Essays on Economic Growth

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by
Minhyeon Jeong

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Minhyeon Jeong
in the Olin Library, May 2018
To my parents, Jaegeuk Jeong and Myoungsoon Kim, my family and my true love, Jeongwon Yoon
My dissertation investigates how economic growth is determined in the long run. To do this, I mainly focus on culture and institutions as fundamental growth factors and develop a general theoretical framework in which culture, institutions and growth are endogenously determined. Using the framework, I highlight the crucial role of the interaction between culture and institutions in the long-term growth.

Chapter 1. Endogenous Financial Friction and Growth

The first chapter investigates the role financial frictions play in economic growth. Most existing studies consider an exogenous form of borrowing constraint, the tightness of which is arbitrarily fixed at a certain value over time. As a result, a sufficiently tight constraint has, if any, a permanent growth effect. The main concern of the chapter is that tightness can vary by economic circumstance, and hence, it will change in different stages of economic development. If borrowers’ profits increase along with economic growth, the tightness will be relaxed, leading to an endogenous relaxation of financial frictions. I develop a theory that has this property as a novel feature; I then provide empirical evidence by applying OLS and dynamic panel GMM regressions to support the endogenous relaxation hypothesis. The endogenous relaxation implies that the growth effects of financial frictions are, at best, temporary; therefore
financial factors may not be fundamental determinants that underlie the long-term economic growth.

Chapter 2. Culture, Institutions and Growth

The second chapter deals with culture and institutions as fundamental engines of growth. To this end, I develop a theoretical framework in which culture (individualism vs. collectivism), institutions (protection of property rights over innovations), and growth are endogenously and jointly determined. I first summarize and corroborate empirical facts about culture, institutions, and growth to make the model consistent with the empirical findings. The primary mechanism that drives the main conclusions of the chapter is that culture and institutions are strategic complements. I show that there are two steady states with opposing properties. One is a good steady state characterized by strong individualism and secure protection of property rights with high long-run growth. The other one is associated with strong collectivism and poor protection of property rights. The strategic interaction between culture and institutions leads to not only multiple static equilibria (multiple steady states) but also to multiple dynamic equilibria with contrasting properties. The second chapter demonstrates the significance of institutions for the economic prosperity of society, especially over the long run. This conclusion provides motivation for investigating other roles of institutions, which is the focus of the last chapter.

Chapter 3. Rent-Seeking, Institutions and Morality

While the second chapter deals with public predation by the government, the third chapter pays attention to rent-seeking as private predation, which has long been an impediment to economic growth. It starts with this question: If rent-seeking is indeed bad for an economy, why does it still prevail in many countries? That is, why did those countries not eliminate rent-seeking activities long ago? This question leads to a consideration of the factors underlying rent-seeking behavior. In this
context, I develop a model where rent-seeking activities result in long-lasting poverty through culture (morality of society) and institutions (effectiveness of enforcement of laws), both of which are endogenously determined. I show that society with healthier morality tends to establish more effective enforcement of laws that diminishes rent-seeking, achieving higher income and faster growth.
Chapter 1

Endogenous Financial Friction and Growth

1.1 Introduction

Financial friction is one of central topics in economics. Accordingly, many researchers have examined the role financial frictions play, especially, on resource misallocations caused by imperfect risk sharing and forgone profit opportunities. Not surprisingly, a large volume of literature has addressed various growth effects of financial frictions.

However, most of the previous studies have adopted an ad-hoc borrowing constraint without any theoretical and/or empirical justification as follows:

\[ b_t \leq \lambda a_t \]

where \( a \) is asset, \( b \) is debt, and \( \lambda \) is the reciprocal of the tightness of the borrowing constraint, which is arbitrarily given.\(^1\)

This *exogenous* financial friction ensures analytical simplicity. Therefore, the previous studies have revealed valuable insights on impacts of financial frictions at

various angles. However, the exogenous borrowing constraint does not capture some important implications arising from financial frictions in reality. For instance, the loan-to-value (LTV) ratio for a borrowing-constrained individual is exogenously given and arbitrarily fixed at $\lambda$. This implies that the LTV ratio is irrelevant to any economic situations including the stage of development, which seems implausible. Also, since the tightness of the credit constraint $1/\lambda$ determines growth effects, if the tightness is fixed and the constraint is assumed to be binding, a growth effect of the exogenous financial friction persists forever. Consequently, one needs to endogenize the tightness of financial frictions to assess its role in growth, especially, over the very long run.

Based on this motivation, we consider the financial friction, the tightness of which varies endogenously. In our framework, the tightness is implied by economic circumstances rather than exogenously given. That is, we view the tightness as an equilibrium value determined by other variables, which are more fundamental. To do that, this chapter presents a simple endogenous growth model with the financial friction led by asymmetric information and costly state verification as in Townsend (1979), Williamson (1986, 1987), Bernanke and Gertler (1989).

In our model, the economy grows through entrepreneurs’ innovations that enhance productivity. Since the productivity-enhancing investment — R&D investment — is risky, entrepreneurs’ ability to borrow for R&D investment is limited under the information asymmetry. More specifically, in the model, an entrepreneur borrows money to invest in R&D. However, even when the investment is successful, he may avoid paying his debt by falsely reporting that he failed in the R&D: the moral hazard problem. When an entrepreneur reneges on his debt, the lender confiscates the borrower’s net worth after auditing, which incurs monitoring costs. For the incentive constraint compatible, any entrepreneur borrowing for R&D investment should pay a higher interest rate than the market deposit rate. This wedge discourages R&D

---

2 This is common in the recent endogenous growth literature, e.g., Aghion and Howitt (1992), Morales (2003), Aghion, Howitt and Mayer-Foulkes (2005), Berentsen, Rojas Breu and Shi (2012), Doepke and Zilibotti (2013) among others.
investments, resulting in lower growth. We call this a direct growth effect.\textsuperscript{3}

Notably, this negative growth effect disappears as the economy grows in our model. This is because the credit market becomes asymptotically efficient although an entrepreneur borrowing for R\&D investment is assumed to be credit constrained.\textsuperscript{4} This is because the incentive constraint is endogenously relaxed as the borrower’s profit (entrepreneurial profit) increases along with growth of the economy.\textsuperscript{5} This implies that the negative growth effect emerges during the transition phase only, and thus, the growth effect is temporary. We call this asymptotic irrelevance to growth.

This finding contradicts the permanent growth effect of financial frictions. If financial frictions adversely affect growth, then this negative impact is permanent in the previous literature adopting the exogenous borrowing constraint as long as it is assumed to be binding. This is simply because the tightness of the financial friction is arbitrarily given and fixed at a certain level. In contrast, the tightness is endogenously determined in our framework, and thereby, the negative growth effect disappears gradually as the economy grows.\textsuperscript{6}

Hence, the growth effect of the financial friction is heterogeneous across countries

---

\textsuperscript{3} Similar growth effects of financial frictions, led by the inefficient resource allocation, can be found in, for example, de Gregorio (1996), Berentsen, Rojas Breu and Shi (2012), Bencivenga and Smith (1991) Aghion, Howitt and Mayer-Foulkes (2005), Trew (2014) and Laeven, Levine and Michalopoulos (2015) among others. In line with this theoretical result, there is extensive empirical evidence that consistently shows negative growth effects of financial frictions. Some prominent examples include King and Levine (1993) and Levine, Loayza and Beck (2000).

\textsuperscript{4} Recently, Azariadis, Kaas and Wen (2016) develop a parsimonious model that features endogenous credit friction led by limited commitment. In the model, the credit market operates efficiently provided that the reputation value is large enough for a borrower to honor his debt voluntarily.

\textsuperscript{5} This intuition seems plausible from the observation that the average firm size correlates positively with income per capita (the average income of a country), e.g., Poschke (2014). Since a firm’s profit is increasing, on average, in its size, the average firm will have higher profits in a richer country. Similarly, Donovan (2014) empirically shows that there are more entrepreneurs who are “poor” (subsistence entrepreneurs) in a poorer country.

\textsuperscript{6} This endogenous relaxation is equivalent as the model-implied tightness decreases, converging asymptotically to the steady state value that guarantees the first-best optimum. Looked at another way, we theoretically show that the interest rate spread, which proxies the severity of the financial friction, decreases along with growth due to the endogenous relaxation of the financial friction. Galor and Zeira (1993) also derive the spread of borrowing and lending rates led by the borrower’s moral hazard problem and connect the interest rate spread to the degree of the financial friction severity. However, the interest rate spread in their model is essentially exogenous since it is independent of the state of the economy, and therefore, the negative effect of the financial friction persists forever in their framework.
due to the asymptotic irrelevance, and this feature is not captured in most of the previous studies dealing with the exogenous financial friction. However, we present evidence supporting the asymptotic irrelevance of financial frictions to growth by showing that the negative effect of the financial friction gets alleviated as the average income of a country increases.

Our model also predicts that the financial friction interplays with aggregate uncertainty through the endogenous determination of the tightness. More precisely, we show that the financial friction amplifies the negative impact of volatility on growth. We refer to this as an indirect growth effect, which also asymptotically disappears due to the endogenous relaxation of the credit constraint. When aggregate volatility increases, the expected profit from R&D investment gets smaller. Then, the credit constraint becomes tighter, leading to a further decrease in R&D investment. This sheds light on the role financial frictions play in the negative growth effect of aggregate uncertainty. As demonstrated by many empirical studies, countries with higher volatility have lower long-term growth, e.g., Ramey and Ramey (1995). We provide a theory that reveals how financial frictions amplify the negative growth impact of aggregate uncertainty by allowing the credit constraint tightness to vary endogenously.

Thirdly, regarding transition dynamics, we will show that the endogenous relaxation of the financial friction replicates various stylized transitional features including the increasing trend of the investment rate in the development process, while they are not always replicated in the exogenous credit constraint setting.

Finally, we can easily understand the main mechanism that causes the direct and indirect growth effects thanks to the parsimony of the model. The degree of the financial friction severity is measured by the interest rate spread, which is easy-to-access. Hence, we can easily test three empirical hypotheses from the theory: the direct, indirect growth effects and the asymptotic irrelevance. We conduct not only cross-country OLS regressions, but also the dynamic panel GMM ones to address the shortcomings of the OLS regressions. The estimation results support all the theoretical results.
**Empirical Evidence on Credit Constraints over R&D Investment**

In the finance literature, information friction has been regarded as a primary culprit that restricts borrowing of firms, e.g., Myers (1977), Stiglitz and Weiss (1981) and Myers and Majluf (1984). Our model reflects this intuition. That is, information asymmetry limits a firm’s borrowing for R&D investment in the model. This intuition seems plausible since many empirical studies suggest that R&D investment is subject to financial restrictions. For example, Carpenter and Petersen (2002) find that small high-tech companies in the U.S. obtain little debt financing. Similarly, from a large sample of European firms, Brown, Martinsson and Petersen (2012) offer evidence that credit constraints adversely affect R&D activities. Brown, Fazzari and Petersen (2009) also show that the U.S. recently experienced a finance-driven cycle in R&D from 1994 to 2004. The evidence in the U.S. and Europe suggests that R&D investment indeed depends largely on financial factors, especially in poor countries where financial markets are primitive.

**Related Literature**

This research is closely related to a few papers adopting the credit constraint with costly state verification. Among others, Azariadis and Smith (1996), Huybens and Smith (1997, 1999) and Aghion, Banerjee and Piketty (1999) consider the overlapping generations model with the same type of credit constraint to investigate various implications in the determinacy of equilibrium, endogenous volatility or the rate of money growth. Recently, Greenwood, Sánchez and Wang (2010) demonstrate how technological progress in financial intermediation affects the economy by using the model where a costly state verification framework is embedded. In contrast, our research offers a more simple and tractable model that allows endogenous growth, which is depending on the severity of the financial friction. That is, our model deals with *growth* effects of the financial friction, not *scale* effects that the aforementioned papers mainly focus on.\(^7\)

\(^7\) Notably, Khan (2001) investigates growth effects of financial development with the costly state
On the indirect growth effect, this chapter is related to previous literature that investigates the relationship between volatility and growth, e.g., Aghion, Angeletos, Banerjee and Manova (2010) (AABM) and the references therein. In the AABM model, the financial friction interacts with aggregate volatility through the substitution between the short- and long-term investments although the tightness of the borrowing constraint is exogenously given. Our model differs in that the financial friction is endogenized. That is, we show that financial frictions interplay with volatility through the endogenous determination of the tightness, not through the cyclical composition of investments as in AABM.

Regarding the R&D investment as a prime engine for economic growth under financial frictions, this chapter is closely linked to Aghion, Howitt and Mayer-Foulkes (2005) (AHM). They investigate growth effects of financial developments during convergence phase in a parsimonious Schumpeterian model. They show that the financial friction has the permanent negative growth effect provided that the financial condition gets worse than a certain threshold.

The rest of the chapter is organized as follows. Section 1.2 introduces the model and describes the equilibrium. Section 1.3 derives the main theoretical results. Section 1.4 presents empirical evidence supporting the theoretical results. Section 1.5 concludes with some remarks.

Verification as in our model. He shows that a reduction in the financial intermediation cost leads to a decline in the spread between borrowing and lending rates, resulting in faster growth. We consider the same type of financial friction, while uncovering other roles of the financial friction on growth, namely the indirect growth effect and asymptotic irrelevance. We also provide empirical evidence supporting the results.

8 Similarly, Morales (2003) also presents a Schumpeterian growth model in which growth is driven by entrepreneurial innovations, which are subject to the exogenous credit constraint. He shows that financial intermediation can mitigate the moral hazard in the R&D sector, raging the growth rate.

9 Berentsen, Rojas Breu and Shi (2012) develop a monetary model where the economy endogenously grows through R&D innovations. Instead of focusing on the growth effects of financial frictions, they investigate the negative growth effect of inflation through its impact on R&D activities under the search friction. Bencivenga and Smith (1991) develop an endogenous growth model with the Diamond-Dybvig type banking contract to show that financial intermediaries can boost growth when they encourage productive investments, the profits of which are realized over a long period of time, by pooling "liquidity risks."
1.2 The Model

We consider a small open economy that consists of two period-overlapping generations. Obviously, the small open economy assumption ensures analytical simplicity since the interest rate $r_t$ is exogenously given. However, it has more important implication in our framework that will be discussed with details later.\footnote{We also look at the case of a closed economy briefly in Appendix A.1.}

In this economy, there are two types of heterogeneous agents: entrepreneurs of unit mass and workers of mass $L_t$. Here, we interpret $L_t$ broadly such that it includes human capital, i.e., $L_t$ is the effective labor supplied by workers. For simplicity, $L_t$ is exogenously given in the model.

1.2.1 Entrepreneur’s Problem

In each period $t$, entrepreneurs are born with a skill/ability $z \in [z_L, z_H]$ drawn from a time-invariant continuous c.d.f. $F(x)$. We assume that $f(x) > 0 \forall x \in [z_L, z_H]$ where $f(x)$ is a p.d.f..\footnote{This implies that $F(x)$ is strictly increasing in $z$.} In the following period $t+1$, each entrepreneur is endowed one unit of labor and can produce through his own productivity augmented to the social common Cobb-Douglas production technology $f(k, \ell) = (k^\alpha \ell^{1-\alpha})^\nu$ that is homogeneous of degree $\nu < 1$. More specifically, an entrepreneur $i$ augments his own productivity $A_i$ such that $y_i = A_i^{1-\nu}f(k_i, \ell_i)$ where $y_i$ denotes individual output. $\nu$ is the span of control parameter of an entrepreneur as in Lucas (1978).

There are two types of investments: R&D and capital investments. R&D investment is risky, long-term and productivity-enhancing investment, which requires borrowings. Hence, in presence of asymmetric information, this investment comes with a credit constraint as shown later. Meanwhile, capital investment is risk-free and short-term investment. Such investments are free of credit restrictions.\footnote{These contrasting features are similar to those of AABM.}

R&D investment takes one period to be completed. This means that, in the first period after elder entrepreneurs produce, a young entrepreneur should determine
whether or not invest in R&D in order to enhance his own productivity level $A_i$ after drawing his individual ability $z_i$. Investing in R&D requires $x(z, A)$ amount of consumption goods. We assume that $x(z, A_t) \geq 0$, $x_z(z, A_t) < 0$, $\lim_{z \to z_L} x(z, A_t) = \infty$, $\lim_{z \to z_H} x(z, A_t) = 0$ and $x_A(z, A_t) > 0$ where $A_t > 0$ denotes the total factor productivity (TFP) in period $t$. The last assumption reflects that it is more costly to conduct R&D innovations as the economy develops further. We denote $x_t(z) \equiv x(z, A_t)$ for convenience.

R&D investment is risky. Once investment has begun, he succeeds in the R&D project with probability $p \in (0, 1)$ and obtains $\gamma A_t$ as individual productivity where $\gamma > 1$. For simplicity, we assume that the entrepreneur obtains zero productivity if he failed in his R&D project, i.e., $A_i = 0$, so that produces and consumes nothing in the following period $t + 1$ (he goes underground). Meanwhile, an entrepreneur who does not invest has the same individual productivity with the TFP given by $A_t$.

Finally, entrepreneurs are risk-neutral, and they consume at the end of second period. Then, an entrepreneur with $z$ solves the following problem:

$$
\pi_t(z) \equiv \pi(z, A_t) \equiv \max \{p [\pi_{1, t+1} - (1 + i_{t+1}) x_t(z)], \pi_{0, t+1}\}
$$

s.t. $\pi_{1, t+1} \equiv \max_{\{k_{1,t+1} \geq 0, \ell_{1,t+1} \geq 0\}} (\gamma A_t)^{1-v} f(k_{1,t+1}, \ell_{1,t+1}) - r_{t+1}k_{1,t+1} - w_{t+1}\ell_{1,t+1}$,

$$
\pi_{0, t+1} \equiv \max_{\{k_{0,t+1} \geq 0, \ell_{0,t+1} \geq 0\}} A_t^{1-v} f(k_{0,t+1}, \ell_{0,t+1}) - r_{t+1}k_{0,t+1} - w_{t+1}\ell_{0,t+1}.
$$

\[13\] Jones (2008) shows with empirical evidence that successive generations of innovators have a heavier educational burden as technology develops further. Similarly, the last condition reflects that it is harder to imitate or adopt more advanced technologies from other countries as the economy develops since the TFP, $A_t$, increases over time through successful R&D projects. Technically, this assumption is necessary for stationary growth and, in fact, widely adopted in many other studies. For instance, the R&D cost function is assumed to be linear in $A_t$ for stationarity in AHM and Acemoglu, Aghion and Zilibotti (2006). As shown later, the linearity is a sufficient condition for stationary growth in our model; see Proposition 1.3.

\[14\] One can introduce project varieties such that $G = \{\gamma_1, \gamma_1, \cdots, \gamma_N\}$ where $\gamma_i > \gamma_j$, $i > j$, and $p_i = \Pr(\text{ith project is done successfully}) < 1$. Here, for simplicity, we consider the simplest case, $N = 1$, since there is no significant change in the qualitative results.

\[15\] This assumption can be easily relaxed such that the entrepreneur can still produce some amount of consumption goods. However, there is no change in the results qualitatively.
We denote the gross borrowing rate by $1 + i$. The subscript 1 indicates that the producer is a (successful) R&D investor while the subscript 0 is for the one who did not invest. The gross borrowing rate $1 + i_{t+1}$ is required to be equal or larger than the gross return to capital $1 + r_{t+1} - \delta > 0$; if not, all entrepreneurs can make profits arbitrarily by borrowing consumption goods at $1 + i_{t+1}$ from a lender and lending them at $1 + r_{t+1}$ after depreciation by $\delta$. That is, the net borrowing rate $i$ has the lower bound at $r - \delta$ in any equilibrium to preclude the arbitrage opportunity. Then, entrepreneur $i$’s borrowing is fixed at $x(z_i, A)$, and therefore, only inputs are choice variables.

Also, the above problem indicates that the borrowing rate $i$ is independent of the skill $z$, and every successful borrower honors his gross debt $(1 + i) x(z)$ to the lender. These features will be verified by deriving the optimal financial contract presented later. Now, from the assumptions on $f(k, \ell)$ and $x(z, A)$, we have the following result.\(^{16}\)

**Lemma 1.1 Unique Existence of Skill Threshold:** Given any price vector, $(r, w, i) \in \mathbb{R}^3_{++}$, there exists a unique skill threshold $z^* \in (z_L, z_H)$ if, and only if, $p \gamma > 1$ such that an entrepreneur with $z \geq z^*$ invests in R&D and does not invest otherwise.

From Lemma 1.1, we assume the following condition for the unique skill threshold:

**Assumption 1** $p \gamma > 1$.

Note that there always exist entrepreneurs investing R&D in any equilibrium since $z^*$ is an interior solution, i.e., $z^* \in (z_L, z_H)$, and $f(z) > 0 \ \forall z \in (z_L, z_H)$ where $f(\cdot)$ is the p.d.f. of the skill $z$.

**Financial Contract**

\(^{16}\) All proofs are relegated in Appendix A.4.
There are financial intermediaries (FI, hereafter) that lend consumption goods to entrepreneurs. Whether there is success or failure in a R&D project is assumed to be private information. Hence, when a borrower defaults, so that he does not honor his debts, FI randomly monitor (or audit) with probability $\eta$ where each monitoring incurs $\mu > 0$ amount of consumption goods as the monitoring cost.

Note that the monitoring cost is time-invariant. We will discuss the case of time-varying monitoring costs in Appendix A.1. However, we show that all the theoretical results do not change with the time-varying monitoring cost as long as it grows slower than the borrower’s net worth $\pi_1$, which is the entrepreneurial profit.\footnote{See Proposition A.1 in Appendix A.1. It is worth noting that although the tightness of the credit constraint is endogenously determined in our model, the monitoring cost varies exogenously over time. Meanwhile, \textit{Laeven, Levine and Michalopoulos (2015)} explore impacts of endogenous innovations in the financial intermediation technology on long-term growth. In their model, given the exogenous cost of innovations in the financial intermediation technology, a financial intermediary tries to improve its intermediation technology to reduce the intermediation cost, which is analogous to the monitoring cost in our model. They show that growth is not sustainable in the long run if the innovation cost, which is exogenously given, increases to infinity, and thereby, there is no innovation in the financial intermediation. This result is similar to the one that we obtain when the monitoring cost grows faster than the entire economy, diverging to infinity; see Case (2-iii) of Proposition A.1. Hence, their conclusion can be obtained as a specific case in our model, implying that there is no loss of generality in our reduced-form-style approach that lets the monitoring cost be given exogenously over time.} In Appendix A.1, we will argue that this is indeed the realistic assumption by showing empirical evidence consistent with the assumption.\footnote{To do that, we use panel data from \textit{Demirgüç-Kunt, Laeven and Levine (2004)}. The data provides abundant information on financial conditions of more than 1,400 banks from 72 developed and developing countries over the period of 1995–1999.} Hence, let us assume that the monitoring cost is time-invariant, which is a particular case of the assumption, for a clearer and simpler exposition of the main idea without any qualitative change in the theoretical results.\footnote{Finally, one can interpret the monitoring cost as the material cost determined by legal and accounting systems in a country. Obviously, the monitoring cost gets smaller as the institutional systems are efficient. According to \textit{Levine, Loayza and Beck (2000)}, the cross-country differences in legal and accounting systems can explain differences in financial development. They show that exogenous components of the development in the financial intermediation correlate positively with economic growth, a finding that is consistent with our theoretical and empirical conclusions.}

Once the audit reveals that a borrower defaulted even though he succeeded in his R&D, and hence, could repay debt, the lender (financial intermediary) confiscates
the borrower’s entire wealth. This environment reflects the financial friction arising from information asymmetry between a lender and a borrower that prevents efficient resource allocations, and hence, leads to welfare loss as we will see clearly later. Under the environment, the terms of contract are derived by solving the following problem in period $t$:\footnote{Recall that the amount of borrowing is fixed at $x(z)$ for an entrepreneur with the ability $z$ since the no-arbitrage condition given by $i \geq 1 + r - \delta$. Also, in our framework, every entrepreneur conducting R&D is borrowing-constrained simply because an entrepreneur is born without asset. In the same spirit, Greenwood, Sánchez and Wang (2010) also assume that any producer producing consumption goods must borrow capital goods from a financial intermediary, and hence, any active producer is borrowing-constrained by construction. Of course, the borrowing constraint can be binding for other reasons; for example, impatience (Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997) or linear gains in the capital investment (Buera and Shin, 2011, 2013; Moll, 2014).}

$$\max_{\{i_{t+1}(z)\geq r_{t+1}-\delta, \eta_{t+1}(z)\geq 0\}} p \left\{ \pi_{1,t+1} - (1 + i_{t+1}(z)) x_t(z) \right\}$$

s.t. \[ p (1 + i_{t+1}(z)) x_t(z) - (1 - p) \eta_{t+1}(z) \mu \geq (1 + r_{t+1} - \delta) x_t(z) \] \[ \pi_{1,t+1} - (1 + i_{t+1}(z)) x_t(z) \geq (1 - \eta_{t+1}(z)) \pi_{1,t+1} \] \( \text{PC} \) \( \text{IC} \) \( \text{PC} \) \( \text{IC} \)

Where $PC$ is the participation compatibility, and $IC$ is the incentive constraint. Prior to characterizing the terms of contract $\{i(z), \eta(z)\}$, we assume that the following restriction is satisfied in any equilibrium, which seems natural and plausible.

**Assumption 2** $p \pi_{1,t} > (1 - p) \mu \forall t \geq 0$.

As demonstrated in the proof of Lemma 1.2 in Appendix A.4, Assumption 2 is a necessary and sufficient condition for guaranteeing the no-arbitrage condition $1 + i_t \geq 1 + r_t - \delta$ in equilibrium. This assumption is easily satisfied when $\gamma$ is sufficiently large.\footnote{More specifically, suppose that the exogenous interest rate $r_t$ is given in the interval $[-\infty, \bar{r}]$ $\forall t \geq 0$. From (10) and (12) presented later, it can be easily shown that:}

$$\min \left\{ \pi_{1,t} \right\}_{t=0}^{\infty} = \gamma (1 - v) (\alpha v)^{\frac{\alpha v}{1-\alpha v}} L_0^{\frac{1}{1-\alpha v}} A_0^{\frac{1-v}{1-\alpha v}} \psi(z_L)^{-\frac{v(1-\alpha)}{1-\alpha v}} \bar{r}^{-\frac{\alpha v}{1-\alpha v}}$$

where $A_0$ and $L_0$ are the initial productivity and effective labor, respectively, which are given in the initial period $t = 0$. Also, as will be presented later, $\psi(z_L) \equiv p \gamma \{1 - F(z_L)\} + F(z_L) = p \gamma$ since
Lemma 1.2 Financial Contract: Given any \((r, \pi_1, x(z)) \in \mathbb{R}_+^2 \times \mathbb{R}_+\), the terms of contract \((i(z), \eta(z))\) are given by:

\[
1 + i_{t+1} = \frac{1 + r_{t+1} - \delta}{p} \frac{p \pi_{1,t+1}}{p \pi_{1,t+1} - (1 - p) \mu} = \frac{1 + r_{t+1} - \delta}{p} \Delta_{t+1}, \tag{1.1}
\]

\[
\eta_{t+1}(z) = \frac{(1 + r_{t+1} - \delta) x_t(z)}{p \pi_{1,t+1} - (1 - p) \mu}. \tag{1.2}
\]

From equation (1.1), \(\Delta_{t+1} > 1\) if, and only if, \(\mu > 0\). Note that \(\Delta_{t+1} > 1\) is the mark-up upon the fair insurance rate \((1 + r_{t+1} - \delta)/p\). This is a wedge caused by the information asymmetry, and the wedge precludes the efficient allocation as seen clearly later. For this reason, the financial friction gets more severe as it increases. For example, if \(\mu = 0\), i.e., monitoring is free, then \(\Delta\) is always unity, implying the asymmetric information is “ineffective” in the sense that it is irrelevant to the real allocation of the economy. Formally, we define the interest rate spread as follows:

Definition 1.1 Interest Rate Spread: \(\Delta_t \equiv p (1 + i_t) / (1 + r_t - \delta)\), or like the common definition in reality, define \(\tilde{\Delta}_t \equiv \log \Delta_t - \log \rho \simeq i_t - r_t\).

Finally, one can show that the participation compatibility always binds; see the proof of Lemma 1.2. Then, the law of large numbers leads to the following condition that should be satisfied in any equilibrium:

\[
p (1 + i_{t+1}) \int_{z_L}^{z_H} x_t(z) dF(z) - (1 - p) \mu \int_{z_L}^{z_H} \eta_{t+1}(z) dF(z) \equiv (1 + r_{t+1} - \delta) \int_{z_L}^{z_H} x_t(z) dF(z) \tag{1.3}
\]

\[\overbrace{E_t[\text{revenue from lending}]}^{F(z_L) = 0}. \]

Then, Assumption 2 is satisfied if the following condition holds:

\[
\gamma > \left( \frac{\rho}{\alpha u} \right)^{\frac{1 - \alpha}{1 - \nu}} \left[ \frac{(1 - p) \mu}{1 - \nu} \right]^\frac{1 - \alpha}{1 - \nu} L_0^{-\frac{1 - \alpha}{1 - \nu}} (p A_0)^{-1}.
\]
Equation (1.3) guarantees no-arbitrage for FI, and hence, can be interpreted as the financial market clearing condition.

### 1.2.2 Worker’s Problem

In each period \( t \), each young worker, born in the same period, solves the following problem:

\[
\begin{align*}
\max_{\{\hat{c}_t^y, \hat{c}_{t+1}^o \geq 0\}} & \quad \beta u(\hat{c}_t^y) + (1 - \beta) u(\hat{c}_{t+1}^o) \\
\text{s.t.} & \quad \hat{c}_t^y + \frac{\hat{c}_{t+1}^o}{1 + r_{t+1} - \delta} = \hat{w}_t \equiv w_t \frac{L_t}{N_t} \quad \cdots \quad (BC) .
\end{align*}
\]

where BC is the budget constraint, and \( \hat{c}_t \) and \( \hat{w}_t \) denote consumption and wage per worker, respectively. \( N_t \) is the population of workers. Then, the total population of the economy is given by \( \hat{N}_t = 2 + N_{t-1} + N_t \). We denote income per capita by \( \hat{y}_t \), which is given by \( \hat{y}_t = Y_t / \hat{N}_t \) where \( Y_t \) is aggregate output. We assume that the growth rate of \( N_t \) is exogenously given, so that the growth rate of income per capita is increasing in the growth rate of \( Y_t \).

Under the strict concavity of the utility function \( u(\cdot) \), we have a unique solution of optimal \((\hat{c}_t^y, \hat{s}_t)\) where \( \hat{s}_t \) is a worker’s saving in period \( t \). Then, the domestic saving in period \( t \), say \( S_t \), is defined by \( S_t = \hat{s}_t N_t \). Now, we denote saving per efficiency unit of labor by \( s_t \) such that \( S_t \equiv s_t L_t \), implying \( s_t = \hat{s}_t N_t / L_t \). Similarly, we denote the consumption per efficiency unit of labor \( \hat{c}^i \) (\( i = y, o \)) such that \( \hat{c}_t^y = \hat{c}_t^y N_t / L_t \) and \( \hat{c}_{t+1}^o = \hat{c}_{t+1}^o N_t / L_t \).
1.2.3 Equilibrium

In each period $t$, all quantity variables and the skill threshold solve each agent’s problem given the prices that clear every market in the economy as follows:

\[ L_t = \int \ell_t (z) \, dF (z) \]  
\[ s_{t-1} L_{t-1} + B_t^f = \int k_t (z) \, dF (z) + \int_{z_{t-1}^H}^{z_t^H} x_{t-1} (z) \, dF (z) \]  
\[ \equiv K_t \]  
\[ \equiv X_{t-1} \]  

\[ p (1 + i_t) \int_{z_{t-1}^H}^{z_t^H} x_{t-1} (z) \, dF (z) - (1 - p) \mu \int_{z_{t-1}^H}^{z_t^H} \eta_t (z) \, dF (z) \]
\[ = (1 + r_t - \delta) \int_{z_{t-1}^H}^{z_t^H} x_{t-1} (z) \, dF (z) \]  
\[ \equiv Y_t \]  

\[ c_t^o L_{t-1} + c_t^y L_t + K_{t+1} + C_t^e + (1 - p) \mu \int_{z_{t-1}^H}^{z_t^H} \eta_t (z) \, dF (z) \]
\[ = \int y_t (z) \, dF (z) + (1 - \delta) K_t - (1 + r_t - \delta) B_t^f + B_{t+1}^f \]  
\[ \equiv Y_t \]  

where aggregate consumption of entrepreneurs $C_t^e$ is given by:

\[ C_t^e = \int_{z_{t-1}^H}^{z_t^H} p \{ \pi_{1,t} - (1 + i_t) x_{t-1} (z) \} \, dF (z) + \int_{z_L}^{z_{t-1}^L} \pi_{0,t} dF (z) \]
\[ = [p \gamma (1 - F (z_{t-1}^*) - F (z_t^*))] \pi_{0,t} - p (1 - i_t) X_{t-1} \]

Equations (1.4) and (1.5) are the labor and capital market clearing conditions, respectively, where $B_t^f$ is net capital flow from foreign countries in period $t$. Equations (1.6) and (1.7) are the financial and goods market clearing conditions, respectively. One can show that once equations (1.4)–(1.6) and (BC) are satisfied, the last goods market clearing condition, equation (1.7), is automatically satisfied: Walras’s law.
1.3 Main Results

Now, we can describe how the aggregate output and TFP are determined as follows:

**Proposition 1.1 Aggregation:** The aggregate output is given by:

\[ Y_t = A_t^{1-v} (K_t^\alpha L_t^{1-\alpha})^v \]

where the TFP, \( A_t \), is given by:

\[ A_t = [p\gamma \{1 - F(z^*_{t-1})\} + F(z^*_{t-1})] A_{t-1} = \int A_i(z) dF(z). \]

Also, \( w_t = v(1 - \alpha) \frac{Y_t}{L_t} \) and \( r_t = \frac{\alpha Y_t}{K_t}. \)

Intuitively, the TFP level in time \( t \), \( A_t \), is determined by the mass of entrepreneurs who succeeded in R&D innovations. This is given by the function \( \psi(z^*) \equiv p\gamma \{1 - F(z^*)\} + F(z^*). \) Here, to avoid the scale issues in the empirical analysis in Section 1.4, we redefine the TFP such that \( \tilde{A}_t \equiv A_t^{1-v} \), which, in turn, yields \( Y_t = \tilde{A}_t (K_t^\alpha L_t^{1-\alpha})^v. \)

From Proposition 1.1, defining capital and output per efficiency unit of labor by \( k_t \equiv K_t/L_t \) and \( y_t \equiv Y_t/L_t \) yields:

\[ y_t = (A_{t-1}/L_t)^{1-v} \psi(z^*_{t-1})^{1-v} k_t^\alpha \]  
(1.8)

\[ \pi_{0,t} = (1 - v) (A_{t-1}/L_t)^{1-v} \psi(z^*_{t-1})^{-v} L_t k_t^\alpha \]  
(1.9)

\[ \pi_{1,t} = \gamma(1 - v) (A_{t-1}/L_t)^{1-v} \psi(z^*_{t-1})^{-v} L_t k_t^\alpha \]  
(1.10)

\[ w_t = (1 - \alpha) v (A_{t-1}/L_t)^{1-v} \psi(z^*_{t-1})^{1-v} k_t^\alpha \]  
(1.11)

\[ r_t = \alpha v (A_{t-1}/L_t)^{1-v} \psi(z^*_{t-1})^{1-v} k_t^\alpha - 1 \]  
(1.12)

where, by the linearity of the model, \( \mathbb{E}[\pi_t] = \int \pi_t(z) dF(z) = (1 - v)Y_{t+1}. \)

---

22 Proposition 1.1 also holds in the closed economy. That is, Proposition 1.1 is robust to the assumption of the small open economy.

23 Note also that \( l_{0,t} = \psi(z^*_{t-1})^{-1} L_t \) and \( k_{0,t} = \psi(z^*_{t-1})^{-1} K_t \) where \( \ell_{1,t} = \gamma \ell_{0,t} \) and \( k_{1,t} = \gamma k_{0,t} \).
1.3.1 No-Growth Case

To identify the model mechanism that drive the growth effects of the financial friction more clearly, we first consider the case of no-growth that uncovers the scale effect, and then turn to the case of long-term growth that reveals the growth effect. For the no-growth case, we assume that newly developed knowledge from R&D innovations disappears at the end of each period after production is completed, i.e., the productivity enhancement is perfectly excludable over generations. Then, an entrepreneur who does not conduct R&D has a “fundamental” TFP denoted by $A_0$, which is given exogenously in the initial period $t = 0$. Also, the efficiency unit of labor is not changed over time, and hence, fixed at its initial value $L_0$, which is also exogenously given. Then, we can obtain the following result.

**Proposition 1.2 No-Growth Steady State:** The no-growth steady state is (i) unique and (ii) $z^*$ is increasing in $\mu$ while $k$ is decreasing in $\mu$, so that (iii) $y$, $\hat{y}$, $Y$ and $\pi_i$ ($i = 0, 1$) are also decreasing in $\mu$.

From Proposition 1.2 and the definition of the interest rate spread $\Delta$ given by equation (1.1), we have an additional result as follows:

**Corollary 1.1 Interest Rate Spread and Monitoring Cost:** The interest rate spread is strictly increasing in $\mu$. That is, the wedge strictly increases as the monitoring cost $\mu$ rises.

Proposition 1.2 and Corollary 1.1 are intuitive by the following logic. First note that the incentive constraint, (IC), tightens whenever the monitoring cost rises. This results in a higher interest rate spread, which, in turn, causes less investment in R&D, leading to smaller TFP and lower stock of capital $k$. Hence, it also results in a smaller output $y$ as well as lower profits $\pi_i$ ($i = 0, 1$). More importantly, from Corollary 1.1 and Proposition 1.2, we can define the degree of financial friction severity as follows:
**Definition 1.2 Financial Friction Severity:** The degree of financial friction severity arising from the information asymmetry is measured by the interest rate spread $\Delta$. More specifically, it increases as $\Delta$ rises.

The definition of the financial friction severity is in accordance with that the financial friction is more harmful as the monitoring cost $\mu$ increases as argued by Proposition 1.2 and the wedge (or the interest rate spread), $\Delta$, is strictly monotonic in $\mu$ as in Corollary 1. This has an empirical implication that it is much harder to observe $\mu$ than observing the adjusted interest rate spread $\hat{\Delta} \equiv \log \Delta - \log p \simeq i - r$, which is also strictly monotonic in $\mu$. That is, one can use the adjusted interest rate spread $\hat{\Delta} \simeq i - r$ as a proxy for the financial friction severity, and hence, we will use $\hat{\Delta}$ in our empirical analysis in Section 1.4.  

**Closed Economy**

The small open economy assumption guarantees analytical simplicity in the model. However, more importantly, the assumption allows unlimited accessibility of funds for both R&D and capital investments through borrowings from foreign countries denoted by $B^f_t$. In contrast, in a closed economy, available funds are limited to the aggregate domestic saving, i.e. $B^f_t = 0$ for all $t \geq 0$.

Therefore, even when the monitoring cost gets higher in the closed economy, the output can even increase when the domestic saving increases a lot enough to cause a sufficiently large increase in the capital investment. Actually, due to this general equilibrium effect — or price effect, in other words, — we can analytically show that the output can even increase when the monitoring cost $\mu$ gets higher with sufficiently high marginal production of capital: the interest rate.  

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24 The credit-to-output ratio has been commonly used as a proxy for the financial friction severity. We also conduct the same empirical analysis with the credit-to-output ratio in Appendix A.6. The estimation results are also consistent with all the theoretical findings in this chapter.

25 See Appendix A.2 for details.
In contrast, the interest rate is exogenously given in the small open economy without regard to the domestic saving, and therefore, the general equilibrium effect does not emerge.

Consequently, the negative effect of the financial friction given by Proposition 1.2 is ambiguous in the closed economy a priori.\textsuperscript{26} It is, however, revealed that the output is always decreasing in $\mu$ in the closed economy setting from numerical simulations with various functional specifications and plausible parameterizations. This suggests that all results of Proposition 1.2 and Corollary 1.1 are robust to the closed economy setting.\textsuperscript{27} Accordingly, we keep the small open economy assumption for more clear understandings of the model mechanism.

1.3.2 Permanent Growth Case

To investigate the growth effect of the financial friction, we consider the economy that grows permanently. We denote the gross growth rate of a variable $q$ in period $t$ by $g_{q,t} \equiv q_{t+1}/q_t$ and define a steady state as follows:

\textbf{Definition 1.3 Long-Run Growth Steady State:} The long-run growth steady state is characterized by a stationary growth such that $g_r = g_c = g_w = 1$ with $g_{z^*} = 1$ and $g_k = g_y$ and $g_A = g_L = g_Y$ are time invariant where $y$ and $k$ denote output and capital per efficiency unit of labor, respectively.

That is, in the steady state, all prices and the mass of entrepreneurs investing in R&D are fixed over time, while all aggregate quantities, as well those per efficiency unit of labor, grow at the respective same rate. For such a steady state where the economy grows permanently, we assume that the efficiency unit of labor $L$ is exogenously adjusted such that $g_{L,t} = g_{A,t}$ for all $t \geq 0$ where $g_{A,t}$ is endogenously

\textsuperscript{26}See also de Gregorio (1996), Jiang, Wang and Wu (2009) and Buera, Kaboski and Shin (2014) for this sort of general equilibrium effects in the closed economy led by limited accessibility of funds from foreign countries.

\textsuperscript{27}Details are available upon request.
Hence, qualitatively when \( \xi \) cannot, then \( \xi \) that only a fraction of entrepreneurs who do not invest in R&D can use the TFP while the other same period, \( A \) rewritten such that \( \bar{A} \) productivity of entrepreneurs, i.e., \( \bar{A} \). Lucas (1988) and a recent application in Doepke and Zilibotti (2013). Letting \( \bar{A} \) effect" in the production, which is standard in the endogenous growth literature, e.g., Romer (1986), determining the environment of the model. The TFP evolves as the following fashion:

\[
A_{t+1} = [p \gamma (1 - F (z_t^*)) + F (z_t^*)] A_t = \psi (z_t^*) A_t. \tag{1.13}
\]

which implies that \( g_{A,t} = \psi (z_t^*) \) and \( g_{\bar{A},t} = (1 - v) \psi (z_t^*) \); recall that \( \bar{A} \equiv A_1^{1-v} \). One can interpret equation (1.13) such that newly-developed knowledge from successful R&D can be utilized from the following period without any permission. Hence, this specification reflects that the patent duration is just one-period (or one generation here). Also, one can control the speed of intergenerational knowledge spillover by assuming that \( A_{t+1} = [\psi (z_t^*)]^{\xi} A_t \) where \( \xi \in [0, 1] \). Of course, the spillover is perfect in the baseline case, i.e., \( \xi = 1 \). Obviously, the no-growth case corresponds to \( \xi = 0 \). Now, we are ready to provide a sufficient condition that guarantees the existence of a unique balanced growth path and its properties as follows:

**Proposition 1.3 Growth Steady State and Properties:** Suppose that \( x(z, A) \) is linear in \( A \) such that \( x(z, \phi A) = \phi x(z, A) \) \( \forall \phi \in \mathbb{R}_+ \). Then, the economy has a (i) unique asymptotic long-run growth steady state such that (ii) \( g_K = g_y = 1 \), (iii) \( g_Y = g_A = g_L = \psi (z^*) \), (iv) \( g_{\pi_i} = \psi (z^*) (i = 1, 2) \) where \( \psi (z^*) > 1 \) and (v) \( g_{\Delta} = 1 \).

From Proposition 1.3 (and its proof), we notice the following asymptotic property

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28 This assumption can be interpreted as intragenerational knowledge spillover or an “external effect” in the production, which is standard in the endogenous growth literature, e.g., Romer (1986), Lucas (1988) and a recent application in Doepke and Zilibotti (2013). Letting \( \bar{A} \) be the average productivity of entrepreneurs, i.e., \( \bar{A} \equiv \int A_{i,t} (z) dF (z) \), suppose that an entrepreneur \( i \) produces consumption goods using the production function such that \( y_{i,t} = \bar{A}^{(1-\alpha)} A^{\alpha} (k_{i,t}^\alpha n_{i,t}^{1-\alpha})^\alpha \) where \( n_{i,t} \) is the number of workers employed by the entrepreneur. Then, the production function is rewritten such that \( A_t^{1-v} (k_{i,t}^\alpha n_{i,t}^{1-\alpha})^\alpha \) where \( \ell_{i,t} \equiv A_t n_{i,t} \) denotes the efficiency unit of labor. If we assume that the number of workers is fixed at \( N \) over time, the labor market clearing condition is rewritten as \( N = \int n_t (z) dF (z) \) \( \forall t \geq 0 \). Then, similarly to Proposition 1.1, we have the aggregate production function such that \( Y_t = A_t^{1-v} [K_t^\alpha (A_t N)^{1-\alpha}]^\alpha = A_t^{1-v} (K_t^\alpha L_t^{1-\alpha})^\alpha \) where \( L_t \equiv A_t N \). Hence, \( g_{L,t} = g_{A,t} \) \( \forall t \geq 0 \). Note that, in this case, \( g_{y,t} = g_{\pi,t} \) \( \forall t \geq 0 \) since \( N_t \equiv N \) \( \forall t \geq 0 \).

29 This is simply because any entrepreneur born in period \( t \) can utilize the aggregate TFP in the same period, \( A_t \), as his productivity even when he does not invest in R&D. Instead, if we assume that only a fraction of entrepreneurs who do not invest in R&D can use the TFP while the other cannot, then \( \xi \) should be smaller than one. All the theoretical results in this chapter are not changed qualitatively when \( \xi \) is assumed to be less than one as long as it is larger than zero.
of the growth effects from the financial friction:

**Corollary 1.2 Asymptotic Irrelevance to Growth:** The financial friction has no effect on the asymptotic growth rate of the economy.

Corollary 1.2 indicates that the growth effect of the financial friction becomes negligible as the economy grows. This is because the wedge arising from the financial friction $\Delta$ converges to unity as the borrower’s profit is growing over time. That is, the financial friction becomes endogenously relaxed as a borrower’s profit increases along with the growth of the economy. Hence, the asymptotic threshold $z^*$ achieves the first-best optimum, which is obtained in the economy without the financial friction, or equivalently, without the monitoring cost, i.e., $\mu = 0$.

However, during the transition to the long-run steady state, the growth effect of the financial friction remains. To investigate this, we need to focus on the transition to the asymptotic long-run growth steady state to obtain the following result:

**Corollary 1.3 Transitional Growth Effect:** On the transition to the long-run growth steady state, the gross growth rate of the TFP $\psi(z^*_t) > 1$ is (i) uniquely determined and (ii) decreasing in $\mu$. Also, (iii) $g_{Y,t}$, $g_{\pi,t}$ and $g_{\pi_i,t+1}$ ($i = 0, 1$) are decreasing in $\mu$ while $\Delta_{t+1}$ is increasing in $\mu$.

Corollary 1.3 indicates that the financial friction has a negative growth effect, which is analogous to the negative scale effect in the no-growth economy, although the negative growth effect disappears as the economy grows. Also, as in the no-growth case, the interest rate spread captures the negative impact of the financial friction on growth since the growth rate is decreasing in $\mu$ while $\tilde{\Delta}$ is increasing in $\mu$. Hence, one can use the (adjusted) interest rate spread $\tilde{\Delta}$ to verify the negative growth effect.

We will use the interest rate spread in the empirical analysis in Section 1.4.

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30 Corollary 1.3 holds without the restriction that $g_{L,t} = g_{A,t}$ $\forall t$, which we assumed for the stationary permanent growth. Without the assumption, one can show that $k_{t+1}$ depends on $\psi(z^*_t) = g_{A,t}$, and it is decreasing in $\mu$. $\pi_{i,t+1}$ ($i = 0, 1$) is, therefore, decreasing in $\mu$ as well.
We now know that the more severe financial friction, the lower growth rate as in Corollary 1.3. However, the asymptotic irrelevance from Corollary 1.2 argues that the negative growth effect is negligible in the long run since the borrower’s net worth $\pi_1$ becomes too high in the long-run, especially compared to the source of the financial friction $\mu$, to renege on his debt; recall that when the lender audits the borrower falsely reporting he failed in R&D, the lender confiscates the borrower’s net worth. Hence, the borrower’s incentive to renege on his debt will be inelastic when the monitoring cost is varying. This is why the asymptotic irrelevance emerges in the model.\footnote{Hence, when the monitoring cost also grows along with economic growth, the asymptotic irrelevance still holds as long as the monitoring cost grows slower than the borrower’s net worth; see Proposition A.1 in Appendix A.1. In the same appendix, we also provide evidence showing the monitoring cost indeed grows slower than the average income of a country by using a panel data for financial conditions of many individual banks in various countries.}

Since the borrower’s net worth $\pi_1$ is strictly increasing in the total income $Y$, and thereby, in the average income $\hat{y}$, the negative growth effect of the financial friction gets smaller as the economy grows, and eventually, it disappears in in the long run. That is, Corollary 1.2 and 1.3 imply the following empirical hypothesis: the negative growth effect of the financial friction will be smaller as the country is richer. In Section 1.4, we will verify the asymptotic irrelevance by showing evidence supporting this empirical hypothesis.

**Transition Dynamics**

From Proposition 1.3, we know that the balanced growth path defined in Definition 1.3 is uniquely determined. Also, from Corollary 1.3, the growth rate decreases in the monitoring cost during the transition phase. This decrease implies that the transition speed to the stationary state will be slower (faster) in a country where the financial friction is more severe (milder). In other words, our model predicts that transition speeds will correlated negatively with the financial friction severity. In other words, the model can describe different transition speeds across countries.
Proposition 1.4 Transition Dynamics: Suppose that $r_t$ is fixed. Then,

(i) the equilibrium threshold $z^*_t$ is decreasing over time, converging to the asymptotic level $z^*$, i.e., $z^*_t \downarrow z^*$.

(ii) the growth rate of GDP and the TFP $\psi(z^*_t)$ is increasing over time, converging to the asymptotic level $\psi(z^*)$, i.e., $\psi(z^*_t) \uparrow \psi(z^*)$.

(iii) the investment-to-output ratio is increasing over time, converging to the asymptotic level, i.e., $\iota_t \uparrow \iota$ where $\iota_t \equiv I_t/Y_t = \{K_t - (1 - \delta) K_{t-1}\}/Y_t$.

(iv) the R&D expenditure-to-output ratio is increasing over time, converging to the asymptotic level, i.e., $\rho_t \uparrow \rho$ where $\rho_t \equiv X_t/Y_t$.

(v) the external debt-to-output ratio is increasing over time, converging to the asymptotic level, i.e., $\theta_t \uparrow \theta$ where $\theta_t \equiv B^f_t/Y_t$.

According to Proposition 1.3 and 1.4, as well as Corollary 1.2 and 1.3, our model predicts gradual increases of TFP and the investment rate during the transition phase.
as depicted in Figure 1.1. All these transitional characteristics, some of which cannot be explained by the neoclassical growth model, are found in many growing economies in the real world, especially in the so-called miracle economies (see Buera and Shin, 2013).

For example, Figure 1.2 depicts the actual transition dynamics of China, Korea, Malaysia, Singapore, Taiwan and Thailand where thick solid lines denote unweighed averages. Note that dynamics of income per capita, the TFP (relative to the U.S.) and the investment rate are consistent with the model predictions. Moreover, sustained TFP growth requires only a sustained fraction of GDP spent on R&D as in the endogenous Schumpeterian growth model (see the third and seventh panels of Figure 1.1 from upper left) although the proposed model does not share the essential feature of the Schumpeterian growth model. This feature is also empirically plausible (see Ha and Howitt, 2007).

Meanwhile, in the bottom right panel of Figure 1.2, one notices that the interest rate spread is on average stable after 1980. Since the data is unavailable before 1980 for those countries from the data we use in this chapter, we guess the dynamics of the interest rate spread for that period. From the increasing investment rate during the same period in the upper right panel, it is natural to infer a decreasing trend of the interest rate spread as represented by the thick dashed line, which is also consistent with the model prediction.

Finally, the transition speed slows as the financial system worsens, i.e., the monitoring cost \( \mu \) is higher so is the interest rate spread. For example, if the monitoring is free, i.e., \( \mu = 0 \), then the model boils down to the standard endogenous growth model, and there is no transition given any fixed interest rate as illustrated in Figure 1.1. This means that the investment rate is not increasing over time, contradicting

---

33 This is not a simulation; but a simple illustration of transition dynamics based on the theoretical conclusions.

34 Greenwood, Sánchez and Wang (2012) calculate the interest rate spread from 1970 to 2005 of the U.S. and Taiwan from various datasets. According to Figure 1.1 in the paper, the interest rate spreads for both countries are similar to our model prediction.
the stylized fact. Indeed, the investment rate is decreasing in the closed economy or constant in the small open economy in the standard neoclassical growth model. These convergence properties are all inconsistent with the actual transition in reality.\footnote{In line with this, Buera and Shin (2013) point out that even the “economic miracles” seems three times slower compared with a calibrated neoclassical model, implying that a mechanism that retards the transition to the steady state is required.}

### 1.3.3 Volatility and Financial Friction

To investigate the amplification effect of the financial friction on the negative growth effect of volatility, we add uncertainty to our model. Suppose that the economy has an aggregate productivity shock $s_t$ in each period such that $\tilde{A}_t = A(A_t, s_{t+1}) > 0$ where $s_t$ is drawn from a c.d.f. $G(x)$ in each period $t$. Then, it is natural to define higher volatility as the mean-preserving spread of $G(x)$. Also, by assuming that $\mathbb{E}_t[\tilde{A}_t] = A_t$, we can regard $\psi(z^*_t) \equiv A_{t+1}/A_t$ as a growth trend of the TFP, or equivalently, the mean growth rate of the TFP.

To make the uncertainty effective, entrepreneurs should determine whether or
not invest in R&D before the aggregate shock is revealed. We assume that the R&D investment cost $x(z,A)$ is independent of the noise $s$. That is, the cost function is deterministic in the sense that every individual knows the exact R&D cost given a level of the TFP and his ability $z$ without regard to the random shock on $A_t$.

