Essays on Marketing Strategy

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Essays on Marketing Strategy
by
Chang Liu

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Chang Liu

Washington University in St. Louis

August 2019
Dedicated to my beloved wife and my parents.
ABSTRACT OF THE DISSERTATION

Essays on Marketing Strategy

by

Chang Liu

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Associate Professor Baojun Jiang, Co-Chair

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In this dissertation, I apply game-theoretical methods in the context of marketing research and investigate the effects of stylized facts in behavioral economics. Chapter 1 studies the effects of managerial optimism on firms’ performance. Research has shown that many managers and entrepreneurs tend to be optimistic and are inclined to believe that negative shocks happen to them less frequently than to others. However, there is also evidence suggesting that such optimism is often inaccurate in reality and managerial optimism can lead to the failure of a company. We develop a game-theoretic model to investigate the impact of managerial optimism on firms’ performance in a competitive market. Our analysis shows that a manager’s optimism about demand can increase the firm’s profit. Moreover, only one firm having managerial optimism can be win-win for both firms in a duopoly, because it can increase the level of product quality differentiation between the firms, alleviating price competition. However, if both firms have optimistic managers, the benefit of increased differentiation disappears, and firms are weakly worse off, compared with the case of both firms having realistic managers. Our research suggests that a firm should hire a realistic manager when managerial optimism is already pervasive in a competitive market.
Chapter 2 studies the effects of different supply chains on firms’ profitability. In many supply chains, the downstream retailer designs product quality and decides retail price, but outsources the production to an upstream manufacturer. This practice is referred to as “contract manufacturing” (CM). Sometimes, in addition to production outsourcing, the retailer also outsources the design process to the manufacturer. This is referred to as “original design manufacturing” (ODM). This chapter compares these two different outsourcing practices and develops a game-theoretical model to investigate the effects of quality design outsourcing on quality level, price, and the profits of both the retailer and the manufacturer in a market with demand uncertainty. Our analysis reveals that in ODM, the product has lower quality level, lower wholesale price, and lower retail price than in CM. The retailer is better off with ODM when demand uncertainty is low, and better off with CM when demand uncertainty is high. Moreover, when demand uncertainty is high, the manufacturer’s profit may increase with demand uncertainty. In Chapter 3, using data from a Chinese textile manufacturer that supplies a major U.S. retailer, we estimate a logit model and demonstrate that, consistent with our prediction, the retailer is more likely to choose ODM and outsource quality design under low high demand uncertainty.
Chapter 1

Managerial Optimism in a Competitive Market

Co-authored with Baojun Jiang

1.1 Introduction

Entrepreneurs and executives are often afflicted with optimistic bias, believing that negative events are less likely to happen to themselves than to others. A survey conducted by Cooper et al. (1988) found that entrepreneurs generally perceived the prospects of their businesses as favorable, with 81% of the respondents seeing their odds of success as 7 out of 10 or better. More remarkably, 33% of them believe that the probability of success is 100%. By contrast, their perceived odds of success are much lower for other businesses. Liang and Dunn (2010) suggest that optimism is positively correlated with other entrepreneurial characteristics, including independence, creativity and risk tolerance. Many scholars have argued that important business decisions, such as starting a new company, entering an existing market, making investments in new projects, or acquiring another firm, are often made under the influence of optimism (Zajac and Bazerman, 1991; Camerer and Lovallo, 1999; Dunne et al, 1988; Malmendier and Tate, 2005; Hayward and Hambrick, 1997).

Sometimes even biased or “irrational optimism” is applauded among entrepreneurs and executives. In an interview with the New York Times, Shafqat Islam, the chief executive of the content-marketing platform called NewsCred, stressed the importance of irrational optimism: “I
feel like you need to be irrationally optimistic about barriers you can break through, or things you can get accomplished, or projects that you can deliver in a certain amount of time.”¹ However, he also admitted that this philosophy has its tradeoffs, as it can make him “being way too aggressive, too ambitious.”

While optimism is pervasive among entrepreneurs, it does not always deliver the best business results. According to Headd (2003), 50% of newly founded firms in the U.S. cease to exist within four years of startup. Frankish et al. (2010) report a similar four-year closure rate of 53% for the UK. Half of the time, entrepreneurs’ optimistic perceptions of their businesses do not come true. Moreover, a report from Startup Genome suggests that most startup companies fail due to premature scaling as they built up capacities too early, i.e., biased beliefs about market demands can directly lead to a company’s failure.²

Given that managerial optimism is not always beneficial to firms, one may wonder under what circumstances optimism is likely to benefit firms. Managerial optimism can be optimism about the manager’s own ability or about some external market conditions. This paper develops an analytical framework to study the latter type of managerial optimism. More specifically, we focus on analyzing how a manager’s optimism about market demand affects the firms’ product quality and pricing decisions. Moreover, we examine whether firms should hire realistic or optimistic managers in a competitive market.

In our model, two competing firms produce products that are differentiated both horizontally and vertically. Each firm has a manager who chooses the quality level and price of its product. A realistic manager has an objective assessment of the market demand—the number of consumers

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¹ [http://www.nytimes.com/2014/02/14/business/shafqat-islam-chief-of-newscred-on-irrational-optimism.html?_r=0](http://www.nytimes.com/2014/02/14/business/shafqat-islam-chief-of-newscred-on-irrational-optimism.html?_r=0)
that have high valuations for quality—whereas an optimistic manager has a biased belief about demand, believing the number of high-valuation consumers to be higher than it actually is. We also analyze several different types of managerial optimism in model extensions and show that our main insight remains robust. We address the following research questions. How does optimism affect a manager’s choice of quality and price? Can a firm make a higher profit with an optimistic manager than with a realistic manager? How does a manager’s optimism affect her competitor’s optimal pricing and quality decisions and its profit?

Our analysis reveals the following main findings. First, we show that when one of the two firms has an optimistic manager, managerial optimism can make the firm better off. This is because even though the firm’s quality and price will be suboptimal conditional on the competitor’s quality and price choices, the manager’s optimism leads to her choosing higher quality and higher price than otherwise, which serves to increase the quality differentiation between the firms’ products and alleviate price competition. In essence, the benefit of the manager’s optimism comes partly from its role as a credible commitment to a higher-than-otherwise quality level and a higher-than-otherwise price, which induces the competing firm to lower its quality and increase its price, alleviating competition in the market.

Second, we find that under some conditions, one firm’s manager’s optimism can also make the competitor better off, because of alleviated price competition. That is, one firm having an optimistic manager can lead to a win-win outcome for both firms in equilibrium, in which both firms make higher profits than when they both have realistic managers. We also show in model extensions that this win-win outcome can occur regardless of whether product quality is endogenous, whether firms have a unit cost or a fixed cost of production, or whether the manager’s optimism is about consumers’ valuations for quality, the distribution of horizontal preferences, or
the quality level of her product.

Third, we find that firms are weakly worse off when both of them hire optimistic managers, compared with the case in which both hire realistic managers. Bilateral optimism makes both firms choose higher quality and price in order to appeal to high-valuation consumers. As the symmetry of managerial optimism will no longer lead to an increase in quality differentiation between the firms’ products, both firms’ profits suffer from not targeting the low-valuation consumers in the market. Thus, although one firm’s managerial optimism can make both firms better off, two managers’ shared optimistic bias will not benefit any firm. This phenomenon may help to explain the extensive managerial failure caused by widespread entrepreneurial optimism.

Lastly, we also study the effects of managerial pessimism, where a pessimistic manager believes the number of high-valuation consumers to be lower than it actually is. We find that although the effects of managerial optimism is ambiguous, a manager’s pessimism is always detrimental to her firm. Note that while optimism can serve as a commitment to charging a higher price and thus mitigate competition, pessimism makes managers choose a lower price, which exacerbates rather than alleviates competition, hence reducing the firm’s profit. However, when the optimistic manager and the pessimistic manager coexist, they can aim to target different segments of the market, and both of them can benefit from their differentiated biases.

In Section 1.2, we review the related literature. Section 1.3 introduces the core model framework and discusses the benchmark case in which optimism does not exist. Section 1.4 presents the main results. Section 1.5 extends our core model and checks the robustness of our results in some alternative settings. Section 1.6 concludes the paper with some discussions for future research. All proofs are provided in the Online Appendix.

1.2 Literature Review
An extensive body of literature on entrepreneurial optimism explores both its causes and implications. Optimism often leads to people making biased decisions or assumptions. Fischhoff et al. (1977) demonstrate through experiments that people are often wrong when they are certain that they know the answer to a question. For questions covering a variety of topics, including history, music, geography, nature and literature, subjects were asked to choose the most likely answer and indicate the degree of certainty that their chosen answer was indeed correct. Answers assigned with a probability of 100% being correct were actually correct less than 30% of the time. However, subjects are sufficiently comfortable risking money on their judgements as they consistently overestimate their accuracy. Mahajan (1992) finds that such effects of overconfidence are even stronger when people are asked to make predictions pertaining to events inside their domain of expertise.

As shown empirically by Cooper et al. (1988), entrepreneurs’ levels of optimism are not correlated with their personal backgrounds or the nature of their firms. In other words, it is not those who are more likely to succeed that are more optimistic. Entrepreneurs who are poorly prepared are just as optimistic as those who are well-prepared. Sharot et al. (2011) argue that people are more likely to update their beliefs in response to better-than-expected information and that unrealistic optimism is caused by diminished neural coding of undesirable information regarding the future. In an experimental study, Proeger and Meub (2013) find that even individual subjects with realistic confidence levels demonstrate a much higher level of overconfidence when they can observe other subjects’ decisions, which implies that overconfidence may be a social rather than an individual bias.

Given that unrealistic optimism is pervasive among entrepreneurs, it is worth considering how it affects their business decisions and welfare. Camerer and Lovallo (1999) investigate the
effect of optimistic bias on firms’ entry decisions. Through an experimental approach, they argue that optimism is part of the explanation for excess entry and subsequent failure. Specifically, when subjects’ post-entry payoffs are based on the subjects’ skills measured by how many questions they answer correctly on a sample of 10 logic puzzles, rather than based on random processes, the subjects tend to overestimate their chance of success and enter the market more frequently. Koellinger et al. (2007) also show that entrepreneurial confidence is positively correlated with business creation and negatively correlated with approximate survival chances of their businesses.

Managerial optimism plays an important role in innovation. Hirshleifer et al. (2012) show that CEOs who postpone the exercise of vested options in their firms, which is an indication of their optimism about the long-term performance of their firms, tend to invest more in innovation. They obtain more patents and patent citations, and achieve greater innovation successes as a result of research and development expenditures, indicating that optimism may help CEOs exploit innovative growth opportunities.

By contrast, Lowe and Ziedonis (2006) find that startup companies continue unsuccessful development efforts of innovative projects for longer periods of time than do established firms. Given that entrepreneurs of startups are more overoptimistic than managers of established firms, they argue that optimistic entrepreneurs may be in denial about the diminishing prospect of their innovations. Similarly, Simon and Houghton (2003) find managers’ overconfidence to be positively correlated with the riskiness of new product introduction, and negatively correlated with the likelihood of success of the new products. Simon and Shrader (2007) also find that optimistic overconfidence is positively related to introducing pioneering products and entering competitive markets. However, Herz et al. (2014) points out that while over-optimism—the tendency of individuals to overestimate their abilities or chances of success—is positively correlated with
innovative activities, judgmental overconfidence—the tendency of individuals to overestimate the precision of their information—is negatively associated with innovation. This result indicates that the two types of overconfidence bias should be treated separately.

In financial markets, overconfidence of fund managers and traders lead to higher trade volumes and potential asset bubbles, as investors who think that they are above average in terms of making sound investments tend to trade more (Glacer and Weber, 2007; Statman et al, 2006; Scheinkman and Xiong, 2003; Odean, 1998). Moreover, overconfidence can partially explain volatility in financial markets, as Abbes (2013) argues that overconfidence bias contributed to the 2008 financial crisis since volatility has been shown to be positively related to trading volume caused by overconfidence bias.

Research has identified cases in which naïve optimism can be a strategic advantage. Johnson and Fowler (2011) argue that “believing you are better than you are in reality” can increase one’s payoff by making one more willing to engage in a competition, which is opposite to our analysis that managerial optimism can benefit a manager by mitigating competition. They show that overconfident players act aggressively in competition and are more likely to enter a contest over resources. Although sometimes they will lose contests due to their over-optimistic perception of their own ability, optimism is still beneficial as it not only allows them to exploit the cautiousness of their opponent and win the unclaimed resources, but also keeps them from walking away from conflicts that they could win. In their model, optimism benefits one player at the cost of her opponent, and never leads to a win-win outcome, which is in contrast to our prediction that both firms can benefit from one firm’s optimism. Similarly, Kyle and Wang (1997) show that in a duopoly model of informed speculation, overconfidence may strictly dominate rationality since an overconfident trader may not only generate higher expected profit and utility than his rational
opponent, but also higher profit than if he were also rational. In their model, overconfidence serves as a commitment device in a standard Cournot duopoly. As a result, the game may end up in a prisoner’s dilemma, as both funds select overconfident managers in equilibrium, lowering both funds’ profits.

Given that managerial optimism does not always improve a firm’s profit, it may be puzzling why firms often hire optimistic executives. Goel and Thakor (2006) provide an explanation for why managers who overestimate their abilities are often promoted to become the CEO. In a two-period model, they show that overconfident managers have higher probability of being promoted to second-period CEO than rational managers. While firms appoint the manager with highest perceived ability as the second-period CEO, overconfident managers—who are optimistic about the future—underestimate project risks and may achieve better results. Moreover, moderately overconfident managers generate higher profits than rational managers. de la Rosa (2011) proposes an alternative explanation: optimistic agents who overestimate the chance of success put in more effort and require lower success-contingent payments. Thus, taking moral hazard into consideration, firms may find it optimal to hire optimistic managers, since they work harder and require less monetary compensation. Hilary et al. (2016) confirms this theory by showing that empirically, over-optimistic managers exert great effort, which can improve firm’s welfare.

Given that naïve optimism can be viewed as a lack of strategic capability, it is noteworthy that Zhou et al. (2015) show that less strategic players can earn more profits than strategic firms in oligopolistic competition. Specifically, strategic players may raise prices to capitalize on their loyal customers’ willingness to pay, and thus drive price-sensitive consumers to nonstrategic firms. With enough loyal customers, a strategic firm’s expected profit can be lower than a nonstrategic firm’s profit. Consequently, both firms can benefit from one firm’s naïveté. Similarly, Li et al.
(2017) find that the overconfident newsvendor, which believes the uncertain demand distribution to be less variable than it really is, may actually make a higher profit than its less biased competitor.

This paper contributes to the literature on managerial optimism in multiple ways. First, the extant research on optimism mainly examines an agent’s optimism over her own ability, whereas we study a manager’s optimism about demand in the market, which has an uncertain proportion of consumers who have high valuations for the product. We show that the latter type of optimism of a manager can benefit both her own firm and its competitor, leading to a win-win outcome—a result that is different from the typical win-lose outcome (i.e., optimism can benefit a player at the cost of her opponents) shown in the existing literature.

Second, we analyze the effects of optimism on firms’ optimal quality decisions as well as prices, whereas previous studies (e.g., Johnson and Fowler 2011, Zhou et al. 2015) mostly focus on firms’ entry decisions and price choices. We show that while optimism makes a manager increase product quality, it tends to induce her competitor to decrease product quality, increasing product differentiation and alleviating price competition in the market.

Third, we also analyze equilibria of the game in which firms decide which type of managers to hire, and show that managerial optimism can be endogenously sustained in equilibrium. We provide suggestions to firms on when to choose optimistic managers.

1.3 Model

Let us consider a market with two competing firms, labeled $a$ and $b$, selling products that are differentiated both horizontally and vertically. Each firm $i \in \{a, b\}$ sells one product, and each

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3 In the literature, the terms “overconfidence” and “optimism” are sometimes used interchangeably. In this paper, similar to Herz et al. (2014), we define optimism as a decision maker’s overestimate of the market demand, and overconfidence as the decision maker’s overestimate the precision of her demand estimate. We focus on the former type of bias.
consumer buys at most one product, based on which product gives the consumer a higher positive utility.

There is a unit mass of consumers in the market. Consumers are heterogeneous in terms of their willingness to pay for quality; there are two types of consumers, denoted by $j \in \{H, L\}$. Let $\theta_j$ denote type-$j$ consumer’s willingness to pay for quality, where $\theta_L < \theta_H$. Without loss of generality, we normalize $\theta_H$ to 1, so we have $0 < \theta_L < 1$. The consumer’s valuation for firm $i$’s product is $q_i \theta_j$, where $q_i$ is the quality of firm $i$’s product. For example, the quality of a smartphone can be interpreted as a combination of its processor speed, storage capacity, screen resolution, and battery life. Product quality is a “vertical” attribute, in that all consumers have the same order of preferences if prices are the same. Let $\alpha \in (0,1)$ denote the fraction of consumers having high valuations for quality, and $1 - \alpha$ the fraction of consumers having low valuations.

Consumers can also have different taste or horizontal preferences. We assume that consumers’ horizontal preferences are uncorrelated with their valuations for quality and are uniformly distributed on the line segment between zero and one. In this horizontal preference dimension, firm $a$ is located at 0 and firm $b$ at 1. So, a consumer’s net utility from firm $i$’s product is $U(i,j) = q_i \theta_j - t d_i(x) - p_i$, where $x$ represents the ideal location of the consumer’s horizontal preference, $d_i(x)$ is the horizontal mismatch between the firm and the consumer’s ideal horizontal preference, $t > 0$ is the consumers’ sensitivity to horizontal mismatch, and $p_i$ is firm $i$’s price. Each consumer will buy the product that gives her the highest non-negative utility, or choose the outside option, whose utility is normalized to zero.

Firm $i$’s unit cost of producing one unit of the product of quality $q_i$ is $c_i = k q_i^2$, where $k > 0$; that is, for a higher-quality product, each unit will cost more to produce. Any fixed cost of production is assumed sunk and normalized to zero. Thus, firm $i$’s unit profit margin is $p_i - k q_i^2$. 
Each firm is managed by a manager, who makes the pricing and quality decisions for the firm. There are two types of managers: the “realistic” type and the “optimistic” type. The manager’s goal is to maximize her firm’s profit based on her belief about the competition and the market. In other words, we implicitly assume that the manager is compensated based on a fraction of the firm’s profit, so there is no moral hazard in our model. When making product quality and pricing decisions, the managers may not know the true $\alpha$. The realistic manager has an unbiased belief about $\alpha$, whereas the optimistic manager is biased and believes that the fraction of high-valuation consumers is $\hat{\alpha}$, where $\hat{\alpha} \in (\alpha, 1]$.\footnote{Note that we have used the term “belief” to describe the managers’ knowledge about $\alpha$. In a more general model, the belief may not be singleton, but in our stylized model, the manager’s belief is just one specific value (rather than a distribution), i.e., the manager knows or expects the fraction of high-valuation consumers to be a specific value.} We first investigate the boundary case in which $\hat{\alpha} = 1$, i.e. an optimistic manager believes that all consumers have high willingness-to-pay for quality. We relax this assumption in a later extension and show that our results are qualitatively the same when $\hat{\alpha} \in (\alpha, 1)$.

Each manager’s type is common knowledge. That is, an optimistic manager knows that her perceived fraction of high-valuation consumers is larger than that of a realistic manager, and vice versa. Note that even though an optimistic manager knows that she has a different belief about $\alpha$ than a realistic manager, she does not know or think that her belief is biased, and thus her profit-maximization decisions are based on her own belief and her knowledge that the realistic manager has a different belief about the market. This is a reasonable assumption since oftentimes decision makers with biased beliefs do not think they are biased even though they know that they have different beliefs than others.

The game has three stages. In the first stage, managers, with their respective beliefs about $\alpha$, simultaneously choose their quality levels. In the second stage, having observed each other’s
quality choices, the managers simultaneously choose their respective prices. In the third stage, consumers make purchase decisions, firms’ profits are realized, and the true $\alpha$ is thereby revealed.

**Benchmark: No Optimism**

First, we analyze the benchmark case in which both firms have realistic managers. We focus on the case i.e. the parameter region in which with no optimism the market is fully covered in equilibrium, i.e., in equilibrium all consumers in both quality-valuation segments will buy a product. We will study how optimism might change the market coverage in the next section.

We solve for the equilibrium outcome by backward induction. In the last stage of the game, the type-$j$ consumer who is indifferent between the firms’ products has a horizontal preference (or location) $x_j$ that is determined by $q_a \theta_j - tx_j - p_a = q_b \theta_j - t(1-x_j) - p_b$. So, $x_j = \frac{t + (q_a - q_b) \theta_j - (p_a - p_b)}{2t}$.

In the second (price-setting) stage, firm $i$’s manager maximizes its expected profit $\pi_i = (p_l - kq_i^2) \frac{t - (p_i - p_{-i}) + (q_l - q_{-l}) \bar{\theta}}{2t}$, where “$-i$” indicates the competitor of firm $i$, and $\bar{\theta} \equiv \alpha \theta_H + (1 - \alpha) \theta_L = \alpha + (1 - \alpha) \theta_L$, where $\theta_H = 1$. Note that $\bar{\theta}$ represents the consumers’ average valuation for quality and we can use $\bar{\theta}$ in this expression due to linearity of the demand function.

The equilibrium price is easily derived: $p_l = t + \frac{1}{3} (q_l - q_{-l}) \bar{\theta} + \frac{k}{3} (q_l^2 + q_{-l}^2)$. Hence, in the first stage, firm $i$’s profit function is $\pi_i = \frac{1}{2t} \left( t + \frac{1}{3} (q_l - q_{-l}) \bar{\theta} + \frac{k}{3} (q_{-l}^2 - q_l^2) \right)^2$. The manager of each firm chooses its quality to maximize its expected profit, and we can calculate the equilibrium outcome $q_i^*, p_i^*$, and $\pi_i^*$ accordingly.

Note that the market is fully covered in equilibrium only when $\sum_i \frac{q_i^* \theta_j - p_i^*}{t} \geq 1$. Since a high-valuation consumer always has a higher utility than a low-valuation consumer with the same
horizontal preference, the high-valuation segment will always be fully covered if the low-valuation segment is fully covered. Thus, only $\sum_i q_i^l \theta^l - p_i^l \geq 1$ is binding in equilibrium. One can readily determine the parameter conditions of full market coverage in the benchmark case, which is given in Lemma 1, together with the equilibrium outcome.

**Lemma 1.** When both firms have realistic managers, the market is fully covered in equilibrium if $\theta_L^2 - \alpha^2 (1 - \theta_L)^2 \geq 6kt$. The equilibrium outcome is $q^*_a = q^*_b = \frac{\theta}{2k}$, $p^*_a = p^*_b = t + \frac{\theta^2}{4k}$, and $\pi^*_a = \pi^*_b = \frac{t}{2}$.

Note that the equilibrium is unique and symmetric. This is true as long as $t > 0$. Lemma 1 shows that the market is fully covered in equilibrium when the low-type consumer’s willingness-to-pay for quality is high enough (i.e., $\theta_L$ is above some threshold), or alternatively when the consumer’s sensitivity ($t$) to horizontal mismatch is relatively low, or the fraction ($\alpha$) of high-valuation consumers is low. For the remainder of this paper, we will assume the condition specified in Lemma 1.

**1.4 Analysis with Optimism**

In this section, we first analyze the case in which only one firm has an optimistic manager, who perceives the market to have more high-valuation consumers than it does. We investigate the effects of managerial optimism on each firm’s optimal product quality, price, and profit. Again, we solve for the equilibrium outcome by backward induction. Without loss of generality, we assume that firm $a$’s manager is optimistic and firm $b$’s manager is realistic.

Even though the market is fully covered in the case without optimism, it may not always be fully covered in equilibrium when a firm’s manager is optimistic. We will analyze both the full-coverage and the partial-coverage scenarios.
Recall that in this part, we focus on the case in which the optimistic manager’s bias is the strongest and assume $\hat{\alpha} = 1$, i.e. she believes that all consumers have high valuation for quality. In the extension, we will show that when $\hat{\alpha} < 1$, our results are qualitatively the same. When both the high-valuation and the low-valuation segments are fully covered under optimism, firm $a$’s manager maximizes $\pi_a = (p_a - kq_a^2)\frac{t-(p_a-p_b)+(q_a-q_b)\theta_H}{2t}$ and firm $b$’s manager maximizes $\pi_b = (p_b - kq_b^2)\frac{t-(p_b-p_a)+(q_b-q_a)\theta_H}{2t}$, where $\theta_H = 1$. When the low-valuation segment is not fully covered, $^5$ firm $b$ ’s manager maximizes $\pi_b = (p_b - kq_b^2)(\alpha \frac{t-(p_a-p_b)+(q_b-q_a)\theta_H}{2t} + (1 - \alpha) \frac{q_b\theta_L-p_b}{t})$. Since firm $a$’s manager believes that all consumers have high valuations, she still maximizes $\pi_a = (p_a - kq_a^2)\frac{t-(p_a-p_b)+(q_a-q_b)\theta_H}{2t}$. Lemma 2 gives the conditions under which the market is fully covered in equilibrium under unilateral optimism.

**Lemma 2.** When firm $a$ has an optimistic manager and firm $b$ has a realistic manager, the market is fully covered in equilibrium if

$$-(11 - 20\alpha + 11\alpha^2) + 2(11 - 20\alpha + 11\alpha^2)\theta_L - (9 - 20\alpha + 11\alpha^2)\theta_L^2 \geq 12kt.$$

$^5$ Note that in this paper we have focused on the parameter region where the market is fully covered in equilibrium of the benchmark case. One can show that in that parameter region, under unilateral optimism, the high-valuation segment is always fully covered (please see the proof of Lemma 2 in the Online Appendix for details). So, here we only need to consider two cases: both segments are fully covered, or the high-valuation segment is fully covered but the low-valuation segment is not.
When one firm has an optimistic manager, the market will in equilibrium be fully covered if \( \theta_L \) is relatively large, or if consumers’ sensitivity \((t)\) to horizontal mismatch is low. However, the effect of the number of high-valuation consumers is non-monotonic: the left-hand side (LHS) of the inequality in Lemma 2 increases with \( \alpha \) when \( \alpha < \frac{10}{11} \) and decreases with \( \alpha \) when \( \alpha > \frac{10}{11} \). Hence, Lemma 2 says that the fraction of high-valuation consumers must be neither too large nor too small for both segments to be fully covered. When \( \alpha \) is very high, there are not many consumers in the low-valuation segment, so even the realistic manager will ignore them and set the high price to target only high-valuation consumers. When \( \alpha \) is very low, the equilibrium price in the benchmark case is low, so the increase of \( p^*_a \) with optimism is large, and the low-valuation consumers located close to 0 may not be served, as they find firm \( a \)’s product too expensive and

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\(^6\) This figure is drawn with \( k = 0.1 \) and \( t = 0.1 \).
firm b’s too far from their ideal horizontal preferences. Figure 1.1 illustrates the equilibrium market coverage. We will analyze both the full-coverage and the partial-coverage scenarios to determine the effects of managerial optimism.

