Spring 5-15-2018

Essays on Financial and Monetary Economics

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Essays on Financial and Monetary Economics

by

Xi Wang

A dissertation presented to
The Graduate School
of Washington University in
partial fulfillment of the
requirements for the degree
of Doctor of Philosophy

May 2018
St. Louis, Missouri
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Acknowledgments

I am particularly grateful to my main advisor Costas Azariadis for his inspiration, advice and patience. I am also in debt to Michele Boldrin and Gaetano Antinolfi for their enlightening discussion and training. I appreciate the conversation with the faculty members and students from the Department of Economics and Finance at Washington University in St. Louis, Saint Louis University and University of Missouri–St. Louis. Meanwhile, I really appreciate the supports from my parents, my wife Xiao Hu and Manjushri.

Certainly, I offer special thanks to Washington University Economic Department for its rigorous training and financial funding. Moreover, I really appreciate the patience and love from my parents and wife.

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May 2018
ABSTRACT OF THE DISSERTATION

Essays on Financial and Monetary Economics

by

Xi Wang

Doctor of Philosophy in Economics

Washington University in St. Louis, 2018

Professor Costas Azariadis, Chair

The first part of this dissertation explores an empirical relevance to understand the equity premium puzzle. Since only the wealthiest people invest significant amounts in the stock market (limited participation), it is reasonable to combine the consumption data of the wealthy, instead of aggregate data, with observed asset returns to estimate the risk aversion coefficient (RRA). I approximate the consumption by the rich from two angles: one explores the income and wealth data to back out synthetic consumption directly, and the other explores the sales data to approximate the expenditure by the rich. By using the created indices, the lowest RRA estimate is around three for the first approach, and slightly below ten for the second one. Furthermore, when I use my indices to fit more moments besides excess return, the estimate of RRA increases modestly, e.g., to fit returns of 25 size and book-to-market portfolios, estimates of RRA are between 2.16 and 18. I conclude that these indices, especially the top consumption processes, provide a useful vantage point from which we can reassess the theory of consumption-based asset pricing. When I used these newly constructed indices in a factor model, my factor model explains cross-sectional excess returns better than CAPM and CCAPM model with aggregate consumption.

The latter two parts are to re-evaluate the Quantity Theory of Money (QTM) using, to the extent possible, the same statistical and economic criteria but a much larger data set covering both a longer period and many more countries. I investigate whether QTM breaks
across countries and I find Lucas’ result fragile. It appears that the period 1955-1980 is the only period during which QTM fits data well in most of our sample countries. It starts to break down when we go beyond this period. Furthermore, the recent breaking down of QTM is not global when I truncate the sample before the crisis since QTM is not a tight rule across countries. To explain the breaking down for the U.S during Pre-crisis Period (1980-2007), the second part shows $M_2$ is a more robust monetary index by investigating the historical performance of $M_1$. Under the view of endogenous money. Namely, broad money($M_2$) is generated from loan issuing. I decompose the structure of loans for the U.S. I found that real estate is the major collateral asset for Household and Firms. I thus propose money is after real estate and final goods. To confirm our theory, we investigate a historical nominal price index of U.S and find that (long-run) growth of nominal house price co-moves with(leads) growth of broad money more robustly. Furthermore, the timing of recent financial innovation matches with breaking data. I thus propose a channel through which financial innovation can affect the estimation of QTM.
Chapter 1

Quality Consumption and Asset Pricing

This paper explores an empirical relevance to understand the equity premium puzzle. Since only the wealthiest people invest significant amounts in the stock market (limited participation), it is reasonable to combine the consumption data of the wealthy, instead of aggregate data, with observed asset returns to estimate the risk aversion coefficient (RRA). I approximate the consumption by the rich from two angles: one explores the income and wealth data to back out synthetic consumption directly, and the other explores the sales data to approximate the expenditure by the rich.

And by using the created indices, the lowest RRA estimate is around three for the first approach, and slightly below ten for the second one. Furthermore, when I use my indices to fit more moments besides excess return, the estimate of RRA increases modestly, e.g., to fit returns of 25 size and book-to-market portfolios, estimates of RRA are between 2.16 and 18. I conclude that these indices, which are approximations to the consumption of the rich, provide a useful vantage point from which we can reassess the theory of consumption-based asset pricing. When I used these newly constructed indices in a factor model, my factor
model explains cross-sectional excess returns better than CAPM and CCAPM model with aggregate consumption.

1.1 Introduction

The equity premium puzzle has been at the heart of financial economics since Mehra and Prescott (1985) (afterward, MP). MP adopts a representative agent framework with time-separable CRRA utility, and calibrates the model to match the average excess return of the stock, i.e., the difference between stock market returns and risk-free rate\(^1\). It turns out that this model requires an unreasonably high risk aversion coefficient to match the equity premium which is annually 6% on average.

The intuition follows a general idea: people invest or save for future consumption. Assets that offer insurance have high prices and low returns. An example is Treasury bonds or life insurance. Contrastly, risky assets have high returns, e.g., stocks. As the old saying goes, the greater the risk, the greater the reward. An average 6% excess return (reward) implies stock should be a risky asset. However, when one turns to data, she will find a low co-variance\(^2\) between the consumption growth and stock return. The consumption data states that stock is not a bad hedge against consumption risk, though the stock is still mildly risky. To justify these two phenomena, investors should be extremely risk averse to require a high (6%) excess return as compensation for this mild risk. In other words, the risk aversion coefficient (RRA afterward) is high. However, the required RRA is too high to fit in any reasonable range by any literature in other fields.

In addition, high risk aversion involves another problem: the “risk-free rate puzzle” (Weil (1989)). High RRA implies an unreasonably high risk-free rate under the CRRA

\(^1\)In this article, I use one month T-bill rate as the risk-free rate. The puzzle prevails even if one uses returns on treasury bonds with other maturities.

\(^2\)It is worth pointing out that low covariance between consumption and risky return comes from the low volatility of consumption data.
utility specification. Risk aversion has implications for peoples’ intertemporal behaviors under the CRRA setup: the elasticity of intertemporal substitution (EIS afterward) equals to the reciprocal of RRA. A high risk aversion means a low EIS. A representative agent with low EIS prefers a smooth intertemporal consumption path. However, U.S consumption per capita grows stably, not a smooth path at all. A high risk-free interest rate is thus necessary to justify this consumption deferral. Alternatively the agent values future consumption more: \( \beta > 1 \), e.g., Kocherlakota (1996).

In this paper, I provide an angle to understand this “puzzle”: to provide another empirical relevant measure for the risk imposed by the stock. First, I present evidence of limited participation. I show that not all Americans invest in the stock market. Only the wealthiest invest significantly in this market, echoing the evidence in Mankiw and Zeldes (1990, 1991). Therefore, the consumption of the rich should be the relevant consumption sequence one should use to justify this 6% annual premium. To approximate the consumption of the rich, I use two approaches: one explores the data from Piketty, Saez and Zucman (2016) on the income and wealth of the rich to back out a synthetic consumption by the rich, and the other uses sales data of luxury goods and services to approximate the expenditure by the rich; To be more precise, I create three quality consumption indices (1960-2015) by following the second method: U.S sales of luxury brands, sales of luxury lodging service and sales of premium grocery. During 1960-2014, the correlation coefficient between personal consumption expenditure and average individual income is 0.76. During the same period, the correlation coefficient between my quality service (goods) and the average income of the top 10% of the rich is 0.630 (0.368).

By using these indices, I reestimate RRA under an endowment economy with the CRRA preference. Estimates are much lower than the ones in the previous empirical literature with a similar setup. For example, the lowest estimate of RRA is 3.8 (7.6 if using the quality index), lower than 13.9 in Ait-sahalia, Parker and Yogo (2004), and lower than 17 in Savov
Previous successful literature has low calibrated RRA ranging from 1 to 10 and above, but the most successful ones are all from the calibration side. For example, Boldrin, Christiano and Fisher (2001) use habit formation and the log utility setup. Barro (2009) combines rare disaster with Epstein-Zin preferences and calibrates RRA to be $3 - 4$. Finally, Bansal and Yaron (2004) combine Epstein-Zin preferences with long-run risk and stochastic volatility and calibrate RRA to be 10 or above.$^{3}$ I summarize this comparison in Table-1.1 and leave a detailed review of the literature to next section. My estimate of RRA is the lowest among empirical studies.

<table>
<thead>
<tr>
<th>Preference Specification</th>
<th>Additional Assumptions or Data</th>
<th>Calibrated Or Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mehra and Prescott (1985)</td>
<td>CRRA</td>
<td>n.a</td>
</tr>
<tr>
<td>Campbell and Cochrane (1999)</td>
<td>CRRA</td>
<td>Habit Formation</td>
</tr>
<tr>
<td>Boldrin, Christiano and Fisher (1997)</td>
<td>CRRA</td>
<td>Habit Formation</td>
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<tr>
<td>Barro (2006)</td>
<td>CRRA</td>
<td>Rare Disaster</td>
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<td>Barro (2009)</td>
<td>E-P</td>
<td>Rare Disaster</td>
</tr>
<tr>
<td>Bansal and Yaron (2004)</td>
<td>E-P</td>
<td>Persistent Consumption$^c$</td>
</tr>
<tr>
<td>Constantinides et.al. (2002)</td>
<td>CRRA</td>
<td>Credit market imperfection</td>
</tr>
<tr>
<td>Mankiw and Zeldes (1990)</td>
<td>CRRA</td>
<td>Consumption of Shareholders</td>
</tr>
<tr>
<td>Ait-sahalia et.al. (2004)</td>
<td>CRRA</td>
<td>Luxury Cars and Durables</td>
</tr>
<tr>
<td>Savov (2011)</td>
<td>CRRA</td>
<td>Garbage Data</td>
</tr>
<tr>
<td>Kronencke (2017)</td>
<td>CRRA</td>
<td>Unfilter</td>
</tr>
</tbody>
</table>

$^a$ However, the de facto risk aversion still range from 60 to 80 (Page 243).
$^b$ With a RRA valued at 4, Barro (2006) can justify an excess return 0.16% (Page 843).
$^c$ Stochastic Volatility and Large EIS are also necessary to justify a 6% equity premium. Without stochastic volatility, a RRA valued at 10 can only justify a excess return less than 5%.

In the following sections, I will first briefly review relevant literature and link my procedure and results to that. In section III, I lay out a parsimonious model to nest several preference specifications and to accommodate composition risks. In section IV, we use aggregate data and explore Euler Equations to estimate RRA $\gamma$ under different specifications. In section V, I provide two pieces of time series evidence to show limited participation of the stock market. In section VI, I start to create consumption indices for the rich. And RRA will be re-estimated by using these created indices in section VII. In section VIII, I explore $^3$Depends on whether they include stochastic volatility or not. See following literature review for details.
the cross-sectional implication of my newly constructed pricing kernels. A conclusion section will then follow.

1.2 Literature Review

Researchers try to solve the Equity Premium puzzle in various ways. Solutions can be categorized into model- and data-oriented approaches. The model approach considers more assumptions, while the data approach investigates and explores relevant indices to price assets, because of issues of NIPA consumption data, which we will elaborate later.

For the model approach, different researchers introduce various additional elements (assumptions) into the classical Lucas-tree model. There are at least four groups of successful and elegant literature: rare disaster, habit formation, long-run risk and imperfect credit market.

The rare disaster model, first introduced by Rietz (1988) and further developed by Barro (2006) and Gabaix (2012), states that there is a small but positive probability of a rare disaster. This rare event reduces consumption significantly when it happens, e.g., 15% Barro (2006). Though this rare event does not happen in postwar datasets which is a typical sample period people are investigating, it did happen and this rare event is so important (significant consumption cut) that investors cannot ignore their existence (a peso problem, as stated in Rietz (1988)). With this tail-event, cautious investors will require a relative high return on stock and other risky assets, though Epstein and Zin preference is still necessary to lower the RRA to a reasonable range, e.g., Under a CRRA specification, $\gamma = 4$ can only justify 0.16% excess return (Page 843, Barro (2006)).

Epstein and Zin preference (E-Z preference afterward) is invented by Epstein and Zin (1989). Under the E-Z preference, the RRA is separated with EIS$^4$. Under this preference,

$^4$see Epstein and Zin (1989, 1991) for further discussion of recursive utility
high risk aversion does not necessarily imply a low EIS or a high risk-free rate. However, the E-Z preference alone cannot generate a high equity premium without the help of a high RRA. This preference cannot deliver a reasonable answer even when one may add in composition risks, for example, Yogo (2006). A high RRA is still necessary to justify 6% equity premium even with this composition risk. With E-Z preferences, Yogo (2006) estimates RRA to be 174 to 199 (Table II, Page 552). Additional elements are thus necessary to lower the estimate.

There is another strand of models that successfully takes the advantage of the E-Z preference: the long-run risk model, e.g., the pioneering work Bansal and Yaron (2004). Under the E-Z preferences, people care about future uncertainty, and under certain parameter restrictions (RRA > \( \frac{1}{\text{EIS}} \)) people are willing to pay for early resolution of that uncertainty. In other words, people need to be compensated for future risks. The required compensation positively depends on the magnitude of risk people are facing. There are thus two indispensable elements to make the long-run model consistent with both a low RRA and a 6% equity premium: (1) A persistent component in consumption process. Since other components of consumption are deterministic, any consumption shock is long-lasting, like a \( I(1) \) Process. Hence this model is named as “long-run” risk model. (2) Stochastic Volatility; the volatility of this “long-run” risk is also random. In other words, there are two layers of future uncertainty to resolve, if anyone can do so. When one of these two elements is absent, the

---

5Yogo (2006) differentiates durable goods with nondurable goods and services. The ratio between these two is varying over time. Yogo (2006) names this as composition risk.

6Persistent shocks may have potential surprising implications on the time-varying property of Equity Premium as pointed out in Boldrin, Christiano and Fisher (1997). Good Shock may bring negative return. The intuition goes as follows: In a life-cycle model, consumption is determined by permanent income. If shocks of income are signaling even higher future income, a good shock means higher permanent income, which causes higher consumption than dividends. People thus have the incentive to sell the tree. To clear asset market, the return of stock should go up and price of the stock goes down. Negative return is generated. Under a (un)carefully calibrated model, RRA can be negatively associated with implied excess return.

7However, it is hard on the basis of a finite sample of observation to test whether a process contains a persistent component or is merely a white noise; This problem is two-sided:For any given sample size (1) for a process contains a persistent component there exists a white noise process such that it is almost indistinguishable from the previous candidate.(2) for any white noise process, we can write down a process containing a persistent component. And this constructed process is indistinguishable from the white noise. See Shephard and Harvey (1990).
long run risk model cannot justify the 6% equity premium with an RRA less than 10, for example, Table-II in Page 1492 of Bansal and Yaron (2004).

Another successful preference-based approach is offered by habit forming preference, e.g., Boldrin, Christiano and Fisher (1997, 2001) and Campbell and Cochrane (1999). Even with a constant risk-free interest rate and random walk consumption, this setup can generate a large equity premium and volatile stock price. The main mechanism is through “endogenous” risk aversion. When consumption \( C \) falls behind habit \( X \), the “endogenous” risk aversion increases \( \left( \gamma \frac{C}{X} \right) \), driving up the equity premium and decreasing stock price. But the risk aversion coefficient is de facto high, “Risk aversion is about 80 at steady state,..., and is still as high as 60 at the maximum surplus consumption ratio... (Campbell and Cochrane (1999), Page 244).”

The last strand explores additional market structure, e.g., borrowing constraint. Under imperfect credit specification, there are at least two working channels increasing the implied excess return, e.g., the elegant setup of Constantinides, Donaldson and Mehra (2002). (1) Precautionary saving channel lowers the risk-free rate. One problem with CRRA utility specification is that increasing RRA boosts risk-free rate, excess return thus increases insignificantly, e.g., Page 842–843 Barro (2006). (2) Worsen hedging channel increases required the return of stock directly and indirectly. When borrowing constraint binds, people want to consume more but they cannot. There is an incentive to sell the asset.

---

8Recently, Yang (2016) tests habit formation model against long-run risk model, the former is preferred
9To be more precise, one needs a special kind of habit formation preference, for example, ratio habit formation in Abel (1990) does not work well. The habit formation here is in the form of difference.
10This statement follows Campbell-Shiller decomposition. Campbell and Shiller (1988) prove that higher expected excess return means lower current price. It is also worth pointing out that Yogo (2008) derives a framework by using reference-dependent preferences. This setup is similar to habit formation model since habit itself serves as a natural type of reference point. The calibrated RRA is 1 in Yogo (2008) (Page 137).
11There is also a huge literature combing incomplete market(market structure), heterogeneity with habit formation or Epstein and Zin preference to study the Equity premium. Examples are Constantinides and Duffie (1996), Heaton and Lucas (1996), Krusell and Smith (1998), Constantinides et al. (2002), Storesletten et al. (2004), Basak and Cuoco(1998) and He et al. (1990,1991). Alvarez and Jermann(2001) and Lustig et al. (2005) use an endogenous incomplete market framework and argue that the observed distribution of wealth justifies a 3% premium.
market, the return of stock has to go up.

Furthermore, there is an indirect channel: limited borrowing makes consumption smoothing more difficult. Though borrowing constraint may be slack currently, risky assets, like stock, can potentially make this constraint binding in the future, especially when there is a collateral constraint, e.g., Wang (2017). Borrowing limit thus makes the positive co-movement between consumption and risky return closer.

However, most of these modified models can perform well quantitatively but not empirically, since all of their critical assumptions are hard to test, Constantinides, Donaldson and Mehra (2002) is an elegant exception, it explores the limited participation structure of the stock market and made a realistic model specification. To be more precise: (1) Though E-P preferences with low RRA can generate a high equity premium when the possibility of rare disaster is added, the model suffers from sample biases. It is hard to estimate the probability of this “rare” event out of several data points. (2) For the long-run risk model, it is hard to test a consumption process with a persistent component against a white noise. The magnitude of EIS is another issue. But the implications out of these two processes are different. A model with white noise consumption process implies a nearly zero equity premium, while a small but persistent component can generate a significantly higher equity premium. (3) For the habit formation model, we do not have direct measures or observation of “habit” term, let alone the statistical properties of the process of this habit term. Hence, the success of these three models comes from quantitative exercises, instead of empirical evidence. Additionally, rare disaster model is fragile under learning framework: Chen, Joslin and Tran (2012) show that under heterogeneous belief setting, the rare disaster model can only deliver

\footnote{A successful calibration requires an EIS larger than 1, much higher than the number in literature focusing on estimating this parameter, e.g., EIS is estimated to be 0.4 in Chirinko and Mallick(2017). See footnote-21 for more details.}

\footnote{Equity premium is 4.20% without stochastic volatility for RRA=10, EIS=1.5. Table II, Page 1492, Bansal and Yaron (2004).}

\footnote{It is worth mentioning that the unfiltered process in Kronencke (2017) has a similar form of habit formation, e.g., the formula in Page 54.
a 2% equity premium when optimists own 10% wealth of the economy.

On the other hand, the literature, using data-oriented approach, reminds us that the aggregate consumption data released by NIPA are not satisfying ones to determine the pricing kernel since NIPA uses filtering and interpolation methods to smooth out fluctuations in consumption\(^{15}\). Therefore, the resulting pricing kernel is not volatile enough\(^{16}\) unless with a high RRA. There are papers trying to address this problem by using other data sources or modifying raw data. Parker and Julliard (2005) adopt consumption growth during a longer horizon instead of the annual growth rate: three years consumption growth is used. Jagannathan and Wang (2007) use fourth quarter to fourth quarter consumption growth. Savov (2011) uses garbage data to approximate the “true consumption. As a work closet to mine, Ait-sahalia, Parker and Yogo (2004) argues that since normal consumption is essential to people’s lives, what is important for asset prices is non-subsistence consumption. Luxury consumption, on the other hand, has no issues on subsistence. Ait-sahalia, Parker and Yogo (2004) collect luxury cars, luxury brand goods, luxury housing and wine data to price assets. More recently, Kronencke (2017) explores the implication of reversing a forward Kalman filter, “un-filtering” the filtered consumption data\(^{17}\) and use the “unfiltered” data to estimate RRA.

All these alternative (approximate) consumption processes are more volatile and correlated with stock returns than the canonical measures; researchers can reduce the estimated risk aversion into a lower range, say under 20.

In this paper, I follow another angle to understand this asset puzzles: limited participation in the stock market. I create more comprehensive and longer quality consumption indices to approximate the consumption of the rich. And I use these created indices to reassess the

\(^{15}\)This contributes to the low volatility of raw consumption data.

\(^{16}\)As pointed out in Hansen and Jagannathan (1991), a nonvolatile pricing kernel generates low equity premium.

\(^{17}\)This method only works if NIPA uses forward Kalman filter only. It cannot recover the original data if NIPA additionally employs a Kalman smoothing process.
Equity Premium Puzzle.

Limited market participation refers to the fact that when not all agents in our economy invest substantially in stocks. This pattern has more profound implication of pricing kernel: the aggregate consumption sequence provides little evidence on the risk aversion coefficient of the actual stock investors\textsuperscript{18}, as emphasized in Brav, Constantinides and Geczy (2002). There are theoretical papers exploring this pattern, e.g., Constantinides\textsuperscript{2002}. In sum, the risk of stock assets should be measured by its co-movement with active investors’ consumption; aggregate consumption is not an appropriate process we should use to estimate RRA.

Mankiw and Zeldes (1990, 1991) explore the Panel Study of Income Dynamics (PSID) dataset to recover the consumption of shareholders. Though they obtain a high estimator (35) of RRA with shareholder’s consumption process, the RRA implied by consumption of all family (100) and non-shareholders (261.9) are much higher. And this high estimate (35) may result from the poor quality of PSID in certain dimensions, as is well-documented in the literature, e.g., Aguiar and Bils (2015) suggest that wealth and consumption data in PSID has low quality, especially during the years before 1999\textsuperscript{19}. For example, PSID measures only the consumption of food and housing, undersampling the wealthy and reporting wealth data infrequently. In a related paper, Malloy, Moskowitz and Vissing-Jorgensen (2009) lay out a long run risk model but focusing on stockholders’ consumption risk. VissingJrgensen (2002) explores Consumer Expenditure Survey(CEX) dataset. However, CEX has also low-quality data of income and wealth. CEX top-codes both consumption and wealth. Furthermore, CEX is only available on a continuous basis since 1980\textsuperscript{20}. If one focuses on the CRRA setting,

\textsuperscript{18}If we focus on PSID data, at most one-fourth of Americans hold stock assets(1991 PSID survey).
\textsuperscript{19}But PSID offers an excellent panel data source for income. This is the reason PSID is so popular in labor literature. As documented in Attanasio and Pistaferri (2016), the motivation of the PSID was to study income dynamics between and across generations. Consumption data collection was thus considered ancillary: Before 1997 wave, PSID collected information only on a few consumption items: food (at home and away from home), home rent, and (occasionally) utility payments. However, since the 1999 wave, PSID began to collect a broader range of consumption items, covering 70% – 90% percent of the spending covered by Consumer Expenditure Survey(CEX).
\textsuperscript{20}But it is a dataset used by BLS(Bureau of Labor Statistics) in the computation of overall Consumption Price Index.
risk aversion coefficient estimates of VissingJrgensen (2002) range from 25 to 33 implicitly\textsuperscript{21}. On the other hand, SCF (Survey of Consumer Finance, offered by Board of Governors of Fed) provides a high-quality data on wealth, but limited consumption data. None of these datasets gives good measure to both of wealth and consumption.

In this paper, I present time series data to show that stock assets are held by the rich (c.f data appendix of Piketty, Saez and Zucman (2016)). Limited participation is robust even when currently more and more people are entering the stock market.

This paper constructs several indices, one of which directly approximate the consumption growth and the others of which use the sales to approximate consumption (indirectly). The later approach extends and refines the idea and sequence in Ait-Sahalia, Parker and Yogo (2004). Certainly, I am not the first one to approximate consumption by rich. For example, consumption used by Mankiw and Zeldes (1990) can be viewed as an index. Instead, I focus on quality consumption to approximate the consumption of this particular group. The closest work to us, Ait-Sahalia et al. (2004) also touches the U.S retails of some luxury brands. However, Ait-Sahalia et al. (2004) focus on the subsistence property part of luxury consumption. In other words, luxury brands are a luxury to the representative agent and the indices used in Ait-Sahalia et al. (2004) are most (luxury) durable.

However, we focus on another property of luxury brands—quality. Consumption quantity of the rich does not necessarily consist of more goods than the general public. But their consumption generally has a higher quality. After all, Iranian caviar is different with Russian Osetra Caviar. Moreover, because of price effect, the rich are major consumers of luxury brands. This fact motives me to use U.S sales of luxury brands (goods and service) to approximate consumption of quality goods. Furthermore, my indices avoid the durability issue, covering a longer horizon and a more extensive brand set. Furthermore, my indices

\textsuperscript{21} Since the focus of VissingJrgensen (2002) is to estimate elasticity of intertemporal Substitution. And from here, one can notice how small the estimator of EIS is. As stated in previous footnotes, EIS of long-run risk is above 1, significantly higher than the number in VissingJrgensen (2002).
are more correlated with the income of the rich, especially my quality service index.

As being pointed out that goods of quality consumption here are not necessarily equivalent to luxury goods defined as Equation-(10) in Ait-Sahalia, Parker and Yogo (2004). For my purposes, “luxury brands” merely means high-quality consumption. For example, men’s suits are a common kind of normal goods. One can pick up a Tommy suit located in Macy’s department store. Or he can pick up a Brioni or Kiton suit (made in Italy), from Bergdorf Goodman or Saks, or go to a professional tailor, for example, Anderson-Sheppard or H-Huntsman, in Savile Row (London) and have a bespoke one made by cashmere\textsuperscript{22} from Lora Piana, Holland Sherr or Harrisons\textsuperscript{23}. Luxury brands stand for quality, texture, and design, not just a famous name or fashion. One can obviously feel the difference between a woven silk tie by Hermes and a common silk tie made in China. Quality groceries in premium groceries, for example, Wholefood, are also different from those in Walmart. Luxury brands signal higher quality and have nothing to do with income elasticity. A belt made by Louis Vuitton may be “luxury” to normal people, but “normal” to the rich. Organic olive oil may seem expensive, but “necessary” to the rich.

In next two sections, I will first lay out a parsimonious model specification to combine Epstein and Zin preferences with kinds of composition risk. Then two pieces of evidence on the limited participation of the stock market follow.

\textsuperscript{22}Alternatively pashimina, shahpashm, Capra-Hircus, Vicuna and Guanaco will feel better. From the perspective of wildlife protection, I will not recommend the last two.

\textsuperscript{23}New money prefers Italian style texture. Scabal is not as popular as before nowadays. Other top brands include W Bill, Smith Woollens, Scabal, Harrisons of Edinburgh, H Lesser, Dormeuil, Zegna, Carlo Babera
1.3 A parsimonious setup with general preference specification

In this section, we lay out and solve a parsimonious setup of a representative agent model. A brief numerical solution section is left in Appendix.

Generally, there can be several kinds of goods, and several sub-kinds of goods within each kind. For an illustration purpose, I set out a setup with three kinds of goods $C$, $D$ and $H$. $c_t$, $d_t$ and $h_t$ represent consumption vectors of $C$, $D$ and $H$. Within each category, there exist an homogeneous degree one aggregator $f(.)$, $g(.)$ and $l(.)$ to aggregate vector up:

$$
C_t = f(c_t) \\
D_t = g(d_t) \\
H_t = l(h_t)
$$

where $C_t$, $D_t$ and $H_t$ are scales, representing aggregate consumptions of corresponding kinds, I interpret them as consumption flow of nondurable goods and services, durable goods and housing services. $c_t$, $d_t$ and $h_t$ represent consumption vectors. For example, $c_t$ represents a supermarket shopping list. And sum or weighted sum can serve as an aggregator: $f(c) \equiv c^T 1$. I define a flow utility function on $C_t$, $D_t$ and $H_t$ through a CES aggregator:

$$
U(C_t, D_t, H_t) = \left[ [C_t^\alpha + aD_t^\alpha]^\frac{\rho}{\epsilon} + bH_t^\rho \right]^\frac{1}{\rho}
$$

Thus $\epsilon = \frac{1}{1-\alpha}$ is the elasticity of substitution between $C$ and $D$ consumption. $\xi = \frac{1}{1-\rho}$ is the elasticity of substitution between $H$ and the bundle goods combining $C$ and $D$. $a$ and $b$ are their corresponding weight. I later calibrate $a$ and $b$ to match expenditure share of nondurables and expenditures other than house services.
To lay out an non-expected utility specification, I follow and extend Epstein and Zin (1989), specifications in Bansal and Yaron (2004) and Yogo (2006). Let $V_t$ and $V_{t+1}$ denote life time utility (value function) at period $t$ and $t+1$. Instead of assuming time separability, I specify\(^\text{24}\):

$$V_t = \{(1 - \beta)U(C_t, D_t, H_t)^{\frac{1-\gamma}{\theta}} + \beta E[V_{t+1}^{1-\gamma}]^{\frac{1}{\theta}}\}^{\frac{\theta}{1-\gamma}}$$  \hspace{1cm} (1)$$

Where $\beta < 1$ is the rate of time preference, $\gamma$ is the risk aversion coefficient. Elasticity of intertemporal substitution $\psi = \frac{\theta}{\theta + \gamma - 1}$ governs agents’ inter-temporal behavior. If one set $\theta = 1$, this specification degenerates into a CRRA framework. And the relative magnitude of $\gamma$ and $\frac{1}{\psi}$ determines whether people prefer an early resolution\(^\text{25}\) of future uncertainty\(^\text{26}\) is preferred when $\gamma > \frac{1}{\psi}$.\(^\text{27}\)

The representative agent maximizes lifetime value function subject to a budget constraint:

$$C_t + q^d_t D_t + q^h_t H_t + p^e_t s^e_t + (p^d_t - q^d_t) s^d_t + (p^h_t - q^h_t) s^h_t \leq W_t$$  \hspace{1cm} (2)$$

$W_t \equiv (p^e_t + d_t) s^e_{t-1} + p^d_t (1 - \delta^d) s^d_{t-1} + p^h_t (1 - \delta^h) s^h_{t-1}$

where $C_t$ denotes nondurable consumption in period-$t$, $D_t$ and $H_t$ are service flow from durable\(^\text{28}\) goods and house. And I further assume that there is a perfect rent market, people

\(^{24}\)A more general setup can be written as $V_t = A(U_t, \mu(V_{t+1}))$, where $A$ is an aggregator function. $U_t$ is the utility flow for period $t$ only. $\mu(V_{t+1})$ denotes the certainty equivalent of future life value. From here, one know why we will later interpret $\gamma$ in specification-1 as risk aversion coefficient.

