1/x$ and pays 0 otherwise). In this paper, everything is under risk-neutral probabilities or market valuation, and we ignore transaction costs of participating in gambling, i.e., the costs in searching for gambling counterparties, writing gambling contracts, transportation, etc.

Superpriority makes feasible larger scales of gambling. Absent superpriority, the owners at most lose all the cash flow π, because the gambling counterparties know that the firm cannot reliably promise more than the sure cash. However, superpriority makes available other assets in the firm to be pledged as collateral and then the owners can gamble down to $-L$. We show an example to illustrate the assumptions: if the firm has \$100 in cash and uses it to gamble, a fair gamble should be worth exactly \$100. Without being able to pledge assets (as in the case absent superpriority), the largest value the firm can lose is \$100. Knowing the firm's limit, the gambling counterparty would not gamble with the firm if the firm promises to pay \$200, unless the firm can pledge its other assets with market value of \$100 (if superpriority makes it feasible). Specifically, not to gamble ($\mathbf{p} \equiv \pi$) can be regarded as a special case of gambling.

The purpose of gambling – to get quick cash when the firm is likely at the edge of bankruptcy– also suggests that gambling, unlike hedging by firms, is more likely to be short-term. Being short-term also indicates a hidden assumption of independence for gambling and randomness in the firm. It also makes sense for the firms' gambling counterparties who are more certain about firm's condition in the short run and thus are more willing to participate in gambling.

The firm owner maximizes the expected payoff to equity, and the firm owners' problem is formally stated as follows.

2.2.1 Firm's problem

Given the cash flow π, the continuation value C, the liquidation value L, and the face value of the debt F, the firm's problem is to choose a fair gamble $\mathbf{p}(\tilde{x})$, where $\tilde{x}$ is the underlying randomness: $\tilde{x} \sim_d U(0,1)$, maximizing[45]

$$\mathrm{E}\big[\big(\mathbf{p}(\tilde{x}) - F\big)^+ + \big(\mathbf{p}(\tilde{x}) \geq F\big)C\big], \tag{2.1}$$

[45]The notations we use are defined as follows:

$$\begin{aligned}
(aRb) &\equiv \begin{cases} 1, & \text{for } aRb \\ 0, & \text{otherwise} \end{cases} \\
a \wedge b &\equiv \min\{a,b\}, \\
(A)^+ &\equiv \begin{cases} A, & \text{for } A \geq 0 \\ 0, & \text{otherwise} \end{cases}
\end{aligned}$$

subject to the gamble being fair,

$$\mathrm{E}[\mathbf{p}(\tilde{x})] = \pi, \tag{2.2}$$

and the constraint of feasible gambling outcome

$$-\sigma L \leq \mathbf{p}(\tilde{x}) \leq \bar{\pi}\ (\rightarrow +\infty). \tag{2.3}$$

Given any choice of gambling $\mathbf{p}$, we have

$$\textit{bond value} = \mathrm{E}\Big[(\mathbf{p}(\tilde{x}) \geq F)F + (\mathbf{p}(\tilde{x}) < F)\Big(\big((1-c)(\mathbf{p}(\tilde{x})+L)\big) \wedge F\Big)\Big].$$

Available liquidation value L may be small because of the nature of the firm and its capital, but we are mostly interested in its availability for gambling. In our simple model, we think of L being the liquidation value of the firm and also the available liquidation value for gambling. In our examples, we focus on gambling only cash (without superpriority) and gambling with all the liquidation value (with superpriority).

2.2.2 Example 1: cannot gamble much (no superpriority)

In general, "gambling for redemption" happens when continuing the firm has net benefit to the owners: the face value of the debt to be repaid for the firm to stay in business is less than the continuation value lost in bankruptcy. We show in the simplest form that *gambling for redemption* minimizes bankruptcy costs for the owners and bondholders and makes everyone better off, while *gambling for ripoff* maximizes both costs with a transfer from bondholders to the owners.

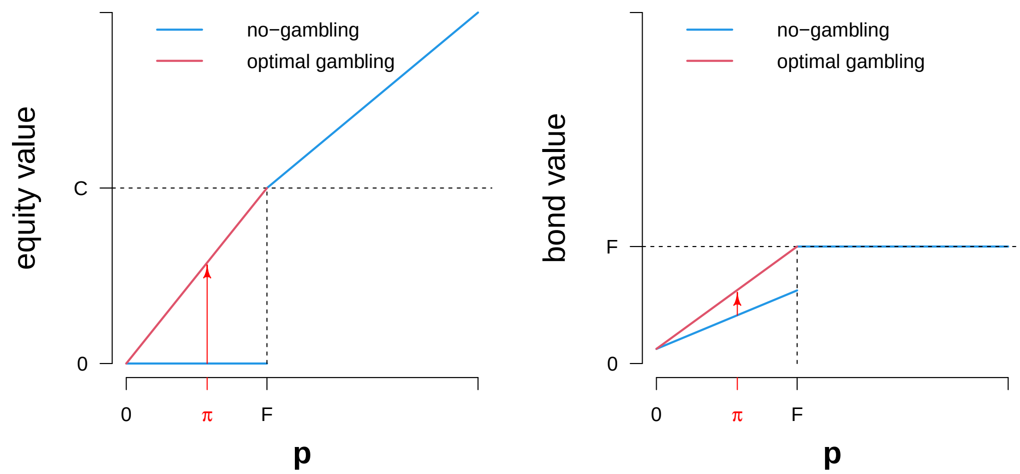


Figure 2.1: ***When $F < C$ (Gambling for redemption.) Dependence of equity value and bond value on cash flow* p.** The optimal gambling depicted by the red lines "concavifies" the value functions. The red arrow shows the increment of the expected value through optimal gambling given cash flow π. Absent bankruptcy costs for bondholders, gambling for redemption using derivatives is precise and stays on a linear segment of the bond payoff (between zero and the face value of debt) where the bondholders are indifferent about gambling.

Figure 2.1 demonstrates the equity value and bond value as functions of cash flow when face value of debt is less than the equity's continuation value. The blue lines represent the values without gambling: if cash flow π is below F, the owner loses all the continuation value in bankruptcy and bondholders lose a fraction $1-c$ of the remaining assets $\pi+L$; if cash flow π is above F, the owner keeps the continuation value and bondholders are paid the face value of debt.

Fair gambling in expectation achieves convex combinations of the gambling outcomes. To reach the maximal expected equity value, an optimal gambling strategy, shown by the red lines, should "concavify" the blue curves. In Figure 2.1, when $\pi \geq F$, the sound firm will only gamble along the 45 degree segment, which is equivalent to no gambling; yet, when $\pi < F$ the equity is out of money, the firm is inclined to be in the money since it is more profitable to obtain the continuation value and payoff the debt ($C > F$). An optimal gambling should retain the continuation value as often as possible, and hence the gambling randomizes payoffs between F, with probability $\frac{\pi}{F}$, and

0, with probability $1-\frac{\pi}{F}$. The bondholders will obtain full repayment of debt F with probability $\frac{\pi}{F}$ and $(1-c)L$ with probability $1-\frac{\pi}{F}$. In effect, the firm owners achieve positive expected equity value instead of zero, and the bondholders collect

$$\frac{\pi}{F}\times F+(1-\frac{\pi}{F})\times(1-c)L=(1-c)L+\frac{\pi}{F}(F-(1-c)(L+F)),$$

which is bigger than the amount $(1-c)(\pi+L)$ if $c>\frac{L}{F+L}$. We assume that bankruptcy cost for bondholders is big enough so that they receive less than the face value of debt when the firm goes bankrupt. To conclude, gambling for redemption adds value to the firm owners and the bondholders due to less frequent value loss in bankruptcy. When there is no bankruptcy costs for bondholders, $c=0$, then

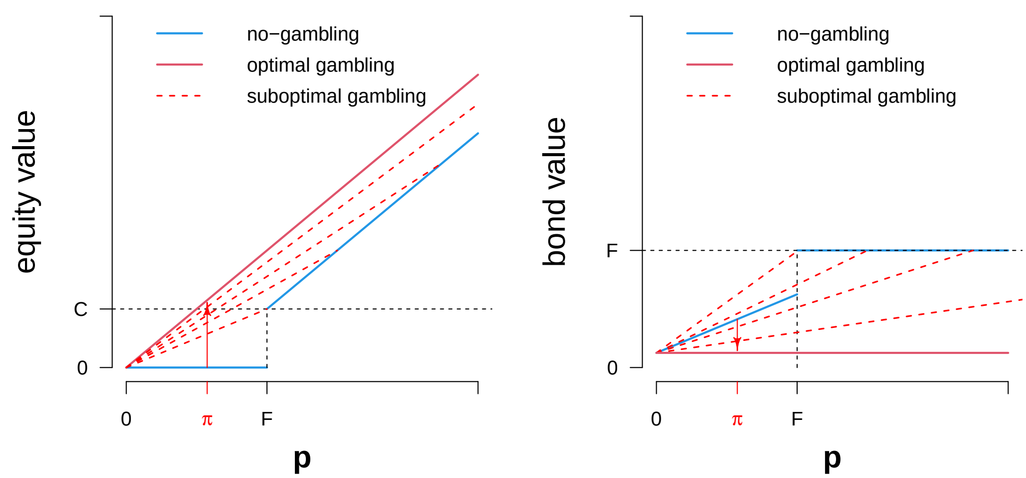


Figure 2.2: ***When $F>C$ (Gambling for ripoff.) Dependence of equity value and bond value on cash flow* p.** The concavification as optimal gambling of the utility function is a linear segment shown by the solid red line. In this case, equity value increases when the firm take bigger gambles and the bond value decreases accordingly. It happens even if $\pi>F$.