In this environment, the terms of contract $\{i, \eta(z)\}$ need to be state-contingent to satisfy the no-arbitrage condition for any $s^t$; see (3). Then, the gross borrowing rate and the probability of being monitored are given by:

$$1 + i(s^{t+1}) = \frac{1 + r_{t+1} - \delta}{p} \frac{p \pi_1(s^{t+1})}{p \pi_1(s^{t+1}) - (1 - p) \mu} \equiv 1 + r_{t+1} - \delta \frac{\Delta(s^{t+1})}{p},$$

$$\eta(z,s^{t+1}) = (1 + r_{t+1} - \delta) \frac{x_t(z)}{p \pi_1(s^{t+1}) - (1 - p) \mu},$$

where $s^{t+1}$ is the history up to and including $s_{t+1}$ such that $s^{t+1} = \{s_0, s_1, \ldots, s_{t+1}\}$. Also, the entrepreneur’s problem is rewritten as follows:

$$\pi_t(z) \equiv \max \{p[\mathbb{E}_t[\pi_{1,t+1}] - \mathbb{E}_t[(1 + i_{t+1})]x_t(z)], \mathbb{E}_t[\pi_{0,t+1}]\}.$$

where $\mathbb{E}_t[q_{t+1}] \equiv \mathbb{E}[q_{t+1}|s^t]$. Now, we can provide the following result:

**Proposition 1.5 Amplification Effect:** *The financial friction amplifies the negative effect of volatility on the mean growth rate.*

This amplification effect is intuitive since the incentive constraint endogenously tightens whenever the borrower’s expected profit decreases. If the economy experiences higher volatility, then $\mathbb{E}_t[\pi_{0,t+1}]$ decreases, so that the threshold increases from $z^*$ to $z^*_N$, as depicted in Figure 1.3, which means R&D investments are discouraged.\(^{37}\)

Once the expected profit decreases, the credit constraint becomes tighter, i.e., $\mathbb{E}_t[\Delta_{t+1}]$

---

\(^{36}\) We use the assumption that $g_{L,t+1} = g_{A,t+1}$ in the proof of Proposition 1.5. However, one can easily show that Proposition 1.5 still holds without this assumption. The only difference is that the LHS of equation (A.10) in the proof of Proposition 1.5 is no longer independent of $z$, but strictly increasing in $z$.

\(^{37}\) In the proof of Proposition 1.5, we show that an increase in the volatility deteriorates growth regardless whether the financial friction is effective or not. This is because entrepreneurial profits are decreasing return to scale.
increases. Therefore, R&D investments get smaller once again, arriving at the new threshold $z_F^*$. That is, the negative effect of volatility on growth is amplified by the financial friction, which is represented by $z_F^* - z_N^*$.

However, it is obvious that this amplification effect also disappears as the wedge (the interest rate spread) decreases along with the growth of the economy. This is because, as just described, the indirect growth effect arises from fluctuations in the interest rate spread. In the extreme case where the interest rate spread is unity, i.e., $1 + i = (1 + r - \delta)/p$, the indirect effect (the amplification effect) does not emerge.

### 1.4 Empirical Evidence

One can examine the direct and indirect growth effects of the financial friction with the asymptotic irrelevance by the following econometric framework:

$$
\tilde{g}_{\hat{y},i} = X'_i \theta + \alpha \sigma_i + \beta \Delta_i + \gamma \tilde{\Delta}_i \sigma_i + \delta \tilde{\Delta}_i \log \hat{y}_{1980,i} + \varepsilon_i
$$
where average variables are used for the relationship of the financial friction and growth as in de Gregorio (1996), AHM and AABM. \( \bar{g}_{gi} \) and \( \bar{\Delta}_i \) denote the average annual growth rate (%) and the average annual interest rate spread (%) over 1980–2010, respectively. For volatility, which is denoted by \( \sigma_i \), we use the standard deviation of % annual growth rates over the sample period as in Ramey and Ramey (1995), AHM and AABM. Finally, \( X_i \) is a vector of important controls for a cross-country growth analysis identified by Levine and Renelt (1992).

For \( X_i \), following Ramey and Ramey (1995), AHM and AABM, we consider (i) logged initial GDP per capita, (ii) initial human capital and (iii) the average growth rate of the population. An investment factor such as the average investment rate needs to be controlled but we omit this factor since, in our case, it is controlled indirectly by the average interest rate spread \( \bar{\Delta}_i \). All the data come from the Penn World Table (PWT Ver. 8.0) except the interest rate spread, which is obtained from the World Bank Dataset.

With the specification, \( \beta \) and \( \gamma \) represent the direct and indirect growth effects, respectively. Meanwhile, suppose that \( \delta \) is positive. Then, the growth effect caused by the (adjusted) interest rate spread, \( \tilde{\Delta} \), becomes smaller as the economy has initially higher income per capita, supporting the asymptotic irrelevance hypothesis.

In summary, we have the following three testable predictions:

**[Empirical Hypothesis I] Direct Growth Effect:** \( \beta < 0 \)

---

38 Since data on the interest rate spread is unavailable for many undeveloped countries before 1980, the sample period is chosen to be 1980–2010.

39 From Proposition 1.4, one can notice that the investment rate is strictly monotonic in the threshold \( z_t^* \), which is also strictly monotonic in the interest rate spread. Hence, the investment rate is controlled by the interest spread through the skill threshold.

40 The human capital is available in the PWT: the variable, \( hc \). It is based on the Barro-Lee years of schooling and Mincer returns to education. For more details, see Inklaar and Timmer (2013): p. 37-38. The interest rate spread for the U.S. is unavailable in the World Bank Data, and therefore, the U.S. is excluded in all regressions.

41 We also verify the asymptotic irrelevance hypothesis through separate regression analysis with split samples in Appendix A.5. The estimation results from the separate regression analysis are consistent with those from the baseline regression with the interaction term, consistently supporting the asymptotic irrelevance argument.
[Empirical Hypothesis II] Indirect Growth Effect: \( \gamma < 0 \)

[Empirical Hypothesis III] Asymptotic Irrelevance: \( \delta > 0 \)

Table 1.1 verifies the three empirical hypotheses. From each column of the table, we can find that \( \beta < 0 \), which means that the interest rate spread, the proxy for the financial friction severity, is negatively correlated with the average growth rate of income per capita. This confirms the direct growth effect.

Also, the estimate of \( \gamma \) is significant and negative in each regression, implying that the financial friction indirectly lowers the average growth rate via volatility. For example, suppose that the standard deviation of \% growth rates (volatility) is 3. Now let the average interest rate spread increase by 1\(\%\). Then, according to the first column, the annual growth rate decreases by \( 0.233 + 3 \times 0.018 = 0.287\% \) on average during the period of 1980-2010. In reverse, volatility affects growth via the financial friction. If the standard deviation of \% growth rates increases by 1, the average growth rate of income per capita decreases by \( \Delta \times 0.018\% \). Hence, if the average interest rate spread is 5\(\%\), the growth rate of GDP per capita decreases by 0.09\(\%\) on average during the sample period. That is, the negative effect of volatility is amplified by more severe financial friction. If the financial friction becomes so severe that the interest rate spread doubles, then the negative effect also doubles to 0.18\(\%\) even with the same volatility level.

It is worth noting that the indirect effect is still significant when volatility is controlled. As demonstrated in both the second and the fourth columns, \( \gamma \) is negative and significant, while the coefficients for the volatility term are not significant any longer. This suggests that volatility affects growth primarily via the interest rate spread.

Finally, \( \delta \) is positive and significant as expected, implying that, with the degree of the financial friction severity being controlled, the negative growth effect of the financial friction is smaller in an already developed country, the income per capita of which is initially higher. That is, the negative growth effect gets alleviated as
Table 1.1: Financial Friction and Income Growth (OLS)

Dependent Variable: Average Growth Rate of GDP per Capita (%)

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N = 74</td>
<td>N = 74</td>
<td>N = 74</td>
<td>N = 74</td>
<td>N = 74</td>
</tr>
<tr>
<td>Interest Rate Spread</td>
<td>−.233***</td>
<td>−.180**</td>
<td>−1.072***</td>
<td>−1.055***</td>
</tr>
<tr>
<td></td>
<td>(.061)</td>
<td>(.081)</td>
<td>(.370)</td>
<td>(.369)</td>
</tr>
<tr>
<td>Interest Rate Spread × Volatility</td>
<td>−.018***</td>
<td>−.030**</td>
<td>−.015***</td>
<td>−.030**</td>
</tr>
<tr>
<td></td>
<td>(.006)</td>
<td>(.013)</td>
<td>(.006)</td>
<td>(.013)</td>
</tr>
<tr>
<td>Interest Rate Spread × Initial Income</td>
<td>.237**</td>
<td></td>
<td>.250**</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.103)</td>
<td></td>
<td>(.103)</td>
</tr>
<tr>
<td>Volatility</td>
<td>.116</td>
<td>.116</td>
<td>.145</td>
<td>.113</td>
</tr>
<tr>
<td></td>
<td>(.116)</td>
<td>(.116)</td>
<td>(.113)</td>
<td></td>
</tr>
<tr>
<td>Initial Income</td>
<td>−1.780***</td>
<td>−1.958***</td>
<td>−3.067***</td>
<td>−3.365***</td>
</tr>
<tr>
<td></td>
<td>(.407)</td>
<td>(.444)</td>
<td>(.685)</td>
<td>(.720)</td>
</tr>
<tr>
<td>Initial Human Capital</td>
<td>.723</td>
<td>.897**</td>
<td>.841*</td>
<td>1.065**</td>
</tr>
<tr>
<td></td>
<td>(.513)</td>
<td>(.542)</td>
<td>(.500)</td>
<td>(.528)</td>
</tr>
<tr>
<td>Population Growth</td>
<td>−.159</td>
<td>−.195</td>
<td>−.073</td>
<td>−.113</td>
</tr>
<tr>
<td></td>
<td>(.163)</td>
<td>(.167)</td>
<td>(.163)</td>
<td>(.165)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.512</td>
<td>.519</td>
<td>.548</td>
<td>.558</td>
</tr>
</tbody>
</table>

Note: All regressors are the averages over the sample period of 1980–2010. Volatility is the standard deviation of % growth rates during the same period. Initial income and human capital are taken for 1980. Initial income is logged GDP per capita. We excluded any country from the regressions either when the number of data points of the interest rate spread is smaller than 20, which is two thirds of the total number of data points over the sample period, 1980–2010, or when the average interest rate spread is larger than 20%. Constant terms are not shown. Standard errors are in parenthesis. ***, **, *, and ˆ significant at 1%, 5%, 10% and 11%, respectively.
an economy grows further, and hence, it has higher GDP per capita. Suppose, for instance, that the economy faces a stationary interest rate spread at 5%. According to the third column, the negative growth effect becomes smaller by $5 \times 1 \times 0.237 = 1.185\%$ when the logged GDP per capita increases by 1. This positive income effect implies that the negative growth effect gets smaller as the economy grows, supporting the asymptotic irrelevance.

**Dynamic Panel GMM Regressions**

There are several shortcomings in the OLS regressions. First of all, financial conditions may be endogenously changing depending on the economic development, e.g., higher income may lead to higher demand for financial services, inducing more efficient financial markets in the economy. Then, the interest rate spread that proxies financial market conditions will depend on the economic development. Also, unobserved country-specific differences cannot be fully controlled in our OLS regressions. Finally, the OLS regressions do not utilize the time-series dimension of the data since all variables are averaged over the sample period. To address all these shortcomings, we conduct dynamic panel GMM regressions by using the following 5-year dynamic
panel model as in Beck, Demirgüç-Kunt and Levine (2007):

\[
\log \hat{y}_{it} = (\theta_1 + 1) \log \hat{y}_{it-1} + X'_{it-1} \theta + \alpha \sigma_{it} + \beta \Delta_{it} + \gamma \overline{\Delta}_{it} \sigma_{it} + \delta \overline{\Delta}_{it} \log \hat{y}_{1980,i} + \nu_{it}
\]

where \( X'_{it-1} \equiv [h_{it-1} \ g_{N,it-1}] \); \( h_{it-1} \) is human capital in the previous period; \( g_{N,it-1} \equiv \log N_{it} - \log N_{it-1} \) is the growth rate of the total population. Note that the specification is rewritten as follows:

\[
\log \hat{y}_{it} - \log \hat{y}_{it-1} = \theta_1 \log \hat{y}_{it-1} + X'_{it-1} \theta + \alpha \sigma_{it} + \beta \Delta_{it} + \gamma \overline{\Delta}_{it} \sigma_{it} + \delta \overline{\Delta}_{it} \log \hat{y}_{1980,i} + \nu_{it}
\]

where the LHS is 5-year growth rate of income per capita. Basically, the specification for this dynamic panel GMM regression is the same as the one for the OLS regression, while all variables are averaged over each 5-year period not the entire sample period, 1980–2010.

Since a lagged dependent variable \( \log \hat{y}_{it-1} \) is used as an independent variable in the regression, one can use two specific GMM techniques in order to obtain the consistency of estimators. One is the first-difference GMM developed by Arellano and Bond (1991), and the other is the system GMM developed by Arellano and Bover (1995) and Blundell and Bond (1998). The main idea of both techniques is to use lagged dependent and independent variables as instruments, assuming that the error term \( \nu_{it} \) is not serially correlated.\(^{42}\)

Blundell and Bond (1998) show that the system GMM yields more precise estimators than the first-difference GMM in the case that variables in time series are highly persistent, causing the first-differences to be weak instruments.\(^{43}\) We will report the system GMM regression estimators as our baseline since it is reasonable to think both income and the TFP growth carries persistence.\(^{44}\) Finally, there are

\(^{42}\) Specifically, the first-difference GMM regression uses lagged variables in levels only, while the system GMM regression uses lagged variables both in levels and differences. Hence, the system GMM has more moment conditions than the the first-difference GMM.

\(^{43}\) This is because, if so, the first-differences become almost uncorrelated with the lagged variables.

\(^{44}\) Halter, Oechslin and Zweimüller (2014) explore the relationship between inequality and growth. Since inequality — the Gini index — is highly persistent similarly to growth, they also use the system...
one- and two-step estimators for the two GMM techniques. The two-step estimator is asymptotically more efficient although efficiency gains are known typically small. We report estimation results of both variants.\textsuperscript{45}

The system GMM estimation results are summarized in Table 1.2. As shown in the table, an increase in the interest rate adversely affects growth, confirming the direct growth effect. Also, the coefficient of the interaction term between the interest rate spread and volatility is negative and statistically significant in regressions (1), (3), (5) and (7), which do not have the volatility regressor. However, it turns out to be insignificant once the volatility is added as a regressor; see regressions (2), (4), (6) and (8). We found that the coefficient of the volatility term becomes negative and statistically significant once the interaction term between the interest rate spread and volatility is dropped although we do not report the result here. This suggests that the financial friction channel may not be a primary route through which aggregate uncertainty affects growth negatively in the short run (five years). Finally, the coefficient of the interaction term between the interest rate spread and initial income turns out to be positive in every case. This implies that the negative growth effects become smaller as the initial income increases, and thereby, supports the asymptotic irrelevance. Consequently, the dynamic panel IV regressions corroborate the empirical findings from the OLS regressions.\textsuperscript{46}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{GMM method as a baseline.} & \textbf{Arellano-Bond robust estimators} & \textbf{Windmeijer bias-corrected estimators} \\
\hline
\end{tabular}
\end{table}

\textsuperscript{45} The Arellano-Bond robust estimators are calculated for the one-step estimation, and the Windmeijer bias-corrected estimators for the two-step.

\textsuperscript{46} We report the Hansen statistics instead of the Sargan statistics to check if the model is over-identified since the Sargan statistic is not robust to heteroskedasticity (or autocorrelation) in the error term; see Arellano and Bond, 1991. As shown in the table, the $p$-values on the Hansen test of over-identifying restrictions suggest that the null of validity of all instruments cannot be rejected. It is also important to verify if there is no autocorrelation in the second-differenced error term in checking the validity of instruments. To do this, we provide the Arellano-Bond test for the second-order autocorrelation in the first-differenced errors $\Delta v_{t,t}$. Rejecting the null hypothesis of no-autocorrelation at the second order implies that the moment conditions for the GMM regressions are invalid. As shown in the table, the $p$-values do not reject the null hypothesis at order two, suggesting that there is no evidence that the model is misspecified.
### Table 1.2: Financial Friction and Income Growth (GMM)

**Dependent Variable:** 5-year Growth Rate of GDP per capita

<table>
<thead>
<tr>
<th><strong>Independent Variables</strong></th>
<th>One-step System GMM</th>
<th></th>
<th></th>
<th>Two-step System GMM</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
</tr>
<tr>
<td>Interest Rate Spread</td>
<td>−0.012***</td>
<td>−0.016***</td>
<td>−0.098**</td>
<td>−0.075***</td>
<td>−0.011***</td>
<td>−0.015***</td>
<td>−0.093***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.005)</td>
<td>(0.039)</td>
<td>(0.026)</td>
<td>(0.004)</td>
<td>(0.005)</td>
<td>(0.042)</td>
</tr>
<tr>
<td>Interest Rate Spread × Volatility</td>
<td>−0.080***</td>
<td>0.036</td>
<td>−0.200***</td>
<td>0.003</td>
<td>−0.085***</td>
<td>0.030</td>
<td>−0.193***</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(0.079)</td>
<td>(0.039)</td>
<td>(0.116)</td>
<td>(0.023)</td>
<td>(0.080)</td>
<td>(0.040)</td>
</tr>
<tr>
<td>Interest Rate Spread × Initial Income</td>
<td>0.011**</td>
<td>0.007**</td>
<td></td>
<td></td>
<td>0.010**</td>
<td>0.006**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.003)</td>
<td></td>
<td></td>
<td>(0.005)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td>−1.046 (1.014)</td>
<td>−0.712 (1.014)</td>
<td>−1.052 (0.784)</td>
<td>−0.699 (0.950)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged Log Income Per Capita</td>
<td>.904***</td>
<td>.914***</td>
<td>.783***</td>
<td>.834***</td>
<td>.911***</td>
<td>.922***</td>
<td>.796***</td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(0.031)</td>
<td>(0.075)</td>
<td>(0.048)</td>
<td>(0.035)</td>
<td>(0.031)</td>
<td>(0.085)</td>
</tr>
<tr>
<td>Initial Human Capital</td>
<td>.448**</td>
<td>.406**</td>
<td>.667**</td>
<td>.566***</td>
<td>.425*</td>
<td>.375**</td>
<td>.664**</td>
</tr>
<tr>
<td></td>
<td>(0.205)</td>
<td>(0.176)</td>
<td>(0.278)</td>
<td>(0.194)</td>
<td>(0.219)</td>
<td>(0.187)</td>
<td>(0.313)</td>
</tr>
<tr>
<td>Population Growth</td>
<td>.003</td>
<td>.006</td>
<td>.008</td>
<td>.005</td>
<td>.002</td>
<td>.005</td>
<td>.009</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.028)</td>
<td>(0.028)</td>
<td>(0.027)</td>
<td>(0.028)</td>
<td>(0.029)</td>
<td>(0.027)</td>
</tr>
<tr>
<td># of Instruments</td>
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<td>56</td>
<td>54</td>
<td>55</td>
<td>55</td>
<td>56</td>
<td>54</td>
</tr>
<tr>
<td># of Groups (Countries)</td>
<td>74</td>
<td>74</td>
<td>74</td>
<td>74</td>
<td>74</td>
<td>74</td>
<td>74</td>
</tr>
<tr>
<td># of Obs.</td>
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<td>360</td>
<td>360</td>
<td>360</td>
<td>360</td>
<td>360</td>
</tr>
<tr>
<td>The Hansen Statistics (p-val)</td>
<td>.118</td>
<td>.115</td>
<td>.233</td>
<td>.184</td>
<td>.118</td>
<td>.115</td>
<td>.233</td>
</tr>
</tbody>
</table>

**Note:** All regressors are the 5-year averages over the sample period of 1980–2010. Volatility is the standard deviation of % growth rates during the same period. Initial income is logged GDP per capita in 1980. All dependent and independent variables are used as instruments; the lagged log income per capita and interest rate spread are instrumented in the GMM style, while the other controls are instrumented in the IV style; we restrict the GMM instrument set to lags 2 and deeper for regressions (4), (5), (7) and (8) to make sure that the number of instruments is smaller than the number of groups. We excluded any country from the regressions either when the number of data points of the interest rate spread is smaller than 20, which is two thirds of the total number of data points over the sample period, 1980–2010, or when the average interest rate spread is larger than 20%. Constant terms are not shown. Standard errors are in parenthesis. ***, ** and * significant at 1%, 5% and 10%, respectively.
1.4.1 Productivity and Financial Friction

The main mechanism that the financial friction adversely affects growth is quite simple. The financial friction increases the wedge (the interest rate spread), and hence, the borrowing rate too, with the interest rate \( r \) being fixed. Consequently, some R&D projects are not initiated, while they would be “profitable” without the financial friction. That is, the negative growth effects of the financial friction in our theory are caused by a decrease in TFP growth led by suboptimal R&D investments, and thus, the three empirical hypotheses must be relevant to TFP growth as well.

To verify this empirically, we first note that \( \tilde{A}_{i,t} \) is given by:

\[
\log \tilde{A}_{i,t} = \log Y_{i,t} - v_{i,t} \{ \alpha_{i,t} \log K_{i,t} + (1 - \alpha_{i,t}) \log L_{i,t} \}
\]

Following the convention, e.g., Benhabib and Spiegel, 2005; Caselli, 2005, we pick \( \alpha_{i,t} = 1/3 \) for all countries and for any \( t \) in the 1980–2010 period. Similarly, the span of control parameter \( v_{i,t} \) is also homogeneous across countries and time-invariant. There are several estimates of \( v \) for the U.S. Atkeson and Kehoe (2005)’s estimate is 0.85, while it is 0.79 in Buera and Shin (2011).\(^{47}\) Here, we set \( v = 0.80.\)\(^{48}\) We define the efficiency unit of labor such that \( L_{i,t} \equiv h_{i,t}N_{i,t} \) where \( h_{i,t} \) is human capital per worker and \( N_{i,t} \) is total population.\(^{49}\) Again, data on \( Y_{i,t}, K_{i,t} \) and \( L_{i,t} \) come from the PWT. Then, we can calculate the productivity \( \tilde{A}_{i,t} \) and its average growth rate \( \bar{g}_{\tilde{A},i} \).

Now, to check if the financial friction adversely affect growth of the TFP, we can regress the following specification, which is the same as the one for income growth:

\[
\bar{g}_{\tilde{A},i} = X_i' \theta + \alpha \bar{\sigma}_i + \beta \bar{\Delta}_i + \gamma \bar{\Delta}_i \bar{\sigma}_i + \delta \bar{\Delta}_i \log \hat{y}_{1980,i} + \nu_i
\]

\(^{47}\) Hsieh and Klenow (2009) estimate \( v \) from a different approach and suggest 0.50 for its estimate, which is quite low.

\(^{48}\) Although we do not report here, the estimation results summarized in Table 1.3 and 1.4 presented below turn out robust to a small change of the value of \( v \).

\(^{49}\) Following Benhabib and Spiegel (2005), we regard \( N_{i,t} \) as the total population rather than the number of workers. When we use the number of workers, which is also provided in the PWT, the results about \( \beta \) and \( \delta \) are not changed qualitatively. However, the coefficient for the indirect growth effect \( \gamma \) is not significant for some cases.
According to the theory, $\beta$ and $\gamma$ should be negative and statistically significant. Meanwhile, for the asymptotic irrelevance, $\delta$ should be positive and statistically significant, and therefore, the negative growth effects on the TFP must be smaller in already developed countries with controlling for the degree of the financial friction severity $\bar{\Delta}_i$.

Table 1.3 shows estimation results from the OLS regression. As expected, the growth effects of the financial friction still hold for the TFP, meaning that the financial friction deteriorates growth of the TFP both directly and indirectly, although the magnitudes are smaller than those for income growth.50 Table 1.3 also shows that an increase in GDP per capita mitigates the negative effect on growth of the TFP although the significance is weak. Hence, one can argue that the asymptotic irrelevance exists for growth of the TFP too, but less confident than the asymptotic irrelevance for income growth. In overall, the results confirm the validity of the model mechanism underlying the main theoretical conclusions.

Table 1.3 also shows that there exists a strong positive effect of human capital on TFP growth; the initial human capital correlates positively with the average growth rate of the TFP. Intuitively, it seems plausible to think that a larger stock of human capital encourages productivity-enhancing investments, raising the growth of the TFP.
### Table 1.3: Financial Friction and TFP Growth (OLS)

**Dependent Variable:** Average Growth Rate of the TFP (%)

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>(1) N = 74</th>
<th>(2) N = 74</th>
<th>(3) N = 74</th>
<th>(4) N = 74</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest Rate Spread</td>
<td>(-.140^{***})</td>
<td>(-.115^*)</td>
<td>(-.602^{**})</td>
<td>(-.594^{**})</td>
</tr>
<tr>
<td></td>
<td>(.044)</td>
<td>(.059)</td>
<td>(.274)</td>
<td>(.274)</td>
</tr>
<tr>
<td>Interest Rate Spread \times Volatility</td>
<td>(-.013^{***})</td>
<td>(-.018^*)</td>
<td>(-.015^{**})</td>
<td>(-.018^*)</td>
</tr>
<tr>
<td></td>
<td>(.004)</td>
<td>(.010)</td>
<td>(.004)</td>
<td>(.010)</td>
</tr>
<tr>
<td>Interest Rate Spread \times Initial Income</td>
<td></td>
<td>(.057^*)</td>
<td>(.060^*)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.044)</td>
<td>(.033)</td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td>.055</td>
<td></td>
<td>.071</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.085)</td>
<td></td>
<td>(.084)</td>
<td></td>
</tr>
<tr>
<td>Initial Income</td>
<td>(-.661^{***})</td>
<td>(-.698^{***})</td>
<td>(-.969^{***})</td>
<td>(-1.032^{***})</td>
</tr>
<tr>
<td></td>
<td>(.129)</td>
<td>(.141)</td>
<td>(.220)</td>
<td>(.233)</td>
</tr>
<tr>
<td>Initial Human Capital</td>
<td>1.112^{***}</td>
<td>1.194^{***}</td>
<td>1.177^{***}</td>
<td>1.286^{***}</td>
</tr>
<tr>
<td></td>
<td>(.374)</td>
<td>(.397)</td>
<td>(.371)</td>
<td>(.394)</td>
</tr>
<tr>
<td>Population Growth</td>
<td>.140</td>
<td>.123</td>
<td>.187</td>
<td>.168</td>
</tr>
<tr>
<td></td>
<td>(.119)</td>
<td>(.122)</td>
<td>(.121)</td>
<td>(.123)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>(.479)</td>
<td>(.482)</td>
<td>(.478)</td>
<td>(.506)</td>
</tr>
</tbody>
</table>

**Note:** All regressors are the averages over the sample period of 1980–2010. Volatility is the standard deviation of % growth rates during the same period. Initial income and human capital are taken for 1980. Initial income is logged GDP per capita. We excluded any country from the regressions either when the number of data points of the interest rate spread is smaller than 20, which is two thirds of the total number of data points over the sample period, 1980–2010, or when the average interest rate spread is larger than 20%. Constant terms are not shown. Standard errors are in parenthesis. ***, ** and * significant at 1%, 5% and 10%, respectively.
Quantitatively, comparing the third and the fourth columns of Table 1.1 and 1.3 indicates that the direct effect for TFP growth is about a half of that for income growth, while the indirect growth effects for both of them are similar to each other. This implies that the direct growth effect emerges not only from the R&D investment but also from the capital investment, while the indirect growth effect arises mainly from the R&D investment.

To address the shortcomings of the OLS regressions, we conduct the dynamic panel GMM regressions for 5-year TFP growth as we did for income growth. Table 1.4 summarizes the estimation results. Similarly to the results in Table 1.2 for income growth, the direct and indirect growth effects are negative for the TFP growth, while the indirect effect appears insignificant when the volatility term is added in the regression; see regressions (2), (4), (6) and (8). The asymptotic irrelevance turns out even less statistically significant than the OLS regressions for the period of 30-year. This suggests that the asymptotic irrelevance for the TFP growth does not emerge in short run, for example, over five years.

1.5 Conclusion

Financial frictions have been viewed as playing an important role in growth of a nation. Motivated by the intuition that the degree of the financial friction severity will be different in different development stages, we construct a tractable endogenous growth model where entrepreneurial activities are constrained by financial frictions. Our model has a novel feature; the tightness of the financial friction varies endogenously by economic circumstance. This feature reveals important aspects of financial frictions that the previous literature has not discover.

Among others, we show that the credit constraint is endogenously relaxed along with economic growth, and hence, the negative growth effects disappear asymptotically. This suggests that impacts of the financial friction on growth are, at best, temporary. This finding also leads to identifying heterogeneous impacts of the finan-
cial friction across countries. In the transition phase, the negative growth effect of the financial friction gets more severe in a poorer country.

However, the simplicity of our model entails some important caveats. Our two-period model does not allow entrepreneurs’ self-financing. As shown by Buera and Shin (2011), self-financing could be an effective substitute for well-functioning financial markets. As a result, the negative impact on growth will be somewhat diminished quantitatively. Also, other types of financial frictions possibly matter in reality. Although we consider a particular type of financial friction, which is driven by information symmetry, another type of financial friction may explain the spread of long-term growth across countries as well as explaining the negative growth effect of “too much finance”; see Arcand et al. (2015).

Regarding policy implications, financial friction still has a considerable impact on output level even in very long run although the growth effects gradually disappear. That is, cross-country differences in the national income caused by different financial market conditions persist. Related, the economy converges slower to the steady state as it suffers from more severe financial friction. The asymptotic irrelevance, therefore, does not mean that policy makers do not need to enhance financial conditions. Obviously, policies mitigating credit constraints are required to minimize output losses.

Finally, the asymptotic irrelevance does not necessarily imply growth acceleration over time. This is because there are many other important growth determinants, for instance, human capital, culture, institutions and geography.\footnote{Our model, however, predicts growth acceleration with other things being constant, which seems, in fact, true in reality once an economy takes off after a period of long-lasting poverty; see, for example, Fact 2 in Jones and Romer (2010).} The asymptotic irrelevance rather argues that financial factors may not be “fundamental determinants” underlying long-term economic growth of society, although they do matter for growth in the short run. Hence, it would be also interesting to analyze those fundamental factors that determine growth even in very long run.
Table 1.4: Financial Friction and TFP Growth (GMM)

Table 1.4 Dynamic Panel IV Regressions for TFP Growth

*Dependent Variable: 5-year Growth Rate of the TFP*

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>One-step System GMM</th>
<th>Two-step System GMM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Interest Rate Spread</td>
<td>−.007*</td>
<td>−.010**</td>
</tr>
<tr>
<td></td>
<td>(.004)</td>
<td>(.004)</td>
</tr>
<tr>
<td>Interest Rate Spread × Volatility</td>
<td>−.106**</td>
<td>−.028</td>
</tr>
<tr>
<td></td>
<td>(.050)</td>
<td>(.079)</td>
</tr>
<tr>
<td>Interest Rate Spread × Initial Income</td>
<td>.006</td>
<td>.005</td>
</tr>
<tr>
<td></td>
<td>(.006)</td>
<td>(.004)</td>
</tr>
<tr>
<td>Volatility</td>
<td>−.667</td>
<td>−.131</td>
</tr>
<tr>
<td></td>
<td>(.599)</td>
<td>(.741)</td>
</tr>
<tr>
<td>Lagged TFP</td>
<td>.070</td>
<td>.133</td>
</tr>
<tr>
<td></td>
<td>(.435)</td>
<td>(.407)</td>
</tr>
<tr>
<td>Lagged Log Income Per Capita</td>
<td>.631*</td>
<td>.588*</td>
</tr>
<tr>
<td></td>
<td>(.326)</td>
<td>(.307)</td>
</tr>
<tr>
<td>Initial Human Capital</td>
<td>.257**</td>
<td>.246**</td>
</tr>
<tr>
<td></td>
<td>(.118)</td>
<td>(.109)</td>
</tr>
<tr>
<td>Population Growth</td>
<td>.006</td>
<td>.008</td>
</tr>
<tr>
<td></td>
<td>(.024)</td>
<td>(.025)</td>
</tr>
<tr>
<td># of Instruments</td>
<td>56</td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>(.024)</td>
<td>(.025)</td>
</tr>
<tr>
<td># of Groups (Countries)</td>
<td>74</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>(.024)</td>
<td>(.025)</td>
</tr>
<tr>
<td># of Obs.</td>
<td>360</td>
<td>360</td>
</tr>
<tr>
<td></td>
<td>(.324)</td>
<td>(.324)</td>
</tr>
<tr>
<td>2nd Order Autocorrelation (p-val)</td>
<td>.449</td>
<td>.383</td>
</tr>
<tr>
<td></td>
<td>(.324)</td>
<td>(.324)</td>
</tr>
<tr>
<td>The Hansen Statistics (p-val)</td>
<td>.146</td>
<td>.132</td>
</tr>
<tr>
<td></td>
<td>(.146)</td>
<td>(.132)</td>
</tr>
</tbody>
</table>

**Note:** All regressors are the 5-year averages over the sample period of 1980–2010. Volatility is the standard deviation of % growth rates during the same period. Initial income is logged GDP per capita in 1980. All dependent and independent variables are used as instruments; the lagged TFP and interest rate spread are instrumented in the GMM style, while the other controls are instrumented in the IV style; we restrict the GMM instrument set to lags 2 and deeper for regressions (4), (5), (7) and (8) to make sure that the number of instruments is smaller than the number of groups. We excluded any country from the regressions either when the number of data points of the interest rate spread is smaller than 20, which is two thirds of the total number of data points over the sample period, 1980–2010, or when the average interest rate spread is larger than 20%. Constant terms are not shown. Standard errors are in parenthesis. ***, ** and * significant at 1%, 5% and 10%, respectively.
Chapter 2

Culture, Institutions and Growth

2.1 Introduction

Everything changes and nothing remains still ... and ... you cannot step twice into the same stream.  

Heraclitus, *Cratylus*

One of the most important and interesting questions in economics is “What is the secret of economic growth?” Numerous economists have tried to uncover the secret to economic prosperity. Thanks to their efforts, we have begun to answer critical questions, but many gaps in our understanding of economic prosperity remain.

In particular, we do not know much about the role that culture and institutions play in the development of prosperity. This gap in understanding persists, even though economists have long regarded those factors as fundamental to determining long-term growth. Notably, North (1991) defines institutions as “humanly devised constraints that structure political, economic and social interactions,” and emphasizes interactions between the evolution of institutions and the development of societies to understand various development paths across countries.

North distinguishes the institutions according to formal and informal constraints. Formal constraints represent rules such as constitutions, laws and property rights, while informal ones are restrictions such as customs, codes of conduct, taboos and
traditions. For the sake of exposition, one can generally refer to the formal and informal constraints as *institutions* and *culture*, respectively. Looking at different historical episodes, North argues that the interaction between formal and informal constraints, or institutions and culture in our terms, plays a crucial role in the determination of the development of society. Ultimately, he raises the following question: “What is the relationship between formal and informal constraints?” (North, 1991, p. 111).¹

In this chapter, we try to understand how the interplay between culture and institutions determines economic prosperity. To this end, we develop a dynamic model in which culture, institutions and long-term growth are jointly determined. Since culture and institutions are highly abstract and broad notions, we focus on a specific dimension of culture and institutions in this chapter. Namely, we investigate interactions between culture, specified as the degree of individualism, and institutions, defined by protection of property rights over innovations.²

To achieve concreteness in the analysis, we interpret individualism and collectivism as agents’ types that are transmitted over generations.³ Based on the empirical evidence on individualism/collectivism and innovation, we assume that an individualist is a type that has comparative and absolute advantages in innovation, while a collectivist has comparative and absolute advantages in the production of goods. The transmission of the two types over generations is partly determined by parents’ altruistic care for their children. A parent is altruistic in that he wants to maximize his child’s consumption. Since a child’s consumption in the next period is affected

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¹ In the same vein, Tabellini (2008a) recently raises a similar question that asks how the interaction between individual values and formal feature of institutions — culture and institutions in our terms, respectively — affects economic outcomes. In particular, he asks the following question: “How do values interact with economic incentives and with formal features of institutions to influence economic and political behavior?” (Tabellini, 2008a, p. 258). In this regard, Tabellini (2008b) investigates how values (trustworthiness) and formal institutions (the quality of legal enforcement) interact with each other in a dynamic setting.


³ Similarly, Galor and Michalopoulos (2011) interpret entrepreneurship as a risk-neutral type and explore how cultural transmission of the entrepreneurial trait over a risk-averse type affects long-term growth.
by the child’s culture type, a parent tries to transmit the culture type that leads to larger expected consumption for his child.

To answer North’s longstanding question on the interaction between culture and institutions, it is critical to understand a strategic complementarity between the altruistic parental efforts on the transmission of cultural traits and the endogenous determination of the degree to which property rights over innovations are secured. The intuition is simple. If the government heavily taxes innovations in the future, parents will try to raise children as collectivists, not individualists. This is because an individualist’s competitive edge in innovation is no longer valuable under the high risk of government expropriation of innovation. Consequently, the society becomes collectivist in the subsequent period, i.e., there are few individualists, which, in turn, implies that there will be a small number of innovators since an individualist is more likely to be an innovator based on his competitive edge in innovation.

With this dynamics in mind, suppose that the degree of protection of property rights is determined by a certain political decision rule, for instance, majority voting, that gives stronger political power to an interest group that has a larger number of members. Then, if society is collectivist, the government tends to tax innovations severely. This is because an individualist is more likely to be an innovator, so that collectivist society that consists of few individualists will not have enough innovators to impose a lower rate of tax on innovation. This is the expropriation regime where innovators are few, and hence, politically impotent. By the same logic, if the government does not tax innovations in the future, parents will try to raise children as individualists, causing the society to become individualistic. Since an individualist is more inclined to be an innovator than a collectivist, the number of innovators would be enough to enforce a lower tax rate on innovation. This is the non-expropriation regime where innovators are many, and hence, politically strong. Consequently, there are two different steady states with opposing properties: individualistic society with strong enforcement of property rights and collectivist society with weak enforcement of property rights.
What are the growth implications of cross-country differences in institutional qualities in terms of protection of property rights over individual innovations? In the model, long-term economic growth is determined by the mass of innovations. Poor protection of property rights undermines long-term growth since it lowers the incentive to innovate. This is a direct effect of institutions on growth. Also, recall that poor protection of property rights accompanies collectivism. Then, long-term growth is hampered again since collectivist society has fewer competent innovators compared to individualistic society; this contrast owes to the different advantages in innovation between a collectivist and an individualist. This is an indirect effect of institutions on growth through a change in the state of culture. In sum, collectivist society operating in the expropriation regime falls into the low-growth trap, which is a bad steady state. In contrast, individualistic society enjoys high growth in the long run since there are more competent innovators and individual inventions are encouraged by a government which does not expropriate individual achievements. This is a good steady state.

The strategic complementarity provides not only multiple static equilibria (multiple steady states) but also multiple dynamic equilibria with contrasting properties. This is the fundamental economic instability arising from endogenous regime switching over time. As we will later observe, the model admits a region where multiple equilibria emerge in every period and an equilibrium is solely determined by self-fulfilling expectations. Hence, if this region is near the bad steady state (low-growth trap), a collectivist society will suffer from not only low growth but also excess volatility. The intuition behind the multiple dynamic equilibria is similar to the one for the multiple steady states. In the model, parents must forecast how much the government will tax innovations in the next period in order to maximize children's consumption.\footnote{Technically, this forecasting behavior translates to forward-looking properties in the dynamics of culture; see equation (15) in Section 3.} If parents form negative expectations about the protection of property rights in the next period, i.e., the government will tax innovations heavily, then the expected return of
being an individualist will be small. Hence, they try to raise children as collectivists, resulting in too few innovators to implement secure property rights. This causes the government to tax innovations severely, fulfilling the pessimistic beliefs. On the other hand, if they form positive expectations about the protection of property rights, i.e., the government will not expropriate innovations, then society will be individualistic. This makes the fraction of innovators large enough to implement strong protection of property rights, confirming the optimistic beliefs.

Finally, by considering culture and institutions, our theoretical framework provides broader perspectives on policy implications for growth miracles. We show that once the economy escapes from the low-growth trap, it may experience high growth over a long period of time as if it had recovered “growth potential,” which we interpret as a growth miracle. The thing is that, in our framework, such a growth miracle is possible not only by changing economic factors, e.g., tax policies, but also by upgrading political factors throughout political reforms or improvement of individual rights to private ownership. Accordingly, our framework uncovers more comprehensive policy prescriptions that encompass not only sound economic policies but also desirable reform packages in political environments.

Related Literature

Our motivation is based on the idea that culture and institutions are the most fundamental determinants of long-term growth, a vision apparent in recent empirical studies such as Acemoglu et al. (2001) and Gorodnichenko and Roland (2017). However, Acemoglu et al. (2001) mostly focus on institutions, so that the study is silent on the interactions between culture and institutions although it acknowledges those interactions are important. Gorodnichenko and Roland (2017) also investigate a role of individualism on the dispersion of income across countries. Our research complements Gorodnichenko and Roland (2017) in that we provide a theoretical framework

\[5\] There is another view focusing on other fundamental factors such as genetic or geographical aspects, e.g., Diamond (1997), Sachs (2001), Ashraf et al. (2010), Vollrath (2011), Ashraf and Galor (2013), among others.
for growth implications with culture and institutions being interconnected.\(^6\)

Regarding the strategic complementarity between culture and institutions, our research is closely related to the seminal contribution by Azariadis and Drazen (1990) that theoretically establishes the possibility of the group convergence by proposing the concept of threshold externalities. Although there are many subsequent papers that investigate sources of such externalities, e.g., Galor and Zeira (1993), Durlauf (1993), Laing et al. (1995), Redding (1996), Sorger (2002), Antinolfi et al. (2007) and Azariadis et al. (2016), at least to the best of our knowledge, this research is the first attempt to lend another source of threshold externality arising from the interaction between culture and institutions. Also, the low-growth trap in the model conceptually resembles the poverty trap, e.g., Azariadis (1996) and Azariadis and Stachurski (2005). It can also be reconciled with the well-known empirical findings such as “The Great Divergence” over the very long run, e.g., Maddison (1995), and the spread of growth in recent decades, e.g., Jones and Romer (2010) and Jones (2015).

The endogenous regime-switching property in our model is related to Azariadis and Smith (1998), Aghion et al. (1999) and Matsuyama (2007, 2013). In those models, the economy features endogenous business cycles arising from regime switching affiliated with tight or slack credit constraints. Here, we show that the interaction between culture and institutions generates regime switching, and hence, endogenous growth cycles driven by self-fulfilling expectations.\(^7\)

\(^6\) Galor and Michalopoulos (2011), Doepke and Zilibotti (2013) and Chakraborty et al. (2016) investigate the culture-growth relationship focusing on entrepreneurship. de la Croix and Delavallade (2016) try to understand how religions affect growth via fertility decisions. Barro and McClearly (2003) demonstrate interesting empirical evidence on contrasting growth effects of believing in and belonging to religions. In a more general perspective, Kim and Lee (2015) discuss various roles of non-productive social values on growth dynamics. These studies, however, abstract from any concern on institutions or on interactions between culture and institutions. Meanwhile, Tabellini (2010) applies novel instrumental variables to identify a causal effect of culture, which is independent of institutional factors, on economic development of the European regions.

\(^7\) There are, of course, other sources of endogenous cycles. Matsuyama (1999), for instance, demonstrates endogenous switching between the no-innovation and innovation regimes, which he calls the Solow and Romer regimes, respectively. This arises from the strategic complementarity between the market size and the incentive to innovate. See also Boldrin and Woodford (1990) for
Finally, the transmission of culture traits over generations in the model shares essential features of the one in Bisin and Verdier (2000, 2001). In particular, Bisin and Verdier (2015) provide a general theoretical framework that jointly determines development paths of culture and institutions as in our model. We investigate specific growth implications consistent with empirical evidence. To do that, we develop a novel growth theory featuring endogenous determination of culture and institutions in an otherwise standard growth model.

The rest of the chapter is organized as follows. Section 2.2 presents empirical facts on individualism, protection of property rights and growth. Section 2.3 introduces the model. Section 2.4 investigates culture dynamics and its growth implications with institutions being fixed. In Section 2.5, we let institutions be endogenously determined by majority voting. We then explore how culture and institutions interact with each other in Section 2.6. Section 2.7 analyzes growth with culture and institutions being endogenously determined. Section 8 explores policy implications for growth miracles at various angles. Section 2.9 concludes with some remarks.

2.2 Empirical Analysis

We briefly explore empirical facts about some notable relationships between individualism, innovation, protection of property rights and growth. Prior to presenting the facts, we briefly discuss the Hofstede individualism index (or score), which we will intensively use, to describe how it was constructed and what it measures. Details of each part can be found in Appendix B.1.

The Hofstede individualism index

The Hofstede individualism index was constructed from an international employee attitude survey program for IBM, a large multinational corporation. The survey was conducted between 1967 and 1973 in two survey rounds covering 72 countries with an excellent survey of models featuring endogenous business cycles.
20 languages. The analysis focused on differences in answers on questions about employee values across countries.

To be concise, the individualism index measures the extent to which it is believed that individuals are supposed to take care of themselves as opposed to being strongly integrated and loyal to a cohesive group (Gorodnichenko and Roland, 2017). A collectivist society assumes a broad responsibility for their members, leading to goal congruency and conformity to organizations (Hofstede, 2001). Accordingly, one of the key criteria identifying individualism and collectivism is how much people are willing to conform to the organizations they are involved in.

The main advantage of Hofstede’s individualism index is that it has been validated in many subsequent studies and widely accepted in relevant academic fields such as psychology, sociology, and political science — see, for examples, Chapter 5 of Hofstede (2001) for a review — and recently in economics as well.

**Culture-Innovation relationship**

There are many empirical studies using the Hofstede individualism index that reveal individualism is more beneficial for innovation compared to collectivism. Among others, Gorodnichenko and Roland (2017) and Acemoglu et al. (2015) conduct comprehensive cross-country empirical analyses and find that individualism encourages innovation more than collectivism both quantitatively and qualitatively.\(^8\)

However, the cross-country analyses have a critical shortcoming that they may not fully control for differences in institutional factors or unobserved country-specific differences. To tackle this problem, following Gorodnichenko and Roland (2017),

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\(^8\) In individualistic societies, people are autonomous and independent from their in-groups: family, tribe, company, nation, etc. Individualists give priority to their personal goals over the goals of their in-groups and behave primarily on the basis of their attitudes rather than the norms of their in-groups. On the other hand, collectivist cultures give priority to group goals of their in-groups, shape their behavior primarily on the basis of in-group norms, and behave in a communal way (Triandis, 2001).

\(^9\) To be more specific, Gorodnichenko and Roland (2017) focus relatively more on innovation *quantities*, while Acemoglu et al. (2015) pay attention to innovation *qualities*. For more details, see Appendix B.1.
we examine the effect of culture on innovations within a given country, and thereby, holding institutional differences and other country-specific factors constant. From the public micro data (IPUMS) for the USA, we find evidence that, controlling for other factors relevant to occupation choices, the propensity to choose research-oriented jobs increases when the respondent’s self-reported country of ancestry is more individualistic in terms of the Hofstede individualism index. This suggests that an individualist (a more individualistic person) has a comparative advantage in innovation compared to a collectivist (a less individualistic person) in the sense of Roy (1951): self-selection based on comparative advantages. Although this evidence itself does not necessarily mean that an individualist also has an absolute advantage in innovation, the empirical fact that individualism induces more and better innovations than collectivism suggests an individualist also has an absolute advantage.

The same logic for this identification method can be applied to Italian immigrants to the USA. Many scholars, e.g., Putnam (1994), argue that northern and southern Italy are vastly different culturally and historically, and the northern part is more individualistic. The estimation result from the regression that controls for many relevant factors to occupation choices shows consistently that an immigrant whose father came from northern Italy is more inclined to have a research-oriented job than the one whose father came from southern Italy.

**Culture-Growth relationship**

In the previous part, we find that individualism is more beneficial for innovation, which has been regarded as the driving force behind virtuous growth dynamics. Hence, it is natural to predict that a more individualistic society will grow faster at least in long run. Gorodnichenko and Roland (2017) provide evidence that individualism, in fact, encourages growth over very long periods of time such as 1500–2000 and 1820–2000 using the Maddison data. We corroborate this observation by providing similar evidence for relatively shorter periods of time with more controls relevant to growth. We also conduct the same regression with the Penn World Table data

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(PWT) over a recent period. The estimation results with the PWT also indicate the same pattern. One advantage of using the PWT is that we can regress growth of the TPF on individualism. The TFP growth rate increases with stronger individualism as well, which is consistent with the observation that individualism is more productive in innovation than collectivism.

**Culture-Institutions relationship**

Many scholars, e.g., North (1990), Posner (1997), Tabellini (2008a, 2008b) and Greif and Tabellini (2010), have argued that culture (values) and institutions (formal rules) are interconnected, and especially, they are mutually reinforced over time. For example, Max Weber noted a constructive role for individualism in the development of the English economy through the rise of capitalism, an economic system that generally guarantees individual property rights. He argued that one of the critical ingredients in the rise of capitalism in England was the spread of Calvinism that emphasized “individualistic motives of rational legal acquisition by virtue of one’s own ability and initiative” and “had the psychological effect of freeing the acquisition of goods from the inhibitions of traditionalistic ethics” (Ball, 2001). For the reverse direction, Choi et al. (2015) provide evidence that experiences of a certain type of institutions affect the development of preferences toward a market economy. They find that North Korean refugees settled in South Korea exhibit weaker support for market economy than native-born South Koreans, implying that institutions also affect preferences (culture).

Related to this, Gorodnichenko and Roland (2017) find empirical evidence that suggests mutual reinforcement relationships between individualism and protection against expropriation risk. We conduct a similar empirical test. In addition to the protection against expropriation risk taken from the International Country Risk Guide, we also use another measure of protection of property rights from Gwartney et al. (2015).\textsuperscript{10} As expected, the Hofstede individualism index correlates positively

\textsuperscript{10} The past version of this dataset is used in Acemoglu et al. (2001).
with both the protection against expropriation risk and the protection of property rights. The result from regressions where the endogeneity problem is alleviated by proper instrument variables shows that individualism induces better protection of property rights, and in reverse, better protection of property rights also induces more individualistic culture. This suggests that individualism and protection of property rights take a form of strategic complementarity.

**Summary**

We have three empirical findings and summarize them as follows:

**Fact 1. Individualism and Innovation:** an individualist has comparative and absolute advantages in innovation compared to a collectivist.

**Fact 2. Individualism and Growth:** societies with more individualistic cultures enjoy higher economic growth in long run.

**Fact 3. Individualism and Property Rights:** the interaction between individualism and protection of property rights takes a form of strategic complementarity.

We now build a theoretical framework that starts with Fact 1 as an assumption and proves Fact 2 and 3 as conclusions. The reason why Fact 1 holds can be, at least partially, found in some academic fields such as social psychology.\(^\text{11}\) From an economics point of view, a more individualistic culture would provide larger incentives to innovate in line with North (1990)’s argument that institutions are all about incentive systems.\(^\text{12}\) If this is true, the cultural dimension of individualism/collectivism can be characterized by a specific incentive system such as social status rewards for innovation, punishments for dissenting or standing out, and etc, which will endogenously change over time — and hence will be an interesting topic to delve into.\(^\text{13}\) In Section

\(^\text{11}\) For instance, one can refer to Niu and Sternberg (2001), Goncalo and Staw (2006) and Černe et al. (2013).

\(^\text{12}\) “Institutions are the rules of the game ··· In consequence they structure incentives in human exchange, whether political, social, or economic ···.” (North, 1990, p. 3).

\(^\text{13}\) Specifically, Friedrich Hayek points out some virtues of individualism that may promote innovation as follows: “It is true that the virtues which are less esteemed and practised now —
3, we adopt a reduced form approach that captures the idea that individualism can be interpreted as an incentive system for innovation, resulting in higher efficiency of overall innovation. By doing that, we can focus on examining why Fact 2 and Fact 3 hold while simply postulating Fact 1 as an assumption since Fact 2 and Fact 3 are far more important to understand in economics than Fact 1. We hope that future research will address incentive systems in an endogenous fashion in order to explain Fact 1 as an economic proposition as well.\(^\text{14}\)

### 2.3 A Theoretical Framework

We consider a simple overlapping generations model in which individuals’ culture traits are endogenously determined. Since we are focusing on a specific dimension of culture, namely individualism verses collectivism, an individual’s culture type \( S \) is either \( I \) or \( C \) where \( I \) stands for an “individualist” and \( C \) does for a “collectivist,” i.e., \( S \in \{I, C\} \). Each \( t = 0, 1, \ldots \) generation consists of a continuum of risk-neutral agents with unit measure. Also, there are homogeneous producers (firms) with unit measure producing consumption goods. Each agent of generation \( t \) is born in the middle of period \( t - 1 \) and socialized by his parent and colleagues. He, then, earns income in the second period and consumes in the last period as in Figure 2.1. In the second period, each agent chooses to be either a worker or an innovator depending on his own culture type and individual innovation skill, indexed by \( z \in [0, 1] \). Workers supply labor in the competitive labor market in order to earn labor income, and innovators create new ideas that enhance productivity of the consumption goods producers. Innovators own property rights on the new ideas and sell the rights to the producers competitively.

\(^{14}\) Regarding the evolution of institutions, interacting with cultural aspects, there are papers adopting game-theoretical approaches for deeper microfoundations. Especially, one can consult Greif (1994) for individualism verses collectivism.
Culture traits transmission

In each period $t$, every agent of generation $t$ belonging to one of the culture groups, $S \in \{I, C\}$, gives birth to one child and makes an effort $d^S_t \in [0, 1]$ to transmit his own culture trait $S$ to his child. At the same time, outside home, individuals’ culture traits are affected by their friends, colleagues, and etc.\footnote{Dohmen et al. (2012) deliver comprehensive evidence that advocates the intergenerational transmission process of individual attitudes that we are now considering. More precisely, they show that risk and trust attitudes are affected by both their parents and prevailing attitudes in their local environment.} To capture this idea, we define the transition probability $P^S_{t+1} \in [0, 1]$ which denotes a probability that a child born by a $S$-type parent becomes $\tilde{S}$ type in $t + 1$ period ($S, \tilde{S} \in \{I, C\}$) as follows:

$$P^{SS}_{t+1} = d^S_t + (1 - d^S_t)\pi^S_t,$$  \hspace{1cm} (2.1)

$$P^{\tilde{S}S}_{t+1} = (1 - d^S_t)\pi^S_t \hspace{1cm} \tilde{S} \neq S.$$  \hspace{1cm} (2.2)

where $\pi^S_t \in [0, 1]$ is the fraction of $S$-type agents of generation $t$, and hence, $\pi^I_t + \pi^C_t = 1$ for every $t \geq 0$. In our framework, for example, $P^{II}_{t+1} = d^I_t + (1 - d^I_t)q_t$ and $P^{IC}_{t+1} = (1 - d^I_t)(1 - q_t)$ where $q_t$ denotes the fraction of individualists in generation $t$, and $1 - q_t$ is that of collectivists. Apparently, it is natural to parameterize the state of culture in the economy by the fraction of individualists, $q_t$; an economy with a larger $q$ is more individualistic (or less collectivist).

Given the transition probabilities $P^{SS}_{t+1}$ and $P^{\tilde{S}S}_{t+1}$, the fraction of individualists $q_t$...
Figure 2.2: Culture Dynamics

evolves by the following law of motion:

\[ q_{t+1} = q_t + q_t(1 - q_t)(d^I_t - d^C_t) \]  \hspace{1cm} (2.3)

There are some notable comments on this framework. First, generation \( t+1 \)'s culture traits are solely determined by generation \( t \), and therefore, the culture parameter, \( q_{t+1} \), is predetermined in view of generation \( t + 1 \). Also, a parent cannot entirely determine his own child's culture type as long as \( d_S^t \) is smaller than one; the social environment plays a role as well.