\(^{25}\)One can consider two consumption streams. The first process draws a level of consumption from a certain distribution for each period, while the second one draws a level of consumption from the same distribution for the first period and consumption in later periods will be fixed at the value realized in the first period. If the second consumption stream is preferred, people are named to have early resolution preference.

\(^{26}\)It is not obvious whether an early resolution is preferred by a normal people. It may depend on situations in reality: people may hate an early resolution of some rare disaster, late cancer for example. Barro (2009) can be understood in this way. However, early consumption resolution is preferred seems to be a reasonable ex-ante assumption. This possibility arises a future extension to add in behavior modification: since a resolution of some events may result in a modification of the original optimization problem.

\(^{27}\)The long-risk model needs a high EIS to have an early resolution preference. Hence, future risk (two layers) increases the required return on risky assets.

\(^{28}\)One can set $\delta^d$ or $\delta^d$ or both to be 1, if one or both is nondurable.
thus can enjoy an amount of service unmatched with her durable stock holding or house size \( s_t^D \) and \( s_t^H \), similar assumptions are also adopted in Piazzesi and Schneider (2016). This assumption will simplify our calculation since we can view \( s_t^D \) and \( s_t^H \) as another two kinds of investment except for stock, \( s_t^e \).

There are three kinds of consumption goods and services: \( C_t, D_t \) and \( H_t \). There is a competitive market for each of them. Furthermore, \( s_t^e, s_t^d \) and \( s_t^h \) are stock level of equity, durable goods, and house. All of them are determined in period-\( t \) and will be predetermined for period-\( t + 1 \). The dividend from stock is denoted as \( d_t \). Durable stock and house \( s_t^d \) and \( s_t^h \) provide service flows \( s_t^d \) and \( s_t^h \). Service flow comes in the same amount as stock. Furthermore, the return of \( s_t^e, s_t^d \) and \( s_t^h \) can be represented as \( \frac{p_{t+1}^{d_h} - q^d_t}{p_t^d} \) and \( \frac{p_{t+1}^{h} - q^h_t}{p_t^h} \).

From Equation-(1) and the budget constraint, one can tell that value function is homogeneous degree on in wealth. I thus denote \( V_t = \phi_t W_t \), and \( \phi \) can be represented recursively. Substitute \( V_t = \phi_t W_t \) into Equation-(1), I can express \( \phi_t^{1-\gamma} \) as\(^{29}\):

\[
\phi_t^{1-\gamma} = (1 - \beta)U^{1-\gamma}(x_t, y_t, z_t) + \beta E^{b_t}[\phi_{t+1}^{1-\gamma}(1 - q_t^d y_t - q_t^h z_t)^{1-\gamma}]
\]

where \( x_t \equiv \frac{C_t}{W_t}, y_t \equiv \frac{D_t}{W_t} \) and \( z_t \equiv \frac{H_t}{W_t} \) stand for consumption tendencies out of wealth. \( q_t^d, q_t^h \) are the real prices of service flow from durable goods and house. \( R_{t+1,m} \equiv \frac{W_{t+1}}{W_t - C_t - q_t^d D_t - q_t^h H_t} \)

\(^{29}\)This representation can be proved as

\[
\phi_t = \{(1 - \beta)U(C_t, D_t, H_t, W_t)\}^{1-\gamma} + \beta E[\phi_{t+1}^{1-\gamma}(\frac{W_t}{W_{t+1}})^{1-\gamma}]^{1-\gamma} \\
= \{(1 - \beta)U(x_t, y_t, z_t)\}^{1-\gamma} \\
+ \beta E[\phi_{t+1}^{1-\gamma}(\frac{W_{t+1}}{W_t - C_t - q_t^d D_t - q_t^h H_t})^{1-\gamma}]^{1-\gamma} (1 - q_t^d y_t - q_t^h z_t)^{1-\gamma} \\
= \{(1 - \beta)U^{1-\gamma}(x_t, y_t, z_t) + \beta E^{b_t}[\phi_{t+1}^{1-\gamma} R_{t+1,m}^{1-\gamma}]^{1-\gamma}(1 - q_t^d y_t - q_t^h z_t)^{1-\gamma}\}^{1-\gamma} \]
denotes the total return of wealth\(^{30}\).

The first order condition of \(x_t, y_t\) and \(z_t\) can be written as:

\[
(1 - \beta)U(x_t, y_t, z_t) \frac{\partial U}{\partial x} = \beta E^\frac{1}{\gamma} \left[ \phi_{t+1}^{1-\gamma} R_{t+1,m} \right] (1 - x_t - q_t^d y_t - q_t^h z_t) \frac{1-\gamma}{\theta} 
\]

\[
(1 - \beta)U(x_t, y_t, z_t) \frac{\partial U}{\partial y} = \beta E^\frac{1}{\gamma} \left[ \phi_{t+1}^{1-\gamma} R_{t+1,m} \right] (1 - x_t - q_t^d y_t - q_t^h z_t) \frac{1-\gamma}{\theta} q_t^d 
\]

\[
(1 - \beta)U(x_t, y_t, z_t) \frac{\partial U}{\partial z} = \beta E^\frac{1}{\gamma} \left[ \phi_{t+1}^{1-\gamma} R_{t+1,m} \right] (1 - x_t - q_t^d y_t - q_t^h z_t) \frac{1-\gamma}{\theta} q_t^h 
\]

After some algebra manipulation\(^{31}\), we can simplify Equation-\(*\) into following equation, where we denote \(U'_x \equiv \frac{\partial U}{\partial x} :\)

\[
\phi_t^{1-\gamma} = (1 - \beta)U \frac{1-\gamma}{\theta} (x_t, y_t, z_t) U'_x 
\]

\(^{30}\)I did not include human capital here. To take human capital into account, Bansal, Kiku and Yaron (2007) and Dittmar, Palomino and Yang (2016)

\(^{31}\)Furthermore, for the second term in Equation-\(*\), we have:

\[
\begin{align*}
\beta E^\frac{1}{\gamma} \left[ \phi_{t+1}^{1-\gamma} R_{t+1,m} \right] (1 - x_t - q_t^d y_t - q_t^h z_t) \frac{1-\gamma}{\theta} \\
= \beta E^\frac{1}{\gamma} \left[ \phi_{t+1}^{1-\gamma} R_{t+1,m} \right] (1 - x_t - q_t^d y_t - q_t^h z_t) \frac{1-\gamma}{\theta} \\
- \beta E^\frac{1}{\gamma} \left[ \phi_{t+1}^{1-\gamma} R_{t+1,m} \right] (1 - x_t - q_t^d y_t - q_t^h z_t) \frac{1-\gamma}{\theta} q_t^d y_t \\
- \beta E^\frac{1}{\gamma} \left[ \phi_{t+1}^{1-\gamma} R_{t+1,m} \right] (1 - x_t - q_t^d y_t - q_t^h z_t) \frac{1-\gamma}{\theta} q_t^h z_t \\
= (1 - \beta)U \frac{1-\gamma}{\theta} \frac{\partial U}{\partial x} - (1 - \beta)U \frac{1-\gamma}{\theta} (x U'_x + y U'_y + z U'_z) \\
= (1 - \beta)U \frac{1-\gamma}{\theta} (x, y, z) \frac{\partial U}{\partial x} - (1 - \beta)U \frac{1-\gamma}{\theta} (x, y, z) 
\end{align*}
\]

The third equality comes from Equation-(3) (5) and (6). The last equality comes from the fact that our period utility function is homogeneous degree one. And \(U'_x = \frac{\partial U(x,y,z)}{\partial x}, U'_y = \frac{\partial U}{\partial y} \) and \(U'_z = \frac{\partial U}{\partial z} \)
On the other hand, we can manipulate Equation-(*) in the following way:

\[
\phi_t^{1-\gamma} = (1 - \beta) U^{1-\gamma} (x_t, y_t, z_t) + \beta E^R [\phi_{t+1}^{1-\gamma} R_{t+1,m}^{1-\gamma} (1 - x_t - q_t^{d} y_t - q_t^{h} z_t)]^{1-\gamma} \bigg|_{x_t,y_t,z_t=x^*,y^*,z^*} \\
= (1 - \beta) U^{1-\gamma} (x_t, y_t, z_t)(xU'_x + yU'_y + xU'_t) \\
+ \beta E^R [\phi_{t+1}^{1-\gamma} R_{t+1,m}^{1-\gamma} (1 - x_t - q_t^{d} y_t - q_t^{h} z_t)]^{1-\gamma} \\
= \beta E^R [\phi_{t+1}^{1-\gamma} R_{t+1,m}^{1-\gamma} (1 - x_t - q_t^{d} y_t - q_t^{h} z_t)]^{1-\gamma} \\
+ \beta E^R [\phi_{t+1}^{1-\gamma} R_{t+1,m}^{1-\gamma} (1 - x_t - q_t^{d} y_t - q_t^{h} z_t)]^{1-\gamma} \\
= \beta E^R [\phi_{t+1}^{1-\gamma} R_{t+1,m}^{1-\gamma} (1 - x_t - q_t^{d} y_t - q_t^{h} z_t)]^{1-\gamma} \\
(8)
\]

Substitute Equation-(7) into (8), we will get the final asset pricing formula \(1 = E[M_{t+1} R_{t+1}]\) with a nesting pricing kernel\(M_{t+1}\):

From Equation-(8):

\[
\phi_t^{1-\gamma} = \beta^\theta E[\phi_{t+1}^{1-\gamma} R_{t+1,m}^{1-\gamma} (1 - x_t - y_t q_t^{d} - z_t q_t^{h})^{1-\gamma}] \\
\Rightarrow \\
1 = \beta^\theta E[(\phi_{t+1}^{1-\gamma} R_{t+1,m}^{1-\gamma})(1 - x_t - y_t q_t^{d} - z_t q_t^{h})^{1-\gamma}] \\
= \beta^\theta E[(U_{t+1}^{1-\gamma} (x_{t+1}, y_{t+1}, z_{t+1}) U_{x,t}^{\theta} R_{t+1,m}^{1-\gamma})(1 - x_t - y_t q_t^{d} - z_t q_t^{h})^{1-\gamma}] \\
= \beta^\theta E[(U_{t+1}^{1-\gamma} (C_{t+1}, D_{t+1}, H_{t+1}) U_{C,t}^{\theta} W_{t+1}^{\gamma})(1 - x_t - y_t q_t^{d} - z_t q_t^{h})^{1-\gamma}] \\
\times (1 - x_t - y_t q_t^{d} - z_t q_t^{h})^{1-\gamma} \\
= \beta^\theta E[(U_{t+1}^{1-\gamma} (C_{t+1}, D_{t+1}, H_{t+1}) U_{C,t}^{\theta} W_{t+1}^{\gamma})(1 - x_t - y_t q_t^{d} - z_t q_t^{h})^{1-\gamma}] \\
\times (1 - C_t W_t - q_t^{d} D_t - q_t^{h} H_t)^{1-\gamma} \\
= \beta^\theta E[(U_{t+1}^{1-\gamma} (C_{t+1}, D_{t+1}, H_{t+1}) U_{C,t}^{\theta} W_{t+1}^{\gamma})(1 - x_t - y_t q_t^{d} - z_t q_t^{h})^{1-\gamma}] \\
\times (1 - C_t W_t - q_t^{d} D_t - q_t^{h} H_t)^{1-\gamma} \\
(8')
\]
where the third equality comes from Equation-(7). The fourth and last equation comes from Euler’s theorem

Combined with Equation-(1), we have pricing kernel $M_{t+1}$ represented as following,

\[ \max_{\omega_i} E[f_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma}] \]

\[ \text{s.t} \]

\[ R_{m,t+1} = \sum \omega_i R_{i,t+1}, \sum \omega_i = 1, \text{ and } R_{N,t+1} = R_f \]

From the first order conditions for $i = 1, \ldots, N - 1$ and $N$, for any $i = 1, \ldots, N - 1$, we have

\[ E[f_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma}(R_{i,t+1} - R_f)] = 0 \Leftrightarrow E[f_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma} R_{i,t+1}] = E[f_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma} R_{f,t+1}] \]

Hence,

\[ E[f_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma}] = \sum_{i=1}^{N} E[f_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma} \omega_i R_{i,t+1}] = \sum_{i=1}^{N} E[f_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma} \omega_i R_f] = E[f_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma} R_f] \]

The first equality comes from the definition of $R_{m,t+1}$, the second is because the first order conditions and the last one follows $\sum \omega_i = 1$. From Equation-(8'), we know $1 = \beta^\theta \frac{(1 - x_t - q_t^y y_t - q_t^z z_t)^{1-\gamma-\theta}}{\phi_t} E[f_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma}]$, hence

\[ 1 = \beta^\theta \frac{(1 - x_t - q_t^y y_t - q_t^z z_t)^{1-\gamma-\theta}}{\phi_t} E[f_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma} R_f] \]

\[ = E[M_{t+1} R_f] \]

\[ = E[M_{t+1} R_{i,t+1}] \]

The first and last two equalities is from equation-(8'), the second one is from first order condition for $\omega_i$ and $\omega_N$. Thus, $M_{t+1}$ is a pricing kernel.
\[ M_{t+1} = \beta^\theta \left( \frac{A_{t+1}}{A_t} \right)^{1-\gamma-\theta \rho} \left( \frac{B_{t+1}}{B_t} \right)^{\theta (\rho-\alpha)} \left( \frac{C_{t+1}}{C_t} \right)^{\theta (\alpha-1)} R_{m,t+1}^{\theta-1} \]  

(9)

where

\[ A_t \equiv \left\{ [C^{\alpha} + aD^{\alpha}]^{\frac{\theta}{\gamma}} + bH^\rho \right\}^{\frac{1}{\theta}} \quad \text{and} \quad B \equiv [C^{\alpha} + aD^{\alpha}]^{\frac{1}{\theta}} \]

Kernel – 8’ or 9 combine the preference in Epstein and Zin (1989) with more general composition risks. As one set \( a = 0 \) and \( b = 0 \), utility function then become \( U(C_t, D_t, H_t) = C_t \), then from Equation-8’, we can have:

\[ M_{t+1} = \beta^\theta R_m^{\theta-1} \left( \frac{C_{t+1}}{C_t} \right)^{1-\gamma-\theta} \]
\[ = \beta^\theta R_m^{\theta-1} \left( \frac{C_{t+1}}{C_t} \right)^{1-\gamma-\theta} \]
\[ = \beta^\theta R_m^{\theta-1} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} \]  

(10)

Under different specification of \( A, B \), my specification can degenerate into CRRA and original setup of E-Z preference. Here, through more general aggregator, composition of \( C, D \) and \( H \) will play a role in asset pricing, namely composition risk. For example, from Equation-(10), we can tell that when \( \theta = 1 \), EIS \( \psi \) becomes \( \psi = \frac{\theta}{\theta+\gamma-1}|_{\theta=1} = \frac{1}{\gamma} \), general formula degenerates into CRRA specification. This observation is confirm as pricing kernel now becomes \( M_{t+1} = \beta\left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} = \beta\left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \). Compared to CRRA case, we need a restriction to take the advantage of E-Z preference: \( \theta > 1 \), in this case \( 1-\theta-\gamma < -\gamma \). Pricing kernel is thus more volatile and more correlated with risky return (\( R_m \) part) than CRRA case.

Further more, when \( \gamma > 1 \), \( \theta > 1 \) is equivalent to \( \gamma > \frac{\theta+\gamma-1}{\theta} \equiv \frac{1}{\psi} \). As stated in previous section, E-Z can improve models’ explanation power against equity premium, when people prefer early resolution.
1.4 Estimate the RRA with Aggregate Data

To study the equity premium, I focus on two versions of asset pricing Euler equations. One from a simplified version of the general framework, and I will use this simple version to illustrate the implications out of covariance of aggregate consumption and returns. The other one follows Equation – (8’) or (9), I adopt this one as granting the model enough freedom to fit the data. In this general pricing kernel, I calibrate the parameter $a$ and $b$ to match the expenditure shares of nondurable goods and other expenditure except house services. The empirical results out of these two specifications are summarized in Table-1.2.

Under the setup in the previous section, I am testing the following Euler equations:

\[(EZ)\quad 0 = E[\beta^\theta (\frac{A_{t+1}}{A_t})^{1-\theta} (\frac{B_{t+1}}{B_t})^\theta \cdot (\frac{C_{t+1}}{C_t})^\theta (R_{m,t+1} - R_f)]\]

\[(CRRA)\quad 0 = E[\beta (\frac{C_{t+1}}{C_t})^{-\gamma} (R_t - R_f)].\]

where CRRA case comes from specifying $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$ or $\gamma = \frac{1}{\psi}$, pricing kernel follows Equation-follows Equation-(8’), without resort to Epstein-Zin or other non-traditional preference parameterization. EZ case follows Equation-(9), with $A_t \equiv \left\{ (C^\alpha + aD^\alpha)^{\frac{\alpha}{\gamma}} + bH^\theta \right\}^{\frac{1}{\gamma}}$ and $B \equiv [C^\alpha + aD^\alpha]^{\frac{1}{\gamma}}$.

If only the excess return equation is to be used in estimation, I can write the estimation process of CRRA case into a more explicitly form by approximate the formula by a Taylor expansion:

\[E[R_t - R_f] \approx \gamma \text{Cov}(g_c, R_t - R_f)\]

---

34 For the intuition; one can focus on CRRA case. In general case, the only extra risk is from the composition. For example, the ratio between different kinds of consumption.

35 GMM with one moment delivers a similar value of $\gamma$

36 Alternatively, one can explore property of lognormal distribution: if $\ln(x) \sim N(\mu, \sigma)$, then $E[x] = \exp(\mu + \frac{\sigma^2}{2})$. Here, I just use Taylor expansion and the fact that $g_c$ and $R - R_f$ are relatively small numbers.
where $g_c \equiv \ln(C_{t+1}) - \ln(C_t)$ and $R - R_f$ is the excess return of equity. Hence, we can estimate $\gamma$ well from raw data or $\{C_t\}$ and $R_t - R_{f,t}$, without resorting to any regression method:

$$\gamma = \frac{E[R - R_f]}{\text{Cov}(g_c, R - R_f)}$$

Where $E[\cdot]$ and Cov$(\cdot, \cdot)$ can be estimated as sample average and covariance under the ergodic assumption. One can tell explicitly that covariance between (relevant) consumption growth with excess rate is the critical term for equity premium puzzle: since $E[R - R_f] \approx 6\%$ during our sample (1961-2016). And covariance between two sequences is determined by correlation coefficient and individual variance. One can tell from this formula that under CRRA case, the equity premium implies unreasonably RRA when covariance between the excess return and consumption growth is low. For example, if we use annual NIPA consumption process, we will get an estimator around 100, as summarized in Table-1.2. Throughout my whole estimation process, I follow Campbell (1999) and use the beginning-of-period timing convention as it gives aggregate consumption data the best chance to fits stock returns.

Since consumption data from NIPA has been filtered and interpolated, the variance of these sequences got dampened. From the first two rows of Table-1.2, one can tell the correlation between NIPA consumption sequences with the excess return is not low. High implied RRA is a result of low individual variance and thus low covariance between consumption and excess return. Hence, Savov (2011) use the garbage data to approximate true consumption

Equation-10 can be proved as following:

$$0 = E[G_e^{-\gamma(R_t - R_f)}]$$

$$\Rightarrow E[\exp(-\gamma g_c)]E[R - R_f] = -\text{Cov}(\exp(-\gamma g_c), R - R_f)$$

$$\Rightarrow (1 - \gamma g_c + O(1))E[R - R_f] = -\text{Cov}(1 - \gamma g_c + O(1), R - R_f)$$

$$\Rightarrow (E[R - R_f] - \gamma g_c E[R - R_f] + O(1)) = -\text{Cov}(1 - \gamma g_c + O(1), R - R_f)$$

$$\Rightarrow E[R - R_f] \approx \text{Cov}(\gamma g_c, R - R_f)$$
process. However, garbage disposable is not a consumption index at all. Moreover, garbage
is measured by weight. Garbage resulting from nondurable goods, like grocery, receives much
less weight than durable goods, e.g., an abandoned running machine.

As an alternative to measuring consumption, I use a more precise index to approximate
aggregate consumption: annual total retail sales (1992-2016). I can get a close estimator
around 25 out of this retail data, while Savov (2011) has 17. Under CRRA specification,
I also try branches of other sequences to represent (approximate) consumption process, for
example, dividend process, auto, Jewelry and watch expenditure from NIPA. As one can
tell from Table-1.2, durable expenditure from NIPA performs relatively better to fit stock
returns, because of higher volatility. To avoid this durability issue, in later sections I exclude
luxury brands focusing on durable goods and try to back out consumption from income and
wealth data of the rich. For example, Richemont is the second largest\textsuperscript{37} luxury group, I
will not consider its sale when I create my quality goods index since this group focuses on
durables like watches, e.g., IWC Schaffhausen, and writing materials, such like Montblanc.

Beside filtering issuance, all consumption data are time aggregated, and there is a poten-
tial time-aggregation bias: As shown by Breeden, Gibbons and Litzenberger (1989), using
time-aggregate consumption can bias the estimated covariance downward by a factor 0.5.
Hence, given an estimate of RRA, an optimistic adjustment can simply divide it by 2. As
summarized in Table-1.2, the lowest estimator of RRA when we use NIPA consumption data
would be 50, still significantly larger than the upbound 10 in Mehra and Prescott (1985).
And it is worth pointing out that aggregation bias is one reason why calibration results differ
with estimation results.

Furthermore, I include estimator under the EZ-specification in the last row of Table-1.2
with $a$ and $b$ matching the expenditure shares. As one can tell from the table, the estimate
of RRA is 29, which is much less than Hansen and Singleton (1983)\textsuperscript{38}. If one would like to

\textsuperscript{37}LVMH is the largest one and stays in our sample.

\textsuperscript{38}I compare my results with Hansen and Singleton (1983), since they use similar estimation process, GMM.
consider more portfolios as extra moments, for example, Fama’s SMB and HML factors, one will get a risk aversion parameter with value 55, significantly lower than Yogo (2006). And if one is more ambitious to fits Fama-French’s 25 portfolios, I will get a risk aversion with value 58, as a benchmark Yogo (2006) gets 191. However, House alone, will not generate a low risk aversion estimator, for example, Davis and Martin (2005)\textsuperscript{39}. Without additional help, it seems impossible to address equity premium puzzle.

\textbf{Table 1.2: Risk Aversion Implied by Aggregate Data}

<table>
<thead>
<tr>
<th>Relevant Consumption Series</th>
<th>Period</th>
<th>Risk Aversion (s.t.d)</th>
<th>RRA-adjusted (s.t.d)</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCE(Total)</td>
<td>1930-2016</td>
<td>85.6</td>
<td>42.8</td>
<td>0.0872</td>
</tr>
<tr>
<td></td>
<td>(90.2)</td>
<td>(45.1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCE(Non-durable)</td>
<td>1930-2016</td>
<td>95.54</td>
<td>47.77</td>
<td>0.0836</td>
</tr>
<tr>
<td></td>
<td>(97.7)</td>
<td>(48.85)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCE(Durable)</td>
<td>1930-2016</td>
<td>51.57</td>
<td>25.78</td>
<td>0.0629</td>
</tr>
<tr>
<td></td>
<td>(58.1)</td>
<td>(29.1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCE(Durable: Jewelry and Watches)</td>
<td>1961-2016</td>
<td>-114.26</td>
<td>-57.13</td>
<td>-0.053</td>
</tr>
<tr>
<td></td>
<td>(122.1)</td>
<td>(61.05)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCE(Durable:auto)</td>
<td>1961-2016</td>
<td>36.84</td>
<td>18.42</td>
<td>0.121</td>
</tr>
<tr>
<td></td>
<td>(42.1)</td>
<td>(21.05)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retail sale(Grocery Stores)</td>
<td>1992(Jan)-2016(Dec)</td>
<td>24.98</td>
<td>12.48</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(31.5)</td>
<td>(15.75)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dividend(S&amp;P 500)</td>
<td>1930-2016</td>
<td>101.96</td>
<td>50.98</td>
<td>0.0333</td>
</tr>
<tr>
<td></td>
<td>(109.8)</td>
<td>(54.9)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real GDP</td>
<td>1930-2016</td>
<td>55.3</td>
<td>27.65</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>(61.7)</td>
<td>(30.85)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Durable, Nondurable consumption and service flow from House \textsuperscript{b}</td>
<td>1930-2016</td>
<td>29.2</td>
<td>14.6</td>
<td>n.a</td>
</tr>
<tr>
<td></td>
<td>(35.1)</td>
<td>(17.55)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{a} The table reports the implied RRA for consumption growth using aggregate (approximated) consumption data. These results are estimators out of GMM with one moment condition of excess return under different preference specification, all rows except the last one are testing $E[G_t^{1/R}(R_t - R_f)] = 0$. Two step of optimal weighting matrix algorithm and Newey and West (1987) adjustment with 5 Lags were adopted through all estimation processes.

\textsuperscript{b} All other specification are under CRRA. Here we adopt a general pricing kernel $M_{t+1} = \beta^\theta \left[ \frac{A_{t+1}}{A_t} \right]^{1-\rho-\theta} \left[ \frac{B_{t+1}}{B_t} \right]^{\theta(\rho-\alpha)} \frac{C_{t+1}}{C_t}^{\theta(\alpha-1)} P_{\text{m},t+1}^{-1}$, with $A_t \equiv \{ [C^\alpha + aD^\alpha]^{\frac{1}{\alpha}} + bH^\alpha \}^{\frac{1}{\alpha}}$ and $B \equiv \{ [C^\alpha + aD^\alpha]^{\frac{1}{\alpha}} \}^{\frac{1}{\alpha}}$.

1.5 Motivation Evidence: Limited Participation in the Stock Market

Mankiw and Zeldes (1990) investigates 1984 Panel Study of Income Dynamics (PSID) survey

\textsuperscript{39}Piazzesi, Schneider and Tuzel (2006) adopt a calibration with a tuned process, it is a quantitative success.
data and concludes that at most one-fourth American invest in stock market\(^{40}\). However, one may doubt that one year sample may not suffice to establish that most of the people are not in the stock market. I thus extend Mankiw and Zeldes (1990) and adopt PSID in year 1984, 1989, 1994, 1999-2015 (Bi-yearly Survey since 1999)\(^{41}\). PSID is available on an annual basis from 1968 to 1997, and on a biannual basis since then, but starts to offer data on equity wealth since 1984. During 1984-1999, PSID only offers equity wealth data in 1984, 1989 and 1994. I summarize the sample in Table-1.3. As one can tell from the table, Mankiw and Zeldes (1990) gives an optimistic estimator of investor fraction, averagely there are 18% American have an investment in equity market directly and indirectly. The fraction of American investing in the stock market has a declining trend, and this is not a problem with PSID sample.

One can tell the dividend and stock fractions occupied by rich starts to increase since the mid-1980s. According to Table-1.3, in 2015 only 11.17% of the American families has positive equity! If one is willing to investigate the fraction of American investing in the stock market with at least $1000, this number will be much lower than the numbers in the last column of Table-1.3, for example, Table-1.6 in Appendix.

However, the pattern I find from PSID seems to disagree the recent popularity of mutual fund. More and more people are entering this market. To address this doubt, I explore a longer and more comprehensive dataset.

I explore a more comprehensive micro-survey dataset – IRS dataset (administrative tax records). Though, there is a large gap among national accounts\(^{42}\), the survey, and tax data. For example, when we investigate PSID, we may conclude that there is one-fourth American

\(^{40}\) According to my calculation, there are 17.7% out of the whole sample have positive stock holding, as Table-1.3 shows. Here, positive stock holding is a mild restriction; a family has stock of the value of $1 will be counted as a shareholder under our conservative assumption.

\(^{41}\) I use variable “Equity in stock” (includes shares of stock in publicly held corporations, mutual funds, and investment trusts), variable Index of PSID: S110, S210, S310, S410, S510, S610, S710, S810, ER46952, ER52356, ER58169, ER65366

\(^{42}\) For example, national income, such as Census bureau estimates
### Table 1.3: Sample Observation of Shareholder, PSID:1984-2015

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Family without Investment in Equity</th>
<th>Number of Family with positive Investment in Equity</th>
<th>Total number of Family in Survey</th>
<th>Fraction of family Staying in Stock Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>5689</td>
<td>1229</td>
<td>6918</td>
<td>17.77%</td>
</tr>
<tr>
<td>1989</td>
<td>5614</td>
<td>1500</td>
<td>7114</td>
<td>21.09%</td>
</tr>
<tr>
<td>1994</td>
<td>6476</td>
<td>2181</td>
<td>8657</td>
<td>25.19%</td>
</tr>
<tr>
<td>1999</td>
<td>5492</td>
<td>1504</td>
<td>6996</td>
<td>21.50%</td>
</tr>
<tr>
<td>2001</td>
<td>5729</td>
<td>1677</td>
<td>7406</td>
<td>22.64%</td>
</tr>
<tr>
<td>2003</td>
<td>6261</td>
<td>1561</td>
<td>7822</td>
<td>19.96%</td>
</tr>
<tr>
<td>2005</td>
<td>6538</td>
<td>1462</td>
<td>8002</td>
<td>18.28%</td>
</tr>
<tr>
<td>2007</td>
<td>6809</td>
<td>1477</td>
<td>8289</td>
<td>17.83%</td>
</tr>
<tr>
<td>2009</td>
<td>7302</td>
<td>1384</td>
<td>8686</td>
<td>15.93%</td>
</tr>
<tr>
<td>2011</td>
<td>7776</td>
<td>1128</td>
<td>8904</td>
<td>12.67%</td>
</tr>
<tr>
<td>2013</td>
<td>7970</td>
<td>1090</td>
<td>9060</td>
<td>12.03%</td>
</tr>
<tr>
<td>2015</td>
<td>8035</td>
<td>1010</td>
<td>9045</td>
<td>11.17%</td>
</tr>
</tbody>
</table>

*The table reports the number of family with positive equity, including positive shares of stock in publicly held corporations, mutual funds, and investment trusts.*

Hold stock, but this one-fourth may not reflect the true ranking. Fortunately, Piketty, Saez and Zucman (2016) provide a dataset which combines tax\(^{43}\), survey, and national accounts data. Piketty, Saez and Zucman (2016) use tax data, which is critical to capture the top part of income and wealth distribution, and supplement it with survey data to capture the income not captured in tax data, for example, fringe benefits and tax-exempt transfers to match national income accounts. It is more comprehensive dataset than PSID.

