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Operations Management under Financial Frictions

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Operations Management under Financial Frictions

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Fasheng Xu
Washington University in St. Louis
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Dedicated to my family.

Abstract of the Dissertation

Operations Management under Financial Frictions

by

Fasheng Xu

Doctor of Philosophy in Business Administration

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Professor Panos Kouvelis, Chair

Professor Fuqiang Zhang, Co-Chair

The main purpose of this dissertation is to study the emerging operations issues under financial frictions, in the contexts of supply chain finance and crowdfunding platform; and to identify the implications for individuals and businesses.

In Chapter 1, "A Supply Chain Theory of Factoring and Reverse Factoring", we develop a supply chain theory of factoring (recourse and non-recourse) and reverse factoring showing when these post-shipment financing schemes should be adopted and who really benefits from the adoption. Factoring is a financial arrangement where the supplier sells accounts receivable to the factor against a premium, and receives cash for immediate working capital needs. Reverse factoring takes advantage of the credit rating discrepancy between small supplier and large retailer, and enables supplier's factoring at the retailer's rate. Given the supplier's credit rating and the trade credit term, recourse factoring is preferred when the supplier's cash investment return rate is relatively high; non-recourse factoring is preferred within certain medium range; otherwise, factoring should not be adopted. Both factoring schemes, if adopted, benefit both the supplier and the retailer, and thus the overall supply chain. Further, we find that reverse factoring may not be always preferred by suppliers among other short-term financing options (bank loans, recourse and non-recourse factoring). Retailers should only offer reverse factoring to suppliers with low, but above a threshold, to medium cash investment return rates. The optimally designed reverse factoring program can always increase the retailer's profit,

but it may leave the supplier indifferent to his current financing option when followed by aggressive payment extension. Interestingly, our results suggest that it is often preferable for the retailer to extend reverse factoring to certain suppliers without any request for payment extension, and leverage the supplier's willingness to carry extra inventory that increases the overall supply chain efficiency.

In Chapter 2, "Crowdfunding under Social Learning and Network Externalities", we investigate how the presence of both social learning and network externalities affects the strategic interaction between a crowdfunding firm and forward-looking consumers. In rewards-based crowdfunding, a firm (campaigner) pre-sells a new product and solicits financial contributions from the crowd (consumers) to cover production costs. When a crowdfunding product with uncertain quality is first introduced, consumers may choose to strategically delay their purchase in anticipation of product quality reviews. Our research yields three main insights. First, we find that in the presence of social learning and strong network externalities, an upward-sloping demand curve may arise. This so-called *Veblen effect* occurs due to the interaction between social learning and strong network externalities. Second, we show that network externalities have important implications for the optimal crowdfunding reward choice. In particular, under strong network externalities, the optimal reward will induce all consumers to either adopt the product early or adopt the product late; whereas under weak network externalities, the consumers will possibly adopt the products in different periods. Third, we characterize the optimal reward strategy under financial constraints and quantify its impact on the optimal reward choice and the induced purchase pattern from consumers. These insights provide useful guidance on how firms can exploit the benefits of crowdfunding.

In Chapter 3, "Crowdfunding vs. Bank Financing: Effects of Market Uncertainty and Word-of-Mouth Communication", we investigate a firm's optimal funding choice when launching an innovative product to the market with both market uncertainty and word-of-mouth (WoM) communication. Bank financing is a traditional source of capital for small businesses, whereas crowdfunding has recently emerged as an alternative fund-raising solution to support innovative ideas and entrepreneurial ventures. Conceivably, crowdfunding could potentially replace some of the conventional roles of bank financing, but puzzles linger over when crowdfunding is a better funding choice. We characterize the firm's optimal pricing strategies under the two funding choices (i.e., bank financing and crowdfunding), compare their performances, and investigate the corresponding implications on social welfare. Among other results, we find that the firm's optimal funding choice and pricing strategy depend critically on the market uncertainty, the WoM, and the initial investment requirement. More specifically, the firm would adopt intertemporal pricing under crowdfunding, where the exact format is determined by the WoM and market uncertainty; under bank financing, however, the firm should always charge a fixed price invariant to those parameters. Moreover, market uncertainty has a non-monotonic effect on the optimal funding choice: Bank financing is preferred only when the market uncertainty is within an intermediate range. The impact of initial investment requirement on the choice of funding schemes shares qualitatively a similar trend. Finally, contrary to the conventional wisdom, we find that more active social interactions in crowdfunding, although beneficial to the firm, may hurt consumers and even reduce social welfare.

1. A Supply Chain Theory of Factoring and Reverse Factoring

1.1 Introduction

[1] Trade credit, as a common industry practice, is the credit extended by a firm's suppliers when the supplier sells the firm goods or services on account. Instead of paying for the goods and services with cash immediately, the firm pays its suppliers with a time lag which creates the equivalent of a loan (i.e., trade credit) from the suppliers to the firm. Since most downstream firms demand 30 to 90 days to pay after goods are delivered, those small and medium-sized enterprise (SME) suppliers will find it difficult to finance their production cycle or miss other attractive investment opportunities. A potential approach to free up cash flow locked in their accounts receivable positions is factoring. Factoring is a type of short-term financing in which suppliers sell their accounts receivable at a discount and receive immediate cash from the factor (typically a bank or a financial service firm). In recent years, the use of factoring has increased dramatically on a global scale as an effective, and relatively low-risk and low-cost, way of expanding access to working capital finance. Since 1996, the global factoring industry has been growing at a relatively fast pace, increasing on average nearly 9% per annum, and the total volume amounts to EUR 2,376 billion in 2016 [2].

Factoring can be done on a recourse basis, especially when it is difficult to assess the default risk of the underlying accounts, meaning that the factor has a claim (i.e., recourse) against its client (the borrower) for any account payment deficiency. The alternative is non-recourse factoring, where the factor not only assumes title to the accounts, but also assumes most of the default risk because the factor does not have recourse against the supplier if the accounts default. Hence, non-recourse factoring removes the receivables from the supplier's books (off-balance sheet), and thus formally separates the accounts receivable from borrowers' other assets in the event of bankruptcy. That said, non-recourse

[1] This paper is based on the author's early work [1] jointly with Panos Kouvelis.

factoring is classified as a sale of receivables, whereas recourse factoring is classified as borrowing, since the eventual receivables recoup is a potential liability to the supplier. The difference between recourse and non-recourse factoring raise a natural question for the cash-constrained supplier: which is a better choice?

It is worth noting that financial constraints of SME suppliers are strengthened by common cash-management practices of retailers (buyers) in the supply chain. Retailers try to collect their account receivables as quick as possible while postponing payments to suppliers. This is not only true for companies experiencing worrisome cash flow problems, like small start-up companies with limited access to credit, but also for big, robust companies who are imposing new schedules on suppliers as a business strategy. For example, during the heart of the recession in January 2009, beverage giant Anheuser-Busch InBev extended its payment terms from 30 days to 120 days with less than a month's notice, giving suppliers no time to prepare. Around the same time, global beverage giant Diageo went from 30 days to 60 days payment, with no offsetting compensation for its suppliers [3].

However, the above cash-to-cash cycle optimization of large buyers by extending payment terms on SME suppliers leads to a suboptimal solution from a supply chain perspective. The highest inventory financing burden is borne by the weakest shoulders, thus increasing overall finance costs in the value chain with serious implications for its smooth performance (deterioration of relationships, price increases, lower service/quality, continuity issues, bankruptcy, etc). [4] and [5] document that extending trade credit with unfavorable payment terms is especially costly for financially constrained firms, who must curtail investments and take on liquidity risk in order to do so. [5] empirically demonstrate that slow payment terms for matched pairs of large customers and small suppliers result in significant underinvestment by the suppliers.

Concerned about the rising financing costs for SME suppliers, retailers are stepping up to help alleviate suppliers' cash flow stress. In many cases retailers employ the new financing scheme of *reverse factoring* (also often referred as *supply chain finance program*[2]). Under this new scheme, the retailer essentially acts as an underwriter of the suppliers' accounts receivable risk, and works with a third-party bank to ensure the lend-

[2] Although the terms SCF and reverse factoring are often used interchangeably, especially by practitioners, the definition of SCF provided by Global Supply Chain Finance Forum in 2016 positions it as a general concept that encompasses reverse factoring and many other financial supply chain solutions.

ing of money to their suppliers with a relatively lower interest rate. A great example of such a practice is the one used by *Jingdong*, China's second-largest e-commerce company. Starting in October 2013, Jingdong has cooperated with banks to streamline procedures for its suppliers to get loans. A Jingdong supplier could ask the company to verify its accounts receivable and use this to borrow from a cooperating bank much more quickly and at a lower rate than usual. Such supply chain finance program is particularly attractive to SMEs in emerging markets, as they can borrow based on their buyer's superior credit rating.

Procter & Gamble's decision in April 2013 to extend its payment terms for all suppliers by 30 days was complemented by the firm's reverse factoring program helping suppliers finance their increased working capital requirements. Using a similar approach, Unilever has been able to achieve a $2 billion working capital reduction in a three-year time span [6]. Similarly, Philips uses reverse factoring to obtain preferred-buyer status with its suppliers and reduce the risk of disruption in times of shortage. Reverse factoring programs have also been initiated in response to disruptions in the financial markets. For example, WalMart's "Supplier Alliance Program", was offered to more than a thousand of its apparel suppliers, many of which SMEs, in the aftermath of the 2009 Chapter 11 bankruptcy filing by CIT Group Inc., an established commercial lender [7].

Reverse factoring is often presented as a "win-win approach" for both the supplier and the buyer [6, 8]. The supplier can finance its working capital needs using the creditworthiness and commitments from the retailer, thus resulting in lower financing costs. They also have the option to improve their cash flow through early payment, should they choose to execute it. The retailer is able to use its credit rating to infuse cash efficiencies into the supply chain, and often leverages such programs for extended payments and cash-to-cash cycle optimization. However, in most cases the reality is nuanced, with large buyers aggressively using reverse factoring to extend the payment terms to suppliers, with the benefits of the reverse factoring program questionable under such terms for suppliers [9, 10]. Our study investigates the performance implications of reverse factoring for all players in the supply chain and the chain's overall efficiency.

In this paper, we study a pull-structure supply chain with a cash-constrained SME supplier and a large credit-worthy retailer. The newsvendor-like supplier has a single opportunity to produce and stock inventory to satisfy future uncertain retail demand, with the wholesale price determined by the retailer. Within this setting, we model different financing alternatives, and we investigate the optimal structures for each financing scheme. We also compare those financing schemes both from a supplier and a supply chain perspective. Our main goal is to shed light on how different financing alternatives impact the strategic interactions within the supply chain, and how they should be designed to create value for firms in the supply chain. Our work contributes to the literature in the following three ways.

First, we develop a pull-structure supply chain model in the presence of liquidity constraints and financing frictions to advance our understanding of how short term financing schemes (factoring and reverse factoring) interface with operational decisions in the chain (wholesale price, production quantity, and payment extension). However, extant literature has mostly relied on push-structure supply chains (i.e., financing the newsvendor-like retailers) to model the role of trade credit on supply chain decisions. Our emphasis on understanding retailer (buyer) motivated financing programs necessitates the use of pull-structure supply chain modeling.

Second, we explicitly model the role of factoring schemes within supply chains, and are able to explain the differing implications of factoring structures, recourse vs. non-recourse, in supply chain efficiency and benefits for each firms in the chain. This allows us to provide conditions under which recourse vs. non-recourse factoring are preferred by capital constrained suppliers. In doing so, we explicitly model "seniority rights" of banks in collecting their loans with priority over factored accounts in the case of bankruptcy, a financing friction that is necessary to explain the advantages of non-recourse factoring. From the perspective of the liquidity constrained supplier, the trade-offs in choosing factoring schemes are twofold: the advanced cash amount for accounts receivable (favors recourse factoring with the supplier receiving higher cash advance as he remains liable for the accounts receivable risk) and the incurred financing cost during factoring (favors non-recourse factoring as it avoids the financing friction of bank seniority). We show that non-recourse factoring is a preferred option for suppliers with low (but above a threshold)-to-moderate investment opportunities for the cash received in advance. However, for suppliers with reasonable credit ratings (above a threshold) and rather high investment return rates, the increased liquidity benefits of recourse factoring exceed any financing cost concerns.

Finally, our work offers a clear supply chain argument, which explicitly recognizes the strategic interactions of the buyer and the supplier in the chain, in explaining the extent of reverse factoring adoption versus other short term financing schemes (bank financing, recourse/non-recourse factoring) of inventories (accounts receivable) in the chain. Reverse factoring has often been argued as an effort by the credit-worthy buyers to compensate smaller capital constrained suppliers for an aggressive lengthening of payment terms. Our work shows that reverse factoring is adopted by suppliers that have non-recourse factoring as a preferable short term financing option. However, reverse factoring might not be preferable for suppliers with low investment return rates, or for suppliers of moderate-to-high credit ratings and rather high investment return rates. In the latter case, the liquidity benefit of reverse factoring might be overcome by the increased financing costs of an extended payment term. For this latter case, an optimally designed reverse factoring scheme is less aggressive in extending payment terms for the retailer, if at all, and fully capitalizes on the supplier's willingness to assume higher inventory risk due to the lower financing costs afforded by reverse factoring. Our work is the first to fully model the strategic choice of suppliers among all available short-term financing options in response to a proposed reverse factoring program by the buyer, and as a result points out that reverse factoring may not be always adopted by suppliers. We also show that buyers (retailers) may not want to extend these programs to certain suppliers.

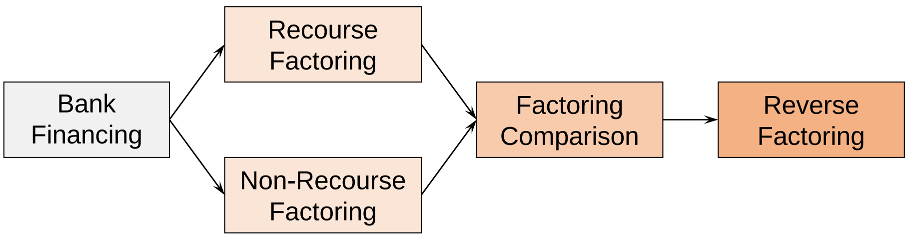


Figure 1.1.: The Structural Outline of the Main Models

The rest of the paper is organized as follows (The structural outline of main models in this paper is given in Figure 1.1). Section 2 reviews the related literature. Section 3 lays out the model framework, the assumptions, the notations, and the benchmark equilibrium solution for the pure bank financing case. In section 4, we investigate the strategic interactions between four parties (retailer, supplier, bank and factor) under recourse and non-recourse factoring, and characterize the Stackelberg equilibrium for these factoring schemes. Then, in section 5 we study the optimal design of the reverse factoring contract accounting for other available short-term financing options. We characterize the conditions under which suppliers will adopt the reverse factoring programs, and retailers will offer such programs. We place emphasis on understanding when successful reverse factoring adoptions are complemented by payment term extensions. We conclude with summary remarks, managerial insights, and model extensions. All proofs are given in the appendix.

1.2 Literature Review

There are three primary streams of research our work is related to: short-term financing in supply chains (in particular, pure bank financing and trade credit), operations management under financial frictions, and the study of factoring and reverse factoring.

Among the first to bring capital constraints and financing issues into operations management, [11] discuss asset-based financing in an inventory context, and investigate the situation that a retailer borrows from a profit-maximizing bank who controls both the loan interest rate and credit limit. In the context of a push supply chain, i.e., selling to the newsvendor, [12] study a Stackelberg game without liquidity constraints, with the supplier offering a wholesale price contract to a retailer facing uncertain demand, and then the retailer placing a single order prior to the selling season. The subsequent "financing the newsvendor" literature builds upon this model by adding liquidity constraints and exploring the effectiveness of various financing schemes to alleviate them. [13] suggest that the strategic supplier prefers to offer cheap trade credit to better price the wholesale price contract and have the retailer order larger order quantities for the same price. Optimally designed trade credit leads to higher order quantities and enhances supply chain efficiency. [14] shows that the inventory decisions of a retailer with multi-items are distorted by debt financing, and the distortion can be mitigated by trade credits with terms contingent on the retailer's order. When risk in a trade relationship originates from the buyer, [15] study how a firm's own riskiness impacts its sourcing diversification strategy. We refer the reader to [16] for a comprehensive discussion of recent contributions and future directions in this area.

Under bankruptcy related frictions, [17] examine how a firm's financial distress and the legal environment of bankruptcy reorganization can alter product market competition and supplier-buyer relationships. [18] present an exhaustive analysis of newsvendor like decisions under liquidity constraints, bankruptcy risks and bankruptcy costs. Further, [19] discuss contract design and supply chain coordination issues under working capital constraints and bankruptcy costs. In some cases, coordination is possible via contracts that appropriately reallocate working capital among firms, but not possible in other cases. [20] study the dynamics of purchase order financing under market frictions of the supplier's credit limit and informational transparency, and show how the financial characteristics of the supplier influence the operational decisions and profits of both firms. Taking a different perspective, we study the financing schemes of factoring and reverse factoring in the pull supply chain setting, and incorporate two types of financial frictions: credit rating discrepancy between supplier and retailer, and the "seniority rights" of banks in collecting their loans with priority over factors in the case of bankruptcy. This latter friction is instrumental in explaining the advantage of non-recourse factoring.

In addition to the work referenced above, our research is related to the emerging literature that deals with various facets of factoring and reverse factoring. Using a case study approach, [21] describe the usual objectives and barriers to the adoption of reverse factoring. Through system dynamics modeling, [22] identify the market factors (e.g., competition, interest rates, receivables volumes, etc.) determining the benefits of reverse factoring, and find that such benefits are highly sensitive to market conditions. On the empirical side, [23] provides an econometric analysis of the benefit of both factoring and reverse factoring as a means of financing SMEs and emphasizes the importance of economic development and growth. Early analytical works on factoring schemes focused on the information asymmetry between the supplier (the seller of accounts receivable) and the bank (or the factor) [7, 24, 25]. [24] examines the optimal factoring contract and finds that the preference of recourse over non-recourse factoring depends on the credit quality of the seller's accounts receivable, the seller's solvency, and the seller's reputation. [26] propose an analytical framework to optimize the payment extension term in reverse factoring, and argue that the financial implications of payment extension need careful assessment in stochastic settings.

[25] shows that from a supplier perspective reverse factoring is beneficial when the spread in deadweight financing costs is high, nominal payment periods are long, the demand volatility is high, and the SME employs an aggressive working capital policy. [7]

analytically study the impact of reverse factoring on SME suppliers in a stochastic multi-period setting, and they find that reverse factoring considerably improves a supplier's operational performance while providing the potential to unlock more than 10 percent of the supplier's working capital. Adopting the Bass diffusion model, [27] find that initial payment terms and procurement volume strongly affect the optimal timing of reverse factoring introduction and optimal payment term extensions. [28] analyze a model with the buyer specifying both wholesale price and order quantity and employing reverse factoring to reduce financing costs for its suppliers. They find empirically that these financing practices generate higher profits for both the supplier and the retailer by inducing higher stocking levels.

Our work explicitly models reverse factoring as one of the short-term financing choices (bank loans, recourse/non-recourse factoring) of the supplier in a strategic interaction game with the buyer. So, optimally designed reverse factoring programs have to balance financing cost advantage of the supplier via use of differential credit ratings (buyer's superior rating over the supplier's) with the penalizing extended payment terms. Using a pull-structure supply chain, we offer a clear supply chain rationale of the advantages of reverse factoring programs when adopted: non-recourse structure reduces importance of "seniority rights" of banks at the collection time of accounts receivable in low demand conditions; suppliers lower financing costs by dealing with factors/banks that use the buyer's superior credit rating in assessing financing terms; and finally potential benefits of extended payment terms for the buyer. In the presence of these trade-offs, our work is the first to provide a comprehensive treatment of a supply chain equilibrium model on the supplier's and retailer's choices on wholesale price, production quantity, and short-term financing scheme, fully accounting for the ensued financing and operational costs.

1.3 Modeling Framework

Building upon the classic *buying-from-the-newsvendor* model, i.e., pull supply chain setting [29], we study a two-firm supply chain where a large downstream retailer determines the wholesale price and then the SME supplier decides how much inventory to produce and stock at the retailer's location, and owns that inventory prior to sales. As mentioned in [29], often encountered situations represented by a pull-structure supply chain are: Vendor Managed Inventory with consignment contract (the supplier decides how much inventory to stock at the retailer and owns that inventory), or drop shipping (the supplier holds the inventory and ships directly to consumers, even though customer orders are placed through the retailer's online channel). Pull supply chain structure makes the supplier fully responsible for the inventory risk, and the uncertainty nature of the supplier's accounts receivable intensifies the interplay between operational and financial decisions in the supply chain.

The SME supplier is capital constrained (i.e., the supplier's initial capital may be insufficient to produce what is needed in support of the retail demand) and in need of short-term financing, which is provided by a third-party bank (or a factor). To fully understand and illustrate the benefits of different short-term financing schemes, we first analyze the benchmark case of pure bank financing. We then model and analyze the equilibrium outcomes under recourse factoring, non-recourse factoring and reverse factoring, and investigate the structural differences, equilibrium solutions and supply chain consequences. For all cases, the bank offers a fairly priced loan for relevant risks. Failure of the supplier's loan repayment leads to bankruptcy. For the rest of the paper, we will refer the retailer as *she*, the supplier as *he*, and the band and factor as *it*.

1.3.1 Notations and Assumptions

The decision variables are the retailer's wholesale price w, the supplier's production quantity q and the bank's/factor's interest rate r. Exogenous parameters are r_f, the risk-free interest rate for unit time period (normalized to 0 without loss of generality); p, the retail price; c, the unit production cost for supplier at time 0; t_1, the supplier's production lead-time and t_2, the retailer's deferral payment period (also referred to as payment term or trade credit term). Let $t_c = t_1 + t_2$ be the total cash conversion cycle (also referred to as cash-to-cash cycle). The timing of events (both operations and finance) is outlined in Figure 1.2. (Note: AR and AP are abbreviations for "accounts receivable" and "accounts payable", respectively.)

To capture the timing effect of the contract terms, we adopt in our study the continuous compounding for all interest related calculation. Because the salvage value of unsold items and goodwill loss for unmet demand do not change the nature of the problem, without loss of generality, they are normalized to zero in our model. The supplier's and retailer's expected profits are denoted by π_i and Π_i, with $i \in \{\mathcal{B}, \mathcal{F}, \mathcal{S}, \mathcal{R}\}$ representing

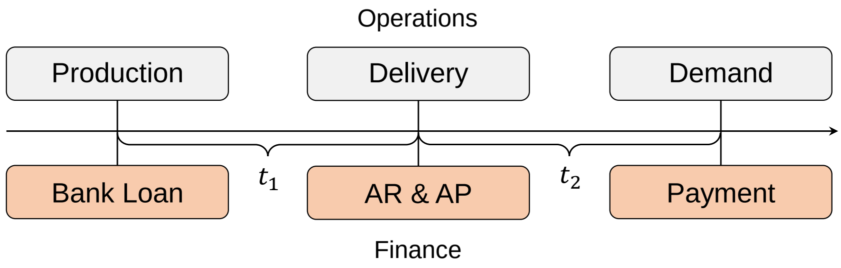


Figure 1.2.: General Timing of Events

pure bank financing, recourse factoring, non-recourse factoring, and reverse factoring, respectively. Hereafter, for a generic decision variable $\mathcal{V}$, we will use $\mathcal{V}_i$, to denote the best response given the Stackelberg leader's (retailer's) decision under financing alternative i, and use $\mathcal{V}_i^*$ to denote the corresponding equilibrium outcomes. Let $\vee$ ($\wedge$) denote the maximizing (minimizing) operator, i.e., $x \vee y = \max(x, y)$ and $x \wedge y = \min(x, y)$. $\mathbb{1}\{\cdot\}$ is used to represent the indicator function and $(x)^+ = \max(x, 0)$. Monotonic relations (increasing, decreasing) are used in the weak sense unless otherwise stated.

We assume, for ease of exposition, that the uncertain product market demand D is realized at a single time t_c, the end-of-the-sales season. Let demand D be a non-negative random variable with p.d.f. $f(\cdot)$, c.d.f. $F(\cdot)$, and complementary c.d.f. $\bar{F}(\cdot) = 1 - F(\cdot)$. We make the following assumptions about demand distribution F throughout the paper: (i) it has a finite mean μ and a continuous p.d.f., with $f(\xi) > 0$ in $[0, \Xi]$ ($\Xi \leq +\infty$); (ii) its failure rate function $z(\xi) = f(\xi)/\bar{F}(\xi)$ is strictly increasing in ξ, i.e., (strictly) IFR. Our other modeling assumptions are summarized in Table 1.1.

1.3.2 Cash Investment Return

The supplier has access to a new investment opportunity at time t_1, with a deterministic *investment output function* $\mathcal{I}_s(x, t) = xe^{\alpha_s t}$ if x amount of cash is invested for a time period of t. If the supplier receives cash x immediately after delivery at time t_1, he can invest it with a return $\mathcal{I}_s(x, t_2)$ at time t_c, where t_2 is retailer's deferral payment period, i.e., open account payment term. We refer to α_s as the supplier's *investment return rate* (per unit time) and t the *investment horizon*. This new investment opportunity captures the supplier's need of unlocking the cash flow trapped in the trade credit obligation (accounts receivable), and it is used a modeling artifice to reflect the working capital needs

Table 1.1: A Summary of Modeling Assumptions

(A1)	The bank, factor, retailer, and supplier are all risk-neutral.
(A2)	All parameters and the pull supply chain structure are common knowledge.
(A3)	The capital market has no taxes and bankruptcy costs. The factor and the bank competitively price risk with a premium based upon the borrower's credit rating.
(A4)	The bank and factor face no bankruptcy risk, but the supplier may claim bankruptcy, depending on whether his cash inflow (sales revenue and investment output) can cover the cash outflow (loan obligations to the bank and factor) at time t_c.
(A5)	The bank is senior to the factor under recourse factoring, i.e., the supplier first repays the bank loan and then repays the factor after the bank loan has been paid in full.
(A6)	Credit ratings are set by an independent rating agency prior to the analyzed supply chain transaction, and the ratings are common knowledge to all three parties.
(A7)	The supplier has zero initial cash position and needs bank financing to start production.

in support of other current or new business opportunities. Similarly, we can define the retailer's cash investment output function $\mathcal{I}_r(x,t) = xe^{\alpha_r t}$. Note here, the linearity of investment output function with respect to x is mainly assumed for convenience of exposition, and our results hold for general concave increasing investment output functions as widely adopted in finance literature [30].

1.3.3 Credit Rating and Interest Rate Premium

Our modeling of credit ratings and implications for supply chain firms follows [31]. A firm's credit rating represents a rating agency's evaluation of a company's overall creditworthiness. The evaluation describes rating agency's opinion of the firm's ability to pay back the debt and the likelihood of default. In our study, credit rating is adopted to reflect default risks for the firm exogenous to the immediate supply chain transactions. We capture such risk via a Bernoulli random default of a firm, with occurrence probability depending on the firm's credit rating. That is, at time t_c, we use two exogenous independent Bernoulli random variables, which are independent of the random demand, to model the supplier's and retailer's potential default due to risk factors captured in credit ratings, but outside their current supply chain transactions. The bank decides the bank loan interest rates, r_r and r_s, charged for the retailer and supplier, respectively, for a unit borrowing period. For $j = r$ (retailer) and s (supplier), let $C_j \geq 0$ be the borrower i's credit rating score. Since we focus on the supply chain with a large credit-worthy retailer and a cash-constrained SME supplier, we assume the retailer credit rating is higher than the supplier's, i.e., $C_r > C_s$. We shall denote by $\rho(C_j)$ the *default probability* of a firm with credit rating C_j, due to exogenous default events. We assume that $\rho(\cdot)$ is a general decreasing function.

Bank loans are competitively priced, but with risk premium reflecting the firm's credit risk. Based on the short-term credit practices described in [32] and similar to the setting of [31], we assume that $\eta(C_j) \geq 0$ is the *interest rate premium* charged by the bank/factor based on the borrower's credit rating and $\eta(\cdot)$ is a general decreasing function, which corresponds to current credit rating practices [**?**, see, e.g.,]]tirole2010theory, babich2012managing. We also assume such premium are the same across all banks due to a transparent credit rating process and a fully competitive banking sector. For notational convenience, we will simply denote $\rho(C_j)$ as ρ_j, and $\eta(C_j)$ as η_j.

Empirical research on small business finance confirms that the financing costs faced by large retailers are significantly lower than what their suppliers can realize. For instance, [33] estimate that marginal equity flotation costs for large firms start at 5.0%, while the corresponding figure for small firms is 10.7%; bankruptcy costs amount respectively to 8.4% or 15.1% of capital. In a novel data set on almost 30,000 trade credit contracts, [34] observe that most suppliers are much smaller and less well rated than their buyers, and are unlikely to have access to cheaper financing.

1.3.4 Pure Bank Financing Benchmark

Now, we present the equilibrium result under pure bank financing when the supplier has no access to other financing alternatives (i.e., cash advance opportunities with factoring and reverse factoring). Let us first consider the supplier's borrowing activity at time $t = 0$. Since both the supplier and the retailer may default with probability ρ_s and

ρ_r due to the exogenous default risk, the supplier may claim bankruptcy in which case the bank receives zero repayment for the short-term loan issued to the supplier. With probability $(1-\rho_s)(1-\rho_r)$, the bank has only demand risks associated with the current supply chain transaction affecting the supplier's ability to repay the loan. Due to the risk premium, the bank expects a return of $e^{[r_f+\eta_s]t_c}$ (i.e., $e^{\eta_s t_c}$ as $r_f = 0$) through the loan transaction with the SME supplier over a time period of t_c. Then, the bank's interest rate $r_{\mathcal{B}}$ for supplier is chosen so that he is indifferent to issuing the loan to the supplier and earning a rate of return $e^{\eta_s t_c}$,

$$cqe^{\eta_s t_c} = (1-\rho_s)(1-\rho_r)\mathbb{E}[\min(wD, cqe^{r_{\mathcal{B}} t_c})]. \tag{1.1}$$

We refer to this equation as the "*competitive credit pricing equation in pure bank financing.*"

Let $b_{\mathcal{B}}(q; w, C_s, C_r)$ be the supplier's *bank financing bankruptcy threshold*, representing the minimal realized demand level under which the supplier is still able to repay the loan obligation. Then, $b_{\mathcal{B}}(q; w, C_s, C_r) = cqe^{r_{\mathcal{B}} t_c}/w$, where the dependence of C_s and C_r is contingent on equation (2.2). It is expositionally convenient to use $b_{\mathcal{B}}(q; w, C_s, C_r)$ instead of $r_{\mathcal{B}}(q; w, C_s, C_r)$, as the fundamental decision variables of the bank. Hereafter, for notational convenience, we also denote $b_{\mathcal{B}}(q; w, C_s, C_r)$ and $r_{\mathcal{B}}(q; w, C_s, C_r)$ as $b_{\mathcal{B}}(q)$ and $r_{\mathcal{B}}(q)$ or simply $b_{\mathcal{B}}$ and $r_{\mathcal{B}}$ for a given w. Let $S(q) = \mathbb{E}[\min(D, q)] = \int_0^q \bar{F}(\xi)d\xi$. Then, equation (2.2) can be rewritten as

$$c_{\mathcal{B}} q = w\mathbb{E}[\min(D, b_{\mathcal{B}})] = wS(b_{\mathcal{B}}), \tag{1.2}$$

where $c_{\mathcal{B}} = \frac{ce^{\eta_s t_c}}{(1-\rho_s)(1-\rho_r)}$ is the supplier's *effective unit production cost* under pure bank financing. From the definition of bankruptcy threshold $b_{\mathcal{B}}$, we know the probability $F(b_{\mathcal{B}})$ captures the the supplier's bankruptcy risk. With the SME supplier the weak link in the chain, we will further adopt $F(b_{\mathcal{B}})$ as a measure for the *supply chain default risk.* Note that the supply chain default risk defined here is separated from firms' exogenous default risk conditioned on credit ratings, and only captures the supply chain inherent risk due to demand uncertainty.

Without exogenous default shock, the supplier's expected revenue at time t_c is $wS(q)$. To calculate the supplier's expected cost, we note that if the realized demand at the retailer side is less than or equal to the bankruptcy threshold $b_{\mathcal{B}}$, the supplier cannot repay the loan obligation and has to declare bankruptcy. In this case, the supplier loses his sales revenue $w\xi$. Hereafter, all sales revenue and cost are calculated with the equivalent time t_c value. However, if the realized demand is larger than $b_{\mathcal{B}}$, the supplier repays the commercial loan, and his total cost is $cqe^{r_{\mathcal{B}}t_c} = wb_{\mathcal{B}}$. In summary, the supplier's expected cost is $\mathbb{E}[\min(wD, cqe^{r_{\mathcal{B}}t_c})] = \mathbb{E}[\min(wD, wb_{\mathcal{B}})] = wS(b_{\mathcal{B}})$. Then, the supplier's expected profit is $\pi_{\mathcal{B}}(q; w) = (1-\rho_s)(1-\rho_r)w[S(q) - S(b_{\mathcal{B}})]$, where $1-\rho_s$ and $1-\rho_r$ reflect the supplier's and the retailer's exogenous default risk, respectively. Since $wS(b_{\mathcal{B}}) = c_{\mathcal{B}}q$ from equation (1.2), the supplier's expected profit can be simplified as

$$\pi_{\mathcal{B}}(q; w) = (1-\rho_s)(1-\rho_r)(wS(q) - c_{\mathcal{B}}q). \tag{1.3}$$

Two points bear mentioning here. First, the supplier's expected profit does not depend on the bankruptcy threshold $b_{\mathcal{B}}$ (equivalently, the bank's interest rate $r_{\mathcal{B}}$), but depends on the credit ratings of both parties in the supply chain. This implies that the supplier's optimal production quantity is not influenced by his financial constraints (without loss of generality, we assume zero initial cash position), but is affected by his financial status (credit ratings). When market frictions (credit rating and interest rate premium) exist, the capital-constrained supplier's operational decisions can not be decoupled from financing decisions. Second, based on the supplier's effective unit production cost $c_{\mathcal{B}}$, the supplier's unit production cost now consists of two parts: the original production cost and the financing cost. This can be directly reflected if we rewrite the effective unit production cost as

$$c_{\mathcal{B}} = \frac{ce^{\eta_s t_c}}{(1-\rho_s)(1-\rho_r)} = c + \underbrace{\frac{e^{\eta_s t_c} - (1-\rho_s)(1-\rho_r)}{(1-\rho_s)(1-\rho_r)}c}_{\text{unit financing cost}}. \tag{1.4}$$

In other words, the effective unit production cost increases as the credit rating of either the supplier or the retailer decreases, and increases in the supplier's cash conversion cycle t_c. Meanwhile, the retailer's profit function is

$$\Pi_{\mathcal{B}}(w) = (1-\rho_r)(p-w)S(q_{\mathcal{B}}(w)), \tag{1.5}$$

where $q_{\mathcal{B}}(w)$ is the supplier's optimal production quantity given a wholesale price w under pure bank financing. The equilibrium outcome is similar to the case of traditional pull supply chain [29], with the adjustment of the supplier's *effective unit production cost* $c_{\mathcal{B}}$. Therefore, the optimal production quantity $q_{\mathcal{B}}$ and wholesale price $w_{\mathcal{B}}$ can be formally summarized in the next proposition. Henceforth, for notational convenience, we denote $j(q) = S(q)/\bar{F}(q)$.

Proposition 1.3.1 *There exists a unique equilibrium* $(w_{\mathcal{B}}^*, q_{\mathcal{B}}^*)$ *under pure bank financing:*

(i) $(w_{\mathcal{B}}^*, q_{\mathcal{B}}^*)$ *can be derived from the equation system:* $p\bar{F}(q) = c_{\mathcal{B}}[1 + z(q)j(q)]$ *and* $w = c_{\mathcal{B}}/\bar{F}(q)$*;*

(ii) In equilibrium, as the credit rating of either party increases or the payment term t_2 *decreases, the wholesale price* $w_{\mathcal{B}}^*$ *decreases, while the production quantity* $q_{\mathcal{B}}^*$ *and both profits increase.*

1.4 Factoring

Factoring is a short-term financing scheme for suppliers with the supplier's accounts receivable serving as the traded asset for immediate cash or serving as a loan collateral. The factor will typically advance less than 100% of the face value of the receivable even though it takes ownership of the entire receivable. The remaining balance is paid to the seller when the receivables are received, less interest and related service fees. For example, most factors offer sellers financing up to 70% of the value of an account receivable and pay the remaining 30% – less interest and service fees – when payment is received from the retailer. The difference between the advance amount and the invoice amount serves as a reserve held by the factor. If and when the invoice is paid in full, the reserve amount is remitted by the factor to its client.

Factoring can be done either on a "non-recourse" or "recourse" basis. If the receivable is sold without recourse, then the supplier is not liable for any delinquency in the payment of the underlying receivable (retailer's payment). If the receivable is sold with recourse, then the supplier may be accountable for all of the uncollected amount. As the value of the supplier's accounts receivable is uncertain due to the demand risk, the factor limits the cash advance amount to the supplier to better control it. In this section, we will provide quantitative examination of the structures of optimal factoring contracts under both recourse and non-recourse factorings, and investigate the corresponding equilibrium outcomes and their implications for supply chain performance.

1.4.1 Timing of Events under Factoring

The chronology of events, notations, and decision structure are summarized as follows, where we use case (a) and (b) to represent recourse factoring and non-recourse factoring, respectively.

(1) At time 0, the retailer proposes the wholesale price w. Then, the supplier responds with the production quantity q, and requests the bank loan cq, with the interest rate based on the supplier's own credit rating C_s and the maturity date at time t_c.

(2) At time t_1, the supplier delivers the products to the retailer and records accounts receivable from the retailer, which are payable at time t_c with face value wq. Then, the supplier either

Case (a) : factors the accounts receivable and receives immediate cash $\mathcal{L}_{\mathcal{F}}$; or

Case (b) : directly sells the accounts receivable to the factor and obtains immediate cash $\mathcal{L}_{\mathcal{S}}$.

For both cases, the received cash is used to invest for a period of t_2 with a deterministic investment output $\mathcal{I}_s(\mathcal{L}_i, t_2) = \mathcal{L}_i e^{\alpha_s t_2}$;

(3) At time t_c, the retailer pays the supplier $w\xi$ for sold items. Then,

Case (a) : the supplier repays the bank first and then the factor, and may claim bankruptcy if the realized sales plus the investment output is less than the total loan obligation;

Case (b) : the factor directly collects payment from the retailer, with the supplier having no liabilities if the collected repayment can't cover the supplier's cash advance amount.

We would like to note that in the case of recourse factoring, the supplier is simultaneously liable at time t_c to both the bank, for the original loan for production purposes, and the factor, for the advanced cash for the factored receivables. As is common in the finance literature [32], the bank has "seniority" during the collection process, or in other words, collects ahead of the factor. Therefore, it is to be expected that the factor charges a higher interest rate then the bank, under the assumption that all have access to the same supplier's credit rating information. Hence, the supplier will use the received cash from the factor at time t_1 to invest in a profitable opportunity (modeled via the investment output function) rather than repay the bank loan. Furthermore, when the supplier requests for bank financing to start production at time 0, the bank knows the supplier's factoring activities at time t_1, which together with the bank's seniority rights, guarantees the supplier's full repayment of the bank loan regardless of the demand realization.

1.4.2 Recourse Factoring

At time 0, the bank offers a competitively priced bank loan under the supplier's credit rating, using the same pricing equation as in the bank financing case, i.e., $c_{\mathcal{B}}q = wS(b_{\mathcal{B}})$. Hence, we have the same bank financing bankruptcy threshold $b_{\mathcal{B}}$, and the interest rate $r_{\mathcal{B}}$ satisfies $cqe^{r_{\mathcal{B}}t_c} = wb_{\mathcal{B}}$. Then, at time t_1 the supplier may choose to factor his accounts receivable, with the factor offering a cash advance (loan) $\mathcal{L}_{\mathcal{F}} = wS(q)$, which is competitively priced under the supplier's credit rating C_s and equal to the expected value of the accounts receivable. Denote $r_{\mathcal{F}}$ to be the interest rate charged by the factor.

When factoring is adopted, the supplier will invest the cash advance $\mathcal{L}_{\mathcal{F}}$ he receives at time t_1 in a new project, with an investment horizon t_2 and a deterministic investment output $\mathcal{I}_s(\mathcal{L}_{\mathcal{F}}, t_2) = \mathcal{L}_{\mathcal{F}}e^{\alpha_s t_2}$. Due to the recourse nature of the factoring, the supplier is responsible for the uncollected amount at time t_c. If the realized demand is too low, the supplier may be liable for the loan obligations to the bank and the factor. The supplier's problem is to optimally choose the production quantity q to maximize the expected net cash flow at time t_c, by considering the bank's loan interest, the factor's interest for cash advance, and the supplier's own cash investment return.

Due to the bank's senior collection rights relative to the factor, the bank's loan is guaranteed to be fully repaid as long as no exogenous default of the supplier. Hence, the bank's interest rate $r_{\mathcal{B}}$ only prices in the exogenous default risk and satisfies $(1-\rho_s)cqe^{r_{\mathcal{B}}t_c} = cqe^{\eta_s t_c}$. Let $b_{\mathcal{F}}$ be the supplier's *recourse factoring bankruptcy threshold*, then $b_{\mathcal{F}}$ should satisfy $wb_{\mathcal{F}} + \mathcal{L}_{\mathcal{F}}e^{\alpha_s t_2} = \mathcal{L}_{\mathcal{F}}e^{r_{\mathcal{F}}t_2} + cqe^{r_{\mathcal{B}}t_c}$, where $\mathcal{L}_{\mathcal{F}} = wS(q)$. Taking into consideration the bank's interest rate decision $r_{\mathcal{B}}$, the recourse factoring bankruptcy threshold can be further derived as[3]

$$b_{\mathcal{F}} = \frac{\mathcal{L}_{\mathcal{F}}(e^{r_{\mathcal{F}}t_2} - e^{\alpha_s t_2}) + cqe^{r_{\mathcal{B}}t_c}}{w} = (e^{r_{\mathcal{F}}t_2} - e^{\alpha_s t_2})S(q) + \frac{cqe^{\eta_s t_c}}{w(1-\rho_s)}. \tag{1.6}$$

This equation builds up the connection between the recourse factoring bankruptcy threshold $b_{\mathcal{F}}$ and the factor's interest rate for the supplier's cash advance $r_{\mathcal{F}}$.

Lemma 1 *For a given $b_{\mathcal{F}}$ and the pull contract (w, q), the factor's expected total repayment $\Omega(b_{\mathcal{F}}; w, q)$ from the supplier under recourse factoring is as follows:*

$$\Omega(b_{\mathcal{F}}; w, q) = (1-\rho_s)(1-\rho_r)wS(b_{\mathcal{F}}) + (1-\rho_s)wS(q)e^{\alpha_s t_2} - cqe^{\eta_s t_c}. \tag{1.7}$$

A risk-neutral factor will offer a fairly priced interest rate $r_{\mathcal{F}}$ to the supplier, which will imply a bankruptcy threshold $b_{\mathcal{F}}$ so that the factor is indifferent between providing the recourse factoring service to the supplier and earning the interest rate with premium $e^{\eta_s t_2}$ adjusted according to the supplier's credit rating. Hence, the competitive credit pricing equation for the factor is as follows:

$$(1-\rho_s)(1-\rho_r)wS(b_{\mathcal{F}}) + (1-\rho_s)wS(q)e^{\alpha_s t_2} - cqe^{\eta_s t_c} = wS(q)e^{\eta_s t_2}. \tag{1.8}$$

Without exogenous default shock, the supplier's expected *cash inflow* (sales revenue plus cash investment output, if any) at time t_c is $wS(q) + \mathcal{L}_{\mathcal{F}}e^{\alpha_s t_2} = wS(q) + wS(q)e^{\alpha_s t_2}$ as the cash advance at time t_1 is $\mathcal{L}_{\mathcal{F}} = wS(q)$ under recourse factoring. To calculate the supplier's expected *cash outflow* (repayment of loan obligations), we note that if the realized demand is less than or equal to the recourse factoring bankruptcy threshold $b_{\mathcal{F}}$, the supplier cannot fully repay the loan obligation and has to declare bankruptcy. In this case, the supplier loses his sales revenue $w\xi$ plus the investment output $\mathcal{L}_{\mathcal{F}}e^{\alpha_s t_2} = wS(q)e^{\alpha_s t_2}$. However, if the realized demand is larger than $b_{\mathcal{F}}$, the supplier repays the loan from the bank at time 0 and the one from the factor at time t_1, and his total cost is $cqe^{r_{\mathcal{B}}t_c} + wS(q)e^{r_{\mathcal{F}}t_2}$. In summary, the supplier's expected *cash outflow* is

$$\mathbb{E}[\min(wD + wS(q)e^{\alpha_s t_2}, cqe^{r_{\mathcal{B}}t_c} + wS(q)e^{r_{\mathcal{F}}t_2})] = wS(q)e^{\alpha_s t_2} + wS(b_{\mathcal{F}}). \tag{1.9}$$

Therefore, the supplier's expected profit can be written as $\pi_{\mathcal{F}}(q; w) = (1-\rho_s)(1-\rho_r)w[S(q) - S(b_{\mathcal{F}})]$, where $(1-\rho_s)(1-\rho_r)$ reflects the supplier's and the retailer's exogenous

[3] When α_s is extremely high, $b_{\mathcal{F}}$ will become zero as demand is non-negative, i.e., no bankruptcy risk for the supplier. A complementary discussion of this special case is given in Appendix C.

default risk. Since $S(b_{\mathcal{F}})$ can be derived from the competitive credit pricing equation (3.4), we further have

$$\pi_{\mathcal{F}}(q;w) = \left[(1-\rho_s)(1-\rho_r) + (1-\rho_s)e^{\alpha_s t_2} - e^{\eta_s t_2}\right] wS(q) - cqe^{\eta_s t_c}. \tag{1.10}$$

Based on the structure of supplier's profit function, we can write out the effective unit production cost under recourse factoring as

$$c_{\mathcal{F}} = \frac{ce^{\eta_s t_c}}{(1-\rho_s)(1-\rho_r) + (1-\rho_s)e^{\alpha_s t_2} - e^{\eta_s t_2}}. \tag{1.11}$$

In other words, the effective unit production cost $c_{\mathcal{F}}$ incorporates both the original production cost c and the associated financing cost of recourse factoring for the supplier, and effectively determines the supplier's best response of production quantity for a given wholesale price.

We are now ready to analyze the optimal wholesale price proposed by the retailer. The retailer's profit function under recourse factoring is $\Pi_{\mathcal{F}}(w) = (1-\rho_r)(p-w)S(q_{\mathcal{F}})$. The equilibrium outcome $(w^*_{\mathcal{F}}, q^*_{\mathcal{F}})$ under recourse factoring is given in the next proposition.

Proposition 1.4.1 *Recourse factoring is adopted by the supplier if and only if (iff)* $\alpha_s > \bar{\alpha}_{\mathcal{F}} = \bar{\alpha}_{\mathcal{F}}(C_s, t_2) := \eta_s - t_2^{-1}\ln(1-\rho_s)$*, and the equilibrium* $(w^*_{\mathcal{F}}, q^*_{\mathcal{F}})$ *can be uniquely derived from:* $p\bar{F}(q) = c_{\mathcal{F}}[1 + z(q)j(q)]$, $w = c_{\mathcal{F}}/\bar{F}(q)$*. Moreover,* $w^*_{\mathcal{F}} < w^*_{\mathcal{B}}$, $q^*_{\mathcal{F}} > q^*_{\mathcal{B}}$, $\pi^*_{\mathcal{F}} > \pi^*_{\mathcal{B}}$*, and* $\Pi^*_{\mathcal{F}} > \Pi^*_{\mathcal{B}}$.

Several remarks are in order. First, the Stackelberg equilibrium is obtained by solving a similar system of equations as in pure bank financing (as shown in Proposition 1.3.1), but with a different effective unit production cost $c_{\mathcal{F}}$ as defined in equation (2.6). Second, by definition we know $\bar{\alpha}_{\mathcal{F}} = \eta_s - t_2^{-1}\ln(1-\rho_s) > \eta_s$, as $0 < 1-\rho_s < 1$. This indicates that recourse factoring is not feasible for those suppliers with $\alpha_s < \eta_s$, which makes intuitive sense. If the outside investment return is lower than the interest rate charged by the bank, there is no value for the supplier to obtain cash at t_1 for investment reasons. Lastly, recourse factoring is only adopted if the supplier is better off ($\pi^*_{\mathcal{F}} > \pi^*_{\mathcal{B}}$). Proposition 3 further points out that once recourse factoring is adopted, the retailer's profit also increases ($\Pi^*_{\mathcal{F}} > \Pi^*_{\mathcal{B}}$). Interestingly, when recourse factoring is adopted in equilibrium, the retailer offers a lower wholesale price ($w^*_{\mathcal{F}} < w^*_{\mathcal{B}}$), while the supplier responds with a higher production quantity ($q^*_{\mathcal{F}} > q^*_{\mathcal{B}}$), as the effective unit production cost reduces through the cash advance benefit of recourse factoring.

Corollary 1 *The threshold $\bar{\alpha}_{\mathcal{F}}$ decreases in the supplier's credit rating C_s and the payment term t_2.*

The threshold $\bar{\alpha}_{\mathcal{F}}$ depends on both the supplier's credit rating C_s (through ρ_s and η_s) and the payment term t_2. As presented in the corollary 1, $\bar{\alpha}_{\mathcal{F}}$ decreases in C_s and t_2. That said, recourse factoring is preferred by suppliers with relatively high credit ratings, or relative long payment terms, as illustrated in Figure 1.3. Moreover, as the trade credit payment term t_2 decreases, the threshold $\bar{\alpha}_{\mathcal{F}}$ will dramatically increase (approaches infinity as t_2 goes to zero). Our findings agree with industry practices that recourse factoring is typically adopted by suppliers with relatively long trade credit payment terms[4].

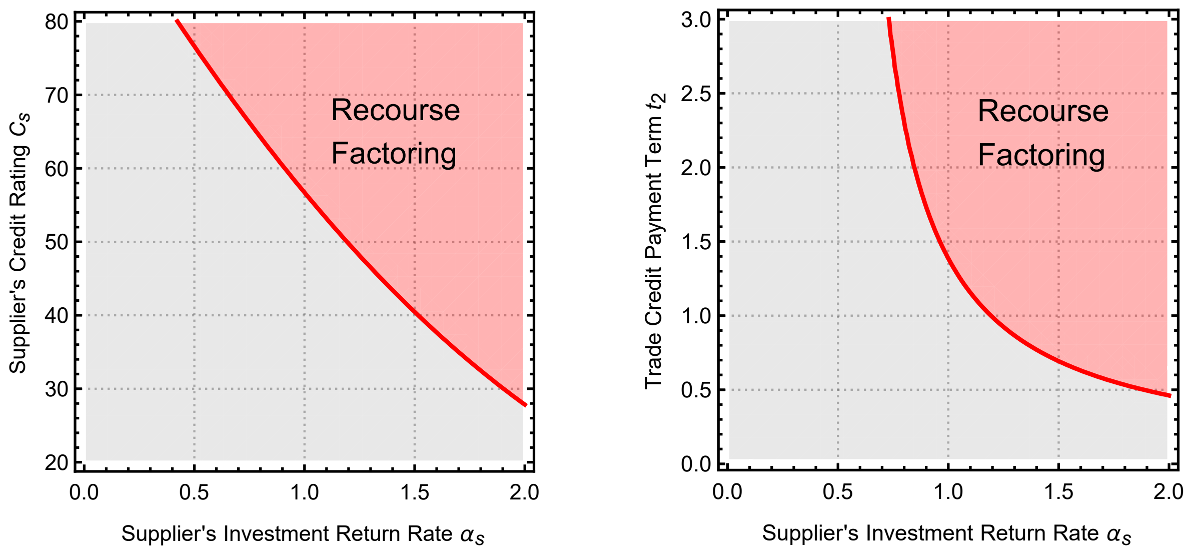


Figure 1.3.: When to Choose Recourse Factoring (Red Region)

1.4.3 Non-Recourse Factoring

In a non-recourse factoring contract, the factor can only collect payment from the retailer, but cannot consider the supplier liable for any uncollected accounts receivable amount. In pull-structure supply chains, such underpayment of the accounts receivable may happen for low demand realization since the retailer only pays for sold items. Therefore, in non-recourse factoring, the supplier's bank loan has a borrowing period of t_1, and the supplier ends its obligation with the factor before the demand risk is resolved. Under

[4] 2014 Supply Chain Finance Payable and Receivable Solutions Guide, The Global Business Intelligence.

the assumption of a fully competitive factoring environment, the supplier's cash advance obtained from selling the accounts receivable to the factor is given in the following lemma.

Lemma 2 *Under non-recourse factoring, the supplier's cash advance at time t_1 through selling accounts receivable is $\mathcal{L}_{\mathcal{S}} = e^{-\eta_s t_2}(1-\rho_r)wS(q)$.*

Recall that the supplier's cash advance in recourse factoring is $\mathcal{L}_{\mathcal{F}} = wS(q)$, which is larger than $\mathcal{L}_{\mathcal{S}}$ here since $e^{-\eta_s t_2}(1-\rho_r) < 1$. The advance rate in non-recourse factoring is lower because the factor bears more risk and has to price in the risk by a deeper discount (lower advance rate).

Under non-recourse factoring, the competitively priced bank loan under credit rating should satisfy $cqe^{\eta_s t_c} = (1-\rho_s)cqe^{r_{\mathcal{B}} t_c}$. Note that the bank's initial loan to the supplier does not depend on the demand uncertainty. The bank is informed about the supplier's factoring activity at time t_1, which guarantees the full repayment of the bank loan. The supplier will have $\mathcal{L}_{\mathcal{S}}$ amount of cash available (as given in Lemma 2) for investment at time t_1, with a investment output $\mathcal{L}_{\mathcal{S}} e^{\alpha_s t_2}$ for the investment period of t_2. Hence, the supplier's expected profit at time t_c can be derived as

$$\pi_{\mathcal{S}}(q; w) = (1-\rho_s)(\mathcal{L}_{\mathcal{S}} e^{\alpha_s t_2} - cqe^{r_{\mathcal{B}} t_c}) = e^{(\alpha_s - \eta_s)t_2}(1-\rho_r)(1-\rho_s)wS(q) - cqe^{\eta_s t_c}. \quad (1.12)$$

Similar as in preceding cases, we shall derive (see proof of Proposition 3.4.2 for details) the effective unit production cost under non-recourse factoring as

$$c_{\mathcal{S}} = \frac{ce^{\eta_s t_c + (\eta_s - \alpha_s)t_2}}{(1-\rho_s)(1-\rho_r)}. \quad (1.13)$$

Hence, for a given wholesale price w, the supplier's problem is to choose the optimal production quantity $q_{\mathcal{S}}$ that maximizes $\pi_{\mathcal{S}}(q; w)$. Given the supplier's best response, the retailer's profit function is $\Pi_{\mathcal{S}}(w) = (1-\rho_r)(p-w)S(q_{\mathcal{S}})$. We formally derive, in the next proposition, the Stackelberg equilibrium $(w^*_{\mathcal{S}}, q^*_{\mathcal{S}})$ under non-recourse factoring.

Proposition 1.4.2 *Non-recourse factoring is adopted by the supplier iff $\alpha_s > \eta_s = \eta(C_s)$, and the equilibrium $(w^*_{\mathcal{S}}, q^*_{\mathcal{S}})$ can be uniquely derived from: $p\bar{F}(q) = c_{\mathcal{S}}[1 + z(q)j(q)]$, $w = c_{\mathcal{S}}/\bar{F}(q)$. Moreover, $w^*_{\mathcal{S}} < w^*_{\mathcal{B}}$, $q^*_{\mathcal{S}} > q^*_{\mathcal{B}}$, $\pi^*_{\mathcal{S}} > \pi^*_{\mathcal{B}}$, and $\Pi^*_{\mathcal{S}} > \Pi^*_{\mathcal{B}}$.*

Similar to the case of recourse factoring, and as illustrated in Figure 1.4, non-recourse factoring is adopted when the supplier's investment return rate is higher than a threshold

η_s that depends on his credit rating C_r (as $\eta_s = \eta(C_s)$ is a decreasing function of C_s). However, different from recourse factoring, the threshold doesn't depend on the payment term t_2, as the supplier is not liable after t_1. Similar to recourse factoring, the equilibrium wholesale price is lower and the production quantity is higher relative to pure bank financing.

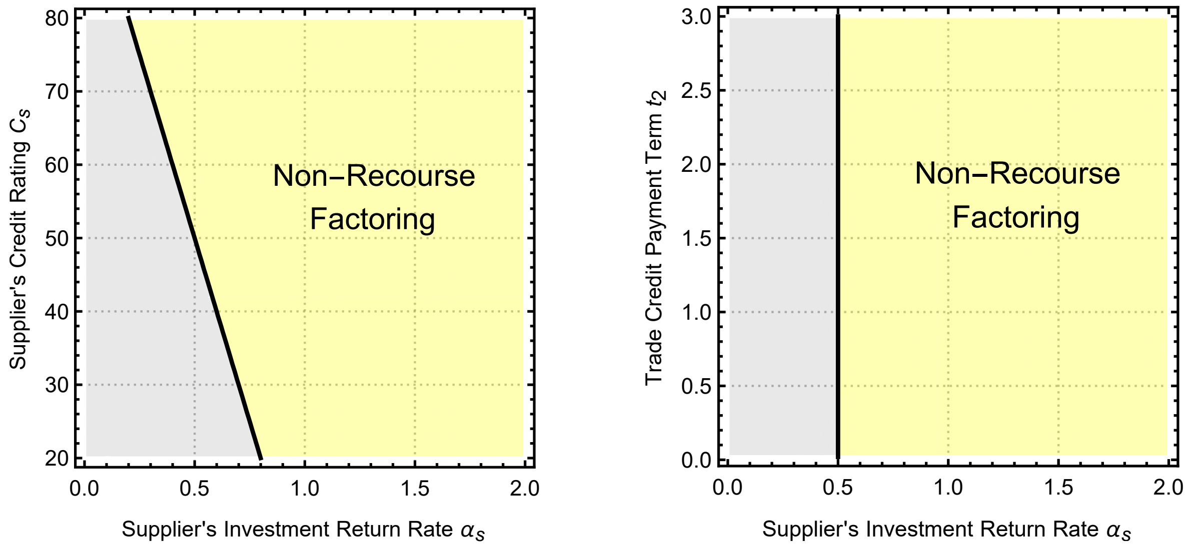


Figure 1.4.: When to Choose Non-Recourse Factoring (Yellow Region)

Non-recourse factoring has two important financial roles. First, it completely eliminates the supplier's bankruptcy risk due to the demand risk (also called, *bankruptcy remoteness*). [23] also points out non-recourse factoring may be particularly useful in countries with weak contract enforcement, inefficient bankruptcy systems, and imperfect records of upholding seniority claims, because receivables factored without recourse are not part of the estate of a bankrupt supplier. Second, similar to recourse factoring, non-recourse factoring helps the supplier free up cash flow at an earlier stage and seize profits through investments (liquidity benefit). It benefits the supplier when his credit rating or investment return rate are relatively high.

1.4.4 Factoring Comparisons and Implications

We now compare the equilibrium outcomes under two factoring schemes when both are available to the supplier, and summarize the conditions under which one is preferred over the other one. We start by summarizing the supplier's expected profit under pure bank financing, recourse factoring and non-recourse factoring as follows:

$$\pi_i(q; w) = \lambda_i w S(q) - cqe^{\eta_s t_c}, \quad i \in \{\mathcal{B}, \mathcal{F}, \mathcal{S}\}, \tag{1.14}$$

where the coefficients λ_i of the expected revenue part are given as

$$\lambda_i = \left\{ \begin{array}{ll} (1-\rho_s)(1-\rho_r), & i = \mathcal{B} \\ (1-\rho_s)(1-\rho_r) + (1-\rho_s)e^{\alpha_s t_2} - e^{\eta_s t_2}, & i = \mathcal{F} \\ (1-\rho_r)(1-\rho_s)e^{(\alpha_s - \eta_s)t_2}, & i = \mathcal{S} \end{array} \right\}.$$

In general, the supplier's expected profit under different financing schemes depends on the key parameter λ_i, which further relates to the credit ratings of both the supplier and the retailer, supplier's investment return rate α_s and payment term t_2. We further show (in Lemma 12 in appendix) that the supplier's preference over the three different financing schemes becomes a simple comparison of the coefficient λ_i. We characterize, in the next proposition, how the equilibrium outcome depends on the supplier's investment return for a given credit rating.

Proposition 1.4.3 *Let $\bar{\alpha}_{\mathcal{S}}$ be a threshold for supplier's investment return rate α_s,*

$$\bar{\alpha}_{\mathcal{S}} = \bar{\alpha}_{\mathcal{S}}(C_s, C_r, t_2) := \frac{1}{t_2} \ln \left[\frac{e^{\eta_s t_2} - (1-\rho_s)(1-\rho_r)}{(1-\rho_s)\left[1 - (1-\rho_r)e^{-\eta_s t_2}\right]} \right]. \tag{1.15}$$

When both factoring schemes are available to the supplier:

(i) Non-recourse factoring is adopted iff $\eta_s < \alpha_s \leq \bar{\alpha}_{\mathcal{S}}$;

(ii) Recourse factoring is adopted iff $\alpha_s > \bar{\alpha}_{\mathcal{S}}$.

Figure 1.5 illustrates the supplier's choice among the three financing options we have discussed so far under different regions along the dimensions of credit rating, investment return rate, and trade credit payment term. The left panel is plotted based on the supplier's investment return rate α_s and credit rating C_s. The dotted line indicates the region separation between pure bank financing and recourse factoring when non-recourse option is not available. For a given credit rating and payment term, the supplier should choose non-recourse factoring when the investment return rate is in a medium range (regions 2 & 3). When the investment return rate is relatively low (region 1), pure bank financing is the preferred option. Recourse factoring is preferred when the supplier has

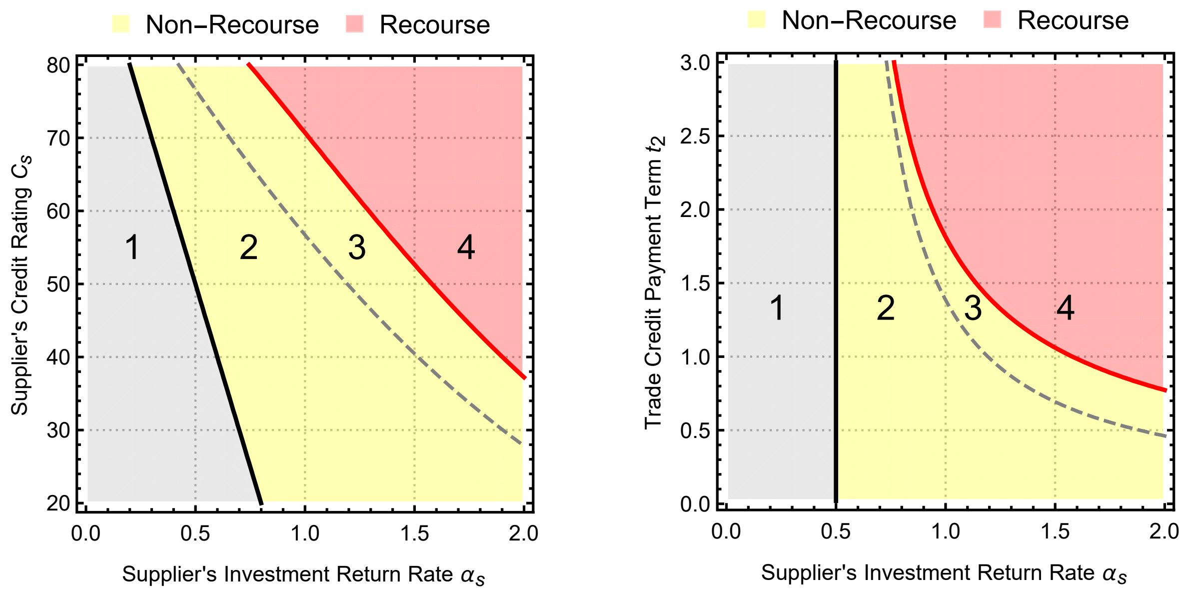


Figure 1.5.: Supplier's Choice among Pure Bank Financing, Recourse and Non-Recourse Factoring

a rather high investment return rate for reasonable (above a threshold) credit ratings (region 4).

The result conveys the intuition of the trade-offs between the benefits of recourse factoring and non-recourse factoring. The key advantage of non-recourse is the lower financing cost, since it can avoid the friction of seniority rights (the bank is senior in collection rights, and thus the factor will charge a higher interest rate) present in recourse factoring. One the other hand, recourse factoring has the advantage of the higher advance rate and the associated liquidity benefit, since the demand risk of the accounts receivable is not directly sold to the factor. For suppliers with reasonable credit ratings and high investment return rates, the liquidity benefit of recourse factoring dominates the benefit of lower financing cost in non-recourse factoring. Our findings agree with the empirical evidence in [24], showing that as a supplier's probability of bankruptcy increases (i.e., supplier's credit rating decreases in our model), his propensity to recourse factoring decreases.