However, when the face value of the debt F is greater than the value lost in bankruptcy C, *"gambling for redemption"* is no longer optimal. The dashed red lines in Figure 2.2 give the payoffs of fair Bernoulli gambles

$$\begin{tikzcd}[row sep=tiny] & \bar{\pi} \\ \mathbf{p} = \arrow[ur, "\pi/\bar{\pi}"] \arrow[dr, "1-\pi/\bar{\pi}"'] & \\ & 0 \end{tikzcd}$$

As the payoff $\bar{\pi}$ increases, the probability of winning declines but the owners benefit more because not paying F is more important to them than not receiving C. The maximum to owners is achieved in the limit as $\bar{\pi}$ approaches infinity. In the limit, gambling will allow the firm obtain $\bar{\pi}$ with probability $\frac{\pi}{\bar{\pi}}$, and 0 with probability $1-\frac{\pi}{\bar{\pi}}$ which in the firm value is $\lim_{\bar{\pi}\uparrow\infty}\left(\frac{\pi}{\bar{\pi}}(\bar{\pi}-F+C)\right)=\pi$. In the limit, the bond value is $\lim_{\bar{\pi}\uparrow\infty}\left(\frac{\pi}{\bar{\pi}}F+(1-\frac{\pi}{\bar{\pi}})(1-c)L\right)=(1-c)L$, i.e., the bondholders almost always only receive part of the liquidation value. Interestingly, gambling for ripoff is optimal in this example even if the cash flow is enough to repay the debt. If the cash flow does not cover the face value of the debt, *gambling for redemption* would increase the total value of bond and equity because the continuation value would be preserved as often as possible, but the owners would rather choose a larger gamble to transfer value from the bondholders. Even worse, with cash surplus the firm would have survived and preserved continuation value for sure if gambling were not allowed, but now the continuation value is lost almost surely.

2.2.3 Example 2: can gamble a lot (superpriority)

Positive available liquidation value to gamble will change the shape of gambling if $C>F>C-L$, as illustrated in Figure 2.3. Superpriority makes the liquidation value available for gambling, allowing firm owners to gamble down to $-L$ instead of 0. In Figure 2.3, the continuation value C is greater than the face value of the debt F, so that absent superpriority the firm will gamble its cash flow for redemption of the face value and obtain $\frac{\pi}{F}C$, depicted by the dashed red line. By allowing gambling with superpriority claims, firms can gamble down to $-L$, and superpriority gambling yields greater benefits when the firm *"gambles for ripoff,"* as shown by the solid red line in the

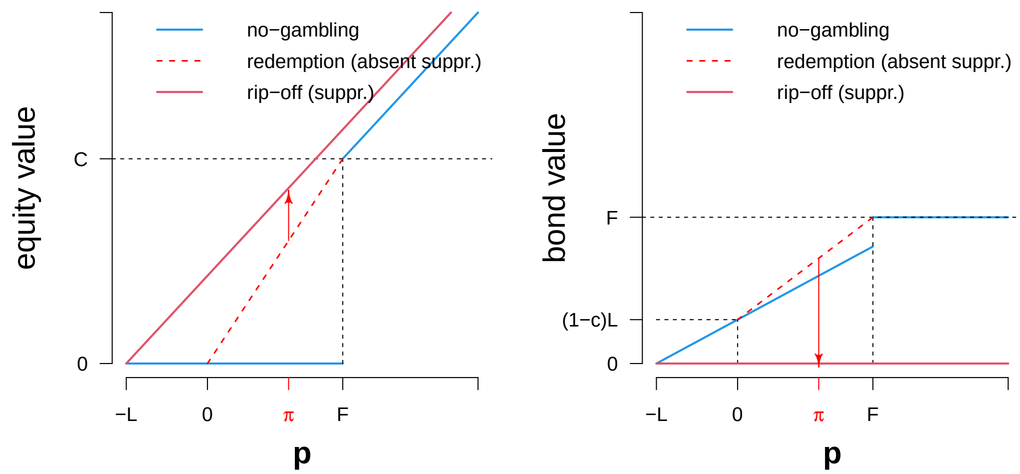


Figure 2.3: ***(Can gamble a lot)*** when $C > F > C - L$, firms "gambles for redemption" absent superpriority (dashed red line), but will "gamble for ripoff" with superpriority (solid red line). The red arrow represents the increase in equity value.

figure. It is straightforward in the graph that the determinant of gambling for redemption or ripoff falls to the comparison of $L+F$ and C, or F and $C-L$, where $C-L$ is the value lost in bankruptcy.

These graphic observations are formally stated by the following propositions:

PROPOSITION 8. *when $F < C - \sigma L$ (the face value of the debt is less than the value lost in bankruptcy), it is optimal to* ***gamble for redemption****. Under this parameter restriction, gambling strictly increases the value of both bond and equity when $\pi < F$, and leaves both unchanged when $\pi \geq F$. Specifically,*

(1) If $\pi \geq F$, $\mathbf{p}^*(\tilde{x}) \equiv \pi$ is one solution. Under linear utility, adding noise such that $\mathbf{p}^*(\tilde{x}) \geq F$ with probability one is also optimal. Otherwise,

(2) If $\pi < F$, the optimal gamble is

$$\mathbf{p}^*(\tilde{x}) = \begin{cases} F, & \text{for } 0 < x \le \frac{\pi+\sigma L}{F+\sigma L} \\ -\sigma L, & \text{for } \frac{\pi+\sigma L}{F+\sigma L} < x < 1 \end{cases} \tag{2.4}$$

(3) The payoffs are

$$\text{equity value} = \begin{cases} \pi - F + C, & \text{for } \pi \ge F \\ \frac{\pi+\sigma L}{F+\sigma L} C, & \text{for } \pi < F \end{cases} \qquad \text{bond value} = \begin{cases} F, & \text{for } \pi \ge F \\ \frac{\pi+\sigma L}{F+\sigma L} F, & \text{for } \pi < F \end{cases}$$

$$\text{bond+equity} = \begin{cases} \pi + C, & \text{for } \pi \ge F \\ \frac{\pi+\sigma L}{F+\sigma L}(C+F), & \text{for } \pi < F \end{cases}$$

Proof. See Appendix B. □

The gambling outcome is a function of a continuum of states and does not have to be binary. However, any optimal gambling can have a binary equivalence because otherwise we can always find an improvement.

In Proposition 8(1), if we believe that gambling is costly, or if we are not using risk neutral probabilities (the owners are risk averse), then $\mathbf{p}^*(\tilde{x}) \equiv \pi$ (no gambling) should be the unique solution.

PROPOSITION 9. *when $F > C - \sigma L$, it is optimal for the firm to* ***gamble for ripoff.*** *Under this parameter restriction, gambling purely transfers value from bondholders to firm owners when $\pi < F$, and also destroys continuation value when $\pi \ge F$. Specifically,* *(1) The optimal gambling is*

$$\mathbf{p}^*(\tilde{x}) = \begin{cases} \bar{\pi}, & \text{for } 0 < x \leq \frac{\pi+\sigma L}{\bar{\pi}+\sigma L} \\ -\sigma L, & \text{for } \frac{\pi+\sigma L}{\bar{\pi}+\sigma L} < x < 1 \end{cases} \tag{2.5}$$

(2) The payoff of the firm is $\frac{\pi+\sigma L}{\bar{\pi}+\sigma L}(\bar{\pi}-F+C)$*, which increases to* $\pi+\sigma L$ *as* $\bar{\pi}\to\infty$*. The value of the debt is* $\frac{\pi+\sigma L}{\bar{\pi}+\sigma L}F$*, and declines to* 0 *as* $\bar{\pi}\to\infty$*. For any* $\pi > 0$*, the total value of bond and equity is always* $\pi+\sigma L$ *when* $\bar{\pi}\to\infty$*.*

Proof. See Appendix B. □

There is also a knife-edge case when $F = C - \sigma L$. Then any fair gamble with outcomes distributed long the 45-degree linear segment would yield the same expected value. That is to say, *gambling for redemption* and *gambling for ripoff* give the same outcome for the owners, and anything in between the two polar cases is also optimal, so that there are uncountable many solutions. Though these optimal gambles generate different values for the bondholders (for example, we still have *gambling for redemption* makes the bondholders better-off and *gambling for redemption* worse-off), we don't want to go into the details of what the equilibrium (equilibria) is (are) because it is reasonable to believe that $F = C - \sigma L$ almost never happen in application.

These results suggest that having maturing debt larger than the firm value lost in bankruptcy is bad because the firm would be more likely to evade liability by taking on high risk gambling. Even if the firm has enough liquidity, it does not necessarily save the firm from going bankrupt. On the contrary, the worst situation happens when the the cash is enough to cover the debt: if not for gambling, the continuation value would not be lost. Superpriority worsen the situation by making bankruptcy more appealing to the firm owners and results in gambling for ripoff.

In the trade-offs between *gambling for redemption* and *gambling for ripoff*, superpriority also plays an important role. It transfers owners the liquidation value which should be grabbed by bondholders, making *gambling for ripoff* more appealing to the owners. With more “ripoff” cases, continuation value can be more easily destroyed.

2.3 The Dynamic Model with Endogenous Debt and Continuation Value

Thus far, we have examined the conditions for *"gambling for redemption"* and *"gambling for ripoff"*, depending on the net loss for the owners in bankruptcy. In the stripped-down framework, gambling for redemption and ripoff are determined by exogenous values. This ex post analysis also shows owners' risk taking when the superpriority law is a surprise. However, it does not tell to what extent gambling for redemption and ripoff are to occur in equilibrium, if anticipated. Is gambling for redemption always efficient enough so that it always tends to increase the firm value, or does the possibility of gambling for ripoff destroy more value? What is the impact of superpriority law on the firm's ability of fundraising? In this section we study the general multi-period model and welfare of gambling with superpriority.

Figure 2.4 provides a representative picture of the result of the dynamic model. If the firm can gamble away assets due to superpriority, the bond value will reduce. This suggests that the new laws make it harder to borrow at first place and lower the market value of equity.

We consider a setting in which both the firm and the potential bondholders understand the game and know the possible payoffs of gambling. In each period t, the firm enters with capital K_t and a maturing debt F_t. The capital pays a cash flow $vK_t > 0$ and is subject to an i.i.d. shock $\tilde{\delta} > 0$ with $\mathrm{E}[\tilde{\delta}] = 1$, so capital is $\widehat{K}_t = \tilde{\delta} K_t$ after the shock.[46] This assumption reflects the fluctuation of firm's asset value under economic uncertainty: in good times, the asset value increases and so does the firm's going concern value; when the economy goes south, the negative shock of asset value may

[46] Any depreciation has been included in $\tilde{\delta}$. We assume i.i.d. distribution for simplicity. It may be interesting to model correlated shocks or temporary shocks to better mirror the reality, but a case is i.i.d. In our model, cash flow in this period is not subject to the capital shock immediately, but will adjust in the later period. Of course, we can also assume that the shock affects the cash flow, or the cash flow has a shock correlated with the capital shock. These changes do not alter the basic results.

