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Essays on Macroeconomics

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Essays on Macroeconomics

by
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To Sujin Yong for her love and support.

ABSTRACT OF THE DISSERTATION

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My dissertation investigates how technological progress shapes economy. Technological changes have heterogeneous effects on economic agents as they are often biased toward certain tasks or sectoral activities. The dissertation aims at understanding the sources of heterogeneity and their impacts on aggregate outcomes, focusing on economic growth and labor allocation.

The first chapter investigates a bi-directional relation between technology and occupational structure (job allocation). Jobs have polarized in the U.S. since at least the 1980s, but the growth of high-skill jobs has been stagnated since 2000s (skill demand reversal). I document that software innovation has increased compared to equipment innovation and relate this changes in the direction of innovation to the skill demand reversal, based on a novel empirical observation: The intensity of software- and equipment-use by occupation represents the cognitive- and routine-task intensity, respectively. I then propose a general equilibrium model that endogenously explains both employment share and software innovation trends. The productivity growth in the equipment-producing sector replaces the middle-skill occupations which use equipment more intensively. Thus the demand for equipment declines, resulting in more software innovation than equipment innovation. This, in turn, leads to a skill demand

reversal by enhancing the productivity of high-skill occupations. Quantitative analysis shows that the model explains approximately 70 to 80% of the rise in software and skill demand reversal in the data.

The second chapter, joint work with Tim Lee and Yongseok Shin, investigates the role of differential productivity growth across jobs (routinization) and industries to explain a slowdown in aggregate growth in the U.S. since the 2000s. In the model, complementarity across jobs and industries in production leads to aggregate productivity slowdowns, as the relative size of those jobs and industries with high productivity growth shrinks. We find that this effect was countervailed by the evolution of computer industry: Its productivity growth was extraordinarily high during the 1980s and 1990s and, at the same time, computer output became an increasingly more important input in production across all industries (computerization). It was only as the productivity growth in the computer industry slowed down in the 2000s that the negative effect of differential productivity growth across jobs became apparent for aggregate productivity.

In the third chapter, Dongya Koh, Raul Santaeulalia-Llopis, and I document a rise of intellectual property products (IPP) captured by up-to-date national accounts in 31 OECD countries. These countries gradually adopt the new system of national accounts (SNA2008) that capitalizes IPP—which was previously treated as an intermediate expense in the pre-SNA1993 accounting framework. We examine how the capitalization of IPP affects stylized growth facts and the big ratios (Kaldor, 1957; Jones, 2016). We find that the capitalization of IPP generates (a) a decline of the accounting labor share, (b) an increase in the capital-to-output ratio across time, and (c) an increase in the rate of return to capital across time. The key accounting assumption behind the IPP capitalization implemented by national accounts is that the share of IPP rents that are attributed to capital, χ, is equal to one. That is, national accounts assume that IPP rents are entirely owed to capital. We argue that this assumption is arbitrary and extreme. More reasonable assumptions about the split of IPP rents between capital and labor—for example, based on the cost structure of R&D—generate a secularly trendless labor share, a constant capital-to-output ratio, and a constant rate of return across time. We discuss the implications of these new measures of IPP capital—conditional on χ—for cross-country income per capita differences using standard development and growth accounting exercises.

Chapter 1

The Rise of Software and Skill Demand Reversal

1.1 Introduction

The employment shares of high- and low-skill occupations grew relative to that of middle-skill occupations in the United States since at least the 1980s. While many studies have focused on the long-run trend of this process of job polarization, less attention has been given to its shorter-run dynamics. When broken down decade-to-decade, the rise of high- and low-skill occupations has shown distinct patterns: The rise of high-skill occupations has stagnated, while that of low-skill occupations has accelerated since the late 1990s. We will refer to this phenomena as skill demand reversal following Beaudry et al. (2016) (figure 1.2).

This paper provides a technology-based explanation for skill demand reversal. We first document that skill demand reversal was accompanied higher growth in software innovation relative to other types of innovation (figure 1.4), and argue that this change in the direction of innovation was closely related to changes in the occupational structure of the U.S. economy. This is done by combining two datasets—the National Income and Product Account (NIPA) and O*NET Tools and Technology Database. The newly merged dataset shows that the average amount of investments in software and/or equipment by occupation are strongly linked to the tasks of each occupation. Namely, software is used intensively by cognitive (high-skill) occupations, while equipment is used intensively by routine (middle-skill) occupations.

We then provide a unified framework that can explain both skill demand reversal *and* the rise of software relative to equipment endogenously. The model has three novel features. First, the model features workers of heterogeneous skill sorting into heterogeneous tasks, and also different types of capital (software and equipment). Second, technological changes in the model are embodied into different types of capital, and innovators endogenously choose which type of technology (or capital) to improve. These two features enable us to simultaneously analyze the static and dynamic implications of interactions between technology and the labor market. Last but not least, technological changes (embodied in different types of capital) alter job allocations because all workers use both types of capital, but with different intensities depending on their occupations. This departs from the typical assumption that only particular kinds of occupations are affected by a specific type of technical change, and also implies that impact of one type of technological change on the labor market can change the subsequent direction of technological change.

In the model, the intensities at which each occupation use software and equipment can be measured directly from the newly merged dataset mentioned above. Equipment and software are modeled as a composite of infinitesimal varieties provided by innovators who are free to choose a type of capital to innovate, so the amount of innovation toward each type of capital can also be directly mapped into capital investment data in the National Accounts, facilitating quantification of the model.

After characterizing the equilibrium, we prove a series of comparative statics in response to one exogenous change: an increase in the productivity of the equipment-producing sector.[1] Increased productivity in equipment production lowers the price of equipment, which leads to job polarization if different occupations are complementary in production. Since middle-skill occupations use equipment most intensively, labor flows out from these jobs and into high- and low-skill jobs. But the decline in middle-skill employment also means that the demand for equipment declines, inducing innovators to shift their focus away from equipment and more toward enhancing software. In turn, the rise of software leads to skill demand reversal if jobs are complementary: Middle-skills jobs were already declining (job-polarization), the employment share of high-skill jobs decelerates since they use software most intensively, and consequently skill demand becomes concentrated in low-skill jobs.

We verify the empirical validity of the model's mechanism using the fact that the decline in the relative price of equipment to software varies across industries. The model predicts: i) A negative relationship between the speed of decline in the relative price of equipment to software and the growth of middle-skill employment relative to high-skill employment; and ii) A positive correlation between the speed of decline in the relative price of equipment to software and the relative growth of software innovation to R&D other than software. We confirm significant correlations in both cases.

Confident of the mechanism, we use the model directly to quantify its importance. Our quantitative analysis shows that the channel of directed technical change can account for more than two-thirds of the rise of software and skill demand reversal. The former is measured by the relative size of software investment to equipment investment, and the latter is measured by a gap between the actual series and the level implied by the linear trend of the 1980s.

[1] We also document a faster increase in the productivity of the equipment-producing sector in the data.

The results have two important implications. First, software and equipment capital measured in the National Accounts is a good proxy for the technological changes shaping the structure of the labor market. Since technological changes have significant impacts on many economic variables, careful investigation of the composition of capital investment can be fruitful in understanding economic phenomena other than job polarization as well.

Second, a technological change that directly affects a particular group of occupations could lead to other types of technological change that eventually affect other occupations. Hence, innovation policy targeting a specific group of products may have to consider this dynamic general equilibrium effect. This paper also shows that recent technical changes reduce cognitive intensive occupations as long as those occupations use software intensively. Moreover, while not explicitly analyzed here, changes in the demand for high-skill occupations will also change the expected returns to skill acquisition, consequently altering individuals' education decisions and labor supply.

Related Literature The relationship between polarization and increases in the productivity of middle-skilled occupations, which are intensive with respect to routine tasks, is well documented in the literature (e.g., Autor et al., 2006; Autor and Dorn, 2013; Goos et al., 2014, among others). Though fewer, there are also studies that have discussed the flattening of the demand for high-skilled workers around 2000, such as Beaudry et al. (2016) and Valletta (2016). This paper contributes to this literature by analyzing both polarization and skill demand reversal in a unified framework, and extends it by linking labor market phenomena to changes in the composition of capital investment.

Several papers analyze the consequences of task-specific technological change on the labor market with an assignment model (Costinot and Vogel, 2010; Lee and Shin, 2017; Michelacci and Pijoan-Mas, 2016; Stokey, 2016; Cheng, 2017, among others). We include a similar assignment feature, but characterize tasks by their different uses of two types of capital, and also introduce endogenous task-specific technological change generated from innovations on each type of capital. By doing so, we obtain a direct mapping of two distinctive task-specific technological changes to observed data.[2] Also, we explain why a particular type of technology may or may not change.

Recent studies by Bárány and Siegel (forthcoming) and Lee and Shin (2017) show that either task-specific technological change or sector-specific technological change can lead to both job polarization and structural change. Since a single type of technological change can result in both phenomena, it is not easy to conclude whether the source of technological change has been task- or sector-specific. Our paper implies that the technological change embodied in a particular type of capital could be a source of task-specific technological change that can generate both phenomena.

Acemoglu and Restrepo (2016) and Hémous and Olsen (2016) also analyze the interaction between technological change and the labor market with the directed technical change framework of Acemoglu (2002). While they provide new insights on how automated technology evolves and affects labor market outcomes, the interpretation of the technology with respect to the observable data is not straightforward. Our technological changes are directly measured from investment in software and equipment in the National Accounts, so the changes are easy to interpret. Our tasks also have a clear interpretation as they are mapped directly to different occupations in the data.

[2] Cheng (2017) also obtains the routine-biased technological change from the data by measuring different capital intensities across occupations. Different from ours, Cheng (2017) measures the capital intensities across occupations from industry level capital share and the variations in the composition of occupations across industries, and confirms that the middle-skill occupations are capital intensive. Summing the equipment and software, our dataset also shows that the total capital is intensively used in the middle jobs. We show, however, that the distinction between equipment and software is important as the software is not used intensively by the middle jobs.

A seminal paper by Krusell et al. (2000) links changes in the price of equipment capital to skill-biased technical change to analyze the effects of technological change on labor market outcomes. They emphasize that skill-capital complementarity (capital substitutes low-skill labor *more* than high-skill labor) is key to understanding how a rise in the productivity of capital leads to higher demand for high-skill workers. In contrast to Krusell et al. (2000), in our model, the substitutability between labor and capital is same across occupations. Instead, we assume that occupations vary in how intensively they use different types of capital, and that the occupations are complementary to one another.

The work by Krusell et al. (2000) and our paper do not contradict each other, as the worker classifications are essentially different.[3] They classify workers by education, and we classify workers by occupation. Low-educated workers may well be able to do what high-educated workers usually do (though less efficiently), whereas workers in certain occupations may not be able to do what workers of other occupations usually do. Indeed, recent papers such as Goos et al. (2014) and Lee and Shin (2017) highlight complementarity between tasks as a key to understanding task-level employment changes (i.e., polarization). In this regard, our paper complements Krusell et al. (2000) by linking capital to task-based employment.

Another important feature of this paper is distinguishing software capital from equipment capital. Software investment is becoming increasingly important, as evidenced by its rapid rise as a share of total investment. Aum et al. (2017) analyzes the role of computer capital (hardware and software) in shaping the dynamics of aggregate productivity. Koh et al. (2018) emphasizes the importance of software capital (more broadly, intellectual property products capital) in accounting for the declining labor share in the US. Whereas their analysis focuses on the relation between total labor and capital, we emphasize the separate roles of software and other types of investment in shaping the distribution of occupational demand. Though

[3]They also classify workers into two types, while we consider more.

not a primary focus of this paper, our model also generates a decline in the labor share caused by higher software investment, and we also show that there is a significant correlation between labor share declines and software intensity at the industry-level.

The rest of the paper is organized as follows. In section 1.2, we summarize the relevant empirical facts. In section 1.3, we present the model and characterize its equilibrium. In section 1.4, we conduct analytical comparative statics and in section 1.5, verify that the model's predictions hold empirically across industries. In section 1.6, we calibrate the model to quantify how important its mechanism is for accounting for the rise of software and skill demand reversal. Section 1.7 concludes.

1.2 Key Facts

We document several data observations in this section. First, equipment-producing industries have experienced much faster TFP growth than that of software-producing industries. Second, the pattern of polarization shows that the rise of high-skill occupations slowed with a greater increase in low-skill occupations since the late 1990s. Third, software development expenditures have increased relative to other R&D expenditures. Meanwhile, a share of software investment in total investment has also increased whereas that of equipment investment has decreased. Fourth, most importantly, we show that middle-skill occupations use equipment intensively, whereas high-skill occupations use software intensively. Moreover, the intensity of equipment and software across tasks is closely correlated with routine task intensity and cognitive task intensity. Again, our main hypothesis is that the first observation – together with the fourth observation – can generate both the second and the third observations.

1.2.1 Productivity of Equipment / Software Production

The input-output table published by BEA reports the industrial composition of equipment and intellectual property products (IPP) investment, where the IPP investment consists of software, R&D, and others. From the table, we can obtain the weights on detailed industries producing equipment and software investment goods. On the basis of these weights, we compute the total factor productivity (TFP) of equipment- and software-producing industries according to the Törnqvist index.

Using Industry Accounts from BEA, we first compute an industry i's TFP growth between time u and t as

$$\log(TFP_{i,t}/TFP_{i,u}) = \log(y_{i,t}/y_{i,u}) - \frac{\alpha_{i,t} + \alpha_{i,u}}{2} \log(k_{i,t}/k_{i,u}),$$

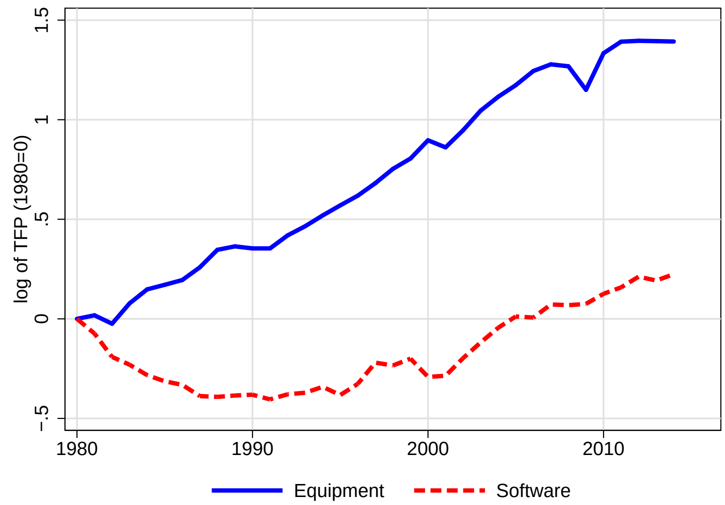
where y is the real value added per employment, k is the real non-residential capital divided by the number of employment, and α is one minus the labor share.

From the input-output table of each year (t), we obtain the share of each industry commodity in equipment investment ($\omega^e_{i,t}$) and software investment ($\omega^s_{i,t}$). Then, the TFPs of the equipment- and software-producing industries are computed by

$$\log(TFP_{e,t}/TFP_{e,t-1}) = \sum_i \frac{\omega^e_{i,t} + \omega^e_{i,t-1}}{2} \log(TFP_{i,t}/TFP_{i,t-1}),$$
$$\log(TFP_{s,t}/TFP_{s,t-1}) = \sum_i \frac{\omega^s_{i,t} + \omega^s_{i,t-1}}{2} \log(TFP_{i,t}/TFP_{i,t-1}).$$

The results are presented in figure 1.1, which shows that equipment-producing sector has experienced much faster increase in the productivity than software-producing sector.

Figure 1.1: TFP of equipment / software producing industries



1.2.2 The Pattern of Job Polarization

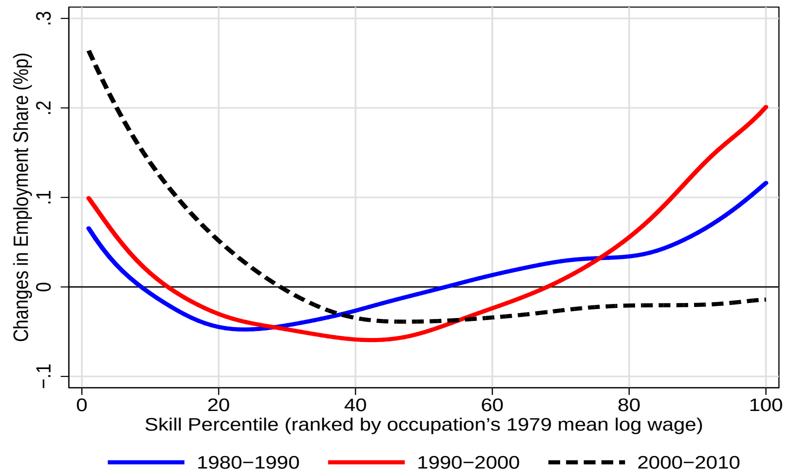
Figure 1.2 shows the changes in employment share across skill percentile by decade from 1980, computed from Census/ACS data. Each point in the skill percentile represents a group of occupations representing 1% of the labor supply in 1980, sorted by average log hourly wage in 1979.

The figure shows clear U-shaped changes in employment share from 1980 to 2010. By assessing the three lines separately, however, we see that the rise in high-skill occupations is strongest in the first two decades while that of low-skill occupations accelerates during 2000-2010. Moreover, the range of shrinking occupations shifts toward the right across decades.

Similar observations are also in the annual data from CPS when occupations are classified into three groups: cognitive (high-skill), routine (middle-skill), and manual (low-skill)[4]. We

[4]The classification of occupations is based on one-digit SOC. The cognitive occupations are management, professionals, and technicians. The routine occupations are machine operators, transportation, sales and office, mechanics, and miners and production.

Figure 1.2: Changes in employment structure in the US by decade



Note: Each point on the horizontal axis is a group of occupations composing 1% of total employment in 1980, sorted by 1979 average log wage.

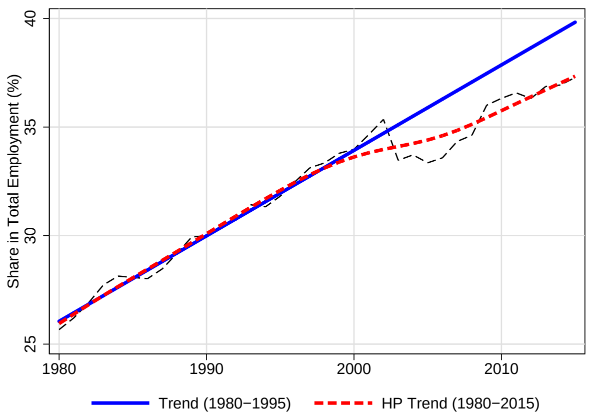
compare two different trends – a linear trend from 1980 to 1995 and an HP trend including all data points – of the employment share of each occupational group. Figure 1.3 confirms that there are breaks in the trends of employment shares of cognitive (high-skill) occupations and manual (low-skill) occupations in the late 1990s. Interestingly, the decline in routine (middle-skill) occupations follows similar trends before and after 1995.

1.2.3 Rising Software Innovation and Investment

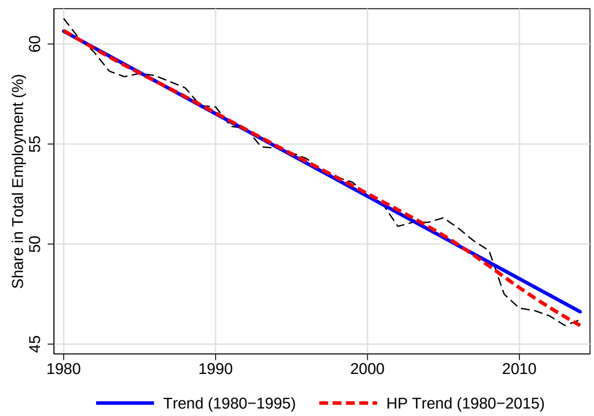
We now turn to the R&D composition in the US. NIPA does not report expenditures on software development directly but the series can be obtained from Crawford et al. (2014) or from differences between R&D in NIPA excluding software development and R&D recorded in the innovation satellite account which includes software development. Figure 1.4 shows the size of software development relative to R&D expenditures from manufacturing industries,

Figure 1.3: Employment share of cognitive, routine, and low-skill services occupations

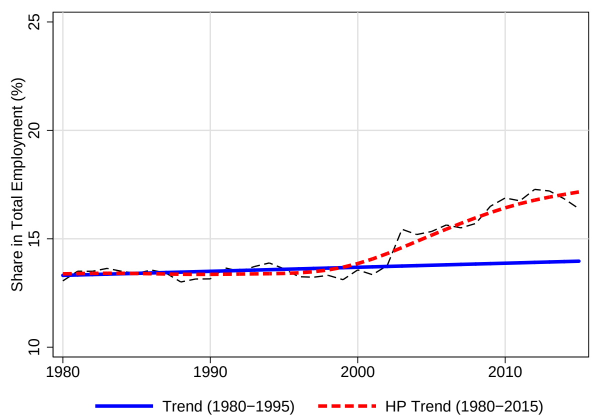
(a) Cognitive (high-skill)



(b) Routine (middle-skill)



(c) Manual (low-skill)



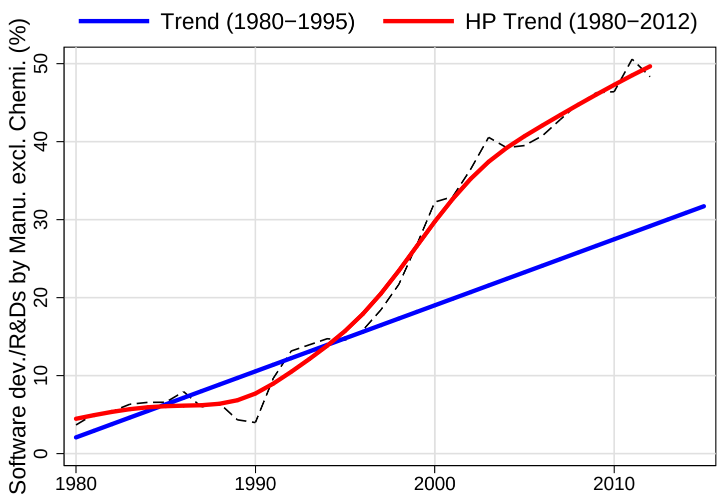
Note: 1) Cognitive occupations are management, professionals, and technicians. Routine occupations include office and sales, transportation, machine operators, mechanics, construction and production workers.
2) The blue line is the linear trend of 1990 to 1995, and the red (dash) line is the HP trend with smoothing parameter 100. All vertical axes represent 15%p of the range.

with and without chemical-related R&D's [5], across years. Both show an increasing pattern, especially during the late 1990s, suggesting that the changes in the pattern of polarization could be related to increasing software innovation.

[5] We view R&D expenditures funded by manufacturing industries except chemicals-related industries as the expenditures most related to R&Ds on equipment.

Another observation to note is that software investment, as well as software innovation, has also increased faster than other types of investment. From NIPA, we compute the share of software investment and equipment investment of total non-residential investment and plot the results in figure 1.5. Figure 1.5a shows an increasing trend of software investment while figure 1.5b shows a decreasing trend of equipment investment. Moreover, the downward trend of equipment share has accelerated since the mid-1990s.

Figure 1.4: Software innovation compared to equipment related R&D[1)]



Note: 1) R&D expenditures funded by manufacturing industries excluding chemical-related industries.
2) The blue line is the linear trend of 1990 to 1995, and the red (dash) line is the HP trend with smoothing parameter 100.

1.2.4 Capital Use by Occupation

We provide data evidence that documents strong connections between the use of different types of capital across occupations. Specifically, we construct capital use by occupation data by combining two data sources – NIPA and O*NET Tools and Technology Database.