The right panel presents a similar comparison from the lens of trade credit payment term t_2 and investment return rate α_s for a given credit rating. (The dotted line has the same interpretation as for the left panel.) An important observation is that when the investment return rate is not too low (region 2-4) and for reasonable credit ratings, an increase in payment term may switch the supplier's preference from non-recourse factoring to recourse factoring. A general observation is that non-recourse factoring is adopted for a wider parameter region, especially for SME suppliers whose credit rating and investment return rate are rather moderate. But recourse factoring may still be preferred in a narrow parameter region, and mostly for suppliers with rather high credit ratings and investment return rates. Our results are supported to some extent by empirical findings that in developed countries factoring is more frequently done on a non-recourse basis. In Italy, for example, 69% of all factoring is done on a non-recourse basis [35]. Similarly, a study of publicly traded firms in the US found that 73% of firms factored their receivables on a non-recourse basis, but higher quality suppliers were more likely to factor with recourse [24]. In Figure 2.5, we plotted the factoring volume data overtime as reported by Factor Chain International (FCI)[5]. The volume growth pattern for recourse and non-recourse factoring also supports the higher popularity of non-recourse factoring globally.

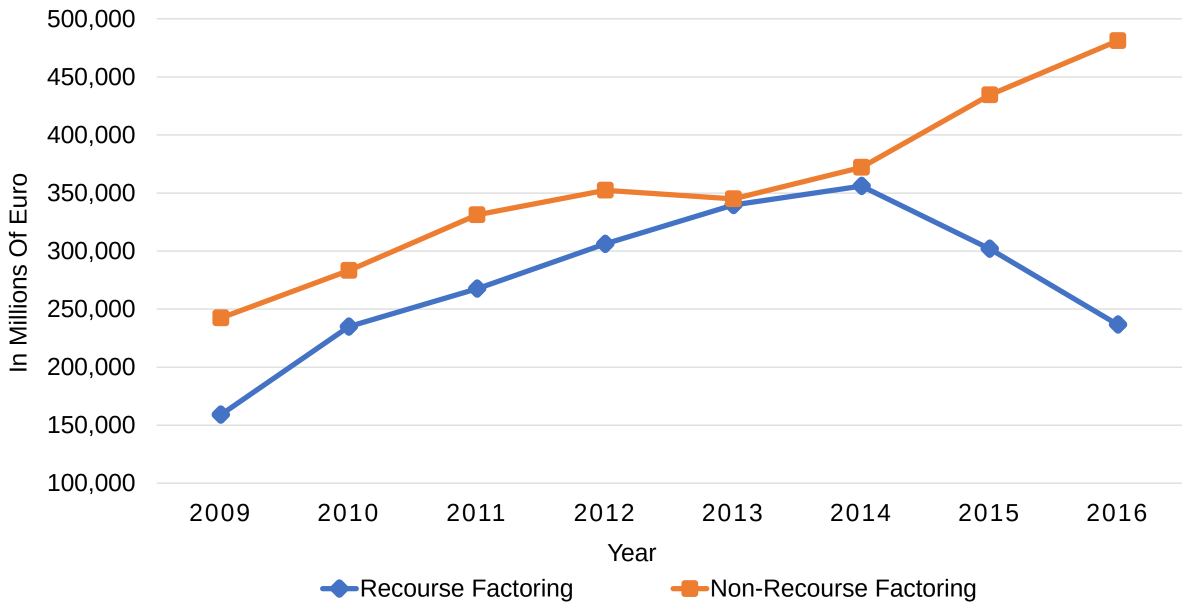


Figure 1.6.: Global Factoring Volume: Recourse v.s. Non-Recourse (Data from FCI)

Now, we turn our attention to the comparative statics of the equilibrium outcomes and analyze how the changes in the both parties' credit ratings and the supplier's cash investment return rate affect the optimal wholesale price and production quantity, as well as different parties' profits.

[5]Factor Chain International (FCI) is the main global network for factoring with more than 400 member companies engaged in factoring activities in over 90 countries.

Proposition 1.4.4 *When either factoring scheme is adopted, as the credit rating of either party C_j or the supplier's investment return rate α_s increases, the optimal wholesale price w_i^* decreases, while the optimal production quantity q_i^* and both parties' profits increase.*

The increase of credit rating reduces the supplier's financing cost, and the increase of investment return rate increases the liquidity benefit of cash advance in factoring, subsequently reducing the effective unit production cost of the supplier. Hence, in the Stackelberg game, the retailer reduces the wholesale price in anticipation of the supplier's best response of increasing production quantity. Essentially, better short-term financing scheme in the supply chain will mitigate to some extent the double-marginalization problem in supply chains, and end up benefiting both firms.

1.5 Reverse Factoring

Reverse factoring programs allow high-risk suppliers to lower the financing costs of their accounts receivable. Typically, these programs have the suppliers factor their accounts receivable at the buyer's higher credit rating. Under reverse factoring, suppliers' accounts receivable are factored on a non-recourse basis. Often in practice, in return for offering the reverse factoring program, the buyer asks for an extension to the payment delay to the supplier. Denote this payment extension, beyond the original (prior to reverse factoring) trade credit terms, as $\tau \geq 0$.

It is often argued that the associated benefits of higher days payable outstanding (DPO) is the motivation for large retailers to initiate the reverse factoring program. While the availability of an alternative form of low-cost financing makes reverse factoring attractive to SMEs, the assessment of the trade-offs between lower cost of financing and payment-term extension requires careful analysis that accounts for strategic interactions of the supply chain players and considers both operational and financing decisions.

The chronology of events, notations, and decision structure are as follows:

(1) At time 0, the retailer offers the reverse factoring program to the supplier with a payment extension τ, and then the supplier decides whether to join the program or not. If yes, similar production and borrowing decision protocol is followed as in factoring.

(2) At time t_1 and t_c, activities and decision protocol are similar as in non-recourse factoring, with the only difference that the interest rate charged by the factor is now based on the *retailer's* credit rating C_r.

(3) At time $t_c + \tau$, the retailer directly pays the factor $w\xi$, and the supplier is not liable to the factor even if the collected amount from the retailer can't cover the supplier's cash advance.

1.5.1 Supplier's Cash Advance and Production Quantity

Under reverse factoring, the factor's expected repayment from the supplier's accounts receivable is $(1-\rho_r)wS(q)$ after a time period of $t_2+\tau$, where τ is the retailer's payment extension under reverse factoring. Suppose the supplier's cash advance at time t_1 is $\mathcal{L}_{\mathcal{R}}$, then the factor competitively prices it, but with the risk premium η_r, adjusted according to the retailer's credit rating C_r. Then, under competitive credit pricing, the factor offers cash advance $\mathcal{L}_{\mathcal{R}}$ to the supplier as summarized in the next lemma.

Lemma 3 *Under reverse factoring, the supplier's cash advance through selling accounts receivable to the factor is* $\mathcal{L}_{\mathcal{R}} = e^{-\eta_r(t_2+\tau)}(1-\rho_r)wS(q)$.

We proceed by solving for the supplier's optimal production decision q, given the retailer's wholesale price w and payment extension τ. Since the supplier will cash out at time t_1 by selling the accounts receivable to the factor, bank's interest rate $r_{\mathcal{B}}$ is then determined through $cqe^{\eta_s t_c} = (1-\rho_s)cqe^{r_{\mathcal{B}} t_c}$. Hence, the supplier's expected profit at time t_c can be derived as

$$\pi_{\mathcal{R}}(q; w, \tau) = (1-\rho_s)(\mathcal{L}_{\mathcal{R}} e^{\alpha_s t_2} - cqe^{r_{\mathcal{B}} t_c}) = e^{\alpha_s t_2 - \eta_r(t_2+\tau)}(1-\rho_s)(1-\rho_r)wS(q) - cqe^{\eta_s t_c}.$$

Hence, for a given wholesale price w and trade credit extension τ, the supplier's problem is to choose optimal production quantity q to maximize $\pi_{\mathcal{R}}(q; w, \tau)$. And the supplier's optimal production quantity under reverse factoring is summarized in the following proposition.

Lemma 4 *Given the retailer's offer of reverse factoring with payment extension* τ, *the supplier's optimal production quantity* $q_{\mathcal{R}}$ *satisfies*

$$w\bar{F}(q_{\mathcal{R}}) = c_{\mathcal{R}}(\tau) := \frac{ce^{\eta_s t_c + \eta_r(t_2+\tau) - \alpha_s t_2}}{(1-\rho_s)(1-\rho_r)}. \tag{1.16}$$

Comparing to the effective unit production cost under pure bank financing $c_{\mathcal{B}} = \frac{ce^{\eta_s t_c}}{(1-\rho_s)(1-\rho_r)}$, the effective unit production cost under reverse factoring $c_{\mathcal{R}}(\tau)$ also depends on the retailer's interest rate premium η_r (i.e., retailer's credit rating C_r) and payment extension τ. It's not clear which one is smaller. Note that when $\eta_r(t_2+\tau) - \alpha_s t_2 = 0$, i.e., $\tau = \frac{\alpha_s - \eta_r}{\eta_r} t_2$, the supplier's effective unit production cost is the same as in the pure bank financing, i.e., $c_{\mathcal{R}}(\tau) = c_{\mathcal{B}}$. Hence, as long as $\alpha_s > \eta_r$, the retailer can still extend payment terms (i.e., $\tau > 0$) and the supplier may still prefer reverse factoring to pure bank financing. However, the supplier may have already adopted or have access to factoring schemes (recourse or non-recourse). The retailer has to take into consideration the supplier's choice among all available short-term financing schemes in structuring the right reverse factoring program the supplier may be willing to adopt.

1.5.2 Optimal Payment Extension in Reverse Factoring

Under the assumption of symmetric information, the retailer is aware of the supplier's response $q_{\mathcal{R}}(w,\tau)$ when deciding on the wholesale price w and payment extension τ to maximize her expected profit. With wholesale price w and payment extension τ, the retailer's profit function becomes

$$\Pi_{\mathcal{R}}(w,\tau) = (1-\rho_r)(p - e^{-\alpha_r \tau} w) S(q_{\mathcal{R}}), \tag{1.17}$$

where $q_{\mathcal{R}}$ is the supplier's optimal production quantity under reverse factoring as shown Lemma 22. Note that in this case, the payment extension τ generates cash flow benefits of delayed payment. Thus, when the retailer offers a wholesale price w but with a payment delay τ, the *effective wholesale price* becomes $e^{\alpha_r \tau} w$.

Suppose the supplier has access to bank financing, and also recourse and non-recourse factoring as post-shipment financing alternatives and has been adopting the optimal one among them. Based on Proposition 1.4.3, the existing equilibrium before introducing the reverse factoring program can be summarized as follows:

$$\begin{Bmatrix} (w^*_{\mathcal{B}}, q^*_{\mathcal{B}}) \text{ in Pure Bank Financing,} & \text{if } \alpha_s \le \eta_s \\ (w^*_{\mathcal{S}}, q^*_{\mathcal{S}}) \text{ in Non-Recourse Factoring,} & \text{if } \eta_s < \alpha_s \le \bar{\alpha}_{\mathcal{S}} \\ (w^*_{\mathcal{F}}, q^*_{\mathcal{F}}) \text{ in Recourse Factoring,} & \text{if } \alpha_s > \bar{\alpha}_{\mathcal{S}} \end{Bmatrix}.$$

Now, the retailer initiates the reverse factoring program where the retailer proposes a new extended payment term and enables the supplier to sell accounts receivable at a better (interest) rate backed by the retailer's better credit rating. We assume the retailer offers the reverse factoring program without adjusting the existing equilibrium wholesale price with the supplier[6]. Hence, the key problem for the retailer is whether to offer the reverse factoring for a specific supplier, and if to offer it, with what payment extension. This problem can be formalized as follows:

$$\max_{\tau\geq 0} \Pi_{\mathcal{R}}(\tau) = \max_{\tau\geq 0} \left\{(1-\rho_r)(p - e^{-\alpha_r \tau} w) S(q_{\mathcal{R}})\right\}, \tag{1.18}$$

$$s.t. \qquad w\bar{F}(q_{\mathcal{R}}) = c_{\mathcal{R}}(\tau), \tag{IC}$$

$$\pi^*_{\mathcal{R}} \geq \max\{\pi^*_{\mathcal{B}}, \pi^*_{\mathcal{F}}, \pi^*_{\mathcal{S}}\}. \tag{IR}$$

For easy of exposition, we define a function $\Xi_{[0,z]}(x)$ as the projection of x onto the line segment $[0, z]$. That is, $\Xi_{[0,z]}(x) = z$, if $x > z$; $\Xi_{[0,z]}(x) = 0$, if $x < z$; otherwise $\Xi_{[0,z]}(x) = x$. Let $\bar{\alpha}_{\mathcal{R}}$ be a new threshold of α_s and $\psi(\alpha_s)$ be a new function of α_s, which are defined as

$$\bar{\alpha}_{\mathcal{R}} := \frac{1}{t_2} \ln \left[\frac{e^{\eta_s t_2} - (1-\rho_s)(1-\rho_r)}{(1-\rho_s)\left[1-(1-\rho_r)e^{-\eta_r t_2}\right]}\right], \tag{1.19}$$

$$\psi(\alpha_s) := \frac{1}{\eta_r} \ln \left[1 + \frac{e^{\alpha_s t_2}}{1-\rho_r} - \frac{e^{\eta_s t_2}}{(1-\rho_s)(1-\rho_r)}\right]. \tag{1.20}$$

We now summarize the retailer's reverse factoring strategy in the next proposition.

Proposition 1.5.1 *The retailer's optimal reverse factoring strategy is as follows:*

(i) If $\alpha_s \leq \eta_r$, then reverse factoring is dominated by pure bank financing $(w^*_{\mathcal{B}}, q^*_{\mathcal{B}})$;

(ii) If $\alpha_s \geq \bar{\alpha}_{\mathcal{R}}$, then reverse factoring is dominated by recourse factoring $(w^*_{\mathcal{F}}, q^*_{\mathcal{F}})$;

(iii) If $\eta_r < \alpha_s < \bar{\alpha}_{\mathcal{R}}$, then reverse factoring should be offered with the payment extension $\tau^*_{\mathcal{R}} = \Xi_{[0,\tau_s]}(\tau^*_0)$, where τ^*_0 is solved from $\{w_s(\alpha_r j(q)z(q) + \eta_r) = p\eta_r e^{\alpha_r \tau}, w_s\bar{F}(q) = c_{\mathcal{R}}(\tau)\}$, and

$$(w_s, \tau_s) = \begin{cases} (w^*_{\mathcal{B}}, (\alpha_s/\eta_r - 1)t_2), & \text{if } \eta_r < \alpha_s \leq \eta_s \\ (w^*_{\mathcal{S}}, (\eta_s/\eta_r - 1)t_2), & \text{if } \eta_s < \alpha_s \leq \bar{\alpha}_{\mathcal{S}} \\ (w^*_{\mathcal{F}}, (\alpha_s/\eta_r - 1)t_2 - \psi(\alpha_s)), & \text{if } \bar{\alpha}_{\mathcal{S}} < \alpha_s < \bar{\alpha}_{\mathcal{R}} \end{cases}. \tag{1.21}$$

[6]This assumption reflects the typical industry practice of reverse factoring program and helps deliver clean insights. An extension is provided in Appendix C where the retailer could adjust both payment term and wholesale price.

To better understand this proposition, it is helpful to first go over some key notations. τ_s in (2.10) represents the retailer's *maximal* payment extension due to the supplier's IR constraint. τ_0^* is the optimal payment extension for the retailer's maximization problem in (1.18) without the non-negativity constraint $\tau \geq 0$. w_s is the equilibrium wholesale price in the existing best financing scheme as summarized in Proposition 1.4.3.

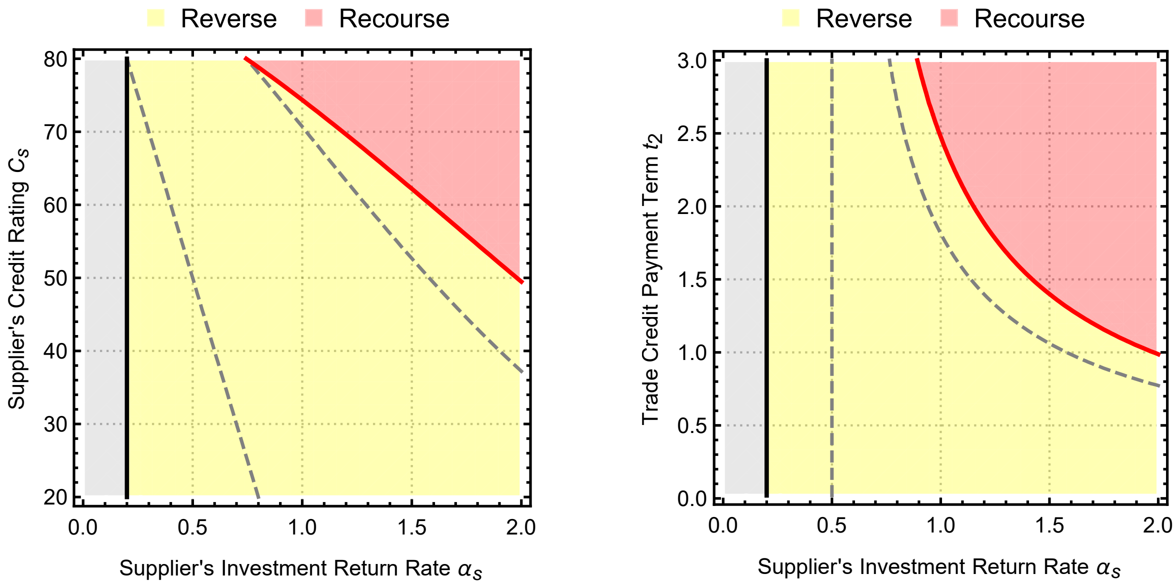


Figure 1.7.: When to Offer Reverse Factoring (Yellow Region)

Proposition 3.6.1, and an illustrative Figure 2.4 of a relevant numerical example, offer insights on reverse factoring. First, parts (*i*) and (*ii*) of Proposition 3.6.1 specify the conditions under which reverse factoring should not be offered as it is dominated by the existing best financing scheme (either pure bank financing or recourse factoring). The yellow region in Figure 2.4 reflects the parameter regions where reverse factoring is preferable to suppliers. The dotted lines, starting from the left, refer to the threshold separating pure bank financing and non-recourse factoring, and the one separating non-recourse and recourse factoring. As reverse factoring is a form of non-recourse factoring, it will Pareto dominate traditional non-recourse factoring through its reliance on the superior buyer's credit rating. It is both implied in Proposition 3.6.1 and also can be seen in Figure 2.4. The yellow region in the middle represents the parameter region where reverse factoring should be offered. It is straightforward to see that the yellow region in Figure 2.4 expands the one in Figure 1.5 (the boundaries are illustrated as the two dashed lines in Figure 2.4). In other words, reverse factoring not only dominates non-recourse factoring, but also limits the preferred regions for pure bank financing and recourse factoring.

Part (iii) of Proposition 3.6.1 fully characterizes the retailer's optimal choice of payment extension for a specific supplier profile. Notice that the reverse factoring program is contingent on the existing financing schemes adopted by the supplier. The payment term extension in reverse factoring differs by regions, reflecting the comparison to the alternative financing scheme of special appeal to the supplier in the parameter region. The formula of the optimal payment extension $\tau_{\mathcal{R}}^* = \Xi_{[0,\tau_s]}(\tau_0^*)$ indicates that the retailer should not extend payment when $\tau_0^* \leq 0$ (more discussion in the next proposition), and that there exists a maximal payment extension τ_s in each region. This maximal payment extension τ_s represents the retailer's *aggressiveness* in payment terms. Intuitively, one expects the retailer to be more aggressive as the supplier's investment return rate increases. Interestingly, we find, however, the retailer's aggressiveness is not monotonic in the supplier's investment return rate, as presented in equation (2.10). Proposition 3.6.2 further summarizes several implications regarding to the retailer's optimal payment extension choice.

Proposition 1.5.2 *When reverse factoring is adopted, i.e., $\eta_r < \alpha_s < \bar{\alpha}_{\mathcal{R}}$:*

(i) *There exists a threshold α_r^τ, such that the retailer should not extend payment term if $\alpha_r \leq \alpha_r^\tau$.*

(ii) *The retailer's profit always increases, but the supplier's may remain unchanged when $\tau_0^* \geq \tau_s$.*

Part (i) of Proposition 3.6.2 states that, as expected, it is not optimal to extend the payment term in reverse factoring when the retailer's own investment return is relatively low. It offers a clear threshold for the retailer's rate to be below, with detailed derivation of the threshold α_r^τ in the proof of Proposition 3.6.2. Optimally designed reverse factoring program, once adopted, can always improve the retailer's profit but do not necessarily require an increase in the payment term. Part (ii) of Proposition 3.6.2 suggests that when reverse factoring is adopted, it is always a "win" option for the large retailer, but may not be a "win-win" solution as typically claimed by industry practitioners since the supplier's profit may remain unchanged.

We now summarize the benefits of reverse factoring compared to the best among other three financing schemes (pure bank financing, recourse factoring and non-recourse factoring) in Table 1.2. Case 1 represents the scenarios in Proposition 3.6.1 (i) and (ii) (i.e., $\alpha_s \leq \eta_r$, or $\alpha_s \geq \bar{\alpha}_{\mathcal{R}}$), where reverse factoring is dominated by either pure bank financing or recourse factoring (the gray and red regions, respectively, in Figure 2.4). Case 2 represents the case where the optimal payment extension reaches the maximum τ_s, and thus the supplier's production quantity and profit remain unchanged. In this case, even though the retailer's profit increases, the supplier's profit is the same as when reverse factoring was not offered. Case 3 is the win-win situation where the retailer is not aggressive on payment extension, leaving some profit gain to the supplier after the adoption of reverse factoring.

Table 1.2: The Benefits of Reverse Factoring (Compared to the Best of Other Schemes)

Three Cases	Case 1: No-Win ($\alpha_s \leq \eta_r$ or $\alpha_s \geq \bar{\alpha}_{\mathcal{R}}$)	Case 2: Retailer-Win ($\tau_0^* \geq \tau_s$)	Case 3: Win-Win ($\tau_0^* < \tau_s$)
Wholesale Price $w_{\mathcal{R}}^*$	$=$	$=$	$=$
Production Quantity $q_{\mathcal{R}}^*$	$\downarrow$	$=$	$\uparrow$
Supplier's Profit $\pi_{\mathcal{R}}^*$	$\downarrow$	$=$	$\uparrow$
Retailer's Profit $\Pi_{\mathcal{R}}^*$	$\downarrow$	$\uparrow$	$\uparrow$

1.6 Conclusion

We consider a pull-structure supply chain with a large retailer and an SME supplier: A newsvendor-like supplier has a single opportunity to produce and stock inventory to satisfy future uncertain demand at the retailer's side, with the wholesale price determined by the retailer. The SME supplier is in need of short-term bank financing for the pre-season production activity. The bank offers a competitively priced loan for relevant risks but with interest rate premium based on the supplier's credit rating. Failure of loan repayment leads to supplier's bankruptcy. The strategic interaction between the supplier and the retailer are modeled and analyzed under a Stackelberg game setting, with the retailer proposing the wholesale price and the supplier deciding on the production quantity. We first derive the unique equilibrium in pure bank financing of the pull supply chain as our benchmark, which has similar structure to the traditional pull contract but with the bank financing cost added into the supplier's effective unit production cost.

Factoring (recourse or non-recourse) is often used by suppliers as post-shipment financing scheme to obtain advanced cash in selling their accounts receivables from retailers and using it to finance their immediate working capital needs or other available investment opportunities. We carefully model, analyze and compare the equilibria of different factoring schemes. Our results show that the benefit of recourse and non-recourse factoring depends on three important financial parameters of the supplier: credit rating, cash investment return rate, and payment delay offered to the retailer. For a given supplier's credit rating, the recourse factoring is adopted only when the supplier has rather high investment return rate; while the non-recourse factoring is adopted when the investment return rate is within certain intermediate (low, but above a threshold, to moderate) range. For a given supplier's investment return rate, in the moderate-to-high level, suppliers that extend longer payment terms favor recourse factoring. The choice between recourse and non-recourse factorings is driven by the trade-off between the higher cash advance in recourse factoring and the lower financing cost in non-recourse factoring.

By engaging suppliers in reverse factoring programs, retailers work with banks or factors to help finance the suppliers' accounts receivable on the retailer's orders through a non-recourse way of factoring, but with the factor calculating financing costs using the superior credit ratings of the retailers. We find that reverse factoring might not be always adopted by suppliers when other short-term financing options are available. Retailers should offer such programs only to suppliers with low, but above a threshold, to medium cash investment returns and relatively low credit ratings. Suppliers with high credit ratings and investment return rates will opt for traditional recourse factoring. We also fully characterize the optimal payment extension strategy in reverse factoring programs. We find that such programs do increase the retailer's profit (due to the retailer's superior credit rating and potential payment extension), but may leave the supplier's profit indifferent to other financing options (due to the retailer's aggressive payment extension). Interestingly, it may be preferable for the retailer not to extend the payment term in reverse factoring, and the retailer can still leverage the supplier's willingness to bear higher inventory risk and benefit from the increased supply chain efficiency. This contradicts with conventional wisdom in the supply chain finance industry that a longer payment term is always better for the downstream buyer and is often regarded as the driving force for the implementation of reverse factoring programs.

Most of current supply chain finance research revolves around post-shipment approved invoice financing. But, purchase order financing, where the under-capitalized supplier has to finance raw material procurement and intermediate components or work-in-process, is an important application area deserving further research. Big banks such as Bank of America and Wells Fargo, large Fintech players such as GT Nexus, a cloud supply chain platform, are heavily involved in advancing pre-shipment financing practices. We consider it a promising future research direction for quantitative modeling of supply chain finance practices.

2. Crowdfunding under Social Learning and Network Externalities

2.1 Introduction

[1] Crowdfunding is the practice of funding a project or venture by raising many small amounts of money from a large number of people, typically via the Internet. Crowdfunding may be used for a variety of purposes, from disaster relief to citizen journalism to artists seeking support from fans to political campaigns. In this paper, we focus on *rewards-based crowdfunding*, where production-based firms pre-sell a product to launch a business without incurring debt or sacrificing equity/shares [2]. In such an environment, a firm solicits financial contributions from the crowd, mostly in the form of pre-buying a product. And these funds could then be used to cover production costs.

Rewards-based crowdfunding is the most prolific form of crowdfunding currently taking place in the U.S. The rising popularity of crowdfunding sites such as *Kickstarter* and *Indiegogo* has promoted crowdfunding as a modern fund-raising solution. It has changed the face of investing, giving startups a chance to prove product-market-fit without the massive costs of going into mass production. Crowdfunding has gone beyond simply being a tool specifically utilized by small businesses and independent entities to get a given project off the ground. Even larger businesses are now turning to crowdfunding as a way to obtain funds to develop new products.

Quality uncertainty is an inevitable issue for crowdfunding firms when they first introduce an innovative product under development (often just a prototype of a complex product such as an electronic vehicle or a smart watch) to the market. A common feature of crowdfunding is that over time, consumers learn more about the product and gain a better sense of its value, through channels such as professional product reviews

[1] This chapter is based on the author's early work [36] jointly with Fuqiang Zhang.

[2] In contrast, another primary type of crowdfunding is *equity-crowdfunding* in which the backer receives shares of a company, usually in its early stages, in exchange for the money pledged.

from websites and magazines, the reviews of fellow consumers (e.g., from online retailers such as Amazon), and the experiences of friends and family who may have purchased the same product. Hence, consumers may recognize that future learning will occur and may choose to delay a purchase until they have more information about a product's value. In this paper, we consider a two-sided learning model where both sides of the market, the firm and the consumer population, exhibit learning behavior: They both learn the true quality of a new product through consumer reviews. Hence, strategic consumers make their decisions on both whether and when to purchase the product based on their anticipation of product quality and their own personal preference.

Another feature of crowdfunding products is that they often exhibit positive network externalities. An *externality* occurs in any situation in which the welfare of an individual is affected by the actions of other individuals, without a mutually agreed-upon compensation [37]. For example, the value of a social networking site to a person depends on the total number of people who use the site. When other people join the site, they increase the welfare even though there is no explicit compensation involved. As concrete examples of such goods, consider online games (e.g., League of Legends, Pokémon Go, etc.) and social networking communities (e.g., Facebook, LinkedIn, etc.). Network externalities may come from multiple sources, such as learning/herding [38, 39] and social influence/peer effects [40, 41]. Specifically for the crowdfunding setting, recent research has documented empirical evidence for network externalities [42]. Hence in this paper, we incorporate the positive network externalities into our model to better reflect the consumer's decision framework. The focus of this research is to investigate the impact of different levels of overall strength of externalities on consumers' purchasing behaviors and its interaction with the social learning feature.

In the spirit of these characteristics, we develop a two-period model (crowdfunding stage and retail stage) that captures three salient features – *network externalities*, *social learning*, and *financial constraints* – as a unified framework to study the strategic interactions between the firm and consumer population. Crowdfunding is a cornucopia of important research questions, of which we mainly consider three in this paper: (*i*) What is the impact of network externalities on the purchasing game dynamics in the "crowd" with strategic consumer behaviors? (*ii*) What is the firm's optimal crowdfunding reward choice in the presence of social learning and network externalities? (*iii*) What is the impact of the firm's financial constraints? Our analysis of these research questions yields three main sets of insights, which we summarize below.

First, we find that an upward-sloping demand curve (documented as the *Veblen effect*) may arise where a higher price induces more demand in crowdfunding. The existing literature has principally focused on either social-related benefits, or consumers' uniqueness-seeking feature (i.e., negative externalities, snobs) to explain the Veblen effect. In contrast, we model the consumers as forward-looking social learners, and provide a novel explanation for the Veblen effect: It may arise due to the presence of both social learning and strong positive network externalities, even in the absence of uniqueness-seeking consumers. The intuition is that the forward-looking consumers may strategically lower the purchasing threshold to lock in a higher social utility from more demand as the crowdfunding price increases. Due to the strong network externalities, the social utility increase from lowering the purchasing threshold will compensate the decrease in personal preference value and the increase in crowdfunding price, eventually leading to a new equilibrium in the firm-consumer game.

Second, we show that network externalities have significant structural implications for the optimal crowdfunding reward choice: Under weak network externalities, the crowdfunding sales induced by optimal reward can be any fraction of the population; by contrast, in the presence of strong network externalities, only very high or very low reward should be offered to induce either adoption inertia (none-purchasing) or adoption frenzy (all-purchasing) in the crowdfunding stage. Such equilibrium purchasing patterns can be explained by the trade-off between two countervailing forces: the *immediate network benefit* and the *potential learning benefit*. When there are weak network externalities, the two benefits are close to each other, so the optimal strategy is to take advantage of both benefits, resulting in any dispersed adoption. However, as the network externalities become very strong, one benefit tends to dominate the other. When the immediate network benefit is dominant, the optimal reward strategy is to induce adoption frenzy; otherwise, adoption inertia is preferred to take advantage of the potential learning benefit.

Third, we characterize the optimal rewarding strategy when facing financial constraints (a fundraising target to meet in the crowdfunding stage) and quantify the impact of the fundraising target on optimal reward choice and the induced purchasing pattern in consumer population. Comparing to the situation without financial constraints, the optimal reward choice can be either higher or lower under each network externalities scenario, depending on the product's cost structure and the prior uncertainty in quality. Furthermore, we observe that as the fundraising target increases, the optimal reward should increase under weak network externalities but decrease under strong network externalities. However, the equilibrium purchasing fraction in the crowdfunding stage always increases. This result offers insights into how financially constrained firms should exploit the "funding" role of crowdfunding.

2.2 Literature Review

This paper is related to four streams of research in the literature: strategic consumer behavior, advance selling, social operations management, and crowdfunding.

There has been a growing body of literature that explores the role of strategic customer behavior in various operations settings [43–48]. This stream of research explicitly captures the strategic response of customers to a seller's pricing and inventory policies; however, none of the studies in this stream considers a unified framework that incorporates quality uncertainty, social learning, and network externalities. As a result, both the model setting and insights are quite different in our paper.

Crowdfunding is similar to advance selling because both encourage customers to place orders before the product is released. A major benefit of advance selling is to help firms plan for inventory by using early demand information [49–51]. Another reason for firms to adopt advance selling is to exploit consumers' uncertainty about their valuations. [52] demonstrate that selling to consumers who have uncertain consumption utility can substantially increase the firm's profit. [53] find that capacity rationing in advance selling can be an effective device to signal quality. Crowdfunding differs from advancing selling because products are typically at a much earlier stage, and thus social learning about product characteristics plays an important role. In addition, network externalities and financial constraints are often present in crowdfunding, but are rarely considered in the advance selling literature.

Apart from the aforementioned papers, our work contributes to the emerging literature on *social operations management*, which studies how firms' operational decisions can influence the formation of collective consumer behaviors under social interactions. For example, [54] and [55] examine the impact of firms' pricing policy on consumer purchasing decisions when consumers can learn from from early adopters' product reviews. [56] study a monopolist firm's product quality strategy under review-based social learning. Our paper considers two social interactions (*social learning* and *network externalities*) and investigates their interaction with firm's crowdfunding strategy.

Besides social learning, our paper is also related to the literature on network goods and network externalities. Models that incorporate network externalities find their origins in the works of [57] and [58]. Many papers have examined important strategic issues involving the network effects, such as pricing [59, 60], advertising strategy [61], and product line design [62]. Recently, [63] study the operations and marketing policies under social influence in a stream of consumers who arrive sequentially. [64] investigate the influence of sales information revelation on a firm's profitability under market size uncertainty. However, these studies do not consider social learning and fundraising and are therefore different from our paper.

Crowdfunding has received increasing attention from researchers in recent years. [65] study how the crowdfunding mechanism affects a firm's product and pricing decisions in a two-period game. [66] explore how private quality information impacts rewards-based crowdfunding. Focusing on the entrepreneurial moral hazard problem, [67] argues that the optimal crowdfunding mechanism should implement deferred payments to project creators. [68] provide a dynamic model of crowdfunding and [69] propose several contingent stimulus policies to mitigate the "cascade effect" on backers' pledging. [70] show that crowdfunding alters interactions between the entrepreneur and the traditional financing sources (banks and VCs), and may either benefit or hurt entrepreneurs. Our study focuses on the role of social learning and network externalities in crowdfunding reward strategy when facing strategic consumers.

2.3 Model Framework

In this section, we introduce the model framework in three steps. First we describe the firm's characteristics and the consumers' utility components; then we introduce the two-sided Bayesian learning framework; finally, we present the consumers' expected utility functions.

2.3.1 The Firm and the Crowd

We consider a crowdfunding campaign where a firm markets a new product of unknown quality to a potential population of heterogeneous consumers. There are two selling periods: the *crowdfunding stage* (period 1) and the *retail stage* (period 2). Hereafter, we use subscripts 1 and 2 to denote these two periods, respectively. We assume both the consumers and the firm are *rational Bayesian* decision makers who aim to maximize their expected utilities. The product is released at the beginning of period 2, but the firm may accept pre-orders in period 1. The firm charges two different prices for the product, *crowdfunding price* p_1 and *retail price* p_2 ($p_1 \leq p_2$), in the two different stages. We assume the retail price p_2 is exogenously determined due to competitive market[3]. Then, the firm's problem is to determine the crowdfunding price, or equivalently, the crowdfunding reward $r_p = p_2 - p_1$. The firm behaves as a Stackelberg leader by announcing the pricing scheme $\{p_1, p_2\}$ at the beginning of the crowdfunding stage. The marginal production cost is c ($p_2 \geq p_1 \geq c > 0$).

The total potential market size is fixed and denoted by M. Due to uncertain social learning on product quality, the actual demand (a portion of market size M) is random. Each customer purchases at most one unit of the product. The sequence of events is as follows. At the beginning of the crowdfunding stage, all consumers arrive and make their purchasing-or-waiting decisions depending on their own utilities. Then in the retail stage, the consumers waiting from period 1 (if any) make their purchasing-or-leaving decision based on the social learning outcome. The social learning could be based on the product reviews generated by those consumers who already purchased in the crowdfunding stage. Finally, any consumers remaining in the market purchase a unit provided that the expected utility from doing so is non-negative. Figure 3.1 summarizes the timing of events.

[3]For strategic consumers, purchase behaviors are mainly driven by the intertemporal price difference. Hence, we assume the retail price is exogenous and focus on the impact of reward choice on consumers' strategic purchase behaviors. This assumption helps maintain analytical tractability and deliver clean insights. We relax this assumption by endogenizing the decision of retail price p_2 and our main insights remain valid (see Appendix B for details).

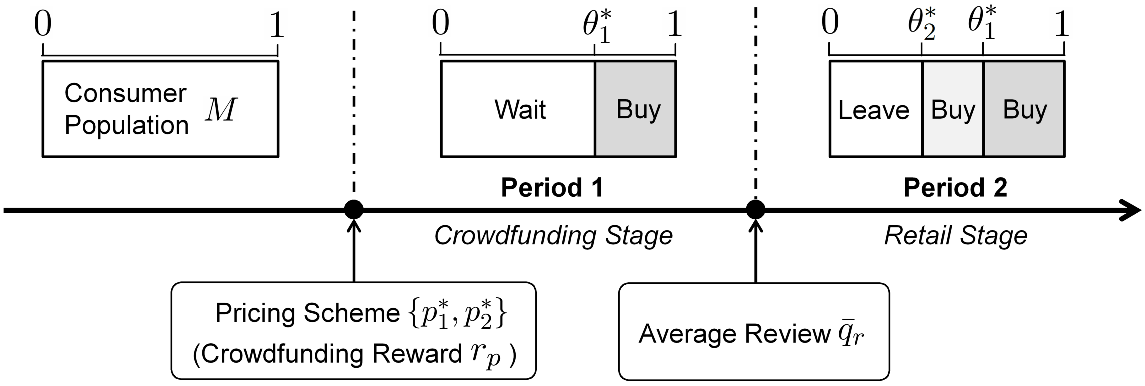


Figure 2.1.: Timing of Events

Each consumer's *utility* for purchasing the new product is $v+q+s(\rho)$, which comprises three components as explained below [55, 64]:

(1) *Preference Value* v: This reflects a customer's idiosyncratic preference over the product's *observable* attributes (e.g., brand, exterior design). It is known to the consumer ex ante, but the value could be different among consumers. For simplicity, we assume v is distributed according to the uniform distribution $U[0, 1]$. Relaxing the uniform assumption will not affect the qualitative results.

(2) *Quality Perception* q: The component q represents the utility derived from *unobservable* attributes before purchase (e.g., product usability) and can be referred to as the product's quality. The value of q is unknown ex ante but will be realized once the product is experienced by the consumer. We use Q to denote the random quality in both periods (i.e., q is the realization of Q). Although the exact value of Q is unobservable, its distribution is the subject of social learning for both the firm and consumers. We assume that the distribution of ex post quality perception is Normal, $Q \sim N(\mu_q, \sigma_q^2)$, where μ_q is the product's *true quality* and σ_q captures the heterogeneity in post-purchase quality perception or illustrates the noise in product reviews. To simplify analysis, we follow the literature to assume that the probability of a negative quality perception is negligible. The firm and consumers share a common prior belief over μ_q. This belief is modeled by a Normal prior $\mu_q \sim N(\mu_0, \sigma_0^2)$, where we set $\mu_0 = 0$ without loss of generality[4]. The social learning process of μ_q will be explained in more detail later.

[4]When $\mu_0 \neq 0$ (e.g., a positive prior mean), we can always normalize this term to zero by adjusting the price components (retail price and crowdfunding price) accordingly.

(3) *Social Utility* $s(\rho)$: $s(\rho)$ represents the network effect on consumer's utility, where $\rho \in [0, 1]$ is the fraction of consumers who have purchased the product. For clarity of analysis, we assume the social utility has a linear functional format: $s(\rho) = k\rho$, $k \geq 0$. The coefficient k may be called the *network effect intensity*, which represents the increase in consumers' willing-to-pay when the whole population joins the consumption network. Three remarks are worth mentioning regarding this social utility function. First, by the social utility function, we specifically consider the *global positive network externalities*. Second, since the market size M is fixed, this function is equivalent to the one that replaces the population fraction ρ with the network size ρM (see [71] and [72]). Third, it is also equivalent to the general linear function $s(\rho) = a + k\rho M$ adopted by [73], because the fixed intercept can be accounted as the corresponding shift in the price components. Now, we categorize the network structures into three different scenarios depending on the network effect intensity parameter.

Definition 2.3.1 (Structure of Network Externalities) *We define the following three scenarios based on the network effect intensity k: (i) weak network externalities (WN) for $0 < k < 1$; (ii) boundary scenario (BS) for $k = 1$; (iii) strong network externalities (SN) for $k > 1$.*

Similar categorization of network externalities based on product types has been discussed in the literature (see, e.g., [63] for similar argument). For some crowdfunding products, consumers would assign more weight to their intrinsic preferences (i.e., $k < 1$), whereas for others, social utility plays a crucial role and can even dominate their weight on personal preference (i.e., $k > 1$). It will be shown later that the equilibrium consumer behavior varies significantly across the three scenarios.

Finally, we make the following assumption to facilitate model analysis. When writing a quality review, the consumers report their quality perceptions rather than the net utility. Alternatively, one may argue that the reviews should reflect the net utility experienced by consumers, which depends on all three utility components as well as the price [74, 75]. In such alternative model, the reviews could be biased because, for example, the consumers with higher preference values are more likely to purchase early and submit a positive review. Nevertheless, this model can be shown to be equivalent to ours if the consumers are rational in the sense that they are able to "debias" or "correct" the reviews to only report quality perceptions. Therefore, for simplicity, we assume that the reviews are already debiased [54, 55].

2.3.2 Bayesian Social Learning

All customers who purchase the product in the first period report their experienced product quality through product reviews (e.g., via an online review platform). In the second period, the firm and consumers observe the reviews of the first-period buyers and update their belief over the product's true quality from μ_0 to μ_1 using Bayes' rule. Given the purchasing fraction ρ_1 in the first period and the average quality review $\bar{q}_r$, the *posterior of true quality* is normally distributed, $\mu_q|\bar{q}_r \sim N(\mu_1, \sigma_1^2)$, with

$$\mu_1 = \frac{\rho_1 M\gamma}{\rho_1 M\gamma + 1}\bar{q}_r, \qquad \sigma_1^2 = \frac{\sigma_0^2}{\rho_1 M\gamma + 1}, \tag{2.1}$$

where the ratio $\gamma = \frac{\sigma_0^2}{\sigma_q^2}$ measures the degree of ex ante quality uncertainty relative to the uncertainty in individual product reviews. We refer to γ as the *learning effectiveness*, since a larger γ means that the social learning process is more effective in shaping the quality perceptions of future buyers.

Now consider a customer's decision to purchase or not during the crowdfunding stage. Suppose the consumer forms a *rational belief* over her second-period *expected quality perception* $\mathbb{E}^1_{\mathbb{Q}}(\widetilde{Q}) = \mu_1$, which is the *expected* posterior mean of the true quality μ_q (hereafter, we use $\widetilde{Q}$ to denote the consumer's belief of ex post quality perception). Since in the second period, consumers still cannot observe the true quality due to review noises, this *posterior expected quality* μ_1 will be critical in the consumer's purchase decision. The ex ante *rational belief* of μ_1 is defined as its *prior predictive distribution*. It is also referred to in the literature as the ex ante distribution of the ex post mean belief, or the *preposterior distribution* [55].

Lemma 5 *Given the purchasing fraction ρ_1 in the crowdfunding stage, the rational belief over the second-period expected quality perception $\mathbb{E}^1_{\mathbb{Q}}(\widetilde{Q})$ can be represented by the following distribution:*

$$\mathbb{E}^1_{\mathbb{Q}}(\widetilde{Q}) = \mu_1 \sim N\left(0, \frac{\rho_1 M\gamma}{\rho_1 M\gamma + 1}\sigma_0^2\right) \tag{2.2}$$

The proofs of all lemmas and propositions are presented in Appendix A. Expressions in (2.1) represent the Bayesian updating process about the true quality perception after reviews are observed, and Lemma 5 further shows the consumer's rational belief over the posterior mean. These results demonstrate the uncertainty structure related to the decision of strategic waiting and help to calculate the expected value of the "waiting" action. However, the consumer's expected value of purchasing in period 1 depends on the rational belief of the post-purchase true quality perception during the second period, and the consumer's ex ante expectation of post-purchase quality perception is zero (see Lemma 14 in Appendix A). The expected quality perception in the crowdfunding stage is zero, and this result will be used to derive the consumer's first-period expected utility later on. We denote the p.d.f. of the Normal distribution in (2.2) as $f(\mu_1|\rho_1)$, and denote the associated c.d.f. and c.c.d.f. as $F(\mu_1|\rho_1)$ and $\bar{F}(\mu_1|\rho_1)$, respectively.

2.3.3 Consumer Expected Utility

Based on the above learning framework, we can examine consumers' utility function componentwise. The social utility is given by $s(\rho) = k\rho$, where $\rho \in [0, 1]$ is the fraction of total consumers who purchase in the two periods. We denote by $\tilde{\rho}$ the consumer's ex ante belief of this total purchasing fraction. Then, it can be decomposed as $\tilde{\rho} = \tilde{\rho}_1 + \tilde{\rho}_2$, where $\tilde{\rho}_j$ is the belief of the purchasing fraction in period j. Note that the expected social utility can be calculated as the social utility under the expected purchasing fraction due to the linear structure of the function $s(\rho)$. In this regard, we shall focus on how consumers form expectation of the total purchasing fraction $\tilde{\rho}$.

In addition, it is worth noting that there is an internal connection between $\tilde{\rho}_1$ and $\tilde{\rho}_2$, as how many consumers choose to purchase in period 2 depends on both how many have purchased in period 1 and the quality learning through reviews. Fewer consumers purchasing in period 2 could be a result of either more consumers purchasing in period 1, or a lower posterior quality expectation. We clarify this interrelation by denoting $\tilde{\rho}_2$ as $\tilde{\rho}_2(\mu_1|\tilde{\rho}_1)$, a function of second-period expected quality perception μ_1, conditional on the belief of first-period purchasing fraction $\tilde{\rho}_1$. Detailed functional form of $\tilde{\rho}_2(\mu_1|\tilde{\rho}_1)$ will be introduced later in Proposition 2.4.1 when we characterize the equilibrium for the P-L game. Hence, the consumer's expected social utility can be written as $\mathbb{E}^0_{\mathbb{Q}}[s(\tilde{\rho})|\tilde{\rho}_1] = \mathbb{E}^0_{\mathbb{Q}}[k(\tilde{\rho}_1 + \tilde{\rho}_2(\mu_1|\tilde{\rho}_1))] = k\tilde{\rho}_1 + k\mathbb{E}^0_{\mathbb{Q}}[\tilde{\rho}_2(\mu_1|\tilde{\rho}_1)]$, where the expectation $\mathbb{E}^0_{\mathbb{Q}}$ is taken over the quality uncertainty (uncertainty in μ_1 stemming from both reviews and post-purchase quality perceptions) based on the prior predictive distribution in Lemma 5.

The driver of a consumer's expected social utility in period 1 is two-fold: the belief of first-period purchasing fraction $\tilde{\rho}_1$, which only depends on the first-period decision, and the belief of second-period purchasing fraction $\tilde{\rho}_2$, which relies on the realization of reviewed quality information $\bar{q}_r$ at the beginning of period 2. If a consumer chooses to wait, her net utility becomes $[v + \mu_1 + k(\tilde{\rho}_1 + \tilde{\rho}_2(\mu_1|\tilde{\rho}_1)) - p_2]^+$ and waiting comes with the option of leaving with zero utility if the posterior quality is too low. For notational convenience, hereafter we simply use ρ_i to denote $\tilde{\rho}_i$. A consumer's *expected net utility* from purchasing the product in the crowdfunding stage and from waiting in the market until the retail stage can be summarized as follows:

$$u_1(v, \rho_1) = v + \mathbb{E}^0_{\mathbb{Q}}[k(\rho_1 + \rho_2(\mu_1|\rho_1))] - p_1, \tag{2.3}$$

$$u_2(v, \rho_1) = \mathbb{E}^0_{\mathbb{Q}}\{[v + \mu_1 + k(\rho_1 + \rho_2(\mu_1|\rho_1)) - p_2]^+\}. \tag{2.4}$$

We use the following notations in the rest of the paper. Let ϕ (Φ) denote the standard Normal probability density function (cumulative distribution function). Let $\vee$ ($\wedge$) denote the maximizing (minimizing) operator, i.e., $x \vee y = \max(x, y)$ and $x \wedge y = \min(x, y)$. $\mathbb{1}\{\cdot\}$ is used to represent the indicator function and $(x)^+ = \max(x, 0)$. Monotonic relations (increasing, decreasing) are used in the weak sense unless otherwise stated.

2.4 Game Analysis

In this section, we use the fulfilled expectations equilibrium (FEE) concept to characterize the game outcome. For easy reference, we denote the purchasing game in periods 1 and 2 as *P-W game* (Purchasing-Waiting) and *P-L game* (Purchasing-Leaving), respectively. The fulfilled expectations equilibrium concept was proposed by [57], and since then has been widely adopted in the literature involving network externalities [62]. It is also called *self-fulfilling expectations equilibrium* [37] or *rational expectations equilibrium* [76–78].

Definition 2.4.1 (Fulfilled Expectations Equilibrium) *A fulfilled expectations equilibrium (FEE) is defined as follows: The consumers form a common belief that the fraction of population using the product is z, and if each consumer makes her purchasing decision based on this expectation, then the consumer fraction who actually purchases is z.*

Consider any crowdfunding price p_1 and the retail price p_2. The consumers need to decide whether to purchase in the first period. Specifically, a consumer with a preference value v compares between the expected surplus gained from immediate purchase and the expected surplus associated with waiting. It can be shown that for any crowdfunding price offered by the firm and any purchasing behavior of the other consumers in the market, it is optimal for an individual consumer to adopt a *threshold policy* in both periods (see the appendix for the proof). Namely, the consumer will purchase a unit if $v \geq \theta_1$; otherwise, the consumer will wait for the retail stage (and purchase if $v \geq \theta_2$). Note here, θ_1 can be obtained at the beginning of period 1 (details will be in P-W game analysis), while θ_2 is only known after the quality reviews are revealed in period 2.

2.4.1 Analysis of P-L Game

We first analyze the P-L game among the consumers in period 2. Given the threshold policy θ_1 in the P-W game (i.e., the purchasing fraction $\rho_1 = 1 - \theta_1$) and the average rating of quality reviews $\bar{q}_r$, a customer first forms the expectation of threshold policy θ_2 adopted by the newly arrived population in period 2, and purchases the product iff

$$v + \frac{(1-\theta_1)M\gamma}{(1-\theta_1)M\gamma+1}\bar{q}_r + k[(1-\theta_1) + (\theta_1-\theta_2)^+] - p_2 \geq 0, \tag{2.5}$$

where $(\theta_1 - \theta_2)^+$ is the belief of the purchase fraction from the first-period waiting consumers. Now define an *outcome function* $G_{[0,1]}(x;\theta_1)$ as follows:

$$G_{[0,1]}(x;\theta_1) = \Xi_{[0,1]}\left\{p_2 - \frac{(1-\theta_1)M\gamma}{(1-\theta_1)M\gamma+1}\bar{q}_r - k[(1-\theta_1)+(\theta_1-x)^+]\right\}, \tag{2.6}$$

where $\Xi_{[0,1]}\{y\}$ is the projection of y onto the line segment $[0,1]$. That is, $\Xi_{[0,1]}\{y\} = 1$, if $y > 1$; $\Xi_{[0,1]}\{y\} = 0$, if $y < 1$; otherwise $\Xi_{[0,1]}\{y\} = y$. Under the FEE paradigm, if every consumer expects a threshold policy x adopted in period 2, then the threshold policy becomes $G_{[0,1]}(x;\theta_1)$. Thus, the FEE in the P-L game is essentially the fixed point of the outcome function. For a given θ_1, the outcome function $G_{[0,1]}(x;\theta_1)$ can be depicted as in Figure C.1.

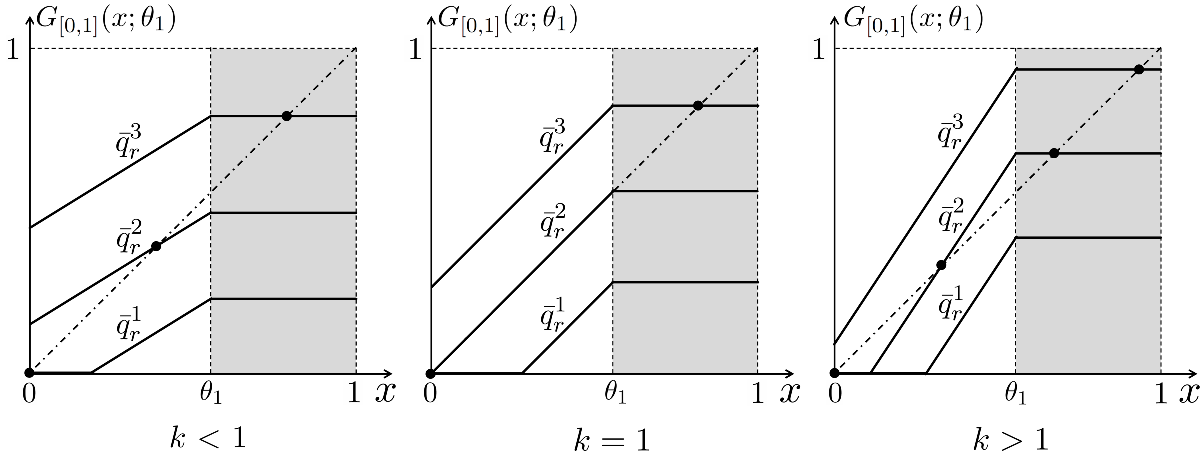


Figure 2.2.: Structure of Outcome Function $G_{[0,1]}(x;\theta_1)$ with $\bar{q}_r^1 > \bar{q}_r^2 > \bar{q}_r^3$

A few observations are worth highlighting. First, for a given threshold policy θ_1 adopted in period 1, all the remaining consumers in the P-L game must have valuations $v < \theta_1$ since the fraction of consumers with higher preference values have already purchased and left the market. Thus, none of the consumers buys in period 2 if $\theta_2 \geq \theta_1$, i.e., $(\theta_1 - \theta_2)^+ = 0$ (as in the shadowed region in the figure). We may view the threshold policy as $\theta_2 = \theta_1$ when $\theta_2 \geq \theta_1$. Second, there exists at least one feasible fixed point for the outcome function $G_{[0,1]}(x;\theta_1)$ for any network effect intensity k and quality review $\bar{q}_r$ (see the dots in the figure). Third, when $k < 1$, there exists exactly one fixed point, or a unique FEE. However, when $k \geq 1$, there may be multiple fixed points for the outcome function, which means we need equilibrium refinement. Fourth, the threshold policy under the P-L game FEE depends on the average quality review $\bar{q}_r$. When the realized quality reviews are relatively low, there is no purchase in period 2. When the realized quality reviews are relatively high, there exists an FEE and its property depends on the strength of network externalities.

When there are multiple FEEs under $k \geq 1$, we focus on the smallest one ($\theta_2 = 0$), which we call *preferred FEE*. There are two reasons for this equilibrium refinement. First, a smaller FEE θ_2 leads to more purchases by consumers, and thus more surplus for the customers (more social utility), and higher profit for the firm. So, it is a Pareto dominant outcome. Second, the smallest equilibrium point under strong network externalities corresponds to the stable equilibrium, according to the network analysis literature [79]. Therefore, we will focus on the smallest FEE as the unique equilibrium in the P-L game in period 2, the form of which depends on the network effect intensity k. We define a new function

$$\mathcal{P}(\theta_1, \bar{q}_r) = p_2 - \frac{(1-\theta_1)M\gamma}{(1-\theta_1)M\gamma + 1}\bar{q}_r, \tag{2.7}$$

which represents the consumer's effective retail price after subtracting the posterior expected quality perception. Hereafter, we use superscript w, b, and s to indicate the three scenarios: WN, BS, and SN, respectively. Now, we characterize the unique FEE in the P-L game.

Proposition 2.4.1 *Given the threshold policy θ_1 adopted in period 1 and the realized average quality review $\bar{q}_r$, there exists a unique FEE in the P-L game that can be characterized as follows:*

$$\theta_2^{i*} = \left\{\begin{array}{ll} \theta_1 \wedge \dfrac{(\mathcal{P}(\theta_1, \bar{q}_r) - k)^+}{1-k}, & i = w \\ \theta_1 \mathbb{1}\{\mathcal{P}(\theta_1, \bar{q}_r) - k \leq 0\}, & i = b, s \end{array}\right\}. \tag{2.8}$$

Based on the FEE threshold θ_2^{i*} in the P-L game, we obtain the purchasing patterns under different network structures shown in Figure 2.3, where $\mu_1 = \frac{(1-\theta_1)M\gamma}{(1-\theta_1)M\gamma+1}\bar{q}_r$ is the posterior expected quality after observing the average review $\bar{q}_r$. There are two thresholds $z(\theta_1)$ and $z(0)$, where the function $z(\theta_1)$ is given by $z(\theta_1) = (k-1)\theta_1 + p_2 - k$. The left plot shows the scenario of weak network externalities ($0 < k < 1$). When $\mu_1 \leq z(\theta_1)$, there is $\theta_2^{w*} = \theta_1$, which means none of the waiting consumers will buy due to the low quality review. When $\mu_1 \geq z(0)$, the quality review is relatively high, and all waiting consumers will buy with $\theta_2^{w*} = 0$. As in the medium range of posterior quality, there is a positive fraction of consumers who will buy in the second period (i.e., $0 < \theta_2^{w*} < \theta_1$). Since θ_2^{w*} is decreasing in μ_1 within this range, it means more consumers will purchase as the quality review improves.

However, when the network externalities are relatively strong ($k \geq 1$), there are only two extreme cases that would happen (middle and right plots). In one case, none of the waiting consumers will buy if the average quality review is lower than a threshold. In the other case, all remaining consumers will buy due to the relatively high quality review. The boundary scenario (BS) can be viewed as the limit of the strong network externalities scenario (SN) and does not add new insights. Hence, the subsequent analysis will focus on the WN and SN scenarios. More detailed analysis of the BS case is relegated to Appendix C.

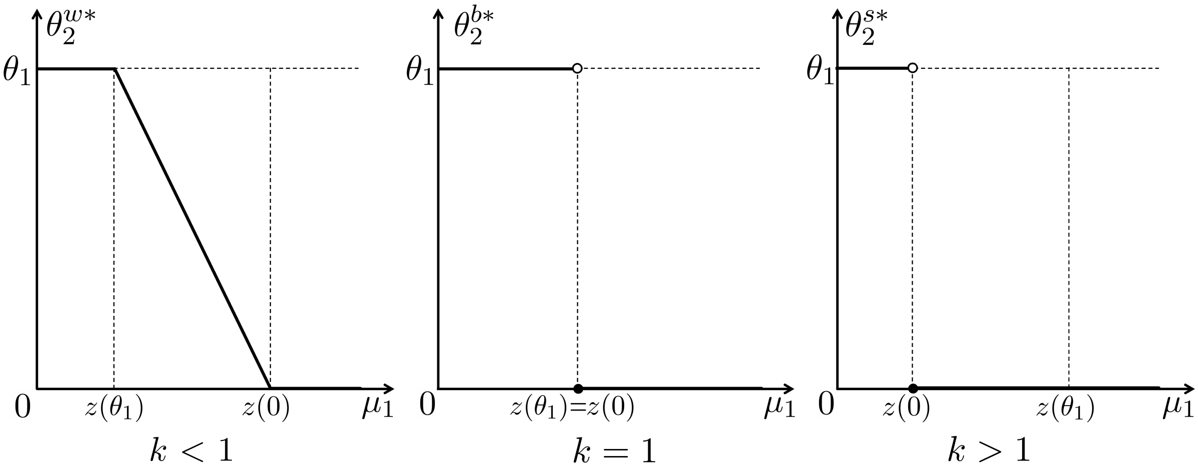


Figure 2.3.: FEE Characterization in the P-L Game

2.4.2 Analysis of P-W Game

Based on the outcome of the P-L game in period 2, now we consider the P-W game in period 1. For any crowdfunding reward r_p (or crowdfunding price p_1) offered by the firm, the consumers need to decide whether to purchase in period 1 or to wait until period 2. Specifically, a consumer compares the expected surplus gained from immediate purchase and the expected surplus associated with waiting. However, the expected surplus associated with waiting depends on both the purchasing policy adopted in period 1 and the realization of posterior expected quality. As the posterior expected quality is ex ante uncertain (depending on the realization of quality reviews), the consumers in the P-W game need to form rational beliefs over the equilibrium outcome in the second-period P-L game, conditioned on their beliefs of the purchasing policy θ_1 adopted in period 1. Then, the first-period consumers' conditional expectation of the total social utility $\mathbb{E}_{\mathbb{Q}}^{0}[s(\rho)|\theta_1]$ can be derived based on those rational beliefs. Since the FEE in the P-L game depends on the network externalities, we will separate the discussion of the P-W game into the two scenarios (WN, SN) accordingly.

First, consider the weak network externalities scenario (WN) with $k < 1$. From Proposition 2.4.1, we know when the posterior expected quality is low (i.e., $\mu_1 \leq z(\theta_1)$), none of the waiting consumers will buy and hence the total social utility becomes $k(1-\theta_1)$. When the posterior expected quality is high (i.e., $\mu_1 \geq z(0)$), all waiting consumers will buy and the total social utility is k. In the medium range, i.e., $z(\theta_1) \leq \mu_1 \leq z(0)$, θ_2 is determined through the equation $k(1-\theta_2)+\theta_2 = p_2-\mu_1$. Thus, we have $\theta_2 = \frac{p_2-\mu_1-k}{1-k}$ and the total social utility is $k(1-\theta_2) = k\big(1-\frac{p_2-\mu_1-k}{1-k}\big)$. Therefore, the consumer's expected total social utility conditional on the belief of threshold policy θ_1 in period 1 can be calculated as follows:

$$\mathbb{E}^0_{\mathbb{Q}}[s(\rho)|\theta_1] = \underbrace{k(1-\theta_1)F\big(z(\theta_1)|1-\theta_1\big)}_{\text{social utility under low quality}} \quad + \quad \underbrace{k\bar{F}\big(z(0)|1-\theta_1\big)}_{\text{social utility under high quality}}$$
$$+ \underbrace{\int_{z(\theta_1)}^{z(0)} k\left(1-\frac{p_2-\mu_1-k}{1-k}\right) f(\mu_1|1-\theta_1)d\mu_1}_{\text{social utility under medium quality}}, \tag{2.9}$$

where $f(\cdot|1-\theta_1)$ and $F(\cdot|1-\theta_1)$ are the p.d.f. and c.d.f. of the second-period expected quality perception μ_1. Based on Lemma 5, we know $\mu_1 \sim N\big(0, \frac{(1-\theta_1)M\gamma}{(1-\theta_1)M\gamma+1}\sigma_0^2\big)$.

Consider the case when $\theta_1^* \in (0,1)$, i.e., excluding the extreme cases where either all consumers purchase or no consumer purchases in the first period. Detailed discussion about the boundary cases will be presented later. By the definition of FEE, the threshold policy $\theta_1^* \in (0,1)$ in the P-W game in period 1 can be solved through the following FEE equation:

$$\theta_1+\mathbb{E}^0_{\mathbb{Q}}[s(\rho)|\theta_1]-p_1 = \int_{z(\theta_1)}^{+\infty}\left[\mu_1+\theta_1+k\left(1-\frac{(p_2-\mu_1-k)^+}{1-k}\right)-p_2\right]f(\mu_1|1-\theta_1)d\mu_1. \tag{2.10}$$

The above condition demonstrates how the firm's pricing scheme $\{p_1,p_2\}$ affects the consumers' rational expectation of the threshold policy in period 1, and thus the purchasing behavior in the P-W game. Plugging equation (2.9) into equation (2.10), after simplification we have

$$\theta_1 + k(1-\theta_1)F\big(z(\theta_1)|1-\theta_1\big) - p_1 = \int_{z(\theta_1)}^{+\infty}(\mu_1+\theta_1-p_2)f(\mu_1|1-\theta_1)d\mu_1. \tag{2.11}$$

Now, define the *potential regret function* $H_w(\theta_1)$ under weak network externalities as

$$H_w(\theta_1) = \int_{z(\theta_1)}^{+\infty}\mu_1 f(\mu_1|1-\theta_1)d\mu_1 + z(\theta_1)F\big(z(\theta_1)|1-\theta_1\big). \tag{2.12}$$

Then, the FEE equation (2.11) is equivalent to $H_w(\theta_1) = p_2-p_1 = r_p$. We call $H_w(\theta_1)$ the *potential regret function* since it essentially represents the consumer's *potential regret* of early purchasing if she waits to see the realized product quality and makes the purchase decision afterwards. Hence, the crowdfunding reward is offered to induce consumers' purchases in the crowdfunding stage. It is more intuitive if we rewrite the function as

$$H_w(\theta_1) = \int_{-\infty}^{z(\theta_1)} \{\underbrace{p_2 - [\theta_1 + k(1-\theta_1) + \mu_1]}_{\text{net disutility under buy-mistake}}\} f(\mu_1|1-\theta_1)d\mu_1 \tag{2.13}$$

$$= \int_{-\infty}^{z(\theta_1)} \{\underbrace{p_2 - k(1-\theta_1)}_{\text{hedonic price}} - \theta_1 - \mu_1\} f(\mu_1|1-\theta_1)d\mu_1. \tag{2.14}$$

The first equality shows the consumers' potential regret function, which essentially is the *expected* net disutility under the wrong purchase decision. In the second equality, we refer to $p_2 - k(1-\theta_1)$ as the *hedonic retail price* (following [57]), i.e., the retail price adjusted for the benefit of network externalities. Note that the hedonic retail price depends on both the network effect intensity k and the equilibrium threshold θ_1, and it will in turn determine the equilibrium threshold θ_1 as shown in the potential regret function. The consumers will strategically make use of this important feature to form the FEE in the crowdfunding stage.