1.4.1 Effect of Optimism on Managers’ Quality and Price Decisions

Solving the game by backward induction, we can obtain the equilibrium outcome, which is provided in the Online Appendix. Our analysis shows that when one firm’s manager is optimistic, both firms will choose a different quality level from their optimal quality in the benchmark case.

**Proposition 1.** When firm a has an optimistic manager and firm b has a realistic manager, in equilibrium, firm a’s quality increases and firm b’s quality decreases relative to the benchmark case.

As shown in Proposition 1, firm a will choose higher quality than it does in the benchmark case, since its manager believes that all consumers have high valuations for quality whereas only α of the consumers have high valuations in the benchmark case. By contrast, firm b’s realistic manager will in equilibrium reduce its product quality to better serve the low-valuation consumers.

**Proposition 2.** When firm a has an optimistic manager and firm b has a realistic manager, in equilibrium, firm a’s price will increase and firm b’s price may either increase or decrease, relative to when both firms have realistic managers.

Given that firm a’s managerial optimism increases firm a’s quality and lowers its competitor’s quality, one may intuit that managerial optimism will also induce firm a to choose a higher price and firm b to choose a lower price than their respective price in the benchmark case (where both firms have realistic managers). However, this intuition may not be true. Although an optimistic manager will choose a high price to exploit the higher willingness-to-pay of high-valuation consumers, managerial optimism can have two effects on firm b’s price. On one hand,
firm \( b \) will reduce its quality due to firm \( a \)'s optimism, which tends to put downward pressure on its price. On the other hand, firm \( a \)'s optimism can alleviate price competition between the two firms, which can induce firms to raise prices. Depending on which effect dominates, \( p_b^* \) may either increase or decrease.

Specifically, both firms’ prices will increase when \( t \) is low, i.e. when consumers’ horizontal preferences are not very strong. Note that when \( t \) is low, the level of product horizontal differentiation between firm \( a \) and firm \( b \) is low, and price competition is fierce in the benchmark case where there is no vertical differentiation. Under the influence of managerial optimism, firm \( a \)'s manager will choose higher quality and price, which alleviates price competition in the market, compared with the intense competition in the benchmark case. As a result, both \( p_a^* \) and \( p_b^* \) will increase under optimism if \( t \) is low enough, even though firm \( b \) has lower quality than that in the benchmark case.
The effects of $\alpha$ and $\theta_L$ on equilibrium prices are less straightforward. As depicted in Figure 1.2, both firms’ prices increase when $\alpha$ is high and $\theta_L$ is neither too high nor too low. When the fraction ($\alpha$) of high-valuation consumers is large, the realistic manager of firm $b$, though having unbiased beliefs about the market, will charge a price higher than that in the benchmark case, as she knows that there are sufficiently many high-valuation consumers who will buy the product. On the other hand, firm $b$ mainly serves the low-valuation consumers, and will raise its price only if $\theta_L$ is high enough such that the low-valuation consumers will accept the increased price, but not too high as to make too many consumers switch to firm $a$’s high-quality product. Consequently, when $\alpha$ is high and $\theta_L$ is moderate, both firms’ prices will increase relative to the benchmark case.

**1.4.2 Effect of Optimism on Firms’ Profits**

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This figure is drawn with $k = 0.1$ and $t = 0.1$. 

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Given the equilibrium quality levels and prices, we can calculate both firms’ equilibrium profits under unilateral optimism.

**Proposition 3.** Suppose that firm a’s manager is optimistic and firm b’s manager is realistic. There exists $t^* = f(\theta_L, \alpha, k)$ such that if $t < t^*$, both firms make higher profits than when both firms have realistic managers.

Figure 1.3 illustrates the effects of unilateral optimism on the firms’ profits (relative to the benchmark of both firms having realistic managers). Note that in region 1, both firms can benefit from the optimism of firm a’s manager. The underlying reason why unilateral optimism can benefit both firms lies in the effect of optimism on the managers’ quality decisions. While both firms’ (realistic) managers choose the same equilibrium quality and price in the benchmark case, if firm a is the only firm having an optimistic manager, firm a’s quality will increase whereas firm b’s quality will decrease. That is, unilateral optimism will increase the level of vertical differentiation between the firms’ products, alleviating price competition in the market, which can under some conditions (in the shaded regions in Figure 1.3) make both firms de facto “local monopolists” to the low-valuation consumers ($\theta_L$), resulting in partial equilibrium coverage in that consumer segment. This is particularly beneficial to firm b, since it mainly targets low-valuation consumers by choosing lower quality and price than firm a. So, within the shaded parameter region, firm b tends to be better off when the willingness-to-pay of low-valuation consumers is high (i.e., $\theta_L$ is relatively high).
As for firm $a$, its product quality is higher than that of the competitor and it aims to target mainly the high-valuation consumers. When the willingness-to-pay of high-valuation consumers is high (i.e., $\theta_L$ is relatively low), firm $a$ will also benefit from its manager’s optimism. Consequently, both firms will earn higher expected profits under unilateral optimism when $\theta_L$ is neither too high nor too low (i.e., in region 1 in Figure 1.3).

Note that this win-win outcome can occur only if the consumer’s sensitivity to horizontal mismatch is relatively weak (i.e., $t < t^*$). Essentially, if $t$ is low, the level of horizontal differentiation is small and the competition is therefore fierce in the benchmark case (in which both firms have realistic managers). As unilateral optimism introduces a higher level of vertical differentiation compared with the benchmark case, competition will be largely alleviated, benefiting both firms and making a win-win outcome more likely. To summarize, although

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8 This figure is drawn with $k = 0.1$ and $t = 0.1$.  

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optimism gives the manager biased beliefs about the market ($\alpha$) and leads to biased decisions about product quality and price, it can also mitigate competition since the optimistic manager will choose a higher quality and higher price than otherwise. When the alleviation of competition is significant enough, both firms can benefit from unilateral optimism.

### 1.4.3 Endogenous Optimism and Firms’ Hiring Decisions

We have so far assumed that a firm’s manager’s type is exogenous. In practice, firms may choose which type of manager—optimistic or realistic—to hire. In this section, we extend our model to allow firms to choose the type of managers to hire; we will analyze firms’ optimal hiring decisions. Let $R$ and $O$ denote the type of the realistic and the optimistic manager, respectively. Again, a manager’s type is common knowledge, and the manager will maximize her firm’s profit based on her belief.

The new game has four stages. In the first stage, the two firms, $a$ and $b$, with unbiased knowledge of $\alpha$, decide which types of managers to hire. This process can be interpreted as each firm’s board of directors’ selecting its CEO. We assume that both firms (i.e., their boards of directors) have unbiased realistic perception of the market. Each firm’s board will maximize the firm’s expected profit by choosing which type of manager to hire. In the second stage, the hired managers, with their respective beliefs about $\alpha$, simultaneously choose their product quality. In the third stage, the managers simultaneously choose the prices for their products. In the final stage, consumers make purchase decisions, firms’ profits are realized, and the true $\alpha$ is thereby revealed.

There are four possible pure-strategy equilibrium outcomes in the first stage of the game: $(R, R)$, $(O, R)$, $(R, O)$, and $(O, O)$, where the first letter denotes the type of firm $a$’s manager and the second letter denotes the type of firm $b$’s manager. We start our analysis by examining the $(O, O)$ outcome, i.e. when both firms hire optimistic managers.
PROPOSITION 4. When both firms have optimistic managers, both firms will make an expected profit that is less than or equal to their respective profit in the benchmark case (with both having realistic managers).

When both firms’ managers are optimistic, they will increase their product quality and prices. This bilateral optimism does not increase the (quality) differentiation between the two firms’ products, and hence will not help alleviate price competition. As a result, neither firm will benefit from this bilateral optimism. Moreover, under bilateral optimism, if both firms increase their prices to a level at which some low-valuation consumers would no longer purchase any product, both firms would be worse off. Proposition 4 shows that if both firms hire optimistic managers, firms can both end up worse off than if they both hire realistic managers.

Further analysis reveals that \((O,O)\) cannot be an equilibrium outcome (please refer to the Online Appendix for details). Under the condition specified in the proof of Proposition 3 \((t < t^*_a)\), the asymmetric equilibrium of \((O,R)\) or \((R,O)\) will be the equilibrium outcome, i.e., one firm hires an optimistic manager and the other hires a realistic manager.\(^9\) Otherwise, \((R,R)\) is the unique equilibrium outcome.

In summary, only \((R,R)\), \((R,O)\), and \((O,R)\) equilibrium outcomes can exist. When optimism makes a firm worse off, both firms will hire realistic managers. When optimism is beneficial, only one firm will hire an optimistic manager. Both firms hiring optimistic managers is not an equilibrium, because one firm can always improve its profit by switching to a realistic manager. This final scenario provides an important suggestion regarding the firms’ hiring decisions. Our

\(^9\) One can interpret the equilibrium of \((O,R)\) and \((R,O)\) as the outcome of a different model setting where the firms’ managers have heterogeneous beliefs about market demand (i.e., one manager has higher belief about \(\alpha\) while the other has lower belief) and they rationally choose not to ensure the accuracy of their beliefs, even if it is costless to do so, before they make quality and pricing decisions. In the corresponding parameter region for that setting, optimistic bias can be endogenously sustained in equilibrium (i.e., the manager with the higher belief will rationally stay optimistic). We thank an anonymous reviewer for this comment.
analysis shows that even though an optimistic manager has a biased belief about the market, having an optimistic manager can be advantageous to the firm if its competitor’s manager is realistic. This is because, anticipating the optimistic manager’s quality and pricing decision, the unbiased manager will rationally reduce its product quality, which can be beneficial to the firm with the optimistic manager. However, if both firms are run by optimistic managers, their shared optimism does not benefit any firm. So, our result suggests that a firm should hire a realistic manager when managerial optimism is already pervasive in the market.  

1.5 Extensions

1.5.1 The Case of Exogenous Quality under Optimism

Note that since managers choose prices after their quality decisions, managerial optimism can affect a firm’s price in two ways. First, it can directly change a manager’s price choice through changing her belief on the fraction of high-valuation consumers. Second, optimism can influence the manager’s pricing decision indirectly, through its effect on her choice of product quality. This leads to ambiguous effects of managerial optimism on prices, as shown in Proposition 2. Thus, in order to separate the direct effect of managerial optimism on price from its indirect effect, we now take product quality as exogenously given, and check the robustness of our results.

When product quality is exogenously given, the game will have only two stages. First, managers, with their respective beliefs about $\alpha$, simultaneously choose prices for their products. Second, consumers make purchase decisions, firms’ profits are realized, and the true $\alpha$ is thereby revealed. Since firm $a$ is assumed to have an optimistic manager, she will maximize $\pi_a = (p_a - \ldots$

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10 Note that even if it is more costly to hire realistic managers, as long as the additional cost is small enough (lower than the loss from hiring an optimistic manager), a firm should still hire a realistic manager. The additional cost does not change the qualitative insights of our results.
\[ kq_a^2 \frac{t-(p_a-p_b)+(q_a-q_b)\theta_H}{2t} \] \[ kq_b^2 \frac{t-(p_b-p_a)+(q_b-q_a)\theta_L}{2t} \]

Our analysis reveals that even if quality levels are exogenously given, under some circumstances, managerial optimism may still mitigate price competition and in turn increase both firms’ profits. Specifically, when \( q_a > q_b \), firm a mainly serves the high-valuation segment consumers since its quality advantage results in a larger advantage in valuation among high-valuation consumers. In this case, if firm a’s manager believes that all consumers have high valuation for quality, she will increase firm a’s price to exploit its advantage, and the realistic manager of firm b anticipates this and will also increase her price. Price competition is thus mitigated and both firms’ prices increase under optimism, making a win-win outcome possible.

As firm a mainly serves high-valuation consumers while firm b mainly serves low-valuation ones, to have a win-win outcome in equilibrium, it is also necessary for both market segments to be lucrative. When \( \alpha \) is large, the number of high-valuation consumers is large, which benefits firm a. When \( \theta_L \) is large, low-valuation consumers’ willingness to pay is high, which benefits firm b. As a result, one manager’s optimism can benefit both firms if \( \alpha \) and \( \theta_L \) are both high.

**1.5.2 The Case of Fixed Cost for Quality**

In our main analysis, we have assumed that, to produce products of higher quality, a firm has to incur a higher unit cost. This assumption fits well the case of tangible goods. For example, in the smartphone context, this means that a firm has to incur a higher unit cost when producing a higher quality smartphone (e.g., with a faster processor or a higher resolution display). By contrast, when a firm produces and sells intangible or digital products, its fixed cost will likely be much...
more prominent than its unit cost. For example, a software firm may have to incur a higher R&D cost in order to develop a software with a higher quality, but after the product is developed, there is typically not much additional cost to deliver the software to an incremental customer. To capture the case of intangible goods, we study an alternative model in which firms incur fixed costs rather than unit costs to produce the products.

In this model, we assume that to produce a product of quality $q_i$, firm $i$ has to incur a one-time fixed cost $c_i = kq_i^2$, which can be interpreted as the R&D cost to design and develop the product. In order to focus on the effects of fixed costs on firms’ quality and pricing decisions, we assume that the unit cost is zero in this case. Consumers’ utility functions are as given before; firm $i$’s manager maximizes its expected profit $\pi_i = p_i \left( \frac{t-(p_i-p_{-i})+(q_i-q_{-i})\theta}{2t} \right) - kq_i^2$. In the benchmark case in which both firms’ managers are realistic, one can show that both firms will have the same equilibrium quality, price, and profit: $q_a^* = q_b^* = \frac{\theta}{6k}, p_a^* = p_b^* = t, \pi_a^* = \pi_b^* = \frac{t}{2} - \frac{\theta^2}{36k}$.

As in the core model, the manager of firm $a$ is assumed to be optimistic and believes that the market has a high fraction of high-valuation consumers (i.e. $\hat{\alpha} = 1$). Our analysis reveals that, similar to the case with unit cost, the optimistic manager chooses higher quality and price than the realistic manager, and both firms may earn higher profits. The intuition is the same: although managerial optimism entails biased perception of the market and induces the manager to make biased quality and pricing decisions, it can also increase the level of vertical differentiation and serve as a commitment to charging a higher price, which can alleviate competition and benefit both firms. So, we have shown that managerial optimism can lead to a win-win outcome even for the case of digital goods (with fixed costs for quality and negligible unit cost of production).

1.5.3 Optimism over Consumers’ Willingness-To-Pay
In our core model, we have assumed that the optimistic manager overestimates the fraction of high-valuation consumers. In practice, optimistic bias may also come from a manager’s overestimation of consumers’ willingness-to-pay for the firm’s product. In this section, we analyze a model in which the optimistic manager believes that consumers have higher product valuations than believed by the realistic (i.e. unbiased) manager.

There is a unit mass of consumers in the market. Buying firm $i$’s product gives a consumer utility $U(i) = q_i \theta_j - td_i(x) - p_i$, where $\theta_j \in \{\theta_H, \theta_L\}$ is the consumer’s willingness-to-pay for quality, and other notations are the same as in the benchmark case. As in the benchmark case, we normalize $\theta_H = 1$ and assume $0 < \theta_L < 1$. While the realistic manager has an objective belief about both $\theta_H$ and $\theta_L$, the optimistic manager overestimates the high-valuation consumers’ willingness-to-pay for quality and has a biased belief $\hat{\theta}_H > \theta_H = 1$. To focus on the bias about the consumers’ willingness-to-pay, we assume that both managers know the fraction of high-valuation consumers.

Under these assumptions, we find that our main result is robust; one manager’s optimism about consumers’ willingness-to-pay can make both firms better off. Moreover, the win-win outcome can occur as long as the optimistic manager’s belief ($\hat{\theta}_H$) is not too high. Otherwise, the optimistic manager will charge so high a price that very few consumers will buy her firm’s product, hence decreasing her profit. As a result, the win-win outcome can be achieved only when firm $a$’s manager’s optimistic bias is not too strong.

1.5.4 Optimism over Consumers’ Horizontal Preference

An optimistic manager may also have a biased belief about the distribution of the consumers’ horizontal preferences (taste) for the firm’s product. More specifically, in our model framework, an optimistic manager may believe consumers to be more densely populated close to her firm’s
location on the Hotelling line. Note that in this case, the manager’s biased belief about consumers’
taste preferences has asymmetric effects on the two firms. In this section, we analyze such a model
and show that similar results and insights will manifest.

Again, we assume that the manager of firm $a$ is optimistic. Consumers are uniformly
distributed on interval $[0,1]$, and firm $a$ and firm $b$ are located at 0 and 1, respectively. The
optimistic manager believes that the probability density function of consumer distribution is
$F(x) = 2(1 - x)$, rather than the uniform distribution. That is, the optimistic manager has a biased
belief that consumers are distributed in her firm’s favor and she has a competitive advantage,
whereas the realistic manager of firm $b$ has an objective (unbiased) belief about the distribution of
the consumers’ horizontal preferences. Note that we analyze only one specific example of the
biased belief about the consumer distribution, to show that the win-win outcome from the main
model can still occur.

Given that managerial optimism is about the consumers’ horizontal preferences, we assume
that all consumers have the same valuations ($\theta$) for quality and that the market is fully covered in
equilibrium. Similar to the core model, in this extension, if both firms have realistic managers,
both firms will in equilibrium have the same quality, price, and expected profit. Proposition 5
shows the effects of this new type of managerial optimism.

**Proposition 5.** In equilibrium, when firm $a$’s manager is optimistic about consumers’
horizontal preferences and firm $b$’s manager is realistic, both firms’ quality will remain
unchanged, and both firms’ prices and profits will increase.

As this type of managerial optimism does not concern the consumers’ valuation for quality
and there is no heterogeneity in the consumer’s valuation for quality, it does not influence firms’
optimal quality choices, and the level of vertical differentiation does not change in this case.
However, both firms can be better off because optimism can help mitigate price competition. While the manager of firm \(a\) believes that consumers are more densely located close to her, she finds it optimal to charge a higher price to extract more surplus from the loyal consumers, and forgo some consumers who find her product less horizontally matched. Knowing that firm \(a\) will increase its price, firm \(b\)’s realistic manager will also choose a price higher than that in the benchmark case, but lower than firm \(a\)’s price, so that it can extract higher surplus while under-cutting firm \(a\) to acquire more of its customers. Therefore, when one manager is optimistic about the horizontal distribution of consumers, managerial optimism can still serve as a commitment for the manager’s choosing a higher price, which can alleviate price competition and benefit both firms. In this case, alleviated price competition allows the firm with an optimistic manager to make more profits (than the benchmark case) by charging a higher unit profit margin without losing too many customers, and the firm with a realistic manager will benefit because her equilibrium unit profit margin and market share will both increase relative to the benchmark case.

### 1.5.5 Optimism over Quality Level

In reality, managerial optimism may also be about a firm’s own product quality as perceived by consumers. That is, an optimistic manager may overestimate the customers’ perception of the quality of her product; in other words, the optimistic manager believes that consumers consider her product’s quality to be higher than its objective quality (e.g. due to the manager’s overestimate of her firm’s positive brand perception). As before, we assume that the manager of firm \(a\) is optimistic and that of firm \(b\) is realistic. The objective (or true) quality level of firm \(a\) is \(q_a\), but its manager overestimates that quality level, perceiving it to be \(\hat{q}_a = (1 + \delta)q_a\) to consumers, where \(\delta > 0\). The optimistic manager is assumed to not overestimate her competitor’s product quality, which is \(q_b\). Firm \(b\)’s manager, being realistic, has an unbiased belief about both \(q_a\) and
Our analysis shows that when a manager is optimistic about the quality of her product, her optimistic bias can make both firms better off. In other words, our main result from the core model is robust in this alternative model. Specifically, we find that in this setting, both managers increase their prices under unilateral optimism, alleviating price competition and thus benefiting both firms. However, this win-win outcome can occur only if the optimistic bias is not too strong, i.e. $\delta$ not too large, so that the optimistic manager can enjoy the benefit of mitigated competition without having to suffer too severe a consequence from biased decisions on quality and price.

1.5.6 Moderate Optimism

Our core model has assumed that the optimistic manager believes that all consumers have high valuation, i.e. $\hat{\alpha} = 1$. Managerial optimism may take a more moderate form as well; for example, an optimistic manager may know the existence of low-valuation consumers but underestimate the size of that segment of consumers. In this model extension, we relax the assumption of extreme optimism ($\hat{\alpha} = 1$) and consider $\hat{\alpha} \in (\alpha, 1)$, i.e. the optimistic manager’s perceived fraction of high-valuation consumers can take any value between the true fraction ($\alpha$) and 1. In Figure 1.4, we compare the parameter regions in which unilateral optimism makes both firms better off (relative to no optimism) for $\hat{\alpha} = 1$ versus $\hat{\alpha} = 0.8$. Note that in the case of $\hat{\alpha} = 0.8$, the meaningful range of $\alpha$ in Figure 1.4 is only $\alpha < 0.8$ since by definition optimism means $\hat{\alpha} > \alpha$.

As shown in Figure 1.4, when $\hat{\alpha} < 1$, moderate unilateral optimism can still lead to a win-win outcome—both firms can make higher profits than in the benchmark case. Moreover, when $\hat{\alpha} < 1$, this win-win equilibrium is more likely to occur, since when $\hat{\alpha} = 1$, firm $a$ will be worse off if the low-valuation consumer’s willingness-to-pay for quality is low (region 3 in Figure 1.4),
but when $\hat{\alpha} = 0.8$, both firms can be better off. This is because when managerial optimism is strong, the increase of firm $a$’s equilibrium price compared with that in the benchmark case is large, and its profit will suffer if low-valuation consumers do not have high enough willingness-to-pay. By contrast, when managerial optimism is moderate, the price increase by firm $a$ is small. Even if $\theta_L$ is low (region 3), many low-valuation consumers are still willing to buy firm $a$’s product, so that both firms can in equilibrium benefit from the optimism of firm $a$’s manager.

Figure 1.4 Effect of the Degree of Managerial Optimism

Figure 1.5 also demonstrates similar results. For a given number of high-valuation consumers ($\alpha = 0.5$), as the perceived number by the optimistic manager ($\hat{\alpha}$) increases, the parameter region in which both firms are better off tends to shrink, which is consistent with the fact that when managerial optimism is moderate, a win-win equilibrium outcome is more likely to occur.

$^{11}$ This figure is drawn with $k = 0.1$ and $t = 0.1$. 
Therefore, our results from the core model are qualitatively the same even when the degree of managerial optimism is not extreme.

Figure 1.5 Effect of Moderate Managerial Optimism$^{12}$

1.5.7 Managerial Pessimism

Given that evidence shows that managers and entrepreneurs tend to be optimistic, we have focused our study on identifying the effects of managerial optimism. However, under certain circumstances, such as during economic recessions, managerial pessimism might also become pervasive among managers. In this section, we investigate the effects of managerial pessimism.

We first investigate the scenario in which one manager is realistic and the other is pessimistic. Without loss of generality, we assume that firm $a$’s manager is realistic and firm $b$’s manager is pessimistic. Recall that there is a unit mass of consumers and the fraction of consumers with high

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$^{12}$ This figure is drawn with $\alpha = 0.5$, $k = 0.1$ and $t = 0.1$. 

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valuations for quality is $\alpha$ and that with low valuations is $1 - \alpha$. While the realistic manager has an unbiased (objective) belief about $\alpha$, the pessimistic manager believes that the fraction of high-valuation consumers is $\tilde{\alpha} < \alpha$. We focus our analysis here on the boundary case of $\tilde{\alpha} = 0$, i.e. the pessimistic manager believes that all consumers have low valuations for quality.\(^\text{13}\)

Our analysis shows that, while unilateral optimism can make both firms better off, unilateral pessimism cannot lead to such a win-win outcome, because the firm with a pessimistic manager will always be (at least weakly) worse off. Note that in the case of optimism, firm a’s optimistic manager increases her price because she believes that all consumers have high valuations for quality. As a result, managerial optimism serves as the optimistic manager’s implicit commitment to charging a higher price, which can mitigate price competition and benefit both firms. By contrast, in the case of pessimism, firm b’s pessimistic manager decreases the price, which exacerbates rather than alleviates price competition. Our findings suggest that while the effect of managerial optimism on firms’ profits is ambiguous, managerial pessimism is unambiguously detrimental to the firm. Similarly, no firms can benefit from bilateral pessimism (i.e. when both firms have pessimistic managers). As a result, in equilibrium, firms will not choose pessimistic managers over realistic managers.

Furthermore, sometimes people can have highly divided opinions on an issue. In this case, it is possible for one manager to be optimistic and the other to be pessimistic. Interestingly, we find that when both managers are biased but in different directions, both firms can make higher profits than when the managers are biased in the same direction (i.e. both optimistic or both pessimistic). When one manager is optimistic and the other is pessimistic, the optimistic manager specializes in

\(^{13}\) We have also analyzed a model where $0 < \tilde{\alpha} < \alpha = 1$. The results and insights are qualitatively similar, i.e. managerial pessimism is detrimental to the firm.
serving the high-valuation consumers and the pessimistic manager specializes in serving the low-
valuation consumers. The asymmetric bias can mitigate price competition by increasing vertical
differentiation and can thus benefit both firms. As a result, \((O, P)\) can be an equilibrium outcome.

1.6 Conclusion

Evidence has shown that entrepreneurs are an optimistic group, though their optimism often has a
negative impact on their companies’ performance. This paper examines the effects of managerial
optimism on the managers’ product quality and pricing decisions as well as the firms’ profits and
the situations in which firms should hire optimistic managers. Specifically, we develop a model to
investigate competition between two firms whose products are differentiated both horizontally and
vertically. Our analysis provides several interesting findings.

First, we show that a manager’s optimism increases her firm’s product quality level and
decreases that of the competitor. When the optimistic manager overestimates the number of
consumers who have high valuations for quality, she increases the quality level for her firm’s
product, whereas the realistic manager chooses lower product quality to better target the low-
valuation consumers. This effect increases the level of product quality differentiation between the
two firms, which can mitigate competition.