The O*NET Tools and Technology database provides information about the types of tools and technology (software) used by each occupation. One caveat of this dataset is that it does not provide information about the value of each item. To address this shortcoming, we attempt to link capital items in O*NET Tools and Technology to NIPA data obtained from the Bureau of Economic Analysis (BEA).

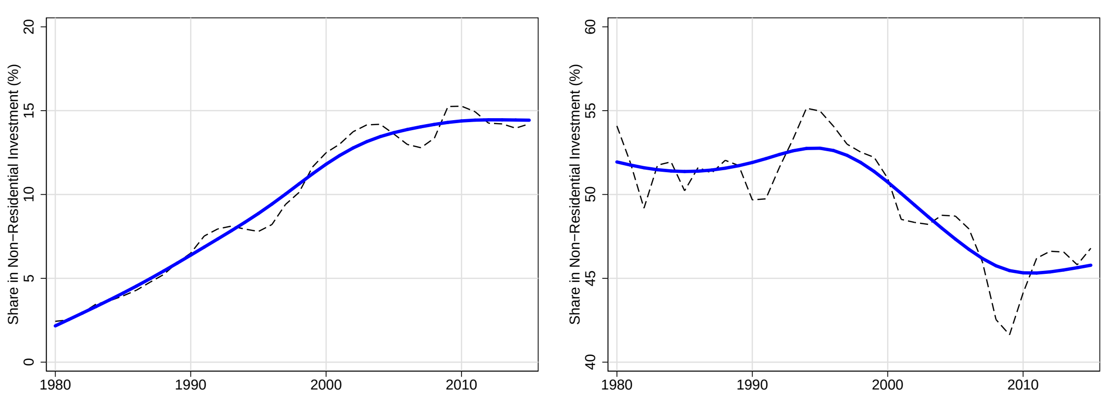
Specifically, we make a naive concordance between the UN Standard Product and Services Code (UNSPSC), a product classification system used in the O*NET database, and 25 categories of non-residential equipment in NIPA table 5.5 (details can be found in the appendix A.1). Then, we distribute the total amount of a particular type of equipment investment to each occupation by means of the number of tools included in the investment category according to the concordance.

For example, suppose that firms have invested USD 20 billion in metalworking machinery in the NIPA table. According to the constructed concordance, metalworking machinery

Figure 1.5: Investment share in private non-residential investment

(a) Software

(b) Equipment



Note: The blue line shows the HP trend with smoothing parameter 100. All vertical lines represent 20 % of the range.

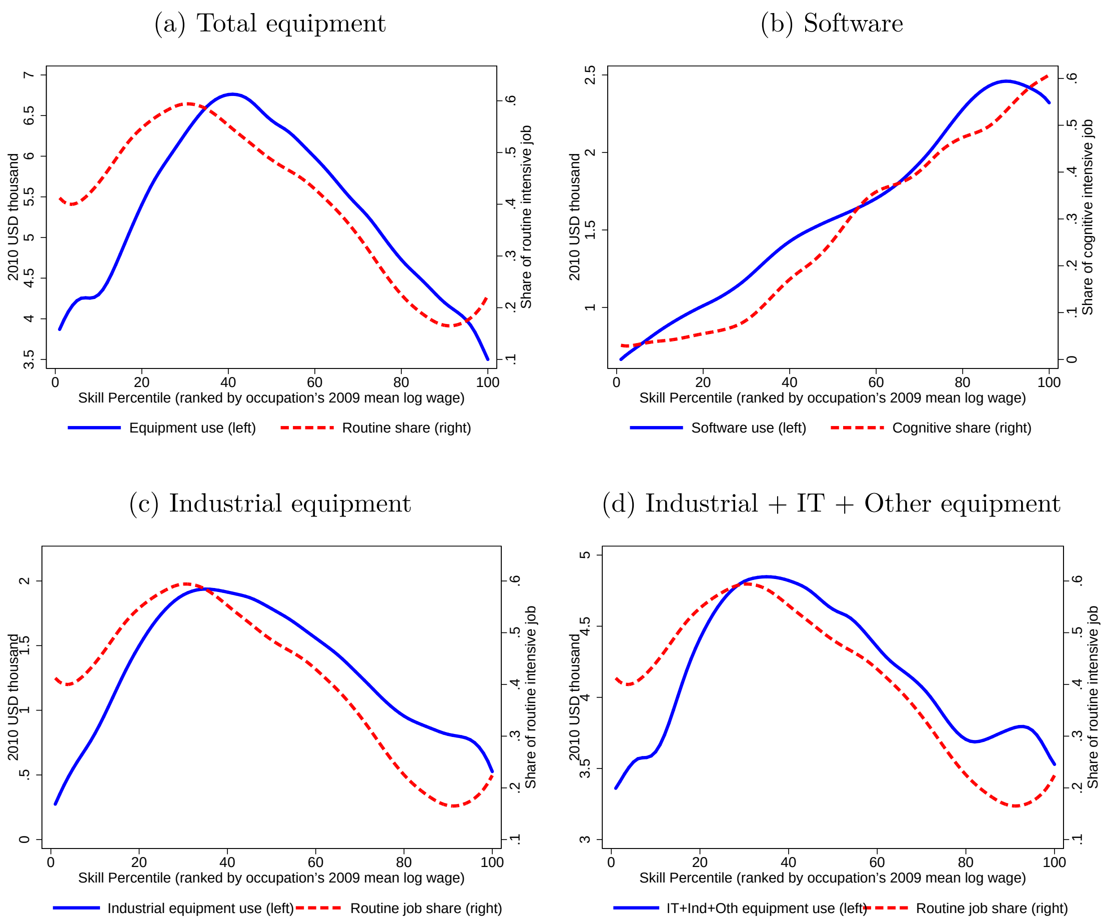
includes a total of 139 commodities in UNSPSC. Some occupations use none of the 139 commodities, and other occupations use various numbers of the commodities in the category. Because we know the number of employment by occupation, we can calculate the total number of metalworking machinery items used by all workers in a given year. Then, we can approximate the amount attributed to an individual occupation by distributing the total USD 20 billion investment according to the number of items used by the occupation. Subsequently, dividing by the number of employees provides an estimate of the per capita investment in metalworking machinery by occupation.

The per capita investment in equipment by occupational skill group is shown in figure 1.6a, where an occupational skill group is defined as a group representing 1% of total employment among all occupations ranked by mean hourly wages. We also plot the routine-intensive task share – a share of routine-intensive employment out of total employment within the skill group – in the same figure. Here, routine-intensive employment is defined as employment in occupations with the highest one-third routine task index of all occupations, where the routine task index is computed using the O*NET task database following Acemoglu and Autor (2011).

In figure 1.6b, we plot software investment per capital across the same wage percentile, and the cognitive-intensive task share defined similarly to the routine-intensive task share. Again, the cognitive task index is computed following Acemoglu and Autor (2011).

We can see from the figures that middle-skill workers use equipment more intensively, whereas high-skill workers use software more intensively. Moreover, the use of equipment closely follows the routine task share while the use of software is closely related to the cognitive task share. We further illustrate the use of equipment subitem by occupation in figures 1.6c (industrial equipment) and 1.6d (industrial and information processing equipment). Among

Figure 1.6: Use of equipment and software across skill percentile



Note: Detailed information on the data is provided in appendix A.1.

the equipment subitems, industrial equipment is most strongly correlated with routine task intensity.

1.3 Model

There is a continuum of individuals endowed with human capital $h \in [1, \bar{h}]$ drawn from a measure $\mathcal{M}(h)$. Specifically, we assume that:

Assumption 1 (distribution) *The measure of skill, $\mathcal{M} : [1, \bar{h}] \mapsto [0, 1]$ is a cumulative distribution function with a differentiable probability distribution function, $\mu : [1, \bar{h}] \mapsto \mathbb{R}+$.*

There is a continuum of tasks $\tau \in [0, \bar{\tau}]$, and final goods are produced by combining task output $T(\tau)$ according to:

$$Y = \left(\int_{\tau} \gamma(\tau)^{\frac{1}{\epsilon}} T(\tau)^{\frac{\epsilon-1}{\epsilon}} d\tau \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{1.1}$$

The task output is produced by integrating human capital specific task production $y(h, \tau)$ across all skill levels used for the production of task τ:

$$T(\tau) = \int_{h \in \mathcal{L}(\tau)} y(h, \tau) dh. \tag{1.2}$$

The human capital specific task production, $y(h, \tau)$, depends not only on worker human capital h but also on task τ that the worker is performing. Specifically, the functional form of $y(h, \tau)$ is given by

$$y(h, \tau) = \left[\left\{ \alpha_h(\tau) \left(b(h, \tau) l \right)^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_e(\tau) E^{\frac{\sigma_e - 1}{\sigma_e}} \right\}^{\frac{\sigma_e(\sigma_s - 1)}{(\sigma_e - 1)\sigma_s}} + \alpha_s(\tau) S^{\frac{\sigma_s - 1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s - 1}}, \tag{1.3}$$

where $l(h)$ represents the level of employment of workers with human capital h, S and E represent software and equipment, respectively.

The function $b(h, \tau)$ captures the productivity of a worker with human capital h when she performs task τ. We assume that $b(h, \tau)$ is strictly log supermodular.

Assumption 2 *The function $b(h, \tau) : [1, \bar{h}] \times [0, \bar{\tau}] \mapsto \mathbb{R}^+$ is differentiable and strictly log supermodular. That is,*

$$\log b(h', \tau') + \log b(h, \tau) > \log b(h, \tau') + \log b(h', \tau),$$

for all $h' > h$ and $\tau' > \tau$.

As shown in Costinot and Vogel (2010), assumption 2 helps to ensure positive assortative matching (PAM). In other words, the higher human capital h is, the higher τ task she will perform in equilibrium. Not only how each occupation utilizes a worker's human skill, but tasks are also different in to which intensities they use two types of capital. This second feature is essential to understanding the differential effects of capital-embodied technical change on various occupations. [6]

The software and equipment available for workers are given by

$$S = \left(\int_0^{N_s} s(k)^{\nu_s} dk \right)^{\frac{1}{\nu_s}} \text{ and } E = \left(\int_0^{N_e} e(k)^{\nu_e} dk \right)^{\frac{1}{\nu_e}}, \tag{1.4}$$

where each variety of capital ($s(k)$ and $e(k)$) is provided by a permanent patent owner under monopolistic competition.

[6] Many studies following Krusell et al. (2000) consider a production technology in which capital substitute a certain group of workers more or less than others. In our model, the substitutability between capital and human skill is same across occupations. We still have differential effects of capital-embodied technical change on various occupations for three reasons. First, each occupation utilizes human skill differently. Second, occupations rely on capital with various intensities. Third, any changes in the relative productivity in occupation-level alter relative demand for occupations through the final production, combining all tasks.

The production technology of software or equipment is

$$s(k) = A_s x, \quad e(k) = A_e x,$$

where x is the amount of final goods used to produce software or equipment. The production technology implies that the marginal costs of producing software and equipment are given by the inverse of productivity, $q_s := 1/A_s$ and $q_e := 1/A_e$.

New software and equipment are created from R&D expenditures Z_s and Z_e, and the laws of motion for total varieties follow

$$\dot{N}_s = Z_s/\eta_s \text{ and } \dot{N}_e = Z_e/\eta_e. \tag{1.5}$$

Finally, the representative household has CRRA preference given by

$$\int_s^\infty e^{-\rho t} \frac{C(t)^{1-\theta} - 1}{1-\theta} dt,$$

and the resource constraint in the economy is

$$C + q_e \int_0^{N_s} s(k) dk + q_s \int_0^{N_e} e(k) dk + Z_e + Z_s \leq Y. \tag{1.6}$$

1.3.1 Static Equilibrium

To characterize the static equilibrium, we take the total varieties of software and equipment, N_e and N_s, as given. We first define the equilibrium.

Definition 1 (Static equilibrium) *The static equilibrium consists of the price function $p(\tau)$, $w(h)$, $p_s(k)$, and $p_e(k)$, the quantity function $T(\tau)$, $l(h,\tau)$, $s(k,\tau)$, $e(k,\tau)$, and the quantity Y such that:*

1. *Given $p(\tau)$, final goods producer solves*

$$\max \; Y - \int_{\tau} p(\tau)T(\tau)d\tau,$$

given equation (1.1).

2. *For each task, the task output is produced to solve*

$$\max \; p(\tau)T(\tau) - \int_{h} w(h)l(h,\tau)dh - \int_{0}^{N_s} p_s(k)s(k,\tau)dk - \int_{0}^{N_e} p_e(k)e(k,\tau)dk,$$

given equation (1.2)*, $w(h)$, $p_s(k)$, and $p_e(k)$.*

3. *A capital provider solves*

$$\max \; \pi_s(k) = \int_{\tau} \left[p_s(k)s(k,\tau) - q_s s(k,\tau)\right] d\tau,$$
$$\max \; \pi_e(k) = \int_{\tau} \left[p_e(k)e(k,\tau) - q_e e(k,\tau)\right] d\tau,$$

given the marginal cost q_s and q_e.

4. *All workers choose the highest-paying occupation (task).*

5. *The labor market clears $\mu(h) = \int_{\tau} l(h,\tau)d\tau$.*

From the final goods production, the demand for task output $T(\tau)$ is given by

$$p(\tau) = \left(\frac{\gamma(\tau)Y}{T(\tau)}\right)^{\frac{1}{\epsilon}}, \tag{1.7}$$

and the price function $p(\tau)$ satisfies $\int_{\mathcal{T}} \gamma(\tau)p(\tau)^{1-\epsilon}d\tau = 1$.

Since we assume that the capital producer maximizes profit under monopolistic competition, we obtain the price of the software and equipment as

$$p_s(k) = \frac{1}{A_s\nu_s} \text{ and } p_e(k) = \frac{1}{A_e\nu_e}, \text{ for all } k.$$

By substituting this result into the first-order conditions from task output production, we can show that the wage function $w(h)$ satisfies

$$w(h) \geq \underbrace{\left[\left\{p(\tau)^{1-\sigma_s} - \left(\frac{\alpha_s(\tau)^{\frac{\sigma_s}{1-\sigma_s}}}{A_s N_s^{\varphi_s}\nu_s}\right)^{1-\sigma_s}\right\}^{\frac{1-\sigma_e}{1-\sigma_s}} - \left(\frac{\alpha_e(\tau)^{\frac{\sigma_e}{1-\sigma_e}}}{A_e N_e^{\varphi_e}\nu_e}\right)^{1-\sigma_e}\right]^{\frac{1}{1-\sigma_e}} \alpha_h(\tau)^{-\frac{\sigma_e}{1-\sigma_e}}}_{:=\omega(\tau)}$$
$$\times b(h,\tau), \tag{1.8}$$

with equality when $l(h,\tau) > 0$.

Equation (1.8) shows that the wage function $w(h)$ can be expressed as a product of terms depending only on τ ($\omega(\tau)$) and human capital task-specific productivity $b(h,\tau)$. The existence of PAM between h and τ follows.

Lemma 1 (Positive assortative matching) *Under assumptions 1 and 2, there exists a continuous and strictly increasing assignment function* $\hat{h} : [0,\bar{\tau}] \mapsto [1,\bar{h}]$ *such that* $\hat{h}(0) = 1$ *and* $\hat{h}(\bar{\tau}) = \bar{h}$.

The proof is same as the proof of Lemma 1 in Costinot and Vogel (2010) and is omitted.

The equilibrium assignment $\hat{h}$ is characterized by:

Lemma 2 (Equilibrium assignment function) *The equilibrium assignment function $\hat{h}(\tau)$, price function $p(\tau)$, and the wage rate $\omega(\tau)$ satisfy the following system of differential equations.*

$$\frac{d\log\omega(\tau)}{d\tau} = -\frac{\partial \log b(\hat{h}(\tau),\tau)}{\partial\tau}, \tag{1.9}$$

$$\hat{h}'(\tau) = \frac{\gamma(\tau)p(\tau)^{\sigma_s-\epsilon}\alpha_h(\tau)^{\sigma_s}\psi(\tau)^{\sigma_e-\sigma_s}Y}{\omega(\tau)^{\sigma_e}b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))}, \tag{1.10}$$

$$p(\tau) = \left[\psi(\tau)^{1-\sigma_s} + \alpha_s(\tau)^{\sigma_s}\left(\nu_s A_s N_s^{\varphi_s}\right)^{\sigma_s-1}\right]^{\frac{1}{1-\sigma_s}}, \tag{1.11}$$

with $\hat{h}(0)=1$, $\hat{h}(\bar{\tau})=\bar{h}$, *and* $\int\gamma(\tau)p(\tau)^{1-\epsilon}d\tau = 1$,
$\psi(\tau) := \left[\alpha_h(\tau)^{\sigma_e}\omega(\tau)^{1-\sigma_e} + \alpha_e(\tau)^{\sigma_e}\left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e-1}\right]^{\frac{1}{1-\sigma_e}}$, $\varphi_e := \frac{1-\nu_e}{\nu_e}$, *and* $\varphi_s := \frac{1-\nu_s}{\nu_s}$.

Proof *In appendix A.3.* ∎

After the assignment function $\hat{h}$ is obtained, all the equilibrium quantities and prices can be computed.

1.3.2 Dynamic Equilibrium

Now consider a dynamic equilibrium where technology evolves endogenously. The HJB equations for innovators are given by

$$r(t)V_s(k,t) - \dot{V}_s(k,t) = \pi_s(k,t), \tag{1.12}$$

$$r(t)V_e(k,t) - \dot{V}_e(k,t) = \pi_e(k,t), \tag{1.13}$$

with profit functions,

$$\pi_s(k) = \int_\tau [p_s(k)s(k,\tau) - q_s s(k,\tau)]d\tau = \frac{1-\nu_s}{\nu_s A_s}\int_\tau s(k,\tau)d\tau, \tag{1.14}$$

$$\pi_e(k) = \int_\tau [p_e(k)e(k,\tau) - q_e e(k,\tau)]d\tau = \frac{1-\nu_e}{\nu_e A_e}\int_\tau e(k,\tau)d\tau. \tag{1.15}$$

The free entry condition ensures that

$$V_e \leq \eta_e, \text{ with equality if } Z_e > 0, \quad \text{and} \quad V_s \leq \eta_s, \text{ with equality if } Z_s > 0.$$

If both R&D's are positive, we have $\eta_e V_e = \eta_s V_s$, and from equations (1.12) and (1.13),

$$r(t) = \pi_e(t)/\eta_e = \pi_s(t)/\eta_s. \tag{1.16}$$

Finally, from the household's problem, we have a standard Euler equation:

$$\frac{\dot{C}(t)}{C(t)} = \frac{r(t)-\rho}{\theta}, \tag{1.17}$$

and the transversality condition:

$$\lim_{t\to\infty}\left[e^{-\int_0^t r(s)ds}\left(N_e(t)V_e(t) + N_s(t)V_s(t)\right)\right] = 0.$$

Now, we have a characterization of the steady state equilibrium in the following lemma.

Lemma 3 (Steady state equilibrium) *There exist $\nu_e < 1$ and $\nu_s < 1$ sufficiently large that are compatible with the unique steady state equilibrium, i.e.,*

$$\pi_e/\eta_e = \pi_s/\eta_s = \rho, \tag{1.18}$$

and every variable remains constant. Moreover, when $\sigma_s = \sigma_e = 1$, $\max\left\{\frac{1-\nu_s}{\nu_s}\frac{\alpha_s(\tau)}{\alpha_h(\tau)} + \frac{1-\nu_e}{\nu_e}\frac{\alpha_e(\tau)}{\alpha_h(\tau)}\right\} < 1$ *ensures the existence of the steady state equilibrium.*

Proof *In appendix A.3.* ∎

Intuitively, high enough ν_e and ν_s ensure profits by providing additional variety not too large, which makes the rate of return on increasing variety strictly decreasing on the total varieties. As the rate of return is strictly decreasing in the size of varieties, we have a certain level of varieties that equates the rate of return and time preference (ρ), leading to the existence of the steady state.

We consider only a case with no growth steady state as no standard balanced growth path exists when the task production is a general CES function. Note that the source of growth (increasing variety) is a capital-augmented technological change in our model. It is well-known that no balanced growth path would exist for a capital-augmented technical change if the production function is not of the Cobb-Douglas form (e.g. Grossman et al., 2017). [7]

A detailed analysis of the transitional dynamics is not the primary focus of this paper. Instead, we focus on the differences between the static equilibrium (where N_s and N_e are

[7] In the Cobb-Douglas task production case ($\sigma_s = \sigma_e = 1$); however, a sustained growth can be obtained by assuming strictly positive population growth, as in Jones (1995). We still have every task growing at a different rate, so the most labor intensive task (the slowest-growing task) would dominate the economy in the limit under complementarity between tasks ($\epsilon < 1$), which is similar to the results in Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008).

fixed) and the steady state throughout the paper. We confirm numerically that the obtained steady state is saddle-path stable in the quantitative analysis. When the steady state is saddle-path stable, the transitional dynamics will be similar to that of the Neo-classical growth model, as a key is that the production function is strictly concave in the varieties.

Exogenous vs Endogenous Productivity Our model has both exogenous and endogenous productivity for equipment and software. Exogenous productivity is augmented in capital production, A_e or A_s, and captures how well one can produce equipment or software that has already been introduced into the economy. For example, when equipment production became faster as a result of using a faster computer in the production process, this shift would be captured in the increase in A_e. Instead, endogenous productivity, N_e or N_s, captures an introduction of new types of capital to the economy. For example, the development of the Uber application supporting drivers would be captured by an increase in N_s.

1.4 Comparative Statics

In this section, we restrict our attention to the case with $\sigma_e = \sigma_s = 1$, $\eta_e = \eta_s$ and $\nu_e = \nu_s$ to obtain analytical comparative statics. Specifically, we assume:

Assumption 3 *The elasticities of substitution between labor and equipment/software are one, i.e.,* $\sigma_s = \sigma_e = 1$*. The individual task production function is then*

$$y(h, \tau) = (b(h, \tau)l(h))^{\alpha_h(\tau)} E^{\alpha_e(\tau)} S^{\alpha_s(\tau)}.$$

Additionally, we put some structures on the intensity functions $\alpha_h(\tau)$, $\alpha_e(\tau)$, and $\alpha_s(\tau)$ to reflect the fact that high-skill workers use software intensively and middle-skill workers use equipment intensively, i.e.,

Assumption 4 (intensities) *The functions* $\alpha_h(\tau) : [0, \bar{\tau}] \mapsto (0, 1]$, $\alpha_s(\tau) : [0, \bar{\tau}] \mapsto (0, 1]$ *and* $\alpha_e(\tau) : [0, \bar{\tau}] \mapsto (0, 1]$ *satisfy the following.*

2.1 $\alpha_s(\tau)$ *is differentiable and increasing on* $[0, \bar{\tau}]$.

2.2 $\alpha_e(\tau)$ *is differentiable, increasing on* $[0, \tau_e]$ *and decreasing on* $[\tau_e, \bar{\tau}]$.

2.3 $\alpha_e(\tau_e) > \alpha_s(\tau_e)$, $\alpha_s(\bar{\tau}) > \alpha_e(\bar{\tau})$, *and* $\alpha_e(0) = \alpha_s(0)$.

Now, we show that an increase in the productivity of equipment production ($A_e \uparrow$) leads to polarization and the rise of software and skill demand reversal when the tasks are complementary. Specifically, we focus on three main predictions of the model: (1) the polarization induced by the rise of equipment-producing productivity in the static equilibrium, and (2) the subsequent rise of software innovation, and (3) the decreasing demand for high-skilled employment in the steady state.