Note that this potential regret function plays a key role in understanding the purchasing dynamics in the P-W game in the crowdfunding period. Similarly, we can define the potential regret function for the scenarios of strong network externalities (SN). Hereafter, we use $\sigma^2(\theta_1) = \frac{(1-\theta_1)M\gamma\sigma_0^2}{(1-\theta_1)M\gamma+1}$, and $\sigma(\theta_1) = \sqrt{\frac{(1-\theta_1)M\gamma\sigma_0^2}{(1-\theta_1)M\gamma+1}}$ to denote the variance and standard deviation of μ_1. We summarize the preliminary technical results about the potential regret functions in the next lemma.

Lemma 6 *The FEE equations can be summarized as $H_i(\theta_1) = r_p$, where*

$$H_i(\theta_1) = \begin{Bmatrix} \sigma(\theta_1)\phi[z(\theta_1)/\sigma(\theta_1)] + z(\theta_1)\Phi[z(\theta_1)/\sigma(\theta_1)], & i = w \\ \sigma(\theta_1)\phi[z(0)/\sigma(\theta_1)] + z(\theta_1)\Phi[z(0)/\sigma(\theta_1)], & i = s \end{Bmatrix}. \tag{2.15}$$

As stated in Lemma 6 and illustrated in Figure 2.4, the potential regret functions under different network externalities exhibit different structural properties, and induce different equilibrium patterns. From the left figure, we can identify the strictly decreasing pattern for the potential regret function under WN, which is formally shown in Lemma 15. The monotonicity of the potential regret function ensures the uniqueness of the FEE under WN. When the FEE equation has no solution, either $H_w(\theta_1) > r_p$ for $\forall\theta_1$ in which case $\theta_1 = 1$ is the FEE, or $H_w(\theta_1) < r_p$ for $\forall\theta_1$ in which case $\theta_1 = 0$ is the FEE. The existence of FEE can be guaranteed by including the boundary case where $\theta_1 = 0,\ 1$.

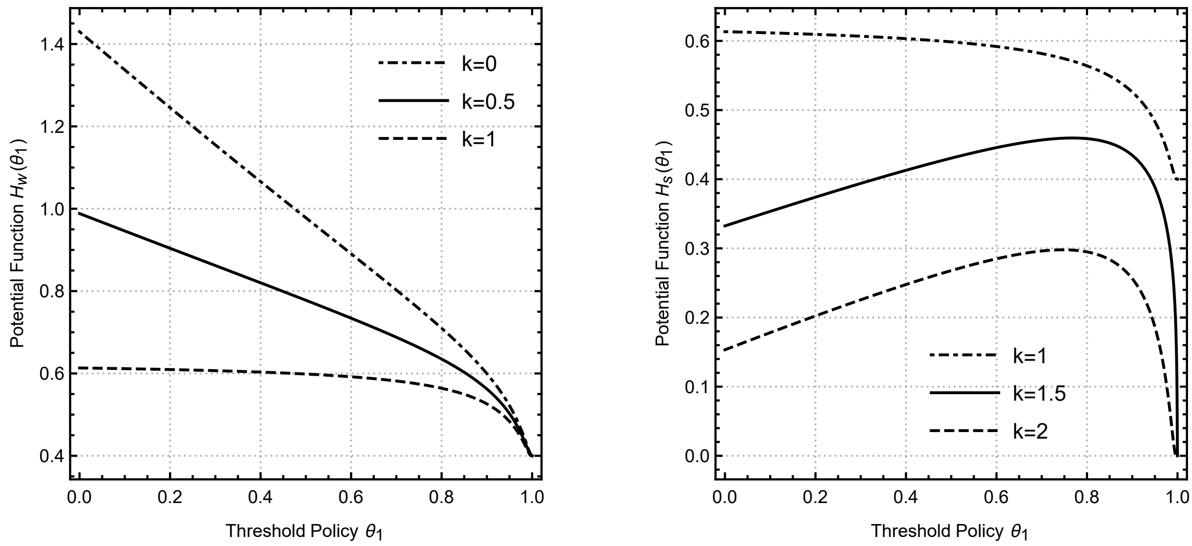


Figure 2.4.: An illustration of the Potential Regret Function $H_i(\theta_1)$

However, under SN (see the right plot in Figure 2.4), the potential regret function is not monotonic in general, which results in multiple solutions to the FEE equation (i.e., multiple equilibria) for a given crowdfunding reward. So multiple FEEs may exist and we apply similar equilibrium refinement as in the P-L game analysis, by focusing on the smallest FEE.

The following proposition highlights some important implications of the P-W game dynamics based on the structure of potential regret function $H_i(\theta_1)$ and the FEE equation $H_i(\theta_1) = r_p$.

Proposition 2.4.2 *(i) The crowdfunding market always exhibits adoption inertia under zero crowdfunding reward. (Formally, $H_i(\theta) > 0$, $\forall \theta_1 \in [0, 1)$, $i \in \{w, s\}$.)*
(ii) Under WN, the crowdfunding market experiences adoption inertia if the crowdfunding price is larger than or equal to 1. (Formally, when $k < 1$, $H_w(1) \geq (p_2 - 1)^+$.)
(iii) Under WN, for a given crowdfunding reward, more consumers choose to wait as the network effect intensity decreases, market size increases, prior quality uncertainty increases, or review noise decreases. (Formally, for a given r_p, θ_1^w increases in M and σ_0^2, while decreasing in k and σ_q^2.)

Part (i) indicates an important pricing structure in crowdfunding under any network externalities structure, i.e., the crowdfunding price should always be lower than the retail price. The consumers are tempted to exploit social learning of the uncertain quality, which entails deferring purchase. In fact, this tendency can be so overwhelming that every consumer defers adoption when no reward is offered in crowdfunding. We call such a market outcome *adoption inertia*. This well explains why we always need a reward in crowdfunding. If there is no discount in the crowdfunding price, it is clearly a better option for consumers to wait until more quality information is disclosed. Product quality uncertainty in crowdfunding puts consumers in a risky position, and hence a reward is needed to compensate for the potential regret of early purchase in the crowdfunding stage. Part (*ii*) and (*iii*) focus on the WN case (as equilibrium is unique) and offer additional insights about firms' feasible pricing range and the impact of some key parameters.

Part (*ii*) reveals an interesting phenomenon: The crowdfunding price must be less than 1 in order to avoid adoption inertia in the crowdfunding market. When the retail price is relatively low ($p_2 \leq 1$), this result is intuitive as part (*i*) has shown the crowdfunding price must be less than the retail price to avoid adoption inertia. When the retail price is relatively high ($p_2 > 1$), the feasible crowdfunding price should be less than 1. To better understand the rationale behind this result, we consider the consumer j with the highest valuation $v_j = 1$. Suppose the consumer j is the only consumer who adopts the product early in crowdfunding (i.e., $\theta_1 = 1$). As one consumer's review is negligible in the social learning process (i.e., $\sigma(1) = 0$), the quality perception component remains unchanged (zero) in period 2, and so does the consumer j's purchase decision. The consumer j's expected net utility in period 1 becomes $u_1(v, \rho_1) = u_1(1, 0) = 1 - p_1$, which indicates the crowdfunding price should be less than 1.

Part (*iii*) provides a summary of how changes in network effect intensity, market size, prior quality uncertainty, and review noise may affect the equilibrium purchasing patterns. Since the network effect intensity has a direct impact on consumers' utility, it is straightforward to see fewer consumers purchase as the network externalities become weaker. For the other three parameters, the intuition is that as the market size increases, the prior quality uncertainty increases, or the review noise decreases, the potential value of social learning increases, hence more consumers choose to wait.

2.5 Optimal Reward Choice

Optimizing the crowdfunding reward choice is a convoluted task for the firm owing to the interaction between its pricing decisions, the adoption decisions of strategic consumers, and the ex ante uncertain effects of the two-sided social learning process on product quality perceptions. The firm's pricing policy will affect consumers' purchasing incentives, which, in turn, will affect sales at different stages and total profitability. The analysis of this section centers around two main questions. The first pertains to the optimal crowdfunding pricing policy: How should the firm adjust its crowdfunding reward r_p to accommodate the strategic purchasing behavior under different scenarios of network externalities? The second question concerns the impact of several important parameters on the equilibrium outcomes: What are the impacts of production cost, quality uncertainty, and social learning on the firm's optimal reward choice and profitability?

2.5.1 Optimal Reward Choice under Perfect Learning

For transparent analysis and clean insights, the subsequent analysis will focus on perfect learning, which is equivalent to the following condition: $M\gamma = M\sigma_0^2/\sigma_q^2 \to +\infty$. This assumption holds when the potential market size M, or the relative ratio of prior quality uncertainty σ_0 over the review noises σ_q, is large enough. In crowdfunding practice, the potential market is indeed very large; for example, there are millions of potential contributors on platforms like Kickstarter and Indiegogo. In addition, the quality uncertainty of the product in crowdfunding is typically large since most crowdfunding products are in an early development stage.

Hereafter, we denote by $\ell(\theta_1)$ the *threshold function*,

$$\ell(\theta_1) = \frac{z(\theta_1)}{\sigma_0} = \frac{(k-1)\theta_1 + p_2 - k}{\sigma_0}. \tag{2.16}$$

Then, $\ell(0) = (p_2 - k)/\sigma_0$ and $\ell(1) = (p_2 - 1)/\sigma_0$. The p.d.f. and c.d.f. of μ_1 can be simply denoted as $f(\cdot)$ and $F(\cdot)$, without dependence on θ_1. Based on the results in Lemma 6, it is straightforward to derive the potential regret functions under perfect learning as follows,

$$H_w(\theta_1) = \sigma_0\phi[\ell(\theta_1)] + z(\theta_1)\Phi[\ell(\theta_1)], \tag{2.17}$$

$$H_s(\theta_1) = \sigma_0\phi[\ell(0)] + z(\theta_1)\Phi[\ell(0)]. \tag{2.18}$$

By investigating the structural property of the potential regret functions, we are able to characterize the impact of the crowdfunding price (i.e., the reward choice) on the crowdfunding demand (i.e., the equilibrium purchasing fraction in the crowdfunding stage), as summarized in the next proposition.

Proposition 2.5.1 *As the crowdfunding price increases, the crowdfunding demand decreases under WN, but increases under SN. (Formally, for* $\forall\theta_1 \in [0, 1]$*,* $H'_w(\theta_1) < 0$*,* $H'_s(\theta_1) > 0$*.)*

Under weak network externalities, a larger fraction of consumers will purchase in the crowdfunding stage if a higher reward is offered. But under strong externalities, it turns out that the crowdfunding demand increases as the crowdfunding reward decreases (i.e., as the crowdfunding price increases). This is an interesting finding because it means demand increases as price goes up. The economics literature documents this effect as the *Veblen effect* [80] and calls the product *Veblen good*. Veblen goods have upward-sloping demand curves, which contradicts the law of demand. By incorporating social considerations into consumer preferences, the economics literature has proposed the nonfunctional demand concept, i.e., the portion of demand not attributable to inherent quality. [80] formalizes *bandwagon*, *snob*, and *Veblen* effects, the last of which implies that consumers will pay a premium for products that convey higher status. [81] argues that the bandwagon effects can explain the pricing strategies observed in restaurants, theater shows, and other social events. [82] study the implications of snob and bandwagon effects on luxury goods taxation, and [83] examines the emergence of fashion cycles. Although their contexts differ, these papers share the idea that status seeking can induce a price premium for a product, regardless of the quality; furthermore, these papers rationalize upward-sloping demand curves under certain conditions. In our study, however, the consumers are fully rational and utility maximizer. Hence, we provide new conditions and novel rationale for the Veblen effect, without the presence of price premium induced by social status seeking.

A number of papers in marketing have analyzed the effect of conspicuous consumption on consumer demand as well as its implications for firm pricing and product line decisions. In a market made up of consumers who desire uniqueness (i.e., snobs or negative network externalities) and some others who desire conformism (i.e., followers or positive network externalities), [76] show that the demand from snobs might increase as the price increases while the overall demand decreases with price. They demonstrate that both snobs and followers must coexist for the existence of an upward-sloping demand curve. Focusing on a horizontally differentiated duopoly, [77] also show that in a market composed of snobs and followers, demand among snobs could increase as the price of a product increases. However, the demand among followers, as well as the total market demand, would decrease as price rises. The intuition for this result is that snobs prefer a higher-priced product if they expect the overall demand to be lower at the higher price, and such an expectation will be rational only if the followers have a downward-sloping demand curve. Therefore, in a market composed of either only snobs or only followers, the demand curve must be downward sloping. More recently, [84] show that the joint increase in price and demand for conspicuous goods may take place due to the change in the underlying durability choice of the firm as long as some snobbish consumers are present.

To summarize, the existing literature has principally focused on either social-related benefits or consumer's uniqueness-seeking feature (i.e., negative externalities, snobs) to explain the Veblen effect. In contrast to this stream of research, we incorporate the consumers as forward-looking social learners, and provide a novel explanation for the Veblen effect: It may be caused by the presence of both social learning and strong positive network externalities, even in the absence of snobbish consumers. The intuition is that forward-looking consumers strategically lower θ_1 to lock a higher social utility from crowdfunding sales in period 1 as the crowdfunding price increases. Due to the strong network externalities, the social utility increase from lowering the threshold θ_1 compensates for the decrease in the preference value v and the increase in crowdfunding price, eventually arriving at a new fulfilled expectations equilibrium point. From another perspective, the function $H_s(\theta_1)$ captures the *potential regret* for the consumer with preference value θ_1, given the purchasing threshold in crowdfunding is also θ_1. It is more intuitive if we rewrite FEE equation as follows:

$$H_s(\theta_1) = \int_{-\infty}^{z(0)} \{\underbrace{p_2 + (k-1)\theta_1 - k - \mu_1]}_{\text{net disutility under buy-mistake}}\} f(\mu_1) d\mu_1 = p_2 - p_1. \tag{2.19}$$

Due to strong network externalities (i.e., $k > 1$), the net disutility under the wrong purchase decision increases in θ_1. Hence, as the crowdfunding price p_1 increases, the consumer's potential regret of purchasing in crowdfunding should decrease, which is attained only by decreasing the equilibrium purchase threshold θ_1. Next we will further show that the Veblen effect in the crowdfunding stage is driven by the interaction between the two important features: social learning and strong positive network externalities, and it does not exist if either of the following two features is absent. Despite its importance, the role of network externalities in the presence of social learning among strategic consumers has not been well explored in the literature. The main objective of Proposition 2.5.2 is to demonstrate how social learning and strong network externalities jointly give rise to the interesting Veblen effect.

Proposition 2.5.2 *The Veblen effect in the crowdfunding stage does not exist if either of the following two features is absent: (i) social learning, (ii) strong positive network externalities.*

Before characterizing the optimal crowdfunding reward choice, we first derive the Pareto choice sets based on the FEEs obtained earlier. There exists a Pareto set $\mathcal{R}_p^w$ and $\mathcal{R}_p^s$ for the crowdfunding reward choice under WN and SN, respectively, which can be characterized as $\mathcal{R}_p^w = \big[H_w(1), H_w(0)\big]$, and $\mathcal{R}_p^s = \big[H_s(0), H_s(1)\big]$. Within these Pareto choice sets, the one-to-one correspondence between θ_1 and r_p can be ensured and characterized by the potential regret function. Suppose consumers' threshold policy in the P-W game is θ_1; it is straightforward to see the crowdfunding sales is $M(1-\theta_1)$, and hence profit in period 1 is $(p_1-c)M(1-\theta_1)$. But the firm's profit in period 2 is not clear as it depends on the uncertain quality learning process. The next lemma establishes the expected sales in period 2 given the equilibrium threshold policy θ_1 in period 1.

Lemma 7 *Given the threshold policy θ_1 adopted in the P-W game, the expected purchasing fraction in the P-L game has the following form:*

$$\rho_{2i}(\theta_1) = \begin{Bmatrix} \theta_1\Phi[-\ell(\theta_1)] + \chi(\theta_1), & i = w \\ \theta_1\Phi[-\ell(0)], & i = s \end{Bmatrix}, \tag{2.20}$$

where

$$\chi(\theta_1) = \frac{p_2-k}{1-k}\Big\{\Phi[\ell(\theta_1)] - \Phi[\ell(0)]\Big\} + \frac{\sigma_0}{1-k}\Big\{\phi[\ell(\theta_1)] - \phi[\ell(0)]\Big\}. \tag{2.21}$$

Then, the firm's expected total profit becomes $\Pi_i(p_1) = M(p_1-c)(1-\theta_1) + M(p_2-c)\rho_{2i}(\theta_1)$, $i \in \{w, s\}$. Since $H_i(\theta_1) = p_2 - p_1 = r_p$ in the P-W game equilibrium and there is a one-to-one mapping between θ_1 and r_p for $\theta_1 \in [0,1]$ (based on Proposition 2.5.1 and

Lemma 16), hereafter we can rewrite $\Pi_i(p_1)$ as a function of θ_1 and ignore the constant M:

$$\Pi_i(\theta_1) = \underbrace{(p_2 - c)[1 - \theta_1 + \rho_{2i}(\theta_1)]}_{\text{profit margin} \times \text{expected total sales}} - \underbrace{(1-\theta_1)H_i(\theta_1)}_{\text{cost of rewarding}}, \quad i \in \{w, s\}. \tag{2.22}$$

The first part of the profit function represents the projected total profit from the total sales but with the retail price p_2, and the second part reflects the profit loss from crowdfunding reward. Then, the analysis will be targeted to obtain the optimal θ_1 that maximizes the profit function $\Pi_i(\theta_1)$. Once we have the optimal θ_1, the optimal reward r_p (and the optimal crowdfunding price p_1) can be uniquely attained through the potential regret function $H_i(\theta_1)$.

In the WN case ($i = w$), the first-order derivative of the firm's profit function becomes $\Pi'_w(\theta_1) = \Phi[\ell(\theta_1)]\mathcal{B}(\theta_1)$, where

$$\mathcal{B}(\theta_1) = c - 2k + 1 + 2(k-1)\theta_1 + \sigma_0 \frac{\phi[\ell(\theta_1)]}{\Phi[\ell(\theta_1)]}. \tag{2.23}$$

After careful investigation (in the proof of the next proposition), we find that $\mathcal{B}(\theta_1)$ is strictly decreasing in θ_1, and thus the profit function $\Pi_w(\theta_1)$ is strictly quasi-concave in θ_1, i.e., the firm's profit function is unimodal on $[0, 1]$ (see the left plot in Figure 2.5). Hence, there exists a unique optimal threshold policy θ_1^{w*}. Denote $\theta_{\mathcal{B}}$ as the solution (if exists) to the first order condition $\mathcal{B}(\theta_1) = 0$. Then, we can obtain the optimal θ_1^{w*} and the corresponding optimal reward choice as presented in the next proposition. In the SN case ($i = s$), we have the profit function $\Pi_s(\theta_1) = (p_2 - c)[1 - \theta_1 + \rho_{2s}(\theta_1)] - (1-\theta_1)H_s(\theta_1)$. Further analysis shows the firm's profit function $\Pi_s(\theta_1)$ is strictly convex on $[0, 1]$ (see the right plot in Figure 2.5). Hence, we can conclude that the optimal θ_1 must be on the boundary of its domain, i.e., either 0 or 1. Also we only need to directly compare the following two values, $\Pi_s(0)$ and $\Pi_s(1)$.

Now, we summarize in the next proposition the firm's optimal crowdfunding reward choice under both weak and strong network externalities, and the induced equilibrium threshold policy in the crowdfunding stage.

Proposition 2.5.3 *The optimal crowdfunding reward is $r_p^{i*} = H_i(\theta_1^{i*})$, $i \in \{w, s\}$, where*

$$\theta_1^{w*} = \begin{Bmatrix} 0, & \text{if } \mathcal{B}(0) \le 0 \\ 1, & \text{if } \mathcal{B}(1) \ge 0 \\ \theta_{\mathcal{B}}, & o.w. \end{Bmatrix}, \quad \theta_1^{s*} = \begin{Bmatrix} 0, & \text{if } \Pi_s(0) \ge \Pi_s(1) \\ & \\ 1, & o.w. \end{Bmatrix}. \tag{2.24}$$

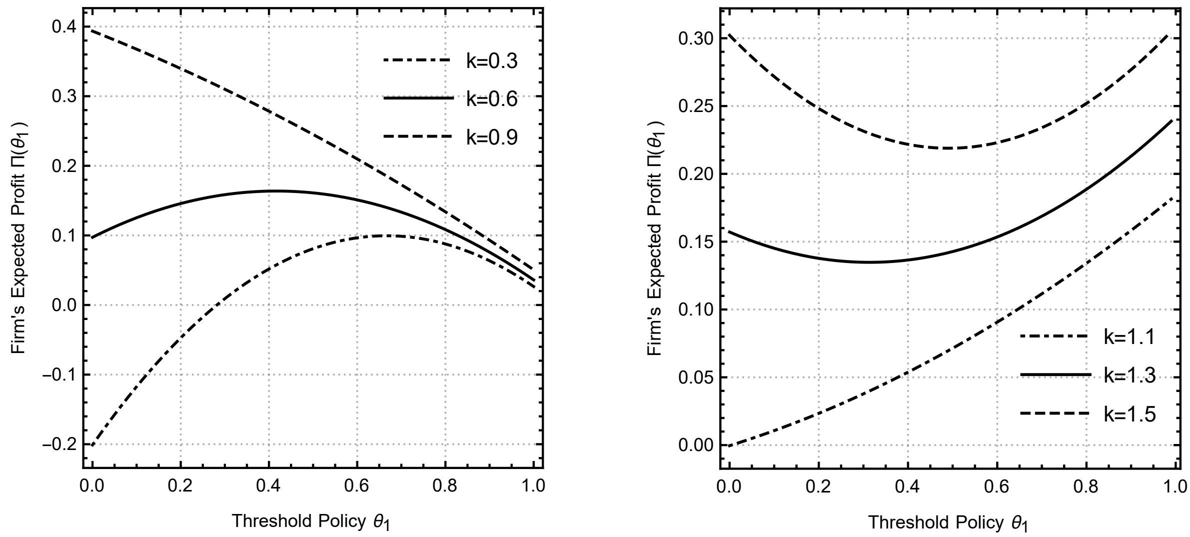


Figure 2.5.: Firm's Profit Function under WN (left) and SN (right)

Proposition 2.5.3 shows that network externalities have significant implications for the optimal crowdfunding reward choice. Under weak network externalities, the crowdfunding sales induced by the optimal reward can be any fraction of the population; by contrast, in the presence of strong network externalities, only extremely low and extremely high reward should be offered to induce either *adoption frenzy*, where all consumers purchase early although true quality is unknown, or *adoption inertia*, where all consumers defer purchase to the retail stage (i.e., there is essentially no crowdfunding). The two extreme patterns under SN in equilibrium are particularly interesting. One may intuit that it is always optimal to charge the highest possible price, inducing adoption frenzy in crowdfunding by leveraging the Veblen effect. However, the proposition points out this is not always an optimal strategy.

To elaborate the above result, we may explore in detail the underlying trade-off in the firm's rewarding decision. There two countervailing forces: the *immediate network benefit* and the *potential learning benefit*. The immediate network benefit represents the additional profit the firm can exploit immediately in the crowdfunding stage due to the positive network externalities, where a higher network effect intensity gives the firm a stronger leverage. The potential learning benefit captures the firm's tendency to limit the crowdfunding sales volume and expose itself to quality uncertainty. Driven by the high retail price and positive network externalities, the benefit of a high quality review would outweigh the loss associated with a low quality review. Note that the immediate network benefit is deterministic and increases in the network effect intensity k, while the potential learning benefit is ex ante uncertain (as it depends on the realization of quality reviews) and increases in k as well.

The equilibrium purchasing patterns in Proposition 2.5.3 can be explained by the trade-off between these two benefits. When the network effect intensity is relatively small (the WN case), the two benefits are close to each other; thus the optimal strategy is to take advantage of both benefits, resulting in any dispersed adoption under weak network externalities. However, as the network effect intensity becomes relatively large (the SN case), one benefit tends to dominate the other. When the immediate network benefit is dominant (e.g., the network effect intensity is high enough), the optimal reward strategy is to induce adoption frenzy; otherwise, adoption inertia is preferred to take advantage of the potential learning benefit (e.g., the prior quality uncertainty is high enough). The following proposition characterizes the conditions under which the adoption inertia strategy would be used by the firm.

Proposition 2.5.4 *The firm should not launch the crowdfunding campaign when:* (i) *the unit production cost is too high;* (ii) *the prior quality uncertainty is too high.* $($*Formally, for* $\forall i \in \{w, s\}$, (i) $\exists\ c^i_{max} \leq p_2$ *s.t.* $\theta_1^{i*} = 1$ *for* $c \geq c^i_{max}$; (ii) $\exists\ \sigma^i_{max} > 0$ *s.t.* $\theta_1^{i*} = 1$ *for* $\sigma_0 \geq \sigma^i_{max}$.$)$

When the potential learning benefit is the dominating one, the firm should not go crowdfunding and wait to sell the product until the quality is perfectly revealed. That means the firm is betting on the huge upside of quality uncertainty while forgoing the moderate immediate network benefit. As pointed out by Proposition 2.5.4, such scenarios arise when the unit production cost is too high (low immediate network benefit as pricing flexibility is limited), or when the prior quality uncertainty is too high (high potential learning benefit).

2.5.2 Effect of Social Learning

In this subsection, we examine the effect of social learning. For this purpose, we introduce a benchmark case without learning. This situation may happen when there is no product quality uncertainty (i.e., $\sigma_0 \to 0$), quality reviews are completely uninformative (i.e., $\sigma_q \to +\infty$), or consumers have no access to quality review information (e.g., Moov runs a crowdfunding campaign on its own custom-built website, without offering any review channel on the website).

Given the increasing pricing pattern, it is clear that consumers have no incentive to wait without the opportunity for social learning. Therefore, in the absence of social learning, consumers will either purchase in the crowdfunding stage or leave the market; hence there are no sales in the retail stage (see Proposition B.0.2 in Appendix A). Then, the firm's profit function without social learning becomes $\Pi_i^n(p_1) = (p_1 - c)(1 - \theta_1)$, $i \in \{w, s\}$. Here, we use the additional superscript n to denote the case without social learning. Without social learning, there is no potential learning benefit for the firm. Hence, the firm's optimal strategy is to exploit the immediate network benefit in the crowdfunding stage. In the next proposition, we characterize the unique equilibrium outcome for the WN and SN cases.

Proposition 2.5.5 *In the absence of social learning, the unique FEE θ_1^{n*} in the crowdfunding stage, the optimal crowdfunding price p_1^{n*}, and the optimal profit Π^{n*} can be characterized as follows:*

$$\left(\theta_1^{n*}, p_1^{n*}, \Pi^{n*}\right) = \left\{ \begin{array}{ll} \left(\dfrac{c+1-2k}{2(1-k)} \wedge 1, \dfrac{c+1}{2}, \dfrac{(1-c)^2}{4(1-k)}\right), & \textit{if } k < \dfrac{1+c}{2} < 1 \\ \left(0, k, k-c\right), & \textit{if } k \geq \dfrac{1+c}{2} \end{array} \right\}. \tag{2.25}$$

Note that the pricing strategy under $k \geq \frac{1+c}{2}$ simply sets the crowdfunding price p_1^{in*} equal to the network effect intensity k, which results in the adoption frenzy as $\theta_1^{in*} = 0$. Proposition 2.5.5 also suggests that the crowdfunding price under WN should satisfy $k \leq p_1^{wn*} = \frac{1+c}{2} \leq 1$ in the absence of social learning. Moreover, the unit production cost c must satisfy $c < 1$ under WN and $c < k$ under SN; otherwise the firm cannot make a positive profit.

Next, we compare the profits between the cases with and without social learning. The majority of literature on strategic customer behavior finds social learning valuable due to the fact that the beneficial *informational effect* of social learning outweighs the detrimental *behavioral effect* [55]. One exception is [56], which shows that the presence of social learning does not necessarily make the firm better off when the consumers are highly strategic. The key driver of the result is the difference in consumers' willingness-to-pay for the product's quality attributes. We find that social learning can have a negative impact on the firm's profit, especially when the unit production cost is low and the network effect intensity is high. The key difference of our model is the consideration of network externalities that introduce the interplay between social learning and network externalities.

2.5.3 Effect of Network Externalities

As introduced in the previous sections, a larger network effect intensity k indicates stronger network externalities. In this subsection, we investigate the impact of the *network effect intensity* k on the optimal reward choice, the crowdfunding sales, and the firm's optimal profit. Intuitively, as the network effect intensity increases, the firm's optimal profit should increase as consumer's gross utility increases. But what is unclear is how the change of network effect intensity affects the firm's reward choice, as well as the resulting consumer purchasing pattern in equilibrium.

Proposition 2.5.6 *As the network effect intensity increases, the optimal reward decreases, the equilibrium purchasing fraction in the crowdfunding stage increases, and the optimal profit increases.* (*Formally, as* k *increases,* r_p^{i*} *decreases,* θ_1^{i*} *decreases, and* Π_i^* *increases,* $i \in \{w, s\}$.)

Proposition 2.5.6 confirms the benefit of higher network effect intensity, i.e., a stronger network effect always improves the expected profit under both WN and SN cases. Another observation is that the change of network effect intensity alters all the equilibrium outcomes in the same direction, regardless of the network externalities scenario. In general, as the network externalities become stronger, the optimal crowdfunding reward can be reduced while still resulting in more consumers purchasing in the crowdfunding stage, and thus a higher expected total profit over two periods.

Further investigation reveals additional insights regarding the role of network externalities, as shown in Figure 3.3. As in the left plot, it is interesting that the firm's optimal reward remains constant when the network effect intensity is relatively small. This insensitivity is good news for the firm because the optimal crowdfunding price is robust to disturbance in the strength of network externalities. Within this region, the equilibrium threshold policy θ_1^* strictly decreases, meaning more crowdfunding sales as network externalities become stronger. When k is larger than some threshold value (see the dash-dot line in Figure 3.3), the crowdfunding market experiences adoption frenzy. Hence, as k

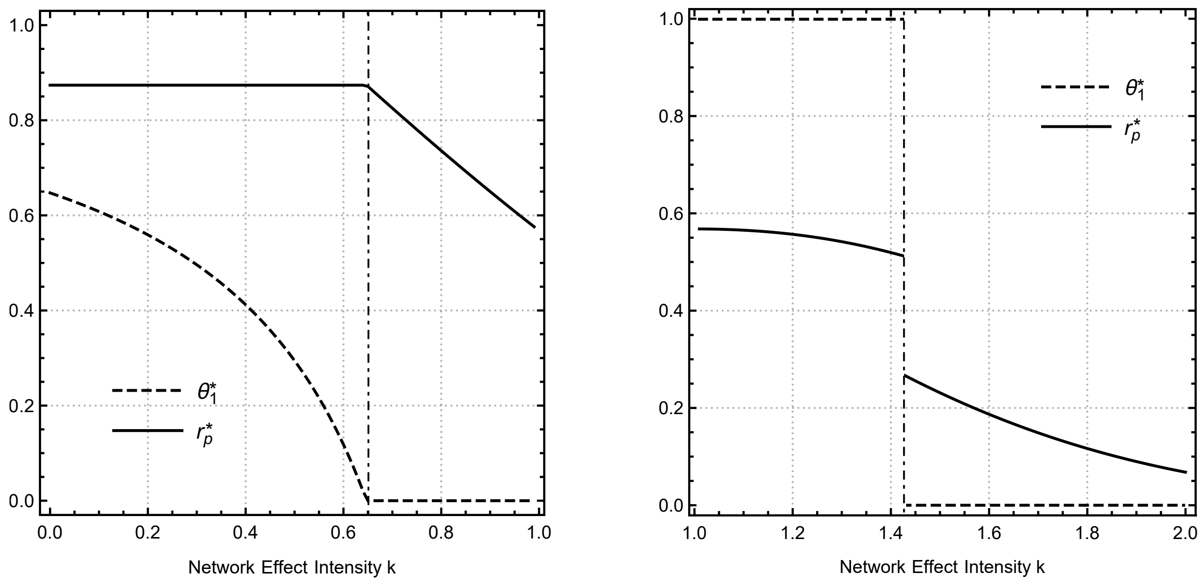


Figure 2.6.: Impact of Network Effect Intensity on Equilibrium Outcome

further increases, the optimal reward decreases. In this case, the firm fully takes advantage of the increasing immediate network benefit and offers the lowest reward that can induce adoption frenzy in the crowdfunding stage. The right plot shows the patterns for equilibrium results under SN. We can see when k is not too high, the optimal strategy is to offer the exact (high) reward (i.e., $r_p^{s*} = H_s(1)$) so that all consumers choose to wait. The intuition is to strategically forsake the immediate network benefit while fully exploiting the dominating potential learning benefit. After the network effect intensity exceeds some threshold value, the firm switches its strategy and offers the precise (low) reward (i.e., $r_p^{s*} = H_s(0)$) so that all consumers choose to purchase earlier, where the firm's strategy is to focus on the the dominating immediate network benefit.

2.6 Financial Constraints

In the previous analysis, it has been assumed that the firm is not subject to any financial constraints. In this section, we introduce the financial constraints and investigate how they affect the firm's crowdfunding strategy. Since crowdfunding originates as a funding tool for startups who have budget constraints, our analysis can provide actionable insights for those financially constrained campaigners on how to fully exploit the "funding" role of crowdfunding. Specifically, we consider the type of financial constraints as *fixed setup costs*, where the variable production cost c is negligible and the funding raised is the only financial resource to support the high manufacturing setup cost S (we call it *fundraising target*). The high fixed setup costs are salient for startup firms, which may include the high expenditures in manufacturing equipments, raw materials and assembly components, and research and development. Without loss of generality, we consider case $c = 0$ for simplicity (see [66] for a similar setting and discussion).

Given the equilibrium θ_1 in the P-W game, the amount of funding raised from the campaign is $p_1(1-\theta_1)M = (p_2-r_p)(1-\theta_1)M = (p_2-H_i(\theta_1))(1-\theta_1)M$, $i \in \{w, s\}$. Then, we can formulate the firm's constrained optimization problem under case i as follows:

$$\max_{\theta_1 \in [0,1]} \Pi_i(\theta_1) = \max_{\theta_1 \in [0,1]} p_2(1 - \theta_1 + \rho_{2i}(\theta_1)) - (1 - \theta_1)H_i(\theta_1), \tag{2.26}$$

$$s.t. \qquad (p_2 - H_i(\theta_1))(1 - \theta_1)M \geq S. \tag{2.27}$$

Define the *fundraising function* to be $\mathcal{Z}_i(\theta_1) = (p_2 - H_i(\theta_1))(1 - \theta_1)$, which represents the normalized fundraising from the campaign when consumers adopt θ_1 as the threshold policy in the P-W game. Then, the financial constraints (2.27) can be written as $\mathcal{Z}_i(\theta_1) \geq S/M$. Now, denote the *feasible set* for θ_1 under constraint (2.27) as $\Theta^i_{\mathcal{F}} = \{\theta_1 | \mathcal{Z}_i(\theta_1) \geq S/M,\ \theta_1 \in [0, 1]\}$. Note that this feasible set may differ in structure since $H_i(\theta_1)$ in function $\mathcal{Z}_i(\theta_1)$ differs under different network externalities.

2.6.1 Optimal Reward Choice under Financial Constraints

To study the impact of a funding target on the optimal reward choice, the key is to understand how the fundraising amount depends on the crowdfunding reward, or equivalently the threshold policy θ_1 adopted in the P-W game. Based on the decreasing pattern of the potential regret function $H_w(\theta_1)$, we know if the crowdfunding price increases (i.e., the reward decreases), then a smaller fraction of the consumer population will purchase in the crowdfunding stage (i.e., the equilibrium threshold θ_1 increases). However, it is still not clear whether the fundraising amount, as a product of profit margin and sales volume, increases or decreases. We start by analyzing the structure of the fundraising function $\mathcal{Z}_i(\theta_1)$ to understand the property of the feasible set $\Theta^i_{\mathcal{F}}$. We can show the feasible set $\Theta^i_{\mathcal{F}}$ is actually convex, as presented in the next lemma.

Lemma 8 *(i) If $\Theta^w_{\mathcal{F}} \neq \emptyset$, there exists θ^w_l and θ^w_u such that $0 \leq \theta^w_l \leq \theta^w_u \leq 1$ and $\Theta^w_{\mathcal{F}} = \left[\theta^w_l, \theta^w_u\right]$. (ii) If $\Theta^s_{\mathcal{F}} \neq \emptyset$, there exists θ^s_u such that $0 \leq \theta^s_u \leq 1$ and $\Theta^s_{\mathcal{F}} = \left[0, \theta^s_u\right]$.*

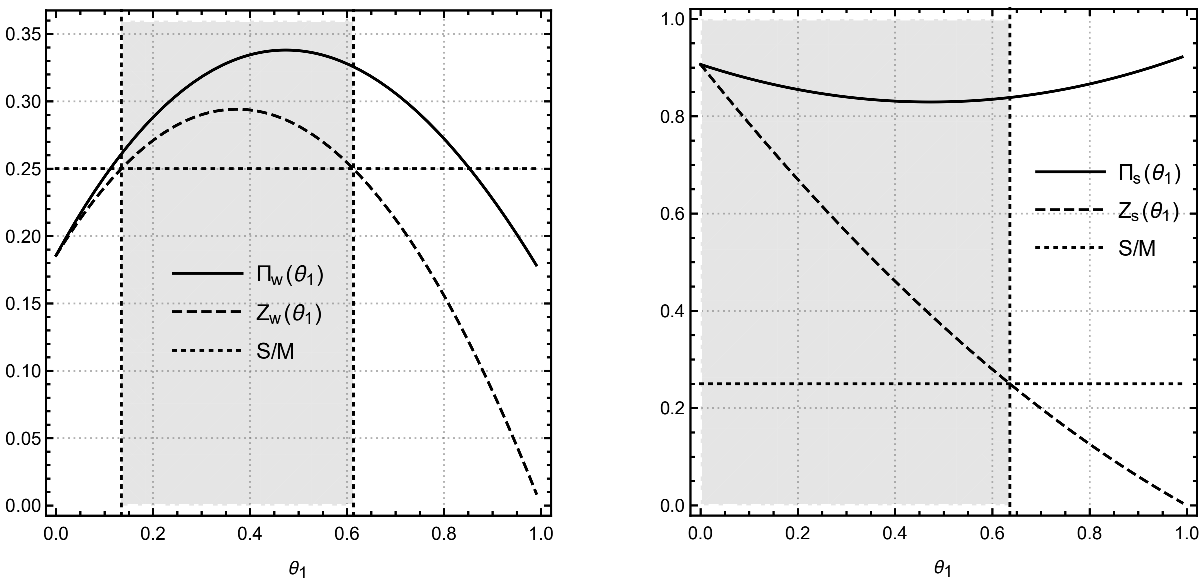


Figure 2.7.: Firm's Profit Function and Feasible Set under Financial Constraints

In Figure 2.7 (left: WN case; right: SN case), we plot the firm's profit function $\Pi_i(\theta_1)$ (solid line), the fundraising function $\mathcal{Z}_i(\theta_1)$ (dashed line) and the normalized fundraising target S/M (dotted line). The shadowed region indicates the new feasible region for θ_1 under financial constraints. Under the WN case, the fundraising function $\mathcal{Z}_w(\theta_1)$ is also strictly quasi-concave in θ_1. By the property of quasi-concave function, we know the feasible set $\Theta^w_{\mathcal{F}}$ is the upper contour set of function $\mathcal{Z}_w(\theta_1)$, which is convex. Under the SN case, we can show the fundraising function $\mathcal{Z}_s(\theta_1)$ is a strictly convex with $\mathcal{Z}_s(1) = 0$. Therefore, the resulting feasible set (if not empty) has the following form $\Theta^s_{\mathcal{F}} = \left[0, \theta^s_u\right]$. Hence, the quasi-concavity of the profit function $\Pi_w(\theta_1)$ and the strict convexity of profit function $\Pi_s(\theta_1)$ preserve over the convex interval $\Theta^i_{\mathcal{F}}$. Then, we can derive the optimal θ_1^{ic*} and r_p^{ic*} for both the WN and SN cases (we use the additional superscript c to represent the equilibrium outcomes under financial constraints). The next proposition summarizes the optimal crowdfunding reward choice and the consequent purchasing threshold policy in equilibrium.

Proposition 2.6.1 *The optimal reward under financial constraints is* $r_p^{ic*} = H_i(\theta_1^{ic*})$, *where*

$$\theta_1^{wc*} = \left\{\begin{array}{ll} \theta^w_u, & \textit{if } \mathcal{B}(\theta^w_u) \geq 0 \\ \theta^w_l, & \textit{if } \mathcal{B}(\theta^w_l) \leq 0 \\ \theta_{\mathcal{B}}, & o.w. \end{array}\right\}, \quad \theta_1^{sc*} = \left\{\begin{array}{ll} 0, & \textit{if } \Pi(0) \geq \Pi(\theta^s_u) \\ & \\ \theta^s_u, & o.w. \end{array}\right\}.$$

2.6.2 Impact of Fundraising Target

We have shown the optimal crowdfunding reward under financial constraints has a similar structure as in the base model. Next, we examine how the optimal crowdfunding reward and the induced purchasing pattern change as the fundraising target increases (i.e., more severe financial constraints).

Proposition 2.6.2 *As the fundraising target increases,* (*i*) *the optimal reward would increase (decrease) under the WN (SN) scenario;* (*ii*) *the equilibrium purchasing fraction in the crowdfunding stage always increases.* (*Formally, as S increases, r_p^{wc*} increases, r_p^{sc*} decreases, and θ_1^{ic*} decreases for $\forall i \in \{w, s\}$.*)

Proposition 2.6.2 suggests that as the fundraising target increases, the firm should always adjust the crowdfunding reward so that more consumers purchase in the crowdfunding stage. This is intuitive for the SN case, because more consumers purchase as the price increases in the crowdfunding stage (due to the Veblen effect), which leads to a higher fundraising amount. However, this finding is less obvious for the WN case since the unit revenue and the sale volume move in opposite directions. Proposition 2.6.2 indicates that it is optimal to increase the crowdfunding reward (decrease the crowdfunding price). This implies that the sales volume increase dominates the unit revenue decrease, and thus a higher reward would yield a higher total revenue in the crowdfunding stage for the firm.

2.7 Conclusion

In this paper, we study the reward choice decision for a crowdfunding campaign, where a single firm sells a product with uncertain quality to heterogeneous consumers. The selling horizon is divided into two stages: the crowdfunding stage and the retail stage. The firm charges two different prices, crowdfunding price and retail price, where the difference is the crowdfunding reward. The retail price is exogenous determined due to competitive market, thus the firm's key decision is the crowdfunding price, or equivalently, the crowdfunding reward choice. Our model of consumer utility consists of three components: a preference component, a quality component, and a social component.

Under this modeling framework, we first identify the consumers' purchasing pattern as a function of the crowdfunding reward. Under weak network externalities, a larger fraction of the consumer population will purchase in the crowdfunding stage if a higher reward is offered. But under strong externalities, interestingly, crowdfunding sales increase as the crowdfunding reward decreases. That is, a higher price may lead to a higher demand, which is called the Veblen effect. We provide a novel explanation of the Veblen effect under the crowdfunding setting, where the underling driving force is a combination of the social learning and strong positive network externalities. Next, we characterize the firm's unique optimal crowdfunding reward choice and the induced equilibrium outcome. We find that network externalities have significant implications for the optimal crowdfunding reward choice. In particular, under strong network externalities, the product adoption behavior may exhibit either the inertia pattern (only late adoption) or the frenzy pattern (only early adoption); whereas under weak network externalities, it exhibits the dispersed pattern (both early and late adoptions). Lastly, we incorporate the financial constraints into our model. Comparing to the situation without financial constraints, the optimal reward choice can be either higher or lower under each network externalities scenario, depending on the product's cost structure and the prior uncertainty in quality. This result offers insights into how financially constrained firms can exploit the "funding" role of crowdfunding.

This research can be extended in several directions. For instance, the current model focuses on a symmetric information structure between the firm and the consumers: Both parties are equally informed about the product quality and hence share a common prior belief about the true quality μ_q. In reality, however, the crowdfunding firm may have a better sense of the product quality. A natural question is how should the firm exploit this information advantage in crowdfunding. Another assumption in this paper is that all consumers are fully rational. It might be more realistic to consider a mixed population, where there are both strategic and myopic consumers. Finally, an alternative financing option for the crowdfunding firm is to borrow from financial institutions like banks. The optimal financing strategy for the firm when multiple options are available is still an open question for future research.

3. Crowdfunding vs. Bank Financing: Effects of Market Uncertainty and Word-of-Mouth Communication

3.1 Introduction

[1] Crowdfunding can be described as "the process of one party financing a project by requesting and receiving small contributions from many parties in exchange for a form of value to those parties" [86]. Crowdfunding is being widely adopted by a variety of industries, such as venture capital (Crowdfunder), real estate (Fundrise), personal loans (Lending Club), and nonprofits (Fundly Pro). In this paper, we focus on *rewards-based crowdfunding*, where production-based firms pre-sell a product to launch a business without incurring debt or sacrificing equity/shares[2]. In such an environment, a firm solicits financial contributions from the crowd, mostly in the form of pre-buying a product. These funds could then be used to cover production costs. Rewards-based crowdfunding is the most prolific form of crowdfunding currently taking place in the U.S. The rising popularity of crowdfunding sites, such as *Kickstarter* and *Indiegogo*, has promoted crowdfunding as a modern fund-raising solution. At present, crowdfunding has gone beyond simply being a tool specifically utilized by small businesses and independent entities to get a given project off the ground. Even larger businesses are now turning to crowdfunding as a way to obtain funds to develop new products, and as a tool to market their products [87].

Besides its important role of financing, rewards-based crowdfunding brings significant operational benefits to firms. The all-or-nothing (AoN) funding mechanism[3] helps firms reduce future demand uncertainty and make better procurement and production deci-

[1]This chapter is based on the author's early work [85] jointly with Xiaomeng Guo, Guang Xiao, and Fuqiang Zhang.

[2]In contrast, another primary type of crowdfunding is *equity crowdfunding* in which the backers receive shares of a company, usually in its early stages, in exchange for the money pledged.

[3]In the all-or-nothing (AoN) model, the firm sets a funding target and gets nothing unless the target is achieved. The most popular example of the AoN model is Kickstarter. In the keep-it-all model, the raised funds are paid immediately to the project initiators, regardless of whether or not the project

sions based on the advance demand information from crowdfunding. For new products, the potential market size is usually hard to predict, yet matching supply with demand is important, especially for capital-constrained firms. What crowdfunding does is to enable product-market-fit experimentation, which has historically been deprived of, in a cost-effective and informative way. If the campaign goes well, not only does the firm have the market validation for its product, it also has capital to fund manufacturing; if it does not, they just spare themselves the pain of spending time and money on an unattractive project. Hence, by crowdfunding the pledge of capital is available to firms before committing to large-scale production, which allows them to do fund raising and market testing simultaneously.

The benefits of rewards-based crowdfunding do not stop after a project is successfully funded. Ordinarily, crowdfunding sites help ventures get a number of natural visits from genuine users, who are likewise essential for effective marketing, as they could further share product news with others. In fact, crowdfunding has become an important "advertising" tool to help spread product/brand awareness, in particular through the word-of-mouth (WoM) communication and recommendation[4]. Typically, crowdfunding platforms encourage and facilitate the use of online WoM through social networks and social media (e.g., Facebook, Instagram), to spread awareness about the crowdfunding campaigns [89]. The popularity of social platforms have enhanced the efficiency and effectiveness of communication, by providing a means to reach a large audience at a low cost. Therefore, WoM is an integral component of the crowdfunding process, and its significant impact has been empirically examined and validated by [90]. Through interviews with entrepreneurs who have launched successful rewards-based crowdfunding campaigns, [91] find the majority of the entrepreneurs considered "expand awareness" as a significant benefit received. For example, a partner of Febtop Tech[5] acknowledges how awareness of the firm has increased as a result of the crowdfunding campaign: "*There are many more that know about us now after our campaign than before*".

reaches its funding goal. Indiegogo and RocketHub are two popular examples of this category. Here, we focus on the more widely adopted AoN funding mechanism.

[4]The influence of WoM is greatest when consumers are buying a product for the first time (typical case in crowdfunding) or when products are relatively expensive [88].

[5]Febtop Tech is campaigner on Indiegogo, who has successfully lunched a crowdfunding project, Optimus, in 2017.

Before the introduction of crowdfunding, bank financing[6] (i.e., bank/business loan) is the traditional source of capital for small businesses. Typically, bank financing requires a good personal and business credit rating, whereas there are no credit rating requirements associated with crowdfunding campaigns. Even if firms qualify for a bank loan, the rate of interest charged by banks may be high, which creates an additional burden on profitability. On the other hand, crowdfunding also bears disadvantages such as failure risk, where the campaign may not reach the funding target under AoN funding mechanism. It is conceivable that crowdfunding could potentially replace some of the conventional roles of bank financing, but it is not clear when the crowdfunding model is a better funding choice for firms.

According to survey data from the Annual Survey of Entrepreneurs[7], the primary sources of initial financing for new businesses in the United States are: personal and family savings (63.9%), business loans from banks (17.9%), and personal credit cards (10.3%), whereas other funding sources are in relatively small scope, such as venture capitalist investment (0.6%) [92]. Similar observation has also been reported in the 2017 year-end economic survey conducted by National Small Business Association. Among all the feasible financing options for small business/start-up firms, various bank loans are still the major choices whereas other innovative funding options, such as crowdfunding and venture capitalist, also receives increasing adoption [93].[8] Although compared to traditional bank financing, crowdfunding, as an innovative funding source, may not be prevalent, but it has drawn wide public attention and is disproportionately important for business growth. As such, understanding the funding choice between crowdfunding and bank financing is practically relevant and of great importance.

Motivated by the above practical observations, we are interested in investigating a firm's optimal choice between crowdfunding and traditional bank financing to fund its innovative product with both WoM communication and market uncertainty. More specif-

[6]In this paper, bank financing refers to any financing activities with the bank/financial institution, including bank business loans, personal credit card financing, business line of credit, etc.

[7]The Annual Survey of Entrepreneurs (ASE), conducted by the U.S. Census Bureau, is the largest annual survey of American entrepreneurs ever done, and exists thanks to a public-private partnership between the Census Bureau, the Kauffman Foundation, and the Minority Business Development Agency. The ASE samples approximately 290,000 employer businesses across all U.S. geographies and demographics, and tells the story of the American entrepreneur.

[8]The percentages of adoption for the surveyed firms are: large bank loan (15%), community bank loan (14%), small business administration loan (4%), ventral capital/angel investors (3%), and crowdfunding (1%).

ically, we ask three main research questions in this paper: (i) What is the impact of WoM communication and market uncertainty on the firm's optimal pricing strategy under different funding schemes? (ii) When is crowdfunding a better choice for fund raising, compared to traditional bank financing? (iii) How do different funding schemes affect consumer surplus and social welfare? To answer these questions, we develop an analytical model in which the firm optimizes its operational decisions (pricing strategy, funding target) while taking into account the financing role (i.e., raising funds to support product development and manufacturing) and the marketing role (i.e., the benefit from WoM communication due to stalled market base in the crowdfunding stage). Our analysis yields the following three main sets of insights.

First, we characterize the firm's optimal pricing strategies under both crowdfunding and bank financing. Specifically, for crowdfunding, the strategic/intertemporal pricing strategy is optimal, which balances the immediate revenue loss from offering crowdfunding rewards to encourage more early adoption for WoM awareness expansion, and the future profit gain from selling the product to the consumer population that would remain otherwise uninformed. In addition, we find that market uncertainty plays a critical role in determining the optimal pricing strategy, and it is not always unfavorable for firms to have a higher market uncertainty under crowdfunding due to the AoN funding mechanism. In contrast, for bank financing, we adopt the risk-based credit pricing model from the supply chain finance literature and find that the optimal pricing strategy and the firm's optimal profit remain fixed even as market uncertainty increases under bank financing.

Second, we endogenize the firm's optimal funding choice and compare the two funding schemes under various market environments, through the lens of both the market uncertainty level and the initial investment requirement. Among other results, we show that the choice between crowdfunding and bank financing is not monotonic in the market uncertainty, with bank financing preferred within an intermediate range. Similar observation holds from the lens of initial investment requirement. The underlying intuition is that with intermediate levels of market uncertainty and initial investment, the firm would target only on high market realization and gives up the project under low market realization due to the AoN funding mechanism. Such profit loss from market shrinkage may outweigh the loss from adopting costly bank financing and, thus, leads to the inferior performance of crowdfunding. Moreover, when the initial investments are relatively high, bank financing could also be adopted as crowdfunding is not feasible.

Third, we find that more active social interactions could hurt consumer surplus and even reduce social welfare, contrary to the conventional wisdom that more informative WoM should be beneficial. The intuition is that, although stronger WoM communication helps inform more consumers in the retail market about the product, it also encourages the firm to strategically increase the crowdfunding price, extracting more consumer surplus. Moreover, the firm's strategic pricing hurts the consumers more compared to the firm's profit gain, resulting in a lower social welfare. A further welfare implication is that crowdfunding is a win-win alternative (benefits both the consumers and the firm) fund-raising solution to bank financing, when market uncertainty is relatively high.

The rest of this paper is organized as follows. We position our paper in the related literature in Section 3.2. Section 3.3 sets up the model. The firm's optimal pricing strategies under crowdfunding and bank financing are investigated in Section 3.4 and Section 3.5, respectively. Section 3.6 discusses the firm's optimal choice of funding schemes. We investigate the impact of the firm's funding scheme choice on the consumer surplus and social welfare in Section 3.8 and conclude the paper in Section 3.10. All proofs and additional results are given in the appendices.

3.2 Literature Review

Our work is mainly related to four streams of research in the literature: crowdfunding, WoM communication, advance selling, and the interface of operations and finance.

There is a rising research stream on crowdfunding [67, 94, 95]. A number of papers, both empirical [96, 97] and theoretical [68, 69], address the dynamics of contribution patterns observed over the duration of crowdfunding campaigns. A class of recent papers have focused on the impact of information asymmetry and moral hazard on the campaign design [67, 98, 99]. In a bargaining game setting, [70] and [100] study the informational role of the crowdfunding campaign and its effect on the entrepreneur's choice between crowdfunding and venture capital. By contrast, our primary focus is on the choice between crowdfunding and bank financing. [101] also compare crowdfunding with traditional bank financing, but focus on the crowdfunding platform design to provide guidelines for how to reduce various types financing frictions resulting from both moral hazard and information asymmetry during the crowdfunding campaign. Different from the above studies, we investigate a firm's funding choice when launching an innovative product to the market with WoM communication and market uncertainty.

Other analytical works on crowdfunding attempt to optimize the operational design (e.g., pricing, rewards, product line) of a crowdfunding campaign. [65] study how the AoN crowdfunding mechanism affects a firm's pricing and product line design strategies under a simple two-period model with one potential buyer in each period. More recently, [102] analyze the optimal design of a crowdfunding campaign as well as the optimal menu of rewards to mitigate the strategic behavior of contributors. [103] develop a revenue management model of crowdfunding dynamics to obtain insights on how to maximize revenues by optimizing both the pledge level and the campaign duration. [36] investigate the optimal choice of crowdfunding rewards in the presence of social learning, network externalities and strategic consumers/contributors. We also characterize the firm's optimal operational (pricing) strategy under crowdfunding, but our focus is the comparison between crowdfunding and traditional bank financing, and examines the impacts of various market parameters on the firm's funding scheme choice and the corresponding social welfare implications.

Our paper also contributes to an emerging literature on the AoN design in the context of equity-based crowdfunding, and its impact on moral hazard [67], information cascade and herding [104], and financing efficiency [105]. [106] compares the choice between fixed vs. flexible funding mechanism to signal project value to backers and finds that a fixed funding commitment can generate higher revenues. [107] show theoretically and find empirically that the size and likelihood of a pledge is affected positively by the size of the most recent pledges, and negatively by the time elapsed since the most recent pledge. Our work departs from these studies in many ways, most notably by considering the interactions between the AoN mechanism and the WoM benefit under market uncertainty, which none of these papers take into consideration. As a cautionary tale, we find that the AoN crowdfunding mechanism entangles the financial and operational risks (as the funding amount and the market size are realized simultaneously), which could make the firm worse-off compared to traditional bank financing.

A growing number of studies have empirically examined WoM communication and shown that the volume and valence of WoM can have a significant impact on consumers' purchase and adoption behaviors [108–111]. On the analytical side, [112] captures social learning via WoM communication and finds the optimal advertising targets are generally not the individuals with the most friends. [113] study the problem of dynamic pricing when selling to a network of consumers. [114] investigates the relationship between product quality and WoM communication. In contrast to all of these works, however, we provide a unified model to study crowdfunding, in addition to its financing role, as a marketing tool to control production information diffusion via WoM communication and to test potential market size.

Crowdfunding is similar to advance selling in certain aspects because both encourage customers to place orders before the product is released. A major benefit of advance selling is to help firms plan for inventory by using early demand information [49,50,115]. Another reason for firms to adopt advance selling is to exploit consumers' uncertainty about their valuations. [52] demonstrate that selling to consumers who have uncertain consumption utility can substantially increase the firm's profit. More recently, [53] find that capacity rationing in advance selling can be an effective device to signal quality. Our model of crowdfunding differs from advancing selling in several key aspects: firms' financing/funding needs, potential project failure (AoN mechanism), and the awareness-expanding benefit via WoM communication. These factors are rarely considered in the advance selling literature.

More broadly, our paper is also related to research at the interface of operations and finance, studying how firm's financial decisions affect its operations [1,11,14,28,116–118]. Closely related are those study the financing and operations strategies of startups. [119] examine the benefits of IPO under product market competition and demand uncertainty. [120] consider the impact of capital market imperfections (fixed bankruptcy cost) on firm technology choice, whereas the main financing friction in our model is the credit rating (financing accessibility) and our focus is the funding choice between crowdfunding and bank financing. More recently, [121] study the extent to which risk-averse entrepreneurs can transfer venture risk to fully diversified investors under ICO financing[9]. We refer the reader to [126] for a review of this literature and [16] for a discussion of recent contributions and future directions, which highlights crowdfunding as one of the main research

[9]ICO (initial coin offering), a new type of crowdfunding based on blockchain technology, allows companies and entrepreneurs to raise money through cryptocurrencies, in exchange for a "token" that can be sold on the Internet or used in the future to gain products or services [121–125].

areas at the interface of Operations Management and Finance. Our paper focuses on the comparisons of two financing schemes (traditional capital markets, i.e., bank financing, and an innovative alternative: crowdfunding) for startup firms that have received little attention in the operations management literature. Our work extends this stream of literature by showing how market uncertainty and WoM communication affect the funding choice between bank financing and crowdfunding.

3.3 The Model

This section introduces the model by describing the firm's decisions and the market environment.

3.3.1 The Firm

A firm (she) wants to launch an innovative product in the marketplace with quality q, which is exogenous and publicly known[10]. The firm is cash constrained. To develop and produce the product, the firm needs to raise capital, either via crowdfunding or bank financing, to meet an *initial investment* requirement z. Note that z is an indicator of the severity of the firm's financial constraint, and can be interpreted as the funding needs, or simply the fixed setup cost when there is no internal funding source. For expositional convenience, we normalize the marginal production cost to zero[11]. High fixed setup costs are not uncommon for launching new products, which may include the expenditures on early stage R&D, prototyping, and manufacturing capacities. In many situations, a capital-constrained firm can choose between two funding schemes: crowdfunding and bank financing. If the firm chooses crowdfunding, she needs to sell the product through two stages: the crowdfunding stage with *crowdfunding price* p_1 and the retail stage with *retail price* p_2. In this paper, we focus on the commonly used AoN funding mechanism: The firm sets a funding target T, together with a pricing path $\{p_1, p_2\}$. Surviving the crowdfunding stage requires $p_1 d_1 \geq T$, where d_1 denotes the number of consumers purchasing in crowdfunding. If the funding target is successfully

[10]Quality uncertainty can be incorporated into our model without changing the main results (see Appendix C).

[11]The zero marginal cost is commonly assumed in the crowdfunding literature for expositional convenience [98–100]. Our model can easily extended to the positive marginal cost without affecting the main insights. See Appendix C for a detailed discussion.

reached, then the firm proceeds with the production in period 1 and continues selling the product in period 2; otherwise the crowdfunding project fails without further production activities.

Alternatively, the firm could use bank financing as the source of capital. Given the funding target z and the firm's credit rating C_s, the bank evaluates the market risk and decides the interest rate r_s to charge (the bank may directly reject the loan request by setting a prohibitively high interest rate). After securing the loan, the firm will produce the product and sell to the market by setting a retail price p_b. More details about bank financing will be provided in Section 3.5.

3.3.2 The Market

Since it is a new product, the firm is uncertain about the market response. We use a variable $\mathcal{I}$, referred to as *market interest*, to capture the uncertainty about consumers' interest in the product, or equivalently the potential market size. The firm does not know the exact market interest before introducing the product, but knows that it follows a two-point distribution:

$$\mathcal{I} = \left\{ \begin{matrix} 1-\beta, & \text{with probability} & 1/2 \\ 1+\beta, & \text{with probability} & 1/2 \end{matrix} \right\}, \tag{3.1}$$

where $\beta \in [0, 1]$ reflects the *market uncertainty*. We normalize the expected market size to 1.

The consumer population is divided into two segments. A fraction $\alpha \in (0, 1)$ (called *crowdfunding consumer segment*) of consumers is active in the crowdfunding market[12], and the rest of consumers are present in the retail market. Each consumer's valuation of the product is θq, where q is the exogenously given product quality, and θ represents the consumer's willingness-to-pay for quality (i.e., consumer's marginal value of quality). We assume θ follows uniform distribution on $[0, 1]$. It is worth pointing out that products in crowdfunding typically exhibit high quality uncertainty. Our assumption that the product quality is perfectly known to the consumer is mainly for expositional clarity, and

[12]It is straightforward to show that even consumers in the crowdfunding market are also present in the retail market, they have no incentive to delay their purchase decisions due to the optimal pricing strategy discussed in Section 3.4.

the model can be extended to incorporate quality uncertainty without affecting the main results (see Appendix C for details).

To conclude this section, we introduce the following notations to facilitate the analysis and exposition. Let $\vee$ denote the maximum operator (i.e., $x \vee y = \max(x, y)$) and define $(x)^+ = \max(x, 0)$. Monotonic relations (increasing, decreasing) are used in the weak sense unless otherwise stated. A summary of notations is given in Appendix A.

3.4 Crowdfunding Strategy

In this section, we analyze the crowdfunding strategy. In the crowdfunding stage, the firm chooses the crowdfunding price $p_1 \leq q$, and consumers with $\theta \geq \theta_1 = p_1/q$ will purchase. Note that the price decision p_1 is equivalent to the choice of consumers' purchase threshold θ_1. Similar observation holds for the price decision p_2 in the retail stage. Henceforth, we will use θ_i and p_i interchangeably to represent the pricing strategy in period i, $i \in \{1, 2\}$. Then, the total number of early adopters in the crowdfunding stage is $\rho\mathcal{I} = \alpha(1 - \theta_1)\mathcal{I}$, where the value of $\mathcal{I}$ is ex ante unknown.