Figure 2.2 describes evolution of the state of culture, \( q_t \), given by equation (2.3). Let us denote \( \delta_t \equiv d^I_t - d^C_t \in [-1, 1] \) the intensity of individualism transmission relative to collectivism. Then, \( q_t \) increases over time when \( \delta_t \in (0, 1] \). In a word, society becomes more individualistic as parents transmit individualism relatively stronger than collectivism. The opposite case that \( \delta_t \in [-1, 0) \) leads to decreasing individualism over time. Intuitively, \( q_{t+1} = q_t \) if \( \delta_t = 0 \).

There are two trivial steady states, \( q = 0 \) and \( q = 1 \). When \( q_t = 0 \) (or \( q_t = 1 \)), \( q_{t+s} = 0 \) (or \( q_{t+s} = 1 \) \( \forall s \geq 1 \). That is, in the case of extreme cultural biases where society is composed of only one type of cultures, it does not admit the other type of
culture forever. This is simply because there is no one who can transmit the other culture type in those cases; see equations (1) and (2).

**Agent’s problem**

After the socialization that determines culture types $S \in \{I, C\}$ for generation $t$, each agent draws his own creativity $z \in [0,1]$ from a continuous time-invariant c.d.f., $F(\cdot)$, at the beginning of period $t$. Then, the probability of success in innovation (or individual efficiency level in innovation) is given by $\psi^S(z) \in [0,1]$ for culture type $S$ where $d\psi^S(z)/dz > 0$. Without loss of generality, we assume that $\psi^S(0) = 0$ and $\psi^S(1) = 1$. In this environment, a $S$-type agent of generation $t$ with a skill level $z$ chooses his occupation by maximizing the discounted expected value of family consumption net of effort costs. To do so, he solves the following problem:

$$v^S_t(z) \equiv \max_{o^S_t(z) \in \{W,E\}, d^S_t \in [0,1]} c^S_{t+1}(z) - \kappa_t(d^S_t) + \beta \mathbb{E}_z[c^S_{t+2}(z)|S]$$

s.t. $c^S_{t+1}(z) = \max\{e^S w_t, (1 - \rho_t)\psi^S(z)\lambda^S p_t\} + \theta_t$,

$$\mathbb{E}_z[c^S_{t+2}(z)|S] = P^S_{t+1}\mathbb{E}_z[c^S_{t+2}(z)] + P^S_{t+1}\mathbb{E}_z[c^S_{t+2}(z)].$$

where $P^S_{t+1}$ and $P^S_{t+1}$ are given by (1) and (2).

The family value, $v^S_t(z)$, consists of two parts. The first part is utility from what he consumes when old, $c^S_{t+1}(z)$. He chooses his occupation in order to maximize the consumption. At the same time, he is altruistic in that he cares his child’s utility. Specifically, $v^S_t(z)$ is increasing in his child’s expected consumption, $\mathbb{E}[c^S_{t+2}(z)|S]$. He, thus, tries to get his child’s expected consumption larger at the cost of disutility from shaping her child’s culture type, $\kappa_t(d^S_t)$. We now describe details of each part in $v^S_t(z)$.

**Occupation Choice:** To maximize his own consumption in period $t + 1$, $c^S_{t+1}(z)$,
he chooses his occupation after drawing creativity parameter \( z \). \( o^S_t(z) \in \{W, E\} \) is occupation choice between a “worker” and an “innovator” for a S-type agent with \( z \). If he becomes a worker, he earns \( e^S w_t \) consumption goods as labor income where \( e^S \) denotes culture-specific effort level of labor supply, or roughly speaking, working hours for production of consumption goods. On the other hand, if he becomes an innovator, he succeeds with probability of \( \psi^S(z) \) that strictly increases in \( z \). Meanwhile, \( \lambda^S \) is the culture-specific efficiency level in innovation, and \( p_t \) is the market price of property rights.

Finally, the government randomly expropriates private property rights with a probability of \( \rho_t \in [0, \bar{\rho}] \) where \( \bar{\rho} < 1 \).\(^{17}\) Hence, \( \rho \) captures the institutional quality of society proxied by the “expropriation risk” à la Acemoglu et al. (2001). Equivalently, one can also interpret \( \rho \) as a tax rate on innovation by the law of large numbers. \( \theta_t \) is a lump-sum transfer that is equally distributed to all agents by the government through revenues from expropriations of property rights. Note that the utility from consumption is assumed to be linear, i.e., risk-neutral. Hence, the uncertainty associated with innovation is irrelevant to the occupation choice, and this simplifies the analysis a lot.\(^{18}\)

In the model, culture (individualism/collectivism) plays a role via differences in the efficiency in the production of goods represented by \( e^S \) and in innovation parametrized by \( \lambda^S \). Regarding the culture-specific efficiency levels, we posit the following condition, which is crucially based on the empirical evidence of Fact 1:

\(^{17}\) We will discuss what \( \bar{\rho} \) means in detail when investigating a role of \( \rho \) in making growth miracles in Section 8.

\(^{18}\) More generally, with a utility function \( u(c) \), the agent problem is given by:

\[
v^S_t(z; \beta) \equiv \max_{o^S_t(z) \in \{W, E\}, d^S_t \in [0,1]} u^S_t(z) - \kappa_t(d^S_t) + \beta \mathbb{E}_z[u^S_{t+2}(z)|S]
\]

s.t. \( u^S_{t+1}(z) = \max \left\{ u(c^SW_{t+1}), \mathbb{E}[u(c^SE_{t+1})|z] \right\} \),

\[
c^SW_{t+1} = e^Sw_t + \theta_t, c^SE_{t+1} = \lambda^S p_t + \theta_t,
\]

\[
\mathbb{E}[u(c^SE_{t+1})|z] = (1 - \rho_t) \psi^S(z) u(c^SE_{t+1}) + \rho_s \psi^S(z) u(\theta_t) + [1 - \psi^S(z)] u(\theta_t),
\]

\[
\mathbb{E}_z[u^S_{t+2}(z)|S] = P^{SS}_{t+1}\mathbb{E}_z[u^S_{t+2}(z)] + P^{S\tilde{S}}_{t+1}\mathbb{E}_z[u^S_{t+2}(z)].
\]
Assumption 3 \( \lambda \equiv \lambda^I > \lambda^C \equiv 1; \ e \equiv e^C > e^I \equiv 1. \)

Assumption 1 indicates that individualism is better in innovation than collectivism but worse for the production of goods. Also note that, under Assumption 1, \( e^I \equiv e^I/\lambda^I = 1/\lambda < 1 \) and \( e^C \equiv e^C/\lambda^C = e > 1 \), implying that, with everything else same, an individualist’s opportunity cost of being an innovator is smaller than a collectivist’s opportunity cost of being an innovator. Hence, Assumption 1 implies that an individualist, \textit{ceteris paribus}, has comparative and absolute advantages in innovation, while a collectivist has comparative and absolute advantages in the production of goods; see Fact 1.\(^{19}\)

**Parental Effort:** As already noted, a parent is altruistic in that he wants to raise his child’s consumption that depends on his child’s creativity level, \( z \). Since a child’s creativity level, \( z \), is drawn at the beginning of period \( t+1 \), a parent do not know his child’s ability in period \( t \). Hence, a parent tries to maximize his child’s \textit{expected} consumption over \( z \). For simplicity, we assume that parents’ skill levels do not correlate with children’s skill levels.

Future consumption depends not only on \( z \) but also on children’s culture type \( S' \in \{I,C\} \), which is conditional on parents’ culture type \( S \) through the transition probabilities, \( P^{SSS} \) and \( P^{SS\bar{S}} \). Therefore, a child’s expected consumption is conditioned on his parent’s culture trait as in the agent problem. To increase the expected consumption, each parent makes the altruistic effort \( d_S^t \) in order to transmit his own culture type \( S \) to his child. Meanwhile, the parental care, \( d_S^t \), incurs costs, measured

---

\(^{19}\) For the other part of Assumption 1 such that \( e \equiv e^C > e^I \equiv 1 \), one can think of a situation that collectivists work together, and their production, say \( G \), is super-additive such that, for any positive \( N \), \( G(1,1,\cdots,1) = Ne > N \) when \( N \) collectivist agents co-operate to produce goods. If they equally divide the total production \( Ne \) over \( N \) coworkers, each agent gets \( e > 1 \). This argument is based on the fact that collectivist cultures emphasize goal congruency and conformity to organizations (Hofstede, 2001). Therefore, when working as a team, collectivists are likely to have a tightly integrated and cohesive team that acts as a monolithic unit, resulting in a higher efficiency in the production of goods. Also, there is evidence that collectivists tend not to indulge in social loafing; people exerting less effort to achieve a goal when they work in a group than when they work alone, see e.g., Earley (1989). In a similar spirit, Gorodnichenko and Roland (2017) also posit that collectivists have a competitive edge in the production of final goods since collectivism makes coordinated actions easier.
by $\kappa_t(d^S_t)$. It is worth noting that, as is obvious from the problem, the occupation choice $o^S_t(z)$ and the optimal level of the parental effort $d^S_t$ are independent of each other, and thereby, $d^S_t$ does not depend on the individual creativity level $z$, but only on his culture type $S$. This is simply because $c^S_t+1$ and $c^S_{t+2}$ are separable. This property simplifies the analysis a lot along with the risk-neutrality on consumptions.

**Voting for Institutional Quality:** According to the time line illustrated in Figure 2.1, each agent of generation $t$ votes for the institutional quality in period $t$, $\rho_t$, after the occupation choice. For that, he will be able to choose a more preferred institutional quality given any pair of $(\rho_{1,t}, \rho_{2,t})$ under his optimal choices of $o^S_t(z)$ and $d^S_t$. For a clearer understanding of the model mechanism, for the time being, we take $\rho_t$ as exogenously given for all $t \geq 0$. Later, we will let it be determined by majority voting in each period $t$ given state variables $(A_{t-1}, q_t)$ where $A_{t-1}$ is aggregate productivity and $q_t$ is the state of culture.

**Producer’s problem**

There are homogeneous producers with unit measure producing consumption goods using the following technology:

$$Y_t = A^\alpha_t L^\eta_t \bar{A}^{1-\alpha}_t$$  \hspace{1cm} (2.4)

where $\bar{A}_t$ is the average level of productivity in period $t$, and it is the same as $A_t$ in any symmetric equilibrium. $L_t$ is aggregate labor in efficiency units. As standard in the endogenous growth literature, $\bar{A}_t$ reflects the positive externality of innovations in forms of knowledge diffusion. Technically, this assumption gives rise to the linearity of the model, and thereby, analytical simplicity.\(^{20}\)

\(^{20}\)Note that, in any symmetric equilibrium, equation (4) collapses to $Y_t = A_t L^\eta_t$, the technology linear in knowledge. The main conclusions in this chapter are not changed without the positive externality of knowledge.
The productivity $A_t$ evolves by the following fashion:

$$A_t = \gamma_t A_{t-1} \quad (2.5)$$

We let $\gamma_t = \sigma X_t$ ($\sigma > 0$) where $X_t$ is aggregate innovation created by generation $t$.\footnote{More generally, one can assume that $\gamma_t = G(X_t)$ where $G(\cdot)$ is monotone increasing, but the main results are not changed.} Hence, the long-term growth rate of GDP per capita in a steady state (where occupation choices, parental effort, and prices are stationary, while consumptions, profits and the output are growing at the same rate), say $g_y$, is given by:

$$g_y = \sigma X - 1 \quad (2.6)$$

$g_y$ is strictly increasing in $X$, which is the aggregate innovation in a steady state.

In each period $t$, given any price vector, $(w_t, p_t)$, each producer maximizes profits $\pi_t$ by hiring $L_t$ and purchasing $X_t$ that solve the following problem:

$$\pi_t \equiv \max_{L_t \geq 0, X_t \geq 0} \, Y_t - w_t L_t - p_t X_t$$

where $Y_t$ and $A_t$ are given by equations (2.4) and (2.5), respectively.

**Equilibrium with fixed institutions**

In any equilibrium, producers maximize profit, given the wage $w_t$ and the price of ideas $p_t$, by hiring workers and purchasing ideas until the marginal production of labor and ideas are the same as the wage and the price of innovation, respectively. Then, the prices are determined in a symmetric equilibrium as follows:

$$w_t = \frac{\partial Y_t}{\partial L_t} = \eta A_t^{\alpha} L_t^{\eta-1} A_t^{1-\alpha} = \eta A_t L_t^{\eta-1} = \eta \sigma A_{t-1} X_t L_t^{\eta-1} \quad (2.7)$$

$$p_t = \frac{\partial Y_t}{\partial A_t} = \sigma A_{t-1}^{\alpha} L_t^{\eta-1} A_t^{1-\alpha} = \alpha \sigma A_{t-1} L_t^{\eta} \quad (2.8)$$
which implies that each producer earns the same profit, $\pi_t = (1 - \alpha - \eta)Y_t$ in a symmetric equilibrium.

Meanwhile, each agent of generation $t$ chooses an occupation that yields a higher consumption $c^S_{t+1}(z)$. As said, his own consumption $c^S_{t+1}(z)$ and his child’s consumption $c^S_{t+2}(z)$ are separable. Hence, it is easy to derive the skill threshold $(z^I_t, z^C_t)$ such that a $S$-type agent with $z \geq z^S_t$ becomes an innovator, and a worker otherwise.

Since the probability of success in innovation, $\psi^S(z)$, is monotone increasing in $z$, the skill threshold $z^S_t$ is unique for any price vector $(w_t, p_t)$. For simplicity, we let
$$\hat{\eta}_t \equiv \eta/((\alpha(1 - \rho_t))$$
and
$$x_t \equiv X_t/L_t.$$ Also recall that $\varepsilon^S \equiv e^S/\lambda^S$ is a $S$-type agent’s opportunity cost of being an innovator. Then, using equations (2.7) and (2.8), we have\(^{22}\):

$$z^S_t = \min \left\{ (\psi^S)^{-1}(\varepsilon^S \hat{\eta}_t x_t), 1 \right\} \quad \forall S \in \{I, C\} \quad (2.9)$$

To derive the optimal level of parental effort $d^S_t$ explicitly, we let $\kappa_t(d^S) \equiv \kappa(d^S, A_t)$ as a quadratic function in $d^S$ such that $\kappa_t(d^S) = \kappa A_t(d^S)^2/2$. Note that $\kappa_t(d^S)$ is linear in $A_t$. This guarantees stationarity of $d^S_t$ against permanent growth of the economy. The optimal effort level $d^S_t$ is then derived as follows:

$$d^I_t = \begin{cases} \beta \kappa^{-1}(1 - q_t)\Delta_{t+1}, & \Delta_{t+1} \geq 0 \\ 0, & o.w. \end{cases} \quad (2.10)$$

$$d^C_t = \begin{cases} \beta \kappa^{-1} q_t(-\Delta_{t+1}), & \Delta_{t+1} \leq 0 \\ 0, & o.w. \end{cases} \quad (2.11)$$

where $\Delta_{t+1} \equiv \mathbb{E}_z[c^I_{t+2}(z) - c^C_{t+2}(z)]/A_t$. That is, $\Delta_{t+1}$ is the premium of being an individualist relative to a collectivist for generation $t + 1$, adjusted by the current productivity level, $A_t$, for its stationarity against permanent growth. $\kappa > 0$ is a constant that measures disutility from $d^S_t$. Later, we will show that if $\kappa > 0$ is sufficiently large, the premium of being an individualist $\Delta_{t+1}$ is bounded both above

\(^{22}\) Equation (2.9) implies that the skill threshold is increasing in the opportunity cost of being an innovator, $\varepsilon^S$. Hence, if $\psi^I = \psi^C$, then $z^I_t \leq z^C_t$, which indicates that an individualist is more likely to be an innovator than a collectivist. This is consistent with the empirical fact that an individualist tends to be a researcher more than a collectivist with other things being equal.
and below, so that \( d^i_t \in [0, 1] \).

One can notice from (2.10) and (2.11) that the premium plays a crucial role in the transmission of cultural traits. An individualist parent will try to transmit his own culture trait \( S = I \) to his child if, and only if, being an individualist is more valued than being a collectivist. That is, \( d^I_t > 0 \) if, and only if, \( \Delta_{t+1} > 0 \). Similarly, a collectivist parent tries to transmit his own culture trait \( S = C \) to his child in the opposite case, i.e., \( d^C_t > 0 \) if, and only if, \( \Delta_{t+1} < 0 \).

Given the skill thresholds \( z^S_t \), the aggregate labor and innovation, \( L_t \) and \( X_t \), are determined as follows:

\[
L_t = q_t F(z^I_t) + (1 - q_t) eF(z^C_t) \tag{2.12}
\]

\[
X_t = q_t \lambda \int_{z^I_t}^1 \psi^I(z) dF(z) + (1 - q_t) \int_{z^C_t}^1 \psi^C(z) dF(z) \tag{2.13}
\]

The lump-sum transfer through government expropriations, \( \theta_t \), is given by the government budget constraint that equates transfers to the value of expropriated innovator income as follows:

\[
\theta_t = \rho_t p_t X_t \tag{2.14}
\]

Finally, in an equilibrium, the state variables \( A_{t-1} \) and \( q_t \) evolve according to the laws of motion in equations (2.5) and (2.3), respectively, where \( X_t \) is derived from

---

23 This suggests that there is no bias in the parent’s altruistic motivation on the transmission of individualism/collectivism in that parents try to maximize their children’s expected consumptions regardless of whether children have different types of the cultural trait from their parents’ traits. This is because the cultural trait of individualism/collectivism has a significant effect on the measure of economic success (the expected consumption here). On the other hand, parents may be “paternalistic” for “pure” cultural traits that have no effect on the objective economic opportunities in that they wish to transmit their own traits, and do not internalize some measures of economic success for their children, which is called “imperfect empathy.” For example, Bisin and Verdier (2000) posit that parents are happier if their children have the same religious traits as parents’ ones. Similarly, Tabellini (2008b) and Chakraborty et al. (2016) also consider the paternalistic bias on cultural traits toward cooperation and entrepreneurship, respectively. Technically, this paternalistic bias adds backward-looking features to the dynamics of culture presented by equation (15). However, the main theoretical conclusions in this chapter are not changed even when the paternalistic bias is allowed in our model although the evolutionary dynamics of culture, \( q_t \), will be somewhat changed. We will discuss welfare implications when allowing the paternalistic bias in our model in another footnote.
2.4 Culture Dynamics with Fixed Institutions

From now on, we confine our attention on a symmetric equilibrium only. Hence, any equilibrium hereafter refers to a symmetric one even when there is no explicit reference. Before investigating theoretical results, for a more general discussion, we define the economy in a symmetric equilibrium for generation $t$, say $E_t$, which abstracts from culture and institutions as follows:

**Definition 2.1 Economy:** Given the aggregate productivity, the state of culture and institutional quality, the economic equilibrium in period $t$ is a list:

$$E_t = \{(o^S_t(z), c^S_{t+1}(z)), (\Delta_{t+1}, d^S_t), (z^S_t, L_t, X_t), (A_t, Y_t, \pi_t), (w_t, p_t) | S \in \{I, C\}\}$$

where each agent of generation $t$ maximizes the family value $v^S_t(z)$, and every producer maximizes the profit $\pi_t$ given any price vector $(w_t, p_t)$ clearing every market in the economy.

Then, we can prove that $E_t$ is uniquely determined given a vector of state variables, which is argued by the following proposition$^{24}$:

**Proposition 2.1** Given any $(A_{t-1}, q_t, q_{t+1}, \rho_t, \rho_{t+1}) \in \mathbb{R}_+^+ \times [0, 1]^2 \times [0, \bar{\rho}]^2$, $E_t$ is uniquely determined. Specifically, there are continuous functions: $L_t = L(q_t, \rho_t)$, $X_t = X(q_t, \rho_t)$, $A_t = A(A_{t-1}, q_t, \rho_t)$, $\Delta_{t+1} = \Delta(q_{t+1}, \rho_{t+1})$, and $d^S_t = d^S(q_t, q_{t+1}, \rho_{t+1})$. Finally, if $\kappa > 0$ is sufficiently large, $d^S_t \in [0, 1]$ ($S \in \{I, C\}$) $\forall (q_t, q_{t+1}, \rho_{t+1}) \in [0, 1]^2 \times [0, \bar{\rho}]$.

Proposition 2.1 indicates that all allocations and prices in the economy are uniquely determined in any equilibrium once culture and institutions are exogenous. This is

$^{24}$ All proofs are offered in Appendix B.1.
simply because our model without culture and institutions collapses to a standard endogenous growth model with a production technology linear in knowledge (or capital).

We now endogenize \( q_t \) while letting \( \{ \rho_t \}_{t \geq 0} \) exogenously given. From Proposition 2.1, it only remains to determine \( q_{t+1} \) in order to solve for \( E_t \) given state variables in each period \( t \), \((A_{t-1}, q_t)\), along with \( \{ \rho_t \}_{t \geq 0} \). Since \( d_t^S = d_t^S(q_t, q_{t+1}, \rho_{t+1}) \) from Proposition 2.1 and culture dynamics obeys equation (3) in any equilibrium, the law of motion for \( q_{t+1} \) can be rewritten as follows:

\[
\Phi(q_t, q_{t+1}; \rho_{t+1}) = q_t + q_t(1 - q_t)\delta(q_t, q_{t+1}; \rho_{t+1}) - q_{t+1} = 0 \quad (2.15)
\]

where \( \delta_t \equiv d_t^I - d_t^C \). That is, an equilibrium state of culture in period \( t + 1 \), \( q_{t+1} \), is a fixed point that satisfies equation (2.15) given \( q_t \) and \( \rho_{t+1} \). Then, one can define a correspondence \( Q : [0, 1] \rightarrow P([0, 1]) \) that includes such fixed points:

\[
Q(q_t; \rho_{t+1}) = \{ q_{t+1} \in [0, 1] | \Phi(q_t, q_{t+1}; \rho_{t+1}) = 0 \} \quad (2.16)
\]

If \( Q \) has multiple elements for some \((q, \rho) \in [0, 1] \times [0, \bar{\rho}]\), then the model inherits multiple equilibria for \( q_{t+1} \) that satisfy equation (2.15), and thereby, the economic equilibrium \( E_t \) is not unique at least for those \((q, \rho) \in [0, 1] \times [0, \bar{\rho}]\). On the contrary, if the correspondence \( Q \) is a singleton for any \((q, \rho) \in [0, 1] \times [0, \bar{\rho}]\), \( q_{t+1} \) is uniquely pinned down by \((q_t, \rho_{t+1})\), and hence, the entire time path of culture, \( \{q_t\}_{t \geq 1} \), is also uniquely determined for any given \( q_0 \in [0, 1] \) with exogenously chosen \( \{\rho_t\}_{t \geq 0} \).

Here, we provide a sufficient condition on \( \Delta(q_{t+1}; \rho_{t+1}) \) that guarantees that \( Q \) is a

\[25\] We can abstract from any change in the state of culture over time by letting \( \beta = 0 \). In a word, this means that parents’ altruistic motivations are not existing. As is obvious from equations (10) and (11), \( d_t^S = 0 \) for all \( t \geq 0 \) if \( \beta = 0 \). Then, \( q_{t+1} = q_t \) for all \( t \geq 0 \) from equation (3), so that \( q_t \) is constant at the initial state of culture, \( q_0 \). Hence, from Proposition 2.1, the economic equilibrium \( E_t \) is uniquely pinned down for all \( t \geq 0 \) given any initial state variables \((A_{t-1}, q_0)\) and institutions, \( \{\rho_t\}_{t \geq 0} \). Also, once \( \rho_{t+s} = \rho_t \) for all \( s \geq 1 \), the economy immediately converges to the stationary state where the skill threshold \( z^S \) and the prices \((w_1, p_1)\) are constant, while consumption, output per capita and the profit are growing at the same rate of \( \sigma X - 1 \).
singleton with some notable properties on the dynamics of culture \( \{q_t\}_{t \geq 0} \) as follows:

**Proposition 2.2** Suppose that \( \Delta(q; \rho) \) is monotone decreasing in \( q \in (0, 1) \). Then,

(i) **unique culture dynamics:** \( Q \) is a singleton \( \forall \rho \in [0, \bar{\rho}] \), and thus, there is a single-valued function \( \phi : [0, 1] \times [0, \bar{\rho}] \to [0, 1] \) such that \( q_{t+1} = \phi(q_t; \rho_{t+1}) \) that satisfies the law of motion for \( q_{t+1} \) given by (15) with trivial steady states at \( \phi(0; \rho) = 0 \) and \( \phi(1; \rho) = 1 \) \( \forall \rho \in [0, \bar{\rho}] \). Also, \( \phi(q; \rho) \) is continuous at any \( q \in [0, 1] \).

Suppose further that \( \Delta(0; \rho) > 0 \) and \( \Delta(1; \rho) < 0 \). Then,

(ii) **unique interior steady state:** There exists a unique \( \bar{q} \in (0, 1) \) such that \( \phi(q; \rho) = \bar{q} \) where \( \Delta(q; \rho) = 0 \).

(iii) **monotone convergence to the steady state:** \( q_t < \phi(q_t; \rho) < \bar{q} \) \( \forall q_t \in (0, \bar{q}) \), and \( \bar{q} < \phi(q_t; \rho) < q_t \) \( \forall q_t \in (\bar{q}, 1) \).

(iv) **monotonicity:** \( \phi(q_t; \rho) \) is monotone increasing \( \forall q_t \in (0, 1) \setminus \bar{q} \).

Part (i) of Proposition 2.2 shows that, if \( \Delta(q; \rho) \) is monotone decreasing in \( q \), we have a unique time path of culture with \( \{\rho_t\}_{t \geq 0} \) exogenously given. Since \( \Delta(q; \rho) \) denotes the premium of being an individualist relative to a collectivist, it is plausible to assume that \( \Delta(q; \rho) \) is in fact decreasing in \( q \). This is because \( X(q; \rho) \) is more likely to be higher, while \( L(q; \rho) \) tends to be smaller as \( q \) rises. It means that the mass of innovations per worker in effective units, \( x \equiv X/L \), is likely to get larger with a larger fraction of individualists since an individualist is better in innovation than a collectivist. This, in turn, leads to a smaller value of being an innovator since the price of innovation, \( p \), decreases with a higher \( x \); see (2.7) and (2.8). Hence, the premium of being an individualist will be smaller if there are more individualists since the return to innovation, which is more likely to go for an individualists, decreases as the number of innovator increases.

If we further assume that the premium value of being an individualist is positive when there is no individualist in society, i.e., \( \Delta(0; \rho) > 0 \), and it becomes negative when society is full of individualists, \( \Delta(1; \rho) < 0 \), then we have additional results summarized by parts (ii)-(iv) of Proposition 2.2.
From Proposition 2.2, we can describe culture dynamics \( \{q_t\}_{t=0}^{\infty} \) that converges to a unique steady state \( \bar{q} \) given an initial state of culture \( q_0 \) with an exogenous stream of institutional qualities, \( \{\rho_t\}_{t=0}^{\infty} \). For a clearer understanding, we draw an example in Figure 2.3 where the institutional quality, \( \rho_t \) is fixed at \( \hat{\rho} \in [0, \bar{\rho}] \) for the purpose of exposition.

First, from part (i) of Proposition 2.2, we can find a single-valued continuous function \( \phi \) that maps \( (q_t, \rho_{t+1}) \) to \( q_{t+1} \) where \( \rho_{t+1} = \hat{\rho} \) as depicted in Figure 2.3. We know that \( \phi(q, \rho) \) always has two trivial steady states at \( q = 0 \) and \( q = 1 \) so does \( \phi(q, \hat{\rho}) \). Second, from part (ii) of Proposition 2.2, there is a unique interior steady state \( \bar{q} \) that solves \( \phi(\bar{q}, \hat{\rho}) = \bar{q} \). Also, from part (iii) of Proposition 2.2, \( q_{t+1} = \phi(q_t, \hat{\rho}) \) is bigger than \( q_t \) and smaller than \( \bar{q} \) if \( q_t \in (0, \bar{q}) \). Hence, \( \phi(q_t, \hat{\rho}) \) is drawn above the 45 degree line and below \( \bar{q} \) for any \( q_t \in (0, \bar{q}) \). Meanwhile, in the region of \( q_t \in (\bar{q}, 1) \), \( q_{t+1} \) is smaller than \( q_t \) and bigger than \( \bar{q} \), so that \( \phi(q_t, \hat{\rho}) \) is drawn above \( \bar{q} \) and below the 45 degree line for any \( q_t \in (\bar{q}, 1) \). Finally, from part (iv) of Proposition 2.2, \( \phi(q_t, \hat{\rho}) \) increases monotonically with \( q_t \). Hence, it does not have any downward bending character as in Figure 2.3.

Then, as in the lower panel of Figure 2.3, with \( \rho_{t+s} \) being fixed at \( \hat{\rho} \) \( \forall s \geq 0 \), society converges globally (and monotonically) to the unique interior steady state \( \bar{q} \) unless it starts from one of the two trivial steady states, \( q = 0 \) and \( q = 1 \). Since it is obvious that the value of the steady state \( \bar{q} \) depends on the level of the institutional quality \( \rho \), one can define a function \( \bar{q}(\cdot) \) that maps \( \rho \) to \( \bar{q} \) such that \( \bar{q} : [0, \bar{\rho}] \rightarrow [0, 1] \). For instance, \( \bar{q} = \bar{q}(\hat{\rho}) \) in the examples of Figure 2.3.

In summary, \( \bar{q} \) is a non-trivial unique global attractor given that the institutional quality, \( \rho \), is fixed at \( \hat{\rho} \in [0, \bar{\rho}] \). This implies that the world converges to \( \bar{q} \) globally as long as the exogenous institutions \( \rho \) is the same across societies.\(^{26}\) This global

\(^{26}\) Suppose that the social planner maximizes the utilitarian social welfare for generation \( t \) where the Pareto weights are the same between the two culture groups, treating the institutional quality \( \rho \) as given. Then, the steady state is also socially optimal in the \textit{ex ante} sense although the long-term growth rate may not be maximized in the steady state. However, if there is the paternalistic bias as in Bisin and Verdier (2000), Tabellini (2008b) and Chakraborty et al. (2016), the decentralized allocation will be different from the social optimum in general.
Figure 2.3: Culture Dynamics with Institutions Fixed
convergence appears simply because $\Delta(q) > 0 \forall q \in (0, \bar{q})$, so that parents try to raise their children as individualists. Meanwhile, $\Delta(q) < 0$ in the region of $q \in (\bar{q}, 1)$, so that parents try to transmit collectivist cultures into their children. One can easily prove this convergence argument rigorously by using parts (i)-(iii) of Proposition 2.2, and thus, we provide the following corollary without a proof:

**Corollary 2.1 Culture Dynamics with Institutions Fixed:** Suppose that $\Delta(q; \rho)$ is monotone decreasing in $q \in (0, 1)$, and $\Delta(0; \rho) > 0$ and $\Delta(1; \rho) < 0 \forall \rho \in [0, \bar{\rho}]$. Then, with the institutional quality $\rho \in [0, \bar{\rho}]$ being fixed, society globally converges to a unique interior steady state $\bar{q} \in (0, 1)$ where $\Delta(\bar{q}; \rho) = 0$ if, and only if, $q_0 \in (0, 1)$.

Now, we know that the state of culture $q_t$ converges to a unique interior steady state $\bar{q}$, and the steady state $\bar{q}$ solves $\bar{q} = \bar{q}(\rho)$. Then, how will the steady state of culture, $\bar{q}$, change in response to a change of the institutional quality, $\rho$? To be specific, will $q$ increase or decrease if the institutional quality becomes worse? We now investigate the comparative statistics on the steady state of culture with respect to a change in the institutional quality, and its growth implications.

To do this, we first note from Figure 2.3 that, assuming differentiability for the purpose of exposition, the sign of $d\bar{q}(\rho)/d\rho$ is solely determined by how $\Delta(q, \rho)$ responds to a change in $\rho$.27 For example, if $\partial \Delta(q; \rho)/\partial \rho < 0 \forall q \in [0; 1]$, then $\Delta(q; \rho)$ will shift leftward whenever $\rho$ rises, and vice versa. This leads to a lower value of $\bar{q}$, which means that society will converge to a steady state with a smaller number of individualists. In a word, if the premium of being an individualist, $\Delta(q; \rho)$, decreases with a higher rate of tax on innovation, society ends up being less individualistic when the tax rate on innovation increases. This comparative statistics is intuitively straightforward. Suppose that the premium value of being an individualist decreases

27 More formally, we have: $$\frac{d\bar{q}(\rho)}{d\rho} = -\frac{\partial \Delta(\bar{q}; \rho)/\partial q}{\partial \Delta(\bar{q}; \rho)/\partial \rho}$$ by the implicit function theorem. Since we have assumed that $\partial \Delta(q; \rho)/\partial q < 0$, the sign of $d\bar{q}(\rho)/d\rho$ is solely determined by the sign of $\partial \Delta(q; \rho)/\partial \rho$. 
along with a higher tax rate on innovation. Then, the decrease in the premium should be compensated by a smaller $q$ in equilibrium since the premium increases as $q$ decreases. This means that the mass of individualists should be smaller in equilibrium as the government taxes on innovations more heavily.

From now on, we posit that $\Delta$ is monotone decreasing in $\rho$, i.e., $\partial \Delta(q; \rho) / \partial \rho < 0 \forall q \in [0; 1]$ if differentiable. This is a plausible assumption. Intuitively, an innovator’s after-tax payoff is decreasing in the tax rate, $\rho$, and therefore, the premium of being an individualist gets smaller with a higher tax rate with other things constant since an individualist is more likely to be an innovator thanks to his advantages in innovation compared to a collectivist. This assumption also leads to the theoretical conclusion consistent with the empirical evidence of Fact 3 that individualism and the protection of property rights are positively correlated with each other.

We summarize the comparative statistics using the diagrams in Figure 2.4. As suggested by the upper panel of Figure 2.4, we assume that $\Delta(q; \rho)$ is monotone decreasing in $\rho$. In this situation, once the protection of property rights is aggravated from $\rho_L$ to $\rho_H$ where $\rho_H > \rho_L$, the steady state of culture decreases from $\bar{q}_H$ to $\bar{q}_L$. Hence, the society converges to a new steady state characterized by weaker individualism and more expropriations on innovation. The comparative statistics also means that club convergence is possible when we introduce culture and institutions into an otherwise standard growth model. Given different institutions across the world, each society converges to its own steady state depending on its institutional quality.

**Growth under Exogenous Institutions**

Let us now investigate growth implications from the comparative statistics of a change in the institutional quality. To do that, we need to solve for the economic equilibrium in a steady state. We have understood so far how culture dynamics $\{q_t\}_{t=0}^\infty$ is uniquely determined given $q_0$ and $\{\rho_t\}_{t=0}^\infty$. Then, from Proposition 2.1, we can solve for the economic equilibrium in period $t$, $\mathcal{E}_t$, for all $t \geq 0$ once the initial
Figure 2.4: Different Steady States by Different Institutions
aggregate productivity, \(A_{-1}\), is given.

Suppose that we have already solved a sequence of economic equilibrium, \(\{\mathcal{E}_t\}_{t=0}^\infty\). Then, recall that the output and consumption per capita are growing at the same rate of \(\sigma X - 1\) in a steady state where culture and institutions are stable such that \(q \equiv q_{t+s} = q_t\) and \(\rho \equiv \rho_{t+s} = \rho_t\) \(\forall s > 1\). Society, therefore, enjoys higher growth in the long run as \(X\), the aggregate innovation in a steady state, increases. This means that the long-term growth rate depends entirely on how \(X\) is changed in response to a variation in the institutional quality \(\rho\). From Proposition 2.1, one can notice that \(X\) is a function of \(\bar{q}\) and \(\rho\), which denote the steady state values of culture and institutions, respectively. Since, \(\bar{q}\) is a function of \(\rho\) as already seen, we can characterize impacts of institutions on growth over a long period as follows:

\[
\frac{dX}{d\rho} = \frac{\partial X}{\partial \rho} \frac{d\bar{q}}{d\rho} \frac{\partial X}{\partial \bar{q}}
\]

The direct effect in equation (2.17) captures a change in the aggregate innovation by a change in the institutional quality, while the indirect effect in equation (2.17) captures an indirect effect of institutions on the aggregate innovation that emerges through a change in the state of culture. It is straightforward from equation (2.17) that we should determine signs of \(\partial X/\partial \rho\) and \(\partial X/\partial q\) in order to assess the sign of \(dX/d\rho\). However, \(X\) depends crucially on functional assumptions on the skill distribution, \(F(z)\), and the probability of success in innovation, \(\psi^S(z)\), so that how \(X\) will response to a variation in \(\rho\) or \(q\) is not, in general, determined \textit{a priori} without specifying \(F(z)\) and \(\psi^S(z)\).

That said, it is plausible to guess that \(X\) gets larger as the society is more individualistic in the long run, i.e., \(\partial X/\partial q > 0\). This is simply because an individualist is more productive in innovation than a collectivist, and thereby, the more individualists in a society, the more innovations. If so, the sign of the indirect effect turns out to be negative since, as we have already seen, \(d\bar{q}/d\rho < 0\). In other words, when
property rights are less secure, the society becomes less individualistic, resulting in a decrease in $X$. This is the indirect effect of institutions on the aggregate innovation via culture changes.

Similarly, it is also realistic to think that a higher tax rate discourages innovation, i.e., $\partial X/\partial \rho < 0$. This is because, *ceteris paribus*, the return to innovation, $(1 - \rho)p$, decreases with a higher $\rho$, and vice versa. If so, the sign of the direct effect becomes negative same as that of the indirect growth effect. In sum, better institutions in terms of protection of property rights lead to more innovations in the long run, and hence, higher long-term growth both by providing larger incentives to innovate (the direct growth effect) and by shaping cultures more productive in innovation (the indirect growth effect).

Figure 2.5 illustrates the direct and indirect growth effects of institutions. Note that the y-axis of the lower panel is the growth rate in a steady state. The upper panel indicates that $X$ gets smaller with a higher tax rate (higher $\rho$), while it gets bigger with stronger individualism (higher $q$).

Suppose now that the economy is in the steady state where the institutional quality and the state of culture are given by $\bar{q}$ and $\bar{q}(\rho_L)$, respectively. In this state, if the institutional quality is aggravated in that $\rho$ rises from $\rho_L$ to $\rho_H$, then the aggregate innovation immediately declines from $X_1$ to $X_2$. This change is done without any variation in the original state of culture, $\bar{q}(\rho_L)$, and thus, $\sigma(X_2 - X_1)$ in the lower panel identifies the direct growth effect. Meanwhile, the increase in $\rho$ results in the decrease in $\bar{q}$ from $\bar{q}(\rho_L)$ to $\bar{q}(\rho_H)$ over time, leading to a further decrease in $X$ from $X_2$ to $X_3$ after some periods of time. This change emerges along with a change in the state of culture, so that $\sigma(X_2 - X_3)$ corresponds to the indirect growth effect.

We now consider an example of the theory by taking specific functional assumptions on $F(z)$ and $\psi^S(z)$ to solve for $X$ and $\bar{q}$. This will validate the assumptions we have had so far to clarify the growth implications, arising from the interaction of culture and institutions. Details of the derivation are provided in Appendix B.2.
Figure 2.5: Growth Effects of Institutions
Example 2.1 Suppose that \( z \) is uniformly distributed, and \( \psi^I(z) = \psi^C(z) = z^\psi \).\(^{28}\)

Then, \( X \) is given by:

\[
X(\bar{q}; \rho) = \frac{\mu(\bar{q}(\rho))}{1 + \psi + \hat{\eta}(\rho)},
\]

where \( \bar{q} \) solves:

\[
\begin{align*}
\mu(\bar{q}(\rho)) &= \frac{\lambda - 1}{\psi(1 + \psi + \hat{\eta}(\rho))} \\
\chi(\bar{q}(\rho)) &= \bar{q} \lambda (1 - \psi) + (1 - \bar{q}) e^{1 + \frac{1}{\psi}}.
\end{align*}
\]

Then, with Assumption 1, as long as the skill threshold is interior, i.e., \( z^S \in (0, 1) \), we can show that:

\[
\frac{d\bar{q}(\rho)}{d\rho} < 0, \quad \frac{\partial X(\bar{q}; \rho)}{\partial \bar{q}} > 0 \quad \text{and} \quad \frac{\partial X(\bar{q}; \rho)}{\partial \rho} < 0.
\]

As summarized in equation (2.21), society ends up being less individualistic when protection of property rights over innovations becomes worse, i.e., \( d\bar{q}/d\rho < 0 \). Also, the aggregate innovation in the steady state, \( X \), decreases when society is less individualistic with the institutional quality being fixed, i.e., \( \partial X/\partial \bar{q} > 0 \), confirming the negative indirect growth effect of worse protection of property rights. Similarly, the direct growth effect is also negative since \( \partial X/\partial \rho < 0 \).\(^{29}\)

So far, we have investigated how culture evolves over time under exogenously given institutions, and how exogenous institutions affect both culture and the long-

\(^{28}\)The parameter \( \psi \) is the elasticity of the success probability in innovation with respect to the creativity parameter \( z \). This implies that the probability of success in innovation increases with \( z \) more rapidly as the elasticity \( \psi \) gets bigger. Consequently, given the simple skill distribution (uniform distribution), \( \psi \) governs income inequality, which increases with \( \psi \).

\(^{29}\)We also provide sufficient conditions for \( \partial \Delta(q; \rho)/\partial q < 0, \Delta(0, \rho) > 0 \) and \( \Delta(1, \rho) < 0 \) in Appendix B.2.
term growth rate of the economy. We show that $E_t$ is uniquely pinned down given any vector of culture and institutions, and the state of culture converges to a unique interior steady state under the monotonicity of $\Delta(q; \rho)$ in $q$ where $\Delta(0; \rho) > 0$ and $\Delta(1; \rho) < 0$. The interior steady state of culture, $\bar{q}$, depends on the exogenous institutional quality, $\rho$, and institutions affect the long-term growth rate both directly (through incentives to innovate) and indirectly (through cultures of society). Specifically, we argue that more secure property rights induce stronger individualism as well as larger incentives to innovate, both of which contribute to faster growth in the long run.

In Section 5, we let institutions be determined in an endogenous way to explore how endogenous interactions of culture and institutions result in multiple dynamic equilibria near the low-growth steady state, $q_L$, and hence, excess volatility.

### 2.5 Political Equilibrium

In effect, the property rights structure that will maximize rents to the ruler (or ruling class) is in conflict with that would produce economic growth. (North, 1981, p. 28)

Now, we are ready to describe how the society politically determines the institutional quality, $\rho_t$, which measures the expropriation risk by the government or, by the law of large numbers, the tax rate on innovation.

Before we continue, let us try to understand how societies decide the protection of property rights. What factors do underlie Pareto inefficient property rights? Of course, there may be many mechanisms that explain the historical existence of inefficient property rights in a nation.\(^\text{30}\) In particular, conflict of interest among different groups of people has been regarded as a primary reason that may lead to persistently

\(^\text{30}\) North quotes as follows: “Because polities make and enforce economic rules, it is not surprising that property rights are seldom efficient.” (North, 1990, p. 110). As a historical episode, North (1990) compares drastic divergence in institutional qualities in terms of property rights between England and Spain after the 17th century. He also uses this idea, which is employed in Acemoglu et al. (2001) as well, to explain the growth divergence between the North and South colonial regions in America.
inefficient property rights. For example, Douglass North argues that a government tends to favor inefficient property rights since it tends to satisfy interests of a certain group of voters whose political power is strong:

\[
\cdots \text{the ruler will avoid offending powerful constituents. If the wealth or income of groups with close access to alternative rulers is adversely affected by property rights, the ruler will be threatened. Accordingly, he will agree to a property rights structure favorable to those groups, regardless of its effects upon efficiency. (North, 1981, p. 28)}
\]

\[
\cdots \text{argument to account for the obvious persistence of inefficient property rights. These inefficiencies existed because rulers would not antagonize powerful constituents by enacting efficient rules that were opposed to their interests} \cdots. \text{ (North, 1990, p. 52)}
\]

Adam Smith also noted this possibility based on human nature as follows\textsuperscript{31}:

Every independent state is divided into many orders and societies, each of which has its own particular powers, privileges, and immunities. Every individual is naturally more attached to his own particular order or society, than to any other. \cdots He is ambitious to extend its privileges and immunities. He is zealous to defend them against encroachments of every other order or society. \textit{(Smith, 1759, p. 271)}

To reflect this insight on externality — or philosophy in another angle —, we consider conflict of interest among different voting groups that determines \(\rho_t\) endogenously through the majority voting rule. This reveals why inefficient protection of property rights can be built and last for a long period of time.

We consider a typical political environment under majority voting. There are two political parties, \(P_1\) and \(P_2\), that competes with each other in order to take the public authority that determines the tax rate \(\rho_t\).\textsuperscript{32} They announce \(\rho_{1,t}\) and \(\rho_{2,t}\), respectively, at the voting stage in each period \(t\). Given the pair of policies, \((\rho_{1,t}, \rho_{2,t})\), each agent of generation \(t\) votes for either \(P_1\) or \(P_2\) closer to his interest, one of which wins if it gets more votes than the other. If they have the same number of votes, they flip

\textsuperscript{31} For possible connections between Adam Smith and Douglass North in methodology and substance, one can refer to Kim (2007).

\textsuperscript{32} As will be clear, the theoretical results are not changed even when there are many political parties, not just two.
a coin to determine the winner, and hence, win the election with the probability of 1/2.

In a standard dynamic game without commitment, history of a player (political party here), defined by the sequence of its policies up to period $t - 1$, may result in multiple subgame-perfect equilibria through reputation mechanisms that are history-dependent. However, in our setting, the two political parties all aim at taking the power to choose the institutional quality, and there are no interests other than that. Therefore, the hold-up problem from the lack of commitment does not arise here simply because they do not have any incentive to break the commitment. This, in turn, implies that there is no room for history-dependent punishment strategies, and hence, subgame-perfect equilibria coincide to Markov-perfect equilibria, defined by a set of strategies that depend only on the current state of the economy.

According to the timing of events, the occupation choice is already done at the voting stage. This implies that the skill threshold $z_t^S$ is already fixed without regard to $\rho_t$, so that $A_t$ is also pinned down before voting. Hence, $(A_t, q_t)$ are the state variables for $P_i$’s strategy. Then, an equilibrium policy, $\rho_t$, is determined by a map $R$ such that $\{\rho_t\} = R(A_t, q_t)$, which is, of course, endogenous and will be derived below.

Now, let us investigate how the institutional quality $\rho_t$ is determined in an equilibrium. For a $S$-type agent with ability $z$ in generation $t$, the most preferred institutional quality, say $\rho_t^S(z)$, maximizes the family value $v_t^S(z)$ as follows:

$$\rho_t^S(z) \equiv \arg \max_{\rho_t \in [0, \bar{\rho}]} v_t^S(z)$$

$$= \arg \max_{\rho_t \in [0, \bar{\rho}]} c_t^S(z) - \frac{\kappa (d_t^S)^2}{2A_t} + \mathbb{E}[c_{t+2}^S(z) | S]; \ (A_t, q_t) \ \text{are given.} \quad (2.22)$$
First note that the map $\mathcal{R}(A_{t+1}, q_{t+1})$ can be rewritten as follows:

$$
\{\rho_{t+1}\} = \mathcal{R}(A_{t+1}, q_{t+1})
= \mathcal{R}(A(A_{t}, q_{t}, \rho_{t+1}), \phi(q_{t}, \rho_{t+1}))
= \tilde{\mathcal{R}}(A_{t}, q_{t}, \rho_{t+1})
$$

(2.23)

where, in equation (2.23), we use $A_{t+1} = A(A_{t}, q_{t}, \rho_{t+1})$ from Proposition 2.1, and $q_{t+1} = \phi(q_{t}, \rho_{t+1})$ from Proposition 2.2. We can notice from the map $\tilde{\mathcal{R}}$ that $\rho_{t+1}$ is independent of $\rho_{t}$ once $(A_{t}, q_{t})$ are already fixed. Technically, this is because, as is obvious from (2.23), an equilibrium tax rate $\rho_{t+1}$ is a fixed point of the map $\tilde{\mathcal{R}}$ with $(A_{t}, q_{t})$ being given. In a word, this means that generation $t$ cannot affect $\rho_{t+1}$ through $\rho_{t}$. As we will see in the next section, $\rho_{t+1}$ is jointly determined together with $q_{t+1}$ through $d_{t}^{S}$, which is the parental effort on culture traits transmission. That is, $\rho_{t+1}$ is affected by generation $t$ only through $d_{t}^{S}$.

Now recall that $d_{t}^{S} = d^{S}(q_{t}, q_{t+1}, \rho_{t+1})$ from Proposition 2.1. Then, $d^{S}$ can be rewritten as follows:

$$
\begin{align*}
\quad d_{t}^{S} &= d^{S}(q_{t}, q_{t+1}, \rho_{t+1}) \\
&= d^{S}(q_{t}, \phi(q_{t}, \rho_{t+1}), \rho_{t+1}) \\
&= \tilde{d}^{S}(q_{t}, \rho_{t+1})
\end{align*}
$$

We already knew that $\rho_{t+1}$ is independent of $\rho_{t}$ with $(A_{t}, q_{t})$ being fixed, and thus, $d_{t}^{S}$ is also independent of $\rho_{t}$. Similarly, $\mathbb{E}[c_{t+2}^{S}(z)|S]$ is independent of $\rho_{t}$ as long as $(A_{t}, q_{t})$ are already fixed. Technically, this is because $\mathbb{E}[c_{t+2}^{S}(z)|S]$ is a function of the state variables in period $t+1$, $(A_{t}, q_{t+1})$. Since $q_{t+1} = \phi(q_{t}, \rho_{t+1})$, and $\rho_{t+1}$ is irrelevant to $\rho_{t}$ when $(A_{t}, q_{t})$ are already fixed, $\mathbb{E}[c_{t+2}^{S}(z)|S]$ is also independent of $\rho_{t}$. In a word, the current institutional quality, $\rho_{t}$, cannot affect the expected consumption in the future, $\mathbb{E}[c_{t+2}^{S}(z)|S]$, by itself. Rather, $\mathbb{E}[c_{t+2}^{S}(z)|S]$ depends on the future state of culture, $q_{t+1}$.
In summary, once the occupation choice is already done, the institutional quality in time $t$ can no longer affect both the parental effort on children $d^S_t$ and children’s future consumption $E[c^S_{t+2}(z)|S]$. We can, therefore, rewrite equation (2.22) as follows:

$$
\rho^S_t(z) \equiv \arg \max_{\rho_t \in [0, \bar{\rho}]} c^S_{t+1}(z); \ (A_t, q_t) \ \text{are given.}
$$

In equilibrium, the lump-sum transfer $\theta_t$ is determined by equation (2.14), and it is monotone increasing in $\rho_t$ once the occupation choice is done. This is because $(z^S_t, p_t, w_t)$ are already determined and fixed after the occupation choice, and hence, irrelevant to $\rho_t$. Since $\theta_t$ is equally distributed to all agents of generation $t$, it is straightforward that every innovator prefers a lower $\rho_t$, while every worker prefers a higher $\rho_t$. That is, there are two voting groups whose interest is sharply conflicting each other. More formally, we can summarize the following single-peaked preference for the each voting group:

For $\{IW, CW\}$, \( \rho_H \succeq \rho_L \ \forall \rho_H \geq \rho_L \), \hspace{1cm} (2.24)

For $\{IE, CE\}$, \( \rho_L \succeq \rho_H \ \forall \rho_H \geq \rho_L \), \hspace{1cm} (2.25)

where $\{IW, IE, CW, CE\}$ represent individualist workers, individualist innovators, collectivist workers and collectivist innovators, respectively.

As in equation (2.24) and (2.25), all workers and innovators, respectively, have the same ordinality on the policy variable, $\rho \in [0, \bar{\rho}]$, regardless of the heterogeneous skill levels $z$. We can, thus, simply think of a situation that there are only two representative voters having different political power that corresponds to the number of voters in each group. More importantly, from (2.24) and (2.25), we notice that the single-peaks of preferences of the two voting groups are at the opposite corners, respectively; the most preferred policy for $\{IW, CW\}$ is $\bar{\rho}$ while 0 for $\{IE, CE\}$.

In this situation, suppose that innovators have relatively stronger or weaker political power than workers, which is parametrized by $\xi \in [-1/2, 1/2]$, and hence, one of
the political parties wins if it gets innovators’ votes bigger than $1/2 - \xi$ regardless of the number of votes it gets.\textsuperscript{33} Then, by applying the median voter theorem slightly differently, we can provide the following theoretical result on the political equilibrium:

**Proposition 2.3** There is always a unique Nash equilibrium such that $\rho_t = 0$ if the fraction of innovators is larger than $1/2 - \xi$, and $\rho_t = \bar{\rho}$ otherwise.

Proposition 2.3 implies that the fraction of innovators plays a crucial role in the determination of the political equilibrium. Hence, we need to know how the fraction of innovators in period $t$, say $\hat{X}_t$, is determined in the model. From the occupation choice problem, $\hat{X}_t$ is given by:

$$\hat{X}_t = q_t \int_{z_t^I}^1 dF(z) + (1 - q_t) \int_{z_t^C}^1 dF(z) \quad (2.26)$$

Equation (2.26) indicates that one can define a function $\hat{X}$ such that $\hat{X}_t = \hat{X}(q_t, \rho_t)$, which is continuous at any $q_t \in [0, 1]$ $\forall \rho \in [0, \bar{\rho}]$ since $z_t^S (S \in \{I, C\})$ is a continuous function of $(q_t, \rho_t)$ by (2.9) and Proposition 2.1.

Now, recall that voting is done after the occupation choice. Therefore, agents of generation $t$ should forecast the equilibrium tax rate, $\rho_t$, at the occupation choice stage. More formally, suppose that the political party $P_i$’s strategy is given by a map $\mathcal{P}_i (i = 1, 2)$. Then, Proposition 2.3 implies that:

$$\{\rho_{i,t}\} = \mathcal{P}_i(q_t; \rho^e_t) = \begin{cases} 
\{0\}, & \hat{X}(q_t, \rho^e_t) > \frac{1}{2} - \xi \\
\{\bar{\rho}\}, & \hat{X}(q_t, \rho^e_t) \leq \frac{1}{2} - \xi 
\end{cases} \quad (i = 1, 2)$$

where $\rho^e_t$ is an expected institutional quality by generation $t$ at the occupation choice stage. Of course, $\rho^e_t$ is simply given from $P_i$’s perspective, so that it does not care about $\rho^e_t$. Hence, the set of strategy is always a singleton $\forall q_t \in [0, 1]$ and, as in

\textsuperscript{33} Without loss of generality, we assume that innovators lose the election when the fraction of them is equal to $1/2 - \xi$. 

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the proof of Proposition 2.3, it is symmetric, i.e., \( P_1(q_t; \rho_t^c) = P_2(q_t; \rho_t^c) \). Then, one can denote a map that gives an equilibrium policy given \( \rho_t^c \) as \( \{ \rho_t \} = \mathcal{P}(q_t; \rho_t^c) = P_1(q_t; \rho_t^c) = P_2(q_t; \rho_t^c) \).