Second, when only one firm is run by an optimistic manager, its price will increase while its
competitor’s price may either increase or decrease, compared with the case in which both firms
have realistic managers. As an optimistic manager overestimates the number of high-valuation
consumers, she tends to increase her price. Anticipating the optimistic manager’s price increase,
the realistic manager of the competing firm will also tend to raise her price. However, the realistic
manager may also have incentives to lower the price, because she has lowered the product quality.
Depending on which effect dominates, a realistic manager’s optimal price may either increase or
Third, we find that a firm can be better off with managerial optimism. In particular, optimism has multifaceted effects on profits. First, optimism causes the firm’s manager to make biased choices of quality and price. Second, optimism can alleviate competition by inducing increased vertical differentiation between the firms’ products. Third, optimism can also serve as the manager’s commitment to charging a higher price, which mitigates price competition between the two firms. If the effect of alleviated competition is stronger than the potential detriment of the manager’s biased decisions, optimism can increase the firm’s expected profit.

More interestingly, when the benefit of alleviated competition is large enough, one manager’s optimism may even make both firms better off. Research has shown that one may benefit from optimism in chicken-game scenarios, since optimism can be an agent’s commitment to making aggressive moves, and lead to a win-lose outcome. This paper contributes to the existing literature by pointing out that unilateral optimism can even generate win-win results in certain situations. We find that though one manager’s optimism can benefit both firms, neither firm can benefit from bilateral optimism, where both firms have optimistic managers. This scenario suggests that a firm should hire a realistic manager when managerial optimism is already pervasive in the market.

Moreover, we have analyzed some model extensions and show that our results can still hold under some alternative assumptions or models. The benefits of optimism can exist regardless of whether firms have unit cost or fixed cost for their products. Even when product quality is exogenously given and managerial optimism cannot increase the level of vertical differentiation, unilateral optimism can serve to mitigate price competition and benefit both firms. Our main results are robust when managerial optimism is about the consumers’ valuation for quality instead of the fraction of high-valuation consumers. Moreover, we find that both firms can earn higher
expected profits than in the benchmark case even when the manager’s optimism is about the
distribution of the consumers’ horizontal preferences (e.g., believing that consumers are more
densely distributed near her firm’s location than that of the competitor). Lastly, when the
manager’s optimism makes her believe that consumers perceive her firm’s product quality to be
higher than it actually is, the win-win outcome can still occur under unilateral optimism due to the
alleviation of price competition.

Our core model has focused on managerial optimism because research has shown its existence
and pervasiveness. However, we have also examined the impact of managerial pessimism on a
firm’s performance. We find that, in contrast to the case of optimism, managerial pessimism,
whether unilateral or bilateral, will not benefit the firm(s) and can reduce both firms’ profits.
Nonetheless, when one firm has an optimistic manager while the other firm has a pessimistic
manager, both firms can benefit from the differentiated biases of their managers, because they will
aim to target different segments of customers.

We conclude this paper by pointing out some directions for future research. First, our study
has focused on duopolistic competition; future research may study the effect of optimism in other
competitive settings, e.g., firms in decentralized channel settings. Second, our analytical
framework is limited to a one-period game and does not study repeated interactions between firms.
Specifically, in a repeated-game setting with uncertain demand conditions, forecasting future
market conditions based on past experiences may alter managerial optimism. The outcome of such
a dynamic game, where managers make product quality and pricing decisions in each period, will
depend on the correlation between the market demands across periods. If the demands across
periods are perfectly correlated, then both managers will know the true demand after the first
period, and the game will result in the \((R, R)\) equilibrium from the second period on. If the
demands across periods are completely uncorrelated (i.e. independent), the dynamic game will reduce to a series of independent one-period games, where the optimistic manager may stay optimistic in the long run. If the demands across periods are moderately correlated, the optimistic manager may gradually update her belief, and the game may eventually converge to the \((R, R)\) equilibrium. This dynamic game deserves its own separate study, so we leave it to future research to more systematically explore the new insights from such situations. Third, our analysis focuses on a duopoly, while increase of firm numbers may change the effects of optimism. In particular, when there are numerous firms in a competitive market, the effect of the managerial optimism of one firm on the market price is marginal, so managerial optimism is not likely to mitigate the competition and thus may not benefit the firm. Fourth, our study focuses on firms’ profit, while an analysis of optimism’s effects on social welfare can also be interesting. In our model, optimism lowers social welfare because it can leave some customers unserved, whereas they are served in the benchmark cases in which there is no managerial optimism. However, given that there are other types of distortion and market failure in reality, optimism increase social welfare in certain situations. As a simple example, if a voter optimistically believes that her vote will be pivotal in the election, she will be more likely to vote. Optimism will improve voter turnout in this case and may very well increase social welfare. Lastly, similar to an approach in Zou et al. (2018) in the context of anticipated regret, it may also be interesting to examine how managerial optimism will affect a firm’s optimal product-line decisions.
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Chapter 2

Quality Design Outsourcing under Demand Uncertainty: An Analytical Study

2.1 Introduction

Many brand-owning retailers outsource product production to manufacturers and focus primarily on product design, development, and marketing. Typically, the retailers specify the required quality level and the specified quantity of a product in a wholesale price contract with the manufacturer. This practice is referred to as “contract manufacturing” (CM) and has become a well-established business model in a variety of industries, including electronics, pharmaceuticals, automotive, and food and beverage production (Plambeck and Taylor, 2005). For example, Apple, Dell, and Hewlett-Packard design their products but outsource the manufacturing to Foxconn, without which the manufacturing cost may be significantly higher. Similarly, Cisco has established “a virtual supply chain with limitless capacity” by outsourcing the manufacturing of its products to contract manufacturers all over the world.

In addition to production outsourcing, sometimes brand-owning retailers also outsource product design to manufacturers. This practice is referred to as “original design manufacturing” (ODM) and has gained increasing popularity among downstream retailers in the last few decades.

15 https://www.strategy-business.com/article/19984?gko=e4f2f
Large supermarket chains, like Walmart and Target, often carry private label products under their own brand but designed and manufactured by upstream suppliers. Celestica, an electronics manufacturing service company, designs products for customers, including IBM, Sun, and Cisco. Moreover, multiple manufacturers in the pharmaceuticals industry have been undertaking research and development, creating a $30 billion drug-development and manufacturing market (Kaya and Özer, 2009). In the notebook industry, Taiwanese manufacturers, who design and produce laptops for the largest PC companies in the world, play an irreplaceable role in this industry (Yang and Chen, 2013). For example, Quanta Computer, the world’s largest designer and manufacturer of notebooks, designs and produces 25% of all Acer notebooks. Similarly, Compal Electronics, another Taiwan-based firm, designs and manufacturers laptops for Dell, Toshiba, HP and Acer.\(^\text{16}\)

ODM allows a retailer to outsource product design to firms who have a comparative advantage on R&D, potentially lowering the development cost and hence the entry barrier of a market, and focus on marketing and retailing. For example, Kylie Jenner, an American internet celebrity with little professional experience in cosmetics, founded her own cosmetics company, Kylie Cosmetics, in 2015. Thanks to ODM, the company manage to outsource the design and production of its products entirely and solely focuses on marketing. While making sales worth $360 million in 2018, the company only have seven full-time employees. Due to the largely successful Kylie Cosmetics, Kylie Jenner was named the youngest-ever self-made billionaire by Forbes.\(^\text{17}\)

While an ODM supply chain can have many potentials, the downstream retailer may also have quality concerns over the product, since it usually does not get to specify the quality level or to monitor the production process. Thus, one may wonder under what conditions a firm can benefit

\(^{16}\) https://www.npinc.ca/who-makes-dell-hp-toshiba-acer-apple-laptops/

\(^{17}\) https://www.ft.com/content/7935b4da-40e6-11e9-9bee-efab61506f44
from outsourcing its product development as well as production to manufacturers. This paper
develops an analytical framework to analyze the effects of ODM and answer this question.
Specifically, we compare an ODM supply chain with a CM supply chain in terms of product
quality, prices, and firms’ profits.

In our model, a retailer, who sells a product directly to end users, outsources its production to
an upstream manufacturer. The retailer can choose CM and design the product by itself, or choose
ODM and outsource the design process to the manufacturer. We address the following research
questions. How does ODM affect the retailer’s profit as well as the manufacturer’s? Do ODM and
CM produce products with different quality levels and result in different prices? When should the
retailer optimally choose ODM over CM?

Our analysis reveals the following main findings. First, we show that in an ODM supply chain,
where the manufacturer designs the product and chooses the quality level, it tends to choose a
quality level lower than that which the retailer would choose. On the other hand, when the retailer
offers the wholesale price in an ODM supply chain, it chooses a price lower than what the
manufacturer would choose. The different quality levels and wholesale prices are essentially
different forms of double marginalization problem. In ODM, when the manufacturer decreases the
quality level, it lowers both its marginal cost and consumers’ demand for the product at any given
price level. While the former benefits the manufacturer alone, the latter hurts both the manufacturer
and the retailer. Not taking into account the retailer’s loss, the manufacturer would choose a quality
level lower than what the retailer would choose in CM. For the same reason, in CM, the
manufacturer would choose a wholesale price higher than what the retailer would choose in ODM.
As a result, both the quality level and the wholesale price are lower in ODM than in CM.
Furthermore, since ODM produces a product with a lower quality at a lower wholesale price, the
retailer also charges a retail price lower than that in a CM supply chain. As a result, outsourcing the design of product quality to the manufacturer would lead to a product with lower quality level, lower wholesale price, and lower retail price.

Second, we find that ODM is optimal for the retailer when demand uncertainty is low. Conversely, when demand uncertainty is high, the retailer is better off choosing CM. When demand uncertainty is low, the retailer wants to obtain a product that can be accepted by the market in both the high state and the low state. Hence, the retailer would outsource the quality design process to the manufacturer and get a product with relatively low quality at low cost. The retailer would then be able to charge a relatively low retail price, which can be affordable to the consumers in the low state. In contrast, when demand uncertainty is high, the low state is so unprofitable that the retailer would practically give up the low state and maximize its expected profit by mainly focusing on the high state. Thus, the retailer would choose to design product quality by itself in order to obtain a product with a high quality at a high cost, so that it can fulfill the demand in the high state.

Third, our analysis shows that, even if it is more cost efficient for the retailer to design product quality by itself, it may nevertheless choose to outsource the design process to the manufacturer, i.e. ODM, as long as the loss from the higher design cost is not too large. Contrary to many predictions in the literature that retailers outsource product design to manufacturers who have a cost advantage (Gray et al. 2009b; Choi 2007), our analysis reveals that the retailer’s choice between ODM and CM does not completely depend on cost efficiency.

Fourth, we find that when the retailer optimally decides whether to outsource quality design based on demand uncertainty, the manufacturer’s profit is non-monotone with demand uncertainty. Contrary to conventional wisdom that uncertainty in the market usually limits a firm’s profitability,
we show that the manufacturer’s profit first decreases and then increases with demand uncertainty. When demand uncertainty is low, the retailer wants to strike a balance between the low state and the high state. As demand uncertainty increases, the retailer becomes more cautious and chooses a lower quality level and pays a lower wholesale price, which decreases the manufacturer’s profit. However, when demand uncertainty is high enough, the retailer would mainly focus on the high state. In that case, as demand uncertainty increases, the difference between the high state and the low state increases, and the retailer would be willing to pay a higher wholesale price for a higher quality level, in order to better cater to the consumers in the high state. As a result, when demand uncertainty is high enough, manufacturer’s profit will increase with demand uncertainty.

The rest of the paper is organized as follows. In Section 2.2, we review the related literature. Section 2.3 introduces the core model framework and discusses the benchmark case in which there is no demand uncertainty. Section 2.4 presents the main results. Section 2.5 extends our core model and checks the robustness of our results in some alternative settings. Section 2.6 concludes the paper with some discussions for future research. All proofs are provided in the Appendix.

2.2 Literature Review

In this paper, we study the effects of outsourcing, on which there is extensive literature. Specifically, we focus on quality design outsourcing under demand uncertainty. While research has shown that outsourcing has the ability to lower innovation costs and risks, to improve financial performance, and to increase productivity (Michael and Michael 2011), the most prominent reason for firms to outsource the production or design of products is cost saving. As shown by Gray et al. (2009b) both analytically and empirically, firms that place a priority on low cost have a high propensity to outsource. Similarly, Choi (2007) shows that when factor prices are not equalized internationally, a firm may outsource the process which uses its scarce source intensively. For
example, given that labor cost is high in the United States, a U.S. based firm may outsource its labor-intensive manufacturing activities to manufacturers in developing countries, where labor cost can be significantly lower.

However, it is noteworthy that it is also shown in the literature that the cost saving benefit from outsourcing may not be sustainable in the long run. In particular, Anderson and Parker (2002) shows that when the effects of learning over time are taken into consideration, outsourcing decisions can create a path-dependent outsourcing trap in which a firm experiences higher long-run costs after an immediate cost benefit. In this case, partial outsourcing, i.e. outsourcing a fraction of component production, may become optimal and dominate either complete outsourcing or complete insourcing. Similarly, Gray et al. (2009a) investigates a two-period game between a retailer and a manufacturer, wherein both firms can reduce their production costs through learning-by-doing, and finds that the retailer’s outsourcing strategy may be dynamic and change from period to period, and partial outsourcing can be an optimal strategy.

Outsourcing can also benefit firms in ways other than cost saving. For instance, outsourcing may mitigate competition under certain circumstances and lead to a win-win outcome. Cachon and Harker (2002) shows that when competing firms face scale economies, they can mitigate price competition through outsourcing because it reduces a firm’s desire to build scale to lower cost. As a result, outsourcing can lead to a win-win outcome in which both retailers and the manufacturer make higher profits than without outsourcing. Similarly, Gilbert et al. (2006) demonstrates that outsourcing provides a mechanism by which the two competing firms can credibly signal that they will not overinvest in cost reduction, mitigating cost competition and benefiting both firms.

On the contrary, Arya et al. (2008) demonstrates the case in which outsourcing can benefit a firm at the cost of its competitor. When a monopolistic input supplier serves two competing
retailers, a retailer may buy an input from the supplier at a price above its in-house cost of production, in order to limit the supplier’s incentive to provide the input on extremely favorable terms to its retail competitor. Consequently, a retailer may pay a premium to outsource production to a common supplier to raise its competitor’s costs.

The literature has revealed potential risks over product design outsourcing as well, the most prominent of which is quality concerns. Feng and Lu (2010) argues that ODM blurs the boundary between manufacturers and brand-owning retailers, which can subsequently dampen their quality incentives. Naturally, if both the manufacturer and the retailer believe that consumers will blame the other party for potential quality issues, both of them will have less incentive to ensure a good quality product. Furthermore, as shown by Kaya and Özer (2009), quality risk is substantial in outsourcing when product quality is noncontractible, or when the cost to increase quality is private information to the manufacturer.

Nevertheless, under demand uncertainty, outsourcing may actually increase quality under certain circumstances, as shown by Jerath et al. (2017). While demand uncertainty reduces a firm’s optimal product quality (since a lower quality helps to mitigate the risk of overproduction because of lower production cost), this optimal quality level can be higher in a decentralized channel, compared to a centralized channel, because a wholesale price contract shields the manufacturer from inventory risk. Liu et al. (2018) shows similar results that optimal quality for the manufacturer can be higher in a decentralized channel than in a centralized channel.

Given that the literature has shown that outsourcing has an important effect on product quality, it is interesting that quality concerns may not play a significant role in firms’ outsourcing decisions, as demonstrated by Gray et al. (2009b). In other words, a firm that emphasizes quality is as likely to outsource as a firm that does not emphasize quality. However, they also argue that this result is
likely caused by managers’ overconfidence regarding their ability to manage quality across organizational boundaries as well as their negligence of unintended consequences of their outsourcing choices.

Product design outsourcing also raises concerns over diminishing product differentiation. Since a manufacturer may simultaneously serve more than one client, the manufacturer may have incentive to offer similar designs to different clients, which can “dilute” product differentiation and therefore intensify market competition (Carbone 2003; Dedrick and Kraemer 2006; Amaral and Parker 2008). For example, Tzeng and Chang (2003) reports that Quanta Computer, a Taiwan-based notebook computer manufacturer, has offered several clients different notebook models derived from a common prototype.

Moreover, Feng and Lu (2010) points out that the design capability gained by the manufacturer in the ODM process may encourage new competitors to enter the market and hence foster competition. For example, ODM suppliers allow mobile operators like T-Mobile to offer their own cellphones at a relatively low price and directly compete against other cellphone companies that are also clients of the same ODM suppliers. Without the ODM model, mobile operators that have no design experience may have to face a much higher entry barrier when entering the cellphone market. Furthermore, as revealed by Feng and Lu (2010), sometimes an ODM firm itself can eventually become a direct competitor of its clients. For instance, Acer Inc. started from a humble and anonymous original design manufacturer and eventually became the 4th largest PC maker with its own globally recognized brand.

When retailers focus on innovations, including research and development, product design, and marketing, and sell their production facilities to manufacturers, production outsourcing may also have a negative impact on innovation. In particular, as shown by Plambeck and Taylor (2005),

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given the intractability of innovation, retailers will invest less in innovation than is ideal for the industry as a whole. As a result, outsourcing may reduce the profitability of the industry generally by weakening the incentives for innovation. Through an examination of 24 outsourced development projects at Siemens, Cui et al. (2009) identifies five drivers of success in innovation outsourcing, most important among which are project-specific partner competence and in-house competence.

This paper contributes to the literature on supply chain and outsourcing in multiple ways. First, while previous research has investigated the effects of the practice to outsource product manufacturing and design, we establish an analytical model to directly compare these two distinct but closely related forms of outsourcing practice, namely CM and ODM. Second, we study the effects of ODM under demand uncertainty, and reveal that the retailer tends to outsource product design in a low-uncertainty market, and insource in a high-uncertainty one. Third, we innovatively show that the manufacturer’s profit is non-monotone with demand uncertainty. Specifically, an increase in demand uncertainty can lead to an increase in the manufacturer’s profit. Fourth, whereas previous research emphasizes cost saving in design as the major benefit of ODM, we demonstrate that the retailer may be able to benefit from ODM even if it has a lower design cost than the manufacturer. In other words, the retailer may want to outsource product design to the manufacturer, even if the manufacturer does not have a cost advantage.

2.3 Model

Let us consider a supply chain with one retailer and one manufacturer, denoted by $R$ and $M$, respectively. The retailer sells a unit product of quality level $q$ at price $p$ to the end consumers and outsources its production to the manufacturer. The marginal cost for the manufacturer to produce a unit of the product is $c = q^2$, as higher quality will impose a higher cost on the manufacturer for
each unit of the product produced.

The consumers’ utility from consuming the product is $u = \theta q - p$, where $\theta$ represents consumers’ willingness-to-pay for quality and is uniformly distributed on $[0,1]$. Consumers may choose to buy one unit of the product at most, or do not make purchase and choose the outside option, whose utility is normalized to 0. The demand of the product is ex ante uncertain and may be either high or low. Specifically, the size of the market, denoted by $N$, is $1 + \delta$ in the high state and $1 - \delta$ in the low state, where $\delta \in (0,1)$ measures the degree of demand uncertainty. The chance of either state (to take place) is equal, where $Pr[N = 1 + \delta] = Pr[N = 1 - \delta] = \frac{1}{2}$. Note that with demand uncertainty, there may be unsold inventory under optimal pricing. We assume the excess inventory has no resale value and normalize the salvage value of the product to zero. In Section 5.2, we relax this assumption and assumes that the manufacturer allows the retailer to return unsold products at a discounted price.

In this research, we consider two types of supply chains: contract manufacturing (CM) and original design manufacturing (ODM). In the CM supply chain, the retailer designs the product quality $q$ and requests that the manufacturer produce the product. The manufacturer gives the retailer a quote of the wholesale price $w$, and the retailer decides the quantity that it wants to order and stock as inventory, which is denoted by $i$. After the production is completed, the market demand is realized and the retailer chooses retail price $p$ accordingly. In the ODM supply chain, the retailer gives the manufacturer an offer of the wholesale price, and the manufacturer submits the quality level it can provide at this price. Based on the quality level offered by the manufacturer, the retailer decides the quantity to order. Afterwards, the demand is realized and the retailer chooses the retail price. (We will refer to the first type of supply chain as CM and the second type as ODM in the rest of the paper.)
Thus, there are four stages to the game. In CM, in the first stage, the retailer decides quality level; in the second stage, the manufacturer decides the wholesale price; in the third stage, the retailer decides inventory level; and in the fourth stage, market demand is realized and the retailer sets the retail price accordingly. Retailer’s profit is realized in the fourth stage. In ODM, in the first stage, the retailer decides the wholesale price; in the second stage, the manufacturer decides quality level; in the third stage, the retailer decides inventory level; and in the fourth stage, market demand is realized and the retailer sets the retail price accordingly. Retailer’s profit is realized in the fourth stage. Figure 2.1 shows the timelines of the game in both CM and ODM.

Figure 2.1 Timelines in CM and ODM

Note that in our model, the retailer has to decide on the inventory level before the demand is realized, but can choose the retail price afterwards. This assumption represents the fact that retailers oftentimes have to stockpile commodities before the selling season, but can adjust the retail price relatively easily by offering discounts during the selling season. Moreover, the retailer moves first in both CM and ODM, so the retailer always enjoys first-mover advantage in this model, which is consistent with the fact that the brand-owning retailers oftentimes have a larger
bargaining power than their upstream suppliers in a supply chain.

**Benchmark: No Demand Uncertainty**

First, we consider the benchmark case in which there is no demand uncertainty in the market. In this case, we normalize the market size to 1 and make it common knowledge to both the retailer and the manufacturer.

We solve for the equilibrium outcome through backward induction. In CM, in the last stage of the game, the retailer sets the retail price based on the quality level and wholesale price of the product. Since there is no demand uncertainty, the retailer does not face the possible problem of unsold inventory and does not have to specify inventory level before demand is realized. The retailer’s profit function is: $\pi_R^C = (p^C - w^C)s^C$, where $s$ represents the sales volume at a given retail price. When consumers make a purchase, the threshold consumer who is indifferent between making a purchase and the outside option has utility: $\theta^C q^C - p^C = 0$, which gives us the demand: $d^C = \left(1 - \frac{p^C}{q^C}\right)$, where $d$ represents the total demand of the product at a given retail price. Note that the sales volume cannot exceed either the demand or the inventory level, so we have $s^C = \min\{d^C, i^C\}$. In the benchmark case, since there is no demand uncertainty, the retailer can fully anticipate the demand, so we always have $s^C = i^C = d^C = \left(1 - \frac{p^C}{q^C}\right)$. Maximizing the retailer’s profit function with respect to $p^C$ gives us $p^C = \frac{q^C + w^C}{2}$.

In the second-to-last stage of the game, the manufacturer chooses the wholesale price. Its profit function is: $\pi_M^C = (w^C - c^C)d^C = (w^C - q^C)^2\left(1 - \frac{q^C + w^C}{2q^C}\right)$. Maximizing the manufacturer’s profit function with respect to $w^C$ gives us $w^C = \frac{q^C + q^C}{2}$.

In the first stage of the game, the retailer decides the quality level. Plugging $p^C$ and $w^C$ into
the retailer’s profit function, we have $\pi_R^C = \frac{q^C(1+q^C)}{2}$. Maximizing $\pi_R^C$ gives us $q^C* = \frac{1}{3}$, and we can solve for other variables accordingly.

Similarly, we can solve for the equilibrium outcome through backward induction for ODM. Lemma 1 shows the equilibrium outcomes for both supply chains. Numerical form of the results are collected in Table 2.1.

Lemma 1. When the market size is 1, the equilibrium outcomes are: in CM, $q^C* = \frac{1}{3}$, $w^C* = \frac{2}{9}$, $p^C* = \frac{5}{18}$, $\pi_R^C* = \frac{1}{108}$, $\pi_M^C* = \frac{1}{54}$; in ODM, $q^O* = \frac{1}{6}(7\sqrt{33} - 39)$, $w^O* = \frac{1}{6}(17\sqrt{33} - 97)$, $p^O* = \frac{1}{3}(6\sqrt{33} - 34)$, $\pi_R^O* = \frac{1}{18}(11\sqrt{33} - 63)$, $\pi_M^O* = \frac{2}{9}(339 - 59\sqrt{33})$.

Note that players of the game, namely $R$ and $M$, are labeled with subscripts, while types of supply chains, namely OM and CDM, are labeled with superscripts. We will use this notation consistently throughout this paper.

Table 2.1 Equilibrium Outcome in Benchmark Case

<table>
<thead>
<tr>
<th></th>
<th>Integrated</th>
<th>ODM</th>
<th>CM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality</td>
<td>0.3333</td>
<td>0.2019</td>
<td>0.3333</td>
</tr>
<tr>
<td>Wholesale Price</td>
<td>-</td>
<td>0.1095</td>
<td>0.2222</td>
</tr>
<tr>
<td>Retail Price</td>
<td>0.2222</td>
<td>0.1557</td>
<td>0.2778</td>
</tr>
<tr>
<td>Demand</td>
<td>0.3333</td>
<td>0.2287</td>
<td>0.1667</td>
</tr>
<tr>
<td>Retailer Profit</td>
<td>-</td>
<td>0.0105</td>
<td>0.0093</td>
</tr>
<tr>
<td>Retailer Share</td>
<td>-</td>
<td>40.18%</td>
<td>33.33%</td>
</tr>
<tr>
<td>Manufacturer Profit</td>
<td>-</td>
<td>0.0157</td>
<td>0.0185</td>
</tr>
<tr>
<td>Manufacturer Share</td>
<td>-</td>
<td>59.82%</td>
<td>66.67%</td>
</tr>
<tr>
<td>Channel Profit</td>
<td>0.0370</td>
<td>0.0263</td>
<td>0.0278</td>
</tr>
</tbody>
</table>

Lemma 1 demonstrates that when there is no demand uncertainty in the market, i.e. $\delta = 0$, the equilibrium quality level, wholesale price, and retail price are all higher in CM than in ODM. In other words, when the retailer decides quality level and the manufacturer decides wholesale price,
the manufacturer produces products with a higher quality and charges a higher wholesale price in equilibrium. As a result, the retailer also charges a higher retail price in equilibrium.

The difference of quality level and wholesale price in CM and ODM is actually a different form of double marginalization. A typical double marginalization problem is caused by the negligence of the other party’s loss of profit when one party in the supply chain is maximizing its profit. When the retailer raises the retail price, it increases its unit profit but decreases its demand. While the increasing unit profit benefits the retailer alone, the decreasing demand hurts both the retailer and the manufacturer. Not taking the loss of the manufacturer into consideration, the retailer would choose a retail price higher than what is optimal for the channel as a whole. Similarly, in ODM, when the manufacturer decreases quality level, it lowers both the manufacturer’s marginal cost and the consumers’ demand for the product at any given price level. While the lower marginal cost benefits the manufacturer alone, the lower demand hurts both the manufacturer and the retailer. Not taking the loss of the retailer into consideration, the manufacturer would set a quality level lower than what the retailer would choose. As a result, quality level is lower in ODM than in CM. Similarly, in CM, when the manufacturer increases wholesale price, it increases the unit profit of the manufacturer, but also forces the retailer to increase the retail price, which in turn decreases demand of the product. While the increasing unit profit benefits the manufacturer alone, the decreasing demand hurts both the manufacturer and the retailer. Not taking the loss of the retailer into consideration, the manufacturer would choose a wholesale price than what the retailer would choose. As a result, wholesale price is higher in CM than in ODM. Since both quality level and wholesale price are lower in ODM than in CM, the retailer also chooses a lower retail price in ODM, and thus we have different equilibrium outcomes in ODM and CM.
Moreover, the manufacturer’s profit is higher in CM than in ODM, whereas the retailer’s profit is higher in ODM than in CM. Therefore, the retailer is better off setting the wholesale price itself and letting the manufacturer design the quality level when there is no demand uncertainty.