3.4.1 Word-of-Mouth Communication

Consumers active in the crowdfunding market are aware of the firm's innovative product, while consumers in the retail market get informed about the product with some probability associated with the word-of-mouth (WoM) communication. During the retail stage, consumers buy the product if and only if they are aware of it and their utility exceeds the retail price. We define the *awareness function* $A(\rho)$ to represent the probability that consumers in the retail market become aware of the innovation product, where $\rho \in [0, \alpha]$ is the fraction of consumer population who has already purchased in the crowdfunding stage. For expositional convenience, we assume a simple form of the awareness function as follows,

$$A(\rho) = (\rho_0 + k\rho) \wedge 1, \tag{3.2}$$

where $\rho_0 \in (0, 1)$ is the *awareness base*, $k > 0$ is the *WoM intensity*, and $\wedge$ denotes the minimum operator (i.e., $x \wedge y = \min(x, y)$). Note that the awareness function $A(\rho) \in [\rho_0, 1]$ increases in ρ, meaning the larger the fraction of consumers purchasing in the crowdfunding stage, the more likely a consumer in the retail market will be informed about the product. In contrast, in the bank financing case, the firm sells directly to the entire market in the retail stage with product awareness $A(0) = \rho_0$. Additional discussion on the assumption of awareness base under bank financing is provided in Appendix C.

The function $A(\rho)$ has the flexibility to capture different market scenarios along two dimensions: the awareness base ρ_0 and the WoM intensity k. The awareness base ρ_0 represents the firm's extant brand awareness or the awareness of the particular crowdfunding product before the crowdfunding campaign. For example, an established brand typically has a higher ρ_0 compared to a new startup when both choose to launch a product through crowdfunding. The WoM intensity k captures how actively the consumers are communicating with each other, or how much buzz the product can generate. A product's WoM intensity also reflects the propensity that its information disseminates among consumers via social forces. More specifically, a product with more chat-worthy features shall have a higher WoM intensity[13].

To first highlight the role of WoM communication, we start our exposition by providing a closed-form characterization of the optimal pricing strategy $\{\theta_1^*, \theta_2^*\}$ and corresponding profits in the base model without financial constraint in Section 3.4.2. In Section 3.4.3, we further incorporate the financial constraint and analyze the optimal pricing problems in different scenarios based on the project features we defined later. In Section 3.4.4, we present our sensitivity analysis regarding market uncertainty, WoM intensity, and the crowdfunding consumer segment, respectively.

3.4.2 Crowdfunding without Financial Constraint

Without financial constraint, the crowdfunding project can be guaranteed to succeed by simply setting a zero funding target, even under the AoN funding mechanism. We solve the pricing problem backwards, starting with the retail stage. The firm needs to choose the retail price p_2 at the beginning of period 2 given the installed market base

[13]The extant literature models WoM communication in three ways. The first type is called "informative WoM" [114], where WoM communication drives the amount of information in the market about the product's existence and conditional on knowing about the product, the customer knows the quality perfectly. The second type is "persuasive WoM" [127], where the larger the installed base, the more likely the remaining consumers will adopt. Such models mostly examine aggregate market behavior, while ignoring the adoption decision of individual consumers. The third type is "learning WoM" [114, 128], where the larger the installed base, the more likely the remaining consumers will discover their quality evaluations, by learning from consumer-generated quality information (e.g., reviews). The WoM communication studied in this paper falls into the first category.

$\rho = \alpha(1 - \theta_1)$ through crowdfunding. Suppose the firm chooses the retail price p_2, and the consumers in the retail market with $\theta \geq \theta_2 = p_2/q$ will purchase. The expected revenue in retail stage can be written as $\pi_2(\theta_2; \theta_1) = (1 - \alpha)\mathcal{A}(\theta_1)(1 - \theta_2)\theta_2 q$, where $\mathcal{A}(\theta_1) = A[\alpha(1 - \theta_1)] = [\rho_0 + k\alpha(1 - \theta_1)] \wedge 1$. The optimal solution is $\theta_2^* = 1/2$, i.e., $p_2^* = q/2$. Notice that the optimal retail price is always $p_2^* = q/2$, which is not affected by either the funding target or the WoM effect. Hence, the optimal revenue in the retail stage becomes $\pi_2^*(\theta_1) = \pi_2(\theta_2^*; \theta_1) = q(1 - \alpha)\mathcal{A}(\theta_1)/4$. Back to the crowdfunding stage, the expected revenue is $\pi_1(\theta_1) = q\alpha\theta_1(1 - \theta_1)$. If the firm myopically focuses only on maximizing the crowdfunding revenue, then the *myopic pricing* should be $p_1^* = p_2^* = q/2$. Considering the initial investment requirement z, the expected total profit over the two periods becomes

$$\pi(\theta_1) = \pi_1(\theta_1) + \pi_2^*(\theta_1) - z = \frac{q}{4}\left[4\alpha\theta_1(1 - \theta_1) + (1 - \alpha)\mathcal{A}(\theta_1)\right] - z. \tag{3.3}$$

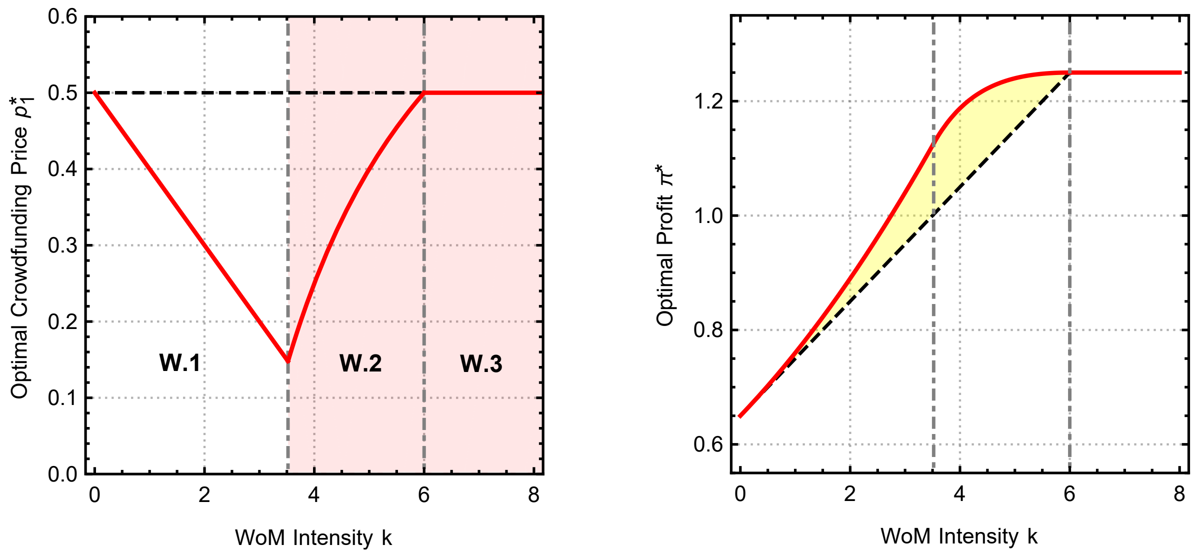
Through analysis, we find the firm's optimal (unconstrained) pricing strategy $\{\theta_1^u, \theta_2^u\}$ exhibits different structures under different ranges of the WoM intensity k, which is formally summarized in the following proposition and depicted in Figure 3.1 accordingly.

Proposition 3.4.1 *Without financial constraint, the optimal pricing strategy $\{\theta_1^u, \theta_2^u\}$ is:*

$$\theta_1^u = \left\{\begin{array}{ll} \dfrac{(4 - k + k\alpha)^+}{8}, & \text{if } 0 \leq k < k_m \\ \dfrac{(k\alpha - 1 + \rho_0)^+}{k\alpha}, & \text{if } k_m \leq k < k_n \\ \dfrac{1}{2}, & \text{if } k \geq k_n \end{array}\right\}, \quad \theta_2^u = \frac{1}{2}, \tag{3.4}$$

where $k_n = 2(1 - \rho_0)/\alpha$ and $k_m > 0$ is the unique positive solution to $k\alpha(4 - k + k\alpha) = 8(k\alpha - 1 + \rho_0)$.

We emphasize several points in Proposition 3.4.1 (Figure 3.1). First, when the WoM intensity is weak (W.1: $0 \leq k < k_m$) or medium (W.2: $k_m \leq k < k_n$), it is optimal to adopt strategic/intertemporal pricing, i.e., the crowdfunding price is lower than the retail price ($p_1 < p_2$). The intuition is that the lower price in crowdfunding can induce higher installed market base, which helps increase the product awareness in the retail stage. However, when the WoM intensity is high (W.3: $k \geq k_n$), it is optimal to adopt myopic/uniform pricing, i.e., setting the crowdfunding price equal to the retail price. The reason is that the strong WoM intensity can already guarantee the highest (full) product



(a) Optimal Crowdfunding Price (b) Optimal Profit

Note: The dashed lines represent the results with myopic/uniform pricing, i.e., $\theta_1^* = \theta_2^* = 1/2$.

Figure 3.1.: Optimal Crowdfunding Price p_1^* and Profit without Financial Constraint

awareness in the retail market (i.e., $\rho_0 + k\alpha/2 \geq 1$, $\mathcal{A}(1/2) = 1$) even without offering any crowdfunding rewards.

Second, under medium and high WoM intensities, the optimal crowdfunding price will drive the full product awareness in the retail market, while this is not the case for low WoM intensity. In Figure 3.1(a), we plot the optimal crowdfunding price as a function of the WoM intensity k. When the WoM intensity k is relatively low as in region (W.1), the optimal crowdfunding price decreases in k since it is set to well balance the *immediate revenue loss* from offering crowdfunding rewards to encourage the product information diffusion through WoM, and the *future profit gain* from selling the product to the additional consumer population that would remain otherwise uninformed without WoM. As the WoM intensity k increases to a higher level as in region (W.2), the optimal crowdfunding price is set to induce full product awareness in the retail market (i.e., $\mathcal{A}(\theta_1^u) = \rho_0 + k\alpha(1 - \theta_1^u) = 1$). When the WoM intensity k is even higher as in region (W.3), a myopic/uniform pricing strategy is adopted, because no crowdfunding reward is needed as the product awareness already reaches the maximum. Figure 3.1(b) presents the optimal profit as an increasing function of the WoM intensity k. That said, more active social interactions always benefit the firm, as the WoM benefit becomes stronger.

The gap between the solid and dashed lines indicates the profit increase with strategic pricing, compared to the case with myopic pricing. This difference can be interpreted as the pure WoM benefit, or the *awareness-expanding* benefit of crowdfunding.

3.4.3 Crowdfunding with Financial Constraint

After understanding the marketing role of crowdfunding, we proceed to incorporate the funding needs and investigate the financing role of crowdfunding, as well as the strategic interaction between these two roles. Due to the AoN funding mechanism, the funding target is an important decision as it will affect the success probability under market uncertainty. We start by showing the optimal funding target should always be equal to the actual fund needed.

Lemma 9 *The optimal funding target is equal to the initial investment, i.e., $T^* = z$.*

With the funding target $T^* = z$ and the crowdfunding price $p_1 = q\theta_1$, the project succeeds if the realized market size satisfies $\mathcal{I} \geq \frac{z}{\alpha p_1(1-\theta_1)} = \frac{z}{\alpha q \theta_1(1-\theta_1)}$. Since $\mathcal{I} \in \{1-\beta, 1+\beta\}$, the project success probability given θ_1 can be summarized as

$$\mathcal{P}(\theta_1) = \left\{ \begin{array}{ll} 1, & \text{if } z \leq \alpha q(1-\beta)\theta_1(1-\theta_1) \\ 1/2, & \text{if } \alpha q(1-\beta)\theta_1(1-\theta_1) < z \leq \alpha q(1+\beta)\theta_1(1-\theta_1) \\ 0, & \text{if } z > \alpha q(1+\beta)\theta_1(1-\theta_1) \end{array} \right\}. \tag{3.5}$$

Note that the success probability $\mathcal{P}(\theta_1)$ depends on the crowdfunding price and achieves its maximum at $\theta_1 = 1/2$. We further distinguish the projects with different success probabilities, by categorizing them into two groups: *risky project* with $\mathcal{P}(\theta_1) = 1/2$ and *safe project* with $\mathcal{P}(\theta_1) = 1$.

Now, we introduce a key metric $\mathcal{F}$ for crowdfunding projects, defined as

$$\mathcal{F} = \frac{\alpha q}{4z}. \tag{3.6}$$

The value of $\mathcal{F}$ captures the *investment feasibility* of a project, which increases in the crowdfunding consumer segment α and the product quality q, but decreases in the initial investment z. Depending on the value of $\mathcal{F}$, we further define two investment feasibility levels as follows.

Definition 3.4.1 (Investment Feasibility Level) *(I.1) Low Investment Feasibility:* $1/2 \leq \mathcal{F} < 1$*; and (I.2) High Investment Feasibility:* $\mathcal{F} \geq 1$*.*

The key difference is that under low initial investment, we have the maximal success probability $\mathcal{P}(1/2) \in \{0, 1/2\}$; under high investment feasibility, we have $\mathcal{P}(1/2) \in \{1/2, 1\}$. With uniform pricing, the project with $\mathcal{F} \geq 1$ is always feasible under any market uncertainty $\beta \in [0, 1]$, while the project with $1/2 \leq \mathcal{F} < 1$ could be potentially infeasible. Moreover, the project is always infeasible if $\mathcal{F} < 1/2$.

(I.1) Low Investment Feasibility Case. We first consider the case of low investment feasibility. By definition of $\mathcal{F}$, the reason for the low investment feasibility could be: The crowdfunding consumer segment is relatively small and thus limits the firm's capital-raising capability; or the initial investment is relatively high compared to the output product quality. In this case, the crowdfunding campaign will succeed only if the realized market size is high ($\mathcal{I} = 1 + \beta$). Because of the funding target z, the feasible pricing strategy θ_1 should also satisfy

$$\alpha q(1 + \beta)\theta_1(1 - \theta_1) \geq z. \tag{3.7}$$

All feasible projects, with θ_1 satisfying equation (3.7), have a success probability of 1/2. Notice that some pricing strategies are always Pareto dominated, as illustrated in the following lemma.

Lemma 10 *The Pareto set for pricing strategy* θ_1 *is* $[\theta_l, 1/2]$*, where* θ_l *is the smaller root of the quadratic equation* $\alpha q(1 + \beta)\theta_1(1 - \theta_1) = z$*. Moreover,* θ_l *increases in* z *and* $0 < \theta_l \leq 1/2$*.*

Denote the firm's profit function in this case as $\pi^r(\theta_1)$, where the superscript r is mnemonic for "risky". Then, the firm's problem becomes $\max_{\theta_1 \in [\theta_l, 1/2]} \pi^r(\theta_1)$, where

$$\pi^r(\theta_1) = \frac{q}{8}(1 + \beta)[4\alpha\theta_1(1 - \theta_1) + (1 - \alpha)\mathcal{A}(\theta_1)] - \frac{z}{2}. \tag{3.8}$$

Compared to the profit function (3.3) without financial constraint, the two profit functions share a similar structure with two differences: the coefficient outside the square brackets and the constant term. By incorporating the constraint of the feasible domain $[\theta_l, 1/2]$, it is straightforward to see the optimal pricing strategy is $\theta_1^* = \theta_1^u \vee \theta_l$ and $\theta_2^* = \theta_2^u = 1/2$, where θ_1^u is given in (3.4). The crowdfunding price formula $\theta_1^* = \theta_1^u \vee \theta_l$ clearly demonstrates the key trade-off: the immediate financing needs and the potential marketing benefits. A further observation is that the financial constraint has no impact under extremely high or low WoM intensity, since the corresponding unconstrained optimal profit is already the highest possible for the firm in the crowdfunding stage. Otherwise, the financial constraint would limit the firm's leverage of the WoM communication effect.

(I.2) High Investment Feasibility Case. We continue to analyze the more complicated case of high investment feasibility (I.2). Different from the previous case (I.1), the crowdfunding is always feasible regardless of the market uncertainty realization. But, the choice of the crowdfunding price could affect the project success probability. The project is ensured to be successful if the pricing strategy θ_1 satisfies

$$\alpha q(1-\beta)\theta_1(1-\theta_1) \geq z. \tag{3.9}$$

Let θ_h be the smaller root of the quadratic equation $\alpha q(1-\beta)\theta_1(1-\theta_1) = z$. Similar argument as in Lemma 10 shows that the Pareto set of θ_1 is $[\theta_h, 1/2]$, where θ_h increases in z and satisfies $0 < \theta_l \leq \theta_h \leq \frac{1}{2}$. Under high investment feasibility, it is still feasible to choose the pricing strategy within the range $[\theta_l, \theta_h]$, under which the project becomes risky with a success probability $1/2$. Hence, we categorize the two pricing regimes: (1) safe pricing scheme: $\theta_1 \in [\theta_h, 1/2]$ and $\mathcal{P}(\theta_1) = 1$ (safe project); (2) risky pricing scheme: $\theta_1 \in [\theta_l, \theta_h]$ and $\mathcal{P}(\theta_1) = 1/2$ (risky project).

Under low investment feasibility (I.1), the safe pricing scheme is not achievable, since the maximal success probability $\mathcal{P}(1/2) = 1/2 < 1$. Under high investment feasibility (I.2), crowdfunding could be either safe or risky, depending on the adoption of either a safe or risky pricing scheme. Note that the firm's profit is different under the two pricing schemes. When a safe pricing scheme is adopted, the profit function $\pi^s(\theta_1)$ is the same as in the base model without financial constraint, which is given in (3.3). When a risky pricing scheme is adopted, the profit function $\pi^r(\theta_1)$ is given in (3.8). Then, for projects with high investment feasibility, the firm's problem becomes:

$$\max_{\theta_1 \in [\theta_l, 1/2]} \pi(\theta_1) = \max \left\{ \max_{\theta_1 \in [\theta_l, \theta_h)} \pi^r(\theta_1), \max_{\theta_1 \in [\theta_h, 1/2]} \pi^s(\theta_1) \right\}. \tag{3.10}$$

The safe pricing scheme ensures the success of the crowdfunding project (and thus the ability to profit from both high and low market sizes), but at the expense of limited pricing flexibility to fully explore the WoM benefit of crowdfunding (and thus a revenue loss in the retail stage). It is not clear which pricing scheme is more beneficial. An intuitive conjecture would be that it should depend on the market uncertainty and the WoM intensity.

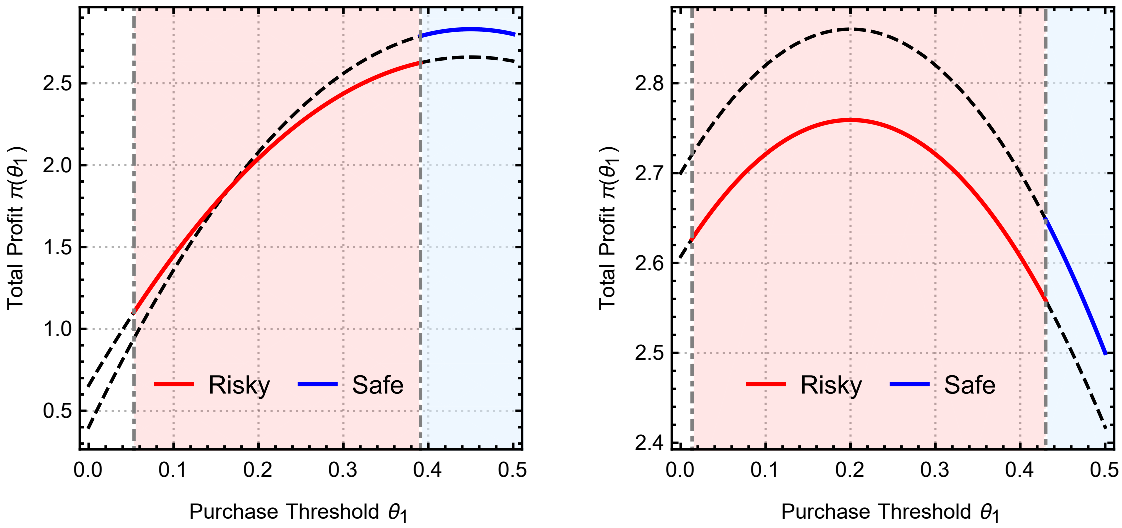


Figure 3.2.: (Color Online) An illustration of Profit Functions under Safe and Risky Pricing Regimes

Comparing the two pricing schemes, the profit functions share similar structure with two differences: the coefficient outside the square brackets and the feasible domain. Figure 3.2 provides a pictorial interpretation of these differences, where the red (blue) region represents the risky (safe) pricing scheme. Essentially, the startup needs to compare the optimal profit in each region and choose the pricing strategy with the higher profit. When the two pricing schemes generate the same expected profit, we assume the safe pricing is preferred. A first observation is that safe pricing is more likely to dominate in a broader parameter regions, but both safe and risky pricing scheme could be optimal. As illustrated in the two plots, the left one has a profit maximizer in the safe pricing regime, while the right one prefers the risky pricing scheme.

To fully characterize the firm's optimal crowdfunding pricing strategy under different investment scenarios, we define the following two auxiliary functions:

$$\mathcal{H}^r(\beta) = \pi^r(\theta_1^u \vee \theta_l \wedge \theta_h),\ \beta \in [0,1] \text{ and } \mathcal{H}^s(\beta) = \pi^s(\theta_1^u \vee \theta_h),\ \beta \in [0, \beta_m], \tag{3.11}$$

where $\beta_m = 1 - \mathcal{F}^{-1} = 1 - \frac{4z}{\alpha q}$. Note that β_m could be either positive or negative, depending on the value of $\mathcal{F}$. Essentially, the two auxiliary functions $\mathcal{H}^r(\beta)$ and $\mathcal{H}^s(\beta)$ represent the optimal profits under risky and safe pricing schemes, respectively, as a function of the market uncertainty β. Then, we can further denote the following threshold β_τ: β_τ is the unique solution to $\mathcal{H}^r(\beta) = \mathcal{H}^s(\beta)$ if $\mathcal{H}^s(\beta_m) < \mathcal{H}^r(\beta_m)$; otherwise, $\beta_\tau = \beta_m$. The optimal pricing strategy under different investment scenarios is fully characterized by the following proposition.

Proposition 3.4.2 *The optimal crowdfunding pricing strategies are as follows:*

(*i*) *Under low investment feasibility, the project is feasible if and only if (iff) $\beta \geq -\beta_m$, and it is optimal to adopt the risky pricing scheme with $\theta_1^* = \theta_1^u \vee \theta_l$ and $\theta_2^* = 1/2$.*

(*ii*) *Under high investment feasibility, if $\beta \leq \beta_\tau$, it is optimal to adopt the safe pricing scheme with $\theta_1^* = \theta_1^u \vee \theta_h$, $\theta_2^* = 1/2$; otherwise, it is optimal to adopt the risky pricing scheme in (i).*

Proposition 3.4.2 characterizes the firm's optimal pricing strategy and identifies the scenarios under which different pricing schemes should be adopted through the lens of market uncertainty level β. Part (*i*) characterizes the condition under which the project with low investment feasibility is feasible, and the corresponding optimal pricing strategy. Since $\mathcal{F} < 1$ in this case, we have $\beta_m < 0$. The condition $\beta \geq -\beta_m$ implies that the crowdfunding project is feasible if and only if the market uncertainty level is higher than the threshold $-\beta_m$. Part (*ii*) provides a clean solution to the decision puzzle for projects with high investment feasibility. The choice between the safe and the risky pricing schemes critically depends on the market uncertainty level β. The general structure remains consistent with one's intuition: A safe pricing scheme should be adopted when the market is relatively "safe", i.e., market uncertainty is lower than a certain threshold; otherwise, risky pricing is optimal. But the meaning of the threshold β_τ or the mechanism driving such results is not unambiguous. We have more detailed discussion in Section 3.4.4, where the impact of market uncertainty is carefully investigated.

3.4.4 Impact of Market Uncertainty

In the following analysis, we conduct sensitivity analysis to understand the impacts of key model parameters, e.g., market uncertainty, WoM intensity, and crowdfunding consumer segment, on the firm's crowdfunding strategy and corresponding profit. We start our investigation from the market uncertainty β, which plays a key role in evaluating the risk and the performance of a specific project, as it affects both the feasibility regime given a firm's profile and the optimal pricing schemes. It is clear that as market uncertainty increases, the firm may not be able to ensure the project success and the safe pricing scheme may become infeasible. The following proposition formalizes such results.

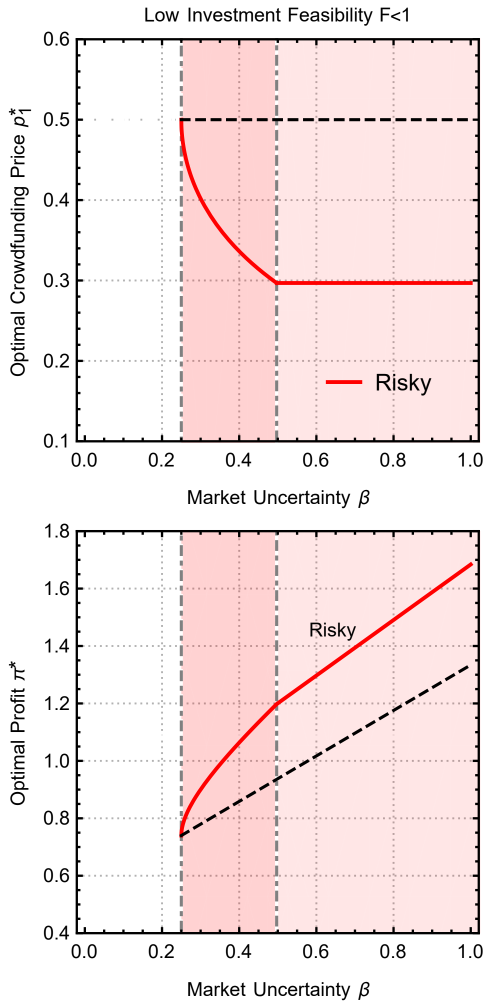
Proposition 3.4.3 *As the market uncertainty β increases in $[0, 1]$:*

- (i) *Under low investment feasibility, crowdfunding is infeasible on $[0, -\beta_m)$; while the optimal crowdfunding price decreases on $[-\beta_m, 1]$, and the optimal profit increases on $[-\beta_m, 1]$.*

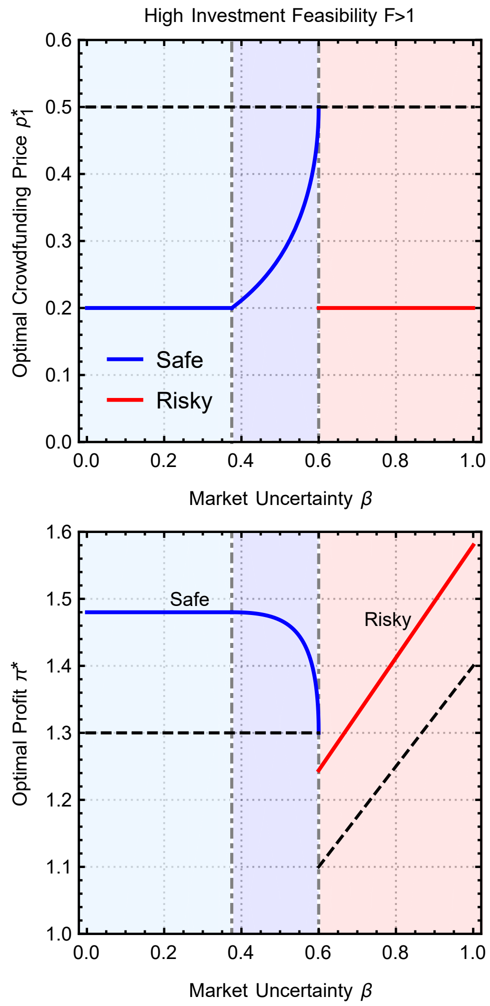
- (ii) *Under high investment feasibility, the optimal crowdfunding price increases on $[0, \beta_\tau]$ and remains fixed on $(\beta_\tau, 1]$; while the optimal profit decreases on $[0, \beta_\tau]$ and increases on $(\beta_\tau, 1]$.*

The change of market uncertainty would affect the pricing strategy and profitability of a given project. More specifically, part (i) focuses on the low investment feasibility case, i.e., $1/2 \leq \mathcal{F} < 1$, and shows that there is a threshold $(-\beta_m)$ (which is positive as $\beta_m < 0$) of market uncertainty, below which the project is not feasible. Recall that with low investment feasibility, the firm can only bet on the "good-enough outcome", which occurs when market uncertainty is high given the fixed expected market size. When the market uncertainty is higher than β_m, the optimal profit increases in the uncertainty level β, since the success probability is always 1/2 and the firm benefits from an increased high demand realization. In contrast, the optimal price always decreases as the firm strategically prices low to exploit the WoM benefit of crowdfunding. As β increases further, the optimal crowdfunding price may remain constant; the reason could be either the product awareness in the retail market has reached one at the current price, or the WoM intensity is relatively low, making it not worthwhile to further reduce the price. Figure 3.3 depicts the impact of market uncertainty on the optimal profit as stated in Proposition 3.4.3.

On the other hand, part (ii) focuses on the high investment feasibility case, i.e., $\mathcal{F} \geq 1$, and shows that the market uncertainty may have some opposite impacts compared to its counterpart discussed previously. In particular, as illustrated in Figure 3.3(b), when β is low, it is optimal to adopt the safe pricing scheme and the project is ensured to be



(a) Low Investment Feasibility $\frac{1}{2} \leq \mathcal{F} < 1$



(b) High Investment Feasibility $\mathcal{F} \geq 1$

Note: The dashed lines represent cases with myopic/uniform pricing, i.e., $\theta_1^* = \theta_2^* = 1/2$.

Figure 3.3.: (Color Online) The Impact of Market Uncertainty on Optimal Profit

successful. When β is in the intermediate range, the optimal crowdfunding price should increase as β increases, which is mainly driven by the firm's preference to stay within the safe pricing regime and take advantage of the high success probability. This preference shrinks the feasible domain of the pricing strategy θ_1, and hence leads to downward revision in expected profits. It is also important to notice that the increase of market uncertainty does not necessarily always bring down the profit of those safe projects. As we can see from Figure 3.3(b), the market uncertainty increase may have no impact within a certain low range.

Proposition 3.4.3 provides a useful managerial implication: The increase of market uncertainty is a double-edged sword. On one hand, it always benefits risky projects. On the other hand, it hurts some (not all) of the safe projects that face intermediate market uncertainty. The important driving force of such phenomenon is the AoN mechanism: The funding scheme conditions the firm's initial investment on the realized market size, and thus enables product-market-fit experimentation in a cost-effective way. We may call such a benefit the "*loser's blessing*": Because firms are somewhat hedged against the downside of the uncertain market outcome, they respond with more aggressive participation in crowdfunding.

A general prediction of our model is that crowdfunding encourages highly risky projects and provides an important funding channel for high-risk ventures, which is typically the case for innovative ideas and projects. Our model prediction is consistent with empirical evidence that entrepreneurs entering Kickstarter shift to projects that face higher uncertainty when crowdfunding becomes relatively more costly [129]. Such favor for high-risk projects can also help explain why the average success rate for crowdfunding projects on Kickstarter is only 36% across all categorizes[14], and why the majority of unsuccessful projects raise no more than 20% of their funding targets (i.e., extremely high or low market size).

3.4.5 Impact of WoM Intensity

An important stage during the introduction of a new product is the time when information about the product is diffusing via WoM through the population, and crowdfunding could function as an indispensable tool to stimulate such viral marketing. The WoM effect is a salient feature for the crowdfunding business model. In preceding analysis, we have seen the WoM effect has significant structural implications for the optimal pricing scheme in crowdfunding, which could be categorized into the three scenarios (weak, medium, strong) according to the strength of the WoM intensity. The WoM communication initiated by early adopters in crowdfunding stage drives the amount of information in the retail market about the product's existence. Conditional on knowing about the product, customers would possibly buy the product if their utility exceeds the retail price.

[14] Data retrieved from https://www.kickstarter.com/help/stats?ref=about_subnav, on March 14, 2018

As WoM expands the awareness of the product, it essentially expands the retail market size. A natural conjecture is that a higher WoM intensity, representing a more powerful expanding of product awareness, would benefit the startup.

Proposition 3.4.4 *As the WoM intensity increases, the optimal profit always increases; the optimal crowdfunding price decreases on* $[0, k_m]$*, increases on* $[k_m, k_n]$*, and remains fixed on* $[k_n, +\infty)$*.*

Proposition 3.4.4 confirms that a more intense WoM communication helps the startup increase profit under both medium and high investment feasibility. As the WoM intensity increases, the optimal crowdfunding price follows a pattern of decreasing-increasing-unchanged, while the optimal profit always increases. See Figure 3.1 for an illustration. The key intuition behind this observation is that it is always better to have a higher WoM intensity since it increases the product awareness and essential expand the potential market in the retail stage. For the crowdfunding price, it is optimal to fully explore the benefit of WoM effect by offering lower price as the WoM intensity increases but within a low range. As it further increases to a high level, a higher price can already guarantee the maximum level of awareness enhancement.

It is important to point out this result draws an interesting contrast with the extant literature of "learning WoM" (social learning) which has highlighted that a higher WoM intensity typically drives down the firm's profit [128]. This difference between "informative WoM" and "learning WoM" is intuitive to understand. The WoM communication in our model framework is always beneficial as it helps spread the product information and inform more consumers about the product, where each consumer's quality valuation is perfectly known. However, the WoM communication in social learning helps consumers to discover their true valuation of the product, and stronger WoM intensity generally leads to more heterogeneous consumer valuation, which undermines the firm's surplus extraction. From the formula of θ_1^* in Proposition 3.4.1 and 3.4.2, it is also straightforward to see that without the WoM effect (i.e., $k = 0$), it is always optimal to adopt the uniform pricing strategy (i.e., no rewards should be offered in crowdfunding, $p_1^* = p_2^* = q/2$). This reflects that taking advantage of the WoM effect in crowdfunding is the key rationale behind the price discount or crowdfunding rewards.

3.4.6 Impact of Crowdfunding Market Share

Lastly, the impact of the crowdfunding market share α is worth investigating, since it not only affects the investment feasibility $\mathcal{F}_s$ but also influences the boundaries of different WoM intensity scenarios (k_m and k_n in Proposition 3.4.1 & 3.4.2). By definition, the investment feasibility $\mathcal{F}_s = \frac{\alpha q}{4 z_s}$ increases in α. Hence, as the crowdfunding market share increases, the project may switch from medium to high investment feasibility, and thus change the optimal pricing scheme from risky pricing to safe pricing. It is not not immediately obvious how the optimal price should be adjusted according to change of market share. However, it is intuitively clear that the increased share of crowdfunding market shall benefit the startup, since it essentially increases the product awareness due to more consumers directly knows the existence of the product from the crowdfunding platform even without any WoM communication. This observation is formally presented in the next proposition.

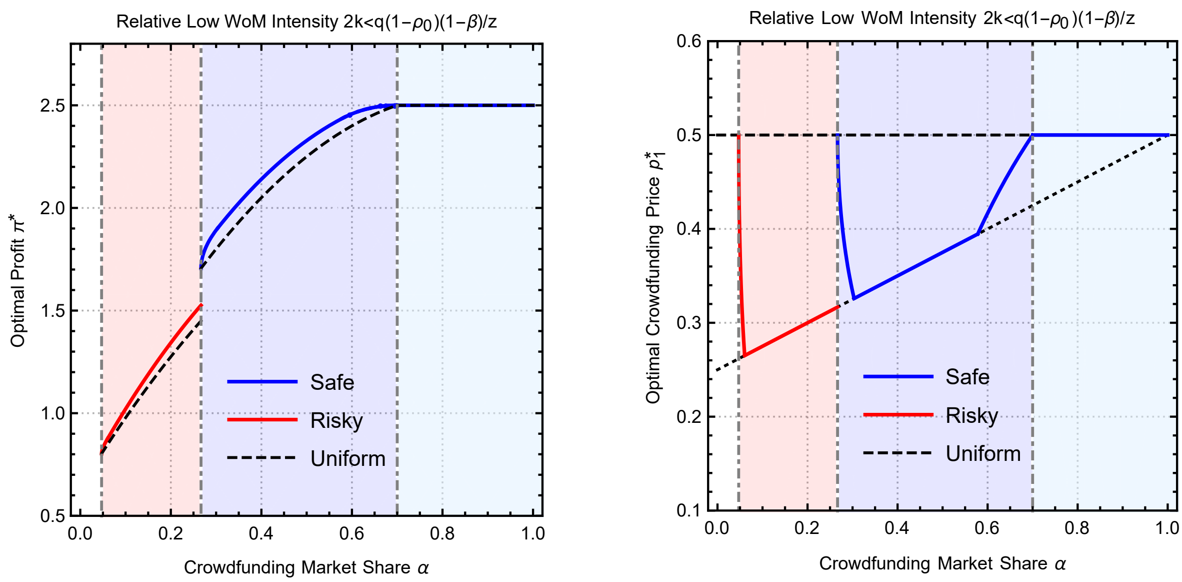


Figure 3.4.: (Color Online) The Impact of Crowdfunding Market Share on Optimal Profit and Crowdfunding Price

Proposition 3.4.5 *The startup's optimal profit always increases in the crowdfunding market share.*

The core result is that larger crowdfunding market share indeed benefits the startup. Although this may be consistent with one's ex ante intuition, it is critical to appreciate that the pricing mechanism driving it may be somewhat less intuitive. As we can see from Figure 3.4, when the crowdfunding market is too small, i.e., $\alpha < \frac{4z_s}{q(1+\beta)}$, it is not feasible to go crowdfunding. As the crowdfunding market share increases, the project investment feasibility first becomes medium (with risky pricing scheme), and then switches to high (with safe pricing scheme). A jump in profit is observed at the threshold point $\alpha = \frac{4z_s}{q(1-\beta)}$, where the investment feasibility changes (project success probability changes). The optimal crowdfunding price, in general, would first decrease from 1/2, and then increases to 1/2. In general, the optimal price increases as the market share increases, with the lower bound $\frac{4-k+k\alpha}{8}$ (the dotted line) linearly increases in α.

Consequently, the startup should always prefer the crowdfunding platform with the highest traffic rank, while adjusting the crowdfunding price accordingly with consideration of other parameters, e.g., WoM intensity, awareness base. Note that Figure 3.4 only gives the example pattern when the WoM intensity is relatively low, i.e., $2k < q(1-\rho_0)(1-\beta)/z$. For the other case, the price pattern is slight different. See Figure 3.5 below.

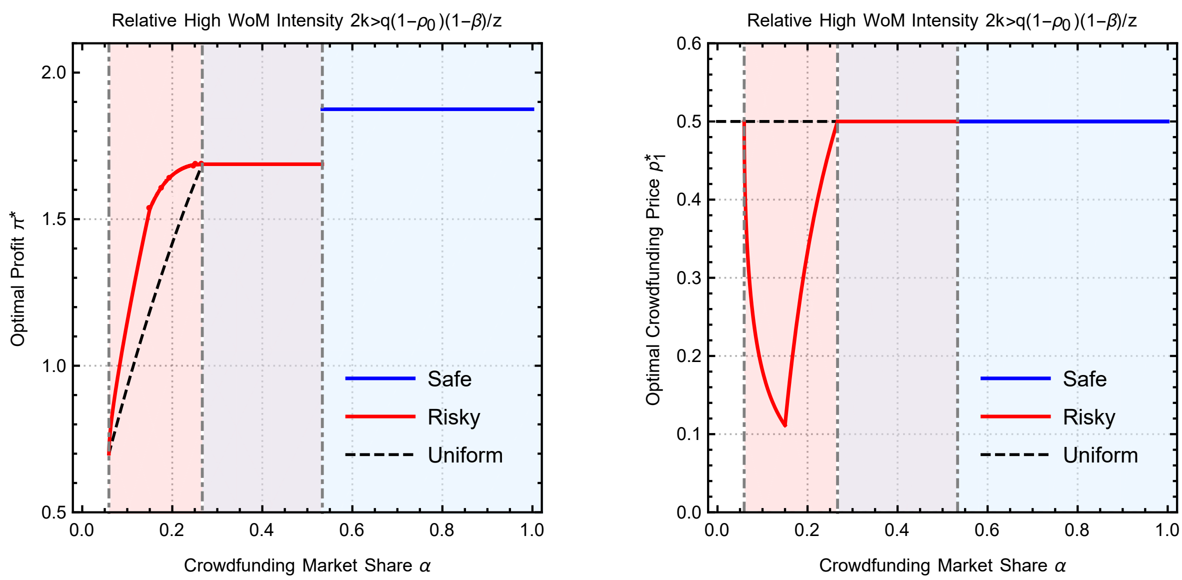


Figure 3.5.: (Color Online) The Impact of Crowdfunding Market Share on Optimal Profit and Crowdfunding Price

3.5 Bank Financing Strategy

Besides crowdfunding, the firm may also have access to short-term financing provided by a bank, and directly sell the product to the retail market. This section studies the firm's optimal pricing strategy under bank financing. We assume all consumers are present in the retail market. This assumption helps disentangle the benefits of crowdfunding from the consumer segment advantage and ensure fair comparisons. In the subsequent analysis, we first characterize the bank's interest rate decision given the firm's profile in the bank loan application, and then derive the firm's optimal pricing decision and its corresponding profit.

Compared to crowdfunding, bank financing has two main differences. One is the product awareness. Crowdfunding has an advantage in this respect, as all consumers in crowdfunding market are informed about the product, and crowdfunding sales can further generate awareness expansion in the retail market. The other key difference is the funding risk. Bank financing entails two risks: One is the firm's *credit risk*, reflected by the credit rating. Note that the credit rating is exogenously given, which could potentially depend on the firm's historical performance. For extremely low credit rating, the firm may not have access to bank financing. The other is the *demand risk*, and the bank will charge a higher interest rate or even reject the loan application if the current business is too risky. But for crowdfunding, the funding risk purely depends on the demand uncertainty. If the realized demand is high, the firm can raise enough funds to continue business. See Figure 3.6 for a graphical illustration of the sequence of risk realization.

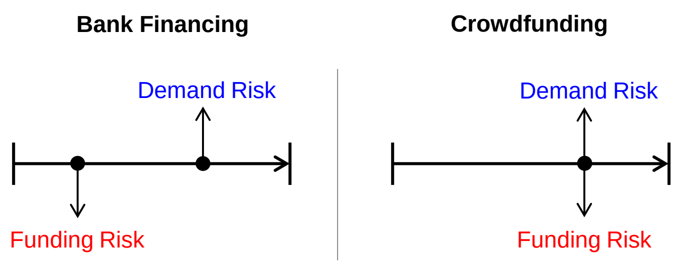


Figure 3.6.: Funding Risk Difference between Bank Financing and Crowdfunding

When bank financing is adopted, the bank loan will be competitively priced but with a premium reflecting the firm's credit risk. The risk-free interest rate is normalized to zero for expositional brevity. Based on the short-term credit practices described in [32] and following the literature [1,31], we assume that $\eta(C_s) > 0$ is the *interest rate premium* charged by the bank based on the borrower's credit rating C_s and $\eta(\cdot)$ is a general decreasing function. Extant empirical work in corporate finance substantiates the assumption by reporting that credit ratings are negatively correlated to the borrowing costs of the firm [130]. In our study, credit rating is adopted to reflect the heterogeneous financing costs across firms and capture the firm's accessibility to traditional bank financing.

Suppose the firm chooses the retail price p_b, then the consumers with $\theta \geq \theta_b = p_b/q$ will purchase. Without the WoM communication, the probability that consumers in the market are aware of the innovation product is $A(0) = \rho_0$. Then, the bank's interest rate r_s for the firm would be chosen so that it is indifferent between issuing the loan to the firm and earning a rate of return $\eta(C_s)$, i.e., $z(1+\eta(C_s)) = \mathbb{E}_{\mathcal{I}}[\min\{q\theta_b(1-\theta_b)\rho_0\mathcal{I}, z(1+r_s)\}]$. We refer to this interest rate pricing equation as the *competitive credit pricing equation.* Now, we define the *financing cost coefficient* $\delta(C_s)$ as $\delta(C_s) = 1+\eta(C_s)$, which is a decreasing function with $\delta(C_s) > 1$. Then, the competitive credit pricing equation becomes

$$z\delta(C_s) = \mathbb{E}_{\mathcal{I}}[\min\{q\theta_b(1-\theta_b)\rho_0\mathcal{I}, z(1+r_s)\}]. \tag{3.12}$$

Note that such competitive credit pricing is widely adopted in finance and operations management literatures [?, see, e.g.,]]burkart2004kind, chod2016inventory, kouvelis2012financing, kouvelis2018should. The essential idea is that the banking sector is assumed to be perfectly competitive, with prices fully reflecting all relevant risk information. Default risks are priced at markups above the risk-free rate; that is, the expected repayment the bank gets equals the loan principal plus the risk-free interest and an added premium related to the borrower's credit rating. Based on such risk pricing mechanism, we can further derive the explicit formula of the bank's interest rate r_s, and its dependence on the firm's profile as well as the market condition, as shown in the following lemma.

Lemma 11 *Given the firm's profile* (q, z, C_s, θ_b) *and the market condition* (β, ρ_0)*:*

(i) When $z\delta(C_s) > q(1+\beta)\rho_0\theta_b(1-\theta_b)$*, the bank will reject the loan request.*

(ii) When $q(1-\beta)\rho_0\theta_b(1-\theta_b) < z\delta(C_s) \leq q(1+\beta)\rho_0\theta_b(1-\theta_b)$*, the bank offers the interest rate*

$$r_s = 2\delta(C_s) - \frac{1}{z}q\theta_b(1-\theta_b)\rho_0(1-\beta) - 1. \tag{3.13}$$

(*iii*) *When* $z\delta(C_s) \leq q(1-\beta)\rho_0\theta_b(1-\theta_b)$*, the bank offers the lowest interest rate* $r_s = \delta(C_s) - 1$*.*

Lemma 11 summarizes the bank's decision on the interest rate and reflects three different levels of risk associated with the firm's borrowing activity. Part (*i*) of Lemma 11 represents the *high financing risk* situation where the firm's credit rating is too low relative to her operational risk, and the bank will always decline her credit application since it cannot break even in high realization of the market size. Hence, bank financing is not a feasible option for the firm in this extreme case. At the other extreme with *low financing risk* as in part (*iii*), the firm may have a favorable position (high credit rating) relative to her low operational risk, and could obtain the needed funding from the bank with a low interest rate competitively priced based on her credit rating. The firm faces no bankruptcy risk since even the low market size could guarantee her full repayment (with interest) of the bank loan. Therefore, the bank charges an interest rate of $r_s = \delta(C_s) - 1$, which is the lowest interest rate for the firm (it only prices in the credit risk reflected by the firm's credit rating). Part (*ii*) represents the *medium financing risk* case, in which the firm will claim bankruptcy if the market size turns out to be low, and thus the interest rate of bank financing is higher than part (*iii*) to hedge against such operational risk.

We further highlight several observations. First, the bank's interest rate decision depends not only on the firm's credit rating C_s and the loan size z, but also on her operational decision θ_b. Second, suppose the objective is to minimize the interest rate r_s of bank financing, it is straightforward to derive the optimal pricing strategy as $\theta_b^* = 1/2$. Third, consistent with our intuition, the bank's interest rate r_s is increasing in the market uncertainty β according to equation (3.13).

Now, we proceed to analyze the firm's expected profit function. Under medium financing risk, the firm goes bankrupt (zero profit) with probability 1/2, and makes a positive profit with probability 1/2. Under low financing risk, the firm's expected profit is the profit over the mean market size, and the bank's interest rate is $r_s = \delta(C_s) - 1$. We can further derive the firm's profit function under bank financing as follows (see Lemma 22):

$$\pi(\theta_b) = q\rho_0\theta_b(1-\theta_b) - z\delta(C_s). \tag{3.14}$$

It is interesting to observe that the profits under low and medium financing risk are the same, even though the interest rates charged by the bank are different according to Lemma 11. This implies that, different from crowdfunding, the firm's price strategy under bank financing is not influenced by the market uncertainty level. Such a result is driven by the fact that the bank loan is competitively priced, and the bank only claims the time value of its investment, with credit rating adjusted risk premium. From equation (3.13), we can see the interest rate increases in the level of market risk, leading to an increase in financing cost. But as market risk increases, the firm's expected revenue also increases due to the firm's limit liability of the bank loan (i.e., claim bankruptcy when realized demand is low). By the competitive pricing nature, the cost increase and the revenue increase are well balanced, resulting in the constant net profit. We further define two threshold values of the credit rating, $C_l(\beta) < C_h(\beta)$, which explicitly depend on the market uncertainty β, and are derived from the equations $4z\delta(C_l) = q\rho_0(1+\beta)$ and $4z\delta(C_h) = q\rho_0(1-\beta)$, respectively. The following proposition characterizes the final equilibrium under bank financing.

Proposition 3.5.1 *When the firm's credit rating is low ($C_s < C_l(\beta)$), bank financing is not accessible; otherwise, the optimal pricing strategy is $\theta_b^* = 1/2$ and the bank's interest rate is*

$$r_s^* = \left\{ \begin{array}{ll} \delta(C_s) - 1, & \text{if } \ C_s \geq C_h(\beta) \\ 2\delta(C_s) - \delta(C_h(\beta)) - 1, & \text{if } \ C_l(\beta) \leq C_s < C_h(\beta) \end{array} \right\}, \tag{3.15}$$

under which the optimal expected profit is $\pi_b^* = q\rho_0/4 - z\delta(C_s)$.

Proposition 3.5.1 states that, in contrast to crowdfunding, the optimal pricing and profit under bank financing are independent of the market uncertainty β. That is, the firm's expected profit only depends on the expected market size (one) and the optimal price is always $q/2$. This is due to the expected profit function in equation (3.14) as discussed earlier. However, the firm's credit rating C_s and the inherent market risk β jointly determine the firm's accessibility to bank financing. If the credit rating is relatively low while the market uncertainty is relatively high, bank financing is not accessible. Moreover, from the firm's perspective, bank financing could be adopted only when the optimal expected profit is positive, i.e., $\delta(C_s) < \frac{q\rho_0}{4z}$. Compared to the threshold $C_l(\beta)$, bank financing is accessible as long as it is feasible for the firm.

3.6 Optimal Funding Choice

In this section, we proceed to analyze the main research question regarding the firm's optimal choice of funding schemes: whether crowdfunding is more attractive than bank financing. Particularly, we investigate the impact of market uncertainty β and initial investment z on the optimal funding choice, and discuss the strengths and weaknesses of both funding schemes.

3.6.1 Impact of Market Uncertainty

It is clear that market uncertainty plays an important role when comparing crowdfunding with bank financing since it affects the optimal profit in crowdfunding but not in bank financing. This suggests that one may dominate the other within a certain region of market uncertainty. We conduct detailed comparisons between the two financing schemes in the next proposition.

Proposition 3.6.1 *Given the firm's profile* $(\alpha, \rho_0, q, z, C_s)$ *and the WoM intensity* k*:*

(i) *Under low investment feasibility, the following statements hold:*

- (a) *When* $\pi_b^* < \mathcal{H}^r(-\beta_m)$*, crowdfunding is preferred iff* $\beta \in [-\beta_m, 1]$.
- (b) *When* $\pi_b^* \geq \mathcal{H}^r(-\beta_m)$*, there exists a* β_r^b*, such that crowdfunding is preferred iff* $\beta \in (\beta_r^b, 1]$.

(ii) *Under high investment feasibility, the following statements hold:*

- (a) *When* $\pi_b^* \leq \mathcal{H}^r(\beta_m)$*, crowdfunding is always preferred.*
- (b) *When* $\mathcal{H}^r(\beta_m) < \pi_b^* < \mathcal{H}^s(\beta_m)$*, there exists a* β_r^b*, such that crowdfunding is preferred iff* $\beta \in [0, \beta_m] \cup (\beta_r^b, 1]$.

Recall that $\mathcal{H}^r(\beta) := \pi^r(\theta_1^u \vee \theta_l \wedge \theta_h)$ with $\beta \in [0, 1]$, and $\mathcal{H}^s(\beta) := \pi^s(\theta_1^u \vee \theta_h)$ with $\beta \in [0, \beta_m]$, are the auxiliary functions defined in equation (3.11). The threshold β_r^b is uniquely derived from equation $\mathcal{H}^r(\beta_r^b) = \pi_b^*$. Proposition 3.6.1 identifies the conditions of the market uncertainty under which crowdfunding is preferred over bank financing. Under low investment feasibility (I.1), bank financing is only favored when market uncertainty is relatively low (i.e., $\beta < -\beta_m$ or $\beta < \beta_r^b$), whereas crowdfunding is always preferred under high market uncertainty (i.e., $\beta > -\beta_m$ or $\beta > \beta_r^b$). Crowdfunding cannot dominate bank financing since the latter is accessible at low market uncertainty ($\beta < -\beta_m$) whereas crowdfunding is not. This result calls attention to the important advantage of bank financing as it is inertial to the level of market uncertainty as reflected in Proposition 3.5.1. In contrast, crowdfunding would gain a competitive edge under high market uncertainty.

Part (*ii*) presents the comparison for projects with high investment feasibility (I.2). In this case, bank financing could be dominated as in case (*a*) due to two potential reasons: a weak awareness base, or a poor credit rating that leads to high financing cost from the bank's competitive pricing. When crowdfunding is not dominating, case (*b*) provides a range of market uncertainty under which one is preferred over the other. It is of particular interest and importance to see that under high investment feasibility, bank financing could only be favored when the market uncertainty is within a certain intermediate range (i.e., $\beta_m < \beta < \beta_r^b$), whereas crowdfunding is always preferred by either safe projects (i.e., $0 \leq \beta \leq \beta_m$) or risky projects with high market uncertainties (i.e., $\beta_r^b < \beta \leq 1$).

The intuition behind Proposition 3.6.1 (*ii*)(*b*) can be explained by the underlying trade-off between two countervailing forces of crowdfunding: the *capital-raising benefit* and the *demand-validating benefit*. The capital-raising benefit represents the free capital injection through monetary contributions from the crowd, without any financing cost (no interest charged). For safe projects, such benefit is relatively high due to the ensured project success. For risky projects, such benefit is weakened due to the lower success probability. The demand-validating benefit comes from the fact that the firm can condition its production (and relevant costs) on high demand realization. Moreover, crowdfunding possesses a third benefit of *awareness-expanding*. However, the magnitude of this benefit, unlike the previous two, is independent of the market uncertainty. Then, it is straightforward to see with extremely low (high) market uncertainty, the capital-raising (demand-validating) benefit is prominent while the demand-validating (capital-raising) benefit is negligible. When the market uncertainty is intermediate, both benefits are mediocre, leaving crowdfunding potentially inferior to bank financing.

Next, we decouple the capital-raising and the awareness-expanding benefits of crowdfunding, and compare the two funding schemes to focus solely on the demand-validating benefit of crowdfunding.

Corollary 2 *When $\rho_0 \to 1$ and $\delta(C_s) \to 1$, crowdfunding strictly dominates bank financing iff $\beta \in (\beta_r^b, 1]$ under both low and high investment feasibilities.*

The conditions $\rho_0 \to 1$ and $\delta(C_s) \to 1$ in Corollary 2 indicate a perfect credit position in the banking system and a perfect marketing position (full product awareness) in the retail market. That is, under such extreme conditions, crowdfunding has no advantage in either its marketing benefit via WoM or its financial benefit (both accessibility and cost of financing). But, even in such an ideal case, bank financing is still strictly dominated by crowdfunding over the high market uncertainty range. Such decoupling of the financing (capital-raising) and marketing (awareness-expanding) benefits helps identify the ambiguous operational advantage (demand-validating benefit) of crowdfunding - that it can hedge against demand risk by leveraging the AoN funding mechanism.

The above analysis provides a rationale for the recent practices of established companies launching products via crowdfunding platforms. A well-known example of such a successful crowdfunding project was run by FirstBuild, a subsidiary of General Electric. In July 2015, FirstBuild launched a campaign on Indiegogo[15] for a countertop nugget ice maker, the Opal. While the funding goal was only \$150,000, the firm raised \$2.8 million from over 6,000 backers within a month [87]. A director of FirstBuild commented: "*The benefits of launching a new product like Opal using the Indiegogo crowdfunding platform allows us immediate feedback on market acceptance.*" Following its success on Indiegogo, FirstBuild soon made the Opal available for purchase in regular retail stores. That is, even without capital-raising and awareness-expanding needs, General Electric can still use crowdfunding as a market test to validate the potential demand before committing to large-scale production and distribution, as pointed out by Corollary 2.

3.6.2 Impact of Initial Investment

The level of initial investment z (i.e., funding target) has a critical role in both crowdfunding and bank financing. For a given product quality q, the initial investment z essentially captures the investment efficiency of the project (the smaller, the better). The following proposition captures the firm's optimal funding scheme choice through the lens of initial investment z.

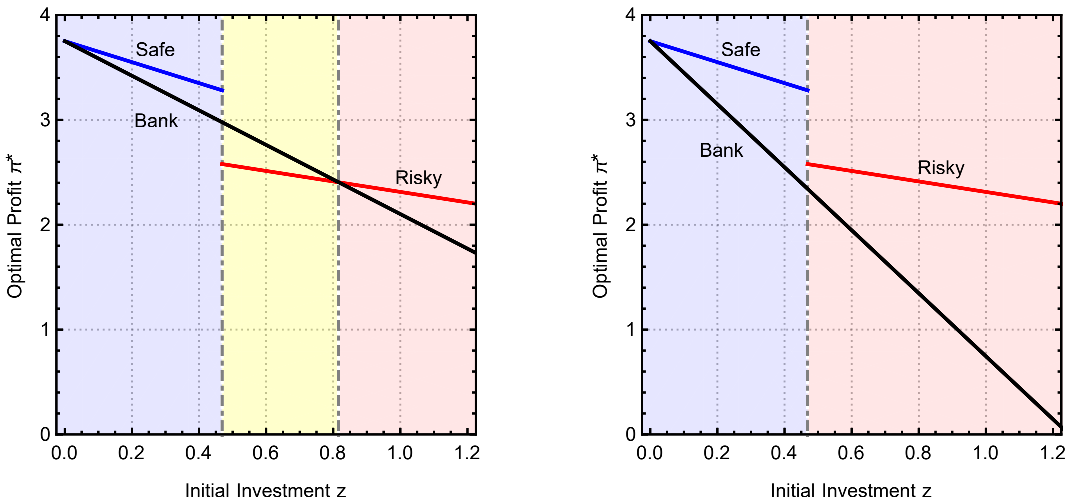
[15]Indiegogo has recently announced a new service called *Enterprise Crowdfunding*, targeting on large companies that are interested in crowdfunding their innovative projects.

Proposition 3.6.2 *Suppose $\rho_0 \to 1$ and C_τ satisfies $\alpha(1-\beta)(2\delta(C_\tau)-1) = 2\rho_0 - 1 - \beta$.*

(*i*) *When $z \leq \frac{(1-\beta)\alpha q}{4}$, crowdfunding is always preferred.*

(*ii*) *When $\frac{(1-\beta)\alpha q}{4} < z \leq \frac{(1+\beta)\alpha q}{4}$: If $C_s \geq C_\tau$, there exists a $z_b \in \left(\frac{(1-\beta)\alpha q}{4}, \frac{(1+\beta)\alpha q}{4}\right]$, such that bank financing is preferred iff $z \in \left(\frac{(1-\beta)\alpha q}{4}, z_b\right)$. Otherwise, crowdfunding is always preferred.*

Proposition 3.6.2 characterizes the firm's preference over the two financing schemes at different levels of initial investments. When the initial investment is low, the crowdfunding project is always successful as the target can be achieved even with low demand realization. For bank financing, even without bankruptcy risk in such a situation, the bank's capital injection is not free, but at a cost of interest rate $r_s = \delta(C_s) - 1 > 0$, which reduces the firm's total profit. Therefore, crowdfunding is always preferred in this case as shown in part (*i*), which corresponds to the left region in Figure 3.7. We further remark that in Proposition 3.6.2, the marketing (awareness-expanding) benefit of crowdfunding has already been decoupled (as $\rho_0 \to 1$); hence the comparison only accounts for the operational (demand-validating) benefit and financial (capital-raising) benefit. In addition, since the crowdfunding project is safe, there is no demand-validating benefit. Hence, the pure capital-raising benefit drives the preferred position of crowdfunding.



(a) $\rho_0 = 1,\ \beta = 0.5,\ \delta(C_s) = 1.5$ (b) $\rho_0 = 1,\ \beta = 0.5,\ \delta(C_s) = 2.1$

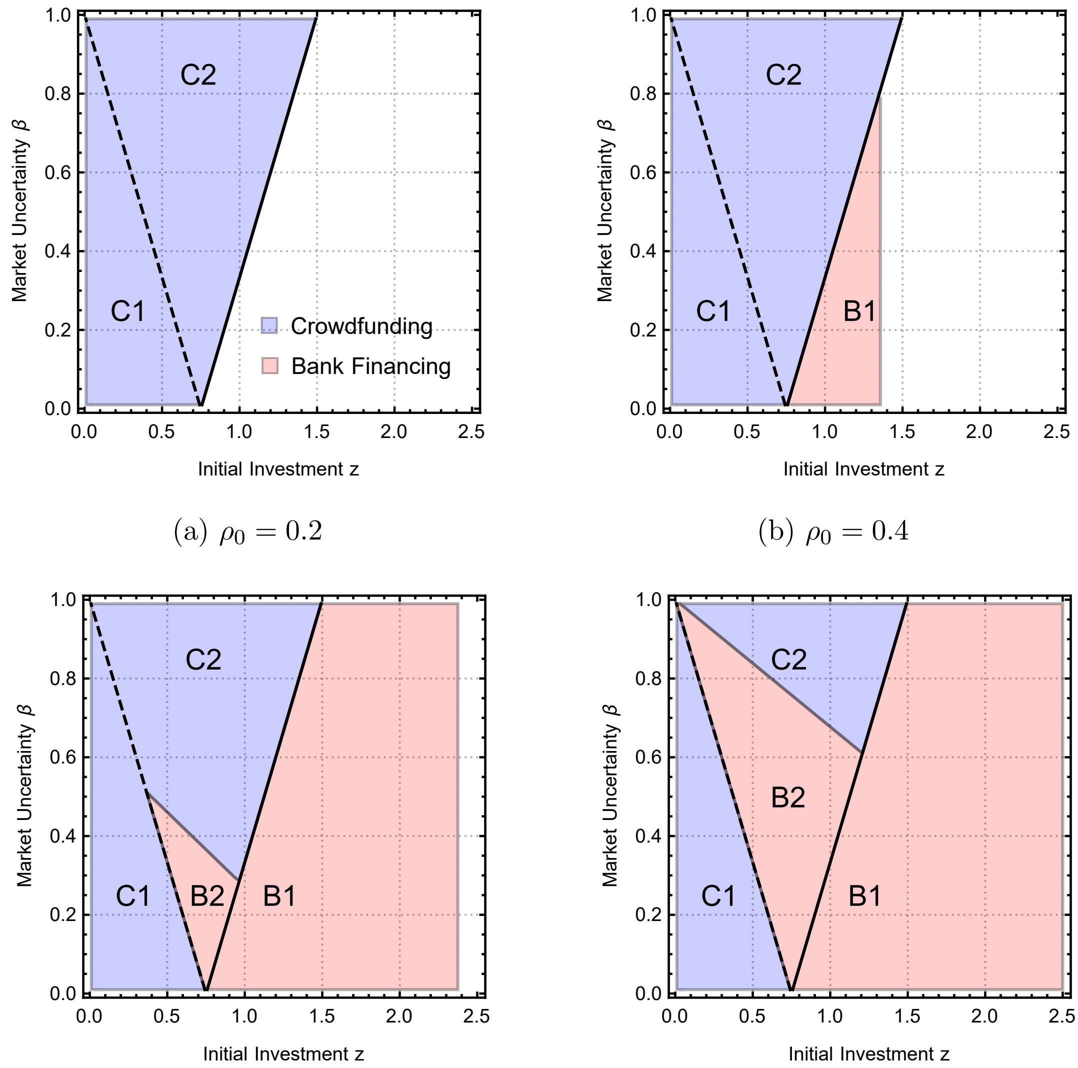
Figure 3.7.: (Color Online) The Impact of Initial Investment on Profits under Different Funding Schemes

As the initial investment increases to the range of $\frac{(1-\beta)\alpha q}{4} < z \leq \frac{(1+\beta)\alpha q}{4}$, the crowdfunding project becomes risky. Part (*ii*) considers two scenarios: If the firm's credit rating is too low (Figure 3.7(b)), crowdfunding still dominates bank financing since the corresponding interest rate charged by the bank is too high. Otherwise, bank financing could outperform crowdfunding in some medium range of the initial investment ($\frac{(1-\beta)\alpha q}{4} < z < z_b$). This result is driven by the downside of the AoN mechanism, i.e., the expected market shrinkage as the firm could only target on high demand realization under a risky crowdfunding project. Given the market uncertainty level, the demand-validating benefit of the AoN mechanism increases in the initial investment. When the initial investment is relatively small, the loss from market shrinkage outweighs the gain from demand validating, leading to a worse performance of crowdfunding compared to that of bank financing, when the latter has a relatively low financing cost.