Job Polarization

First, we show the impact of an increase in the equipment productivity (A_e) on equilibrium assignment function $\hat{h}(\tau)$ in the static equilibrium (i.e., when N_e and N_s are fixed). We consider $A_{1e} < A_{2e}$ and denote the equilibrium assignment functions corresponding to A_{1e} and A_{2e} as $\hat{h}_1$ and $\hat{h}_2$, respectively.

Proposition 1 (Polarization) *Consider $A_{1e} < A_{2e}$. Suppose $\epsilon < 1$ and assumptions 1 to 4. For sufficiently small $\alpha_h'(\tau)$, we have $\tau^* \in (0, \bar{\tau})$ such that $\hat{h}_1(\tau^*) = \hat{h}_2(\tau^*)$, $\hat{h}_1(\tau) < \hat{h}_2(\tau)$ for $\tau \in (0, \tau^*)$, and $\hat{h}_1(\tau) > \hat{h}_2(\tau)$ for $\tau \in (\tau^*, \bar{\tau})$.*

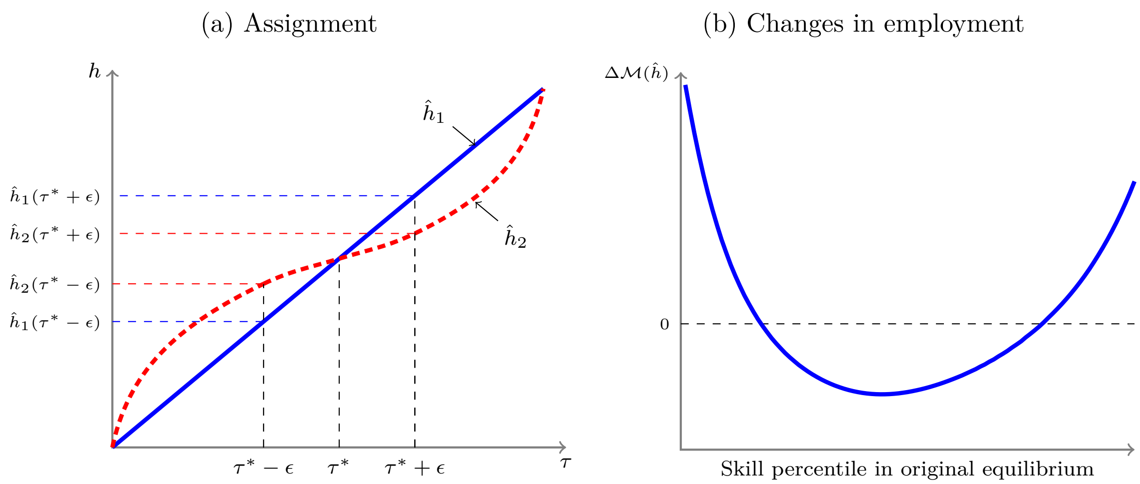
Proof *In appendix A.3.* ∎

Proposition 1 states that there will be a shrinking employment of task around τ^* where corresponding equipment intensity $\alpha_e(\tau^*)$ is relatively higher than $\alpha_e(0)$ and $\alpha_e(\bar{\tau})$. Figure 1.7 illustrates the change in the assignment function with A_{1e} (blue solid line) and $A_{2e} > A_{1e}$ (red dashed line). For a given task $\tau \in [\tau^* - \epsilon, \tau^* + \epsilon]$, we can see that employment decreases because we have higher $\hat{h}_2(\tau)$ on the left side of τ^* and lower $\hat{h}_2(\tau)$ on the right side of τ^*.

As shown in section 1.2, tasks with higher equipment intensities are consistent with routine-intensive tasks; hence, the proposition states that decreasing routine employment can be caused by a decrease in the price of equipment.

The condition of sufficiently small $\alpha_h'(\tau)$ is assumed because the impact of the change in equipment price on human capital depends on the relative size of α_e to α_h, not α_e alone. The

Figure 1.7: Equilibrium comparison: A_{1e} vs $A_{2e} > A_{1e}$



condition is sufficient but not necessary. As we show via numerical examples in appendix A.4, $\alpha_h'(\tau)$ need not be too small.

The intuition under the proposition is as follows. An increase in the equipment productivity (A_e) leads to a decrease in the price of equipment (q_e). This increases the productivity of all tasks but to a greater extent for tasks with higher equipment intensities. When the production is complementary in the tasks ($\epsilon < 1$), the rise of relative productivity causes factors to flow out to other tasks, which results in polarization.

The intuition is similar to other papers in the literature, for example, Lee and Shin (2017), Goos et al. (2014), and Cheng (2017): The technological change making the middle-skill tasks more productive reduces demand for the middle jobs when the tasks are relatively more complement than the relation between workers and technology. Our model enables a more direct mapping to the data due to the intensity function $\alpha_e(\tau)$ and $\alpha_s(\tau)$, meaning that it is not the technology affects a certain occupational group only but affects occupations to different extents. Cheng (2017) also introduces heterogeneous intensities across occupations. Ours differs from Cheng (2017) that we focus on the equipment among capital, which is a specific component used by middle-skill occupations intensively. We also highlight that the changes in the occupational structure itself can lead to another type of task-specific technological change, which we explore in the following propositions.

The Rise of Software

The profits from providing software and equipment are proportional to the demand, which in turn, is proportional to the task output times the factor intensity of the task. Hence, changes in the relative size of task production result in changes in the profit from providing each type of capital according to the corresponding factor intensity.

We know from proposition 1 that the employment share around τ^* (in the middle) shrinks. As long as $\alpha_h'(\tau)$ is small, the share of task production around τ^* also has to decrease. Meanwhile, $\alpha_e(\tau^*) > \alpha_s(\tau^*)$, together with $\alpha_e(\bar{\tau}) < \alpha_s(\bar{\tau})$ and $\alpha_e(0) = \alpha_s(0)$ (assumption 4), imply that a decrease in production share around τ^* actually decreases e more than s, and an increase in production share around $\bar{\tau}$ increases s more than e. Therefore, providing software becomes more profitable for innovators. Innovators then focus innovation toward software, resulting in higher N_s/N_e in the new steady state.

Although this prediction is valid for most reasonable quantifications, we have to impose tight restrictions on the structures of the intensities over the entire range of $\tau \in [0, \bar{\tau}]$ to prove the analytical proposition as we are comparing the ratio of two integrations over all τ ($\pi_e/\pi_s \propto \int \alpha_e(\tau)p(\tau)T(\tau)d\tau / \int \alpha_s(\tau)p(\tau)T(\tau)d\tau$). To express the analytical proposition in a simple way, we consider an approximation with three discrete tasks ($j = 0, 1, 2$ for low, middle, and high) in this subsection. Specifically, consider a production technology given by

$$Y = \left(\sum_j \gamma_j^{\frac{1}{\epsilon}} T_j^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \quad \text{for } j = 0, 1, 2, \tag{1.19}$$

with $T_j = (b(h,j)l(h))^{\alpha_{h,j}} E^{\alpha_{e,j}} S^{\alpha_{s,j}}$. The detailed derivation of the equilibrium conditions for this approximation can be found in appendix A.2.

With this approximation, assumptions 1 and 4 are replaced by the following.

Assumption 5 (distribution-II) *The measure $\mathcal{M} : [1, \bar{h}] \to [0, 1]$ has a differentiable p.d.f. $\mu(h)$, where $\mu(h)$ is sufficiently small everywhere.*

Assumption 6 (intensities-II) *The discrete intensities satisfy the following.*

6.1 $\frac{\alpha_{e,1}}{\alpha_{h,1}} > \frac{\alpha_{e,0}}{\alpha_{h,0}} \approx \frac{\alpha_{e,2}}{\alpha_{h,2}}$.

6.2 $\alpha_{e,0} \approx \alpha_{s,0}$*,* $\alpha_{e,1} > \alpha_{e,2}$*, and* $\alpha_{s,2} > \alpha_{s,1}$*.*

In assumption 5, we add the requirement for $\mu(h)$ to be sufficiently small to consider the discretization as an approximation of continuous tasks matched with a continuum of skills[8].

Assumption 6.1 states that task 1 is the most equipment intensive, relative to labor, compared to task 0 and task 2. Assumption 6.2 states that middle-skill tasks use equipment more than software, high-skill tasks use software more than equipment, and low-skill tasks use software and equipment similarly.

Again, consider an exogenous increase in the productivity of equipment, $A_{1e} < A_{2e}$. Denote the total varieties in the previous steady state as N_{s1} and N_{e1} and those in the new steady state as N_{s2} and N_{e2}. Then, we have:

Proposition 2 (Rise of software) *Consider* $A_{1e} < A_{2e}$ *with discretized tasks* (1.19)*, where equipment variety is at least as large as software variety in the original equilibrium (*$N_{e1} \geq N_{s1}$*). Suppose* $\epsilon < 1$*,* $\nu_e = \nu_s$*, assumptions 2, 5, and 6. In the new steady state, software variety increases more than equipment variety, i.e.,* $N_{s2}/N_{e2} > N_{s1}/N_{e1}$*.*

Proof *In appendix A.3.* ∎

[8]For discretized tasks, the assignment function $\hat{h}(\tau)$ becomes a sequence of threshold human capital $\hat{h}_j$. When the $\hat{h}_j$'s change, it not only affects demand for labor around the threshold level but also the total labor supply given to each task, $\int_{\hat{h}_{j-1}}^{\hat{h}_j} \mu(h)dh$. We ignore indirect effects resulting from changes in $\int_{\hat{h}_{j-1}}^{\hat{h}_j} \mu(h)dh$ by assuming $\mu(\hat{h}_j)$ and $\mu(\hat{h}_{j-1})$ are sufficiently small.

Skill Demand Reversal

We now show that an increase in N_s results in skill demand reversal (i.e., a decrease in the demand for high-skilled labor). We consider $N_{s2} > N_{s1}$, and denote $\hat{h}_1$ and $\hat{h}_2$ as the equilibrium assignment corresponding to N_{s1} and N_{s2}, respectively.

Proposition 3 (Skill demand reversal) *Consider $N_{s2} > N_{s1}$ and suppose $\epsilon < 1$ and assumptions 1 to 4. With sufficiently small $\alpha_h'(\tau)$, the matching function shifts upward everywhere, i.e., $\hat{h}_2(\tau) > \hat{h}_1(\tau)$ for all $\tau \in (0, \bar{\tau})$.*

Proof *In appendix A.3.* ∎

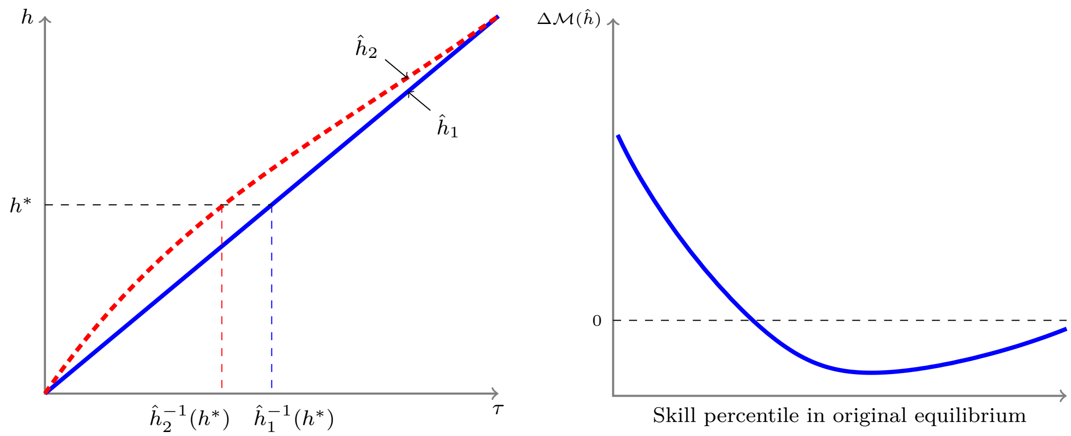
Note that an increase in variety increases the productivity of software-intensive tasks more than that of other tasks (equation (1.11)). Following the same intuition as in the case of polarization, this would lead to a reallocation of labor from high-skill tasks to lower-skilled tasks under complementarity ($\epsilon < 1$). The change in assignment function is depicted in figure 1.8, which shows that all workers downgrade their tasks.

This proposition, together with proposition 2, impies that skill demand reversal results from the increase in software innovation induced by the increase in A_e. Note that, when A_e increases, N_e should also increase; otherwise, $\pi_e/\eta_e > \rho$. The proposition 2, however, confirms that the variety of software would increase *more* than that of equipment. Accordingly, we have the following transition dynamics.

First, increases in A_e lead to immediate polarization according to proposition 1. Second, N_s jumps to equate π_e and π_s. N_s and N_e rise from then on until N_e and N_s reach the new steady state. Since increasing N_e will lead to polarization, the resulting steady state

Figure 1.8: Equilibrium comparison: N_{s1} and $N_{s2} > N_{s1}$

(a) Assignment (b) Changes in employment



equilibrium itself would be a mix of polarization *and* skill demand reversal. As N_s increases more quickly, the skill demand reversal effect becomes stronger.

Again, the technical assumption of sufficiently small $\alpha_h'(\tau)$ is required for proof, but it does not need be that small quantitatively. We provide numerical examples, including the case with a general CES task production, in appendix A.4 to illustrate the comparative statics.

1.5 Empirical Evidence

This section checks a validity of the model's predictions using industry data. Specifically, we test two predictions. First, the model predicts a negative relationship between changes in the relative price of software to equipment and changes in middle-skill employment relative to high-skill employment. Second, the model implies a positive correlation between changes in the relative price of software to equipment and changes in software innovation relative to other innovation. Note that, in our model, the prices of equipment and software are inversely related to productivity in the equipment- and software-producing sectors, respectively.

We measure the relative price of equipment to software by dividing nominal investment by real investment, provided by BEA. They are different by industry as each industry uses a different combination of subitems within the category of equipment or software. For the relative employment of middle-skill to high-skill occupations, we use the employment of routine occupations divided by the employment of cognitive occupations by industry, computed from Census data. Finally, the relative size of software innovation to other innovation is measured by own account software investment (in-house software investment by firms) divided by R&D excluding software. Details of the data construction are presented in appendix C.1.

Figure 1.9a shows the differences in the growth of middle-skill and high-skill employment against changes in software price relative to equipment price. Figure 1.9b shows the changes in software innovation net of R&D expenditures excluding software against changes in the relative price. The first has a negative relation, and the second has a positive relation, consistent with the model's predictions.

To determine whether these relations are statistically significant, we estimate the following regression:

$$\Delta \log y_{i,t} = a + c_t + \Delta \log(q_{s,i,t}/q_{e,i,t}) + \varepsilon_{i,t},$$

where $y_{i,t}$ is either the ratio of routine (middle-skill) employment to cognitive (high-skill) employment or the ratio of in-house software investment to R&D expenditures excluding software. The estimation results, which show significant relations between the two variables, are given in table 1.1.

Table 1.1: Estimation results

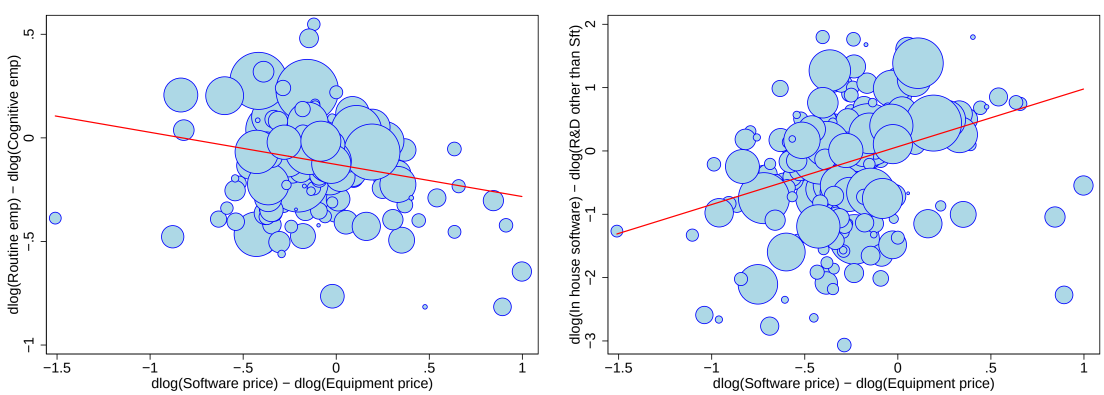
	Routine/Cognitive		Sft/R&D (excl. sft.)	
Sft price/ Eqp price	-0.220*** (0.000)	-0.152** (0.014)	+0.747** (0.016)	+0.717*** (0.001)
Fixed Effects	Yes	No	Yes	No
R^2	0.172	0.054	0.117	0.064

p-values in parentheses.

Figure 1.9: Changes in the relative price and employment / innovation

(a) Relative employment vs Relative price

(b) Relative innovation vs Relative price



1.6 Quantitative Analysis

In this section, we use the discretized model (appendix A.2) to map the tasks to ten occupational groups consistent with one-digit SOC code (as in table 1.3). Specifically, the following production technology is used for the quantitative analysis.

$$Y = \left(\sum_j \gamma_j^{\frac{1}{\epsilon}} T_j^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \text{ and} \tag{1.20}$$

$$T_j = M_j \left[\left(\alpha_{h,j} \left(\int_{\hat{h}_{j-1}}^{\hat{h}_j} b(h,j)\mu(h)dh \right)^{\frac{\sigma_e-1}{\sigma_e}} + \alpha_{e,j} E_j^{\frac{\sigma_e-1}{\sigma_e}} \right)^{\frac{\sigma_e(\sigma_s-1)}{(\sigma_e-1)\sigma_s}} + \alpha_{s,j} S_j^{\frac{\sigma_s-1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s-1}}.$$

Sources of Exogenous Variation Note that we have two types of exogenous productivity (A_i's and M_j's), as well as endogenous productivity (N_i's). The changes in exogenous productivities (A_i's and M_j's) are sources of exogenous variation in this section. What do these different productivities represent?

First, A_e and A_s capture how well a given technology produces already-introduced equipment or software. In other words, increases in A_i capture improvements in the production process, not the varieties of tools that occupations can utilize. To map A_i's as data, we use the TFP of equipment- or software-producing industries.

Second, M_j's are additional task-specific productivities introduced to match changes in the employment share of routine occupations in the model with data exactly. Recall that the changes in A_e or A_s also act as task-specific productivity in our model. A natural question is how much of the changes in the employment share between occupations can be captured only through A_e and A_s. We answer this question in one of the exercises in this section.

Third, N_e and N_s are endogenous productivities that capture varieties of software or equipment that workers can use to do their tasks. We view changes in N_e or N_s roughly correspond to R&D expenditures on each kind of capital in the data.

One may wonder how we can distinguish the productivity embodied in capital (A_i's) from dis-embodied technical change (N_i's). Regarding identification, increases in A_e and A_s result in the decline in the price of equipment and software, whereas changes in N_e and N_s do not alter the price of capital. Indeed, the price of equipment decreases more quickly than that of software, and the TFP of equipment-producing industries increases faster than that of software. To the contrary, software development expenditures rise more quickly than other types of R&D. These observations are consistent with our distinction between A_i's and N_i's.

Scenarios Given the changes in exogenous productivities, we perform two main exercises. The first is to investigate the extent to which endogenous innovation of software explains the *changes in the pattern* of polarization. For this exercise, we match the changes in the employment share of middle-skill occupations with A_i's and M_j's and look at the employment dynamics of high- and low-skill occupations generated from the model with innovation and without innovation.

The next exercise aims to determine the extent to which changes in the productivity of the equipment- and software-producing sectors only account for the shifts in the employment share between occupations. To address this question, we repeat the simulation with all the other parameters fixed, assuming constant M_j for all middle-skill occupations.

Finally, as a sensitivity analysis, we test how the simulation results change by varying the elasticity of substitution between tasks, mark-ups, and alternative measures for the productivity of the equipment- and software-producing sectors. Importantly, we confirm that observed changes in the price of equipment and software are consistent with the changes in the TFPs.

Following the literature, we label high-, middle-, and low-skill occupations as cognitive, routine, and manual occupations, respectively. Cognitive occupations include management, professionals, and technicians. Routine occupations are administrative, machine operators, transportation, sales, mechanics, and production workers. Finally, manual occupations are low-skilled services occupations. The reason we use ten occupational groups rather than three occupational groups is that we use the changes in the payroll share of each occupational group to calibrate the elasticity of substitution between tasks, which is the most crucial parameter driving the mechanism.

1.6.1 Calibration

We calibrate most of the parameters according to the 1980 data assuming a steady state. For the functional forms, we set the productivity function $b(h, j)$ as

$$b(h, j) = \begin{cases} \bar{h} & \text{if } j = 0 \\ h - \chi_j & \text{if } j \geq 1 \end{cases}$$

and the skill distribution $\mathcal{M}(h)$ as

$$\mathcal{M}(h) = 1 - h^{-a}.$$

The weight parameters in the final production (γ_j's) are taken from the employment share by occupation in 1980. The χ_j's and a are determined to match income share across occupational groups in 1980. Between-factor intensities by task ($\alpha_h,\ \alpha_e,\ \alpha_s$) are matched to equipment and software investment by occupational group[9] and labor share in 1980. For the benchmark analysis, we map the equipment in the model to industrial equipment in the data as it has the closest relation with the routineness of occupations (figure 1.5).

There are two categories of parameters that are difficult to identify from only 1980 data: (1) the elasticity of substitution (ϵ, σ_e and σ_s), and (2) markup-related parameters (ν_e and ν_s). We use various methods to identify these parameters.

For the elasticity of substitution between tasks, we set ϵ to minimize the root-mean-squared error of the changes in payroll share between 1980 and 2010 by occupation. That is, we set ϵ to minimize $\left[\sum_{\tau=1}^{J}\left[(w^m_{\tau,2010}-w^m_{\tau,1980})-(w^d_{\tau,2010}-w^d_{\tau,1980})\right]^2/J\right]^{\frac{1}{2}}$, where ω_τ is a payroll share of occupational group τ. Intuitively, occupations are complementary when changes in quantity share (employment share) and changes in the relative price (relative wage) move in the same direction. Figure 1.10 shows that this is the case as relative wages of cognitive and manual occupations to routine occupations both have increased while the employment share of routine occupations has decreased (figure 1.3). The resulting parameter value for the elasticity of substitution between tasks is 0.301, which confirms the complementarity between tasks. We also perform a robustness check by varying the value of ϵ in subsection 1.6.4.

For the elasticity of substitution between factors in task production (σ_e and σ_s), we match linear trends of aggregate labor share and labor share only with equipment capital. To illustrate the identification process, note that factor share in a given task τ can be derived as follows:

[9]We assume that the number of commodities used by each occupation is the same and attribute the capital investment in 1980 to each occupation to get occupational use of equipment and software in 1980.

$$LS_{-s} = \frac{wL}{wL + p_e\tilde{E}} = \frac{1}{1 + \left(\frac{\alpha_e}{\alpha_h}\right)^{\sigma_e} \left(\nu_e A_e N_e^{\varphi_e}\omega\right)^{\sigma_e - 1}}, \tag{1.21}$$

$$\begin{aligned} LS &= \frac{wL}{wL + p_e\tilde{E} + p_s\tilde{S}} \\ &= \frac{1}{1 + \left(\frac{\alpha_e}{\alpha_h}\right)^{\sigma_e} \left(\nu_e A_e N_e^{\varphi_e}\omega\right)^{\sigma_e - 1} + \frac{\alpha_s^{\sigma_s}\left(\nu_s A_s N_s^{\varphi_s}\right)^{\sigma_s - 1}}{\omega^{1-\sigma_e}\alpha_h^{\sigma_e}\left(\alpha_h^{\sigma_e}\omega^{1-\sigma_e} + \alpha_e^{\sigma_e}\left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e - 1}\right)^{\frac{\sigma_e - \sigma_s}{1-\sigma_e}}}}, \end{aligned} \tag{1.22}$$

where LS_{-s} is the labor share without software and LS is the standard labor share. From equation (1.21), it is straightforward to see that the labor share without software does not directly depend on the elasticity of substitution between labor and software, σ_s.