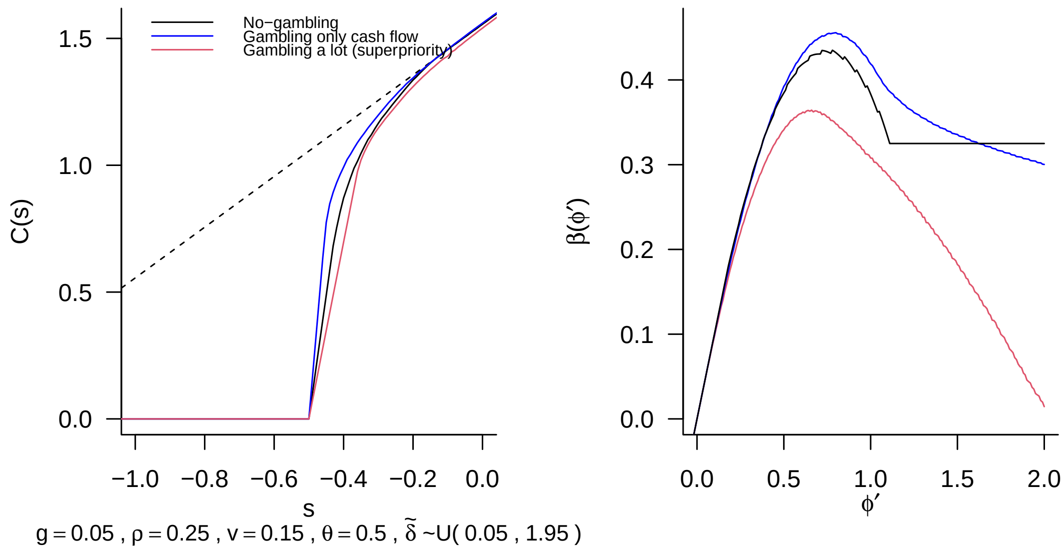


Figure 2.4: ***Left: Continuation value/Equity value*** $C(s)$ ***as a function of cash surplus*** s***. Right: Bond value*** $\beta(\phi')$ ***as a function of face value*** ϕ'***.*** (Everything is measured by per unit of capital.) The second figure shows that gambling with superpriority reduces the maximum borrowing (red curve) compared to gambling absent superpriority (blue curve), and decreases equity value as depicted in the first figure.

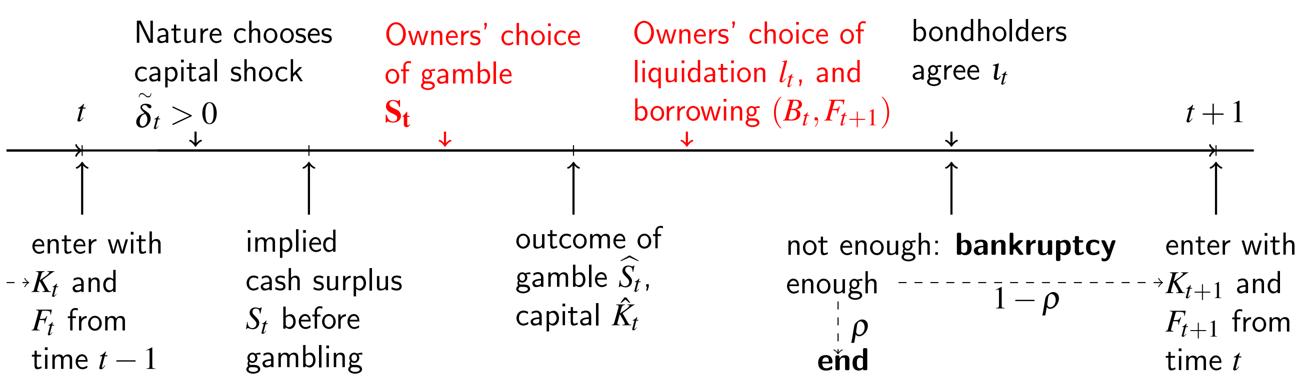
lower the firm's going concern value down below the face value of the maturing debt. The shock can also be industry or firm-specific. Good news such as a change of corporate tax legislation or regulation that favors the industry may increase firm value, but a financial market crash, a war or a pandemic that hampers the business for some firms but thrives other firms can impose different shocks to firms. In either case the firm can gamble, liquidate, borrow and invest.

Under the original bankruptcy law, the firm can purchase fair claims to gamble in a frictionless competitive market using only the cash flow, but can deploy cash flow plus liquidation value if there is superpriority. The liquidation value is θ per unit of capital. With a fraction $l_t \in [0,1]$ liquidated, the firm acquires $\theta l_t \widehat{K}_t$ in cash and remains productive capital $(1-l_t)\widehat{K}_t$. The firm's new borrowing has a face value F_{t+1} and amount B_t priced as the expected value determined in equilibrium, where $B_t < 0$ is interpreted as risk-free lending or savings. If there is a cash surplus after clearing all the debt, the firm may also increase the capital at a growth rate capped by g per period, and we require $(1+g)(1-\rho) < 1$ to avoid Ponzi scheme.[47] If the firm's shortfall is cover by asset liquidation and new borrowing, the firm is saved.

The firm owners care about the expected net payoff at firm's termination without discount, i.e., the shadow risk-free interest rate is zero. The life of the firm can be determined in one of the two ways: (1) if the state of nature is unfavorable, the owners can abandon the firm at any time and deserts all assets, or (2) the game ends with an exogenous probability ρ (the hazard ratio), in which case the payoff to the owners is $K_{T+1} - B_T$, where K_{T+1} is the capital at the end of the firm's life. The timeline is shown below:

2.3.1 The timeline

We formally describe the information sets and the trajectory of events in Appendix C for reference.



The implied cash surplus $S_t \equiv vK_t - F_t$ is the net cash before gambling. Choice of gambling $\mathbf{S_t}$ randomizes the cash surplus S_t and is fair. Similar to the single period model, superpriority allows

[47] It is a reasonable assumption because acquisitions are usually time consuming, and firms usually have limited capacity to expand within a period of time. We also need this assumption to rule out infinite borrowing.

gambling also the liquidation value $\theta\widehat{K}_t$, so that

$$\mathrm{E}\big[\mathbf{S_t}\big] = S_t, \text{ and}$$
$$\mathbf{S_t} \geq -\sigma\theta\widehat{K}_t - F_t. \tag{*}$$

The outcome of gambling $\widehat{S}_t$ and capital after the shock $\hat{K}_t$ are the state variables. The case absent gambling can be seen as a special case in which $\widehat{S}_t = \mathbf{S_t} = S_t$. Gambling in this model has short duration. We think that it is optimal for the owners to use short-maturity derivative since they have to acquire current information about the various positions at the time the derivative matures.[48] After the realization of the gamble, the firm chooses a fraction of liquidation $l_t \in [0,1]$ and a debt contract (B_t, F_{t+1}) given $\widehat{S}_t$ and $\hat{K}_t$, and the bondholders decide a probability $\iota_t \in [0,1]$ of granting the debt. We can assume $\iota_t \equiv 1$ in equilibrium if the bondholders always accept a fairly priced debt.[49] If liquidation and borrowing cannot cover the shortfall, the firm files for bankruptcy and all the assets are liquidated to repay the debt. If enough, the firm continues with probability $1-\rho$, the new capital K_{t+1} is expressed as the result of liquidation, borrowing, debt repayment and new investment:

$$K_{t+1} = ((1-l_t)\widehat{K}_t + l_t\theta\widehat{K}_t + \widehat{S}_t + B_t)(l_t\theta\widehat{K}_t + \widehat{S}_t + B_t \geq 0).$$

In bankruptcy (when $l_t\theta\widehat{K}_t + \widehat{S}_t + B_t < 0$), bondholders obtains $(1-c)(\widehat{S}_t + F_t + \theta\widehat{K}_t)$, a fraction of the remaining assets value in the firm, and a fraction c is lost in bankruptcy.

The firm's problem in a form of Bellman equation is stated below.

[48] Firms' motivations for holding derivatives (either to gamble or to hedge) might be distinguished by duration of derivatives. Long-dated derivatives are probably for hedging, while short-dated derivatives are more likely for gambling. We thank Harold Zhang for raising this discussion.

[49] If the firm does not borrow, ι_t is out of the path. In equilibrium, if the firm needs borrowing to stay alive, it must be true that the loan is fairly priced and is granted. Otherwise the firm defaults, any debt contract is equivalent to offering $(0,0)$ and is accepted by the bondholders. In either case, assuming $\iota_t = 1$ does not change value.

2.3.2 Bellman equation

Given capital $\widehat{K}_0$ and cash surplus after gambling $\widehat{S}_0$, the firm chooses adapted liquidation fraction $\{l_t\}_{t=0}^{T} \in [0,1]$, debt contracts $\{(B_t, F_{t+1})\}_{t=0}^{T} \geq 0$, and gambling $\{\mathbf{S_{t+1}}(\widetilde{x})\}_{t=0}^{T}$, to maximize expected net capital at T, $\mathrm{E}_0\left[K_{T+1} - B_T\right]$. The value function at time t satisfies[50]

$$C_t(\widehat{K}_t, \widehat{S}_t) = \left\{ \max_{l_t, B_t, F_{t+1}, \mathbf{S_{t+1}}(\widetilde{x})} \mathrm{E}_t\left[\rho(K_{t+1} - B_t) + (1-\rho) C_{t+1}(\widehat{K}_{t+1}, \mathbf{S_{t+1}}(\widetilde{x}))\right]\right\}^+,$$

subject to the constraints of fair gambling

$$\mathrm{E}\big[\mathbf{S_{t+1}}(\widetilde{x})\big] = S_{t+1}, \quad \mathbf{S_{t+1}}(\widetilde{x}) \geq -\sigma\theta\widehat{K}_{t+1} - F_{t+1},$$

and the bondholders' willingness to lend[51]

$$\mathrm{E}_t\left[\Big(l_{t+1}\theta\widehat{K}_{t+1} + \mathbf{S_{t+1}}\big(\widetilde{x}\big) + B_{t+1} < 0\Big)\Big((1-c)\Big(\mathbf{S_{t+1}}(\widetilde{x}) + \theta\widehat{K}_{t+1} - cF_{t+1}\Big)\Big)^- + F_{t+1}\right] \geq B_t, \quad (2.6)$$

[50] We assume the hazard ratio ρ is constant and independent of other variables. Then the firm's value function can be written as the following:

$$\begin{aligned}
C_0(\widehat{K}_0, \widehat{S}_0) &= \max_{\{l_t, B_t, F_{t+1}, \mathbf{S_{t+1}}(\widetilde{x})\}_{t=0}^{T}} \mathrm{E}_0\left[K_{T+1} - B_T\right] \\
&= \max_{\{l_t, B_t, F_{t+1}, \mathbf{S_{t+1}}(\widetilde{x})\}_{t=0}^{T}} \mathrm{E}_0\left[\rho \sum_{t=0}^{\infty} (1-\rho)^t (K_{t+1} - B_t)\right] \\
C_t(\widehat{K}_t, \widehat{S}_t) &= \max_{\{l_t, B_t, F_{t+1}, \mathbf{S_{t+1}}(\widetilde{x})\}} \mathrm{E}_t\left[\rho(K_{t+1} - B_t) + (1-\rho) C_{t+1}(\widehat{K}_{t+1}, \mathbf{S_{t+1}}(\widetilde{x}))\right]
\end{aligned}$$

If the value is negative, the owners can simply run away and the value is bounded below by 0. We write $C_t(\widehat{K}_t, \widehat{S}_t)$ with $\{\cdot\}^+$ indicating running away as a plausible choice.