Meanwhile, as said, generation \( t \) should foresight \( \rho_t \) at the occupation choice stage, and the perfect foresight assumption means that:

\[
\{ \rho_t^c \} = \mathcal{P}(q_t; \rho_t^c) \quad \forall q_t \in [0, 1]
\]

where \( \rho_t^c \in \{ 0, \bar{\rho} \} \). Then, one can define a set-valued function that gives the political equilibrium at the occupation choice stage, which is consistent with the perfect foresight assumption, as follows:

\[
\{ \rho_t \} = \mathcal{R}(q_t) = \left\{ \rho \in \{ 0, \bar{\rho} \} \mid \bar{\rho}^{1/2} \frac{1}{2} \leq \frac{1}{2} - \xi \right\}
\] (2.27)

In a word, the perfect foresight assumption means that, given the state of culture \( q_t \), each agent of generation \( t \), before choosing her occupation, predicts an equilibrium policy \( \rho_t \) through the map \( \mathcal{R}(q_t) \), i.e., \( \rho_t^c \in \mathcal{R}(q_t) \).

Here are a few remarks. First, the map \( \mathcal{R} \) depends only on \( q \), while it is independent of \( A \). This means that \( q_t \) is the only payoff-relevant state variable for the determination of \( \rho_t \), while \( A_t \) is not. Technically, this is because an agent’s choices, \((o^{S}(z), d^{S})\), are scale-independent. As one can find it from equation (2.9) and Proposition 2.1, the occupation choice is independent of \( A \). \( d^{S} \) is also irrelevant to \( A \) in a similar way. This property, of course, simplifies the analysis a lot since we need to focus only on the state of culture to determine the political equilibrium. More importantly, this implies that the way of determining the political outcome under majority voting is irrelevant to the wealth of a nation, and hence, the political equilibrium is determined by the same map \( \mathcal{R}(q) \) regardless of weather the country is rich or poor.
That is, in our theory, the only factor underlying the determination of the political outcome is the state of culture, not the wealth of a nation.
Another remark is that an equilibrium policy \( \rho \) always gets only 0, \( \bar{\rho} \), or both. This is because the policy preferences of the two voting groups, given by (24) and (25), are so sharply conflicting that the most preferred policies of the two groups are persistently incompatible. Technically, this property simplifies the analysis further since we need to focus only on \([0, 1] \times \{0, \bar{\rho}\}\), not \([0, 1] \times [0, \bar{\rho}]\), for the domain of \( \hat{X}(q, \rho) \). Lastly, the image of the map \( \mathcal{R}, \{\rho_t\} \), is not an empty set \( \forall q_t \in [0, 1] \); it is either \( \{0\}, \{\bar{\rho}\} \) or \( \{0, \bar{\rho}\} \).

To investigate how the state of culture \( q_t \) determines the political equilibrium \( \rho_t \) through the map \( \mathcal{R}(q_t) \) given by (2.27), we need to know how \( \hat{X}(q, \rho) \) varies with respect to \( q \) given \( \rho \in \{0, \bar{\rho}\} \). However, it is not definite \textit{a priori} since \( \hat{X}(q, \rho) \) depends crucially on functional assumptions on \( F(z) \) and \( \psi^S(z) \) as shown in equation (2.26). To facilitate the analysis, we rely on some assumptions on \( \hat{X}(q, \rho) \). These restrictions are not only plausible but also useful in that they make the analysis on the endogenous determination of culture and institutions a lot simpler.

First, we assume that \( \hat{X}(q, \rho) \) has an inverse U-shaped relationship in \( q \). This non-monotonic relationship arises from two different effects of \( q \) on \( \hat{X}(q, \rho) \), opposing each other. When \( q \) increases, \( \hat{X}(q, \rho) \) also increases since an individualist is more inclined to be an innovator. This is a positive effect. At the same time, the more individualists in society, the more able innovators it has. Hence, self-selection for innovators becomes severe, i.e, the skill threshold \( z^S \) rises. This severe self-selection makes the number of innovators smaller, and this is a negative effect.

The positive effect dominates the negative one when the degree of individualism \( q \) is smaller than a certain value. Once \( q \) gets bigger than the value, the self-selection effect dominates, and thus, \( \hat{X} \) starts to decrease. These conflicting effects render an inverse U-shaped relationship between \( q \) and \( \hat{X} \) for any given \( \rho \in \{0, \bar{\rho}\} \). We present empirical evidence supporting this relationship in Appendix B.3.\textsuperscript{34}

Second, we posit that \( \hat{X}(q, 0) > \hat{X}(q, \bar{\rho}) \) \( \forall q \in [0, 1] \). In a word, there are fewer

\textsuperscript{34} We show that the fraction of workers employed in R&D sectors has an inverse U-shaped relationship with the Hofstede individualism index by using the RDS dataset that includes information on R&D activities across the OECD and other major economies more than 30 years from 1980.
innovators whenever the government taxes innovators heavily, which is quite straightforward. The incentive to be an innovator will decrease with other things constant as the tax rate on innovation increases from the minimum level to the maximum, $\bar{\rho}$. This leads to fewer innovators in society. Since this restriction is extensively used to facilitate the analysis, we take an example to check whether or not it is validate. Details of the derivation are provided in Appendix B.2.

**Example 2.2** Suppose that $z$ is uniformly distributed and $\psi^I(z) = \psi^C(z) = z^\psi$ as in Example 1. Then, $\hat{X}$ is given by:

$$
\hat{X}(q, \rho) = 1 - \varphi(q) [\hat{\eta}(\rho) x(q, \rho)]^{\frac{1}{\psi}}
$$

where

$$
\varphi(q) = q \lambda^{-\frac{1}{\psi}} + (1 - q) e^{\frac{1}{\psi}},
$$

$$
x = x(q, \rho) = \left[ \frac{1}{\hat{\eta}(\rho)} \frac{\mu(q)}{(1 + \psi + \hat{\eta}(\rho)) \chi(q)} \right]^{\frac{\psi}{1 + \psi}},
$$

$x \equiv X/L$, and $\hat{\eta}(\rho)$, $\mu(q)$ and $\chi(q)$ are given by (18), (19) and (20), respectively. Then, with Assumption 1, as long as the skill threshold is interior, i.e., $z^S \in (0, 1)$, we can show that:

$$
\frac{\partial \hat{X}(q, \rho)}{\partial \rho} < 0 \quad \forall q \in [0, 1].
$$

Hence, equation (2.28) validates the last assumption that $\hat{X}(q, 0) > \hat{X}(q, \bar{\rho}) \forall q \in [0, 1]$ as expected.35

Figure 2.6 illustrates how the political equilibrium, given by $R(q_t)$, is determined under the two restrictions on $\hat{X}(q, \rho)$; the fraction of innovators, $\hat{X}(q, \rho)$, takes an inverse U-shaped relationship with $q_t$ and $\hat{X}(q, 0) > \hat{X}(q, \bar{\rho}) \forall q \in [0, 1]$. First note that $\hat{X}(q_t, 0)$ is smaller than $1/2 - \xi$ in the region of $q_t \in [0, \hat{q}_L]$. This implies

---

35 Sufficient conditions for an inverse U-shaped relationship between $\hat{X}(q, \rho)$ and $q$ are provided in Appendix B.2.
that \( \hat{X}(q, \hat{\rho}) \) is also smaller than \( 1/2 - \xi \) in this region since \( \hat{X}(q, 0) > \hat{X}(q, \hat{\rho}) \) \( \forall q \in [0, 1] \). Hence, according to the map \( \mathcal{R}(q_t) \) given by equation (2.27), the political equilibrium is uniquely determined in this region such that \( \rho_t = \hat{\rho} \). We call this political equilibrium “expropriation regime” since the government taxes innovators heavily for sure.

In the region of \( q_t \in (\tilde{q}_H, 1] \), the fraction of innovators is larger than \( 1/2 - \xi \) even under the maximum tax rate, i.e., \( \hat{X}(q_t, \hat{\rho}) > 1/2 - \xi \), and therefore, the political equilibrium is definite, which is given by \( \rho_t = 0 \). We call this equilibrium “non-expropriation regime” since the tax rate on innovation is minimized for sure in this region.

Meanwhile, in the region of \( q_t \in (\tilde{q}_L, \tilde{q}_H] \), \( \hat{X}(q_t, 0) \) is larger than \( 1/2 - \xi \), while \( \hat{X}(q_t, \hat{\rho}) \) is smaller than \( 1/2 - \xi \). We, therefore, have multiple political equilibria such that \( \{\rho_t\} = \{0, \hat{\rho}\} \). In other words, both the minimum and maximum tax rates are consistent with the conditions for the political equilibrium.
We now provide an useful result for the political equilibrium, $\mathcal{R}(q)$, with the restriction that $\hat{X}(q,0) > \hat{X}(q,\bar{\rho}) \ \forall q \in [0,1]$. One can easily prove the result using Proposition 2.3, so that we present the following corollary without a proof:

**Corollary 2.2 Political Equilibrium:** Suppose that $\hat{X}(q,0) > \hat{X}(q,\bar{\rho}) \ \forall q \in [0,1]$. Then, the political equilibrium is given by:

$$
\{\rho_t\} = \mathcal{R}(q_t) = \begin{cases} 
\{0\}, & \hat{X}(q_t,\bar{\rho}) > \frac{1}{2} - \xi \\
\{\bar{\rho}\}, & \hat{X}(q_t,0) \leq \frac{1}{2} - \xi \\
\{0,\bar{\rho}\}, & \text{o.w.}
\end{cases}
$$

(2.29)

Here is an implication from the political equilibrium worth noting. The degree of individualism correlates positively with the institutional quality. Recall that $\rho_t = \bar{\rho}$ for sure when $q_t$ is small enough, while $\rho_t = 0$ for sure when $q_t$ is large enough. For an intermediate degree of $q_t$, characterized by $q_t \in (\hat{q}_L, \hat{q}_H]$, society has either the minimum or the maximum tax rate in equilibrium. This monotone property is led by the first assumption on $\hat{X}(q,\rho)$ that $\hat{X}$ increases with a bigger $q$ although it starts to decrease as $q$ gets to the point high enough. As in Figure 2.6, if the society is barely individualistic ($q_t \in [0,\hat{q}_L]$), there are few individualists, who are more likely to be innovators compared to collectivists. Hence, the minimum tax rate cannot be enforced since, even under the minimum tax rate on innovation, the number of innovators is not enough high to win the election. Then, the only possible equilibrium is the maximum tax rate, $\bar{\rho}$. On the contrary, if the society is highly individualistic ($q_t \in (\hat{q}_H,1]$), there are many people having the competitive edge in innovation. Then, the minimum tax rate is a unique political equilibrium since, even under the maximum tax rate on innovation, there are enough innovators to win the election.
2.6 Joint Determination of Culture and Institutions

We now investigate how culture and institutions are jointly determined. In each period $t$, an agent of generation $t$ chooses $d^S_t$, the intensity of culture transmission, that incurs disutility, $\kappa_t(d^S_t)$. Then, $\delta_t \equiv d^I_t - d^C_t$, the relative strength of individualism transmission, shapes the state of culture for the next generation through the law of motion of $q_t$ given by equation (2.15).

Since $d^S_t$ is a function of $(q_t, q_{t+1}, \rho_{t+1})$ as in Proposition 2.1, generation $t$ should forecast $(q_{t+1}, \rho_{t+1})$ given $q_t$ for the optimal choice of $d^S_t$. In the perfect foresight setting, they predict $(q_{t+1}, \rho_{t+1})$ correctly as they do for $\rho_t$. This implies that they know how $(q_{t+1}, \rho_{t+1})$ are determined in equilibrium, and mathematically, this is equivalent with that they solve the following system of equations:

$$
q_{t+1} = \phi(q_t, \rho_{t+1}) \quad (2.30)
$$

$$
\rho_{t+1} \in \mathcal{R}(q_{t+1}) \quad (2.31)
$$

where $\mathcal{R}(\cdot)$ is given by equation (2.29), or more generally, by (2.27). Let us denote the set of solutions as $\{(q_{t+1}, \rho_{t+1})\}$ of which an element, $(q_{t+1}, \rho_{t+1})$, jointly solves (2.30) and (2.31).

One can easily notice from equations (2.30) and (2.31) that culture and institutions are interconnected, and they are jointly determined given a current state of culture, $q_t$; $q_{t+1}$ is determined by institutions $\rho_{t+1}$, which is also determined by $q_{t+1}$ at the same time. Then, what type of interactions can describe the joint determination of culture and institutions from equations (2.30) and (2.31)?

Intuitively, if innovations are expected to be taxed heavily ($\rho_{t+1} = \bar{\rho}$), parents will not raise their children as individualists, leading to weak individualism in the future. This implies that equation (2.30) can be characterized by decreasing $q_{t+1}$ with a higher $\rho_{t+1}$. In reverse, if society becomes less individualistic, the number of innovators will
be small since an individualist is better in innovation than a collectivist. This causes
the tax rate on innovation to be higher as suggested by Corollary 2.2. Hence, one
can infer from (2.31) that \( \rho_{t+1} \) tends to increase along with a lower \( q_{t+1} \).

Consequently, equations (2.30) and (2.31) indicate a strategic complementarity
between higher individualism and better protection of property rights in terms of the
tax rate on innovation, and vice versa. As apparent below, this strategic interaction in
the joint determination of culture and institutions permits multiple equilibria in each
period, which we refer to as multiple dynamic equilibria. The equilibria consist of
the two different combinations of culture and institutions with contrasting properties.
Before presenting a formal proposition, we will investigate the intuition underlying
this feature with more details by using an example depicted in Figure 2.7.

There are two panels in Figure 2.7. The upper panel illustrates the political
equilibrium, which is given by equation (2.31), while the lower one depicts the law of
motion for \( q_{t+1} \), which is given by equation (2.30). Since the tax rate is always chosen
to be either 0 or \( \bar{\rho} \) in equilibrium, in the upper panel, there are only two \( \dot{X}(q_{t+1}, \rho_{t+1}) \),
both of which correspond to \( \rho_{t+1} = 0 \) and \( \rho_{t+1} = \bar{\rho} \), respectively. Similarly, in the
lower panel, there are only two laws of motion for \( q_{t+1} \) that correspond to \( \rho_{t+1} = 0 \)
and \( \rho_{t+1} = \bar{\rho} \), respectively.

We first note from Corollary 2.2 that there are three cases for the political equi-
librium depending on the level of \( q_{t+1} \). The three cases are characterized by two
thresholds, \( \hat{q}_L \) and \( \hat{q}_H \). The government can impose the maximum tax rate, \( \bar{\rho} \), in
time \( t+1 \) when the future individualism is not high enough that \( q_{t+1} \) is smaller than
the high threshold, \( \hat{q}_H \), i.e., \( q_{t+1} \in [0, \hat{q}_H] \). Meanwhile, the tax rate can be minimized
in time \( t+1 \) when the future individualism is not low enough that \( q_{t+1} \) is higher than
the low threshold, \( \hat{q}_L \), i.e., \( q_{t+1} \in (\hat{q}_L, 1] \). Hence, there is a region where both the
minimum tax rate (\( \rho_{t+1} = 0 \)) and the maximum tax rate (\( \rho_{t+1} = \bar{\rho} \)) are possible at
the same time. This region is for an intermediate degree of individualism such that
\( q_{t+1} \in (\hat{q}_L, \hat{q}_H] \). In contrast, we have the region where only the maximum tax rate is
imposed for sure (the expropriation regime) when the future individualism is weak

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enough that \( q_{t+1} \in [0, \hat{q}_L] \). Similarly, only the minimum tax rate is enforced for sure (the non-expropriation regime) when the future individualism is strong enough that \( q_{t+1} \in (\hat{q}_H, 1] \).

We then move to the lower panel to investigate how the culture parameter, \( q_t \), evolves with institutions being endogenously determined. To do that, we first map the two political thresholds, \( \hat{q}_L \) and \( \hat{q}_H \), from the upper panel to the lower panel. Since \( q_{t+1} \) is on the vertical axis in the lower panel while it is on the horizontal axis in the upper panel, we use the 45 degree line in the lower panel in order to map the two thresholds. As depicted in the graph, \( \hat{q}_L \) and \( \hat{q}_H \) are marked on the vertical axis in the lower panel.

Now, recall first that the minimum tax rate cannot be imposed if \( q_{t+1} \) is smaller than the low threshold, \( \hat{q}_L \). Hence, given the current state of culture, \( q_t \), if \( q_{t+1} \) is lower than the low threshold even under the minimum tax rate, then the minimum tax rate cannot be enforced for sure in time \( t+1 \). This means that \( q_t \) cannot evolve along with \( \phi(q, 0) \), the law of motion associated with the minimum tax rate, as long as \( q_{t+1} = \phi(q_t, 0) \) is smaller than the low threshold, \( \hat{q}_L \), since the minimum tax rate cannot satisfy the equilibrium condition represented by equation (2.31) in this case. In other words, the law of motion, \( \phi(q, 0) \), must not be used for culture dynamics if \( q_{t+1} = \phi(q_t, 0) \) is smaller than the low threshold, \( \hat{q}_L \). On the contrary, the minimum tax rate can be enforced whenever \( q_{t+1} \) is larger than the low threshold, \( \hat{q}_L \). Hence, the equilibrium condition (2.31) is satisfied for the minimum tax rate in this case, and this implies that the law of motion, \( \phi(q, 0) \), can be applied to the dynamics of culture to satisfy the other condition represented by equation (2.30). In sum, the law of motion, \( \phi(q, 0) \), can be used if \( q_{t+1} = \phi(q_t, 0) \) is larger than the low threshold, \( \hat{q}_L \). This is why \( \phi(q, 0) \) gets thick in the lower panel of Figure 2.7 for the case that \( q_{t+1} \) is larger than the low threshold, \( \hat{q}_L \), in order to indicate that \( \phi(q, 0) \) can be applied in that case.

Similarly, recall that the maximum tax rate, \( \bar{\rho} \), cannot be imposed when \( q_{t+1} \) is higher than the high threshold, \( \hat{q}_H \). Hence, the maximum tax rate cannot be imposed
Figure 2.7: Culture and Institutions Jointly Determined
in period \( t + 1 \) if, given \( q_t \), \( q_{t+1} \) is higher than the high threshold even under the maximum tax rate, \( \bar{\rho} \). That is, if \( q_{t+1} = \phi(q_t, \bar{\rho}) \) is larger than the high threshold, \( \hat{q}_H \), the maximum tax rate, \( \bar{\rho} \), cannot satisfy the equilibrium condition given by equation (2.31). In contrast, if \( q_{t+1} = \phi(q_t, \bar{\rho}) \) is smaller than the high threshold, the maximum tax rate can be imposed, implying that \( q_t \) can follow the law of motion with \( \bar{\rho} \), which is given by \( \phi(q, \bar{\rho}) \), to satisfy the other equilibrium condition represented by equation (2.30). In sum, the law of motion, \( \phi(q, \bar{\rho}) \), can be used if \( q_{t+1} = \phi(q_t, \bar{\rho}) \) is smaller than the high threshold, \( \hat{q}_H \). This is why \( \phi(q, \bar{\rho}) \) gets thick only when \( q_{t+1} \) is smaller than \( \hat{q}_H \).

We are now ready to explain how culture and institutions are jointly determined in equilibrium through generation \( t \)'s expectations about \((q_{t+1}, \rho_{t+1})\) for the optimal choice of \( d_t^s \). Suppose first that \( q_t \) is smaller than \( \tilde{q}_L \) where \( \tilde{q}_L \) solves \( \hat{q}_L = \phi(\tilde{q}_L, 0) \). This is the case that society is so collectivist in period \( t \) that the number of innovators in period \( t + 1 \) is smaller than \( 1/2 - \xi \) even under the minimum tax rate, and hence, the minimum tax rate cannot be enforced in time \( t + 1 \). Knowing this, generation \( t \) forecasts that \( \rho_{t+1} \) will be chosen to be \( \bar{\rho} \) for sure. Then, \( q_{t+1} \) is determined only by \( \phi(q_t, \bar{\rho}) \). This means that the set of equilibria that consists of equilibrium culture and institutions for generation \( t + 1 \), which is given by \( \{(q_{t+1}, \rho_{t+1})\} \), is a singleton, i.e., \( \{(q_{t+1}, \rho_{t+1})\} = \{\phi(q_t, \bar{\rho})\} \forall q_t \in [0, \tilde{q}] \).

On the contrary, suppose that \( q_t \) is high enough that \( q_t \in (\hat{q}_H, 1] \) where \( \hat{q}_H \) solves \( \hat{q}_H = \phi(\hat{q}_H, \bar{\rho}) \). In this region, society is so individualistic in period \( t \) that the number of innovators in period \( t + 1 \) is larger than \( 1/2 - \xi \) even under the maximum tax rate, and therefore, the maximum tax rate cannot be imposed in time \( t + 1 \). Then, generation \( t \) anticipates correctly that \( \rho_{t+1} \) will be minimized for sure, so that \( q_{t+1} \) is determined only by the law of motion with the minimum tax rate, which is given by \( \phi(q_t, 0) \). Since \( (q_{t+1}, \rho_{t+1}) \) is uniquely determined, the set of equilibrium, \( \{(q_{t+1}, \rho_{t+1})\} \), is a singleton, i.e., \( \{(q_{t+1}, \rho_{t+1})\} = \{(\phi(q_t, 0), 0)\} \forall q_t \in (\hat{q}_H, 1] \).

For an intermediate degree of individualism in period \( t \) such that \( q_t \in (\tilde{q}_L, \hat{q}_H] \), the equilibrium, \( (q_{t+1}, \rho_{t+1}) \), is not uniquely pinned down. This is because both the
laws of motion, $\phi(q, 0)$ and $\phi(q, \bar{\rho})$, are consistent with the two political equilibria, $\rho_{t+1} = 0$ and $\rho_{t+1} = \bar{\rho}$, in this region, and in reverse, both institutional qualities, $\rho_{t+1} = 0$ and $\rho_{t+1} = \bar{\rho}$, are supported by two different $q_{t+1}$ from the two laws of motion, both of which correspond to $\rho_{t+1} = 0$ and $\rho_{t+1} = \bar{\rho}$, respectively. That is, we have multiple equilibria for generation $t + 1$ in this region, which are given by $\{(q_{t+1}, \rho_{t+1})\} = \{(\phi(q_t, 0), 0), (\phi(q_t, \bar{\rho}), \bar{\rho})\}$.

Suppose that society lies in this indeterminacy region. If parents predict that the government will expropriate property rights on innovation in the next period by maximizing tax rate on innovation, then the premium of being an individualist will decrease. This makes parents less passionate to transmit the individualist culture to their children than they would do under the minimum tax rate. This, in turn, leads to weaker individualism in the next period than that would be formed under the minimum tax rate. Consequently, there are too few innovators in the next period to enforce the minimum tax rate on innovation, and the government imposes the maximum tax rate, fulfilling the pessimistic prophecy of generation $t$.

On the contrary, if generation $t$ believes that the government will enforce the minimum tax rate in period $t+1$, then the premium of being an individualist increases, resulting in stronger individualism than that would be formed under the maximum tax rate. Strong individualism in period $t + 1$ period, in turn, renders a large fraction of innovators enough to minimize the tax rate, confirming the optimistic belief.

In summary, multiple equilibria are likely to emerge when society is neither highly collectivistic nor individualistic, i.e., $q_t \in (\tilde{q}_L, \tilde{q}_H]$, so that any belief on the maximum or minimum tax rate is rational. In this situation, equilibrium selection depends entirely on self-fulfilling prophecies by generation $t$. The dynamic complementarity between culture and institutions gives rise to multiple dynamic equilibria with contrasting properties. Recall that strong individualism in period $t + 1$ can support better protection of property rights characterized by the minimum tax rate on innovation. In reverse, the minimum tax rate in period $t + 1$ provides the rationale behind the strong individualism since it raises an innovator’s expected payoff, which is more
likely to go for an individualist. Similarly, weak individualism and bad protection of property rights featured by the maximum tax rate are mutually re-enforcing each other.

For a more general discussion about the role that the interaction between culture and institutions plays in society, we define equilibrium of society after voting, which includes culture and institutions in equilibrium as well as the economic equilibrium in production, occupation choice and consumption.

**Definition 2.2** Society: Given \((A_{t-1}, q_t)\), the social equilibrium after voting in period \(t\) is a list:

\[ S_t = \{(q_t, \rho_t), \mathcal{E}_t\} \]

where \(\rho_t\) is an element of the map \(\mathcal{R}(\cdot)\) given by equation (2.27). \(\mathcal{E}_t\) is the economic equilibrium in period \(t\), defined by Definition 2.1, that corresponds to \((A_{t-1}, q_t, \rho_t, q_{t+1}, \rho_{t+1})\) where \((q_{t+1}, \rho_{t+1})\) is an element of \(\{(q_{t+1}, \rho_{t+1})\}\), the set of solutions for the system of equations given by equations (2.30) and (2.31).

\(S_t\) includes \((q_t, \rho_t)\) and one economic equilibrium in period \(t\). Since \(S_t\) is defined as an equilibrium of society after voting in period \(t\), the tax rate in period \(t\), \(\rho_t\), is simply given from the set of political equilibria such that \(\rho_t \in \mathcal{R}(q_t)\). This implies that, given \((A_{t-1}, q_t)\), \(S_t\) is uniquely pinned down if, and only if, \(\mathcal{E}_t\) is unique. From Proposition 2.1, \(\mathcal{E}_t\) is always uniquely determined for any given vector \((A_{t-1}, q_t, \rho_t, q_{t+1}, \rho_{t+1})\). Hence, given \(q_t\), if \(\{(q_{t+1}, \rho_{t+1})\}\) is a singleton, there is only one economic equilibrium, \(\mathcal{E}_t\). This, in turn, implies that \(S_t\) is uniquely pinned down if, and only if, \(\{(q_{t+1}, \rho_{t+1})\}\) is a singleton.

Suppose that it is not a singleton, then it should have two elements at most such that \(\{(q_{t+1}, \rho_{t+1})\} = \{(\phi(q_t, 0), 0), (\phi(q_t, \bar{\rho}), \bar{\rho})\}\). The first element in the set, \((\phi(q_t, 0), 0)\), represents the rosy belief of generation \(t\) that results in strong individualism with solid protection of property rights over innovations for generation \(t+1\). The other one, \((\phi(q_t, \bar{\rho}), \bar{\rho})\), on the other hand, stands for the dismal belief that leads
to weak individualism with the severe expropriation on innovation. Let $E_t^O$ denote the economic equilibrium under the optimistic prophecy, and $E_t^P$ for the one under the pessimistic prophecy. More precisely, $E_t^O$ is determined under the vector of state variables, $(A_{t-1}, q_t, \rho_t, q_{t+1}, \rho_{t+1}) = (A_{t-1}, q_t, \rho_t, \phi(q_t, 0), 0)$, while $E_t^P$ is pinned down under $(A_{t-1}, q_t, \rho_t, q_{t+1}, \rho_{t+1}) = (A_{t-1}, q_t, \rho_t, \phi(q_t, \bar{\rho}), \bar{\rho})$; $E_t^O$ and $E_t^P$ differ from each other in $((\Delta_{t+1}, d_t^S))$. Then, it is natural to define $S_t^O \equiv \{(q_t, \rho_t), E_t^O\}$ that denotes society in equilibrium governed by the optimistic belief. Similarly, the one governed by the pessimistic belief is given by $S_t^P \equiv \{(q_t, \rho_t), E_t^P\}$.

We know that there may be multiple social equilibria for some $q_t \in [0, 1]$. In the example illustrated in Figure 2.7, when $q_t$ lies in $(\tilde{q}_L, \tilde{q}_H]$, $S_t$ can be either $S_t^O$ or $S_t^P$, depending on generation $t$’s beliefs. More formally, we provide the following theoretical results:

**Proposition 2.4** Suppose that $\Delta(q, \rho)$ is monotone decreasing in $q$, and $\Delta(q, 0) > \Delta(q, \bar{\rho}) \forall q \in [0, 1]$. Also, suppose that $\hat{X}(q, 0) > \hat{X}(q, \bar{\rho}) \forall q \in [0, 1]$ where $\hat{X}(q, 0)$ is either a weakly increasing or an inverse U-shaped function with respect to $q$ such that $\hat{X}(0, 0) \leq \hat{X}(1, 0)$, and there is at most one $q^*_t \in [0, 1]$ such that $\hat{X}(q^*_t, 0) = 1/2 - \xi$. Then,

(i) **existence of the social equilibrium:** $S_t$ exists $\forall t = 0, 1, 2, \ldots$.

Suppose further that $\hat{X}(q, \rho)$ is either a weakly increasing or an inverse U-shaped function with respect to $q$ such that $\hat{X}(0, \bar{\rho}) \leq \hat{X}(1, \bar{\rho})$, and $\xi \in [-1/2, 1/2]$ satisfies $\max_{q \in [0, 1]} \hat{X}(q, 0) > 1/2 - \xi \geq \min_{q \in [0, 1]} \hat{X}(q, \rho)$.

Now define $\hat{q}_L$ that solves $\hat{X}(\hat{q}_L, 0) = 1/2 - \xi$. If there is no such $\hat{q}_L$, $\hat{q}_L = 0$. Similarly, $\hat{q}_H$ solves $\hat{X}(\hat{q}_H, \bar{\rho}) = 1/2 - \xi$. If there is no such $\hat{q}_H$, $\hat{q}_H = 1$. Also define $\tilde{q}_L$ and $\tilde{q}_H$ that solve $\phi(\tilde{q}_L, 0) = \hat{q}_L$ and $\phi(\tilde{q}_H, \bar{\rho}) = \hat{q}_H$, respectively. Then,

(ii) **multiple dynamic equilibria:** there always exists an interval $(\tilde{q}_L, \tilde{q}_H]$ where society with $q_t \in (\tilde{q}_L, \tilde{q}_H]$ has multiple dynamic equilibria such that $\{S_t\} = \{S_t^O, S_t^P\}$, one of which is selected by self-fulfilling expectations about $(q_{t+1}, \rho_{t+1})$. Otherwise, society always has a unique equilibrium. More precisely, $\{S_t\} = \{S_t^O\} \forall q_t > \tilde{q}_H$, and
\{S_t\} = \{S_t^P\} \forall q_t \leq \bar{q}_L.

Furthermore, suppose that \(\Delta(0, \rho) > 0\) and \(\Delta(1, \rho) < 0\) \(\forall \rho \in \{0, \bar{\rho}\}\). Then,

(iii) multiple steady states: There are two interior steady states \(\bar{q}_L \in (0, 1)\) and \(\bar{q}_H \in (0, 1)\) such that \(\bar{q}_H > \bar{q}_L\) where \(\bar{q}_H\) solves \(\phi(\bar{q}_H, 0) = \bar{q}_H\), and \(\bar{q}_L\) solves \(\phi(\bar{q}_L, \bar{\rho}) = \bar{q}_L\).

Finally, suppose that there is at most one \(q^*_p \in [0, 1]\) such that \(\bar{X}(q^*_p, \bar{\rho}) = 1/2 - \xi\), and \(\xi\) is intermediate, so that neither \(\bar{q}_L > \bar{q}_H\) nor \(\bar{q}_H < \bar{q}_L\). Then, we obtain the following stability property that the multiple dynamic equilibria emerge only around \(\bar{q}_L\), which is the steady state characterized by poor protection of property rights and strong collectivism:

(iv) stability of the steady states: \(\bar{q}_H\) is always locally stable but \(\bar{q}_L\) is neither stable nor unstable if \(\bar{q}_H\) is high enough that \(\bar{q}_H > \bar{q}_H\)\(^{36}\) and \(\bar{q}_L\) is not so small that \(\bar{q}_L \geq \bar{q}_L\)\(^{37}\).

In summary, although the economic equilibrium is always uniquely determined given any vector of culture and institutions, the model admits multiple equilibria through different combinations of culture and institutions in equilibrium. The multiplicity stems from the strategic complementarity between culture and institutions. Each social equilibrium consists of an economic equilibrium along with culture and institutions, and equilibrium selection depends entirely on self-fulfilling expectations. Looked at another angle, borrowing Karl Marx’s terms, we developed a theoretical framework in which multiple equilibria arise through considering superstructure (culture and institutions) on top of substructure (the economy).

We lastly remark one implication from the theory. One can interpret part (ii) of Proposition 2.4 as the path dependence (or history dependence) of expectations for society where the culture is extreme. For example, suppose that \(q_L > 0\). If society has strong enough collectivism today such that \(q_t \leq \bar{q}_L\), the social equilibrium is associ-

\(^{36}\) This is a case where \(\frac{\lambda}{e}\) is not too small.

\(^{37}\) This is a case where \(\bar{\rho}\) is not too large. If \(\bar{\rho}\) is large enough to make sure that \(\bar{q}_L \geq \bar{q}_L\), then the steady state, \(\bar{q}_L\), is also locally stable as the other steady state, \(\bar{q}_H\), is. In this case, the region admitting multiple dynamic equilibria arises between the two steady states, \(\bar{q}_L\) and \(\bar{q}_H\).
ated with pessimistic beliefs only, i.e., \( \{S_t\} = \{S_t^P\} \). This leads to strong collectivism tomorrow, and hence, the society is likely to have \( \{S_{t+1} = \{S_{t+1}^P\} \), implying that the dismal beliefs persist. Similarly, if \( q_H < 1 \) and society has strong enough individualism today such that \( q_t > \bar{q}_H \), then optimistic beliefs on the institutional quality in period \( t \) must continue over period \( t+1 \). This suggests that expectations about future policies are history-dependent in economies governed by extreme cultures.

### 2.7 Growth under Endogenous Culture and Institutions

Here, we investigate growth implications of the endogenous interaction between culture and institutions. For a better understanding, we use the example illustrated in Figure 2.8, which consists of three panels. In the top panel, there are two political thresholds, \( \hat{q}_L \) and \( \hat{q}_H \), that characterize the political equilibrium. In the middle panel, there are two different \( \Delta(q, \rho) \), the premium of being an individualist, both of which correspond to the maximum and minimum tax rate on innovation, respectively. Lastly, in the bottom panel, we have two laws of motion, \( \phi(q_t, \rho_{t+1}) \), both of which correspond to \( \Delta(q, \bar{\rho}) \) and \( \Delta(q, 0) \), respectively, and thereby, the maximum and minimum tax rates.

Note from the bottom panel that the society has two different steady states depending on the quality of institutions in terms of the tax rate on innovation. Since the premium of being an individualist is larger when the government does not expropriate innovator, i.e., \( \Delta(q, \bar{\rho}) > \Delta(q, 0) \forall q \in [0, 1] \), as we have seen in Section 4, the steady state without expropriation features strong individualism, denoted as \( \bar{q}_H \), while \( \bar{q}_L \) is the other steady state that features weaker individualism and severe expropriation on innovation. If the aggregate innovation, \( X(q, \rho) \), is increasing in \( q \) but decreasing in \( \rho \) as in Example 1, we have \( X(\bar{q}_H, 0) > X(\bar{q}_L, \bar{\rho}) \). In a word, innovations are more abundant in an individualistic society without any expropriation than a collectivist
society with severe expropriation. Since the long-term growth rate is given by $\sigma X - 1$, society at $\bar{q}_H$ enjoys higher growth than the one at $\bar{q}_L$; recall the negative direct and indirect growth effects of less secure property rights. For this reason, one can call the steady state at $\bar{q}_H$ a “good steady state” in terms of the long-term growth rate. We then call the other steady state at $\bar{q}_L$ a “bad steady state.”

Meanwhile, from the top panel of Figure 2.8, we can notice that society has three different regions for political equilibrium depending on the degree of individualism. When the current individualism is so weak that $q_t \in [0, \bar{q}_L]$, generation $t$ anticipates that next period individualism will be weak enough to cause severe expropriation on innovation for sure. Then, we have a unique equilibrium characterized by $\rho_{t+1} = \bar{\rho}$ and $q_{t+1} = \phi(q_t, \bar{\rho})$. This equilibrium emerges in the dark gray region. On the contrary, when current individualism is so strong that $q_t \in (\bar{q}_H, 1]$, generation $t$ predicts that the next period individualism will be strong enough to enforce the minimum tax rate for sure. Then, the equilibrium is unique such that $\rho_{t+1} = 0$ and $q_{t+1} = \phi(q_t, 0)$, which appears in the white region. Finally, when the current individualism is between $\bar{q}_L$ and $\bar{q}_H$, there are multiple social equilibria from Part (ii) of Proposition 2.4. In period $t + 1$, individualism can be strong enough for the government to minimize the tax rate on innovation (the non-expropriation regime), i.e., $S_t = S_t^O$. At the same time, individualism in period $t + 1$ can be weak enough for the government to impose the maximum tax rate on innovation, $\bar{\rho}$ (the expropriation regime), i.e., $S_t = S_t^P$. This case corresponds to the light gray region where multiple dynamic equilibria emerge. Equilibrium selection between the two regimes depends solely on generation $t$’s self-fulfilling expectations.

Suppose that society is initially in the white region, where it enforces efficient property rights characterized by the minimum tax rate on innovation. Then, the state of culture, $q_t$, evolves along with the law of motion, $\phi(q_t, 0)$, denoted by the thick black line in the white region. The society, thus, converges monotonically to the good steady state, enjoying a higher growth rate over a long period of time.

Suppose now that society starts in the dark gray region for some reasons —
Figure 2.8: Growth Implications
maybe historical and/or geographical reasons. Then, it will have bad protection of property rights characterized by the maximum tax rate on innovation, so that $q_t$ evolves along with the law of motion, $\phi(q_t, \bar{\rho})$, depicted by the thick black line in the dark gray region. After a while, it reaches the light gray region where the social equilibrium is indeterminate between $S^O_t$ and $S^P_t$. If agents in the society keep believing that the government will expropriate innovators severely as in the current period, i.e., $\rho_{t+1} = \rho_t = \bar{\rho}$, then the pessimistic belief leads the society to be collectivist enough to impose the maximum tax rate on innovation in period $t + 1$. Accordingly, the economy converges to and stays at the bad steady state, $\bar{q}_L$, growing at the rate of $\sigma X(\bar{q}_L, \bar{\rho}) - 1$, which is smaller than the one at the good steady state, $\bar{q}_H$. We, therefore, call the bad steady state the “low-growth trap” as well since society cannot escape from the trap unless people expect government’s commitment to the non-expropriation.\footnote{If $\hat{q}_L$ is larger than $\hat{q}_L$, the bad steady state $\hat{q}_L$ belongs to the dark gray region, so that $\hat{q}_L$ has a unique political equilibrium that $\rho_{t+1} = \hat{\rho}$. In this case, society near $\hat{q}_L$ converges to the bad steady state for sure, implying that $\hat{q}_L$ is locally stable. That is, any society near the low-growth trap ends up being trapped, suffering from low growth permanently as long as there is no attempt or shock that makes the society get out of the trap. The main message in this chapter, however, is not changed: stronger (weaker) individualism tends to be associated with stronger (weaker) protection of property rights, and such an interaction induces economic prosperity (disaster) in the long run.}

It may experience expectations-driven growth cycles in this region instead of staying at the low-growth trap, $\hat{q}_L$. Figure 2.8 shows an example of two-period limit cycles that alternates between two states of culture, say $\bar{q}_0$ and $\bar{q}_1$ where $\bar{q}_1 > \bar{q}_0$. This implies that society has two steady states in the long run. The steady state at $\bar{q}_1$ displays strong individualism and non-expropriation, while the one at $\bar{q}_0$ is described by weaker individualism and severe expropriation. Hence, the two steady states are opposite in terms of institutional qualities, and one can interpret the growth cycles as political instability. That is, the society in the light gray region suffers from excess volatility both in economy and politics caused by the endogenous regime switching. What is important is that the society may not escape from the indeterminacy region even when it does not stay at the low-growth trap, $\bar{q}_L$. This is because the society
should lead agents’ beliefs to the optimistic direction persistently for some periods of time in order to escape from this region and converge to the good steady state at $\bar{q}_H$. However, there is no systematic way to steer agents’ expectations to a certain direction, and hence, society may lie in the indeterminacy region for indefinite periods of time, suffering from low growth with excess volatility.

2.8 Making Growth Miracles

In this section, we investigate policy implications from the theory to explore how society can get out of the low-growth trap and converge to the good steady state. Since our theory deals with endogenous culture and institutions, policy prescriptions for growth miracles include not only economic tools such as tax policies but also political reform packages presented below.

**Political reform**

Suppose that society is in the indeterminacy region. As described by the arrows between the two laws of motion for $q_t$ in the bottom panel of Figure 2.9 (a), this society has not been able to escape from the region, suffering from expectations-driven volatility both in growth rates and in institutional qualities. In this situation, suppose that the society goes through a political reform that shifts the political power from workers to innovators. This political reform can be parameterized by an increase in $\xi$. For example, if $\xi$ increases to $\xi'$ as in Figure 2.9 (b), then $\hat{X}(q, \hat{\rho})$ becomes larger than $1/2 - \xi'$ whenever the degree of current individualism, $q_t$, is higher than $\hat{q}'_H$. Since the threshold $\hat{q}'_H$ is smaller than the original one, $\hat{q}_H$, the indeterminacy region gets much smaller as in the bottom panel of Figure 2.9 (b).\(^{39}\) Society then converges to the good steady state, $\bar{q}_H$, regardless of the initial state of culture as depicted in Figure 2.9 (b). Consequently, the economy grows at a

\(^{39}\) Also notice that the expropriation regime does not exist since $\hat{X}(q, 0)$, the number of innovators under the zero tax, is larger than $1/2 - \xi'$ $\forall q \in [0, 1]$, and hence, $\hat{q}'_L = 0$. 97
Figure 2.9: Political Reform and Growth Miracles
higher rate given by $\sigma X(\bar{q}_H, 0) - 1$ over a long period of time as if it had recovered its "growth potential." This can be interpreted as a growth miracle led by a policy reform that strengthens the political power of innovators whose inventions determine the aggregate productivity of the economy.

**Improvement of individual rights**

In the theoretical model, one can interpret the maximum tax rate, $\bar{\rho} \in (0, 1)$, as the maximal degree of the public predation on the private ownership. It is then natural to think that $\bar{\rho}$ will be determined by, for example, an independent judiciary, constraints on power of authority and the role given to decision makers on taxation. Not only from those “formal” factors, more familiar in economics, one can also regard that $\bar{\rho}$ reflects the value of outside option for innovators; if an innovator can hide his wealth $W$ at the expense of $\bar{\rho}W$ in order to avoid the public predation, the government will not set the rate of expropriation above $\bar{\rho}$.

In any interpretation for $\bar{\rho}$, we can technically show that a decrease in $\bar{\rho}$ (improving individual rights) may make society escape from the low-growth trap. Suppose again that society has been stuck in the indeterminacy region near the low-growth trap as in Figure 2.10 (a). In this situation, if $\bar{\rho}$ decreases to $\bar{\rho}'$, the mass of innovators under the expropriation regime will increase $\forall q \in [0, 1]$ as in the top panel of Figure 2.10 (b); for example, see equation (28) to check $\partial \hat{X}(q, \rho)/\partial \rho < 0$. Same as the case of an increase in $\xi$, this makes the threshold $\bar{q}'_H$ smaller than the original one, $\bar{q}_H$. The indeterminacy region therefore becomes much smaller as in the bottom panel of Figure 2.10 (b).

In addition to this, the premium value of being an individualist under the expropriation regime increases $\forall q \in [0, 1]$, and this makes the law of motion for $q_t$ under the expropriation regime shift upward; compare $\Delta(q_t, \bar{\rho})$ in the middle panel of Figure 2.10 (a) and that of Figure 2.10 (b). This makes indeterminacy region even smaller. Summing up all those impacts, regardless of the initial state of culture, society converges to the good steady state, $\bar{q}_H$, where it grows at a higher rate over a long period
Figure 2.10: Individual Rights and Growth Miracles

(a) Before Improvement of Individual Rights

(b) After Improvement of Individual Rights
of time. This is a growth miracle from improvement of individual rights to private ownership parameterized by a decrease in $\bar{\rho}$.

**Tax policies**

We now know that increasing the mass of innovators under the expropriation regime is a key to make society get out of the low-growth trap. Then, one can think of tax policies that induce more innovators in the economy. To do so, the skill threshold $z^S (S \in \{I, C\})$ needs to be lower given $q$ and $\bar{\rho}$. From equation (9), this can be done through a decrease in the opportunity cost of being an innovator given by $\hat{\varepsilon}^S \equiv e^S/\lambda^S$. However, it is, in general, difficult for the government to do simply because it is hard to distinguish each individual’s culture type. Instead, the government can change the relative price of innovation, $p/w$, by using subsidies and taxes.

Suppose that, after the voting in period $t$, the government announces that it will subsidize innovation in period $t+1$ by $s_{t+1} > 0$. Then, the price of innovation in period $t+1$ is given by $(1+s_{t+1})p_{t+1}$. The government imposes lump-sum tax $\tau_{t+1} > 0$ equally on each individual of generation $t$ in order to fund the subsidy. Note that this fiscal policy can be implemented without any political conflict after the voting in period $t$ since generation $t$ has no longer the right to vote. The skill thresholds in period $t+1$ is then determined as follows:

$$z^S_{t+1} = \min \left\{ (\psi^S)^{-1}[\hat{\varepsilon}^S(s_{t+1})\hat{\eta}_{t+1}\hat{x}_{t+1}], 1 \right\} \quad \forall S \in \{I, C\}$$

where $\hat{\varepsilon}^S(s_{t+1}) \equiv e^S/\lambda^S (1+s_{t+1})$ denotes the tax-adjusted opportunity cost of being an innovator. That is, changing the relative price of innovation $p/w$ through the tax policy is essentially equivalent with changing the opportunity cost, $e^S/\lambda^S$, to affect $z^S$, the skill threshold.

This tax policy increases the number of innovators, $\hat{X}(q, \rho)$, for any $q$ and $\rho$; see equation (26). To see this more clearly, note that $\hat{X}(q, \rho)$ in Example 2 increases
whenever \( \lambda \) rises to \( \lambda(1 + s) \). Similarly, \( \Delta(q, \bar{\rho}) \) also increases since being an individualist is more valued with the subsidy, which is more likely to go for individualists. Hence, the fiscal policy can make society escape from the low-growth trap as in the case of a decrease in \( \bar{\rho} \), and this can be referred to as a growth miracle from tax policies.\footnote{The difference is that both \( \bar{X}(q, 0) \) and \( \Delta(q, 0) \) also increase in the case of the tax policy. An increase in \( \bar{X}(q, 0) \) further shrinks both the indeterminacy and expropriation regimes since \( q_L \) also decreases as in the case of the political reform. This fiscal policy, however, is limited in that it may not be applied \textit{repeatedly} over generations since new generations will learn from the history that the government will tax them when they are old. Workers knowing the government will tax them in the next period will want to eliminate the subsidy when they are young, and the political parties will reflect this policy preference provided that the fraction of workers is large enough.}

## 2.9 Conclusion

We develop a dynamic model in which culture and institutions evolve jointly, and the interaction between them determines economic growth. The interaction takes the form of a strategic complementarity: individualism and strong enforcement of property rights are mutually reinforced, which is consistent with our empirical findings. For example, a vicious mutual reinforcement dynamically causes decades—or even centuries—of stagnation, which we call the low-growth trap. In addition to low growth, economies near the trap may suffer from excess volatility driven by self-fulfilling beliefs, which are independent of fundamental factors of the economy.

We also show that improvement of political factors greatly helps society in the low-growth trap. Alongside sound fiscal policies, improvement of the political environment such as strengthening innovators’ political power or improving individual rights to private ownership can break the vicious cycles—perhaps, for instance, the recent experiences in the Soviet Union, China and Vietnam. Society then departs from the expropriation regime and enjoys the fruits of high growth over a long period of time as a nation shifts toward individualism.

The novel feature of the model is that it considers “society,” which incorporates
culture and institutions on top of the “economy.” Multiple equilibria possibly arise once culture and institutions are endogenously determined in an otherwise standard growth model without aggregate uncertainty where the economy itself is complete in that there is no missing market. In the case of multiple equilibria, self-fulfilling beliefs matter since equilibrium selection depends solely on those beliefs. This leaves an important research question open: “What factors underlie the self-confirming beliefs?” If we provide a plausible answer to this question, we may devise a systematic means to form the self-fulfilling optimism which helps to steer society towards economic prosperity.

41 Azariadis (1981) demonstrates that multiple equilibria, consistent with the rational expectations equilibrium, emerge when markets are incomplete. In this setting, self-confirming beliefs also matter in equilibrium selection.
Chapter 3

Rent-Seeking, Institutions and Morality

3.1 Introduction

“Because that’s where the money is.”
Willie Sutton on why he robbed banks

We study rent-seeking. Rent-seeking is defined by any type of redistribution activity that takes up resources from others. One can classify it into two categories: private and public rent-seeking. The private rent-seeking is done by a private agent without public authority, which includes, for example, theft, robbery, looting, trickery, fraud, scam, embezzlement, appropriation and takeover. The public rent-seeking is executed under public authority, and thereby, usually executed by government officials, which includes, for example, expropriation through severe taxation, poor protection of property rights, abuse of authority (corruption), and confiscation of private ownership. Then, it is clear that rent-seeking is a part of our daily life although the extent of rent-seeking varies by nation. In this chapter, we investigate why the degree of rent-seeking varies across societies and how rent-seeking can explain continuing income gap over the globe focusing on its redistribution aspect.
This research is motivated by three questions. First, is rent-seeking bad for an economy? and, if so, why is it bad? Second, if it is indeed bad, why it still prevails in many countries? That is, why those countries did not eliminate rent-seeking activities long ago? Third, what is the most profound factor underlying rent-seeking behaviors? The last question is particularly relevant if one wants to provide a fundamental solution for eradicating “ingrained” rent-seeking in society. To answer these three questions, we examine how rent-seeking results in long-lasting poverty by investigating the interaction between institutions — the effectiveness of law enforcement — and culture — morality of society.

To define the morality of society, we divide economic activities into two different types: productive and unproductive activities. The productive activity represents any kind of activity that increases one’s wealth through “hard work” without hurting anyone else. For example, it includes (higher) education, diligent work, innovation, entrepreneurship — becoming an entrepreneur, or more narrowly, launching his own business — and etc. On the contrary, the unproductive activity is purely redistributive in that it takes resources from one to another.\footnote{In this sense, one can label the productive and unproductive activities as profit-seeking and rent-seeking activities, respectively.} In our model, each individual is allowed to conduct the productive or unproductive activities (or both at the same time). Hence, individuals engaging in the productive and unproductive activities are determined by their abilities in the productive and unproductive activities.

Then, it would be natural to define a society is morally more sound than another if it has a smaller fraction of people who have the comparative advantage in the unproductive activity. Since the incentive to be a rent-seeker is dependent on the individual comparative advantage between productive and unproductive activities, the social morality turns out to be a significant factor underlying how much rent-seeking prevails in society.

However, the extent that rent-seeking prevails in society does not only depend on the morality. It is also dependent on how effective the law enforcement is, which we
call the institutional quality. For example, if the government operates an inefficient investigation system and enforcers are not trained, not motivated, and even corrupt, the probability to be punished for rent-seeking will be low. Consequently, we need to take into account the two fundamental factors — the morality and institutional quality — in the examination of rent-seeking activities, and hence, we delve into the two factors by developing theoretical model where they are playing a crucial role determining rent-seeking behavior.

We now note a somewhat obvious fact to answer the first question: “is rent-seeking bad for an economy? and, if so, why is it bad?” The fact is that, a victim’s actual loss from any rent-seeking tends to be larger than the benefit that a rent-seeker obtains from the rent-seeking. This is simply because every productive project in progress is “profitable,” which means that a victim’s actual loss from rent-seeking will be larger than a rent seeker’s benefit in the rent-seeking. Imagine a situation that there is an entrepreneur who has a creative idea that leads to some profits once it is proceeded with, for example, $1,000,000 of the initial investment. If the initial investment is expropriated by a rent-seeker, e.g., thief or corrupt friend, then the profits from the idea cannot be realized, so that profits are forgone. The forgone profits are the net-loss in the society although the initial investment is simply redistributed to the rent-seeker. In other words, the wedge between a rent-seeker’s benefit and a victim’s loss leads to deadweight loss in rent-seeking, reducing the aggregate income of the economy.

In addition to the wedge, there is one more channel through which rent-seeking negatively affects the economy. Namely, the higher threat of rent-seeking, the smaller incentive to engage in the productive activity. That is, you are less willing to engage in the profit-seeking activity as you are more likely to meet a rent-seeker while you are doing the productive activity. Hence, the extensive margin of people who are willing to engage in the profit-seeking activity gets smaller as the fraction of rent-seekers increases. This further decreases the aggregate output. Looked at another side, one can interpret this negative impact of rent-seeking as that society gets a smaller
fraction of people who believe “economic success through hard work and education” as the threat to be victimized by rent-seekers increases.

Rent-seeking harms growth if the productive activity entails positive externalities for the economy. This is based on the insight that even when the productive activity (which is successfully done) makes the entire economy better off through, for example, learning-by-doing and knowledge diffusion, a rent-seeker, in general, does not care about the positive externalities. Rather, he cares only about how much resources he can take away from rent-seeking and how strict he will be punished once his rent-seeking activity is exposed, epitomized as the quote by Willie Sutton, a legendary bank robber, at the beginning of this chapter. In summary, using economics jargon, the negative impacts of private rent-seeking both on the aggregate income and growth emerge because a rent-seeker does not internalize any external impact of rent-seeking.

The second question deals with why rent-seeking prevails in many countries even in very long-run. If rent-seeking is indeed bad for economic prosperity, why so many countries, especially, poor countries, did not eliminate rent-seeking long ago? One can answer to this puzzling question using a simple economic intuition — and jargon again. That is, there will be a mechanism that results in an increasing return to scale for rent-seeking, which many economists have already argued a lot. We add another mechanism that induces such an increasing return to scale by focusing on politics.² More precisely, we propose a political factor that determines the incentive to do rent-seeking as an important factor that creates an increasing return to scale.

² In the standard discourse in economics, an individual is modeled as a globule which is perfectly consonant with Leibniz’ “windowless monad” (or monas); it behaves independently by its own principle — optimization —, without any explicit relationship with others, but, at the same time, those respective principles are perfectly harmonized throughout the entire economy — equilibrium. This philosophical point of view explains individual behaviors in reality to some extent as well as simplifying the economic analysis a lot by insulating economic aggregates — equilibrium prices and allocations — from any impact of a single individual’s behavior. However, in the real world, individuals have their own “will” that makes them together in order to realize the will collectively. The collective choice, of course, can change aggregates. For example, workers form a labor union to raise their bargaining power, and hence, wage, just like that firms form a cartel to seek rents. Hence, society is not always harmonized by the windowless monads, rather it usually encompasses various conflicts among different interest groups, and society should equip a systematic and influential vehicle that accommodates people’s various conflicts: politics.
Figure 3.1: Time Line

for rent-seeking. The intuition is simple. As there are more people who are more likely to be rent-seekers, their political power will be stronger. Hence, the chances are pretty low that society establishes institutions against rent-seeking. This is how an increasing return to scale for rent-seeking activities can emerge. In a word, the more people who are more likely to be rent-seekers, the poorer institutions, characterized by less effective enforcement of laws for rent-seeking, resulting in a higher (ex ante) return to rent-seeking.