For this dataset, I plot out the fraction of equity and dividend flow owned by Top 10% wealthy, in Figure-1.1. In Figure-1.1, equities held through pension plans are counted as equity holding too\(^{44}\). I thus view this estimate as a conservative one. During 1913-2015, averagely, 88.1% (89.1%) of equity (dividend) is held by top 10% wealthy. The wealthy are thus a group who own, operate and manage the firms; they bear most of the risk of the stock market. This pattern echoes the finding documented in Table-1.6: Only the risk invest a significant amount in the stock market.

However, there exist no dataset documenting the consumption of rich, I will try several

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\(^{43}\)Raw data is available after 1962 from annual public-use micro-files created by the Statistics of Income division of the IRS. Piketty, Saez and Zucman (2016) supplement this dataset with the internal use Statistics of Income (SOI) Individual Tax Return Sample files after 1979. 1916-1962, Statistics of Income, U.S Treasury Department, Internal Revenue Service(Piketty and Saez (2003))

\(^{44}\)Data Appendix, Piketty, Saez and Zucman (2016), Page 25.
ways to “back out” or approximate the consumption of the rich and use the “backed” or approximated consumption to estimate RRA. In next section, I lay out a simple exercise to illustrate this idea.

1.6 A Quantitative Exercise

As illustrated in previous sections, aggregate data cannot explain equity return well. One reason is that of filtering and interpolation issue. However, as a more appropriate index was adopted, e.g., with retail data I estimate RRA to be around 25, the fifth row of Table-1.2. In another word, one is still unable to explain the equity return even with an aggregate consumption approximate without filtering issue.

As we have documented in motivation evidence section, not all people are involved in the stock market. This phenomenon may be a result of high fixed costs of participating in the stock market or relative high subsistence level. Poor are not willing to incur this cost to invest or incur this risk. Since the risk is too large for them to hold the risky

\[45\text{In reality, investment in the stock market involves a lower bound investment amount, even ignoring other fees like transaction fee.}\]
asset, especially when income is close to the subsistence level. Or both of these two factors contributes, for example when non-rich households have relatively low income compared to their subsistence:\footnote{Under subsistence assumption, the pricing kernel under EZ preference will not be as simple as Equation-(9) and derivation method should change accordingly, since value function is not homogeneous of degree one anymore.}

$$U(c) = \begin{cases} \frac{(c-x)^{1-\gamma}}{1-\gamma} & \text{if } c > x \\ -\infty & \text{if otherwise} \end{cases}$$

A mile volatile, risky return means a damaging event to the family, as $c \to x$, where $x$ is subsistence level, marginal Utility will become infinity. Put in another way, current equity premium is not high enough to attract poor. Hence, the stock market is mainly occupied by the wealthy and rich. Limited participation thus predicts a low covariance between the aggregate consumption and stock returns, since an average American does not even bear the risk from the stock market. Aggregate consumption, no matter how we are to measure it, includes the consumption of all families. Using this aggregate sequence potentially contributes to the Equity Premium Puzzle since the covariance is down-biased.

As one can tell from Table-1.3 and Figure-1.1, top 10\% wealthy is the player in stock market. I thus make a simplifying assumption to facilitate our exploration exercise: a representative agent is assumed to have ln utility function. Without any market structure friction, consumption will be a fix fraction of rich’s wealth. The growth rate of consumption of rich can be then represented by the growth rate of the wealth of the rich. Then we use this “backed” consumption process to replicate the exercise of Mehra and Prescott (1985).

This structure “back-out” method can be extended to a general case, using only income and wealth data without the information on investment portfolios: for any given(guessed) RRA, one can solve a consumption policy function. Then we “back out” rich’s consumption data from income and wealth data. Furthermore, this backed out data can be used to
calibrate RRA again, as Mehra and Prescott (1985) does. And the true estimator should be the one at which guessed value agrees with the calibrated one. I will leave general case exploration for later sections. Here we merely explore the backed-consumption process of top income under log preference specification.

The exercise of Mehra and Prescott (1985) can be implemented as following: calibrate the income process by a two-state Markovian process, matching the expected, variance and first-order auto-correlation of income growth. And under CRRA framework, earning/price ratio(policy function) will thus be a function of the only state variable(income growth). Expected risky return can thus be calculated. I used policy function iteration algorithm as stated in the Numerical appendix.

Given a backed-out consumption process, we take it to be the “true” consumption process and parameters except time discounting rate and RRA are calibrated to match its sample
average, variance and first-order autocorrelation. The time discounting rate is fixed at 0.98, then implied excess return is plotted against varying RRA, as in Figure-1.2. To justify a 6% equity premium, rich people in top 10% quantile will need an RRA around 10, rich people in top 5% quantile will need an RRA around 7. The mechanism here is that top incomes are more volatile than aggregate income (GDP). The relatively high volatility of rich’s “consumption” can be justified by risk-sharing mechanism: workers get a fixed salary, and share-holder afford whatever the firm earns or losses. Furthermore, rich’s income is more persistent than GDP, echoing the mechanism in Bansal and Yaron (2004). It is worth pointing out that the persistent should not be too large. As well-illustrated in Boldrin, Christiano and Fisher (1997), too persistent consumption process will deliver a wired pattern: when RRA increases the implied excess return decreases. If I use the backed consumption process to estimate RRA, estimate are 11.34 for Top 10%, 10.8 for Top 5%, 9.67 for Top 1%.

In next two sections, I start to design and create a consumption index to approximate rich’s consumption. The data we use in previous and later sections will also be documented in next section.

1.7 Data

To have an index to measure quality goods, we construct sale data of Tiffany, Sakes, Gucci\textsuperscript{47}, Neiman Marcus, LVMH, Hermes, Burberry, Bulgari\textsuperscript{48}, Gucci\textsuperscript{49} and Kering\textsuperscript{50}. Many groups here own branches of brands. For example, LVHM owns 56 brands, not only Louis Vuitton, but LVHM also owns Loro Piana, Kenzo, Givenchy, and Berluti. Bering\textsuperscript{51} maybe not as

\textsuperscript{47}Now Belongs to Bering, collected sales in U.S area from Annual Reports
\textsuperscript{48}independent until 2011, now belongs to LVMH, collected sales in U.S area from Annual Reports
\textsuperscript{49}independent until 2004, belongs to Kering now
\textsuperscript{50}There is another famous luxury group, Richemont, which mainly focuses on jewellery, watches, leather products, and writing instruments. But as we discussed in the early sections, we try our best to focus on the luxury brands not concentrate on durables.
\textsuperscript{51}Formerly named Pinault-Printemps-Redoute
famous as LVMH. It transforms into a luxury retailer after 2004. However, it owns well-
known luxury brands like Brioni, YSL. All sales data are restricted to turnover in U.S area
and taken from their original annual reports.

Since most brands have products like leather products and clothes, durability is a po-
tential issue, though BEA includes them in non-durable goods. A baby cashmere\textsuperscript{52} sweater
by Loro Piana or a handbag by Hermes may last for over ten years. However, this is one
character of quality goods. Not only they provide tremendous consumption experience, such
like texture and fitting, but also they can offer them consistently and during a more extended
period.

As a solution to this durability. One can argue that fashion is fickle. Their products are
updated seasonly. However, as we are focusing on the quality part of luxury brands, fashion
is not our concern. Hence, I instead resort to two other indices and another approach: (1)
Quality Grocery, I estimate the sale of Premium Grocers by aggregate the sales of four
public grocery firms: Whole Foods Market, The Fresh Market, Sprouts Farmers Market,
and Fairway Group Holdings. I choose premium grocers as they focus on providing grocery
with high qualities. (2) Luxury lodging service. According to "J.D. Power 2015 North
America Hotel Guest Satisfaction Index". I selected out the most luxury hotels listed\textsuperscript{53}.
I then deleted those who has minor or even no operation in U.S, for example Gran Melia
hotel. They focus their business in Spain. Fortunately, most of these hotels belong to several
hotel groups: Marriott(formly name Hot Shoppes,then Marriott Hotel corporation), Hilton,

\textsuperscript{52}Gathered only from the under fleece of Hyrcus goat kids, Baby Cashmere is exceptionally fine, at just
13 microns. It takes the fleeces of 19 child goats to make a single sweater.

\textsuperscript{53}Listed hotels: Bulgari Hotels and Resorts (Marriott Hotel Development), Park Hyatt, Mandarin Orient-
tal Hotel Group(a member of the Jardine Matheson Group), St. Regis Hotels & Resorts(Starwood Hotels
System), Ritz-Carlton Hotel Company(belongs to Marriott International), Waldorf Astoria Hotels & Res-
orts(Owmed by Hilton Worldwide), Four Seasons Hotels,The Peninsula Hotels, Luxury Collection(Marriott)
,Sofitel :French luxury hotel chain brand under AccorHotels, Rosewood Hotels & Resorts, Conrade Hotels
(Hilton Group), JW Marriott(Marriott Group), Andaz (Hyatt), W Hotel (S.P.G Marriot), Renaissance Ho-
tels (Marriot), Autograph Collection Hotels (Marriot), Edition (Marriot), Curio(Hilton), Westin(Starwood
Hotels and Resorts Worldwide Or Marriot), Le Meridien (SPG), InterContinental (InterContinental Hotels
Group), Marriott, Hilton Hotels & Resorts, Hyatt Gecency (Hyatt), Sheraton(Starwood Hotels and Resorts
Worldwide), Crowne Plaza(InterContinental Hotels Group)

For the other approach, I explore the data on income, wealth and investment portfolio of the rich to estimate their consumption. Data on after-tax real income per capita, labor income share, real wealth, investment portfolios, equity wealth distribution and dividends distribution is from data appendix II of Piketty, Saez and Zucman (2016)\textsuperscript{54} . Return data of relevant categorized assets is from data appendix I of the same paper.

For returns, the stock return is measured as the return on the value-weighted NYSE-AMEX portfolio from the Center for Research in Security Prices (CRSP). I follow Fama and French (1993) to define excess return as stock return minus one-month Treasury bill rate. Along with three FamaFrench factors, return data and one-month Treasury bill rate are available from French’s Website\textsuperscript{55}. The three FamaFrench factors are excess returns on the market portfolio, returns on the SMB portfolio(small minus big), and returns on the HML portfolio(high(book-to-market) minus low). The excess market return is the return on a value-weighted portfolio of NYSE, AMEX, and Nasdaq stocks minus the one-month T-bill rate. The SMB and HML portfolios are based on the six FamaFrench benchmark portfolios sorted by size (breakpoint at the median) and book-to-market equity (breakpoints at the 30th and 70th percentiles). The SMB return is the difference in average returns between three small and three big stock portfolios. The HML return is the difference in average returns between two high and two low book-to-market portfolios. The 25 FamaFrench portfolios are constructed from an independent sort of all NYSE, AMEX, and Nasdaq stocks into quintiles based on size (i.e., market equity) and book-to-market equity. Data on the FamaFrench factors and portfolios are again obtained from Kenneth Frenchs web page.

\textsuperscript{54}Available through http://gabriel-zucman.eu/usdina/\textsuperscript{55}http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
1.7.1 Synthetic Consumption by the Rich, Backed from Wealth and Income Data

In this section, I explore the dataset provided by Piketty, Saez and Zucman (2016) to estimate the consumption by the rich. In the following sections, I will explore data from public firms to create another three indices to approximate the consumption by rich.

From data Appendix II of Piketty, Saez and Zucman (2016), I get data on income, labor share, investment portfolio, and per capita wealth data of the Top 10%, 5% and 1% wealthy. I make an empirical conjecture that the wealthy people are the same as people with highest after-tax income. Since this assumption holds just approximately, I view my estimate of consumption of the rich as an approximate. And since this conjuncture may not hold for the top 1%, in the main text, I only report the estimates of 10% and 5%.

One can view my process as an inverse procedure in Saez and Zucman (2016). Section VI of Saez and Zucman (2016) tries to estimate the saving rate of rich. Here, I try to estimate the synthetic consumption of rich.
Furthermore, I back out consumption by the following formula:

\[ W_{t+1} = R_{m,t+1} \cdot [W_t + Y_t - C_t] \]

where \( Y_t \) and \( C_t \) denote the labor income and consumption during period \( t \), and \( W_t \) represents the wealth the end of period \( t \). \( R_{m,t+1} \) is the real return of wealth. Here I use the assumption that labor income comes before capital income. Alternatively, in appendix I used \( W_{t+1} = R_{m,t+1} \cdot [W_t - C_t] + Y_t \) to replicate the whole estimate process here. Our results are robust to this modification.

Since I need data from \( W_t \) and \( W_{t+1} \) to estimate \( C_t \), hence the estimated Consumption ranges from 1927-2012. And to estimate the wealth return of the rich, I also make an approximation assumption: people obtain the same return of the same kinds of assets.

To estimate the real return of wealth, I append my data with the average real return of various kinds of asset: Home, Equity, Fixed income asset, Pension, and Business. Return data comes from data Appendix Table-I. To estimate the wealth return of the rich, I also make an approximation assumption: people obtain the same return of the same kinds of assets because of data restriction.

Following the formula, I estimate the consumption by top 10% and 5%. The growth rate of these consumption process is plotted against excess return from 1927-2012 in Figure-1.3. One can tell that these sequences comove with excess return closely.

The GMM RRA estimate out of a single moment (1927-2012) are 4.88 and 3.23 for the top 10% and 5%. For period 1960–2012, the RRA are 3.011 and 2.941.

To show the low estimate of RRA is not a result of this approach, I next focus on sales data to create three indices to approximate the consumption by the rich in following sections.

\[56\] Available through http://gabriel-zucman.eu/usdina/ or https://sites.wustl.edu/xiwang/research/.
1.7.2 Quality Goods and Equity Premium

In this section, I start to use my constructed indices to approximate the rich’s consumption and estimate RRA. One uses the sales of luxury brands in the U.S. Another one adopts quality grocery to price the asset. Another uses luxury lodging service. As a robustness check, I estimate and use the stock of quality goods, instead of sales to approximate consumption flow. The estimators we get are summarized in Table-1.4. As one can tell at first glance, all these estimates are below ten after adjustment even though we are using annual series.

Sales of Quality Goods

I plot the growth rate of constructed quality goods index against excess return in Figure-1.4. Since the goods sold by each group are close substitutes, we merely sum them up to get an aggregate index. As one can tell from Figure-1.4, there is a positive co-movement over time between excess stock returns and sales growth for quality goods. And this co-movement becomes more significant when it enters the late-1980s, when we include more and more brands in our quality goods index, before late-1980s, Tiffany is the only one in our sample.
The overall correlation (covariance) between quality sales and excess return is 0.285(0.731), while the correlation is 0.208(0.409) during 1961-1989. Furthermore, our index is much more correlated with rich's income (Top 10%) than NIPA nondurable consumption index: 0.368 against 0.113 during 1961-2014, while the correlation between aggregate consumption and income is 0.301 during the same period. Our index thus offers a good approximate to rich's consumption.

With sale index of quality goods, we estimate RRA by GMM with one moment condition under CRRA specification, hence we restrict our ability to fit the data. Estimate of RRA out of this process is 9.48, without any adjustment. This estimate of risk aversion is an order of magnitude less than estimator by using (non)durable process, which is above 50 as summarized in Table-1.2.

However, there are still two issues about our constructed index: (1) Durability. Quality goods, by definition, lives longer than there peer product. (2) Before the late-1980s, Tiffany is the sole company in our sample. In next subsections, we tend to adopt another two datasets to attack these two problems and confirm that our estimator is robust if one investigates a nondurable and longer quality consumption data.
Premium Groceries

Since the quality goods, I list in the previous section includes clothes, Jewelry, and handbags, durability is an issue. The ideal index should be nondurable goods or service. I adopt and construct two indexes to approximate them. Focus here is the same with previous sections: quality goods, for example, organic fruits, olive oil and luxury lodging service like Ritz-Carlton.

As an approximate quality nondurable goods and comparable series to the retail data we adopt in the previous section, we adopt annual(quarterly) sale of Premium Grocers. I estimate the sale of Premium Grocers by aggregate the sales of four public grocery firms\footnote{There are several large premium groceries stay private, for example, Trader Joe's and HEBs Central Market.}: Whole Foods Market, The Fresh Market, Sprouts Farmers Market, and Fairway Group Holdings. I choose premium grocers as they focus on providing grocery with high qualities. For example, Whole Foods Market is the first certified organic grocer in U.S, which means it ensures, to National Organic Program standards, organic integrity of the heterogeneous products from the time they reach stores until they are placed in a shopping cart. Even though organic foods do not have a higher content of nutrients, studies have found they have a lower level of anti-nutrients, cadmium and pesticide residues. The sales and excess return are plotted in Fig-1.5. Estimate of RRA with GMM(one moment) is 17.47, without any adjustment. Since whole food has the longest historical data, we adjust our return according to its fiscal year, which ends in September. The estimator of RRA by using whole food alone is 19.38.

\footnote{Though Safeway, Kroger, and even Walmart have expanded their premium and organic offerings in recent years. They used not to focus on premium grocery market.}
Luxury Lodging

In my main index, Tiffany is the only sample we have before the late-1980s. And as we can tell from Table-1.2, Jewelry did a not bad job to explain stock return. One may doubt the explanation power to some degree, at least before the 1980s, may come from this durability. To address this issue, we adopt luxury lodging service to approximate consumption flow. From "J.D. Power 2015 North America Hotel Guest Satisfaction Index". I then further restrict our sample to those hotel brands focusing in U.S. For example, Bulgari Hotels, and Resorts, Park Hyatt, St. Regis (Starwood Hotels System), Ritz-Carlton Company(Marriott), Waldorf Astoria Hotels & Resorts(Owned by Hilton Worldwide). Most of these hotels belong to four hotel group: Marriott (formerly name Hot Shoppes, then Marriott Hotel corporation), Hilton, Four Season(not Public), Hyatt. Hence, we collect sales of full-service and luxury sales from their annual report and construct a luxury lodging service index. The sales and excess return are plotted in Fig-1.6. Since there is a potential reservation lag, people use to book a room in advance; they plan this consumption in advance. I thus take a one-year lag to implement estimation process. Estimate of RRA with GMM(one moment) is 15.05,
Robust check: Add in Depreciation

By adopting different sources of data, we tried to address durability issue in previous subsections. In this subsection, we provide further evidence that durability is not the reason driving our results. Parker and Julliard (2005) argue that the stock of durables should be cointegrated with durables expenditure. In other words, in long-run durable expenditure can represent the stock of durables. No adjustment is necessary, as long as we use a growth rate of a longer horizon, e.g., three years. Besides this long-horizon growth rate, we estimate the stock of durable by assuming a depreciation rate.

By using the quality goods index we have constructed, we back-out the stock of quality goods, $S_t$, from the flow, $F_t$ by assume a depreciation rate$^{59}$:

$$S_t = S_{t-1}(1 - \delta) + F_t$$

$^{59}$If $\delta = 1$, then quality goods are perishable goods, and sales date can represent consumption flow.
Where \( \delta \) is the rate at which the durable good depreciates. Then \( 1/\delta \) is the expected lifetime of durable. By setting expected durability to be 1.25, 2 and 2.5 years, we can back out a sequence of stock. The RRA Estimates out of this sequence are 14.01, 17.40 and 19.9 without any adjustment. And these stocks data are plotted against excess return in Fig-1.7. I summarize our empirical finding by using quality goods sequences in Table-1.4. I lay out several summary tables with more moments conditions: Fama-French’s three factors and 25 portfolios. In brief, we have a much smaller estimator of RRA when comparing with previous literature.

Furthermore, as another kind of robustness check, we implement estimation with more moment conditions and explore the cross-sectional average return in next sections.

<table>
<thead>
<tr>
<th>Relevant Consumption Series</th>
<th>Period</th>
<th>Risk Aversion (s.t.d)</th>
<th>RRA-adjusted (s.t.d)</th>
<th>Correlation with excess return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption by Top 10%</td>
<td>1927-2012</td>
<td>4.88</td>
<td>2.06</td>
<td>0.367</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.11)</td>
<td>(2.06)</td>
<td></td>
</tr>
<tr>
<td>Consumption by Top 5%</td>
<td>1927-2012</td>
<td>3.23</td>
<td>1.62</td>
<td>0.243</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.97)</td>
<td>(1.49)</td>
<td></td>
</tr>
<tr>
<td>Quality Goods(Total)</td>
<td>1961-2016</td>
<td>9.48</td>
<td>4.74</td>
<td>0.285</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(12.2)</td>
<td>(6.1)</td>
<td></td>
</tr>
<tr>
<td>Quality Goods(1.25 Years)</td>
<td>1971-2016</td>
<td>14.01</td>
<td>7.01</td>
<td>0.164</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(16.7)</td>
<td>(8.35)</td>
<td></td>
</tr>
<tr>
<td>Quality Goods(2 Years)</td>
<td>1971-2016</td>
<td>17.40</td>
<td>8.70</td>
<td>0.1524</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(22.1)</td>
<td>(11.1)</td>
<td></td>
</tr>
<tr>
<td>Quality Goods(2.5 Years)</td>
<td>1971-2016</td>
<td>19.90</td>
<td>9.95</td>
<td>0.1417</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(25.3)</td>
<td>(12.65)</td>
<td></td>
</tr>
<tr>
<td>Premium Grocery</td>
<td>1993-2016</td>
<td>12.91</td>
<td>8.735</td>
<td>0.1207</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(15.1)</td>
<td>(9.55)</td>
<td></td>
</tr>
<tr>
<td>Quality Lodging</td>
<td>1969-2016</td>
<td>15.05</td>
<td>7.53</td>
<td>0.246</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(19.7)</td>
<td>(9.85)</td>
<td></td>
</tr>
</tbody>
</table>

*The table reports the implied RRA for consumption growth using quality goods consumption data. These results are estimators out of GMM with one moment condition of excess return under different preference specification, all rows are testing \( E[G_c, \gamma (R - R_f)] = 0 \). Two step of optimal weighting matrix algorithm and Newey and West(1987) adjustment with 5 Lags were adopted through all estimation processes.*
Figure 1.8: CCAPM can explain almost nothing cross-sectionally

Figure 1.9: 1927-2012, Realized versus predicted excess returns (25 Portfolio), Top Consumption
1.8 More Moment Conditions

In previous sections, the estimation burden we imposed on our CRRA model is mild: only one asset pricing moment, requiring our model to fit equity premium data. However, an asset pricing Euler equation is more than a moment restriction on market excess return. For example, there are Euler equations restricting the moment condition of risk-free rate, three Fama French factors and more extremely 25 or 100 portfolios formed on Size and Book-to-Market, as defined in Fama and French (1993).

I thus adopt two-step procedure of GMM method with Newey and West (1987) adjustment (5 Lags). To better assess the performance of our constructed indexes, we mainly lay out another three strings of estimation. In one string, we impose four moment conditions: Euler equations of Market excess return, risk-free rate (one-month T-bill rate), SMB and HML excess return. The last two are as defined in Fama and French (1993), for completeness of our article, we rephrase the definition as follows: SMB (Small Minus Big) is the average return on the three small portfolios minus the average return on the three big portfolios. HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios. One can view them as two different returns of two pairs of portfolios.

Another two strings impose 27 moments condition: Euler equations of Market excess return, risk-free rate (one-month T-bill rate) and pricing formula for 25 portfolios formed on Size and Book-to-Market (Value and Equally weighted), definitions and summary statistics are reserved in Appendix.

Furthermore, all these estimation processes include three parts: (1) unconditional moment estimation (2) Conditional moment estimation with lag consumption growth as instruments (3) Conditional moment estimation with lag consumption growth and risk-free rate as instruments. I adopt our two quality index: quality goods index and luxury lodging index. As

\footnote{Our results are robust if one would like to adjust more lags.}
a benchmark, we adopt nondurable consumption and service and total consumption as the benchmark.

Estimation results are summarized in Table-1.7, 1.8 and 1.9. As one can tell from these tables, adding more moments does not increase our estimation of RRA if we use our quality index, but standard deviation got reduced. For example, the first row of Table-1.8, the lowest RRA estimate is 2.17 with s.t.d 0.80, within the range pointed out in Mehra and Prescott (1985). However, when we adopt aggregate consumption index, equity premium persists. For example, the last row of Table-1.8, RRA is estimated to be 61.6 with s.t.d 12.34, significantly higher than 10.

1.9 Cross-Sectional Expected Return

In this section, we evaluate the ability of quality indices to price the average returns on different portfolios of stocks besides the excess market return. If our quality indices approximate the rich’ consumption, stocks that have more exposure to this consumption risk should have a higher expected return. In this section, we consider the ability of our quality indices to explain the cross-sectional difference of expected return across portfolios.

I am considering 25/100 portfolios formed on Size and Book-to-Market as test portfolios. As a benchmark, we compare our fitting to Fama-French three sector, CCAPM, and CAPM models. In sum, Fama-French three sector model dominates. Quality indices perform better(has lower MAE) than the other two models.

Cross section specification emerges from the linearization of pricing kernel. Suppose $M_{t+1}$ is a pricing kernel, which is a linear function of some factors, e.g. consumption growth rate in
CCAPM Model.

\[
E[M_{t+1}(R_{i,t+1} - R_f)] = 0, \quad \text{for } i = 1, 2, \ldots, N
\]

\[
\frac{M_{t+1}}{E[M_{t+1}]} = -[\alpha + \theta^T f_{t+1}].
\]

Combine these two equations, we have a cross sectional relationship:
$E\left[(R_{i,t+1} - R_f)\right] = -\frac{\text{Cov}(M_{t+1}, R_{i,t+1} - R_f)}{E[M_{t+1}]}, \quad \text{for} \quad i = 1, 2, \ldots, N$

$= \text{Cov}(\theta^T f_{t+1}, R_{i,t+1} - R_f))$

$\equiv \theta^T \Sigma_{f,i}$

$= \theta^T \Sigma_{f,f} \Sigma_{f,i}^{-1} \Sigma_{f,f} \equiv \beta_i$

$= \text{Factor Risk Premium} \cdot \beta_i$

Portfolios with higher risk exposure($\beta_i$) should enjoy a higher excess return. I visualize our results in Figure-(1.8) to (1.12). In Figure-1.8, we plot the predicted average return implied by CCAPM model to actual returns, left panel is the case of 25 portfolios and right
for 100 portfolios. You can tell from the figure that CCAPM can almost justify nothing in the difference between portfolios.

On the other hand, Fama-French model(FF) is (one of) the best among our candidate to fit cross-sectional returns. From 1960 – 2015, one can tell from upper left panel of Figure-(1.11) and (1.12), predicted returns almost lines up with actual ones. Moreover, though our quality indices cannot beat FF, it beats CAPM and CCAPM. For example, MAE of quality goods index to fit 100 portfolios is 0.026, while MAE of CCAPM and CAPM are both 0.028. From 1927 – 2012, one can tell from Figure-1.9 and 1.10 that my top consumption is not dominated by FF method.
1.10 Conclusion

In this article, we reviewed the fact that equity premium persists under the model without the help of tuned process or modified data. I argue that the equity premium puzzle is partly because we are using a polluted consumption data. I layout two pieces of evidence to show that the stock market involves only a small fraction of people. They are the top wealthy people in U.S. As a way to approximate their consumption process, we discuss and create indices of quality goods and services.

By using several quality indices, we find that marginal utility of the investors (the wealthy) co-moves significantly with the return on equity. The covariance of quality indices and excess returns implies a coefficient of relative risk aversion estimate lower than the one estimated by using aggregate consumption. To avoid the issue of durability issue, we restrict our brands sampling during the process of creating quality goods index. Moreover, we also provide two sequences of nondurable and service indices as alternative quality goods and service indices. In general, our estimators of RRA can be reduced in a reasonable range according to Mehra and Prescott (1985) and lower than those estimators in existing literature. Furthermore, our indices provide a better cross-sectional fitting than CCAPM and CAPM.
1.11 Appendix

1.11.1 An Empirical Estimation for Habit Formating Framework

It is hard to find a empirical corresponds for habit formation models. All papers based on Habit formation are using quantitative assumptions to characterize the property of habit term.

On the other hand, utility with a varying subsistence level can also be viewed as a habit formation preference with exogenous varying habit. Put in another way, subsistence level approximates habit term. To approximate this subsistence level, we use poverty thresholds, which can be understood as the lowest income to stay alive.

This subsection will explore the successful specification being used in previous literature: Habit difference. In the remaining text, I will explain the data source. Then, a model with habit formation are laid out and moments conditions are derived, And I summarize the estimation results out of a two stage GMM with HAC adjustment.

There are two slightly different versions of the federal poverty measure: poverty thresholds and poverty guidelines.