The fact that the aggregate labor share and labor share with equipment capital only show different trends from 1980 to 2010 makes this strategy even more useful (figure 1.14a). The labor share with equipment capital only has an increasing trend, whereas the aggregate labor

Figure 1.10: Log of relative wage

(a) Cognitive to Routine (b) Manual to Routine

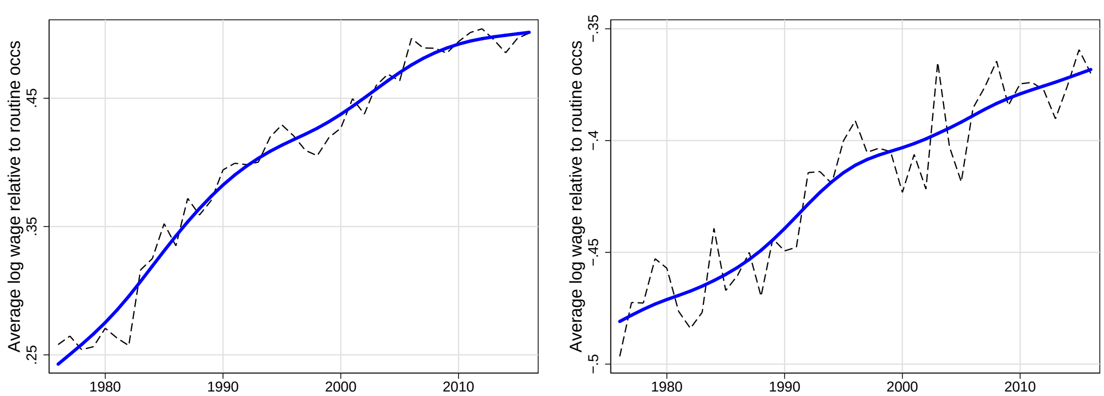


Table 1.2: Estimation results: markup

	Equipment[1)]	Software[2)]
b	.228	.473
	(.000)	(.113)
N	333	37

Note: 1) Industries 331, 332, 333, 334, 335, 3361MV, 3364TO, 337, and 339.
2) Industry 511. 3) p-values in parentheses.

share has a declining trend. [10] It is easy to predict $\sigma_e < 1$ and $\sigma_s > 1$ on the basis in the trends of total labor share and increasing productivity (both exogenous and endogenous) of capital.

We estimate the markup-related parameters ν_e and ν_s separately using the Industry Account and Fixed Asset Table from BEA, following Domowitz et al. (1988). Specifically, we estimate

$$\Delta \log q_{it} - \alpha_{Lit} \Delta \log l_{it} - \alpha_{mit} \Delta \log m_{it} = c_i + b \Delta \log q_{it} + \varepsilon_{it},$$

where q is gross output/capital, l is employment/capital, m is intermediate input/capital, and α_{Lit} and α_{Mit} are the labor and intermediate shares, respectively. We estimate this relation for the equipment-producing industry (industry 3 in the BEA industry codes) and software-producing industry (industry 511). To control for endogeneity, GDP growth is used as an instrumental variable. Once estimated, ν_e and ν_s can be obtained by calculating $1 - b$. The estimation results are presented in table 1.2.

[10] We compute the labor share with equipment capital only following Koh et al. (2018). To be specific, a standard asset pricing formula gives $R_i = (1+r)q_i - q_i'(1-\delta_i)$, where R_i is the gross return on capital type i, q_i is the relative price of capital type i, δ_i is a depreciation rate of capital type i, and r is the net rate of return. The no arbitrage condition implies that the net rate of return, r, is common across i. Using the fact that one minus labor share is equal to $\sum_i R_i K_i / Y$ under the CRS production technology, we can impute the gross rate of return on equipment, R_e. The labor share with equipment capital only then can be computed by $CE/(CE + R_e K_e)$, where CE is the compensation of employees and K_e is the equipment capital stock.

Table 1.3: Parameters by occupation

	α_e	α_s	α_h	γ	χ
Low-skilled services	0.190	0.009	0.801	0.004	
Administrative	0.060	0.186	0.754	0.711	0.000
Machine operators	0.644	0.017	0.339	0.077	0.002
Transportation	0.551	0.016	0.433	0.037	0.027
Sales	0.084	0.019	0.898	0.004	0.029
Technicians	0.265	0.020	0.714	0.002	0.071
Mechanics	0.696	0.020	0.285	0.135	0.071
Production	0.528	0.022	0.450	0.022	0.096
Professionals	0.133	0.012	0.854	0.005	0.097
Management	0.019	0.013	0.969	0.004	0.097
Target	Equipment, software, and labor share			Employment	Income share

Table 1.4: Remaining parameters

	Value	Obtained from
σ_s	1.425	Labor share with and without software in 2010
σ_e	0.974	
ν_e	0.772	Estimation (table 1.2)
ν_s	0.527	
ϵ	0.301	Changes in average wage by occupation

Table 1.3 and 1.4 summarizes all the calibration results. Detailed calibration procedure is in appendix A.6.

1.6.2 Simulation Results: With Changes in A_i's and M_j's

We assume the economy was in a steady state in 1980 and compute a new steady state corresponding to the exogenous changes (A_e, A_s, M_j's). We then assess how well the model explains the shifts in the trends of high-skilled and low-skilled employment with and without endogenous software innovation.

The Pattern of Occupational Employment Figure 1.11a displays the annualized changes in employment during the first two decades (blue bar) and last decade (light blue bar) by occupational group. Figures 1.11b and 1.11c show the same series generated from the model with endogenous innovation (varying N_e and N_s) and without innovation (no changes in N_e and N_s), respectively.

The blue bar (changes during 1980-2000) is higher than the light blue bar (changes during 2000-2010) for cognitive occupations and lower for manual occupations, as highlighted in section 1.2. As shown in Figures 1.11b and 1.11c, these changes in the pattern appear only in the simulation with endogenous innovation, i.e., with increases in N_s/N_e. The increase in cognitive occupation during the last decade in the data is 0.31%p lower than the average of the first two decades, whereas in the model, it is 0.26%p lower with endogenous innovation and only 0.05%p lower without endogenous innovation. For manual occupations, the change in the increases between 2000-2010 and 1980-2000 is +0.26%p in the data and +0.22%p in the full model. By contrast, the model without innovation shows a change of +0.01%p only.

Figure 1.12 shows a decadal pattern from 1980 to 2010. The deviation from the initial trend in cognitive occupation in the model captures 75% of the actual deviation in the data (figure 1.12a and 1.12b), where the deviation from the initial trend in manual employment in the model is 70% of that in the data (figure 1.12c and 1.13a). The model captures not only the magnitude but also the timing of changes in the trends, as it produces much larger changes during 2000-2010 than during the first two decades. Without endogenous innovation, the simulation generates almost no variation in the trends of high- and low-skill employment.

The Rise of Software The ratio of software investment to industrial equipment investment increases from 0.16 to 1.7 in the data, a more than tenfold increase. Since we match the initial level of relative investment 0.16 exactly by calibration, we compare the level of the ratio in 2010 to determine how well the model explains the rise of software. The full model with innovation explains 63% of the rise of software investment relative to that of equipment (figure 1.13a). If we remove the endogenous innovation channel (i.e., no changes in N_s and N_e), the model generates only 19% of the change in the software to equipment ratio (green dashed line).

In figure 1.13b, we plot the ratio of software variety to equipment variety (N_s/N_e). There is no clear counterpart for the varieties in the data as we do not have data of R&D on equipment. As a crude measure, we compare the varieties N_e/N_s to the cumulated software development and the cumulated R&D funded by manufacturing industries, excluding chemicals. Both show an increasing pattern, and the ratio between varieties in the model increases faster in the last decade.

The Decline of Labor Share Although the labor share dynamics are not a goal of this exercise, they merit further discussion. Note that we use labor share trend as a target variable to calibrate the elasticity of substitution (σ_e and σ_s); therefore, it is not surprising that the labor share in the model exactly matches the labor share trend in the data. What is new is

Figure 1.11: Simulation results – changes in employment shares

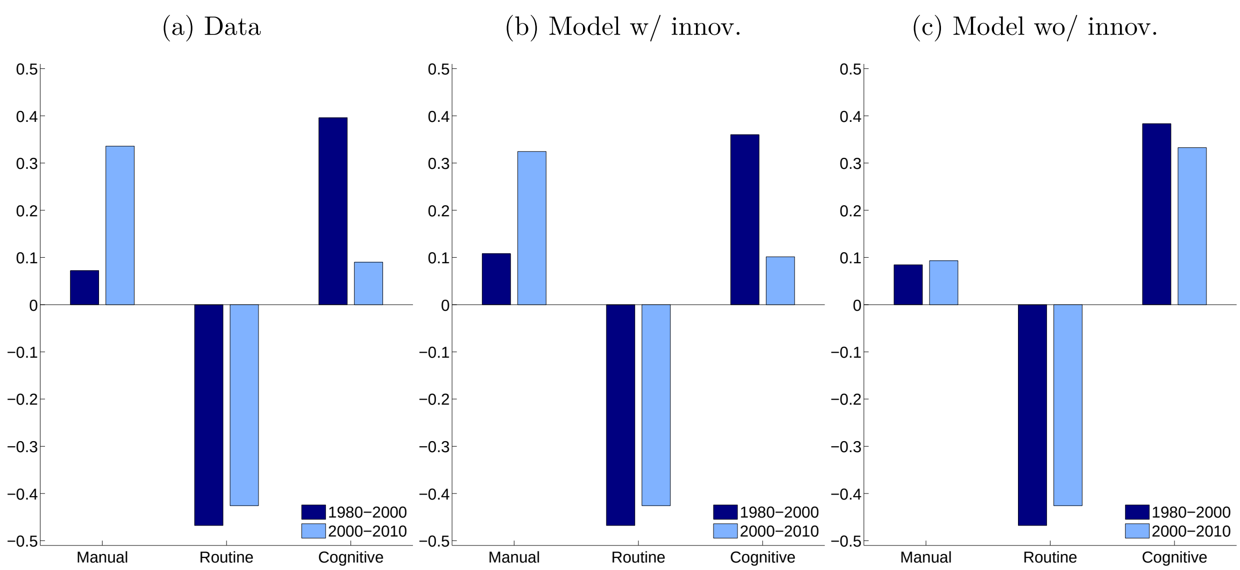
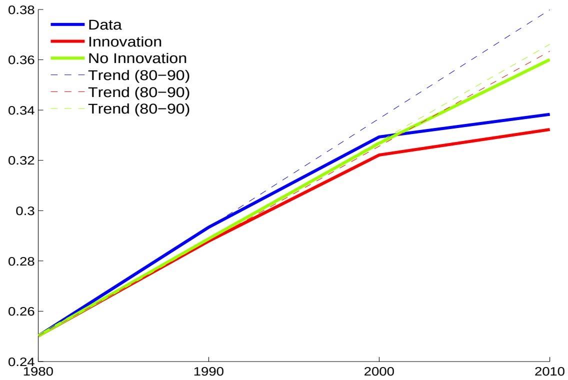
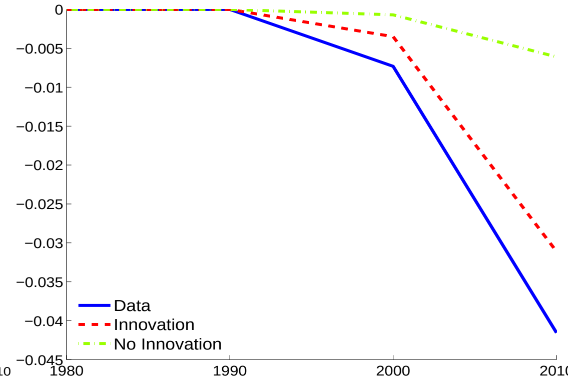


Figure 1.12: Simulation results – employment shares by decade

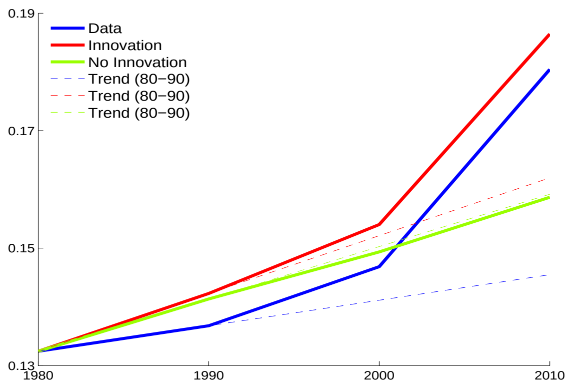
(a) Cognitive employment share



(b) Gaps from trend



(c) Manual employment share



(d) Gaps from trend

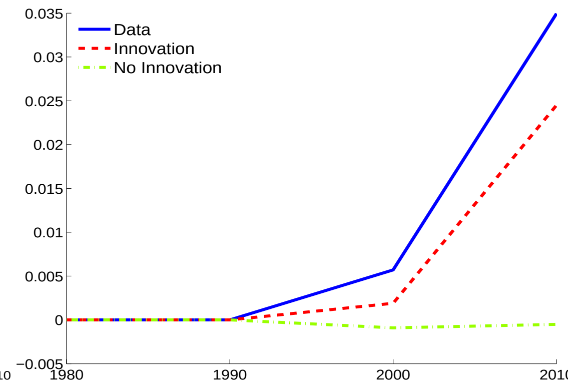
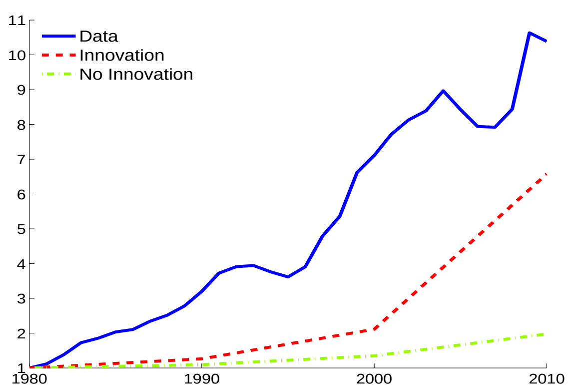
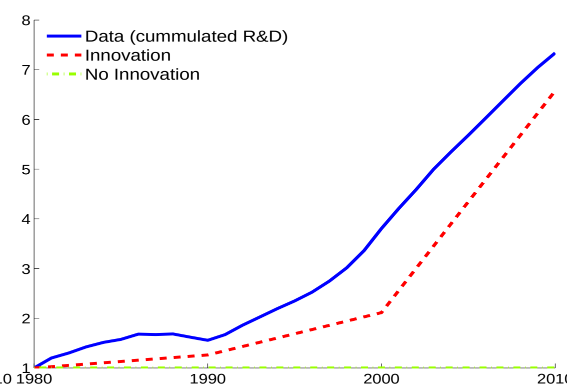


Figure 1.13: Simulation results – relative investment and labor share

(a) Software/Equipment investment



(b) Software/Equipment varieties



that the simulation without endogenous software innovation produces an almost flat labor share (figure 1.14b).

This occurs because an elasticity of substitution between equipment and labor (σ_e) is close to one, and hence exogenous variation does not generate declining labor share without software innovation. Therefore, the declining labor share in our model is mostly a result of endogenously increasing software investment. We highlight a negative correlation between software investment and labor share not only in the time series (figure 1.14a) but also in the industrial variation, especially during 2000-2010 (figures 1.14c and 1.14d). We believe that a detailed investigation of the relation between labor share and software capital is meaningful future research.

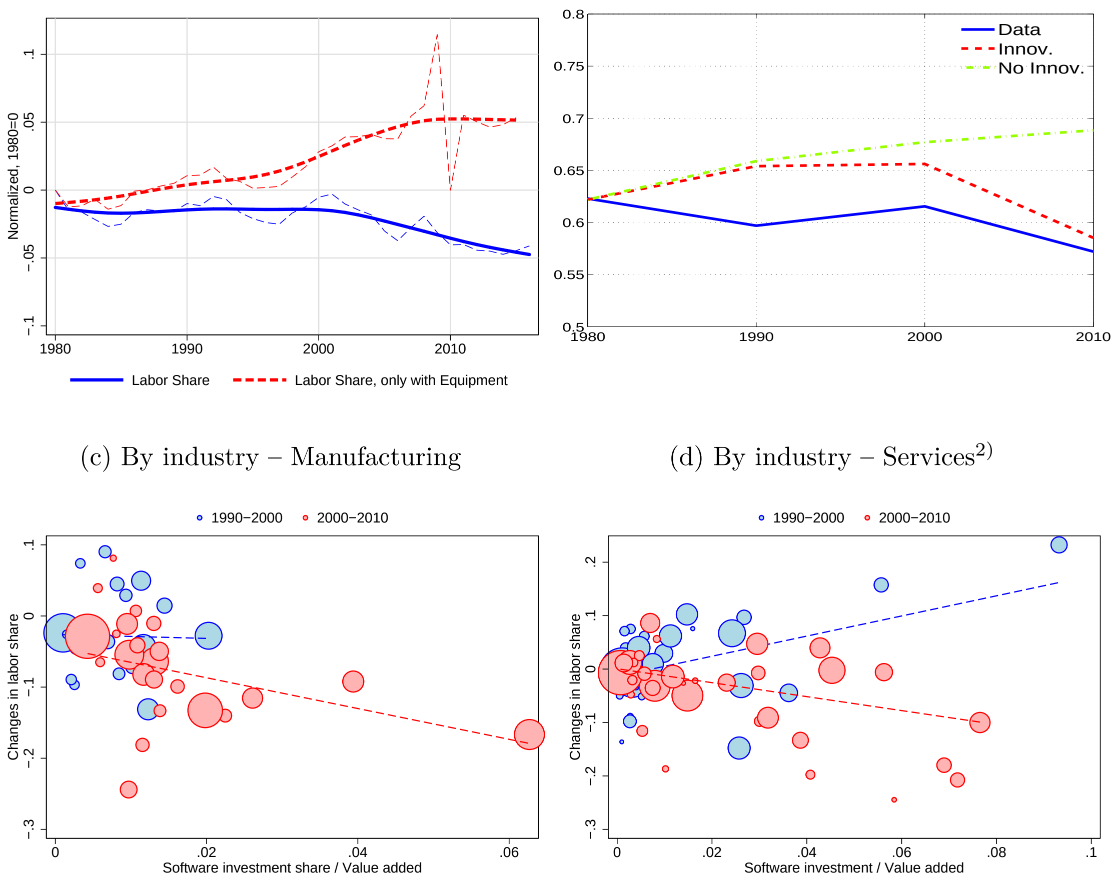
1.6.3 Simulation Results: With Changes in A_i's Only

Recall that we the use exogenous variation in the middle-skill specific technical change (changes in M_j) in addition to the evolution of the productivities of capital to match the changes in the employment share of the middle-skill occupations exactly. The natural question is how much of the observed changes in the productivities alone explain the variation in the share of employment by occupation.

The results are shown in figure 1.15. The changes in equipment price explain 78%, 75%, and 69% of the changes in cognitive, routine and low-skilled services employment. Two characteristics are noteworthy.

First, *all* occupational groups move in the same direction as the data, meaning that the differential growth of sectoral productivities – together with differences in the use of capital – captures routine-biased technical change quite well. The analysis suggests that a

Figure 1.14: Labor share and software



(a) Labor share trend1)

(b) Labor share simulation

(c) By industry – Manufacturing

(d) By industry – Services2)

Note: 1) The labor share only with equipment capital is constructed following Koh et al. (2018). The solid lines are HP trend with smoothing parameter 100. They are normalized to 0 in 1980. 2) Industry 514 (with changes in labor share greater than 1 in both periods) has been excluded from this figure.

differential productivity growth on the sector level (among capital-producing sectors) could be an underlying source of routine biased technological change.

Second, the decadal pattern of changes in occupational employment is also similar to that of the data, even without additional task-specific technical change (figure 1.15a and 1.15b). Moreover, changes in TFP generates 75% of job polarization (declines in routine occupations) in magnitude. We conclude that the evolution of the productivities embodied in equipment and software has been crucial in generating a pattern consistent with the data.

The analysis suggests that two further studies would be helpful in understanding the changes in the occupational structure caused by capital-embodied technical change. The first is to look at heterogeneity in sector-level production more closely. The second is an attempt to obtain a better productivity measure for various capital items.

1.6.4 Sensitivity

We assess how the results vary by the elasticity of substitution between tasks (ϵ), markups (ν_e and ν_s), and measures for A's.

The Elasticity of Substitution between Tasks Regarding the elasticity of substitution between tasks, we expect the model's explanatory power to increase as ϵ decreases as the model mechanism is amplified when the tasks are more complementary. Table 1.5 confirms this intuition.

Markups As can be seen in figure 1.16, the price-to-cost margins of equipment- and software-producing industries exhibit different trends. The changes in market structure also affect innovational incentives. To examine the importance of the time-varying markups, we map the variations in the price-to-cost margin into changes in ν_e and ν_s. With time-varying markups, explanation for the pattern of cognitive employment share decreases and that of manual employment share increases.

Alternative Measures for A_e and A_s Another way to measure the capital-embodied productivity is to compute the inverse of the price of equipment and software. We compare the simulation results with the inverse of the price of equipment and software as A_e and A_s.

The sensitivity analysis shows that the price series give a bit lower explanatory power for changes in employment share but higher explanation for the increase in software investment than a case with the benchmark.