3.6.3 A Holistic View

So far, we have shown how the firm's optimal funding choice depends on market uncertainty and initial investment, separately. We now conclude this section by discussing their joint effect from a holistic view. More importantly, the funding scheme comparison in this subsection is extended to a broader parameter region, with the inclusion of the region where crowdfunding is not feasible (i.e., $\mathcal{F} < 1/2$).

Figure 3.8 depicts the firm's optimal funding scheme choice in the space of market uncertainty and initial investment under different awareness bases. There are several noteworthy observations. First, when the firm's awareness base is limited as in Figure 3.8(a), bank financing is always infeasible. Hence, crowdfunding could dominate bank financing when its marketing benefit predominates (low awareness base) or bank financing is prohibitively expensive (low credit rating). The crowdfunding-feasible region is represented by regions C1 and C2, where C1 (C2) comprises those safe (risky) projects. Second, as the awareness base increases, bank financing becomes complementary to crowdfunding and it could be adopted in a certain parameter region (i.e., region B1 in Figure 3.8(b)), where the initial investments are relatively high and crowdfunding is not feasible. This crowdfunding-infeasible region contains projects with relatively low market uncertainties and/or high initial investments. Third, as the awareness base increases further in Figure 3.8(c), the advantage of bank financing becomes more promi-



(a) $\rho_0 = 0.2$ (b) $\rho_0 = 0.4$

(c) $\rho_0 = 0.7$ (d) $\rho_0 = 1.0$

Figure 3.8.: (Color Online) The Impact of Initial Investment and Market Uncertainty on Funding Choice

nent, as it covers a broader crowdfunding-infeasible region (B1) and also substitutes part of the crowdfunding-feasible region (B2). The reason is that although bank financing is costly and takes away a portion of the firm's profit, such loss could be less significant than that from the market shrinkage under crowdfunding as the firm could only target on high market realization. As the awareness base fully expands, bank financing could be feasible even for those projects with extremely high initial investments (expanding region B1 to the right side), regardless of the market uncertainty. It is also preferred by the firm in more crowdfunding-feasible region (B2) with medium levels of market uncertainty and initial investment. Finally, an important overall observation is that crowdfunding is always preferred for safe projects (C1), i.e., those projects with relatively low levels of both initial investment and mark uncertainty.

To sum up, our model is endowed with the flexibility and the capability to compare the two funding schemes under different business environments. The advantage of adopting traditional bank financing is two-fold: First, compared to crowdfunding, it has a broader feasibility, especially for capital-intensive projects (large initial investments). Second, it enables full market exploitation, covering both high and low demand realizations. The advantage of crowdfunding can be explicitly categorized into the following three underlying benefits: (1) *Capital-raising benefit*, which represents the free (zero-interest) capital injection through monetary contributions from the crowd; (2) *Awareness-expanding benefit*, which means early adopters in the crowdfunding stage can help expand product awareness, and thus increase the potential market size in the retail stage; and (3) *Demand-validating benefit*, which comes from the fact that the firm can condition its production (and relevant cost) on high demand due to the AoN mechanism. As such, the optimal funding choice is involved as the firm needs to carefully balance the ensued benefits of each funding scheme in accordance with detailed market conditions and project attributes.

3.7 Advance Selling

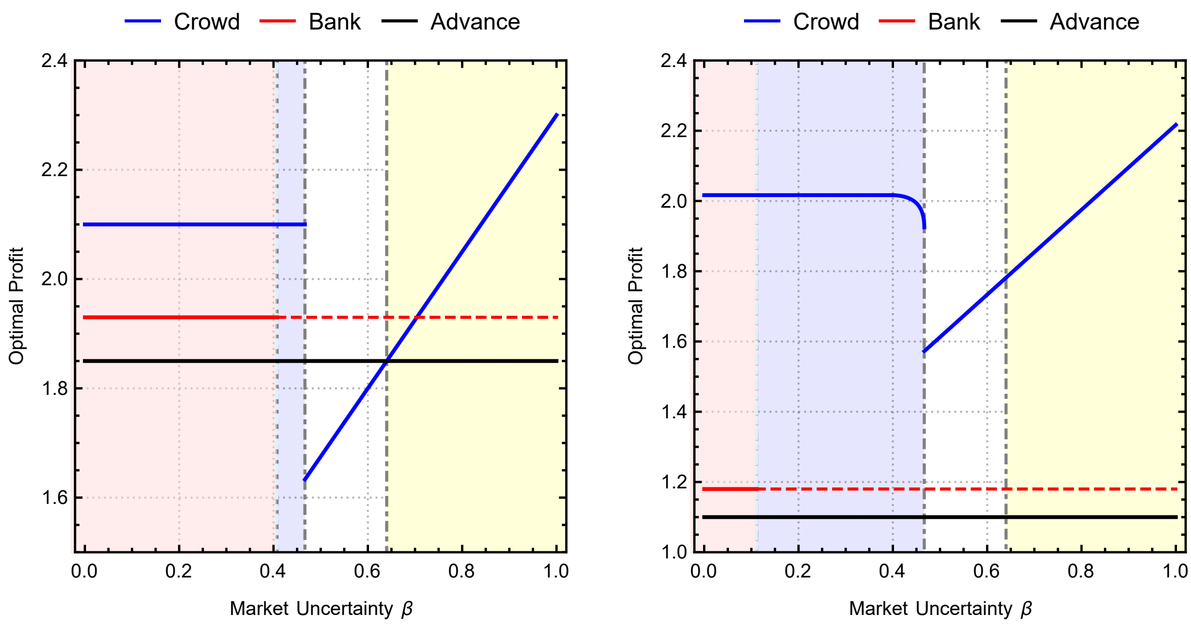
In this section, we study the equilibrium outcomes when the startup can adopt advance selling with bank financing, and to further compare with the results we get in crowdfunding. Different from the crowdfunding case, consumers active in the advance selling market and in the retail market get informed about the product with same probability ρ_0. That said, there is WoM benefit in advance selling. In the advance selling stage, the expected revenue is $\pi_1(\theta_1) = q\alpha\rho_0\theta_1(1-\theta_1)$. The expected revenue in the retail stage can be written as $\pi_2(\theta_2) = (1-\alpha)\rho_0(1-\theta_2)\theta_2 q$. Then, it is straightforward to see the optimal solution is always uniform pricing $\theta_1^* = \theta_2^* = 1/2$. Hence, the total profit is

$$\pi^* = \frac{q}{4}[\alpha\rho_0 + (1-\alpha)\rho_0] - z = \frac{\rho_0 q}{4} - z. \tag{3.16}$$

However, the key difference between advance selling and crowdfunding is that there should not exist any product development and delivery risk in advance selling, which means the funding collected in any demand realization must be able to support the initial investment z. Mathematically, the additional constraint is

$$\frac{q\alpha\rho_0(1-\beta)}{4} \geq z. \tag{3.17}$$

The constraint indicates that advance selling is only feasible for projects with relatively low market uncertainty, i.e., relatively small β. Compared with bank financing, the advance selling has the capital-raising advantage as crowdfunding, since there is not interest tied with the funding collected. But, the downside is the feasibility issue as advance selling needs the condition of less market uncertainty.



(a) $\rho_0 = 0.9$ (b) $\rho_0 = 0.6$

Note: The shadowed red region is where advance selling is feasible.

Figure 3.9.: (Color Online) Funding Choice: Crowdfunding, Bank Financing, or Advance Selling

Proposition 3.7.1 *Advance selling is only feasible when the market uncertainty is less than a threshold. Moreover, advance selling may outperform bank financing, but is always dominated by crowdfunding.*

In Figure 3.9, we plot the optimal profit in crowdfunding, bank financing, and advance selling. The shadowed red region is where advance selling is feasible. We can observe the red line (advance selling) is always above the black line (bank financing), but bank financing is feasible for all range of market uncertainty. Compared with crowdfunding, advance selling is always dominated. The intuition is that when advance selling is feasible, the firm can also adopt crowdfunding with safe pricing scheme, which has the same capital raising benefit with additional WoM benefit.

3.8 Consumer Surplus and Welfare Implications

The previous sections analyze the optimal pricing strategy and the corresponding profit for the firm, under both crowdfunding and traditional bank financing. In this section, we investigate the impact of the firm's funding choice on the consumer surplus, which is measured by the utility that people gain from consuming the product, and the social welfare, which is defined as the sum of consumer surplus and firm profit.

For the crowdfunding scheme, given the firm's optimal pricing strategy θ_1^*, we can derive the consumer surplus as follows. For risky projects, let $\mathcal{C}_1^r$ ($\mathcal{C}_2^r$) be the surplus in the crowdfunding stage (the retail stage). Then, it is straightforward to get $\mathcal{C}_1^r(\theta_1^*) = \frac{1}{4}(1+\beta)\alpha q(1-\theta_1^*)^2$ and $\mathcal{C}_2^r(\theta_1^*) = \frac{1}{16}(1+\beta)(1-\alpha)q\mathcal{A}(\theta_1^*)$. Taken together, the total consumer surplus can be formulated as

$$\mathcal{C}^r(\theta_1^*) = \frac{1}{16}q(1+\beta)\left[4\alpha(1-\theta_1^*)^2 + (1-\alpha)\mathcal{A}(\theta_1^*)\right]. \tag{3.18}$$

Similarly, for safe projects, we can obtain the consumer surplus as

$$\mathcal{C}^s(\theta_1^*) = \frac{1}{8}q\left[4\alpha(1-\theta_1^*)^2 + (1-\alpha)\mathcal{A}(\theta_1^*)\right]. \tag{3.19}$$

The social welfare can be written as $\mathcal{S}^i(\theta_1^*) = \pi^i(\theta_1^*) + \mathcal{C}^i(\theta_1^*)$, $i \in \{r, s\}$, where $\pi^i(\theta_1^*)$ is given in Section 3.4.3. For bank financing, based on the results in Section 5, we can derive the expected consumer surplus $\mathcal{C}_b$ and the total social welfare $\mathcal{S}_b$, respectively, as follows:

$$\mathcal{C}_b = \frac{1}{8}\rho_0 q, \quad \mathcal{S}_b = \pi_b^* + \mathcal{C}_b = \frac{3}{8}\rho_0 q - z\delta(C_s). \tag{3.20}$$

Now, we are ready to analyze and answer several important questions pertaining to consumer surplus and social welfare. In the following proposition, we first investigate the effects of both the WoM intensity and the market uncertainty under crowdfunding, as those factors will not affect the consumer surplus and the social welfare under bank financing.

Proposition 3.8.1 *For feasible crowdfunding projects, the following statements hold:*

(i) *As WoM intensity increases, the consumer surplus and the social welfare first increase, then decrease, and remain fixed afterwards.*

(ii) *As market uncertainty increases, the consumer surplus and the social welfare increase for risky projects, but decrease for safe projects.*

Interestingly, Proposition 3.8.1 (i) shows that consumer surplus and social welfare may decrease as the WoM intensity increases in the medium range. This is contrary to the conventional wisdom that stronger informative WoM should benefit consumers [**?**, e.g.,]]jing2011social. The underlying reason is that, although more active WoM communication helps diffuse product information and make more consumers aware of the product in the retail market, it also encourages the firm to strategically increase the crowdfunding price and extract more consumer surplus. More importantly, such strategic firm behavior hurts consumers more compared to the firm's profit gain, resulting in a lower social welfare. Part (ii) suggests that both the consumer surplus and the social welfare are influenced by the market uncertainty in a similar fashion as the firm's profit, due to the similar functional structures with respect to β.

Proposition 3.8.2 *For feasible crowdfunding projects, when bank financing is not dominated, i.e.,* $\pi_b^* > \mathcal{H}^r(|\beta_m|)$*, there exist two thresholds* β_s^b *and* β_c^b *such that:*

(i) *Bank financing offers a higher consumer surplus when* $\beta \in (|\beta_m|, \beta_c^b)$.

(ii) *Bank financing offers a higher social welfare when* $\beta \in (|\beta_m|, \beta_s^b)$.

Proposition 3.8.2 provides the preference over bank financing and crowdfunding, from the perspectives of both consumer surplus and social welfare. Similar to the discussion from the firm's perspective, as stated in Proposition 3.6.1, we find bank financing could only be preferred when the market uncertainty is relatively low while there always exists a range of high market uncertainty where crowdfunding is a better choice. We also numerically observe that the threshold from the firm's profit perspective (β_r^b) could be smaller than the one from the social welfare perspective (β_s^b), and the one from the consumer surplus perspective (β_c^b) is the largest. Consequently, there may exist an intermediate range of market uncertainty, where the firm prefers crowdfunding but bank financing is favored by consumers. Moreover, a further implication based on Propositions 3.6.1 and 3.8.2 is that crowdfunding is a win-win alternative fund-raising solution to bank financing when market uncertainty is relatively high (i.e., $\beta > \beta_c^b \vee \beta_r^b$).

3.9 Endogenous Quality Design

In the new Section 8, we endogenize the product quality decision and investigate the startup's optimal quality choice under both crowdfunding and bank financing. The analysis of this section centers around two main questions. The first pertains to the impact of financing sources (crowdfunding v.s. bank financing) on the startup's optimal quality choice and the corresponding consumer surplus and social welfare: Should firms increase or decrease product quality when they go crowdfunding? Which financing option could increase the consumer surplus or the social welfare? The second question concerns the impact of several important exogenous parameters on the optimal quality choices: What happens to the optimal quality as the awareness base, the WoM intensity, the crowdfunding market share, or the credit rating increases? Should firms always offer better products when consumers interact more frequently?

3.9.1 Optimal Quality under Crowdfunding

We assume that a quality design with target q incurs a quadratic fixed setup/investment cost $z_s = c(q) = q^2$. The crowdfunding failure threshold $\mathcal{I}_m(\theta_1)$ can be written as

$$\mathcal{I}_m(\theta_1) = \frac{z_s}{\alpha q \theta_1(1-\theta_1)} = \frac{q}{\alpha \theta_1(1-\theta_1)}, \tag{3.21}$$

and the minimal value is $\mathcal{I}_m(1/2) = 4q/\alpha$. Hence, the investment feasibility of the startup's crowdfunding project becomes a function of q, $\mathcal{F}_s(q) = \frac{\alpha}{4q}$, indicating a higher quality choice dampens the investment feasibility and would potentially switch the project investment feasibility from high to medium scenario. Depending on the choice of quality q, there are three different scenarios as follows:

(I.1) Low Investment Feasibility: $\mathcal{F}_s(q) < (1+\beta)^{-1}$, i.e., $4q > \alpha(1+\beta)$;

(I.2) Medium Investment Feasibility: $(1+\beta)^{-1} \leq \mathcal{F}_s(q) < (1-\beta)^{-1}$, i.e., $\alpha(1-\beta) < 4q \leq \alpha(1+\beta)$;

(I.3) High Investment Feasibility: $\mathcal{F}_s(q) \geq (1-\beta)^{-1}$, i.e., $4q \leq \alpha(1-\beta)$.

Taking this into account, we can start by analyzing the quality design and pricing problem independently for the case (I.2) and case (I.3), within which the optimal pricing problem given the quality decision has been analyzed in previous sections. The only difference is the boundary value θ_l and θ_h of the feasible range, which becomes a function of q now. Let $\theta_l(q)$ and $\theta_h(q)$ be functions of q that are derived from $\alpha(1+\beta)\theta_l(1-\theta_l) = q$, and $\alpha(1-\beta)\theta_h(1-\theta_h) = q$, respectively.

(I.2) Medium Investment Feasibility

Given the optimal pricing policy, the quality design problem ($\mathbb{P}_1$) can be formulated as

$$\max_{q \in \mathcal{Q}_1} \pi_{(1)}(q) = \max_{q \in \mathcal{Q}} \left\{ \psi^*(q, \beta) q - q^2 \right\}, \tag{3.22}$$

where

$$\mathcal{Q}_1 = \big(\alpha(1-\beta)/4, \alpha(1+\beta)/4\big], \tag{3.23}$$

$$\psi_1^*(q, \beta) = \frac{(1+\beta)}{8}[4\alpha\theta_1^*(q)(1-\theta_1^*(q)) + (1-\alpha)\mathcal{A}(\theta_1^*(q))], \tag{3.24}$$

and

$$\theta_1^*(q) = \left\{ \begin{array}{ll} \dfrac{(4-k+k\alpha)^+}{8} \vee \theta_l(q), & \text{if } \ 0 \le k < k_m \\ \dfrac{(k\alpha - 1 + \rho_0)^+}{k\alpha} \vee \theta_l(q), & \text{if } \ k_m \le k < k_n \\ \dfrac{1}{2}, & \text{if } \ k \ge k_n \end{array} \right\}. \tag{3.25}$$

(I.3) High Investment Feasibility

Given the optimal pricing policy, the quality design problem ($\mathbb{P}_2$) can be formulated as

$$\max_{q \in \mathcal{Q}_2} \pi_{(2)}(q) = \max_{q \in \mathcal{Q}} \left\{ \psi^*(q, \beta) q - q^2 \right\}, \tag{3.26}$$

where

$$\mathcal{Q}_2 = \big[0, \alpha(1-\beta)/4\big], \tag{3.27}$$

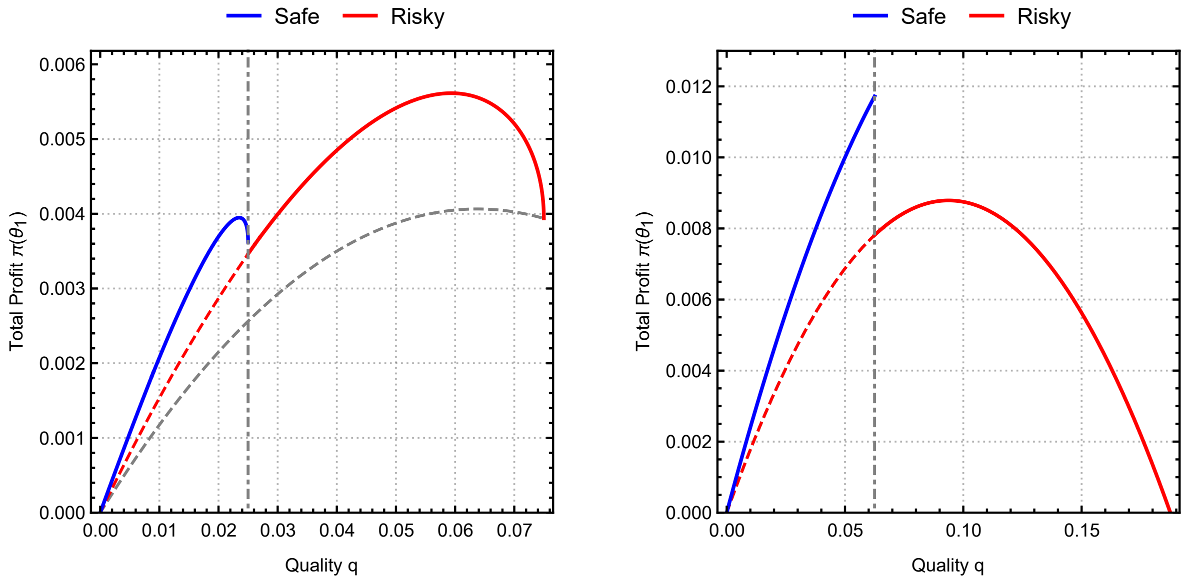
$$\psi_2^*(q, \beta) = \frac{1}{4}[4\alpha\theta_1^*(q)(1-\theta_1^*(q)) + (1-\alpha)\mathcal{A}(\theta_1^*(q))] \tag{3.28}$$

and

$$\theta_1^*(q) = \left\{ \begin{array}{ll} \dfrac{(4-k+k\alpha)^+}{8} \vee \theta_h(q), & \text{if } \ 0 \le k < k_m \\ \dfrac{(k\alpha - 1 + \rho_0)^+}{k\alpha} \vee \theta_h(q), & \text{if } \ k_m \le k < k_n \\ \dfrac{1}{2}, & \text{if } \ k \ge k_n \end{array} \right\}. \tag{3.29}$$

Proposition 3.9.1 *There exists a unique optimal solution q_i^* for Problem $\mathbb{P}_i$ and the optimal quality choice under crowdfunding is*

$$q^* = \arg\max\{\pi_{(1)}(q_1^*), \pi_{(2)}(q_2^*)\}.$$



(a) $\alpha = 0.2$ (b) $\alpha = 0.5$

Note: The dashed lines represent the results with myopic/uniform pricing, i.e., $p_1^* = p_2^* = q/2$.

Figure 3.10.: (Color Online) Optimal Profit w.r.t. Quality Choice q

3.9.2 Optimal Quality under Bank Financing

The optimal price is $p_b^* = q/2$ and the corresponding optimal profit is

$$\pi_b^* = q\rho_0/4 - z_s\delta(C_s) = q\rho_0/4 - \delta(C_s)q^2. \tag{3.30}$$

Note the constraint for quality choice is $4q\delta(C_s) \leq \rho_0(1+\beta)$, since the bank financing is not accessible/feasible otherwise. Hence, the quality design problem can be formulated as

$$\max_q \pi_b(q) = \max_q \left\{ \frac{\rho_0}{4} q - \delta(C_s)q^2 \right\}, \tag{3.31}$$

$$s.t. \quad 0 < q \leq \frac{\rho_0(1+\beta)}{4\delta(C_s)}. \tag{3.32}$$

It is straightforward to derive the optimal quality is

$$q_b^* = \frac{\rho_0}{8\delta(C_s)} \wedge \frac{\rho_0(1+\beta)}{4\delta(C_s)} = \frac{\rho_0}{8\delta(C_s)}. \tag{3.33}$$

Proposition 3.9.2 *For any credit rating C_s, the firm will choose the optimal pricing and quality as follows,*

$$p_b^* = \frac{q}{2}, \quad q_b^* = \frac{\rho_0}{8\delta(C_s)},$$

under which bank financing is always accessible, and the optimal expected profit is

$$\pi_b^* = \frac{\rho_0^2}{64\delta(C_s)}.$$

It is worth noticing from Proposition 3.9.2 that when the quality is chosen optimally, the startup can always get access to bank financing, even with low credit rating. As the credit rating decreases, the optimal quality choice also decreases.

3.10 Conclusion

In this paper, we investigate a firm's optimal funding choice when launching an innovative product to the market with demand uncertainty and WoM communication. The firm could fund its product by either running a crowdfunding campaign, or applying for a traditional bank loan. We build an analytical model to study the firm's optimal pricing strategies under each funding scheme, and then compare them to investigate the firm's optimal choice of funding scheme and the corresponding impacts on consumer surplus and social welfare.

We report several main results from this paper. First, in crowdfunding, the firm should use intertemporal pricing, which depends on both WoM and market uncertainty. Such a pricing strategy is driven by the trade-off between the immediate revenue loss from offering crowdfunding rewards to induce more early adoption and expand product awareness via WoM, and the future profit gain from selling the product to the consumer population that would remain otherwise uninformed. For bank financing, we find that the firm's credit rating and the market uncertainty jointly determine the feasibility of bank financing, but the optimal pricing strategy is independent of them. Next, the choice between crowdfunding and bank financing is not monotonic in the market uncertainty, with bank financing preferred only within an intermediate range. Similar observation holds for initial investment requirement. Although featured with benefits of capital-raising, awareness-expanding, and demand-validating, crowdfunding has never been perfect for every type of project. Bank financing could be either a complementary financing option when crowdfunding is not feasible, or a more appropriate funding choice that substitutes crowdfunding. Lastly, we find that, contrary to conventional wisdom, more active social interactions could hurt consumers and even social welfare under crowdfunding.

This paper is among the first to study a firm's funding scheme choice between crowdfunding and traditional bank financing, and to rationalize the advantage of crowdfunding by explicitly demonstrating the three underlying benefits: capital-raising, awareness-expanding, and demand-validating. As such, our research can help explain several phenomenal real-world puzzles, such as why established companies have began to launch products via crowdfunding platforms. The key contribution of our work is to provide thorough comparisons (firm profit, consumer surplus, and social welfare) between crowdfunding and bank financing under different scenarios along three key dimensions: market uncertainty, WoM communication, and initial investment. The managerial insights provide useful guidance on whether firms (including startups and established firms) should go crowdfunding, and (if yes) how to jointly exploit the operational, financial, and marketing benefits of crowdfunding. Our model also provides welfare implications on when crowdfunding is a win-win alternative fund-raising solution to support entrepreneurship and innovation.

This research can be extended in several directions to address other open questions about crowdfunding. First, the current model studies the optimal pricing strategies in crowdfunding and bank financing, but leaves the product quality decision as exogenously given. One interesting extension would focus on investigating the effect of different funding schemes on the firm's optimal product quality design. Second, besides the WoM communication through crowdfunding, the firm can directly invest in informative advertising to spread the product information and increase consumer awareness. It is worth studying when the firm might find it beneficial to directly invest in advertising. Finally, the firm is assumed to be a monopolist in the marketplace. It would also be interesting to explore the effect of market competition on the firm's funding scheme choice and pricing strategies.

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APPENDICES

A. Appendix for Chapter 1

Appendix A: Summary of Model Notations

To help the reader keep track of different components of our model, we summarize in Table A.1 the notations that we use throughout the paper.

Table A.1: Summary of Model Notations

Parameters	
p	retail price of the product
c	supplier's unit production cost at time 0
t_1	supplier's production lead-time
t_2	retailer's deferral payment period, i.e., trade credit payment term
t_c	supplier's current cash conversion cycle $t_c = t_1 + t_2$
α_j	investment return rate per unit time, $j = r$ (retailer) and s (supplier)
C_j	borrower j's credit rating score, $j = r$ (retailer) and s (supplier)
$\rho_j = \rho(C_j)$	default probability of a borrower with credit rating C_j
$\eta_j = \eta(C_j)$	interest rate premium charged by the bank/factor based on credit rating C_j
Decisions	
w_i	retailer's wholesale price, $i \in \{\mathcal{B}, \mathcal{F}, \mathcal{S}, \mathcal{R}\}$
q_i	supplier's production quantity, $i \in \{\mathcal{B}, \mathcal{F}, \mathcal{S}, \mathcal{R}\}$
r_i	bank's interest rate (per time unit) for the supplier's loan, $i \in \{\mathcal{B}, \mathcal{F}, \mathcal{S}, \mathcal{R}\}$
τ	retailer's payment extension in reverse factoring
$\mathcal{L}_i$	supplier's cash advance obtained from the factor at t_1, $i \in \{\mathcal{F}, \mathcal{S}, \mathcal{R}\}$
Derived	
$b_{\mathcal{B}}$	supplier's bank financing bankruptcy threshold
$b_{\mathcal{F}}$	supplier's recourse factoring bankruptcy threshold
c_i	supplier's effective unit production cost, $i \in \{\mathcal{B}, \mathcal{F}, \mathcal{S}, \mathcal{R}\}$
$\bar{F}(\cdot)$	The complementary c.d.f. of the demand distribution
$z(\cdot)$	the failure rate of the demand distribution $z(\xi) = f(\xi)/\bar{F}(\xi)$
$S(q)$	$S(q) = \mathbb{E}[\min(D, q)] = q - \int_0^q F(\xi)d\xi = \int_0^q \bar{F}(\xi)d\xi$
$j(q)$	$j(q) = S(q)/\bar{F}(q)$
$\bar{\alpha}_{\mathcal{F}}$	threshold for supplier's investment return rate $\bar{\alpha}_{\mathcal{F}} := \eta_s - t_2^{-1}\ln(1-\rho_s)$
$\bar{\alpha}_{\mathcal{S}}$	threshold for supplier's investment return rate defined in equation (1.15)
$\bar{\alpha}_{\mathcal{R}}$	threshold for supplier's investment return rate defined in equation (1.19)
$\psi(\alpha_s)$	a function of supplier's investment return rate α_s defined in equation (2.11)

Appendix B: Proofs of the Statements

We put all the proofs of the Lemmas and Propositions in Appendix B.

Proof of Proposition 1.3.1:

(i) First, we derive the supplier's best response of production quantity to the retailer's wholesale price. For a given w, we have shown the supplier's expected profit is $\pi_{\mathcal{B}}(q; w) = (1-\rho_s)(1-\rho_r)(wS(q) - c_{\mathcal{B}}q)$. Then,

$$\frac{\partial \pi_{\mathcal{B}}(q; w)}{\partial q} = (1-\rho_s)(1-\rho_r)(w\bar{F}(q) - c_{\mathcal{B}}),$$

which is strictly decreasing in q. Hence, $\pi_{\mathcal{B}}(q; w)$ is quasi-concave in q, and the supplier's optimal production quantity $q_{\mathcal{B}}$ given a wholesale price w can be solved from the first-order condition, $w\bar{F}(q_{\mathcal{B}}) = c_{\mathcal{B}}$.

Next, we analyze the retailer's profit function given the supplier's best response derived above. We have shown $\Pi_{\mathcal{B}}(w) = (1-\rho_r)(p-w)S(q_{\mathcal{B}})$. Taking derivatives with respect to w, we have

$$\frac{d\Pi_{\mathcal{B}}(w)}{dw} = (1-\rho_r)\Big[-S(q_{\mathcal{B}}) + (p-w)\bar{F}(q_{\mathcal{B}})\frac{dq_{\mathcal{B}}}{dw}\Big].$$

Taking derivatives for both sides of the supplier's best response equation $w\bar{F}(q_{\mathcal{B}}) = c_{\mathcal{B}}$ with respect to w, we now have

$$-wf(q_{\mathcal{B}})\frac{dq_{\mathcal{B}}}{dw} + \bar{F}(q_{\mathcal{B}}) = 0, \quad \frac{dq_{\mathcal{B}}}{dw} = \frac{\bar{F}(q_{\mathcal{B}})}{wf(q_{\mathcal{B}})} = \frac{1}{wz(q_{\mathcal{B}})} > 0.$$

Under our assumption, $z(q_{\mathcal{B}})$ increases in $q_{\mathcal{B}}$. Hence, $dq_{\mathcal{B}}/dw$ is positive and decreasing in w. Thus, we can conclude that $d\Pi_{\mathcal{B}}(w)/dw$ strictly decreases in w, and thus $\Pi_{\mathcal{B}}(w)$ is strictly concave in w. Here, the range for wholesale price is $w \in [c_{\mathcal{B}}, p]$. And it's straightforward to check that

$$\frac{d\Pi_{\mathcal{B}}(w)}{dw}\bigg|_{w=c_{\mathcal{B}}} > 0, \quad \frac{d\Pi_{\mathcal{B}}(w)}{dw}\bigg|_{w=p} < 0.$$

Therefore, there exists a unique optimal $w_{\mathcal{S}}$ that can be solved from

$$S(q_{\mathcal{B}}) = (p-w)\bar{F}(q_{\mathcal{B}})\frac{dq_{\mathcal{B}}}{dw} = \frac{(p-w)\bar{F}(q_{\mathcal{B}})}{wz(q_{\mathcal{B}})},$$

$$p\bar{F}(q_{\mathcal{B}}) = w\bar{F}(q_{\mathcal{B}})[1 + z(q_{\mathcal{B}})\jmath(q_{\mathcal{B}})].$$

Then, the result follows as $w\bar{F}(q_{\mathcal{B}}) = c_{\mathcal{B}}$.

(ii) As the credit rating C_j of either party increases or the payment term t_2 decreases, it is straightforward to see the effective unit production cost $c_{\mathcal{B}} = \frac{ce^{\eta_s t_c}}{(1-\rho_s)(1-\rho_r)}$ decreases. Then, based on the system of equations it is direct to show the wholesale price $w^*_{\mathcal{B}}$ decreases, the production quantity $q^*_{\mathcal{B}}$ and both parties' profits increase. ■

Proof of Lemma 9:

(1) Case 1: No exogenous default happens, with probability $(1-\rho_s)(1-\rho_r)$. In this case, the factor's expected repayment from the supplier can be derived as follows,

$$\begin{aligned}
\Omega_1(b_{\mathcal{F}}, r_{\mathcal{F}}; w, q) &= wS(q)e^{r_{\mathcal{F}} t_2}\bar{F}(b_{\mathcal{F}}) + \int_0^{b_{\mathcal{F}}} \underbrace{\left[wS(q)e^{\alpha_s t_2} - \frac{cqe^{\eta_s t_c}}{1-\rho_s} + w\xi\right]}_{\text{factor's repayment under supplier's bankruptcy}} dF(\xi) \\
&= wS(q)e^{r_{\mathcal{F}} t_2}\bar{F}(b_{\mathcal{F}}) + w\int_0^{b_{\mathcal{F}}} (S(q)e^{r_{\mathcal{F}} t_2} - b_{\mathcal{F}} + \xi)dF(\xi) \\
&= wS(q)e^{r_{\mathcal{F}} t_2}\bar{F}(b_{\mathcal{F}}) + w(S(q)e^{r_{\mathcal{F}} t_2} - b_{\mathcal{F}})F(b_{\mathcal{F}}) + wS(b_{\mathcal{F}}) - wb_{\mathcal{F}}\bar{F}(b_{\mathcal{F}}) \\
&= wS(q)e^{r_{\mathcal{F}} t_2} - wb_{\mathcal{F}} + wS(b_{\mathcal{F}}).
\end{aligned}$$

By definition of the supplier's recourse factoring bankruptcy threshold, we have $b_{\mathcal{F}} = (e^{r_{\mathcal{F}} t_2} - e^{\alpha_s t_2})S(q) + \frac{cqe^{\eta_s t_c}}{w(1-\rho_s)}$. Thus, we can write the $\Omega_1(b_{\mathcal{F}}, r_{\mathcal{F}}; w, q)$ as a function of only $b_{\mathcal{F}}$, which gives

$$\Omega_1(b_{\mathcal{F}}; w, q) = wS(q)e^{\alpha_s t_2} - \frac{cqe^{\eta_s t_c}}{1-\rho_s} + wS(b_{\mathcal{F}}).$$

(2) Case 2: The retailer defaults due to exogenous default, but the supplier does not, with probability $\rho_r(1-\rho_s)$. In this case, the factor's expected repayment from the supplier is simply the investment output $wS(q)e^{\alpha_s t_2}$ minus the full repayment of bank's loan $cqe^{r_{\mathcal{B}} t_c}$. That gives,

$$\Omega_2(b_{\mathcal{F}}; w, q) = wS(q)e^{\alpha_s t_2} - cqe^{r_{\mathcal{B}} t_c} = wS(q)e^{\alpha_s t_2} - \frac{cqe^{\eta_s t_c}}{1-\rho_s}.$$

Summarizing, the factor's expected total repayment from the supplier can be derived as follows,

$$\begin{aligned}
\Omega(b_{\mathcal{F}}; w, q) = (1-\rho_s)(1-\rho_r)&\left[wS(q)e^{\alpha_s t_2} - \frac{cqe^{\eta_s t_c}}{1-\rho_s} + wS(b_{\mathcal{F}})\right] \\
&+ \rho_r(1-\rho_s)\left[wS(q)e^{\alpha_s t_2} - \frac{cqe^{\eta_s t_c}}{1-\rho_s}\right] \\
&= (1-\rho_s)wS(q)e^{\alpha_s t_2} - cqe^{\eta_s t_c} + (1-\rho_s)(1-\rho_r)wS(b_{\mathcal{F}}).
\end{aligned}$$

Remark: The factor's expected total repayment from the supplier depends not only the supplier's credit rating (exogenous default), but also the retailer's. The reason is the retailer's default will directly cause the supplier's default, by the nature of the supply chain relationship. ■

Proof of Proposition 3:

For a given w, the supplier's expected profits under pure bank financing and recourse factoring are

$$\pi_{\mathcal{B}}(q;w) = (1-\rho_s)(1-\rho_r)wS(q) - cqe^{\eta_s t_c},$$
$$\pi_{\mathcal{F}}(q;w) = \left[(1-\rho_s)(1-\rho_r) + (1-\rho_s)e^{\alpha_s t_2} - e^{\eta_s t_2}\right] wS(q) - cqe^{\eta_s t_c}.$$

Then, recourse factoring is adopted when $(1-\rho_s)(1-\rho_r) + (1-\rho_s)e^{\alpha_s t_2} - e^{\eta_s t_2} \geq (1-\rho_s)(1-\rho_r)$, which can be simplified as $(1-\rho_s)e^{\alpha_s t_2} \geq e^{\eta_s t_2}$, $\alpha_s > \bar{\alpha}_{\mathcal{F}} := \eta_s - t_2^{-1}\ln(1-\rho_s)$.

By similar argument as in proof of Proposition 1.3.1, the equilibrium $(w^*_{\mathcal{F}}, q^*_{\mathcal{F}})$ can be uniquely derived from: $p\bar{F}(q) = c_{\mathcal{F}}[1 + z(q)j(q)]$, $w = c_{\mathcal{F}}/\bar{F}(q)$, where $c_{\mathcal{F}}$ is defined in equation (2.6). Lastly, the relationships $w^*_{\mathcal{F}} < w^*_{\mathcal{B}}$, $q^*_{\mathcal{F}} > q^*_{\mathcal{B}}$, $\pi^*_{\mathcal{F}} > \pi^*_{\mathcal{B}}$, and $\Pi^*_{\mathcal{F}} > \Pi^*_{\mathcal{B}}$ are direct from the comparisons of effective unit production costs: $c_{\mathcal{F}} < c_{\mathcal{B}}$. ■

Proof of Corollary 1:

By definition, we have $\bar{\alpha}_{\mathcal{F}} = \bar{\alpha}_{\mathcal{F}}(C_s, t_2) := \eta_s - t_2^{-1}\ln(1-\rho_s)$, where η_s and ρ_s are decreasing in C_s. Then, it is straightforward to see the threshold $\bar{\alpha}_{\mathcal{F}}$ decrease in C_s and t_2. ■

Proof of Lemma 2:

The factor's expected value of the supplier accounts receivable is $(1-\rho_r)wS(q)$, where $(1-\rho_r)$ indicates the retailer's default risk. The factor's expected cost of lending $\mathcal{L}_{\mathcal{S}}$ to the supplier, whose credit rating is C_s, becomes $\mathcal{L}_{\mathcal{S}}e^{\eta_s t_2}$. Hence, we have the following competitive pricing equation, $\mathcal{L}_{\mathcal{S}}e^{\eta_s t_2} = (1-\rho_r)wS(q)$, from which we further obtain $\mathcal{L}_{\mathcal{S}} = e^{-\eta_s t_2}(1-\rho_r)wS(q)$. ■

Proof of Proposition 3.4.2:

Under the competitive credit pricing, we have $cqe^{\eta_s t_c} = (1-\rho_s)cqe^{r_{\mathcal{B}} t_c}$. From Lemma 2, we know the supplier will have $\mathcal{L}_{\mathcal{S}} = e^{-\eta_s t_2}(1-\rho_r)wS(q)$ amount of cash available for investment at time t_1, with the investment period of t_2. Hence, the supplier's profit at time t_c can be derived as follows:

$$\begin{aligned}
\pi_{\mathcal{S}}(q;w) &= (1-\rho_s)[\mathcal{L}_{\mathcal{S}} e^{\alpha_s t_2} - cqe^{r_{\mathcal{B}} t_c}] \\
&= (1-\rho_s)\left[(1-\rho_r)wS(q)e^{(\alpha_s-\eta_s)t_2} - \frac{cqe^{\eta_s t_c}}{1-\rho_s}\right] \\
&= e^{(\alpha_s-\eta_s)t_2}(1-\rho_r)(1-\rho_s)wS(q) - cqe^{\eta_s t_c}
\end{aligned}$$

Compared to the pure bank financing case, where $\pi_{\mathcal{B}}(q;w) = (1-\rho_s)(1-\rho_r)wS(q) - cqe^{\eta_s t_c}$. If $\alpha_s \leq \eta_s$, then non-recourse factoring is not adopted and $(w^*_{\mathcal{S}}, q^*_{\mathcal{S}}) = (w^*_{\mathcal{B}}, q^*_{\mathcal{B}})$. Then,

$$\frac{\partial \pi_{\mathcal{S}}(q;w)}{\partial q} = e^{(\alpha_s-\eta_s)t_2}(1-\rho_r)(1-\rho_s)w\bar{F}(q) - ce^{\eta_s t_c}.$$

Note that $\partial \pi_{\mathcal{S}}(q;w)/\partial q$ is strictly decreasing in q, hence $\pi_{\mathcal{S}}(q;w)$ is quasi-concave in q, and the supplier's optimal production quantity given a wholesale price w can be solved from the following equation,

$$w\bar{F}(q) = \frac{ce^{\eta_s(t_c+t_2)-\alpha_s t_2}}{(1-\rho_s)(1-\rho_r)} = c_{\mathcal{S}}.$$

In addition, the retailer's profit function is $\Pi_{\mathcal{S}}(w) = (1-\rho_r)(p-w)S(q_{\mathcal{S}})$. By similar argument as in the proof of Proposition 1.3.1, the equilibrium $(w^*_{\mathcal{S}}, q^*_{\mathcal{S}})$ can be derived from the equation system: $p\bar{F}(q) = c_{\mathcal{S}}[1 + z(q)j(q)]$ and $w = c_{\mathcal{S}}/\bar{F}(q)$. Lastly, the relationships $w^*_{\mathcal{S}} < w^*_{\mathcal{B}}$, $q^*_{\mathcal{S}} > q^*_{\mathcal{B}}$, $\pi^*_{\mathcal{S}} > \pi^*_{\mathcal{B}}$, and $\Pi^*_{\mathcal{S}} > \Pi^*_{\mathcal{B}}$ are direct from the comparisons of effective unit production costs: $c_{\mathcal{S}} < c_{\mathcal{B}}$. ■

Lemma 12 *Consider a Stackelberg game with follower's profit function* $\pi(q;w) = \lambda wS(q) - cq$ *where* $\lambda > 0$, *and the leader's profit function* $\Pi(w) = (p-w)S(q)$. *Denote the Stackelberg equilibrium outcome as* (w^*, q^*, π^*, Π^*). *Then, as* λ *increases,* w^* *decreases;* q^*, π^* *and* Π^* *increase.*

Proof of Lemma 12:

The follower's best response q_w for a given w can be derived from the following first-order condition:

$$\frac{\partial \pi(q;w)}{\partial q} = \lambda w\bar{F}(q) - c = 0.$$

For the leader's profit function, taking derivatives with respect to w, we have

$$\frac{d\Pi(w)}{dw} = -S(q) + (p-w)\bar{F}(q)\frac{dq}{dw}.$$

From $\lambda w \bar{F}(q) = c$, we have

$$-\lambda w f(q)\frac{dq}{dw} + \lambda \bar{F}(q) = 0, \quad \frac{dq}{dw} = \frac{1}{wz(q)} > 0.$$

Under our assumption, $z(q)$ increases in q. Hence, dq/dw is positive and decreasing in w. Thus, we can conclude that $d\Pi(w)/dw$ strictly decreases in w, and thus $\Pi(w)$ is strictly concave in w. Therefore, in equilibrium (w^*, q^*) can be solved from $\lambda p \bar{F}(q) = c[1 + z(q)j(q)]$ and $\lambda w \bar{F}(q) = c$. Then, it is straightforward to see that as λ increases, q^* increases (from the first equation), and thus $\lambda \bar{F}(q^*)$ increases, which indicates w^* decreases from the second equation. Moreover, $\pi^* = \lambda w^* S(q^*) - cq^* = c(j(q^*) - q^*)$ increases in q^* since $j'(q) = 1 + j(q)z(q) > 1$. Lastly, $\Pi^* = (p - w^*)S(q^*)$ increases as w^* decreases and q^* increases. ■

Proof of Proposition 1.4.3:

The supplier's expected profit under pure bank financing, recourse factoring and non-recourse factoring can be summarize as $\pi_i(q; w) = \lambda_i w S(q) - cqe^{\eta_s t_c}$, $i \in \{\mathcal{B}, \mathcal{F}, \mathcal{S}\}$, where the coefficients of the expected revenue part λ_i are defined as

$$\lambda_i = \begin{cases} (1-\rho_s)(1-\rho_r), & i = \mathcal{B} \\ (1-\rho_s)(1-\rho_r) + (1-\rho_s)e^{\alpha_s t_2} - e^{\eta_s t_2}, & i = \mathcal{F} \\ (1-\rho_r)(1-\rho_s)e^{(\alpha_s - \eta_s)t_2}, & i = \mathcal{S} \end{cases}.$$

Based on Lemma 12, we know the supplier's preference is simply based on λ_i.

(1) Bank financing is preferred by the supplier when $\lambda_{\mathcal{B}} > \lambda_{\mathcal{F}}$ and $\lambda_{\mathcal{B}} > \lambda_{\mathcal{S}}$. That is α_s should satisfy the following two conditions:

$$\begin{cases} (1-\rho_s)(1-\rho_r) \geq (1-\rho_s)(1-\rho_r) + (1-\rho_s)e^{\alpha_s t_2} - e^{\eta_s t_2}, \\ (1-\rho_s)(1-\rho_r) \geq (1-\rho_r)(1-\rho_s)e^{(\alpha_s-\eta_s)t_2}. \end{cases}$$

Simplifying the above conditions gives $\alpha_s \leq \eta_s$.

(2) Non-recourse factoring is better than pure bank financing and recourse factoring when $\lambda_{\mathcal{S}} > \lambda_{\mathcal{B}}$ and $\lambda_{\mathcal{S}} > \lambda_{\mathcal{F}}$. The former is equivalent to $\alpha_s > \eta_s$. The latter is equivalent to,

$$(1-\rho_s)\left[1 - (1-\rho_r)e^{-\eta_s t_2}\right] e^{\alpha_s t_2} \leq e^{\eta_s t_2} - (1-\rho_s)(1-\rho_r),$$
$$e^{\alpha_s t_2} \leq \frac{e^{\eta_s t_2} - (1-\rho_s)(1-\rho_r)}{(1-\rho_s)\left[1 - (1-\rho_r)e^{-\eta_s t_2}\right]},$$
$$\alpha_s \leq \bar{\alpha}_{\mathcal{S}} = \frac{1}{t_2} \ln\left[\frac{e^{\eta_s t_2} - (1-\rho_s)(1-\rho_r)}{(1-\rho_s)\left[1 - (1-\rho_r)e^{-\eta_s t_2}\right]}\right].$$

Hence, when $\eta_s < \alpha_s \leq \bar{\alpha}_{\mathcal{S}}$, the non-recourse factoring is adopted, with equilibrium $(w^*_{\mathcal{S}}, q^*_{\mathcal{S}})$ given in Proposition 3.4.2.

(3) Lastly, when $\alpha_s > \bar{\alpha}_{\mathcal{S}}$, we have $\lambda_{\mathcal{F}} > \lambda_{\mathcal{B}}$ and $\lambda_{\mathcal{F}} > \lambda_{\mathcal{S}}$. Hence, in this case recourse factoring is adopted, with equilibrium $(w^*_{\mathcal{F}}, q^*_{\mathcal{F}})$ as given Proposition 3. ■

Proof of Proposition 1.4.4:

The results are straightforward based on Lemma 12, hence details are omitted. ■

Proof of Lemma 11:

The factor's expected value of the supplier accounts receivable is $(1-\rho_r)wS(q)$, where $(1-\rho_r)$ indicates the retailer's default risk. The factor's expected cost of lending $\mathcal{L}$ to the supplier, whose credit rating is C_s, becomes $\mathcal{L}_{\mathcal{R}} e^{\eta_s t_2}$. Hence, we have the following competitive pricing equation, $\mathcal{L}_{\mathcal{R}} e^{\eta_s (t_2+\tau)} = (1-\rho_r)wS(q)$, from which we further obtain $\mathcal{L}_{\mathcal{R}} = e^{-\eta_r (t_2+\tau)}(1-\rho_r)wS(q)$. ■

Proof of Lemma 22:

Given the retailer's offer of (w,τ) and the bank's competitively priced loan, the supplier's problem is to choose optimal production quantity q to maximize $\pi_{\mathcal{R}}(q; w, \tau)$, where

$$\begin{aligned}\pi_{\mathcal{R}}(q; w, \tau) &= (1-\rho_s)(\mathcal{L}_{\mathcal{R}} e^{\alpha_s t_2} - cqe^{r_{\mathcal{B}} t_c}) \\ &= e^{\alpha_s t_2 - \eta_r (t_2+\tau)}(1-\rho_s)(1-\rho_r)wS(q) - cqe^{\eta_s t_c}.\end{aligned}$$

Taking derivative with respect to q, we have

$$\frac{\partial \pi_{\mathcal{R}}(q; w, \tau)}{\partial q} = e^{\alpha_s t_2 - \eta_r (t_2+\tau)}(1-\rho_s)(1-\rho_r)w\bar{F}(q) - ce^{\eta_s t_c}.$$

Note that $\partial \pi_{\mathcal{R}}(q; w, \tau)/\partial q$ is strictly decreasing in q, hence $\pi_{\mathcal{R}}(q; w, \tau)$ is strictly concave, and the optimal production quantity can be solved from the first-order condition, $\partial \pi_{\mathcal{R}}(q; w, \tau)/\partial q = 0$, i.e.,

$$e^{\alpha_s t_2 - \eta_r (t_2+\tau)}(1-\rho_s)(1-\rho_r)w\bar{F}(q) - ce^{\eta_s t_c} = 0,$$

$$w\bar{F}(q) = \frac{ce^{\eta_s t_c + \eta_r (t_2+\tau) - \alpha_s t_2}}{(1-\rho_s)(1-\rho_r)} = c_{\mathcal{R}}(\tau),$$

where $c_{\mathcal{R}}(\tau)$ is the effective unit production cost under reverse factoring. ■

Proof of Proposition 3.6.1:

First, consider the retailer's decision of payment extension τ. For a given wholesale price w, the retailer's profit function (ignoring the constant term $(1-\rho_r)$ for now) can be written as a function of the payment extension τ: $\Pi_{\mathcal{R}}(\tau) = (p - e^{-\alpha_r \tau} w) S(q_{\mathcal{R}})$, where $q_{\mathcal{R}}$ is the supplier's optimal production quantity under reverse factoring and satisfies $w\bar{F}(q_{\mathcal{R}}) = c_{\mathcal{R}}(\tau)$ as shown in Lemma 22. Hence,

$$\Pi'_{\mathcal{R}}(\tau) = \alpha_r w e^{-\alpha_r \tau} S(q_{\mathcal{R}}) + (p - e^{-\alpha_r \tau} w)\bar{F}(q_{\mathcal{R}}) \frac{dq_{\mathcal{R}}}{d\tau}.$$

Since $w\bar{F}(q_{\mathcal{R}}) = c_{\mathcal{R}}(\tau)$, we have

$$\frac{dq_{\mathcal{R}}}{d\tau} = -\frac{\eta_r c_{\mathcal{R}}(\tau)}{w f(q_{\mathcal{R}})} = -\frac{\eta_r \bar{F}(q_{\mathcal{R}})}{f(q_{\mathcal{R}})} = -\frac{\eta_r}{z(q_{\mathcal{R}})} < 0. \tag{A.1}$$

Hence,

$$\begin{aligned}
\Pi'_{\mathcal{R}}(\tau) &= \alpha_r w e^{-\alpha_r \tau} S(q_{\mathcal{R}}) - (p - e^{-\alpha_r \tau} w)\bar{F}(q_{\mathcal{R}}) \frac{\eta_r}{z(q_{\mathcal{R}})} \\
&= \frac{\bar{F}(q_{\mathcal{R}})}{z(q_{\mathcal{R}})} [\alpha_r w e^{-\alpha_r \tau} j(q_{\mathcal{R}}) z(q_{\mathcal{R}}) - \eta_r (p - w e^{-\alpha_r \tau})] \\
&= \frac{\bar{F}(q_{\mathcal{R}})}{z(q_{\mathcal{R}})} [(\alpha_r j(q_{\mathcal{R}}) z(q_{\mathcal{R}}) + \eta_r) w e^{-\alpha_r \tau} - \eta_r p].
\end{aligned}$$

As τ increases, $q_{\mathcal{R}}$ decreases based on derivative in A.1, and thus $j(q_{\mathcal{R}})z(q_{\mathcal{R}})$ decreases. Hence, $\Pi_{\mathcal{R}}(\tau)$ is strictly quasi-concave in τ. Therefore, given the unchanged wholesale price w, the retailer's (unconstrained) optimal payment extension τ_0^* and the supplier's optimal production quantity in equilibrium can be solved from: $\{w(\alpha_r j(q) z(q) + \eta_r) = p\eta_r e^{\alpha_r \tau},\ w\bar{F}(q) = c_{\mathcal{R}}(\tau)\}$.

Next, we consider when should the retailer offer reverse factoring and how to ensure it is acceptable to the supplier. The supplier's expected profit under different financing alternatives can be summarized as follows:

$$\begin{aligned}
\pi_i(q; w) &= \lambda_i w S(q) - cqe^{\eta_s t_c}, \quad i \in \{\mathcal{B}, \mathcal{F}, \mathcal{S}\}; \\
\pi_{\mathcal{R}}(q; w, \tau) &= \phi_{\mathcal{R}}(\tau) w S(q) - cqe^{\eta_s t_c},
\end{aligned}$$

where the coefficients λ_i and $\phi_{\mathcal{R}}(\tau)$ are defined as

$$\lambda_i = \left\{\begin{array}{ll}
(1-\rho_s)(1-\rho_r), & i = \mathcal{B} \\
(1-\rho_s)(1-\rho_r) + (1-\rho_s)e^{\alpha_s t_2} - e^{\eta_s t_2}, & i = \mathcal{F} \\
(1-\rho_r)(1-\rho_s)e^{(\alpha_s - \eta_s)t_2}, & i = \mathcal{S}
\end{array}\right\};$$

$$\phi_{\mathcal{R}}(\tau) = (1-\rho_s)(1-\rho_r)e^{\alpha_s t_2 - \eta_r (t_2 + \tau)}.$$

It is straightforward to see the supplier's preference is simply based on the comparison between λ_i, $i \in \{\mathcal{B}, \mathcal{F}, \mathcal{S}\}$ and $\phi_{\mathcal{R}}(\tau)$.

(1) Bank financing is better than reverse factoring with zero payment extension when $\lambda_{\mathcal{B}} > \phi_{\mathcal{R}}(0) \geq \phi_{\mathcal{R}}(\tau)$, i.e., $\alpha_s < \eta_r$. Thus, the reverse factoring should not be offered to the supplier, since the best reverse factoring program is the one without payment extension, which is still dominated by the pure bank financing from the supplier's perspective.

(2) When $\eta_r < \alpha_s \leq \eta_s$, the supplier is adopting pure bank financing before the retailer introduced the reverse factoring program. Hence, the retailer needs to choose the payment extension so that the supplier is indifferent between reverse factoring and pure bank financing, i.e., $\lambda_{\mathcal{B}} = \phi_{\mathcal{R}}(\tau)$. Therefore, the maximal payment extension $\tau_s = (\alpha_s/\eta_r - 1)t_2$.

(3) Based on Proposition 1.4.3, we know non-recourse factoring is better than pure bank financing and recourse factoring when $\eta_s < \alpha_s \leq \bar{\alpha}_{\mathcal{S}}$. Hence, in this case non-recourse factoring should be the one adopted by the supplier before introducing the reverse factoring program. To induce the supplier to adopt reverse factoring, the retailer needs to choose the payment extension so that the supplier is indifferent between reverse factoring and non-recourse factoring, i.e., $\lambda_{\mathcal{S}} = \phi_{\mathcal{R}}(\tau)$. Therefore, the maximal payment extension $\tau_s = (\eta_s/\eta_r - 1)t_2$.

(4) Recourse factoring is better than reverse factoring with zero payment extension from the supplier's perspective when $\lambda_{\mathcal{F}} > \phi_{\mathcal{R}}(0) \geq \phi_{\mathcal{R}}(\tau)$. Equivalently,

$$(1-\rho_s)(1-\rho_r) + (1-\rho_s)e^{\alpha_s t_2} - e^{\eta_s t_2} > e^{(\alpha_s - \eta_r)t_2}(1-\rho_s)(1-\rho_r),$$
$$e^{\alpha_s t_2} > \frac{e^{\eta_s t_2} - (1-\rho_s)(1-\rho_r)}{(1-\rho_s)\left[1-(1-\rho_r)e^{-\eta_r t_2}\right]},$$
$$\alpha_s > \bar{\alpha}_{\mathcal{R}} = \frac{1}{t_2}\ln\left[\frac{e^{\eta_s t_2} - (1-\rho_s)(1-\rho_r)}{(1-\rho_s)\left[1-(1-\rho_r)e^{-\eta_r t_2}\right]}\right].$$

Hence, when $\alpha_s > \bar{\alpha}_{\mathcal{R}}$, the reverse factoring should not be offered to the supplier (since the supplier wouldn't adopt it even without payment extension).

(5) When $\bar{\alpha}_{\mathcal{S}} < \alpha_s < \bar{\alpha}_{\mathcal{R}}$, the supplier is adopting recourse factoring before the retailer introduced the reverse factoring program. Hence, the retailer needs to choose the payment extension so that the supplier is indifferent between reverse factoring and recourse factoring, i.e., $\lambda_{\mathcal{F}} = \phi_{\mathcal{R}}(\tau)$. Equivalently,

$$(1-\rho_s)(1-\rho_r) + (1-\rho_s)e^{\alpha_s t_2} - e^{\eta_s t_2} = e^{\alpha_s t_2 - \eta_r(t_2+\tau)}(1-\rho_s)(1-\rho_r),$$
$$1 + \frac{e^{\alpha_s t_2}}{1-\rho_r} - \frac{e^{\eta_s t_2}}{(1-\rho_s)(1-\rho_r)} = e^{\alpha_s t_2 - \eta_r(t_2+\tau)}.$$

Hence, the maximal payment extension τ_s is

$$\tau_s = \left(\frac{\alpha_s}{\eta_r} - 1\right)t_2 - \frac{1}{\eta_r}\ln\left[1 + \frac{e^{\alpha_s t_2}}{1-\rho_r} - \frac{e^{\eta_s t_2}}{(1-\rho_s)(1-\rho_r)}\right].$$

We complete the proof. ■

Proof of Proposition 3.6.2:

(i) We have shown in proof of Proposition 3.6.1 that $\Pi_{\mathcal{R}}(\tau)$ is strictly quasi-concave in τ, and

$$\Pi'_{\mathcal{R}}(\tau) = \frac{\bar{F}(q_{\mathcal{R}})}{z(q_{\mathcal{R}})}[(\alpha_r j(q_{\mathcal{R}})z(q_{\mathcal{R}}) + \eta_r)we^{-\alpha_r \tau} - \eta_r p].$$

Then, it is optimal to choose $\tau = 0$ if $\Pi'_{\mathcal{R}}(0) \leq 0$. Note that $\Pi'_{\mathcal{R}}(0)$ is increasing in α_r. Hence, the threshold α_r^τ can be solved from: $\{w(\alpha_r j(q)z(q)+\eta_r) = p\eta_r,\ w\bar{F}(q) = c_{\mathcal{R}}(0)\}$. The retailer should not extend the payment term if $\alpha_r \leq \alpha_r^\tau$.

(ii) A direct result from the definition of the maximal payment extension τ_s. ■

Appendix C: Complementary Results

In this appendix, we provide several complementary results to the main paper.

C.1 High Cash Investment Return Rate in Recourse Factoring

The recourse factoring bankruptcy threshold we derived in equation (3.7) is

$$b_{\mathcal{F}} = (e^{r_{\mathcal{F}} t_2} - e^{\alpha_s t_2})S(q) + \frac{cqe^{\eta_s t_c}}{w(1-\rho_s)}.$$

If the supplier's cash investment return rate α_s is high enough, the recourse factoring bankruptcy threshold will become zero ($b_{\mathcal{F}} = 0$) as demand is non-negative, which means there is no bankruptcy risk for the supplier. Note that in such cases the formula defined above becomes negative, i.e.,

$$w(1-\rho_s)(e^{r_{\mathcal{F}} t_2} - e^{\alpha_s t_2})S(q) + cqe^{\eta_s t_c} < 0. \tag{A.2}$$

Now, the factor's expected total repayment $\Omega(r_{\mathcal{F}}; w, q)$ from the supplier under recourse factoring can be written as a function of $r_{\mathcal{F}}$:

$$\begin{aligned}\Omega(r_{\mathcal{F}}; w, q) &= \underbrace{(1-\rho_s)(1-\rho_r)wS(q)e^{r_{\mathcal{F}} t_2}}_{\text{Case 1: no exogenous default}} + \underbrace{\rho_r(1-\rho_s)wS(q)e^{r_{\mathcal{F}} t_2}}_{\text{Case 2: retailer defaults}} \\ &= (1-\rho_s)wS(q)e^{r_{\mathcal{F}} t_2}.\end{aligned}$$

Note that even when the retailer defaults in Case 2, the supplier can still fully repay both the bank and the factor (as long as no supplier's own default) according to the condition (A.2).

Hence, the competitive credit pricing equation for the factor becomes $(1-\rho_s)wS(q)e^{r_{\mathcal{F}} t_2} = wS(q)e^{\eta_s t_2}$. Without supplier's exogenous default, at time t_c the supplier's expected *cash inflow* is $(1-\rho_r)wS(q) + wS(q)e^{\alpha_s t_2}$ and expected *cash outflow* is $cqe^{r_{\mathcal{B}} t_c} + wS(q)e^{r_{\mathcal{F}} t_2}$. Therefore, the supplier's expected profit can be written as

$$\begin{aligned}\pi_{\mathcal{F}}(q; w) &= (1-\rho_s)[(1-\rho_r)wS(q) + wS(q)e^{\alpha_s t_2} - cqe^{r_{\mathcal{B}} t_c} - wS(q)e^{r_{\mathcal{F}} t_2}] \\ &= \left[(1-\rho_s)(1-\rho_r) + (1-\rho_s)e^{\alpha_s t_2} - e^{\eta_s t_2}\right] wS(q) - cqe^{\eta_s t_c},\end{aligned}$$

where the second equality is based on the two competitive pricing equations $(1-\rho_s)cqe^{r_{\mathcal{B}} t_c} = cqe^{\eta_s t_c}$ and $(1-\rho_s)wS(q)e^{r_{\mathcal{F}} t_2} = wS(q)e^{\eta_s t_2}$. Therefore, we have shown in this special case, the supplier's expected profit function remains the same as in (3.8).