Poverty thresholds are the first version of poverty measure, being updated by Census Bureau on yearly base since 1959. It is the number used by the Census Bureau to determine the poverty status of families. Thus, all official poverty population are calculated by using this sequence. This data sequence is available from Census Bureau’s website, 1959-2016.

The other index, poverty guidelines, is created by Department of Health and Human Services(HHS). This index is a simplification of the former one, and used for administrative purposes, e.g. determining eligibility for certain federal programs. This data sequence is available from the website of HHS, 1982-2016.

Consumption per capita and inflation data are from BEA’s website.

To estimate the subsistence (habit) term $X$ in Utility function $\frac{(C-X)^{1-\gamma}}{1-\gamma}$, I use Poverty
Line (Guide) data, consumption per capita, and H-P filter.

To estimate long-run consumption per capita \( \{C_t^{\text{trend}}\}_{t=1}^{T} \), I use \( \lambda = 100 \) (annual data) to extract the long run trend from \( \{\ln(C_t)\}_{t=1}^{T} \), then long-run consumption can be backed by taking an exponential transformation of the resulting sequence.

Subsistent level or habit term should be a fraction of this long-run term. To pin down this fraction, I take the average of the ratio between Poverty line (Guide) and consumption per capita. The fractions are pinned down as 0.461 and 0.376.

Hence \( \{X_t^i\}_{t=1}^{T} \) are estimated as:

\[
X_t^i = \theta_1 \cdot C_t^{\text{trend}}
\]

where \( C_t^{\text{trend}} \) is the long-run component of consumption per capita in period \( t \). \( \theta_1 = 0.461 \) and \( \theta_2 = 0.376 \).

Thus RRA (\( \gamma \)) estimate is from GMM with the moment:

\[
E[\beta(C_{t+1} - X_{t+1})^{-\gamma}(R - R_f)] = 0
\]

To show my results are robust to my choice of \( \lambda = 100 \) in HP filter, I document the results with other value of \( \lambda \) in following table.

1.11.2 Numerical Algorithm: Policy Iteration

In this section, we layout a general version of our numerical algorithm we used in this article to solve our model. This method can be used to solve Long run risk model or Habit formation
Table 1.5: Estimated RRA with Habit formatting specification

<table>
<thead>
<tr>
<th>λ in HP filter</th>
<th>RRA₁</th>
<th>RRA₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>35.91</td>
<td>38.75</td>
</tr>
<tr>
<td>120</td>
<td>36.07</td>
<td>38.77</td>
</tr>
<tr>
<td>150</td>
<td>36.20</td>
<td>38.79</td>
</tr>
<tr>
<td>180</td>
<td>36.29</td>
<td>38.84</td>
</tr>
<tr>
<td>300</td>
<td>36.82</td>
<td>38.98</td>
</tr>
<tr>
<td>500</td>
<td>37.37</td>
<td>39.14</td>
</tr>
<tr>
<td>80</td>
<td>35.89</td>
<td>38.72</td>
</tr>
<tr>
<td>60</td>
<td>35.77</td>
<td>38.75</td>
</tr>
<tr>
<td>30</td>
<td>35.97</td>
<td>38.92</td>
</tr>
</tbody>
</table>

method too. To fix idea, we lay out several notation first: For any period \( t \), exogenous state variables are \( z_t \), endogenous state variable is \( s_t \). In general, \( z_t \) and \( s_t \) can be vectors. In our case, \( z_t \) is the growth rate of consumption, \( s_t \) is the stock holding. In Long-run risk setup, \( z_t \) will be the volatility and consumption.

Given any realization of exogenous state variables, our goal is to find a optimal choice variable and endogenous state variable for next period, namely Policy function. Since \( s_t \) here is restricted to be 1, as required by market clearing condition. I do not really have an endogenous state variable. Hence, the goal here is to find policy function for consumption and asset price.

Given any realization of \( z_t \) and policy functions from last iteration \( (k) \), \( c^k(z, F, B), q^k(z, F, B) \):

The equations we are to explore under CRRA setting is:

\[
\begin{align*}
    c_t(z_t, s_{t+1}) &= (U')^{-1}(E^{\mathbb{F}_t}[\beta U'(c^k(z_{t+1}, s_{t+1}))R_f]) \\
    q_t(z_t) &= (E^{\mathbb{F}_t}[^{\mathbb{F}_t}[\beta \frac{U'(c^k(z_{t+1}))}{U''(c^k(z_t))}(q_t(z_{t+1}, s_{t+1}) + d(z_{t+1}))])
\end{align*}
\]

(A.1)

where \( (U')^{-1} \) is inverse function of \( U'(.) \) and \( c^k(z, s_{t+1}) \) is policy function from iteration \( k \).
$E^{F_t}$ denotes an expectation operator conditional on information set at period $t$. Since our model is Markovian, $F_t$ is sufficient for agents to form an expectation. Generally, policy function is defined on state variable space. It is worth pointing out that $c_t(z_t, s_{t+1})$ and $q_t(z_t, s_{t+1})$ is not a policy function yet, since $B'$ is not a state variable, at least not for period $t$. And generally, we can choose $z_{t+1}$ instead of searching the optimum at period $t$, following endogenous grid method.

Under endowment economy setting, numerical problem is dramatically simplified by two reason: (1) $s_t$ is set to 1 exogenously, because of asset market clearing condition. Thus endogenous grid method can be avoided (2) consumption policy function can be avoid, since all the consumption comes from dividend, which will be assumed exogenously. Hence the only relevant policy function is Equation-(A.1).

Policy iteration process goes as following:

(1) Specify a initial guess of policy function, which defined on exogenous state space. I denote it as $q^0(z)$;

(2) Given any policy function $q^k(z)$ from k-th iteration. the policy function for next iteration is defined as

$$q^{k+1}(z_t) = E[\beta M_{t+1}(q^k(z_{t+1}) + d(z_{t+1}))|z_t]$$

For example, under E-Z setting $M_{t+1} = \beta^0 (\frac{A_{t+1}}{A_t})^{1-\rho-\theta} (\frac{B_{t+1}}{B_t})^{\theta (\rho-\alpha) (\frac{C_{t+1}}{C_t})^\theta (\alpha-1) R_{m,t+1}^{\theta-1}}$, with $A_t \equiv \left\{[C^\alpha + aD^\alpha]^\frac{1}{\alpha} + bH^\rho\right\}^{\frac{1}{\rho}}$ and $B \equiv [C^\alpha + aD^\alpha]^\frac{1}{\alpha}$. $A$, $B$, $C$ will be all pre-specified function of exogenous variables. Under CRRA setting $M_{t+1} = \frac{U'(c^k(z_{t+1}))}{U'(d(z_t))}$. And again $d(.)$ is exogenous specified by our model.

(3) Calculate the distance between the new policy function and the old one. Keep ite-
ating until the distance drops below a prior setting, for example $10^{-6}$.

This procedure can also be used to solve long-run risk and habit formation model. As long as within a endowment economy, extension is straightforward.
### Table 1.6: Sample Observation of Shareholder, PSID:1984-2015

<table>
<thead>
<tr>
<th>Year</th>
<th>Total number of Family in Survey</th>
<th>Number of Family with more than $1000 Investment in Equity$^a$</th>
<th>Fraction of family staying inside</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>6918</td>
<td>960</td>
<td>13.88%</td>
</tr>
<tr>
<td>1989</td>
<td>7114</td>
<td>1267</td>
<td>17.81%</td>
</tr>
<tr>
<td>1994</td>
<td>8657</td>
<td>1924</td>
<td>22.22%</td>
</tr>
<tr>
<td>1999</td>
<td>6996</td>
<td>1376</td>
<td>19.67%</td>
</tr>
<tr>
<td>2001</td>
<td>7406</td>
<td>1481</td>
<td>20.00%</td>
</tr>
<tr>
<td>2003</td>
<td>7822</td>
<td>1387</td>
<td>17.73%</td>
</tr>
<tr>
<td>2005</td>
<td>8002</td>
<td>1329</td>
<td>16.61%</td>
</tr>
<tr>
<td>2007</td>
<td>8289</td>
<td>1360</td>
<td>16.41%</td>
</tr>
<tr>
<td>2009</td>
<td>8686</td>
<td>1238</td>
<td>14.25%</td>
</tr>
<tr>
<td>2011</td>
<td>8904</td>
<td>1020</td>
<td>11.46%</td>
</tr>
<tr>
<td>2013</td>
<td>9060</td>
<td>1005</td>
<td>11.09%</td>
</tr>
<tr>
<td>2015</td>
<td>9045</td>
<td>937</td>
<td>10.36%</td>
</tr>
</tbody>
</table>

$^a$ The table reports the number of family with positive equity, including positive shares of stock in publicly held corporations, mutual funds, and investment trusts.
Table 1.7: Risk Aversion Estimation with Four Moment Condition

<table>
<thead>
<tr>
<th>Relevant Consumption Series</th>
<th>Period</th>
<th>Risk Aversion with $g_{c,t-1}$ (s.t.d)</th>
<th>Instruments with $r_{t-1}^{f}$ and $g_{c,t-1}$ (s.t.d)</th>
<th>Instruments $^{abc}$ (s.t.d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality Index</td>
<td>1961-2016</td>
<td>7.838725 (2.935593)</td>
<td>7.765891 (2.893989)</td>
<td>7.550789 (2.694529)</td>
</tr>
<tr>
<td>Top 10% Consumption</td>
<td>1961-2016</td>
<td>2.168263 (.8017915)</td>
<td>2.740748 (.6622103)</td>
<td>2.666241 (.2561315)</td>
</tr>
<tr>
<td>Top 5% Consumption</td>
<td>1961-2016</td>
<td>1.3628 (.6635609)</td>
<td>1.875603 (.363467 )</td>
<td>1.942435 (.2496861)</td>
</tr>
<tr>
<td>Nondurable goods and Service</td>
<td>1961-2016</td>
<td>9.312636 (2.53162)</td>
<td>7.765891 (2.893989 )</td>
<td>7.603876 (2.801075)</td>
</tr>
<tr>
<td>Total Consumption</td>
<td>1961-2016</td>
<td>18.72309 (2.288341)</td>
<td>17.72594 (3.631274 )</td>
<td>17.45517 (2.607822)</td>
</tr>
</tbody>
</table>

$^{a}$ The table reports the implied RRA for consumption growth using quality goods consumption data. These results are estimators out of GMM with four moment condition of excess return under RRA specification. Two step of optimal weighting matrix algorithm and Newey and West(1987) adjustment with 5 Lags were adopted through all estimation processes.

$b$ Four moments includes pricing risk free rate, pricing equity premium, SMB and HML Fama-French 3 factors.

$c$ Instruments estimation is essentially estimating conditional moment conditions.

Table 1.8: Risk Aversion Estimation with 25 Size to Book Portfolio(Value Weight)

<table>
<thead>
<tr>
<th>Relevant Consumption Series</th>
<th>Period</th>
<th>Risk Aversion with $g_{c,t-1}$ (s.t.d)</th>
<th>Instruments with $r_{t-1}^{f}$ and $g_{c,t-1}$ (s.t.d)</th>
<th>Instruments $^{abc}$ (s.t.d)</th>
</tr>
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<tr>
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<td>1961-2016</td>
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</tr>
<tr>
<td>Nondurable goods and Service</td>
<td>1961-2016</td>
<td>9.312636 (2.53162)</td>
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</tr>
<tr>
<td>Total Consumption</td>
<td>1961-2016</td>
<td>18.72309 (2.288341)</td>
<td>17.72594 (3.631274 )</td>
<td>17.45517 (2.607822)</td>
</tr>
</tbody>
</table>

$^{a}$ The table reports the implied RRA for consumption growth using quality goods consumption data. These results are estimators out of GMM with Twenty seven moment conditions of excess return under RRA specification. Two step of optimal weighting matrix algorithm and Newey and West(1987) adjustment with 5 Lags were adopted through all estimation processes.

$b$ Twenty seven moments includes pricing risk free rate, pricing equity premium, and 25 Portfolios(Value Weighted) Formed on Size and Book-to-Market.

$c$ Instruments estimation is essentially estimating conditional moment conditions.
Table 1.9: Risk Aversion Estimation with 25 Size to Book Portfolio (Equally Weighted)

<table>
<thead>
<tr>
<th>Relevant Consumption Series</th>
<th>Period</th>
<th>Risk Aversion</th>
<th>Instruments with $g_{c,t-1}$ (s.t.d)</th>
<th>Instruments with $r_{f,t-1}$ and $g_{c,t-1}$ (s.t.d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top 10% Consumption</td>
<td>1961-2016</td>
<td>2.340826</td>
<td>2.909688 (.8048266)</td>
<td>2.872546 (.4408074)</td>
</tr>
<tr>
<td>Top 5% Consumption</td>
<td>1961-2016</td>
<td>1.444308</td>
<td>1.968068 (.636315)</td>
<td>1.869321 (.2714227)</td>
</tr>
<tr>
<td>Quality Index</td>
<td>1961-2016</td>
<td>11.3465</td>
<td>11.30186 (.665057)</td>
<td>10.9427 (.3265988)</td>
</tr>
<tr>
<td>Luxury lodging</td>
<td>1961-2016</td>
<td>18.87187</td>
<td>17.0533 (2.024994)</td>
<td>17.25982 (1.093398)</td>
</tr>
<tr>
<td>Nondurable goods and Service</td>
<td>1961-2016</td>
<td>-53.03731</td>
<td>-37.7818 (-0.8420869)</td>
<td>-35.20731 (-1.033398)</td>
</tr>
<tr>
<td>Total Consumption</td>
<td>1961-2016</td>
<td>61.18114</td>
<td>60.37082 (23.91825)</td>
<td>62.31743 (1.303017)</td>
</tr>
</tbody>
</table>

a The table reports the implied RRA for consumption growth using quality goods consumption data. These results are estimators out of GMM with Twenty seven moment conditions of excess return under RRA specification. Two step of optimal weighting matrix algorithm and Newey and West(1987) adjustment with 5 Lags were adopted through all estimation processes.
b Twenty seven moments includes pricing risk free rate, pricing equity premium, and 25 Portfolios (Equally Weighted) Formed on Size and Book-to-Market.
c Instruments estimation is essentially estimating conditional moment conditions.
Chapter 2

The Quantity Theory of Money: An Empirical and Quantitative Reassessment

2.1 Introduction

The Quantity Theory of Money (QTM) has been at the heart of Monetary Economics since its birth. The QTM states that the general price level should, over the long-run, co-move with the quantity of money available in the economy. Hence general inflation should co-move with the growth rate of money, and such movement should be one-to-one. This means that the QTM is both a theory of money (it says what ”money really is”) and a theory of how markets for monetary exchanges function. In fact, the QTM begins with a well-known accounting identity

\[ M \cdot V = P \cdot Y \]
and turns into a theory of how the price level $P$ is determined as a function of the available quantity of $M$ by making assumptions about

- the way in which $Y$ (GNP or any other pile of goods traded in monetary exchanges) is determined;

- the way in which $V$ (velocity of money) moves over time and is or is not affected by $Y, M$ and other economic variables such as interest rates and what not;

The QTM is a few centuries old and, from a technical vantage point, it has assumed a variety of forms. After Friedman’s classical spelling out of its modern version (Friedman (1956)), there is no doubt that Lucas has become - both on theoretical and empirical grounds - the point of reference for the contemporary research on this subject. In fact, Lucas (Lucas (1980)) was able to show that, in US data covering the years 1955-1977, M1 growth and CPI inflation moved together when short-run movements had been reasonably filtered out. Following in the steps of Lucas, many other researchers have also contributed to strengthen the view that ...the central prediction of the quantity theory are that, in the long run, money growth... should affect the inflation rate on a one-for-one basis... the application of the quantity theory of money is not limited to currency reforms and magical thought experiments. It applies, with remarkable success, to co-movements in money and prices generated in complicated, real-world circumstances...\(^1\)

In the present paper, I examine recent data from the US and from a group of advanced economies over the half-century 1955-2016 to evaluate if this statement still stands. The

\(^1\)There are bunches of paper confirming this observation with different measure and from different angles. From a cross-country standpoint, McCandless Jr and Weber (1995) uses 30 years (1960-1990) averages of annual inflation and growth rates of $M_2$ across 110 countries to show that they line up almost perfectly along a 45-degree line. Lucas (1996) views this as a great success of QTM and Monetary Economics. From now on, I would like to refer methods analyzing the properties of data in the time domain as the temporal approach, for example, the method in McCandless Jr and Weber (1995), simply adopting sample average of raw data. More recently, Benati et al. (2016) applies co-integration tests to long spanned dataset and propose the existence of long-run money demand. Co-integration test is also a method explore (dynamic) properties of data in time domain. On the other hand, papers like Lucas (1980), Christiano and Fitzgerald (2003) and Sargent and Surico (2011) adopt frequency domain method.
executive summary of my findings is that Lucas’ original formulation works quite well for M1(2) until the early-middle 1980s(early-1990s) but begins to break down after that. By the middle 1990s the one-to-one relationship between M1 (or M2 for that matter) and inflation, which was so stable for about three decades, is all but disappeared. I investigate some, somewhat “natural,” measures of money supply other than M1 (including ”NewM1” as defined in Lucas and Nicolini (2015)) and find that, when the last twenty years or so are taken into consideration, none of them is capable of replicating what M1 used to do. Figures-2.1 and 2.1 report the (raw) time series we are studying, in levels and growth rates respectively; measures of money supply are in the upper panel while prices indices are in the lower panel in both figures.

A number of technical and theoretical issues are involved in the empirical study of the QTM: (i) the definition of what is the ”money” used in transactions; (ii) the definition of what is being transacted, which needs not necessarily be GNP; (iii) a convincing way of measuring the ”long-run movements” of the various variables. These are difficult problems, and I will next describe how I approached them, starting from the last, which is in some
In his pioneering work Lucas (1980) used an elementary band-pass filter to extract the long run\(^2\) signal from M1, CPI and the T-bill rate. This procedure is not necessarily immediate because at least M1 is a non-stationary time series. To make the band-pass filter work and avoid a spurious regression Lucas (1980) used the annual growth rates of \(M_1\) (hence of the CPI) in his statistical analysis. The stationarized sequences are used to estimate “long-run” signal.

After estimating the long run trends of inflation and money growth, Lucas (1980) plots (1) (QTM) long-run(filtered) growth rate of money against inflation rate; (2) (Fisher Effect) long-run(filtered) growth of money against T-bill rate. Lucas (1980) finds that these points line up almost perfectly along a 45-degree line. Hence inflation rate co-moves with money growth on a one-to-one basis. Furthermore, nominal interest rate is sum of real interest rate and expected inflation rate. Lucas (1980) argues that interest rate is determined by the

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\(^2\)Long-run signal, in Spectrum Analysis, refers to the signals with low frequencies, for example, frequencies around \(\omega = 0\). But it is practically impossible to identify the spectrum on \(\omega = 0\). For more details, one can refer to Sargent (1987), Hamilton (1994) and Ltkepohi (2007).
status of the real economy, which is in long run irrelevant with money stock. The long-run variation of nominal interest thus reflects variations of expected inflation. The relationship between money and the nominal interest rate is thus determined by the effect of money onto (expected) inflation. As a result, QTM reveals itself with direct and indirect evidence from its influence over inflation. In this article, we have no intent to defend or attack whether money neutrality, we instead focus on the validity of QTM. Thus, all the empirical findings below are all adjusted for real income.

Recent evidence of U.S presents a counterexample to QTM, no matter real income adjusted or not. Inflation reacts to money growth, at best, in a drawling way, certainly not on a one-for-one basis anymore. In upper(down) panel of Figure-2.1 and -2.2, we plot out the level(growth) of these normalized index during 1979-2007. In the upper panel, we normalize the level of $M_1(0/2)$ stock and price all to be 100 at 1979. From the figures, one can tell that price of consumption(Grey lines) did not keep pace with any monetary index(Black lines), during 1979 to 2007, no matter what kind of price index we are using to represent the price level. In Figure-2.1 and 2.2, we used CPI index, PCE Chain price index and GDP deflator. One can tell that PCE Chain price has almost the same trend with GDP deflator and neither of them comoves with any money index. In sum, all price index grows at an almost constant rate ignoring fluctuations from money part at the raw data level. I thus investigate whether QTM holds for U.S in a long history.

I am not the first to investigate whether QTM was stable across time for U.S. Benjamin M. Friedman(1988) did a preliminary test\textsuperscript{3} to check the relationship between price and money, and found the collapse of the one-to-one relationship between money and prices in the 1980s. There are also several recent papers take “long run” seriously, documenting breaking down of QTM(after some date). For example: “...For most of the last 25 years, the quantity theory of money has been sleeping...(Sargent and Surico (2011), Page 110). “...in the later

\textsuperscript{3}Friedman (1988) does not take “long-run” title into consideration.
period, the relationship (between money and inflation) turns negative in frequencies 20 years or higher......(Christiano and Fitzgerald (2003)).

Our empirical findings confirm this recent breaking down of QTM in U.S. I find that the period 1953-1977 which is under the investigation of Lucas (1980), is a special period, beyond which QTM is hardly a tight law governing the one-for-one relationship between money and inflation. For example, during the period from 1945-1954, (long run) inflation ran off the track of (long run) growth rate of the money stock, no matter we are using M1 or M2. However, U.S is not alone.

This fragility of QTM is robust across countries. In our robust check, we found QTM is not a global (universal) law: we have countries where QTM never holds and still holds. Furthermore, for more countries, QTM tends to hold for a while, then collapse. Though different countries have different breaking dates and degrees, collapsing QTM is qualitative robust. For example, QTM never holds in Germany and France. And QTM used to exist in Australia, but not after 2000. QTM used to exist in Italy, but not after 1998. Countries deviate away from QTM in their own ways. Before having a unifying theory to explain the breaking down, we believe one should investigate countries’ own monetary history.

Put evidence on one side; researchers have proposed several explanations for collapsing QTM. For example, Sargent and Surico (2011) adopts a general equilibrium framework and proposes disinflation policy as a candidate. Benati (2009) believes monetary velocity shock plays a role. Teles, Uhlig and Valle e Azevedo (2016) adopts the temporal approach to test whether disinflation policy weakens the relationship between money growth and inflation. Following the same approach, McCallum and Nelson (2010) finds QTM deteriorates after disinflation policies. Nevertheless, all supportive evidence provided by the temporal approach is misleading to some degree: they never checked whether QTM exists in the investigated countries or not. For example, they include France in their sample. But it is hard to conclude that QTM used to exist in France. From cross countries long-dated data, we thus believe it
is more useful to investigate in a case-by-case way.

Theoretically, disinflation policy must have a role in the revealing process of QTM, since the central bank is supposed to have the power of controlling the money supply. However, to make disinflation policy to have any effect on QTM or estimated QTM, one has to adopt a framework with uncertainty. If our economy is under a circumstance without any shock, and the central bank has an incentive to change(lower) inflation rate. The Central bank must have channels to make its inflation-rate choice. In another word, central bank needs a policy tool to implement its policy target. The channel there should be QTM or stable money demand function. In other words, QTM lays out a menu, and central bank picks its favorite from this menu. In the end, no matter how central bank dislikes inflation or what policy target central bank chooses, all the realized points should line up according to this menu: QTM. The one-to-one relationship will still hold in a deterministic world. Logically disinflation policy should, therefore, have no effects on the relationship between money growth and inflation without the help of (relative) randomness (of money supply and demand). With the help of randomness, the problem becomes a simultaneous estimation process, classical wisdom applies. Relative volatility of supply and demand play a critical role, the effect of the slope of the supply curve, at best, is second-order, see Wang (2015).

Without the help of randomness, collapsing of QTM should be explained from the angle of money demand. To make QTM be A THEORY out of an identity(or to have a stable money demand), people place restrictions on money velocity, for example, Lucas and Nicolini (2015) links money velocity with interest rate. As long as velocity is not stable, QTM will be gone. Hence, a micro foundation to this changing velocity is necessary to explain collapsing QTM.

4This cannot explain recent breaking of QTM either: points after 1984 all lie under the fitting curve. However, if one would like to link velocity with interest rate. Stationarity of interest rate implies a stable long-run velocity. Hence, a long run money regression still applies. Furthermore, the channel in Lucas and Nicolini (2015) cannot explain breaking QTM: since interest rate starts to become stationary again after 1990. It is hard to conclude that there is a trend in the process of the T-bill rate during 1990-2007. However, QTM still breaks after 1990
To micro-found changing (decreasing) velocity of QTM, we went back to money generation process. However, we have already established that Transaction purpose should not serve as a micro-foundation for the Money demand, i.e., \( M_1 \) does not robustly co-moves with inflation. Instead, \( M_2 \) does. Though \( M_2 \) version of QTM breaks, it sheds lights on the direction of us to investigate: \( M_2 \) is the major liability of depositary financial institution. On the asset side, loan plays a major role. In other words, instead of investigating deposit demand, we look at credit generation or Money generation process. Beside deposit, loan issuing also generates money. In another word, money demand is not only represented by deposit but also by the loans. As taught in Econ 101 class, a large portion of the money is generated by the financial system. I call the process of money generation “money multiplier. And money-multiplier heavily depends on the process of loan issuing. For example, Commercial and Industry loan can be used by firms to purchase intermediary goods and input. Consumer credit can be used to buy final consumption goods. These types of loan can boost prices of final goods. Instead of tracking the demand of deposit, we track the composition of loans. In U.S, major kinds of loans are associated with real estate. For example, nearly 70% of all commercial and industrial loans in the United States are secured by collateral assets (Berger and Udell (1990)). And real estate is an important tangible asset for small and large firms (from Z.1 Tables and Liu, Wang and Zha (2013)). In another word, Real estate, as an important collateral, generates significant loans. Meanwhile, in U.S the fraction of real estate loan out of total loans stays high since the beginning of available data. In sum, Real estate not only serves as collateral generates loans, but it also generates loans directly by being a transaction target. This fact inspires us to propose that (long-run) demand of money and real estate are intertwined.

To confirm our judgment, we investigate the historical nominal price of house during pre-crisis period (S&P CoreLogic Case-Shiller Home Price Indices, available from Robert Shiller’s website; Longest dataset we can find) for U.S. And we found that in U.S (filtered) growth
rate of nominal house price co-moves nearly perfectly with (filtered) growth rate of broad money during 1955-2007, not only during Lucas period(1955-1980). Put in other words, index of house price moves more robust with the growth rate of money than CPI. Hence, the demand for money or money generation should be closely linked to housing purchase. Then here comes another question: can we explain the valid QTM during 1955-1980(90)? Can we have a general theory to nest the classical QTM? Both answers turn out to be yes.

During 1955-1990s, (filtered) price growth of house tracks CPI(PCE price) inflation well, or one can think reversely: CPI(PCE) price index tracks the price of the house well. However, after a certain date, (filtered) inflation fails to track the price of the house. As notes by Davis and Heathcote (2007), cost of land plays a more and more important role in determining the price of house. One can think house as a bundle, comprising consumption goods(reproducible structure) and non-reproducible plots of land. If consumption goods contribute a constant fraction, there will be a constant fraction of money used to purchase consumption goods. However, if the fraction going into final goods decreases, in other words, the fraction of land increases, we should expect a larger fraction of money going after land, leaving the price of consumption goods growing more slowly than money growth. Put in another words, the real price of land increases, thus creates larger money demand. With this endogenous money generating process, we then explore the effect of a potential explanation of breaking QTM: financial innovation. Since financial innovation also starts around mid-1980s. I am thus to propose a broader money generating process: borrowing collateral by land also generates money. Under this endogenous money generation process, we then explore the implication of financial innovation.

Concerning to empirical method, we adopt the original method of Lucas (1980) in the main body of this article. As a robustness check, we also used bandpass filter like Christiano and Fitzgerald (2003) and (window) spectrum at like Benati (2009). Though the idea behind Lucas (1980) is to estimate long-run signal out of original data, it is worth to repeat that the
estimation of the spectrum at \( \omega = 0 \) is notorious of unstable property. Furthermore, we do not have a criterion or would like to take a stand on how long should be named to be “Long run, 30 years or over or 40 years or above? For example, if we would like to explore dataset of U.S is from 1955-2015 (post-Lucas period), 30 years or above will also have the similar disadvantage of a spectrum estimation at \( \omega = 0 \). And laying out the gain estimator would be a misleading part, high value of this estimator tends to have a high variance. Hence, when we have high gain estimator, we tend to accept that QTM is still alive. However, an estimator significant higher than one should be viewed as a rejection of QTM. And it is also straight forward to extend our analysis to adopt a Fourier Analysis approach, for example one can take \( \{ \sum_{j<5} \left[ \alpha_j \cos(\omega_j(t-1)) + \delta_j \sin(\omega_j(t-1)) \right] \}_{t=1}^{T} \) to be the estimated long-run signal. Then one must have a criterion on which frequencies should be chosen. Fourier Analysis is available from the author upon request; we closely follow Lucas (1980) in our main text.

In the related literature, various empirical methods were used. For example, Benati (2009) uses nonparametric spectrum estimation and found that QTM is not stable. Rolnick and Weber (1997) adopts the temporal approach and finds that correlation between inflation and money growth was weaker during the standard Gold Period. Sargent and Surico (2011) uses time-varying VAR to establish the recent failure of QTM. However, Sargent and Surico (2011) adopts filter of Lucas (1980) with 16 quarters window length. Benati et al. (2016) explore several co-integration tests(temporal approach) to prove the existence of long-run money demand.