[51] Bond value given gambling outcome $\widehat{S}_{t+1}$ is

$$\begin{aligned}
&\Big(l_t\theta\widehat{K}_{t+1} + \widehat{S}_{t+1} + B_{t+1} < 0\Big)\Big[\Big((1-c)(\widehat{S}_{t+1} + F_{t+1} + \theta\widehat{K}_{t+1})\Big) \wedge F_{t+1}\Big] + \Big(l_t\theta\widehat{K}_{t+1} + \widehat{S}_{t+1} + B_{t+1} \geq 0\Big) F_{t+1} \\
=&\Big(l_t\theta\widehat{K}_{t+1} + \widehat{S}_{t+1} + B_{t+1} < 0\Big)\Big((1-c)(\widehat{S}_{t+1} + F_{t+1} + \theta\widehat{K}_{t+1}) - F_{t+1}\Big)^- + F_{t+1}
\end{aligned}$$

The bond price is composed of the face value and the value lost in the case of bankruptcy, which in equilibrium happens when $l_{t+1}\theta\widehat{K}_{t+1} + S_{t+1} + B_{t+1} < 0$.

and the capital growth rate not exceeding g

$$K_{t+1} \leq \widehat{K}_t(1+g), \tag{2.7}$$

where

$$\begin{aligned}
&K_{t+1} = ((1-l_t)\widehat{K}_t + l_t\theta\widehat{K}_t + \widehat{S}_t + B_t)(l_t\theta\widehat{K}_t + \widehat{S}_t + B_t \geq 0), \\
&\widehat{K}_{t+1} = \widetilde{\delta}_{t+1}K_{t+1}, \text{ and} \\
&S_{t+1} = vK_{t+1} - F_{t+1}.
\end{aligned} \tag{2.8}$$

Again, the firm's problem without gambling is a special case when $\widehat{S}_t = \mathbf{S_t}(\tilde{x}) = S_t$, for every $t \geq 0$. We next simplify the firm's problem to only one state variable by dividing $\widehat{K}_t$, so everything can be expressed as value per unit of capital.

2.3.3 Normalization

The firm's problem is homogeneous in $\widehat{K}_t$, so the problem and the solution can be expressed in terms of per unit of capital by dividing $\widehat{K}_t$.

Given $l_t\theta\widehat{K}_t + \widehat{S}_t + B_t \geq 0$, we define

$$\begin{gathered}
s \equiv \frac{\widehat{S}_t}{\widehat{K}_t},\ b \equiv \frac{B_t}{\widehat{K}_t};\quad s' \equiv \frac{\widehat{S}_{t+1}}{\widehat{K}_{t+1}},\ b' \equiv \frac{B_{t+1}}{\widehat{K}_{t+1}}; \\
\beta \equiv \frac{B_t}{K_{t+1}},\ \phi' \equiv \frac{F_{t+1}}{K_{t+1}};\quad l \equiv l_t,\ l' \equiv l_{t+1}.
\end{gathered}$$

The "prime" on any variable refers to the variable at $t+1$. For example, s is the cash surplus per unit of capital after gambling in this period, while s' is cash surplus per capital after gambling in the next period. The choice variables are l and (β, ϕ'), representing liquidation fraction and debt per unit of capital. We can compute other variables accordingly.[52]

Now we restate the firm's problem:

Firm's problem: *Given an s (cash surplus per unit of capital after gambling), the owners choose adapted liquidation fraction $l \in [0,1)$, debt contract (β, ϕ') and gambling for the next period $\mathbf{s}(\tilde{x}, \phi', \tilde{\delta}') \in [-\frac{\phi'}{\tilde{\delta}'}, +\infty)$, (where $\tilde{x} \sim_d U(0,1)$ is the underlying randomness,) to maximize expected net capital. The value function has the similar form*

$$C(s) = \left\{ \max_{\mathbf{s}(\tilde{x},\phi',\tilde{\delta}'),l,\beta,\phi'} \left(\rho \cdot (1+s-l(1-\theta)) + (1-\rho) \times \frac{1+s-l(1-\theta)}{1-\beta} \mathrm{E}\big[\tilde{\delta}' \cdot C(\mathbf{s}(\tilde{x},\phi',\tilde{\delta}'))\big] \right) \right\}^{+}, \tag{2.9}$$

subject to the constraints of fair gambling

$$\begin{aligned} &\mathbf{s}(\tilde{x},\phi',\tilde{\delta}') \in [-\sigma\theta - \frac{\phi'}{\tilde{\delta}'}, +\infty), \ \textit{and} \\ &\mathrm{E}[\mathbf{s}(\tilde{x},\phi',\tilde{\delta}') | \tilde{\delta}'] = \frac{v-\phi'}{\tilde{\delta}'}, \end{aligned} \tag{2.10}$$

[52] We have

$$\frac{\widehat{K}_{t+1}}{\widehat{K}_t} = \tilde{\delta}'(1+s+b-l(1-\theta)), \ \frac{K_{t+1}}{\widehat{K}_t} = 1+s+b-l(1-\theta);$$

$$\beta = \begin{cases} 1, \text{ if } 1+s-l(1-\theta)=0, \\ \frac{b}{1+s+b-l(1-\theta)}, \text{ otherwise.} \end{cases} \quad b = \frac{(1+s-l(1-\theta))\beta}{1-\beta}, (\beta \neq 1).$$

and the bondholders' willingness to lend

$$\mathrm{E}\big[\big(\mathbf{s}(\tilde{x},\phi',\tilde{\delta}') < \underline{s}\big)\big((1-c)(\mathbf{s}(\tilde{x},\phi',\tilde{\delta}')+\theta)\tilde{\delta}' - c\phi'\big)^{-}\big] + \phi' \geq \beta, \tag{2.11}$$

and the constraints of borrowing

$$-\frac{s+l\theta}{1-l} \leq \beta \leq \frac{g-s+l(1-\theta)}{1+g} \tag{2.12}$$

In (2.11), $\underline{s}$ is a presumed threshold of cash flow, and below which the firm defaults and declares bankruptcy. In equilibrium, there will be a set $\mathscr{S}$ of values of s such that the firm will borrow enough to avoid bankruptcy. We conjecture that $\mathscr{S}$ is of the form $\mathscr{S} = \{s | s \geq \underline{s}\}$.

Without gambling, $s = \frac{v-\phi}{\delta}$ and $\mathbf{s}(\tilde{x},\phi',\tilde{\delta}') \equiv \frac{v-\phi'}{\tilde{\delta}'}$ is a special case of the above firm's problem. The difference between with and without superpriority is also implicit in the constraints of borrower's willingness to lend. Without superpriority, the bondholders can at least redeem the liquidation value which would be taken by gambling counterparties with superpriority.

2.4 Equilibrium and Graphic Illustration

The firm's problem does not have a closed form solution because of the inter-reliance of gambling and (continuation) value function, but the equilibrium properties and the numerical results provide useful implications for understanding gambling by firm in a dynamic setting. We begin with propositions which are immediate derivations from the model setting.

PROPOSITION 10. *In all three cases, the firm will not liquidate capital and grow capital at the same time. Then, conditional on $l\theta + s + b \geq 0$,*

$$\frac{K_{t+1}}{\widehat{K}_t} = \begin{cases} \left(0 \vee \left(\frac{\theta+s}{\theta-\beta} \wedge \frac{1+s}{1-\beta}\right)\right), & \text{for } \beta \neq \theta \\ 1-l, & \text{if } \beta = \theta \text{ and } s = -\theta \\ \left(0 \vee \frac{1+s}{1-\theta}\right), & \text{if } \beta = \theta \text{ and } s \neq -\theta \end{cases} \tag{2.13}$$

$$l(s,\beta) = \begin{cases} \left(\left(0 \vee \frac{\beta+s}{\beta-\theta}\right) \wedge 1\right), & \text{for } \beta \neq \theta \\ l \in [0,1], & \text{for } \beta = \theta \text{ and } s = -\theta \\ 0, & \text{for } \beta = \theta \text{ and } s \neq -\theta \end{cases} \tag{2.14}$$

Proof. The firm will not liquidate and grow capital at the same time, otherwise the firm bears loss in liquidation, which is avoidable if the firm keeps the capital.

If the firm liquidates, $l\theta + s + b = 0 \Rightarrow l = -\frac{s+b}{\theta}$. Then

$$\begin{aligned} \frac{K_{t+1}}{\widehat{K}_t} &= 1 + s + b - l(1-\theta) = 1 - l \\ &= 1 + \frac{s+b}{\theta} = \begin{cases} 1-l, & \text{if } \beta = \theta \text{ and } s = -\theta \\ \text{(cannot have liquidation)}, & \text{if } \beta = \theta \text{ and } s \neq -\theta \\ \frac{\theta+s}{\theta-\beta}, & \text{otherwise.} \end{cases} \end{aligned}$$

If the firm grows capital, $\frac{K_{t+1}}{\widehat{K}_t} = \frac{1+s}{1-\beta}$. □

This proposition claims that ϕ' and l are functions of β and s in equilibrium, and the firm's choice variables can be reduced to only β. This alternative is useful to simplify the problem when computing the numerical solutions.