Table 1.5: Sensitivity

The elasticity of substitution between tasks (ϵ)

		Data	Innov.		No innov.	
Cognitive	benchmark	-4.16	-3.11	(.75)	-0.61	(.15)
(dev. from trend)	$\epsilon = .1$	-4.16	-3.63	(.87)	-0.60	(.14)
	$\epsilon = .5$	-4.16	-2.97	(.72)	-0.64	(.15)
	$\epsilon = .7$	-4.16	-2.57	(.62)	-0.63	(.15)
Low skilled	benchmark	3.50	2.45	(.70)	-0.05	(-.01)
(dev. from trend)	$\epsilon = .1$	3.50	2.97	(.85)	-0.06	(-.02)
	$\epsilon = .5$	3.50	2.31	(.66)	-0.02	(-.01)
	$\epsilon = .7$	3.50	1.90	(.54)	-0.03	(-.01)
Soft/eqp	benchmark	1.68	1.06	(.63)	0.32	(.19)
(lev. in 2010)	$\epsilon = .1$	1.68	1.08	(.65)	0.31	(.19)
	$\epsilon = .5$	1.68	1.21	(.72)	0.34	(.21)
	$\epsilon = .7$	1.68	1.30	(.78)	0.37	(.22)

Markup-related parameters (ν_e and ν_s)

		Data	Innov.		No innov.	
Cognitive	benchmark	-4.16	-3.11	(.75)	-0.61	(.15)
(dev. from trend)	time-varying	-4.16	-2.72	(.65)	-0.55	(.13)
Low skilled	benchmark	3.50	2.45	(.70)	-0.05	(-.01)
(dev. from trend)	time-varying	3.50	2.64	(.76)	-0.44	(-.13)
Soft/eqp	benchmark	1.68	1.06	(.63)	0.32	(.19)
(lev. in 2010)	time-varying	1.68	0.97	(.58)	0.33	(.20)

Alternative measure for A_e and A_s

		Data	Innov.		No innov.	
Cognitive	Ind eqp	-4.16	-2.05	(.49)	-0.39	(.09)
(dev. from trend)	Total eqp	-4.16	-1.75	(.42)	0.33	(-.08)
	Ind+IT	-4.16	-2.24	(.54)	-0.11	(.03)
Low skilled	Ind eqp	3.50	1.39	(.40)	-0.27	(-.08)
(dev. from trend)	Total eqp	3.50	1.09	(.31)	-0.99	(-.28)
	Ind+IT	3.50	1.57	(.45)	-0.55	(-.16)
Soft/eqp	Ind eqp	1.68	1.06	(.63)	0.42	(.25)
(lev. in 2010)	Total eqp	0.35	0.63	(1.83)	0.12	(.35)
	Ind+IT	0.59	0.76	(1.27)	0.19	(.33)

Figure 1.15: Simulation results: no task-specific technological changes (constant M_j's)

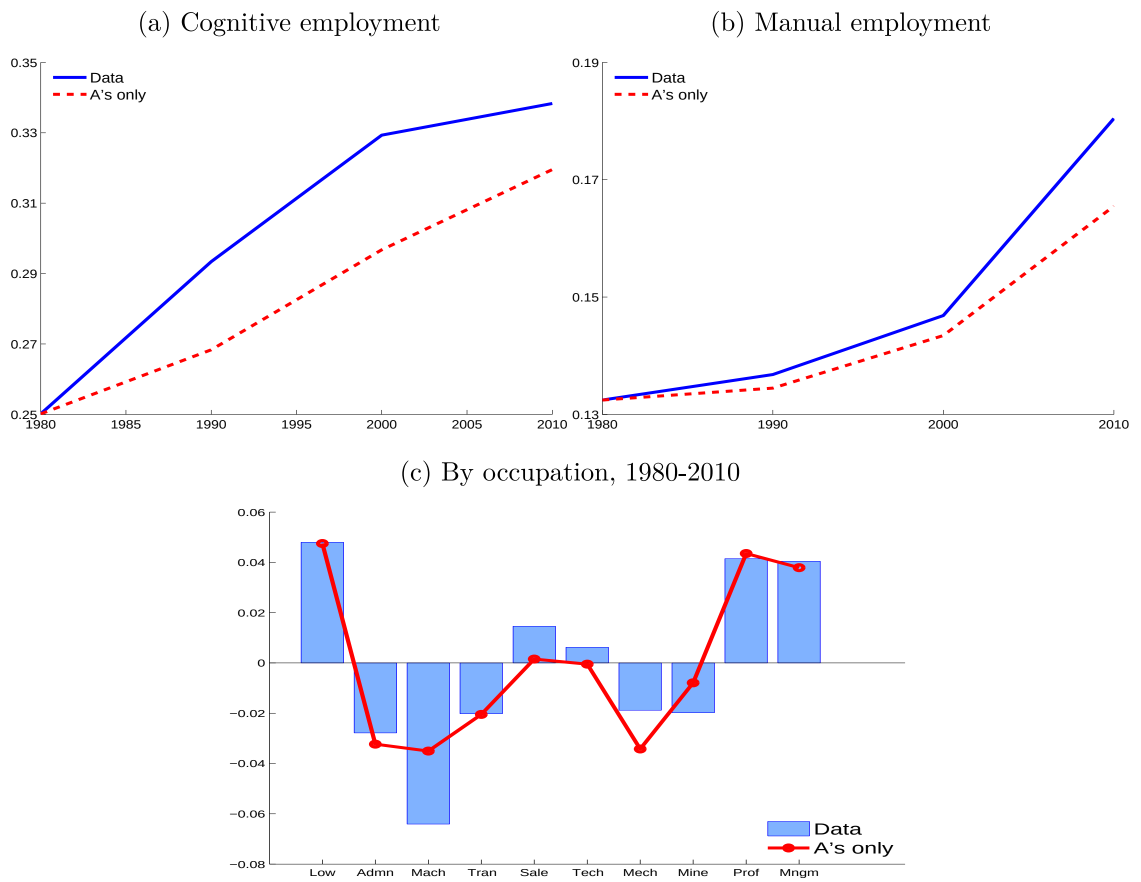
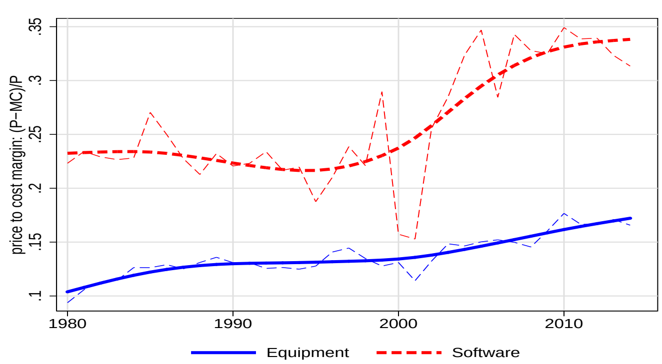


Figure 1.16: Price-to-cost margin[1)] of the equipment- and software-producing industries[2)]



Notes: 1) (Gross output-intermediates-compensation of employees)/gross output. 2) The equipment-producing industries are 331, 332, 333, 334, 335, 3361MV, 3364TO, 337, and 339. The software-producing industry is 511.

1.7 Conclusion

We provided a model with heterogenous tasks and two types of capital whose varieties are determined endogenously through a firm's innovation. We showed both analytically and quantitatively that the mechanism in the model is important in understanding the impact of capital-augmented technical change on the structure of the labor market.

One important implication is that two types of capital – software and equipment– measured in National Accounts, provide a good proxy for recent technological changes. Understanding the impact of a technical change on the economy has always been an important topic. One of the main difficulties is that technological change is not easy to measure, especially in aggregate analyses. This paper shows that the investigation of different types of capital can be a meaningful process to capture recent technological changes.

Our paper also implies that a technological change affecting a small group of occupations leads to other types of innovation, eventually affecting a broader set of occupations. Note that the same intuition applies to sectoral technical changes. This paper analyzes the technical change in the context of task-biased technological change, but a task-biased technical change is strongly linked to a sector-biased technical change, as emphasized in Lee and Shin (2017) and Bárány and Siegel (forthcoming).

Our model has many useful extensions that can be implemented easily. For example, further decomposition of equipment capital into subcategories will be helpful in understanding more detailed changes in occupational structure through technological changes embodied in capital. Further, integrating a multi-sector structure would provide additional interesting implications with respect to the relation between polarization and structural changes and the evolution of task-specific and sector-specific productivity, as in Aum et al. (2017).

Though not as straightforward, the analysis herein could also lead to many interesting future research topics. For example, by using firm-level software and equipment investment data, we may generate interesting implications on the impact of technological change on firm-level heterogeneity and occupation-level heterogeneity. Many countries are attempting to broaden the types of capital measured in National Accounts (System of National Account 2008). A multi-country extension would also be meaningful, enabling the analysis of trade or offshoring in addition to technological changes.

Chapter 2

Computerizing Industries and Routinizing Jobs: Explaining Trends in Aggregate Productivity

This chapter was coauthored with Tim Lee and Yongseok Shin.

2.1 Introduction

Amid the sluggish recovery following the Great Recession, much attention has been given to the slowdown in productivity growth in the United States economy (sometimes referred to as "secular stagnation"). We dissect this trend in aggregate productivity by developing a model in which technological progress is both sector- and occupation-specific,[11] to better understand

[11]Throughout the text, we will use "sector" and "industry" interchangeably, as well as "occupations" and "jobs."

which sectors and occupations contribute most to trends in aggregate productivity.[12] In particular, we pay special attention to the computer sector (hardware and software), which enjoyed an impressive rise in productivity even as the rest of the economy lagged behind. Computers have become an important factor of production for all other sectors, especially since the 1990s (which we call "computerization"), so we separate them from other machinery equipment as a distinct type of capital. Using the model, we quantify the importance of the computer sector and compare it against "routinization" (i.e., faster technological progress specific to occupations that involve routine or repetitive tasks) in explaining trends in aggregate productivity.

We find that a downward trend in aggregate productivity growth was already present since the 1970s, but that this was more than compensated for by the extraordinary productivity growth of the computer sector in the 1980s and 1990s. It was only when the computer sector's productivity growth came down to normal levels in the 2000s that the deceleration in aggregate productivity became abruptly apparent. This generated the illusion that the aggregate productivity slowdown has its roots in the 2000s, even though the slowdown had already been underway in the preceding decades.

In our analysis, the driving force of the aggregate productivity slowdown is complementarity across occupations and across industries in production: Those occupations and industries with above-average productivity growth shrink in terms of value-added and employment shares, and their contributions toward aggregate productivity growth becomes smaller even when their productivity continues to grow fast. This is related to "Baumol's disease," i.e., that aggregate productivity growth can slow down because sectors with high productivity

[12]Our model will admit an aggregate productivity that is distinct from conventional measures of total factor productivity (TFP), which assumes a homogeneous of degree one (HD1) production function in the two factors of capital and labor. When distinction is necessary, we will refer to our version with three factors (capital, labor and computers) simply as "productivity," and the two-factor residual as "TFP."

growth may decline in importance (e.g., manufacturing). However, our results show that it is the shrinkage of *occupations* with fast occupation-specific productivity growth, not *sectors*, that accounts for most of the downward trend in aggregate productivity growth.

Another novel element of our analysis is the computer sector. When sectors are complementary to one another, the extraordinarily high productivity growth of the computer sector should reduce its relative importance, and hence its contribution to aggregate productivity growth over time (Baumol's disease). However, because we model the computer sector's output as a distinct type of capital used in the production of all sectors (including itself), its productivity growth and the accompanying fall in its price boost the demand for computers from all sectors. Consequently, the computer sectors's contribution to aggregate productivity remained important for a prolonged period of time, more than offsetting the negative effect of routinization on aggregate productivity growth for over two decades. We also show that computerization accounts for most of the decline in the labor income share since the 1980s.

In our model, individuals inelastically supply labor to differentiated jobs. Each sector uses all these jobs, but with different intensities. Sectors are complementary across one another for the production of the final good. Within each sector, jobs are also complementary to one another, and labor is combined with capital for sectoral production. Most important, we divide capital into computer capital (including software) and the rest (i.e., all capital not produced from the computer sector), and assume that the substitutability between labor and computer capital may differ across sectors. We model computer and software as capital used by all other sectors rather than an intermediate input, because the computer share of all investment is substantially larger than its share of all intermediates (14 vs. 2 percent, averaged between 1980 and 2010).

It should be noted that computerization and routinization are empirically distinct phenomena. Computer and software usage increased the most for high-skill or cognitive occupations, not middle-skill or routine occupations, justifying our choice to model productivity growth in both dimensions (sector- and occupation-specific). We then estimate the degree of complementarity across sectors, and calibrate the growth rates of the sector- and occupation-specific productivities, substitutability/complementarity across jobs, and substitutability between computer capital and labor, using detailed data on employment shares and computer capital by industry and by occupation. Our estimation and calibration verify that as long as productivity growth rates are positive, (i) sectors are complementary to one another for final good production;[13] (ii) jobs are complementary to one another within sectors; and most important, (iii) computer capital is in fact substitutable with labor in all sectors.

Given the structure of our model and estimated/calibrated parameters, we find that when sector- and occupation-specific productivities grow at constant but different rates, aggregate productivity growth declines over time due to the two types of complementarity (across jobs within sectors, and across sectors in final good production). Jobs and sectors with highest productivity growth shrink in terms of employment and value-added. Then low-growth jobs and sectors gain more weight when computing aggregate productivity growth, resulting in its slowdown. As productivity growth slows down, output growth slows down even more.

The mechanics of our model is consistent with our empirical findings: Since the 1980s, sectors that rely heavily on routine jobs experienced the highest growth in their TFP's, as measured by conventional growth accounting.[14] These occupations, and the sectors that rely relatively more on them, also saw their employment shares fall.

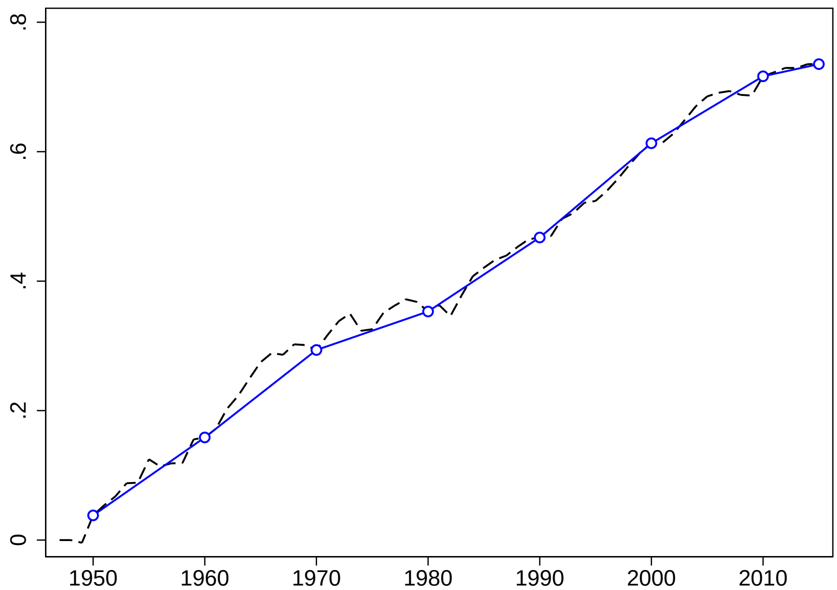
[13] Or consumption, which we do not model.

[14] That is, assuming an HD1 production function with two factors, capital and labor. By "measured," we mean productivity or TFP obtained directly from the data by growth accounting, as opposed to being computed from our model.

Next, we find that the fall in aggregate productivity growth in the longer run is more due to the differential growth across occupations (i.e., routinization) rather than the differential growth across sectors. In fact, if all occupation-specific productivities had grown at a common rate from 1980, holding all else equal, aggregate productivity growth rates would have stayed nearly constant through 2010. This contrasts with Baumol's disease, which emphasizes the differential sector-specific productivity growths, especially the slow productivity growth of the service sector.

The natural question is then why the downward trend in aggregate productivity growth did not manifest itself until the 2000s. In our model, the slowdown in aggregate productivity growth can be temporarily arrested and even reversed if certain sectors or jobs experience faster-than-usual technological progress. We find that this is exactly what happened during the 1990s, when the computer sector recorded impressive productivity growth. Without the technological progress specific to the computer industry, aggregate productivity growth during the 1990s would have been 0.5 percent per year, instead of 0.8 percent. It is only after the subsequent slowdown in the computer sector's productivity growth in the 2000s that the longer-run downward trend in aggregate productivity growth became apparent. Our analysis confirms that if productivity growth in the computer sector had been completely absent, aggregate productivity growth would have declined monotonically since 1980. In fact, although our focus is on the slowdown toward the end of the sample period in Figure 2.1, a slowdown is also apparent in the 1970s to early 1980s.

In the data, sectors with higher measured TFP growth saw their employment shares decline, *except* for the computer sector. The same happens in our model because all sectors use computer capital in production. Then, as the computer sector's productivity growth reduces the price of computer capital, all sectors use more computers, which contributes to output growth in addition to the computer sector's direct contribution to aggregate productivity



Period	$d\,$logTFP
1950-	1.20
1960-	1.35
1970-	**0.59**
1980-	1.14
1990-	1.46
2000-	**1.03**
2010-	**0.38**

Figure 2.1: Log TFP, Aggregate

Source: National Income and Product Accounts (NIPA) from the Bureau of Economic Analysis (BEA). TFP is measured as the Solow residual assuming a homogeneous of degree one production function with two factors, capital and labor.

growth.[15] Indeed, if there had been no productivity growth in the computer sector and hence no computerization, output per worker growth would have been 1.5 percent per year during the 1990s, rather than the 3.5 percent observed in the data. In other words, the sluggish growth of aggregate productivity and output in the 2000s was not abnormal. It was the faster-than-trend growth during the 1990s driven by the outburst of the computer sector's productivity that was extraordinary.

Treating computer capital as a separate production factor as we do also has implications for the measurement of aggregate productivity. We find that conventional TFP accounting with only two factor inputs, with all types of capital being summed up into a single category, overstates aggregate productivity growth by 0.4 percentage points per year when averaged

[15] As discussed earlier, this model element is also important for understanding why the direct contribution of the computer sector to aggregate productivity growth did not dwindle in importance despite the complementarity across sectors. The computer sector's production share has been stable over time: 3.1 percent in the 1980s, 3.4 percent in the 1990s, 3.9 percent in the 2000s, and 3.4 percent in the 2010s.

between 1980 and 2010. That is, ignoring different types of capital, which differ in their rental and depreciation rates, can bias productivity measurements upward.

Lastly, we relate computerization to the decline in the labor income share. In our model, the labor share decline is caused by the substitutability between labor and computer capital, as the computer sector becomes more productive. We find that computerization during the 1990s accounts for most of the decline in the labor share between 1980 and 2010 (4 out of 5 percentage points), even the model does not target the labor share at all. This implies that computer capital alone is more important than all other machinery and equipment in explaining the decline in the labor share.

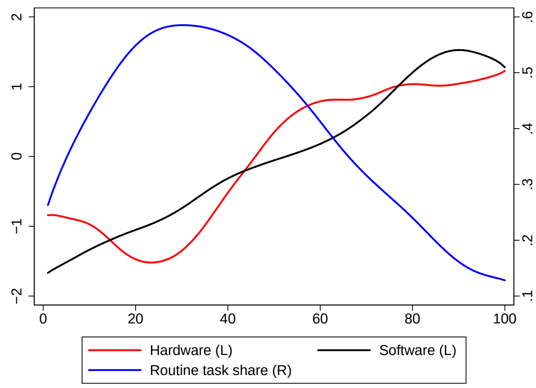
Related literature In our model, employment shifts across sectors—or "structural change"—occur due to differential sector- and occupation-specific productivity growth as in Lee and Shin (2017). Most studies in the structural change literature that consider sector-specific productivity growth, e.g., Ngai and Pissarides (2007), have paid little attention to its implications for changes in aggregate productivity. In fact, most were interested in obtaining balanced growth. However, since as far back as Baumol (1967), it was well known that complementarity between industries can lead to an increase in the employment share of the low productivity growth sector, consequently leading to a slowdown in aggregate productivity. A recent study by Duernecker et al. (2017) is a notable exception. They explicitly consider Baumol's disease in a multi-sector model, and evaluate whether structural change is quantitative important for explaining the aggregate productivity slowdown. In our analysis, we model differential progress across occupation-specific technologies in addition to heterogeneous sector-specific productivity growth, and find that it was the dispersion of occupation-specific productivities that was more important for the aggregate productivity slowdown in the United States.[16]

[16] Aum et al. (2017) document occupation-specific and sector-specific shocks at a higher frequency—during and after the Great Recession.

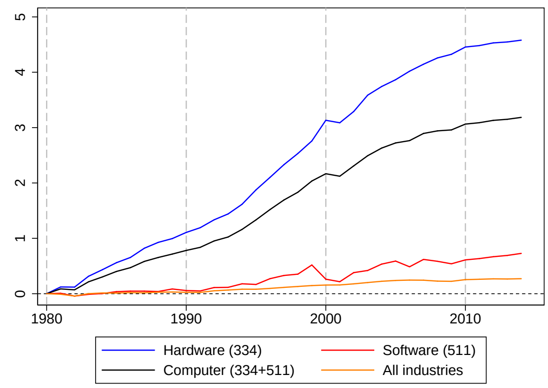
Our work also relates to studies on the importance of information technology (IT) in explaining the evolution of productivity (e.g., Byrne et al., 2016; Syverson, 2017). In particular, Acemoglu et al. (2014) investigate the relationship between productivity growth and IT capital intensity by industry, and conclude that IT usage has little impact on productivity. While we emphasize the role of computerization, our analysis is consistent with theirs. Computerization is important for shaping aggregate productivity growth in our analysis, but there is no direct effect of computerization on the productivity of other industries. Instead, computerization affects industry level output and value-added through an increase in the use of computer capital.

In many empirical analyses related to routinization, the price of information and communication technology (ICT) capital is often used as a proxy for routine-biased technological change (e.g., Goos et al., 2014; Cortes et al., 2017). However, when we break down computer usage by occupation, we find that computerization and routinization are two distinct phenomena, with different implications for the macroeconomy. Related, the first chapter of this dissertation analyzes increasing investment in software in a model that also features routinization. While the first chapter focuses on its impact on changes in occupational employment, the second chapter focuses on its implications for aggregate productivity.

Finally, Karabarbounis and Neiman (2014) suggest that the decline in the labor income share could be due to a decline in the price of capital. Since the decline in the price of capital is mostly driven by the price of computer-related equipment, and it mirrors the productivity increase in the computer industry, our analysis concurs with their explanation of the declining labor share. Furthermore, our results show that a specific component of capital—computer hardware and software—can be more important than all other types of capital. This is in line with Koh et al. (2016), who emphasize the importance of intellectual property products capital (including software) for the decline of the labor share.



(a) Routinization and computerization



(b) log-TFP of computer industry

Figure 2.2: PC use by occupation and PC industry TFP
Source: (a) IPUMS Census, BEA NIPA and O*NET. (b) BEA Industry Accounts. The computer industry includes industries 334 and 511 (for hardware and software, respectively). See footnote 18 and text for the data and accounting behind the graphs.

2.2 Empirical Evidence

We begin by establishing that routinization and computerization are two distinct phenomena. For the empirical analysis, occupational data is from the decennial censuses and industrial data from the BEA industry accounts. We consider industries at the 2-digit level, resulting in 60 industries. In particular, we label industry 334 (computers and electronic products) the "hardware" industry and 511 (publishing industries including software) the "software" industry. The combination of both is the "computer sector."

In Figure 2.2(a), the horizontal axis is occupational employment shares (percentile), in ascending order of each occupation's 1980 average wage.[17] The figure shows that the routine-task intensity (RTI) of occupations (Autor and Dorn, 2013) is high for middle-wage occupations, as is well known in the routinization/polarization literature, but that high-wage occupations tend to use computers more.[18] So at the occupational level, an increase in the

[17]The ordering of occupational mean wages barely changes from 1980 to 2010.

[18]Computer usage is approximated from 2010 NIPA Tables 5.5.5 (Private Fixed Investment in Equipment by Type), 5.6.5 (Private Fixed Investment in Intellectural Properties by Type), and the O*NET Tools and

use of computers (i.e., computerization) should be distinguished from routinization, which is typically understood as faster productivity growth among middle-wage or routine-intense tasks.