2.4 Analysis

As demonstrated in the benchmark case, CM and ODM induce different strategic behaviors for both players in the game, and hence can lead to different quality levels and prices in equilibrium. As demand uncertainty changes, the optimal quality and price may also change, so the retailer may want to choose a different supply chain accordingly. In this section, we first study how changes in demand uncertainty can affect the retailer’s choice between CM and ODM.

Proposition 1. There exist $\delta^* \in (0,1)$ such that when $\delta < \delta^*$, the retailer is better off choosing ODM than CM; when $\delta > \delta^*$, the retailer is better off choosing CM than ODM.

Proposition 1 reveals that the retailer’s choice between ODM and CM is directly dependent on demand uncertainty in the market. This is consistent with the fact that, in the benchmark case, when there is no demand uncertainty, the retailer would choose ODM over CM.

Recall that in an ODM supply chain, the retailer sets the wholesale price and lets the manufacturer design the quality level of the product. As discussed in the last section, the retailer has an incentive to offer a relatively low wholesale price, and the manufacturer would design a relatively low quality level. On the other hand, in a CM supply chain, the retailer designs the quality level and lets the manufacturer decide the wholesale price. In that case, the retailer would desire a product with a higher quality, and the manufacturer would want to charge a higher wholesale price as well. Consequently, both the quality level and the wholesale price are lower in an ODM supply chain when demand uncertainty is the same.

Note that when demand uncertainty is low, the difference in the market size between the high
state and the low state is not very large. In the low state, there are still a large number of consumers from whom the retailer can make a significant profit. Therefore, the retailer, when making its decisions on quality level and wholesale price, wants to take into account not only the high state but also the low state. Given that in the case of low state, the retailer must charge a retail price low enough due to the low demand, it would prefer to choose a product with relatively low quality and low cost so that it can afford to charge a relatively low retail price in the low state. Thus, the retailer is better off choosing ODM, which provides a product with a lower quality level and a lower wholesale price than CM.

On the contrary, when demand uncertainty is high enough, the gap between the high state and the low state becomes so large that the retailer would find it optimal to practically give up the low state and mainly focus on the potential high state when deciding quality level and wholesale price. After all, the market size is too small in the low state for the retailer to make much profit anyway. In this case, the retailer would like to have a product with high quality and high cost, so it can charge a high retail price in the high state and fully utilize the large size of the market. Hence, the retailer is better off choosing CM, which provides a product with a higher quality level and a higher wholesale price.

Depending on the level of demand uncertainty, the retailer would choose a different supply chain to induce the optimal quality level and wholesale price. When demand uncertainty is low, it chooses ODM to induce low wholesale price at the cost of low quality and, when demand uncertainty is high, it chooses CM to induce high quality at the cost of high wholesale price. As a result, the manufacturer’s profit also largely depends on demand uncertainty, which is pointed out in Proposition 2.

Proposition 2. Given that the retailer chooses supply chains optimally, the manufacturer’s
profit is non-monotone with demand uncertainty. It first decreases with demand uncertainty when \( \delta < \delta^* \) and then increases with demand uncertainty when \( \delta > \delta^* \).

Conventional wisdom tells us that a firm usually makes a lower expected profit when demand is uncertain in the market. However, contrary to popular belief, Proposition 2 reveals that the manufacturer’s profit first decreases and then increases with demand uncertainty. This relationship is demonstrated in Figure 2.2.

**Figure 2.2 Manufacturer’s Profit**

Note that when \( \delta < \delta^* \), the retailer would choose ODM and decide the wholesale price by itself. As demand uncertainty increases, the demand in the low state decreases, so the retail price charged by the retailer in the low state has to decrease, too. In order to be able to afford a lower retail price, the retailer has to choose a lower wholesale price in the first stage of the game. As a result, when demand uncertainty increases, the wholesale price offered by the retailer decreases, which reduces the manufacturer’s profit. Thus, when \( \delta < \delta^* \), the manufacturer’s profit decreases with \( \delta \).

On the other hand, when \( \delta > \delta^* \), the retailer forgoes the low state and primarily focuses on the
high state to maximize its expected profit. When demand uncertainty increases, the demand in the high state increases, so the retailer would choose a higher quality to fulfil that demand and be willing to accept a higher wholesale price. As a result, when demand uncertainty increases, the wholesale price charged by the manufacturer also increases, which increases the manufacturer’s profit. Thus, when $\delta > \delta^*$, the manufacturer’s profit increases with $\delta$. Consequently, the manufacturer’s profit is non-monotone with demand uncertainty.

So far, we have assumed that the manufacturer has to incur a marginal cost to produce each unit of the product and normalize any fixed cost incurred in the process of development to zero. In practice, firms often have to incur a fixed cost in the process of R&D. In order to investigate how the cost to design a product affects the retailer’s choice of supply chain, we assume that the development process of a product induces a fixed cost, $F$, on the firm. The retailer can either choose CM and incur the design cost $F_R$ by itself, or choose ODM and pay $F_M$ to hire the manufacturer to design the product. Proposition 3 reveals the effects of fixed development cost.

Proposition 3. The retailer may choose ODM and let the manufacturer design product quality, even if the manufacturer has a higher design cost.

It is often argued that downstream firms should outsource product design to upstream suppliers who can design the product at a lower cost, due to their experience, technological expertise, and economy of scale. However, as shown in Proposition 3, development cost is not the retailer’s only concern when deciding whether to outsource product design. Since outsourcing product design changes both players’ behavior in the game, the retailer would like to choose different supply chains in order to induce the optimal quality level and wholesale price under different demand uncertainties. If the benefit gained from choosing the supply chain that corresponds to the demand uncertainty faced by the retailer exceeds the loss resulting from the extra cost in product
development, the retailer may very well choose a supply chain that incurs a higher fixed development cost.

When demand uncertainty is low, the retailer would like to choose ODM to induce a low wholesale price. Even if the retailer has the advantage in designing the product, as long as the advantage is not too large, the retailer would still optimally choose to let the manufacturer design the product so that it can pay a relatively low wholesale price. On the other hand, when demand uncertainty is high, the retailer would like to choose CM to induce high quality level. Even if the manufacturer has the advantage in designing the product, the retailer would still choose to design the product by itself to induce a higher quality level. Thus, our analysis shows that when deciding whether to outsource product design, the retailer takes into account not only the design cost but also demand uncertainty and strategic dynamics in different supply chains.

2.5 Extensions

2.5.1 Uncertainty of Consumers’ Willingness-To-Pay

In our analysis, we have assumed that the total market size is uncertain while the distribution of consumers’ willingness-to-pay is known to the firms. In some circumstances, it is also possible that the total market size is known but the distribution of the consumers’ willingness-to-pay for quality is uncertain. In this section, to investigate this type of demand uncertainty, we study an alternative model where firms ex ante do not know what fraction of consumers have high valuations versus low valuations.

In this model, in order to focus on the uncertainty of consumers’ willingness-to-pay for quality, we assume that the total market size is common knowledge and normalized to 1. Moreover, there are two types of consumers, denoted by $j \in \{H, L\}$. Let $\theta_j$ denote type-$j$ consumer’s willingness to
pay for quality, where \( \theta_L < \theta_H \). For simplicity, we assume \( \theta_H = 1 \) and \( \theta_L = \frac{1}{4} \). Although the size of the market is fixed, the proportion of high-type consumers is ex ante uncertain. In particular, there are two possible states of the market, the low state and the high state. In the low state, the fraction of consumers having high willingness-to-pay is \( \alpha \), where \( \alpha \in \left( 1, \frac{1}{2} \right) \), and the fraction of consumers having low willingness-to-pay is \( 1 - \alpha \); in the high state, the fraction of consumers have high willingness-to-pay is \( 1 - \alpha \), and the fraction of consumers having low willingness-to-pay is \( \alpha \). The chance of either state (to take place) is equal, where \( Pr[low \ state] = Pr[high \ state] = \frac{1}{2} \). Note that the smaller \( \alpha \) is, the larger the difference between the low state and the high state is. Hence, \( \alpha \) is the reverse indicator of demand uncertainty in this model, in the sense that the higher \( \alpha \) is, the lower demand uncertainty is.

Given that the size of the market is fixed in this model, we omit the stage of retailer deciding inventory level and assume that the retailer can choose the quantity of its order freely after the state of the market is realized.

Thus, there are three stages to the game. In CM, in the first stage, the retailer decides quality level; in the second stage, the manufacturer decides the wholesale price; in the third stage, market demand is realized and the retailer sets the retail price accordingly. Retailer’s profit is realized in the third stage. In ODM, in the first stage, the retailer decides the wholesale price; in the second stage, the manufacturer decides quality level; in the third stage, market demand is realized and the retailer sets the retail price accordingly. Retailer’s profit is realized in the third stage. Figure 2.3 shows the timelines of the game when consumers’ willingness-to-pay is ex ante uncertain.
Recall that in the previous section, we show that when demand uncertainty is low, the retailer makes higher expected profit in ODM; when demand uncertainty is high, the retailer makes higher expected profit in CM. We find that our result is qualitatively the same in this model. Similar to the core model, when the difference of the fractions of high-type consumers between the low state and the high state is large, the retailer makes a higher expected profit if it chooses CM than if it chooses ODM; when the difference is small, the retailer makes a higher expected profit if it chooses ODM than if it chooses CM.

Thus, when the total market size is known but the distribution of the consumers’ willingness-to-pay for quality is uncertain, retailer should still optimally choose ODM when the uncertainty is low, and choose CM when the uncertainty is high.

2.5.2 Return Policy

In our analysis, we have assumed that any excess inventory that remains unsold after consumers make purchase decisions have no resale value and normalized the salvage value of the product to zero. In some cases, particularly when the retailer has relatively large bargaining power, the manufacturer may allow the retailer to return the unsold inventory at a discounted price. Given
that such a return policy potentially lowers the retailer’s risk of over-stocking, it may also change the influence of demand uncertainty and the retailer’s choice of supply chains. Hence, in the section, we investigate the effects of CM and ODM when the manufacturer offers a return policy.

In this alternative setting, we assume that the manufacturer allows the retailer to return all unsold products at a discount ratio, \( \rho \). Hence, after consumers make purchase decisions, the retailer returns unsold products to the manufacturer and generates revenue \( \min\{\rho w(i - s), 0\} \), where \( w \) is the wholesale price, \( i \) is the inventory level, and \( s \) is the sales volume. We assume that the returned products have no resale value to the manufacturer. Other settings and assumptions remain unchanged from the core model.

Our analysis shows that the effects of the return policy critically depends on the discount ration, \( \rho \). If \( \rho \) is small, our result from previous sections remain unchanged: the retailer is better off with ODM when demand uncertainty is low, and better off with CM when demand uncertainty is high. However, if \( \rho \) is large, ODM can always generated a higher expected profit for the retailer, and therefore is the optimal supply chain for the retailer regardless of demand uncertainty.

Note that the return policy lowers the loss caused by unsold inventory, and therefore essentially acts as an insurance against demand uncertainty, in the sense that the potential loss of the retailer in the state of low demand is partly compensated by the manufacturer. Thus, the return policy lowers the retailer’s risk of unsold inventory under demand uncertainty, and may change the retailer’s optimal choice of supply chain under a given level of demand uncertainty.

When \( \rho \) is small, the compensation in the low state is small, so the effects of the insurance is limited and does not change the retailer’s incentive significantly. Consequently, the retailer’s optimal choice of supply chain remains unchanged from the core model.

On the other hand, when \( \rho \) is large, the insurance is very favorable to the retailer and can
significantly lower the risk of unsold inventory under demand uncertainty. Recall that without the return policy, the retailer is ex ante better off with CM only if demand uncertainty is large enough. However, given that the manufacturer provides a generous return policy which insures the retailer against demand uncertainty, the risk of demand uncertainty faced by the retailer is lowered to a degree that the retailer no longer finds CM optimal. Essentially, the return policy and CM work as substitutes, since both of them are tools for the retailer to protect itself under high demand uncertainty. If when one of them, namely the return policy in this case, is powerful enough, the retailer does not need the other one, namely CM, anymore. As a result, when the return policy is favorable enough, the retailer will always be better off with ODM.

2.6 Conclusion

Outsourcing has been a common business practice in various industries for a long time, and different forms of outsourcing have developed in recent years. Nowadays, firms can outsource not only the production of a product but also the development and design of it to manufacturers. This paper compares original design manufacturing (ODM), in which the retailer outsources both the production and the development of a product to the manufacturer, with contract manufacturing (CM), in which the retailer only outsources the production process. Specifically, we develop a model to investigate the retailer’s choice of supply chains and its effect on the manufacturer’s profit under demand uncertainty. Our analysis provides several interesting findings.

First, we show that, in ODM, the product has lower quality, lower wholesale price, and lower retail price than in CM. The different equilibrium outcomes are essentially different forms of double marginalization problem in different supply chains. The classic double marginalization problem is caused by the fact the when one party in the supply chains maximizes its profit, it neglects the negative externality incurred upon the other party in the supply chain, and makes a
decision suboptimal for the channel as a whole. In ODM, if the manufacturer lowers the quality level, it decreases both its marginal cost and consumers’ demand for the product at any given price level. The former benefits only the manufacturer, whereas the latter hurts both the manufacturer and the retailer. Since the manufacturer does not take into consideration the retailer’s loss of profit due to a lower quality, it will choose a quality level lower than what the retailer would choose in CM. As a result, the product has a lower quality level in ODM than in CM. Similarly, in CM, if the manufacturer increases the wholesale price, it increases its own unit margin, but forces the retailer to choose a higher retail price and decreases the sales volume. The former benefits only the manufacturer, while the latter hurts both the manufacturer and the retailer. Since the manufacturer does not take the retailer’s loss of profit into account when choosing the wholesale price, it will choose a wholesale price higher than when the retailer would choose in ODM. Consequently, the equilibrium wholesale price is lower in ODM than in CM. Moreover, given that the product has lower quality and lower cost in ODM, the retailer charges a lower retail price in this case. As a result, the product’s quality level, wholesale price, and retail price are all lower in ODM than in CM.

Second, we find that the retailer is better off outsourcing product design to the manufacturer when demand uncertainty is low, and not outsourcing when demand uncertainty is high. When demand uncertainty is low, the difference between the high state and the low state is small, so the retailer wants to keep a balance between its effort to satisfy the low state and that to satisfy the high state, because it can make considerable profit in both states. In that case, the retailer wants to choose ODM and outsource the product design to induce a low wholesale price, so that it can charge a retail price low enough in the low state. On the other hand, when demand uncertainty is high, the gap between the low state and the high state is large and the low state becomes so
unprofitable to retailer that the retailer would practically forgo the low state and focus on the potential high state. The retailer chooses CM to obtain a product with high quality so that it can fully utilize the high demand in the high state. Thus, the retailer chooses ODM when demand uncertainty is low and chooses CM when demand uncertainty is high.

This result does not depend on the form of demand uncertainty. Specifically, no matter whether the uncertainty is about the total size of the market or consumers’ willingness-to-pay for quality, the retailer always makes a higher expected profit with ODM when the degree of uncertainty is low, and makes a higher expected profit with CM otherwise. Moreover, if the manufacturer allows the retailer to return the unsold products at a discounted price, as long as the return price is not too high, the result remains qualitatively the same, that the retailer is better off with ODM when demand uncertainty is low, and better off with CM when demand uncertainty is high. Given that the return policy essentially shields the retailer against demand uncertainty, if the return price is too high, the actual risk caused by demand uncertainty is always low, and the retailer will always better off with ODM.

Third, given that the retailer chooses supply chain optimally, we find that the manufacturer’s profit is non-monotone with demand uncertainty. Intuitively, one may expect a firm to make lower profit when demand uncertainty is higher. Our analysis shows otherwise. When demand uncertainty is low, the retailer takes both the high state and the low state into account when making decisions. As demand uncertainty increases, market size in the low state decreases, so the retailer selects a lower wholesale price so that it can charge a lower retail price in the low state. As a result, the manufacturer’s profit decreases as well. On the other hand, when demand uncertainty is high, the retailer mainly focuses on the high state. As demand uncertainty increases, market size in the high state also increases, so the retailer selects a higher quality level and is willing to accept a
higher wholesale price. Consequently, the manufacturer’s profit increases. Therefore, the manufacturer’s profit first increases and then decreases with demand uncertainty.

Fourth, our study shows that the retailer may, based on demand uncertainty, choose to outsource product design to the manufacturer even if it can design the product at a lower cost. In ODM, the retailer outsources product design and pays a lower wholesale price than in CM, which is desirable when demand uncertainty is low. If the benefit gained from lower wholesale price is higher than the extra design cost, the retailer may choose to outsource product design, even if it is less cost efficient than designing the product by itself. In contrast to popular belief that a retailer should outsource product design to a manufacturer who can design the product at lower cost due to technological expertise or economy of scale, our analysis reveals that, even if the manufacturer does not have the cost advantage in R&D, the retailer may still optimally choose to outsource the product design.

We conclude this paper by pointing out some directions of future research. First, our study has focused on a monopoly; future research may study the effect of quality design outsourcing in competitive settings, e.g., duopolistic competition. Second, our analytical framework is limited to a one-period game and does not study repeated interactions between the retailer and the manufacturer. This dynamic game deserves its own separate study, so we leave it to future research to more systematically explore the new insights from such situations. Third, while our research assumes that the firms only produce products at one quality level, it may also be interesting to investigate the case in which firms produce products with different quality, and see how quality design outsourcing will affect firms’ optimal product-line decisions.
References


Chapter 3

Quality Design Outsourcing under Demand Uncertainty: An Empirical Study

3.1 Introduction

Using data from a Chinese manufacturer that supplies a major U.S. textile retailer, we conduct an empirical investigation of ODM in a labor-intensive industry. In particular, using the cushion industry as a representative category, we study how demand uncertainty affects the retailer’s quality design outsourcing decisions, which in turn affects the manufacturer’s profit.

During the sample period from 2011 to 2018, the manufacturer worked as a supplier of the retailer in two types of contracts simultaneously. In the first type of contracts, the retailer designed a product, specified the quality requirements (density of the fabric, components of the dye, etc.), and requested the manufacturer to produce it. The manufacturer gave the retailer a quote of wholesale price, and the retailer decided the quantity to order. In the second type of contracts, the retailer offered a wholesale price to the manufacturer, and requested the manufacturer to design and produce a product based on the wholesale price. The manufacturer were allowed to choose the quality level with a degree of freedom, and the retailer decided the quantity to order accordingly.

The retailer chooses different supply chains for different products within the cushion category. According to the manufacturer, the retailer chooses the first type of contracts for innovative products with fashionable designs, for which the retailer is less sensitive to the price but demands
the manufacturer to provide products with the required features; the retailer chooses the second type of contracts for products with conventional designs, for which the retailer is sensitive about the wholesale price since these products are less differentiable from competitors’ products and face harsh price competition. We categorize the first type of contracts as CM, and the second type as ODM. We compare how different contracts performed under different demand uncertainty.

We estimate a logit model constructed based on our game-theoretical analysis. We find that demand uncertainty is a significant predictor of the retailer’s choice of supply chains. More specifically, we find that adjusted shipment in CM has a statistically significantly higher coefficient of variation than that in ODM. In other words, the retailer has a higher chance of choosing CM with products that have higher demand uncertainty.

We also estimate a linear regression model to investigate the effects of demand uncertainty on the manufacturer’s profit in both CM and ODM. However, we find that the effects are not statistically significant. It is possible that the insignificance of the results is due to limited data size. This requires further research based on larger sample size to confirm.

The rest of the paper is organized as follows. In Section 3.2, we present the data that we present the data, discuss the variables that we construct for this study, and provides some summary statistics. In Section 3.3 we present the conceptual framework. In Section 3.4, we demonstrate the estimation results. Section 3.5 concludes the paper with some discussions for future research. All proofs are provided in the Appendix.

3.2 Data

We use data collected from a Chinese manufacturer that supports a U.S. textile retailer. The production of textile products is highly labor-intensive, which is a typical case in many heavily outsourced industries. We focus on the cushion category. The dataset includes the transaction
records of the cushion products between the manufacturer and the retailer from 2011 to 2018, including wholesale price, cost, shipment, revenue, and profit.

To better understand and classify the outsourcing practices of the retailer, we acquire information from the manufacturer about how each order was put by the retailer and fulfilled by the manufacturer. In the dataset, there are two types of orders: CM order and ODM order. For an order in which the retailer set the desired features of the product (for example, density of the fabric, components of the dye, etc.) at the beginning of the negotiation and the manufacturer proposed a wholesale price accordingly, we classify it as a CM order. For an order in which the retailer proposed a practically non-negotiable wholesale price and allowed the manufacturer a degree of freedom to figure out the details of the product (i.e., the manufacturer could use relatively low-end fabrics), we classify it as ODM order. Thus, each order is assigned to either the CM group or the ODM group based on the negotiation process preceding that transaction.

Our analysis in the previous sections demonstrate that different demand uncertainty can lead to different outsourcing practices of the retailer and different profits of the manufacturer. Thus, we construct demand uncertainty as a potential relevant explanatory variable.

In this study, demand uncertainty is measured by the coefficient of variation of adjusted monthly shipment. First, we aggregate the transaction records into monthly data. Second, note that the difference of shipment in each month is partly caused by the fact that some months are high season and some are low season (for example, February usually has low shipment, and December usually has high shipment), we calculate the adjusted shipment with the month controlled. Third, we calculate the standard deviation of adjusted monthly shipment with in each year. Fourth, we calculate the coefficient of variation of adjusted monthly shipment, which is measured by the ratio of the standard deviation to the mean. The coefficient of variation is a better measurement of
demand uncertainty than the standard deviation in this case, because two variables with the same extent of variation but different means may have different standard deviations. In our dataset, ODM orders have higher average shipment than CM, so they may also have higher standard deviation even if the extent of variation is low in ODM than in CM. In order to avoid such ambiguity, we measure the demand uncertainty of a year by the coefficient of variation of adjusted monthly shipment in that year in this study.

Summary statistics of all the aforementioned variables for CM and ODM are provided in Table 3.2.1 and 3.2.2 respectively. Due to the limited space, only part of the sampled periods are covered in the tables.

**Table 3.2.1 Transactions in CM**

<table>
<thead>
<tr>
<th></th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shipment Mean</td>
<td>36516</td>
<td>72832</td>
<td>98967</td>
<td>61748</td>
<td>134250</td>
<td>91925</td>
</tr>
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<td>Shipment SD</td>
<td>35862</td>
<td>56130</td>
<td>103113</td>
<td>58082</td>
<td>96265</td>
<td>86516</td>
</tr>
<tr>
<td>Wholesale Price Mean</td>
<td>15.40</td>
<td>14.35</td>
<td>17.04</td>
<td>12.76</td>
<td>13.55</td>
<td>14.07</td>
</tr>
<tr>
<td>Wholesale Price SD</td>
<td>3.68</td>
<td>2.50</td>
<td>7.53</td>
<td>1.07</td>
<td>1.20</td>
<td>2.65</td>
</tr>
<tr>
<td>Unit Cost Mean</td>
<td>12.55</td>
<td>10.73</td>
<td>13.65</td>
<td>9.67</td>
<td>10.08</td>
<td>11.00</td>
</tr>
<tr>
<td>Unit Cost SD</td>
<td>3.37</td>
<td>1.30</td>
<td>6.07</td>
<td>0.74</td>
<td>0.83</td>
<td>2.19</td>
</tr>
<tr>
<td>Profit Mean</td>
<td>103477</td>
<td>272621</td>
<td>305129</td>
<td>211686</td>
<td>484861</td>
<td>283864</td>
</tr>
<tr>
<td>Profit SD</td>
<td>99778</td>
<td>265356</td>
<td>279859</td>
<td>225143</td>
<td>354082</td>
<td>278011</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>1.0974</td>
<td>0.5837</td>
<td>0.7846</td>
<td>0.8464</td>
<td>0.4536</td>
<td>0.7686</td>
</tr>
</tbody>
</table>

**Table 3.2.2 Transactions in ODM**

<table>
<thead>
<tr>
<th></th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shipment Mean</td>
<td>110695</td>
<td>157676</td>
<td>175894</td>
<td>149675</td>
<td>188937</td>
<td>165545</td>
</tr>
<tr>
<td>Shipment SD</td>
<td>88949</td>
<td>93467</td>
<td>136571</td>
<td>66031</td>
<td>90293</td>
<td>102983</td>
</tr>
<tr>
<td>Wholesale Price Mean</td>
<td>12.13</td>
<td>11.83</td>
<td>14.03</td>
<td>10.52</td>
<td>11.22</td>
<td>11.64</td>
</tr>
<tr>
<td>Wholesale Price SD</td>
<td>4.35</td>
<td>2.46</td>
<td>5.36</td>
<td>1.15</td>
<td>2.21</td>
<td>2.72</td>
</tr>
<tr>
<td>Unit Cost Mean</td>
<td>10.25</td>
<td>8.97</td>
<td>11.56</td>
<td>8.12</td>
<td>8.42</td>
<td>9.27</td>
</tr>
<tr>
<td>Unit Cost SD</td>
<td>3.68</td>
<td>1.73</td>
<td>4.79</td>
<td>0.72</td>
<td>1.35</td>
<td>2.22</td>
</tr>
<tr>
<td>Profit Mean</td>
<td>229142</td>
<td>504567</td>
<td>419039</td>
<td>396621</td>
<td>609451</td>
<td>439601</td>
</tr>
<tr>
<td>Profit SD</td>
<td>247670</td>
<td>431499</td>
<td>271900</td>
<td>239396</td>
<td>432413</td>
<td>351465</td>
</tr>
</tbody>
</table>
As one can see from Table 3.2.1 and 3.2.2, consistent with our expectation, the average wholesale price and unit cost (which measures the quality level) both have higher means in CM than in ODM. Although ODM has higher average standard deviation of adjusted shipment, this is mostly due to the fact that ODM oftentimes have higher shipment than CM. On the other hand, CM has higher average coefficient of variation, which indicates that demand uncertainty is higher in CM than in ODM. We will formally establish the correlation in the next section.