C.2 Wholesale Price Adjustment in Reverse Factoring

The retailer's and supplier's profit under reverse factoring can be formulated as follows:

$$\Pi_{\mathcal{R}}(q,\tau) = (1-\rho_r)[pS(q) - e^{-\alpha_r\tau}c_{\mathcal{R}}(\tau)j(q)],$$
$$\pi_{\mathcal{R}}(q,\tau) = e^{\alpha_s t_2 - \eta_r(t_2+\tau)}(1-\rho_s)(1-\rho_r)c_{\mathcal{R}}(\tau)j(q) - cqe^{\eta_s t_c}.$$

Let $\mathcal{N}(q,\tau) = \Pi_{\mathcal{R}}(q,\tau) + \pi_{\mathcal{R}}(q,\tau)$ be the overall supply chain profit. Then,

$$\begin{aligned}\mathcal{N}(q,\tau) &= (1-\rho_r)pS(q) + [e^{\alpha_s t_2 - \eta_r(t_2+\tau)}(1-\rho_s) - e^{-\alpha_r\tau}](1-\rho_r)c_{\mathcal{R}}(\tau)j(q) - cqe^{\eta_s t_c}\\ &= (1-\rho_r)pS(q) + \left[(1-\rho_s)e^{\eta_s t_c} - e^{\eta_s t_c + (\eta_r - \alpha_s)t_2 + (\eta_r - \alpha_r)\tau}\right]\frac{cj(q)}{1-\rho_s} - cqe^{\eta_s t_c}.\end{aligned}$$

With wholesale price w and payment extension τ, the retailer's profit function (ignoring the constant term $(1-\rho_r)$ for now) can be written as $\Pi_{\mathcal{R}}(w,\tau) = (p - e^{-\alpha_r\tau}w)S(q_{\mathcal{R}})$, where $q_{\mathcal{R}}$ is the supplier's optimal production quantity under reverse factoring and satisfies $w\bar{F}(q_{\mathcal{R}}) = c_{\mathcal{R}}(\tau)$ as shown Lemma 22. First, for a given payment extension τ, we have

$$\frac{\partial \Pi_{\mathcal{R}}(w,\tau)}{\partial w} = -e^{-\alpha_r\tau}S(q_{\mathcal{R}}) + (p - e^{-\alpha_r\tau}w)\bar{F}(q_{\mathcal{R}})\frac{\partial q_{\mathcal{R}}}{\partial w}.$$

Since $w\bar{F}(q_{\mathcal{R}}) = c_{\mathcal{R}}(\tau)$, we have

$$\frac{\partial q_{\mathcal{R}}}{\partial w} = \frac{1}{wz(q_{\mathcal{R}})} > 0.$$

Hence,

$$\frac{\partial \Pi_{\mathcal{R}}(w,\tau)}{\partial w} = -e^{-\alpha_r\tau}S(q_{\mathcal{R}}) + (p - e^{-\alpha_r\tau}w)\frac{\bar{F}(q_{\mathcal{R}})}{wz(q_{\mathcal{R}})}.$$

As w increases, $q_{\mathcal{R}}$ increases. Thus, both the first term and the second term in the above equation strictly decrease in w. Therefore, we can conclude that $\frac{\partial \Pi_{\mathcal{R}}(w,\tau)}{\partial w}$ is strictly decreasing in w. Hence, for a given τ, there exists a unique optimal $(w_{\mathcal{R}}, q_{\mathcal{R}})$ which can be obtained from the following equation system

$$\begin{cases} wz(q)j(q) = pe^{\alpha_r\tau} - w, \\ w\bar{F}(q) = c_{\mathcal{R}}(\tau). \end{cases} \tag{A.3}$$

Now, the retailer's profit function can be written as a function of τ,

$$\Pi_{\mathcal{R}}(\tau) = (p - e^{-\alpha_r\tau}w)S(q),$$

where (w, q) depends on τ through the equation system A.3. Plugging the second equation of A.3 into the first one gives

$$j(q) = \frac{e^{\alpha_r \tau} p}{w z(q)} - \frac{1}{z(q)} = \frac{e^{\alpha_r \tau} p \bar{F}(q)}{c_{\mathcal{R}}(\tau) z(q)} - \frac{1}{z(q)},$$

$$e^{-\alpha_r \tau} c_{\mathcal{R}}(\tau) = \frac{p \bar{F}(q)}{1 + j(q) z(q)}. \tag{A.4}$$

Then, we can rewrite the profit function as a function of q,

$$\Pi_{\mathcal{R}}(q) = pS(q) - e^{-\alpha_r \tau} c_{\mathcal{R}}(\tau) j(q) = pS(q) - \frac{p j(q) \bar{F}(q)}{1 + j(q) z(q)} = \frac{pS(q) j(q) z(q)}{1 + j(q) z(q)} = \frac{pS(q)}{\frac{1}{j(q) z(q)} + 1}.$$

Since $z(q)$, $S(q)$ and $j(q)$ are strictly increasing in q, it's straightforward to see $\Pi_{\mathcal{R}}(q)$ strictly increases in q. Hence, the retailer's problem becomes find the maximal q subject to the constraint A.4. Since the right-hand side of equation A.4, as function of q, strictly decreases in q, the problem is to minimize the left-hand side of equation A.4, which is a function of τ and denoted as $\mathcal{H}(\tau)$. Rearranging the terms gives

$$\mathcal{H}(\tau) = e^{-\alpha_r \tau} c_{\mathcal{R}}(\tau) = \frac{c e^{\eta_s t_c + \eta_r (t_2 + \tau) - \alpha_s t_2 - \alpha_r \tau}}{(1 - \rho_s)(1 - \rho_r)} = \frac{c e^{\eta_s t_c + \eta_r t_2 - \alpha_s t_2 + (\eta_r - \alpha_r) \tau}}{(1 - \rho_s)(1 - \rho_r)},$$

which is strictly decreasing (increasing) in τ if $\eta_r < \alpha_r$ ($\eta_r > \alpha_r$). Therefore, when $\eta_r > \alpha_r$, it is optimal not to extend payment term, i.e., $\tau^*_{\mathcal{R}} = 0$. When $\eta_r < \alpha_r$, it is optimal to choose the maximal possible payment term. Suppose there exists a longest term the retailer could potentially adopted in reverse factoring, denoted as τ_{max}. Then, $\tau^*_{\mathcal{R}} = \tau_{max}$. With the optimal payment extension $\tau^*_{\mathcal{R}}$, the retailer's optimal strategy is to offer the wholesale price such that the supplier's IR constraint is satisfied, i.e., $\pi^*_{\mathcal{R}} \geq \max\{\pi^*_{\mathcal{B}}, \pi^*_{\mathcal{F}}, \pi^*_{\mathcal{S}}\}$.

B. Appendix for Chapter 2

Appendix A: Proofs of Statements

We present the proofs of the Lemmas and Propositions in Appendix A. We use ϕ (Φ) to denote the probability density function (p.d.f.) and the cumulative distribution function (c.d.f.) of the standard Normal distribution, with the following properties that are extensively used throughout the proofs:

$$\phi'(x) = -x\phi(x), \quad \phi(-x) = \phi(x), \quad \Phi(-x) = \bar{\Phi}(x).$$

For a general normal distribution with mean μ and standard deviation σ, the p.d.f. f and the c.d.f. F can be written as

$$f(x|\mu, \sigma^2) = \frac{1}{\sigma}\phi\Big(\frac{x-\mu}{\sigma}\Big), \quad F(x|\mu, \sigma^2) = \Phi\Big(\frac{x-\mu}{\sigma}\Big).$$

Lemma 13 *Given the purchasing fraction ρ_1 in the crowdfunding stage and the average quality review $\bar{q}_r$, the posterior of true quality is normally distributed, $\mu_q|\bar{q}_r \sim N(\mu_1, \sigma_1^2)$, with*

$$\mu_1 = \frac{\rho_1 M\gamma}{\rho_1 M\gamma + 1}\bar{q}_r, \qquad \sigma_1^2 = \frac{\sigma_0^2}{\rho_1 M\gamma + 1}, \qquad \gamma = \frac{\sigma_0^2}{\sigma_q^2}.$$

Proof of Lemma 13:

We assume that the distribution of ex post quality perceptions in the population is Normal, $Q \sim N(\mu_q, \sigma_q^2)$, where μ_q is the product's *unobservable true quality*, or the unknown parameter to learn. Both the firm and consumers share a common and public Normal prior belief $\mu_q \sim N(\mu_0, \sigma_0^2)$. From the Bayesian statistics theory, we know Normal is a conjugate prior for Normal distribution with unknown mean and known variance. The posterior distribution and its derivation can be found from standard textbooks [131]. Let $d = \rho_1 M$ denote the crowdfunding sales volume. After the reviews $q = (q_1, ..., q_d)$ are observed, the posterior distribution of μ_q is the product of the prior and the likelihood function. Hence, we have $f(\mu_q|q) \propto f(\mu_q)f(q|\mu_q) = f(\mu_q)\prod_{i=1}^{d} f(q_i|\mu_q)$, where

$$f(\mu_q) = \frac{1}{\sqrt{2\pi}\sigma_0}\exp\Big\{-\frac{1}{2\sigma_0^2}(\mu_q-\mu_0)^2\Big\},$$
$$f(q_i|\mu_q) = \frac{1}{\sqrt{2\pi}\sigma_q}\exp\Big\{-\frac{1}{2\sigma_q^2}(q_i-\mu_q)^2\Big\}.$$

Furthermore,

$$f(\mu_q|q) \propto \prod_{i=1}^{d} \exp\left\{-\frac{1}{2\sigma_q^2}(q_i-\mu_q)^2\right\}\exp\left\{-\frac{1}{2\sigma_0^2}(\mu_q-\mu_0)^2\right\}$$
$$\propto \exp\left\{-\frac{1}{2}\left(\frac{d}{\sigma_q^2}+\frac{1}{\sigma_0^2}\right)\left(\mu_q-\frac{\left(\frac{d\bar{q}_r}{\sigma_q^2}+\frac{\mu_0}{\sigma_0^2}\right)}{\left(\frac{d}{\sigma_q^2}+\frac{1}{\sigma_0^2}\right)}\right)^2\right\}.$$

Therefore, the conditional distribution of μ_q given q is $N(\mu_1, \sigma_1^2)$ with

$$\mu_1 = \frac{\sigma_q^2}{d\sigma_0^2+\sigma_q^2}\mu_0 + \frac{d\sigma_0^2}{d\sigma_0^2+\sigma_q^2}\bar{q}_r = \frac{d\gamma}{d\gamma+1}\bar{q}_r + \frac{1}{d\gamma+1}\mu_0, \quad \sigma_1^2 = \frac{\sigma_0^2\sigma_q^2}{d\sigma_0^2+\sigma_q^2} = \frac{\sigma_0^2}{d\gamma+1},$$

where $\gamma = \frac{\sigma_0^2}{\sigma_q^2}$. When $\mu_0 = 0$, we obtain the result in Lemma 13. ■

Proof of Lemma 5:

The posterior of true quality μ_q, is normally distributed, $\mu_q|\bar{q}_r \sim N(\mu_1, \sigma_1^2)$, with $\mu_1 = \frac{d\gamma}{d\gamma+1}\bar{q}_r$, and $\sigma_1^2 = \frac{\sigma_0^2}{d\gamma+1}$, where $d = \rho_1 M$ is the crowdfunding sales volume. In the first period, the posterior mean μ_1 is a random variable, since it depends on the unobservable realization of product quality $\bar{q}_r$. Since $\bar{q}_r = \frac{1}{d}\sum_{i=1}^{d} q_i$, and $q_i \sim N(\mu_q, \sigma_q^2)$, we know $\bar{q}_r|\mu_q \sim N(\mu_q, \frac{\sigma_q^2}{d})$. Then,

$$\mu_1|\mu_q \sim N\left(\frac{d\gamma}{d\gamma+1}\mu_q, \frac{d\gamma^2\sigma_q^2}{(d\gamma+1)^2}\right).$$

Since $\mu_q \sim N(0, \sigma_0^2)$, we can further calculate $f(\mu_1)$ according to equation

$$f(\mu_1) = \int f(\mu_q, \mu_1)d\mu_q = \int f(\mu_q)f(\mu_1|\mu_q)d\mu_q.$$

After tedious but straightforward algebra, we can obtain

$$f(\mu_1) = \left(\frac{2\pi d\gamma\sigma_0^2}{d\gamma+1}\right)^{-\frac{1}{2}} \exp\left\{-\frac{d\gamma+1}{2d\gamma\sigma_0^2}\mu_1^2\right\}.$$

Thus, we have shown $\mu_1 \sim N\big(0, \frac{d\gamma}{d\gamma+1}\sigma_0^2\big)$, i.e., $\mu_1 \sim N\big(0, \frac{\rho_1 M\gamma}{\rho_1 M\gamma+1}\sigma_0^2\big)$. ■

Lemma 14 *Given the purchasing fraction ρ_1 in the crowdfunding stage, the rational belief of true quality perception, defined as its prior predictive distribution, has the following form: $\widetilde{Q} \sim N\big(0, \sigma_0^2 + \frac{\sigma_p^2}{\rho_1 M}\big)$.*

Proof of Lemma 14:

The rational belief of the quality distribution can be represented by the prior predictive distribution of average quality review, $\bar{q}_r$. Let $d = \rho_1 M$. Since $\bar{q}_r = \frac{1}{d}\sum_{i=1}^{d} q_i$, and $q_i \sim N(\mu_q, \sigma_q^2)$, we know $\bar{q}_r|\mu_q \sim N(\mu_q, \frac{\sigma_q^2}{d})$. Since $\mu_q \sim N(0, \sigma_0^2)$, we can further calculate $f(\bar{q}_r) = \int f(\mu_q)f(\bar{q}_r|\mu_q)d\mu_q$. After some algebra we can show $\widetilde{Q} \sim N\big(0, \sigma_0^2 + \frac{\sigma_p^2}{d}\big)$. ■

Proposition B.0.1 *Consider a consumer with the preference value v. For any pricing scheme $\{p_1, p_2\}$, and any purchasing rules adopted by all other consumers, it is optimal for the consumer to follow a threshold policy. Namely, the consumer in the crowdfunding stage will purchase a unit if $v \geq \theta_1$; otherwise, the consumer will wait for the retail stage and will purchase if $v \geq \theta_2$.*

Proof of Proposition B.0.1:

In the second period, after observing the actual crowdfunding sales volume fraction ρ_1 and the average rating of product quality reviews $\bar{q}_r$, a customer purchases the product if and only if $v + \frac{\rho_1 M\gamma}{\rho_1 M\gamma+1}\bar{q}_r + k(\rho_1 + \tilde{\rho}_2) - p_2 \geq 0$, where $\tilde{\rho}_2$ is the consumer's belief of consumer population fraction in period 2. Then, for any given beliefs $\tilde{\rho}_2$, a consumer's expected net utility is strictly increasing in v. Thus, the threshold policy is optimal in period 2.

In the first period, a consumer's *expected net utility* from purchasing the product in each stage can be written as:

$$\begin{aligned} u_1(v, \tilde{\rho}_1) &= v + \mathbb{E}^0_{\mathbb{Q}}[k(\tilde{\rho}_1 + \tilde{\rho}_2(\mu_1|\tilde{\rho}_1))] - p_1, \\ u_2(v, \tilde{\rho}_1) &= \mathbb{E}^0_{\mathbb{Q}}\big\{[v + \mu_1 + k(\tilde{\rho}_1 + \tilde{\rho}_2(\mu_1|\tilde{\rho}_1)) - p_2]^+\big\} \\ &= \int_{-\infty}^{+\infty} [v + \mu_1 + k(\tilde{\rho}_1 + \tilde{\rho}_2(\mu_1|\tilde{\rho}_1)) - p_2]^+ f(\mu_1|\tilde{\rho}_1) d\mu_1. \end{aligned}$$

Define $\Delta(v, \tilde{\rho}_1) = u_1(v, \tilde{\rho}_1) - u_2(v, \tilde{\rho}_1)$ as the expected net utility difference between purchasing and waiting. Then we have

$$\Delta(v, \tilde{\rho}_1) = v + \mathbb{E}^0_{\mathbb{Q}}[k(\tilde{\rho}_1 + \tilde{\rho}_2(\mu_1|\tilde{\rho}_1))] - p_1 - \int_{-\infty}^{+\infty} [v + \mu_1 + k(\tilde{\rho}_1 + \tilde{\rho}_2(\mu_1|\tilde{\rho}_1)) - p_2]^+ f(\mu_1|\tilde{\rho}_1) d\mu_1.$$

To prove the threshold policy, it suffices to show the strict monotonicity of $\Delta(v, \tilde{\rho}_1)$ in v for any belief $\tilde{\rho}_1$. By *Leibnitz's Rule*, the derivative of $\Delta(v, \tilde{\rho}_1)$ with respect to v is

$$\frac{\partial \Delta(v, \tilde{\rho}_1)}{\partial v} = 1 - \int_{p_2 - v - k(\tilde{\rho}_1 + \tilde{\rho}_2(\mu_1|\tilde{\rho}_1))}^{+\infty} f(\mu_1|\tilde{\rho}_1) d\mu_1 = 1 - \bar{F}\big[p_2 - v - k(\tilde{\rho}_1 + \tilde{\rho}_2(\mu_1|\tilde{\rho}_1))|\tilde{\rho}_1\big] > 0.$$

Since the above strict monotonicity holds for any arbitrary belief $\tilde{\rho}_1$, it follows that the threshold policy is also optimal in period 1. ■

Proof of Proposition 2.4.1:

The FEE in the P-L game is essentially a fixed point of the outcome function

$$G_{[0,1]}(x; \theta_1) = \Xi_{[0,1]}\big\{\mathcal{P}(\theta_1, \bar{q}_r) - k[(1 - \theta_1) + (\theta_1 - x)^+]\big\},$$

where $\mathcal{P}(\theta_1, \bar{q}_r) = p_2 - \frac{(1-\theta_1)M\gamma}{(1-\theta_1)M\gamma+1}\bar{q}_r$ by definition. When $k < 1$, it is straightforward to see for any quality review $\bar{q}_r$, there is a unique fixed point for the outcome function (there is a unique solution θ_2^{w*} to the equation $G_{[0,1]}(x; \theta_1) = x$). When the fixed point $\theta_2^{w*} \in (0, \theta_1)$, it can be solved from $\mathcal{P}(\theta_1, \bar{q}_r) - k(1 - \theta_2) = \theta_2$, which yields $\theta_2^{w*} = \frac{\mathcal{P}(\theta_1,\bar{q}_r)-k}{1-k}$. Using the two boundary cases and the fact that any threshold policy $\theta_2 \geq \theta_1$ is equivalent to $\theta_2 = \theta_1$, we can obtain $\theta_2^{w*} = \theta_1 \wedge \frac{[\mathcal{P}(\theta_1,\bar{q}_r)-k]^+}{1-k}$. When $k \geq 1$, and the quality review $\bar{q}_r$ is high such that $\mathcal{P}(\theta_1, \bar{q}_r) - k \leq 0$, then the preferred FEE should be $\theta_2^{i*} = 0$, $i = b, s$. Otherwise, the FEE threshold policy should be θ_1. ■

Proof of Lemma 6:

(1) Weak Network Externalities (WN)

The FEE equation under weak network externalities can be rearranged to be:

$$\begin{aligned}
\theta_1 + k(1-\theta_1)F\big(z(\theta_1)|1-\theta_1\big) - p_1 &= \int_{z(\theta_1)}^{+\infty} \mu_1 f(\mu_1|1-\theta_1)d\mu_1 + (\theta_1 - p_2)\bar{F}\big(z(\theta_1)|1-\theta_1\big),\\
\theta_1 + k(1-\theta_1) - p_1 &= \int_{z(\theta_1)}^{+\infty} \mu_1 f(\mu_1|1-\theta_1)d\mu_1 - z(\theta_1)\bar{F}\big(z(\theta_1)|1-\theta_1\big),\\
-z(\theta_1) + p_2 - p_1 &= \int_{z(\theta_1)}^{+\infty} \mu_1 f(\mu_1|1-\theta_1)d\mu_1 - z(\theta_1)\bar{F}\big(z(\theta_1)|1-\theta_1\big),\\
r_p = p_2 - p_1 &= \int_{z(\theta_1)}^{+\infty} \mu_1 f(\mu_1|1-\theta_1)d\mu_1 + z(\theta_1)F\big(z(\theta_1)|1-\theta_1\big).
\end{aligned}$$

Therefore, the FEE equation (2.11) is equivalent to $H_w(\theta_1) = r_p$, by the definition of $H_w(\theta_1)$. Then, we have the following simplification:

$$\begin{aligned}
H_w(\theta_1) &= \int_{z(\theta_1)}^{+\infty} \mu_1 f(\mu_1|1-\theta_1)d\mu_1 + z(\theta_1)F\big(z(\theta_1)|1-\theta_1\big)\\
&= \int_{z(\theta_1)}^{+\infty} \frac{\mu_1}{\sigma(\theta_1)}\phi\left(\frac{\mu_1}{\sigma(\theta_1)}\right)d\mu_1 + z(\theta_1)\Phi\left(\frac{z(\theta_1)}{\sigma(\theta_1)}\right)\\
&= \sigma(\theta_1)\phi[z(\theta_1)/\sigma(\theta_1)] + z(\theta_1)\Phi[z(\theta_1)/\sigma(\theta_1)].
\end{aligned}$$

(2) Strong Network Externalities (SN)

When $k > 1$, we have $z(\theta_1) = (k-1)\theta_1 + p_2 - k \geq z(0) = p_2 - k$. Based on the discussion of the P-L game, we can obtain the first period consumer's expected total social utility $\mathbb{E}^0_{\mathbb{Q}}[s(\rho)|\theta_1]$ conditional on the belief of threshold policy θ_1,

$$\begin{aligned}
\mathbb{E}^0_{\mathbb{Q}}[s(\rho)|\theta_1] &= \underbrace{k(1-\theta_1)F\big(z(0)|1-\theta_1\big)}_{\text{social utility under low quality}} + \underbrace{k\bar{F}\big(z(0)|1-\theta_1\big)}_{\text{social utility under high quality}}\\
&= k - k\theta_1 F\big(z(0)|1-\theta_1\big).
\end{aligned}$$

By the definition of FEE, the threshold policy $\theta_1^* \in (0,1)$ in the P-W game in period 1 can be solved using the following equation:

$$\theta_1 + k - k\theta_1 F\big(z(0)|1-\theta_1\big) - p_1 = \int_{z(0)}^{+\infty} (\mu_1 + \theta_1 + k - p_2) f(\mu_1|1-\theta_1) d\mu_1. \tag{B.1}$$

The FEE equation under strong network externalities can be rearranged to be:

$$\begin{aligned}
\theta_1 + k(1-\theta_1)F\big(z(0)|1-\theta_1\big) - p_1 &= \int_{z(0)}^{+\infty} (\mu_1 + \theta_1 - p_2) f(\mu_1|1-\theta_1) d\mu_1, \\
\theta_1 + k(1-\theta_1) - p_1 &= \int_{z(0)}^{+\infty} \mu_1 f(\mu_1|1-\theta_1) d\mu_1 - z(\theta_1)\bar{F}\big(z(0)|1-\theta_1\big), \\
-z(\theta_1) + p_2 - p_1 &= \int_{z(0)}^{+\infty} \mu_1 f(\mu_1|1-\theta_1) d\mu_1 - z(\theta_1)\bar{F}\big(z(0)|1-\theta_1\big), \\
r_p = p_2 - p_1 &= \int_{z(0)}^{+\infty} \mu_1 f(\mu_1|1-\theta_1) d\mu_1 + z(\theta_1)F\big(z(0)|1-\theta_1\big).
\end{aligned}$$

Similarly, we define the potential regret function $H_s(\theta_1)$ as

$$H_s(\theta_1) = \int_{z(0)}^{+\infty} \mu_1 f(\mu_1|1-\theta_1) d\mu_1 + z(\theta_1)F\big(z(0)|1-\theta_1\big).$$

Therefore, the FEE equation (B.1) is equivalent to $H_s(\theta_1) = r_p$, by the definition of $H_s(\theta_1)$. After further simplification, we have $H_s(\theta_1)$. ■

Lemma 15 *The potential regret function $H_w(\theta_1)$ is strictly decreasing in θ_1 on $[0,1]$.*

Proof of Lemma 15:

Let $y(\theta_1) = z(\theta_1)/\sigma(\theta_1)$. Then, the potential regret function can be written as $H_w(\theta_1) = \sigma(\theta_1)\phi[y(\theta_1)] + z(\theta_1)\Phi[y(\theta_1)]$. Taking derivative of $H_w(\theta_1)$, we have

$$\begin{aligned}
H_w'(\theta_1) &= \sigma'(\theta_1)\phi[y(\theta_1)] + \sigma(\theta_1)\phi'[y(\theta_1)]y'(\theta_1) + z'(\theta_1)\Phi[y(\theta_1)] + z(\theta_1)\phi[y(\theta_1)]y'(\theta_1) \\
&= \sigma'(\theta_1)\phi[y(\theta_1)] - \sigma(\theta_1)y(\theta_1)\phi[y(\theta_1)]y'(\theta_1) + z'(\theta_1)\Phi[y(\theta_1)] + z(\theta_1)\phi[y(\theta_1)]y'(\theta_1) \\
&= \sigma'(\theta_1)\phi[y(\theta_1)] - z(\theta_1)\phi[y(\theta_1)]y'(\theta_1) + z'(\theta_1)\Phi[y(\theta_1)] + z(\theta_1)\phi[y(\theta_1)]y'(\theta_1) \\
&= \sigma'(\theta_1)\phi[y(\theta_1)] + z'(\theta_1)\Phi[y(\theta_1)].
\end{aligned}$$

Since $z(\theta_1) = (k-1)\theta_1 + p_2 - k$, we have $z'(\theta_1) = k - 1 < 0$ as $k < 1$. It is straightforward to show $\sigma'(\theta_1) < 0$, which leads to $H_w'(\theta_1) < 0$. ■

Proof of Proposition 2.4.2:

(i) In preparation for the proof, we first prove the following claim.

Claim: $\underline{\mathcal{H}(x) = \phi(x) + x\Phi(x) > 0 \text{ for } x \in (-\infty, +\infty).}$

Since $\mathcal{H}'(x) = -x\phi(x) + \Phi(x) + x\phi(x) = \Phi(x) > 0$, we know that for $x \in (-\infty, +\infty)$, $\mathcal{H}(x) > \lim_{x\to-\infty} \mathcal{H}(x) = \lim_{x\to-\infty} \phi(x) + x\Phi(x) = 0$, which completes the proof of the claim.

Under WN, the potential regret function can be written as

$$\begin{aligned} H_w(\theta_1) &= \sigma(\theta_1)\left[\phi\left(\frac{z(\theta_1)}{\sigma(\theta_1)}\right) + \frac{z(\theta_1)}{\sigma(\theta_1)}\Phi\left(\frac{z(\theta_1)}{\sigma(\theta_1)}\right)\right] \\ &= \sigma(\theta_1)\big[\phi\big(\hbar(\theta_1)\big) + \hbar(\theta_1)\Phi\big(\hbar(\theta_1)\big)\big], \end{aligned}$$

where $\hbar(\theta_1) = z(\theta_1)/\sigma(\theta_1)$. Since $\sigma(\theta_1) = \sqrt{\frac{(1-\theta_1)M\gamma\sigma_0^2}{(1-\theta_1)M\gamma+1}} > 0$ for $\theta_1 \in [0,1)$, $\hbar(\theta_1) \in (-\infty, +\infty)$. Then, applying the claim we directly have $H_w(\theta_1) > 0$ for $\forall\theta_1 \in [0,1)$. Under SN, the potential regret function satisfies

$$\begin{aligned} H_s(\theta_1) &= \sigma(\theta_1)\left[\phi\left(\frac{z(0)}{\sigma(\theta_1)}\right) + \frac{z(\theta_1)}{\sigma(\theta_1)}\Phi\left(\frac{z(0)}{\sigma(\theta_1)}\right)\right] \\ &\geq \sigma(\theta_1)\left[\phi\left(\frac{z(0)}{\sigma(\theta_1)}\right) + \frac{z(0)}{\sigma(\theta_1)}\Phi\left(\frac{z(0)}{\sigma(\theta_1)}\right)\right], \end{aligned}$$

where the inequality comes from the fact that $z(\theta_1) \geq z(0)$ when $k > 1$. Applying similar argument as in the WN case yields $H_s(\theta_1) > 0$ for $\theta_1 \in [0,1)$.

(ii) When $\theta_1 < 1$, the crowdfunding price under WN satisfies $p_1 = p_2 - r_p = p_2 - H_w(\theta_1)$. Since $z(1) = p_2 - 1$ and $\sigma(1) = 0$, we have

$$\lim_{\theta_1\to1} H_w(\theta_1) = \lim_{\sigma\to0} \sigma\phi\left(\frac{p_2-1}{\sigma}\right) + (p_2-1)\Phi\left(\frac{p_2-1}{\sigma}\right) = (p_2-1)^+.$$

Since $H_w(\theta_1)$ is strictly decreasing in θ_1, we know $p_1 \leq p_2 - \lim_{\theta_1\to1} H_w(\theta_1) = p_2 - (p_2 - 1)^+ = p_2 \wedge 1$. Therefore, the crowdfunding price p_1 under WN must be less than 1 in order to avoid non-purchase.

(iii) The potential regret function can be written as a function of both θ_1 and k: $H_w(\theta_1, k) = \sigma(\theta_1)\phi[y(\theta_1, k)] + z(\theta_1, k)\Phi[y(\theta_1, k)]$, where $z(\theta_1, k) = p_2 - \theta_1 - k(1-\theta_1)$ and $y(\theta_1, k) = z(\theta_1, k)/\sigma(\theta_1)$. Then, taking partial derivative with respect to k gives

$$\begin{aligned} \frac{\partial H_w(\theta_1,k)}{\partial k} &= \sigma(\theta_1)y(\theta_1,k)\phi[y(\theta_1,k)]\frac{1-\theta_1}{\sigma(\theta_1)} - (1-\theta_1)\Phi[y(\theta_1,k)] - z(\theta_1,k)\phi[y(\theta_1,k)]\frac{1-\theta_1}{\sigma(\theta_1)} \\ &= y(\theta_1,k)\phi[y(\theta_1,k)](1-\theta_1) - (1-\theta_1)\Phi[y(\theta_1,k)] - y(\theta_1,k)\phi[y(\theta_1,k)](1-\theta_1) \\ &= -\Phi[y(\theta_1,k)](1-\theta_1) \leq 0. \end{aligned}$$

Since we have shown $\partial H_w(\theta_1, k)/\partial\theta_1 < 0$, by the Implicit Function Theorem, we have

$$\frac{\partial\theta_1}{\partial k} = -\frac{\partial H_w(\theta_1,k)/\partial k}{\partial H_w(\theta_1,k)/\partial\theta_1} \leq 0.$$

Hence, for a given r_p, $\theta_1 = H_w^{-1}(r_p)$ decreases in k.

Since $\sigma(\theta_1)$ increases in M and σ_0^2, while decreases in σ_q^2, it is equivalent to show for a given θ_1, $H_w(\theta_1)$ increases in $\sigma(\theta_1)$ (implicitly by the fact that $H_w(\theta_1)$ decreases in θ_1). Rewriting $H_w(\theta_1)$ as a function of $\sigma(\theta_1)$ gives

$$H(x) = x\phi\left(\frac{z(\theta_1)}{x}\right) + z(\theta_1)\Phi\left(\frac{z(\theta_1)}{x}\right),$$

and we are left to show $H'(x) > 0$. Taking derivative with respect to x, we have

$$\begin{aligned} H'(x) &= \phi\left(\frac{z(\theta_1)}{x}\right) - z(\theta_1)\phi\left(\frac{z(\theta_1)}{x}\right)\left(\frac{z(\theta_1)}{x}\right)' + z(\theta_1)\phi\left(\frac{z(\theta_1)}{x}\right)\left(\frac{z(\theta_1)}{x}\right)' \\ &= \phi\left(\frac{z(\theta_1)}{x}\right) > 0. \end{aligned}$$

We complete the proof. ■

Proof of Proposition 2.5.1:

The derivatives of the potential regret functions can be written as

$$\begin{aligned} H_w'(\theta_1) &= -\sigma_0\phi[\ell(\theta_1)]\ell(\theta_1)\ell'(\theta_1) + z'(\theta_1)\Phi[\ell(\theta_1)] + z(\theta_1)\phi[\ell(\theta_1)]\ell'(\theta_1) \\ &= (k-1)\Phi[\ell(\theta_1)], \\ H_s'(\theta_1) &= (k-1)\Phi[\ell(0)]. \end{aligned}$$

Then, $H_w'(\theta_1) < 0$ ($H_s'(\theta_1) > 0$) is a direct result of $k < 1$ ($k > 1$). ■

Proof of Proposition 2.5.2:

In the absence of social learning of uncertain quality, we can first show that none of the consumers will purchase in the retail stage (see Proposition B.0.2). The two-period problem now reduces to a single period problem, which is equivalent to the P-L game as we have discussed, with p_2 replaced by p_1. Without social learning, the quality component in the utility function can be treated as a fixed constant, or setting to be zero without loss of generality. Then, based on the purchasing patterns in the P-L game in Proposition 2.4.1, we know all consumers will purchase if the crowdfunding price is lower than a threshold; otherwise none of them will purchase under SN. Therefore, the Veblen effect will not happen in this case.

In the absence of strong positive network externalities, it is straightforward to show the nonexistence of the Veblen effect by using the properties of the potential regret functions $H_w(\theta_1)$ and $H_b(\theta_1)$ as described in Proposition 2.5.1. ■

Lemma 16 *There are the Pareto sets $\mathcal{R}_p^w$ and $\mathcal{R}_p^s$ for the reward choice under WN and SN, respectively, which can be characterized as $\mathcal{R}_p^w = \big[H_w(1), H_w(0)\big]$ and $\mathcal{R}_p^s = \big[H_s(0), H_s(1)\big]$.*

Proof of Lemma 16:

We need to show that any reward choice outside the set $\mathcal{R}_p$ is (weakly) dominated by the reward choices in the set. A reward choice is dominating if it generates the same FEE in period 1 but yields a (weakly) higher expected profit for the firm.

First, consider the WN case. Note the function $H_w(\theta_1)$ is strictly decreasing and the profit in period 1 is $(p_1 - c)M(1-\theta_1)$. So, any reward $r_p' > H_w(0)$ is strictly dominated by $r_p = H_w(0)$, since $\theta_1 = 0$ in both cases and a higher reward r_p' results in a lower profit, i.e., $(p_1' - c)M < (p_1 - c)M$. For any reward $r_p' < H_w(1)$, the resulting profit is the same as $r_p = H_w(1)$ (both zero profit) since $\theta_1 = 1$ in equilibrium. Therefore, the Pareto set under WN can be characterized as $\mathcal{R}_p^w = \big[H_w(1), H_w(0)\big]$.

Next, consider the SN case. Note the function $H_s(\theta_1)$ is strictly increasing and the profit in period 1 is $(p_1 - c)M(1-\theta_1)$. So, any reward $r_p' > H_s(1)$ is strictly dominated by $r_p = H_s(0) < r_p'$, since $\theta_1 = 0$ in equilibrium in both cases but a higher reward r_p' results in a lower profit, i.e., $(p_1' - c)M < (p_1 - c)M$. For any $r_p' < H_s(0)$, the resulting profit is the same as $r_p = H_s(1)$ (both zero profit) since $\theta_1 = 1$ in equilibrium. Therefore, the Pareto set under SN can be characterized as $\mathcal{R}_p^s = \big[H_s(0), H_s(1)\big]$. ■

Proof of Lemma 7:

When $k < 1$, from Proposition 2.4.1, we have

$$\begin{aligned}
\rho_{2w}(\theta_1) &= \int_{z(\theta_1)}^{+\infty} \left(\theta_1 - \frac{(p_2 - \mu_1 - k)^+}{1-k}\right) f(\mu_1) d\mu_1 \\
&= \theta_1 \bar{F}(z(0)) + \int_{z(\theta_1)}^{z(0)} \left(\theta_1 - \frac{p_2 - \mu_1 - k}{1-k}\right) f(\mu_1) d\mu_1 \\
&= \theta_1 \bar{F}(z(0)) + \left(\theta_1 - \frac{z(0)}{1-k}\right)\big[F(z(0)) - F\big(z(\theta_1)\big)\big] + \frac{1}{1-k}\int_{z(\theta_1)}^{z(0)} \mu_1 f(\mu_1) d\mu_1 \\
&= \theta_1 - \theta_1 \Phi\left(\frac{z(\theta_1)}{\sigma_0}\right) - \frac{z(0)}{1-k}\left[\Phi\left(\frac{z(0)}{\sigma_0}\right) - \Phi\left(\frac{z(\theta_1)}{\sigma_0}\right)\right] \\
&\quad + \frac{\sigma_0}{1-k}\left[\phi\left(\frac{z(\theta_1)}{\sigma_0}\right) - \phi\left(\frac{z(0)}{\sigma_0}\right)\right] \\
&= \theta_1 \Phi[-\ell(\theta_1)] + \frac{p_2 - k}{1-k}\Big\{\Phi[\ell(\theta_1)] - \Phi[\ell(0)]\Big\} + \frac{\sigma_0}{1-k}\Big\{\phi[\ell(\theta_1)] - \phi[\ell(0)]\Big\}.
\end{aligned}$$

When $k > 1$, from Proposition 2.4.1, we have $\rho_{2s}(\theta_1) = \theta_1 \bar{F}\big(z(0)\big) = \theta_1 \Phi[-\ell(0)]$. ■

Proof of Proposition 2.5.3:

(1) Weak Network Externalities ($i = w$).

When $0 \le k < 1$, the firm's expected profit can be written as $\Pi_w(\theta_1) = (p_2 - c)[1 - \theta_1 + \rho_{2w}(\theta_1)] - (1-\theta_1)H_w(\theta_1)$. Hence, $\Pi'_w(\theta_1) = (p_2 - c)[\rho'_{2w}(\theta_1) - 1] + H_w(\theta_1) - (1-\theta_1)H'_w(\theta_1)$. We already have the expected purchasing fraction in the second period under WN (Lemma 7). Taking derivative of $\rho_{2w}(\theta_1)$ gives

$$\begin{aligned}
\rho'_{2w}(\theta_1) &= \Phi[-\ell(\theta_1)] - \theta_1\phi[\ell(\theta_1)]\ell'(\theta_1) + \frac{p_2 - k}{1-k}\phi[\ell(\theta_1)]\ell'(\theta_1) - \frac{\sigma_0}{1-k}\ell(\theta_1)\phi[\ell(\theta_1)]\ell'(\theta_1) \\
&= \Phi[-\ell(\theta_1)] - \theta_1\phi[\ell(\theta_1)]\ell'(\theta_1) + \frac{p_2 - k}{1-k}\phi[\ell(\theta_1)]\ell'(\theta_1) \\
&\quad - \frac{\sigma_0}{1-k}\left[\frac{p_2 - k}{\sigma_0} + \frac{(k-1)\theta_1}{\sigma_0}\right]\phi[\ell(\theta_1)]\ell'(\theta_1) \\
&= \Phi[-\ell(\theta_1)].
\end{aligned}$$

Now, we plug the above simplified $\rho'_{2w}(\theta_1)$ into $\Pi'_w(\theta_1)$:

$$\begin{aligned}
\Pi'_w(\theta_1) &= (p_2 - c)[\rho'_{2w}(\theta_1) - 1] + H_w(\theta_1) - (1-\theta_1)H'_w(\theta_1) \\
&= -(p_2 - c)\Phi[\ell(\theta_1)] + \sigma_0\phi[\ell(\theta_1)] + z(\theta_1)\Phi[\ell(\theta_1)] - (1-\theta_1)(k-1)\Phi[\ell(\theta_1)] \\
&= \Phi[\ell(\theta_1)]\big[c + 2(k-1)\theta_1 - 2k + 1\big] + \sigma_0\phi[\ell(\theta_1)] \\
&= \Phi[\ell(\theta_1)]\left\{c - 2k + 1 + 2(k-1)\theta_1 + \sigma_0\frac{\phi[\ell(\theta_1)]}{\Phi[\ell(\theta_1)]}\right\}.
\end{aligned}$$

Since $\mathcal{B}(\theta_1) = c - 2k + 1 + 2(k-1)\theta_1 + \sigma_0\frac{\phi[\ell(\theta_1)]}{\Phi[\ell(\theta_1)]}$, we have $\Pi'_w(\theta_1) = \Phi[\ell(\theta_1)]\mathcal{B}(\theta_1)$. Next, we need to show $\mathcal{B}'(\theta_1) < 0$.

$$\begin{aligned}
\mathcal{B}'(\theta_1) &= 2(k-1) + \sigma_0\frac{-\ell(\theta_1)\phi[\ell(\theta_1)]\ell'(\theta_1)\Phi[\ell(\theta_1)] - \phi^2[\ell(\theta_1)]\ell'(\theta_1)}{\Phi^2[\ell(\theta_1)]} \\
&= (k-1)\left\{2 - \frac{\ell(\theta_1)\phi[\ell(\theta_1)]\Phi[\ell(\theta_1)] + \phi^2[\ell(\theta_1)]}{\Phi^2[\ell(\theta_1)]}\right\}.
\end{aligned}$$

Now, define another function $\mathcal{C}(x) = \frac{x\phi(x)\Phi(x) + \phi^2(x)}{\Phi^2(x)}$ and let $y = -x$. Then, we have

$$\mathcal{C}(y) = \frac{-x\phi(x)\bar{\Phi}(x) + \phi^2(x)}{\bar{\Phi}^2(x)} = \left[\frac{\phi(x)}{\bar{\Phi}(x)}\right]' = h'(x),$$

where $h(x) = \phi(x)/\bar{\Phi}(x)$ is the failure rate of the standard Normal distribution. Since the failure rate function is increasing and convex, we can further obtain $h''(x) = -\mathcal{C}'(y) > 0$, which means $\mathcal{C}'(y) < 0$. Thus, we have the following relationships:

$$
\begin{aligned}
\mathcal{C}(x) > \lim_{x\to-\infty}\mathcal{C}(x) &= \lim_{x\to-\infty}\frac{x\phi(x)\Phi(x)+\phi^2(x)}{\Phi^2(x)}\\
&= \lim_{x\to-\infty}\frac{x\phi(x)}{\Phi(x)} + \lim_{x\to-\infty}\left[\frac{\phi(x)}{\Phi(x)}\right]^2\\
&= \lim_{x\to-\infty}\frac{\phi(x)-x^2\phi(x)}{\phi(x)} + \lim_{x\to-\infty}\left[\frac{-x\phi(x)}{\phi(x)}\right]^2\\
&= \lim_{x\to-\infty} 1-x^2+x^2 = 1;\\
\mathcal{C}(x) < \lim_{x\to+\infty}\mathcal{C}(x) &= \lim_{x\to+\infty}\frac{x\phi(x)\Phi(x)+\phi^2(x)}{\Phi^2(x)} = \lim_{x\to+\infty} x\phi(x) = 0.
\end{aligned}
$$

Therefore, $0 < \mathcal{C}(x) < 1$. Figure B.1 below provide an illustration of the functions $h(y)$ and $\mathcal{C}(x)$.

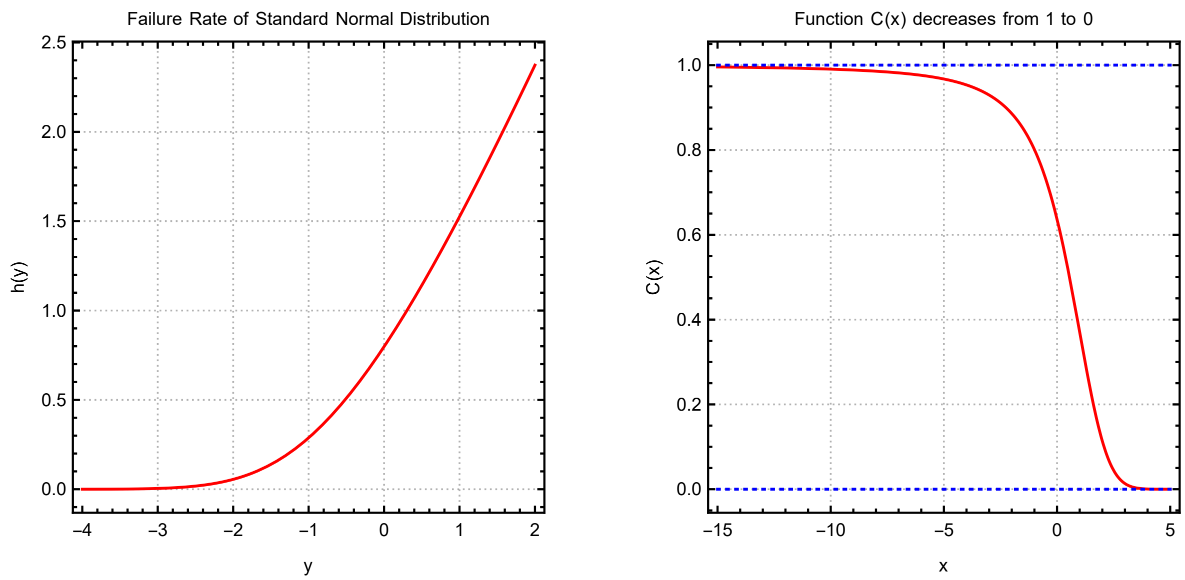


Figure B.1.: Illustration of functions $h(y)$ and $\mathcal{C}(x)$

Now, we know $\mathcal{C}[\ell(\theta_1)] \leq 1$ for any θ_1. Since $0 < k < 1$, we can further obtain

$$
\mathcal{B}'(\theta_1) = (k-1)\left\{2 - \frac{\ell(\theta_1)\phi[\ell(\theta_1)]\Phi[\ell(\theta_1)] + \phi^2[\ell(\theta_1)]}{\Phi^2[\ell(\theta_1)]}\right\} < 0.
$$

Therefore, we can conclude $\Pi_w(\theta_1)$ is strictly quasi-concave in θ_1, i.e., the firm's profit function is unimodal on $[0, 1]$.

(2) Strong Network Externalities ($i = s$).

When $k > 1$, we have $\rho_{2s}(\theta_1) = \theta_1\Phi[-\ell(0)]$. Taking derivatives gives $\rho'_{2s}(\theta_1) = \Phi[-\ell(0)]$. Now, we plug the above simplified $\rho'_{2s}(\theta_1)$ into $\Pi'_s(\theta_1)$:

$$\begin{aligned}\Pi'_s(\theta_1) &= (p_2 - c)[\rho'_{2s}(\theta_1) - 1] + H_s(\theta_1) - (1-\theta_1)H'_s(\theta_1)\\ &= -(p_2 - c)\Phi[\ell(0)] + \sigma_0\phi[\ell(0)] + z(\theta_1)\Phi[\ell(0)] - (1-\theta_1)(k-1)\Phi[\ell(0)]\\ &= \Phi[\ell(0)][2(k-1)\theta_1 + c - 2k + 1] + \sigma_0\phi[\ell(0)],\end{aligned}$$

and $\Pi''_s(\theta_1) = 2(k-1)\Phi[\ell(0)] > 0$ as $k > 1$. Therefore, the firm's profit function $\Pi_s(\theta_1)$ is strictly convex on $[0, 1]$. We can conclude that the optimal θ_1 must be on the boundary of its domain, i.e., either 0 or 1. We only need to directly compare the following two values,

$$\begin{aligned}\Pi_s(0) &= (p_2 - c) - \sigma_0\phi[\ell(0)] - (p_2 - k)\Phi[\ell(0)],\\ \Pi_s(1) &= (p_2 - c)\Phi[-\ell(0)].\end{aligned}$$

If $\Pi_s(0) \geq \Pi_s(1)$, then $\theta_1^{s*} = 0$; otherwise, $\theta_1^{s*} = 1$. The optimal crowdfunding reward can be derived accordingly as $r_p^{s*} = H_s(\theta_1^{s*})$. ■

Proof of Proposition 2.5.4:

First, we prove the following three claims.

Claim 1: $x + \phi(x)/\Phi(x) > 0$ for $\forall x \in (-\infty, +\infty)$.

In proof of Proposition 2.5.3, we find

$$\mathcal{C}(x) = \frac{\phi(x)}{\Phi(x)}\left[\frac{\phi(x)}{\Phi(x)} + x\right] \in (0, 1), \quad \forall x.$$

Thus, we know $x + \phi(x)/\Phi(x) > 0$ for $\forall x \in (-\infty, +\infty)$.

Claim 2: $x + \phi(x)/\Phi(x)$ strictly increases in x on $(-\infty, +\infty)$.

Taking derivatives gives

$$\left[x + \frac{\phi(x)}{\Phi(x)}\right]' = 1 - \frac{\phi(x)}{\Phi(x)}\left[\frac{\phi(x)}{\Phi(x)} + x\right] = 1 - \mathcal{C}(x) > 0,$$

where the last inequality follows from the fact $\mathcal{C}(x) \in (0, 1)$ as shown in the preceding proof.

Claim 3: $\phi(x)/\Phi(x)$ is strictly decreasing in x on $(-\infty, +\infty)$.

By taking derivative, we have

$$\left[\frac{\phi(x)}{\Phi(x)}\right]' = \frac{-x\phi(x)\Phi(x) - \phi^2(x)}{\Phi^2(x)} = -\frac{\phi(x)}{\Phi(x)}\left[\frac{\phi(x)}{\Phi(x)} + x\right] < 0,$$

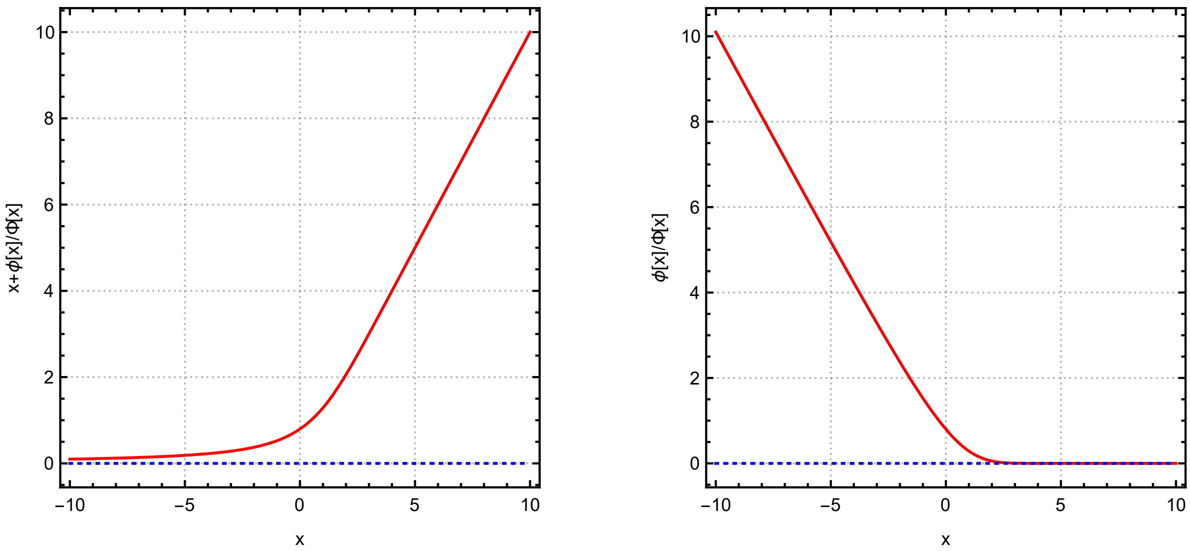


Figure B.2.: Illustration of function $x + \phi(x)/\Phi(x)$ (left) and $\phi(x)/\Phi(x)$ (right)

where the last inequality follows from Claim 1.

See Figure B.2 below for an illustration of Claim 1-3.

Now, we proceed to present the proof of this proposition.

(1) Under WN, we have $\Pi'_w(\theta_1) = \Phi[\ell(\theta_1)]\mathcal{B}(\theta_1)$, where $\mathcal{B}(\theta_1) = c - 2k + 1 + 2(k-1)\theta_1 + \sigma_0 \frac{\phi[\ell(\theta_1)]}{\Phi[\ell(\theta_1)]}$, $\ell(\theta_1) = \frac{z(\theta_1)}{\sigma_0} = \frac{(k-1)\theta_1 + p_2 - k}{\sigma_0}$. We have shown $\Pi_w(\theta_1)$ is strictly quasi-concave and $\mathcal{B}(\theta_1)$ is strictly decreasing in θ_1. It is straightforward to see $\mathcal{B}(\theta_1)$ is strictly increasing in c. To prove statement (i), it suffices to show $\mathcal{B}(1) > 0$ at $c = p_2$. When $\theta_1 = 1$ and $c = p_2$, we have

$$\mathcal{B}(1) = p_2 - 1 + \sigma_0 \frac{\phi\left(\frac{p_2-1}{\sigma_0}\right)}{\Phi\left(\frac{p_2-1}{\sigma_0}\right)} = \sigma_0 \left[\varpi_p + \frac{\phi(\varpi_p)}{\Phi(\varpi_p)}\right],$$

where $\varpi_p = (p_2 - 1)/\sigma_0$. From Claim 1, we know $\mathcal{B}(1) > 0$ for any p_2 and σ_0. Hence, $\exists$ $c^w_{max} \leq p_2$ s.t. $\theta_1^{w*} = 1$ for $c \geq c^w_{max}$.

To prove statement (ii), it suffices to show $\exists$ σ^w_{max} s.t.

$$\mathcal{B}(1) = c - 1 + \sigma_0 \phi\left(\frac{p_2 - 1}{\sigma_0}\right) / \Phi\left(\frac{p_2 - 1}{\sigma_0}\right) > 0$$

for $\sigma_0 \geq \sigma^w_{max}$. When $p_2 > 1$, i.e., $p_2 - 1 > 0$, by Claim 3 we know $\mathcal{B}(1)$ strictly increases in σ_0. When $p_2 = 1$, $\mathcal{B}(1)$ is simply a linear function of σ_0 and it is strictly increasing in σ_0. Thus, the statement holds for $p_2 \geq 1$. When $p_2 < 1$, by taking derivative with respect to σ_0, we have

$$\begin{aligned}\frac{d\mathcal{B}(1)}{d\sigma_0} &= \frac{\phi\left(\frac{p_2-1}{\sigma_0}\right)}{\Phi\left(\frac{p_2-1}{\sigma_0}\right)} - \sigma_0 \frac{\phi\left(\frac{p_2-1}{\sigma_0}\right)}{\Phi\left(\frac{p_2-1}{\sigma_0}\right)} \left[\frac{\phi\left(\frac{p_2-1}{\sigma_0}\right)}{\Phi\left(\frac{p_2-1}{\sigma_0}\right)} + \frac{p_2-1}{\sigma_0}\right]\left(-\frac{p_2-1}{\sigma_0^2}\right)\\ &= \frac{\phi\left(\frac{p_2-1}{\sigma_0}\right)}{\Phi\left(\frac{p_2-1}{\sigma_0}\right)}\left[1 + \underbrace{\frac{p_2-1}{\sigma_0}\frac{\phi\left(\frac{p_2-1}{\sigma_0}\right)}{\Phi\left(\frac{p_2-1}{\sigma_0}\right)}}_{①} + \underbrace{\left(\frac{p_2-1}{\sigma_0}\right)^2}_{>0}\right].\end{aligned}$$

Since $\phi(x)/\Phi(x)$ is strictly decreasing in x on $(-\infty, +\infty)$, we know $x\phi(x)/\Phi(x)$ strictly increases in x on $(-\infty, 0)$. Thus, term ① is strictly increasing in σ_0. Let σ_m be the solution to

$$1 + \frac{p_2-1}{\sigma_0}\frac{\phi\left(\frac{p_2-1}{\sigma_0}\right)}{\Phi\left(\frac{p_2-1}{\sigma_0}\right)} = 0.$$

Then, we can conclude that $\frac{d\mathcal{B}(1)}{d\sigma_0} > 0$ for $\sigma_0 \geq \sigma_m$. Therefore, $\exists\ \sigma_{max}^w$ s.t. $\theta_1^{w*} = 1$ for $\sigma_0 \geq \sigma_{max}^w$.

(2) Under SN, from Proposition 2.5.3, we have $\theta_1^{s*} = 1$ if $\Pi_s(0) < \Pi_s(1)$, where $\Pi_s(0) = (p_2 - c) - \sigma_0\phi[\ell(0)] - (p_2 - k)\Phi[\ell(0)]$, $\Pi_s(1) = (p_2 - c)\Phi[-\ell(0)]$, and $\ell(0) = (p_2 - k)/\sigma_0$. Since

$$\Pi_s(1) - \Pi_s(0) = -(p_2 - c)\Phi[\ell(0)] + \sigma_0\phi[\ell(0)] + (p_2 - k)\Phi[\ell(0)] = \sigma_0\phi[\ell(0)] + (c - k)\Phi[\ell(0)],$$

we know $\Pi_s(1) - \Pi_s(0)$ is strictly increasing in c. In addition, when $c = p_2$,

$$\Pi_s(1) - \Pi_s(0) = \sigma_0\phi[\ell(0)] + (p_2 - k)\Phi[\ell(0)] = \sigma_0\Phi[\ell(0)]\left(\frac{\phi[\ell(0)]}{\Phi[\ell(0)]} + \ell(0)\right) > 0,$$

where the last inequality comes from Claim 1. Therefore, $\exists\ c_{max}^b \leq p_2$ s.t. $\theta_1^{b*} = 1$ for $c \geq c_{max}^b$.

Now, to prove statement (ii) under SN, we need to show $\exists\ \sigma_{max}^s$ s.t. $\Pi_s(1) - \Pi_s(0) > 0$ for $\sigma_0 \geq \sigma_{max}^s$. The proof proceeds by considering the following three cases. **(a)** When $c \geq k$, it is clear that $\Pi_s(1) - \Pi_s(0) > 0$ for $\forall \sigma_0 > 0$. **(b)** When $c < k \leq p_2$, $\ell(0) = (p_2 - k)/\sigma_0$ decreases in σ_0. Now, rewrite $\Pi_s(1) - \Pi_s(0)$ as

$$\Pi_s(1) - \Pi_s(0) = \sigma_0\Phi[\ell(0)]\underbrace{\left(\frac{\phi[\ell(0)]}{\Phi[\ell(0)]} + \frac{c-k}{\sigma_0}\right)}_{②},$$

and we know ② strictly increases in σ_0, as the first part in ② increases in σ_0 because of Claim 3, and the second part increases in σ_0 because $c < k$. Also, as $\sigma_0 \to +\infty$,

② $\rightarrow \phi(0)/\Phi(0) > 0$. Hence, in this case $\exists\ \sigma^s_{max}$ s.t. $\Pi_s(1) - \Pi_s(0) \geq 0$ for $\sigma_0 \geq \sigma^s_{max}$.
(c) When $c \leq p_2 < k$, $\ell(0) = (p_2 - k)/\sigma_0$ increases in σ_0. Now, rewrite $\Pi_s(1) - \Pi_s(0)$ as follows:

$$\Pi_s(1) - \Pi_s(0) = \sigma_0 \Phi[\ell(0)] \Bigg(\underbrace{\frac{\phi[\ell(0)]}{\Phi[\ell(0)]} + \ell(0) + \frac{c - p_2}{\sigma_0}}_{③} \Bigg).$$

We know ③ strictly increases in σ_0, as the first two parts in ③ increase in σ_0 because $\ell(0)$ increases in σ_0, and the last part also increases in σ_0 as $c \leq p_2$. So the statement is also true in this case. Combining **(a)**-**(c)**, we conclude that $\exists\ \sigma^s_{max}$ s.t. $\Pi_s(1) - \Pi_s(0) > 0$ for $\sigma_0 \geq \sigma^s_{max}$. ■

Proposition B.0.2 *In the absence of social learning, consumers will either purchase in the crowdfunding stage or leave the market, and hence there is no sales in the retail stage.*

Proof of Proposition B.0.2:

In the absence of social learning, a consumer's expected net utility from purchasing the product in the crowdfunding stage and from waiting in the market can be written as $u_1(v, \tilde{\rho}_1, \tilde{\rho}_2) = v + k(\tilde{\rho}_1 + \tilde{\rho}_2) - p_1$ and $u_2(v, \tilde{\rho}_1, \tilde{\rho}_2) = v + k(\tilde{\rho}_1 + \tilde{\rho}_2) - p_2$. Since $p_2 \geq p_1$, $u_1(v, \tilde{\rho}_1, \tilde{\rho}_2) \geq u_2(v, \tilde{\rho}_1, \tilde{\rho}_2)$, we know none of the consumers will purchase in the retail stage, which means there is no sales in the retail stage. ■

Proof of Proposition 2.5.5:

In the absence of social learning, the consumers only choose between purchasing at price p_1 or leaving the market. Hence, the FEE characterization is similar to the case of P-L game in Proposition 2.4.1, with $\mathcal{P}(\theta_1, \bar{q}_r)$ replaced by p_1 and θ_1 replaced by 1. Then, we can conclude that there exists a unique FEE $\theta_1^{in}(p_1)$ for a given crowdfunding price p_1, which has the following form:

$$\theta_1^{in}(p_1) = \begin{Bmatrix} 1 \wedge \dfrac{(p_1 - k)^+}{1 - k}, & i = w \\ \mathbb{1}\{p_1 - k \leq 0\}, & i = s \end{Bmatrix}.$$

When $i = w$, $\theta_1^{wn}(p_1)$ and the firm's profit $\Pi_w^n(p_1)$ can be rewritten as

$$\theta_1^{wn}(p_1) = \begin{Bmatrix} 0, & p_1 \leq k \\ \dfrac{p_1 - k}{1 - k}, & k < p_1 < 1 \\ 1, & p_1 \geq 1 \end{Bmatrix};$$

$$\Pi_w^n(p_1) = \begin{cases} p_1 - c, & p_1 \leq k \\ (p_1 - c)\left(1 - \dfrac{p_1 - k}{1 - k}\right), & k < p_1 < 1 \\ 0, & p_1 \geq 1 \end{cases}.$$

Then, we can obtain the optimal crowdfunding price p_1^{wn*} to maximize the piecewise profit function $\Pi_w^n(p_1)$ and the corresponding optimal profit Π_w^{n*} as follows:

$$p_1^{wn*} = \begin{cases} k, & \dfrac{c+1}{2} \leq k < 1 \\ \dfrac{c+1}{2}, & k < \dfrac{c+1}{2} < 1 \end{cases};$$

$$\Pi_w^{n*} = \begin{cases} k - c, & \dfrac{c+1}{2} \leq k < 1 \\ \dfrac{(1-c)^2}{4(1-k)}, & k < \dfrac{c+1}{2} < 1 \end{cases}.$$

Next, when $i = s$, $\theta_1^{sn}(p_1) = \mathbb{1}\{p_1 - k \leq 0\}$. Then, the optimal crowdfunding price is $p_1^{sn*} = k$ and the corresponding optimal profit Π_s^{n*} is $k - c$. Therefore, the unique FEE θ_1^{n*} in the crowdfunding stage and the corresponding optimal crowdfunding price p_1^{n*} and optimal profit Π^{n*} can be summarized as follows:

$$\left(\theta_1^{n*}, p_1^{n*}, \Pi^{n*}\right) = \begin{cases} \left(\dfrac{c+1-2k}{2(1-k)} \wedge 1, \dfrac{c+1}{2}, \dfrac{(1-c)^2}{4(1-k)}\right), & k < \dfrac{1+c}{2} < 1 \\ \left(0, k, k-c\right), & k \geq \dfrac{1+c}{2} \end{cases}.$$

We now complete the proof. ■

Proof of Proposition 2.5.6:

(1) Weak Network Externalities ($i = w$)

The firm's expected profit can be written as a function both θ_1 and k, $\Pi_w(\theta_1, k) = (p_2 - c)[1 - \theta_1 + \rho_{2w}(\theta_1, k)] - (1-\theta_1)H_w(\theta_1, k)$, where $H_w(\theta_1, k) = \sigma_0\phi[\ell(\theta_1, k)] + \sigma_0\ell(\theta_1, k)\Phi[\ell(\theta_1, k)]$, and

$$\rho_{2w}(\theta_1, k) = \theta_1\Phi[-\ell(\theta_1, k)] + \frac{p_2 - k}{1-k}\Big\{\Phi[\ell(\theta_1, k)] - \Phi[\ell(0, k)]\Big\} + \frac{\sigma_0}{1-k}\Big\{\phi[\ell(\theta_1, k)] - \phi[\ell(0, k)]\Big\}.$$

First, let us consider the case with $0 < \theta_1^{w*} < 1$. Then, θ_1^{w*} satisfies the following first-order condition:

$$c - 2k + 1 + 2(k-1)\theta_1^{w*} + \sigma_0 \frac{\phi[\ell(\theta_1^{w*}, k)]}{\Phi[\ell(\theta_1^{w*}, k)]} = 0,$$

$$c - 2k + 1 + 2[\sigma_0\ell(\theta_1^{w*}, k) + k - p_2] + \sigma_0 \frac{\phi[\ell(\theta_1^{w*}, k)]}{\Phi[\ell(\theta_1^{w*}, k)]} = 0,$$

or equivalently,

$$2\ell(\theta_1^{w*}, k) + \frac{\phi[\ell(\theta_1^{w*}, k)]}{\Phi[\ell(\theta_1^{w*}, k)]} = \frac{2p_2 - c - 1}{\sigma_0}.$$

We have shown $x + \phi(x)/\Phi(x)$ strictly increases in x on $(-\infty, +\infty)$ (see Claim 2 in the proof of Proposition 2.5.4). Then, it is straightforward to see that $2x + \phi(x)/\Phi(x)$ also strictly increases in x. Hence, we know in equilibrium, $\ell(\theta_1^{w*}, k)$ remains fixed as k changes, i.e., $p_2 - \theta_1^{w*} - k(1 - \theta_1^{w*}) = C_1$, where C_1 is a constant. Therefore, $\frac{d\theta_1^{w*}}{dk} = -\frac{1-\theta_1^{w*}}{1-k} < 0$, which means θ_1^{w*} decreases in k. Moreover, we have the following relationship: $\ell(1, k) \leq \ell(\theta_1^{w*}, k) \leq \ell(0, k)$.