Computerization in our model is a consequence of the fast productivity growth of the computer industry. We first employ conventional accounting to measure each industry's TFP growth: the growth rate of real value-added net of the growth of capital and labor inputs, weighted by the income share of each factor. Specifically, industry i's measured TFP growth between time s and t is

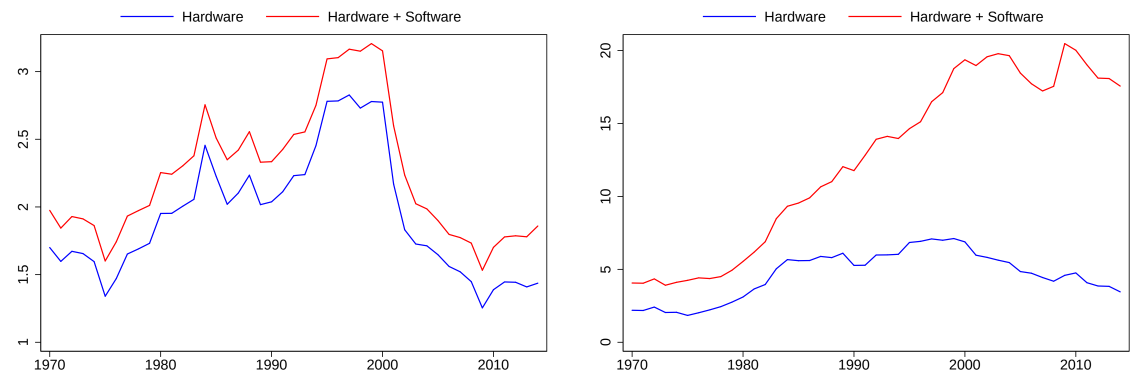
$$\log \frac{TFP_{it}}{TFP_{is}} = \log \frac{Y_{it}}{Y_{is}} - \frac{\alpha_{is} + \alpha_{it}}{2} \cdot \log \frac{L_{it}}{L_{is}} - \frac{2 - \alpha_{is} - \alpha_{it}}{2} \cdot \log \frac{K_{it}}{K_{is}},$$

where Y is real value-added, L is employment, K is the net real stock of non-residential fixed capital, and α is the labor share (compensation of employees divided by value-added).[19]

Figure 2.2(b) depicts the log-TFP of computer-related industries (BEA industry code 334 for hardware and 511 for software) and the average of the log-TFP of all industries excluding agriculture and government (weighted according to the Törnqvist index). The TFP of hardware shows an average annual growth rate of 16 percent, far higher than the average across all industries. Software also features higher TFP growth compared to the average. The TFP of the "computer industry"—the value- added weighted average of hardware and

Technology database as follows. In NIPA Table 5.5.5, we assume that "computers and peripheral equipment" are produced by industry 334, and in Table 5.6.5, that "software" are produced by industry 511. O*NET lists all the tools and technology that are used for each occupation. O*NET occupation codes can be easily mapped to the census, and tools and technology are coded using the UNSPSC commodity system. We assume that 4321xxxx corresponds to "hardware," which includes all computers and peripheral equipment, and that 4323xxxx corresponds to "software." Then we count the number of distinct commodities needed in each occupation, multiply it by the employment share of that occupation, and assume that hardware and software investment is allocated across occupations proportionately to this number. Finally, we standardize this measure of computer investment by occupation to have a mean of zero and standard deviation of 1. While this may be a crude measure for computer usage, it is highly correlated with data from the CPS, which reports computer use intensity by occupation. See Appendix A for more details.

[19] Later when we separately consider computer capital, TFP computed here would be a misspecification.



(a) Computer share of intermediates (%)

(b) Computer share of non-residential investment (%)

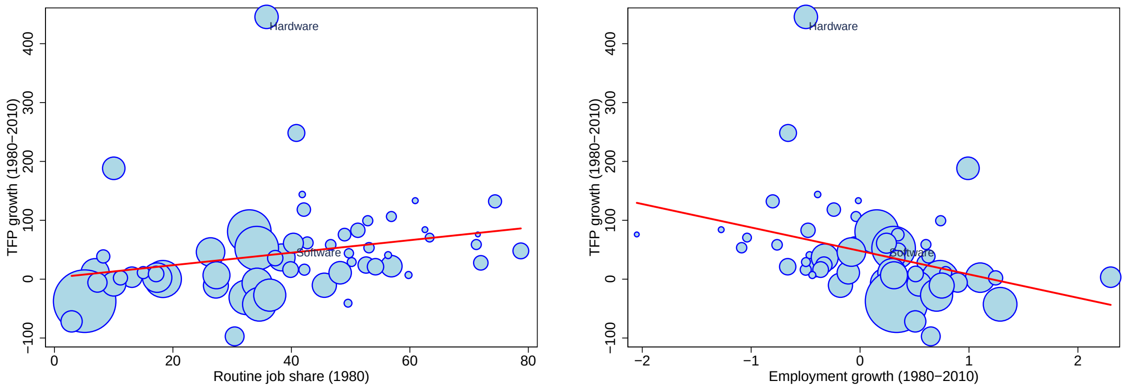
Figure 2.3: Computer use in production over time
Source: BEA Input-Output Tables and Fixed Asset Tables (FAT). In panel (a), hardware and software are industries 334 and 511. In panel (b), hardware and software are investments into "computers and peripheral equipment" and "software" in the FAT.

software—shows that the hardware industry mostly determines the TFP of the computer industry. Note that the exceptionally fast growth of the computer industry's TFP slowed down since around the early 2000s.

Reflecting the fast growth of the computer industry's measured TFP, the use of computer and software also rose substantially until the late 1990s. Figure 2.3(a) shows the computer and software share of total intermediates over time. Figure 2.3(b) plots the share of computers and software in total non-residential investment. In both figures, it is clear that there was a steep rise in the importance of computers in the 1980s to 1990s, which stagnated starting in the 2000s.[20]

We now turn to disaggregated evidence at the industry level, which will support our hypotheses of heterogeneous growth rates and complementarity across jobs and industries. Because job or occupation-level productivity is not directly measurable, we first establish two new empirical patterns, utilizing the fact that industries differ in the composition of

[20]The data behind Figures 2.3(a) and (b) come from BEA's Input-Output Tables and Fixed Assets Tables, respectively.



(a) Routine occupation share and TFP growth (b) TFP and employment growth across industries

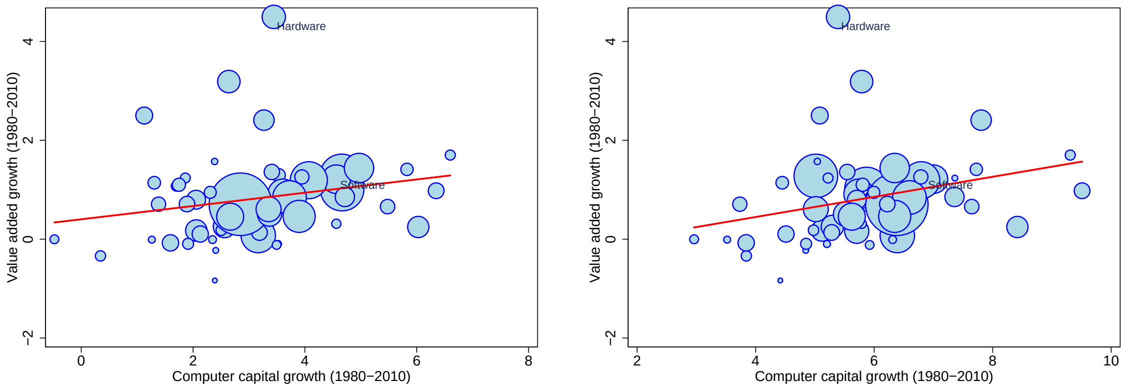
Figure 2.4: Routinization and industry TFP and employment

Source: IPUMS Census and BEA Industry Accounts. Hardware and Software are industries 334 and 511, respectively. In panel (a), routine jobs are defined as occupations above the 66 percentile in terms of the RTI index (Autor and Dorn, 2013). In panel (b), FTPT is full-time plus part-time workers.

their workers' occupations. Figure 2.4(a) shows that the routine job share of an industry is positively correlated with its measured TFP growth (log difference) between 1980 and 2010 (consistent with routinization), where routine jobs are defined as occupations that are above the 66 percentile in terms of the RTI index following Autor and Dorn (2013). Figure 2.4(b) shows that TFP growth and employment growth are negatively correlated across industries, consistent with complementarity across jobs and/or industries.[21]

However, note that the computer industry is a conspicuous outlier. In Figure 2.4(a), despite having a routine job share around the median, not only is the computer industry's TFP growth 10 times larger than other industries at similar levels of routineness, it is in fact 2 to 4 times larger than the next two industries with the highest levels of TFP growth overall. Despite this, as shown in Figure 2.4(b), its employment barely fell. With complementarity across industries, a high productivity growth sector should lose value-added and employment shares. A possible explanation is that other industries depend heavily on the computer

[21]Employment in this figure is full-time plus part-time workers (FTPT). Full-time equivalent (FTE) employment shows similar patterns, but is only available by industry from 1997 onward. For this period, there are level differences between the two measures, but dynamic patterns are similar for both.



(a) Hardware

(b) Software

Figure 2.5: Growth of Value-added Output and Computer Capital
Source: BEA Industry Accounts and FAT Nonresidential Detailed Estimates by Industry and Type. Hardware and Software are industries 334 and 511, respectively. Hardware capital is the net stock of "computers and peripheral equipment" and software capital the net stock of "software" by industry.

industry, so that even as its productivity grows the size of this sector would not shrink as long as other industries rely on it more. If so, those industries with faster growth in computer capital should grow faster than those that use computers less intensively *in terms of output*: since computer capital is a factor in production, it would not necessarily increase productivity. Figure 2.5 confirms the positive relationship between the growth of computer capital (total investment into hardware and software from 1980-2010) for an industry and its value-added growth between 1980 and 2010.

2.3 Model

The model for our quantitative analysis builds on those in Goos et al. (2014) and Lee and Shin (2017), both of which simultaneously analyze an economy's occupational and industrial structure. In particular, the latter explicitly models how workers of heterogeneous skill sort into different occupations, and also industries that differ in the intensity with which they combine workers of different occupations for production. Here we ignore selection on skill, but instead expand previous models by letting all industries use output from the computer sector as a capital good in production, an important channel through which the productivity gains of the computer industry affect aggregate production.

Environment A representative household maximizes its discounted sum of utility

$$\sum_{t=0}^{\infty} \beta^t u(C_t)$$

subject to the sequence of budget constraints,

$$C_t + I_t + p_{I,t} F_t \leq Y_t,$$

where I is investment in traditional capital (machinery and equipment excluding computer hardware and software), F investment in computer capital, and p_I the price of computers. The final good is the numeraire, which can be used for consumption and traditional capital investment. The law of motion for each type of capital satisfies

$$K_{t+1} = I_t + (1 - \delta_K) K_t, \quad S_{t+1} = F_t + (1 - \delta_S) S_t,$$

where (K, S) are traditional and computer capital, respectively, and (δ_K, δ_S) their depreciation rates. In what follows, we drop the time subscript unless necessary, and simply denote next period variables with a prime.

Within the representative household is a unit mass of identical individuals who supply labor inelastically to one of J occupations, indexed by $j \in \{1, \ldots, J\}$. The final good is produced by combining products from I sectors, which we index by $i \in \{1, \ldots, I\}$. To be specific, final good production combines industrial output using a CES aggregator with the elasticity of substitution ϵ:

$$Y = \left[\sum_{i=1}^{I} \gamma_i^{\frac{1}{\epsilon}} Y_i^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}.$$

In each sector, a representative firm organizes the J occupations to produce sectoral output Y_i according to

$$Y_i = A_i K_i^{\alpha_i} Z_i^{1-\alpha_i}, \tag{2.1}$$

where A_i is industry i's exogenous sector-specific productivity and Z_i a computer-labor composite that combines computer capital S_i with an occupation composite X_i:

$$Z_i = \left[\omega_i^{\frac{1}{\rho_i}} S_i^{\frac{\rho_i-1}{\rho_i}} + (1-\omega_i)^{\frac{1}{\rho_i}} X_i^{\frac{\rho_i-1}{\rho_i}} \right]^{\frac{\rho_i}{\rho_i-1}}, \quad X_i = \left[\sum_{j=1}^{J} \nu_{ij}^{\frac{1}{\sigma}} \left(M_j L_{ij} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}.$$

Each L_{ij} is the number of occupation j labor (i.e., workers) used in sector i, and M_j is the exogenous occupation-specific productivity of job j that differs across occupations but not sectors. The parameters ω_i and ν_{ij} are CES weights that differ by sector, as well as ρ_i, the elasticity of substitution between computers and labor in sector i. However, we assume that the elasticity of substitution across occupations, σ, is identical across sectors. There are several reasons we let the ρ's vary across sectors but not σ, which we discuss in Section 2.4.2.

Since each industry uses all types of occupations but with different intensities ν_{ij}, changes in M_j would have differential effects on the occupation composite X_i, and thus on Z_i, the computer-labor composite. Ultimately, it will manifest itself as differential effects on sectoral productivity and output. In contrast, changes in A_i affects sectoral productivity and output directly.

Computer capital S_i is also used in all sectors, and without loss of generality we will assume that the computer industry is industry $i = I$. So the total amount of computer capital in the economy is $S = \sum_{i=1}^{I} S_i$ and F is the total amount of newly produced computers. While the model assumes that computer capital is required for production in all industries, there is no other input-output linkage among the rest. Each industry rents traditional capital and computer capital at rates R_K and R_S.

Equilibrium The final good firm takes prices p_i as given and solves

$$\max \left\{ Y - \sum_{i=1}^{I} p_i Y_i \right\}. \tag{2.2}$$

Each sector i firm takes all prices as given and chooses capital, computer capital and labor to solve

$$\max \left\{ p_i Y_i - R_K K_i - R_S S_i - w \sum_{j=1}^{J} L_{ij} \right\}, \tag{2.3}$$

where p_i is the price of the sector i good, R_K the rental rate of traditional capital, R_S the rental rate of computer capital, and w the wage rate—which is equal across jobs since individuals do not differ in skill. In a competitive equilibrium,

1. Final good producers choose Y_i to maximize profits (2.2), so

$$\gamma_i Y / Y_i = p_i^{\epsilon} \quad \text{for } i \in \{1, \ldots, I\}. \tag{2.4}$$

Since we normalized the final good price to 1,

$$\sum_{i=1}^{I} \gamma_i p_i^{1-\epsilon} = 1^{\frac{1}{1-\epsilon}} = 1$$

is the ideal price index.

2. All sector i firms maximize profits (2.3). The first-order necessary conditions are

$$R_K = \alpha_i p_i Y_i / K_i, \tag{2.5a}$$

$$R_S = (1-\alpha_i) \cdot (p_i Y_i / Z_i) \cdot (\omega_i Z_i / S_i)^{\frac{1}{\rho_i}}, \tag{2.5b}$$

$$w = (1-\alpha_i) \cdot (p_i Y_i / Z_i) \cdot [(1-\omega_i) Z_i / X_i]^{\frac{1}{\rho_i}} \cdot \left[\nu_{ij} \tilde{M}_j X_i / L_{ij}\right]^{\frac{1}{\sigma}} \tag{2.5c}$$

where $\tilde{M} := M^{\sigma-1}$.

3. Capital, computer and labor markets clear:

$$K = \sum_{i=1}^{I} K_i, \quad S = \sum_{i=1}^{I} S_i, \quad L = \sum_{i=1}^{I} L_i = \sum_{i=1}^{I} \left[\sum_{j=1}^{J} L_{ij}\right] \tag{2.6}$$

where $L_i := \sum_j L_{ij}$ is the total amount of labor used in sector i.

4. The rental rates satisfy

$$\frac{u'(C)}{\beta u'(C')} = 1 + r = R'_K + (1-\delta_K) = [R'_S + (1-\delta_S) p'_I] / p_I, \tag{2.7}$$

and the transversality conditions hold.

$$\lim_{t\to\infty} \beta^t u'(C_t) K_t = 0, \quad \lim_{t\to\infty} \beta^t u'(C_t) S_t = 0.$$

Equilibrium Characterization From (2.4) and (2.5a), we find that

$$\begin{aligned} & \alpha_i p_i Y_i / \alpha_I p_I Y_I = K_i / K_I = (\alpha_i / \alpha_I) (\gamma_i / \gamma_I)^{\frac{1}{\epsilon}} \cdot (Y_i / Y_I)^{\frac{\epsilon-1}{\epsilon}} \\ \Rightarrow \quad & \alpha_i p_i y_i / \alpha_I p_I y_I = k_i / k_I = (\alpha_i / \alpha_I) (\gamma_i / \gamma_I)^{\frac{1}{\epsilon}} \cdot (y_i / y_I)^{\frac{\epsilon-1}{\epsilon}} \cdot (L_i / L_I)^{-\frac{1}{\epsilon}}, \end{aligned}$$

where $y_i := Y_i/L_i$ is output per worker and $k_i := K_i/L_i$ is capital per worker in sector i. So using (2.1), we can write

$$\frac{A_i}{A_I} = \left(\frac{\alpha_I}{\alpha_i}\right)^{\frac{\epsilon}{\epsilon-1}} \cdot \frac{k_i^{\frac{\epsilon}{\epsilon-1}-\alpha_i}}{k_I^{\frac{\epsilon}{\epsilon-1}-\alpha_I}} \cdot \frac{z_I^{1-\alpha_I}}{z_i^{1-\alpha_i}} \cdot \left(\frac{\gamma_I L_i}{\gamma_i L_I}\right)^{\frac{1}{\epsilon-1}} \tag{2.8}$$

where (z_i, s_i) is the labor productivity and computer per worker in sector i. From (2.5c), holding i fixed we obtain $L_{ij}/L_{i1} = \nu_{ij}\tilde{M}_j/\nu_{i1}\tilde{M}_1$ for all j, so

$$L_{ij} = \left(\tilde{V}_i^{1-\sigma} \cdot \nu_{ij}\tilde{M}_j\right) \cdot L_i \text{ and } X_i = \tilde{V}_i L_i, \quad \text{where } \tilde{V}_i := \left(\sum_{j=1}^{J} \nu_{ij}\tilde{M}_j\right)^{\frac{1}{\sigma-1}}. \tag{2.9}$$

Then the equilibrium allocations of (L_{ij}, Z_i) can be expressed as

$$L_{ij}/L_i = \nu_{ij}\tilde{M}_j\tilde{V}_i^{1-\sigma}, \quad \text{and} \tag{2.10}$$

$$Z_i = \left[\omega_i^{\frac{1}{\rho_i}} S_i^{\frac{\rho_i-1}{\rho_i}} + V_i^{\frac{1}{\rho_i}} L_i^{\frac{\rho_i-1}{\rho_i}}\right]^{\frac{\rho_i}{\rho_i-1}} \;\Rightarrow\; z_i := Z_i/L_i = \left[\omega_i^{\frac{1}{\rho_i}} s_i^{\frac{\rho_i-1}{\rho_i}} + V_i^{\frac{1}{\rho_i}}\right]^{\frac{\rho_i}{\rho_i-1}} \tag{2.11}$$

where $V_i := (1-\omega_i)\tilde{V}_i^{\rho_i-1}$. Plugging these expressions into (2.5b)-(2.5c) we obtain

$$R_S = (1-\alpha_i) \cdot (p_i y_i/z_i) \cdot (\omega_i z_i/s_i)^{\frac{1}{\rho_i}}, \tag{2.12a}$$

$$w = (1-\alpha_i) \cdot (p_i y_i/z_i) \cdot (V_i z_i)^{\frac{1}{\rho_i}}, \tag{2.12b}$$

and taking the wage-computer rent ratio (w/R_S) across all sectors, we can express all other sectors' computer capital per worker relative to the computer sector's:

$$(V_i/\omega_i) \cdot s_i = \left[(V_I/\omega_I) \cdot s_I\right]^{\frac{\rho_i}{\rho_I}} \tag{2.13}$$

and plugging this expression into the definition of z_i in (2.11), we obtain

$$z_i = V_i^{\frac{1}{\rho_i - 1}} \left[1 + (\omega_i / V_i) \left[(V_I / \omega_I) \cdot s_I \right]^{\frac{\rho_i - 1}{\rho_I}} \right]^{\frac{\rho_i}{\rho_i - 1}}. \tag{2.14}$$

Thus, all z_i's can be obtained given s_I, the computer sector's computer capital per worker, and exogenous parameters. Similarly, taking the wage-capital rent ratio (w/R_K) across all sectors using (2.5a) and (2.12b), we obtain

$$\frac{(1-\alpha_i)\alpha_I}{(1-\alpha_I)\alpha_i} \cdot \frac{k_i}{k_I} = \left(z_i^{\frac{\rho_i - 1}{\rho_i}} / z_I^{\frac{\rho_I - 1}{\rho_I}} \right) \Big/ \left(V_i^{\frac{1}{\rho_i}} / V_I^{\frac{1}{\rho_I}} \right), \tag{2.15}$$

and since all z_i's are functions of s_I, all k_i's can be obtained given s_I and k_I's, the computer sector's traditional capital per worker. So the equilibrium allocation can be found from (2.8) subject to the market clearing conditions (2.6).

Discussion In our model, sector- and occupation-specific productivities are exogenous (A_i and M_j, respectively). In particular, sector-specific productivities A_i are distinct from "sectoral productivity" which refers to the productivity of a sector in an accounting sense. And since the occupation-specific productivities affect sectoral productivity through $V_i := (1-\omega_i)(\sum_j \nu_{ij} \tilde{M}_j)^{\frac{\rho_i - 1}{\sigma - 1}}$, sectoral productivity depends on M_j's as well as A_i. Specifically, sectoral productivity in our model is obtained by decomposing output into factors:

$$\hat{y}_i = \underbrace{\left[\hat{A}_i + (1-\alpha_i) \frac{1}{\rho_i - 1} \frac{V_i^{\frac{1}{\rho_i}}}{z_i^{\frac{\rho_i - 1}{\rho_i}}} \hat{V}_i \right]}_{\text{Sectoral Productivity}} + \underbrace{\alpha_i}_{K \text{ share}} \hat{k}_i + \underbrace{(1-\alpha_i) \frac{\omega_i^{\frac{1}{\rho_i}} s_i^{\frac{\rho_i - 1}{\rho_i}}}{z_i^{\frac{\rho_i - 1}{\rho_i}}}}_{S \text{ share}} \hat{s}_i, \tag{2.16}$$

where $\hat{x} := d \log x$.

The above is our definition of productivity in the subsequent quantitative analysis, which is distinct from traditional measures of TFP.[22] A rise in M_j, the occupation-specific productivity of job j, raises sectoral productivity through changes in V_i. In this case, all sectoral productivities would move in the same direction (either up or down), but their growth rates will differ depending on the sector-specific parameters included in the expression for sectoral productivity in (2.16), as well as the endogenous response of z_i. And since the production technology is homogeneous of degree one (HD1), aggregate productivity is a sectoral output-weighted average of the sectoral productivities. Hence changes in the exogenous productivities A_i or M_j affect aggregate productivity both directly by changing all sector's sectoral productivities, but also indirectly by altering sectoral output shares.

Last but not least, changes in A_I, the computer industry's sector-specific productivity, has further repercussions on aggregate output. As other industries, changes in A_I alter aggregate productivity both directly (by increasing the computer sector's sectoral productivity) and indirectly (by altering the output share of the computer industry). But in addition, it lowers the price of computers (p_I) and consequently the rental rate of computer capital (R_S), leading to a rise in the use of computers for industries whose elasticity of substitution between computers and labor (ρ_i) is larger than one. Consequently, not only because it raises aggregate productivity, but also because it increases the use of computers in all sectors, a rise in A_I contributes more to an increase in aggregate output than any other sector-specific productivity does.

[22]The difference is that conventional TFP measurements separate only capital and labor, while we are taking out computers as a distinct type of capital with its own income share.

Industry	BEA industry code
Mining	211, 212, 213
Construction	23
Durable goods manufacturing	311FT, 313TT, 315AL, 322, 323, 324, 325, 326
Non-durable goods manufacturing	321, 327, 331, 332, 333, 335, 3361MV, 3364OT, 337, 339
FIRE	521CI, 523, 524, 531, 532RL
Health	621, 622HO
Other high-skill services	512, 513, 514, 5411, 5412OP, 5415, 55, 61
Trade (Retail & Wholesale)	42, 44RT
Other low-skill services	22, 481, 482, 483, 484, 485, 486, 487OS, 493, 561, 562, 624, 711AS, 713, 721, 722, 81
Computer	334, 511

Table 2.1: Industry classification

Refer to BEA Industry Accounts for names of industries. The computer industry comprises hardware (computer and electronic products) and software (publishing industries).