Note that the fraction of people who are more likely to be rent-seekers is monotone in the fraction of people who have the comparative advantage in the unproductive activity, which is defined as the social morality. Hence, this intuition helps us answer to the third question about the role of the morality as a fundamental factor underlying rent-seeking activities. Since the morality is more fundamental than institutions in our framework, this theoretical result implies further that the social morality needs to be revived if one desires fundamentally solves rent-seeking problem. If not, the good institutions (effective enforcement of laws) will not be established, raising the incentive to seek rents, and hence, rent-seeking will prevail again. This calls for an important role for the government to somehow revive the morality.
3.2 The Model

3.2.1 Overview

We consider an OLG economy where each agent lives for two and a half periods. Time is discrete and extends from zero to infinity; \( t = 0, 1, 2, \cdots \). Each generation \( t = 0, 1, 2, \cdots \) is born at the middle of period \( t - 1 \) and enters at the beginning of \( t \) as depicted in Figure 3.1. Each generation consists of a continuum of individuals, which is unit measure, and the total population is not changed for all \( t \). Consumption occurs only in the second period of life. The utility from consumption is linear, and there is no time discount.

At the beginning of the first period, each agent \( i \) of generation \( t \) enters the economy and draws \( x_i \equiv z_i / \rho_i \in \mathcal{X} \subset \mathbb{R}_+ \) from a time-varying continuous distribution \( \tilde{F}_t (x) \) defined on \( \mathcal{X} \). Here, \( z_i \) is his own trait or skill for “productive activity,” for example, diligence, cooperation, earnestness and creativity, while \( \rho_i \) is the one for “unproductive activity,” for example, deceit, craftiness, cruelty and selfishness. Specifically, we interpret the productive activity as any type of activity that increases his wealth through “hard work” without hurting anyone else. For example, it includes (higher) education, diligent work, innovation, entrepreneurship — becoming an entrepreneur, or more narrowly, launching his own business — and etc. We posit that the productive activity entails some risk by its nature, and hence, we refer to the productive activity as “innovation” (or “entrepreneurship”). On the contrary, the unproductive activity is purely redistributive in that it takes some resources from others. We refer to the unproductive activity as rent-seeking.

After drawing the skill (personal trait), each individual of generation \( t \) votes for institutional quality that determines the effective punishment level for rent-seeking activities. Then, he gives birth one child, so that the life-cycle of generation \( t + 1 \) begins. After the voting, each agent chooses whether or not to engage in the productive activity. If he engages in the productive activity, he becomes an “entrepreneur” so that we refer to this decision as occupation choice. After the occupation choice, a
pair of two agents randomly meets with each other at the matching stage where rent-seeking can occur. After the matching, the first period of life ends, and generation \( t \) proceeds to the second period of life. At the beginning of the period \( t+1 \), innovation (the productive activity) conducted in period \( t \) is realized, and each of generation \( t \) consumes his entire wealth, ending his life-cycle.

### 3.2.2 Skills in Productive and Unproductive Activities

Now, let us investigate how \( z_i \in [0,1] \) and \( \rho_i \in [0,1] \) are modeled in the economy. Each agent of generation \( t \) earns labor income \( w_t \) in the first-period. After that, he can be an entrepreneur doing the productive activity. If he decides to be an entrepreneur, he invests his own\(^3\) wealth \( \omega \leq w_t \). He succeeds in the innovation (entrepreneurship) with probability \( p(z) \in [0,1] \) where \( p'(z) > 0 \) and obtains \( \lambda \omega \) at the beginning of the second period of life. That is, innovation requires both resources and time; you cannot create something new right now with empty hands. Of course, \( \lambda > 1 \).

He fails in the innovation with probability \( 1 - p(z) \). Then, he gets nothing, and thereby, consumes nothing in the second period. If he decides not to be an entrepreneur, he stays idle and keeps his own wealth \( w_t \). We let \( E \) and \( I \) denote the occupation choice of an agent where \( E \) and \( I \) stand for “an entrepreneur” and “an inactivist,” respectively.

After the occupation choice, each agent meets another agent at the matching stage. The one-to-one matching is anonymous, so that each player \( i \) does not know his opponent \( j \)’s type \( x_j \). At this matching, each player \( i \) can take \( \rho_i \) fraction of \( j \)’s wealth \( w_j \). The rent-seeking is exposed by the government with probability \( \pi_t \). Then, the rent-seeker \( i \) returns what he has stolen, which is given by \( \rho_i w_j \), to the victim \( j \). In addition, the government confiscates \( \theta \) fraction of an arrested rent-seeker’s wealth.

\(^3\) We assume that there is no financial market. This is simply because agents conducting the productive activity will be those whose \( z \) are highest under the perfect financial market due to the linear return to the productive activity. However, regardless whether the financial market is perfect or imperfect, the main results will not be changed as long as the set of agents doing the productive activity is not measure zero, and hence, the fraction of them gets smaller along with the extent of rent-seeking.
at the beginning of the second period of life. For simplicity but without any change in the main results, we assume that the government identifies the rent-seeking activities case by case. Hence, an agent’s rent-seeking activity may not be exposed, so that he may keep the stolen goods, while his own loss by his opponent’s rent-seeking is recovered once the rent-seeker is caught by the government.

3.2.3 Destructive Features of Rent-Seeking

What is the primary source that results in negative impacts of rent-seeking on the economy? That is, what “nature” of rent-seeking makes it destructive? One immediate answer, which is well explained by Grossman and Kim (1995, 1996), is that the possibility of rent-seeking induces people invest resources in sharpening predatory power and in protecting private properties, both of which are purely unproductive, and hence, the deadweight loss in economics point of view.

Our intuition suggests another destructive nature of rent-seeking. Namely, there is a potential gap between a rent-seeker’s benefit from rent-seeking and his victim’s loss from the rent-seeking. Since the productive activity already executed must be “profitable”, and at the same time, rent-seekers are not likely to conduct productive activities according to the self-selection, the chances are high that a rent-seeker’s benefit from rent-seeking is smaller than his victim’s loss from the rent-seeking. This wedge, in turn, leads to the deadweight loss, decreasing the aggregate income.

The second destructive feature is that rent-seeking lowers the number of people who want to engage in the productive activity. We will show that the number of entrepreneurs gets smaller as the number of rent-seekers increases. This is simply because the probability for an entrepreneur to meet a rent-seeker increases as the fraction of rent-seeker increases. That is, the incentive to conduct innovation or entrepreneurship decreases along with a higher threat of rent-seeking (higher probability to be victimized).

Not only income loss, the two destructive features can also aggravate growth. This
is because some of the productive activity may have had positive externalities that promote economic growth through, for example, learning-by-doing and knowledge diffusion. Based on this argument, we will show that the two destructive features themselves decrease both the aggregate income and growth even when there is no waste of resources for sharpening offensive and defensive skills.\(^4\) In our model, an agent invests in neither predatory nor defensive skills, but rent-seeking reduces the total output and growth because of the two destructive features.

### 3.2.4 Morality of Society

It is clear that \(x_i \equiv z_i/\rho_i\) measures the comparative advantage in the productive activity against the unproductive one. That is, an agent \(i\) is relatively better in the productive activity compared to an agent \(j\) if, and only if, \(x_i > x_j\). Looked at another way, one can interpret \(x_i \equiv z_i/\rho_i\) as individual morality since \(z_i\) and \(\rho_i\) represent “good” and “bad” personal traits, respectively. That is, an individual \(i\) is more morally sound as \(x_i\) increases. Then, one would be able to rank the morality between different societies using the distribution of the individual morality as follows:

**Definition 3.1 Morality** Society \(A\) is morally more sound than society \(B\) if \(\tilde{F}^A(x)\) dominates \(\tilde{F}^B(x)\) at first-order, i.e., \(\tilde{F}^A(x) < \tilde{F}^B(x) \forall x \in \mathcal{X}\).

The definition is quite intuitive; the morality of society \(A\) is worse than society \(B\) if, at any cumulative probability \(\tilde{F}(x)\), individuals in society \(B\) have worse individual morality, \(x\), than those of society \(A\). For analytical simplicity in the rent-seeking game, which will be presented below, we set \(\rho \equiv \rho_i \in (0, 1) \forall i \in [0, 1]\). This does not lose generality of the analysis, at least, for the purpose of this research since, as in the definition of the morality, what is important is the relative magnitude of \(z_i\) to \(\rho_i\). That is, with \(\rho_i\) being fixed across agents, the comparative advantage is completely

\(^4\)As will be shown, in our model, potential victims rely on the “public fortification” by enhancing the institutional quality in order to protect their private properties. Then, one should notice that any costs for establishing a higher quality of institutions are also economic losses.
recovered through comparison of $z_i$ and $z_j$. Now, let $F(z)$ be a transformation of $\tilde{F}(x)$ with $\rho_i$ being fixed at $\rho \in (0, 1) \forall i \in [0, 1]$. Then, Definition 3.1 needs to be rewritten such that society $A$ is morally more sound than society $B$ if $F^A(z)$ dominates $F^B(z)$ at first-order, i.e., $F^A(z) < F^B(z) \forall z \in (0, 1)$. Figure 3.1 depicts an example of moral decay and revival with $F(z)$.

We first let the institutional quality $\pi_t$ and the morality $F_t$ being fixed for the time being to clarify the model mechanism. the institutional quality will be endogenized later.

### 3.2.5 Rent-Seeking and Aggregate Output

Here, we briefly investigate how rent-seeking affects aggregate output of the economy in the model environment. Recall the first destructive feature that a rent-seeker’s benefit from rent-seeking is smaller than his victim’s actual loss. In the model, this gap does not emerge as long as a rent-seeker’s victim is an inactivist, who does not conduct the productive activity. In this case, the rent-seeker takes $\rho w$, and the

\[\text{Figure 3.2: Moral Decay and Revival}\]

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5 In another note below, we will argue that this simplifying assumption is indeed not restricted for the main results. This is because the solution of rent-seeking game with $\rho_i$ being fixed $\forall i \in [0, 1]$ is qualitatively the same as the one with varying $\rho_i$. 

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inactivist loses $\rho w$, so that the benefit and loss are exactly the same. Hence, rent-seeking victimizing an inactivist has no impact on the aggregate output; it is simply redistributive, and there is no output loss.

However, if a rent-seeker predates an entrepreneur, the wedge emerges. Recall that the return to the entrepreneurship (innovation) is linear. Hence, anyone who chooses to be an entrepreneur invests his entire wealth $w_t$. That is, $\omega = w_t \forall z \in [0,1]$ such that $o_t(z) = E$. Then, his expected wealth without any interrupt (rent-seeking) is given by $p(z)\lambda w_t$. Since the utility from consumption is linear, if there is no rent-seeking, any agent whose skill is high enough that $p(z)\lambda \geq 1$ will chooses to be an entrepreneur, and an inactivist otherwise. Let us define the skill threshold $z^*$ such that anyone whose $z$ is higher than $z^*$ becomes an entrepreneur, and an inactivist otherwise. Then, $z^*$ should be at least $\bar{z}$ where $\bar{z}$ solves $p(\bar{z})\lambda = 1$.

Suppose now that the entrepreneur’s skill is $z_0 \geq z^*$. Then, although the rent-seeker takes $\rho w$ amount of resources in any rent-seeking incidence, the entrepreneur actually losses $p(z_0)\lambda \rho w$, which increases along with his skill $z_0$. The gap between the benefit and cost of rent-seeking victimizing an entrepreneur is given by $[1 - p(z_0)\lambda]\rho w$, which is strictly negative almost surely since the skill threshold $z^*$ is bounded below by $\bar{z}$ where $p(\bar{z})\lambda = 1$. That is, although rent-seeking is simply redistributive, it decreases the aggregate output due to the gap between the benefit and the cost. We will investigate the deadweight loss arising from this wedge between the benefit and the cost of rent-seeking with details after deriving the rent-seeking equilibrium of the model.

In addition, rent-seeking further decreases the output since it increases the skill threshold $z^*$. To examine this negative feature more clearly, we first consider the aggregate income of the economy without rent-seeking, which is given by:

$$\bar{Y}_t = w_{t-1} \left[ F_{t-1}(\bar{z}) + \lambda \int_{\bar{z}}^{1} p(z) dF_{t-1}(z) \right] \quad (3.1)$$

where $\bar{Y}_t$ is the first-best output before the production of generation $t$ given insti-
tutions and morality \((\pi_{t-1}, \theta_{t-1}, F_{t-1} (\cdot))\). Here, \(\bar{z}\) is the minimum level of the skill threshold that solves \(p(\bar{z}) = 1\).

Suppose now that society \(A\) and \(B\) are exactly the same, except only that \(A\) is morally more sound than \(B\), i.e., \(F^A (z) \leq F^B (z) \forall z \in (0, 1)\). Then, \(Y^A_t\) should be larger than \(Y^B_t\) since \(p(z) \lambda \geq 1 \forall z \geq \bar{z}\). Later, we will show that the threshold \(z^*\) increases when there are more rent-seekers, meaning more severe rent-seeking decreases \(Y_t\) since it reduces the fraction of people willing to engage in the profitable productive activity. This is the second channel through which rent-seeking reduces social welfare through the threat of victimized by a rent-seeker.

### 3.3 Rent-Seeking Decision

We first examine the rent-seeking game as a backward induction. After deriving the optimal rent-seeking decision, we will investigate the optimal occupation choice. From now on, we drop the time subscript unless it makes confusion.

According to the timing, the occupation choice is already done, so that the threshold \(z^*\) is common knowledge at the one-to-one matching where rent-seeking occurs. However, since the one-to-one matching is anonymous, each player does not know his opponent’s type \(z\). To solve this type of incomplete information games, one can use the Bayesian Nash equilibrium (BNE), which maximizes each type’s expected payoff for an agent \(i\), given any type’s optimal strategy for an agent \(j \neq i\). Here, we focus only on a pure strategy equilibrium. Then, BNE is a profile of optimal strategies, \(\{a_i | i \in [0, 1]\}\), where \(a_i \equiv a(z_i) \in \{N, R\}\) is an optimal strategy for a player \(i\) whose type is \(z_i \in [0, 1]\). \(N\) indicates “no rent-seeking” while \(R\) represents “rent-seeking.”

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\(^6\) That is, the morality matters even without rent-seeking. This is simply because, in society \(A\), there are more “active” agents who are self-motivated, and hence, willing to work hard in the productive activity in order to achieve higher income (or “better life”) in future. For this reason, one can interpret the fraction of people who chose \(E\), given by \(1 - F(z^*)\), as the fraction of people who believe “economic success through hard work and education.”
Mathematically, \( a_i \) solves:

\[
\int_0^1 u_{a_i,a(z)}(z_i, z) \, dF(z) \geq \int_0^1 u_{\tilde{a}_i,a(z)}(z_i, z) \, dF(z) ; \quad a(z) \text{ is given } \forall z \in [0, 1]
\]

where \( a_i, \tilde{a}_i \in \{N, R\} \) and \( a_i \neq \tilde{a}_i \). \( u_{a_i,a(z)}(z_i, z) \) is \( i \)'s expected payoff when he meets a player \( j \) whose type is \( z \) and they choose \( a_i \) and \( a(z_j) \), respectively.\(^7\) For simplicity, we assume that a player choose \( N \) over \( R \) when the two choices are indifferent.

Although the types are infinitely many, it is simple to solve the game. This is mainly because we let \( \rho \equiv \rho_i \in (0, 1) \forall i \in [0, 1] \),\(^8\) and every agent has the same initial endowment \( w \) in the game. That is, an agent’s loot from rent-seeking is given by \( \rho w \) regardless of a victim’s types, and hence, the optimal strategy on the rent-seeking behavior is independent of the opponent’s type. We summarize the BNE for the rent-seeking game as follows:

**Lemma 3.1 Solution of the Game:** \( \forall z^* \in [0, 1] \), there always exists a unique BNE that solves the rent-seeking game as follows:

\[
a(z) = \begin{cases} 
N & \text{if } \pi \theta \geq (1 - \pi) \rho \quad \forall z < z^*, \\
R & \text{otherwise}.
\end{cases}
\]

\[
a(z) = \begin{cases} 
N & \text{if } \pi \theta p(z) \lambda \geq (1 - \pi) \rho \quad \forall z \geq z^*, \\
R & \text{otherwise}.
\end{cases}
\]

Lemma 3.1 is quite straightforward. First, an inactivist, i.e., \( \forall z < z^* \), does

\(^7\) This shortcut for solving the BNE is available since the matching is totally random between a type \( i \) and a type \( j \); the probability of matching between types \( i \) and \( j \) is i.i.d.. Hence, maximizing a player \( i \)'s expected payoff over the type space \( z \in [0, 1] \) is identical with maximizing the expected utility of each type of \( z \in [0, 1] \).

\(^8\) This simplifying assumption is justifiable since, as already mentioned, what is important for the main results is the relative magnitude of \( z_i \) to \( \rho_i \). Although we can solve the game with varying \( \rho_i \) across agents, there is no significant gain in the analysis while the algebra becomes complicated a lot more since the solution depends on the distribution \( \tilde{F}(x) \). More precisely, with some simplifying assumptions, one can show that a player \( i \) is more likely to be a rent-seeker as \( x_i \equiv z_i / \rho_i \) is small; details are available upon request. This is the same as, at least qualitatively, that a player \( i \) is more likely to be a rent-seeker as \( z_i \) is small with \( \rho_i = \rho \forall i \in [0, 1] \), which coincides with Lemma 3.1 below.
not seek rent as long as the expected punishment $\pi \theta w$ is larger than the expected benefit of rent-seeking $(1 - \pi) \rho w$. This logic also applies to an entrepreneur, i.e., $\forall z \geq z^*$, but slightly differently. This is because an entrepreneur’s expected wealth is given by $p(z) \lambda w$ not by $w$. Since $p(z) \lambda \geq 1 \forall z \geq z^*$, Lemma 3.1 means that an inactivist is more likely to be a rent-seeker than an entrepreneur. This is simply because an inactivist has the comparative advantage in the unproductive activity, i.e., an inactivist has a smaller $x_i \equiv z_i/\rho_i$ than an entrepreneur with $\rho_i$ being fixed at $\rho \forall i \in [0, 1]$.

For convenience, we let $\hat{\pi} \equiv \pi \theta/(1 - \pi)$ denotes the effective (ex-ante) degree of punishment for rent-seeking. Note also that $\rho(z) \equiv \rho/[p(z) \lambda]$ represents the comparative advantage in the unproductive activity against the productive activity. It is obvious that $\rho(z)$ decreases as he is more able in the productive activity. With $\hat{\pi}$ and $\rho(z)$, one can rewrite Lemma 3.1 as follows:

$$a(z; z^*) = \begin{cases} N & \text{if } \hat{\pi} \geq \rho \mathbb{1}_{\{z < z^*\}} + p(z) \mathbb{1}_{\{z \geq z^*\}} \\ R & \text{o.w.} \end{cases} \quad (3.2)$$

### 3.4 Occupation Choice

It is clear that the occupation choice depends on the number of rent-seekers. This is because the loss from rent-seeking is proportionately larger for an entrepreneur than for an inactivist although a rent-seeker’s benefit from rent-seeking is $\rho w$ regardless of a victim’s type. Hence, we first need to understand how the number of rent-seekers, say $\psi^* \in [0, 1]$, is determined. Specifically, from Lemma 3.1, we have:

$$\psi^* = \psi(z^*; \pi, \theta, F(\cdot)) = \frac{F(\max\{z^*, \hat{z}\}) - F(z^*) + F(z^*) \mathbb{1}_{\{\hat{z} < \rho\}}}{\underbrace{F(\max\{z^*, \hat{z}\}) - F(z^*)\mathbb{1}_{\{\hat{z} < \rho\}}}_{\text{entrepreneur rent-seekers}}} + \underbrace{F(z^*) \mathbb{1}_{\{\hat{z} < \rho\}}}_{\text{inactivist rent-seekers}} \quad (3.3)$$

where $\hat{z}$ solves $\rho(\hat{z}) = \hat{\pi}$. We will show that there is no entrepreneur rent-seeker as long as there is no inactivist rent-seekers since an inactivist rent-seeker gets higher (ex ante) net-benefit from rent-seeking than an entrepreneur rent-seeker due the
difference in the comparative advantages between them. Note also that $\psi^*$ is a function of the morality of society $F(\cdot)$ along with institutions of society $(\pi, \theta)$.

The final remark is related to the strategic relationship between entrepreneurship and rent-seeking. As is obvious from (3.3), the number of rent-seekers in equilibrium, $\psi^*$, is a function of the equilibrium threshold $z^*$. An agent regards $\psi^*$ as given when he chooses his occupation since he is infinitesimal (measure zero). However, he should expect $\psi^*$ in order to choose his occupation optimally.\(^9\) As we will see, compared to an inactivist’s expected utility, an entrepreneur’s expected utility decreases more largely as $\psi^*$ increases. Hence, $z^*$ increases when agents expect that $\psi^*$ is larger, i.e., the more rent-seekers, the smaller entrepreneurs. Meanwhile $\psi^*$ is also (weakly) increasing in $z^*$ as in (3.3) because an inactivist has the comparative advantage in rent-seeking, i.e., the smaller entrepreneurs, the more rent-seekers. This strategic complementarity between $z^*$ and $\psi^*$ admits multiple equilibria for $(z^*, \psi^*)$ as long as $\psi^*$ is strictly larger than zero, i.e., as long as there are rent-seeking activities in equilibrium.

We now consider the expected utility for each occupation to examine the optimal occupation choice. Given any $(z; \psi^*, \pi, \theta)$,

$$v (z; \psi^*) \equiv \max_{o(z) \in \{E, I\}} \left\{ v^I (\psi^*), v^E (z; \psi^*) \right\}$$

(3.4)

s.t. $v^I (\psi^*) \equiv [1 - (1 - \pi) \rho \psi^*] + \max \{(1 - \pi) \rho - \pi \theta, 0\}$

(3.5)

$v^E (z; \psi^*) \equiv [1 - (1 - \pi) \psi^*] p (z) \lambda + \max \{(1 - \pi) \rho - \pi \theta p (z) \lambda, 0\}$

(3.6)

Interpretation of (3.4)–(3.6) is quite straightforward. If a rent-seeker is matched with only either a no-rent-seeker or another rent-seeker, the probability to meet a rent-

\(^9\) The expectation about $\psi^*$ may vary by individual. Suppose that an agent $i$’s expectation on $\psi^*$ is given by $\psi^*_i$. Then, the rationality implies that $\psi^*_i = \psi^* \ \forall i \in [0, 1]$ in any equilibrium. That is, in equilibrium with the perfect foresight assumption, individuals are rational in that each of them forms a correct expectation on $\psi^*$.  

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seeker should depend on whether or not he is a rent-seeker. However, in our model, an agent can be matched with a rent-seeker regardless whether he chooses $I$ or $E$, and $N$ or $R$, i.e., the matching is i.i.d. Hence, by the law of large numbers, the probability to be matched with a rent-seeker is the same as the fraction of the rent-seekers $\psi^*$ for all agents.\(^{10}\) That is, the probability that an entrepreneur is matched with a rent-seeker is the same as the probability that an inactivist is matched with a rent-seeker. Likely, the probability is independent of whether he is a rent-seeker or not. Hence, both $v^I(\psi^*)$ and $v^E(z; \psi^*)$ are function of the same $\psi^*$ as in (3.5) and (3.6). One can notice that (3.5) and (3.6) are consistent with the optimal rent-seeking behavior given by Lemma 3.1 — or equation (3.2).

Notably, equations (3.5) and (3.6) imply that $v^I$ is independent of $z$ while $v^E(z)$ is strictly increasing in $z$ since $p(z)$ is monotone increasing in $z$. Also, $v^I$ and $v^E(z)$ are all strictly decreasing in $\psi^*$ but $v^E(z)$ decreases proportionately more than $v^I$ due to the comparative advantage in the model. To investigate the occupation choice, it is convenient to define the following expected utilities:

$$
\begin{align*}
v^I_N(\psi^*) &\equiv [1 - (1 - \pi) \rho \psi^*] \\
v^I_R(\psi^*) &\equiv [1 - (1 - \pi) \rho \psi^*] + (1 - \pi) \rho - \pi \theta \\
v^E_N(z; \psi^*) &\equiv [1 - (1 - \pi) \psi^*] p(z) \lambda \\
v^E_R(z; \psi^*) &\equiv [1 - (1 - \pi) \psi^*] p(z) \lambda + (1 - \pi) \rho - \pi \theta p(z) \lambda
\end{align*}
$$

where the subscript $R$ (or $N$) stands for the case where an agent chooses (no) rent-seeking at the matching stage.

For a clearer understanding, we draw $v^I$ and $v^E(z)$ in Figure 3.3 for the case of

\(^{10}\) As an illustrative example, suppose that the total number of people is $n \in \mathbb{N}$, and the total number of rent-seekers is $r \leq n$. Then, the fraction of rent-seekers is $\psi^* = r/n \in [0, 1]$. In this situation, for a no-rent-seeker, the probability to be matched with a rent-seeker is given by $r/(n-1) = \psi^* n/(n-1) \forall n > 1$. Hence, with the fraction of rent-seekers fixed at a certain level, the probability converges to $\psi^*$ as $n$ goes to infinity. Meanwhile, for a rent-seeker, the probability to be matched with another rent-seeker is given by $(r-1)/(n-1) = (\psi^* n - 1)/(n-1) \forall n > 1$, and therefore, the probability also converges to $\psi^*$ as $n$ goes to infinity with the fraction of rent-seekers fixed.
\( \hat{\pi} < \rho \). One can show that \( \hat{z} \), the break-even threshold between “no rent-seeking” and “rent-seeking” for a potential entrepreneur, is always larger than the first-best threshold \( \bar{z} \) if, and only if \( \hat{\pi} < \rho \). Hence, when \( \hat{\pi} < \rho \), there are some entrepreneur seeking rent if \( z^* < \hat{z} \); see equation (3.3). On the contrary, \( z^* \) can be larger than \( \hat{z} \) as in Figure 3.3. This case is where only highly able agents in the productive activity become entrepreneurs, so that they do not want to seek any rent since they are confiscated a lot when their rent-seeking activity is caught by the government. Therefore, every entrepreneur is not a rent-seeker while every inactivist is a rent-seeker when \( \hat{\pi} < \rho \).

For convenience and a more general discussion, we define the rent-seeking equilibrium as follows:

**Definition 3.2 Rent-seeking with Exogenous Institutions:** For any given institutions and morality \((\pi, \theta, F(\cdot))\), a vector \((z^*, \psi^*)\) is the rent-seeking equilibrium if it solves the occupation choice problem given by (3.4)-(3.6), satisfying equation (3.3).

Then, a close investigation of equations (3.4)–(3.6) and Lemma 3.1 provides the following theoretical result that describes the optimal occupation choice and the fraction of rent-seekers in equilibrium.
Proposition 3.1 (i) **existence:** There always exists at least one rent-seeking equilibrium such that \((z^*, \psi^*) \in [\tilde{z}, 1] \times [0, 1]\).

(ii) **utopia equilibrium:** There is only one equilibrium such that \((z^*, \psi^*) = (\tilde{z}, 0)\) if, and only if \(\hat{\pi} \geq \rho\).

(iii) **real world equilibrium:** Suppose that \(\hat{\pi} < \rho\). Then, any vector of \((z_H^*, \psi_H^*)\) where \(z_H^* \in [\hat{z}, 1]\) and \(\psi_H^* = F(z_H^*)\) is a rent-seeking equilibrium if it satisfies the following:

\[
v^I_R(\psi_H^*) \geq v^E_N(z_H^*, \psi_H^*). \tag{3.11}
\]

Also, if the following condition holds:

\[
\frac{\rho}{\hat{\pi}} > \frac{1 - \pi \theta + (1 - \pi)(1 - F(\hat{z})) \rho}{1 - (1 - \pi) F(\hat{z})}, \tag{C}
\]

then \(\exists! (z_L^*, \psi_L^*)\) where \(\psi_L^* = F(\hat{z})\), and \(z_L^* \in (\tilde{z}, \hat{z})\) solves the following:

\[
v^I_R(\psi_L^*) = v^E_N(z_L^*, \psi_L^*). \tag{3.12}
\]

(iv) **welfare implications:** \(z_H^* \geq \hat{z} > z_L^* > \tilde{z}\) and \(\psi_H^* > \psi_L^* > 0\).

We have three implications from Proposition 3.1. First, there always exists at least one rent-seeking equilibrium for any given vector of institutions and morality, \((\pi, \theta, F(\cdot))\).

Second, the economy achieves the first-best outcome given a vector of institutions and the morality \((\pi, \theta, F(\cdot))\) if, and only if the institutional quality \((\pi, \theta)\) is sufficiently high that the effective punishment \(\hat{\pi}\) is higher than \(\rho\). In this case, the morality \(F(\cdot)\) has no impact on the rent-seeking equilibrium although it still matters for the aggregate output as already seen before.

When the institutional quality is not sufficiently good, i.e., \(\hat{\pi} < \rho\), there are possibly infinitely many rent-seeking equilibria due to the strategic complementarity between \(z^*\) and \(\psi^*\), which is already mentioned before. That said, the first-best allocation cannot be achieved in any equilibrium since \(z_H^* \geq \hat{z} > z_L^* > \tilde{z}\) and \(\psi_H^* >
\(\psi^*_L > 0\). In this situation, the morality does matter in rent-seeking equilibrium. First note that \(\psi^*_H = F(z^*_H)\) and \(\psi^*_L = F(\tilde{z})\). Hence, the number of rent-seekers in equilibrium increases as the morality of society gets worse following Definition 3.1. Since both \(z^*_H\) and \(z^*_L\) are increasing along with \(\psi^*_H\) and \(\psi^*_L\), unhealthy morality also increases the skill threshold, and hence, makes some people, who would engage in the productive activity without rent-seeking, to become inactivists. This is because the more rent-seekers, the higher threat (probability) of rent-seeking.

The last implication is about the relationship between institutions and rent-seeking. One can show that both \(z^*_H\) and \(z^*_L\) decreases as either \(\pi\) or \(\theta\) increases. This, in turn, implies that both \(\psi^*_H\) and \(\psi^*_L\) are also decreasing in \(\pi\) and \(\theta\), i.e., the better institutional quality, the smaller number of rent-seekers. If the institutional quality improves a great deal, so that \(\hat{\pi} > \rho\), then rent-seeking disappears in the society.

We summarize these implications in the following corollary:

**Corollary 3.1**  
(i) Suppose that institutions are good enough that \(\hat{\pi} \geq \rho\). Then, the morality has no impact on the rent-seeking equilibrium.  
(ii) Suppose that institutions are not good enough that \(\hat{\pi} < \rho\). Then,  
**Morality and Rent-Seeking:** Rent-seeking becomes more severe as the morality is decayed in that both \(\psi^*_H\) and \(\psi^*_L\) increases whenever the morality worsens.  
**Institutions and Rent-Seeking:** Rent-seeking becomes more severe as the institutional quality is aggravated in that both \(\psi^*_H\) and \(\psi^*_L\) are decreasing in \(\pi\) and \(\theta\).

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11 One can show that \(z^*_L = \tilde{z}\) when the probability of success in innovation is still the same at \(p(z)\) even if the rent-seeker is not caught, so that any loss from rent-seeking is not recovered; details are available upon request. However, all of the main results are not changed since there are always rent-seekers in equilibrium, i.e., \(\psi^* \geq \psi^*_L = F(\tilde{z}) > 0\), and it is still possible that the economy obtains an equilibrium such that \(z^* = z^*_H > \tilde{z}\) due to the multiplicity of the rent-seeking equilibrium. Specifically, if there are infinitely many \(z^*_H\), then the economy has \(z^*_H > \tilde{z}\) almost surely.
3.5 Welfare Implications

In this section, we investigate economic consequences of rent-seeking, and hence, welfare implications of the morality as well. From Proposition 3.1, we knew that the first-best outcome \((z^*, \psi^*) = (\bar{z}, 0)\) is obtained when the institutional quality is sufficiently high, so that \(\hat{\pi} \geq \rho\). Hence, we consider only the case \(\hat{\pi} < \rho\), in which there are always rent-seekers in equilibrium, so that the first-best allocation is not obtainable. We first examine the aggregate resources that rent-seekers take, say \(\Psi^B\), which is given by:

\[
\Psi^B = w\psi^* \rho
\]

Meanwhile, the actual loss from rent-seeking, say \(\Psi^L\), is given by:

\[
\Psi^L = w\psi^* \left[ \int_0^{z^*} \rho dF(z) + \int_{z^*}^{1} p(z) \lambda dF(z) \right]
\]

Then, the net-benefit of rent-seeking is given by:

\[
\Psi^B - \Psi^L = -w\psi^* \int_{z^*}^{1} \{p(z) \lambda - \rho\} dF(z) < 0
\]

since \(p(z) \lambda > 1 > \rho \forall z > z^*\). Note that \(\Psi^B - \Psi^L\) is the net-benefit of rent-seeking prior to the government’s investigation upon rent-seeking activities. Since each rent-seeking case is exposed with the probability of \(\pi\), only \(1 - \pi\) fraction of the net-benefit becomes “permanent.” Hence, the real net-benefit of rent-seeking for society, say \(\Psi^*\), is given by:

\[
\Psi^* \equiv (1 - \pi) (\Psi^B - \Psi^L)
\]

\[
= - (1 - \pi) w\psi^* \left[ 1 - F(z^*) \right] \left( \lambda \mathbb{E}[p(z) | z \geq z^*] - \rho \right) < 0
\]
That is, the real net-benefit is always negative as long as \( \psi^* > 0 \) and \( \pi < 1 \). Then, the aggregate output before the production of generation \( t \), which is analogous to equation (3.1), is given by:

\[
Y_t = w_{t-1} \left[ F_{t-1} (z_{t-1}^*) + \lambda \int_{z_{t-1}^*}^{1} p (z) dF_{t-1} (z) \right] + \Psi_{t-1}^*
\]

(3.13)

which is strictly smaller than the first-best output, \( \bar{Y}_t \). More precisely, \( Y_t \) is smaller than \( \bar{Y}_t \) even with the same morality \( F (\cdot) \) since \( z_{t-1}^* > \bar{z} \) and \( \Psi_{t-1}^* < 0 \).

We now consider the growth effect of rent-seeking. Formally, we assume that the labor income \( w_t \) is given by:

\[
w_t = h (A_t)
\]

where \( h' (\cdot) > 0 \), and \( A_t \) denotes aggregate productivity of the economy. We assume that there is a positive externality in the productive activity, which is not excludable over generations, at least, to some extent. Formally, \( A_t \) evolves as the following fashion:

\[
g_t = \frac{A_{t+1}}{A_t} = g (X_t)
\]

(3.14)

where \( g' (\cdot) > 0 \), and \( X_t \) is aggregate innovation successfully done, which is given by:

\[
X_t \equiv \lambda \left[ 1 - (1 - \pi_t) \psi_t^* \right] \int_{z_t^*}^{1} p (z) dF_t (z)
\]

(3.15)

Any innovation trial interrupted by a rent-seeker fails for sure with the probability of \( 1 - \pi_t \) since we assume that the probability of success in innovation becomes

12 As indicated in the previous note, we can show that \( z_L^* = \bar{z} \) when the probability of success in innovation is not changed and stays at \( p (z) \) even if the loss from rent-seeking is not recovered. However, even when \( z_L^* \) is chosen as an equilibrium instead of \( z_H^* > \bar{z} \), the real net-benefit of rent-seeking, \( \Psi^* \), is still negative since \( \psi_L^* = F (\bar{z}) > 0 \). More precisely, for this case, we have:

\[
\Psi^* = - (1 - \pi) w \psi^* [1 - F (z^*)] \rho \left( \lambda E [p (z) | z \geq z^*] - 1 \right) < 0.
\]

13 Careful readers may have noticed that equation (14) implies the confiscated wealth from arrested rent-seekers is not thrown away. The confiscated wealth is already summed in (3.13). \( Y_t \) becomes even smaller if the wealth is thrown away.
zero when it is interrupted and the loss is not recovered.\textsuperscript{14} We notice from (3.15) that rent-seeking lowers the rate of growth in two ways as it does for the aggregate output. First, it decreases $X_t$ directly through $\psi^*_t$, that lowers the aggregate volume of successful innovation. Also, $z_t^*$ increases as $\psi^*_t$ rises, so that rent-seeking decreases the extensive margin of people who are willing to conduct the productive activity that causes the positive externality.

3.6 Endogenizing Institutions

In this section, we endogenize the institutional quality through majority voting. Recall that any rent-seeking activity is exposed with the probability $\pi \in (0, 1)$. Once being caught by the government with the probability $\pi$, the rent-seeker returns what he has stolen to the victim of his rent-seeking. In addition, the government confiscates $\theta \in (0, 1)$ fraction of an arrested rent-seeker’s wealth.

Hence, $\pi$ generally represents the quality of institutions while $\theta$ represents the degree of punishment for people already arrested for anti-social behavior at large. Then, a natural question arises: what determines the level of $\pi$? It will depend on qualitative and quantitative features of public administration. For example, it will increase along with a higher quantitative measure of human resources and physical equipment such as the number of patrols, investigators, prosecutors and public officers as well as the number offices, computers, patrol cars and police weapons. Qualitatively, it will depend on the efficiency of the government administration called “institutional infrastructure.” For example, if the government operates an inefficient investigation system and enforcers are not trained, not motivated, and even corrupt, then $\pi$ will

\textsuperscript{14} Again, growth is reduced in presence of rent-seeking without this assumption. This is because even when $z_L^* = \bar{z}$, there are rent-seeking activities in the economy, i.e., $\psi_L^* = F(\bar{z}) > 0$. Hence, the aggregate innovation $X$ decreases. More precisely, $X$ is rewritten without this assumption as follows:

$$X \equiv \rho\lambda [1 - (1 - \pi_t) \psi^*] \int_{z^*}^1 p(z) dF(z)$$

which decreases with a higher $\psi^*$. 
be low even with a large number of enforcers and equipment.

It is then plausible to guess that increasing \( \pi \) requires some pecuniary costs. For example, the government should spend money and time in order to hire and train enforcers. Also, it seems plausible to assume that \( \pi \in [\underline{\pi}, \bar{\pi}] \) where \( 0 < \underline{\pi} < \bar{\pi} < 1 \). Here, \( \bar{\pi} \) represents “national capacity” that limits society to have extremely sophisticated institutions.

In this research, we endogenize the institutional quality \( \pi \) rather than endogenize \( \theta \). This is because adjusting the institutional quality \( \pi \) itself is a more fundamental way to control rent-seeking activities than raising \( \theta \), the intensity of punishment for those already arrested. Notice that even when \( \theta \) is approximately at its maximum, i.e., \( \theta \approx 1 \), it is meaningless for a rent-seeker as long as he is not arrested by the government for almost sure, i.e., \( \pi \approx 0 \). Hence, a stricter punishment does not reduce the incentive to be a rent-seeker. In reverse, even when the punishment \( \theta \) is very low (but non-zero), it is effective for a rent-seeker as long as he is highly likely to be arrested by the government. From the same logic, a victim does not care about the degree of punishment, at least economically, as long as the rent-seeker is caught by the government with a high probability, so that his loss from rent-seeking is recovered. That is, the effectiveness of law enforcement both in controlling the incentive of rent-seeking and the incentive of being an entrepreneur depends primarily on the institutional quality \( \pi \) rather than on \( \theta \) (as long as \( \theta > 0 \)).

For this reason, we endogenize \( \pi \) while fixing \( \theta \) at a certain level strictly larger than zero. We let \( T(\pi_t, w_t) = \tau(\pi_t) w_t \) denote the aggregate pecuniary cost to establish a

---

\(^{15}\) Actually, although the developed countries have, in general, better institutions compared to those of undeveloped and developing countries, it is commonly observed that punishments for anti-social behaviors are rather stricter in undeveloped and developing countries. For example, corruption in the public sector is rampant in China compared to other developed countries while China applies heavy sentences to public officers involved in a corruption affair, which possibly reaches to a sentence of death. Also, it is legal for the government to execute corporal punishments such as caning and flogging in many undeveloped and developing countries, for example, Botswana, Brazil, Guatemala, Malaysia, Nepal, Peru, and so on, while those corporal punishments are usually banned in many developed countries except for a few countries like Singapore. Although the punishments are severe, as said, what are economically meaningful for both of a rent-seeker and a victim are the possibility to get punished and the probability to be recovered from rent-seeking, respectively.
certain level of $\pi_t \in [\bar{\pi}, \bar{\pi}]$.\textsuperscript{16} Of course, $\tau' (\pi) > 0$. We also assume that $\tau'' (\pi) > 0$, $\lim_{\pi \to \bar{\pi}} \tau' (\pi) = 0$ and $\lim_{\pi \to \bar{\pi}} \tau' (\pi) = \infty$. This cost is financed through a lump-sum tax $T (\pi_t, w_t)$ upon generation $t$, and hence, the after-tax labor income is given by $[1 - \tau (\pi_t)] w_t$. That is, if generation $t$ agrees to set the institutional quality at a certain level $\pi_t$ through majority voting, each agent of generation $t$ pays the lump-sum tax $T (\pi_t, w_t)$, or equivalently, the income tax $\tau (\pi_t) w_t$ to the government.\textsuperscript{17} Finally, we assume that $\hat{\pi} (\bar{\pi}; \theta) \equiv \bar{\pi} \theta / (1 - \bar{\pi}) < \rho$. Otherwise, $\bar{\pi}$ will be always chosen in any political equilibrium, and there is no rent-seeking (see Proposition 3.1), which is an unrealistic and uninteresting case.

**Majority voting**

We consider majority voting as a device determining $\pi$ endogenously. The setting is standard. There are two political parties, $P_1$ and $P_2$, that competes with each other in order to take the power that determines $\pi_t$, or equivalently, the income tax rate $\tau (\pi_t)$. They announce $\pi_{1,t}$ and $\pi_{2,t}$, respectively, at the voting stage in each period $t$. Given the pair of policies, $(\pi_{1,t}, \pi_{2,t})$, each agent of generation $t$ votes for either $P_1$ or $P_2$ closer to her interest, one of which wins if it gets more votes than the other. If they have a same number of votes, they flip a coin to determine the winner, and hence, win the election with the probability of $1/2$.

According to the timing of events, voting is done before the occupation choice, and hence, $(z^*, \psi^*)$, the rent-seeking equilibrium defined by Definition 3.2, is yet determined at the voting stage. We now let $\pi (z)$ denote the most preferred policy for a type $z$ agent. Then, $\pi (z)$ solves the following problem given the morality $F(\cdot)$:

$$\pi (z) \equiv \arg \max_{\pi \in [\bar{\pi}, \bar{\pi}]} v(z; \psi^*, \pi) [1 - \tau (\pi)]$$  \hspace{1cm} (3.16)\textsuperscript{127}

\textsuperscript{16} Mathematically, the linearity of the cost $T (\pi, w)$ in $w$ is for stationarity; since $w$ grows over time, the pecuniary cost also needs to grow at the same rate. Economically, one can interpret the linearity such that the government pays a fraction of the aggregate labor income $w$ for enforcers’ labor income.

\textsuperscript{17} Due to the linearity of the cost $T (\pi, w)$ in $w$, the incidence of the lump-sum tax $T (\pi, w)$ is identical with that of an income tax. Hence, the political equilibrium of the model is independent of whether the government finances the cost through a lump-sum tax or an income tax.
where $v(z; \psi^*, \pi)$ is given by (3.4)–(3.6), and $\psi^* \equiv \psi(z^*; \pi)$ is given by (3.3). That is, choosing $\pi(z)$ is equivalent with choosing $(z^*, \psi^*)$ that maximizes $v(z; \psi^*, \pi) [1 - \tau(\pi)]$ given the morality $F(\cdot)$.

To facilitate the derivation of $\pi(z) \forall z \in [0, 1]$, we postulate the following assumptions:

\begin{align*}
\bar{\pi} &\leq \rho / (\rho + \theta), \quad (A1) \\
z^*(\pi) &\equiv \min \{z^*_H(\pi)\}, \quad (A2) \\
\psi^*(\pi) &= F(z^*(\pi)) \text{ is twice differentiable}, \quad (A3) \\
\frac{d^2 \psi^*}{(d\pi)^2} &\geq 0. \quad (A4)
\end{align*}

The first assumption (A1) precludes the utopia equilibrium where there is no rent-seeking. That is, we only focus on the real world equilibrium where some people become rent-seekers, and the fraction of rent-seekers is controlled by the morality and the institutional quality. Then, from Proposition 3.1, we know that there are possibly many equilibria $(z^*, \psi^*)$ for any $\pi \in [\bar{\pi}, \bar{\pi}]$. Hence, we need to fix $(z^*, \psi^*)$ in order to derive $\pi(z)$. Therefore, as indicated by (A2), we assume that each agent believes $z^*(\pi)$ will be chosen as the minimum value of $z^*_H(\pi)$.\(^{18}\) (A3) and (A4) jointly guarantee a single-peaked preference over the policy variable $\pi$ $\forall z \in [0, 1]$. More precisely, (A3) and (A4) make sure that $\pi(z)$ is uniquely determined, and $v(z; \psi^*, \pi) [1 - \tau(\pi)]$ decreases monotonically as $\pi$ differs from $\pi(z)$.\(^{19}\)

Then, we can provide the following result:

**Lemma 3.2** Suppose that (A1)-(A4) hold. Then, (i) for any given $F$, there are two institutional qualities $\pi^E(F)$ and $\pi^I(F)$, each of which is the most preferred institu-

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\(^{18}\) One can easily notice from the proof of Lemma 3.2 that (A2) is not critical for the political equilibrium summarized by Proposition 3.2. For example, one can assume that $z^*(\pi) \equiv \mathbb{E} [z^*_H(\pi) | \pi]$, but the political equilibrium is not changed qualitatively in that $\pi^E > \pi^I$ still holds.

\(^{19}\) Of course, (A3) implicitly requires that the distribution $F(z)$ and the probability of success $p(z)$ are twice differentiable. Then, (A3) is always satisfied if $\exists z^*_H \in [\hat{z}, 1]$ when $\pi = \bar{\pi}$. Since (A2) guarantees that there exists $z^*_H \forall \pi \in [\bar{\pi}, \bar{\pi}]$, the twice differentiability of $F(z)$ and $p(z)$ are necessary and sufficient conditions for (A3).
tional quality for an entrepreneur and an inactivist, respectively, and they satisfy the following inequalities:
\[
\bar{\pi} \leq \pi^I (F) < \pi^E (F) < \bar{\pi}.
\]

Also, (ii) \( v(z; \psi^*, \pi) [1 - \tau(\pi)] \) decreases monotonically as \( \pi \) differs from \( \pi^E (F) \) and \( \pi^I (F) \) for an entrepreneur and for an inactivist, respectively.

Lemma 3.2 indicates that an entrepreneur prefers better institutional quality than an inactivist. This is quite intuitive since an entrepreneur loses more than an inactivist from rent-seeking when he is a victim, and therefore, an entrepreneur is more willing to pay for a better institutional quality than an inactivist. Importantly, both \( \pi^E (F) \) and \( \pi^I (F) \) are independent of the individual skill \( z \). Since an inactivist’s payoff \( v^I \) is independent of \( z \), \( \pi^I (F) \) must be independent of \( z \) as well. The reason that \( \pi^E (F) \) is also independent of \( z \) is simple: for an entrepreneur, the marginal benefit and cost arising from better institutional quality increases at the same rate in his ability \( z \).\(^{20}\) Hence, \( \pi^E (F) \) is independent of \( z \) as \( \pi^I (F) \) is. This independence, of course, simplifies the analysis for the political equilibrium a lot. Using this property, we can obtain the following political equilibrium by applying the median voter theorem with the property that both \( \pi^E (F) \) and \( \pi^I (F) \) are independent of \( z \).

**Proposition 3.2 Political Equilibrium:** Let \( z^M \) denote the median voter such that \( F (z^M) = 1/2 \). Then, the equilibrium policy, say \( \pi^* \), is given by:

\[
\pi^* (F) = \begin{cases} 
\pi^E (F) & \text{if } z^M \geq z^I, \\
\pi^I (F) & \text{if } z^M < z^E, \\
\pi^E (F) \text{ or } \pi^I (F) & \text{if } z^M \in [z^E, z^I),
\end{cases}
\]

\(^{20}\) Note that an entrepreneur pays \( \tau(\pi) p(z) \lambda w \) as the tax, which increases as \( z \) rises. Although the marginal benefit of a higher institutional quality increases with a higher \( z \), the marginal cost also increases, and they exactly cancel out each other.
where $z^E$ and $z^I$, respectively, solve the following:

$$z^E = \min\{z_H^*(\pi^E)\}; \quad z^I = \min\{z_H^*(\pi^I)\},$$

and hence:

$$\bar{z} < z^E < z^I.$$

Now, we are ready to define the rent-seeking equilibrium with endogenous institutions as follows:

**Definition 3.3 Rent-Seeking with Endogenous Institutions:** For any given morality $F(\cdot)$, the equilibrium institutional quality $\pi^*(F)$ is determined by (3.17).\(^{21}\) Then, the rent-seeking equilibrium with endogenous institutions, $(z^*, \psi^*)$, is given by:

$$z^*(F) = \min\{z_H^*(\pi^*(F))\},$$
$$\psi^*(F) = F(z^*(F)).$$

Proposition 3.2 indicates that $\pi^*(F)$ tend to increase as $z^M$ is larger. This, in turn, implies that $\pi^*(F)$ tend to be higher (lower) as the morality is healthy (unhealthy). This is because $z^M$ hets larger as $F(z)$ is small $\forall z \in (0, 1)$. Then, from Proposition 3.1, Corollary 3.1 and Definition 3.2, we have the following observations on the role of morality in the determination of the institutional quality, and thereby, the aggregate output and the growth rate as well.

**Corollary 3.2 Morality and Institutions:** The institutional quality is more likely to be poorer as the morality worsens in that $z^M$ decreases as the morality worsens.

**Morality and Economy:** Since unhealthy morality tends to induce a poorer institutional quality in equilibrium, both the aggregate output and the growth rate are more likely to be reduced as the morality worsens.

\(^{21}\) More precisely, when $z^M \in [z^E, z^I)$, $\pi^*(F) = \pi^E(F)$ if $v(z^M; \psi^*, \pi^E) [1 - \tau(\pi^E)] \geq v(z^M; \psi^*, \pi^I) [1 - \tau(\pi^I)]$, and $\pi^*(F) = \pi^I(F)$ otherwise.
3.7 Concluding Remarks

We examine rent-seeking behavior through the lens of culture (morality) and institutions (effective enforcement of laws). We show that rent-seeking harms both the aggregate income and growth due to the wedge between a rent-seeker’s benefit and his victim’s loss in the rent-seeking. However, it is, in general, hard for the economy to eliminate rent-seeking behavior since, to do that, voters have to agree to establish effective enforcement of laws suppressing the incentive to be a rent-seeker. The social morality solely determines whether or not they vote for such effective enforcement of laws, highlighting the fundamental role that the culture plays, interconnected with what the institutions do, in the determination of the rent-seeking equilibrium.

Consequently, this implies that the government is possibly able to do a crucial role in dealing with the ingrained rent-seeking problem by properly manipulating the social morality. As some brilliant previous literature already discussed, e.g., Grossman and Kim (2000), one needs to study how the government (or a group of people) can revive the morality to eradicate long-lasting rent-seeking in society.
Appendix A

Appendix for Chapter 1

A.1 Time-Varying Monitoring Cost

We set the monitoring cost to be time-invariant in the theory part of the chapter. We find that the negative growth effects of the financial friction disappear as the economy grows. As one can easily notice, for the asymptotic irrelevance to growth, it is crucial to assume that the monitoring cost is fixed at a certain level. This is simply because the monitoring cost \( \mu \) becomes relatively smaller as the borrower’s net worth \( \pi_1 \) (the entrepreneurial profit) increases along with the growing economy. Then, the borrower’s net worth becomes too high in the long-run, compared to \( \mu \), to renege on his debt. Hence, the borrower’s incentive to renege on his debt becomes inelastic regardless whether the monitoring cost raises or lowers. This is the intuition behind the asymptotic irrelevance.

However, if the monitoring cost also increases along with growth of the economy as well, the asymptotic irrelevance would not hold for sure. This is simply because, if so, the monitoring cost would not be trivial even in very long run unless the growth rate of the monitoring cost, \( g_{\mu_t} \), is strictly smaller than that of the borrower’s profit, \( g_{\pi_{1,t}} \). This means that the financial friction may adversely affect growth permanently in some cases.

To show this clearly, we redefine the monitoring cost by \( \mu_t \equiv \mu (w_t, A_t) \) where, obviously, \( \mu_w (w_t, A_t) > 0 \). The growth rate of the monitoring cost in the steady state
$g_\mu$ will be determined by $\mu_A(w_t, A_t)$ since the wage is constant in the steady state; recall that $w_t$ is wage per efficiency unit of labor, and it is constant on the balanced growth path from Proposition 1.3. For example, suppose that $\mu_A < 0$, then the monitoring cost is diminishing over time, i.e., $g_\mu < 1$, since the TFP $A$ is continually growing, while $w$ is fixed on the balanced growth path.\footnote{One can interpret this case such that the financial intermediation technology evolves relatively faster than an entrepreneur’s defection technology.} We can summarize the negative growth effects with the time-varying monitoring cost as follows:

**Proposition A.1 Growth Effect with Time-Varying Monitoring Cost:** Suppose that $\mu = \mu(w, A)$ is continuously differentiable in $A$. Then, we have the following results:

**Case (1) $\mu_A(w, A) \leq 0$:** The growth effects of the financial friction are temporary.

**Case (2) $\mu_A(w, A) > 0$:**

(i) If $\mu(w, \phi A) = \phi^n \mu(w, A) \forall \phi \geq 1$ with $n \in (0, 1)$, then the growth effects of the financial friction are temporary.

(ii) If $\mu(w, \phi A) = \phi \mu(w, A) \forall \phi \geq 1$, then the growth effects of the financial friction are permanent.

(iii) If $\mu(w, \phi A) = \phi^n \mu(w, A) \forall \phi \geq 1$ with $n > 1$, then the economy stops growing in the long run, i.e., $g_{Y,t} \to 1$.

In summary, Case (1) of Proposition A.1 indicates that the growth effects of the financial friction are temporary if the monitoring cost never grows in the long run, i.e., $g_\mu \leq 1$. This is because the growth rate of the monitoring cost is smaller than the growth rate of the borrower’s profit $g_{\pi_i}$; recall that $g_{\pi_i}$ is larger than one. The financial friction, however, can be still irrelevant to growth even when the monitoring cost continually grows. More precisely, this is Case (2-i) of Proposition A.1 where the monitoring cost grows slower than the borrower’s profit, i.e., $g_{\pi_i} > g_\mu > 1$, and hence, it becomes negligible in the long run. In contrast, if it grows as fast as the entrepreneurial profit, i.e., $g_\mu = g_{\pi_i}$, then the negative growth effects are permanent, meaning the growth rate in the stationary state gets smaller as the financial friction is worse. This results, which corresponds to Case (2-ii) of Proposition A.1, is quite intuitive since the monitoring cost will not be trivial any longer even in the long run when it grows as fast as the entire economy, affecting the economy permanently.
Finally, if the monitoring cost grows even faster than the entire economy, and hence, than the borrower’s net worth, the borrowing rate diverges to infinity as time goes on. Hence, there will be no one willing to invest in the risky but productivity-enhancing project, so that the economy will stop growing in the long-run. This is Case (2-iii) of Proposition A.1. Based on this result, we can provide an example of the time-varying monitoring cost and its growth implications as follows:

**Example: Linear Monitoring Costs**

Suppose that the monitoring cost is linear in the efficiency unit of labor as follows:

\[ \mu_t = w_t \ell_m (A_t) \]

where \( \ell_m \) is the efficiency unit of labor required for the monitoring, which is determined by the current TFP level such that \( \ell_m : \mathbb{R}_{++} \to \mathbb{R}_+ \). Assume that \( \ell_m \) is twice continuously differentiable. Then, from Proposition A.1, we have the following results:

**Case (1)** \( \ell'_m (A_t) \leq 0 \): The monitoring cost converges to zero, i.e., \( \mu_t \to 0 \), and therefore, the negative growth effects are temporary.