3.3 Statistical Model

Our previous analysis provides two major insights to the ODM under demand uncertainty. First, the retailer tends to choose ODM if demand uncertainty is low and choose CM if demand uncertainty is high. Second, In ODM, the manufacturer’s profit decreases with demand uncertainty; in CM, its profit increases with demand uncertainty. We validate these two predictions with different statistical models.

For the first prediction, we model the retailer’s outsourcing decisions using a logit model. In each transaction \( i \), the retailer makes the decision whether to choose ODM or CM. We denote the two possible values of the transaction by \( odm_j = 1 \) and \( odm_j = 0 \), respectively. Let \( cv_j \) be the coefficient of variation of adjusted shipment, which measures demand uncertainty in the model, in transaction \( j \). The logit model is represented using variable \( l_j \) as follow:

\[
l_j = cv_j \beta + u_j;
\]

\[
odm_j = \begin{cases} 
1, & l_j > 0 \\
0, & l_j < 0 
\end{cases}
\]
Note that we do not include cost or wholesale price as explanatory variables. In our analytical model, product quality (which determines cost) and wholesale price are decided after the retailer’s choice of supply chain. Thus, cost, wholesale price, and the manufacturer’s profit are dependent on $odm_j$ and cannot be explanatory variables of $odm_j$.

As predicted by the analytical model, the retailer is expected to respond to different demand uncertainty with different outsourcing decisions. Specifically, the retailer may choose ODM for products with low demand uncertainty and choose CM for products with high demand uncertainty. Thus, we expect demand uncertainty to be negatively correlated with $odm_j$, and its coefficient to be negative.

For the second decision, we model the manufacturer’s profit with a linear regression model. Beside demand uncertainty, we include wholesale price, cost, shipment as explanatory variables, which are denoted by $w_j$, $c_j$, $i_j$ respectively. Thus, the manufacturer’s profit function is as follows:

$$
\pi_j = \gamma_1 w_j + \gamma_2 c_j + \gamma_3 i_j + e_j
$$

Intuitively, the manufacturer’s profit should increase with wholesale price and shipment, and decrease with cost. Thus, we expect $\gamma_2$ and $\gamma_4$ to be positive and $\gamma_3$ to be negative. Moreover, given that our analysis predicts that the manufacturer’s profit will decrease with demand uncertainty in ODM and increase with it in CM, we expect $\gamma_1$ to be negative in ODM and positive in CM. We estimate the coefficients in ODM and CM separately.

3.4 Estimation Results

Table 3.3 presents the estimation results of the logit model of the retailer’s outsourcing practices.
Consistent with our expectation, the coefficient of demand uncertainty is negative and significant, i.e., the retailer is less likely to choose ODM for products with high demand uncertainty. This result demonstrates that in practice, the retailer may be, at least to some extent, aware that it has to choose the supply chain of each order based on demand uncertainty in order to maximize its expected profit. In times of low uncertainty, it can go with the “safer choice” of ODM, which leads to a product with low risk of overstocking in low state and limited profitability in high state; in times of high uncertainty, it has to take more risk and choose CM, which leads to a product with higher quality and higher cost, and can result in either large profit or large loss depending on the realized demand.

The estimation results of the linear regression model of the manufacturer’s profit in CM and ODM are provided in Table 3.4.1 and 3.4.2 respectively.

Table 3.4.1

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Standard Error</th>
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</thead>
<tbody>
<tr>
<td>Demand Uncertainty</td>
<td>-116898.4</td>
<td>77985.8</td>
</tr>
<tr>
<td>Wholesale Price</td>
<td>90869.6***</td>
<td>10276.6</td>
</tr>
<tr>
<td>Cost</td>
<td>-90629.7***</td>
<td>12529.4</td>
</tr>
<tr>
<td>Quantity</td>
<td>2.3902***</td>
<td>0.1492</td>
</tr>
<tr>
<td>Constant</td>
<td>-127225.7</td>
<td>85945.8</td>
</tr>
</tbody>
</table>

18 “*” for $p < 0.1$, “**” for $p < 0.05$, “***” for $p < 0.01$.

19 “*” for $p < 0.1$, “**” for $p < 0.05$, “***” for $p < 0.01$.  

75
Table 3.4.2

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Uncertainty</td>
<td>-156980.8</td>
<td>163998.1</td>
</tr>
<tr>
<td>Wholesale Price</td>
<td>155933.3***</td>
<td>13620.9</td>
</tr>
<tr>
<td>Cost</td>
<td>-145006.6***</td>
<td>16301.6</td>
</tr>
<tr>
<td>Quantity</td>
<td>2.0815***</td>
<td>0.1690</td>
</tr>
<tr>
<td>Constant</td>
<td>-323690.0***</td>
<td>77070.1</td>
</tr>
</tbody>
</table>

Consistent with our expectations, the manufacturer’s profit is positively correlated with wholesale price and shipment, and negatively correlated with cost. However, we do not find a statistically significant effect of demand uncertainty on the manufacturer’s profit. Given that the dataset is aggregated at monthly level, it is possible that the insignificance of the results is caused by limit of data size. This requires further research with a more comprehensive dataset to confirm.

3.5 Conclusion

Using data from a Chinese manufacturer that designs and produces cushions for a major U.S. textile retailer, we conduct an empirical investigation and study how demand uncertainty affects the retailer’s quality design outsourcing decisions, which in turn can affect the manufacturer’s profit.

Our analysis shows that demand uncertainty is a significant predictor of the retailer’s choice of supply chains. Specifically, we find that adjusted shipment in CM has a statistically significantly higher coefficient of variation than that in ODM. In other words, the retailer has a higher chance of choosing CM for products with higher demand uncertainty. On the other hand, we find that the effects of demand uncertainty on the manufacturer’s profit are not statistically significant. It is

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20 “*” for $p < 0.1$, “**” for $p < 0.05$, “***” for $p < 0.01$.  

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possible that the insignificance of the results is due to limited data size. This requires further research based on larger sample size to confirm.

Due to limited access to data, our empirical analysis has focused on the manufacturer’s side. With access to the retailer’s data, one may be able to conduct a more comprehensive analysis based on the analytical framework of this paper.
References


Appendix A

Additional Proofs and Analysis

A.1 Managerial Optimism in a Competitive Market

A.1.1 Proof of Lemma 1:

In the benchmark case, we focus only on the parameter region in which the market is fully covered in equilibrium and every consumer will buy a product. We solve for the equilibrium outcome by backward induction.

In the last stage of the game, the consumer located at $x_j$ in segment $j$ who makes a purchase from firm $i$ has a utility of

$$U(i, j) = q_i \theta_j - t d_i(x_j) - p_i.$$  

For the consumer who is indifferent between firm $a$ and firm $b$, her location $x_j$ is determined by

$$U(a, j) = U(b, j),$$

which is equivalent to:

$$q_a \theta_j - tx_j - p_a = q_b \theta_j - t (1 - x_j) - p_b.$$  

This equality gives us the point at which the two competing firms divide up the market:

$$x_j = \frac{t + (q_a - q_b) \theta_j - (p_a - p_b)}{2t}.$$  

In the previous stage (the second stage of the game), firm $i$’s manager maximizes profit:

$$\pi_i = (p_i - k q_i^2) \left( \alpha \frac{t + (q_a - q_b) \theta_H - (p_a - p_b)}{2t} + (1 - \alpha) \frac{t + (q_a - q_b) \theta_L - (p_a - p_b)}{2t} \right).$$
Due to the linearity of the demand function, it can be simplified to:

\[ \pi_i = (p_i - kq_i^2) \frac{t-(p_i-p-\bar{q})+(q_i-q-\bar{q})\bar{\theta}}{2t}, \]

where \( \bar{\theta} = \alpha \theta_H + (1 - \alpha) \theta_L = \alpha + (1 - \alpha) \theta_L. \)

Note that managers choose their prices in the second stage, so we take the derivative of firm \( i \)'s profit function with respect to \( p_i \) and set it equal to 0, which gives us:

\[ \frac{t-(2p_i-p-\bar{q})+(q_i-q-\bar{q})\bar{\theta}-kq_i^2}{2t} = 0. \]

Solving the two first-order conditions simultaneously gives us:

\[ p_i = t + \frac{1}{3}(q_i - q-\bar{q})\bar{\theta} + \frac{k}{3}(2q_i^2 + q_i^2). \]

This is how managers choose their prices conditional on their quality choices in the first stage.

Thus, in the first stage, firm \( i \)'s manager maximizes the profit function:

\[ \pi_i = (p_i - kq_i^2) \frac{t-(p_i-p-\bar{q})+(q_i-q-\bar{q})\bar{\theta}}{2t} = \frac{1}{2t} \left( t + \frac{1}{3}(q_i - q-\bar{q})\bar{\theta} + \frac{k}{3}(q_i^2 - q_i^2) \right)^2. \]

Note that firm \( i \)'s profit is given by its unit profit margin multiplied by the demand, where firm \( i \)'s unit profit margin is \( p_i - kq_i^2 = t + \frac{1}{3}(q_i - q-\bar{q})\bar{\theta} + \frac{k}{3}(q_i^2 - q_i^2) \) and its demand is \( \frac{(t-(p_i-p-\bar{q})+(q_i-q-\bar{q})\bar{\theta})}{2t} = \frac{t+\frac{1}{3}(q_i-q-\bar{q})\bar{\theta}+\frac{k}{3}(q_i^2-q_i^2)}{2t} \). Given that the firm’s unit profit margin and its demand (quantity sold) must be nonnegative at equilibrium, to maximize \( \pi_i \), we should restrict firm \( i \)'s quality choices (\( q_i \)) such that the term inside of quadratic of the profit function, i.e. \( t + \frac{1}{3}(q_i - q-\bar{q})\bar{\theta} + \frac{k}{3}(q_i^2 - q_i^2) \), must be positive. Thus, we just need to find the \( q_i \) value that gives a positive maximum for \( t + \frac{1}{3}(q_i - q-\bar{q})\bar{\theta} + \frac{k}{3}(q_i^2 - q_i^2) \). This gives us the unique equilibrium outcome: \( q_i^* = \frac{\bar{\theta}}{2k}, p_i^* = t + \frac{\bar{\theta}^2}{4k}, \) and \( \pi_i^* = \frac{t}{2} \).

Now we examine the conditions under which the market is fully covered in equilibrium in
the benchmark case. The consumer in segment $j$ who is indifferent from firm $i$ and the outside option has a utility function

$$U(i, j) = q_i \theta_j - td_i(x_j) - p_i = 0,$$

which leads to:

$$d_i(x_j) = \frac{q_i \theta_j - p_i}{t}.$$ 

For segment $j$ to be fully covered, we need

$$\sum_i d_i(x_j) = \sum_i \frac{q_i \theta_j - p_i}{t} \geq 1.$$ 

Note that $\sum_i \frac{q_i \theta_H - p_i}{t} \geq \sum_i \frac{q_i \theta_L - p_i}{t}$, so the high-valuation segment must also be fully covered if the low-valuation segment is fully covered, and only $\sum_i \frac{q_i \theta_L - p_i}{t} \geq 1$ is binding.

Applying $q^*_i = \frac{\theta}{2k}$ and $p^*_i = t + \frac{\theta^2}{4k}$ into the inequality, we get

$$\overline{\theta}(2\theta_L - \overline{\theta}) \geq 6kt,$$

which is equivalent with $(\alpha \theta_H + (1 - \alpha)\theta_L)(1 + \alpha)\theta_L - \alpha \theta_H \geq 6kt$. Given that $\theta_H = 1$, we can simplify the inequality to $\theta_L^2 - \alpha^2(1 - \theta_L)^2 \geq 6kt$. It is easy to see that the LHS is increasing with $\theta_L$ and decreasing with $\alpha$, and the RHS is increasing with $k$ and $t$.

A.1.2 Proof of Lemma 2:

Note that we assume $\tilde{\alpha} = 1$ here. When firm $a$’s manager is optimistic and firm $b$’s manager is realistic, if the market is fully covered, firm $a$’s manager maximizes:

$$\pi_a = (p_a - kq^2_a) \frac{t - (p_a - p_b) + (q_a - q_b)\theta_H}{2t},$$

where $\theta_H = 1$, and firm $b$’s manager maximizes:

$$\pi_b = (p_b - kq^2_b) \frac{t - (p_b - p_a) + (q_b - q_a)\overline{\theta}}{2t}.$$
Again, we solve for the equilibrium by backward induction. In the second stage of the game, managers simultaneously choose their respective prices taking both firms’ quality levels as given, which gives us

\[ p_a = t + \frac{1}{3} (q_a - q_b)(2\theta_H - \overline{\theta}) + \frac{k}{3} (2q_a^2 + q_b^2), \]
\[ p_b = t + \frac{1}{3} (q_b - q_a)(2\overline{\theta} - \theta_H) + \frac{k}{3} (2q_b^2 + q_a^2). \]

We then plug the price expressions into firms’ objective functions and solve for the optimal quality levels. Firm a’s manager maximizes

\[ \frac{(t + \frac{1}{3} (q_a - q_b)(2\theta_H - \overline{\theta}) + \frac{k}{3} (2q_a^2 + q_b^2) )^2}{2t} \]

and firm b’s manager maximizes

\[ \frac{(t + \frac{1}{3} (q_b - q_a)(2\overline{\theta} - \theta_H) + \frac{k}{3} (2q_b^2 + q_a^2) )^2}{2t}. \]

Similar to the proof of Lemma 1, since both unit profit margin and demand are non-negative, we can solve for the optimal quality level by maximizing

\[ t + \frac{1}{3} (q_a - q_b)(2\theta_H - \overline{\theta}) + \frac{k}{3} (2q_a^2 + q_b^2) \]
\[ t + \frac{1}{3} (q_b - q_a)(2\overline{\theta} - \theta_H) + \frac{k}{3} (2q_b^2 + q_a^2). \]

The equilibrium quality levels are:

\[ q_a^* = \frac{2\theta_H - \overline{\theta}}{2k}, \]
\[ q_b^* = \frac{2\overline{\theta} - \theta_H}{2k}. \]

The equilibrium prices are

\[ p_a^* = t + \frac{7\theta_H - 10\theta_H \overline{\theta} + 4\overline{\theta}^2}{4k}, \]
\[ p_b^* = t + \frac{7\overline{\theta}^2 - 10\theta_H \overline{\theta} + 4\theta_H^2}{4k}. \]

The corresponding equilibrium profits are:

\[ \pi_a^* = \frac{t^2 - (\frac{3}{4k} (\theta_H - \overline{\theta}))^2}{2t}, \]
\[ \pi_b^* = \frac{t^2 - (\frac{3}{4k} (\overline{\theta} - \theta_H))}{2t}. \]
\[
\pi^*_b = \frac{\left( t + \frac{3}{4k}(\theta_H - \tilde{\theta}) \right)^2}{2t}.
\]

Same as in the benchmark case, for the market to be fully covered in equilibrium, it is sufficient and necessary for the low-valuation segment of the market to be fully covered. This requires \( \sum_i q_i^0 \theta_L - p_i^0 \geq 1 \). Applying \( q_a^* = \frac{2\theta_H - \tilde{\theta}}{2k} \), \( q_b^* = \frac{\tilde{\theta} - \theta_H}{2k} \), \( p_a^* = t + \frac{7\theta_H^2 - 10\theta_H \tilde{\theta} + 4\tilde{\theta}^2}{4k} \) and \( p_b^* = t + \frac{\tilde{\theta}^2 - 10\theta_H \tilde{\theta} + 4\theta_H^2}{4k} \) into the inequality, we find it to be equivalent to:

\[
\frac{(\theta_H + \tilde{\theta}) \theta_L}{2k} - \frac{11\theta_H^2 - 20\theta_H \tilde{\theta} + 11\tilde{\theta}^2}{4k} \geq 3t.
\]

Simplification of this inequality gives us:

\[-(11 - 20\alpha + 11\alpha^2) + 2(11 - 20\alpha + 11\alpha^2)\theta_L - (9 - 20\alpha + 11\alpha^2)\theta_L^2 \geq 12kt.\]

Clearly, the RHS increases with \( k \) and \( t \). Moreover, differentiation of the LHS over \( \theta_L \) gives us \( 2(11 - 20\alpha + 11\alpha^2) + 2(-9 + 20\alpha - 11\alpha^2)\theta_L \), which is linear to \( \theta_L \). When \( \theta_L = 0 \), it equals to \( 22 - 40\alpha + 22\alpha^2 > 0 \); when \( \theta_L = 1 \), it equals to \( 4 > 0 \). Thus, given \( 0 < \theta_L < 1 \), we have \( 2(11 - 20\alpha + 11\alpha^2) + 2(-9 + 20\alpha - 11\alpha^2)\theta_L > 0 \) and the LHS is increasing with \( \theta_L \). Thus, the market will be fully covered in equilibrium if \( \theta_L \) is relatively large, or if the consumers’ sensitivity \( (t) \) to horizontal mismatch is low.

Taking the derivative of the LHS over \( \alpha \) gives us \( 20(1 - 2\theta_L + \theta_L^2) - 22\alpha (1 - 2\theta_L + \theta_L^2) > 0 \), so the LHS increases with \( \alpha \) when \( \alpha < \frac{10}{11} \) and decreases with \( \alpha \) when \( \alpha > \frac{10}{11} \). For the market to be fully covered, \( \alpha \) should be neither too high nor too low.

Note that we only consider two cases: both segments are fully covered, or the high-segment is fully covered but the low-segment is not. This is because given that the market is fully covered in the benchmark case, the high-valuation segment will in equilibrium always be fully covered under unilateral optimism. We show this by contradiction.
Suppose that the high segment is not fully covered under unilateral optimism. Then, the low-valuation segment must not be fully covered, either. As a result, both firms become local monopolists in both market segments.

Thus, firm $a$’s optimistic manager maximizes:

$$\pi_a = (p_a - kq_a^2)\frac{q_a\theta_H - p_a}{t},$$

where $\theta_H = 1$, and firm $b$’s realistic manager maximizes:

$$\pi_b = (p_b - kq_b^2)\frac{q_b\theta_H - p_b}{t}.$$ 

This gives us the equilibrium outcome:

$$q_a = \frac{1}{2k}, q_b = \frac{\theta}{2k};$$

$$p_a = \frac{3}{8k}, p_b = \frac{3\theta^2}{8k}.$$ 

Thus, the total coverage of the high-valuation segment is $\frac{q_a\theta_H - p_a}{t} + \frac{q_b\theta_H - p_b}{t} = \frac{1+4\alpha-3\alpha^2+2(1-\alpha)(2-3\alpha)\theta_L-3(1-\alpha)^2\theta_L^2}{8kt} < 1$. Note that for the market to be fully covered in the benchmark case (with both firms having realistic managers), we need $\frac{\theta_L^2 - \alpha^2(1-\theta_L)^2}{6kt} \geq 1$, so we have:

$$\frac{1+4\alpha-3\alpha^2+2(1-\alpha)(2-3\alpha)\theta_L-3(1-\alpha)^2\theta_L^2}{8kt} < 1 < \frac{\theta_L^2 - \alpha^2(1-\theta_L)^2}{6kt},$$

$$3(1 + 4\alpha - 3\alpha^2 + 2(1 - \alpha)(2 − 3\alpha)\theta_L − 3(1 − \alpha)^2\theta_L^2) < 4(\theta_L^2 − \alpha^2(1 − \theta_L)^2),$$

$$3 + 12\alpha − 5\alpha^2 + 2(6 − 15\alpha + 5\alpha^2)\theta_L − (1 − \alpha)(13 − 5\alpha)\theta_L^2 < 0.$$ 

Note that $\frac{\partial LHS}{\partial \alpha} = 2(1 - \theta_L)(6 - 5\alpha - (9 - 5\alpha)\theta_L) = 0$ if $\alpha = \frac{3(2-3\theta_L)}{5(1-\theta_L)}$. When $\theta_L \geq \frac{2}{3},$

$$\frac{\partial LHS}{\partial \alpha} \leq 0,$$ 

so the LHS reaches its minimum at $\alpha = 1$, where $LHS = 10 - 8\theta_L > 0$. When $\theta_L < \frac{2}{3},$

$$\frac{\partial LHS}{\partial \alpha} > 0 \text{ if } \alpha < \frac{3(2-3\theta_L)}{5(1-\theta_L)} \text{ and } \frac{\partial LHS}{\partial \alpha} < 0 \text{ if } \alpha > \frac{3(2-3\theta_L)}{5(1-\theta_L)}.$$ 

Thus, the LHS reaches its minimum at $\alpha$ = 86.
0, where \( LHS = 3 + 12\theta_L - 13\theta_L^2 > 0 \) or at \( \alpha = 1 \), where \( LHS = 10 - 8\theta_L > 0 \). Therefore, we always have \( LHS > 0 \), which leads to a contradiction. As a result, in equilibrium, the high-valuation segment must always be fully covered under unilateral optimism.

**A.1.3 Proof of Proposition 1:**

As shown in the proof of Lemma 2, if the market is fully covered under unilateral optimism, the optimistic manager of firm \( a \) will choose the quality level:

\[
q^*_a = \frac{2\theta_H - \theta}{2k} = \frac{2 - \theta}{2k} > \frac{\theta}{2k}.
\]

The realistic manager of firm \( b \) chooses:

\[
q^*_b = \frac{2\theta_H - \theta}{2k} = \frac{2\theta - \theta_H}{2k} < \frac{\theta}{2k}.
\]

If only the high-valuation segment is fully covered under optimism and the low-valuation segment is partially covered, firm \( b \)’s manager maximizes the profit function:

\[
\pi_b = (p_b - kq_b^2) \left( \alpha \left( \frac{t - (p_b - p_a) + (q_b - q_a)\theta_H}{2t} \right) + (1 - \alpha) \frac{q_b\theta_L - p_b}{t} \right).
\]

Firm \( a \)’s manager, who believes that all consumers have high valuations, will maximize:

\[
\pi_a = (p_a - kq_a^2) \left( \frac{t - (p_a - p_b) + (q_a - q_b)\theta_H}{2t} \right).
\]

Applying the backward induction again gives us

\[
q^*_a = \frac{\theta_H}{2k} = \frac{1}{2k};
\]

\[
q^*_b = \frac{\alpha\theta_H + 4(1 - \alpha)\theta_L}{2(4 - 3\alpha)k} = \frac{\alpha + 4(1 - \alpha)\theta_L}{2(4 - 3\alpha)k}.
\]

While it is apparent that \( \frac{1}{2k} > \frac{\theta}{2k} \), one can also readily show that \( \frac{\alpha + 4(1 - \alpha)\theta_L}{2(4 - 3\alpha)k} - \frac{\theta}{2k} = \frac{3\alpha(1 - \alpha)(\theta_L - 1)}{2(4 - 3\alpha)k} < 0 \), so when firm \( a \) has an optimistic manager and firm \( b \) has a realistic manager, firm \( a \)’s quality level will increase and firm \( b \)’s quality level will decrease, compared with the
benchmark case with both firms having realistic managers.

**A.1.4 Proof of Proposition 2:**

Recall that in the benchmark case, \( p^*_a = p^*_b = t + \frac{\theta^2}{4k} \).

When firm \( \alpha \)'s manager is optimistic, if the market is fully covered in equilibrium, we have:

\[
p^*_a = t + \frac{7\theta^2_{H} - 10\theta_H\bar{\theta} + 4\bar{\theta}^2}{4k} = t + \frac{7-10\bar{\theta} + 4\bar{\theta}^2}{4k},
\]

\[
p^*_b = t + \frac{7\theta^2_{L} - 10\theta_L\bar{\theta} + 4\bar{\theta}^2}{4k} = t + \frac{7\theta^2_{L} - 10\bar{\theta} + 4\bar{\theta}^2}{4k},
\]

If the market is not fully covered:

\[
p^*_a = \frac{1}{8-5\alpha}(4 - \alpha)t + \frac{1}{4(4-3\alpha)^2k}((192 - 448\alpha + 350\alpha^2 - 91\alpha^3) - 2(1 - \alpha)(64 - 106\alpha + 43\alpha^2)\theta_L + 32(3 - 2\alpha)(1 - \alpha)^2\theta_L^2).
\]

\[
p^*_b = \frac{1}{8-5\alpha}(3\alpha t + \frac{1}{4(4-3\alpha)^2k}(-\alpha(16 - 36\alpha + 17\alpha^2) + 2\alpha(1 - \alpha)(34 - 25\alpha)\theta_L + 32(1 - \alpha)^2(5 - 3\alpha)\theta_L^2).
\]

We first consider the case of full coverage.

The change of firm \( \alpha \)'s price is:

\[
\Delta p^*_a = (t + \frac{7\theta^2_{H} - 10\theta_H\bar{\theta} + 4\bar{\theta}^2}{4k}) - \left( t + \frac{\bar{\theta}^2}{4k} \right) = \frac{(7\theta_H - 3\bar{\theta})(\theta_H - \bar{\theta})}{4k} = \frac{(7-3\bar{\theta})(1-\bar{\theta})}{4k} > 0.
\]

So, firm \( \alpha \)'s price will increase unambiguously under unilateral optimism.

The change of firm \( b \)'s price is:

\[
\Delta p^*_b = (t + \frac{7\theta^2_{L} - 10\theta_L\bar{\theta} + 4\bar{\theta}^2}{4k}) - \left( t + \frac{\bar{\theta}^2}{4k} \right) = \frac{(4\theta_L - 6\bar{\theta})(\theta_L - \bar{\theta})}{4k} = \frac{(4-6\bar{\theta})(1-\bar{\theta})}{4k}.
\]
Given $\bar{\theta} < 1$, firm $b$’s price will increase if $\bar{\theta} < \frac{2}{3}$ and decrease otherwise. However, recall that $\bar{\theta} = \alpha + (1 - \alpha)\theta_L$, so $\bar{\theta} < \frac{2}{3}$ is equivalent to $\theta_L < \frac{2 - 3\alpha}{3(1 - \alpha)}$. As demonstrated in Lemma 2, the condition for the market to be fully covered under unilateral optimism is:

$$-(11 - 20\alpha + 11\alpha^2) + 2(11 - 20\alpha + 11\alpha^2)\theta_L - (9 - 20\alpha + 11\alpha^2)\theta_L^2 \geq 12kt.$$  

Recall that the LHS is increasing with $\theta_L$, so substituting $\theta_L < \frac{2 - 3\alpha}{3(1 - \alpha)}$ into the LHS gives us:

$$-(11 - 20\alpha + 11\alpha^2) + 2(11 - 20\alpha + 11\alpha^2)\theta_L - (9 - 20\alpha + 11\alpha^2)\theta_L^2 < \frac{3 + 7\alpha}{9(\alpha - 1)} < 0 < 12kt.$$  

Thus, given that the market is fully covered under unilateral optimism, firm $b$’s price will not increase.