2.4 Quantitative Analysis

For the quantitative analysis, we classify industries into ten groups as summarized in Table 2.1. We exclude the agricultural sector and government. In Table 2.2, we classify occupations into ten groups which broadly correspond to one-digit occupation groups in the census. We then fit the model exactly to the data for 1980, and let only the exogenous occupation- and sector-specific productivities (M_j, A_i) grow at a constant rate. Thus, a major test of the model is how well it replicates the data in 2010, or equivalently, the growth of sectoral and aggregate variables from 1980 to 2010.

2.4.1 Calibration

Aggregate production function The parameters of the final good production function are estimated outside of the model using real and nominal value-added data by industry.

Occupation	Occupation code
High skill	
Management	4 - 37
Professionals	43 - 199
Middle skill	
Mechanics & Construction	503 - 599
Miners & Precision workers	614 - 699
Technicians	203 - 235
Sales	243 - 283
Transportation	803 - 889
Machine operators	703 - 799
Administrative support	303 - 389
Low skill services	405 - 498

Table 2.2: Occupation classification

Consistent occupation code (`occ1990dd`) constructed following Autor and Dorn (2013).

Specifically, we estimate the sectoral weights γ_i and complementarity parameter ϵ from

$$\log(p_i Y_i / p_I Y_I) = \frac{1}{\epsilon}(\gamma_i/\gamma_I) + \frac{\epsilon - 1}{\epsilon}\log(Y_i/Y_I), \text{ for } i = 1, \cdots, I-1.$$

This system of equations is estimated by iterated feasible generalized nonlinear least squares method. To reflect constraints on the parameters ($\epsilon > 0$ and $0 < \gamma_i < 1$), we estimate the unconstrained coefficents b and c_i's in

$$\log(p_{i,t}Y_{i,t}/p_{I,t}Y_{I,t}) = (1+e^b)c_i + e^b \log(Y_{i,t}/Y_{I,t}) + \varepsilon_{i,t},$$

where $\epsilon = 1/(1+e^b)$ and $\gamma_i = e^{c_i}/(1+\sum e^{c_i})$.

Each sector i in the model consists of several industries in the BEA Industry Accounts, to which we apply the Törnqvist index to obtain the price index of sector i. Real quantities Y_i are similarly aggregated up from the detailed BEA data. The aggregate price index is normalized to 1 in 1963, the initial year in the data. The sample period for the estimation

Table 2.3: Estimation results

Parameters	Estimates
ϵ	0.765^{***} (0.002)
γ_1	0.084^{***} (0.001)
γ_2	0.159^{***} (0.002)
γ_3	0.099^{***} (0.003)
γ_4	0.124^{***} (0.002)
γ_5	0.142^{***} (0.001)
γ_6	0.087^{***} (0.002)
γ_7	0.057^{***} (0.002)
γ_8	0.094^{***} (0.003)
γ_9	0.117^{***} (0.002)
AIC	-1001.432

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

covers 1980 to 2010, which is our main interest. The point estimates for ϵ and γ_i are presented in Table 2.3.

Parameters calibrated without simulation In the calibration, we fix the traditional capital share of *only* the computer industry (α_I) from the data. Though computing the total capital share is straightforward (i.e., 1 minus labor share), computing the traditional capital share according to our model is not. To obtain this number for the computer industry, we follow Koh et al. (2016), which we briefly describe below.

We begin by specifying an empirical no-arbitrage condition for rental prices. The return on both types of capital must be equal to the interest rate $1 + r'$, so

$$[R'_K + (1 - \delta'_K)p'_K]/p_K = [R'_S + (1 - \delta'_S)p'_I]/p_I \tag{2.17}$$

where p_K is the price of traditional capital and p_I the price of computers. Note that this is different from the model's no-arbitrage condition (2.7) in that we have included the price of capital, which in the model we had normalized to be equal to the price of the final

Parameters	Value	Obtained from
σ	0.815	Mean absolute distance of the changes in the employment share
$r + \delta_S$	0.300	Average depreciation rate of computer capital from FAT

Table 2.4: Calibrated Parameters

consumption good. Next, since sectoral production is HD1 in all factor inputs (traditional and computer capital, and labor), for the computer industry we have

$$1 - \text{labor share}_I = \frac{R_K K_I}{p_I Y_I} + \frac{R_S S_I}{p_I Y_I}.$$

We solve for R_K and R_S from these two equations assuming a steady state ($R'_K = R_K, R'_S = R_S$ and $p_K = p'_K, p_I = p'_I$), plugging in for all other variables using data on the quantities, prices and depreciation rates of each type of capital (from BEA FAT Nonresidential Estimates by Industry and Type); and the computer industry's real and nominal value-added, and its labor share (from BEA Industry Accounts).[23] Once we know R_S, we can set $\alpha_I = R_S S_I / p_I Y_I$ since all other variables are recovered directly from the data. We rely only on data from 1980.

Although the above procedure can be used for all industries, in our calibration we only use it to compute the computer industry's traditional capital share. All other industries' traditional capital shares are calibrated directly from the model as explained below. Appendix Figure B1(a) compares the traditional capital shares obtained using the above procedure against those predicted by the calibration, which confirms that they are generally consistent.

Method of Moments The rest of parameters are recovered from simulating model moments to match corresponding data moments. To be precise, we plug the data for L_{ij} (from the IPUMS Census), and (k_i, s_i) (from the BEA FAT Nonresidential Estimates by Industry

[23] We take the weighted average across industries 334 and 511 (software and hardware) to obtain this value for the computer industry, which in our quantitative model comprises both. For each industry, computer capital is the sum of the net stock of "computers and peripheral equipment" and "software."

and Type) directly into the equilibrium equations, assuming a steady state in both 1980 and 2010, respectively. The detailed procedure is as follows:

1. Guess σ.

   (a) Fix α_I as above, and guess $A_{I,1980}$ and ρ_i's.

      i. For 1980: obtain $(\nu_{ij}, \omega_i, \alpha_i, A_{i,1980})$ given guess.

      - Normalize $M_j = 1$ for all j. Then the industry-specific occupation weights ν_{ij}'s and $\tilde{V}_i$ are recovered from (2.9)-(2.10) using data on 1980 employment shares..

      - From (2.12a) of industry I, and replacing for y_i using (2.1) and z_i using (2.11), ω_I must solve

$$R_S = (1-\alpha_I) \cdot A_I k_I^{\alpha_I} \cdot \left[\omega_I^{\frac{1}{\rho_I}} s_I^{\frac{\rho_I - 1}{\rho_I}} + (1-\omega_I)^{\frac{1}{\rho_I}} \tilde{V}_I^{\frac{\rho_I - 1}{\rho_I}} \right]^{\frac{1-\rho_I \alpha_I}{\rho_I - 1}} \cdot (\omega_I / s_I)^{\frac{1}{\rho_I}},$$

        given data on k_I and s_I in 1980. The solution $\omega_I \in (0,1)$ if $1 < (1-\alpha_I) A_I (k_I/s_I)^{\alpha_I}$.

      - Given ω_I, obtain all other ω_i's from (2.13) (since $V := (1-\omega_i)\tilde{V}_i^{\rho_i - 1}$).

      - For all $i \neq I$, compute α_i's from (2.15) by replacing for z_i using (2.14), and plugging in data on (k_i, s_i).

      - Exogenous sector-specific productivities $A_{i,1980}$'s are recovered from (2.8) and $A_{I,1980}$.

      ii. For 2010: obtain $M_{j,2010}$ and updated guesses for the substitutability between computers and workers, ρ_i^{new}.

- Choose the M_j's that yields the best fit of (2.10) across all i given 2010 employment shares:

$$\frac{M_j}{M_1} = \left[\sum_i \left(L_i \cdot \frac{L_{ij}}{L_{i1}} \cdot \frac{\nu_{i1}}{\nu_{ij}}\right)\right] \Big/ \sum_i L_i,$$

  Using this we can compute $\tilde{V}_i$ for 2010 using (2.9).

- From (2.15), we set ρ_I^{new} to get the best fit of

$$\rho_I^{new} \cdot I = \sum_i \left\{ \frac{\log(\omega_I \tilde{V}_I) - \log((1-\omega_I)s_I)}{\log\left[\left(1 - \frac{\alpha_I(1-\alpha_i)k_i}{\alpha_i(1-\alpha_I)k_I}\right)\tilde{V}_I\right] - \log\left(\frac{s_I\alpha_I(1-\alpha_i)k_i}{\alpha_i(1-\alpha_I)k_I} - s_i\right)} \right\}$$

  given data on (k_i, s_i) in 2010.

  Note that we need $s_i/s_I < (1-\alpha_i)\alpha_I k_i/(\alpha_i(1-\alpha_I)k_I) < 1$ or $s_i/s_I > (1-\alpha_i)\alpha_I k_i/(\alpha_i(1-\alpha_I)k_I) > 1$ for ρ_I^{new} to be a real number. We exclude those industries with (k_i, s_i) for which this condition is not satisfied *only* when we compute ρ_I^{new}.

- Compute the implied ρ_i^{new}'s that are consistent with the 2010 s_i's, i.e.,

$$\rho_i^{new} = \frac{\rho_I^{new} \log\left(\frac{1-\omega_i}{\omega_i s_i \tilde{V}_i}\right)}{\rho_I^{new} \log\left(\frac{\tilde{V}_I}{\tilde{V}_i}\right) + \log\left(\frac{1-\omega_I}{\omega_I s_I \tilde{V}_I}\right)}$$

(b) Iterate over ρ_i's till $\rho_i \approx \rho_i^{new}$.

(c) Set $A_{I,1980}$ so that y_I equals the computer industry's real value-added per worker in the data. Iterate over $A_{I,1980}$ till convergence.

2. Iterate over σ to minimize $\sum_j |\ell^d_{j,2010} - \ell^m_{j,2010}|$, where ℓ_j is the employment share of occupation j.

In the outermost loop of the above procedure, note that we use occupation employment shares *in aggregate*. The industry-specific occupation weights ν_{ij}'s were recovered only from *within industry* employment shares by occupation. Once we have recovered all the parameters,

1. Get $A_{i,2010}$'s to match measured productivity by sector in (2.16) to 2010 data.[24]

2. Between 1980 and 2010, we assume that the $M_{j,t}$'s, and all $A_{i,t}$'s except A_I, grow at constant rates, so:

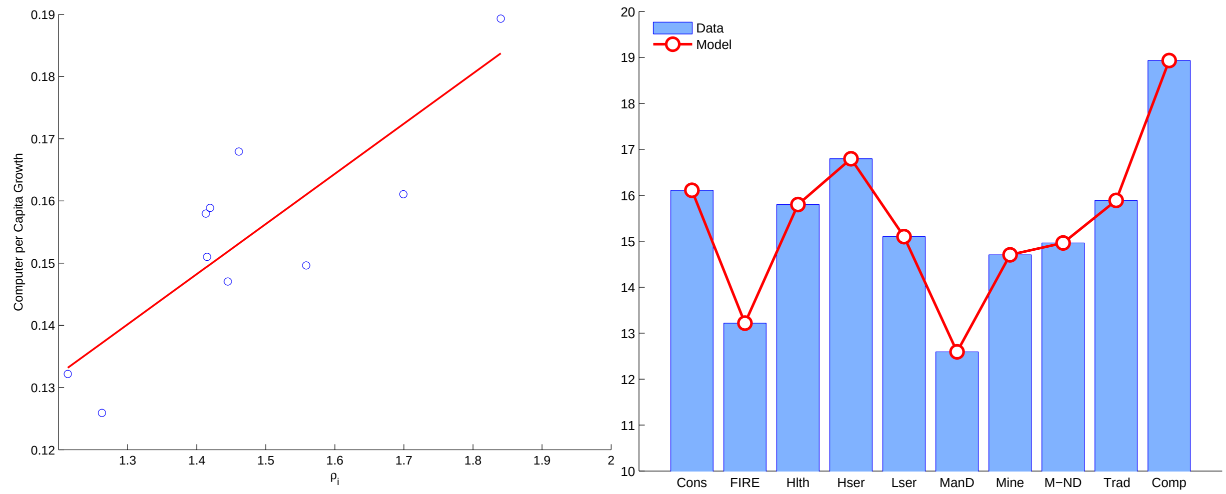
$$M_{j,t} = M_{j,1980}(M_{j,2010}/M_{j,1980})^{(t-1980)/30},$$
$$A_{i,t} = A_{i,1980}(A_{i,2010}/A_{i,1980})^{(t-1980)/30}.$$

3. The computer sector's exogenous productivity (A_I) for other years are chosen so that the sectoral productivity of the computer sector in (2.16) is equal in the data and model.

2.4.2 Properties of the Benchmark Model

The calibration results are summarized in Tables 2.5 to 2.7. Since changes in M_j affect occupational employment across all industries, we can identify occupation-specific productivities separately from the sector-specific productivities. Specifically, occupational employment data alone gives enough information to identify the M_j's, from Equation (2.10). Given this, we can identify the sector-specific A_i's to fit measured sectoral productivities from the data using (2.16). The calibrated values for M_j's show that routine intensive occupations, such as machine operators or mechanics, indeed experienced much faster growth in their

[24] We compute traditional and computer capital income shares, and measure sectoral productivity directly from the data. Hence, the model's sectoral output may differ from the data.



(a) Data against computer-worker substitutability ρ_i

(b) Computer capital per worker

Figure 2.6: Computer per worker growth between 1980 and 2010

Source: BEA Industry Accounts and FAT. See Table 2.1 for details of the industry classification. Computer capital is measured as the sum of "computers and peripheral equipment" and "software" by industry, available in FAT Table 3.1.

occupation-specific productivities. And as expected, the sector-specific productivity of the computer industry (A_I) grew exceptionally fast especially during the 1990s.

It is also noteworthy that the ρ_i's are identified from how computer capital per worker (s_i) and traditional capital per worker (k_i) evolve differently across industries. Roughly speaking, when an industry that increases computers per worker more than other industries also uses more traditional capital per worker, the elasticity of substitution ρ_i tends to be greater than one (Equation 2.15). But since traditional capital is a constant share of production in our model, our model admits $\rho_i > 1$ for sectors whose output per worker increases with computers per worker. Since this is indeed the case for most industries in the data, as we saw in Figure 2.5, all calibrated ρ_i's are larger than 1.[25] This also implies that computerization leads to a decline in the labor share both at the sector and aggregate levels.

[25] Figure 2.5 shows that some small industries have a negative relationship in the data, but this is no longer the case once we aggregate the 60 industries into 10 more broadly defined sectors.

Param/Target		Const	FIRE	Health	High serv.	Low serv.	Dur	Mine	Non-durable	Trade	Comp-uter
γ	outside	0.084	0.159	0.099	0.124	0.142	0.087	0.057	0.094	0.117	0.037
ρ	$s_{i,2010}$	1.699	1.213	1.413	1.461	1.415	1.263	1.445	1.559	1.419	1.840
ω	$s_{i,1980}$	0.001	0.094	0.003	0.025	0.006	0.028	0.020	0.009	0.008	0.020
α	$k_{i,1980}$	0.167	0.374	0.301	0.454	0.475	0.333	0.793	0.402	0.186	0.322

Table 2.5: Industry specific parameters

Industry weights γ_i and the computer industry's traditional capital income share α_I are estimated directly from the data using the BEA Industry Accounts and FAT, while the rest are calibrated according to a method of moments. See text for details.

In turn, sectors with higher computer per worker growth would also have higher values of ρ_i, as in Figure 2.6. This is illustrated in Figure 2.6(a), which plots computer per worker growth in the data against the ρ_i's. While panel (a) makes it clear how the relative values of ρ_i are identified across sectors, note that the relationship is not exactly linear, even though the model fits computer capital per worker exactly by assumption as shown in panel (b)—since their empirical values are directly fed into step 1.ii of our calibration. This is because computers are not substituting labor directly, but only indirectly through the occupation composite X_i.[26]

Model Fit The model-implied employment share changes fit the data better by occupation than by industry (Figure 2.7). This is because the M_j's directly affect occupational employment through (2.10), and once we match sectoral productivity growth by industry using (2.16), employment by industry is pinned down by (2.8).

This is also an indirect consequence of assuming constant σ's across all industries. Note that nowhere in our calibration did we separately target 2010 traditional capital per worker, nor employment share changes by industry. Our calibration step 1.ii and Equation (2.15)

[26]Related, since computers substitute a composite of labor rather than each occupation separately, the values of the substitutability parameters ρ_i's are potentially sensitive to σ, which measures the complementarity across occupations. We find that this is not the case for a wide range of values for σ lower than its benchmark value, as shown in Appendix Table B1. While ρ_i's are sensitive to much larger values of σ, then it becomes impossible to fit other moments in the data (employment shares and TFP by industry).

	L serv.	Admin.	Mach	Sales	Trans	Tech	Mech	Mine.	Prof.	Mngm
Const	0.009	0.058	0.027	0.009	0.218	0.016	0.564	0.015	0.021	0.061
FIRE	0.048	0.444	0.005	0.225	0.015	0.014	0.013	0.004	0.021	0.211
Health	0.328	0.172	0.005	0.004	0.006	0.122	0.009	0.011	0.293	0.050
H serv.	0.109	0.222	0.010	0.020	0.022	0.043	0.037	0.007	0.420	0.110
L serv.	0.375	0.143	0.025	0.041	0.129	0.012	0.080	0.023	0.070	0.101
Durable	0.022	0.115	0.372	0.021	0.102	0.027	0.081	0.136	0.049	0.076
Mining	0.017	0.103	0.047	0.010	0.195	0.051	0.121	0.311	0.065	0.080
Non-dur	0.028	0.118	0.386	0.039	0.135	0.023	0.050	0.106	0.036	0.079
Trade	0.025	0.150	0.022	0.406	0.152	0.005	0.066	0.042	0.022	0.110
Computer	0.016	0.165	0.310	0.059	0.042	0.062	0.041	0.070	0.124	0.111

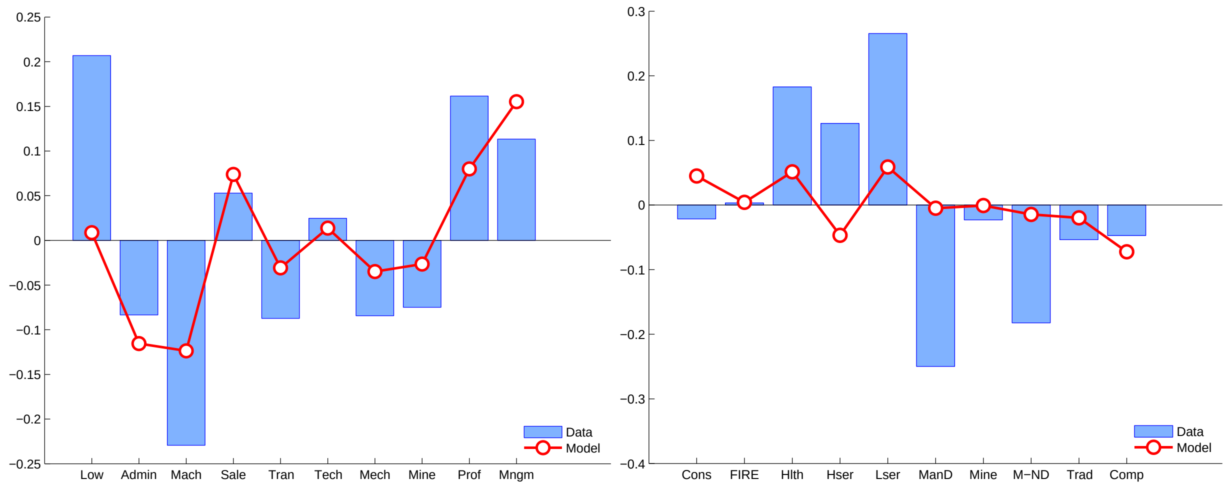
Table 2.6: Industry-occupation specific weights on labor (ν_{ij})

Calibration results for ν_{ij} from a method of moments. Empirical targets are within-industry employment shares by occupation in 1980.

Target: emp. share by ind. and occ. in 2010					Target: measured productivity in 1980 and 2010				
M_j	1980	1990	2000	2010	A_i	1980	1990	2000	2010
Low serv.	1.000	1.000	1.000	1.000	Const	14.125	10.394	7.648	5.628
Admin.	1.000	1.384	1.914	2.649	FIRE	17.924	17.267	16.633	16.023
Machine	1.000	2.273	5.168	11.749	Health	6.155	6.460	6.780	7.115
Sales	1.000	0.590	0.348	0.205	High serv.	1.385	1.624	1.904	2.232
Trans	1.000	1.263	1.595	2.014	Low serv.	0.050	0.053	0.057	0.060
Tech	1.000	0.736	0.542	0.399	Durable	0.198	0.191	0.185	0.179
Mechanics	1.000	1.610	2.591	4.171	Mining	3.048	3.104	3.161	3.219
Mine.	1.000	1.444	2.085	3.010	Non-durable	0.701	0.708	0.716	0.724
Prof.	1.000	0.553	0.306	0.169	Trade	0.269	0.373	0.516	0.714
Mngm	1.000	0.461	0.212	0.098	Computer	1.945	3.667	13.624	26.618

Table 2.7: Occupation- and sector-specific productivity

Occupation-specific productivities are normalized to 1 in 1980. For 2010, we minimize the distance between the model and data on within-industry employment shares by occupation averaged across all industries in the IPUMS Census. The computer industry's 1980 sector-specific productivity is chosen to minimize the distance between model and data on its real value-added per worker in the BEA Industry Accounts, while all other industries' productivities are implied by the model and data on capital and labor data relative to the computer sector from the Industry Accounts and FAT. All sector-specific productivities in 2010 are recovered from our expression for sectoral productivity in (2.16), using the Industry Accounts data and our calibrated parameters. Except for the computer sector-specific productivity A_I, all A_i's are assumed to grow at a constant rate from 1980 to 2010.



(a) By occupation (b) By industry

Figure 2.7: Changes in employment shares between 1980 and 2010
Data Source: occupation data are from IPUMS Census, and industries from BEA Industry Accounts. See Table 2.2 for details of the occupation classification.

exploit all three factors at once, per industry, using only data on 2010 computer capital per worker by industry. This makes it clear that we can only let one of ρ or σ vary by sector.[27] Both would affect how factor input ratios, and in particular computer capital per worker s_i, change across sectors in response to changes in M_j's. But one of our major goals is to quantitatively compare how aggregate productivity is affected by complementarity across occupations (shifts in M_j through σ) relative to complementarity across industries (shifts in A_i through ϵ). How to implement such a comparison becomes less obvious if σ's vary across sectors.

More important, letting the elasticity of substitution between computers and labor (ρ_i) vary across sectors directly captures how computer capital per worker evolves differentially across sectors, as we discussed above. If we were to instead let σ vary, the effect is only indirect since computer-labor substitution would differ across sectors only due to differential shifts in relative labor demand. That is, unlike the clear relationship between ρ_i and the

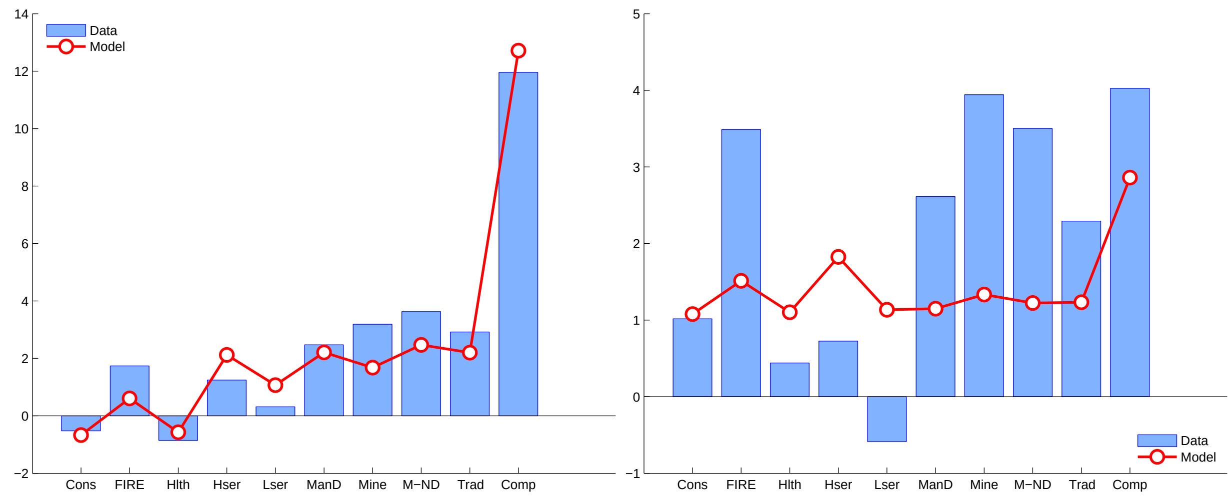
[27]Since the V_i's are functions of σ.

growth of computer per worker s_i as seen in Figure 2.6(a), there would be no systematic relationship between σ and s_i since it would also depend on the sector-specific occupation weights ν_{ij}'s.