**Case (2-i)** \( \ell'_m (A_t) > 0, \ell''_m (A_t) < 0 \): The gross growth rate of the monitoring cost converges to unity, i.e., \( g_{\mu_t} \to 1 \), since \( A_t \) increases to infinity. Hence, the negative growth effects are temporary.

**Case (2-ii)** \( \ell'_m (A_t) > 0, \ell''_m (A_t) = 0 \): The gross growth rate of the monitoring cost is the same as the growth rate of \( A_t \), resulting permanent negative growth effects.

**Case (2-iii)** \( \ell'_m (A_t) > 0, \ell''_m (A_t) > 0 \): Growth stops in the long run.

**Which Case is Empirically Plausible?**

Then, which one is the case in the real world? First, Case (2-iii) seems not the case simply because the interest rate spread increases forever if it is true (see the proof
of Proposition A.1) while, in the real world, the interest rate spread decreases, and then, stabilizes in the long run (see Figure 1.2).

Similarly, Case (2-ii) seems not empirically plausible. If it is true, the theory predicts that the growth rate, the investment rate and the interest rate spread will be fixed at a constant level over time, respectively, once $r_t$ is fixed; see the proofs of Proposition 1.4 and A.1. This implies that the financial friction severity is not endogenously relaxed, contradicting the actual transition features depicted in Figure 1.2. As shown in the figure, the averaged growth rate and investment rate increase along with a decreasing trend of the interest rate spread, and this reflects the endogenous relaxation of the financial friction. Therefore, this case does not support the asymptotic irrelevance property, which is verified by our empirical analysis; recall that $\delta$ turned out strictly positive, not zero.

Then, only Case (1) and Case (2-i) remain. Both of the cases are empirically plausible since they are consistent with the essential features of transition dynamics in reality, and therefore, we have assumed that the monitoring cost is fixed at a certain level over time, which is a specific example of Case (1) for a clearer exposition of the main idea without any change in the theoretical results.

Consequently, for the theory, it seems plausible to assume that either the monitoring cost decreases as the economy grows, or that it grows slower than the entire economy. To make this assumption more attracting, we provide another empirical evidence that the financial intermediation cost, in fact, grows slower than the economy, and hence, the constant monitoring cost assumption we have had so far is indeed realistic. This, in turn, implies that the asymptotic irrelevance holds in the real world based on our theoretical conclusion; see Corollary 1.3.

For the evidence, we will use comprehensive data, provided by Demirgüç-Kunt, Laeven and Levine (2004) (DLL). This data includes abundant information on financial conditions of more than 1,400 banks across 72 developed and developing countries over the period of 1995–1999. One of advantages of using this data is that we can control for country-level differences in the banking environment as well as bank-level differences in various banking-related factors.

In virtue of this advantage, DLL examined impacts of bank regulations, market structure and national institutions on the financial intermediation cost proxied by the
net interest margin and overhead expenditure ratio. The net interest margin is equal to interest income minus interest expense divided by interest-bearing assets, and thus, it is analogous to the interest rate spread in our theoretical model. Therefore, if the net interest margin becomes smaller over time, it indicates that the monitoring cost (the financial intermediation cost) grows slower than the economy from the theoretical conclusion of our model; recall that the interest rate spread decreases when the monitoring cost grows slower than GDP per capita by equation (1.1). Meanwhile, the overhead expenditure ratio is equal to bank overhead costs divided by total assets. That is, it measures overhead costs with controlling for the bank size at a constant level, and therefore, it proxies the monitoring cost in our model.

In the empirical analysis, we control for other relevant differences in individual banking activities by utilizing the virtue of the data. Not only bank specific differences, we also control for nationwide banking environments, institutional factors and some macro variables. We drop an observation associated with negative economic growth of a given country since we want to verify the argument that the monitoring cost gets relatively smaller as an economy grows, although we have similar results with the full sample.

We use a simple identification strategy. We first regress each of both financial intermediation costs (the net interest margin and overhead ratio) on a rich set of control variables in order to identify error terms that represent idiosyncratic shocks over the sample period. Then, we can obtain fitted growth rates of the two dependent variables orthogonal to the identified non-systematic exogenous shocks, and we can check if the fitted intermediation costs, in fact, are implying either that the monitoring cost decreases along with economic growth, or that it grows slower than the economy.

For the empirical analysis, we use the following specification similar to the one used in DLL:

\[
\ln \text{cost}_{ict} = \alpha + \mathbf{X}_{ict}'\beta + \mathbf{X}_{ic}'\gamma + \mathbf{X}_{ct}'\delta + \mathbf{X}_c'\zeta + \eta D_t + u_i + \varepsilon_{ict}
\]

where \(i, c\) and \(t\) index a bank, a country and year, respectively. \(\mathbf{X}_{ict}\) and \(\mathbf{X}_{ic}\) are time-varying and time-invariant bank-specific controls, respectively. \(\mathbf{X}_{ct}\) and \(\mathbf{X}_c\)
are country-specific controls, both of which are time-changing and time-constant, respectively. Details of them are provided in the note of Table A.1. $D_t$ is time dummy, and $u_i$ is either the fixed or random effect for an individual bank. The dependent variable $\ln \text{cost}_{ict}$ is the log of either the net interest margin (the interest rate spread) or the overhead expenditure ratio (the monitoring cost).

Consider first the logged net interest margin as the dependent variable, i.e., $\ln \text{cost}_{ict} = \ln \Delta_{ict}$. Also, let $\hat{\text{cost}}_{ict}$ denote the fitted value (predicted value) of the dependent variable, $\ln \text{cost}_{ict}$. Then, one can calculate the net growth rate of the interest rate spread, which is orthogonal to idiosyncratic random shocks, by calculating $\gamma_{\Delta_{ict}} = \ln \hat{\text{cost}}_{ict+1} - \ln \hat{\text{cost}}_{ict}$. If $\gamma_{\Delta_{ict}} < 0$, it means that the interest rate spread decreases when the economy grows. This, in turn, implies that the monitoring cost grows slower than the entire economy based on our theory.

Consider second that the logged overhead expenditure ratio as the dependent variable, i.e., $\ln \text{cost}_{ict} = \ln \mu_{ict}$. Similarly, the net growth rate of the monitoring cost is computed by $\gamma_{\mu_{ict}} = \ln \hat{\text{cost}}_{ict+1} - \ln \hat{\text{cost}}_{ict}$. Since the overhead ratio can be interpreted as the monitoring cost in our model, the monitoring cost grows slower than the economy, confirming the hypothesis again, if $\gamma_{\mu_{ict}} - \gamma_{\hat{y}_{ict}} < 0$.

Table A.1 summarizes estimation results from the panel regression with either the fixed effect (FE) or random effect (RE). As indicated, the estimated growth rate of the interest rate spread, $\gamma_{\Delta_{ict}}$, is negative at 99% confidence level in each sample year. This pattern is also the same for the other estimated growth rate, $\gamma_{\mu_{ict}} - \gamma_{\hat{y}_{ict}}$, the growth rate of the monitoring cost minus the growth rate of GDP per capita. Consequently, these findings support the argument that the monitoring cost, which represents the financial intermediation cost, in fact, grows slower than the economy on average, and thereby, advocate the use of the assumption in the theory part that the monitoring cost is time-invariant.$^2$

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$^2$ We have similar results with more controls for banking regulation although we do not report the estimation results here.
Table A.1: Growth Rate of the Financial Intermediation Cost (Panel)

<table>
<thead>
<tr>
<th></th>
<th>( \gamma_{\Delta t} )</th>
<th></th>
<th>( \gamma_{\mu t} - \gamma_{y t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>(-.012)</td>
<td>(-.011)</td>
<td>(-.036)</td>
</tr>
<tr>
<td>1995</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>99% c.i.</td>
<td>([-0.020; -0.003])</td>
<td>([-0.020; -0.002])</td>
<td>([-0.048; -0.023])</td>
</tr>
<tr>
<td># of Obs.</td>
<td>1,059</td>
<td>1,059</td>
<td>1,059</td>
</tr>
<tr>
<td>mean</td>
<td>(-.043)</td>
<td>(-.042)</td>
<td>(-.048)</td>
</tr>
<tr>
<td>1996</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>99% c.i.</td>
<td>([-0.052; -0.034])</td>
<td>([-0.052; -0.033])</td>
<td>([-0.063; -0.032])</td>
</tr>
<tr>
<td># of Obs.</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>mean</td>
<td>(-.034)</td>
<td>(-.033)</td>
<td>(-.021)</td>
</tr>
<tr>
<td>1997</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>99% c.i.</td>
<td>([-0.043; -0.024])</td>
<td>([-0.042; -0.023])</td>
<td>([-0.035; -0.007])</td>
</tr>
<tr>
<td># of Obs.</td>
<td>987</td>
<td>987</td>
<td>987</td>
</tr>
<tr>
<td>mean</td>
<td>(-.024)</td>
<td>(-.024)</td>
<td>(-.042)</td>
</tr>
<tr>
<td>1998</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>99% c.i.</td>
<td>([-0.034; -0.014])</td>
<td>([-0.035; -0.014])</td>
<td>([-0.055; -0.029])</td>
</tr>
<tr>
<td># of Obs.</td>
<td>1,026</td>
<td>1,026</td>
<td>1,026</td>
</tr>
</tbody>
</table>

Note: We conduct panel regressions with the fixed effect (FE) or random effect (RE). The set of controls includes bank-specific variables, bank environment, institutional factors and some macro variables. For the bank-specific variables, we use bank size, return on average assets, cost to income ratio, bank equity, liquidity, fee income, bank risk and market share. The bank environment variables consist of the number of banks and bank concentration in a nation. The institutional factors include property rights, economic freedom and the KKZ quality index of a country. Finally, for the macro variables, we use GDP per capita, total trade value, and the average inflation rate and growth rate of total population. Details of each variable are given in Appendix of Demirgüç-Kunt, Laeven and Levine (2004).
Figure A.1: Mean Growth Rate of the Financial Intermediation Cost
A.2 The Closed Economy

Switching to recursive notation where primes denote next period variables, we can rewrite the equation that determines the threshold of the entrepreneurial ability \(z^\ast\) as follows:

\[
p [\pi'_1 - (1 + i') x (z^\ast)] = \pi'_0
\]

Since \(\pi_1 = \gamma \pi_0\) and \(1 + i = (1 + r - \delta) \Delta / p\), the equation is rewritten as follows:

\[
p \frac{\gamma - 1}{\gamma} \{p \gamma \pi'_0 - (1 - p) \mu\} = (1 + r' - \delta) x (z^\ast)
\]

where we use \(\Delta \equiv p \gamma \pi_0 / [p \gamma \pi_0 - (1 - p) \mu]\). From equations (1.8) and (1.11), we can define \(\pi'_0\) and \(r'\) as follows: \(\pi'_0 \equiv \pi (z^\ast, k')\); \(r' \equiv r (z^\ast, k')\). Then, we have:

\[
p \frac{\gamma - 1}{\gamma} \{p \gamma \pi (z^\ast, k') - (1 - p) \mu\} = \{1 + r (z^\ast, k') - \delta\} x (z^\ast)
\]

From (8), it is obvious that \(\pi_z > 0\) and \(\pi_{k'} > 0\). This implies that \(\gamma\) needs to be sufficiently large since the condition that \(p \gamma \pi (z, k') - (1 - p) \mu > 0\) should be satisfied at any equilibrium, or equivalently, \(i \geq r + \delta\) for any given \(z^\ast\) and \(k' > 0\). Hence, suppose that \(\gamma\) is sufficiently large, so that \(p \gamma \pi (z, k') - (1 - p) \mu = 0\) for \(k \approx 0\). Then, one can easily verify that, for any \(k' \geq k \approx 0\), \(z^\ast \in [z_L, z_H]\) is uniquely determined since the RHS is decreasing in \(z^\ast\) from infinity to zero (note that \(r_z < 0\)), while the LHS is increasing in \(z^\ast\) under the assumption, \(p \pi (z^\ast, k') - (1 - p) \mu > 0\) in any equilibrium. Hence, we can define \(z^\ast \equiv z (k'; \mu)\), which is straightforward that \(z_k < 0\) and \(z_\mu > 0\).

To pin down the capital investment in equilibrium, we can rewrite the capital market clearing condition, given by equation (1.5), by letting \(B^f_t = 0\). Then, we have:

\[
k' + \frac{X (z (k'; \mu))}{L} = s (z (k; \mu), z (k'; \mu), k, k')
\]

where we use \(w \equiv w (z (k; \mu), k)\) from equation (1.10), \(r' \equiv r (z (k'; \mu), k')\) from equation (1.11) and \(s \equiv s (w, r')\). The LHS is the demand for funds and the RHS is the supply of funds. Now, let us confine our attention on a steady state. Then, the
capital market clearing condition is rewritten as follows:

\[ k + \frac{X(z(k; \mu))}{L} = s(z(k; \mu), k) \]

Note first that \( dX(z(k; \mu))/dk > 0 \) since \( z_k < 0 \) and \( x_z < 0 \). Now let us specify the R&D investment cost function \( x(z, A) \) as follows:

\[ x(z, A) \equiv \zeta A \left( \frac{1}{z - z_L} - \frac{1}{z_H - z_L} \right) \]

where \( \zeta > 0 \) measures the efficiency in the R&D investment. It is straightforward that the cost function satisfies all of the assumptions on \( x(z, A) \): \( x_z(z, A) < 0 \), \( \lim_{z \to z_L} x(z, A) = \infty \), \( \lim_{z \to z_H} x(z, A) = 0 \) and \( x_A(z, A) > 0 \). With this cost function, one can show that \( \lim_{k \to 0} dX(z(k; \mu))/dk = 0 \),\(^3\) and \( d^2X(z(k; \mu))/(dk)^2 > 0 \).

Similarly, \( ds(z(k; \mu), k)/dk > 0 \) with the log utility function \( u(c) = \ln c \) since, with the log utility function, \( s(z(k; \mu), k) = Q\psi(z(k; \mu))k^{\alpha_v} \) where \( Q \) is a positive

---

\(^3\) From the entrepreneur’s problem, we have:

\[
\frac{dz}{dk} = \frac{r_k(z(k), k) x(z(k)) - p\pi_k(z(k), k)}{C_1 k^{\alpha_v-2} x(z(k)) - C_2 k^{\alpha_v-1} - \left[C_3 k^{\alpha_v-1} x(z(k)) + C_4 k^{\alpha_v} + C_5\right]} \]

where \( C_i \) (\( i = 1, 2, \ldots, 5 \)) is a constant. Then,

\[
\frac{dX}{dk} = \frac{dX}{dz} \frac{dz}{dk} = \frac{C_2 k^{\alpha_v-1} x(z(k)) - C_1 k^{\alpha_v-2} [x(z(k))]^2}{C_2 k^{\alpha_v-1} - \left[C_3 k^{\alpha_v-1} x(z(k)) + C_4 k^{\alpha_v} + C_5\right]} \]

since \( dX(z)/dz = -x(z) \). Hence, given any \( z(k) \), if \( x(z(k)) \) satisfies that \( x(z(k)) \geq k \forall k \in [0, \varepsilon] \), then \( \lim_{k \to 0} dX(z(k))/dk = 0 \). This condition is easily satisfied. Since \( x(z, A) \equiv \zeta A\{1/(z - z_L) - 1/(z_H - z_L)\} \) where \( \zeta > 0 \) and \( A > 0 \), the condition requires:

\[ z(k) - z_L \leq \frac{1}{k/A + 1/(z_H - z_L)} \forall k \in [0, \varepsilon] \]

Note that, for any \( k \approx 0 \), \( 1/[k/A + 1/(z_H - z_L)] \approx z_H - z_L = \max\{z(k) - z_L\} \geq z(k) - z_L \) as desired.

\(^4\) \( d^2X/(dk)^2 = -[z_kk(k; \mu)]^2\{x(z) + x'(z)\} \). Hence, \( d^2X/(dk)^2 > 0 \) if, and only if, \( -x'(z) > x(z) \), which always holds with the cost function, \( x(z) \equiv \zeta A\{1/(z - z_L) - 1/(z_H - z_L)\} \).
constant. Then, it is easy to check that \( \lim_{k \to 0} ds/dk = \infty. \) Hence, a sufficient condition for the uniqueness of \( k^* > k \approx 0 \) is that \( d^2s (z (k; \mu), k) / (dk)^2 < 0, \) which is, unfortunately, not trivial to verify. However, if the c.d.f. of the skill, given by \( F(z) \), is assumed to be highly concave, so that the p.d.f. \( f(z) \) is decreasing in \( z \) fast, i.e., \( f'(z) \ll 0 \), it is likely to have \( d^2s (z (k; \mu), k) / (dk)^2 < 0, \) and then, \( k^* \) is uniquely determined.

Suppose now that \( k^* \) is uniquely pinned down for analytical simplicity. Then, we have:

\[
\frac{dk^*}{d\mu} = \frac{z_\mu (k^*) \left\{ \frac{X'(z^*)}{L} + s_z (z^*, k^*) \right\}}{1 - \left\{ s_z (z^*, k^*) z_k (k^*) + s_k (z^*, k^*) - \frac{X'(z^*)}{L} \right\}}
\]

where \( s_z (z^*, k^*) < 0 \) with the log utility function, so that the numerator is negative. Hence, \( dk^*/d\mu > 0 \) if, and only if, the denominator is negative. Since \( X'(z^*) < 0 \), the denominator is negative when \( ds (k^*) / dk = s_z (z^*, k^*) z_k (k^*) + s_k (z^*, k^*) > 0 \) is sufficiently large. One possible case is that \( k^* \) is sufficiently small, so that the marginal product of capital is sufficiently large; recall that we have assumed \( d^2s (z (k; \mu), k) / (dk)^2 < 0 \) for the uniqueness of \( k^* \).

This implies that an increase in the monitoring cost may result in a huge increase in the supply of funds (the domestic saving) so does the capital stock \( k \). The equilibrium threshold \( z^* \), then, decreases due to the fact that \( z_k (k; \mu) < 0 \), and this leads to an increase in the output. Therefore, the negative scale effect of the financial friction presented in Proposition 1.1 is ambiguous \textit{a priori} in the closed economy.

---

\(^5\) \( Q \) is equal to \((1 - \beta) (1 - \alpha) \nu (A/L)^{1 - \nu} > 0.\)

\(^6\) \( s'(k) = Q \left[ \psi'(k) k^{\alpha v} + \alpha \psi(k) k^{\alpha v - 1} \right] \) where \( f(z) \) is the p.d.f. of the skill/ability \( z \). Since \( \psi'(k) = -(pr - 1) f(z) z_k > 0 \forall k > 0 \) (recall that \( f(z) > 0 \forall z \in [z_L, z_h] \)), \( \lim_{k \to 0} \psi'(k) k^{\alpha v} \equiv \xi \geq 0. \) Hence, \( \lim_{k \to 0} s'(k) = Q \xi + Q \lim_{k \to 0} \alpha \psi(k) k^{\alpha v - 1} = \infty. \)

\(^7\) Note that \( s''(k) = Q \left[ \psi''(k) k^{\alpha v} + 2\alpha \psi'(k) k^{\alpha v - 1} - \alpha \psi(k) k^{\alpha v - 2} \right] \) where \( \psi'(k) = -(pr - 1) f(z) z_k > 0 \) and \( \psi''(k) = -(pr - 1) [f'(z) z_k + f(z) z_{kk}], \) the sign of which is not determined \textit{a priori}. However, if \( f'(z) \ll 0 \), then it is likely to have \( \psi''(k) \ll 0 \) so is \( s''(k). \)
A.3 The Case of Exogenous Borrowing Constraint

In this appendix, we consider the exogenous borrowing constraint adopted in most of the previous literature in order to point out its limitations by comparing it with the endogenous credit constraint. This will clarify the importance of using the endogenous credit constraint in the growth analysis. The exogenous credit constraint, adopted in most of previous literature, is given by:

\[ b_t \leq \lambda a_t \]

where \( a \) and \( b \) denote asset and debt, respectively, and \( \lambda \) is exogenously given.\(^8\)

In our model, every prospect borrower (an entrepreneur) is born without any asset, and hence, \( a_t = 0 \) for all \( t \geq 0 \). We, thus, assume that an entrepreneurs born in period \( t \) is endowed one unit of labor as a worker. He, then, earns labor income in the first period of his life. We let \( L_t \) be the efficiency unit of labor supplied by entrepreneurs. Recall that, we have assumed that \( g_{L,t} = g_{A,t} \) for the stationary growth. Similarly, we assume that \( g_{L,e,t} = g_{L,t} \). Then, \( g_{a,t} = g_{L,e,t} \) since the total population of entrepreneurs born in period \( t \) is given by unity \( \forall t \).

For simplicity, we assume that the asset \( a_t \) is illiquid in period \( t \), and hence, cannot be invested in R&D in the same period. Then, the exogenous borrowing constraint is rewritten as follows:

\[ x(z, A_t) \leq \lambda a_t \]

and one can easily prove that the skill threshold under this exogenous borrowing constraint, say \( \hat{z}_t \), is uniquely determined for all \( t \geq 0 \).\(^9\)

Then, the exogenous financial friction does not affect the economy as long as the

---

\(^8\) The exogenous borrowing constraint can be interpreted as a collateral constraint. Suppose that a financial intermediary can seize the entire asset of a borrower \( a \) if the borrower renege on his debt \( b \). Suppose also that a borrower can abscond with fraction of \( 1/\lambda \) of the loan \( b \). Then the financial intermediary must lend asset only to the extent that no borrower can defect: \( b/\lambda \leq a \).

\(^9\) The determination of the threshold \( \hat{z}_t \) is the same as that in the benchmark case with the endogenous credit constraint except that the borrowing limit is exogenously given by \( \lambda a_t \) and the gross borrowing rate \( 1 + i_t \) is equal to the fair insurance rate \( (1 + r_t - \delta) / p \). To see this more clearly, let \( \mu = 0 \) in the financial market clearing condition given by equation (1.3).
following condition holds:

\[ x(z^*, A_t) \leq \lambda a_t \]

where \( z^* \) denotes the asymptotic level of \( z_t^* \), or equivalently, the skill threshold that guarantees the first-best optimum. The more interesting case is when the exogenous borrowing constraint is binding as in the previous literature, i.e., \( x(z^*, A_t) > \lambda a_t \), so that \( \hat{z}_t > z^* \), meaning it is infeasible to achieve the first-best allocation.\(^{10}\)

Same as the argument applied to the main section, one can easily show that the growth rates of GDP and the TFP are the same under the exogenous borrowing constraint, and the rate is given by \( \psi(\hat{z}) \) where the threshold \( \hat{z} \) increases as \( \lambda \) decreases. Hence, the severity of the financial friction is measured by \( 1/\lambda \), which is exogenously given at a constant level \( \forall t \).

\textbf{Limitations of the Exogenous Borrowing Constraint}

First note that we cannot define and describe the interest rate spread with the exogenous borrowing constraint. This is because \( 1 + i_t = (1 + r_t - \delta) / p \) by the financial market clearing condition; see equation (1.3), letting \( \mu = 0 \).

Also, it is straightforward that the amplification effect (the indirect growth effect) of the financial friction is not captured. This is because the tightness of the borrowing constraint, which is measured by \( \lambda \), does not change regardless of whether the aggregate uncertainty rises or lowers.

Third, note that \( \hat{z}_{t+1} = \hat{z}_t \) if the growth rate of \( a_t \) is equal to the growth rate of \( x(\hat{z}_t, A_t) \). For purpose of comparison, assume that \( x(\hat{z}, A) \) is linear in \( A \) as in the baseline case. Then, the growth rate of the R&D cost becomes equal to \( g_{A,t} \), which is also equal to the growth rate of the entrepreneurial net worth \( a_t \). Hence, with the exogenous borrowing constraint, \( \hat{z}_t \) is time-invariant given any constant interest rate \( r \). This, in turn, implies that the negative growth effect is permanent since the skill threshold \( \hat{z}_t \) is time-invariant even when the economy continually grows.

The permanence of the growth effect is reminiscent of the well-known conclusion in the previous literature with the exogenous borrowing constraint, where \( \lambda \) is

\(^{10}\) This is because any entrepreneur with \( z \in [z^*, \hat{z}_t] \) cannot invest in R&D although the investment is profitable.
fixed at a certain level. This is inconsistent with empirical evidence supporting the asymptotic irrelevance of the financial friction provided in Section 1.4.

One implication of the exogeneity in the financial friction is that the maximal loan-to-value (LTV) ratio, or the LTV ratio for a borrowing-constrained individual, is also arbitrarily given and fixed, which seems clearly unrealistic. This implies further that, under the exogenous credit constraint, the convergence is immediate, and therefore, the convergence speed is irrelevant to the financial friction. In fact, the investment rate is fixed over time, which is inconsistent with the actual transition dynamics in the real world as illustrated by Figure 1.2.

In contrast, the maximal LTV ratio, say \( \lambda_t^* \equiv x(z_t^*, A_t)/a_t \), is endogenously determined in the economy with the endogenous financial friction. Recall that the skill threshold \( z_t^* \) increases as the financial friction is more severe. Then, the maximal LTV ratio \( \lambda_t^* \) is monotone decreasing in \( \mu \) since \( \lambda_t^* \) is monotone decreasing in \( z_t^* \). Consequently, \( 1/\lambda_t^* \), which is endogenously determined in equilibrium, can be interpreted as the implied credit constraint tightness that reflects the financial friction severity.

Note also that the implied tightness \( 1/\lambda_t^* \) is time-varying along with various economic circumstances, while the economy converges to the stationary growth steady state. In particular, \( \lambda_t^* \) increases during the transition phase, converging to its asymptotic level, i.e., \( \lambda_t^* \uparrow \lambda^* \). Since the credit constraint is binding tightly at the beginning of economic development, the maximal LTV ratio \( \lambda_t^* \) is low, but it increases over time as the economy grows. Hence, less able entrepreneurs can finance funds for productivity-enhancing investments along with the endogenous relaxation of the credit constraint, and this causes an increasing trend of the investment rate consistent with what we observe from data.\(^{13}\)

\(^{11}\) This is simply because \( \partial x_t / \partial z < 0 \).

\(^{12}\) This is easily verified. Note that:

\[
\frac{\lambda_{t+1}^*}{\lambda_t^*} = \frac{x(z_{t+1}^*, A_{t+1})}{x(z_t^*, A_t)} \frac{a_t}{a_{t+1}} = \frac{x(z_{t+1}^*, A_{t+1})}{x(z^*_t, A_t)} \frac{\psi(z_t^*)}{\psi(z^*_t)} = \frac{x(z_{t+1}^*, A_{t+1})}{x(z_t^*, A_t)} > 1
\]

where the inequality comes from \( z_t^* > z_{t+1}^* \) (see (i) of Proposition 1.4). The convergence holds by the continuity of \( \lambda_t^* \) in \( z_t^* \). Obviously, this is consistent with the endogenous relaxation, represented by \( \lim_{t \to \infty} \Delta_t = -\log p > 0 \) (see Figure 1.2).

\(^{13}\) It is worth noting that the increasing investment rate is obtainable under the exogenous
**Aggregate LTV Ratio**

We now think of aggregate debt-to-net worth ratio, or equivalently, aggregate LTV ratio, say $\Lambda_t$, which is defined by:

$$
\Lambda_t \equiv \frac{\int_{\hat{z}_t}^{z_H} x_t(z) dF(z)}{\int a_t dF(z)} = \frac{X_t}{Y_t} \frac{Y_0}{a_0} = \rho_t a_0.
$$

where $\rho_t$ is the R&D expenditure rate in period $t$ as defined in Proposition 1.4. In the economy with the exogenous borrowing constraint, the aggregate LTV ratio is fixed over time, i.e., $\Lambda_t = \Lambda$, as long as the interest rate $r$ is constant. This is because the threshold $\hat{z}_t$ does not change as we have seen. However, under the endogenous credit constraint considered in the 3 chapter, the aggregate LTV ratio, say $\Lambda^*_t$, increases and converges to its steady state value $\Lambda^*$ since $\rho_t$ is increases and converges to its steady state value; see part (iv) of Proposition 1.4.

### A.4 Proofs

**Proof of Lemma 1.1:**

Let us suppress the time subscript for convenience. First, from the first-order conditions for $(k_0, \ell_0)$, one can notice that $\pi_0 > 0$ for any given price vector, $(r, w) \in \mathbb{R}^{2+}$, since $A > 0$. Then, $k_0 > 0$ and $\ell_0 > 0$, so that the non-negative constraints for $(k_0, \ell_0)$ are not binding. Also, the first-order conditions for $(k_0, \ell_0)$ and $(k_1, \ell_1)$ imply that $k_1 = \gamma k_0$ and $\ell_1 = \gamma \ell_0$, and therefore, $\pi_1 = \gamma \pi_0$. Since $x'(z) < 0$, $\lim_{z \to z_L} x(z) = \infty$ and $\lim_{z \to z_H} x(z) = 0$, there is a unique interior solution, $z^* \in (z_L, z_H)$, which solves $p\pi_1 - \pi_0 = (p \gamma - 1) \pi_0 = p (1 + i) x(z^*)$ for any given $i \geq 0$ if, and only if, $p \gamma - 1 > 0$, or equivalently, $p \gamma > 1$. ■

**Proof of Lemma 1.2:**

Let us suppress the time subscript for convenience. Since the objective function is borrowing constraint. For example, if an entrepreneur is allowed to self-finance through his own saving, then he will be eventually able to conduct profitable R&D investment by accumulating necessary funds for the investment. This results in the realistic trend of the investment rate, which is increasing along with economic development; see Buera and Shin (2013) for details.
linear in the choice variables, the optimal borrowing rate $1 + i$ should be minimized while satisfying both (PC) and (IC). Note that $\pi_1 - [1 + i(z)] x(z) \geq \pi_0 \forall z \in [z^*, z_H]$ and $\pi_0 > 0$ since $A > 0$. This implies that $\pi_1 > 0$. Also, $1 + i(z) \geq 1 + r - \delta$ should be satisfied, so that $1 + i(z) > 0$ since $1 + r - \delta > 0$. Then, we can replace (PC) and (IC) with the following inequality:

$$\frac{p}{(1 - p) \mu} x(z) (1 + i(z)) - \frac{1 + r - \delta}{(1 - p) \mu} x(z) \geq \frac{x(z)}{\pi_1} (1 + i(z)) \quad (A.1)$$

It is easily shown that if inequality (A.1) is satisfied, there exists a $\eta(z)$ satisfying (IC). More specifically, if the LHS is equal to the RHS, there exists a unique $\eta(z)$ such that (IC) holds with equality. Rearranging inequality (A.1) yields:

$$[p\pi_1 - (1 - p) \mu] (1 + i) \geq \pi_1 (1 + r - \delta) \quad (A.2)$$

Hence, we must assume that $p\pi_1 - (1 - p) \mu > 0$ as in Assumption 2 since both the RHS of inequality (A.2) and $1 + i(z)$ are strictly positive. In reverse, once $1 + i \geq 1 + r - \delta$ is satisfied, Assumption 2 must be satisfied; note that $\pi_1 > 0$. Then, to minimize $(1 + i)$, inequality (A.2) should hold with equality, resulting in equation (1.1). Since inequality (A.2) holds with equality, (IC) also holds with equality, yielding equation (1.2). Then, $\eta \geq 0 \forall z \in [z_L, z_H]$ by Assumption 2. Finally, we must check whether or not $1 \geq \eta \forall z \in [z^*, z_H]$ from equations (1.1) and (1.2). Since $p\{\pi_1 - (1 + i)x\} \geq \pi_0 \forall z \in [z^*, z_H]$ and $\pi_0 > 0$, $1 \geq \eta$. In sum, $\eta \in [0, 1) \forall z \in [z^*, z_H]$ as desired. 

Proof of Proposition 1.1:

Note first that $k_{1,t} = \gamma k_{0,t}$ and $l_{1,t} = \gamma l_{0,t}$. Then, from the law of large numbers, we have:

$$K_t \equiv \int k_{i,t}(z) dF(z) = [p\gamma \{1 - F(z_{t-1}^*)\} + F(z_{t-1}^*)] k_{0,t}$$

$$L_t \equiv \int \ell_t(z) dF(z) = [p\gamma \{1 - F(z_{t-1}^*)\} + F(z_{t-1}^*)] \ell_{0,t}$$
Hence,

\[(K_t^\alpha L_t^{1-\alpha})^v = [p\gamma \{1 - F(z_{t-1}^*)\} + F(z_{t-1}^*)]^v (k_{0,t}^\alpha \ell_{0,t}^{1-\alpha})^v \quad (A.3)\]

Since \( Y_t = \int y_t(z) dF(z) = [p\gamma \{1 - F(z_{t-1}^*)\} + F(z_{t-1}^*)] A_{t-1}^{1-v} (k_{0,t}^\alpha \ell_{0,t}^{1-\alpha})^v \), equation (A.3) yields:

\[Y_t \equiv \left( [p\gamma \{1 - F(z_{t-1}^*)\} + F(z_{t-1}^*)] A_{t-1} \right)^{1-v} (K_t^\alpha L_t^{1-\alpha})^v\]

Finally, from the perfect competition in the factor markets, we have:

\[w_t = v (1 - \alpha) A_{t-1}^{1-v} k_{0,t}^{\beta} L_{0,t}^{1-\alpha} \ell_{0,t}^{-1}\]

and observe that:

\[
\frac{\partial Y_t}{\partial L_t} = v (1 - \alpha) \left[ p\gamma \{1 - F(z_{t-1}^*)\} + F(z_{t-1}^*) \right]^{1-v} A_{t-1}^{1-v} (K_t^\alpha L_t^{1-\alpha})^v L_t^{-1}
\]

\[= v (1 - \alpha) A_{t-1}^{1-v} k_{0,t}^{\beta} L_{0,t}^{1-\alpha} \ell_{0,t}^{-1} = w_t\]

where we use equation (A.3) for the second equality. Note also that \( \partial Y_t/\partial L_t = v (1 - \alpha) Y_t/L_t \). The same argument can be applied for the remaining part. ■

**Proof of Corollary 1.1:**

Since \( \pi_1 \) is decreasing in \( \mu \) (see (iii) of Proposition 1.2), the interest rate spread, \( \Delta \equiv p\pi_1 / [p\pi_1 - (1 - p) \mu] \), is increasing in \( \mu \). ■

**Proof of Proposition 1.2:**

Without loss of generality, we let \( L_0 = 1 \) for simplicity. Since \( r_t \) is fixed at \( r \) in a steady state, from equation (1.12), we have:

\[r = \alpha v [A_{-1} \psi (z^*)]^{1-v} k^{\alpha v - 1}\]

Then, the capital per efficiency unit of labor in the steady state, \( k \), is given by:

\[k = \left( \frac{\alpha v}{r} \right)^{\frac{1}{1-\alpha v}} [A_0 \psi (z^*)]^{\frac{1-v}{1-\alpha v}} \quad (A.4)\]
From the entrepreneur’s profit maximization problem, we have the following fundamental equation that determines $z^*$ for any given $(A_0, r)$:

$$
\frac{p\gamma - 1}{p\gamma} \left[ p\gamma (1 - v) \left( \frac{\alpha v}{r} \right)^{\frac{1 - \alpha v}{1 - \alpha v - v(1 - \alpha)}} A_0^{\frac{1 - \alpha v}{1 - \alpha v}} [\psi(z^*)]^{-v(1 - \alpha)} - (1 - p) \mu \right] = (1 + r - \delta) x(z^*, A_0)
$$

(A.5)

The RHS is continuous at any $z \in [z_L, z_H]$ while strictly decreasing from infinity to zero; recall that $x_z(z, A) < 0$, $\lim_{z \to z_L} x(z, A) = \infty$ and $\lim_{z \to z_H} x(z, A) = 0$. Meanwhile, $\psi'(z^*) < 0$ and $\psi(z_L) = p\gamma > 1$ and $\psi(z_H) = 1$, implying that the LHS is continuous at any $z \in [z_L, z_H]$ while strictly increasing in $z^*$. Now, note that $z^* > z_L$ since $\lim_{z \to z_L} x(z, A) = \infty$. Meanwhile, $z^* \leq z_H$ if, and only if,

$$
p\gamma (1 - v) \left( \frac{\alpha v}{r} \right)^{\frac{1 - \alpha v}{1 - \alpha v - v(1 - \alpha)}} A_0^{\frac{1 - \alpha v}{1 - \alpha v}} [\psi(z^*)]^{-v(1 - \alpha)} = p\pi_1 \big|_{z^* = z_H} \geq (1 - p) \mu
$$

The inequality is satisfied by the assumption that $p\pi_1 - (1 - p) \mu > 0$ in any equilibrium to guarantee $i \geq r - \delta$. Hence, $z^* \in (z_L, z_H)$. Finally, the LHS shifts downward when $\mu$ increases, so that $z^*$ increases in $\mu$, so does $\psi$. Then, both $k$ and $y$ decrease in $\mu$, respectively, from equations (A.4) and (1.8). Since $Y = yL$, $Y$ is also decreasing in $\mu$. This means that $\hat{y} = Y/\hat{N}$ is also decreasing in $\mu$. Finally, from equations (1.9) and (1.10), $\pi_i (i = 1, 2)$ is decreasing in $\mu$. ■

**Proof of Proposition 1.3:**

We let $\varphi_t \equiv (A_t/L_t)^{1 - v}$ for convenience. First, from equation (1.12), we have:

$$
r_t = \alpha v \varphi_t k_t^{\alpha v - 1}
$$

Hence, $k_t$ is given by:

$$
k_t = \left( \frac{\alpha v \varphi_t}{r_t} \right)^{\frac{1}{1 - \alpha v}}
$$

(A.6)

First note that $\varphi_t \equiv (A_0/L_0)^{1 - v} \equiv \varphi$ where $\varphi$ is a constant due to the assumption that $g_{L,t} = g_{A,t} \forall t \geq 0$ for stationary growth. Then, $g_k = 1$ since $g_r = 1$ in the stationary state; see Definition 1.3. To show $g_w = 1$, observe equation (1.11) to
have:

\[ w_t = (1 - \alpha) v \varphi^a k_t^{av} \]

which means that \( g_w = 1 \) in the long run since \( g_k = 1 \), and this is consistent with the definition of long-run growth steady state. Also, from equation (1.8), \( y = \varphi^a k^{av} \), and therefore, \( g_y = 1 \), proving (ii). To prove (iii), suppose that \( g_{z^*} = 1 \); we will verify this later. Then, \( g_A = \psi (z^*) \) from equation (1.13), and hence, \( g_L = \psi (z^*) \). Now, recall that \( Y_t \equiv y_t L_t \), implying that \( g_Y = g_L = \psi (z^*) \). To prove (iv), note that:

\[ \pi_{0,t} = (1 - v) \varphi_t \left[ \psi \left( z_{t-1}^* \right) \right]^{-1} L_t k_t^{av} = (1 - v) \varphi_t L_{t-1} k_t^{av} \]

which is obtained from equation (1.9). Hence, \( g_{\pi_i} = \psi (z^*) \) \( (i = 1, 2) \) since \( \pi_{1,t} = \gamma \pi_{0,t} \) from equation (1.10). Now, we will check that \( g_i = 1 \) in the long run. From equations (1.1) and (1.2), we have:

\[ g \left( 1 + i \right) = \frac{1}{1 + i} = \frac{p r \pi_i' p r \pi_0 - \left( 1 - p \right) \mu}{p r \pi_0 p r \pi_0 - \left( 1 - p \right) \mu} = \psi (z^*) \frac{p r \pi_0 - \left( 1 - p \right) \mu}{p r \pi_0 - \left( 1 - p \right) \mu} \approx \psi (z^*) \frac{1}{\psi (z^*)} = 1 \]

where the prime denotes variables in the following period. The last approximation holds for large enough \( \pi_1 \), and it must hold in the long-run if \( \pi_1 \) grows over time. We will prove later that this is true in the long run by showing that \( \psi (z^*) > 1 \); recall that the growth rate of \( \pi_i \) is equal to \( \psi (z^*) \) \( (i = 1, 2) \). Hence, asymptotically, \( 1 + i = 1 + i' \) that implies \( g_i = 1 \), and therefore, \( g_A = 1 \) from equation (1.1). We should prove that \( g_{z^*} = 1 \) too. From the entrepreneur’s problem, the skill threshold \( z^* \) is given by:

\[ (p r - 1) \pi_0 = p (1 + i) x (z^*, A) \]

which implies that:

\[ \frac{1 + i'}{1 + i} = \frac{\psi (z^*) x (z^*, A)}{x (z^*, A')} = \frac{\psi (z^*) x (z^*, A)}{x (z^*, \psi (z^*) A)} \]

since \( A' = \psi (z^*) A \) and \( g_{\pi_0} = \psi (z^*) \) in the steady state. Since \( g_i = 1 \) in the long run, we have:

\[ x (z^*, \psi (z^*) A) = \psi (z^*) x (z^*, A) \]

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Since \( x(z, A) \) is linear in \( A \) while strictly decreasing in \( z \), \( z^* \) should be equal to \( z^* \), implying that \( g_{z^*} = 1 \) in the long run. Now, we will prove the uniqueness of the balanced growth path. First note that \( 1 + i = (1 + r - \delta) / p \) in the asymptotic stationary state, which means that \( \lim_{t \to \infty} \Delta_t = 1 \). The asymptotic threshold \( z^* \) is, therefore, determined by the following equation from the entrepreneur’s profit maximization problem:

\[
(p \gamma - 1) \pi_0' = (1 + r - \delta) x(z^*, A)
\]

From equation (1.8), knowing that \( g_{L,t} = g_{A,t} \forall t, \pi_0' = (1 - v) \varphi Lk^{\alpha v} \). Then, from equation (1.9), we have:

\[
(p \gamma - 1) (1 - v) \left( \frac{\alpha v}{r} \right) \frac{A}{L-1} \frac{1}{1 - \alpha v} L = (1 + r - \delta) x(z^*, A)
\]

Finally, using the linearity of \( x(z, A) \) in \( A \) and \( g_{L,t} = g_{A,t} = \psi(z_t^*) \), we have:

\[
(p \gamma - 1) (1 - v) \left( \frac{\alpha v}{r} \right) \frac{A}{L-1} \frac{1}{1 - \alpha v} = (1 + r - \delta) x(z^*, A-1)
\]

Equation (A.7) is similar to equation (A.5) but different in that the monitoring cost \( \mu \) is irrelevant to the determination of \( z^* \). Then, it immediately follows that \( z^* > z_L \) since \( \lim_{z \to z_L} x(z, A_t) = \infty \). Also, \( z^* \) is below its upper bound, i.e., \( z^* < z_H \). This is because the LHS of equation (A.7) is \( p\pi_1 - (1 - p) \mu \) where \( \mu = 0 \), while \( p\pi_1 - (1 - p) \mu > 0 \) for guaranteeing \( i \geq r - \delta \). Hence, \( z^* \) is uniquely pinned down, and it is interior of \( [z_L, z_H] \). That is, \( z^* \in (z_L, z_H) \forall (A_0, L_0, r) \). Finally, it is straightforward to show that \( \psi(z^*) > 1 \) since the p.d.f. of skill distribution, \( f(\cdot) \) satisfies \( f(x) > 0 \forall x \in [z_L, z_H] \).

**Proof of Corollary 1.2:**

The asymptotic skill threshold \( z^* \) is determined by equation (A.7). Note that \( z^* \) is independent of \( \mu \) since equation (A.7) does not include \( \mu \). Note now that the gross growth rate \( \psi(z_t^*) \) is solely dependent on \( z_t^* \), the asymptotic growth rate is independent of \( \mu \).
Proof of Corollary 1.3:

On the transition phase, \( z^*_t \) is determined by the following equation, which is analogous to (A.5):

\[
\left( \frac{p \gamma - 1}{p \gamma} \right) \left[ \frac{p \gamma (1 - v)}{r_{t+1}} \right] \left( \frac{\alpha v}{r_{t+1}} \right)^{\frac{1}{1 - \alpha v}} \left( \frac{A_t}{L_{t+1}} \right)^{\frac{1 - v}{1 - \alpha v}} L_{t+1} \left[ \psi (z^*_t) \right]^{\frac{1 - (1 - v)}{1 - \alpha v}} - (1 - p) \mu \\
= (1 + r_{t+1} - \delta) x (z^*_t, A_t)
\]

which is rewritten as follows:

\[
\left( \frac{p \gamma - 1}{p \gamma} \right) \left[ \frac{p \gamma (1 - v)}{r_{t+1}} \right] \left( \frac{\alpha v}{r_{t+1}} \right)^{\frac{1}{1 - \alpha v}} \left( \frac{A - 1}{L - 1} \right)^{\frac{1 - v}{1 - \alpha v}} L_t - (1 - p) \mu = (1 + r_{t+1} - \delta) x (z^*_t, A_t)
\]

since \( g_A = \psi (z^*_t) \), and \( g_{L,t} = g_{A,t} \). From Lemma 1.1, \( z^*_t \) is uniquely pinned down given any vector of \( (A_t, L_t, r_{t+1}) \). Then, one can notice that an increase in \( \mu \) shifts the LHS downward, so that \( z^*_t \) is increasing in \( \mu \), implying that the gross growth rate in period \( t \), \( \psi (z^*_t) \), is decreasing in \( \mu \). Meanwhile, we have the following equation from (A.6):

\[
k_{t+1} = \left( \frac{\alpha v \varphi}{r_{t+1}} \right)^{\frac{1}{1 - \alpha v}}
\]

which implies that \( k_{t+1} \) is independent of \( \mu \). Also, from equations (1.9) and (1.10), \( \pi_{i,t+1} (i = 0, 1) \) is decreasing in \( z^*_t \) and hence, decreasing in \( \mu \) as well; recall that \( z^*_t \) is increasing in \( \mu \). This proves (iii). Therefore, \( \Delta_t \) is increasing in \( \mu \) by equation (1.1). From Proposition 1.1, \( g_{Y,t} = \psi (z^*_t) \), so that \( g_{Y,t} \) is decreasing in \( \mu \). Hence, \( g_{\hat{y},t} \) is also decreasing in \( \mu \) since it is monotone increasing in \( g_{Y,t} \).  

Proof of Proposition 1.4:

(i) The asymptotic convergence is trivial from the proof of Proposition 1.3. To prove the decreasing property, one can use the fact from the proof of Corollary 1.3 that \( z^*_t \) is determined by the following equation for all \( t \geq 0 \):

\[
\left( \frac{p \gamma - 1}{p \gamma} \right) \left[ \frac{p \gamma (1 - v)}{r_{t+1}} \right] \left( \frac{\alpha v}{r} \right)^{\frac{1}{1 - \alpha v}} \left( L_t^{(1-\alpha)} A_t^{1-v} \right)^{\frac{1}{1 - \alpha v}} - (1 - p) \mu = (1 + r - \delta) x (z^*_t, A_t)
\]

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where we let \( r_t \) be constant at \( r \forall t \geq 0 \). Then, we have:

\[
\psi(z_t^*) x(z_{t+1}^*, A_t) = x(z_{t+1}^*, A_{t+1})
\]

\[
= (1 + r - \delta)^{-1} \left( \frac{p \gamma - 1}{p \gamma} \right) \left[ p \gamma (1 - v) \left( \frac{\alpha v}{r} \right)^{\frac{1}{1-\alpha v}} \left( L_{t+1}^{v(1-\alpha)} A_{t+1}^{1-v} \right)^{\frac{1}{1-\alpha v}} - (1 - p) \mu \right]
\]

\[\equiv R^{-1} X_1 \left[ C_2 \left( L_{t+1}^{v(1-\alpha)} A_{t+1}^{1-v} \right)^{\frac{1}{1-\alpha v}} - (1 - p) \mu \right]\]

where \( R \equiv 1 + r - \delta, C_1 \equiv (p \gamma - 1) / (p \gamma) \) and \( C_2 \equiv p \gamma (1 - v) \). Hence, we obtain:

\[
x(z_{t+1}^*, A_t) = \left\{ \psi(z_t^*) \right\}^{-1} R^{-1} C_1 \left[ C_2 \left( L_{t+1}^{v(1-\alpha)} A_{t+1}^{1-v} \right)^{\frac{1}{1-\alpha v}} - (1 - p) \mu \right]
\]

\[\equiv \left\{ \psi(z_t^*) \right\}^{-1} R^{-1} C_1 \psi(z_t^*) \left[ C_2 \left( L_{t}^{v(1-\alpha)} A_{t}^{1-v} \right)^{\frac{1}{1-\alpha v}} - \left\{ \psi(z_t^*) \right\}^{-1} (1 - p) \mu \right]
\]

\[> R^{-1} C_1 \left[ C_2 \left( L_{t}^{v(1-\alpha)} A_{t}^{1-v} \right)^{\frac{1}{1-\alpha v}} - (1 - p) \mu \right]
\]

\[= x(z_t^*, A_t)
\]

where the second equality holds since \( g_{L,t} = g_{A,t} = \psi(z_t^*) \). Then, we have:

\[
x(z_{t+1}^*, A_t) > x(z_t^*, A_t) \quad \forall t \geq 0
\]

which implies \( z_{t+1}^* < z_t^* \forall t \geq 0 \).

(ii): This is straightforward from (i) of Proposition 1.4. The convergence holds by the continuity of \( \psi \) in \( z^* \).

(iii): Note that, given any constant \( r \) over time, \( g_{k,t} = 1 \forall t \geq 0 \) from equation (A.8).
Hence, let us denote \( k_t \) by \( k \forall t \geq 0 \). Then, we have:

\[
\iota_t = \frac{I_t}{Y_t} = \frac{K_t - (1 - \delta) K_{t-1}}{Y_t} = \frac{L_t}{Y_t} \left( 1 - \frac{1 - \delta}{\psi \left( z^*_{t-1} \right)} \right) k
\]

\[
= \left( \frac{A_{t-1}}{L_{t-1}} \right)^{v-1} \left( 1 - \frac{1 - \delta}{\psi \left( z^*_{t-1} \right)} \right) k^{1-\alpha v}
\]

\[
< \left( \frac{A_{t-1}}{L_{t-1}} \right)^{v-1} \left( 1 - \frac{1 - \delta}{\psi \left( z^*_{t} \right)} \right) k^{1-\alpha v} = \iota_{t+1}
\]

where we use \( L_{t-1}/L_t = 1/\psi \left( z^*_{t-1} \right) \), and \( k_t = k_{t-1} = k \) in the second equality. The third equality holds since \( Y_t/L_t = (A_t/L_t)^{1-v} k^{\alpha v} \) from Proposition 1.1 with the assumption that \( g_{A,t} = g_{L,t} \forall t \geq 0 \). Now, note that \( 1 - (1 - \delta)/\psi \left( z^*_{t-1} \right) > 0 \forall t \geq 0 \) since \( \psi \left( z^*_{t-1} \right) > 1 \forall t \geq 0 \); from Lemma 1.1, \( z^*_t \) is an interior solution \( \forall t \geq 0 \), implying that \( \psi \left( z^*_t \right) > 1 \forall t \geq 0 \) since \( f(z) > 0 \forall z \) where \( f(z) \) is the p.d.f. of the skill \( z \). Also, we know from part (i) of Proposition 1.4 that \( z^*_t \) decreases over time, and therefore, \( \psi \left( z^*_t \right) > \psi \left( z^*_{t-1} \right) \), proving that \( \psi \left( z^*_t \right) \) increases \( \forall t \geq 0 \). The convergence is obvious by the continuity of \( \iota \) in \( z^* \); recall that \( \psi(z) \) is continuous at any \( z \).

(iv): Note first that:

\[
\rho_{t+1} = \frac{X_{t+1}}{Y_{t+1}} > \frac{X_t}{Y_t} = \rho_t \Leftrightarrow g_{X,t} \equiv \frac{X_{t+1}}{X_t} > \frac{Y_{t+1}}{Y_t} \equiv g_{Y,t}
\]

From Proposition 1.1 with the result from the proof of (iii) of Proposition 1.4 such that \( g_{k,t} = 1 \forall t \geq 0 \) under a constant interest rate \( r \), one can show that: \( g_{Y,t} = \psi \left( z^*_t \right) \forall t \geq 0 \). Meanwhile, we also have:

\[
g_{X,t} \equiv \frac{X_{t+1}}{X_t} = \frac{\int_{z^*_{t+1}}^{z^*_t} x(z, A_{t+1}) \, dF(z)}{\int_{z^*_{t}}^{z^*_{t+1}} x(z, A_t) \, dF(z)} = \frac{\psi \left( z^*_t \right) \int_{z^*_{t+1}}^{z^*_t} x(z, A_t) \, dF(z)}{\int_{z^*_{t}}^{z^*_{t+1}} x(z, A_t) \, dF(z)} > \psi \left( z^*_t \right) = g_{Y,t}
\]

where the third equality comes from the linearity of \( x(z, A) \), and the last inequality comes from \( z^*_{t+1} < z^*_t \) from the part (i). Hence, \( \rho_{t+1} > \rho_t \forall t \). The convergence holds
by the continuity of \( \rho \) at any \( z \).

(v): From equation (1.13) and \( g_{A,t} = g_{L,t} = \psi \left( z^*_t \right) \), we have:

\[
y_t = \frac{A}{L} k^{\alpha v}
\]

Notice first from the proof of (iii) of Proposition 1.4 that \( k \) is time-invariant when \( r \) is time-invariant. Hence, \( y_t \) is also time-invariant. Also, from Proposition 1.1, \( w_t = v (1 - \alpha) Y_t / L_t = v (1 - \alpha) y_t \), implying that \( w_t \) is also time-invariant. Then, the saving, which can be defined by the saving function, \( s_t \equiv s \left( r_{t+1}, w_t \right) \), is time-invariant too. Knowing it all, from equation (1.5), we obtain:

\[
\theta_t \equiv \frac{B}{Y_t} = k + \rho_t - \frac{s(k)}{y \psi \left( z^*_t \right)} < k + \rho_{t+1} - \frac{s(k)}{y \psi \left( z^*_t \right)} \equiv \frac{B}{Y_{t+1}} = \theta_{t+1}
\]

where \( \rho_t \equiv X_t / Y_t \). We know from (ii) and (iv) of Proposition 1.4 that both \( \rho_t \) and \( \psi \left( z^*_t \right) \) are increasing over time, converging to its respective asymptotic value. Hence, \( \theta_t \equiv B / Y_t \) is increasing \( \forall t \geq 0 \). The convergence is trivial by the continuity.