PROPOSITION 11. *In all three cases, given the growth rate g, cash flow v per unit of capital, and ending of the firm at a rate* ρ*, the firm's value per unit of capital satisfies* $C(s) \le 1 + s + \frac{(1-\rho)(1+g)}{\rho - g + \rho g} v$*, and* $\lim_{s\to\infty} C(s) = 1 + s + \frac{(1-\rho)(1+g)}{\rho - g + \rho g} v$*.*

Proof. Since firm's growth is bounded by g in each period, the value is capped by growing at maximum in each period perpetually, equivalently $\frac{(1-\rho)(1+g)}{\rho-g+\rho g} v$. When the firm's cash surplus s is approaching infinity, the firm achieves (or tends to achieve) the maximum perpetual growth, and an increment of s raises firm's value at a one-for-one rate. □

2.4.1 Bond pricing

For further analysis we assume $\tilde{\delta}'$ follows a uniform distribution in $(\bar{\delta}, \underline{\delta})$, without loss of generality.

The firm maximizing the form owners' wealth should always offer a fair bond price in equilibrium that equals to the face value subtracts the loss from bankruptcy. Without gambling, it is explicitly a function of face value ϕ':

$$\beta(\phi') = \phi' + \mathrm{E}\big[\big(\frac{v - \phi'}{\tilde{\delta}'} < \underline{s}\big)\big((1-c)(v + \theta\tilde{\delta}') - \phi'\big)^{-}\big] \tag{2.15}$$

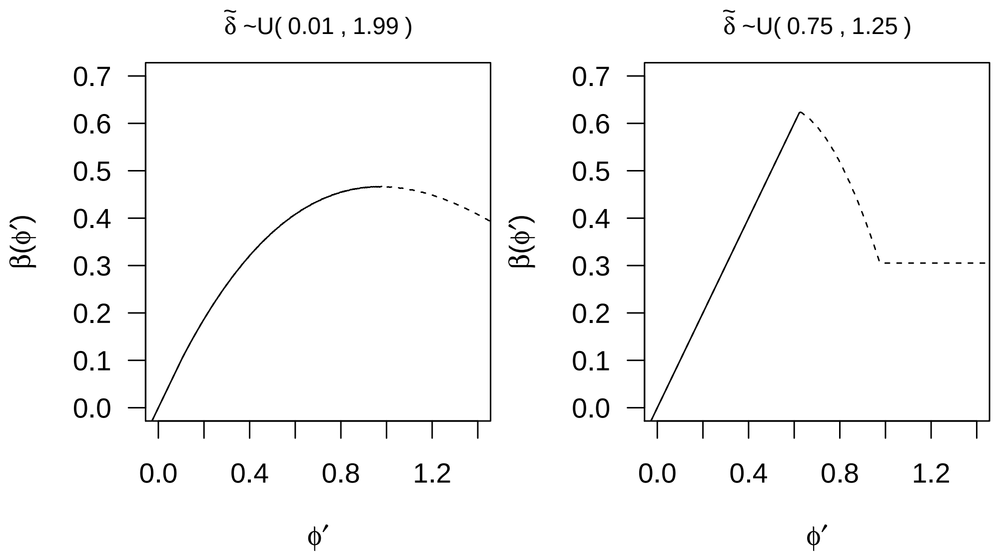


Figure 2.5: ***Bond pricing without gambling*** ($\theta = 0.5$, $c = 0.5$, $v = 0.1$, $\theta = 0.5$, $g = 0.05$, $\rho = 0.25$, $\underline{s} = -0.7$) bond value as a function of face value. Feasible bond pricing (solid curves) represents the rational choice set of the bond contracts that satisfy the pricing function (2.15).

Figure 2.5 shows two possibilities: risky borrowing and risk free borrowing (only). The curves represent the balance between borrowing and risk of bankruptcy. When the face value (compared to capital level) is small, borrowing can be less risky and the bondholders obtain full repayment more often and is demonstrated by the linear (or close to linear) part of the curve. As the face value becomes greater, the risk of default becomes bigger and bond value has smaller increment and can be decreasing when the face value is high enough. The black solid upward curves are efficient borrowing frontiers which represent the set of the bond contracts that the firm would choose in equilibrium. The firm can never write a contract beyond the maximum borrowing and the upper bound is an endogenous borrowing constraint. Comparing the two graphs, when capital shocks are relatively volatile (Figure 2.5 left graph), firms may choose risky debt; when capital is "stable," borrowing is always safe.[53]

[53] Safe borrowing is probably not an interesting case, therefore we mainly focus on the situations in which capital volatility is large.

g	0.05
ρ	0.25
c	0.5
$\tilde{\delta}$	$U(0.05, 1.95)$
v	$\{0.05, 0.1, 0.2\}$
θ	$\{0, 0.5, 0.7\}$

Table 2.1: Parameter values for the numerical exercise.

Bond pricing with gambling requires explicit functional expression of gambling and the continuation value. Figure 2.4 at the beginning of Section 3 provides an example. We can observe some sensible features: when face value is small, the upward linear segment of each curve implies a risk-free bond absent bankruptcy risk; as face value is getting bigger, the growing possibility of bankruptcy generates higher costs that offset the promise of higher repayment. As can be seen, there is a maximum amount that the firm can borrow, and with greater promise of repayment, bankruptcy costs are also greater. The discrepancy between gambling with and without superpriority becomes larger when face value is getting bigger. This is likely due to the higher occurrence of "gambling for ripoff" with superpriority that depletes value.

2.4.2 Equity value

Table 2.1 shows the parameter values we use for the numerical exercise in this section:[54] We fix g, ρ and $\tilde{\delta}$ and vary v and θ. The result is shown in Figure 2.6.

We can observe the effects of pure gambling as well as superpriority in the figures with different profitability (v) and liquidation value of assets (θ). The comparison of the black curves and the blue curves shows the pure gambling effect without superpriority: gambling tends to be more

[54] The hazard ratio ρ is set large because we want the curves to converge quicker. The capital shock is also set with a large range because we are more interested in the case with risky borrowing.

damaging with greater liquidation value per unit of capital. The first row depicts cases in which the firm's assets have no liquidation value and hence superpriority does not impact gambling. In this case, gambling is actually value enhancing because gambling for redemption dominates. The bondholders are also more willing to lend and the borrowing constraints are relaxed. All the value of gambling is reflected in the equity value (since bond is fairly priced). The benefit of gambling is enlarged as the cash flow is greater - the idea is that higher cash flow increases the chance of winning a fair gamble, further alleviating borrowing and improves equity value. For greater liquidation values, the black and blue curves are getting closer, indicating that the advantage of gambling tends to be smaller. Gambling plays a smaller roll in affecting equity value probably because borrowing is more determined by asset value instead of gaining from gambling. However, firm value is largely impaired when gambling assets is plausible with superpirority. Compared to gambling without superpriority, two effects may contribute to the dissipation: first, the assets are diluted because the ability of asset redeployment by the firm owners; second, "gambling for ripoff" would be more likely to prevail.

The effect of superpriority can be shown by comparing the blue solid curves and red dashed curves. Both curves coincide in the first row, because the owners' problems look exactly the same with and without superpriority when $\theta = 0$. When $\theta > 0$, equity value with superpriority tends to be smaller than that without superpriority. Of course, when the cash surplus s is very negative (for example in some graphs $s < -0.5$), a firm has no value with and without superpriority. With larger cash surplus (for example in some graphs $s > 0$), the blue curves and red curves tend to converge since the risky borrowing and gambling would have a smaller proportion. This is also shown by the convergence to the black (no gambling) curves as well as the dashed black lines which represents the maximum firm values.

2.4.3 Optimal gambling

The optimal gambling can be found following the same procedure as in the single-period model, that is, by "concavifying" the value function $C(s)$ for each contingency. Figure 2.7 shows an example. Notice that s' is the pre-gambling cash surplus in the next period, and it can be used to target where can the firm gamble to. For example, given any s' and $\frac{\phi'}{\delta'}$, the firm can use s' to gamble down to $-\frac{\phi'}{\delta'}$ without gambling assets (or to $-\frac{\phi'}{\delta'} - \theta$ with gambling assets) and up to a tangent point of $C(s)$ and a linear line going through the point $(-\frac{\phi'}{\delta'}, 0)$ (or $(-\frac{\phi'}{\delta'} - \theta, 0)$ with gambling assets). By using the optimal gambling and the value function, we can compute the pre-gambling value functions of equity value and bond value, denoted as $\widehat{C}(s', \frac{\phi'}{\delta'})$ and $\widehat{\phi}(s', \frac{\phi'}{\delta'})$, respectively.

We first define

$$s_0 \equiv -1 - \frac{(1-\rho)(1+g)}{\rho - g + \rho g} v, \quad s_1 \equiv \underline{s} - \frac{C(\underline{s})}{C'(\underline{s})},$$

where s_0 is the interception of the upper bound of value function (see Prop. 11) and the x axis, and s_1 is the smallest s on the x axis through which the tangent point on the value function $C(s)$ is the "kink." (see Figure 2.7.) Also define

$$i(-\frac{\phi'}{\delta'}) \equiv -\frac{\phi'}{\delta'} - \sigma\theta$$

and

$$I(-\frac{\phi'}{\delta'}) \equiv \begin{cases} +\infty, & \text{if } i(-\frac{\phi'}{\delta'}) \leq s_0 \\ \arg\max_s \frac{C(s)}{s - i(-\phi'/\delta')}, & \text{if } s_0 < i(-\frac{\phi'}{\delta'}) \end{cases}$$

where $i(-\frac{\phi'}{\delta'})$ is the minimum value that the firm can gamble. Absent superpriority, the firm can only gamble down to $-\frac{\phi'}{\delta'}$, while the firm can gamble down to $-\frac{\phi'}{\delta'} - \theta$ if there is superpriority. $I(-\frac{\phi'}{\delta'})$ is the tangent point of $C(s)$ and the straight line going through $i(-\frac{\phi'}{\delta'})$, if not infinite.