Now, we consider the case of partial coverage. In this case, note that the difference between the two firms’ prices is:

$$p_a^* - p_b^* = \frac{1}{4(4 - 3\alpha)^2(8 - 5\alpha)k} (1 - \alpha) \left( 8(4 - 3\alpha)^2kt + \left( 37 - 17\theta_L - 16\theta_L^2 \right) \alpha^2 - 24(5 - 3\theta_L - 2\theta_L^2)\alpha + 32(3 - 2\theta_L - \theta_L^2) \right) > 0.$$  

So, as long as firm $b$’s price under optimism is higher than that in the benchmark case, firm $a$’s price is higher than that in the benchmark case as well. Thus, in order to show that both firms’ prices can increase under unilateral optimism, we only have to show that firm $b$’s price can increase.

When firm $a$ has an optimistic manager and the market is not fully covered under optimism, the change of firm $b$’s price is:

$$\Delta_{p_b'} = \frac{1}{8 - 5\alpha} \left( 3at + \frac{1}{4(4 - 3\alpha)^2k} \left( -\alpha(16 - 36\alpha + 17\alpha^2) + 2\alpha(1 - \alpha)(34 - 25\alpha)\theta_L + 32(1 -$$
\[
\alpha^2(5 - 3\alpha)\theta_L^2) - (t + \frac{\theta}{4k}) = \frac{1}{\alpha - 5\alpha} \left(3\alpha t + \frac{1}{4(4-3\alpha)^2k} (-\alpha(16 - 36\alpha + 17\alpha^2) + 2\alpha(1 - \\
\alpha)(34 - 25\alpha)\theta_L + 32(1 - \alpha)^2(5 - 3\alpha)\theta_L^2) - (t + \frac{(\alpha + (1 - \alpha)\theta_L^2)}{4k}).
\]

For firm b’s price to increase under optimism, we need \(\Delta_{p_b} > 0\), which is equivalent to
\[
(1 - \alpha)(15(1 - \theta_L)^2\alpha^3 - (12 - 38\theta_L - 37\theta_L^2 + 288kt)\alpha^2 - 4(4 - \theta_L + 45\theta_L^2 - 192kt)\alpha + 128(\theta_L^2 - 4kt)) > 0.
\]

Solving this equation gives us:
\[
k > 0,
\]
\[
0 < \alpha < \frac{1}{21} (32 - 8\sqrt{2}),
\]
\[
\frac{-2\alpha - 19\alpha^2 + 15\alpha^3 + \sqrt{2048\alpha - 1340\alpha^2 - 3412\alpha^3 + 3685\alpha^4 - 945\alpha^5}}{128 - 180\alpha + 37\alpha^2 + 15\alpha^3} < \theta_L < 1,
\]
\[
0 < t < \frac{(-16 + 12\alpha - 15\alpha^2) + (4 + 38\alpha - 30\alpha^2)\theta_L - (180 - 37\alpha - 15\alpha^2)\theta_L^2)\alpha}{(512 - 768\alpha + 288\alpha^2)k},
\]

Note that the set of parameters satisfying these inequalities and the coverage conditions is non-empty. We have shown that firm b’s price, and thus firm a’s price, can both increase under unilateral optimism when the market is not partially covered. This completes the proof of Proposition 2.

A.1.5 Proof of Proposition 3:

Note that according to Proof of Lemma 2, when the market is fully covered under unilateral optimism, firm a’s profit is:
\[
\pi^*_a = \frac{t^2 - \left(\frac{3}{4k}(\theta_H - \theta)^2\right)^2}{2t} < \frac{t}{2}.
\]

Firm a’s profit in this case is less than that in the benchmark case \(\frac{t}{2}\), so there cannot be a win-win equilibrium outcome if the market is fully covered under optimism.
Next, we consider the case in which the market is partially covered under optimism.

Note that firm $a$’s total profit includes two parts: profits from high-valuation consumers, and profits from low-valuation consumers. The profits from high-valuation segment are $\alpha (p_a - kq_a^2)^{t - (p_a - p_b) + (a - q_b)} = \frac{\alpha}{2t} \left( \frac{1}{8 - 5\alpha} \left( (4 - \alpha)t + \frac{1}{4(4 - 3\alpha)^2 k} (2(3 - 2\alpha)(32 - 56\alpha + 25\alpha^2) - 2(1 - \alpha)(64 - 106\alpha + 43\alpha^2)\theta_L + 64(2 - \alpha)(1 - \alpha^2)\theta_L^2) \right) \right)^2$. Thus, we have:

$$\pi_a^* \geq \frac{\alpha}{2t} \left( \frac{1}{8 - 5\alpha} \left( (4 - \alpha)t + \frac{1}{4(4 - 3\alpha)^2 k} (2(3 - 2\alpha)(32 - 56\alpha + 25\alpha^2) - 2(1 - \alpha)(64 - 106\alpha + 43\alpha^2)\theta_L + 64(2 - \alpha)(1 - \alpha^2)\theta_L^2) \right) \right)^2.$$

Hence, for firm $a$ to be better off under optimism, it is sufficient to have $\frac{\alpha}{2t} \left( \frac{1}{8 - 5\alpha} \left( (4 - \alpha)t + \frac{1}{4(4 - 3\alpha)^2 k} (2(3 - 2\alpha)(32 - 56\alpha + 25\alpha^2) - 2(1 - \alpha)(64 - 106\alpha + 43\alpha^2)\theta_L + 64(2 - \alpha)(1 - \alpha^2)\theta_L^2) \right) \right)^2 > \frac{t}{2}$, which is equivalent to:

$$\frac{1}{8 - 5\alpha} \left( (4 - \alpha)t + \frac{1}{4(4 - 3\alpha)^2 k} (2(3 - 2\alpha)(32 - 56\alpha + 25\alpha^2) - 2(1 - \alpha)(64 - 106\alpha + 43\alpha^2)\theta_L + 64(2 - \alpha)(1 - \alpha^2)\theta_L^2) \right) > t\sqrt{\alpha},$$

$$2(3 - 2\alpha)(32 - 56\alpha + 25\alpha^2) - 2(1 - \alpha)(64 - 106\alpha + 43\alpha^2)\theta_L + 64(2 - \alpha)(1 - \alpha^2)\theta_L^2 >$$
\[4(4 - 3\alpha)^2 \left( (8 - 5\alpha)\sqrt{\alpha} - (4 - \alpha) \right) kt,\]
\[t < t^*_a = \frac{2(3-2\alpha)(32-56\alpha+25\alpha^2)-2(1-\alpha)(64-106\alpha+43\alpha^2)\theta_L + 64(2-\alpha)(1-\alpha)^2 \theta_L^2}{4(4-3\alpha)^2 \left( (8-5\alpha)\sqrt{\alpha} - (4 - \alpha) \right) k} .\]

Firm \( b \)'s profit is:
\[\pi^*_b = \frac{2-\alpha}{2t} \left( \frac{1}{8-5\alpha} \left( 3\alpha t + \frac{1}{(4-3\alpha)k} (-\alpha(1 - \alpha) + 2\alpha(1 - \alpha)\theta_L + 4(1 - \alpha)^2 \theta_L^2) \right) \right)^2 ,\]

For firm \( b \) to be better off, we need:
\[\Delta \pi_b = \frac{2-\alpha}{2t} \left( \frac{1}{8-5\alpha} \left( 3\alpha t + \frac{1}{(4-3\alpha)k} (-\alpha(1 - \alpha) + 2\alpha(1 - \alpha)\theta_L + 4(1 - \alpha)^2 \theta_L^2) \right) \right)^2 - \frac{t}{2} > 0.\]

This is equivalent to:
\[\frac{1}{8-5\alpha} \left( 3\alpha t + \frac{1}{(4-3\alpha)k} (-\alpha(1 - \alpha) + 2\alpha(1 - \alpha)\theta_L + 4(1 - \alpha)^2 \theta_L^2) \right) > \frac{t}{\sqrt{2-\alpha}},\]
\[\frac{1}{(4-3\alpha)k} (-\alpha(1 - \alpha) + 2\alpha(1 - \alpha)\theta_L + 4(1 - \alpha)^2 \theta_L^2) > \left( \frac{8-5\alpha}{\sqrt{2-\alpha}} - 3\alpha \right) t ,\]
\[t < t^*_b = \frac{(-\alpha(1-\alpha)+2\alpha(1-\alpha)\theta_L+4(1-\alpha)^2 \theta_L^2)\sqrt{2-\alpha}}{(4-3\alpha)(8-5\alpha-3\alpha\sqrt{2-\alpha})k} .\]

Let \( t^* = \min\{t^*_a, t^*_b\} \), then we have both firms will be better off under optimism if \( t < t^* \).

**A.1.6 Proof of Proposition 4:**

When both firms have optimistic managers, firm \( i \)'s manager maximizes:
\[\pi_i = (p_i - kq_i^2) t-(p_i-p_{-i})+q_i^2)\theta_H = \frac{t-(p_i-p_{-i})+q_i^2}{2t}.\]

Just as in the benchmark case, we solve for the equilibrium through backward induction and get:
\[q_i^* = \frac{1}{2k},\]
\[p_i^* = t + \frac{1}{4k}.\]
If the market is fully covered under bilateral optimism, firm $i$’s realized profit is:

$$
\pi_i = (p_i - kq_i^2) \left( t - (p_i - p_{-i}) + (q_i - q_{-i}) \tilde{g} \right).
$$

Substituting $q_i$ and $p_i$ gives us:

$$
\pi_i^* = \frac{t}{2}.
$$

Thus, firm $i$’s profits are the same as in the benchmark case if the market is fully covered under bilateral optimism.

Next, we consider the case in which the market is partially covered. Since neither firm’s perceived profit function has changed, we still get $q_i^* = \frac{1}{2k}$ and $p_i^* = t + \frac{1}{4k}$ in this case. Firm $i$’s profit is:

$$
\pi_i = (p_i - kq_i^2) \left( \alpha \frac{t - (p_i - p_{-i}) + (q_i - q_{-i}) \theta_H}{2t} + (1 - \alpha) \frac{q_i \theta_L - p_i}{t} \right).
$$

Substituting $q_i^* = \frac{1}{2k}$ and $p_i^* = t + \frac{1}{4k}$ gives us:

$$
\pi_i^* = t \left( \alpha \frac{1}{2k} + (1 - \alpha) \frac{q_i \theta_L - p_i}{t} \right).
$$

Given that the low-valuation segment is not fully covered, we have:

$$
\sum_i \frac{q_i \theta_L - p_i}{t} < 1.
$$

Given the symmetry of the equilibrium outcome, this gives us:

$$
\frac{q_i \theta_L - p_i}{t} < \frac{1}{2}.
$$

Substituting this inequality into the profit function gives us:

$$
\pi_i^* = t \left( \alpha \frac{1}{2k} + (1 - \alpha) \frac{q_i \theta_L - p_i}{t} \right) < t \left( \alpha \frac{1}{2} + (1 - \alpha) \frac{1}{2} \right) = \frac{t}{2}.
$$

Thus, firm $i$’s profit is lower than that in the benchmark case. Both firms will be worse off.

In summary, both firms will be weakly worse off under bilateral optimism.

A.1.7 Analysis of Bilateral Optimism as a Possible Equilibrium:
Here we show that \((O,O)\) cannot be an equilibrium outcome when firms endogenously choose which type of managers to hire.

For \((O,O)\) to be an equilibrium, we need \(\pi_b^*\) to be higher in \((O,O)\) than in \((O,R)\). Note that in \((O,R)\), if the market is fully covered, we have \(\pi_b^* = \frac{(t+\frac{3}{4k}(\theta_H-\bar{\theta}))^2}{2t} > \frac{t}{2}\); meanwhile, Proposition 4 states \(\pi_b^* \leq \frac{t}{2}\) in \((O,O)\). Thus, if the market is fully covered in \((O,R)\), firm \(b\) will be worse off by switching to \((O,O)\), and \((O,O)\) cannot be an equilibrium. Therefore, \((O,O)\) can possibly be an equilibrium outcome only if the market is not fully covered in \((O,R)\). This leaves us with two cases: (1) the market is fully covered in \((O,O)\), and partially covered in \((O,R)\); (2) the market is partially covered in \((O,O)\) and \((O,R)\).

Case 1. Market is fully covered in \((O,O)\), and partially covered in \((O,R)\).

In \((O,O)\), \(q_i^* = \frac{1}{2k}, p_i^* = t + \frac{1}{4k}, \pi_i^* = \frac{t}{2}\). For the market to be fully covered, we need
\[
\frac{q_i^* \theta_L - p_i^*}{t} > \frac{1}{2},
\]
which leads to \(\frac{1}{2} < \theta_L < 1\) and \(0 < kt < \frac{2\theta_L-1}{6}\).

In \((O,R)\), for the market to be partially covered, we need \(\sum_l q_l^* \theta_L - p_l^* t < 1\), which is
\[
\frac{1}{8-5\alpha}
\left(-2(2 + \alpha) + \frac{-96+232\alpha-193\alpha^2+54\alpha^3+2(96-232\alpha+193\alpha^2-54\alpha^3)\theta_L+4(-4+20\alpha-25\alpha^2+9\alpha^3)\theta_L^2}{2kt(4-3\alpha)^2}\right) < 1.
\]
This gives us \(kt > \frac{-96+232\alpha-193\alpha^2+54\alpha^3+2(96-232\alpha+193\alpha^2-54\alpha^3)\theta_L+4(-4+20\alpha-25\alpha^2+9\alpha^3)\theta_L^2}{6(4-3\alpha)^2(4-\alpha)}\). Given that \(kt < \frac{2\theta_L-1}{6}\), we have
\[
\frac{-96+232\alpha-193\alpha^2+54\alpha^3+2(96-232\alpha+193\alpha^2-54\alpha^3)\theta_L+4(-4+20\alpha-25\alpha^2+9\alpha^3)\theta_L^2}{6(4-3\alpha)^2(4-\alpha)} < \frac{2\theta_L-1}{6},
\]
which gives us \(\frac{1}{2} < \theta_L < 2 - \sqrt{2}\) and \(0 < \alpha < \frac{4(11-22\theta_L+8\theta_L^2-\sqrt{31-124\theta_L+183\theta_L^2-318\theta_L^3+28\theta_L^4})}{9(5-10\theta_L+4\theta_L^2)}\).
\[
\frac{\partial}{\partial \theta_L} \left( 4 \left( 11 - 22 \theta_L + 8 \theta_L^2 - \sqrt{31 - 124 \theta_L + 183 \theta_L^2 - 118 \theta_L^3 + 28 \theta_L^4} \right) \right) < 0, \text{ so when } \theta_L = \frac{1}{2}, \text{ LHS takes its maximum where } \frac{4 \left( 11 - 22 \theta_L + 8 \theta_L^2 - \sqrt{31 - 124 \theta_L + 183 \theta_L^2 - 118 \theta_L^3 + 28 \theta_L^4} \right)}{9(5 - 10 \theta_L + 4 \theta_L^2)} = \frac{8 - 2 \sqrt{7}}{9}. \text{ Thus, for the market to be fully covered in } (O, R), \text{ the maximum value } \alpha \text{ can take is } \frac{8 - 2 \sqrt{7}}{9}. 
\]

Moreover, in \((O, R)\), \(\pi^*_b = \frac{2 - \alpha}{2t} \left( \frac{1}{8 - 5 \alpha} \left( 3at + \frac{1}{(4 - 3 \alpha)k} (-\alpha(1 - \alpha) + 2\alpha(1 - \alpha)\theta_L + 4(1 - \alpha)^2 \theta_L^2) \right) \right)^2 \). For firm \(b\) to be better off in \((O, O)\) than in \((O, R)\), we need \(\frac{2 - \alpha}{2t} \left( \frac{1}{8 - 5 \alpha} \left( 3at + \frac{1}{(4 - 3 \alpha)k} (-\alpha(1 - \alpha) + 2\alpha(1 - \alpha)\theta_L + 4(1 - \alpha)^2 \theta_L^2) \right) \right)^2 < \frac{t}{2}, \text{ which (given } \frac{1}{2} < \theta_L < 2 - \sqrt{2}) \text{ gives us } \frac{kt}{6 - 5 \alpha} > \frac{(-\alpha(1 - \alpha) + 2\alpha(1 - \alpha)\theta_L + 4(1 - \alpha)^2 \theta_L^2)\sqrt{2 - \alpha}}{(4 - 3 \alpha)(8 - 5 \alpha - 3 \alpha \sqrt{2 - \alpha})}.

Given that \(kt < \frac{2\theta_L - 1}{6}\), we have \(\frac{(-\alpha(1 - \alpha) + 2\alpha(1 - \alpha)\theta_L + 4(1 - \alpha)^2 \theta_L^2)\sqrt{2 - \alpha}}{(4 - 3 \alpha)(8 - 5 \alpha - 3 \alpha \sqrt{2 - \alpha})} < \frac{kt}{6 - 5 \alpha} < \frac{2\theta_L - 1}{6}. \) Note that \(\frac{\partial \text{LHS}}{\partial \alpha} < 0\), so LHS is decreasing with \(\alpha\). Recall that \(\alpha < \frac{8 - 2 \sqrt{7}}{9}\), so

\[
\frac{(-\alpha(1 - \alpha) + 2\alpha(1 - \alpha)\theta_L + 4(1 - \alpha)^2 \theta_L^2)\sqrt{2 - \alpha}}{(4 - 3 \alpha)(8 - 5 \alpha - 3 \alpha \sqrt{2 - \alpha})} \text{ takes its minimum when } \alpha = \frac{8 - 2 \sqrt{7}}{9}, \text{ where we have}
\]

\[
\frac{(-\alpha(1 - \alpha) + 2\alpha(1 - \alpha)\theta_L + 4(1 - \alpha)^2 \theta_L^2)\sqrt{2 - \alpha}}{(4 - 3 \alpha)(8 - 5 \alpha - 3 \alpha \sqrt{2 - \alpha})} - \frac{2\theta_L - 1}{6} = \frac{1}{18} \left( 3 - 6c - \frac{2\theta_L - 1}{6} \right) \frac{(-\alpha(1 - \alpha) + 2\alpha(1 - \alpha)\theta_L + 4(1 - \alpha)^2 \theta_L^2)\sqrt{2 - \alpha}}{(4 - 3 \alpha)(8 - 5 \alpha - 3 \alpha \sqrt{2 - \alpha})},
\]

which is decreasing with \(\theta_L\). Thus, when \(\theta_L = 2 - \sqrt{2}\), it is at minimum where \(\frac{(-\alpha(1 - \alpha) + 2\alpha(1 - \alpha)\theta_L + 4(1 - \alpha)^2 \theta_L^2)\sqrt{2 - \alpha}}{(4 - 3 \alpha)(8 - 5 \alpha - 3 \alpha \sqrt{2 - \alpha})} - \frac{2\theta_L - 1}{6} = 0.027339 > 0\).
As a result, as long as $\alpha < \frac{8-2\sqrt{7}}{9}$ (which is the necessary condition for the market to be partially covered in $(O,O)$), we have
$$\frac{(-\alpha(1-\alpha)+2\alpha(1-\alpha)\theta_{L}+4(1-\alpha)^2\theta_{L}^2)(\sqrt{2}-\alpha)}{(4-3\alpha)(8-5\alpha+3\alpha\sqrt{2-\alpha})} > \frac{2\theta_{L}-1}{6}$$
and firm $b$ is worse off in $(O,O)$ than in $(O,R)$. Therefore, $(O,O)$ cannot be an equilibrium if the market is fully covered in $(O,O)$, and partially covered in $(O,R)$.

Case 2. The market is partially covered in $(O,O)$ and $(O,R)$.

Note that in this case, when the market is partially covered in $(O,O)$, it is possible that the low-valuation segment is partially covered or not covered at all (low-valuation consumers do not make purchases). We analyze both cases separately.

In the first case, the low-valuation segment is partially covered in $(O,O)$.

In $(O,O)$, for the market to be partially covered, we need $\frac{q_i^\alpha \theta_p - p_i}{t} < \frac{1}{2}$, which leads to $kt > g_1 = \frac{2\theta_{L}-1}{6}$.

In $(O,R)$, for the market to be partially covered, we need $\sum_l \frac{q_i^\alpha \theta_p - p_i}{t} < 1$. As we have already shown, this gives us:

$$kt > g_2 = \frac{-96+232\alpha-193\alpha^2+54\alpha^3+\alpha(16-36\alpha+17\alpha^2)-2\alpha(16-36\alpha+17\alpha^2)\theta_{L}+8(8-12\alpha+3\alpha^2+\alpha^3)\theta_{L}^2}{6(4-3\alpha)(4-\alpha)}.$$ 

Meanwhile, we also need $\frac{q_i^\alpha \theta_p - p_i}{t} < 1$, which is $\frac{1}{8-5\alpha} \left(-3\alpha + \frac{\alpha(16-36\alpha+17\alpha^2)-2\alpha(16-36\alpha+17\alpha^2)\theta_{L}+8(8-12\alpha+3\alpha^2+\alpha^3)\theta_{L}^2}{4\alpha(4-3\alpha)^2}\right) < 1$. This leads to $kt > g_3 = \frac{-96+232\alpha-193\alpha^2+54\alpha^3+\alpha(16-36\alpha+17\alpha^2)-2\alpha(16-36\alpha+17\alpha^2)\theta_{L}+8(8-12\alpha+3\alpha^2+\alpha^3)\theta_{L}^2}{8(4-\alpha)(4-3\alpha)^2}.$

In $(O,O)$, $\pi_b = \alpha\frac{t}{2} + (1 - \alpha) t \left(\frac{2\theta_{L}-1}{4kt} - 1\right) = \frac{-1-2\alpha+2(1-\alpha)\theta_{L}}{4k}$. For firm $b$ to be better off in $(O,O)$ than in $(O,R)$, we need $\frac{-1-2\alpha+2(1-\alpha)\theta_{L}}{4k} > \frac{2-\alpha}{2t} \left(\frac{1}{8-5\alpha} \left(3\alpha + \frac{-96+232\alpha-193\alpha^2+54\alpha^3+\alpha(16-36\alpha+17\alpha^2)-2\alpha(16-36\alpha+17\alpha^2)\theta_{L}+8(8-12\alpha+3\alpha^2+\alpha^3)\theta_{L}^2}{8(4-\alpha)(4-3\alpha)^2}\right)\right)$.
\[
\frac{1}{(4-3\alpha)k} \left( -\alpha(1-\alpha) + 2\alpha(1-\alpha)\theta_L + 4(1-\alpha)^2\theta_L^2 \right) \right)^2. \]

Solving the inequality gives us: (1) when \( \alpha > \frac{4(7-\sqrt{7})}{21} \), \( kt > g_4 = \frac{1}{16(4-3\alpha)(-32+56\alpha-21\alpha^2)} \left( 256 - 512\alpha + 316\alpha^2 - 63\alpha^3 - 2(256 - 512\alpha + 316\alpha^2 - 63\alpha^3)\theta_L + 48\alpha(1-\alpha)(2-\alpha)\theta_L^2 + (8-5\alpha)(1024-2816\alpha + 2640\alpha^2 - 952\alpha^3 + 105\alpha^4)\theta_L + 4(1024-2496\alpha + 1824\alpha^2 - 288\alpha^3 - 63\alpha^4)\theta_L^2 + 64\alpha(40-102\alpha + 83\alpha^2 - 21\alpha^3)\theta_L^3 + 256(1-\alpha)^2(4-8\alpha + 3\alpha^2)\theta_L^4 \right) \) \( < kt < g_6 = \frac{1}{16(4-3\alpha)(32-56\alpha+21\alpha^2)} \left( -256 + 512\alpha - 316\alpha^2 + 63\alpha^3 + 2(256 - 512\alpha + 316\alpha^2 - 63\alpha^3)\theta_L - 48\alpha(1-\alpha)(2-\alpha)\theta_L^2 + (8-5\alpha)(1024-2816\alpha + 2640\alpha^2 - 952\alpha^3 + 105\alpha^4)\theta_L + 4(1024-2496\alpha + 1824\alpha^2 - 288\alpha^3 - 63\alpha^4)\theta_L^2 + 64\alpha(40-102\alpha + 83\alpha^2 - 21\alpha^3)\theta_L^3 + 256(1-\alpha)^2(4-8\alpha + 3\alpha^2)\theta_L^4 \right) \).

Combining these conditions together, we have \( kt > g^* = \max\{g_1, g_2, g_3, g_4\} \) when \( \alpha > \frac{4(7-\sqrt{7})}{21} \), and \( kt > g^{**} = \max\{g_1, g_2, g_3, g_5\} \) when \( \alpha < \frac{4(7-\sqrt{7})}{21} \). However, for the market to be fully covered in the benchmark case, we need \( kt \leq g_0 = \theta_L^2 + \frac{\alpha^2(1-\theta_L)^2}{6} \), where \( g_0 < g^* \) and \( g_0 < g^{**} \). Thus, firm \( b \) is worse off in \( (O, O) \) than in \( (O, R) \) if the low-valuation segment is partially covered in \( (O, O) \). Therefore, \( (O, O) \) cannot be an equilibrium in this case.

In the second case, the low-valuation segment is not covered in equilibrium in \( (O, O) \).
In \((O, O)\), for the low-valuation segment to be not covered, we need \(q_i^\ast \theta_k - p_i^\ast < 0\), which leads to \(kt > h_1 = \frac{2\theta_k - 1}{4}\).

In \((O, R)\), the market coverage conditions still require \(kt > g_2\) and \(kt > g_3\).

In \((O, O)\), firm \(b\)’s profit is \(\pi_i^\ast = \alpha^t\), so for firm \(b\) to be better off in \((O, O)\) than in \((O, R)\), we need \(\frac{2 - \alpha}{2t} \left( \frac{1}{8 - 5\alpha} \left( 3\alpha t + \frac{1}{(4 - 3\alpha)k} \left( -\alpha (1 - \alpha) + 2\alpha (1 - \alpha) \theta_k + 4 (1 - \alpha)^2 \theta_k^2 \right) \right) \right)^2 < \alpha^t\), which gives us \(kt > h_2 = \frac{(-\alpha (1 - \alpha) + 2\alpha (1 - \alpha) \theta_k + 4 (1 - \alpha)^2 \theta_k^2) \sqrt{2 - \alpha}}{(4 - 3\alpha) ((8 - 5\alpha) \sqrt{\alpha - 3\alpha}) \sqrt{2 - \alpha}}\).

Thus, we have \(kt > h^* = \max\{h_1, h_2, g_2, g_3\}\), where \(h^* > g_0\), so firm \(b\) is worse off in \((O, O)\) than in \((O, R)\) if the low-valuation segment is not covered in \((O, O)\). Therefore, \((O, O)\) cannot be an equilibrium in this case, either.