To summarize, in the first part of this article, we check whether QTM ever holds across time in U.S. To robust check my results for U.S, I did a cross-countries analysis. Put in other words, we ask following question: Does Bob get lucky when he explores the data of U.S during 1955-1977? Does QTM hold for developed countries during 1955-1977? As a

\[5\text{Because of cross-correlation, cointegration specification is supposed to be a more appropriate one than the difference specification, which is taken by Lucas (1980) and us}\]
robust check, we also checked the performance of QTM across countries during the years after 1980. I find that for U.S, QTM breaks down. Our results are robust even if we abandon the post-crisis period, namely QTM breaks way before 2007. And later we will focus on this pre-crisis period. In other words, 1953-1977 is a special period. In the robust check, I investigated data of 13 countries during a long historical period, mainly from 1870-1880s or early 1890s to 2016. For each country, we check whether QTM exits 1955-1980 and post-1980. During 1950s-1980, for most of the countries in our sample, the growth rate of money provides a good explanation of inflation. Furthermore, the latest breaking down of QTM happens at a different time for different countries (Monetary measure). For example, the breaking date for U.S is 1991 if we focus on M2, while it is 1984 if we focus on M1. And this breaking date also depends on window length of our filter, for example, Sargent and Surico (2011) used a window length of 5 years and identified 1984 is breaking date.

Logically, if quantity theory of money holds for most of the countries during 1953-1977 but not recently, it is meaningful to ask another question: Why does QTM break down recently? And an interesting policy question would be whether and how the central bank can control inflation. Why is there no co-movement of inflation after expansion of monetary aggregates? However, as shown in data, each country has its pattern, we believe it is beneficial to investigate country by country. I thus investigate U.S case in this article and leave the rest for future research.

In the second half, we explore a possible explanation of the recent breaking down of QTM for U.S during the pre-crisis period. I explore the historical nominal house price of U.S (Robert Shiller's price index). I found that for U.S, in long run, the growth rate of house price co-moves with the growth of M2. Since money is endogenously generated, a higher growth rate of house price generates higher demand of loans, which will generate money

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6 It turn out that Quantity Theory of Money is not a global pattern before the 1950s, but one may doubt the quality of data during the pre-WWII period, we thus focus the results after 1955. Results of pre-1955 are available from the author upon request.
endogenously. Before the 1980s, the relative price of land and final goods stays stable. Hence the fraction of money related to final goods transaction is stable. However, since the mid-1980s, following a sequence of financial innovations, the land becomes better and better collateral, the relative price of land becomes higher. With complementary assumption, this will cause the fraction of money related to final goods decreases. Or the growth of money is not generated by final good inflation but asset inflation. Hence the relationship between the growth of money and inflation breaks. Namely, narrowly defined QTM breaks.

To model financial innovation, we use a simple occasionally binding model with collateral borrowing and Bayesian Learning following Boz and Mendoza (2014), and our new money demand function. Under this framework, we can generate a pattern with growing money stock but with significantly lower inflation. To solve this model, we adopt the method of policy iteration with the endogenous grid point. Then we calibrate our model to U.S economy and check the implied relationship between money growth and inflation.
2.2 Empirical Method

When one stares at the identity $MV = PY$, where $M$ is the stock of money, $V$ is money velocity, $P$ is aggregate price level, and $Y$ is real income. Without any restriction, it is just an identity without any further empirical implications. QTM claims a constant money velocity in the long-run. Then it implies that $\Delta M = \Delta P + \Delta Y$ or $\Delta P = \Delta M - \Delta Y$, with money neutrality assumption, $\Delta Y$ can be viewed as an error term, uncorrelated with $\Delta M$. Hence, when one run a regression $\Delta P = \alpha + \beta \Delta M + \epsilon$, QTM is a hypothesis that $\beta = 1$. Without money neutrality assumption, regression can be run as $\Delta P/Y = \alpha + \beta \Delta M + \epsilon$. Now $\epsilon$ is interpreted as measurement error. QTM is still a hypothesis stating $\beta = 1$.

To extract low frequency signal from raw data, we follow the process of Lucas (1980). First, we take fourth order difference of (quarterly) the raw data to make them into stationary processes:

$$g_m(t) = \log(M_t) - \log(M_{t-4})$$
$$\pi(t) = \log(CPI_t) - \log(CPI_{t-4})$$
$$g_y(t) = \log(y_t) - \log(y_{t-4})$$

Where $CPI_t$ and $y_t$ stand for consumption Price index and real GDP at time $t$. And $M_t$ stands for Monetary aggregate, which can be currency plus reserves $M_0$, $M_1$, $M_2$ even $M3/4$ for some countries. Since the definitions of Monetary index are different across countries. I will try to label monetary stock as narrow or broad money in robust check section, but all empirical process are the same.

After stationarizing the data, Lucas (1980) then uses exponential decreasing sequence as filter coefficient to extract long-run information. To be more precise, Let filter coefficient
ωₚ,L(n) with bandwidth L, for n = 0, 1, 2, ..., T, defined as

\[
\Psi_n(\rho, L) = \begin{cases} 
\rho^{|n|} & |n| \leq L \\
0 & |n| > L 
\end{cases}
\]

I will denote \(x(t)\) and \(x^f(t)\) for \(t = 1, 2, 3, ..., T\) as raw and filtered data respectively. \(x^f(t) = \sum_{s=1}^{T} \Psi_s(\rho, L)x(t - s) = \Psi(L)x(t)\). I explicit write \(\Psi_n\) as function of \(\rho, L\) to emphasize that filter \(\Psi(L)\) depends on discounting rate \(\rho\) and bandwidth \(L\). One can try different bandwidth and discounting rate of this filter, to extract long run signal out of raw data\(^7\). And it is worth to note that here we use zero padding technique\(^8\), which is common in signal analysis. After fixed bandwidth and \(\rho\) to be sixteen years and 0.95, we extract out long-run growth rate of Monetary stock, Inflation and real income. Then we would like to check how long-run inflation correlates with long run growth of money.

The idea of Long-run signal can be understood as following: when one applies a Fourier Transformation to our data, we can decompose the signal, represented by the data, into a sequences of sub-signals with different frequencies (periods) and their corresponding weights\(^9\).

Formally, for any \(\{y_t\}_{t=1}^{T}\), there is \(\{\alpha_j\}, \{\delta_j\}\) such that \(y_t = \bar{y} + \sum_j [\alpha_j \cos(\omega_j(t - 1)) + \delta_j \sin(\omega_j(t - 1))]\), where \(\omega_j = \frac{2i\pi j}{T}, j = 1, 2, ..., \frac{T-1}{2}\). Here, with the help of formula, we can define long-run signal to be those signals with low frequencies \(\omega_j\), for example

\(^7\)when the gain function concentrate on low frequency. Variation of filtered date comes from low frequency variance of original data series; See Sargent (1987) or Stoica and Moses (2005)

\(^8\)To be more precise, we append the original data with infinite 0. Say \(\{Y_t\}_{t=1}^{T}\) is our observation, then we apply analysis to sequence \(\{Y_t\}_{t=-\infty}^{\infty}\), where \(Y_t = 0\) if \(t > T\) or \(T < 1\)

\(^9\)Riesz Fischer theorem.

\(^10\)This perfect fitting comes from orthogonality of sequence \(\{\sin(\omega_j(t - 1))\}, \{\cos(\omega_k(t - 1))\}\)

\(^11\)The formula of \(\{\alpha_j, \delta_j\}_{j=1}^{T-1}(\frac{T}{2})\) are from Residue Theorem.

\(^12\)Whiteman (1984) interprets the filter of Lucas (1980) is one to identify the signal with frequency 0. But as we have stated in previous footnotes, it is mostly practical impossible to accomplish. With the formula here, we can put it in a more explicit way: the longest cycle we can identify from a dataset with length \(T\) is \(T\). It is impossible to torture a dataset to identify a cycle with an infinite period. Furthermore, the gain of spectrum of filter in Lucas (1980) is not a precise delta function on frequency \(\omega = 0\). In the language of spectrum analysis or electronical engineering, It has sidelobe leakage, see Chapter 2, Stoica and Moses (2005)

\(^13\)The so called “long-run risk” finance literature also refers to the lasting effects of a shock to the (small)
\{\alpha_1 \cos(\omega_1(t-1)) + \delta_1 \sin(\omega_1(t-1))\}_{t=1}^T is the long-run signal associated with frequency \(\omega_1\) or period \(\frac{2\pi}{\omega_1}\).

To understand why filter can extract a particular group of frequencies out. It is helpful to step back and view our data on frequency domain. Suppose our demeaned data \(\{x_t\}\) is generated by a stationary and ergodic data generating process\(^{14}\), we denotes auto correlation sequence as \(\{\gamma_k\}_{k=1}^\infty\), where \(\gamma_k = E[x_t x_{t-k}]\). According to inverse Fisher Reitz theorem, there is well-defined function \(g(\omega) = \sum \gamma_k e^{-ik\omega}\), we denote \(s(\omega) = \frac{1}{2\pi} g(\omega)\). \(s(\omega)\) is then named to be spectrum density of \(\{x_t\}\). Under a mild condition, an equivalent definition can be used as \(s(\omega) = \frac{1}{2\pi} \lim_{N \to \infty} E\left[\frac{1}{N} \left| \sum_{t=1}^N x_t e^{-it\omega}\right|^2\right]\)\(^{15}\).

Intuitively, we can view any random variable as a sequence with energy. A constant sequence is one with zero energy, it does not move at all. Similar to physics, a more energetic sequence is more active, in another words, more volatile. To measure volatility, we use variance as an index. Inverse Fourier transformation of \(s(\omega)\) or from the second representation of \(s(\omega)\), one can easily tell that \(Var(x) = \gamma_0 = \int_{-\pi}^{\pi} s(\omega) e^{ik\omega} |_{k=0} d\omega = \int_{-\pi}^{\pi} s(\omega) d\omega\). \(s(\omega)\) thus represents the energy contributed by sub-signal\(^{16}\) with frequency \(\omega\).

\(^{14}\)I implicitly assume it is a one dimensional data process, it is straightforward to extend this intuition into multidimensional case.

\(^{15}\)This mild condition is absolute value of \(\{\gamma_k\}\) decays sufficiently enough, so \(\lim_{N \to \infty} \frac{1}{N} \sum_{k=-N}^{N} |k| |\gamma_k| = 0\). The equivalence can be proved as following:

\[
\lim_{N \to \infty} E\left[\frac{1}{N} \sum_{t=1}^N x_t e^{-it\omega}\right]^2 = \lim_{N \to \infty} E\left[\frac{1}{N} \sum_{t,s} x_t x_s^* e^{-i(t-s)\omega}\right] = \lim_{N \to \infty} \frac{1}{N} E\left[\sum_{k=-N+1}^{N-1} (N - |k|) \gamma_k e^{-i\omega k}\right] = \sum \gamma_k e^{-i\omega k} + \lim_{N \to \infty} \frac{1}{N} \sum_{k=-N}^{N} |k| \gamma_k e^{-i\omega k} = g(\omega)
\]

The last equation follows the fact that mode of last term goes to zero as previous assumption states.

\(^{16}\)As a more general statement, we can represent the data generating process as \(y_t = \mu + \int_0^{\pi} \pi |a(\omega)\cos(\omega t) + \)
When we apply a filter, say $H(L)$, to the raw data. The spectrum density of filtered data can be represented by $H(e^{-iω})s(ω)H(e^{iω})$ or $|H(e^{-iω})|^2s(ω)$, where $s(ω)$ is the spectrum density of raw data. The ideal filter should mimic function shape of delta function: it is zero elsewhere except for one point. As Figure-2.3, we plot out the $|H(e^{-iω})|^2$ of Lucas filter with different $ρ$ and window length. Since, gain values of these six specifications go to zero after $\frac{π}{9}$, we plot out the value from $[0, \frac{π}{9}]^{17}$. One can tell that, though with a different shape, values $|H(e^{-iω})|^2$ of all six specifications peak at zero and then decay, like a delta function, but not perfect delta function. Thus Lucas filter is a filtering extracting signals with low frequencies. Furthermore, it is worth to notice that as $ρ$ or window length $L$ increases, extracting quality improves.

During the period under investigation of Lucas (1980), the growth rate of real GDP can be viewed as a stable series. However, during a long historical period, we are to explore, the growth rate of income cannot be taken as given. Furthermore, we do not tend to defend or

\[\delta(ω)\sin(ωt)\], so signal is a compound by series of sub-signals.

\[17\text{Since } s(ω) \text{ is an even function and periodic, with period } 2\pi. \text{ Normally, we plot out the value of spectrum from } [0, π].\]
attack money neutrality. To take this time-varying growth rate of real GDP into account, we adjust our money growth by the growth rate of real GDP, especially when we analyze European countries in robust check part. Adjusting real income is important for QTM revealing. For example, without adjusting real income, QTM does not show up during 1955-1980 in Switzerland. I thus include and focus on the cases with income adjustment.

Following Lucas (1980), we plot filtered growth rate of money or excess(adjusted) growth rate of money against (filtered) inflation rate. Excess growth rate of money is defined to be growth rate of money minus growth rate of real GDP.

Lucas (1980) implicitly adopt an eyeball metric to illustrate the nearly perfect linear(one-to-one) relationship between (filtered) growth rate of money and inflation. This method, interpreted by Sargent and Surico (2011), can be boiled down to a test whether the coefficient before money growth equals to one if we run a regression of monetary growth onto inflation:

\[ \pi(t) = \alpha + \beta g_m(t) + \epsilon \]

\( H_0 : \beta = 1 \)

Before we move on into the estimation results, it is worthy to note that \( \beta = 1 \) does not necessarily mean quantity theorem holds. One can easily scatter several points along the 45-degree lines to get unite slope. But it is hard to conclude that these two are closely related. Quantity theorem of money does not only requires more than \( \beta = 1 \), but it also states that money should be the *dominating* driving factor of inflation. Put it in another way, \( R^2 \) of this regression should be close to 1. And this is the reason why papers like Benati (2009) includes coherency. In spectrum analysis, coherency can be understood similar to \( R^2 \). However, coherency estimation without any window adjustment will be one by definition. It is reasonable to doubt this measurement even after adjustment. I thus prefer to implement a simple regression method in this section. Next, we turn to discuss the empirical results for
U.S, data description and result for other countries is reserved in Appendix.

### 2.3 Empirical Results

In this section, we present data and extend Lucas’s scatter plots of filtered money growth and inflation as well as the cointegration test of long-run money demand. I find that recent decades experiences collapsing of QTM no matter what kind of empirical methods are adopted.

#### 2.3.1 Data

To avoid data quality issue, we focus on data after WWII, but our results are robust when one would like to extend data. I use quarterly U.S Real GDP(M1/2) data from FRED from 1947Q1(1959Q1). M2/1 data before 1959Q1 are from Appendix B of Balke and Gordon (1986). I plot out the growth rate of raw data in Figure-2.4; the gray area in the Figure...
represents periods identified as the recession by NBER. I can confirm the era of Great moderation, a reduction in the volatility of business cycle fluctuations starting in the mid-1980s\textsuperscript{18}. And this is one of the reasons we insist on adjusting for real income. Not only we can observe a decreasing volatility, but we can also tell that inflation trends down and never came back, no matter how money stock grows or nominal interest rate adjusts.

\subsection{2.3.2 Scatter Graphs: Frequency Approach}

I plot filtered money growth(income-unadjusted case in the upper panel and adjusted one in the down panel) against inflation in Figure-2.5. As we can tell from the figures, the relationship between growth rate of M1/2 and inflation, no matter controlling the growth of income or not, is nearly perfect during Pre-1980 period. To facilitate illustration, we label different periods with different colors. When we compare the graphics in upper panel with their counterparts, one can tell that for U.S under the period we are investigating, controlling the growth rate of income improves the fitting of QTM. However, adjusting income cannot

\textsuperscript{18}This pattern is well documented by Stock and Watson (2003), Bernanke (2004), and Clark (2009)
save QTM from recent breaking down, the relationship between money and price flattens out after some points. After a certain point, the growth of money is associated with inflation in a non-one-to-one way, if they are still related.

Growth rates of M1 and M2 fit inflation perfectly during Lucas Period. For upper panel in Figure-2.5, without controlling real income, $\beta$ and $R^2$ are 1.1(1.3), 0.86(.81), for M1(2). $\beta$ and $R^2$ will become 0.82(1.0075) and 0.82(0.96) for M1(2) when we control for real income. As one will see in robust check section, Lucas period(1955-1980) is a unique interval during which inflation co-moves with monetary indexes closely(or reversely) in most countries. Since we used filtered data for QTM regression, it is hard to use standard deviation to determine whether we can reject hypothesis $\beta = 1$ or $R^2 = 1$. I adopt an ad hoc criterion here$^{19}$, we used $\beta = 0.7, R^2 = 0.8$ as a benchmark$^{20}$. However, for $M_1(2)$, $\beta$ is significantly less than 1 during post-1984(1991). For the points in Figure-2.5, during post-1984(1991) era for $M_1(M_2)$, $\beta = 0.33489(-0.1), R^2 = 0.0889(0.0064)$ if not adjust for real income, $\beta = 0.002(-0.2)$, $R^2 = 0(0.25)$ for adjusted case. It is also worth to note that with different bandwidth, it is possible to find different breaking dates. For example, when we use 16 years as bandwidth, QTM of $M_2$ broke around 1991(1993), while Sargent and Surico (2011) finds QTM broke around mid-1980(1984) by using four years(8 quarters before and eight quarters after) as bandwidth. However, as we showed before, shorter window length means poorer extracting quality.

Additionally, Lucas and Nicolini (2015) link money velocity to interest rate and create another relevant index of money stock, NewM1, M1+MMDA. And it seems natural that they propose nominal interest rate is the opportunity cost of holding money (M1+MMDA), hence $V$ should be increasing in interest rate.

Because (1) Velocity growth is similar to a ignore term in the QTM regression, (2) Velocity is positively related to the interest rate. Hence when interest rate increases, Velocity

$^{19}$These ad hoc criterion are justified from our Mont Carlo experiments.

$^{20}$Or be more precise $\beta$ should not stay too large either, we set $\beta < 1.4$, so $\frac{1}{\beta} < 0.7$. 

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increases. When interest rate decreases, velocity decreases. Hence, if I plot inflation against
the growth of Money-GDP, the slope should be larger when the interest rate is increasing
than the slope when the interest rate is decreasing. As shown in the right panel of 2.6,
If one looks at two time periods in the history: 1915-1928, 1950-1977. The slopes of the
inflation and money regressions are similar, both are close to 1. But the interest rate during
1915-1928 was decreasing while the interest rate was increasing during 1950-1977.

2.3.3 Evidence from Cointegration test: Temporal Approach

I set out a general cointegration specification below, then interpret Benati et al. (2016)
through this setting:

\[ p_t = m_t + v_t \]
\[ m_t = m_{t-1} + \xi_t \]

where \( p_t = \ln(P_t) \), \( v_t = \ln(V_t) \), \( m_t = \ln(M_t) - \ln(y_t) \), \( P_t \), \( y_t \), \( M_t \) and \( V_t \) stand for Price level,
real GDP, level of monetary stock and money velocity. There are two issue here may cause
our original (difference) specification not appropriate: \( E[\xi_t v_t], E[\xi_t v_{t+s}] \neq 0 \). I explore this
point in another paper Wang(2016). Here we merely replicate an exercise in Benati et al.
(2016) and Lucas and Nicolini (2015), and extend it with \( M_2 \) data. Both paper try to use
shoe cost to micro-found money velocity: money velocity is interpreted as frequency people
go to the bank, interest rate serves as an opportunity cost of holding cash(\( M_1 \)). Benati et al.
(2016) explores several co-integration tests to identify a long-run equilibrium relationship
between real money balance and price under different specification. Benati et al. (2016)
ignore the second equation in our specifications, focuses on the first equation and replaces
\( v_t \) as a function of interest rate. Functions to represent \( v_t \) generally vary under different
specifications.

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I replicate Baumol-Tobin specification in Benati et al. (2016). I plot money balance over GDP against interest rate with fitted “long-run money demand” in Figure-2.6, upper panel plots the case of $M_1$, down panel plots $M_2$. As one can tell from Figure-2.6, the long-run demand for money performs well to fit such long historical data (1914-2015), points scatter tightly along the fitted curves. However, if we color different era with different colors, for example, in the upper panel we color the points post-1984 out, in the down panel we color the points post-1991 out, these points lie under the fitted money demand, as demonstrated in Figure-2.6. An eye ball metric can tell that the variation of data after 1984 is too little to justify the validity of this long-run demand function. The same breaks down of “long-run” demand apply to $M_2$ after 1991 as well.

This running off track is confirmed by a cointegration test. Intuition behinds co-integration is to identify a persistent relationship, under which a combination of $I(1)$ processes become stationary, or the residual will be stationary. In Figure-2.7, we plot out residuals of our estimated cointegration relationship\textsuperscript{21}. One can easily tell that after some date, residuals become negative uniformly: the points cannot be justified by historical (reasonable) track. The long-run demand for money fits long historical data well. But recent decades seems to be a totally different era.

\textbf{2.3.4 Cross countries Robust Check}

I summarize the empirical results here. Data and figures are reserved in Appendix. In the third part of this thesis, I conduct a robustness check, I investigate the cases of Australia, Canada, Denmark, France, Germany, Italy, Netherlands, Norway, Spain, Sweden, Switzerland, United Kingdom.

QTM holds, at least for a while (1955–1980), for most of the countries under our investigation. However, QTM is not a universal law. For example, QTM rarely exists in Netherlands,

\textsuperscript{21}I adopt Phillips and Hensen’s fully modified OLS estimates.
France and Germany (except the hyperinflation period). Among those countries where QTM applies, QTM recently collapses in U.S, Sweden, Denmark, Spain, Switzerland at different dates, which do not coincide with their starting dates of disinflation policy. If we take into account the fact that QTM rarely exists in some countries, we can conclude that the recent breaking down of QTM is robust.

2.4 Proposed Channel: Endogenous generated money and financial innovation

As previous evidence shows, the recent breaking down of QTM is robust. To nail down our problem, we focus on the case of U.S: Quantity theory of money works for a while in U.S (1955 to mid-1980s (early-1990s)), then it breaks down. As stated previously, we choose to focus on the demand part of Money and try to find a micro-foundation of changing money
The demand for money or the incentive of holding money balance come from transaction purpose. And this is the reason why Lucas (1980) and Lucas and Nicolini (2015) adopt $M_1$ and similar indexes. However, during 1932-1954, (filtered) growth rate of $M_1$ has no explanation power to inflation ($\beta = -0.17, R^2 = 0.07$), while $M_2$ are valid after controlling income growth ($\beta = .91, R^2 = 0.76$). Furthermore, one should not contribute the uselessness of $M_1$ to Great recession period since $M_1$ cannot explain inflation variation during several years previous to 1929. However, it is hard to believe during 1932-1950, $M_2$ plays any role in implementing transactions. Though $M_2$ version of QTM breaks, it sheds lights on the direction we should investigate: $M_2$ is the major liability of depositary financial institution. On the asset side, loan plays a major role. In other words, instead of investigating deposit demand, we look at credit generation or Money creating process.

$M_2$ stands for a major part of the liability of depositary system, namely banks. On the other side of the bank, there are kinds of assets, the most important of which are

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22 In Wang (2016), I explore the case in which central bank can affect money velocity
23 This may serve as a reason why Sargent and Surico (2011) adopts $M_2$ index.
loans. Furthermore, money can be endogenously generated by loan issuing. When a loan got issued, the bank has facilitated a transaction, no matter how receivers of fund keep it in their checking or saving accounts. Experience of U.S during 1930-1955 justify this understanding, $M_2$ can explain inflation well. I choose to focus on loan issuing instead of deposit demanding, emphasizing that money is generated endogenously.

I made an empirical conjuncture that the purpose of loan represents the transaction purpose of money. Next, to decompose the loan and extract the most important components, we explore the data of flow of fund. For individuals, as Figure-2.15 demonstrates, the largest liability is from mortgage loan since the first available data point. For corporations, collateral loans are also important borrowing. Berger and Udell (1990) reports that nearly 70% of all commercial and industrial loans in the United States are secured by collateral assets. An important collateral asset for both small firms and large corporations is real estate. According to the S.5 of Z.1 tables provided by the Federal Reserve Board, during 1952Q1-2016Q4, averagely real estate represents 63% of the tangible assets held by non-financial corporate firms on their balance sheets. For the period from 1952Q1 to 2016Q4, tangible assets (the sum of real estate, equipment, and intellectual property products) average about 58% of total corporate assets. Furthermore, total corporate assets include financial asset; not all financial assets can serve as collateral, for example, firms’ foreign investment and kinds of deposit accounts. Each of them contributes 20% and 25% of financial asset. For non-farm noncorporate U.S. firms, averagely real estate accounts for 90% of tangible assets (which is in turn about 87% of total assets). And we can tell from Figure-2.9 commercial and industrial loan and mortgage loan plays a major role in loan issuing. Hence, it is now reasonable to propose that real estate serves as another major component in money generating process\textsuperscript{24}.

Since real estate can be viewed as a bundle goods of structure, a kind of final goods, and land. I can view the money as generated by final goods and land transaction. In other

\textsuperscript{24}I even ignore the expenditure affected by house purchase, for example, home-related durables and home improvements sectors. See Benmelech, Guren and Melzer (2017).
words, growth rate of money does not need to track the final goods price inflation, because of the existence of another component.

To confirm this finding, we explore long-run data of house price during the pre-crisis period (S&P CoreLogic Case-Shiller Home Price Indices). In Figure-2.11, we plot raw data of house price index in the upper panel and filtered growth rate of house price with inflation and growth of $M_2$ in down panel. Again we adopt the filter of Lucas (1980). As one can tell from the figure, the filtered growth rate of this price index co-moves with money more closely and robustly, if one follows Lucas (1980) to plot growth rate of $M_2$ against house price, and run a QTM regression the $\beta(R^2)$ will be 0.89(0.93). In the time domain, the nominal price of real estate tracks money throughout the whole pre-crisis sample, including Lucas Period.

One can read the graph as follows: If relative price of land is fixed, real estate price tracks consumption price well. In other words, two transaction component grow by a similar rate. Growth of money tracks both price inflation. QTM hold under this case. To confirm my conjuncture, I use the fraction of real estate loan in the total loans as a parameter to estimate the fraction of $M_2$ goes into real estate. From figure-?, this part of money is mainly generated by land value, especially during the innovation.

However, Financial innovation starts around the mid-1990s. Back to then, Collateralized debt obligation (CDO) was introduced. Then residential mortgage-backed securities (MBS) and collateralized mortgage obligations (CMO) follow. Then credit default swaps follow. Meanwhile, a sequence of acts got passed to facilitate financial innovation, for example, Gramm-Leach-Bliley Act (1999), Commodity Futures Modernization Act (2000)\textsuperscript{25}.

As a result, aggregate leverage ratio and the ratio of market value of residual land over GDP increase since the early-1990s, as shown in Figure-2.10. In other words, the land becomes better and better collateral during the new era. Under the framework of endogenous money creating, the land generates more and more money, or more money and credit are

\textsuperscript{25}GLB Act allows bank holding companies to own other financial companies. CFM Act moves regulation out of OTC market.
Figure 2.9: Ratio of Commercial and Industry loans and mortgage loan over total loans, all commercial banks. Source: H.8 table provided by the Federal Reserve Board, 1945-2016, monthly data

generated because of land. QTM thus reveals itself in a different way.

To be more precise, as one will see in a model with financial innovation, for example, the model in next section, financial innovation promotes the real price of the asset, which will, on the other hand, generate more money endogenously. This fact will thus have a tremendous implication for QTM: more money is after the price of land. Furthermore, if people treat land and final goods are complementary (e.g., in-house production, one need to combine final goods with land), the higher relative price of land means more money will be generated by land but not final goods. Or put in another way, less money is chasing or generated from final goods. Narrowly defined QTM thus breaks down.

2.4.1 Model outline

In this section, we would like to embed a (broader) money demand (generating) function (process) into a framework with financial innovation to explain collapsing QTM for U.S during Pre-crisis period. To model financial innovation, first we would like to outlay our economy in a frictional environment, then financial innovation improves agents’ situation. The friction we choose is borrowing limit, which is a fundamental feature of our financial system. I thus
adopt a framework with endogenous borrowing limit. A critical feature of this limited borrowing framework is that borrowers are subject to an endogenous borrowing limit, which is itself endogenously determined by the status of the economy. This feedback loop has already been explored in financial friction literature, such like Kiyotaki and Moore (1997), Jermann and Quadrini (2012), Bianchi and Mendoza (2010) and Bianchi, Boz and Mendoza (2012). A binding borrowing constraint will be reflected on economic status, which will further affect borrowing limit. In other words, this feedback loop is two-sided: economy status and borrowing limit are determined simultaneously.

For example, under the collateral constraint we are adopting, the degree of consumption smoothing is positively linked to the market value of collateral since people can borrow more when the collateral value is high. Furthermore, as people can have a more stable consumption path, price of collateral will also increase and become higher than its fundamental value. Vice versa, if the price of collateral decreases, borrowing limit will shrink accordingly. People then will face a “sudden stop” and reduced consumption. Lower consumption will further dampen the price of collateral.

I model financial friction as a process alleviating this borrowing friction; people are thus able to keep a higher leverage ratio if they are willing to. To make this innovation endogenous, we model financial innovation as a learning process, following Boz and Mendoza (2014). The main mechanism is that haircut of collateral is random, following a Markovian regime changing process(2 states Markovian process). However, agents in our economy have no perfect information about this regime-changing probability. They are Bayesian learners. Conditional on new observation, the agent will update her subjective belief, optimal planning will be then carried on accordingly. To make our analysis computation efficient, we follow the algorithm in Cogley and Sargent (2008) and Kreps (1998): the optimal decision will be planned as if the belief will not be updated further.