If $i(-\frac{\phi'}{\delta'})$ falls on the left of s_0, we cannot find a tangent point of $i(-\frac{\phi'}{\delta'})$ along the the value function curve $C(s)$, and hence the firm will gamble for ripoff. If $i(-\frac{\phi'}{\delta'})$ falls between s_1 and $\underline{s}$, then the tangent point is exactly the "kink" and the firm will gamble for redemption. Any point in between s_0 and s_1 always has tangent point(s) on $C(s)$, and the tangent point(s) should be the point(s) that the firm gamble towards. Proposition 12 formally states the gambling feature:

PROPOSITION 12. ***(Optimal gambling)*** *Given ϕ', δ',*

1. *the optimal gambling for the firm is*

$$\mathbf{s}^*(\tilde{x}, \phi', \delta') = \begin{cases} I(-\frac{\phi'}{\delta'}) \vee s', & \text{if } 0 < x < w(s', \frac{\phi'}{\delta'}) \\ i(-\frac{\phi'}{\delta'}), & \text{if } w(s', \frac{\phi'}{\delta'}) \leq x < 1 \end{cases}$$

   where $w(s', \frac{\phi'}{\delta'}) \equiv \frac{s' - i(-\phi'/\delta')}{I(-\phi'/\delta') \vee s' - i(-\phi'/\delta')}$ is the probability or weight that the equity value goes up. This result indicates that if $I(-\frac{\phi'}{\delta'}) \leq s'$, then $\mathbf{s}^*(\tilde{x}, \phi', \delta') \equiv s'$ (i.e., the firm does not choose to gamble); otherwise, if $I(-\frac{\phi'}{\delta'}) > s'$, the firm gambles s' up to $I(-\frac{\phi'}{\delta'})$, and down to $i(-\frac{\phi'}{\delta'})$.

2. *for $I(-\frac{\phi'}{\delta'}) > s'$, define*

$$\psi(-\frac{\phi'}{\delta'}) \equiv \sup_{s \geq \underline{s}} \frac{C(s)}{s - i(-\phi'/\delta')}, \quad \gamma(-\frac{\phi'}{\delta'}) \equiv \begin{cases} 0, & \text{if } i(-\frac{\phi'}{\delta'}) \leq s_0, \\ \frac{\phi'}{I(-\phi'/\delta') - i(-\phi'/\delta')}, & \text{if } s_0 < i(-\frac{\phi'}{\delta'}). \end{cases}$$

   Then the value functions of the firm and the bond before gambling are

$$\textit{\textbf{firm:}}\ \widehat{C}(s', \frac{\phi'}{\delta'}) = \begin{cases} \psi(-\frac{\phi'}{\delta'})(s' - i(-\frac{\phi'}{\delta'})), & I(-\frac{\phi'}{\delta'}) > s', \\ C(s'), & I(-\frac{\phi'}{\delta'}) \leq s'. \end{cases} \tag{2.16}$$

$$\textit{\textbf{bond:}}\ \widehat{\phi}(s', \frac{\phi'}{\delta'}) = \begin{cases} \gamma(-\frac{\phi'}{\delta'})(s' - i(-\frac{\phi'}{\delta'})), & I(-\frac{\phi'}{\delta'}) > s', \\ \phi', & I(-\frac{\phi'}{\delta'}) \leq s'. \end{cases} \tag{2.17}$$

or, equivalently,

$$\textit{\textbf{firm:}}\ \widehat{C}(s', \frac{\phi'}{\delta'}) = w(s', \frac{\phi'}{\delta'}) \cdot C\left(I(-\frac{\phi'}{\delta'}) \vee s'\right), \tag{2.18}$$

$$\textit{\textbf{bond:}}\ \widehat{\phi}(s', \frac{\phi'}{\delta'}) = w(s', \frac{\phi'}{\delta'}) \cdot \phi'. \tag{2.19}$$

The optimal gambling has a feature different from many existing literature: the firm does not always choose extreme risks. Rather, gambling has a mixed feature of gambling for redemption and ripoff and is continuous rather than jumping between extremes.

Gambling can easily adjust for different assumptions. For example, if the firm value has a decreased margin when cash flow increases, gambling for extreme ripoff may not happen since we can find a finite tangent point on the flatter segment of the function. Yet the conclusion that gambling is bigger and more damaging with higher level of liability and liquidation value still holds.

2.5 Conclusions

We provided a simple framework to analyze gambling by firms. "Gambling for redemption" is a Pareto improvement and occurs when the firm owners are eager to maintain the firm, whereas "gambling for ripoff" can be socially costly and occurs when continuing a firm is beneficial socially but not to the owners. By making gambling some of the assets possible, superpriority law lowers the value lost to owners in bankruptcy and increases the incentives for the firm owners to gamble for ripoff. In the more realistic intertemporal model with endogenous borrowing and endogenous continuation value, the firm's choices of gambling are intermediate between gambling for redemption and ripoff. We find that superpriority increases the scale of gambling taken by the firm and makes funding more difficult. Our results suggest an interesting empirical question: how do we distinguish "gambling for redemption" and "gambling for ripoff" ex post since they both wipe out the firm's assets in the case of failure? To know the exact gambling, we can instead look at their bets in place, compare the risk of their assets and the amount of matured debt in place.

One possible implication of superpriority law will be the adoption of financing that reduces the scale of superpriority gambling. One possibility is the adoption by bond issuers of more defensive measures that protect against superpriority claims. For example, it may be more common to protect bonds to specific perfected collateral instead of passive covenants claiming the preclusion of asset sales and security transfers. It may also incentivize the firms to issue short-term bonds which have less exposure to a stay in bankruptcy, or even use repos which are also protected against bankruptcy. The substitution away from traditional financing to repo financing can cause an asset grab race which undermines the purposes of bankruptcy law to facilitate an orderly liquidation (or reorganization) and to give breathing space for the firm owners to resolve financial difficulties.

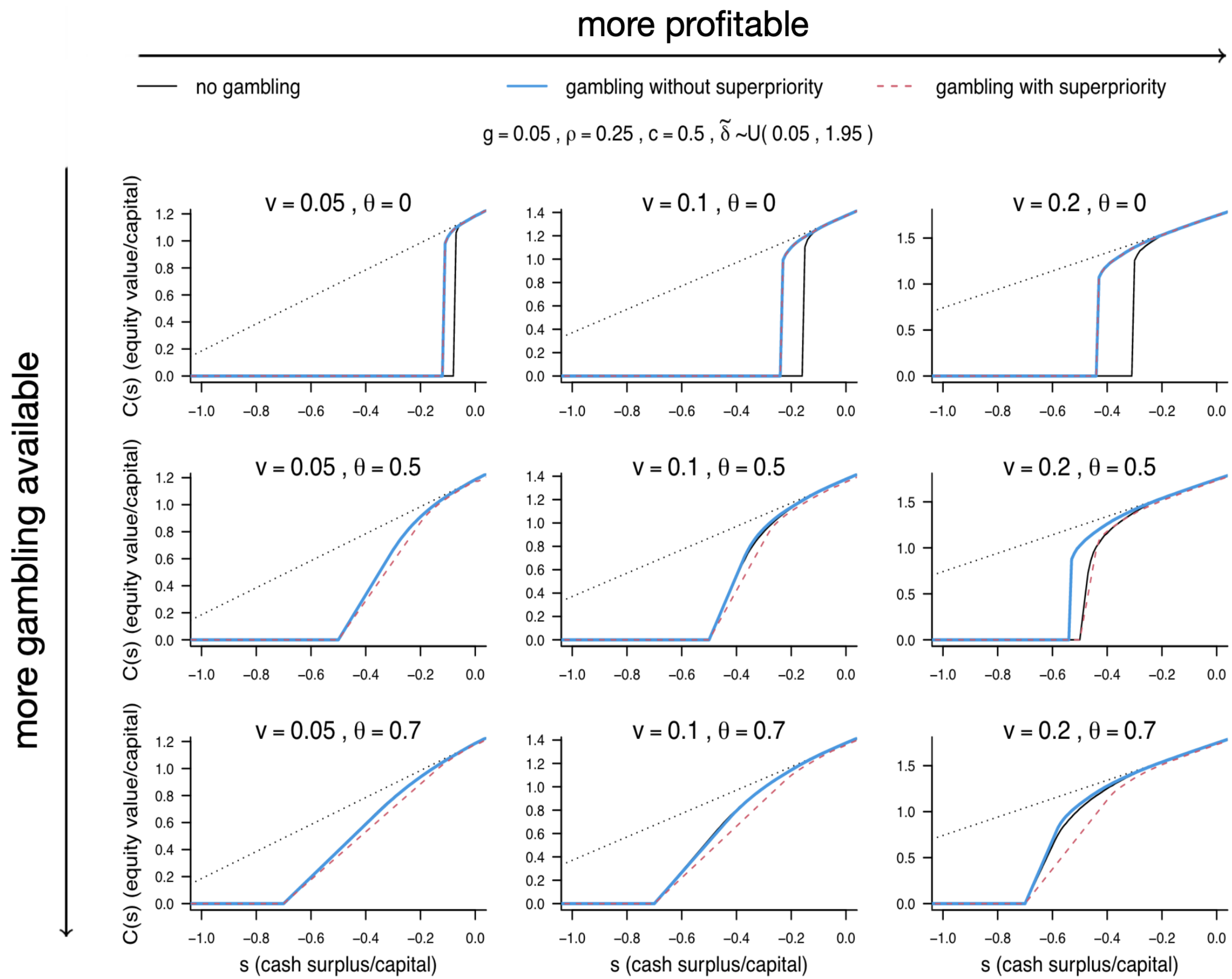


Figure 2.6: ***(Equity value/capital as a function of cash surplus/capital)***
v : cash flow/capital; θ : liquidation value available/capital. If $\theta = 0$, superpriority is irrelevant, and increasing θ implies increasing damage from superpriority, especially when v is large.