As a result, in all the cases discussed above, firm \(b\) is worse off in \((O, O)\) than in \((O, R)\). Therefore, \((O, O)\) cannot be an equilibrium.

**A.1.8 Proof of Proposition 5:**

In the benchmark case, both firms have realistic managers who maximize:

\[
\pi_i = (p_i - kq_i^2)^t - (p_i - p_i^\ast (q_i - q_i^\ast ))^t \theta^t.
\]

The equilibrium outcome is:

\[
q_i^\ast = \frac{\theta}{2k},
\]

\[
p_i^\ast = t + \frac{g^2}{4k},
\]

\[
\pi_i^\ast = \frac{t}{2}.
\]

When firm \(a\) has an optimistic manager who believes that the probability density function of consumer distribution is \(F(x) = 2(1 - x)\), firm \(b\)’s profit function does not change and firm \(a\)’s
manager maximizes:

$$\pi_a = (p_a - kq_a^2) \frac{t-(p_a-p_b)+(q_a-q_b)\theta}{2t} \left[ 2 - \frac{(t-(p_a-p_b)+(q_a-q_b)\theta)}{2t} \right].$$

The equilibrium outcome is:

$$q_a^* = \frac{\theta}{2k},$$

$$q_b^* = \frac{\theta}{2k},$$

$$p_a^* = \frac{\theta^2}{4k} + \frac{2\sqrt{2t}-3}{5} t,$$

$$p_b^* = \frac{\theta^2}{4k} + \frac{1+\sqrt{2t}}{5} t,$$

$$\pi_a^* = \frac{3(7\sqrt{2t}-23)}{50} t,$$

$$\pi_b^* = \frac{11+\sqrt{2t}}{25} t.$$

It is easy to see that $\Delta p_a^* > 0$, $\Delta p_b^* > 0$, $\Delta \pi_a^* > 0$, and $\Delta \pi_b^* > 0$. Both firms’ prices and profits will increase under managerial optimism.

A.1.9 Analysis of Exogenous Quality under Optimism:

If product quality is exogenous, managers maximize firms’ profits through pricing decisions. For simplicity, we assume full equilibrium coverage for this extension. In the benchmark case in which there is no managerial optimism, as shown in the Proof of Lemma 1, firm $i$’s manager maximizes:

$$\pi_i = (p_i - kq_i^2) \frac{t-(p_i-p_{-i})+(q_i-q_{-i})\bar{\theta}}{2t}.$$

Solving for the optimal price gives us firm $i$’s optimal price and profit:

$$p_i^* = t + \frac{1}{3} (q_i - q_{-i}) \bar{\theta} + \frac{k}{3} (2q_i^2 + q_{-i}^2),$$

$$\pi_i^* = \frac{1}{2t} \left( t + \frac{1}{3} (q_i - q_{-i}) \bar{\theta} + \frac{k}{3} (q_{-i}^2 - q_i^2) \right)^2.$$
When firm \(a\) has an optimistic manager and firm \(b\) has a realistic manager, firm \(b\)’s profit function does not change and firm \(a\)’s manager maximizes:

\[
\pi_i = (p_i - k q_i^2) \frac{t-(p_{i-1}-p_i)+(q_{i-1}-q_i)}{2t}.
\]

The equilibrium prices and profits are:

\[
p_a^* = t + \frac{1}{3} (q_a - q_b)(2 - \bar{\theta}) + \frac{k}{3} (2q_a^2 + q_b^2),
\]

\[
p_b^* = t + \frac{1}{3} (q_b - q_a)(2\bar{\theta} - 1) + \frac{k}{3} (2q_b^2 + q_a^2),
\]

\[
\pi_a^* = \frac{1}{2t} \left( t + \frac{1}{3} (q_a - q_b)(2 - \bar{\theta}) + \frac{k}{3} (q_b^2 - q_a^2) \right) \left( t + \frac{1}{3} (q_a - q_b)(2\bar{\theta} - 1) + \frac{k}{3} (q_b^2 - q_a^2) \right),
\]

\[
\pi_b^* = \frac{1}{2t} \left( t + \frac{1}{3} (q_b - q_a)(2\bar{\theta} - 1) + \frac{k}{3} (q_a^2 - q_b^2) \right)^2.
\]

Under optimisn, the changes of both firms’ prices and profits are:

\[
\Delta p_a^* = \frac{2}{3} (q_a - q_b)(1 - \bar{\theta}),
\]

\[
\Delta p_b^* = \frac{1}{3} (q_a - q_b)(1 - \bar{\theta}),
\]

\[
\Delta \pi_a^* = \frac{1}{2t} \left( t + \frac{1}{3} (q_a - q_b)(2 - \bar{\theta}) + \frac{k}{3} (q_b^2 - q_a^2) \right) \left( t + \frac{1}{3} (q_a - q_b)(2\bar{\theta} - 1) + \frac{k}{3} (q_b^2 - q_a^2) \right) - \frac{1}{2t} \left( t + \frac{1}{3} (q_a - q_b)\bar{\theta} + \frac{k}{3} (q_b^2 - q_a^2) \right)^2
\]

\[
\Delta \pi_b^* = \frac{1}{2t} \left( t + \frac{1}{3} (q_b - q_a)(2\bar{\theta} - 1) + \frac{k}{3} (q_a^2 - q_b^2) \right)^2 - \frac{1}{2t} \left( t + \frac{1}{3} (q_b - q_a)\bar{\theta} + \frac{k}{3} (q_a^2 - q_b^2) \right)^2
\]

When \(q_a = q_b\), \(\Delta p_a^* = \Delta p_b^* = \Delta \pi_a^* = \Delta \pi_b^* = 0\), so managerial optimism does not affect firms’ prices and profits.

When \(q_a < q_b\), we have \(\Delta p_a^* < 0\) and \(\Delta p_b^* < 0\), so both firms’ prices will decrease.

When \(q_a > q_b\), we have \(\Delta p_a^* > 0\) and \(\Delta p_b^* > 0\), so both firms’ prices will increase.
Now we examine firms’ profits.

For firm $b$, $\Delta \pi^*_b = \frac{1}{6t}(q_a - q_b)(1 - \theta)\left(\left(t + \frac{1}{3}(q_b - q_a)(2\theta - 1) + \frac{k}{3}(q_b^2 - q_a^2)\right) + \left(t + \frac{1}{3}(q_b - q_a)\theta + \frac{k}{3}(q_b^2 - q_a^2)\right)\right)$.

Note that $\left(t + \frac{1}{3}(q_b - q_a)(2\theta - 1) + \frac{k}{3}(q_b^2 - q_a^2)\right) > 0$ since they are firm $b$’s equilibrium unit profit margins, so $\left(t + \frac{1}{3}(q_b - q_a)\theta + \frac{k}{3}(q_b^2 - q_a^2)\right) > 0$. Thus, we have $\Delta \pi^*_b > 0$ if and only if $q_a - q_b > 0$. Firm $b$’s profit will increase if $q_a > q_b$, and will decrease if $q_b > q_a$.

For firm $a$, $\Delta \pi^*_a = \frac{1}{2t}\left(\frac{1}{3}(q_a - q_b)(1 - \theta)\left(\left(t + \frac{1}{3}(q_a - q_b)\theta + \frac{k}{3}(q_b^2 - q_a^2)\right) - 2\left(\frac{1}{3}(q_a - q_b)(1 - \theta)\right)\right)\right)$, so $\Delta \pi^*_a > 0$ is equivalent to

$$\frac{1}{3}(q_a - q_b)(1 - \theta)\left(\left(t + \frac{1}{3}(q_a - q_b)\theta + \frac{k}{3}(q_b^2 - q_a^2)\right) - 2\left(\frac{1}{3}(q_a - q_b)(1 - \theta)\right)\right) > 0.$$  

If $q_a < q_b$, we need $t + \frac{1}{3}(q_a - q_b)\theta + \frac{k}{3}(q_b^2 - q_a^2) < 2\left(\frac{1}{3}(q_a - q_b)(1 - \theta)\right) < 0$, which cannot be true, since $t + \frac{1}{3}(q_a - q_b)\theta + \frac{k}{3}(q_b^2 - q_a^2)$ is the firm $a$’s equilibrium unit profit margin. Thus, firm $a$’s profit will decrease if $q_a < q_b$.

If $q_a > q_b$, we need $t + \frac{1}{3}(q_a - q_b)\theta + \frac{k}{3}(q_b^2 - q_a^2) > 2\left(\frac{1}{3}(q_a - q_b)(1 - \theta)\right)$, which is equivalent to $t > \frac{k}{3}(q_a^2 - q_b^2) - \frac{1}{3}(q_a - q_b)(3\theta - 2)$. When $t$ is large enough, firm $a$’s profit will increase under managerial optimism.

A.1.10 Analysis of Managerial Pessimism:
We first examine the case of unilateral pessimism. Without loss of generality, we assume that firm a’s manager is realistic and firm b’s manager is pessimistic. We first consider the case in which the market is fully covered in equilibrium under pessimism, where firm a’s manager maximizes:

\[
\pi_a = (p_a - kq_a^2) \frac{\theta - (p_a - p_b) + (q_a - q_b)\bar{\theta}}{2t}. \]

Firm b’s manager maximizes:

\[
\pi_b = (p_b - kq_b^2) \frac{\theta - (p_b - p_a) + (q_b - q_a)\theta_L}{2t}. \]

Solving the problem by backward induction, we obtain the equilibrium quality levels:

\[
q_a^* = \frac{2\bar{\theta} - \theta_L}{2k},
\]

\[
q_b^* = \frac{2\theta_L - \bar{\theta}}{2k}.
\]

The equilibrium prices are:

\[
p_a^* = t + \frac{7\bar{\theta}^2 - 10\theta_L\bar{\theta} + 4\theta_L^2}{4k},
\]

\[
p_a^* = t + \frac{7\theta_L^2 - 10\theta_L\bar{\theta} + 4\bar{\theta}^2}{4k}.
\]

The equilibrium profits are:

\[
\pi_a^* = \frac{\left(t + \frac{3\bar{\theta} - \theta_L}{2k}\right)^2}{2t},
\]

\[
\pi_b^* = \frac{t^2 - \left(\frac{3\bar{\theta} - \theta_L}{2k}\right)^2}{2t}.
\]

It is easy to see that \(\pi_b^* < \frac{t}{2}\), so firm b is always worse off when the market is fully covered under pessimism.

Now we demonstrate that the market is always fully covered under unilateral pessimism. Note
that for the market to be fully covered in equilibrium, we need \( \sum_l q_l \frac{\theta_L - p_l}{t} \geq 1 \), which is equivalent to

\[
\frac{2\alpha + (4 - 2\alpha - 11\alpha^2)\theta_L - 2\alpha(1 - 11\alpha) \theta_L^2 - (2 - 2\alpha + 11\alpha^2) \theta_L^2}{4(1 + 2\theta_L)} \geq kt.
\]

Given that the market is fully covered in the benchmark case, we have \( \frac{\theta_L - \alpha^2(1-\theta_L)^2}{6} \geq kt \).

Note that

\[
\frac{2\alpha + (4 - 2\alpha - 11\alpha^2)\theta_L - 2\alpha(1 - 11\alpha) \theta_L^2 - (2 - 2\alpha + 11\alpha^2) \theta_L^2}{4(1 + 2\theta_L)} - \frac{\theta_L - \alpha^2(1-\theta_L)^2}{6} = \frac{1 - \theta_L}{12(1 + 2\theta_L)} (2\alpha(1 + 3\alpha) + (12 - 31\alpha^2) \theta_L + (10 - 6\alpha + 29\alpha^2) \theta_L^2).
\]

Since \( 1 - \theta_L \frac{1}{12(1 + 2\theta_L)} > 0 \), we have

\[
\frac{2\alpha(1 + 3\alpha) + (12 - 31\alpha^2) \theta_L + (10 - 6\alpha + 29\alpha^2) \theta_L^2}{6} > 0 \text{ if } 2\alpha(1 + 3\alpha) + (12 - 31\alpha^2) \theta_L + (10 - 6\alpha + 29\alpha^2) \theta_L^2 > 0.
\]

Taking the derivative of the LHS with respect to \( \theta_L \), we get

\[
\frac{\partial}{\partial \theta_L} (2\alpha(1 + 3\alpha) + (12 - 31\alpha^2) \theta_L + (10 - 6\alpha + 29\alpha^2) \theta_L^2) = 12 - 31\alpha^2 + 2(10 - 6\alpha + 29\alpha^2) \theta_L.
\]

If \( \alpha < \frac{12}{\sqrt{31}} \),

\[
12 - 31\alpha^2 + 2(10 - 6\alpha + 29\alpha^2) \theta_L \geq 0; \text{ if } \alpha > \frac{12}{\sqrt{31}}, 12 - 31\alpha^2 + 2(10 - 6\alpha + 29\alpha^2) \theta_L < 0
\]

when \( \theta_L < \frac{31\alpha^2 - 12}{2(10 - 6\alpha + 29\alpha^2)} \) and \( 12 - 31\alpha^2 + 2(10 - 6\alpha + 29\alpha^2) \theta_L > 0 \) when \( \theta_L > \frac{31\alpha^2 - 12}{2(10 - 6\alpha + 29\alpha^2)} \).

In the first case, \( 2\alpha(1 + 3\alpha) + (12 - 31\alpha^2) \theta_L + (10 - 6\alpha + 29\alpha^2) \theta_L^2 \) will be at its minimum when \( \theta_L = 0 \), where \( 2\alpha(1 + 3\alpha) + (12 - 31\alpha^2) \theta_L + (10 - 6\alpha + 29\alpha^2) \theta_L^2 = 2\alpha(1 + 3\alpha) > 0 \). In the second case, \( 2\alpha(1 + 3\alpha) + (12 - 31\alpha^2) \theta_L + (10 - 6\alpha + 29\alpha^2) \theta_L^2 \) is at its minimum when \( \theta_L = \frac{31\alpha^2 - 12}{2(10 - 6\alpha + 29\alpha^2)} \), where \( 2\alpha(1 + 3\alpha) + (12 - 31\alpha^2) \theta_L + (10 - 6\alpha + 29\alpha^2) \theta_L^2 = -144 + 240\alpha + 680\alpha^2 + 648\alpha^2 - 729\alpha^4 \)

\[\frac{4(10 - 6\alpha + 29\alpha^2)}{4(10 - 6\alpha + 29\alpha^2)} \]. Given \( \sqrt{\frac{12}{31}} < \alpha < 1 \), we have
\[-\frac{144 + 240\alpha + 680\alpha^2 + 648\alpha^3 - 729\alpha^4}{4(10 - 6\alpha + 29\alpha^2)} > 0.\] Therefore, we always have \(2\alpha(1 + 3\alpha) + (12 - 31\alpha^2)\theta_L + (10 - 6\alpha + 29\alpha^2)\theta_L^2 > 0\), which means \(2\alpha(1 + 3\alpha) + (12 - 31\alpha^2)\theta_L + (10 - 6\alpha + 29\alpha^2)\theta_L^2\) is increasing with \(\theta_L\).

Thus, \(2\alpha(1 + 3\alpha) + (12 - 31\alpha^2)\theta_L + (10 - 6\alpha + 29\alpha^2)\theta_L^2\) takes its minimum when \(\theta_L = 0\), so \(2\alpha(1 + 3\alpha) + (12 - 31\alpha^2)\theta_L + (10 - 6\alpha + 29\alpha^2)\theta_L^2 > 2\alpha(1 + 3\alpha) > 0\).

Therefore, we have \(\frac{2\alpha + (4 - 2\alpha - 11\alpha^2)\theta_L - 2\alpha(1 - 11\alpha)\theta_L^2 - (2 - 2\alpha + 11\alpha^2)\theta_L^3}{4(1 + 2\theta_L)} > \frac{\theta_L^2 - \alpha^2(1 - \theta_L)^2}{6} \geq \frac{kt}{2}\) As a result, the market is always fully covered under unilateral pessimism, and the pessimistic manager is always worse off.

Now we study the case of bilateral pessimism. Given that both managers believe that all consumers have low valuations, the low-valuation segment is always fully covered under bilateral pessimism, so the whole market will be fully covered. Thus, firm \(i\)'s manager maximizes:

\[
\pi_i = (p_i - kq_i^2) \frac{t - (p_i - p_-) + (q_i - q_-)\theta_L}{2t}.
\]

The equilibrium outcomes are:

\[
q_i^* = \frac{\theta_L}{2k},
\]

\[
p_i^* = t + \frac{\theta_L^2}{4k^*},
\]

\[
\pi_i^* = \frac{t}{2}.
\]

Both firms’ profits remain unchanged under bilateral pessimism. Therefore, managerial pessimism always makes the pessimistic manager weakly worse off, and cannot lead to a win-win outcome.

Lastly, we consider the case of \((O, P)\), when firm \(a\) has an optimistic manager and firm \(b\) has a pessimistic manager. We focus on the case in which the market is fully covered in equilibrium.
Firm $a$’s optimistic manager maximizes: $\pi_a = (p_a - kq_a^2) t - (p_a - p_b) + (q_a - q_b) \theta_H / 2t$.  

Firm $b$’s pessimistic manager maximizes: $\pi_b = (p_b - kq_b^2) t - (p_b - p_a) + (q_b - q_a) \theta_L / 2t$.  

The equilibrium outcomes are:  

$$q_a^* = \frac{2 - \theta_L}{2k},$$

$$q_b^* = \frac{2\theta_L - 1}{2k},$$

$$p_a^* = t + \frac{7 - 10\theta_L + 4\theta_L^2}{4k},$$

$$p_b^* = t + \frac{7\theta_L^2 - 10\theta_L + 4}{4k},$$

$$\pi_a^* = \frac{(4kt + 3(1 - \theta_L)^2)(4kt + 3(2\alpha - 1)(1 - \theta_L)^2)}{32k^2t},$$

$$\pi_b^* = \frac{(4kt + 3(1 - \theta_L)^2)(4kt + 3(1 - 2\alpha)(1 - \theta_L)^2)}{32k^2t}.$$  

Given that both firms are weakly worse off in the state of $(O, O)$ and $(P, P)$ compared with in $(R, R)$, we know that they are better off in $(O, P)$ than in $(O, O)$ and $(P, P)$ if their profits are higher than $t^2$, which is the equilibrium profit in $(R, R)$.  

Solving for $\pi_a^* > t^2$, $\pi_b^* > t^2$, and $\sum_i q_i^* \theta_L - p_i^* t \geq 1$ (the condition for the market to be fully covered in $(O, P)$) gives us:  

$$k > 0,$$

$$\frac{11 - \sqrt{22}}{9} < \theta_L < 1,$$

$$0 < t < \frac{-11 + 22\theta_L - 9\theta_L^2}{12k},$$

$$\frac{3(1 - \theta_L)^2}{8kt + 6(1 - \theta_L)^2} < \alpha < \frac{8kt + 3(1 - \theta_L)^2}{8kt + 6(1 - \theta_L)^2}.$$  

When these conditions are satisfied, both firms make higher profits in $(O, P)$ than in $(O, O)$.
and \((P, P)\), and asymmetric managerial bias in the market is better than symmetric bias.

Moreover, neither firm has incentives to deviate from \((O, P)\) to \((O, O)\) or \((P, P)\), so \((O, P)\) is an equilibrium outcome.

**A.2 Quality Design Outsourcing under Demand Uncertainty:**

**An Analytical Study**

**A.2.1 Proof of Lemma 1:**

In the benchmark case, there is no demand uncertainty and the market size is normalized to 1. We solve for the equilibrium outcomes in each supply chain separately and we do it through backward induction.

In CM, in the last stage of the game, the retailer’s profit function is:

\[
\pi_R^C = (p_C - w_C)s_C,
\]

where \(p_C\) is the retail price, \(w_C\) is the wholesale price, and \(s_C\) is the sales volume. Given that there is no demand uncertainty, the retailer can fully anticipate the demand, and therefore will not have any unsold inventory. Thus, in the benchmark case, the sales volume equals to the demand, \(d_C\).

When consumers make a purchase, the threshold consumer who is indifferent between making a purchase and the outside option has utility:

\[
\theta_C q_C^* - p_C = 0.
\]

Consumers whose willingness-to-pay for quality is higher than that of the threshold consumer will make purchase. This gives us the demand:

\[
d_C = \left(1 - \frac{p_C^*}{q_C^*}\right).
\]

Thus, the retailer’s profit function is:
\[ \pi_R^C = (p^C - w^C)s^C = (p^C - w^C)d^C = (p^C - w^C) \left( 1 - \frac{p^C}{q^C} \right). \]

Since the retailer chooses the retail price in the last stage, we take derivative of \( \pi_R^C \) with respect to \( p^C \) and set it equal to 0, which gives us:

\[ \frac{q^C + w^C - 2p^C}{q^C} = 0. \]

Solving the equation gives us the retail price: \( p^C = \frac{q^C + w^C}{2} \).

In the second-to-last stage of the game, the manufacturer chooses the wholesale price. Its profit function is:

\[ \pi_M^C = (w^C - c^C)i^C = (w^C - c^C)d^C = (w^C - q^C)(1 - \frac{q^C + w^C}{2q^C}). \]

We take derivative of \( \pi_M^C \) with respect to \( w^C \) and set it equal to 0 gives us:

\[ q^C + q^C - 2w^C = 0 \]

Solving the equation gives us the wholesale price: \( w^C = \frac{q^C + q^C}{2} \).

In the first stage of the game, the retailer decides the quality level. Plugging \( p^C \) and \( w^C \) into the retailer’s profit function, we get:

\[ \pi_R^C = \frac{q^C(1-q^C)^2}{16}. \]

Taking derivative of \( \pi_R^C \) with respect to \( q^C \) and setting it equal to 0, we get:

\[ \frac{(1-q)(1-3q)}{16} = 0. \]

Thus, in equilibrium, the quality level is:

\[ q^C^* = \frac{1}{3}. \]

Plugging \( q^C \) into \( w^C \) and \( p^C \), we can solve for other equilibrium outcomes in CM accordingly:
\[ w^*_C = \frac{2}{9}, \]
\[ p^*_C = \frac{5}{18}, \]
\[ \pi^*_R = \frac{1}{108}, \]
\[ \pi^*_M = \frac{1}{54}. \]

Similarly, we can solve for the equilibrium outcome in ODM, and get the following results:

\[ q^*_O = \frac{1}{6} (7\sqrt{33} - 39), \]
\[ w^*_O = \frac{1}{6} (17\sqrt{33} - 97), \]
\[ p^*_O = \frac{1}{3} (6\sqrt{33} - 34), \]
\[ \pi^*_R = \frac{1}{18} (11\sqrt{33} - 63), \]
\[ \pi^*_M = \frac{2}{9} (339 - 59\sqrt{33}). \]

We can also calculate the numerical results accordingly:

<p>| Table 2.1 Equilibrium Outcome in Benchmark Case |</p>
<table>
<thead>
<tr>
<th>Integrated</th>
<th>ODM</th>
<th>CM</th>
</tr>
</thead>
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<tr>
<td>Quality</td>
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<td>Wholesale Price</td>
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</tr>
<tr>
<td>Retail Price</td>
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<td>Demand</td>
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<tr>
<td>Retailer Profit</td>
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<td>0.0105</td>
</tr>
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<td>Retailer Share</td>
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<td>40.18%</td>
</tr>
<tr>
<td>Manufacturer Profit</td>
<td>-</td>
<td>0.0157</td>
</tr>
<tr>
<td>Manufacturer Share</td>
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</tr>
<tr>
<td>Channel Profit</td>
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<td>0.0263</td>
</tr>
</tbody>
</table>

**A.2.2 Proof of Proposition 1:**

In order to find the optimal supply chain for the retailer, we need to calculate the retailer’s
expected profit in each supply chain.

In CM, in the fourth stage of the game, the retailer chooses retail price. It has profit function:

$$\pi^C_R = p^C s^C - w^C i^C,$$

Where the sales volume $s^C = \min\{i^C, d^C\}$. Same as in the benchmark case, the threshold customer’s willingness-to-pay for quality is $\frac{p^C}{q^C}$, so demand for the product is:

$$d^C = N \left(1 - \frac{p^C}{q^C}\right).$$

Since the demand is uncertainty, it is possible that the retailer overstocked in the previous stage and cannot sell all the products in the last stage.

If $d^C < i^C$ and the retailer does not sell all the inventory, the retailer’s profit function is:

$$\pi^C_R = p^C d^C - w^C i^C = p^C N \left(1 - \frac{p^C}{q^C}\right) - w^C i^C.$$

Maximizing the retailer’s profit function with respect to $p^C$ gives us: $p^C = \frac{q^C}{2}$ and $d^C = \frac{N}{2}$.

If $d^C \geq i^C$ and the retailer sells out all the inventory, the retailer’s profit function is:

$$\pi^C_R = (p^C - w^C) i^C,$$

s.t. $d^C = N \left(1 - \frac{p^C}{q^C}\right) \geq i^C$.

Maximizing the retailer’s profit function under constraint gives us: $p^C = \left(1 - \frac{i^C}{N}\right) q^C$ and $d^C = i^C$.

Since the retailer’s profit function is different in these two cases, we need to discuss the two cases separately.

Note that it is not optimal for the retailer to choose an inventory level that will not be sold out in both the low state and the high state, because the retailer can always slightly reduce the inventory level to a degree that it still will not be sold out in both states, and obtain the same revenue but at
lower total cost. Hence, we need to look at two cases: (1) the products are sold out in both states, and (2) the products are sold out in the high state but not the low state.

Case 1: the products are sold out in both states.

In the high state, we have \( d^C = N \left( 1 - \frac{p^C}{q^C} \right) = (1 + \delta) \left( 1 - \frac{p^C}{q^C} \right) = i^C \), so the retail price is:

\[
p^C = q^C \left( 1 - \frac{i^C}{1+\delta} \right).
\]

In the low state, we have \( d^C = N \left( 1 - \frac{p^C}{q^C} \right) = (1 - \delta) \left( 1 - \frac{p^C}{q^C} \right) = i^C \), so the retail price is:

\[
p^C = q^C \left( 1 - \frac{i^C}{1-\delta} \right).
\]

Thus, in the third stage, when the retailer chooses the inventory level, its expected profit is:

\[
\pi_R^C = \frac{1}{2} q^C \left( 1 - \frac{i^C}{1+\delta} \right) i^C + \frac{1}{2} q^C \left( 1 - \frac{i^C}{1-\delta} \right) i^C - w^C i^C,
\]

\[
\pi_R^C = \left( q^C \left( 1 - \frac{i^C}{1-\delta^2} \right) - w^C \right) i^C.
\]

Maximizing the retailer’s expected profit with respect to \( i^C \) gives us:

\[
i^C = \frac{(q^C-w^C)(1-\delta^2)}{2q^C}.
\]

In the second stage, the manufacturer chooses wholesale price. Its profit function is:

\[
\pi_M^C = (w^C - c^C) i^C = \left( w^C - q^C^2 \right) \frac{(q^C-w^C)(1-\delta^2)}{2q^C}.
\]

Maximizing the manufacturer’s profit function gives us:

\[
w^C = \frac{q^C+q^C^2}{2}.
\]

Plugging \( p^C, i^C, \) and \( w^C \) into the retailer’s profit function, we get the retailer’s profit function in the first stage of the game:

\[
\pi_R^C = \frac{1}{16} (1 - \delta^2) q^C (1 - q^C)^2.
\]

Maximizing the retailer’s expected profit with respect to \( q^C \) gives us:

\[
q^C^* = \frac{1}{3}.
\]

We can solve for other equilibrium outcomes in the this case accordingly:
\[ w^C = \frac{2}{9}, \]
\[ i^C = \frac{1}{6} (1 - \delta^2), \]
\[ \pi^C_R = \frac{1-\delta^2}{108}, \]
\[ \pi^C_M = \frac{1-\delta^2}{54}. \]

Case 2: the products are sold out in the high state but not the low state.