I will first lay out the basic setup, then briefly go through the numerical method to solve
Basic Setting: Real Economy

Consider a economy in infinite discrete time $t = 1, 2, 3, ..., $ there is a unit measure of agents. Agents face a stochastic endowment flow: she will receive $y_t$ (perishable) every period, where $\{y_t\}$ follows a Markovian process. Agents act atomistically in a competitive market and value consumption $\{c_t\}_{t=0}^{\infty}$ according to a standard time-separable expected utility function as below. The normal assumption on time discounting rate and shape of utility function applies.

$$E\left[ \sum_{T=0}^{\infty} \beta^T U(c_t) \right]$$

There is a risk-neutral bank, and it trades one-period non-state-contingent discount real bonds $b_t$ with the economic agents. I assume the bank is willing to hold any collateral asset directly, e.g., because of asymmetric information. To simplify our setting, we further fix the
price of one period bond as $1/R$, where $R$ is the real interest rate. As explained in Sargent and Ljungqvist (2004), we need restriction $R^\beta < 1$ to ensure the existence of a well-defined long-run distribution of borrowing. Furthermore, this real interest rate $R$ is exogenous to our economy\textsuperscript{26}. For domestic agents, one-period real risk-free bond and land are two kinds of trade-able assets. Later we will allow a role for cash. Furthermore, land can serve as collateral. The period budget constraint is thus:

$$c_t + b_{t+1}/R + l_{t+1}q_t \leq b_t + l_t(d_t + q_t) + L_t$$

where $l_t$, $q_t$ is the land holding and price of land in period $t$, $d_t$ is the dividend flow from land and $L_t$ is labor income. I divide the total production $z_t f(l_t)$ into dividend(rent) from land, $d_t$ and $L_t$. $z_t$ follows a Markovian process ($z_t \in Z_t = \{z_1, z_2, ..., z_N\}$). To emphasize the effect of financial innovation, we assume that people has perfect statistical information about $\{z_t\}_t$ and land is not reproducible and available stock is normalized to 1. Hence it is straightforward to state that $d_t = z_t f'(1)$ and $L_t = z_t [f(1) - f'(1)]$.

Under this incomplete market framework, we add in an endogenous collateral borrowing constraint:

$$b_{t+1} \geq -\psi_t - \phi_t q_t l_{t+1}$$

And as we stated previously, stochastic process of $\psi$ and $\phi$ follows a “true” Markovian process. For simplicity, we adopt a binary support set. Namely, $(\phi_t, \psi_t)$ can take two value $(\phi_h, \psi_h)$ and $(\phi_l, \psi_l)$, where $1 > \phi_h > \phi_l$ and $\psi_h > \psi_l$. The learning process will be elaborated in later in this section. It is straightforward to extend our set into a setup with multiple

\textsuperscript{26}This assumption reflects recent evidence documented in papers like Warnock and Warnock (2006), Bernanke (2005) and Mendoza et al. (2009): during the era of financial globalization, risk free rate has been significantly affected by foreign factors, even for U.S. And we view this assumption as a simplification one.
Furthermore, it is worth pointing out that $\phi_t$ is enough to represent the status of the financial structure if one only concerns about collateral borrowing. Moreover, we would like to give a role to the evolving credit payment system, as pointed out in Wang (2016). Figure 2.12 is borrowed from Wang (2016), plotting the time path of ratios between consumer credit and $M_1$(wealth). Furthermore, credit card balance is the major component in consumer credit. From the figure, one can tell that credit card keeps crowding out $M_1$ or money to implement a transaction. This is another reason we prefer to honor the role of loans and do not want to merely focus on cash or $M_1$. Here we use $\psi_t$ as a shortcut\footnote{This is a shortcut for limited commitment setup to justify the existence of unsecured credit, such as in Azariadis, Kaas and Wen (2016) and Wang (2016)} to represent this “non-collateral” borrowing.

Let $\mu_t$ denote the Lagrange multiplier of the collateral constraint, the Euler Equations
for $b_{t+1}$ and $l_{t+1}$ will be

$$U'(c_t) = E^{B_t}[[\beta RU'(c_{t+1})] + \mu_t$$

$$q_t = \frac{E^{B_t}[\beta U'(c_{t+1})(q_{t+1} + d_{t+1})]}{U'(c_t) - \phi_t \mu_t}$$

Where $E^{B_t} [...]$ represents the expectation according to belief at period $t$, $B_t$. I mark $B(.)$ with subindex $t$ to emphasize belief is evolving.

Equilibrium condition of this model consists of Euler equations, budget constraint, collateral borrowing, complementary slackness condition, and unit normalized stock of land:

$$c_t = b_t + z_t f(1) - b_{t+1}/R$$

$$b_{t+1} \geq -\psi_t - \phi_t q_t l_{t+1} |_{l_{t+1}=1} \quad (\ast)$$

$$l_{t+1} = 1$$

$$\mu_t [b_{t+1} + \psi_t + \phi_t q_t l_{t+1}] = 0$$

Collateral constraint brings a feedback loop into our setup. To see how feedbacks work, it is worth emphasizing that one can express policy functions of consumption(by complementary slackness condition) as the minimum of two branches, one denotes unconstrained case and another for constrained. $c_t = min\{(U')^{-1}(E^{B_t}[\beta RU''(c_{t+1})]), \psi_t + \phi_t q_t l_{t+1} + b_t + z_t g(1)\}$

Through this formula, it is easier for one to understand the feedback between financial structure and consumption: binding agents choose a lower level of consumption than unconstrained case (so min function). Furthermore, reduced consumption will dampen asset price, which is a critical component in collateral borrowing, since in equilibrium asset holding is normalized to be 1. To make this observation more explicit, we take a difference between the
two Euler Equations of risky and risk-free rate\(^{28}\) as following and define \(R_{t+1}^q \equiv \frac{(q_{t+1} + d_{t+1})}{q_t} \):

\[
1 - \mu_t = E^{B_t} [\beta U'(c_{t+1})(R_{t+1}^q - R)]
\]

\[
E^{B_t}[R_{t+1}^q] - R = - \frac{\text{Cov}(U'(c_{t+1}), R_{t+1}^q) - (1 - \phi_t)\mu_t}{E^{B_t}[U'(c_{t+1})]} 
\]

Except \((1 - \phi_t)\mu_t\) term, expected excess return follows a classical formulation:

Payments from a good-hedging security positively correlate with pricing kernel: pays better when people value consumption more. It offers insurance to some degree; people will thus require lower (future) excess return or price of a good hedging security is high. On the other hand, a binding borrowing constraint directly increases excess return, because of positive \(\mu\), e.g., Equation-0. In another word, the price of the asset will be dampened: according to Campbell and Shiller decomposition\(^{29}\), higher expected future excess return means lower current asset price. To make this statement clearer under our setup, one can write \(E^{B_t}[R_{t+1}^q] \equiv E^{B_t}[(q_{t+1} + d_{t+1})/q_t] \) recursively(forward) to express price of asset as a discounted value of a dividend sequence as following. And the discounting rate is general different with

---

\(^{28}\)The intuitions in Aiyagari and Gertler (1999) still applies.

\(^{29}\)I present Campbell and Shiller as following, which one will tell that it is a little different with our later formulation:

\[
R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}
\]

\[
r_{t+1} \equiv \ln(R_{t+1}) = p_{t+1} - p_t + \ln(1 + \frac{D_{t+1}}{P_{t+1}}) = p_{t+1} - p_t + \ln(1 + \exp(d_{t+1} - p_{t+1}))
\]

\[
= p_{t+1} - p_t + \ln(1 + \exp(d - p)) + \frac{\exp(d - p)}{1 + \exp(d - p)}(d_{t+1} - p_{t+1} - (d - p))
\]

\[
= p_{t+1} - p_t + k + (1 - \rho)(d_{t+1} - p_{t+1})
\]

\[
\Rightarrow
p_t = \rho p_{t+1} + k + (1 - \rho)d_{t+1} - r_{t+1}, \quad \rho = \frac{1}{1 + \exp(d - p)}
\]

One can tell from the last equation(Compbell-Shiller decomposition) that \(p_{t+1}\) is decreasing in future return \(r_{t+1}\). Moreover, one can apply the last equation forward recursively to represent \(p_t\) as a discounted value of future dividends.

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risk-free rate unless investors are risk-neutral:

\[ q_t = \frac{E^{\mathcal{B}_t}[d_{t+1} + q_{t+1}]}{E^{\mathcal{B}_t}[R_{t+1}^q]} \]

\[ = E^{\mathcal{B}_t}[(d_{t+1} + q_{t+1}) \frac{1}{E^{\mathcal{B}_t}[R_{t+1}^q]}] \]

\[ = E^{\mathcal{B}_t}[(d_{t+1}) \frac{1}{E^{\mathcal{B}_t}[R_{t+1}^q]}] + E^{\mathcal{B}_t}[(d_{t+2} + q_{t+2}) \frac{1}{E^{\mathcal{B}_t}[R_{t+1}^q]}] \]

\[ = E^{\mathcal{B}_t}[(d_{t+1}) \frac{1}{E^{\mathcal{B}_t}[R_{t+1}^q]}] + E^{\mathcal{B}_t}[(d_{t+2} + q_{t+2}) \frac{1}{E^{\mathcal{B}_t}[R_{t+1}^q]}] = \ldots \]

\[ = E^{\mathcal{B}_t}\left[ \sum_{k=0}^{\infty} \left( \frac{1}{\Pi_{i=0}^{k} E^{\mathcal{B}_t}[R_{t+1+i}^q]} \right) d_{t+1+k} \right] \quad (**) \]

From the formula-**, it is clear that higher future expected excess return, which may be caused by binding borrowing constraint, positive \( \mu \), can dampen the price of the asset. Furthermore, endogenous borrowing constraint makes land an even worse hedging against consumption through an indirect channel: when constraint binds, higher realized return means higher land price, which further implies higher borrowing limit, better consumption. Hence covariance between consumption and realized return would be higher than the cases without borrowing limit. Reduced asset price will further trigger tighter binding of borrowing constraint again. Mathematically, |Cov\((U'(c_{t+1}), R_{t+1}^q)\)| will be larger than frictionless case.

Furthermore, formulation-** give us another self-full filling channel, especially under a framework with endogenous belief. When people expect borrowing constraint to be binding in the future, this expectation will also reduce asset price today, since it will increase \( E^{\mathcal{B}_t}[R_{t+k}] \) for some \( k \). To focus on the purpose of this article, we will leave this sun-spot equilibrium exploration to future research.

To make financial innovation as a smoothly progressive process\(^{30}\), we model it as a updating optimistic belief through Bayesian learning channel and adopt equilibrium compu-

\(^{30}\)Under a smoothly progressive process, real shocks still have a place. Hence, our quantitative results takes care of shocks from real economy.
tation algorithm in Cogley and Sargent (2008). Bayesian learners update their belief to
take new data points into account according to Bayesian law\(^\text{31}\). In our setup, condition
on \((\phi_{t-1}, \psi_{t-1}) = (\phi_h, \psi_h)\), after the realization of \((\phi_t, \psi_t)\), agents update transition prob-
ability of period and form a new belief \(B_t\), which can be summarized in
\[ \Pr((\phi_{t+1}, \psi_{t+1}) = (\phi_h, \psi_h) \mid (\phi_t, \psi_t) = (\phi_h, \psi_h)). \]

Since belief itself evolves, a complete computation is time-consuming. Anticipated Utility
approach in Cogley and Sargent (2008)\(^\text{32}\) adopts a simplifying computation strategy(assumption)
that after observing every new sample, agents update their belief and choose consumption,
investment and borrowing decision according to this updated belief. In another word, they
ignore the possibility that their belief will change in next period. Moreover, to focus on
financial sector part, we assume \((\phi_t, \psi_t)\) can be observed directly without any noise. Under
a framework with observation error, it is straightforward to embed a (forward) Kalman filter
process in this setting to “filtering” noise. Our numerical computation consists of two steps:
(1) After setting the prior belief; we simulate a sequence of observation. Agents’ posterior
belief \(\{B_{T_t} = 1\}\) will be generated according to Bayesian Law. (2) For any period \(t\), given belief
\(B_t\), agents will make their optimal decision according to this belief. Put it in another way;
we solve the model as if belief \(B_t\) will last forever. Policy functions in any period-\(t\) are time
specific, only valid in the particular period. In next subsection, we elaborate how people’s
belief is updated.

\(^{31}\)Generally, Bayesian learner has a prior belief first, say \(q(\theta)\), where \(\theta\) is the parameter learner feel
interested in. For illustration purpose, we assume that \(q(\theta), \theta \in \Theta\) is a probability density function, \(\Theta\) is
the support set of parameters. Bayesian learning still apply with any well defined random variable, not
matter its probability density function exists or not. Posterior belief about \(\theta\) will be updated according to
\[ \Pr(\theta \mid y_t) = \frac{P_r(y_t \mid \theta) q(\theta)}{\int_\Theta P_r(y_t \mid \theta) q(\theta) d\theta}. \] Since the denominator is complicated in most cases, but it is a constant. In
most analysis, researchers write \(\Pr(\theta \mid y_t) \propto P_r(y_t \theta) q(\theta)\) and use MCMC algorithm to sample this posterior
distribution. For more details on MCMC, see Chib (2003)

\(^{32}\)Cogley and Sargent (2008) gave the credit to Kreps(1998).
Figure 2.12: Ratios between consumer credit and $M_1$(wealth), U.S, Source: G.19 and Z.1 Table from Federal Reserve Board

**Bayesian Learning**

Since agents has no perfect information about the transition probability of financial structure parameters $F_t = (\phi_t, \psi_t)$. I further simply assume that there are only two state of $F_t$, hence learning process degenerates to learn the transition probability $p = Pr(F_t = H|F_t = H)$ and $q = Pr(F_t = L|F_t = L)$. I can write the transition probability matrix of $F_t$(a function of time) as:

$$
\Pi = \begin{bmatrix}
    p & 1 - p \\
    1 - q & q
\end{bmatrix}
$$

Furthermore, we set prior of $p, q$ are independent beta distribution. Posterior belief on $p, q$ will be beta distribution too: $p_t \sim \beta(n_t^{HH}, n_t^{HL})$ and $q_t \sim \beta(n_t^{LL}, n_t^{LH})$ where $n_t^{ij}$ is the number of observation of transition from state $i$ to state $j$. For example, $n_t^{LH} - n_t^{0LH}$ denotes the numbers of observation of shrinking lending. Counters $N_0 = [n_0^{HH}, n_0^{HL}, n_0^{LL}, n_0^{LH}]^T$ summarize prior belief, and counter will be updated afterward after any new data sample is
observed, these counters $N_t$ is updated as

$$N_t^{ij} = \begin{cases} 
N_{t-1}^{ij} + 1 & \text{if } F_{t-1} = i \text{ and } F_t = j \\
N_{t-1}^{ij} & \text{otherwise}
\end{cases}$$

It is clear that $F_t$ and $(F_t, N_t)$ is a Markovian process. For our purpose, we do not need to derive out the full transition matrix of $(F_t, N_t)$. I only care about $Pr(F_{t+1} = h|F_t, N_t)$, it can be calculated as

$$Pr(F_{t+1} = H|F_t, N_t) = Pr(F_{t+1} = H|N_t) = \begin{cases} 
\int_0^1 pf(p|N_t)dp & \text{if } F_t = H \\
\int_0^1 (1-q)f(q|N_t)dq & \text{if } F_t = L
\end{cases}$$

The first equality is because $N_t$ summarizes the information of $F_t$. And $N_t$ is also evolving as previously stated. Hence the belief $B_t$ can be summarized as

$$\Pi_t = \begin{bmatrix} 
\int_0^1 pf(p|N_t)dp & 1 - \int_0^1 pf(p|N_t)dp \\
1 - \int_0^1 qf(q|N_t)dq & \int_0^1 qf(q|N_t)dq
\end{bmatrix} = \begin{bmatrix} 
\frac{N_t^{HH}}{N_t^{HH} + N_t^{HL}} & \frac{N_t^{HL}}{N_t^{HH} + N_t^{HL}} \\
\frac{N_t^{LH}}{N_t^{LH} + N_t^{LL}} & \frac{N_t^{LL}}{N_t^{LH} + N_t^{LL}}
\end{bmatrix}$$

The first equality comes from Bayesian learning; the second follows properties of the beta distribution. For prior belief, we set $N_0 \approx 0$, equivalently, $p_0 = q_0 = \frac{1}{2}$, this prior implicitly assume that agents have no prior information about the financial innovation process. Thus no previous transition was observed, $N_0 \approx 0$.

As Cogley and Sargent (2008) show, this updated belief will asymptotically converge to the true process, a numerical example is reserved in Appendix. In previous sections, the expectation operator bases on belief $B_t$, which now can be summarized by $\Pi_t$ and transition matrix of $z_t$. 

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2.5 Endogenous Money Creation: Loan Issuing

As we argued in previous sections, transaction-oriented money demand is not robust enough to justify QTM: $M_1$ is not even a robust index, co-moving with inflation on a one-to-one basis. I can understand $MV = PY$ in another way; money is generated by credit issuing: bank issues loans with the amount proportional to the value of goods, “money” here can be unsecured credit, e.g., credit card, or collateral loans. Surely, there is a cost associated with credit, more precisely, there is a yield curve for credit. Hence, $V$ should represent the level and slope of this yield curve: as interest increases more with longer maturity, agents tend to keep loan shorter and trade more frequently. This is how we interpret velocity under our this money creation process. Along with a stationary interest rate assumption, we view $V$ as a constant in the long run. In another word, we assume the amount of money generated is proportional to the value of goods, and now we have more goods to take into consideration. Surely, this is merely a simplification to model money creation.

To make this simplification aligned with our previous data analysis, we propose a setup with loans, where loans can also implement a transaction. Furthermore, there is another transaction: house purchasing. I make canonical assumptions on this Monetary economy:
people are working but only get paid for the units of cash after all consumption, borrowing (repayment) and asset investment transaction happened\(^{33}\). People cannot enjoy output from own labor; they have to purchase from other people. There is no monitoring system except the bank. People can use cash or borrow from the bank to implement a transaction. Of course, the maximum credit is restricted by collateral constraint. The Central bank will use a helicopter to drop cash at the very beginning of every period. House is necessary for a consumption process to take place, no further utility it will offer. *Homogeneous houses* are offered by a competitive real estate company. I adopt this timeline because of liquidity of collateral asset.

Besides cash, all transactions can be implemented by borrowing too. Borrowing itself is still under collateral constraint. And we will call the balance of cash and “money” created by borrowing as broad money, \(M_2\) for example.

Concerning to house, we assume competitive real estate companies\(^{34}\) are to max \(c_H, l \left[ c_H^p + l^p \right]^{\frac{1}{\rho}} \), \(\rho < 0\) and \(p c_H + p q l \leq \text{Cost}\). Real estate company, owned by economy agents, will offer the houses to households at its cost: \(p_t \left[ 1 + q_t^{\rho - 1} \right]^{\frac{\rho - 1}{\rho}}\), the total quantity of house is \(\left[ c_H^p + l^p \right]^{\frac{1}{\rho}}\), where \(c_H\) is the amount of final goods embedded in-house, i.e house structure. Furthermore, from data we can tell that structure-replacement cost is almost a constant fraction of GDP, as documented in Figure-2.13. I have no government expenditure and private investment in our model; we thus restrict \(c_H\) is a fixed fraction of total consumption, \(\theta\) exogenously: more consumption needs a larger house. To make our micro-foundation simple, we assume that factory are all built underground, and a house is built on the ground. Since agents act atomically, they take the price and size of house as given:

---

\(^{33}\)This is the simplest timeline to solve. One interesting modification to this timeline is to allow asset market open after all transaction happens. In other words, under our original timeline, the only output from labor need to be transacted through money, and there is no inflation risk for asset holding. Under this modified timeline, collateral asset faces inflation risk.

\(^{34}\)Real Estate Companies are owned by households.
\[
\frac{M_t + G_t}{p_t} + b_t + s_t(d_t + q_t) \geq c_t + [1 + q_t^{\rho - 1}]^{\frac{\rho - 1}{\rho}} [\bar{c}_H^{\rho} + \bar{l}^{\rho}]^{\frac{1}{\rho}} + \frac{b_t'}{R} + s_{t+1}q_t
\] (10)

Where \(\bar{c}_H\) and \(\bar{l}\) is the final goods and land used in house construction, people take them as given. \(G_t\) is cash transfer from government. But at equilibrium: \(\bar{l} = 1\), \(\bar{c}_H = \theta c\) and \(\bar{c}_H + c = y\). From this equation, we embed in the pattern we found in data: right hand-side - Loan(broad money) is issued(generated) to purchase real estate, left hand-side- broad money can be generated by using collateral.

Furthermore, new budget constraint will become:

\[
c_t + b_{t+1}/R + l_{t+1}q_t + \frac{M_{t+1}}{p_t} + [1 + q_t^{\rho - 1}]^{\frac{\rho - 1}{\rho}} [\bar{c}_H^{\rho} + \bar{l}^{\rho}]^{\frac{1}{\rho}} \leq b_t + l_t(d_t + q_t) + L_t + \frac{M_t + G_t}{p_t} + \Pi_t
\]

where \(\Pi_t = 0\) is profit from real estate company. Again with borrowing constraints, our equilibrium condition consists of

\[
U'(c_t) = E^{B_t}[\beta U'(c_{t+1})] + \mu_t
\] (1)

\[
q_t = \frac{E^{B_t}[\beta U'(c_{t+1})(q_{t+1} + d_{t+1})]}{U'(c_t) - \phi_t\mu_t}
\] (2)

\[
c_t = b_t + z_t f(1) - b_{t+1}/R
\] (3)

\[
b_{t+1} \geq - (\psi_t + \phi_t q_t l_{t+1})|_{t+1=1}
\] (5)

\[
M_t^b = p_t(1 - \theta)c_t + p_t[(\theta c_t)^\rho + l^\rho]^{\frac{1}{\rho}} [1 + q_t^{\rho - 1}]^{\frac{\rho - 1}{\rho}}
\]

Where \(M_t^b\) represents the stock of broadly defined money including credit.

The main mechanism can be understood through a simplification to Equation-(6). I simplify Equation-(6) to be \(M_t \approx p_t[(\theta c_t)^\rho + l^\rho]^{\frac{1}{\rho}} [1 + q_t^{\rho - 1}]^{\frac{\rho - 1}{\rho}}\), since consumption value is relatively small. I use this specification to emphasize the channel through which QTM
breaks. It is straightforward to see that from this simplified money generating function:

\[
\frac{M_{t+1}^b}{M_t^b} \approx \frac{p_{t+1}}{p_t} \left[ 1 + q_t^\rho \right]^{\frac{\rho - 1}{\rho}} \left[ 1 + \frac{q_t^\rho}{q_{t+1}} \right]^{\frac{\rho - 1}{\rho}}
\]

From this equation, one can tell growth of broad money is not merely associated with inflation of final goods \( \frac{p_{t+1}}{p_t} \), but also with real price inflation of the collateral asset \( \frac{q_{t+1}}{q_t} \). This intuition applies to our original setup too. Not only money is necessary to implement more kinds of transaction, but money can also be generated through (collateral) borrowing. Empirically, to estimate the effect of money growth onto inflation under this framework, one can take a log-difference and read the formula reversely, in a regression form: regress inflation onto growth rate of money. To be more explicit, from period \( t \) to \( t + N \) the coefficient will be:

\[
\beta_{t,t+N} = \frac{\left[ 1 + q_t^\rho \right]^{\frac{\rho - 1}{\rho}}}{\left[ 1 + \frac{q_t^\rho}{q_{t+1}} \right]^{\frac{\rho - 1}{\rho}}}, \quad \rho < 0
\]

This is a formula nesting canonical QTM and broader QTM: when the relative price of collateral asset stays stable \( \frac{q_{t+1}}{q_t} \approx 1 \), as what happens during Lucas’ Period 1955-1980, \( \beta \) will be close to 1. When relative price of land is increasing \( \frac{q_{t+1}}{q_t} > 1 \), as more money is generating from or chasing after asset; we will get a \( \beta \) significant less than 1. The reason is that more money is generated by (because of) collateral asset instead of final goods under financial innovation era.

To solve this model at period \( t \), we take \( B_t \) as given, as if this belief will last forever, all usual optimization process goes through, we will get \( b_{t+1}(b_t, z_t, F_t) \), \( c_t(b_t, z_t, F_t) \) and \( q_t(b_t, z_t, F_t) \) to solve equation-(1) to (6). The solution, policy functions \( b_{t+1}(b_t, z_t, F_t) \), \( c_t(b_t, z_t, F_t) \) and \( q_t(b_t, z_t, F_t) \) thus merely determine the optimal plan for period \( t \). People will act according to these policy function only during period \( t \). People will have another
group of policy functions in following periods. Furthermore, different realization paths of \( \{z_t\} \) and \( \{(\psi_t, \gamma_t)\} \) will form different belief, sequence of policy function will be thus different accordingly.

To summarize, financial innovation is a process in which asset becomes better collateral, alleviating borrowing friction in this economy. To have a progressive process, leave a role for real shocks, we deviate from rational expectation framework and model financial innovation as a learning process, following Boz and Mendoza (2014) and Cogley and Sargent (2008). The financial innovation is a process in which people learn that their asset become better collateral, at least they believe so. One can skip this learning and exogenously change the Markovian process governing parameters \((\psi, \phi)\) in financial innovation regime. According to our calculation, the QTM implications out of these two methods are very similar: both processes generate increasing sequence of real asset price, narrowly defined QTM breaks.

### 2.5.1 Numerical Algorithm and Calibration

In this section, we will go through our numerical method briefly in the first part. Detailed procedure can be found in numerical algorithm Appendix. Then we calibrate our model to match U.S economy and financial innovation process. In the end, we will use our calibrated model to simulate a(bunches of) sequence(s) data and use the simulated data to run a QTM regression. As one will tell, we will have a slope significantly less than 1. Since we have more than one simulation, we have a distribution of the loading of Money onto inflation, \( \beta \).

#### Numerical Solution Method

As we can tell from equilibrium conditions Equation-(1) to (6). The first four equations can be solved by policy functions \( \mu_t(b_t, z_t, F_t), b_{t+1}(b_t, z_t, F_t), c_t(b_t, z_t, F_t) \) and \( q_t(b_t, z_t, F_t) \). Given a exogenous money supply, \( p_t(b_t, z_t, F_t) \) can be solved from the last condition. In this section, we focus on explaining the idea of how to solve the first four equations.
I combine the implication of complementary slackness condition and policy function iteration with endogenous grids. One can still use a usual value function iteration procedure to solve it. From slackness condition, we can have following equation-(\(O\)). And I will use policy function iteration to solve equations-(2) to (5) with equation-(\(O\)).

\[
c_t = \min \{(U')^{-1}(E^{B_t}[\beta R U'(c_{t+1})]), \psi_t + \phi_t q_t s_{t+1} + b_t + z_t g(1)\}
\] (\(O\))

Where \(c_t\) will take the left branch if collateral constraint is loose\((\mu_t = 0)\) since \(U'(c_t) = \beta R U'(c_{t+1}) + \mu_t\), and take the right branch if collateral constraint is binding\((\mu_t > 0)\), then consumption should be solved from budget constraint\(^{35}\). When collateral constraint is loose \(\mu = 0\), normal pricing kernel applies and asset pricing formula goes through. Otherwise, when collateral constraint is binding, \(c_t\) will be determined by \(\psi_t + \phi_t q_t s_{t+1} + b_t + z_t g(1)\). In this case, binding constraint \(b_{t+1} = -\psi_t - \phi_t q_t\) gives us an asset price. Then we can explore asset pricing equation-2 to recover consumption policy function. Policy function \(\mu\) follows equation-1.

I then explore theoretical characteristics of solution: for every given \(z_t\) and \(F_t\), there is a threshold \(\bar{b}\) such that, when \(b_t > \bar{b}\) collateral constraint goes slack, when \(b_t < \bar{b}\) collateral constraint goes binding, and when \(b_t = \bar{b}\) collateral constraint is marginally binding. Hence \(\bar{b}\) is the value of \(b_t\), at which \(c^{binding}(b_t, z_t, F_t) = c^{slack}(b_t, z_t, F_t)\), \(q^{binding}(b_t, z_t, F_t) = q^{slack}(b_t, z_t, F_t)\), and \(\mu^{binding}(b_t, z_t, F_t) = 0\).

On the top of all these procedures, we simulate a realization of \(F_t\) from “True” process. People will then update their belief \(B_t\) and combine it with knowledge of process \(\{z_t\}\) to form the expectation, from which our optimization process starts. More details are left in Numerical algorithm Appendix.

\(^{35}\)This structure can be explored more in a more general context, see Korinek and Mendoza (2014)
Calibration

First, utility function and production technology are specified to be $U(c_t) = c_1^{1-\gamma} - 1$ and $f(l_t) = l_1^\alpha$. I need to set the value of $\gamma, \alpha, \rho, \phi^H, \phi^L, \psi^H, \psi^L$, and prior belief $N_0$ as long as the process of $\{z_t\}$.

I calibrate the model to U.S during 1984-2006, excluding the crisis period. The date $t = 1$ is set to 1984, $t = T$ is set to be 2006 since it is believed that 2007 is the start of financial crisis. I will then set $F_t = H$ during $t = 1$ to $t = T$. Learning period is 23 years.

The value of $\gamma, \alpha, \rho, \phi^L, \psi^L$ will be set to match U.S annual data 1955-1984. And there is only one regime for financial structure for the pre-innovation period. The real interest rate is set to 2% since the average annual real return of three months and one year bill between 1984-2006 is 1.84% and 2.39%. TFP $\{z_t\}$ is approximate use a log-AR(1) process, namely, $\ln(z_t) = \theta \ln(z_{t-1}) + \epsilon_t$. I adopt Tauchen Method to discretize this AR(1) process. Routinely estimation give us value of $\theta = 0.98$ and $\sigma_\epsilon = 0.01575$. For $\gamma$, we set it equal to 5, a middle value of the acceptable range from Mehra and Prescott(1984). I set the mean of output to
Furthermore, during pre-1984 era, residual land accounts for 43% GDP (Davis and Heathcote (2007) Dataset) and borrowing from abroad accounts for 29.7% GDP. Hence, we set $\psi_L = 0.297$, $\phi_L = 0.0001$, $\phi_H = 0.1$ and $\psi_H = 0.35$ to match the leverage ratio at the end of 2006. I set $\beta = 0.95$ and $N_0 = N_0^{HH} = N_0^{HL} = N_0^{LH} = N_0^{LL} = 0.015$ (Boz and Mendoza (2014)). Furthermore, consumption contributes 60.5% of GDP during 1955-1984. Since there is no government and private investment here in our model, we assume there is a lump-sum loss $A$, budget constraint is $c + \frac{b'}{R} + A = b+1$, hence $A = 1 - 0.605 - 0.297(1+2\%-1)/(1+2\%) = 0.389$. And the true process of $F_t$ is not quite important for our purpose, since our calibration process. The calibrated parameter is summarized in Table-2.1. The prior belief parameter is critical. For example, Boz and Mendoza (2014) set this parameter to match option-adjusted spread on Fannie Mae RMBS with 30-year maturity over T-bill. Under our set-up, $N_t$ counts the changes. Since there is no regime change before the mid-1980s, it is no-harmful to set a small $N_0 \approx 0$. I take no stands on which excess return we should match; we thus implement numerical experiments with $N_0 = 0.01, 0.005, 0.001$. Our results are robust to this parametrization.
Quantitative findings

In this section, we first discuss the path of borrowing, consumption and land price. Then we apply our QTM regression to this simulated data. As one can tell from Figure-2.17, asset price is higher under a better financial structure.