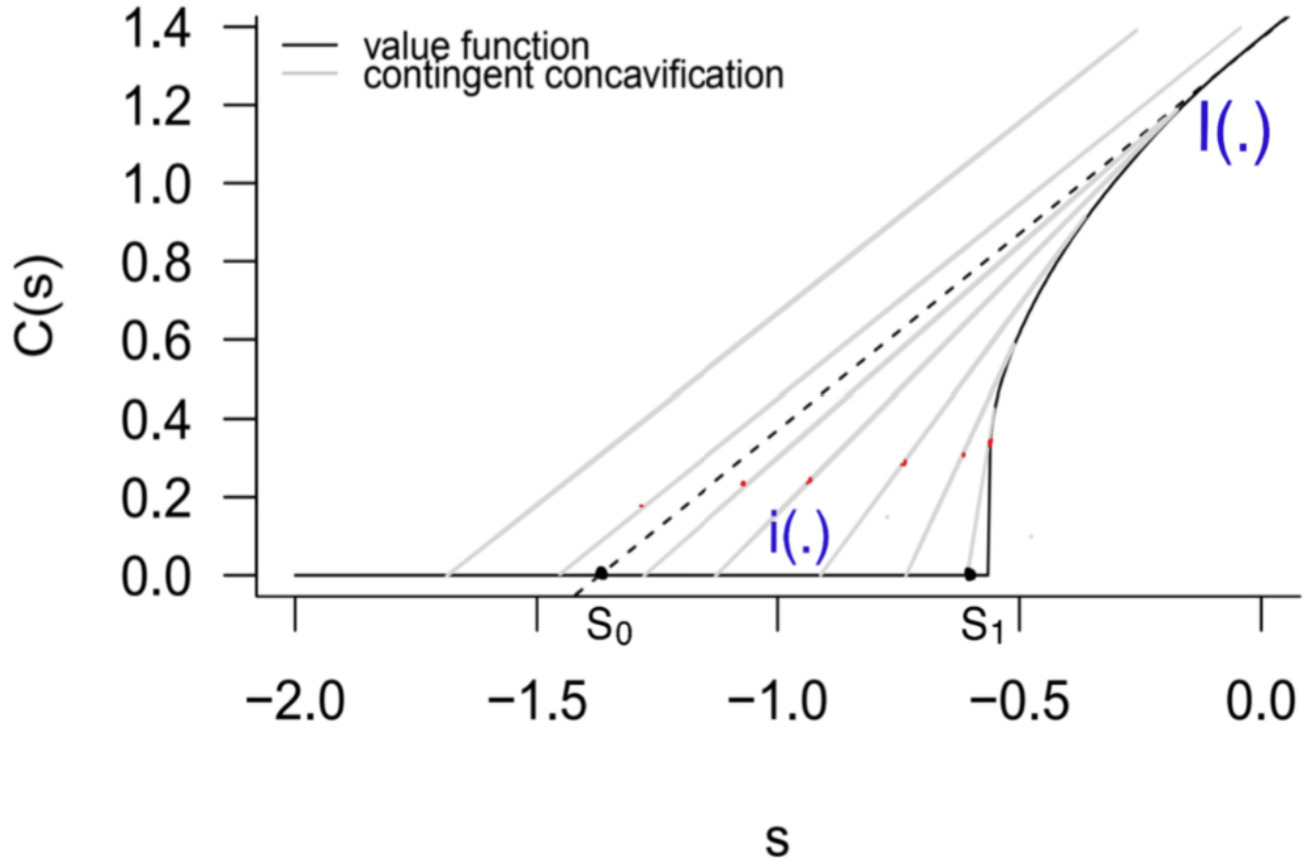


Figure 2.7: ***An example of gambling*** gambling has a mixed feature of "gambling for redemption" and "gambling for ripoff" and is continuous rather than jumping between extremes.

Appendix A

A.1

The firm's problem is to maximize

$$(1-s)Q\Big(m(Q)-d\Big)^{+}+sQm(Q)+R_fQ[W/Q-\big(1+C(s)\big)]$$

The first order conditions are

$$\begin{aligned}
&\text{if } m(Q)^{+} \leq d \\
&\qquad (s) \quad m(Q) = R_f C'(s) \\
&\qquad (Q) \quad m(Q) + m'(Q)Q = R_f(1+C(s))/s \\
&\text{if } m(Q)^{-} \geq d \\
&\qquad (s) \quad d = R_f C'(s) \\
&\qquad (Q) \quad m(Q) + m'(Q)Q - R_f(1+C(s)) - (1-s)d = 0
\end{aligned}$$

When $m(Q) < d$,

$$m(Q_l^*) = \frac{R_f(1+C(s_l^*))/s_l^*}{1-1/e_p(Q_l^*)} \tag{A.1}$$

is derived from the first order condition above.

When $e_p(Q_l^*) > \big[1 - \frac{R_f\left(1+C(s_l^*)\right)}{s_l^* m(Q_s^*)}\big]^{-1}$, we have $Q_l^* < Q^*$.

A.2

The firm's problem is to maximize

$$(1-s)Q\Big(m(Q)+R_f[W/Q-\big(1+C(s)\big)]-d\Big)^+ + sQ\Big(m(Q)+R_f[W/Q-\big(1+C(s)\big)]\Big)$$

The first order conditions are

$$\begin{aligned}
&\text{if } m(Q)+R_f(W/Q-1-C(s)) < d\\
&\qquad (s) \quad m(Q)+R_f(W/Q-1-C(s)) - sR_fC'(s) = 0\\
&\qquad (Q) \quad m(Q)+m'(Q)Q-R_f(1+C(s)) = 0\\
&\text{if } m(Q)+R_f(W/Q-1-C(s)) \geq d\\
&\qquad (s) \quad d-R_fC'(s) = 0\\
&\qquad (Q) \quad m(Q)+m'(Q)Q-R_f(1+C(s))-(1-s)d = 0
\end{aligned}$$

Since $m(Q_j^*)+R_f(W/Q_j^*-1-C(s_j^*)) < d$, $C'(s_j^*)s_j^* < C'(s_s^*)$ indicating that $s_j^* \lesseqgtr s_s^*$. Then $m(Q_j^*)+m'(Q_j^*)Q_j^* < m(Q_s^*)$ and Q_j^* can be either greater or less than Q_s^*.

For the second statement, if $Q_j^* < Q_l^*$, then $m(Q_j^*)+R_f(W/Q_j^*-1-C(s_j^*)) > m(Q_l^*)$, indicating that $C'(s_j^*) > C'(s_l^*)$, thus $s_j^* > s_l^*$.

For (3), d does not enter the first order conditions when $m(Q_j^*) + R_f(W/Q_j^* - 1 - C(s_j^*)) < d$. It affects the threshold of being judgment-proof. However, It is uncertain how W affects the threshold because s and Q would also change.

Specifically, when $W \uparrow$, $s \downarrow$ indicates that $Q \uparrow$, otherwise the FOC for s does not hold equal. This can also be confirmed by the FOC for Q following assumption C. If $s \uparrow$ instead, the FOC for Q suggests that $Q \downarrow$.

A.3

We can do the same calculation for limited liability: when $m(Q_l^*) < d$,

$$
\begin{aligned}
(s_i) &\quad m(Q_l^*) - R_f C'(s_l^*) \\
(Q_i) &\quad m(Q_l^*) + m'(Q_l^*)Q_l^*/N - R_f(1 + C(s_l^*))/s_l^*. \\
\frac{dQ}{dN} &= \frac{I/N}{N + 1 + \frac{m''(Q)Q}{m'(Q)} + \frac{m'(Q)Q}{sR_f C''(s)}} \\
\frac{ds}{dN} &= \frac{m'(Q)}{R_f C''(s)} \frac{dQ}{dN}
\end{aligned}
$$

When N is sufficiently large, $\frac{dQ}{dN} > 0$ and $\frac{ds}{dN} < 0$. This is similar for unlimited liability with judgment-proof investors.

A.4

With limited liability, firm's problem is to maximize

$$(1-\tau)(Q_i p-(1-s_i)\Lambda(Q_i,p,w_i))+R_f[W/N-(1+C(s_i))Q_i]$$

The first order conditions are

$$\begin{aligned}
&\text{if } m(Q)<d\\
&\qquad (s_i) \quad (1-\tau)m(Q)Q-R_fC'(s)Q=0\\
&\qquad (Q_i) \quad (1-\tau)s(m(Q)+m'(Q)Q/N)-R_f(1+C(s))=0\\
&\text{if } m(Q)+R_f(W/Q-1-C(s))\geq d\\
&\qquad (s) \quad d-R_fC'(s)=0\\
&\qquad (Q) \quad m(Q)+m'(Q)Q-R_f(1+C(s))-(1-s)d=0
\end{aligned}$$

Appendix B

Proof of optimal gambling

Notice that this proof works for the superpriority case when liquidation value L is used for gambling, but to prove the no-superpriority case, we can simply assume $L = 0$. Given constants $F, C, \pi \in \mathbb{R}_{++}$, and $L \in \mathbb{R}_{+}$, the question becomes

$$\max_{\mathbf{p}(x)} \int_0^1 \left\{ (\mathbf{p}(x) \geq F)(\mathbf{p}(x) - F + C) \right\} dx$$
$$\text{s.t.} \int_0^1 \mathbf{p}(x) dx = 1, \text{ and } -L \leq \mathbf{p}(x) \leq \bar{\boldsymbol{\pi}}$$

To get the necessary conditions for the solution, we first concavify the function $(\mathbf{p}(\tilde{x}) \geq F)(\mathbf{p}(\tilde{x}) - F + C)$ to make it continuous.

Gambling for redemption: When $C > F + L$, define the concavified function

$$G(\mathbf{p}(x)) \equiv (\mathbf{p}(x) < F) \frac{C}{F + L}(\mathbf{p}(x) + L) + (\mathbf{p}(x) \geq F)(\mathbf{p}(x) - F + C)$$

Assume that $q(x) = \int_0^x \mathbf{p}(t)dt$, then $q'(x) = \mathbf{p}(x)$. For short, we use $\mathbf{p}$ to represent $\mathbf{p}(x)$. Rewrite the concavified problem

$$\begin{aligned}
&\max_{\mathbf{p}} \int_0^1 G(\mathbf{p})dx \\
&\text{s.t. } q(0) = 0, q(1) = \pi, \\
&\quad q' = \mathbf{p}, \text{ and } -L \leq \mathbf{p} \leq \bar{\pi}.
\end{aligned}$$

$G(\mathbf{p})$ is continuous together with its partial derivative $\frac{\partial G(\mathbf{p})}{\partial q} \equiv 0$, and is piece-wise smooth in $\mathbf{p}$. We can assume that $\mathbf{p}(\tilde{x})$ is piece-wise smooth in x, and $q(x)$ is continuous and piece-wise smooth. Thus, we have the necessary conditions for the solution.