Proof of Proposition 1.5:
One can solve for the threshold \( z^*_t \) by the following equation:

\[
(p \gamma - 1) E_t \left[ \pi_{0,t+1} \right] = (1 + r_{t+1} - \delta) x \left( z^*_t, A_t \right) E_t \left[ \Delta_{t+1} \right]
\]

where \( \pi_1 \left( s^{t+1} \right) = \gamma (1 - v) \left( \frac{\alpha v}{\pi_{t+1}} \right) \frac{1}{1 - \alpha v} L^{-1}_{t+1} \psi \left( z^*_t \right) \) and \( \pi_0 \left( s^{t+1} \right) = \gamma^{-1} \pi_1 \left( s^{t+1} \right) \) from equations (1.9), (1.10) and (1.12). Then, equation (A.9) is rewritten as follows:

\[
\left( \frac{p \gamma - 1}{p \gamma} \right) \frac{E_t \left[ A_t^{1 - \alpha v} \right]}{E_t \left[ p \gamma (1 - v) \left( \frac{\alpha v}{\pi_{t+1}} \right) \frac{1}{1 - \alpha v} A_t^{1 - \alpha v} (1 - p) \mu \right]} = (1 + r_{t+1} - \delta) x \left( z^*_t, A_t \right)
\]

First, from equation (A.10), the equilibrium threshold \( z^*_t \) is uniquely pinned down as an interior point on \([z_L, z_H]\) since the LHS of equation (A.10) is strictly positive and
independent of $z$. Meanwhile, the RHS of equation (A.10) is strictly decreasing in $z$ from infinity to zero. Note also that $\pi_{0,t+1}$ is strictly concave in $A_t$, and therefore, the LHS of equation (A.9) is strictly decreasing in mean-preserving spread of the random shock $s$. Now, suppose that there is no financial friction, i.e., $\mu = 0$, and hence, $\Delta_{t+1} = 1$. Then, the RHS of equation (A.9) is equal to $(1 + r_{t+1} - \delta) \psi(z^*_t, A_t)$. In this situation, $z^*_t$ increases along with mean-preserving spread, and therefore, the mean growth rate $\psi(z^*_t)$ decreases with mean-preserving spread. Hence, the mean growth rate of $Y_t$ also decreases as the volatility increases; recall that $g_{Y_{t+1}}$ is strongly monotonic in $\psi(z^*_t)$ by Corollary 1.3. In other words, growth decreases by higher volatility even without the financial friction. Next, to prove the amplification effect, note that the wedge, $\Delta_{t+1} \equiv \frac{p\pi_{1,t+1}}{\pi_{1,t+1} - (1-p)\mu'}$, is strictly convex and decreasing in $\pi_{1,t+1}$, while $\pi_{1,t+1}$ is strictly concave in $A_t$. This implies that $\frac{p\pi_{1,t+1}}{\pi_{1,t+1} - (1-p)\mu}$ is strictly convex in $A_t$. Hence, the RHS of equation (A.9) increases with mean-preserving spread. Hence, $z^*_t$ increases more compared to the case without the financial friction. ■

Proof of Proposition A.1:

Case 1) The case that $\mu_A(w, A) = 0$ is equivalent with the baseline case that the monitoring cost is fixed over time, i.e., $g_{\mu,t} = 1 \forall t$. This is because wage $w_t$ is constant in the stationary state. Hence, the growth effects disappear asymptotically in this case. Also, $\mu_A(w, A) < 0$ implies that $\mu$ decreases in each period in the steady state since $\psi(z^*_t) > 1$ implies that $A$ continuously increases. Hence, $g_{\mu} < 1$, and $\mu$ converges to its lower bound, 0; recall that $\mu$ is assumed to be greater than zero. Hence, the economy asymptotically boils down to the standard growth model without the financial friction, so that the growth effects disappear in the long run.

Case 2) (i) In this case, it is trivial to show that $g_{\mu,t} \in (1, g_{\pi,t}) \forall t \geq 0$. We now let $\epsilon_t \equiv g_{\pi,t}/g_{\mu,t} > 1 \forall t \geq 0$. Then, $\forall M > 0$, $\exists T > 0$ such that $\pi_{1,t}/\mu_{t+1} > M \forall t > T$ since one can choose any $T > 0$ satisfying that $\bar{\epsilon}_{T+1} > M\mu_0/\pi_{1,0}$, where $\bar{\epsilon} \leq \min \{\epsilon_1, \epsilon_2, \cdots \epsilon_{T+1}\}$. Then, by picking an arbitrary large $M$ such that $\mu_{t+1}/\pi_{1,t} = 1/M \simeq 0$, we have the following result for large enough $t > T$ satisfying that $\mu_{t+1}/\pi_{1,t} \simeq 0$:

$$g_{(1+i)} = \frac{p\pi_1^t p\pi_1 - (1-p)\mu}{p\pi_1^t p\pi_1^t -(1-p)\mu'} = \psi(z^*) \frac{p - (1-p)\mu/\pi_1}{p\pi_1^t/\pi_1 - (1-p)\mu'/\pi_1} \simeq \psi(z^*) \frac{1}{\psi(z^*)} = 1$$
which implies $g_i = 1$ asymptotically. Then, we can prove $g_{z^*} = 1$ by using the same method applied to the proof of Proposition 1.3. Finally, $\mu_{t+1}/\pi_{1,t+1} \simeq 0$ for any $t > T$ since $\mu_{t+1}/\pi_{1,t} \simeq 0 \; \forall t > T$ and $\pi_{1,t+1} > \pi_{1,t}$. Hence, the determination of the asymptotic threshold $z^*$ is irrelevant to the monitoring cost.

**Case 2) (ii)** In this case, it is straightforward that $g_{\pi_{1,t}} = g_{\mu,t} \; \forall t \geq 0$. Hence, the equation that determines $z^*$ becomes different from equation (A.7), and it is given by:

$$
p_{\gamma} - 1 \left[ p_{\gamma} (1 - v) \left( \frac{\alpha v}{r} \right)^{\frac{\alpha v}{1-\alpha v}} \left( \frac{A_{-1}}{L_{-1}} \right)^{\frac{1-v}{1-\alpha v}} - (1 - p) \mu_0 \right] = (1 + r - \delta) x (z^*, A_{-1})
$$

where $z^*$ is decreasing in the initial monitoring cost $\mu_0$. Hence, the initial monitoring cost reduces the long-term growth rate $\psi(z^*)$ in the steady state.

**Case 2) (iii)** In this case, $g_{\mu,t} > g_{\pi_{1,t}} = \psi(z_{t}^*) > 1$. Hence, $1+i$ grows faster than $\pi_1$ over time while assuming that Assumption 2 is still satisfied. Then, the equilibrium threshold $z_t^* \in (z_L, z_H)$ must decrease over time, converging to its minimum value, $z_L$. This, in turn, implies that the growth rate of the economy converges to $\psi(z_L) = 1$, meaning no-growth in the stationary state.

### A.5 Empirical Analysis with Split Sample

In Section 1.4, we verify the asymptotic irrelevance hypothesis by using the interaction term between the (adjusted) interest rate spread and initial income. The coefficient of the interaction term is estimated positive, meaning that the negative growth impact of the financial friction gets smaller as the economy is initially richer.

In this appendix, we corroborate this argument through regression with two split samples. We divide the total sample into two sub-samples according to whether a given country has higher or lower GDP per capita than the median of the total sample.

With the two sub-samples, we use the following econometric framework to esti-
mate the direct growth effect of the financial friction for each group:

\[ g_{yi}^G = X_i^G \hat{\theta} + \beta^G \tilde{\Delta}^G_i + \varepsilon_i^G \]

\( G \) is the index for the two sub-samples such that \( G \in \{ P, R \} \) where \( P \) and \( R \) denote the sub-sample for countries whose initial incomes are below (poor) and above (rich) the median, respectively. Then, to be consistent with the asymptotic irrelevance hypothesis, the estimate of \( \hat{\beta}^G \) needs to be statistically significant \( \forall G \in \{ P, R \} \) and the following inequalities must hold: \( \hat{\beta}^P < \hat{\beta}^R < 0 \). In words, the negative growth effect led by the unit change of the interest rate spread \( \tilde{\Delta} \) gets smaller when the economy is richer.

The estimation results are summarized in Table A.2. We notice from regressions (2) and (3) that the estimate of \( \hat{\beta}^G \) is statistically significant \( \forall G \in \{ P, R \} \) and \( \hat{\beta}^P < \hat{\beta}^R < 0 \). This means that the financial friction decreases the long-term growth rate of GDP per capita and this negative impact on growth is more severe when the initial income is relatively low. As already mentioned, this is consistent with the estimation result in Section 1.4 where the empirical analysis is done with the interaction term.

The same estimation approach can be applied to growth of the TFP. That is, we can estimate the direct growth effect on the TFP by using the average growth rate of the TFP for the dependent variable. The estimation results are the same; from regressions (5) and (6), we find that \( \hat{\beta}^G \) is statistically significant \( \forall G \in \{ P, R \} \) and \( \hat{\beta}^P < \hat{\beta}^R < 0 \). Hence, the estimation results from regressions (2), (3), (5) and (6) support the asymptotic irrelevance hypothesis for TFP growth as well.

### A.6 Empirical Analysis with Credit Ratio

The ratio of private credit to GDP has been commonly used in many empirical studies as a measure of financial development.\(^\text{14}\) Although the interest rate spread is

\( ^{14} \) For instance, King and Levine (1993), de Gregorio (1996), Levine, Loayza and Beck (2000), AHM and AABM among others. However, Čihák, Demirgüç-Kunt, Feyen and Levine (2013) argue that this commonly used measure does not provide sufficient information in the assessment of financial development in a country. They, therefore, construct a new cross-country database that
Table A.2: Regressions with Split Sample (OLS)

**Panel A.** *Dependent Variable:* Average Growth Rate of GDP per capita (%)

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Full Sample</th>
<th>Below Median</th>
<th>Above Median</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>74</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>Interest Rate Spread</td>
<td>−.322***</td>
<td>−.317***</td>
<td>−.200**</td>
</tr>
<tr>
<td></td>
<td>(.057)</td>
<td>(.078)</td>
<td>(.008)</td>
</tr>
<tr>
<td>Initial Income</td>
<td>−1.876***</td>
<td>−2.045*</td>
<td>−2.697***</td>
</tr>
<tr>
<td></td>
<td>(.430)</td>
<td>(1.101)</td>
<td>(1.819)</td>
</tr>
<tr>
<td>Initial Human Capital</td>
<td>.975*</td>
<td>1.372</td>
<td>.701</td>
</tr>
<tr>
<td></td>
<td>(.537)</td>
<td>(.929)</td>
<td>(.558)</td>
</tr>
<tr>
<td>Population Growth</td>
<td>−.175</td>
<td>−.804*</td>
<td>.028</td>
</tr>
<tr>
<td></td>
<td>(.173)</td>
<td>(.434)</td>
<td>(.090)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>.444</td>
<td>.584</td>
<td>.345</td>
</tr>
</tbody>
</table>

**Panel B.** *Dependent Variable:* Average Growth Rate of the TFP (%)

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Full Sample</th>
<th>Below Median</th>
<th>Above Median</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>74</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>Interest Rate Spread</td>
<td>−.203***</td>
<td>−.194***</td>
<td>−.130*</td>
</tr>
<tr>
<td></td>
<td>(.041)</td>
<td>(.055)</td>
<td>(.070)</td>
</tr>
<tr>
<td>Initial Income</td>
<td>−.691***</td>
<td>−.810**</td>
<td>−.996***</td>
</tr>
<tr>
<td></td>
<td>(.136)</td>
<td>(.653)</td>
<td>(.435)</td>
</tr>
<tr>
<td>Initial Human Capital</td>
<td>1.291***</td>
<td>1.550**</td>
<td>1.130**</td>
</tr>
<tr>
<td></td>
<td>(.390)</td>
<td>(.653)</td>
<td>(.435)</td>
</tr>
<tr>
<td>Population Growth</td>
<td>.128</td>
<td>−.348</td>
<td>.294**</td>
</tr>
<tr>
<td></td>
<td>(.126)</td>
<td>(.305)</td>
<td>(.131)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>.675</td>
<td>.514</td>
<td>.534</td>
</tr>
</tbody>
</table>

**Note:** All regressors are the averages over the sample period of 1980–2010. Initial income and human capital are taken for 1980. Initial income is logged GDP per capita. “Below (Above) Median” is for a country, the initial income of which is below (above) the median. We excluded any country from the regressions either when the number of data points of the interest rate spread is smaller than 20, which is two thirds of the total number of data points over the sample period, 1980–2010, or when the average interest rate spread is larger than 20%. Constant terms are not shown. Standard errors are in parenthesis. ***\*, ** and * significant at 1%, 5% and 10%, respectively.
a proper proxy for financial conditions based on our theoretical framework, we can test all the theoretical results using the credit-to-output ratio instead of the interest rate spread.

To do that, we can rewrite the econometric specification used in Section 1.4 as follows:

\[ \tilde{g}_{g,i} = X_i \tilde{\theta} + \tilde{\alpha} \sigma_i + \tilde{\beta} \phi_i + \tilde{\gamma} \phi_i \sigma_i + \delta \phi_i \log \hat{y}_{1980,i} + \epsilon_i \]

where \( \phi_i \) is the average of annual credit-to-output ratios (%) over the sample period, 1980–2010. Similarly, the same hypotheses for TFP growth can also be tested. For the credit ratio during the sample period, \( \{ \phi_{i,t} \} \), we use the the World Bank Dataset for the domestic credit to the total output (% of GDP).

In our model, the credit-to-output ratio in period \( t \) is defined such that \( \phi_t \equiv (X_t + I_t) / Y_t \times 100 \) where \( X \) and \( I \) are the R&D and capital investments, respectively. Then, from parts (iii) and (iv) of Proposition 1.4, \( \phi_{i,t} \) is decreasing in the monitoring cost \( \mu \) during the transition phase. Hence, \( \phi_{i,t} \) must negatively correlates with the interest rate spread \( \tilde{\Delta}_{i,t} \) (recall that \( \tilde{\Delta}_{i,t} \) is increasing in \( \mu_i \)). This implies that \( \tilde{\beta} \) and includes a rich set of dimensions on financial conditions.
\(\tilde{\gamma}\) must be positive while \(\tilde{\delta}\) must be negative to be consistent with the theoretical results. To check if it is true, we conduct OLS regressions, and the results are presented in Table A.3. As indicated in the table, both \(\tilde{\beta}\) and \(\tilde{\gamma}\) are positive, while \(\tilde{\delta}\) is negative. All estimates are statistically significant at either 1% or 5%, implying validity of the theoretical conclusions again. Therefore, we have another evidence for the theoretical results from the commonly used proxy for financial conditions: the credit-to-output ratio.
Table A.3: Regressions with Credit-to-Output Ratio (OLS)

<table>
<thead>
<tr>
<th>Dependent Variable: Average Growth Rate (%)</th>
<th>GDP per capita (1)</th>
<th>TFP (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent Variable</td>
<td>N = 109</td>
<td>N = 109</td>
</tr>
<tr>
<td>Credit-to-Output Ratio</td>
<td>.112***</td>
<td>.065**</td>
</tr>
<tr>
<td></td>
<td>(.037)</td>
<td>(.027)</td>
</tr>
<tr>
<td>Credit-to-Output Ratio × Volatility</td>
<td>.006***</td>
<td>.004***</td>
</tr>
<tr>
<td></td>
<td>(.002)</td>
<td>(.001)</td>
</tr>
<tr>
<td>Credit-to-Output Ratio × Initial Income</td>
<td>−.028***</td>
<td>−.007***</td>
</tr>
<tr>
<td></td>
<td>(.009)</td>
<td>(.003)</td>
</tr>
<tr>
<td>Volatility</td>
<td>−.238***</td>
<td>−.170*</td>
</tr>
<tr>
<td></td>
<td>(.071)</td>
<td>(.052)</td>
</tr>
<tr>
<td>Initial Income</td>
<td>−.718</td>
<td>−.408***</td>
</tr>
<tr>
<td></td>
<td>(.483)</td>
<td>(.152)</td>
</tr>
<tr>
<td>Initial Human Capital</td>
<td>.672</td>
<td>981***</td>
</tr>
<tr>
<td></td>
<td>(.430)</td>
<td>(.312)</td>
</tr>
<tr>
<td>Population Growth</td>
<td>−.224</td>
<td>.120</td>
</tr>
<tr>
<td></td>
<td>(.158)</td>
<td>(.113)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.429</td>
<td>.398</td>
</tr>
</tbody>
</table>

Note: All regressors are averages over 1980–2010 period. Volatility is the standard deviation of % growth rates. Initial income and human capital are taken for 1980. Initial income is logged GDP per capita. Constant terms are not shown. Standard errors are in parenthesis. ***, ** and * significant at 1%, 5% and 10%, respectively.
Appendix B

Appendix for Chapter 2

B.1 Details of the Empirical Analysis

The Hofstede individualism index

Given at least 40 respondents in each country for the two international surveys by IBM, Hofstede constructed measures for five different dimensions of culture for 50 countries with three multicountry regions. The five dimensions are Individualism, Power Distance, Uncertainty Avoidance, Masculinity and Long-term Orientation. Most of the 50 countries have more than 200 respondents in the first and second round surveys combined. The culture measure has been updated by new waves of surveys and subsequent studies using a similar set of questionnaires to that used in the original surveys, and it now covers 102 countries.\(^1\)

To construct the individualism index, Hofstede chose 14 survey questions related to “work goals,” which are conceptually consistent with the notion of individualism/collectivism. Among these, only 9 questions were used to calculate the individualism score from the factor analysis; only the 9 questions’ factor loadings — the square root of relative explanatory power — are higher than 0.35 (35%). From the factor analysis results, he categorized the 9 questions into positive and negative groups that are positively and negatively correlated with the notion of individualism,

\(^1\) The most recent version of the data can be found at http://www.geert-hofstede.com.
respectively, in order to calculate the individualism score.\(^2\)

Then, is it reliable as a measure of culture? It has been in fact validated in many subsequent studies. For example, across various studies and measures of individualism, the UK, the USA and Netherlands are consistently among the most individualistic countries, while Pakistan, Nigeria and Peru are among the most collectivist countries (Gorodnichenko and Roland, 2017). More surprisingly, a survey conducted by Chinese scholars — not by Western scholars — in 1979, which overlapping 20 countries with Hofstede’s sample countries, obtained very similar results to the Hofstede individualism index despite the completely different questions, different populations, different time periods, and different mix of countries (Hofstede and Bond, 1988). Furthermore, the IBM surveys were translated into local languages to prevent cultural biases in the way questions are framed (Gorodnichenko and Roland, 2017). For these reasons, the Hofstede individualism index have been widely used in various fields.

\textit{Culture-Innovation relationship}

Gorodnichenko and Roland (2017) conduct cross-country OLS regressions controlling for many relevant explanatory variables, including protection of property rights as a proxy for the institutional quality, to show that the number of patents per million population tends to be bigger as a country is more individualistic. Furthermore, they provide evidence that the same relationship also holds for the innovation performance index, which incorporates information on the number of patents and alternative indicators such as royalty and license fees.\(^3,4\)

\(^2\) The positive and negative groups consist of four and five questions, respectively. One can refer to the Chapter 5 in Hofstede (2001) for more details on how he constructed the individualism index.

\(^3\) Gorodnichenko and Roland (2017) also show that the TFP level is higher in a more individualistic country, which is also consistent with their empirical findings.

\(^4\) Similarly, Shane (1992) investigates the positive relationship between individualism and innovation by using the Hofstede culture indices. He notes that the Power Distance Index (PDI), which is also one of the culture dimensions measured by Hofstede, has the strong negative relationship with the individualism index. Then, he concludes that individualism encourages innovation by showing that the number of patents correlates negatively with PDI. This may be attributable to the tendency that innovation needs “good support” — permissions, authorizations and etc. — from hierarchy in a high PDI society [Shane et al. (1995)].
Acemoglu et al. (2015) define the concept of creative innovation that proxies for qualitative aspects of innovation rather than quantitative ones. It consists of a set of sub-indices: Innovation Quality, Superstar Fraction, Tail Innovation and Generality (or Originality). In the cross-country regressions controlling for various explanatory variables, all of the qualitative measures of innovation are consistently higher in a more individualistic country where the Hofstede individualism score is higher. In sum, innovation is more encouraged by individualism both quantitatively and qualitatively.

For the micro-evidence within a country, as argued by Gorodnichenko and Roland (2017), the USA is a particularly appropriate research object since there are various ethnicities in the country, and occupational opportunities are relatively open for peoples of all origins and cultures. We use 5 percent public micro data (IPUMS) of the U.S. Census in 2000 to obtain the data of ethnicity, age, gender, birth place, educational attainment, marital status, English proficiency, citizenship and school attendance. To avoid potential biases, our sample includes only employed males. We also drop respondents from the sample if they are younger than 25 or older than 65. The identification method is basically the same as the one in Gorodnichenko and Roland (2017). We estimate the following probit in the first step:

\[
ROO_i = \Phi(X_i'\beta + \sum_k \alpha_k D_{ik} + \epsilon_i)
\]

where \(i \) and \(k \) are individuals and ethnic groups, respectively; the ethnicity is the respondent’s self-reported country of ancestry in the 2000 census. \(ROO\) is a dummy variable equal to one if an individual has a research-oriented occupation and zero otherwise. \(D\) is a set of dummies of each ethnicity, and \(X\) includes controls such as age, age squared, education attainment, marital status, citizenship, English proficiency, and school attendance.\(^5\) Also, we consider four sub-samples for a more comprehensive analysis. The number of observations for the sub-samples in the first stage probit regression ranges from 426,492 to 1,759,246.

\(^5\) Following Gorodnichenko and Roland (2017), the ethnicity variable for the UK is dropped since the UK is the second most individualistic country in the sample.
In the second step, we estimate the following specification by OLS:
\[ \hat{\alpha}_k = \gamma + \theta \text{IND}_k + \varepsilon_k \]
where \( \hat{\alpha}_k \) is the estimated coefficient for \( k \) th ethnicity that indicates the explanatory power of \( k \) th ethnicity in the propensity to choose research-oriented jobs. \( \text{IND} \) is the Hofstede individualism score. Table B.1 summarizes the estimation results. As seen in the table, the estimates of \( \theta \) are significant and positive in all regressions.

For the case of Italian immigrants to the USA, we use the IPUMS US census 1980 since it includes information about which province an Italian immigrant came from, one can exploit the same probit regression already used. Since southern Italy can be defined either widely or narrowly in terms of the regions as depicted in Figure B.2, we conduct the probit regression separately for each of the two cases.

As seen in Table B.2, we notice that the same result also holds for the Italian immigrants; an immigrant from northern Italy is more likely to have a research-oriented job than an immigrant from southern Italy. Accordingly, we can argue that an individual is more (less) likely to voluntarily choose a research-oriented job if he is an individualist (a collectivist), and this suggests that an individualist has a comparative advantage in innovation compared to a collectivist.

**Culture-Growth relationship**

The OLS regression results are shown in Table B.3. As seen in the table, individualism increases the average growth rate over the sample period even when we control continent-specific differences by adding continent dummies. Since cultures also change over time according to the development of society, the endogeneity problem needs to be addressed, especially for long periods of time. For this, we use historical pathogen prevalence such as malaria, dengue and etc. as an instrument variables following Gorodnichenko and Roland (2017).\(^6\)

\(^6\) Fincher et al. (2008) argue that pathogen prevalence may require collectivist behaviors such as extended nepotism and in-group care more generally, resulting in strong positive correlations between prevalence and collectivism. One can find details of how the data was constructed from Fincher et al. (2008).

\(^7\) Gorodnichenko and Roland (2017) advocate the use of the historical pathogen prevalence as
As demonstrated in Table B.3, we have the same results in all IV regressions. However, a word of caution follows; the IV turns out “weak” in that the first stage F-stats are less than 10, which is the rule of thumb criterion suggested by Staiger and Stock (1997). In this case, one can employ the Anderson-Rubin (AR) test, which is an inference robust to weak instrumental variables. Using the AR test, we find that all estimates on the instrumented variables are still statistically significant at least at the 10% level.\(^8\)

Table B.4 summarizes the estimation results with the PWT data. The results also indicate the same pattern as that we have already seen using the Maddison data; individualism increases the average growth rate over the sample period. The TFP growth rate increases with individualism, which is consistent with the observation that individualism is more beneficial for innovation than collectivism. The results are the same in the IV regressions using the historical pathogen prevalence. Also, the estimates are significant when using the AR test, and hence, robust to the weak instrument.

**Culture-Institutions relationship**

Table B.5 provides the OLS regression results for the interactions between individualism and property rights. The protection against expropriation risk is taken from the International Country Risk Guide and averaged between 1985 and 2009. Another measure of protection of property rights is taken from Gwartney et al. (2015), which is averaged over 1995 and 2009.\(^9\) To address the endogeneity issue arising from health variables that may or may not affect income, e.g., life expectancy, are not significantly correlated with historical pathogen prevalence. In line with this, Fincher et al. (2008) report that variations in the average life expectancy cannot be attributed to variations in the historical pathogen prevalence.

\(^8\) When IVs are weak under the assumption of normal errors, one can also use the conditional likelihood ratio (CLR) test developed by Cruz and Moreira (2005) to achieve more reliable statistics since it is known that the CLR test is more powerful than the AR test among a class of two-sided tests; see Andrews et al. (2006). Although we do not report the estimation results, we have similar results when using the CLR test, assuming homoskedasticity. However, even when the estimates are significant under the weak instruments robust tests, the finite sample bias of 2SLS estimators becomes severe with weak instruments, and hence, there could be substantial bias in the estimates; see Cruz and Moreira (2005).

\(^9\) This is because the earliest available year for the data is 1995.
from a two-way interaction, we use the historical pathogen prevalence for the effect from individualism to property rights while using the Acemoglu et al. (2001) settler mortality rate for the reverse effect as in Gorodnichenko and Roland (2017). As expected, the Hofstede individualism score correlates positively with both the protection against expropriation risk and the protection of property rights. The results still hold in the IV regressions. The estimates are also significant under the AR test, and thereby, robust to the weak instrument.
Table B.1: Propensity to Choose Research-Oriented Occupations Using IPUMS US 2000 Census (OLS)

<table>
<thead>
<tr>
<th>Extended controls</th>
<th>All Persons</th>
<th>All Persons w/ Bachelor Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Individualism</td>
<td>.004***</td>
<td>.003***</td>
</tr>
<tr>
<td></td>
<td>(.001)</td>
<td>(.001)</td>
</tr>
<tr>
<td>Obs.</td>
<td>83</td>
<td>83</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.167</td>
<td>.134</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>U.S. Born</th>
<th>U.S. Born w/ Bachelor Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
</tr>
<tr>
<td>Individualism</td>
<td>.011***</td>
</tr>
<tr>
<td></td>
<td>(.003)</td>
</tr>
<tr>
<td>Obs.</td>
<td>83</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.104</td>
</tr>
</tbody>
</table>

**Note:** The dependent variable is the set of estimated coefficients associated with the set of nationalities of immigrants’ ancestors. The definition of research-oriented occupations includes Life, Physical, and Social Science Occupations (codes 160-196 in the 2000 census occupational classification system recorded in the IPUMS variable OCC). “Individualism” is the Hofstede individualism index. Controls are age, age squared, and dummies of college graduates and marital status. Extended controls further include dummies of citizenship, English proficiency, and school attendance. Heteroskedasticity robust standard errors in parentheses. *** significant at 1%.
Figure B.1: Provinces of Italy

Figure B.2: Two Different Definitions of Southern Italy
<table>
<thead>
<tr>
<th>Region</th>
<th>Narrow</th>
<th></th>
<th></th>
<th>Wide</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coeff.</td>
<td></td>
<td></td>
<td></td>
<td>Coeff.</td>
<td></td>
</tr>
<tr>
<td>Avg. ME (%)</td>
<td></td>
<td></td>
<td></td>
<td>Avg. ME (%)</td>
<td></td>
</tr>
<tr>
<td>From northern Italy</td>
<td>.396*</td>
<td>.350**</td>
<td>3.55</td>
<td>4.09**</td>
<td>2.30</td>
</tr>
<tr>
<td></td>
<td>(.239)</td>
<td>(.169)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>295 (N = 122; S = 173)</td>
<td>1201 (N = 122; S = 1079)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** The dependent variable is a dummy of research-oriented occupations; if the respondent’s job is a research-oriented occupation, the dependent variable is one, and zero otherwise. The definition of research-oriented occupations includes Life, Physical, and Social Science Occupations (codes 160-196 in the 2000 census occupational classification system recorded in the IPUMS variable OCC). “From northern Italy” is a dummy that is one if the respondent’s father came from northern Italy, and zero for the respondent whose father came from southern Italy. Coefficients associated with controls are omitted. The controls are age, age squared, and dummies of college graduates, marital status, citizenship, school attendance and sex. “Narrow” indicates the narrow definition of southern Italy including only four south provinces — BA, CL, CM and PU. “Wide” is the wide definition including seven south provinces and the Sicily island — AB, BA, CL, CM, MO, PU and SI. Northern Italy consists of eight north provinces of EM, FR, LI, LO, PI, PR, VE and VA. “Avg. ME” indicates average marginal effects at mean values. Heteroskedasticity robust standard errors in parentheses. ** and * significant at 5% and 10%, respectively.
Table B.3: Individualism and Growth Using the Maddison data (OLS)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Continent dummies</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Individualism</td>
<td>.011**</td>
<td>.013***</td>
<td>.013***</td>
<td>.017***</td>
</tr>
<tr>
<td></td>
<td>(.005)</td>
<td>(.004)</td>
<td>(.004)</td>
<td>(.004)</td>
</tr>
<tr>
<td>Obs.</td>
<td>49</td>
<td>49</td>
<td>66</td>
<td>66</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>.420</td>
<td>.614</td>
<td>.368</td>
<td>.535</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Continent dummies</td>
<td>(9)</td>
<td>(10)</td>
<td>(11)</td>
<td>(12)</td>
</tr>
<tr>
<td>Individualism</td>
<td>.034***</td>
<td>.028**</td>
<td>.054*</td>
<td>.029*</td>
</tr>
<tr>
<td></td>
<td>(.012)</td>
<td>(.011)</td>
<td>(.030)</td>
<td>(.015)</td>
</tr>
<tr>
<td>Obs.</td>
<td>48</td>
<td>48</td>
<td>65</td>
<td>65</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>.161</td>
<td>.528</td>
<td>-.387</td>
<td>.489</td>
</tr>
<tr>
<td>1st. stage F-stat.</td>
<td>6.49</td>
<td>4.10</td>
<td>3.17</td>
<td>4.42</td>
</tr>
<tr>
<td>Partial ( R^2 )</td>
<td>.243</td>
<td>.264</td>
<td>.078</td>
<td>.131</td>
</tr>
</tbody>
</table>

**Note:** The dependent variable is the annual average growth rate of income per capita from Maddison (2013). “Individualism” is the Hofstede individualism index. Coefficients associated with controls are omitted. The controls consist of initial logged GDP per capita, and geographical and religion factors. The geographical controls are the absolute values of latitude and longitude, mean distance to the nearest river or coast, the land productivity, and dummies of landlocked countries and small islands. The religion factor is the percentage of population practicing major religions — Christianity, Judaism, Muslim, Hinduism, Buddhism, and Eastern religion — in a given country in 1900 CE for (1)-(4) and (9)-(12), while the same one in 1970 CE is used for the rest. For regressions (7), (8), (15) and (16), the average investment-to-output ratio and the average trade-to-output ratio are further controlled. The instrument is historical pathogen prevalence. All estimates on the instrumented variables are statistically significant at least at the 10% level when using the Anderson-Rubin test, which is an inference robust to weak instrumental variables. Heteroskedasticity robust standard errors in parentheses. ***, ** and * significant at 1%, 5% and 10%, respectively.
Figure B.3: Individualism and Growth (Maddison)
Table B.4: Individualism and Growth Using the PWT (OLS)

<table>
<thead>
<tr>
<th>Continent dummies</th>
<th>Income</th>
<th>TFP</th>
<th>Income</th>
<th>TFP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Individualism</td>
<td>.031***</td>
<td>.019***</td>
<td>.030***</td>
<td>.021***</td>
</tr>
<tr>
<td></td>
<td>(.009)</td>
<td>(.007)</td>
<td>(.007)</td>
<td>(.008)</td>
</tr>
<tr>
<td>Obs.</td>
<td>79</td>
<td>79</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.585</td>
<td>.746</td>
<td>.634</td>
<td>.783</td>
</tr>
<tr>
<td>1st. stage F-stat.</td>
<td>13.71</td>
<td>7.28</td>
<td>8.82</td>
<td>12.27</td>
</tr>
<tr>
<td>Partial $R^2$</td>
<td>.222</td>
<td>.199</td>
<td>.255</td>
<td>.242</td>
</tr>
</tbody>
</table>

Note: The dependent variable is the annual average growth rate of income per capita from the PWT (Ver. 8.0). “Individualism” is the Hofstede individualism index. Coefficients associated with controls are omitted. The controls consist of initial logged GDP per capita, and the average investment and trade to output ratios, and geographical and religion factors. The geographical controls are the absolute values of latitude and longitude, mean distance to the nearest river or coast and the land productivity, and dummies of landlocked countries and small islands. The religion factor is the percentage of population practicing major religions — Christianity, Judaism, Muslim, Hinduism, Buddhism, and Eastern religion — in a given country in 1970 CE. The instrument is historical pathogen prevalence. All estimates on the instrumented variables are statistically significant at least at the 10% level when using the Anderson-Rubin test, which is an inference robust to weak instrumental variables. Heteroskedasticity robust standard errors in parentheses. ***, ** and * significant at 1%, 5% and 10%, respectively.
Figure B.4: Individualism and Growth (PWT)
### Table B.5: Culture and Institutions (OLS)

**Panel A.** Dependent Variable: Institutions Quality; Instrument: Historical Pathogen Prevalence  
**IV Regressions (2SLS)**

<table>
<thead>
<tr>
<th>Continent dummies</th>
<th>Protection Against Exp. Risk</th>
<th>Protection of Prop. Rights</th>
<th>IV Regressions (2SLS)</th>
<th>Protection Against Exp. Risk</th>
<th>Protection of Prop. Rights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>Yes</td>
<td></td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>Individualism</td>
<td>.113***</td>
<td>.086***</td>
<td>.057***</td>
<td>.058***</td>
<td>.242***</td>
</tr>
<tr>
<td></td>
<td>(.022)</td>
<td>(.025)</td>
<td>(.006)</td>
<td>(.007)</td>
<td>(.051)</td>
</tr>
<tr>
<td>Obs.</td>
<td>88</td>
<td>88</td>
<td>85</td>
<td>85</td>
<td>87</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.511</td>
<td>.571</td>
<td>.711</td>
<td>.724</td>
<td>.405</td>
</tr>
<tr>
<td>1st. stage F-stat.</td>
<td>26.69</td>
<td>11.20</td>
<td>25.77</td>
<td>10.20</td>
<td>.331</td>
</tr>
<tr>
<td>Partial $R^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Panel B.** Dependent Variable: Individualism; Instrument: Settler Mortality  
**IV Regressions (2SLS)**

<table>
<thead>
<tr>
<th>Continent dummies</th>
<th>No</th>
<th>Yes</th>
<th>No</th>
<th>Yes</th>
<th>No</th>
<th>Yes</th>
<th>No</th>
<th>Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(9)</td>
<td>(10)</td>
<td>(11)</td>
<td>(12)</td>
<td>(13)</td>
<td>(14)</td>
<td>(15)</td>
<td>(16)</td>
</tr>
<tr>
<td>Prot. against exp. risk</td>
<td>1.870***</td>
<td>1.444***</td>
<td>8.669***</td>
<td>8.043***</td>
<td>7.545*</td>
<td>5.324***</td>
<td>10.33***</td>
<td>10.85***</td>
</tr>
<tr>
<td>Prot. of prop. rights</td>
<td>(.396)</td>
<td>(.492)</td>
<td>(1.193)</td>
<td>(1.321)</td>
<td>(3.939)</td>
<td>(1.961)</td>
<td>(1.744)</td>
<td>(1.893)</td>
</tr>
<tr>
<td>Obs.</td>
<td>88</td>
<td>88</td>
<td>85</td>
<td>85</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>37</td>
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<tr>
<td>$R^2$</td>
<td>.595</td>
<td>.639</td>
<td>.757</td>
<td>.786</td>
<td>.442</td>
<td>.741</td>
<td>.898</td>
<td>.900</td>
</tr>
<tr>
<td>1st. stage F-stat.</td>
<td>1.87</td>
<td>3.86</td>
<td>8.73</td>
<td>8.06</td>
<td>.087</td>
<td>.153</td>
<td>.223</td>
<td>.249</td>
</tr>
<tr>
<td>Partial $R^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** “Protection Against Expropriation Risk” and “Protection of Property Rights” are from the International Country Risk Guide and Gwartney et al. (2015), respectively, which Acemoglu et al. (2001) used to approximate the strength of a country’s institutions. A larger value of the indices corresponds to a greater strength of institutions. “Individualism” is the Hofstede individualism index. Coefficients associated with controls are omitted. The controls consist of geographical and religion factors. The geographical controls are the absolute values of latitude and longitude, mean distance to the nearest river or coast and the land productivity, and dummies of landlocked countries and small islands. The religion factor is the percentage of population practicing major religions — Christianity, Judaism, Muslim, Hinduism, Buddhism, and Eastern religion — in a given country in 1970 CE. The instruments, “Historical Pathogen Prevalence” and “Settler Mortality” are used in and from Gorodnichenk and Roland (2015). All estimates on the instrumented variables are statistically significant at least at the 10% level when using the Anderson-Rubin test, which is an inference robust to weak instrumental variables. Heteroskedasticity robust standard errors in parentheses. *** and ** significant at 1% and 5%, respectively.
Figure B.5: Individualism and Property Rights
Lemma B.1 Given any \((q_t, \rho_t) \in [0,1] \times [0,\bar{\rho}]\), the skill thresholds \(z^S_t\) and efficiency units of workers and ideas \((L_t, X_t)\) are uniquely determined in equilibrium. Also, \((z^I_t, z^C_t) \in (0,1]^2 \setminus (1,1) \forall (q_t, \rho_t) \in [0,1] \times [0,\bar{\rho}]\). Hence, \(\forall \rho_t \in [0,\bar{\rho}], z^I_t \in (0,1)\) if \(q_t = 1\), and \(z^C_t \in (0,1)\) if \(q_t = 0\). Also, \((z^I_t, z^C_t, L_t, X_t)\) are independent of \(A_{t-1}\).

Proof. It is obvious that any solution of \((z^I_t, z^C_t, L_t, X_t)\) is independent of \(A_{t-1}\) since \(A_{t-1}\) is not appeared in (2.9), (2.12) and (2.13). For the rest of arguments, we drop the time subscript for simplicity. First, we can prove that neither \((z^I, z^C) = (0,0)\) nor \((z^I, z^C) = (1,1)\) can be supported in any equilibrium. If \((z^I, z^C) = (0,0), L = 0\) and \(X > 0\) from (2.12) and (2.13). Hence, \(x \equiv X/L > \infty\). Then, \(z^S = 1\) from (2.9), which is a contradiction. If \((z^I, z^C) = (1,1), X = 0\) and \(L > 0\) from (2.12) and (2.13), and thus, \(x \equiv X/L = 0\). Then, \(z^S = 0\) from (2.9), which is a contradiction. Hence, if there are individualists only, i.e., \(q = 1\), then \(z^I\) should be an interior solution, i.e., \(z^I \in (0,1)\). On the contrary, if there are collectivists only, i.e., \(q = 0\), then \(z^C \in (0,1)\).

We now know that, in any equilibrium, \(L > 0\) and \(X > 0\ \forall (q, \rho) \in [0,1] \times [0,\bar{\rho}]\), and hence, \(x \equiv X/L > 0\). To prove that \((z^I, z^C)\) are uniquely determined \(\forall (q, \rho) \in [0,1] \times [0,\bar{\rho}]\), we first notice that \((z^I, z^C) \in (0,1]^2 \setminus (1,1)\) is uniquely determined \(\forall (x, \rho) \in \mathbb{R}_+ \times [0,\bar{\rho}]\) from (2.9). Note also that both \(z^I\) and \(z^C\) are strictly increasing in \(x \equiv X/L \geq 0 \ \forall \rho \in [0,\bar{\rho}]\) as long as both \(z^I\) and \(z^C\) are strictly smaller than one. Finally, it is straightforward from (2.9) that \(z^S (S \in \{I, C\})\) is a continuous function with respect to \(x\). We denote the function as \(z^S = z^S(x; \rho)\) with \(\rho\) being fixed at any point in \([0,\bar{\rho}]\). Then, one can define a continuous function \(z\) that maps from \(z^C(x; \rho) < 1\) to \(z^I(x; \rho)\) such that \(z^I(x; \rho) = z(z^C(x; \rho))\). That is, with the function \(z(z^C)\), we can calculate \(z^I\) given any \(z^C < 1\). Now suppose that \(x\) increases. Then, \(z^C(x; \rho)\) will strictly increase as long as \(z^C(x; \rho) < 1\) since \(z^C\) is strictly increasing in \(x \equiv X/L \geq 0\). Then, one can notice that the function \(z(z^C(x; \rho))\) is strictly increasing in \(z^C\) as long as \(z^I < 1\) since \(z^I(x; \rho) = z(z^C(x; \rho))\) and \(z^I(x; \rho)\) is strictly increasing in \(x\) given \(z^I < 1\). Meanwhile, \(z^I\) will not change once it reaches its maximum. That is, \(z^I\) is independent of \(x\) if \(z^I = 1\), which, in turn, implies that the function \(z(z^C(x; \rho))\) is independent of \(z^C\) if \(z^I = 1\). In sum, \(z^I = z(z^C) \in [0,1]\) is weakly increasing in \(z^C \in [0,1]\).

Now notice from (2.12) and (2.13) that \(L\) and \(X\) are weakly increasing and
Figure B.6: Uniqueness of the Skill Threshold

decreasing in $z^S (S = I, C)$, respectively, so that $x \equiv X/L$ is weakly decreasing in $z^S (S = I, C) \forall q \in [0,1]$. We define a continuous function $x$ such that $x = x(z^I, z^C; q)$. Then, by using the function $z(z^C)$, we can redefine the function $x$ such that $x = \hat{x}(z^C; q) \equiv x(z(z^C), z^C; q) \forall z^C \in [0,1)$. Then, it immediately follows that $\hat{x}$ is weakly decreasing in $z^C \in [0,1) \forall q \in [0,1]$ since we already knew that $z(z^C)$ is weakly increasing in $z^C \in [0,1)$. That is, with letting $z^I \in [0,1]$ vary along with $z^C$ in the case that $z^C \in [0,1)$, $x$ decreases (weakly) from a certain positive value as $z^C$ rises from zero; note that $x > 0$ if $z^C = 0$. Then, if $\lim_{z^C \rightarrow 1} x(z(z^C), z^C; q) < \bar{x}(\rho)$ where $\bar{x}(\rho)$ satisfies $z^C(\bar{x}; \rho) = 1$, there always exists a single point in the interval $(0,1)$ where $z^C(x; \rho)$ and $x(z(z^C), z^C; q)$ crosses; see Figure B.6 for a graphical example. Hence, $z^I$ is also uniquely determined by $z^I = z(z^C(x; \rho))$. On the other hand, if $\lim_{z^C \rightarrow 1} x(z(z^C), z^C; q) \geq \bar{x}(\rho)$, then $z^C = 1$. In this case, an equilibrium threshold, $z^I$, solves $z^I(x; \rho) = x(z^I, 1; q)$. Note that $z^I(x; \rho)$ increases strictly from zero to one, and $x(z^I, 1; q)$ weakly decreases from a certain positive value to zero; $x > 0$ if $z^I = 0$, and since $z^C = 1$, $x = 0$ if $z^I = 1$. Then, there should be a single point that solves $z^I(x; \rho) = x(z^I, 1; q)$. Moreover, it must be strictly smaller than one since $z^C = 1$; as shown in Lemma B.1, $(z^I, z^C) = (1, 1)$ cannot be an equilibrium $\forall (q, \rho) \in [0,1] \times [0, \bar{\rho}]$. Therefore, in any case, the equilibrium thresholds $(z^I, z^C)$ are always uniquely pinned down $\forall (q, \rho) \in [0,1] \times [0, \bar{\rho}]$. Then, given the unique pair of $(z^I, z^C)$ in equilibrium, $(L, X)$ are also uniquely determined by (2.12) and (2.13).
It immediately follows from Lemma B.1 that there are continuous functions $L : [0, 1] \times [0, \bar{\rho}] \to \mathbb{R}_{++}$ such that $L_t = L(q_t, \rho_t)$ and $X : [0, 1] \times [0, \bar{\rho}] \to \mathbb{R}_{++}$ such that $X_t = X(q_t, \rho_t)$. Then, $A_t$ is uniquely determined by (2.5) given $A_{t-1}$. Hence, one can construct a continuous function $A : \mathbb{R}_{++} \times [0, 1] \times [0, \bar{\rho}] \to \mathbb{R}_{++}$ such that $A_t = A(A_{t-1}, q_t, \rho_t)$. With $A_t$, we can obtain $Y_t$. Also, prices $(w_t, p_t) \in \mathbb{R}^2_{++}$ are uniquely determined by (2.7) and (2.8). With the price vector, the government lump-sum transfer $\theta_t$ is determined by (2.14), and hence, we can also solve for $\{c^S_t(z) \mid S \in \{I, C\}, z \in [0, 1]\}$. In the same way, $\{c^S_{t+2}(z) \mid S \in \{I, C\}, z \in [0, 1]\}$ is derived by $A_t$ along with $(q_{t+1}, \rho_{t+1})$, which are exogenously given. Then, we can solve for $\Delta_{t+1}$ by using $A_t$ and $\{c^S_{t+2}(z) \mid S \in \{I, C\}, z \in [0, 1]\}$. Note that $c^S_{t+2}(z)$ is a linear function of prices, $(w_{t+1}, p_{t+1})$, which are also linear in $A_t$ by (2.7) and (2.8); $L_{t+1}$ and $X_{t+1}$ are independent of $A_t$ by Lemma B.1. This implies that $c^S_{t+2}(z)$ is linear in $A_t$. This, in turn, implies that $\Delta_{t+1}$ is independent of $A_t$ since $\Delta_{t+1}$ is a linear function of $c^S_{t+2}(z)$ and divided by $A_t$. However, $c^S_{t+2}(z)$ depends on $q_{t+1}$ and the institutional quality $\rho_{t+1}$, so does $\Delta_{t+1}$. In consequence, there exists a continuous function $\Delta : [0, 1] \times [0, \bar{\rho}] \to \mathbb{R}$ such that $\Delta_{t+1} = \Delta(q_{t+1}, \rho_{t+1})$, where $\Delta_{t+1}$ is scale-invariant with respect to $\forall A_t \in \mathbb{R}_{++}$. Then, $\Delta_{t+1}$ is bounded both above and below $\forall t \geq 0$ since, from (2.12) and (2.13), both $L_{t+1}$ and $X_{t+1}$ are bounded. Given $\Delta_{t+1}$, one can solve for $d^S_t$ from (2.10) and (2.11), meaning that there exist a continuous function $d^S : [0, 1]^2 \times [0, \bar{\rho}] \to \mathbb{R}_{++}$ such that $d^S_t = d^S(q_t, q_{t+1}, \rho_{t+1})$ from (2.10) and (2.11), this implies further that if $\kappa > \beta \bar{\Delta} \equiv \beta \max_{(q, \rho) \in [0, 1] \times [0, \bar{\rho}]} |\Delta(q, \rho)| \geq 0$, $(d^L_t, d^C_t)$ are always in the unit square, i.e., $(d^L_t, d^C_t) \in [0, 1]^2 \forall (q_t, q_{t+1}, \rho_{t+1}) \in [0, 1]^2 \times [0, \bar{\rho}]$. 

**Proof of Proposition 2.2:**

For simplicity, we drop the institutional quality $\rho$ from below, while the argument here is supposed to be hold $\forall \rho \in [0, \bar{\rho}]$. From (2.3), (2.10) and (2.11), one can notice that $q_{t+1}$ solves the following function $\forall q_t \in [0, 1]$:

$$q_{t+1} = \begin{cases} q^+(q_{t+1}; q_t) & \text{if } \Delta(q_{t+1}) \geq 0 \\ q^-(q_{t+1}; q_t) & \text{if } \Delta(q_{t+1}) < 0 \end{cases}$$

where $\bar{\kappa} \equiv \beta \kappa^{-1} > 0$. It is convenient to define the following sets in proving Proposition 2.2:

$$Q^+ = \{q_{t+1} \in [0, 1] \mid \Delta(q_{t+1}) \geq 0\}$$

$$Q^- = \{q_{t+1} \in [0, 1] \mid \Delta(q_{t+1}) < 0\}$$

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where $Q^j (j = +, -)$ can be an empty set. Then, we can first prove the following lemma:

**Lemma B.2** Suppose that $\Delta(q_{t+1})$ is monotone decreasing in $q_{t+1} \in [0, 1]$. Then, $\overline{q}^j \equiv \max_{(q_t,q_{t+1})\in[0,1] \times Q^j} q^j \leq 1$ and $\underline{q}^j \equiv \min_{(q_t,q_{t+1})\in[0,1] \times Q^j} q^j \geq 0$ ($j = +, -$), where the inequalities strictly hold $\forall (q_t, q_{t+1}) \in (0, 1) \times Q^j$. Also, $q^j$ is monotone decreasing in $q_{t+1} \in [0, 1]$ ($j = +, -$).

**Proof.** Recall that $\kappa > \beta \overline{\Delta} \equiv \beta \max_{(q_t,\rho)\in[0,1] \times [0,\beta]} |\Delta(q;\rho)| \geq 0$ to guarantee that $(d^l_t, d^r_t) \in [0,1]^2$. Then, $q_t(1 - q_t)^2 \kappa \Delta(q_{t+1}) \leq 1 - q_t \ \forall (q_t, q_{t+1}) \in [0,1] \times Q^+$ since $\overline{\kappa} \equiv \beta \kappa^{-1} > 0$, and the inequality strictly holds $\forall (q_t, q_{t+1}) \in (0,1) \times Q^+$. Hence, $q^+(q_{t+1};q_t) \leq 1 \ \forall (q_t, q_{t+1}) \in [0,1] \times Q^+$ where the inequality strictly holds $\forall (q_t, q_{t+1}) \in (0,1) \times Q^+$. Similarly, we can prove for the case of $q^-$. Meanwhile, it is straightforward that $q^-(q_{t+1};q_t) \geq 0 \ \forall (q_t, q_{t+1}) \in [0,1]^2$ where the inequality strictly holds $\forall (q_t, q_{t+1}) \in (0,1) \times (0,1]$. Finally, it is obvious that $q^j$ is monotone decreasing $\forall q_{t+1} \in [0,1]$ since $\overline{\kappa} > 0$, and $\Delta(q_{t+1})$ is monotone decreasing in $q_{t+1} \in [0,1]$.

Now, we are ready to prove that the correspondence $Q$ is a singleton if $\Delta(q_{t+1})$ is monotone decreasing in $q_{t+1} \in [0,1]$. Since $\Delta(q_{t+1})$ is monotone decreasing, we need to consider only three different cases as follows:

**Case 1** $\Delta(1) \geq 0$:

In this case, $\Delta(q_{t+1}) \geq 0 \ \forall q_{t+1} \in [0,1]$, and thus, we only consider $q^+(q_{t+1};q_t)$. When $q_t = 0$, only $q_{t+1} = 0$ solves $q_{t+1} = q^+(q_{t+1};0)$. When $q_t = 1$, only $q_{t+1} = 1$ solves $q_{t+1} = q^+(q_{t+1};1)$. When $q_t \in (0,1)$, $q^+(q_{t+1};q_t)$ is monotone decreasing from $1 > \overline{q}^+ \geq q^+(0;q_t)$ to $q^+(0;q_t) \geq q^+ > 0$; see Lemma B.2. Meanwhile, $q_{t+1}$ is monotone increasing from 0 to 1, and hence, there exists a unique fixed point $q_{t+1} \in (0,1)$ such that $q_{t+1} = q^+(q_{t+1};q_t) \ \forall q_t \in (0,1)$.

**Case 2** $\Delta(0) \leq 0$:

In this case, $\Delta(q_{t+1}) \leq 0 \ \forall q_{t+1} \in [0,1]$, and thus, we only consider $q^-(q_{t+1};q_t)$; note that $q^+ = q^-$ if $\Delta(q_{t+1}) = 0$. Similarly to Case 1, when $q_t = 0$, only $q_{t+1} = 0$ solves $q_{t+1} = q^-(q_{t+1};0)$. Also, when $q_t = 1$, only $q_{t+1} = 1$ solves $q_{t+1} = q^-(q_{t+1};1)$. For $q_t \in (0,1)$, we can show that there exists a unique fixed point $q_{t+1} \in (0,1)$ such that $q_{t+1} = q^-(q_{t+1};q_t) \ \forall q_t \in (0,1)$ by applying the same method used for Case 1.

**Case 3** $\Delta(0) > 0$ and $\Delta(1) < 0$:

In this case, it is straightforward that $\exists! \overline{q} \in (0,1)$ such that $\Delta(\overline{q}) = 0$ where $\Delta(q_{t+1}) \geq 0 \ \forall q_{t+1} \in Q^+ = [0,\overline{q}]$ and $\Delta(q_{t+1}) < 0 \ \forall q_{t+1} \in Q^- = (\overline{q}, 1]$. Hence, for any given $q_t \in [0,1]$, we need to consider $q^+(q_{t+1};q_t) \ \forall q_{t+1} \in Q^+$, and $q^-(q_{t+1};q_t)$
otherwise. When \( q_t = 0 \), only \( q_{t+1} = 1 \) solves \( q_{t+1} = q^+(q_{t+1}; 1) \). In contrast, when \( q_t = 1 \), only \( q_{t+1} = 1 \) solves \( q_{t+1} = q^-(q_{t+1}; 1) \). Now, let us consider the case of \( q_t \in (0, 1) \). First, when \( q_t \in (0, 1) \) and \( q_{t+1} \in Q^+ \), it is already proven that there exists a unique fixed point \( q_{t+1} \in (0, 1) \) such that \( q_{t+1} = q^+(q_{t+1}; q_t) \forall q_t \in (0, 1) \); see Case 1. In the other case where \( q_t \in (0, 1) \) and \( q_{t+1} \in Q^- \), it is also already shown that there exists a unique fixed point \( q_{t+1} \in (0, 1) \) such that \( q_{t+1} = q^-(q_{t+1}; q_t) \forall q_t \in (0, 1) \); see Case 2.

That is, if \( \Delta(q_{t+1}) \) is monotone decreasing in \( q_{t+1} \), there is always a unique fixed point \( q_{t+1} \) that solves (2.15) \( \forall q_t \in [0, 1] \) since the correspondence \( Q \) given by (2.16) is a singleton. Then, one can construct a function \( \phi : [0, 1] \rightarrow [0, 1] \) such that \( q_{t+1} = \phi(q_t) \) that satisfies the following law of motion \( \forall q_t \in [0, 1] \):

\[
\phi(q_t) = \begin{cases} 
q^+(\phi(q_t); q_t) = q_t + q_t(1 - q_t)^2 \hat{\kappa} \Delta(\phi(q_t)), & \Delta(\phi(q_t)) \geq 0 \\
q^-(\phi(q_t); q_t) = q_t - q_t^2(1 - q_t) \hat{\kappa} [-\Delta(\phi(q_t))], & \Delta(\phi(q_t)) < 0 
\end{cases}
\]  

(B.1)

Then, one can solve for \( q_{t+1} \) from (B.1) as long as \( q_t \) is given. Also, (B.1) indicates that \( \phi(q; \rho) \) is continuous at any \( q \in [0, 1] \) since \( \Delta(q) \) is continuous at any \( q \in [0, 1] \) from Proposition 2.1. Finally, it is obvious that \( \phi(0) = 0 \) and \( \phi(1) = 1 \).