Since $H_w(\theta_1^{w*}, k) = \sigma_0\phi[\ell(\theta_1^{w*}, k)] + \sigma_0\ell(\theta_1^{w*}, k)\Phi[\ell(\theta_1^{w*}, k)]$ and $\ell(\theta_1^{w*}, k)$ is fixed for $0 < \theta_1^{w*} < 1$, we know $r_p^* = H_w(\theta_1^{w*}, k)$ is also a fixed constant, which means the optimal crowdfunding reward remains fixed when k changes. Since $\ell(\theta_1, k) = \frac{p_2 - \theta_1 - k(1-\theta_1)}{\sigma_0}$, we have $\frac{\partial \ell(\theta_1, k)}{\partial k} = -\frac{1-\theta_1}{\sigma_0}$. Taking partial derivatives with respect to k gives

$$\begin{aligned}
\frac{\partial H_w(\theta_1, k)}{\partial k} &= -\sigma_0\ell(\theta_1, k)\phi[\ell(\theta_1, k)]\frac{-(1-\theta_1)}{\sigma_0} - (1-\theta_1)\Phi[\ell(\theta_1, k)] \\
&\quad + \sigma_0\ell(\theta_1, k)\phi[\ell(\theta_1, k)]\frac{-(1-\theta_1)}{\sigma_0} \\
&= -\Phi[\ell(\theta_1, k)](1-\theta_1),
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \rho_{2w}(\theta_1, k)}{\partial k} &= \theta_1\phi[\ell(\theta_1, k)]\frac{1-\theta_1}{\sigma_0} + \frac{p_2 - 1}{(1-k)^2}\Big\{\Phi[\ell(\theta_1, k)] - \Phi[\ell(0, k)]\Big\} \\
&\quad + \frac{p_2 - k}{(1-k)\sigma_0}\Big\{\phi[\ell(0, k)] - (1-\theta_1)\phi[\ell(\theta_1, k)]\Big\} \\
&\quad + \frac{\sigma_0}{(1-k)^2}\Big\{\phi[\ell(\theta_1, k)] - \phi[\ell(0, k)]\Big\} \\
&\quad + \frac{\sigma_0}{1-k}\Big\{\frac{\ell(\theta_1, k)(1-\theta_1)}{\sigma_0}\phi[\ell(\theta_1, k)] - \frac{\ell(0, k)}{\sigma_0}\phi[\ell(0, k)]\Big\} \\
&= \frac{p_2 - 1}{(1-k)^2}\Big\{\Phi[\ell(\theta_1, k)] - \Phi[\ell(0, k)]\Big\} \\
&\quad + \left[\frac{p_2 - k}{(1-k)\sigma_0} - \frac{\sigma_0}{(1-k)^2} - \frac{\ell(0, k)}{1-k}\right]\phi[\ell(0, k)] \\
&\quad + \left[\frac{\theta_1(1-\theta_1)}{\sigma_0} - \frac{(p_2 - k)(1-\theta_1)}{(1-k)\sigma_0} + \frac{\sigma_0}{(1-k)^2} + \frac{\ell(\theta_1, k)(1-\theta_1)}{1-k}\right]\phi[\ell(\theta_1, k)] \\
&= \frac{p_2 - 1}{(1-k)^2}\Big\{\Phi[\ell(\theta_1, k)] - \Phi[\ell(0, k)]\Big\} + \frac{\sigma_0}{(1-k)^2}\Big\{\phi[\ell(\theta_1, k)] - \phi[\ell(0, k)]\Big\} \\
&= \frac{\sigma_0}{(1-k)^2}\Big\{\ell(1, k)\Phi[\ell(\theta_1, k)] + \phi[\ell(\theta_1, k)] - \ell(1, k)\Phi[\ell(0, k)] - \phi[\ell(0, k)]\Big\},
\end{aligned}$$

$$\begin{aligned}\frac{\partial \Pi_w(\theta_1,k)}{\partial k} &= (p_2-c)\frac{\partial \rho_{2w}(\theta_1,k)}{\partial k} - (1-\theta_1)\frac{\partial H_w(\theta_1,k)}{\partial k}\\ &= \frac{\sigma_0(p_2-c)}{(1-k)^2}\Big\{\ell(1,k)\Phi[\ell(\theta_1,k)] + \phi[\ell(\theta_1,k)] - \ell(1,k)\Phi[\ell(0,k)] - \phi[\ell(0,k)]\Big\}\\ &\quad + \Phi[\ell(\theta_1,k)](1-\theta_1)^2.\end{aligned}$$

Using the Envelope Theorem, we have $\frac{d\Pi_w^*}{dk} = \frac{\partial \Pi_w(\theta_1^{w*},k)}{\partial k}$, and

$$\begin{aligned}\frac{d\Pi_w^*}{dk} &= \frac{\sigma_0(p_2-c)}{(1-k)^2}\Big\{\ell(1,k)\Phi[\ell(\theta_1^{w*},k)] + \phi[\ell(\theta_1^{w*},k)] - \ell(1,k)\Phi[\ell(0,k)] - \phi[\ell(0,k)]\Big\}\\ &\quad + \Phi[\ell(\theta_1^{w*},k)](1-\theta_1^{w*})^2.\end{aligned}$$

To show $\frac{d\Pi_w^*}{dk} > 0$, it is sufficient to show $\ell(1,k)\Phi[\ell(\theta_1^{w*},k)]+\phi[\ell(\theta_1^{w*},k)] \geq \ell(1,k)\Phi[\ell(0,k)]+\phi[\ell(0,k)]$. We have shown $\ell(1,k) \leq \ell(\theta_1^{w*},k) \leq \ell(0,k)$, so it is sufficient to prove $\mathcal{J}_1(x) = \ell(1,k)\Phi(x) + \phi(x)$ decreases in x for $x \geq \ell(1,k)$. Note $\mathcal{J}_1'(x) = \ell(1,k)\phi(x) - x\phi(x) = \phi(x)(\ell(1,k) - x) \leq 0$ as $\ell(1,k) \leq x$. Therefore, we can now conclude that $\frac{d\Pi_w^*}{dk} > 0$.

Next, we further consider the case with $\theta_1^{w*} = 0, 1$. When $\theta_1^{w*} = 0$, the optimal profit function becomes $\Pi_w^* = \Pi_w(0,k) = (p_2 - c) - H_w(0,k)$, and $H_w(0,k) = \sigma_0\phi[\ell(0,k)] + \sigma_0\ell(0,k)\Phi[\ell(0,k)]$, where $\ell(0,k) = (p_2-k)/\sigma_0$. Let $\mathcal{J}_2(x) = x\Phi(x) + \phi(x)$, then $\mathcal{J}_2'(x) = \Phi(x) + x\phi(x) - x\phi(x) = \Phi(x) > 0$. Then, we know $r_p^{w*} = H_w(0,k)$ decreases in k as $\ell(0,k)$ decreases in k. Hence, Π_w^* increases in k.

When $\theta_1^{w*} = 1$, the potential regret function becomes $H_w(1,k) = \sigma_0\phi[\ell(1,k)] + \sigma_0\ell(1,k)\Phi[\ell(1,k)]$ and the optimal profit function becomes $\Pi_w^* = \Pi_w(1,k) = (p_2 - c)\rho_{2w}(1,k)$, where

$$\rho_{2w}(1,k) = \Phi[-\ell(1,k)] + \frac{p_2-k}{1-k}\Big\{\Phi[\ell(1,k)] - \Phi[\ell(0,k)]\Big\} + \frac{\sigma_0}{1-k}\Big\{\phi[\ell(1,k)] - \phi[\ell(0,k)]\Big\}.$$

It is straightforward to see the optimal reward $r_p^{w*} = H_w(1,k)$ remains constant when k increases. Note that $\ell(1,k) = (p_2-1)/\sigma_0$ and $\ell(0,k) = (p_2-k)/\sigma_0$. We have the following derivative of $\rho_{2w}(1,k)$,

$$\begin{aligned}\frac{d\rho_{2w}(1,k)}{dk} &= \frac{p_2-1}{(1-k)^2}\Big\{\Phi[\ell(1,k)] - \Phi[\ell(0,k)]\Big\} + \frac{\ell(0,k)\phi[\ell(0,k)]}{1-k}\\ &\quad + \frac{\sigma_0}{(1-k)^2}\Big\{\phi[\ell(1,k)] - \phi[\ell(0,k)]\Big\} - \frac{\ell(0,k)\phi[\ell(0,k)]}{1-k}\\ &= \frac{\sigma_0}{(1-k)^2}\Big\{\ell(1,k)\Phi[\ell(1,k)] + \phi[\ell(1,k)] - \ell(1,k)\Phi[\ell(0,k)] - \phi[\ell(0,k)]\Big\}.\end{aligned}$$

We have shown $\ell(1,k) \leq \ell(0,k)$, and $\mathcal{J}_1(x) = \ell(1,k)\Phi(x) + \phi(x)$ decreases in x for $x \geq \ell(1,k)$. Hence, we have $\ell(1,k)\Phi[\ell(1,k)] + \phi[\ell(1,k)] \geq \ell(1,k)\Phi[\ell(0,k)] + \phi[\ell(0,k)]$, which means $\frac{d\rho_{2w}(1,k)}{dk} > 0$. Therefore, Π_w^* also increases in k in this case.

(2) Strong Network Externalities ($i = s$)

From Proposition 2.5.3, $\theta_1^{s*} = 1$ if $\Pi_s(0,k) < \Pi_s(1,k)$, and $\theta_1^{s*} = 0$, otherwise. That means firm's optimal profit is chosen from $\Pi_s(0,k) = (p_2 - c) - \sigma_0\phi[\ell(0,k)] - (p_2 - k)\Phi[\ell(0,k)]$ and $\Pi_s(1,k) = (p_2 - c)\Phi[-\ell(0,k)]$. Note that $\ell(0,k) = \frac{p_2 - k}{\sigma_0}$, and thus $\frac{d\ell(0,k)}{dk} = -\frac{1}{\sigma_0}$. Let $\Pi_\Delta(k) = \Pi_s(1,k) - \Pi_s(0,k)$, then we have

$$\begin{aligned}
\Pi_\Delta(k) &= -(p_2 - c)\Phi[\ell(0,k)] + \sigma_0\phi[\ell(0,k)] + (p_2 - k)\Phi[\ell(0,k)] \\
&= \sigma_0\phi[\ell(0,k)] + (c - k)\Phi[\ell(0,k)] \\
&= \sigma_0\Phi[\ell(0,k)]\left(\frac{\phi[\ell(0,k)]}{\Phi[\ell(0,k)]} + \ell(0,k) + \frac{c - p_2}{\sigma_0}\right).
\end{aligned}$$

By Claim 2 in proof of Proposition 2.5.4, we know $x + \phi(x)/\Phi(x)$ strictly increases in x. Then, we have $\ell(0,k) + \phi[\ell(0,k)]/\Phi[\ell(0,k)]$ strictly decreases in k as $\ell(0,k)$ strictly decreases in k. Therefore, there exists a threshold value $\bar{k} \geq 1$ such that $\Pi_\Delta(k) < 0$ for $k > \bar{k}$ and $\Pi_\Delta(k) \geq 0$ as $k \leq \bar{k}$. Equivalently, $\theta_1^{s*} = 1$ for $k \leq \bar{k}$ and $\theta_1^{s*} = 0$ for $k > \bar{k}$, which means θ_1^{s*} decreases in k. The potential regret function under SN is $H_s(\theta_1, k) = \sigma_0\phi[\ell(0,k)] + \sigma_0\ell(\theta_1,k)\Phi[\ell(0,k)]$. Hence,

$$\begin{aligned}
H_s(0,k) &= \sigma_0\{\phi[\ell(0,k)] + \ell(0,k)\Phi[\ell(0,k)]\}, \\
H_s(1,k) &= \sigma_0\{\phi[\ell(0,k)] + \ell(1,k)\Phi[\ell(0,k)]\}.
\end{aligned}$$

We have shown $\mathcal{J}_2(x) = x\Phi(x) + \phi(x)$ increases in x, together with the fact that $\ell(0,k)$ decreases in k, we can conclude $H_s(0,k)$ decreases in k. Similarly, we have shown $\mathcal{J}_1(x) = \ell(1,k)\Phi(x) + \phi(x)$ decreases in x for $x \geq \ell(1,k)$, then we can obtain that $H_s(1,k)$ also decreases in k. Moreover, $H_s(0,k) > H_s(1,k)$ as $\ell(0,k) > \ell(1,k)$. We have shown the optimal threshold θ_1^{s*} will jump from 1 to 0 at some threshold value of k (or possibly remains at 0 for any k). Now, we can conclude $r_p^{s*} = H_s(\theta_1^{s*}, k)$ decreases in k.

Now, we are left to show the optimal profit $\Pi_s(0,k)$ and $\Pi_s(1,k)$ both increases in k. We can derive the following derivatives:

$$\begin{aligned}
\frac{d\Pi_s(0,k)}{dk} &= -\sigma_0\frac{d\ell(0,k)}{dk} - (p_2 - k)\phi[\ell(0,k)]\frac{d\ell(0,k)}{dk} + \Phi[\ell(0,k)] \\
&= 1 + \ell(0,k)\phi[\ell(0,k)] + \Phi[\ell(0,k)], \\
\frac{d\Pi_s(1,k)}{dk} &= -(p_2 - c)\phi[\ell(0,k)]\frac{d\ell(0,k)}{dk} = \frac{(p_2 - c)\phi[\ell(0,k)]}{\sigma_0}.
\end{aligned}$$

It is straightforward to see $\frac{d\Pi_s(1,k)}{dk} > 0$. Now, we need to show $\frac{d\Pi_s(0,k)}{dk} > 0$. Let $\mathcal{J}_3(x) = 1 + x\phi(x) + \Phi(x)$. We can show $\mathcal{J}_3'(x) = \phi(x)(2 - x^2)$ and the minimum point is attained at point $x^* = -\sqrt{2}$. Then, it is straightforward to show $\mathcal{J}_3(x^*) > 0$, as illustrated in Figure B.3 below. ■

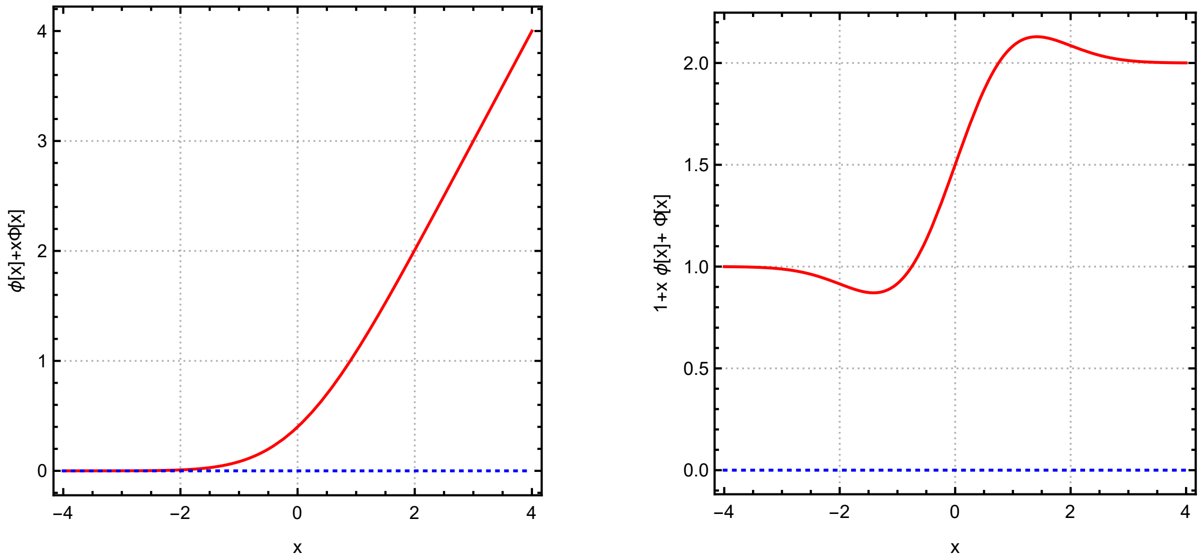


Figure B.3.: An Illustration of Functions $\mathcal{J}_2(x)$ (left) and $\mathcal{J}_3(x)$ (right)

Proof of Lemma 8:

(i) Since $H_w(\theta_1) = \sigma_0\phi[\ell(\theta_1)] + z(\theta_1)\Phi[\ell(\theta_1)]$ and $H_w'(\theta_1) = (k-1)\Phi[\ell(\theta_1)]$ under WN, we have

$$\begin{aligned}
\mathcal{Z}_w'(\theta_1) &= -[p_2 - H_w(\theta_1)] - (1-\theta_1)H_w'(\theta_1) \\
&= \sigma_0\phi[\ell(\theta_1)] + z(\theta_1)\Phi[\ell(\theta_1)] - p_2 - (1-\theta_1)(k-1)\Phi[\ell(\theta_1)] \\
&= \Phi[\ell(\theta_1)][2(k-1)\theta_1 + p_2 - 2k + 1] + \sigma_0\phi[\ell(\theta_1)] - p_2 \\
&= \Phi[\ell(\theta_1)]\left\{2(k-1)\theta_1 + p_2 - 2k + 1 + \sigma_0\frac{\phi[\ell(\theta_1)]}{\Phi[\ell(\theta_1)]} - \frac{p_2}{\Phi[\ell(\theta_1)]}\right\} \\
&= \Phi[\ell(\theta_1)]\left\{\mathcal{B}(\theta_1) - c + p_2 - \frac{p_2}{\Phi[\ell(\theta_1)]}\right\},
\end{aligned}$$

where $\mathcal{B}(\theta_1) = c - 2k + 1 + 2(k-1)\theta_1 + \sigma_0\frac{\phi[\ell(\theta_1)]}{\Phi[\ell(\theta_1)]}$. We have shown in section 5.2 that the function $\mathcal{B}'(\theta_1) < 0$. When $0 \le k < 1$, it is straightforward to see $\Phi[\ell(\theta_1)]$ is decreasing in θ_1. Taken together, we know the term in the braces is strictly decreasing in θ_1. Hence, we can conclude that $\mathcal{Z}_w(\theta_1)$ is strictly quasi-concave in θ_1. By the property of quasi-concave function, we know the feasible set for θ_1, $\Theta_{\mathcal{F}}^w = \{\theta_1 | \mathcal{Z}_w(\theta_1) \ge S/M,\ \theta_1 \in [0,1]\}$ is the upper contour set for function $\mathcal{Z}_w(\theta_1)$ if it is not empty, hence is convex. Therefore, if $\Theta^w_{\mathcal{F}} \neq \emptyset$, there exist θ^w_l and θ^w_u such that $0 \leq \theta^w_l \leq \theta^w_u \leq 1$ and $\Theta^w_{\mathcal{F}} = \left[\theta^w_l, \theta^w_u\right]$.

(ii) From Lemma 15, we know $H_s(\theta_1) = \sigma_0\phi[\ell(0)] + z(\theta_1)\Phi[\ell(0)]$, $H'_s(\theta_1) = (k-1)\Phi[\ell(0)]$, and

$$\begin{aligned}
\mathcal{Z}'_s(\theta_1) &= -[p_2 - H_s(\theta_1)] - (1-\theta_1)H'_s(\theta_1) \\
&= \sigma_0\phi[\ell(0)] + z(\theta_1)\Phi[\ell(0)] - p_2 - (1-\theta_1)(k-1)\Phi[\ell(0)] \\
&= \sigma_0\phi[\ell(0)] + [(k-1)\theta_1 + p_2 - k]\Phi[\ell(0)] - p_2 - (1-\theta_1)(k-1)\Phi[\ell(0)] \\
&= 2(k-1)\Phi[\ell(0)]\theta_1 + \sigma_0\phi[\ell(0)] + (1-2k)\Phi[\ell(0)] - p_2\Phi[-\ell(0)],
\end{aligned}$$

which is strictly increasing in θ_1 as $k > 1$. Thus, $\mathcal{Z}_s(\theta_1)$ is a strictly convex function with $\mathcal{Z}_s(1) = 0$. Therefore, when $\Theta^s_{\mathcal{F}} \neq \emptyset$, there exists a unique θ^s_u such that $\mathcal{Z}_s(\theta^s_u) = S/M$ and the resulting feasible set is $\Theta^s_{\mathcal{F}} = \left[0, \theta^s_u\right]$. ■

Proof of Proposition 2.6.1:

Based on the property of the feasible set obtained in Lemma 8, $\Theta^w_{\mathcal{F}} = \left[\theta^w_l, \theta^w_u\right]$, and the quasi-concavity of the firm's profit function $\Pi_w(\theta_1)$, we can derive the results in this proposition by similar argument used in the proof of Proposition 2.5.3. Based on the property of the feasible set obtained in Lemma 8, $\Theta^b_{\mathcal{F}} = \left[0, \theta^b_u\right]$, and the strict convexity of the firm's profit function $\Pi_s(\theta_1)$ as derived in Proposition 2.5.3, we can derive the results in this proposition by similar argument used in the proof of Proposition 2.5.3. ■

Proof of Proposition 2.6.2:

Due to the one-to-one relationship between r^{ic*}_p and $H(\theta^{ic*}_1)$, it is sufficient to prove statement (ii) of this proposition. Proof of statement (ii) will proceed by analyzing the following scenarios.

(1) Under WN, from the proof of Lemma 8, we have $\Pi'_w(\theta_1) = \Phi[\ell(\theta_1)]\mathcal{B}(\theta_1)$, and

$$\mathcal{Z}'_w(\theta_1) = \Phi[\ell(\theta_1)]\left\{\mathcal{B}(\theta_1) + \underbrace{p_2 - \frac{p_2}{\Phi[\ell(\theta_1)]} - c}_{\oplus}\right\},$$

where $\mathcal{B}(\theta_1) = c - 2k + 1 + 2(k-1)\theta_1 + \sigma_0\frac{\phi[\ell(\theta_1)]}{\Phi[\ell(\theta_1)]}$. It is straightforward to see $\oplus < 0$, and thus $\Pi'_w(\theta_1) > \mathcal{Z}'_w(\theta_1)$ for $\forall\theta_1 \in [0,1]$. From the preceding proof, we know both $\Pi_w(\theta_1)$ and $\mathcal{Z}_w(\theta_1)$ are strictly quasi-concave in θ_1. Then, there are three possible scenarios for the pattern of $\Pi_w(\theta_1)$: (a) strictly increasing, (b) strictly decreasing, and (c) first increasing then decreasing. In case (a), i.e., when $\Pi'_w(\theta_1) > 0$ for $\forall\theta_1 \in [0,1]$, from

Proposition 2.6.1 we know $\theta_1^{wc*} = \theta_u^w$. If $\mathcal{Z}_w'(\theta_1) > 0$ for $\forall \theta_1 \in [0,1]$, then $\Theta_{\mathcal{F}}^w = \emptyset$ since $\mathcal{Z}_w(1) = 0$. If $\mathcal{Z}_w(\theta_1)$ is strictly decreasing, or first increasing then decreasing, the feasible set shrinks as S increases and θ_u^w decreased. In case (b), $\mathcal{Z}_w'(\theta_1) < \Pi_w'(\theta_1) < 0$, then θ_u^w decreases. In case (c), if $\mathcal{Z}_w(\theta_1)$ is strictly decreasing, then as S increases, θ_u^w decreased and the optimal θ_1^{wc*} first remains the same and then decreases as θ_u^w decreases. If $\mathcal{Z}_w(\theta_1)$ is first increasing and then decreasing, since $\Pi_w'(\theta_1) > \mathcal{Z}_w'(\theta_1)$, it is clear to see θ_1^{wc*} first remains the same and then decreases as θ_u^w decreases. Taken together, we can conclude that statement (ii) is true under WN.

(2) Under SN, from Lemma 8, we know θ_u^s decreases as S increases. Then, the optimal θ_1^{sc*} remains unchanged if $\theta_1^{s*} = 0$. Or θ_1^{sc*} decreases as θ_u^s decreases if $\theta_1^{s*} = 1$ and $\Pi_s(\theta_1)$ is strictly increasing on $[0,1]$. Or θ_1^{sc*} first decreases and then jumps to 0 and remains unchanged afterwards, as S increases, if $\theta_1^{s*} = 1$ and $\Pi_s(\theta_1)$ is first decreasing and then increasing on $[0,1]$. Taken together, we can conclude that statement (ii) is true under SN. ■

Appendix B: Endogenous Retail Price

The basic model in the paper have assumed the retail price p_2 is exogenous determined due to a competitive retail market. This assumption helps maintain analytical tractability and deliver clean insights. Now, we relax this assumption by endogenizing the retail price as an additional decision variable for the firm and derive further insights into the crowdfunding problem. We will show the optimal structure of crowdfunding pricing scheme $\{p_1^*, p_2^*\}$ under different network externalities, present some important observations based on numerical experiments. Following the notations in previous sections, we shall define the *threshold function* as

$$\ell(\theta_1, p_2) = \frac{(k-1)\theta_1 + p_2 - k}{\sigma_0}. \tag{B.2}$$

All the functions of θ_1 (equivalently, p_1) shall be written as the functions of (θ_1, p_2).

Weak Network Externalities

When $k < 1$, the firm's profit function can be written as $\Pi_w(\theta_1, p_2) = (p_2 - c)[1 - \theta_1 + \rho_{2w}(\theta_1, p_2)] - (1-\theta_1)H_w(\theta_1, p_2)$. Note that the potential regret function becomes $H_w(\theta_1, p_2) = \sigma_0\phi[\ell(\theta_1, p_2)] + z(\theta_1, p_2)\Phi[\ell(\theta_1, p_2)]$, and the expected purchasing fraction in the retail stage can be written as $\rho_{2w}(\theta_1, p_2) = \theta_1\Phi[-\ell(\theta_1, p_2)] + \chi(\theta_1, p_2)$, where

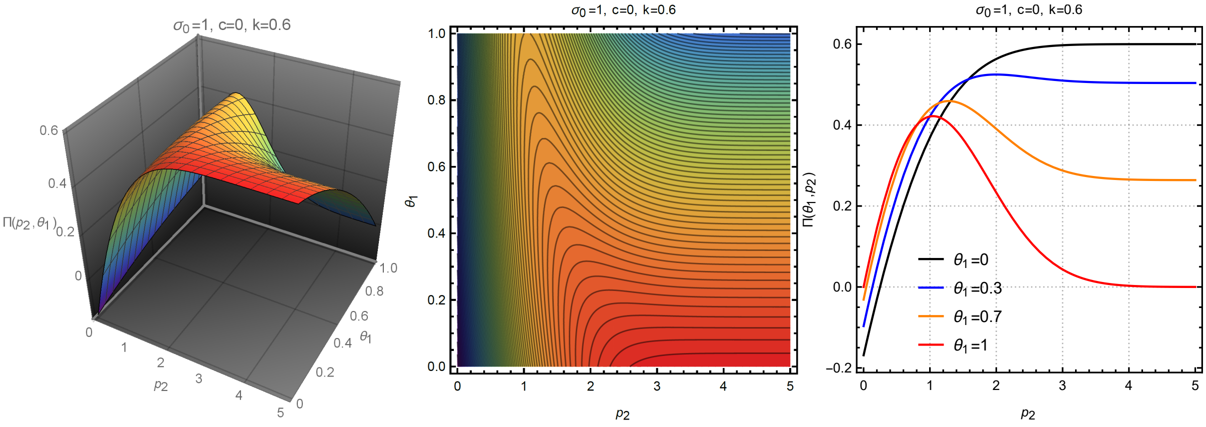
$$\chi(\theta_1, p_2) = \frac{p_2 - k}{1-k}\Big\{\Phi[\ell(\theta_1, p_2)] - \Phi[\ell(0, p_2)]\Big\} + \frac{\sigma_0}{1-k}\Big\{\phi[\ell(\theta_1, p_2)] - \phi[\ell(0, p_2)]\Big\}.$$

In preparation for the next result, we define the following two auxiliary functions,

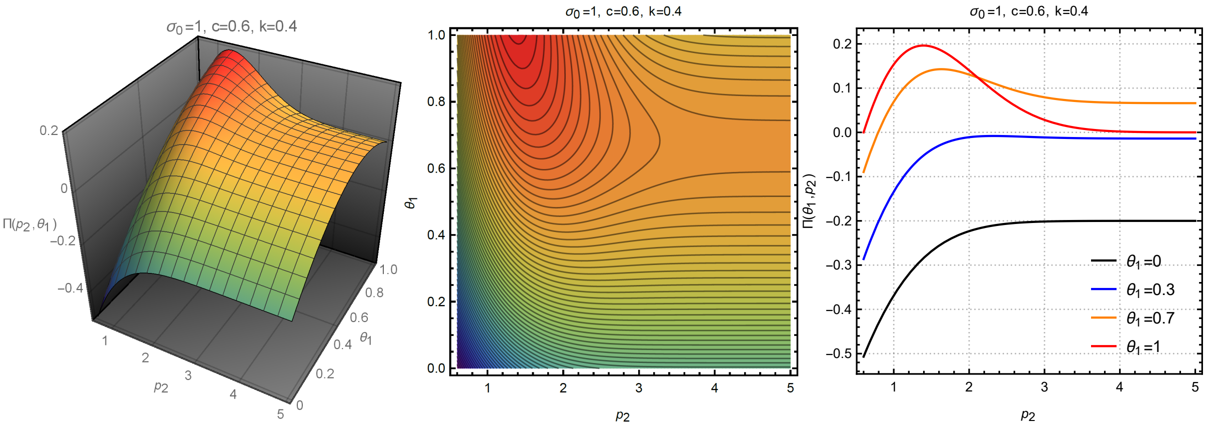
$$\mathcal{T}_1(\theta_1, p_2) = \Phi[\ell(\theta_1, p_2)]\big[c + 2(k-1)\theta_1 - 2k + 1\big] + \sigma_0\phi[\ell(\theta_1, p_2)],$$
$$\mathcal{T}_2(\theta_1, p_2) = (p_2 - c)\big\{\Phi[\ell(\theta_1, p_2)] - \Phi[\ell(0, p_2)]\big\} + (1-k)\big\{\Phi[-\ell(\theta_1, p_2)] + \chi(\theta_1, p_2)\big\}.$$

Proposition B.0.3 *The optimal pricing scheme $\{p_1^{w*}, p_2^{w*}\}$ under WN can be obtained by comparing the following three scenarios:*
(a) Adoption Frenzy: $\theta_1^{w*} = 0$, $p_1^{w*} = p_2^{w*} - H_w(0, p_2^{w*})$, and $p_2^{w*} = +\infty$;
(b) Adoption Inertia: $\theta_1^{w*} = 1$, $p_1^{w*} = p_2^{w*} - H_w(1, p_2^{w*})$, and p_2^{w*} is the solution to $\mathcal{T}_2(1, p_2) = 0$;
(c) Adoption Dispersion: $0 < \theta_1^{w*} < 1$, $p_1^{w*} = p_2^{w*} - H_w(\theta_1^{w*}, p_2^{w*})$, where θ_1^{w*} and p_2^{w*} are solved from $\mathcal{T}_1(\theta_1^{w*}, p_2^{w*}) = 0$ and $\mathcal{T}_2(\theta_1^{w*}, p_2^{w*}) = 0$.

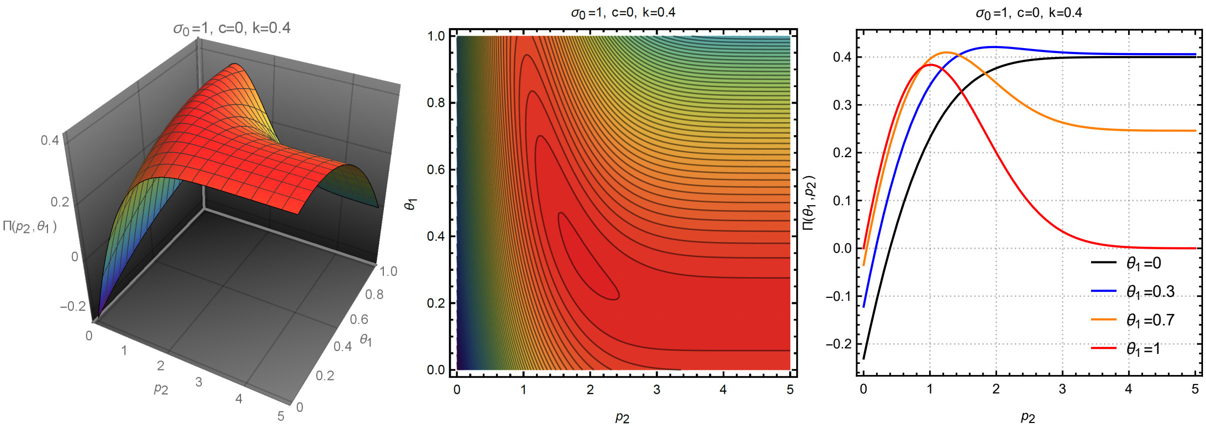
We plot in Figure B.4 the numerical illustrations of the firm's profit function under SN.



(a) Adoption Frenzy: $\theta_1^{w*} = 0$



(b) Adoption Inertia: $\theta_1^{w*} = 1$



(c) Adoption Dispersion: $0 < \theta_1^{w*} < 1$

Figure B.4.: Illustration of Firm's Profit Function with Endogenous Retail Price

Briefly, the proposition implies that the equilibrium purchasing pattern in the consumer population under the firm's optimal pricing strategy is similar to the base model in the main paper, where the crowdfunding purchase can be any fraction of the population. Similar intuition applies as well: When the network effect intensity is relatively small, the immediate network benefit and the potential learning benefit are close to each other, thus the optimal strategy is to take advantage of both benefits, resulting in any dispersed adoption. Note that the proof of Proposition B.0.3 (b) and (c) only provide first-order necessary conditions for optimality, without guarantee of uniqueness. Throughout our extensive numerical experiments, we observe the optimal pricing scheme is unique and Figure B.4 provides an example.

An important observation is that in case (a), the optimal retail price is $+\infty$. The intuition is that when adoption frenzy strategy is optimal, the firm should charge an extremely high retail price. It would decreases the consumer's potential regret, hence increases the firm's crowdfunding price, but still with all-purchase in equilibrium.

Strong Network Externalities

When $k > 1$, the expected purchasing fraction in the second period can be written as $\rho_{2s}(\theta_1, p_2) = \theta_1 \Phi[-\ell(0, p_2)]$, where $\ell(0, p_2) = (p_2 - k)/\sigma_0$. We also have the potential regret function in this case as $H_s(\theta_1, p_2) = \sigma_0 \phi[\ell(0, p_2)] + z(\theta_1, p_2)\Phi[\ell(0, p_2)]$. Hence, the firm's profit function can be written, in terms of θ_1 and p_2, as

$$\Pi_s(\theta_1, p_2) = (p_2 - c)[1 - \theta_1 + \rho_{2s}(\theta_1, p_2)] - (1 - \theta_1)H_s(\theta_1, p_2).$$

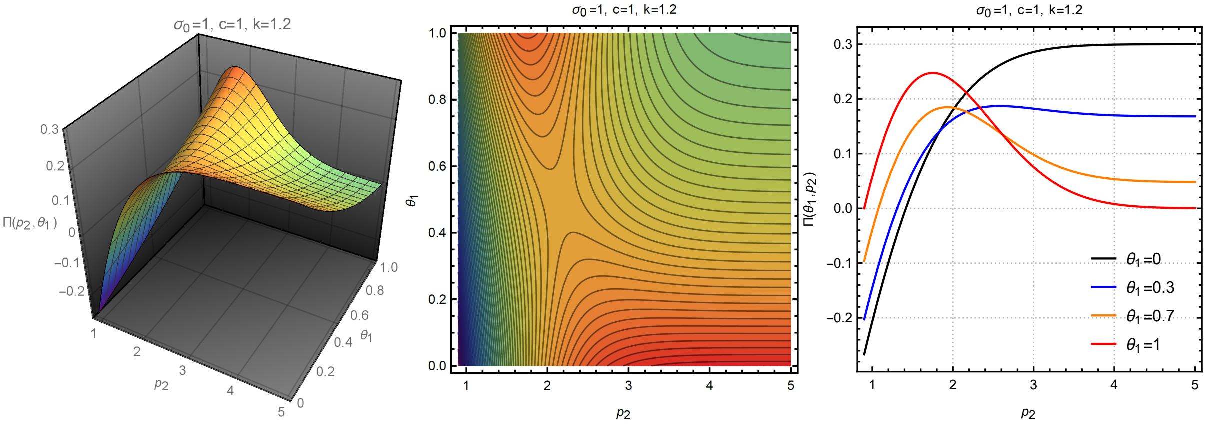
Based on Proposition 2.5.3, we know for any p_2, the optimal θ_1 can only be either 0 or 1. Hence, it is sufficient to focus on the structural property of functions $\Pi_s(0, p_2)$ and $\Pi_s(1, p_2)$. In preparation for the next result, we denote p_2^τ as the solution to $(p_2 - c)h[\ell(0, p_2)] = \sigma_0$, where $h[\cdot]$ is the failure rate function of the standard Normal distribution.

Proposition B.0.4 *The optimal pricing scheme $\{p_1^{s*}, p_2^{s*}\}$ under SN can be summarized as*

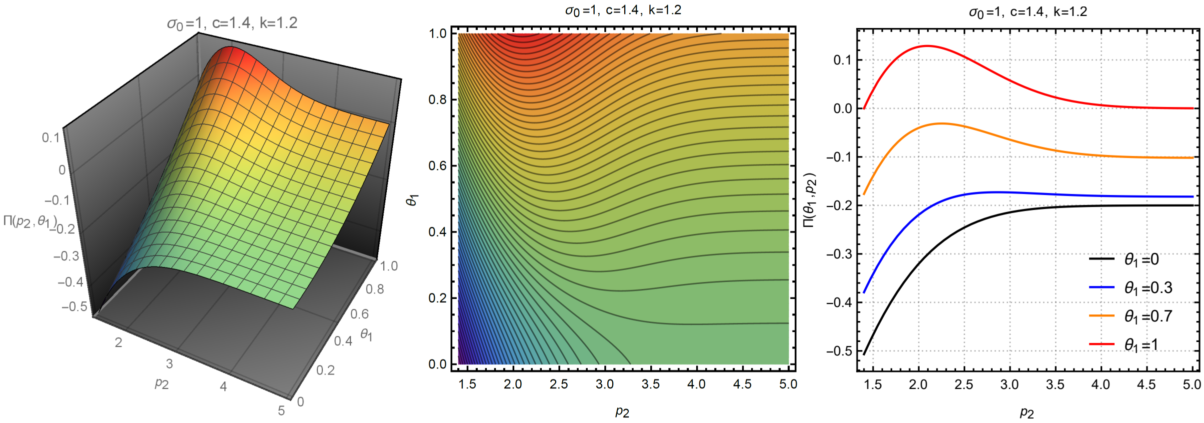
(a) Adoption Frenzy: $\theta_1^{s*} = 0$, $p_1^{s*} = k$, and $p_2^{s*} = +\infty$, if $\Pi_s(1, p_2^\tau) \le k - c$;

(b) Adoption Inertia: $\theta_1^{s*} = 1$, $p_1^{s*} = p_2^\tau - H_s(1, p_2^\tau)$, and $p_2^{s*} = p_2^\tau$, if $\Pi_s(1, p_2^\tau) > k - c$.

Proposition B.0.4 summarizes the optimal pricing strategy under strong network externalities. When we extend the base model to endogenize the retail price p_2, the induced equilibrium purchasing pattern remains the same: the optimal pricing strategy will induce all consumers to either adopt the product early (case (a) adoption frenzy) or adopt the product late (case (b) adoption inertia). We plot in Figure B.5 the numerical illustrations of the firm's profit function under SN.



(a) Adoption Frenzy: $\theta_1^{s*} = 0$



(b) Adoption Inertia: $\theta_1^{s*} = 1$

Figure B.5.: Illustration of Firm's Profit Function with Endogenous Retail Price

Proof of Proposition B.0.3:

When $k < 1$, we have

$$\frac{\partial \ell(\theta_1, p_2)}{\partial p_2} = \frac{\partial \ell(0, p_2)}{\partial p_2} = \frac{1}{\sigma_0}, \quad \frac{\partial z(\theta_1, p_2)}{\partial p_2} = 1.$$

Then, we can further derive the following partial derivatives with respect to p_2:

$$\begin{aligned}\frac{\partial H_w(\theta_1,p_2)}{\partial p_2} &= -\sigma_0\ell(\theta_1,p_2)\phi[\ell(\theta_1,p_2)]\frac{\partial \ell(\theta_1,p_2)}{\partial p_2} + \Phi[\ell(\theta_1,p_2)]\frac{\partial z(\theta_1,p_2)}{\partial p_2}\\ &\quad + z(\theta_1,p_2)\phi[\ell(\theta_1,p_2)]\frac{\partial \ell(\theta_1,p_2)}{\partial p_2}\\ &= \Phi[\ell(\theta_1,p_2)];\end{aligned}$$

$$\begin{aligned}\frac{\partial \rho_{2w}(\theta_1,p_2)}{\partial p_2} &= \frac{\Phi[\ell(\theta_1,p_2)]}{1-k} + \frac{z(\theta_1,p_2)\phi[\ell(\theta_1,p_2)]}{(1-k)\sigma_0} - \frac{\Phi[\ell(0,p_2)]}{1-k} - \frac{p_2-k}{(1-k)\sigma_0}\phi[\ell(0,p_2)]\\ &\quad + \frac{1}{1-k}\Big\{-\ell(\theta_1,p_2)\phi[\ell(\theta_1,p_2)] + \ell(0,p_2)\phi[\ell(0,p_2)]\Big\}\\ &= \frac{1}{1-k}\Big\{\Phi[\ell(\theta_1,p_2)] + \ell(\theta_1,p_2)\phi[\ell(\theta_1,p_2)] - \Phi[\ell(0,p_2)]\\ &\quad - \ell(0,p_2)\phi[\ell(0,p_2)] - \ell(\theta_1,p_2)\phi[\ell(\theta_1,p_2)] + \ell(0,p_2)\phi[\ell(0,p_2)]\Big\}\\ &= \frac{1}{1-k}\Big\{\Phi[\ell(\theta_1,p_2)] - \Phi[\ell(0,p_2)]\Big\};\\ \frac{\partial \Pi_w(\theta_1,p_2)}{\partial p_2} &= [1-\theta_1+\rho_{2w}(\theta_1,p_2)] + (p_2-c)\frac{\partial \rho_{2w}(\theta_1,p_2)}{\partial p_2} - (1-\theta_1)\frac{\partial H_w(\theta_1,p_2)}{\partial p_2}\\ &= 1 + \frac{z(\theta_1,p_2)}{1-k}\Phi[\ell(\theta_1,p_2)] - \frac{p_2-k}{1-k}\Phi[\ell(0,p_2)]\\ &\quad + \frac{\sigma_0}{1-k}\Big\{\phi[\ell(\theta_1,p_2)] - \phi[\ell(0,p_2)]\Big\}\\ &\quad + \frac{p_2-c}{1-k}\Big\{\Phi[\ell(\theta_1,p_2)] - \Phi[\ell(0,p_2)]\Big\} - (1-\theta_1)\Phi[\ell(\theta_1,p_2)]\\ &= \frac{2p_2-c-k}{1-k}\Big\{\Phi[\ell(\theta_1,p_2)] - \Phi[\ell(0,p_2)]\Big\} + \Phi[-\ell(\theta_1,p_2)]\\ &\quad + \frac{\sigma_0}{1-k}\Big\{\phi[\ell(\theta_1,p_2)] - \phi[\ell(0,p_2)]\Big\}.\end{aligned}$$

There are three possible cases to consider:

(a) When $\theta_1^{w*} = 0$, $\frac{\partial \Pi_w(0,p_2)}{\partial p_2} = \Phi[-\ell(0,p_2)] > 0$. Hence, the optimal retail price is $p_2^{w*} = +\infty$.

(b) When $\theta_1^{w*} = 1$, the first-order necessary condition for optimal p_2 is $\frac{\partial \Pi_w(1,p_2)}{\partial p_2} = 0$

(c) When $0 < \theta_1^{w*} < 1$, the first-order necessary conditions are $\frac{\partial \Pi_w(\theta_1,p_2)}{\partial p_2} = 0$, $\frac{\partial \Pi_w(\theta_1,p_2)}{\partial \theta_1} = 0$. From the preceding proofs and the definition of $\mathcal{T}_1(\theta_1,p_2)$ and $\mathcal{T}_2(\theta_1,p_2)$, we have

$$\frac{\partial \Pi_w(\theta_1,p_2)}{\partial \theta_1} = \Phi[\ell(\theta_1,p_2)]\big[c+2(k-1)\theta_1-2k+1\big] + \sigma_0\phi[\ell(\theta_1,p_2)] = \mathcal{T}_1(\theta_1,p_2);$$

$$\frac{\partial \Pi_w(\theta_1,p_2)}{\partial p_2} = \frac{\mathcal{T}_2(\theta_1,p_2)}{1-k}.$$

Therefore, the results in part (b) and (c) follow. ∎

Proof of Proposition B.0.4:

Taking derivatives with respect to p_2, we have

$$\begin{aligned}
\frac{\partial \rho_{2s}(\theta_1,p_2)}{\partial p_2} &= -\theta_1\phi[-\ell(0,p_2)]\frac{\partial \ell(0,p_2)}{\partial p_2} = -\frac{\theta_1}{\sigma_0}\phi[\ell(0,p_2)];\\
\frac{\partial H_s(\theta_1,p_2)}{\partial p_2} &= -\sigma_0\ell(0,p_2)\phi[\ell(0,p_2)]\frac{\partial \ell(0,p_2)}{\partial p_2} + \Phi[\ell(0,p_2)]\frac{\partial z(\theta_1,p_2)}{\partial p_2}\\
&\quad + z(\theta_1,p_2)\phi[\ell(0,p_2)]\frac{\partial \ell(0,p_2)}{\partial p_2}\\
&= -\ell(0,p_2)\phi[\ell(0,p_2)] + \Phi[\ell(0,p_2)] + \ell(\theta_1,p_2)\phi[\ell(0,p_2)]\\
&= \Phi[\ell(0,p_2)] + \frac{(k-1)\theta_1}{\sigma_0}\phi[\ell(0,p_2)].
\end{aligned}$$

Since the profit function is $\Pi_s(\theta_1,p_2) = (p_2-c)[1-\theta_1+\rho_{2s}(\theta_1,p_2)] - (1-\theta_1)H_s(\theta_1,p_2)$, we have

$$\begin{aligned}
\frac{\partial \Pi_s(\theta_1,p_2)}{\partial p_2} &= [1-\theta_1+\rho_{2s}(\theta_1,p_2)] + (p_2-c)\frac{\partial \rho_{2s}(\theta_1,p_2)}{\partial p_2} - (1-\theta_1)\frac{\partial H_s(\theta_1,p_2)}{\partial p_2}\\
&= 1-\theta_1\Phi[\ell(0,p_2)] - \frac{(p_2-c)\theta_1}{\sigma_0}\phi[\ell(0,p_2)]\\
&\quad - (1-\theta_1)\left[\Phi[\ell(0,p_2)] + \frac{(k-1)\theta_1}{\sigma_0}\phi[\ell(0,p_2)]\right]\\
&= 1-\Phi[\ell(0,p_2)] - \frac{\theta_1}{\sigma_0}[(k-1)(1-\theta_1)+p_2-c]\phi[\ell(0,p_2)].
\end{aligned}$$

Based on Proposition 2.5.3, we know for any p_2, the optimal θ_1 can only be either 0 or 1. Hence, it is sufficient to focus on the structural property of $\Pi_s(0,p_2)$ and $\Pi_s(1,p_2)$, and $\Pi_s(0,p_2) = (p_2-c)[1+\rho_{2s}(0,p_2)] - H_s(0,p_2)$, $\Pi_s(1,p_2) = (p_2-c)\rho_{2s}(1,p_2)$. We can show

$$\begin{aligned}
\frac{\partial \Pi_s(0,p_2)}{\partial p_2} &= 1-\Phi[\ell(0,p_2)],\\
\frac{\partial \Pi_s(1,p_2)}{\partial p_2} &= 1-\Phi[\ell(0,p_2)] - \frac{p_2-c}{\sigma_0}\phi[\ell(0,p_2)].
\end{aligned}$$

It is straightforward to see $\partial\Pi_s(0,p_2)/\partial p_2 > 0$ and as $p_2 \to +\infty$, $\partial\Pi_s(0,p_2)/\partial p_2 \to 0$. Then, the optimal p_2 for $\theta_1 = 0$ is $p_2^{\infty} = +\infty$. The corresponding maximal profit for the firm is

$$\begin{aligned}
\Pi_s(0,p_2^{\infty}) &= \lim_{p_2\to+\infty}(p_2-c)[1+\rho_{2s}(0,p_2)] - H_s(0,p_2)\\
&= \lim_{p_2\to+\infty} p_2 - c - \sigma_0\phi[\ell(0,p_2)] - (p_2-k)\Phi[\ell(0,p_2)]\\
&= k-c.
\end{aligned}$$

Next, rewrite $\partial\Pi_s(1,p_2)/\partial p_2$ as

$$\begin{aligned}\frac{\partial\Pi_s(1,p_2)}{\partial p_2} &= \Phi[-\ell(0,p_2)]\left\{1-\frac{p_2-c}{\sigma_0}\frac{\phi[\ell(0,p_2)]}{\Phi[-\ell(0,p_2)]}\right\}\\ &= \Phi[-\ell(0,p_2)]\left\{1-\frac{p_2-c}{\sigma_0}h[\ell(0,p_2)]\right\}.\end{aligned}$$

Since $\ell(0,p_2)$ is strictly increasing in p_2 and Normal distribution has a strictly increasing failure rate function $h[\cdot]$, we know the component within the braces is strictly decreasing in p_2. Thus, $\Pi_s(1,p_2)$ is strictly quasi-concave in p_2, and the optimal retail price for $\theta_1=1$ can be denoted as p_2^τ and solved from $(p_2-c)h[\ell(0,p_2)]=\sigma_0$.

Finally, we can simply compare the two optimal profits $\Pi_s(0,p_2^\infty)$ and $\Pi_s(1,p_2^\tau)$, and find the maximum one as the firm's optimal profit under SN. If $\Pi_s(1,p_2^\tau)\le k-c$, the optimal retail price is $p_2^{s*}=p_2^\infty=+\infty$, and the optimal reward is $H_s(0,p_2^\infty)=p_2^\infty-k$. Therefore, the optimal crowdfunding price is $p_1^{s*}=k$. If $\Pi_s(1,p_2^\tau)>k-c$, we have $p_1^{s*}=p_2^\tau-H_s(1,p_2^\tau)$ and $p_2^{s*}=p_2^\tau$. ■

Appendix C: Complementary Results for Boundary Scenario

In this appendix, we present all the complementary results for the boundary scenario (BS) of network externalities (i.e., the case with $k = 1$).

Analysis of P-W Game

Lemma 17 *The FEE equation under BS can be written as* $H_b(\theta_1) = r_p$, *where*

$$H_b(\theta_1) = \sigma(\theta_1)\phi[z(1)/\sigma(\theta_1)] + z(1)\Phi[z(1)/\sigma(\theta_1)]. \tag{B.3}$$

Proof of Lemma 17:
When $k = 1$, $z(\theta_1) = p_2 - 1 = z(1)$ and we can obtain the consumer's expected total social utility $\mathbb{E}^0_{\mathbb{Q}}[s(\rho)|\theta_1]$ conditional on the belief of threshold policy θ_1 as follows:

$$\begin{aligned}\mathbb{E}^0_{\mathbb{Q}}[s(\rho)|\theta_1] &= \underbrace{(1-\theta_1)F\big((p_2-1)|1-\theta_1\big)}_{\text{social utility under low quality}} + \underbrace{\bar{F}\big((p_2-1)|1-\theta_1\big)}_{\text{social utility under high quality}} \\ &= 1 - \theta_1 F\big((p_2-1)|1-\theta_1\big).\end{aligned}$$

By definition of FEE, the threshold policy $\theta_1^* \in (0, 1)$ in the P-W game in period 1 can be solved through the following FEE equation:

$$\theta_1 + \big[1 - \theta_1 F\big((p_2-1)|1-\theta_1\big)\big] - p_1 = \int_{p_2-1}^{+\infty} (\mu_1 + \theta_1 + 1 - p_2) f(\mu_1|1-\theta_1) d\mu_1.$$

The FEE equation under boundary scenario can be rearranged as follows,

$$\begin{aligned}\theta_1 + \big[1 - \theta_1 F\big((p_2-1)|1-\theta_1\big)\big] - p_1 &= \int_{p_2-1}^{+\infty} (\mu_1 + \theta_1 - p_2) f(\mu_1|1-\theta_1) d\mu_1 \\ &\quad + \bar{F}\big((p_2-1)|1-\theta_1\big), \\ \theta_1 + (1-\theta_1)F\big((p_2-1)|1-\theta_1\big) - p_1 &= \int_{p_2-1}^{+\infty} (\mu_1 + \theta_1 - p_2) f(\mu_1|1-\theta_1) d\mu_1, \\ r_p = p_2 - p_1 &= \int_{p_2-1}^{+\infty} \mu_1 f(\mu_1|1-\theta_1) d\mu_1 + (p_2-1)F\big((p_2-1)|1-\theta_1\big).\end{aligned}$$

Similar to the WN case, we define the *potential regret function* $H_b(\theta_1)$ in BS case,

$$H_b(\theta_1) = (p_2-1)F\big((p_2-1)|1-\theta_1\big) + \int_{p_2-1}^{+\infty} \mu_1 f(\mu_1|1-\theta_1) d\mu_1.$$

Therefore, the FEE equation is equivalent to $H_b(\theta_1) = r_p$, by definition of $H_b(\theta_1)$. After further simplification, we have $H_b(\theta_1) = \sigma(\theta_1)\phi[z(1)/\sigma(\theta_1)] + z(1)\Phi[z(1)/\sigma(\theta_1)]$. ■

Lemma 18 *The potential regret function $H_b(\theta_1)$ is strictly decreasing in θ_1 on* $[0, 1]$.

Proof of Lemma 18:

Since $H_b(\theta_1) = \sigma(\theta_1)\phi\big(\frac{p_2-1}{\sigma(\theta_1)}\big) + (p_2-1)\Phi\big(\frac{p_2-1}{\sigma(\theta_1)}\big)$, taking derivative we have

$$\begin{aligned}
H_b'(\theta_1) &= \sigma'(\theta_1)\phi\left(\frac{p_2-1}{\sigma(\theta_1)}\right) - \sigma(\theta_1)\frac{p_2-1}{\sigma(\theta_1)}\phi\left(\frac{p_2-1}{\sigma(\theta_1)}\right)\left(\frac{p_2-1}{\sigma(\theta_1)}\right)' \\
&\quad + (p_2-1)\phi\left(\frac{p_2-1}{\sigma(\theta_1)}\right)\left(\frac{p_2-1}{\sigma(\theta_1)}\right)' \\
&= \sigma'(\theta_1)\phi\left(\frac{p_2-1}{\sigma(\theta_1)}\right) < 0.
\end{aligned}$$

Hence, we complete the proof. ∎

Optimal Reward Choice

Starting from this subsection, all analysis will be based on the assumption of perfect learning.

Lemma 19 *Under perfect learning, the potential regret function under BS becomes* $H_b(\theta_1) = \sigma_0\phi[\ell(1)] + z(1)\Phi[\ell(1)]$, *and* $H_b'(\theta_1) = 0$.

Proof of Lemma 19:

Under perfect learning, we now have $\sigma(\theta_1) = \sigma_0$ and $\ell(1) = (p_2-1)/\sigma_0 = z(1)/\sigma_0$. Based on the potential regret function $H_b(\theta_1)$ obtained previously, it is straightforward to derive $H_b(\theta_1) = \sigma(\theta_1)\phi[z(1)/\sigma(\theta_1)] + z(1)\Phi[z(1)/\sigma(\theta_1)] = \sigma_0\phi[\ell(1)] + z(1)\Phi[\ell(1)]$, and its derivative $H_b'(\theta_1) = 0$. ∎

Lemma 20 *Suppose consumer's threshold policy in the P-W game is θ_1, then the expected purchasing fraction in second period under BS is* $\rho_{2b}(\theta_1) = \theta_1\Phi[-\ell(1)]$.

Proof of Lemma 20:

When $k = 1$, the expected purchasing fraction in the second period is straightforward to derive based on Proposition 2.4.1,

$$\rho_{2b}(\theta_1) = \int_{z(\theta_1)}^{+\infty}(\theta_1 - 0)f(\mu_1|1-\theta_1)d\mu_1 = \theta_1\bar{F}(p_2-k|1-\theta_1) = \theta_1\Phi[-\ell(1)].$$

We complete the proof. ∎

Now, we analyze the firm's profit and optimal crowdfunding reward choice under the BS case. Based on previous analysis, we have the profit function is $\Pi_b(\theta_1) = (p_2 - c)[1 - \theta_1 + \rho_{2b}(\theta_1)] - (1-\theta_1)H_b(\theta_1)$, where $H_b(\theta_1) = \sigma_0\phi[\ell(1)] + z(1)\Phi[\ell(1)]$, $\rho_{2b}(\theta_1) = \theta_1\Phi[-\ell(1)]$.

Proposition B.0.5 *The optimal reward is* $r_p^{b*} = H_b(\theta_1^{b*})$, *where* $\theta_1^{b*} = \mathbb{1}\{\Pi_b(0) < \Pi_b(1)\}$.

Proof of Proposition B.0.5:

When $k = 1$, $z(\theta_1) = p_2 - 1 = z(1)$, $\ell(\theta_1) = (p_2 - 1)/\sigma_0 = \ell(1)$, $H_b(\theta_1) = \sigma_0\phi[\ell(1)] + z(1)\Phi[\ell(1)]$, $H_b'(\theta_1) = 0$, $\rho_{2b}(\theta_1) = \theta_1\Phi[-\ell(1)]$ and $\rho_{2b}'(\theta_1) = \Phi[-\ell(1)]$. Then, the profit function is $\Pi_b(\theta_1) = (p_2 - c)[1 - \theta_1 + \rho_{2b}(\theta_1)] - (1-\theta_1)H_b(\theta_1)$, and its derivative becomes $\Pi_b'(\theta_1) = (p_2 - c)[\rho_{2b}'(\theta_1) - 1] + H_b(\theta_1) - (1-\theta_1)H_b'(\theta_1) = -(p_2 - c)\Phi[\ell(1)] + \sigma_0\phi[\ell(1)] + (p_2 - 1)\Phi[\ell(1)] = (c-1)\Phi[\ell(1)] + \sigma_0\phi[\ell(1)]$, which is a constant. Hence, the profit function should be (weakly) monotonic, and the optimum shall be attained at the two boundary points. When $\Pi_b'(\theta_1) = 0$, any threshold θ_1 generates the same profit, hence should be optimal. To break this tie, let the smallest one (left boundary point $\theta_1 = 0$) be preferred by the firm, since it yields the same profit without any risk (all purchases happen in the crowdfunding stage). ■

Proposition B.0.6 *The firm should not launch the crowdfunding campaign under BS when: (i) the unit production cost is too high; (Formally,* $\exists\ c_{max}^b \le p_2$ *s.t.* $\theta_1^{b*} = 1$ *for* $c \ge c_{max}^b$*.) (ii) the prior product quality uncertainty is too high. (Formally,* $\exists\ \sigma_{max}^b$ *s.t.* $\theta_1^{b*} = 1$ *for* $\sigma_0 \ge \sigma_{max}^b$*.)*

Proof of Proposition B.0.6:

Under BS, we have $\Pi_b'(\theta_1) = (c-1)\Phi[\ell(1)] + \sigma_0\phi[\ell(1)]$, where $\ell(1) = (p_2 - 1)/\sigma_0$. Then,

$$\Pi_b'(\theta_1) = \Phi\left(\frac{p_2 - 1}{\sigma_0}\right)\left[c - 1 + \sigma_0\frac{\phi\left(\frac{p_2-1}{\sigma_0}\right)}{\Phi\left(\frac{p_2-1}{\sigma_0}\right)}\right] = \Phi\left(\frac{p_2 - 1}{\sigma_0}\right)\mathcal{B}(1).$$

From proof of part (1) of Proposition 2.5.4, we know $\mathcal{B}(1) > 0$ as $c = p_2$. Therefore, it is straightforward to see $\Pi_b'(1) > 0$ and statement (i) holds. If $p_2 \ge k = 1$, by Claim 3 in the proof of Proposition 2.5.4 we know $\mathcal{B}(1)$ strictly increases in σ_0. If $p_2 < k = 1$, in the proof of part (1) of Proposition 2.5.4, we have shown $\exists\ \sigma_m$ s.t. $\mathcal{B}(1)$ strictly increases in σ_0 for $\sigma_0 \ge \sigma_m$. Taken together, $\exists\ \sigma_{max}^w$ s.t. $\mathcal{B}(1) > 0$, i.e., $\theta_1^{w*} = 1$ for $\sigma_0 \ge \sigma_{max}^w$ and statement (ii) holds. ■

Financial Constraints

We can formulate the firm's constrained optimization problem under BS as follows:

$$\max_{\theta_1 \in [0,1]} \Pi_b(\theta_1) = \max_{\theta_1 \in [0,1]} p_2[1-\theta_1+\rho_{2b}(\theta_1)] - (1-\theta_1)H_b(\theta_1)$$
$$s.t. \qquad \mathcal{Z}_b(\theta_1) \geq S/M \tag{B.4}$$

where $\mathcal{Z}_b(\theta_1) = [p_2 - H_b(\theta_1)](1-\theta_1)$ is the *fundraising function*. Then, the *feasible set* for θ_1 under constraint (B.4) as $\Theta^b_{\mathcal{F}} = \{\theta_1 | \mathcal{Z}_b(\theta_1) \geq S/M,\ \theta_1 \in [0,1]\}$. Next, we present the property of the feasible set in the following lemma.

Lemma 21 *If $\Theta^b_{\mathcal{F}} \neq \emptyset$, there exists a θ^b_u such that $0 \leq \theta^b_u \leq 1$ and $\Theta^b_{\mathcal{F}} = [0, \theta^b_u]$.*

Proof of Lemma 21:

From Lemma 19, we know $H_b(\theta_1) = \sigma_0\phi[\ell(1)] + z(1)\Phi[\ell(1)]$ and $H'_b(\theta_1) = 0$ when $k=1$. Thus $\mathcal{Z}'_b(\theta_1) = -[p_2 - H_b(\theta_1)] = \sigma_0\phi[\ell(1)] + z(1)\Phi[\ell(1)] - p_2$, which is a constant. Since $\mathcal{Z}_b(1) = 0$, we know $\mathcal{Z}_b(\theta_1) \leq 0$ if $\mathcal{Z}'_b(\theta_1) \geq 0$. Hence, if $\mathcal{Z}'_b(\theta_1) \geq 0$, $\Theta^b_{\mathcal{F}} = \emptyset$ for any positive fundraising target S. If $\mathcal{Z}'_b(\theta_1) < 0$ and $\Theta^b_{\mathcal{F}} \neq \emptyset$, then there exists a unique θ^b_u such that $\mathcal{Z}_b(\theta^b_u) = S/M$ and the resulting feasible set is $\Theta^b_{\mathcal{F}} = [0, \theta^b_u]$. ■

Proposition B.0.7 *The optimal reward is $r^{bc*}_p = H_b(\theta^{bc*}_1)$, where $\theta^{bc*}_1 = \theta^b_u \mathbb{1}\{\Pi(0) < \Pi(\theta^b_u)\}$.*

Proof of Proposition B.0.7:

Based on the property of the feasible set in Lemma 21, we have $\Theta^b_{\mathcal{F}} = [0, \theta^b_u]$. In addition, the firm's profit function $\Pi_b(\theta_1)$ is (weakly) monotonic as shown in the proof of Proposition B.0.5, and the optimum shall be attained at the two boundary points of the feasible set $\Theta^b_{\mathcal{F}}$. Therefore, the optimal purchasing threshold θ^{bc*}_1 and the corresponding optimal reward choice r^{bc*}_p have the form as presented in this proposition. ■

Proposition B.0.8 *As the fundraising target increases, the optimal crowdfunding reward decreases and the equilibrium purchasing fraction in the crowdfunding stage increases under BS. (Formally, as S increases, r^{bc*}_p decreases and θ^{bc*}_1 decreases.)*

Proof of Proposition B.0.8:

Under BS, from Lemma 21 and Proposition B.0.5, we know both $\mathcal{Z}'_b(\theta_1)$ and $\Pi'_b(\theta_1)$ are

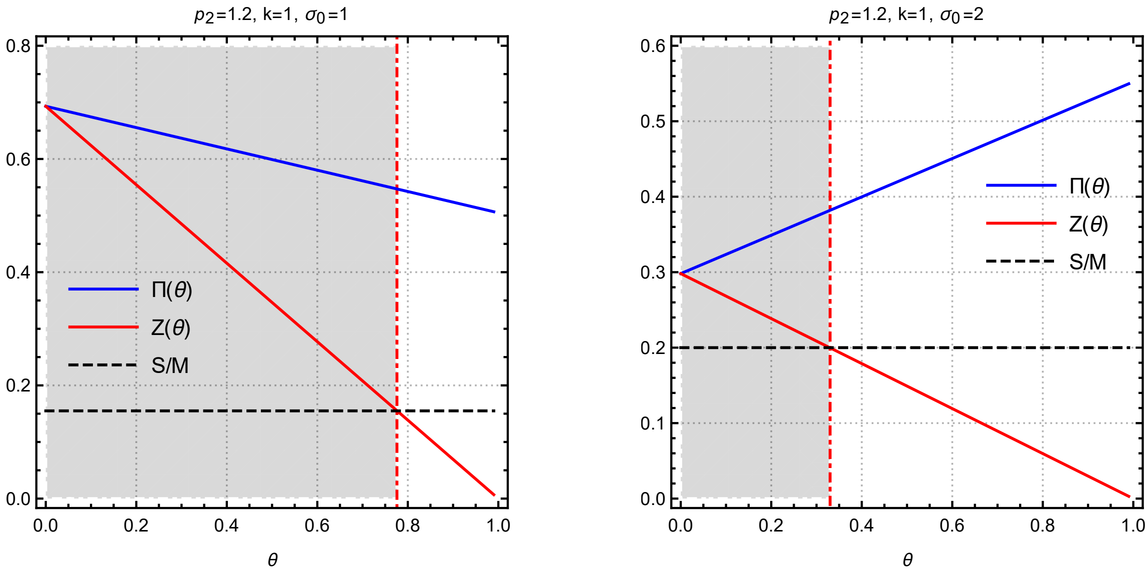


Figure B.6.: Firm's Profit Function and Feasible Set under BS

constant and $\Theta^b_{\mathcal{F}} \neq \emptyset$ only when $\mathcal{Z}'_b(\theta_1) < 0$. Then, it is straightforward to see as S increases, θ^b_u decreases. Also, θ^{bc*}_1 remains unchanged if $\Pi'_b(\theta_1) \leq 0$ and θ^{bc*}_1 decreases as θ^b_u decreases. Taken together, we can conclude that statement (ii) is true under BS. ■

C. Appendix for Chapter 3

Appendix A: Summary of Notations

To keep track of the different components in our model, we summarize the notations throughout the paper in Table C.1.