In the high state, the products are sold out and we have \( d^C = N \left( 1 - \frac{p^C}{q^C} \right) = (1 + \delta) \left( 1 - \frac{p^C}{q^C} \right) = i^C \), so the retail price is:

\[ p^C = q^C \left( 1 - \frac{i^C}{1+\delta} \right). \]

In the low state, the products are not sold out, and we have

\[ p^C = \frac{q^C}{2}, \]
\[ d^C = \frac{N}{2}. \]

In the third stage, when the retailer chooses the inventory level, its expected profit is:

\[ \pi^C_R = \frac{1}{2} q^C \left( 1 - \frac{i^C}{1+\delta} \right) i^C + \frac{1}{2} \star \frac{q^C}{2} \star \frac{N}{2} - w^C i^C. \]

Maximizing the retailer’s expected profit with respect to \( i^C \) gives us:

\[ i^C = \frac{(1+\delta)(q^C-2w^C)}{2q^C}. \]

In the second stage, the manufacturer chooses wholesale price. Its profit function is:

\[ \pi^M = \frac{(1+\delta)(q^C-2w^C)(w^C-q^C^2)}{2q^C}. \]

Maximizing the manufacturer’s profit function gives us:

\[ w^C = \frac{q^C+2q^C^2}{4}. \]

Plugging \( p^C, i^C, \) and \( w^C \) into the retailer’s profit function, we get the retailer’s profit function in the first stage of the game:
π_R = \frac{1}{32} q^C \left( 5 - 3\delta - 4q^C (1 + \delta) + 4q^C (1 + \delta) \right).

Maximizing the retailer’s expected profit with respect to \( q^C \) gives us:

\[ q^C_* = \frac{2 + 2\delta - \sqrt{-11 + 2\delta + 13\delta^2}}{6(1+\delta)}. \]

We can solve for other equilibrium outcomes in the this case accordingly:

\[ w^C_* = \frac{(5 + 5\delta - \sqrt{-11 + 2\delta + 13\delta^2})(2 + 2\delta - \sqrt{-11 + 2\delta + 13\delta^2})}{72(1+\delta)}, \]

\[ i^C = \frac{1 + \delta + \sqrt{-11 + 2\delta + 13\delta^2}}{12}, \]

\[ π^C_R = \frac{(13 - 11\delta + \sqrt{-11 + 2\delta + 13\delta^2})(2 + 2\delta - \sqrt{-11 + 2\delta + 13\delta^2})}{864(1+\delta)}, \]

\[ π^C_M = \frac{(1 + \delta + \sqrt{-11 + 2\delta + 13\delta^2})(2 + 2\delta - \sqrt{-11 + 2\delta + 13\delta^2})}{864(1+\delta)}. \]

One can readily show that \( \frac{(13 - 11\delta + \sqrt{-11 + 2\delta + 13\delta^2})(2 + 2\delta - \sqrt{-11 + 2\delta + 13\delta^2})}{864(1+\delta)} \geq \frac{1 - \delta^2}{108} \) when \( \frac{11}{13} \leq \delta < 1 \). Therefore, in CM, the retailer’s profit function is:

\[ π^C_R = \begin{cases} \frac{1 - \delta^2}{108}, & 0 < \delta < \frac{11}{13} \\ \frac{(13 - 11\delta + \sqrt{-11 + 2\delta + 13\delta^2})(2 + 2\delta - \sqrt{-11 + 2\delta + 13\delta^2})}{864(1+\delta)}, & \frac{11}{13} \leq \delta < 1 \end{cases}. \]

We can find the equilibrium profit of the retailer in ODM in the same way.

Now we compare the retailer’s equilibrium profit in CM and that in ODM, and find that \( π^C_R > π^O_R \) if and only if \( \frac{11}{13} \leq \delta \leq 1 \). Thus, there exists \( \delta^* = \frac{11}{13} \) such that the retailer is better off with ODM when \( \delta < \delta^* \), and better off with CM when \( \delta > \delta^* \).

Given that the retailer chooses the optimal supply chain under any given demand uncertainty, we have the equilibrium profits of both retailer and the manufacturer:
\[\pi_R^* = \begin{cases} \frac{(29-5\sqrt{33})^2(1-\delta^2)}{6(7\sqrt{33}-39)}, & 0 < \delta < \frac{11}{13} \\ \frac{(13-11\delta+\sqrt{11+2\delta+13\delta^2})(2+2\delta-\sqrt{11+2\delta+13\delta^2})}{864(1+\delta)}, & \frac{11}{13} \leq \delta < 1 \end{cases}\]

\[\pi_M^* = \begin{cases} \frac{2(29-5\sqrt{33})(27\sqrt{33}-155)(1-\delta^2)}{3(7\sqrt{33}-39)}, & 0 < \delta < \frac{11}{13} \\ \frac{(2+2\delta-\sqrt{11+2\delta+13\delta^2})(1+\delta+\sqrt{11+2\delta+13\delta^2})^2}{864(1+\delta)^2}, & \frac{11}{13} \leq \delta < 1 \end{cases}\]

### A.2.3 Proof of Proposition 2:

When \(0 < \delta < \frac{11}{13}\), the manufacturer’s profit is:

\[\pi_M^C = \frac{2(29-5\sqrt{33})(27\sqrt{33}-155)(1-\delta^2)}{3(7\sqrt{33}-39)}.\]

Taking derivative of \(\pi_M^C\) with respect to \(\delta\), we get:

\[\frac{d\pi_M^C}{d\delta} = -\frac{4(29-5\sqrt{33})(27\sqrt{33}-155)\delta}{3(7\sqrt{33}-39)} = -0.0314\delta < 0.\]

Thus, when \(0 < \delta < \frac{11}{13}\), the manufacturer’s profit decreases with demand uncertainty.

When \(\frac{11}{13} \leq \delta < 1\), the manufacturer’s profit is:

\[\pi_M^C = \frac{(2+2\delta-\sqrt{11+2\delta+13\delta^2})(1+\delta+\sqrt{11+2\delta+13\delta^2})^2}{864(1+\delta)^2}.\]

Taking derivative of \(\pi_M^C\) with respect to \(\delta\), we get:

\[\frac{d\pi_M^C}{d\delta} = \frac{(1+\delta+\sqrt{11+2\delta+13\delta^2})(25+13\delta^2-23\sqrt{11+2\delta+13\delta^2}+\delta(38-11\sqrt{11+2\delta+13\delta^2})}{864(1+\delta)^2\sqrt{11+2\delta+13\delta^2}}.\]

Note that \(1 + \delta + \sqrt{11 + 2\delta + 13\delta^2} > 0\) and \(864(1 + \delta)^2\sqrt{-11 + 2\delta + 13\delta^2} > 0\), so the sign of \(\frac{d\pi_M^C}{d\delta}\) depends on that of \(25 + 13\delta^2 - 23\sqrt{-11 + 2\delta + 13\delta^2} + \delta(38 - 11\sqrt{-11 + 2\delta + 13\delta^2})\).

When \(\frac{11}{13} \leq \delta < 1\), \(-11 + 2\delta + 13\delta^2\) is increasing with \(\delta\), so we have \(-11 + 2\delta + 13\delta^2 <
4. Thus, we have:

\[ 25 + 13\delta^2 - 23\sqrt{-11 + 2\delta + 13\delta^2} + \delta(38 - 11\sqrt{-11 + 2\delta + 13\delta^2}) \]

\[ = 25 + 38\delta + 13\delta^2 - (23 + 11\delta)\sqrt{-11 + 2\delta + 13\delta^2} \]

\[ > 25 + 38\delta + 13\delta^2 - (23 + 11\delta)\sqrt{4} \]

\[ = -21 + 16\delta + 13\delta^2 \]

\[ > -21 + 16 \frac{11}{13} + 13 \left(\frac{11}{13}\right)^2 \]

\[ = \frac{24}{13} > 0. \]

Hence, we show that \[ 25 + 13\delta^2 - 23\sqrt{-11 + 2\delta + 13\delta^2} + \delta(38 - 11\sqrt{-11 + 2\delta + 13\delta^2}) > 0, \] and thus \( \frac{d\pi^{CM}}{d\delta} \). Therefore, when \( \frac{11}{13} \leq \delta < 1 \), the manufacturer’s profit increases with demand uncertainty.

**A.2.4 Proof of Proposition 3:**

In this case, the retailer needs to pay a fixed cost, \( F \), of research and development. In ODM, when \( 0 < \delta < \frac{11}{13} \), the retailer’s profit is:

\[ \pi^{ODM}_R = \frac{(29-5\sqrt{33})^2(1-\delta^2)}{6(7\sqrt{33}-39)} - F_M. \]

In CM, when \( 0 < \delta < \frac{11}{13} \), the retailer’s profit is:

\[ \pi^{CM}_R = \frac{1-\delta^2}{108} - F_R. \]

The difference in the retailer’s equilibrium profit in two supply chains is:

\[ \Delta\pi^{*}_R = \pi^{ODM*}_R - \pi^{CM*}_R = \left(\frac{(29-5\sqrt{33})^2(1-\delta^2)}{6(7\sqrt{33}-39)} - F_M\right) - \left(\frac{1-\delta^2}{108} - F_R\right) = \left(\frac{30027-5227\sqrt{33}(1-\delta^2)}{108(7\sqrt{33}-39)}\right) - (F_M - F_R). \]
Note that \( \frac{30027 - 5227\sqrt{33}(1 - \delta^2)}{108(7\sqrt{33} - 39)} = 0.0013(1 - \delta^2) > 0 \), so we have \( \Delta \pi^* > 0 \) if:

\[
F_M - F_R < \frac{30027 - 5227\sqrt{33}(1 - \delta^2)}{108(7\sqrt{33} - 39)}.
\]

Therefore, as long as the retailer’s development costs is not significantly lower than that of the manufacturer, the retailer will choose ODM and let the manufacturer design product quality, even if the manufacturer has a higher design cost.

**A.2.5 Analysis of Uncertainty of Consumers’ Willingness-To-Pay:**

In the last stage of the game, a consumer will purchase the product if \( u_j = \theta_j q - p \geq 0 \). High-valuation consumers will make purchase if \( u_H = \theta_H q - p \geq 0 \), and low-valuation consumers will make purchase if \( u_L = \theta_L q - p \geq 0 \). Since \( \theta_H > \theta_L \), high-valuation consumers will always make purchase if low-valuation consumers make purchase. Thus, the retailer has two options when choosing the retail price: to choose a price that both high-valuation consumers will buy, or to choose a price that only the high-valuation consumers will buy.

If the retailer chooses a price that both high-valuation consumers and low-valuation consumers will buy, its profit function is:

\[
\pi_R = p - w \text{ s.t. } u_L = \theta_L q - p \geq 0.
\]

To maximize its profit, the retailer would set \( p = \theta_L q \) and have profit:

\[
\pi_R = \theta_L q - w = \frac{q}{4} - w.
\]

If the retailer chooses a price that only the high-valuation consumers will buy, in the low state, its profit function is:

\[
\pi_R = (p - w)\alpha \text{ s.t. } u_H = \theta_H q - p \geq 0.
\]

To maximize its profit, the retailer would set \( p = \theta_H q \) and have profit:

\[
\pi_R = (\theta_H q - w)\alpha = (q - w)\alpha.
\]
In the low state, the retailer will sell the product to everyone instead of only high-valuation consumers if \( \frac{q}{4} - w \geq (q - w)\alpha \), which is equivalent to \( w \leq \frac{q(1 - 4\alpha)}{4(1 - \alpha)} \).

In the high state, its profit function is:

\[
\pi_R = (p - w)(1 - \alpha) \text{ s.t. } u_H = \theta_H q - p \geq 0.
\]

To maximize its profit, the retailer would set \( p = \theta_H q \) and have profit:

\[
\pi_R = (\theta_H q - w)(1 - \alpha) = (q - w)(1 - \alpha).
\]

In the high state, the retailer will sell the product to everyone instead of only high-valuation consumers if \( \frac{q}{4} - w \geq (q - w)(1 - \alpha) \), which is equivalent to \( w \leq \frac{q(4\alpha - 3)}{4\alpha} \). Given that \( 0 < \alpha < \frac{1}{2} \), we have \( w \leq \frac{q(4\alpha - 3)}{4\alpha} < 0 \), which is impossible. Thus, in the high state when there are more high-valuation consumers, the retailer will choose a price that only the high-valuation consumers will buy.

Note that previous analysis apply to both CM and ODM. Now, we investigate CM and ODM separately.

In CM, in the second stage of the game, the manufacturer chooses \( w \). No matter what \( w^C \) it chooses, in the high state, the retailer will always choose \( p^C = \theta_H q^C = q^C \), so the manufacturer has profit:

\[
\pi_M^C = \left( w^C - q^C^2 \right) \alpha \text{ s.t. } w^C \leq q^C.
\]

In the low state, if the manufacturer chooses \( w^C \leq \frac{q^C(1 - 4\alpha)}{4(1 - \alpha)} \), it will have profit:

\[
\pi_M^C = w^C - q^C^2 \text{ s.t. } w^C \leq \frac{q^C(1 - 4\alpha)}{4(1 - \alpha)}.
\]

To maximize its profit, the manufacturer will choose \( w^C = \frac{q^C(1 - 4\alpha)}{4(1 - \alpha)} \), and its expected profit is:
\[
\pi_M^c = \frac{1}{2} \left( \frac{q^c (1 - 4\alpha)}{4(1 - \alpha)} - q^c \right)^2 + \frac{1}{2} \left( \frac{q^c (1 - 4\alpha)}{4(1 - \alpha)} - q^c \right)^2 \alpha
\]

If the manufacturer chooses \( w^c > \frac{q^c (1 - 4\alpha)}{4(1 - \alpha)} \), it will have profit:

\[
\pi_M^c = (w^c - q^c) \alpha \text{ s.t. } w^c \leq q^c.
\]

To maximize its profit, the manufacturer will choose \( w^c = q^c \), and its expected profit is:

\[
\pi_M^c = (q^c - q^c) \alpha.
\]

The manufacturer will choose \( w^c = \frac{q^c (1 - 4\alpha)}{4(1 - \alpha)} \) if \( \frac{1}{2} \left( \frac{q^c (1 - 4\alpha)}{4(1 - \alpha)} - q^c^2 \right) + \frac{1}{2} \left( \frac{q^c (1 - 4\alpha)}{4(1 - \alpha)} - q^c^2 \right) \alpha \geq (q^c - q^c^2) \alpha \), which gives us:

\[
0 < \alpha < \frac{1}{8} (11 - \sqrt{105}),
0 < q^c \leq \frac{1-11\alpha+4\alpha^2}{4-8\alpha+4\alpha^2}.
\]

When \( \alpha \geq \frac{1}{8} (11 - \sqrt{105}) \), or \( 0 < \alpha < \frac{1}{8} (11 - \sqrt{105}) \) but \( q^c > \frac{1-11\alpha+4\alpha^2}{4-8\alpha+4\alpha^2} \), the manufacturer chooses \( w^c = q^c \); when \( 0 < \alpha < \frac{1}{8} (11 - \sqrt{105}) \) and \( 0 < q^c \leq \frac{1-11\alpha+4\alpha^2}{4-8\alpha+4\alpha^2} \), the manufacturer chooses \( w^c = \frac{q^c (1 - 4\alpha)}{4(1 - \alpha)} \).

In the first stage of the game, the retailer chooses quality level.

When \( \alpha \geq \frac{1}{8} (11 - \sqrt{105}) \), the manufacturer always chooses \( w^c = q^c \), which gives the retailer profit \( \pi_R^c = (p^c - w^c)\alpha = (q^c - q^c)\alpha = 0 \).

When \( 0 < \alpha < \frac{1}{8} (11 - \sqrt{105}) \), if \( q^c > \frac{1-11\alpha+4\alpha^2}{4-8\alpha+4\alpha^2} \), the manufacturer still chooses \( w^c = q^c \), which gives the retailer profit \( \pi_R^c = 0 \). If \( 0 < q^c \leq \frac{1-11\alpha+4\alpha^2}{4-8\alpha+4\alpha^2} \), the manufacturer chooses \( w^c = \frac{q^c (1 - 4\alpha)}{4(1 - \alpha)} \), and the retailer’s profit is:
\[ \pi_R^C = \frac{1}{2} \left( \frac{q^C}{4} - \frac{q^C(1-4\alpha)}{4(1-\alpha)} \right) + \frac{1}{2} \left( q^C - \frac{q^C(1-4\alpha)}{4(1-\alpha)} \right) \alpha = \frac{9q^C\alpha}{8(1-\alpha)} > 0, \text{ s.t. } 0 < q^C \leq \frac{1-11\alpha+4\alpha^2}{4-8\alpha+4\alpha^2}. \]

To maximize its profit, the retailer would set \( q^C = \frac{1-11\alpha+4\alpha^2}{4-8\alpha+4\alpha^2} \), and make profit:

\[ \pi_R^C = \frac{9\alpha(11\alpha+4\alpha^2)}{32(1-\alpha)^3}. \]

Thus, in CM, the retailer’s expected profit is:

\[ \pi_{R}^{C^*} = \begin{cases} \frac{9\alpha(1-11\alpha+4\alpha^2)}{32(1-\alpha)^3}, & 0 < \alpha < \frac{1}{8} (11 - \sqrt{105}) \\ 0, & \frac{1}{8} (11 - \sqrt{105}) \leq \alpha < \frac{1}{2}. \end{cases} \]

In ODM, in the second stage of the game, the manufacturer chooses \( q^O \).

No matter what \( q^O \) it chooses, in the high state, the retailer will always choose \( p^O = \theta_H q^O = q^O \), so the manufacturer has profit:

\[ \pi^O_M = \left( w^O - q^O \right) \alpha \text{ s.t. } q^O = p^O \geq w^O. \]

In the low state, recall that the retailer will sell the product to everyone instead of only high-valuation consumers if \( \frac{q^O}{4} - w^O \geq (q^O - w^O)\alpha \), which is equivalent to \( q^O \geq \frac{4(1-\alpha)w^O}{1-4\alpha} \).

If the manufacturer chooses \( q^O \geq \frac{4(1-\alpha)w^O}{1-4\alpha} \), it will have profit:

\[ \pi^O_M = w^O - q^O \alpha \text{ s.t. } q^O \geq \frac{4(1-\alpha)w^O}{1-4\alpha}. \]

To maximize its profit, the manufacturer will choose \( q^O = \frac{4(1-\alpha)w^O}{1-4\alpha} \), and its expected profit is:

\[ \pi^O_M = \frac{1}{2} \left( w^O - \left( \frac{4(1-\alpha)w^O}{1-4\alpha} \right)^2 \right) + \frac{1}{2} \left( w^O - \left( \frac{4(1-\alpha)w^O}{1-4\alpha} \right)^2 \right) \alpha. \]

If the manufacturer chooses \( q^O < \frac{4(1-\alpha)w^O}{1-4\alpha} \), it will have profit:

\[ \pi^O_M = \left( w^O - q^O \right) \alpha \text{ s.t. } q^O \geq w^O. \]
To maximize its profit, the manufacturer will choose \( q^O = w^O \), and its expected profit is:

\[
\pi^O_M = (w^O - w^{O^2}) \alpha.
\]

The manufacturer will choose \( q^O = \frac{4(1-\alpha)w^O}{1-4\alpha} \) if \( \frac{1}{2}(w^O - \left(\frac{4(1-\alpha)w^O}{1-4\alpha}\right)^2) + \frac{1}{2}(w^O - \left(\frac{4(1-\alpha)w^O}{1-4\alpha}\right)^2) \alpha \geq (w^O - w^{O^2}) \alpha \), which is equivalent to \( 0 < w^O \leq \frac{1-9\alpha+24\alpha^2-16\alpha^3}{16-18\alpha-16\alpha^3} \).

In the first stage of the game, the retailer chooses wholesale price.

If the retailer chooses \( w^O > \frac{1-9\alpha+24\alpha^2-16\alpha^3}{16-18\alpha-16\alpha^3} \), the manufacturer will choose \( q^O = w^O \), which gives the retailer profit \( \pi^O_R = (p^O - w^O) \alpha = (q^O - q^O) \alpha = 0 \).

If the retailer chooses \( 0 < w^O \leq \frac{1-9\alpha+24\alpha^2-16\alpha^3}{16-18\alpha-16\alpha^3} \), the manufacturer will choose \( q^O = \frac{4(1-\alpha)w^O}{1-4\alpha} \), which gives the retailer profit:

\[
\pi^O_R = \frac{1}{2} \left( \frac{1}{4} \cdot \frac{4(1-\alpha)w^O}{1-4\alpha} - w^O \right) + \frac{1}{2} \left( \frac{4(1-\alpha)w^O}{1-4\alpha} - w^O \right) \alpha = \frac{3w^O \alpha}{1-4\alpha} > 0.
\]

To maximize its profit, the retailer would choose \( w^O = \frac{1-9\alpha+24\alpha^2-16\alpha^3}{16-18\alpha-16\alpha^3} \), and make profit:

\[
\pi^O_R = \frac{3\alpha(1-\alpha)(1-4\alpha)}{2(8-9\alpha-8\alpha^3)}.
\]

Thus, in ODM, the retailer’s expected profit is:

\[
\pi^O_R = \frac{3\alpha(1-\alpha)(1-4\alpha)}{2(8-9\alpha-8\alpha^3)}.
\]

The retailer would choose ODM over CM if \( \pi^O_R > \pi^C_R \). If \( \frac{1}{8} (11 - \sqrt{105}) \leq \alpha < \frac{1}{2}, \pi^C_R = 0 < \pi^O_R \). If \( 0 < \alpha < \frac{1}{8} (11 - \sqrt{105}) \), the difference in equilibrium profit is:

\[
\Delta \pi^* = \pi^O_R - \pi^C_R = \frac{3\alpha(1-\alpha)(1-4\alpha)}{2(8-9\alpha-8\alpha^3)} - \frac{9\alpha(1-11\alpha+4\alpha^2)}{32(1-\alpha)^3} = \frac{96\alpha(1-\alpha)^4(1-4\alpha)-18\alpha(1-11\alpha+4\alpha^2)(8-9\alpha-8\alpha^3)}{64(1-\alpha)^2(8-9\alpha-8\alpha^3)} = \frac{3\alpha}{32(1-\alpha)^3(8-9\alpha-8\alpha^3)}(-8 + 163\alpha - 41\alpha^2 -
\]

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\[316\alpha^3 + 8\alpha^4 + 32\alpha^5\].

Note that when \(0 < \alpha < \frac{1}{8}(11 - \sqrt{105})\), we have \(\alpha > 0\), \((1 - \alpha)^3 > 0\), and \(8 - 9\alpha - 8\alpha^3 > 0\), so the sign of \(\Delta \pi^*\) depends on that of \(-8 + 163\alpha - 41\alpha^2 - 316\alpha^3 + 8\alpha^4 + 32\alpha^5\), whose derivative over \(\alpha\) is:

\[
d\frac{d(-8 + 163\alpha - 41\alpha^2 - 316\alpha^3 + 8\alpha^4 + 32\alpha^5)}{d\alpha} = 163 - 82\alpha - 948\alpha^2 + 32\alpha^3 + 160\alpha^4
\]

\[
> 163 - 82\left(\frac{1}{8}(11 - \sqrt{105})\right) - 948\left(\frac{1}{8}(11 - \sqrt{105})\right)^2
\]

\[
= \frac{1}{8}(2689\sqrt{105} - 26379)
\]

\[= 146.8813 > 0.\]

When \(\alpha = 0\), \(-8 + 163\alpha - 41\alpha^2 - 316\alpha^3 + 8\alpha^4 + 32\alpha^5 = -8 < 0\); when \(\alpha = \frac{1}{8}(11 - \sqrt{105})\), \(-8 + 163\alpha - 41\alpha^2 - 316\alpha^3 + 8\alpha^4 + 32\alpha^5 = \frac{9}{16}(101\sqrt{105} - 1023) = 6.7173 > 0\). Since \(-8 + 163\alpha - 41\alpha^2 - 316\alpha^3 + 8\alpha^4 + 32\alpha^5\) increases with \(\alpha\), there must exist \(\alpha^*\) such that when \(0 < \alpha < \alpha^*\), \(\Delta \pi^* < 0\); when \(\alpha^* < \alpha < \frac{1}{8}(11 - \sqrt{105})\), \(\Delta \pi^* > 0\). Moreover, when \(\frac{1}{8}(11 - \sqrt{105}) < \alpha < \frac{1}{2}\), \(\pi^*_{\text{R}} = 0 < \pi^*_{\text{R}}\), so \(\Delta \pi^* > 0\). Therefore, there exists \(\alpha^*\) such that \(\pi^*_{\text{R}} > \pi^*_{\text{O}}\) if \(0 < \alpha < \alpha^*\), and \(\pi^*_{\text{R}} < \pi^*_{\text{O}}\) if \(\alpha^* < \alpha < \frac{1}{2}\).

Note that demand uncertainty is lower when \(\alpha\) is larger, so the retailer is better off with ODM when demand uncertainty is low.

**A.2.6 Analysis of Return Policy:**

We can see the effects of the return policy with two extreme cases.

If \(\rho = 0\), then the game is the same as the core model in which there is no return policy. In
In this case, the result is the same: the retailer is better off with ODM when demand uncertainty is low.

If $\rho = 1$, then the retailer can return any unsold products at the wholesale price and does not have to bear the risk of overstocking. The retailer will always order an amount that can cover the demand in both states, and the game is the same as that in the benchmark case, in which there is no demand uncertainty. In this case, the retailer is always better off with ODM.

Hence, with the return policy, if $\rho$ is low, the result is the same as that in the core model; if $\rho$ is high, the retailer is always better off with ODM.