Now, we prove that (ii)–(iv) of Proposition 2.2. First, from (B.1), it is trivial that \( \phi(\bar{q}) = \bar{q} \) since \( \Delta(\bar{q}) = 0 \). Second, suppose that \( q_t \in (0, \bar{q}) \). Then, from (B.1), \( q^+(\bar{q}; q_t) = q_t < \bar{q} \). Meanwhile, \( q^+(q_t; q_t) > q_t \) since \( q_t < \bar{q} \) that implies \( \Delta(q_t) > 0 \). Then, by continuity of \( q^+ \), \( \exists q_{t+1} \in (q_t, \bar{q}) \) such that \( q^+(q_{t+1}; q_t) = q_{t+1} \). Moreover, such a fixed point \( q_{t+1} \) is unique since \( q^+ \) is monotone decreasing in \( q_{t+1} \). That is, \( \phi(q_t) = q_{t+1} \in (q_t, \bar{q}) \forall q_t \in (0, \bar{q}) \). On the contrary, if \( q_t \in (\bar{q}, 1) \), from (B.1), \( q^-(\bar{q}; q_t) = q_t > \bar{q} \). Also, \( q^-(\bar{q}; q_t) < q_t \) since \( q_t > \bar{q} \) that means \( \Delta(q_t) < 0 \). Then, by continuity of \( q^- \), \( \exists q_{t+1} \in (\bar{q}, q_t) \) such that \( q^-(q_{t+1}; q_t) = q_{t+1} \). Such a fixed point \( q_{t+1} \) is also unique since \( q^- \) is monotone decreasing in \( q_{t+1} \). Hence, \( \phi(q_t) = q_{t+1} \in (\bar{q}, q_t) \forall q_t \in (\bar{q}, 1) \), establishing (ii) and (iii); see Figure B.7 for graphical exposition.

To prove (iv), we first consider \( Q^+ \) region. Notice that \( \hat{\kappa} \Delta(q_{t+1}) \in (0, 1) \forall q_{t+1} \in Q^+ \). Now, note from (B.1) that \( q^+ \) is differentiable \( \forall q_t \in [0, 1] \). Then, we have:

\[
\frac{\partial q^+}{\partial q_t} = 1 + \frac{\hat{\kappa} \Delta(q_{t+1}) (1 - 3q_t)(1 - q_t)}{\min_{q_{t+1} \in Q^+} \min_{q_t \in [0, 1]} \left( -\frac{1}{2} \right)} > 0 \forall (q_t, q_{t+1}) \in (0, 1) \times Q^+ \tag{B.2}
\]

Now, suppose that \( q_t \) increases to \( q_t' \), so that \( q_t' > q_t \). Suppose further that \( q_{t+1} \) remains constant or decreases to \( q_{t+1}' \) in equilibrium, so that \( q_{t+1}' \leq q_{t+1} \). Since \( \Delta(q_{t+1}) \)
Figure B.7: Determination of $q_{t+1} \in (q_t, \bar{q})$

is monotone decreasing, $\Delta(q'_{t+1}) \geq \Delta(q_{t+1})$, and this implies that $q^+(q'_{t+1}; q'_t) > q^+(q_{t+1}; q_t)$ from (B.2). Then, the equilibrium condition (B.1) is violated since $q'_{t+1}$ is supposed to be either equal to or smaller than $q_{t+1}$, which is a contradiction. Hence, $q'_{t+1}$ should be strictly larger than $q_{t+1}$ in equilibrium. For $Q^-$ region, we have the following inequality:

$$\frac{\partial q^-}{\partial q_t} = 1 - \bar{\kappa} \left[ -\Delta(q_{t+1}) \right] \left( 2 - 3q_t \right) q_t > 0 \quad \forall (q_t, q_{t+1}) \in (0, 1) \times Q^-$$

Similarly, one can prove that $q_{t+1}$ must increase (decrease) in equilibrium once $q_t$ increases (decreases). Hence, $\phi(q_t)$ is monotone increasing in $q_t$. ■

**Proof of Proposition 2.3:**

**Proof.** Since the policy is one dimension and every voter has the single-peaked preference given by (2.24) and (2.25), the median voter theorem can be used. Let us denote the strategy of each political party $P_i$ as $\rho_i$ ($i = 1, 2$) where $\rho_i \in [0, \bar{\rho}]$. Suppose first that $X > 1/2 - \xi$. Then, in any Nash equilibrium, neither $\rho_1$ nor $\rho_2$ is larger than 0. This is because, if so, the other party, say $P_{-i}$, can win the election for sure by setting $\rho_{-i} = 0$. Hence, there is at least one party choosing $\rho_i = 0$ in
any Nash equilibrium in this case. If one of them chooses \( \rho_i = 0 \), then the other one should choose \( \rho_{-i} = 0 \) as well in order to raise the probability to win. This is because, if \( \rho_{-i} \neq 0 \), the probability of winning the election for \( P_{-i} \) is zero. On the contrary, if \( \rho_{-i} = 0 \), then the two parties have the same number of votes, precisely 1/2 by the law of large numbers, so that the probability of winning the election is 1/2, which is the probability of winning in flip of a coin. Hence, \( (\rho_1, \rho_2) = (0, 0) \) is a unique Nash equilibrium when \( \bar{X} > 1/2 - \xi \). Similarly, one can show that \( (\rho_1, \rho_2) = (\bar{\rho}, \bar{\rho}) \) is a unique Nash equilibrium when \( \bar{X} \leq 1/2 - \xi \). ■

**Proof of Proposition 2.4:**
The following lemma is useful in proving part (i) of Proposition 2.4.

**Lemma B.3** Suppose that \( \Delta(q, \rho) \) is monotone decreasing in \( q \). Also, for any \( q \in [0, 1] \), if \( \bar{X}(\phi(q, \bar{\rho}) \bar{\rho}) > 1/2 - \xi \) then \( \bar{X}(\phi(q, 0), 0) > 1/2 - \xi \). Finally, for any \( q \in [0, 1] \), if \( \bar{X}(\phi(q, 0), 0) \leq 1/2 - \xi \) then \( \bar{X}(\phi(q, \bar{\rho}) \bar{\rho}) \leq 1/2 - \xi \). Then, the set of solutions, \( \{(q_{t+1}, \rho_{t+1})\} \), where each element \( (q_{t+1}, \rho_{t+1}) \) solves equations (2.30) and (2.31), is not an empty set \( \forall q_t \in [0, 1] \).

**Proof.** If \( \Delta(q, \rho) \) is monotone decreasing in \( q \), there exists a single-valued function \( \phi(q, \rho) \) by part (i) of Proposition 2.2. Also note that \( \mathcal{R}(q) \), which is given by equation (2.29), is not an empty set \( \forall q \in [0, 1] \). To prove the lemma, note first that \( \mathcal{R}(q) = \{0\} \), \( \{\bar{\rho}\} \), or \( \{0, \bar{\rho}\} \). Hence, \( \mathcal{R}(\phi(q_t, \rho)) = \{0\} \), \( \{\bar{\rho}\} \), or \( \{0, \bar{\rho}\} \) \( \forall q_t \in [0, 1] \) and \( \forall \rho \in \{0, \bar{\rho}\} \). Note also that, from equations (2.30) and (2.31), \( \phi(q_t, \rho), \rho \) is a solution vector if \( \mathcal{R}(\phi(q_t, \rho)) \) includes \( \rho \) where \( \rho \in \{0, \bar{\rho}\} \). Now, suppose that \( \bar{X}(\phi(q_t, \bar{\rho}) \bar{\rho}) \leq 1/2 - \xi \). Then, \( \bar{\rho} \in \mathcal{R}(\phi(q_t, \bar{\rho})) \), so that an equilibrium vector for \( q_t \) exists. On the other hand, suppose that \( \bar{X}(\phi(q, \bar{\rho}) \bar{\rho}) > 1/2 - \xi \). Then, \( \bar{\rho} \) cannot be an equilibrium policy in the next period at the current state of culture, \( q_t \), i.e., \( \bar{\rho} \notin \mathcal{R}(\phi(q_t, \bar{\rho})) \). In this situation, if \( \bar{X}(\phi(q, 0), 0) > 1/2 - \xi \), the zero tax can be an equilibrium, and this is a unique equilibrium, i.e., \( \{0\} = \mathcal{R}(\phi(q, 0)) \). Similarly, suppose that \( \bar{X}(\phi(q, 0), 0) > 1/2 - \xi \). Then, \( 0 \in \mathcal{R}(\phi(q, 0)) \), so that an equilibrium vector for \( q_t \) exists. We now have the last case such that \( \bar{X}(\phi(q, 0), 0) \leq 1/2 - \xi \), the zero tax cannot be an equilibrium policy, i.e., \( 0 \notin \mathcal{R}(\phi(q, 0)) \). In this situation, if \( \bar{X}(\phi(q, \bar{\rho}), \bar{\rho}) \leq 1/2 - \xi \), then \( \bar{\rho} \) can be an equilibrium, and this is a unique equilibrium, i.e., \( \{\bar{\rho}\} = \mathcal{R}(\phi(q, \bar{\rho})) \). ■

To prove (i), we first show that \( \bar{X}(q, \rho) \) is continuous at any \( q \in [0, 1] \) \( \forall \rho \in [0, \bar{\rho}] \).

This is because \( z^S(S \in \{I, C\}) \) is continuous at any \( x \equiv X/L \geq 0 \) from (2.9), where
both $X$ and $L$ are continuous at any $q \in [0, 1]$ from Proposition 2.1. Then, by definition of $\hat{X}(q, \rho)$ given by equation (2.26), $\hat{X}(q, \rho)$ is also continuous at any $q \in [0, 1]$ $\forall \rho \in [0, \bar{\rho}]$.

Suppose now that $q_0^* \in [0, 1]$ that solves $\hat{X}(q_0^*, 0) = 1/2 - \xi$. Then, this means either that $\hat{X}(q, 0) > 1/2 - \xi \forall q \in [0, 1]$, or that, $\hat{X}(q, 0) < 1/2 - \xi \forall q \in [0, 1]$. If $\hat{X}(q, 0) > 1/2 - \xi \forall q \in [0, 1]$, then $0 \in \mathcal{R}(\phi(q, 0)) \forall q \in [0, 1]$. Hence, a solution vector exists $\forall q \in [0, 1]$. If $\hat{X}(q, 0) < 1/2 - \xi \forall q \in [0, 1]$, then $\hat{X}(q, \bar{\rho}) < 1/2 - \xi \forall q \in [0, 1]$ since $\hat{X}(q, 0) > \hat{X}(q, \bar{\rho}) \forall q \in [0, 1]$. Then, $\bar{\rho} \in \mathcal{R}(\phi(q, \bar{\rho})) \forall q \in [0, 1]$, implying that a solution vector exists $\forall q \in [0, 1]$.

Suppose now that there is one $q_0^* \in [0, 1]$ that solves $\hat{X}(q_0^*, 0) = 1/2 - \xi$. If $q_0^* = \arg\max_{q \in [0, 1]} \hat{X}(q, 0)$, then $\hat{X}(q, \bar{\rho}) < 1/2 - \xi \forall q \in [0, 1]$ since $\hat{X}(q, 0) > \hat{X}(q, \bar{\rho}) \forall q \in [0, 1]$, and hence, $\bar{\rho} \in \mathcal{R}(\phi(q, \bar{\rho})) \forall q \in [0, 1]$. We now consider the last case such that $q_0^* \neq \arg\max_{q \in [0, 1]} \hat{X}(q, 0)$, which implies that $\hat{X}(q, 0) \leq 1/2 - \xi \forall q \in [0, q_0^*]$, and $\hat{X}(q, 0) > 1/2 - \xi \forall q \in (q_0^*, 1]$. This is because there is only one $q_0^*$ such that $\hat{X}(q_0^*, 0) = 1/2 - \xi$ and $\hat{X}(q, 0)$ is a weakly increasing or an inverse U-shaped function with respect to $q$ that satisfies $\hat{X}(0, 0) \leq \hat{X}(1, 0)$. Then, $\forall q \in [0, 1]$ such that $\phi(q, 0) \in (q_0^*, 1]$, there is a solution vector since $0 \in \mathcal{R}(\phi(q, 0))$. Now, consider $\forall q \in [0, 1]$ such that $\phi(q, 0) \in [0, q_0^*]$. For such $q$, we need to show that $\hat{X}(\phi(q, \bar{\rho}), \bar{\rho}) \leq 1/2 - \xi$. Then, by Lemma B.3, there is a solution vector. To show this, first note that $\phi(q, 0) > \phi(q, \bar{\rho}) \forall q \in [0, 1]$ since $\Delta(q, 0) > \Delta(q, \bar{\rho}) \forall q \in [0, 1]$. Then, $\hat{X}(\phi(q, \bar{\rho}), 0) \leq 1/2 - \xi \forall q$ such that $\phi(q, 0) \in [0, q_0^*]$ since $\hat{X}(q, 0) \leq 1/2 - \xi \forall q \in [0, q_0^*]$. Meanwhile, $\hat{X}(\phi(q, \bar{\rho}), \bar{\rho}) < \hat{X}(\phi(q, \bar{\rho}), 0)$ since $\hat{X}(q, \bar{\rho}) < \hat{X}(q, 0) \forall q \in [0, 1]$. Hence, we have $\hat{X}(\phi(q, \bar{\rho}), \bar{\rho}) < 1/2 - \xi \forall q \in [0, 1]$ such that $\phi(q, 0) \in [0, q_0^*]$, and therefore, there is a solution vector since $\bar{\rho} \in \mathcal{R}(\phi(q, \bar{\rho}))$.

To prove part (ii), first note that $\xi$ satisfies $\max_{q \in [0, 1]} \hat{X}(q, 0) > 1/2 - \xi \geq \min_{q \in [0, 1]} \hat{X}(q, \bar{\rho})$. Hence, it suffices to consider only two cases. The first case is that $\max_{q \in [0, 1]} \hat{X}(q, \bar{\rho}) \geq 1/2 - \xi \geq \min_{q \in [0, 1]} \hat{X}(q, \bar{\rho})$. In this case, we define a set $Q_\rho^* = \{q \in [0, 1] \mid \hat{X}(q, \bar{\rho}) = 1/2 - \xi\}$, which is not an empty set, and denote $q_0^* \in [0, 1]$ such that $Q_\rho^* = \{q \in [0, 1] \mid \hat{X}(q, \bar{\rho}) \leq 1/2 - \xi\}$. We can define a set, $Q_\rho^{**}$, which is a subset of $Q_\rho^*$ that satisfies $Q_\rho^{**} = Q_\rho^* \cap \{q \leq q_0^*\}$. Note that $Q_\rho^{**}$ is not an empty set since $\hat{X}(q, \bar{\rho})$ is either a monotone increasing or an inverse U-shaped function with respect to $q$ such that $\hat{X}(0, \bar{\rho}) \leq \hat{X}(1, \bar{\rho})$. Similarly, there must exist a set, $Q_0^{**} \subset [0, 1]$, such that $Q_0^{**} = \{q \in [0, 1] \mid q \leq q_0^*\}$ and $\hat{X}(q, 0) > 1/2 - \xi \forall q \in [0, 1], Q_0^{**}$ is not an empty set and $\min Q_0^{**} < \max Q_0^{**}$. Since $\hat{X}(q, 0) > 1/2 - \xi \forall q \in Q_0^{**}$, $\rho_{t+1} = 0$ can be supported as an political equilibrium $\forall q \in (q_L, q_0^*)$.
where $\hat{q}_L \equiv \min Q_0^{**}$. Similarly, since $\hat{X}(q, \hat{\rho}) \leq 1/2 - \xi \forall q \in Q_0^{**}$, $\rho_{t+1} = \hat{\rho}$ can be supported as an political equilibrium $\forall q \in [0, \hat{q}_L]$ where $\hat{q}_H \equiv \max Q_0^{**}$. Since $\Delta(q, 0) > \Delta(q, \hat{\rho}) \forall q \in [0, 1]$, $\Delta(q, 0) \neq \Delta(q, \hat{\rho}) \forall q \in (\hat{q}_L, \hat{q}_H]$. This implies that $\phi(q, 0) \neq \phi(q, \hat{\rho}) \forall q \in (\hat{q}_L, \hat{q}_H]$. Similarly, $\Delta(q, 0) > \Delta(q, \hat{\rho}) \forall q \in (\hat{q}_L, \hat{q}_H]$ where $\hat{q}_L$ solves $\phi(\hat{q}_L, \hat{\rho}) = \hat{q}_L$, and $\hat{q}_H$ solves $\phi(\hat{q}_H, \hat{\rho}) = \hat{q}_H$. Consequently, $\phi(q, 0) \neq \phi(q, \hat{\rho}) \forall q \in (\hat{q}_L, \hat{q}_H]$. Finally, note that, $\forall q_t \in (\hat{q}_L, \hat{q}_H]$, $q_{t+1} = \phi(q, 0)$ is consistent with the political equilibrium, $\rho_{t+1} = \hat{\rho}$ since $\hat{\rho} \in R(\phi(q_t), \hat{\rho})$. Hence, $\{S_I\} = \{S_O, S_P\} \forall q_t = (\hat{q}_L, \hat{q}_H]$. We now consider the last case that $\max_{x \in [0, 1]} \hat{X}(q, 0) > 1/2 - \xi > \max_{x \in [0, 1]} \hat{X}(q, \hat{\rho})$. If there exist $q_0^*$ that solves $\hat{X}(q_0^*, 0) = 1/2 - \xi$, then one can easily prove it similarly to above, which is more straightforward, so that we omit the proof here to save space. If not, i.e., $\not\exists q_0^*$ that solves $\hat{X}(q_0^*, 0) = 1/2 - \xi$, then $\hat{X}(q, 0) > 1/2 - \xi \forall q \in [0, 1]$, while $\hat{X}(q, \hat{\rho}) < 1/2 - \xi \forall q \in [0, 1]$. Hence, it is straightforward that $\{0, \hat{\rho}\} = R(q) \forall q \in [0, 1]$, meaning that there are multiple social equilibria $\forall q \in [0, 1]$, and therefore, $\hat{q}_L = 0$ and $\hat{q}_H = 1$.

Finally, parts (iii) and (iv) are rather trivial to prove by using part (ii) of Proposition 2.2, Corollary 2.1 and part (ii) of Proposition 2.4, and hence, omit the proof here to save space. ■

**Derivation of $X(\bar{q}(\rho); \rho)$ and $\bar{q}(\rho)$ in Example 1:**
Given the functional specifications, the skill thresholds given by (9) can be written as follows:

$$z_t^l = \min \left\{ (\hat{\eta}_t \lambda^{-1} x_t)^{\frac{1}{\hat{\psi}}}, 1 \right\} ; \quad z_t^C = \min \left\{ (\hat{\eta}_t e x_t)^{\frac{1}{\hat{\psi}}}, 1 \right\} . \quad (B.3)$$

Recall that $\hat{\eta}_t = \eta_t / (\alpha(1 - \rho_t))$ and $x_t \equiv X_t / L_t$. Here, for simplicity, we consider only the case of $(\hat{\eta}_t e x_t)^{\frac{1}{\hat{\psi}}} < 1$ where $\bar{x} \equiv \max_{(q, \rho) \in [0, 1] \times [0, \rho]} x(q, \rho)$.

Using (2.12) and (B.3), one can rewrite $L_t$ as follows:

$$L_t = q_t z_t^l + (1 - q_t) e z_t^C$$

$$= \chi_t [\hat{\eta}_t x_t]^{\frac{1}{\hat{\psi}}} \quad (B.4)$$

where

$$\chi_t = \chi(q_t) \equiv q_t \lambda^{-\frac{1}{\hat{\psi}}} + (1 - q_t) e^{1 + \frac{1}{\hat{\psi}}} \quad (B.5)$$

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Similarly, using (2.13) and (B.3) yields:

\[
(1 + \psi)X_t = [q_t \lambda (1 - (z^I_t)^{1+\psi}) + (1 - q_t) (1 - (z^C_t)^{1+\psi})] \\
= \mu_t - \chi_t \tilde{\eta}_t x_t^{\frac{1}{\psi}} \tag{B.6}
\]

where

\[
\mu_t = \mu(q_t) \equiv q_t \lambda + (1 - q_t) \tag{B.7}
\]

Note that \(\mu_t\) is the aggregate innovation in period \(t\) when all agents are innovators, and they all succeed in innovation; recall that \(\lambda^C\) is normalized at one, while \(\lambda \equiv \lambda^I > 1\). Hence, \(d\mu(q)/dq > 0\) from (B.7) with Assumption 1. That is, \(\mu_t\) can be interpreted as the maximum level of the aggregate innovation in society given \(q_t\), which gets larger as the society is more individualistic.

Meanwhile, \(X_t = L_t x_t\), and thus, (B.4) implies:

\[
X_t = \chi_t \tilde{\eta}_t x_t^{\frac{1}{\psi}} \tag{B.8}
\]

Then, from (B.6) and (B.8), we obtain:

\[
X_t = X(q_t, \rho_t) = \frac{\mu(q_t)}{1 + \psi + \tilde{\eta}(\rho_t)} \tag{B.9}
\]

In our simple baseline functional specifications, \(X_t\) also takes a simple formula; it is the maximum level of innovations, \(\mu(q_t)\), adjusted by \(\psi\), the elasticity of the success probability in innovation with respect to the skill, and by \(\tilde{\eta}(\rho_t)\), the relative importance of labor in the production of consumption goods adjusted by the expropriation risk.

Notably, since \(d\mu(q)/dq > 0\) and \(d\tilde{\eta}(\rho)/d\rho > 0\), it immediately follows from (B.9) that \(\partial X_t/\partial q_t < 0\) and \(\partial X_t/\partial \rho_t < 0\) with Assumption 1 as expected. Then, it is also obvious that \(X\), the aggregate innovation in the steady state, is monotone decreasing in \(q\). Also note that both \(\partial X_t/\partial q_t < 0\) and \(\partial X_t/\partial \rho_t < 0\) imply that both direct and indirect effects of institutions on long-term growth are all negative, so that \(dX/d\rho < 0\).
Now, we solve for \( \Delta(q; \rho) \). First note that:

\[
\mathbb{E}_z [c^C_{t+1}(z)] = z^C e w_t + \frac{(1 - \rho_t) p_t}{1 + \psi} \left[ 1 - (z^C_t)^{1+\psi} \right]
\]

\[
= z^C e \frac{\eta}{\alpha} x_t p_t - \frac{(1 - \rho_t) p_t}{1 + \psi} (z^C_t)^{1+\psi} + \frac{(1 - \rho_t) p_t}{1 + \psi}
\]  \( \text{(B.10)} \)

where, for (B.10), we use \( w_t = \frac{\eta}{\alpha} x_t p_t \) from equations (2.7) and (2.8).

Also note that:

\[
z^C e \frac{\eta}{\alpha} x_t = \eta^{\frac{1}{\psi}} e^{1+\frac{1}{\psi}} x_t^{1+\frac{1}{\psi}} \frac{\eta}{\alpha}
\]

\[
= \left[ \frac{\eta}{\alpha(1 - \rho_t)} e x_t \right]^{1+\frac{1}{\psi}} (1 - \rho_t)
\]

\[
= (z^C_t)^{1+\psi} (1 - \rho_t)
\]  \( \text{(B.11)} \)

where we use (B.3) to obtain (B.11). Then, equations (B.10) and (B.11) yield:

\[
\mathbb{E}_z [c^C_{t+1}(z)] = \frac{(1 - \rho_t) p_t}{1 + \psi} \left[ \psi(\hat{\eta} e x_t)^{1+\frac{1}{\psi}} + 1 \right]
\]  \( \text{(B.12)} \)

using equation (B.3) again. Similarly, we can obtain:

\[
\mathbb{E}_z [c^I_{t+1}(z)] = \frac{\lambda(1 - \rho_t) p_t}{1 + \psi} \left[ \psi(\hat{\eta} \lambda^{-1} x_t)^{1+\frac{1}{\psi}} + 1 \right]
\]  \( \text{(B.13)} \)

Then, from (B.12) and (B.13), we have:

\[
\mathbb{E}_z [c^I_{t+2}(z) - c^C_{t+2}(z)] = \frac{(1 - \rho_{t+1}) p_{t+1}}{1 + \psi} \left[ (\lambda - 1) - (e^{1+\frac{1}{\psi}} - \lambda^{\frac{1}{\psi}}) \psi(\hat{\eta}_{t+1} x_{t+1})^{1+\frac{1}{\psi}} \right]
\]  \( \text{(B.14)} \)

where, from (B.4) and (B.9), innovations per worker \( x_{t+1} \equiv X_{t+1}/L_{t+1} \) is given by:

\[
x_{t+1} = x(q_{t+1}; \rho_{t+1}) = \left[ 1 - \frac{\mu(q_{t+1})}{\hat{\eta}(\rho_{t+1})^{\frac{1}{\psi}} (1 + \psi + \hat{\eta}(\rho_{t+1})) \chi(q_{t+1})} \right]^{\frac{1}{1+\psi}}
\]  \( \text{(B.15)} \)

Hence, the relative value of being an individualist to a collectivist is increasing in \( \lambda^I \equiv \lambda \) and decreasing in \( e^C \equiv e \) with other things being constant, which are intuitively straightforward. Also, as long as the relative value is positive, it is increasing in the after-tax price of innovations \( (1 - \rho_{t+1}) p_{t+1} \). This is simply because an individualist
is more productive in innovation, so that he is more inclined to be an innovator.

Finally, we can solve for $\Delta(q; \rho)$ as follows:

$$
\Delta_{t+1} = \Delta(q_{t+1}; \rho_{t+1}) = \frac{\alpha\sigma(1 - \rho_{t+1})}{1 + \psi} \left[ (\lambda - 1) - (e^{1+\frac{1}{\psi}} - \lambda^{-\frac{1}{\psi}})\psi(\tilde{\eta}_{t+1}x_{t+1})^{1+\frac{1}{\psi}} \right]
$$

(B.16)

where, from (B.4) and (B.15), $L_{t+1}$ is given by:

$$
L_{t+1} = L(q_{t+1}; \rho_{t+1}) = \left[ \frac{\hat{\eta}(\rho_{t+1})}{1 + \psi + \hat{\eta}(\rho_{t+1})} \mu(q_{t+1})\chi(q_{t+1})^{\psi} \right]^{\frac{1}{1+\psi}}
$$

(B.17)

To prove that $dq(\rho)/d\rho < 0$, first note, from (B.15) and (B.16), that $\bar{q}(\rho)$ solves the following equation:

$$
\frac{\mu(\bar{q}(\rho))}{\chi(\bar{q}(\rho))} = (\lambda - 1)(1 + \psi + \hat{\eta}(\rho)) \equiv \zeta(\rho)
$$

(B.18)

which yields the following equation:

$$
\frac{d\zeta}{d\hat{\eta}} = -\psi(1 + \psi)(\lambda - 1)(e^{1+\frac{1}{\psi}} - \lambda^{-\frac{1}{\psi}}) < 0
$$

(B.19)

Since $d\bar{q}(\rho)/d\rho > 0$, (B.18) and (B.19) imply that $d\zeta(\rho)/d\rho < 0$. Then, given Assumption 1, since $\mu(q)$ is monotone increasing in $q$ from (B.7) while $\chi(q)$ is monotone decreasing in $q$ from (B.5), putting (B.18) together with $d\zeta(\rho)/d\rho < 0$ yields $dq(\rho)/d\rho < 0$ as expected and already depicted in Figure 2.4 and argued in Example 1. That is, better (worse) protection of property rights induces stronger (weaker) individualism in the long run, which confirms the negative indirect growth effect of institutions.

Recall that we have assumed that $\Delta(q; \rho)$ is monotone decreasing in $q$ for the uniqueness of $q_{t+1}$ given $q_t$ and $\rho_{t+1}$. Furthermore, we have also assumed that $\Delta(0; \rho) > 0$ and $\Delta(1; \rho) < 0$ for any given $\rho \in [0, \bar{\rho}]$ for the existence of an interior steady state $\bar{q}(\rho)$. Here, we provide sufficient conditions that guarantee these assumptions.

**Condition for** $\partial\Delta(q; \rho)/\partial q < 0$:

From (B.16), it is straightforward that $\partial\Delta(q; \rho)/\partial q < 0$ if $\partial L(q; \rho)/\partial q < 0$ and $\partial x(q; \rho)/\partial q < 0$. Also, one can notice from (B.15) that $\partial x(q; \rho)/\partial q < 0$, meaning
innovations per worker, \( x \equiv X/L \), is monotone decreasing in \( q \). This is simply because an individualist is more productive in innovation than a collectivist, and in return, a collectivist has a competitive edge in the production of goods.

For \( \partial L(q; \rho)/\partial q < 0 \), note first from (B.17) that:

\[
\frac{\partial L}{\partial q} < 0 \iff \frac{d}{dq} \left[ \mu(q) \chi(q)^\psi \right] < 0 \iff \mu'(q) \chi(q) + \psi \mu(q) \chi'(q) < 0
\]  

(B.20)

Since we already knew that \( \mu'(q) > 0 \) and \( \chi'(q) < 0 \) \( \forall q \in [0,1] \) from (B.5) and (B.7), a sufficient condition that guarantees the inequality in (B.20) is given by:

\[
(\lambda - 1) \bar{\chi} + \psi \mu \left( \lambda^{-\frac{1}{\psi}} + e^{1+\frac{1}{\psi}} \right) < 0
\]  

(B.21)

where \( \bar{\chi} \equiv \max_{q \in [0,1]} \chi(q) = e^{1+\frac{1}{\psi}} \), and \( \mu \equiv \min_{q \in [0,1]} \mu(q) = 1 \). Then, (B.21) is equivalent with the following inequality:

\[
e > \left[ \frac{\psi}{\psi - (\lambda - 1)} \right]^{\frac{\psi}{1+\psi}} \lambda^{-\frac{1}{1+\psi}} \equiv f(\psi)
\]  

(B.22)

Then, one can easily show that \( f'(\psi) < 0 \) \( \forall \psi > \bar{\psi} \) where \( \bar{\psi} \equiv \bar{\psi}(\lambda) \) is a threshold depending on \( \lambda \). Also, \( \lim_{\psi \to \infty} f(\psi) = 1 \) given any \( \lambda < \infty \). Hence, the inequality in (B.22) is satisfied when \( \psi \) is sufficiently large given any \( \lambda \) since \( e \) is assumed bigger than one. Mathematically, \( \Delta(q, \rho) \) is monotone decreasing in \( q \) given any \( (\lambda, e) \in \mathbb{R}^2_{++} \setminus [0,1]^2 \) as long as \( \psi \) is sufficiently large.

**Conditions for \( \Delta(0; \rho) > 0 \) and \( \Delta(1; \rho) < 0 \):**

Since \( \Delta(0; \rho) > 0 \) and \( \Delta(1; \rho) < 0 \), by continuity of \( \Delta \), \( \exists \bar{q} = \bar{q}(\rho) \in (0,1) \) where \( \Delta(\bar{q}; \rho) = 0 \) \( \forall \rho \in [0, \bar{\rho}] \). From (B.16), \( \Delta(0; \rho) > 0 \) \( \forall \rho \in [0, \bar{\rho}] \) if, and only if,

\[
\lambda - 1 > (e^{1+\frac{1}{\psi}} - \lambda^{-\frac{1}{\psi}}) \psi \bar{\eta}(\rho) x(0; \rho)^{1+\frac{1}{\psi}} \quad \forall \rho \in [0, \bar{\rho}]
\]

Using (46), we can rewrite this as follows:

\[
\frac{(\lambda - 1)(1 + \psi + \bar{\eta}(\rho))}{\psi \bar{\eta}(\rho)(e^{1+\frac{1}{\psi}} - \lambda^{-\frac{1}{\psi}})} > \frac{\mu(0)}{\chi(0)} \quad \forall \rho \in [0, \bar{\rho}]
\]
Then, we have:

\[
\bar{\eta}(\rho) \equiv \frac{1 + \psi + \dot{\eta}(\rho)}{\psi \dot{\eta}(\rho)} > \frac{1 - (\lambda e)^{-\frac{1}{\psi}} e^{-1}}{\lambda - 1} \quad \forall \rho \in [0, \bar{\rho}] \quad (B.23)
\]

Since \( \dot{\eta}'(\rho) > 0 \), we have \( \bar{\eta}'(\rho) < 0 \), and hence, (B.23) yields the following condition:

\[
\bar{\eta}(\bar{\rho}) = \frac{1 + \psi + \dot{\eta}(\bar{\rho})}{\psi \dot{\eta}(\bar{\rho})} > \frac{1 - (\lambda e)^{-\frac{1}{\psi}} e^{-1}}{\lambda - 1} \quad (B.24)
\]

Similarly, \( \Delta(1; \rho) < 0 \) \( \forall \rho \in [0, \bar{\rho}] \) if, and only if,

\[
\frac{(\lambda e)^{1 + \frac{1}{\psi} - \lambda}}{\lambda - 1} > \frac{1 + \psi + \dot{\eta}(0)}{\psi \dot{\eta}(0)} = \bar{\eta}(0) \quad (B.25)
\]

Then, (B.24) and (B.25) are conditions that guarantee \( \Delta(0; \rho) > 0 \) and \( \Delta(1; \rho) < 0 \) \( \forall \rho \in [0, \bar{\rho}] \) while assuming \( \partial \Delta(q, \rho) / \partial \rho < 0 \).

Finally, combining (B.23)–(B.25) yields the following sufficient conditions guaranteeing that \( \Delta(q; \rho) \) is monotone decreasing in \( q \in [0, 1] \), and \( \Delta(0; \rho) > 0 \) and \( \Delta(1; \rho) < 0 \) \( \forall \rho \in [0, \bar{\rho}] \) as follows:

**Condition 1** Given \((\alpha, \eta)\), suppose that \((\psi, \lambda, e, \bar{\rho}) \in \mathbb{R}^+ \times (1, \infty)^2 \times (0, 1)\) satisfies the following inequalities:

\[
e > \left[ \frac{\psi}{\psi - (\lambda - 1)} \right]^{\frac{1}{1 + \psi}} \lambda^{-\frac{1}{1 + \psi}},
\]

\[
\frac{1 + \psi + \dot{\eta}(\bar{\rho})}{\psi \dot{\eta}(\bar{\rho})} > \frac{1 - (\lambda e)^{-\frac{1}{\psi}} e^{-1}}{\lambda - 1},
\]

\[
\frac{(\lambda e)^{1 + \frac{1}{\psi} - \lambda}}{\lambda - 1} > \frac{1 + \psi + \dot{\eta}(0)}{\psi \dot{\eta}(0)}.
\]

Then, \( \partial \Delta(q; \rho) / \partial q < 0 \), \( \Delta(0; \rho) > 0 \) and \( \Delta(1; \rho) < 0 \).

**Derivation of \( \dot{X}(q, \rho) \) in Example 2:**

The number of innovators, \( \dot{X}(q, \rho) \), in Example 2 can be easily derived by replacing the skill threshold, \( z^S \), in equation (2.26) with the one given by (B.3). We already solved \( x(q, \rho) \), innovations per worker, which is given by (B.15). Since \( d\dot{\eta}(\rho) / d\rho > 0 \), we can notice that \( \partial x / \partial \rho < 0 \). Then, it is straightforward to show that \( \partial X(q, \rho) / \partial \rho < 0 \) \( \forall q \in [0, 1] \). Recall that we also assumed that \( \dot{X}(q, \rho) \) takes an
inverse U-shaped relationship with $q$, which is supported by the empirical evidence presented in Appendix B.3. Here, we provide sufficient conditions that guarantee this assumption.

**Conditions for an inverse U-shaped relationship between $\hat{X}$ and $q$:**
For convenience, $\hat{X}$ given in Example 2 as follows:

$$\hat{X} = 1 - \hat{\eta}^\frac{1}{\psi} \varphi \hat{x}$$

where

$$\varphi = \varphi(q) = q \lambda^\frac{1}{\psi} + (1 - q)e^\frac{1}{\psi}$$

$$\hat{x} = [x(q)]^\frac{1}{\psi}$$

and we drop the time subscript for simplicity. Then we have:

$$\frac{\partial \hat{X}}{\partial q} = -\hat{\eta}^\frac{1}{\psi} \varphi \hat{x} \left[ \varepsilon_\varphi(q) + \varepsilon_\hat{x}(q) \right]$$

(B.26)

where

$$\varepsilon_\varphi(q) = \frac{\lambda^\frac{1}{\psi} - e^\frac{1}{\psi}}{q \lambda^\frac{1}{\psi} + (1 - q)e^\frac{1}{\psi}}$$

(B.27)

$$\varepsilon_\hat{x}(q) = \frac{1}{1 + \psi} \left[ \frac{\lambda - 1}{q \lambda + (1 - q)} - \frac{\lambda^\frac{1}{\psi} - e^{1 + \frac{1}{\psi}}}{q \lambda^\frac{1}{\psi} + (1 - q)e^{1 + \frac{1}{\psi}}} \right]$$

(B.28)

Recall that $\varepsilon_y(q)$ is the elasticity of a variable $y$ in $q$. Notice also that $\partial \hat{X}(q; \rho)/\partial q$ is independent of the institutional quality, $\rho$. From (B.26), $\partial \hat{X}(q; \rho)/\partial q > 0$ if, and only if, $\varepsilon_\varphi(q) + \varepsilon_\hat{x}(q) < 0$.

We first derive conditions that guarantee $\varepsilon_\varphi(0) + \varepsilon_\hat{x}(0) < 0$ and $\varepsilon_\varphi(1) + \varepsilon_\hat{x}(1) > 0$. If so, $\hat{X}_q(0; \rho) > 0$ and $\hat{X}_q(1; \rho) < 0$, so that $\exists \hat{q} \in (0, 1)$ such that $\hat{X}_q(\hat{q}; \rho) = 0$ by the continuity of $\hat{X}_q(q; \rho)$ in $q$. Then, we provide a condition that guarantees the uniqueness of such $\hat{q}(\rho) \forall \rho \in [0, \bar{\rho}]$.

First note that $\varepsilon_\varphi(q) + \varepsilon_\hat{x}(q) < 0$ is equivalent with the following inequality:

$$f(\lambda) \equiv (1 + \psi - \lambda)\lambda^\frac{1}{\psi} > [(1 + \psi)e - 1]e^{-(1 + \frac{1}{\psi})} \equiv g(e)$$

(B.29)

Hence, to satisfy the inequality given by (B.29), $1 + \psi > \lambda$ should be satisfied since
Figure B.8: Graphical Illustration for $f(\lambda_0; \psi) > g(e; \psi)$.

$g(e) > 0 \ \forall e > 1$. Now note that:

$$
\lim_{x \to 1} f(x) = \lim_{x \to 1} g(x) = \psi \quad \text{(B.30)}
$$

Meanwhile, from (B.29), we have the following inequalities:

$$
f'(x) = -(x - 1)(1 + \frac{1}{\psi})^{-\frac{1}{\psi}} < 0 \ \forall x > 1 \ \text{and} \ \forall \psi > 0 \quad \text{(B.31)}
$$

$$
g'(x) = -(x - 1)(1 + \frac{1}{\psi})^{-\frac{2}{\psi}} < 0 \ \forall x > 1 \ \text{and} \ \forall \psi > 0 \quad \text{(B.32)}
$$

and $0 > g'(x) > f'(x)$. Then, we know that $g(x) > f(x) \ \forall x > 1 \ \text{and} \ \forall \psi > 0$. From (B.30)–(B.32), we can notice that $\exists! \bar{e} > \lambda$ such that $g(\bar{e}) = f(\lambda) \ \forall \lambda > 1 \ \text{and} \ \forall \psi > 0$; see Figure B.8 for an example. For the purpose of exposition, we define the function $\bar{e}(\lambda; \psi)$ as follows:

$$
g(\bar{e}(\lambda; \psi); \psi) = f(\lambda; \psi)
$$

That is:

$$
\bar{e} = \bar{e}(\lambda; \psi) = g^{-1}[f(\lambda; \psi)] \quad \text{(B.33)}
$$

Then, if $e > \bar{e}(\lambda; \psi) > \lambda$ for any given $(\lambda, \psi) \in (1, \infty) \times \mathbb{R}^+$, then $\varepsilon_\varphi(q) + \varepsilon_\hat{x}(q) < 0$, and thus, $\hat{X}_q(0; \rho) > 0$.

Second, note that $\varepsilon_\varphi(1) + \varepsilon_\hat{x}(1) > 0$ is equivalent with:

$$
l(\lambda) \equiv (1 + \psi - \lambda^{-1})\lambda^{-\frac{1}{\psi}} > [1 + \psi - e] e^{\frac{1}{\psi}} \equiv m(e) \quad \text{(B.34)}
$$
Then,

$$\lim_{x \to 1} l(x) = \lim_{x \to 1} m(x) = \psi \quad (B.35)$$

Similarly, from (B.34), we have:

$$l'(x) = -(x - 1)(1 + \frac{1}{\psi})x^{-2-\frac{1}{\psi}} < 0 \quad \forall x > 1 \text{ and } \forall \psi > 0 \quad (B.36)$$

$$m'(x) = -(x - 1)(1 + \frac{1}{\psi})x^{-1-\frac{1}{\psi}} < 0 \quad \forall x > 1 \text{ and } \forall \psi > 0 \quad (B.37)$$

Hence, $0 > l'(x) > m'(x)$. Then, from (B.35)–(37), we know that $l(x) > m(x) \forall x > 1$ and $\forall \psi > 0$. This implies that $l(\lambda) > m(e)$ if $e \geq \lambda \forall \psi > 0$. That is, it suffices to have $e \geq \lambda$ to guarantee that $\varepsilon_\varphi(1) + \varepsilon_\hat{\varepsilon}(1) > 0$, and thus, $\hat{X}_q(1; \rho) < 0 \forall \rho \in [0, \bar{\rho}]$. Since $\bar{e}(\lambda; \psi) > \lambda$, this condition is automatically satisfied once $e > \bar{\varepsilon}(\lambda; \psi)$.

Now, we know that $\hat{X}_q(0; \rho) > 0$ but $\hat{X}_q(1; \rho) < 0$ if $e > \bar{\varepsilon}(\lambda; \psi)$. Then, by the continuity of $\hat{X}_q$, $\exists q^* \in (0, 1)$ such that $\hat{X}_q(q^*; \rho) = 0$. For the uniqueness of such $q^*(\rho) \in (0, 1)$ for each $\rho \in [0, \bar{\rho}]$, it suffices to show that $\hat{X}'(q; \rho)$ is monotone decreasing in $q \in [0, 1]$, i.e., $\hat{X}(q; \rho)$ is strictly concave in $q$. To find a condition for this, note first that $\hat{X}_q(q^*; \rho) = 0$ implies $\varepsilon_\varphi(q^*) + \varepsilon_\hat{\varepsilon}(q^*) = 0$, and this is also equivalent with the following equation:

$$h(q^*) \equiv \frac{\mu(q^*) \chi(q^*)}{\varphi(q^*)} = \frac{\lambda e^{1+\frac{1}{\psi}} - \lambda^{-\frac{1}{\psi}}}{(1 + \psi)(e^{\frac{1}{\psi}} - \lambda^{\frac{1}{\psi}})} \quad (B.38)$$

Since the RHS of (B.38) is independent of $q$, it suffices to find conditions that make $h'(q)$ is monotone decreasing $\forall q \in [0, 1]$. Note that:

$$h'(q) = \frac{\mu(q)}{\varphi(q)} \hat{\varepsilon}_\varphi(q) q e^{\frac{1}{\psi}} (e - 1) + (\lambda - 1) [1 + \sigma(q)] \quad (B.39)$$

where

$$\hat{\varepsilon}_\varphi(q) \equiv -\varepsilon_\varphi(q)(1 - q) - 1 \quad (B.40)$$

$$\sigma(q) \equiv \frac{(1 - q) e^{\frac{1}{\psi}} (e - 1)}{\varphi(q)} \quad (B.41)$$
Note also that:

\[
\max_{q \in [0,1]} \varepsilon_\varphi(q) < -\varepsilon_\varphi(1) - 1 < 0 \quad (B.42)
\]

\[
\max_{q \in [0,1]} \mu(q) < \frac{\mu(1)}{\varphi(1)} \quad (B.43)
\]

\[
\max_{q \in [0,1]} \sigma(q) < \frac{e^\frac{1}{\psi}(e - 1)}{\varphi(1)} \quad (B.44)
\]

Then, using (B.39)–(B.44), we can prove that the following inequality holds:

\[
\max_{q \in [0,1]} h'(q) < \Psi(\psi) \quad (B.45)
\]

where

\[
\Psi(\psi; \lambda, e) \equiv \frac{\mu(1)}{\varphi(1)} \left[ -\varepsilon_\varphi(1) - 1 \right] e^\frac{1}{\psi}(e - 1) + (\lambda - 1) \left[ 1 + \frac{e^\frac{1}{\psi}(e - 1)}{\varphi(1)} \right] = (e - 1)\lambda^{1+\frac{1}{\psi}} \left[ (\lambda e)^{\frac{1}{\psi}} - 1 \right] e^\frac{1}{\psi} + (\lambda - 1) \left[ 1 + (\lambda e)^{\frac{1}{\psi}}(e - 1) \right] \quad (B.46)
\]

One can notice that \(\Psi(\psi; \lambda, e) > 0\) for \(\psi = 0\ \forall (\lambda, e) \in (1, \infty)^2\) from (B.46). Also, we know:

\[
\Psi'(\psi) < 0 \quad \forall \psi > 0, \quad (B.47)
\]

\[
\lim_{\psi \to \infty} \Psi(\psi; \lambda, e) = -\lambda(e - 1) + e(\lambda - 1) \quad (B.48)
\]

(B.47) and (B.48) imply that \(\exists! \hat{\psi} > 0\) such that \(\Psi^{-1}(0; \lambda, e) = \hat{\psi}\) if \(-\lambda(e - 1) + e(\lambda - 1) < 0\), i.e., \(\frac{e}{\lambda} < \frac{e - 1}{\lambda - 1}\). This condition is equivalent with the condition that \(e > \lambda\) given \(\lambda > 1\). This condition is automatically satisfied once the condition that \(e > \bar{e}(\lambda; \psi)\) is satisfied since \(\bar{e}(\lambda; \psi) > \lambda\). Hence, provided that \(\psi > \hat{\psi}\), the inequality given by (B.45) is satisfied, meaning \(h'(q) < 0 \ \forall q \in [0, 1]\). We finally have the following conditions for the unique existence of \(q^* \in (0, 1)\) that guarantees an inverse U-shaped \(\hat{X}(q; \rho)\) in \(q\) for any \(\rho \in [0, \bar{\rho}]\):

**Condition 2** Suppose that \((\psi, \lambda, e) \in \mathbb{R}_{++} \times (1, \infty)^2\) satisfies the following inequality:
ties:

\[ \psi > \hat{\psi} = \Psi^{-1}(0; \lambda, e), \]

\[ e > \bar{e}(\lambda; \psi). \]

where \( \bar{e}(\lambda; \psi) \) and \( \Psi(\psi) \) are given by (B.33) and (B.46), respectively. Then, \( \exists q^* \in (0, 1) \) such that \( \partial \hat{X}(q, \rho)/\partial q > 0 \ \forall q \in [0, q^*), \) while \( \partial \hat{X}(q, \rho)/\partial q < 0 \ \forall q \in (q^*, 1]. \)
B.3 Evidence on Individualism and the Fraction of Innovators

In the theoretical model, we assume an inverse U-shaped relationship between the fraction of innovators and the degree of individualism. Although it is theoretically appealing, considering the general equilibrium effect caused by a change in the relative price, we provide empirical evidence on this theoretical result.

To verify the inverse U-shaped relationship empirically, we need a reliable dataset that gives information on the number of innovators in a given country. For this purpose, we can use the Research and Development Statistics (RDS) data provided by the OECD. The data spans from 1981 to the current date for the OECD countries and other major economies including Argentina, China, Romania, Russia, Singapore, Taiwan and South Africa, which are 41 countries in total. The RDS data contains information on R&D activities such as the number of workers engaged in comprehensive R&D sectors range from natural sciences and engineering to social sciences and humanities, i.e., the number of researchers in a given nation. We will use the number of workers in R&D sectors to test if there exists an inverse U-shaped relationship between the fraction of researchers and individualism.

Prior to conducting the empirical analysis, we need to emphasize that the number of researchers in the R&D sectors of the RDS data is not the same as the total number of innovators, \( \hat{X} \), in the model. The set of innovators in our model is more comprehensive since innovation is defined more generally in our theory that it is any type of ideas that enhances the productivity in the economy, and hence, can be priced in the market. That is, the total number of innovators in our theory includes the set of researchers in the R&D sectors as its subset. Although the RDS data is restrictive in this sense, we will use it by assuming that the number of total innovators is monotonically increasing in the number of researchers in the R&D sectors in a country. That is, we regard the number of researchers in the R&D sectors as a proxy for the total number of innovators in the model.

For the test, we consider the following econometric framework:

\[
y_{it} = \alpha + X_{it}'\beta + \gamma IND_i + \delta IND_i^2 + \theta D_i + u_i + \varepsilon_{it} \tag{B.49}
\]

where \( i \) and \( t \) index a country and year, respectively. The sample period is 1981–2011. \( y_{it} \) is the fraction of researchers either to the total population or to the total number of workers employed in a given country. It is natural to think that the fraction
of researchers \( y_{it} \) depends on country-level variables, especially on those related to development stages. Hence, the controls, \( X_{it} \), include the institutional quality in terms of protection of property rights, the total population, the employment rate, GDP per capita, human capital, and investment and trade to output ratios. \( IND_i \) is the Hofstede individualism index normalized by one. Since an inverse U-shaped relationship is a downward quadratic form, \( \gamma \) needs to be positive, while \( \delta \) needs to be negative. We also use continent dummies \( D_i \) since R&D investments depend on knowledge diffusion in the form of adoptions and imitations of technologies within a region. Finally, \( u_i \) measures country specific random effects, which are time-invariant.

For the statistical assessment of the inverse U-relationship given by (B.49), Lind and Mehlum (2010) show that the following joint null hypothesis can be used:

\[
H_0 : \{ \gamma + 2\delta \min \{ IND_i \} \leq 0 \} \cup \{ \gamma + 2\delta \max \{ IND_i \} \geq 0 \}
\]

against the alternative as follows:

\[
H_1 : \{ \gamma + 2\delta \min \{ IND_i \} > 0 \} \cap \{ \gamma + 2\delta \max \{ IND_i \} < 0 \}
\]

In a word, the null asserts that the slope at the minimum of individualism scores is weakly negative, but weakly positive at the maximum. This means the function is monotone or U-shaped, given that it is either concave or convex. On the contrary, the alternative asserts that the function is inverse U-shaped. To test these hypotheses, we use the likelihood ratio approach developed by Sasabuchi, Lind and Mehlum (SLM), which is employed in Arcand et al. (2015).

Using equation (B.49), we conduct panel regressions as our baseline since it is reasonable to think that there are unobserved (time-invariant) heterogeneities across groups — countries here —, and the error terms may be autocorrelated within a group; recall that the RDS data covers a long period of time (30 years). We also provide estimation results from pooled OLS regressions with clustered standard errors to corroborate our empirical findings from the panel regressions.

Table B.6 summarizes the estimation results from both the panel and pooled OLS regressions. As shown in the table, the estimates of \( \gamma \) are consistently positive in all regressions, while the estimates of \( \delta \) are consistently negative, providing empirical evidence supporting the inverse U-shaped relationship in the theory. Also, the SLM test rejects the null hypothesis, and hence, further confirms our findings. That is, the estimation results show that both the fractions of researchers in the R&D sectors
have the inverse U-shaped relationship with the Hofstede individualism index. The table also reports the Fieller estimate for the extremum. The estimates suggest that the fraction of researchers in the R&D sectors is maximized somewhere between 50 and 80 of the Hofstede individualism index.

\footnote{We also found similar results from the panel and pooled OLS regressions with one-year lagged explanatory variables and time effects being further controlled although we do not report the results here; details are available upon request.}
Table B.6: Individualism and the Fraction of Researchers (Panel)

<table>
<thead>
<tr>
<th>Dependent variables</th>
<th>Panel Regressions</th>
<th>Pooled OLS Regressions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Researchers × 100</td>
<td>Researchers × 100</td>
</tr>
<tr>
<td></td>
<td>Population</td>
<td>Employed</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Individualism</td>
<td>1.680** (.674)</td>
<td>3.404** (1.546)</td>
</tr>
<tr>
<td>Individualism²</td>
<td>−1.457** (.612)</td>
<td>−2.860** (1.377)</td>
</tr>
<tr>
<td>Institutions</td>
<td>.031 (.128)</td>
<td>.054 (.294)</td>
</tr>
<tr>
<td>Population</td>
<td>.034 (.024)</td>
<td>.093* (.052)</td>
</tr>
<tr>
<td>Employment rate (%)</td>
<td>.009*** (.004)</td>
<td>.003 (.007)</td>
</tr>
<tr>
<td>GDP per capita</td>
<td>.143** (.069)</td>
<td>.265* (.153)</td>
</tr>
<tr>
<td>Human capital</td>
<td>.146 (.090)</td>
<td>.347* (.192)</td>
</tr>
<tr>
<td>Investment rate (%)</td>
<td>−.004 (.003)</td>
<td>−.008 (.005)</td>
</tr>
<tr>
<td>Trade rate (%)</td>
<td>.001* (.0005)</td>
<td>.002* (.001)</td>
</tr>
<tr>
<td>Obs.</td>
<td>734</td>
<td>734</td>
</tr>
<tr>
<td>Number of groups</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>R²</td>
<td>.530</td>
<td>.400</td>
</tr>
<tr>
<td>Slope at min {IN}</td>
<td>1.185*** (.472)</td>
<td>2.431** (1.090)</td>
</tr>
<tr>
<td>Slope at max {IN}</td>
<td>−.971** (.471)</td>
<td>−1.802** (1.037)</td>
</tr>
<tr>
<td>SLM test (t value)</td>
<td>2.14** (.471)</td>
<td>1.74** (.671)</td>
</tr>
<tr>
<td>90% Fieller interval</td>
<td>[50, 72]</td>
<td>[50, 84]</td>
</tr>
</tbody>
</table>

Note: The dependent variable is the % fraction of researchers in R&D sectors to the total population or to the total number of workers employed in a given country, which is taken from the RDS. “Individualism” is the Hofstede individualism index, which is normalized at one. “Individualism²” is square value of “Individualism.” “Institutions” refers to the protection of property rights against expropriation risk by ICRG, which is normalized by one. “Population” and “GDP per capita” are logged values of the total population and income per capita, respectively. All the data come from the PWT (Ver. 8.0) except the “Trade rate”, which is taken from the World Bank dataset. “SLM test (t value)” is the t statistics for testing the inverse U-shaped relationship suggested by Sasabuchi, Lind and Mehlum. 80% Fieller interval for †. “R²” is the R-square value for the overall effect for (1) and (2). Heteroskedasticity robust standard errors in parentheses, which are clustered by groups for (3) and (4). ***, ** and * and significant at 1%, 5% and 10%, respectively.
Figure B.9: Individualism and the Fraction of Researchers
Bibliography