Table C.1: Summary of Model Notations

Parameters	
α	crowdfunding market share
z	initial investment/fixed setup cost
$\mathcal{I}$	market size (random variable)
q	product quality
k	word-of-mouth (WoM) intensity
ρ_0	awareness base of the crowdfunding product
β	market uncertainty
C_s	startup's credit rating
$\eta(C_s)$	interest rate premium under bank financing for the credit rating C_s
$\delta(C_s)$	financing cost coefficient, defined as $\delta(C_s) = 1 + \eta(C_s)$
Decisions	
p_1, θ_1	crowdfunding price/consumers' purchase threshold in period 1, $\theta_1 = p_1/q$
p_2, θ_1	retail price/consumers' purchase threshold in period 2, $\theta_2 = p_2/q$
T	funding target of the crowdfunding campaign
Derived	
$\mathcal{F}$	investment feasibility of the crowdfunding project, $\mathcal{F} = \frac{\alpha q}{4z}$
$\mathcal{A}(\theta_1)$	awareness function $\mathcal{A}(\theta_1) = [\rho_0 + k\alpha(1-\theta_1)] \wedge 1$
θ_b	consumers' purchase threshold under bank financing, $\theta_b = p_b/q$
θ_l	smaller root of the quadratic equation $\alpha q(1+\beta)\theta_1(1-\theta_1) = z$
θ_h	smaller root of the quadratic equation $\alpha q(1-\beta)\theta_1(1-\theta_1) = z$
k_m	WoM intensity threshold, satisfying $k\alpha(4-k+k\alpha) = 8(k\alpha - 1 + \rho_0)$
k_n	WoM intensity threshold $k_n = 2(1-\rho_0)/\alpha$
β_r^b	market uncertainty threshold satisfying $\mathcal{H}^r(\beta_r^b) = \pi_b^*$
β_m	market uncertainty threshold $\beta_m = 1 - \frac{4z}{\alpha q}$
β_τ	unique solution to $\mathcal{H}^r(\beta) = \mathcal{H}^s(\beta)$ if $\mathcal{H}^s(\beta_m) < \mathcal{H}^r(\beta_m)$; otherwise, $\beta_\tau = \beta_m$
C_τ	credit rating threshold satisfying $\alpha(1-\beta)(2\delta(C_\tau)-1) = 2\rho_0 - 1 - \beta$

Appendix B: Proofs of Statements

We present all the proofs of the Lemmas, Propositions and Corollaries in this section.

Proof of Proposition 3.4.1:

Case (i): $k \leq (1-\rho_0)/\alpha$. In this case, the awareness function can be written as $\mathcal{A}(\theta_1) = [\rho_0 + k\alpha(1-\theta_1)] \wedge 1 = \rho_0 + k\alpha(1-\theta_1)$, its first-order derivative with respect to θ_1 is $\mathcal{A}'(\theta_1) = -k\alpha$. Hence, the first-order derivative of the profit function $\pi(\theta_1)$ becomes

$$\pi'(\theta_1) = \frac{1}{2}q(1+\beta)\alpha - q(1+\beta)\alpha\theta_1 - \frac{1}{8}q(1+\beta)(1-\alpha)\alpha k.$$

The optimal solution θ_1^* to the unconstrained profit-maximization problem $\max_{\theta_1\in[0,1]} \pi(\theta_1)$ is

$$\theta_1^* = \frac{(4-k+k\alpha)^+}{8}.$$

Case (ii): $k > (1-\rho_0)/\alpha$. In this case, the awareness function could be truncated with $\rho_0 + k\alpha(1-\theta_1) > 1$. Hence, its first-order derivative with respect to θ_1 can be summarized as

$$\frac{d\mathcal{A}(\theta_1)}{d\theta_1} = \begin{Bmatrix} 0, & \text{if} \;\; \theta_1 < 1 - \dfrac{1-\rho_0}{k\alpha} \\ -k\alpha, & \text{if} \;\; \theta_1 > 1 - \dfrac{1-\rho_0}{k\alpha} \end{Bmatrix}.$$

Hence,

$$\pi'(\theta_1) = \begin{Bmatrix} \pi_1'(\theta_1), & \text{if} \;\; \theta_1 \leq \theta_1^\tau = 1 - \dfrac{1-\rho_0}{k\alpha} \\ \pi_2'(\theta_1), & \text{if} \;\; \theta_1 \geq \theta_1^\tau = 1 - \dfrac{1-\rho_0}{k\alpha} \end{Bmatrix},$$

where

$$\pi_1'(\theta_1) = \frac{1}{2}q(1+\beta)\alpha - q(1+\beta)\alpha\theta_1,$$
$$\pi_2'(\theta_1) = \frac{1}{2}q(1+\beta)\alpha - q(1+\beta)\alpha\theta_1 - \frac{1}{8}q(1+\beta)(1-\alpha)\alpha k.$$

Notice that $\pi(\theta_1)$ is continuous but not differentiable at θ_1^τ. Hence, $\pi_1'(\theta_1)$ and $\pi_2'(\theta_1)$ represent the left and right derivative of $\pi_1(\theta_1)$, respectively. Then, it is straightforward to see that $\pi'(\theta_1)$ is strictly decreasing in θ_1, and thus the profit function $\pi(\theta_1)$ is unimodal on $[0,1]$. Then, the optimal solution can be found by investigating the following three cases.

(1) If $\pi_1'(\theta_1^\tau) \leq 0$, then $\pi_2'(\theta_1) < 0$ for $\theta_1 \in [\theta_1^\tau, 1]$, and thus the optimal solution is the first-order condition $\pi_1'(\theta_1^*) = 0$, which gives $\theta_1^* = 1/2$ if $k \geq k_n = 2(1-\rho_0)/\alpha$.

(2) If $\pi_2'(\theta_1^\tau) \geq 0$, then $\pi_1'(\theta_1) > 0$ for $\theta_1 \in [0, \theta_1^\tau]$, and thus the optimal solution is the first-order condition $\pi_2'(\theta_1^*) = 0$, which gives $\theta_1^* = (4-k+k\alpha)/8$ if $(1-\rho_0)/\alpha < k \leq k_m$

and $k_m > 0$ is uniquely derived from $k\alpha[4 - k(1-\alpha)] = 8(k\alpha - 1 + \rho_0)$. Here, it is straightforward to verify $k_m > (1-\rho_0)/\alpha$.

(3) If $\pi_1'(\theta_1^\tau) > 0 > \pi_2'(\theta_1^\tau)$, then the optimal solution is $\theta_1^* = \theta_1^\tau$. Taken together case (1)-(3), the optimal solution θ_1^* can be summarized as follows:

$$\theta_1^* = \left\{ \begin{array}{ll} \dfrac{4-k+k\alpha}{8}, & \text{if } \dfrac{1-\rho_0}{\alpha} < k < k_m \\ \dfrac{k\alpha - 1 + \rho_0}{k\alpha}, & \text{if } k_m \leq k < k_n \\ \dfrac{1}{2}, & \text{if } k \geq k_n \end{array} \right\},$$

where $k_n = 2(1-\rho_0)/\alpha$ and $k_m > 0$ is uniquely derived from $k\alpha[4 - k(1-\alpha)] = 8(k\alpha - 1 + \rho_0)$.

Finally, taken together case (i) and (ii), and adding the non-negativity constraint, we have the following optimal solution:

$$\theta_1^* = \left\{ \begin{array}{ll} \dfrac{(4-k+k\alpha)^+}{8}, & \text{if } 0 \leq k < k_m \\ \dfrac{(k\alpha - 1 + \rho_0)^+}{k\alpha}, & \text{if } k_m \leq k < k_n \\ \dfrac{1}{2}, & \text{if } k \geq k_n \end{array} \right\}. \tag{C.1}$$

We complete the proof. ■

Proof of Lemma 9:

First, due to financial constraints, it's not feasible to set the funding target T less than the fixed setup cost z. Second, any funding target higher than the fixed setup cost is suboptimal, as it only decreases the success probability of the crowdfunding project. ■

Proof of Lemma 10:

Let $\theta_l < \theta_l'$ be the two roots of the quadratic equation $\alpha q(1+\beta)\theta_1(1-\theta_1) = z$. Then, $\theta_l < 1/2 < \theta_l'$, and they generate the same expected profit in period 1, i.e., $\pi(\theta_l) = \pi(\theta_l')$. However, for the profit in period 2, $\pi_2^*(\theta_l) > \pi_2^*(\theta_l')$ due to a higher product awareness under strategy θ_l. Equivalently, we can show $\pi_2^*(\theta_1) = q(1+\beta)(1-\alpha)\mathcal{A}(\theta_1)/8$ is a decreasing function of θ_1. Hence, θ_l is Pareto dominating θ_l', which gives the Pareto set $[\theta_l, 1/2]$. Since θ_l is the smaller root of the quadratic equation $\alpha q(1+\beta)\theta_1(1-\theta_1) = z$, hence increases in z by the property of quadratic function. ■

Proof of Proposition 3.4.2:

Let $\beta_m = 1 - \mathcal{F}^{-1} = 1 - \frac{4z}{\alpha q}$. Two auxiliary functions as we defined are

$$\mathcal{H}^r(\beta) = \pi^r(\theta_1^u \vee \theta_l \wedge \theta_h),\ \beta \in [0,1]; \quad \mathcal{H}^s(\beta) = \pi^s(\theta_1^u \vee \theta_h),\ \beta \in [0, \beta_m].$$

Essentially, they represent the optimal profit under risky and safe pricing, respectively, as a function of β. Note that the startup's problem in the retail stage is not affected by the financial constraint, hence is always $\theta_2^* = 1/2$.

(i) Under low investment feasibility, we have $\mathcal{F} = \frac{\alpha q}{4z} < 1$ by definition. Hence, $\beta_m = 1 - \mathcal{F}^{-1} < 0$. If $\beta < -\beta_m$, then

$$1 + \beta < 1 - \beta_m = \frac{4z}{\alpha q} = \mathcal{I}_m(1/2),$$

hence $\mathcal{P}(1/2) = 0$ and the project is not feasible. If $\beta \geq -\beta_m$, then the project is risky within the feasible pricing strategy regime, i.e., $\mathcal{P}(\theta_1) = 1/2$ for $\theta_1 \in [\theta_l, 1/2]$. Since the optimal solution to the unconstrained problem is θ_1^u as given in Proposition 3.4.1, we know the optimal solution to the constrained problem is simply $\theta_1^* = \theta_1^u \vee \theta_l$ due to the unimodality of the profit function.

(ii) Under high investment feasibility, we have $\mathcal{F} = \frac{\alpha q}{4z} \geq 1$ by definition. Hence, $\beta_m = 1 - \mathcal{F}^{-1} \geq 0$. In this case, both risky pricing with $\theta_1 \in [\theta_l, \theta_h)$ and safe pricing with $\theta_1 \in [\theta_h, 1/2]$ are feasible. The startup's problem becomes:

$$\begin{aligned}
\max_{\theta_1 \in [\theta_l, 1/2]} \pi(\theta_1) &= \max\left\{ \max_{\theta_1 \in [\theta_l, \theta_h)} \pi^r(\theta_1), \max_{\theta_1 \in [\theta_h, 1/2]} \pi^s(\theta_1) \right\} \\
&\stackrel{\star}{=} \max\left\{ \max_{\theta_1 \in [\theta_l, \theta_h]} \pi^r(\theta_1), \max_{\theta_1 \in [\theta_h, 1/2]} \pi^s(\theta_1) \right\},
\end{aligned}$$

where the equality $\star$ comes from the following Claim 1. Essentially, the startup needs to compare the optimal profit in each region and choose the pricing strategy with the higher profit. When the two pricing schemes generate the same expected profit, we assume the safe pricing is preferred. A first observation is that both safe and risky pricing scheme could be optimal, but safe (risky) pricing is more likely to be adopted when the market uncertainty is relatively low (high). As illustrated in Figure C.1, the left one has a profit maximizer in the safe pricing regime, while the right one prefers the risky pricing scheme.

Then, for a given β, we have

$$\mathcal{H}^r(\beta) = \max_{\theta_1 \in [\theta_l, \theta_h]} \pi^r(\theta_1), \quad \mathcal{H}^s(\beta) = \max_{\theta_1 \in [\theta_h, 1/2]} \pi^s(\theta_1).$$

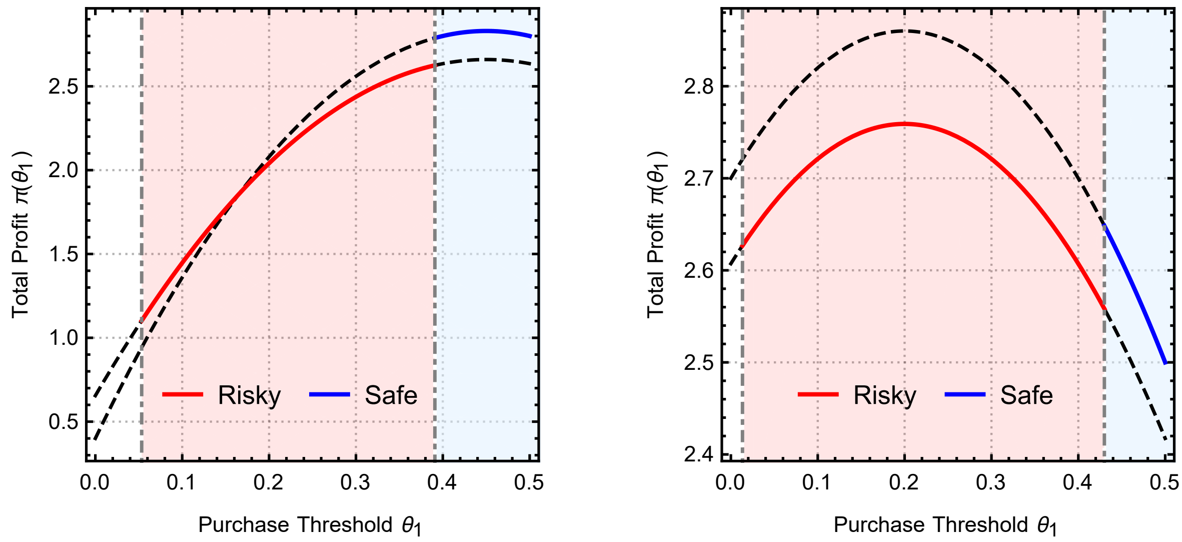


Figure C.1.: Illustration of Profit Functions for Safe and Risky Pricing Schemes

Claim 1: $\pi^r(\theta_h) < \pi^s(\theta_h)$.

Proof of Claim 1: By definition, θ_h satisfies $\alpha q(1-\beta)\theta_h(1-\theta_h) = z$. Then,

$$\pi^r(\theta_h) = \frac{q}{8}(1+\beta)[4\alpha\theta_h(1-\theta_h) + (1-\alpha)\mathcal{A}(\theta_h)] - \frac{z}{2}, \tag{C.2}$$

$$\pi^s(\theta_h) = \frac{q}{4}\left[4\alpha\theta_h(1-\theta_h) + (1-\alpha)\mathcal{A}(\theta_h)\right] - z, \tag{C.3}$$

$$\pi^s(\theta_h) - \pi^r(\theta_h) = \frac{q}{8}(1-\beta)(1-\alpha)\mathcal{A}(\theta_h) + \frac{1}{2}\left[q(1-\beta)\alpha\theta_h(1-\theta_h) - z\right]. \tag{C.4}$$

The second part in the last equality is zero by definition of θ_h, hence $\pi^s(\theta_h) > \pi^r(\theta_h)$.Q.E.D.

For the two auxiliary functions $\mathcal{H}^r(\beta)$ and $\mathcal{H}^s(\beta)$, we can show the following properties.

Claim 2: Given other model parameters, $\mathcal{H}^r(\beta)$ is strictly increasing in β on $[0,1]$; while $\mathcal{H}^s(\beta)$ is weakly decreasing in β on $[0,\beta_m]$.

Proof of Claim 2: Since θ_l is the smaller root of the quadratic equation $\alpha q(1+\beta)\theta_1(1-\theta_1) = z$, θ_l decreases in β. Similarly, θ_h increases in β. The unconstrained optimal solution θ_1^u is independent of β, and thus $\theta_1^u \vee \theta_l \wedge \theta_h$ becomes less constrained as β increases. Moreover, $\pi^r(\theta_1) = \frac{q}{8}(1+\beta)[4\alpha\theta_1(1-\theta_1) + (1-\alpha)\mathcal{A}(\theta_1)] - \frac{z}{2}$ is strictly increasing β for a given θ_1. Therefore, we can conclude that $\mathcal{H}^r(\beta) = \pi^r(\theta_1^u \vee \theta_l)$ is strictly increasing in β on $[0,1]$. As θ_h increases in β, $\theta_1^u \vee \theta_h$ becomes more constrained as β increases on $[0,\beta_m]$. (Note if $\beta > \beta_m$, then θ_h does not exist, and safe pricing is not feasible.) Moreover, $\pi^s(\theta_1) = \frac{q}{4}\left[4\alpha\theta_1(1-\theta_1) + (1-\alpha)\mathcal{A}(\theta_1)\right] - z$ is independent of β for a given θ_1. Therefore, $\mathcal{H}^s(\beta) = \pi^s(\theta_1^u \vee \theta_h)$ is weakly decreasing in β on $[0,\beta_m]$.Q.E.D.

When $\beta = 0$, $\theta_h = \theta_l$, and thus $\mathcal{H}^s(0) = 2\mathcal{H}^r(0) > \mathcal{H}^r(0)$, if $\mathcal{H}^r(0) > 0$. If $\mathcal{H}^s(\beta_m) < \mathcal{H}^r(\beta_m)$, then there exists a unique solution to $\mathcal{H}^r(\beta) = \mathcal{H}^s(\beta)$, denoted as β_τ. If $\mathcal{H}^s(\beta_m) \geq \mathcal{H}^r(\beta_m)$, then let $\beta_\tau = \beta_m$. Finally, we can conclude that under high investment feasibility, if $\beta \leq \beta_\tau$, it is optimal to adopt the safe pricing scheme with $\theta_1^* = \theta_1^u \vee \theta_h$, $\theta_2^* = 1/2$; otherwise, it is optimal to adopt the risky pricing scheme. ■

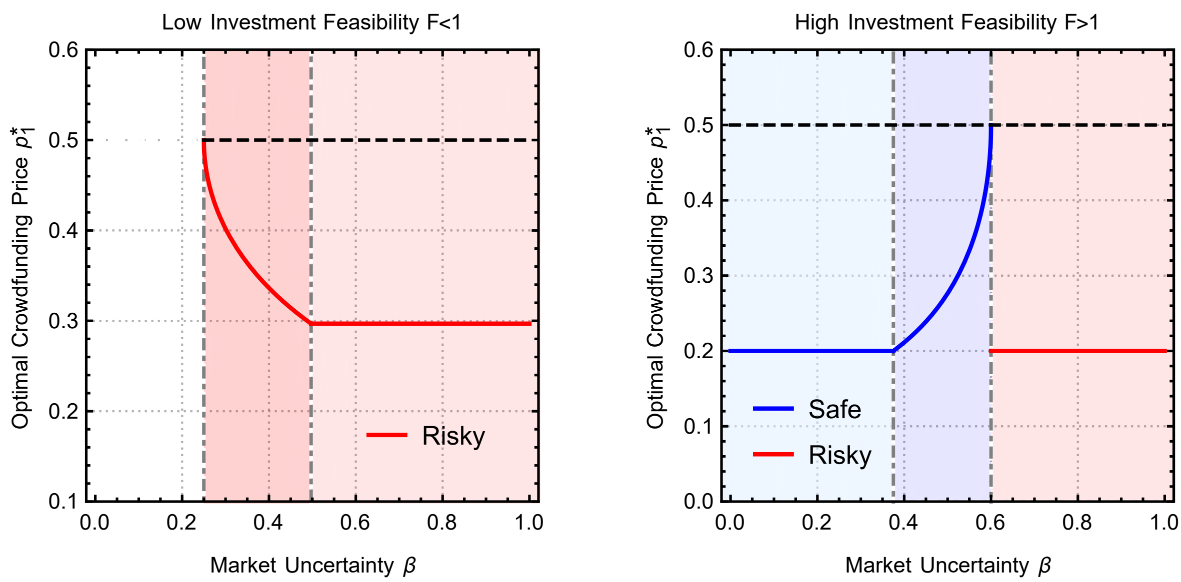
Proof of Proposition 3.4.3:

(i) Under low investment feasibility, crowdfunding is infeasible on $[0, -\beta_m)$, which is directly from Proposition 3.4.2. When $\beta \in [-\beta_m, 1]$, the optimal crowdfunding price is $p_1^* = q\theta_1^* = q(\theta_1^u \vee \theta_l)$, where θ_1^u is independent of β and θ_l decreases in β. Hence, the optimal crowdfunding price decreases on $[-\beta_m, 1]$. Based on Claim 2 in proof of Proposition 3.4.2, $\mathcal{H}^r(\beta)$ is strictly increasing in β, and thus the optimal profit increases on $[-\beta_m, 1]$.

Note that the special case is when the WoM intensity is strong $k \geq k_n$, and the optimal crowdfunding price is always $q/2$, with the awareness function $\mathcal{A}(1/2) = 1$. The optimal profit becomes

$$\pi^{r*} = \pi^r(1/2) = \frac{q}{8}(1+\beta)[\alpha + (1-\alpha)\mathcal{A}(1/2)] - z = \frac{q}{8}(1+\beta) - \frac{z}{2},$$

which is linearly increasing in β. Figure C.2 provides an illustration of the price pattern.



Note: The solid lines represent case with $k < k_n$ and the dashed lines represent case with $k \geq k_n$.

Figure C.2.: The Impact of Market Uncertainty on Optimal Crowdfunding Price

(ii) Under high investment feasibility, it is optimal to adopt the safe pricing scheme with $\theta_1^* = \theta_1^u \vee \theta_h$ when $\beta \le \beta_\tau$. Based on the discussion in proof of Proposition 3.4.2, we know the optimal crowdfunding price increases and the optimal profit decreases on $[0, \beta_\tau]$. Similarly, we can obtain the results for the case with $\beta \in (\beta_\tau, 1]$.

Similarly, for the special case with strong WoM intensity $k \ge k_n$, and the optimal crowdfunding price is always $q/2$, and the optimal profit under risky pricing is the same as in part (1), and the optimal profit under safe pricing is given as

$$\pi^{s*} = \pi^s(1/2) = \frac{q}{4}[\alpha + (1-\alpha)\mathcal{A}(1/2)] - z = \frac{q}{4} - z,$$

which is a constant. See the dashed line in the shadowed blue region in Figure 3.3. ■

Proof of Proposition 3.4.4:

(i) First, we want to show the startup's optimal profit always increases in he WoM intensity. From Proposition 3.4.2, we know the optimal crowdfunding price strategy θ_1^* under safe pricing scheme is

$$\theta_1^* = \left\{ \begin{array}{ll} \dfrac{4-k+k\alpha}{8} \vee \theta_h, & \text{if } \ 0 \le k < k_m \\ \dfrac{k\alpha - 1 + \rho_0}{k\alpha} \vee \theta_h, & \text{if } \ k_m \le k < k_n \\ \dfrac{1}{2}, & \text{if } \ k \ge k_n \end{array} \right\}.$$

Since $\frac{4-k+k\alpha}{8}$ decreases in k, $\frac{k\alpha-1+\rho_0}{k\alpha}$ increases in k, and θ_h is independent of k, we can conclude that the optimal crowdfunding price $p_1^* = \theta_1^* q$ decreases on $[0, k_m]$, then increases on $[k_m, k_n]$, and remains fixed on $[k_n, +\infty)$. The profit function under safe pricing scheme is $\pi^s(\theta_1) = \frac{q}{4}[4\alpha\theta_1(1-\theta_1) + (1-\alpha)\mathcal{A}(\theta_1)] - z$, where $\mathcal{A}(\theta_1) = [\rho_0 + k\alpha(1-\theta_1)] \wedge 1$. Note that $\partial\mathcal{A}(\theta_1)/\partial k \ge 0$ for $\forall \theta_1 \in [0, 1]$. Applying the Envelope Theorem, we have

$$\frac{d\pi^{s*}}{dk} = \frac{\partial \pi^s(\theta_1^*)}{\partial k} = \frac{q(1-\alpha)}{4}\frac{\partial \mathcal{A}(\theta_1^*)}{\partial k} \ge 0.$$

Consider the case where risky pricing is adopted, i.e., Proposition 3.4.1 and Proposition 3.4.2 (1). The profit function under risky pricing scheme is $\pi^r(\theta_1) = \frac{q}{8}(1+\beta)[4\alpha\theta_1(1-\theta_1) + (1-\alpha)\mathcal{A}(\theta_1)] - z/2$, and the optimal pricing scheme can be summarized as:

$$\theta_1^* = \left\{ \begin{array}{ll} \dfrac{4-k+k\alpha}{8} \vee \theta_l, & \text{if } \ 0 \le k < k_m \\ \dfrac{k\alpha - 1 + \rho_0}{k\alpha} \vee \theta_l, & \text{if } \ k_m \le k < k_n \\ \dfrac{1}{2}, & \text{if } \ k \ge k_n \end{array} \right\}. \tag{C.5}$$

Then, similar argument as in the safe pricing case gives the result under risky pricing. It is important to notice that as k changes, the optimal pricing strategy may switch between risky and safe pricing in some special case. But the increasing pattern of optimal profit is always guaranteed by the nature of the startup's problem, i.e., $\max\{\pi^{s*}, \pi^{r*}\}$.

(ii) Next, we prove the startup's optimal profit always increases in the crowdfunding market share. Consider the case where the crowdfunding market share is relatively small, $\alpha < \frac{4z}{q(1-\beta)}$ and hence the project investment feasibility is medium and the risky pricing scheme is adopted. The *effective* profit function in this case is $\pi^r(\theta_1) = 4\alpha\theta_1(1-\theta_1) + (1-\alpha)\mathcal{A}(\theta_1)$, where we ignore the coefficient term $\frac{q}{8}(1+\beta)$ and constant term $z/2$ for notational convenience. The optimal pricing scheme is given in equation (C.5). It is straightforward to see θ_1^* could have four different forms: $1/2$, θ_l, $\frac{4-k+k\alpha}{8}$ and $\frac{k\alpha-1+\rho_0}{k\alpha}$. Then, it suffices to show $\pi^r(\theta_1^*)$ increases in α in each case. A further observation is that we can put those four cases into two group based on whether $\mathcal{A}(\theta_1^*)$ is equal to 1. When $\theta_1^* = \theta_l$, $\frac{4-k+k\alpha}{8}$, then $\mathcal{A}(\theta_1^*) = \rho_0 + k\alpha(1-\theta_1^*) \leq 1$. When $\theta_1^* = 1/2$, $\frac{k\alpha-1+\rho_0}{k\alpha}$, then $\mathcal{A}(\theta_1^*) = 1$. (To be precise, for the special case when $\theta_l = 1/2$, we regard it as the case $\theta_1^* = \theta_l$, not the case $\theta_1^* = 1/2$.)

Case 1: $\theta_1^* = \theta_l$, where θ_l is the smaller root of the quadratic equation $\alpha q(1+\beta)\theta_l(1-\theta_l) = z$. Then, $\pi^{r*} = \pi^r(\theta_l) = 4\alpha\theta_l(1-\theta_l) + (1-\alpha)\mathcal{A}(\theta_l)$, where $\mathcal{A}(\theta_l) = \rho_0 + k\alpha(1-\theta_l)$. Taking derivative with respect to α gives,

$$\begin{aligned}
\frac{d\pi^{r*}}{d\alpha} &= 4\theta_l(1-\theta_l) + 4\alpha(1-2\theta_l)\frac{d\theta_l}{d\alpha} - \mathcal{A}(\theta_l) + (1-\alpha)\frac{d\mathcal{A}(\theta_l)}{d\alpha} \\
&= 4\theta_l(1-\theta_l) + 4\alpha(1-2\theta_l)\frac{d\theta_l}{d\alpha} - \mathcal{A}(\theta_l) + (1-\alpha)\left[k(1-\theta_l) - k\alpha\frac{d\theta_l}{d\alpha}\right] \\
&= \alpha[4 - 8\theta_l - k(1-\alpha)]\frac{d\theta_l}{d\alpha} + (1-\theta_l)[4\theta_l + k(1-\alpha)] - \mathcal{A}(\theta_l).
\end{aligned}$$

From $\alpha q(1+\beta)\theta_l(1-\theta_l) = z$, we have

$$\theta_l(1-\theta_l) + \alpha(1-2\theta_l)\frac{d\theta_l}{d\alpha} = 0, \quad \frac{d\theta_l}{d\alpha} = -\frac{\theta_l(1-\theta_l)}{\alpha(1-2\theta_l)}.$$

Thus,

$$\begin{aligned}
\frac{d\pi^{r*}}{d\alpha} &= -[4 - 8\theta_l - k(1-\alpha)]\frac{\theta_l(1-\theta_l)}{1-2\theta_l} + (1-\theta_l)[4\theta_l + k(1-\alpha)] - \mathcal{A}(\theta_l) \\
&= (1-\theta_l)\left[-4\theta_l + \frac{\theta_l k(1-\alpha)}{1-2\theta_l} + 4\theta_l + k(1-\alpha)\right] - \mathcal{A}(\theta_l) \\
&= k(1-\alpha)\frac{(1-\theta_l)^2}{1-2\theta_l} - \mathcal{A}(\theta_l).
\end{aligned}$$

When $\theta_1^* = \theta_l$, from Proposition 3.4.1 we know $\theta_l \geq \frac{4-k+k\alpha}{8}$, i.e., $k(1-\alpha) \geq 4 - 8\theta_l$. Together with the fact that $\mathcal{A}(\theta_l) \leq 1$ and $\theta_l \leq 1/2$, we have

$$\frac{d\pi^{r*}}{d\alpha} \geq (4-8\theta_l)\frac{(1-\theta_l)^2}{1-2\theta_l} - \mathcal{A}(\theta_l) = 4(1-\theta_l)^2 - \mathcal{A}(\theta_l) \geq 1 - \mathcal{A}(\theta_l) \geq 0.$$

Case 2: $\theta_1^* = \frac{4-k+k\alpha}{8}$. As $\mathcal{A}(\theta_1^*) = \rho_0 + k\alpha(1-\theta_1^*)$, similar calculation as in case 1 gives

$$\frac{d\pi^{r*}}{d\alpha} = \alpha[4 - 8\theta_1^* - k(1-\alpha)]\frac{d\theta_1^*}{d\alpha} + (1-\theta_1^*)[4\theta_1^* + k(1-\alpha)] - \mathcal{A}(\theta_1^*).$$

Since $\theta_1^* = \frac{4-k+k\alpha}{8}$, the term in the above equation is zero. Since $\mathcal{A}(\theta_1^*) \leq 1$, it is sufficient to show $(1-\theta_1^*)[4\theta_1^* + k(1-\alpha)] \geq 1$. Equivalently,

$$\frac{4+k(1-\alpha)}{8}\left[\frac{4-k+k\alpha}{2} + k(1-\alpha)\right] \geq 1, \quad [4+k(1-\alpha)]^2 \geq 16,$$

which is satisfied since $k(1-\alpha) \geq 0$. Therefore, we have shown $\frac{d\pi^{r*}}{d\alpha} \geq 0$.

Case 3: $\theta_1^* = 1/2$. Since $\mathcal{A}(\theta_1^*) = 1$ in this case, we have $\pi^{r*} = \alpha + 1 - \alpha = 1$, and thus $\frac{d\pi^{r*}}{d\alpha} = 0$.

Case 4: $\theta_1^* = \frac{k\alpha-1+\rho_0}{k\alpha}$. Since $\mathcal{A}(\theta_1^*) = 1$ in this case, we have

$$\pi^{r*} = \pi^r\left(\frac{k\alpha - 1 + \rho_0}{k\alpha}\right) = 4\left(1 - \frac{1-\rho_0}{k\alpha}\right)\frac{1-\rho_0}{k} + (1-\alpha),$$
$$\frac{d\pi^{r*}}{d\alpha} = 4\left(\frac{1-\rho_0}{k\alpha}\right)^2 - 1.$$

Since $\theta_1^* = 1 - \frac{1-\rho_0}{k\alpha} \leq 1/2$, we know $\frac{1-\rho_0}{k\alpha} \geq 1/2$, which gives $\frac{d\pi^{r*}}{d\alpha} \geq 0$ in the equation above.

Combining Case 1-4, we can conclude that $\frac{d\pi^{r*}}{d\alpha} \geq 0$. Next, consider the case where the crowdfunding market share is relatively large, $\alpha \geq \frac{4z}{q(1-\beta)}$ and hence the project investment feasibility is high and the safe pricing scheme is adopted. Applying similar arguments, we can show $\frac{d\pi^{s*}}{d\alpha} \geq 0$.

Finally, the global increasing pattern is guaranteed by the nature of the startup's problem as $\max\{\pi^{s*}, \pi^{r*}\}$. ■

Proof of Lemma 11:

The bank prices the interest rate for the startup's loan according to the competitive credit pricing equation (3.12), $z\delta(C_s) = \mathbb{E}_{\mathcal{I}}[\min\{q\rho_0\theta_b(1-\theta_b)\mathcal{I}, z(1+r_s)\}]$.

(1) High Financing Risk: $z\delta(C_s) > q(1+\beta)\rho_0\theta_b(1-\theta_b)$. In this case, it is straightforward to see $\mathbb{E}_{\mathcal{I}}[\min\{q\rho_0\theta_b(1-\theta_b)\mathcal{I}, z(1+r_s)\}] < q(1+\beta)\rho_0\theta_b(1-\theta_b)$, and thus the

competitive credit pricing equation (3.12) can not be satisfied for any choice of r_s. Hence, the bank can not break even by issuing loan to the startup, and thus directly rejects the loan request.

(2) Medium Financing Risk: $q(1-\beta)\rho_0\theta_b(1-\theta_b) < z\delta(C_s) \leq q(1+\beta)\rho_0\theta_b(1-\theta_b)$. In this case, the bank can break even by charging an interest rate of r_s. With probability 1/2, the market interest takes out to be high, the startup can fully repay bank's loan $z(1+r_s)$. With probability 1/2, the startup claims bankruptcy due to the low market realization and the bank can only get back $q\rho_0\theta_b(1-\theta_b)(1-\beta)$. From the competitive credit pricing equation (3.12), we have

$$\frac{1}{2}z(1+r_s) + \frac{1}{2}q\rho_0\theta_b(1-\theta_b)(1-\beta) = z\delta(C_s),$$

$$r_s = 2\delta(C_s) - \frac{q}{z}\rho_0\theta_b(1-\theta_b)(1-\beta) - 1.$$

(3) Low Financing Risk: $z\delta(C_s) \leq q(1-\beta)\rho_0\theta_b(1-\theta_b)$. In this case, there is no bankruptcy risk for the startup, as the bank's loan can be fully repaid even when the market interest turns out to be low. Then, the competitive credit pricing equation (3.12) can be met by charging the interest rate r_s such that $\min\{q\rho_0\theta_b(1-\theta_b)\mathcal{I}, z(1+r_s)\} = z(1+r_s)$, and thus $z\delta(C_s) = z(1+r_s)$, which gives $r_s = \delta(C_s) - 1$. ■

Lemma 22 *The expected profit functions under both low and medium financing risk share the same form as follows:* $\pi(\theta_b) = q\rho_0\theta_b(1-\theta_b) - z\delta(C_s)$.

Proof of Lemma 22:

Under medium financing risk, the startup goes bankrupt (zero profit) with probability 1/2, and makes positive profit with probability 1/2. Hence,

$$\begin{aligned}
\pi(\theta_b) &= \frac{1}{2}\big[q(1+\beta)\rho_0\theta_b(1-\theta_b) - z(1+r_s)\big] \\
&= \frac{1}{2}q(1+\beta)\rho_0\theta_b(1-\theta_b) + \frac{1}{2}q(1-\beta)\rho_0\theta_b(1-\theta_b) - z\delta(C_s) \\
&= q\rho_0\theta_b(1-\theta_b) - z\delta(C_s).
\end{aligned}$$

Similarly, under low financing risk we know there is no bankruptcy risk for the startup, and thus the expected profit is the profit over the mean market interest which is one, and the bank's interest rate is $r_s = \delta(C_s) - 1$. Hence, the expected profit becomes

$$\pi(\theta_b) = q\rho_0\theta_b(1-\theta_b) - z(1+r_s) = q\rho_0\theta_b(1-\theta_b) - z\delta(C_s).$$

Therefore, the expected profit functions have the same form for low and medium financing risk. ∎

Proof of Proposition 3.5.1:

The startup's problem under bank financing can be summarized as follows:

$$\max_{\theta_b \in [0,1]} \pi(\theta_b) = \max_{\theta_b \in [0,1]} q\rho_0\theta_b(1-\theta_b) - z\delta(C_s).$$

Hence, the optimal pricing strategy is $\theta_b^* = 1/2$, under which the optimal profit is $\pi_b^* = \frac{1}{4}q\rho_0 - z\delta(C_s)$.

We further characterize the condition under which bank financing is feasible. From Lemma 11, we know the bank directly rejects the loan request when $z\delta(C_s) > q(1+\beta)\rho_0\theta_b(1-\theta_b) = q(1+\beta)\rho_0/4$. By definition $C_l(\beta)$ satisfies $4z\delta(C_l) = q\rho_0(1+\beta)$, the condition is equivalent to $C_s < C_l(\beta)$ due to the fact that $\delta(\cdot)$ is a decreasing function. Similarly, the bank's interest rate decision is a direct result from Lemma 11 with the optimal pricing strategy $\theta_b^* = 1/2$. ∎

Proof of Proposition 3.6.1:

From Proposition 3.5.1, the optimal profit under bank financing is $\pi_b^* = q\rho_0/4 - z\delta(C_s)$, which is independent of the market uncertainty β.

(i) Under low investment feasibility, crowdfunding is not feasible when $\beta \in [0, -\beta_m)$ and in this case bank yields strictly higher profit. If $0 < \pi_b^* < \mathcal{H}^r(-\beta_m)$, crowdfunding yields strictly higher profit than bank financing when $\beta \in [-\beta_m, 1]$, since $\mathcal{H}^r(\beta)$ is strictly increasing in β, based on Claim 2 in the proof of Proposition 3.4.2. If $\mathcal{H}^r(-\beta_m) \leq \pi_b^*$, we first need to show there always exists a β_r^b, such that $\mathcal{H}^r(\beta_r^b) = \pi_b^*$. Then, it suffices to show $\mathcal{H}^r(1) \geq \pi_b^*$. Note that $\mathcal{H}^r(\beta) = \max_{\theta_1 \in [\theta_l, \theta_h]} \pi^r(\theta_1)$, and thus

$$\mathcal{H}^r(1) \geq \pi^r(\theta_l) = \frac{q}{4}[4\alpha\theta_l(1-\theta_l) + (1-\alpha)\mathcal{A}(\theta_l)] - \frac{z}{2}.$$

Since θ_l satisfies $2\alpha q\theta_l(1-\theta_l) = z$ when $\beta = 1$, $z > \alpha q/4$ under low investment feasibility, and $\rho_0 \leq \mathcal{A}(\theta_l) \leq 1$, we further have

$$\mathcal{H}^r(1) \geq \pi^r(\theta_l) = \frac{q}{4}(1-\alpha)\mathcal{A}(\theta_l) \geq \frac{q}{4}(\rho_0 - \alpha) > \frac{q\rho_0}{4} - z > q\rho_0/4 - z\delta(C_s) = \pi_b^*.$$

Therefore, there always exists a β_r^b, such that $\mathcal{H}^r(\beta_r^b) = \pi_b^*$, and crowdfunding yields strictly higher profit than bank financing when $\beta \in (\beta_r^b, 1]$.

(ii) Under high investment feasibility, we first want to show $\mathcal{H}^s(\beta_m) > \pi_b^*$ always hold. By definition of β_m, we know $\mathcal{H}^s(\beta_m) = \pi^s(1/2)$. Then, we have

$$\mathcal{H}^s(\beta_m) = \pi^s(1/2) = \frac{q}{4}\left[\alpha + (1-\alpha)\mathcal{A}(1/2)\right] - z > \frac{q\rho_0}{4} - z > q\rho_0/4 - z\delta(C_s) = \pi_b^*.$$

Therefore, bank financing could be preferred only when $\mathcal{H}^r(\beta_m) < \pi_b^* < \mathcal{H}^s(\beta_m)$. In such case, crowdfunding achieves lowest profit (the limit of its lowest profit) when $\beta \to \beta_m^+$ based on preceding result. Then, part (a) follows immediately. Part (b) is also straightforward by know the monotonicity of $\mathcal{H}^r(\beta)$ and $\mathcal{H}^s(\beta)$ obtained in Claim 2 in the proof of Proposition 3.4.2. Note that β_r^b is derived from $\mathcal{H}^r(\beta) = \pi_b^*$. ■

Proof of Corollary 2:

When $\rho_0 \to 1$ and $\delta(C_s) \to 1$, the optimal profit under bank financing becomes $\pi_b^* = q/4 - z$ according to Proposition 3.5.1. Then, we want to show there always exists a β_r^b, such that $\mathcal{H}^r(\beta_r^b) = \pi_b^*$, and crowdfunding yields strictly higher profit than bank financing when $\beta \in (\beta_r^b, 1]$. This result follows by similar argument as in the proof of Proposition 3.6.1 , and thus details are omitted here. ■

Proof of Proposition 3.6.2:

First, notice when the initial investment is low, i.e., $z \le \frac{(1-\beta)\alpha q}{4}$, the project is safe under crowdfunding. When initial investment is high, i.e., $\frac{(1-\beta)\alpha q}{4} < z \le \frac{(1+\beta)\alpha q}{4}$, the project is risky under crowdfunding. When $\rho_0 \to 1$, the optimal pricing strategy in crowdfunding is always $\theta_1^* = \theta_2^* = 1/2$, and the optimal profits for risky and safe projects are

$$\pi^r(1/2) = \frac{q}{8}(1+\beta) - \frac{z}{2}, \quad \pi^s(1/2) = \frac{q}{4} - z.$$

The optimal profit under bank financing is $\pi_b^* = q\rho_0/4 - z\delta(C_s) = q/4 - z\delta(C_s)$, as $\rho_0 \to 1$.

(i) When $z \le \frac{(1-\beta)\alpha q}{4}$, the project is safe under crowdfunding. Since $\delta(C_s) > 1$, we know $\pi_b^* < \pi^s(1/2)$, hence crowdfunding is always preferred.

(ii) When $\frac{(1-\beta)\alpha q}{4} < z \le \frac{(1+\beta)\alpha q}{4}$, the project is risky under crowdfunding. Let C_τ be the threshold of startup's credit rating C_s such that $\pi^r(1/2) = \pi_b^*$ at $z = \frac{(1-\beta)\alpha q}{4}$. That gives,

$$\frac{q}{8}(1+\beta) - \frac{(1-\beta)\alpha q}{8} = \frac{q}{4} - \delta(C_s)\frac{(1-\beta)\alpha q}{4},$$

which can be simplified as $\alpha(1-\beta)(2\delta(C_s) - 1) = 2\rho_0 - 1 - \beta$. Note that the credit rating threshold C_τ is unique because the function $\delta(C_s)$ is decreasing in C_s. Further, if

$C_s < C_\tau$, $\pi^r(1/2) > \pi_b^*$ at $z = \frac{(1-\beta)\alpha q}{4}$ and it is straightforward to see that $\pi^r(1/2) > \pi_b^*$ always holds for $\frac{(1-\beta)\alpha q}{4} < z \leq \frac{(1+\beta)\alpha q}{4}$. Thus, crowdfunding is always preferred in this case. If $C_s \geq C_\tau$, as both profits $\pi^r(1/2)$ and π_b^* are line function of z with different slop, we can conclude that there exists a threshold $z_b \in \left(\frac{(1-\beta)\alpha q}{4}, \frac{(1+\beta)\alpha q}{4}\right]$, such that bank financing is preferred when $z \in \left(\frac{(1-\beta)\alpha q}{4}, z_b\right)$. ■

Proof of Proposition 3.8.1:

Ignoring some constant terms, the consumer surplus for risky and safe projects can be written as $\mathcal{C}(\theta_1^*) = 4\alpha(1-\theta_1^*)^2 + (1-\alpha)\mathcal{A}(\theta_1^*)$.

(i) We want to show: As the WoM intensity increases, for risky projects, the consumer surplus and social welfare first increase on $[0, k_l \vee k_m]$, decrease on $[k_l \vee k_m, k_n]$, and remain stable on $[k_n, +\infty)$, where $k_l = \frac{1-\rho_0}{\alpha(1-\theta_l)}$. Then, similar argument gives the results for safe project.

First, consider the case $k_l \vee k_m = k_m$, i.e., $k_l \leq k_m$. For risky projects, the optimal pricing scheme is summarized in in equation (C.5). When $0 \leq k < k_m$, we have $\mathcal{A}(\theta_1^*) = \rho_0 + k\alpha(1-\theta_1) < 1$, and

$$\begin{aligned}\frac{d\mathcal{C}(\theta_1^*)}{dk} &= -8\alpha(1-\theta_1^*)\frac{d\theta_1^*}{dk} + (1-\alpha)\frac{d\mathcal{A}(\theta_1^*)}{dk} \\ &= \alpha(1-\alpha)(1-\theta_1^*) + \frac{1}{8}k(1-\theta_1^*)(1-\alpha)^2 > 0.\end{aligned}$$

When $k_m < k < k_n$, we have $\mathcal{A}(\theta_1^*) = 1$, and

$$\frac{d\mathcal{C}(\theta_1^*)}{dk} = -8\alpha(1-\theta_1^*)\frac{d\theta_1^*}{dk} = -\frac{8}{k^2}(1-\theta_1^*)(1-\rho_0) < 0.$$

When $k > k_n$, we have $\mathcal{A}(\theta_1^*) = 1$ and $\theta_1^* = 1/2$, and thus $\mathcal{C}(\theta_1^*)$ remains fixed.

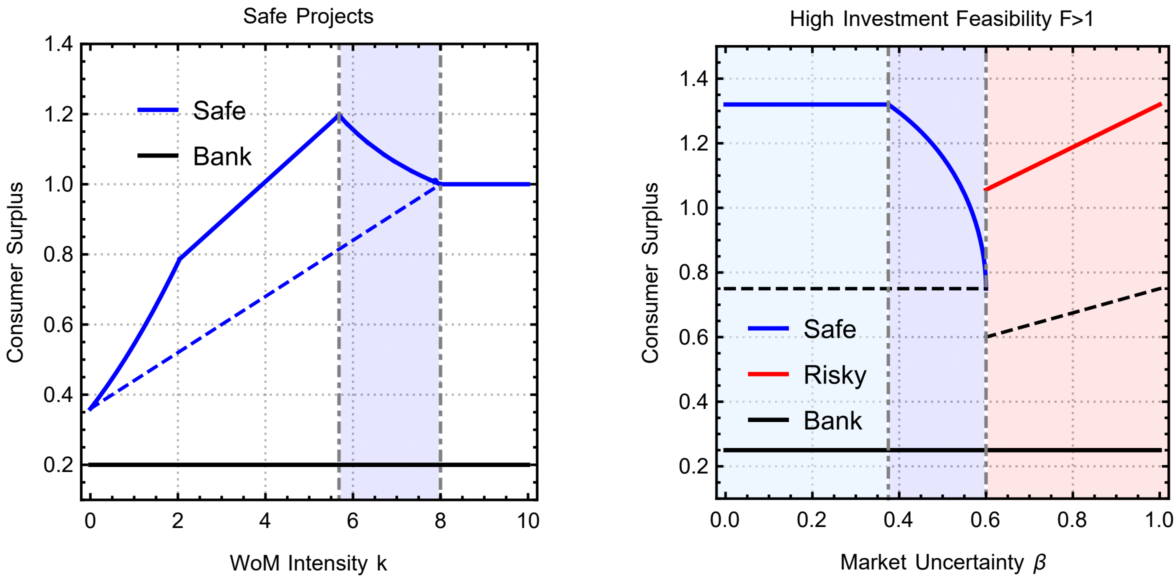
Second, consider the case $k_l \vee k_m = k_l$, i.e., $k_l \geq k_m$. Then, by preceding results, it suffices to show $\mathcal{C}(\theta_1^*)$ increases in k when $\theta_1^* = \theta_l$. Note that θ_l is independent of k and in this case $\mathcal{A}(\theta_1^*) = \rho_0 + k\alpha(1-\theta_1) < 1$. Hence, we have

$$\frac{d\mathcal{C}(\theta_l)}{dk} = (1-\alpha)\frac{d\mathcal{A}(\theta_l)}{dk} = \frac{1}{8}k(1-\theta_l)(1-\alpha)^2 > 0,$$

which completes the proof. Figure C.3 provides an illustration of the impact of WoM intensity and market uncertainty on consumer surplus.

(ii) Due to similar structures between $\mathcal{C}^r(\theta_1^*)$ $(\mathcal{C}^s(\theta_1^*))$ and $\pi^r(\theta_1)$ $(\pi^s(\theta_1))$, the results in the proposition is straightforward to obtain by analogy. Hence, details omitted here.■

Proof of Proposition 3.8.2:



Note: Similar patterns exist for the impact of WoM intensity and market uncertainty on social welfare.

Figure C.3.: The Impact of WoM Intensity and Market Uncertainty on Consumer Surplus

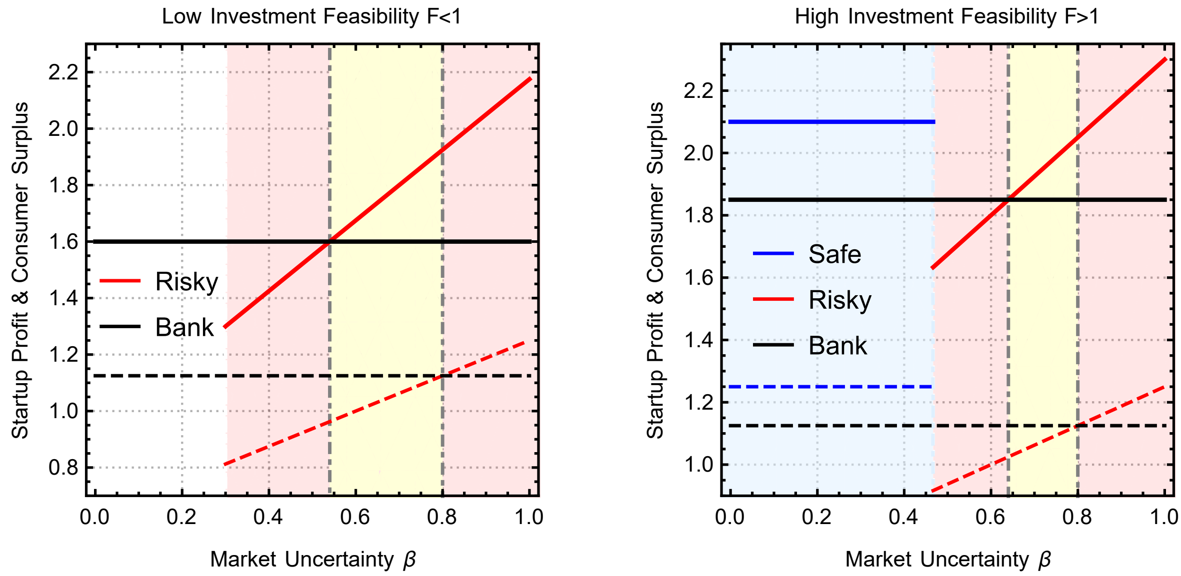
The social welfare $\mathcal{S}^r(\theta_1^*)$ for risky projects under optimal pricing strategy θ_1^* can be derived as

$$\begin{aligned}\mathcal{S}^r(\theta_1^*) &= \pi^r(\theta_1^*) + \mathcal{C}^r(\theta_1^*)\\ &= \frac{1}{16}q(1+\beta)\left[4\alpha(1-\theta_1^*)^2 + (1-\alpha)\mathcal{A}(\theta_1^*)\right]\\ &\quad + \frac{1}{8}q(1+\beta)[4\alpha\theta_1(1-\theta_1) + (1-\alpha)\mathcal{A}(\theta_1)] - \frac{z}{2}\\ &= \frac{3}{16}q(1+\beta)(1-\alpha)\mathcal{A}(\theta_1^*) + \frac{1}{4}q(1+\beta)\alpha(1-\theta_1^{*2}) - \frac{z}{2}.\end{aligned}$$

The three thresholds β_r^b, β_c^b and β_s^b are derived respectively from the following three equations

$$\begin{aligned}&\pi^r(\theta_1^*) = \frac{1}{8}q(1+\beta_r^b)[4\alpha\theta_1^*(1-\theta_1^*) + (1-\alpha)\mathcal{A}(\theta_1^*)] - \frac{z}{2} = \pi_b^* = \frac{1}{4}q\rho_0 - z\delta(C_s),\\ &\mathcal{C}^r(\theta_1^*) = \frac{1}{16}q(1+\beta_c^b)\left[4\alpha(1-\theta_1^*)^2 + (1-\alpha)\mathcal{A}(\theta_1^*)\right] = \mathcal{C}_b = \frac{1}{8}q\rho_0,\\ &\mathcal{S}^r(\theta_1^*) = \frac{3}{16}q(1+\beta_s^b)(1-\alpha)\mathcal{A}(\theta_1^*) + \frac{1}{4}q(1+\beta_s^b)\alpha(1-\theta_1^{*2}) - \frac{z}{2} = \mathcal{S}_b = \frac{3}{8}q\rho_0 - z\delta(C_s).\end{aligned}$$

Since $\mathcal{C}^r(\theta_1^*)$ is increasing in β, we know bank financing is preferred when $\beta \in (|\beta_m|, \beta_c^b)$ from the perspective of consumer surplus. Similarly, for the social welfare, bank financing is preferred when $\beta \in (|\beta_m|, \beta_s^b)$. Figure C.4 depicts an example of the above relationship, where the highlighted yellow region represents the range $[\beta_r^b, \beta_c^b]$, and β_s^b lies somewhere in between. ■



Note: The solid lines represent the startup's profit and the dashed lines represent the consumer surplus.

Figure C.4.: Preference between Crowdfunding and Bank Financing

Appendix C: Complementary Results

In this appendix, we provide complementary results to show that our main results continue to hold when several assumptions are relaxed, and to present additional results of our model.

1. Positive Marginal Cost

Suppose the startup needs to incur a fixed cost z and a marginal cost $0 < c < q$. Then, given the retail price p_2, consumers in the retail market with $\theta \geq \theta_2 = p_2/q$ will purchase, which gives the expected revenue in retail stage $\pi_2(\theta_2; \theta_1) = (q\theta_2 - c)(1-\alpha)\mathcal{A}(\theta_1)(1-\theta_2)$. The optimal pricing strategy becomes

$$\theta_2^* = \frac{q+c}{2q}.$$

Back to the crowdfunding stage, the firm's expected revenue is $\pi_1(\theta_1) = (q\theta_1 - c)\alpha(1-\theta_1)$. The expected total profit for safe ($\mathcal{P} = 1$) and risky ($\mathcal{P} = 1/2$) projects can be written as

$$\begin{aligned}\pi^s(\theta_1) &= \pi_1(\theta_1) + \pi_2(\theta_2^*; \theta_1) - z \\ &= (q\theta_1 - c)\alpha(1-\theta_1) + \frac{(q-c)^2}{4q}(1-\alpha)\mathcal{A}(\theta_1) - z;\end{aligned}$$

$$\pi^r(\theta_1) = \frac{1+\beta}{2}[\pi_1(\theta_1) + \pi_2(\theta_2^*;\theta_1)] - \frac{z}{2}$$
$$= \frac{1+\beta}{2}\left[(q\theta_1 - c)\alpha(1-\theta_1) + \frac{(q-c)^2}{4q}(1-\alpha)\mathcal{A}(\theta_1)\right] - \frac{z}{2}.$$

The unimodality of the above two profit functions can be easily verified. Because of positive marginal cost, the funding target T should cover both the fixed cost and associated variable production cost. For risky project with profit function $\pi^r(\theta_1)$, the feasible pricing strategy θ_1 should satisfy $\theta_1 \in [\theta_l, \frac{q+c}{2q}]$, where θ_l be the smaller root of the quadratic equation $(1+\beta)\alpha(\theta_1 q - c)(1-\theta_1) = z$. Similarly, for safe project with profit function $\pi^s(\theta_1)$, the feasible pricing strategy is $\theta_1 \in [\theta_h, \frac{q+c}{2q}]$, where θ_h be the smaller root of the quadratic equation $(1-\beta)\alpha(\theta_1 q - c)(1-\theta_1) = z$.

Similarly, we define the investment feasibility $\mathcal{F}$ with slightly different formula as follows,

$$\mathcal{F} = \frac{\alpha(q-c)^2}{4qz}.$$

Then, we can categorize projects into low ($1/2 \leq \mathcal{F} < 1$) and high ($\mathcal{F} \geq 1$) investment feasibility, and analyze the profit maximization problem under each case. All the analysis of crowdfunding can carry through here, and similar results can be obtained with different thresholds for k and β as in Propositions 3.4.1-3.4.3. Similar arguments also hold for the case of bank financing.

2. Quality Uncertainty

Suppose the value of q is unknown in the crowdfunding stage, but will be realized in the retail stage. The consumers' prior belief of the quality q follows the same distribution G, with mean denoted as μ_q. For any quality realization, the optimal retail price is $p_2^* = q/2$, inducing the same purchase threshold $\theta_2^* = 1/2$. Hence, the optimal *expected* revenue in the retail stage becomes

$$\mathbb{E}[\pi_2(\theta_2^*;\theta_1)] = \int x(1-\alpha)\frac{\mathcal{A}(\theta_1)}{4}dG(x) = \frac{\mu_q}{4}(1-\alpha)\mathcal{A}(\theta_1).$$

Back to the crowdfunding stage, consumer's purchase decision will be based on the expected value of quality q, i.e., μ_q, and thus the revenue is $\pi_1(\theta_1) = \mu_q\theta_1\alpha(1-\theta_1)$. Therefore, the *expected* total profit over the two periods becomes

$$
\begin{aligned}
\pi(\theta_1) &= \pi_1(\theta_1) + \mathbb{E}[\pi_2(\theta_2^*; \theta_1)] - z \\
&= \frac{\mu_q}{4}\left[4\alpha\theta_1(1-\theta_1) + (1-\alpha)\mathcal{A}(\theta_1)\right] - z,
\end{aligned}
$$

which is essentially the same as the one derived in the main paper, simply by replacing q with μ_q. As such, all the results remain unchanged.

However, it is important to further point out that even facing quality uncertainty, consumers in the crowdfunding stage have no incentive to delay their purchase decision to the retail stage, i.e., no strategic waiting behavior. Note that $\theta_1^* \leq 1/2$, and consumers appearing in the crowdfunding stage can be divided into two groups: $\theta \in [1/2, 1]$ and $\theta \in [\theta_1, 1/2]$. For the first group with $\theta \in [1/2, 1]$, they always purchase in the retail stage if they choose to wait since $\theta_2^* = 1/2$. Hence, their expected net utility from purchasing in two stages are $\theta\mu_q - p_1^* = (\theta - \theta_1^*)\mu_q$ and $\theta\mu_q - p_2^* = (\theta - \theta_2^*)\mu_q$, respectively. They have no incentive to wait just because $\theta_1^* \leq \theta_2^*$. For the second group with $\theta \in [\theta_1, 1/2]$, they won't purchase in the retail stage for any quality realization since $\theta_2^* = 1/2$, which gives zero net utility from waiting. However, if they choose to purchase in the crowdfunding market, their expected net utility is nonnegative. Therefore, the second group has no incentive to wait as well.

3. WoM Communication in Bank Financing

Our model of bank financing differs from crowdfunding in product awareness, and WoM communication only happens in crowdfunding. Hence, crowdfunding has a competitive advantage, as all consumers in crowdfunding market are informed about the product, and crowdfunding sales can further generate awareness expansion in the retail market. Essentially, more consumers are informed about the product in crowdfunding. But, our model can be easily extended to the case where WoM communication also happens in bank financing. One way to incorporate this feature is to change the initial awareness base under bank financing from $A(0)$ to $A(x)$, with additional parameter $x > 0$ reflecting the integrated WoM interaction in the single selling period under bank financing. Such a modification will not affect any of our main results and insights qualitatively.

4. Crowdfunding-Bank Hybrid Financing

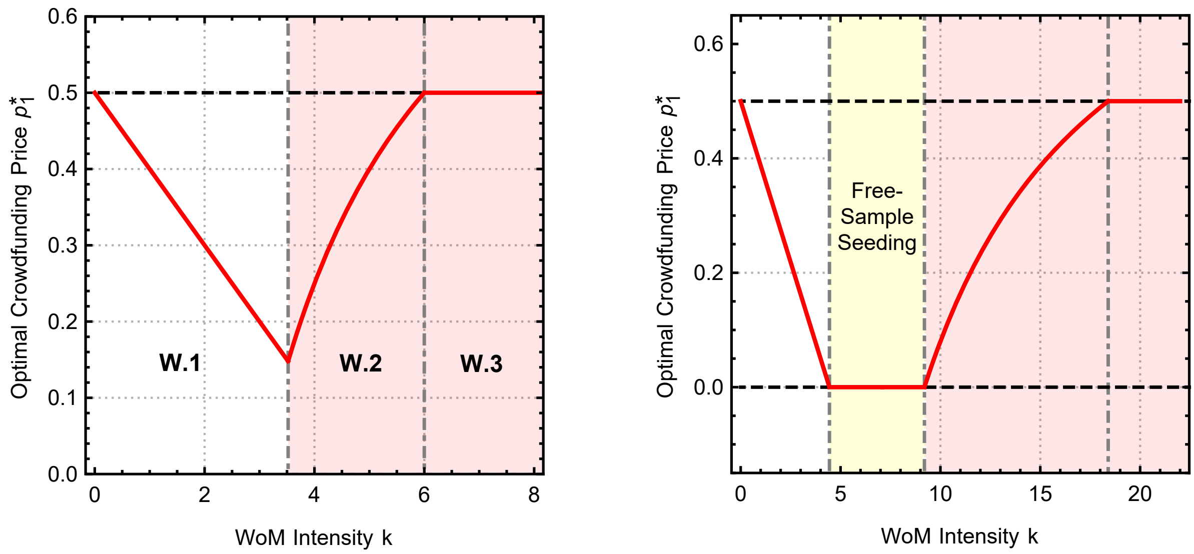
The firm may consider combining multiple financing options to better secure funds for its project. As a first step, our paper aims to provide thorough understandings on the pros and cons of those distinct choices by a comprehensive analysis and comparison of the available funding options. If one option, for sure, dominates the other in certain circumstance, then it could be reasonable to adopt that specific option, rather than combining them and incurring additional administrative and operational complexities. One case would likely happen in practice is that after a crowdfunding failure, the firm may use the demand information collected in crowdfunding to support further bank financing. Note in this case, there is no risk in bank financing as demand is perfectly learned via crowdfunding sales. Hence, the problem will become trivial and easy to solve.

5. Free-Sample Seeding Strategy (Special Case Discussion)

In certain situations, crowdfunding shall work as a device of "informative advertising" and the firm may be willing to offer the product for free to increase awareness. This pricing strategy is often referred to as *free-sample seeding strategy*, which could be optimal when the benefit from WoM communication completely dominates the revenue loss in the crowdfunding stage due to free sample offering. Such a pricing strategy is not uncommon on crowdfunding platforms (Kickstarter, Indiegogo) at the early stage of projects (although such a strategy is not applied for the entire campaign duration, the rationale is quite similar). The highlighted yellow region in Figure C.5(b) represents the case where the free-sample seeding strategy is adopted and the following corollary characterizes the adoption conditions for this strategy.

Corollary 3 *If* $z = 0$ *and* $4/(1-\alpha) \leq k \leq (1-\rho_0)/\alpha$, *then* $p_1^* = 0$.

The premise of such strategy is that by initially targeting on the crowdfunding market (small), giving them free samples of the product, the firm can trigger a cascade of influence by which friends will recommend the product to other friends, and many individuals will ultimately know it and try it in the retail market (large).This strategy is optimal because the benefit from WoM communication completely dominates the revenue loss in the crowdfunding stage due to free sample offering. The conditions for the adoption



(a) Strategic Pricing

(b) Free-Sample Seeding

Figure C.5.: Optimal Crowdfunding Price p_1^* without Financial Constraint

of this free-sample seeding strategy are: the firm does not rely on crowdfunding for capital raising (otherwise funding is the first priority); the initial awareness base of the crowdfunding product is low (otherwise the potential benefit of awareness expanding is limited); the crowdfunding consumer segment is small (otherwise too much revenue loss); and the WoM intensity is medium (otherwise the optimal crowdfunding rewards are low).