In this subsection, we simulated 10000 sequences. For each seed, we draw \{z_t\}_t = 1^{23} from the Markovian process under the parameters in the previous section. \(b_0\) will be set at \(-0.3\), following Boz and Mendoza (2014). Agents’ belief \({\mathcal B}_t\}_t towards \(F_t\) will be the same for each seed. The belief is formulated under the prior in the previous section and a sequence of \(F^H\) realization. Transition matrix of \(F_t\) is a 2-by-2 matrix; it thus can be summarized by two parameters \(\phi_{HH}\) and \(\phi_{LL}\), which denote the probability of future(one period later) parameter stay in high-regime or low-regime conditional on current realization is high or low.
low. I plot this transition matrix in Panel D of Figure-2.14.

For illustration purpose, we pick up a path of $z_t$ and plot out the path of consumption, borrowing and asset price in Figure-2.14 (implied $\beta = 0.5745$). As one can tell from Figure-2.14, as optimism builds up, consumption increase, while borrowing keeps increase though then converge to a steady state. Furthermore, the Land price keeps going up during the whole period. Under some parametrization of “true” process, people borrow too much. This over-borrowing is a result of non-rational expectation and will make the sudden stop more damaging. I can tell that consumption excesses new steady state 7%, at the peak, then converges to steady state quickly (less than ten periods). However, the land price keeps increasing during the whole sample period.

Thus, we then calibrate an ARMA(2,2) process\textsuperscript{36} to match $M_2$ growth of U.S., And we will use equation-5 to solve out the price. Then we run QTM regression to estimate the effect of Money growth onto inflation; we focus on income adjustment here, the results without income adjustment have very close distribution. Under our baseline parameter, from our 10000 simulations, slope estimator $\beta$ of QTM is 0.572, as plotted in Figure-2.16, where we plot out the simulation distribution of $\beta$. As one can tell from Figure-2.16, our money loading ranges from 0.54 to 0.6, significantly less than 1. Hence our mechanism and model can offer an alternative explanation to collapsing of QTM.

\section*{2.6 Conclusion}

In this article, we reviewed and discussed the historical performance of the Quantity Theory of Money (QTM). I re-evaluate the one-to-one relationship between money growth and inflation. By adopting the same statistical and economic criteria as Lucas (1980), with a much larger data set covering both a longer period and many more countries, we found QTM\textsuperscript{36} ARMA(2,2) is used just for a parsimonious setting. One can use an AR(2) process too.
breaks in U.S, and this collapsing of QTM is universal.

It appears that the period 1955-1980 is the only period during which QTM fits data well in most of our sample countries. It starts to break down when we go beyond this period. Furthermore, the recent breaking down of QTM coincides with a process of financial innovation.

To explain this breaking down for U.S, we use a money generating theory instead of money demand. To be more precise, the loan is an important source of money generation. And we decompose the loan structure of U.S market. I found real estate is the major collateral asset of Households and Firms. I thus propose money chases after real estate, a bundle of final goods and land. To confirm our judgment, we use a long historical data of nominal house price and find that (long-run) growth of nominal house price co-moves with(leads) growth of broad money more robustly.

I then propose a framework under which financial innovation can affect our estimation of QTM. And as our quantitative exercise shows our model is capable of explaining the collapsing of QTM.
2.7 Appendix

2.7.1 Data source, Cross-Country

I obtain different measures of money stock from various sources. The main data source of cross country data is from the dataset offered by Schularick and Taylor (2012). They also offer an extraordinary database recently: http://www.macrohistory.net/data/. The original data sources, which I explored, are summarized as follows:

Norway: Historical statistics of Norge Bank \(^{37}\). For M1/2, 1870-1919 are from Jan T. Klovland, Monetary Aggregates in Norway 1819-2003, chapter 5, Page 208-210. 1919-2006 are from Table a2a. Nominal and real GDP are from Table-c6-table5 and table6. For deflator, data are taken from Table-c6-table7. Data from 1940-1945 are missing. For 2004-2015, Statistics Norway(M1 and M2 are from Table-08253\(^{38}\) and Table-10945). Data of real and nominal GDP from 2000-2015 are from Eurostat. CPI data from 1924(1864)-2015 are from table 08184 of Consumption index, issued by Statistics Norway. After 1924, we use the growth rate of CPI to represent inflation.

Switzerland: 1907-2006, M1/3 is from The monetary base and the M1, M2, and M3 monetary aggregates, Swiss National Bank, 2007. Before 1907, we take the data from Schularick and Taylor(2012) For Price, 1870-1992 are from H.1 of historical statistics of Switzerland online. After that, we adopt CPI from Swiss Federal Statistical Office. For GDP, 1870-1913 are from Table Q.1a; 1914-2005 are from Q.16a, b; 1980-2016 are from Eurostat. From 1984-2016, data of M1/2/3 are available from the database of Swiss National Bank.

Sweden: Historical Monetary and Financial Statistics for Sweden, Volume 1-2, Sveriges Riksbank. For 1872-2006, CPI and GDP deflator are taken from Table A8.1, Volume 1, Page 443-447. M0/3 during 1846-2012 are taken from Table A7.2, and 7.3, Volume 1, Page 325-332, since No data on M1 was presented before 1999. After 2012, we use data offered

\(^{38}\)data of M1/2 stops at Feb, 2015
by Statistics Sweden. GDP is taken from Table A4.3, Page 164-169 from 1872-2000. After that, we take data from Statistics Sweden. CPI during 1980-2016 is taken from Statistics Sweden(Consumer Price Index (CPI)/Living Cost Index, July 1914=100).

Australia: M1/3: 18701983 Page 5771 from David Pope, Australian Money and Banking Statistics. M3 after 1960 is from Reserve bank of Australia(RBA, ID: DMAM3N), M1 after 1975 is from RBA(ID: DMAM1N), between 1960-1975, we use data from OECD. CPI from 1922-2016 from RBA(Table G1, GCPIAG)

U.S: Real and nominal GDP (M1, M2 stock) are available from the FRED database since 1947: I (1959: I). Before that growth rates on the real GNP and M1, M2 series constructed by Balke and Gordon (1986). Since there is no M1 data until 1914, we use the growth rate of money base to approximate the growth rate of M1 from 1871-1914.

U.K: M0/1/3/4/4x and nominal GDP is from Money creation in the modern economy, by Michael Mcleay, Amar Radia and Ryland Thomas. Consumption price and Real GDP data is from The UK recession in context what do three centuries of data tell us? by Bank of England. After 1997, we use M4x(excluding intermediate other financial corporations) to represent broad money index. According to Bank of England, “...modify the measurement of UK M4 by excluding the money holdings of some OFCs in order to obtain a better measure of those money holdings that are likely to be used as a medium of exchange.”

Spain: From 1870-1998, M1/2 and Broad index(Disponibilidades liquidas, similar to M2 definition of Fed). Table 9.16, Page697. Carreras and X. Tafunell (eds.), Estadisticas Historicas De Espana, Madrid 2005 http://www.fbbva.es/TLFU/dat/autores.pdf. After that, we get them from IFS. 1936-1940 are missing for Money. Nominal GDP(El PIB a precios corrientes) from 1870-2000, Table 17.7, Page 1339-1340. After that we can have GDP from Eurostat(1995-2006). GDP deflator, from 1870-2000, Table 17.16(Deflactores implcitos del PIB a precios de mercado y sus componentes de gasto). 1995-1997, we can use data from OECD. After 1998, we use (M1/3)data from Table 1.13 from Banco de Espana.
Denmark: From 1995-2016, M1/3 are from Danmarks Nationalbank (1995-2013 from Table DNM1KOR, after 2005 from Table DNMNOGL). From 1970-1995, we use data from OECD. Nominal Gdp is from Barro-Ursua Macroeconomic Data (2010). From 1971-2016, we take from OECD. Real GDP (1995-2016) and Harmonized Index of Consumer Prices (2001-2016) are taken from Eurostat. Monetary data before 1971 are from narrow(broad) monetary index of Schularick and Taylor (2012)\(^{39}\).

Germany: (M0/M2, Price and output, Pre-1960) are from Rolnick and Weber (1997)\(^{40}\). Then for M1/3 during 1969-1998, we take data from IFS, after 1999 from Deutsche Bundesbank (we add "German contribution to the monetary aggregate M3 and its components in the euro area" with “Banknotes in circulation / Deutsche Bundesbank.” ). For real GDP and CPI from 1971-2000, we use data from OECD. Then I use data from Deutsche Bundesbank.

Netherlands (M1/2/3), from 1982-2015, Table 5.4 from De Nederlandsche Bank. They do not directly provide M1, M2. So we use the definition of Monetary stock according to ECB: M1= Currency in circulation+Overnight deposits. M2= M1+Deposits with an agreed maturity up to 2 years+Deposits redeemable at a period of notice up to 3 months\(^{41}\). For pre-1982, we adopt data from Schularick and Taylor (2012).

France (M1/2) are from Banque De France-M1,2\(^{42}\) after 1980, before which we adopt data from Rolnick and Weber (1997).

Italy (M1, M2 plus/3): For the whole period, GDP index is taken from Alberto Baffigii (2011) Italian National Accounts, 1861-2011. M1/2, 1861-1939, Monetary aggregates in Italy 1861-2014, Banca d’Italia. After 1999, we take data from Banca d’Italia. I con-

\(^{39}\)They construct the monetary index by sum liability of the central bank, commercial banks and saving banks, by using data from Hans Chr. Johansen (1986)

\(^{40}\)As robust check we also used data from Schularick and Taylor (2012), they construct M1 based on Deutsche Bundesbank (1976), “Deutsches Geld- und Bankwesen in Zahlen 1876-1975”

\(^{41}\)M3=M2+Repurchase agreements+Money market fund (MMF) shares/units+Debt securities up to 2 years. This debt is restricted to liabilities of the money-issuing sector and central government liabilities with a monetary character held by the money-holding sector

struct M3(M1) by adding “Italian contribution to euro-area M3(M1) excluding currency”\textsuperscript{43} and “currency held by public”\textsuperscript{44}. From 1940-1998, M1\textsuperscript{45} Data is offered by Banca d’Italia. M2plus\textsuperscript{46} offered by Banca d’Italia) was used before 1998 (Historic data offered by Banca d’Italia). After 1998, we use M3.

For Canada, M1/2 series: M1 before 1952 is from Metcalf, Redish, and Shearer (1996) after 1980 we use data from Statistics Canada, M2 before 1967 is from Metcalf, Redish, and Shearer (1996). All data between can be downloaded from Statistics Canada. And Metcalf, Redish, and Shearer (1996) offer monthly data; we convert the monthly series to the annual frequency by taking value in December. GDP and GDP deflator data are from Statistics Canada\textsuperscript{47}.

\textsuperscript{43}BAM-AGGM.M.1020001.M3XC.3.101.EMUBI4.SBI138.1000
\textsuperscript{44}BAM-AGGM.M.1010001.AM01.0.101.WRDBI2.S0.EUR
\textsuperscript{45}SST-STSMB.M.M1ST.101
\textsuperscript{46}SST-STSMB.M.M2PLST.101
\textsuperscript{47}Table 380-0064
2.7.2 Numerical Solution

For completeness, we first derive equation-(1) to (6). Then we went through the solution details.

\[
L = \sum_t \beta^t \{U(c_t) + \lambda_t [(q_t + d_t)s_t + b_t + L_t - s_{t+1}q_t - c_t - \frac{b_{t+1}}{R}] \\
+ \mu_t \frac{b_{t+1}}{R} + \phi_t q_t s_{t+1} + \psi_t \}
\]

First order conditions of \( c_t, s_{t+1} \) and \( b_{t+1} \) will be

\[
U'(c_t) = \lambda_t \\
\frac{\lambda_t}{R} = \frac{\mu_t}{R} + E_t^g[\beta \lambda_{t+1}] \Rightarrow U'(c_t) = R E_t^g[\beta U(c_{t+1})] + \mu_t \\
\lambda_t q_t = E_t^g[\beta \lambda_{t+1}(q_{t+1} + d_{t+1})] + \mu_t \phi_t q_t \Rightarrow q_t = \frac{E_t^g[\beta \lambda_{t+1}(q_{t+1} + d_{t+1})]}{U'(c_t) - \phi_t \mu_t} \tag{A.2}
\]

Combine equation-(A.1) with budget constraint and binding borrowing constraint will have:

\[
c_t = \min \left\{ (U')^{-1}(E_t^g[\beta RU'(c_{t+1})]), \psi_t + \phi_t q_t s_{t+1} + b_t + z_t g(1) \right\} \tag{A.3}
\]

For any period \( t \), exogenous state variables are \( z_t \) and \( F_t \), endogenous state variable is \( b_t \) and \( s_t \). Since market clear condition restricts \( s_t \) to be 1, we simply state that \( b_t \) is the only endogenous variable. Given any realization of exogenous state variables, there exists a \( \bar{b} \), such that borrowing constraint will be binding if \( b_t < \bar{b} \).

Given any realization of \( z_t, F_t, \bar{b} \) and policy functions from last iteration(\( k \)), \( c^k(z, F, B) \), \( q^k(z, F, B) \), and \( \mu^k(z, F, B) \):

When \( b_t \geq \bar{b} \), borrowing constraint is loose, we denote the current case as Case.1. Consumption is thus determined by the left branch of equation-A.3. If agent choose a bond

\[^{48}\text{It is possible that } \bar{b} = \infty, \text{ so borrowing constraint will be binding regardless the realization of exogenous variable}\]
holding $B'$ next period, we denotes the consumption policy function as $c_t(z_t, F_t, B')$, which will defined as following:

$$c_{1,t}(z_t, F_t, B') = (U')^{-1}(E^{B_t}[\beta RU'(c_{t+1}(z_{t+1}, F_{t+1}, B'))])$$  \hspace{1cm} (A.5)

where $(U')^{-1}$ is inverse function of $U'(.)$ and $c^k(z, F, B)$ is policy function from iteration $k$. This policy function is defined on state variable space. It is worthy to point out that $c_{1,t}(z_t, F_t, B')$ is not a policy function yet, since $B'$ is not a state variable, at least not for period $t$. And when $b_t \geq \bar{b}$, borrowing constraint is loose, thus $\mu_t(z_t, F_t, B') = 0$. From equation-(A.2), we can solve out the value of collateral asset given people choose bond holding of next period to be $B'$ as below:

$$q_{1,t}(z_t, F_t, B') = \frac{E^{B_t}[\beta U'(c^k(z_{t+1}, F_{t+1}, B')))(q^k(z_{t+1}, F_{t+1}, B') + d_{t+1})]}{U'(c_{1,t}(z_t, F_t, B'))}$$  \hspace{1cm} (A.6)

where $q^k(z, F, B)$ is policy function from iteration $k$. $d_{t+1}$ is dividend from land. And the exception in equation-(A.5) and (A.6) are based on belief $B_t$ towards $z_{t+1}$ and $F_{t+1}$. And $q_{1,t}(z_t, F_t, B')$ is an increasing function of the first and third input. Furthermore, $b_t$ can be backed out through budget constraint if people choose bond holding of next period to be $B'$, given $s_{t+1} = s_t = 1$:

$$c_t + \frac{B'}{R} = d_t + L_t + b_t \Rightarrow$$

$$b_t(z_t, F_t, B') = d_t + L_t - c_t(z_t, F_t, B') - \frac{B'}{R}$$

Hence, for any $z_t$ and $F_t$, we define policy functions in iteration $k + 1$ for $B \geq \bar{b}(z_t, F_t)$ or
Case 1 as:

\[ c_{1,t}^{k+1}(z, F, B) = c_{1,t}(z, F, d_t(z) + L_t(z) - Rc_t(z, F, B') - RB) \]
\[ q_{1,t}^{k+1}(z, F, B) = q_{1,t}(z, F, d_t(z) + L_t(z) - Rc_t(z, F, B') - RB) \]
\[ \mu_{1,t}^{k+1}(z, F, B) = 0 \]

where \( c_{1,t}(z, F, B') \) and \( q_{1,t}(z, F, B') \) are defined in equation-(A.5) and (A.6).

Otherwise, when \( b_t < \bar{b} \), borrowing constraint is binding, we denote the current case as Case 2. Thus value of collateral asset is thus determined by borrowing constraint.*

Given agent choose a bond holding \( B' \) next period, we denotes asset price policy function as \( q_t(z_t, F_t, B') \), which will defined as following:

\[ q_{2,t}(z_t, F_t, B') = \frac{1}{\phi_t}[-\psi_t - \frac{B'}{R}] \tag{A.7} \]

where both of \( \phi_t \) and \( \psi_t \) are determined by \( F_t \). And we substitute \( mu_{2,t} = U'(c_t) - E^{B_t}[RU''(c_{t+1})] \) into asset pricing equation-A.2 to solve consumption, we will get:

\[ c_{2,t}(z_t, F_t, B') = U'^{-1}(1 - \frac{1}{\phi_t} \left\{ \frac{U'(c^{k}(z_{t+1}, F_{t+1}, B'))(d_{t+1} + q^{k}(z_{t+1}, F_{t+1}, B'))}{q_{2,t}(z_t, F_t, B')} \right\} 
- \phi_t E^{B_t}[RU''(c^{k}(z_{t+1}, F_{t+1}, B'))]) \tag{A.8} \]

where \( q_{2,t}(z_t, F_t, B') \) is determined in Equation-A.8. Furthermore, according to equation-A.1:

\[ \mu_{2,t}(z_t, F_t, B') = U'(c_{2,t}(z_t, F_t, B')) - E^{B_t}[RU''(c^{k}(z_{t+1}, F_{t+1}, B'))] \tag{A.9} \]

where \( (U')^{-1} \) is inverse function of marginal utility \( U'(\cdot) \), \( c^k(z, F, B) \) and \( q^k(z, F, B) \) are
consumption and asset price policy function from the \(k\)th iteration. And finally in following formula, we back out the endogenous state variable \(b_t\) at which given exogenous state variables \(F_t\), agent is to choose \(B'\) as bond holding for next period.

From budget constraint:

\[
b_t(z_t, F_t, B') = d_t + L_t - c_t(z_t, F_t, B') - \frac{B'}{R}
\]

The policy functions for the \(k+1\)th iteration can be defined as following when borrowing constraint is binding:

\[
c_{2}^{k+1}(z, F, B) = c_{2,t}(z, F, d_t(z) + L_t(z) - Rc_t(z, F, B') - RB)
\]

\[
q_{2}^{k+1}(z, F, B) = q_{2,t}(z, F, d_t(z) + L_t(z) - Rc_t(z, F, B') - RB)
\]

\[
\mu_{2}^{k+1}(z, F, B) = \mu_{2,t}(z, F, d_t(z) + L_t(z) - Rc_t(z, F, B') - RB)
\]

In the end, we need to define a \(\bar{b}\) for each \(z_t\) and \(F_t\). As we know, \(\bar{b}\) is the threshold, if \(b_t \geq \bar{b}\) agent stays unconstrained. In another words, \(\bar{b}\) is the level at which agent will be marginal binding: For any realization of \(z_t\) and \(F_t\), say \(z_t = z\), \(F_t = F\), \(\bar{b}(z_t, F_t)^{49}\) is the solution of

\[
c_{2}^{k+1}(z, F, \bar{b}) = c_{1}^{k+1}(z, F, \bar{b})
\]

where \(c_{i}^{k}(z, F, b), i = 1, 2\) represents consumption policy function under case \(i\). As our policy functions converge, threshold level will converge too.

Furthermore, since we are forcing agents to choose \(B'\) for next period, then we should restrict the choice of \(B'\) is a reasonable set. For example, for a borrowing constraint to be bind, \(B' < -\psi_t\), or agent will face no restriction at all, even when the collateral has zero

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49Since \(\bar{b}\) will be different for different exogenous variables. Hence it is a function of \(z_t\) and \(F_t\).
The range of $B'$ is of course given by Equation-*. Under the constraint, $B' \geq -\psi_t - \phi_t q_{t+1} \geq -\psi_t - \phi_t q_t$. Given any realization of $z_t = z$ and $F_t$, $B' \geq -\psi_t - \phi_t q_{t,t}(z_t, F_t, B') \geq -\psi_t - \phi_t q_{t,t}(z_t, F_t, B')$, the second inequality follows from Equation-A.2 with $\mu = 0$. Or we can present borrowing constraint as $\psi_t + \phi_t q_{1,t}(z_t, F_t, B') + B' \geq 0$, the left hand side of this equation is increasing function of $B'$. The lower bound of $B'$ should be thus solved from equation below:

$$\psi_t + \phi_t q_{1,t}(z_t, F_t, B_{lower}) + B_{lower} = 0$$

Hence, we will restrict $B' \in [B_{lower}, -\psi_t]$ and solve case 2. If backed $b_t$ stays below $\bar{b}$, the policy function serves as constraint branch. Similarly, we restrict $B' \geq B_{lower}$ when solving case 1. If backed $b_t$ stays above $\bar{b}$, the policy function serves as unconstrained branch. Since $b_t$ are backed through budget constraints in both cases and consumption and borrowing is lower in case 2, backed $b_t$ has no overlapping. I can patch these two branch together and form the policy function for next iteration.
A numerical solution under parameter $\beta = 0.96$, $R = 1.03$, $\gamma = 3$, $\alpha = .2$, $\phi^{H(L)} = 0.070(.05)$, $\psi^{H(L)} = 0.2(.2)$, $z^{H(L)} = 1.1(.969)$, is displayed in Figure-2.17.
Chapter 3

Quantity Theory of Money is not a Universal Rule: Evidence from Monetary History of Thirteen Countries

3.1 Introduction

The Quantity Theory of Money (QTM) is at the heart of Monetary Economics since its birth. QTM is a long-run statement which provides that inflation rate will be the same with the growth rate of Money in the long-run. Since Money has real effects in the short run, we can only have QTM in the long-run or as a steady state equilibrium, for example Lucas (1980) and Lucas (1996). QTM receives strong rigorous empirical supports since Lucas’ works. Lucas (1980) believes that QTM

“...possess[es] a combination of theoretical coherence and empirical verification shared by no other propositions in Monetary economics... It is hard to imagine a nonvacuous
economic prediction obtaining stronger confirmation than that shown. This is the kind of “empirical verification” of the quantity theory on which economists who assign it a central theoretical role base most of their confidence.” (Page 1007, Lucas (1980))

There is no doubt that, at least since Friedman (1956), Lucas (1980) has been the most influential restatement of the QTM for the elegance and strength of this empirical result. In fact, Bob Lucas shows that, for US during 1955-1977, M1 growth and CPI inflation where, when short-run movements had been reasonably filtered out, essentially one-to-one linked. The co-movements between inflation and the growth rate of M1 were on a one-to-one basis.

Of course, the “Long-run” is not an empty title. The cross-country evidence, which shows up in McCandless Jr and Weber (1995), uses 30 years (1960-1990) average annual inflation rate and average (annual) growth rate of M2 across 110 countries. McCandless Jr and Weber (1995) plots inflation rate against the growth of M2 and finds that these points line up near perfectly along a 45-degree line. Lucas (1996) views this as a great success of QTM and Monetary Economics:

“...The kind of monetary neutrality shown in this figure needs to be a central feature of any monetary or macroeconomic theory that claims empirical seriousness...”

Long-run signal, in Spectrum Analysis, is equivalent to the signals with a range of low frequencies (long periods), say the area around frequency $\omega = 0$. Lucas (1980) uses a band-pass filter (e.g., Christiano and Fitzgerald (2003)) to extract (estimate) long-run signal from raw data and then plots (1) (QTM) filtered growth of money against filtered inflation; (2) (Fisher’s Effect) filtered growth of money against (filtered) T-bill rate. Lucas (1980) finds that in both graphs points almost line up along a 45-degree line. As a result, QTM reveals itself with direct and indirect evidence from Lucas (1980) its influence over inflation.

Researchers use several empirical methods to elaborate(estimate) the title “long run”. A famous cross-country evidence to support QTM, which shows up in McCandless Jr and Weber (1995), uses 30 years (1960-1990) average annual inflation rate and average (annual)
growth rate of $M_2$ across 110 countries. McCandless Jr and Weber (1995) plots (average) inflation rate against (average) growth of M2 and finds that these points line up near perfectly along a 45-degree line. More recently, Benati et al. (2016) applies co-integration test to long spanned dataset. 

From now on, we would like to refer this average of annual rate across some time span as Temporal Approach. In contrast, Lucas (1980) uses a Frequency Approach to exploit supporting time series evidence from U.S data. In the pioneering work of Lucas (1980), the author uses a preliminary band-pass filter to extract (estimate) the long run signal (most persistent part of) from the annual growth rate of M1, CPI and T-bill rate during 1953-1977.

However, does QTM stable across time? Friedman (1988) conducts a preliminary test to check the relationship between Price and Money during 1977-1980s. The inflation rate, realized or expected, stays relatively more stable than, and irrelevant to money growth (in U.S). Along with several others, we also found that QTM is not robust after some date for the US.

For example: “...For most of the last 25 years, the quantity theory of money has been sleeping...” (Sargent and Surico (2011), Page 110).

“...in a later period, the relationship (between money and inflation) turns negative in frequencies 20 years or higher.....” (Christiano and Fitzgerald (2003)). Additionally, this relationship is not even robust across history. In this article, I will show QTM is not even robust across countries.

Furthermore, the latest breaking of QTM seems to be a global pattern, though different countries have different breaking dates and degrees. There are several proposed explanation, say disinflation policy, fiat or commodity Standards. However, we cannot find positive evidence to confirm these proposals.

This article, instead of trying to explain QTM, checks whether QTM ever holds across time and countries. Put in other words, does Bob get lucky when he explores the data of U.S

In this article, I investigated historical data of 13 countries during a long time span, mainly from the late-1880s or early 1890s to 2006. I truncated my data to avoid the late financial crisis. I applied the original method of Lucas (1980) to the cases of these 13 countries. It turns out that the Quantity theory of Money is not a global pattern before the 1950s. But during 1950s-1980, for most of the countries in my sample, the (Long-Run) growth rate of money provides a good explanation of inflation. However, the growth rate of money has nothing to do with the interest rate, except 1950s-1980 for almost all the countries. Most importantly, the latest breaking down of QTM happens at a different time for different country. For example, the breaking date for U.S is 1991 if I use $M_2$ as my main focus, while for most of the European countries, it is 1998.

Additionally, whether QTM breaks hugely depends on the measure of money I am using, echoing the message from Lucas and Nicolini (2015). For example, if one focuses on $M_4$ index for U.K, the QTM still holds. However, $M_2$ and $M_1$ version of QTM breaks for U.K. Hence, it is reasonable to doubt whether QTM still holds for more general definitions of money. However, this goes beyond this scope of this paper.

Concerning to the method, I adopt the original method of Lucas (1980) in the main body of this article. As a robustness check, we also used bandpass filter like Christiano and Fitzgerald (2003) and (rolling over) spectrum at like ?. For U.S and U.K, I also checked time-varying VAR. I agree that idea of Lucas (1980) is to estimate the long-run signal out of original data. However, the estimation of the spectrum at $\omega = 0$ is notorious of unstable property. For more details, one can refer to Sargent (1987), Hamilton (1994) and Lütkepohl (2007), can be viewed as a smoothing(filtering) method to estimate the most persisting identifiable component(s) from raw data. One can also refer it as the long-run
trend. Furthermore, I do not have a criterion or would like to take a stand on how long should be named to be “Long-run” 30 years? 40 years? For example, if we would like to explore dataset of U.S is from 1948-2015(post-WWII period), 30 years or above will also have the similar disadvantage of a spectrum estimation at $\omega = 0$. Additionally, laying out the gain and coherence estimators would be a misleading part. For example, for coherence, though people may have used several kinds of window function adjusted it, high value of this estimator tends to have a high variance\(^1\). And when people have high gain estimator, we should not accept that QTM is still alive. Since QTM refers to a one-to-one relationship, the gain should be far away from 1 if QTM holds.

Lucas (1980) uses an elementary band-pass filter to extract the long run signal from M1, CPI and T-bill rate during 1953-1977(hereafter, as Frequency (domain) Approach\(^2\)).

To make band-pass filter work and avoid Spurious regression\(^3\), Lucas (1980) stationarizes the data by taking annual growth rate of the raw data of $M_1$ and CPI.

I am not the first to arise whether QTM is alive or not. For example, ? uses spectrum estimation and finds that QTM is not stable. Rolnick and Weber (1997) finds that correlation between inflation and money growth was weaker during the standard Gold Period than during the fiat standard. Sargent and Surico (2011) found that QTM does not work recently. Sargent and Surico (2011) use time-varying VAR and the filter of Lucas (1980) to establish the recent failure of QTM. However, Sargent and Surico (2011) used the filter of Lucas(1980) with 16 quarters window length.

In this empirical paper, we will replicate the filter of Lucas (1980) and check whether Quantity theorem of Money hold across countries and cross time. In Wang (2017), I mainly

\(^1\)It worths to point out that ? applies a Mont Carlo Method to simulate the variance of gain and coherence.