Since $G(\mathbf{p}(x)) \geq (\mathbf{p}(\tilde{x}) \geq F)(\mathbf{p}(\tilde{x}) - F + C)$, then any solution of the concavified problem falling into the domain of the original function $(\mathbf{p}(\tilde{x}) \geq F)(\mathbf{p}(\tilde{x}) - F + C)$ is also the solution of the original problem.

Then $\mathbf{p}(\tilde{x})$ is to be chosen at each x to maximize the Hamiltonian

$$\mathscr{H}(\lambda, \mathbf{p}) = G(\mathbf{p}) + \lambda \mathbf{p}, \text{ s.t. } -L \leq \mathbf{p} \leq \bar{\pi}.$$

The Lagrangian, with the multipliers w_1 and w_2, of the new problem, is

$$\mathscr{L}(\lambda, w_1, w_2, \mathbf{p}) = G(\mathbf{p}) + \lambda \mathbf{p} + w_1(\mathbf{p} + L) + w_2(\bar{\pi} - \mathbf{p})$$

When $C > F$, necessary conditions for $\mathbf{p}$ to be maximizing are

$$0 = \mathscr{L}_p(\lambda, w_1, w_2, \mathbf{p}) = \begin{cases} [\frac{C}{F+L} + \lambda + w_1 - w_2, +\infty), & \text{for } \mathbf{p} = -L \\ \frac{C}{F+L} + \lambda + w_1 - w_2, & \text{for } -L < \mathbf{p} < F \\ [1 + \lambda + w_1 - w_2, \frac{C}{F+L} + \lambda + w_1 - w_2], & \text{for } \mathbf{p} = F \\ 1 + \lambda + w_1 - w_2, & \text{for } F < \mathbf{p} < \bar{\pi} \\ [0, 1 + \lambda + w_1 - w_2], & \text{for } \mathbf{p} = \bar{\pi} \end{cases}$$

$$w_1 \geq 0, \quad w_1(\mathbf{p} + L) = 0, \text{ and}$$

$$w_2 \geq 0, \quad w_2(\bar{\pi} - \mathbf{p}) = 0.$$

Further,

$$\lambda' = -\partial \mathscr{H} / \partial q = -\partial (G(\mathbf{p}) + \lambda \mathbf{p}) / \partial q = 0,$$

so that $\lambda(x)$ is a constant, for all x. Then,

$$\lambda(x) = \begin{cases} (-\infty, -w_1 - \frac{C}{F+L}], & \text{for } \mathbf{p} = -L, (w_2 = 0) \\ -\frac{C}{F+L}, & \text{for } -L < \mathbf{p} < F, (w_1 = w_2 = 0) \\ [-\frac{C}{F+L}, -1], & \text{for } \mathbf{p} = F, (w_1 = w_2 = 0) \\ -1, & \text{for } F < \mathbf{p} < \bar{\pi}, (w_1 = w_2 = 0) \\ [-1 + w_2, +\infty), & \text{for } \mathbf{p} = \bar{\pi}, (w_1 = 0) \end{cases}$$

Since $-w_1 - \frac{C}{F+L} \leq -\frac{C}{F+L} < -1 \leq -1 + w_2$, we cannot have $\mathbf{p} < F$ and $\mathbf{p} > F$ at the same time as a solution of $\mathbf{p}$. Then the solution should be either $\mathbf{p}(x) \leq F$, $\forall x$, or $\mathbf{p}(x) \geq F$, $\forall x$.

- If $\pi < F$, we cannot have $\mathbf{p}(x) \geq F$, $\forall x$, otherwise $\int_0^1 \mathbf{p}(x)dx \geq F > \pi$, which does not satisfy the constraint. Therefore, $\mathbf{p}(x) \leq F$, $\forall x$. If we further constrain the value $\mathbf{p}(x) \in \{0\} \cup [F, \bar{\pi}]$, we have $\lambda = -\frac{C}{F+L}$ and the unique solution (we assume decreasing $\mathbf{p}(x)$ with respect to x)

$$\mathbf{p}^*(\tilde{x}) = \begin{cases} F, \text{ if } x \leq \frac{\pi+L}{F+L} \\ -L, \text{ otherwise} \end{cases}$$

  This applies to the original problem when $L = 0$. Proposition 8(2) is proved.

- If $\pi > F$, we cannot have $\mathbf{p}(x) \leq F$ for the same reason. Then any randomization of $\mathbf{p}(\tilde{x})$ above or equal to F would be optimal. Of course, this include $\mathbf{p}(\tilde{x}) \equiv \pi$, which is not to gamble at all. If $\pi = F$, then the only possible solution is $\mathbf{p}(\tilde{x}) = F$, for all x. This proves Proposition 8(1).

Gambling for rip-off: when $C < F + L$, the concavified function is $G(\mathbf{p}) = \frac{\bar{\pi}-F+C}{\bar{\pi}+L}(\mathbf{p}+L)$. The Lagrangian is

$$\mathscr{L}(\lambda, w_1, w_2, \mathbf{p}) = \frac{\bar{\pi} - F + C}{\bar{\pi} + L}(\mathbf{p}+L) + \lambda\mathbf{p} + w_1(\mathbf{p}+L) + w_2(\bar{\pi} - \mathbf{p}).$$

The necessary conditions are

$$\lambda' = 0,$$

$$0 = \mathscr{L}_p(\lambda, w_1, w_2, \mathbf{p}) = \begin{cases} [1 - \frac{L+F-C}{\bar{\pi}+L} + \lambda + w_1 - w_2, +\infty), & \text{for } \mathbf{p} = -L \\ 1 - \frac{L+F-C}{\bar{\pi}+L} + \lambda + w_1 - w_2, & \text{for } -L < \mathbf{p} < \bar{\pi} \\ [0, 1 - \frac{L+F-C}{\bar{\pi}+L} + \lambda + w_1 - w_2], & \text{for } \mathbf{p} = \bar{\pi} \end{cases}$$

$$w_1 \geq 0, \quad w_1(\mathbf{p} + L) = 0,$$

$$w_2 \geq 0, \quad w_2(\bar{\pi} - \mathbf{p}) = 0.$$

Similarly,

$$\lambda(x) = \begin{cases} (-\infty, -w_1 - 1 + \frac{L+F-C}{\bar{\pi}+L}], & \text{for } \mathbf{p} = -L, (w_2 = 0) \\ -1 + \frac{L+F-C}{\bar{\pi}+L}, & \text{for } -L < \mathbf{p} < \bar{\pi}, (w_1 = w_2 = 0) \\ [-1 + \frac{L+F-C}{\bar{\pi}+L} + w_2, +\infty), & \text{for } \mathbf{p} = \bar{\pi}, (w_1 = 0) \end{cases}$$

When $w_1 = w_2 = 0$, any randomization along the line is the optimal solution. Specifically, if we constrain the solution on $\mathbf{p}(\tilde{x}) \in \{-L, \bar{\pi}\}$, which is the set falling into the domain of the original problem. We have $\lambda = -1 + \frac{L+F-C}{\bar{\pi}+L}$ and the unique solution

$$\mathbf{p}^*(\tilde{x}) = \begin{cases} \bar{\pi}, \text{ if } x \leq \frac{\pi+L}{\bar{\pi}+L} \\ -L, \text{ otherwise} \end{cases}$$

This proves Proposition 9.

Appendix C

The trajectory and history

This section provides a rigorous description of the information sets and timing of events for reference.

A trajectory represented as $\mathscr{P}$ is a possible path of events known to both the firm and the prospective lender:

$$\mathscr{P} = (F_0, K_0, \delta_0, S_0, \mathbf{S_0}, \widehat{S}_0, \widehat{K}_0, l_0, B_0, F_1, K_1, \delta_1, S_1, \mathbf{S_1}, \widehat{S}_1, \widehat{K}_1, l_1, B_1, F_2, K_2, \delta_2, S_2, \mathbf{S_2}, \widehat{S}_2, \widehat{K}_2, l_2, B_2, \ldots).$$

where $S_t \equiv vK_t - F_t$ is the implied cash surplus defined as the excessive cash after repaying the maturing debt in full (or more properly called the "shortfall" if smaller than zero). Gambling choice $\mathbf{S_t}$ randomizes the cash surplus S_t, and the realization is $\widehat{S}_t$. Therefore, the case without gambling is a special case in which $\mathbf{S_t} = \widehat{S}_t = S_t$.

The history at time t contains the first $7+9t$ elements in the relevant trajectory, denoted

$$\mathscr{H}^{7+9t} \equiv \mathscr{P}_{1:(7+9t)} = (F_0, K_0, \delta_0, \ldots, F_t, K_t, \delta_t, S_t, \mathbf{S_t}, \widehat{S}_t, \widehat{K}_t).$$

Along the path, we have

$\widehat{S}_t$: the cash surplus after gambling in period t *(state variable)*

$\widehat{K}_t$: capital after the shock in period t *(state variable)*

$l_t = l^*(\mathscr{H}^{7+9t})$: the fraction of liquidation *(choice variable)*

$B_t = B^*(\mathscr{H}^{7+9t})$: new borrowing *(choice variable)*

$F_{t+1} = F^*(\mathscr{H}^{7+9t})$: face value of the debt *(choice variable)*

$K_{t+1} = ((1-l_t)\widehat{K}_t + l_t\theta\widehat{K}_t + \widehat{S}_t + B_t)(l_t\theta\widehat{K}_t + \widehat{S}_t + B_t \geq 0)$

δ_{t+1} : the realized capital shock at $t+1$

$\widehat{K}_{t+1} = \delta_{t+1}K_{t+1}$: capital after the shock at $t+1$

$\mathbf{S_{t+1}}$: gambling choice subject to $\mathrm{E}\left[\mathbf{S_{t+1}}\right] = S_{t+1}$, and $\mathbf{S_{t+1}} \geq -F_{t+1}$ without superpriority, or $\mathbf{S_{t+1}} \geq -\theta\widehat{K}_{t+1} - F_{t+1}$ with superpriority *(choice variable)*

We intentionally place $\widehat{S}_t$ and $\widehat{K}_t$ at the beginning of the cycle and the choice of gambling in the end, so that the timeline coincides with the firm owners' problem. There are different time points in a period at which we can look at the equity value, but we choose the moment after the realization of gambling. The purpose is threefold: firstly, to have only two state variables in the firm's Bellman equation instead of three before gambling; secondly, to have a comparable value function to our benchmark case; thirdly, to mirror the continuation value in the single-period model to firm owners' value function, upon which the gambling strategy should depend.

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