\(^2\)One can view (stationary) data $\{Y_t\}_{t=0}^{\infty}$ as (1)(Time domain) $Y_t = \alpha + \sum_j \phi_j \epsilon_{t-j}$ or (2)(Frequency domain) $Y_t = \alpha + \int_0^\pi a(\omega)\cos(\omega t) d\omega + \int_0^\pi b(\omega)\sin(\omega t) d\omega$. Please notice that $M_1$ and CPI are not stationary process yet.

\(^3\)Since at least $M_1$ is a $I(1)$ process in theory: $M_t = M_{t-1} + \epsilon_t$ where $\epsilon_t$ is the money injection by central bank. One can easily test unit root in $M_1$ and CPI too. Con-integration test, e.g., Benati et al. (2016) is another way out.
focus on the data U.S case.

This article will be organized as follows; I will describe the empirical method, data, and results in the first three sections. Then we will conclude and leave the explanations of this collapse to future research.

3.2 Empirical Method

Following Lucas (1980), I will work on following (quarterly) data transformation:

\[ G_m(t) = \log(M_t) - \log(M_{t-4}) \]
\[ \pi(t) = \log(CPI_t) - \log(CPI_{t-4}) \]
\[ G_y(t) = \log(y_t) - \log(y_{t-4}) \]

Where \( CPI_t \) and \( y_t \) stand for consumption Price index and real GDP at time \( t \). And \( M_t \) stands for monetary aggregate, which can be currency and reserve, \( M_1, M_2 \) even \( M3/4 \) for some countries. Since the definitions of monetary indices are different among countries. I will label monetary stocks as narrow or broad money.

Lucas (1980) uses an exponential decreasing sequence as filter coefficient to extract long-run information. To be more precise, Let window function \( \omega_{\rho,L}(n) \) with bandwidth \( L \), for \( n = 0, 1, 2, .., T \)

\[ \omega_{\rho,L}(n) = \begin{cases} 
\rho^{|n|} & |n| \leq L \\
0 & |n| > L 
\end{cases} \]

I will denote \( x(t) \) and \( x^f(t) \) for \( t = 1, 2, 3, ..., T \) as raw and filtered data respectively. \( x^f(t) = \sum_{s=1}^{T} \omega_{\rho,L}(t-s)x(s) = \Omega(L)x(t) \). One can try different bandwidth and discounting rate of
this filter, to extract long run signal out of raw data\(^4\). And it is worth to note that here we use zero padding technic, which is common in signal analysis: setting the missing data to 0. After fixing bandwidth and \(\rho\) to be sixteen years and 0.95, I extract out the long-run growth rate of monetary stock, inflation and real income. Then I would like to check how long-run inflation correlates with the long-run (income-adjusted) growth of money.

During the period under investigation of Lucas (1980), the growth rate of real GDP can be viewed as a stable series. However, during the long historical period, I am to explore, the growth rate of income cannot be taken as given. And to take this varying pattern of the growth rate of GDP into account, I need to adjust growth rate of real GDP or GNP, especially for the European countries. For example, the average growth rate of GDP of Spain is 6.7\% during 1951-1975, but 2.9\% at most during 1976-2006.

Following Lucas (1980), I plot the filtered growth rate of money or excess growth rate of money against inflation rate. The excess growth rate is defined to be the part of the growth rate of money exceeding the growth rate of real GDP.

Lucas (1980) implicitly adopts an eyeball metric to illustrate the nearly perfect linear relationship between growth rate of money and inflation. This idea, interpreted by Sargent and Surico (2011), can be boiled down to a test whether the coefficient before money growth equals to one if we run a regression of monetary growth onto inflation:

\[
\pi(t) = \alpha + \beta G_m(t) + \epsilon \quad \text{H}_0 : \beta = 1 \quad (3.1)
\]

Before we move on into the estimation results, it is worthy to note that \(\beta = 1\) does not necessarily mean quantity theorem holds. One can easily scatter several points along the 45-degree lines to get unine slope. But it is hard to conclude that these two are closely related.

\(^4\)when the gain function concentrate on low frequency. Variation of filtered date comes from the low-frequency variance of original data series; See Sargent (1987) or Stoica and Moses (2005) for more details.
Quantity theorem of money requires more than $\beta = 1$. It requires that money should be the dominating driving factor of inflation. Put it in another way, $R^2$ of this regression should be close to 1. And this is the reason why $\beta$ includes coherency. In spectrum analysis, coherency can be understood to be the $R^2$. However, coherency estimation without any window adjustment will be one by definition. It is reasonable to doubt this measurement even after adjustment. Hence, we prefer to implement a simple regression method in this section. Next, I turn to discuss the empirical results, data is mainly from the dataset of Schularick and Taylor (2012), Balke and Gordon (1986), Bank of England, Bank of Italy, Bank Australia and European central bank.

### 3.3 Empirical Results

Before getting into details, we have a problem of varying monetary regime: from Gold standard to Bretton Woods system, from Brettonwoods to its breaking down. And as Rolnick and Weber (1997) conclude, the correlation between money and inflation is weaker during commodity standard than fiat standard. But Rolnick and Weber (1997) mainly adopt a cross-country regression, and different countries have final dates of commodity money standard. And I find this weak correlation is a temporal pattern, not necessarily linked to the commodity money standard: Weak correlation happens mainly before 1900. Even when commodity standard got abandoned before 1900, weak correlation generally will continue for most of the countries.

During the period I am to investigate, several countries adopt Inflation target policy, one will find that this seems to cause breaking down of narrowly defined money in these countries. For example, according to Hammond (2012), Canada adopt it since 1991, UK 1993, Australia 1993, Spain 1994, Switzerland and Germany 1970. M1 of U.S lost its explanation power to

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5For example, Teles, Uhlig and Valle e Azevedo (2016)
inflation after 1980, Which is within the period during which Fed adopt disinflation policy. However, there are countries with breaking QTM before the change of monetary policy change.

Recent Breaking down of QTM happens in U.S, Sweden, Netherlands (rarely exists), Denmark, France (rarely exists), Switzerland (QTM is a recent pattern), Australia and Canada.

However, it is hard to conclude that QTM went asleep after some specific year, say 1984 for U.S. (1) Pattern breaking date depends on the window length we are using. Put in another word; exact breaking date may be not robust to window length. (2)The monetary stock is not well defined. There are dates before which M1 works, after which M1 does not but M2/3/4 takes over.

This finding has a negative implication for policy recommendation. It does not matter whether QTM is alive or not. Even if QTM is still alive, one does not have a stable and robust measure to implement QTM. Concerning to learning or predict future inflation rate, people do not have a public signal now.

In this section, we are using 16 years as bandwidth, and 0.95 as discounting rate, in honor of Lucas (1980). In the graphs in appendix, I will color the point during different periods; one will find that QTM is not a universal phenomenon. Whether QTM holds or not is irrelevant with whether country implements fiat money or not.

3.3.1 U.S.A

In Figure-3.1, with a much longer data set, from 1871-2008. I plot the growth rate of M1 and M2 against inflation rate in the upper panel. Down panel gives the pattern with income adjustment. I label five different periods with different colors: 1871-1931(commodity stan-

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6One will conclude different breaking date with a different filter. For example, using Lucas’ filter with different window length.

7In an appendix available upon request, I also implement Fast Fourier Analysis and time-varying Var.
Figure 3.1: Filtered Growth rate of Money stock against Inflation rate, U.S

As we can tell from the figure, the relationship between growth rate of M1 and inflation, no matter controlling the growth of income or not, is nearly perfect. When we compare the upper panel graphics with the ones in down panel, one can tell that for the case of U.S that controlling the growth rate of income improves the fitting of QTM. However, adjusting income will not save QTM from recent breaking down.

Growth rates of M1 and M2 co-moves with inflation perfectly during Lucas Period. Without controlling real income, $\beta$ and $R^2$ are 0.937(1.2), 0.88(.81), for M1(2). With income adjustment, $\beta$ and $R^2$ will become 0.771(0.967) and 0.97(0.96) for M1(2). Additionally, Lucas period 1951-1980 is the period during which monetary indexes move closely with inflation in most countries. Since I am using filtered data to run this regression, it is hard to use standard deviation to determine whether we can reject hypothesis $\beta = 1$ or $R^2 = 1$. I adopt...
an ad-hoc criterion here, we used $\beta = 0.7$, $R^2 = 0.8$ as a benchmark\textsuperscript{8}.

More surprisingly, during 1932-1954, the growth rate of M1 has no explanation power to inflation ($\beta = -0.17$, $R^2 = 0.07$). And this powerless fact cannot be contributed to the Great Recession since M1 starts not to explain inflation variation during several years previous to 1929. One can tell that during the same period, more broad indexes are valid after controlling income growth ($\beta = .91$, $R^2 = 0.76$). QTM failed for M1 completely. However, it is hard to believe during 1932-1950, M2 plays a role in transactions. This is the reason why Wang (2017) focuses on $M_2$ for U.S.

During 1871-1931, if one would like to explore more on monetary standard: explaining powers of M1 are different during this two sub-period ($\beta = 0.29$ for 1872-1899, and 0.91 for 1900-1931), difference persists even after controlling the real income. Additionally, M2 outperforms M1, especially after controlling income. But one will find the fitting of M2 during pre-1900 is still worse than 1900-1931. It is also worth to note that with different bandwidth, it is possible to find different breaking date. For example, when we use 16 years as bandwidth, QTM break around 1991, while Sargent and Surico (2011) found QTM break around mid-1980 by using four years as bandwidth and much lower $R^2$ of the QTM regression.

\subsection*{3.3.2 Australia}

In Figure-3.2, I plot the growth rate of M1 and M2 against inflation in the upper panel. Income adjustment cases are laid out at the down panel. I divide the case of Australia into four periods, 1874-1931, 1932-1954, 1955-1980, 1981-1992, and 1993-2006. I have 1955-1980 as a period is to fit Lucas (1980). And I choose 1931 to be another threshold is because U.K abandons commodity standard at 1931 according to Rolnick and Weber (1997). Additionally, 1993 is the year when Australia adopt Inflation target policy according to Hammond (2012).

\textsuperscript{8}Or be more precise $\beta$ should not stay too large either, we set $\beta < 1.4$, so $\frac{1}{\beta} < 0.7$
For Australia, M1 works well during 1955-1980, all points during this period lie along the 45-degree line, except M2 during 1955-1959. But during 1932-1954, the one-to-one relationship goes away no matter whether we control income and what monetary index one would like to use. Hence, during 1932-1954, I conclude that inflation is robustly un-correlated with money growth. If one would like to explore more, one will find from 1945-1950, the correlation between inflation and money growth is negative.

Concerning to 1874-1931, one will again find that the “poor” performance of money during commodity standard can be contributed to years previous to 1900: $\beta$ and $R^2$ for M1 are 0.6352 and 0.97723 after 1900, while they are 0.27 and 0.097 before 1900. M2 has a similar pattern: poor during years pre-1900 ($\beta = 0.33$, $R^2 = 0.58$), fits better from 1900-1929 ($\beta = 0.98$, $R^2 = 0.87$), then deteriorates during 1930-1954, start to work during later half of 1955-1980 and 1981-2006. On the other hand, M1 lost its explanation to inflation after
1981, much earlier than when Australia adopt Inflation targeting policy, while the growth of $M_2$ still co-moves with inflation.

**3.3.3 Canada**


Again, M1 works well from 1955-1980, $\beta = 0.73, R^2 = 0.94$. Fitting of M2 is as good as M1 during the same period, $\beta = 0.74, R^2 = 0.84$. And these fitting comparison will be reversed when we control for income. However, when I explore the period before 1955,
namely 1930-1954, inflation react insensitively to money growth (M1 and M2) no matter whether I control for income or not.

During the area of the commodity standard, messages are mixing: During the years before 1900, it is hard to conclude that inflation moves with the growth rate of money in a one-to-one way, no matter whether we control income or not, the $R^2$ are all less than 0.2. The situation of 1900-1929 improves a little: inflation starts to react to changing growth rate of money aggregate (M1 and M2), but still not in a one-to-one fashion.

Furthermore, for M2, there is no significant break at 1991, which is the year Canada adopt inflation targeting policy. The breaking happens around 1997.

### 3.3.4 Switzerland

In the case of Switzerland, it is important to adjust the growth of income, especially for the period after 1946 as shown in Figure-3.4. No index except excess growth rate of M2 can fit inflation well. And M2 only starts to work after 1946. Switzerland is a little different from
all the other countries I investigate: (1) it abandons gold standard after 2000; (2) QTM is a recent pattern for Switzerland and still holds. Switzerland is a perfect counter example to the argument in favor of monetary standards.

Hence, I view Switzerland as a counterexample to statement of Rolnick and Weber (1997). The weak correlation between money and inflation is not a necessary result of commodity standard. Instead, the weak correlation seems to be a temporal pattern specific to the years pre-1900. Since 2000 is close to the end of our data sample, it is hard to conclude any result for fiat money standard in this case.

M1 does not work for Switzerland all the time, except when one would like to count the fitting during 1900-1945 as a small success ($\beta = 0.88, R^2 = 0.45$). On the other hand, it seems to be safe to state that M2 after adjustment works well after 1946 until 2000. Furthermore, for Switzerland QTM (controlled for income) is a recent pattern, which did not exist before that.

3.3.5 U.K

U.K is the last country I investigate with filtering the whole sample, because of WWII, the economy may fall under a different regime, prices may not be able to move freely during some
period. It will not give us too much hints on how money works. Wartime is an extreme case for the economy: Government needs to issue a lot of money to finance their war, and this fiat money is not backed by any commodity. People are afraid of government default, which is pretty common phenomenon during the wartime. Then inflation will increase dramatically. But this is not a confirmation of purchasing power of money, but a story of trust. For U.K, I divide the whole sample into six pieces: 1870-1899, 1900-1931, 1932-1952, 1953-1980, 1981-1990 and 1991-2006 as shown in Figure-3.5. 1931 is the year when U.K abandon commodity standard, 1953 gives the best fit for QTM during the Lucas Period, and 1991 is when U.K adopt Inflation target policy.

For United kingdom, generally speaking, quantity theorem works well, except 1932-1952 and recent breaking. For 1932-1952, as we discussed, this can be a result of the war. And more interesting, the recent breaking down of QTM can be saved by using a broader monetary index, M4.

As we found in other countries, inflation reacts to money growth in a detached pattern during pre-1900 years, values of $\beta$ of M1 and M2 are around 0.3. But if one gets into 1900-1931, both $\beta$ go up to 0.9. And during 1900-1931 inflation moves more closely with money growth than the period 1953-1993, if one does not control income growth. However, fittings get worse during 1932-1952.

I choose 1991 as a separating point is because U.K officially adopts inflation target after 1993. As one can tell from figure that the breaking of QTM locates around 1991, if one chooses a narrow monetary measure, say $M_0$ or $M_1$. However, QTM of M4 cannot be counted as broken if one is willing to abandon the sample period after 2006, following Sargent and Surico (2011) to avoid the latest financial crisis. For $M_4$, $\beta = 0.80$, $R^2 = 0.97$ after controlled for income.

In sum, one will find positive historical evidence of QTM for U.K, and it is still alive if
we adopt broad monetary index\(^9\).

### 3.3.6 Germany

Since Germany got involved in two World Wars, I decide to divide the sample into pre-1900, 1900-1913 (commodity standard), 1926-1938 (between WWI and WWII, no hyperinflation period, which locates 1921-1924), 1952-1980 (Lucas period), 1981-2006 as shown in Figure 3.6. Germany abandoned the commodity standard in 1913. I filter commodity standard area as a whole piece and color pre-1900 and post-1900 (1900-1913) by different colors. 1952-2006 are also filtered together but colored differently. Period 1926-1938 is filtered alone, and the hyperinflation period is excluded from my sample.

Without controlling growth or income, neither of the monetary indexes can explain fluc-

\(^9\)It does not matter we adopt M4 or M4(x) during 1998-2006 since the growth rate of two indexes are pretty close.
tuation of inflation. For the case of M1, $\beta$ attains 0.28 during 1884-1899, and stay at the level of 0.12 – 0.16 till 2006. For M3, situation is similar, the maximum $\beta = 0.26$. If one would like to insist on QTM without income adjustment, I have to conclude QTM never (rarely) exists in Germany.

For comparison of pre-1900 and after-1900, a contrast to what happens in other countries, during years pre-1900, M1 and M3 fit better than during 1900-1913.

When one controls income growth, one will find that Lucas period is the only period during which inflation lies closely to M1 growth ($\beta = 0.81, R^2 = 0.86$). After 1981, QTM breaks down for M1 ($\beta = -0.48, R^2 = 0.34$). Furthermore, there is a regime change in Germany during 1952-2006. While M1 works well during Lucas period (1952-1980), the explanatory power of M3 is not as strong as M1 during the same time ($R^2 = 0.72$). However, the situation got reversed after 1980: M3 has better fitting after 1980 ($\beta = 0.7, R^2 = 0.9$), while M1 got divergent with inflation. Since 1970 is the year Germany adopt Inflation target policy, one can also divided the sample into pre-1970 and post-1970, but it is hard to conclude that there is a structure break at this year.

3.3.7 Italy

Italy is a role model for Quantity theorem of Money. And there is even no breaking down of QTM, at least if one restricts herself into the period before crisis as shown in Figure-3.7, as this and other papers for all of the samples. I filtered two pieces of data sample separately: 1881-1939, 1952-2006. Then I further use different colors to label different periods: 1881-1899, 1900-1939, 1952-1980, 1981-1998(before-Euro), and 1999-2006.

Italy abandons commodity standard on 1935. Since this year is close to WWII period, I decide to investigate 1881-1939 as a whole piece for Italy. However, the result here is robust, if one separates the sample at 1935. I further use two different color two eras of 1881-1939: pre-1900 and 1900-1939. As one can tell from Figure 7, generally relationship between
money and inflation is weak during the years pre-1900 (without controlling income $R^2$ for M1(M3) is 0.6(0.68) before 1900, 0.85(0.95) after 1900. similar patterns show up when one controls income). This is in line with my previous findings: so-called weak correlation during commodity standard is a time-specific pattern, only happens before 1900. Furthermore, M1/3 after income adjustment fits inflation during 1900-1939 (for M1, $\beta = 0.84, R^2 = 0.95$) as well as 1952-1980 (for M1, $\beta = 1.2, R^2 = 0.97$). Hence, it is difficult to contribute QTM to fiat money standard.

During 1952-1980 and 1981-1998, the growth of M1 or M3 correlated with inflation well, and almost perfectly when one is willing to control the growth of income. During Lucas period (1952-1980), income-controlled M1 and M2 works well for inflation ($\beta = 1.2, 0.8$ and $R^2 = 0.97, 0.95$). For Italy, the popular breaking down of QTM1 does not happen $\beta = 0.75, R^2 = 0.9$, but explanation of M2 does decrease: $\beta = 1, R^2 = .96$ during 1981-1998, while $\beta = 0.61, R^2 = 0.99$ during 1999-2006.

To sum up, everything works well until late 1990s, before the establishment of Euro. M2
Figure 3.8: Filtered Growth rate of Money stock against Inflation rate, Spain

falls to explain inflation during the late 1990s, and then recover its explain power after 2000. On the other hand, M1 after income adjustment keeps closely correlated with inflation before and after the Euro. For M1, fitting after 1980 is the same good as during Lucas period. For M2, there is a temporal lost.

3.3.8 Spain

Spain offers another example to establish a recent validity of QTM of M1. After control the growth of income, M1 still correlated closely to inflation after Spain join Euro($\beta = 0.8, R^2 = 0.95$). In another word, QTM1 does not break down in Spain. However, this validity is improving during the history(for income controlled M1, $R^2$ increased from 0.78 during period 1943-1980 to 0.96 during 1999-2006). For example, when we controlled income growth, QTM of M1 start to reveal itself around the late 1970s to early 1980s ($R^2$ increases). On the other hand, M2 after controlling income fails to explain inflation after 1981($\beta = 0.48, 0.34$ during 1981-1998 and 1999-2006 respectively).
Spain adopts disinflation policy after 1994 according to Hammond (2012). And 1994 is too close to Euro, if there is a structured break, it is hard to determine whether it is because of Euro or disinflation policy. Like other European countries, our empirical results (not presented here) show that if we only use data before Euro, namely focusing on data before 1998 only, there will be no breaking of Quantity theorem of Money for M2 either.

During 1881-1899, after we control growth rate of income, M1 and M2 work well to explain inflation (without income adjustment $\beta = 0.7(1.07), R^2 = 0.8(0.75)$ for M1(M2)). Then M1 begin to be not sufficient to move inflation around ($\beta = 0.58(0.46)$ with(out) income adjustment), M2 took over the power during 1900-1935 ($\beta = 1.2(0.8)$ with(out) income adjustment). Generally, 1881-1935 is not a bad period for QTM.

During 1943-1975, I divide the sample into pre-1975 and post-1975 is because it is natural to think Spain went through a regime change after Francisco passing away.

To sum up, for Spain, the message of QTM is mixing as shown in Figure-3.8. For M1, it still works, especially after adjusting income growth. However, M2 seems to get divorced with inflation after 1991. But as we discussed earlier, this breaking down depends on whether we take the time after Euro into account.

### 3.3.9 France

I divide France into 1881-1899 (Pre-1900), 1900-1913 (1900-Pre-WWI), 1922-1938 (After WWI), 1946-1954 (after WWII), 1955-1980 (Lucas Period), 1981-1998 and 1999-2006 as shown in Figure-3.9. France abandons commodity standard on 1936, which is too close to WWII, so we group this year into 1922-1938. I filter Pre-1913, 1922-1938, and post-1946 separately.

Generally speaking, we cannot find a robust relationship between inflation and growth of money. One may find that QTM reveals itself during the early part of 1981-1998, leaving alone the breaking down during the later half. But generally speaking, it is even hard to conclude that monetary index has ever correlated with inflation in a robust way in France.
Figure 3.9: Filtered Growth rate of Money stock against Inflation rate, France

Figure 3.10: Filtered Growth rate of Money stock against Inflation rate, Denmark
3.3.10 Denmark

There is no obvious time division for Denmark; we mainly divide Denmark into Pre-WII and After WWII period. I filter these two pieces separately; one is from 1886 to 1938, one is from 1951-2006. Then we label different subsamples with different colors: 1886-1899, 1900-1913, 1914-1938, 1951-1980, 1981-1998 and 1999-2006 as shown in Figure-3.10. I do not have a specific reason to group them in this way; I simply follow the case of Germany. I filter data during 1886-1938 and 1951-2006 separately.

Quantity theorem of money holds perfectly during 1900-1938. Weak correlation still applies to the pre-1900 period. The performance of M1 becomes bad after 1938. The relationship between M3 and inflation is still there, though deteriorates. The breaking happens during late-1990 depends on whether we sample in years after 2000. There is a breaking down of QTM after 1999, for both M1 and M2.

During 1886-1899, the scatty plot is a common pattern across countries: Inflation is only weakly correlated with money growth. After entering the 20th century, money starts to reveal its power onto inflation. During 1900-1913 and 1914-1938, inflation is nearly perfectly correlated with the growth of M1/3: Before 1900, M1(M3) $\beta$ equals 0.28(0.38) if we do not control the income. After 1900, $\beta$ of M1(M2), is around 0.7.

However, after WWII, inflation got divorced with M1, even during 1951-1980(Lucas period), $\beta = 0.92$ but $R^2 = 0.64$. During 1951-1980 and 1981-1991, we have years during which inflation fluctuate by its own without varying M1 growth. But M2 works after 1951 until late-1990($R^2 = 0.77, 0.97$). This kind of regime change happens in Germany too: before a certain year, M1 works, but not after that date. M2 take over M1 after the threshold. Broad money takes over.
3.3.11 Netherlands

Netherlands abandon commodity standard on 1936 and adopt disinflation policy on 1999. 1999 is a specific date for European countries. So we try to label this year out for all European countries. For Netherlands, we separately filter two pieces of data: 1901-1948 and 1949-2006, then labor sub-samples with different colors. In the end, we have 1901-1913 (Pre-WWI), 1914-1936 (Commodity standard), 1937-1948 (WWII), 1949-1980 (Lucas period), 1981-1998 (before Euro) and 1999-2006. I filtered three pieces of sample separately: pre-WWII, WWII and post-WWII. The reason of separating WWII from other sample is because that though Netherlands had proclaimed neutrality when war broke out in September 1939, just as it did during the World War I, it was still invaded by Germany in 1940.

Quantity theorem of money never exists in Netherlands, no matter we use M1 or M2, control income or not. The relationship between the growth of money and inflation was never close to one-to-one, though they move together closely.

During pre-WWII area, one can tell from Figure-3.11, the relationship between money
and inflation during these years are very similar. Inflation reacts to the growth of M1 and M2 similarly. And inflation reacts to money in a very stable way: positive, but significantly less than 1 (the highest $\beta$ is 0.5, M3 during 1914-1936). There seems to be another “quantity theorem of money” in Netherlands, with coefficient less than 1 but closely co-movement between money growth and inflation. Additionally, the performances of money are different during post-1900 than pre-1900: $\beta = 0.35(0.36)$ for M1(M2) during pre-1900, $\beta = 0.49(0.50)$ during post-1900.

When one turns to 1949-1980, during the early stage, neither of varying M1 or M2 can move inflation around, no matter we control income growth or not. Then inflation seems to react to M1 in a one-to-one way for several years. Actually, this one-to-one relationship is between 1955-1980, if we run regression just for this period, $\beta = 0.77(0.72), R^2 = 0.85(0.83)$ for M1 with(out) controlling. After 1981, neither M1 or M3 growth co-moves with inflation well, no matter whether I adjust income. It seems like QTM breaks down after 1981. However, it rarely exists in Netherlands, except Lucas Period.

### 3.3.12 Norway

Norway abandons commodity standard on 1931, which is pretty close to our last year before WWII, 1939. I group 1931 into our Pre-WWII sample. According to Hammond (2012), Norway adopts disinflation policy on 2001. I separately filter data of Norway during Pre-WWII and after-WWII. Then I further use different color to label different sub-samples: 1871-1899, 1900-1939, 1947-1960, 1961-1980, 1981-1998, and 1999-2006 as shown in Figure-3.12. The reason I still choose 1999 is to keep consistent with other European countries, results are robust to this threshold choice.

M3 works more robustly than M1 in Norway. Inflation reacts to the growth of M3 in a one-to-one way generally. To be more precise, QTM works during 1900-1939, 1961-1980,

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10The $R^2$ are pretty similar.
1981-1998 and 1999-2006. On the other hand, M1 works during Lucas Period and 1900-1939. During 1947-1960, the performances of these two indices get worse, no matter whether we adjust income growth.

And it is worth to note that as most of other countries, money performs badly to explain inflation variation during the year before 1900. This weak correlation hence should not be taken as a characteristic of the commodity standard. Again, in Norway, commodity standard got abandoned on 1931, thirty years later. Take M2 as an example, with(out) controlling income, $\beta = 0.72(0.74), R^2 = 0.27(0.58)$ before 1900, $\beta = 0.75(0.80), R^2 = 0.88(0.89)$.

Generally, when I controlled income, M3 fits inflation well. In other words, QTM still holds in Norway. Additionally, QTM holds for most of the European countries, if one stop the data at 1998. And this is the reason why this article tries to separately label pre-1999 era out.
3.3.13 Sweden

Sweden starts to adopt fiat standard from 1931. I divide the sample of Sweden into three pieces and filter them separately pre-1938 (pre-WWII), 1939-1950 (WWII around), post-WWII. I further label the whole sample by different colors: 1872-1899 (pre-1900), 1900-1938 (pre-WWII), 1939-1950 (WWII around), 1951-1980 (Lucas period), 1981-1998 (Pre-Euro), and 1999-2006. To reconfirm that weak correlation is not a result of the commodity standard, we further label years before 1900 by using different colors with other years during 1900-1938. Since Sweden keeps neutral in two World Wars, I filter the sample together.

During 1872-1899, there is no significant correlation between inflation and money growth, $\beta = 0.07(0.88), R^2 = 0.03(0.69)$ for M0(M3). Then M0 starts to positively correlated with inflation from 1900, but less close than M3, $\beta = 0.67(1.06), R^2 = 0.61(0.87)$ for M0(M3) during 1900-1938. During this period, M3 performs well to explain inflation. During 1961-1980, both of M0 and M3 perform well, with income adjustment, $\beta = 0.86(1.17), R^2 = 0.86(0.87)$ for

Figure 3.13: Filtered Growth rate of Money stock against Inflation rate, Sweden
0.92(0.98) for M0(M3).

WWII is notorious for invalidity of QTM, as we have already witnessed in other countries. As one can tell from Figure-3.13, inflation keeps weakly co-move with money growth during 1939-1950, $\beta = 0.085(0.19), R^2 = 0.63(0.56)$ for M0(M3). Additionally, QTM breaks down for Sweden at least since 1999.

3.4 Conclusion

In this paper, we are inspired by recent silent quantity theorem of money(QTM), especially in U.S. I start out to check whether QTM is a robust relationship governing the co-movement between money growth and inflation. I found evidence of recent breaking down of QTM in U.S, confirming the finding of Sargent and Surico (2011) and Christiano and Fitzgerald (2003). Moreover, we try to explore whether QTM is a robust pattern across the history of U.S. The answer turns to be negative for $M_1$ but positive for $M_2$. In another word, QTM is not a tight law across U.S history if one restricts itself to a pre-determined definition of money. It implies that it is preferable to conclude which version of QTM breaks down and more importantly which version of QTM still lives, echoing the message from Lucas and Nicolini (2015). I then use long historical international data to confirm our finding in U.S. By exploiting our international historical data; we found QTM is not robust across countries either. There are countries where QTM exits and continues to exist. There are countries where QTM exists for certain period. Furthermore, there are countries where QTM rarely exists. I thus conclude that each country deserves a closer investigation to their own monetary history to understand the recent universal collapse of QTM.
Bibliography