Thus, our exact fit to computer capital per worker growth, to some extent, comes at the expense of a lesser fit to employment share changes and traditional capital per worker growth by industry. See Figures 2.7(b) and 2.8(b). This indicates that the unit elasticity assumption between traditional capital and other factors, and also the assumption that the elasticity is constant across sectors, may be too stringent. Still, both changes in employment shares and traditional capital per worker by industry are qualitatively consistent with the data.

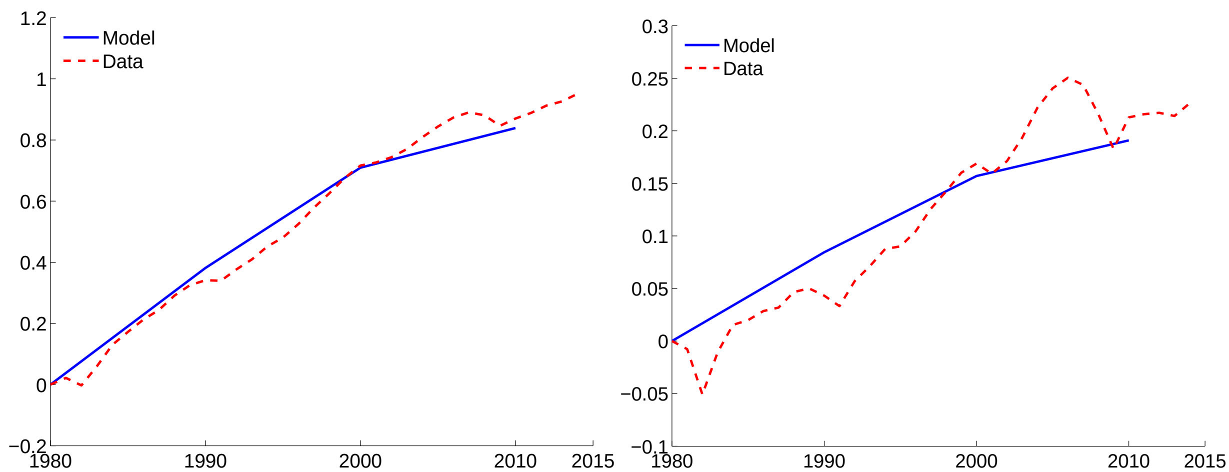
More assuringly, even though we did not use any data on output per worker growth—neither by industry nor in aggregate—nor aggregate productivity, the model prediction of output per worker growth by industry is remarkably close to the data, Figure 2.8(a). Most importantly for our purposes, the model generates a slowdown in aggregate output and productivity growth starting in 2000, similarly as in the data, as shown in Figure 2.9 and tabulated in Appendix Table B2. The fit to aggregate productivity is especially remarkable considering that we assume constant productivity growth rates for M_j and A_i—other than A_I—and do not target any aggregate variables in 2010.

Lastly, the model-implied factor income shares by industry are also generally consistent with the data (Appendix Figure B1). Partly because of this, the aggregate labor share in the model closely tracks the trend in the data, both in direction and magnitude (Figure 2.10), despite not being targeted at all at the sectoral nor aggregate levels. Recall that our production technology assumes that traditional capital's income share is constant by construction. Thus, our results suggest that computer hardware and software, which are a



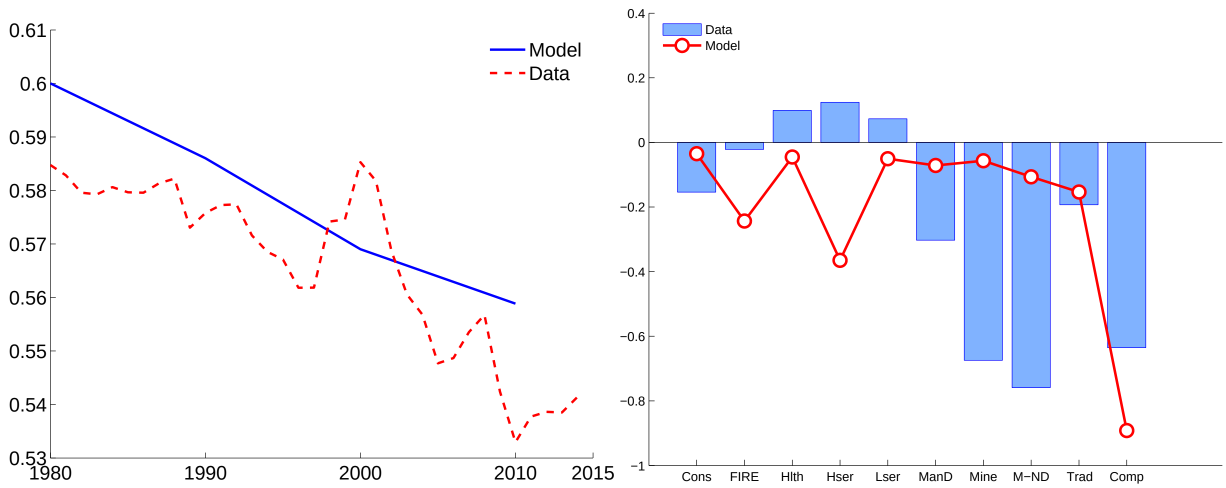
(a) Output per worker (b) Tradition capital per worker

Figure 2.8: Log changes of **y** and **k** between 1980 and 2010
Data Source: BEA Industry Accounts and FAT. See Table 2.1 for details of the industry classification.



(a) Output (b) Productivity

Figure 2.9: Aggregate production
Data Source: BEA NIPA. Exact numbers for the plots are tabulated in Appendix Table B2.



(a) Aggregate (b) By industry

Figure 2.10: Changes in labor share: model vs. data
Data Source: BEA NIPA and Industry Accounts. See Table 2.1 for details of the industry classification.

subset of total capital that accounts for 14 percent of all investment, can be responsible for the vast majority of the fall in the labor share (4 out of 5 percentage points) since 1980.[28]

2.4.3 Counterfactual Analysis

In this section, we investigate the underlying factors that shape aggregate output and productivity, focusing on routinization and computerization. Routinization in our model is a faster increase in the occupation-speciifc productivity, M_j, of certain occupations. Computerization is driven by the computer industry-specific term, A_I, which propagates through all industries because computer capital is used in the production of all industrial goods.

In our model equilibrium, this propagation happens by shifting the price of computer capital. High A_I shrinks the computer sector employment because of complementarity, but

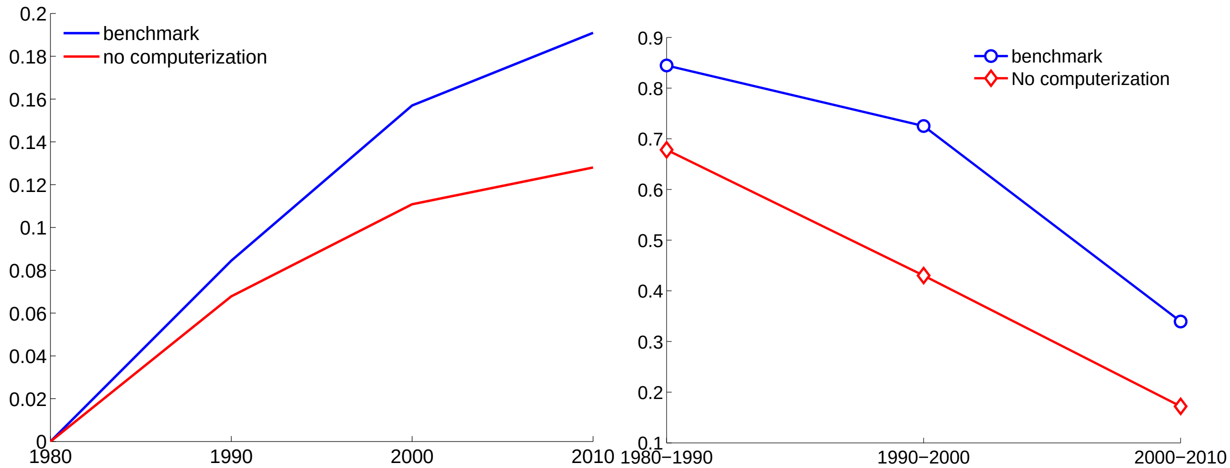
[28] As a direct consequence of not fitting capital per worker growth by sector, the model fit to the fall in labor shares by sector is poorer than in aggregate. Aggregate capital per worker k in the data is directly fed into the model.

also lowers the relative price of computers. This, in turn, leads to a drop in the rental rate of computer capital, which induces all sectors to use more computers. This prevents the computer sector from shrinking.

Aggregate productivity Note that the growth rates of occupation- and sector-specific productivities (M_j and A_i) were assumed to be constant for the entire sample period except for the computer sector's (A_I). Nonetheless, in the benchmark calibration, aggregate TFP increases almost linearly from 1980 to 2000, slowing down in the last decade (Figure 2.11).[29] We now show that the high growth rate of the computer sector-specific productivity (A_I) prevented a potential slowdown in aggregate productivity that would have appeared between 1990 and 2000. Figure 2.11 shows that, if we assume A_I were constant between 1980 and 2010, aggregate productivity growth would have slowed down since 1990. Without the growth in A_I, aggregate productivity would have grown by only 13 percent from 1980 to 2010, one-third lower than the benchmark growth rate of 20 percent over the same period. This magnitude is surprising considering the fact that the computer sector's share of aggregate output is only 3 to 4 percent throughout the observation period.

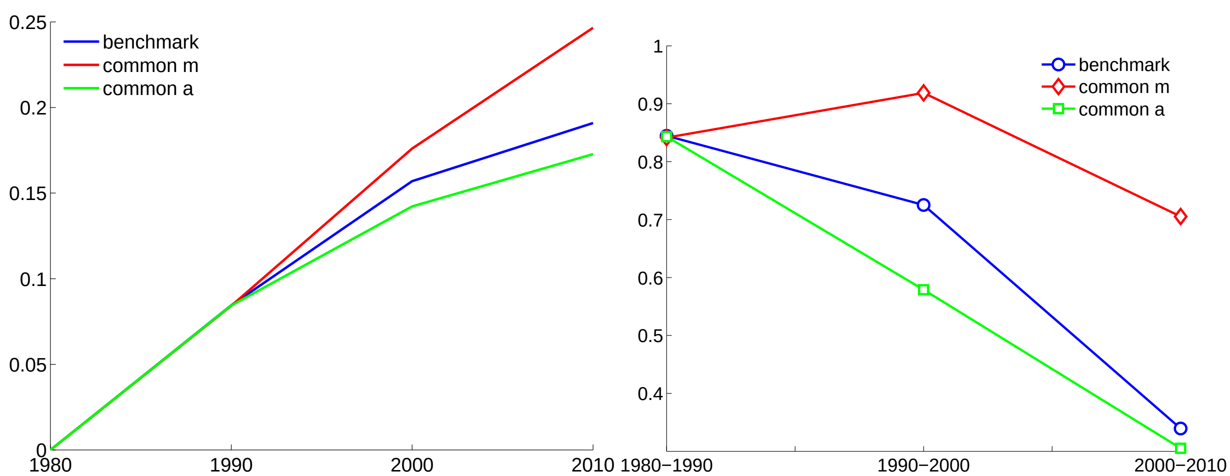
When all occupation- and sector-specific productivities grow at constant rates over time, complementarity across jobs and sectors induces the faster growing jobs and sectors to shrink in relative size, reducing their weights in the computation of aggregate productivity. Hence, as long as occupation- and sector-specific productivities grow at different rates, aggregate productivity growth must slow down over time. So both the dispersions in the growth rates of occupation-specific productivities (M_j) and in sector-specific productivities (A_i's) contribute to the aggregate productivity slowdown. To find out which dispersion is more important for the slowdown, we conduct the following exercises.

[29] Aggregate productivity growth is measured as $d\log(y)$ − (traditional capital share) $\cdot\ d\log(k)$ − (computer share) $\cdot\ d\log(s)$.



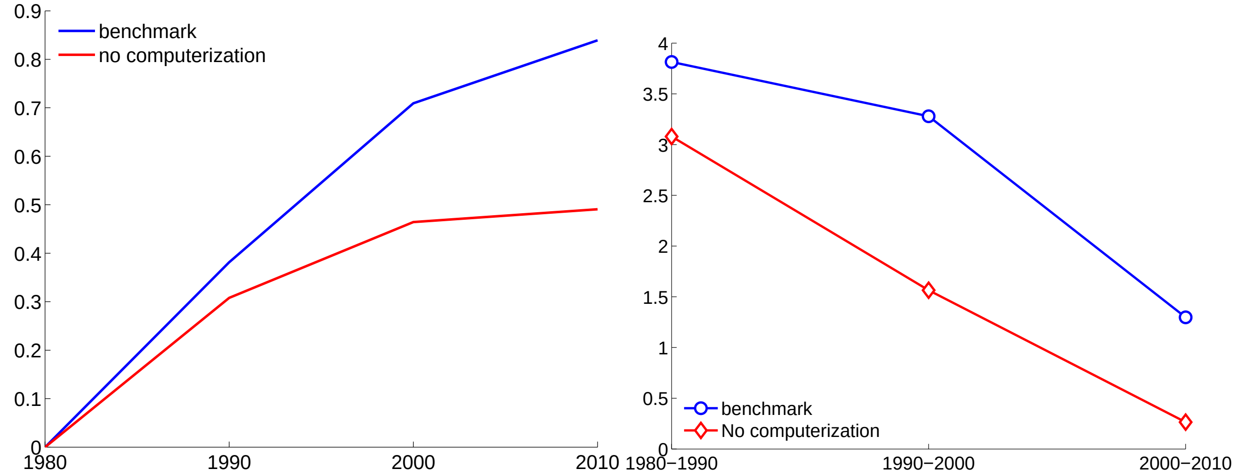
(a) Log level (b) Growth rate

Figure 2.11: Aggregate Productivity without Computerization



(a) Log level (b) Growth rate

Figure 2.12: Aggregate Productivity without Complementarity



(a) Log level

(b) Growth rate

Figure 2.13: Aggregate Output without Computerization

In the first exercise, we force all M_j's to grow at the same rate m for all j (i.e., no routinization) while leaving the growth rates of A_i's to be different from one another as in the benchmark. Second, we force all A_i's to grow at a common rate a while leaving the growth rates of M_j's heterogeneous as in the benchmark. The common growth rates m and a are set so that aggregate productivity grows at the same rate as in the first decade of our benchmark calibration. The results are shown in Figure 2.12, which shows that routinization, or the dispersion in the growth rates of M_j, is more important in explaining the decline in the growth rate of aggregate productivity. Without routinization, the growth rate of aggregate productivity remains near 0.8 percent per year throughout the three decades. In contrast, even when all sector-specific productivities grow at a common rate, aggregate productivity growth falls almost as much as in the benchmark. Of course for the latter exercise, we are also ruling out the faster growth of the computer sector, which partially explains the gap between the benchmark growth rate and this counterfactual growth rate in the 1990s.

Output Fast-growing computer sector-specific productivity directly boosts aggregate productivity, which leads to an acceleration of aggregate output growth. Furthermore, there is

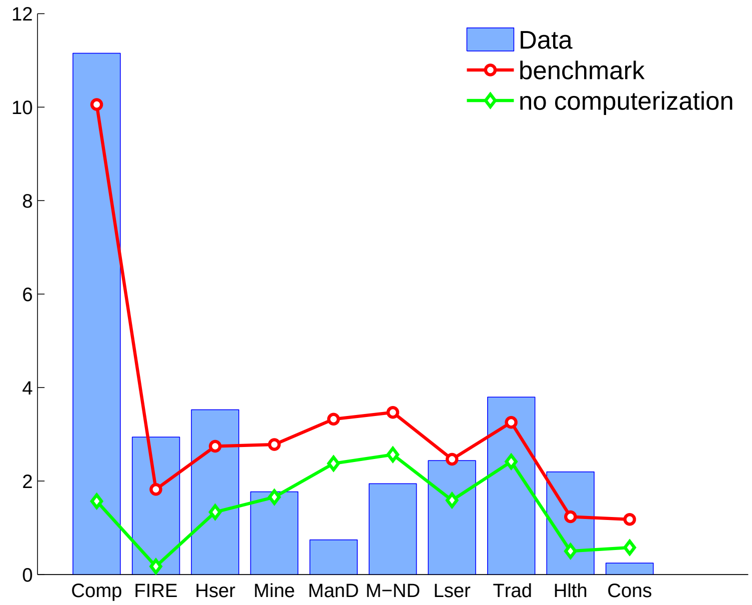


Figure 2.14: Output Growth by Industry without Computerization
In panel (a), we plug in model-simulated income shares and quantities into the accounting equations in (2.18). In panel (b), we plug in data from NIPA and FAT directly. See Table 2.1 for details of the industry classification.

an additional effect on aggregate output, since all sectors use more computer capital. Figure 2.13 shows the total effect of computerization on aggregate output. If A_I were to remain constant between 1980 and 2010, aggregate output growth from 1980 to 2010 would be 63 percent, or only about half of the growth in the benchmark. As expected, this is a larger impact than that on aggregate productivity.

Figure 2.14 shows output growth by industry with and without A_I growth. Due to the substitutability between computer and labor, all industries benefit from computerization. Unsurprisingly, the computer industry itself is affected the most, followed by finance and high-skilled services. The construction industry has the least to gain (in terms of output growth) from computerization.

Labor share Because the model calibration yields sector-specific elasticities of substitution between labor and computer capital (ρ_i) that are larger than 1, computerization results

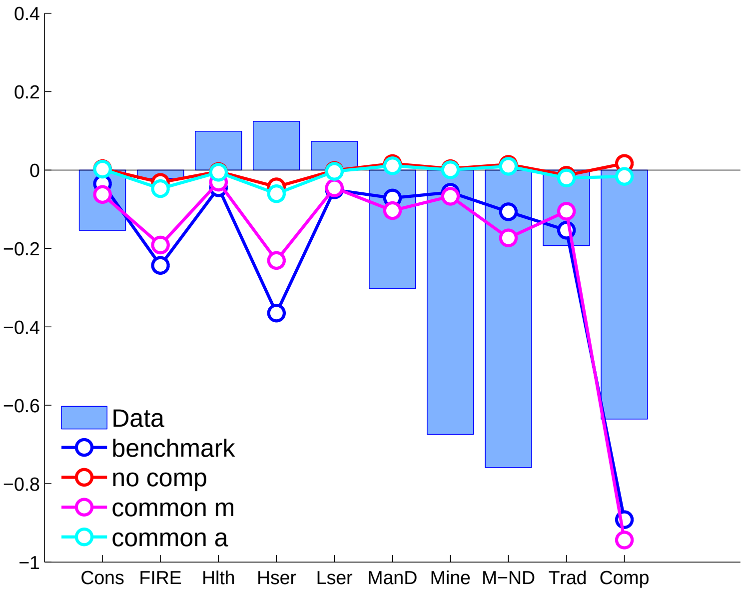


Figure 2.15: Changes in Labor Income Shares by Industry
"no comp": no computerization. "common m": no complementarity across jobs. "common a": no complementarity across industries. See Table 2.1 for details of the industry classification.

in the decline of labor shares in all sectors. Figure 2.15 shows changes in labor shares by industry for various counterfactual exercises. Among all these exercises, the only two that affect labor shares are when we eliminate computerization either explicitly (in red); or by assuming common growth rates across all industries (in sky-blue). So we can conclude that the growth in A_I is the only important driving force behind the decline of the labor share.

Computer capital in the measurement of TFP In our benchmark, we measured aggregate productivity growth between times s and t as follows:

$$\log(A_t/A_s) = \log(Y_t/Y_s) - \frac{1}{2}\left(\frac{LI_t}{Y_t} + \frac{LI_s}{Y_s}\right)\log(L_t/L_s) - \frac{1}{2}\left(\frac{SI_t}{Y_t} + \frac{SI_s}{Y_s}\right)\log(S_t/S_s)$$
$$- \frac{1}{2}\left(\frac{KI_t}{Y_t} + \frac{KI_s}{Y_s}\right)\log(K_t/K_s), \tag{2.18a}$$

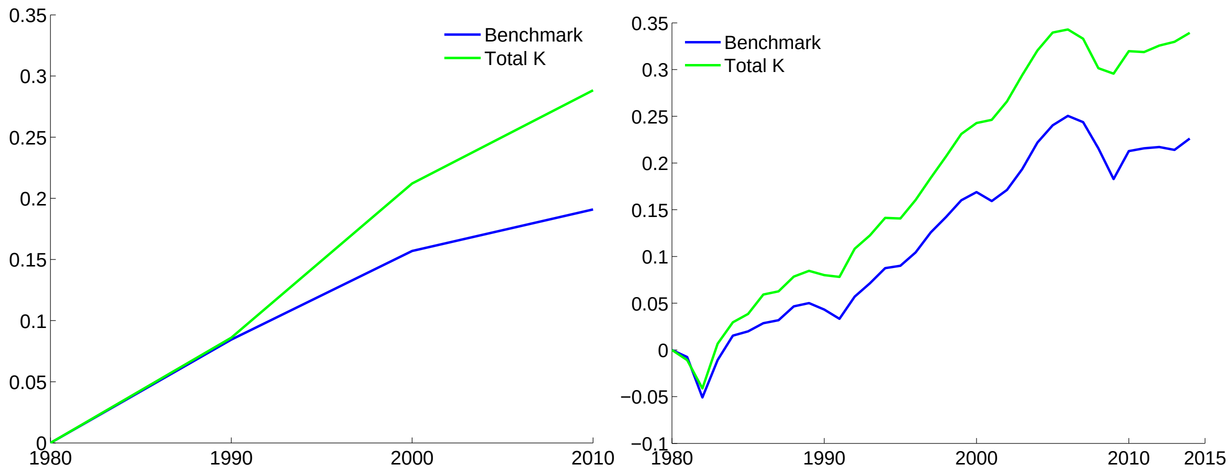
where LI is labor income, SI is computer capital income, and KI is traditional capital income. But typically, the standard way we compute TFP growth (the Solow residual $\hat{A}$) is

$$\begin{aligned}\log(\hat{A}_t/\hat{A}_s) = & \log(Y_t/Y_s) - \frac{1}{2}\left(\frac{LI_t}{Y_t} + \frac{LI_s}{Y_s}\right)\log(L_t/L_s) \\ & - \frac{1}{2}\left(\frac{KI_s + SI_t}{Y_t} + \frac{KI_s + SI_s}{Y_s}\right)\log[(K_t + S_t)/(K_s + S_s)]. \end{aligned} \tag{2.18b}$$

Note that A_t and $\hat{A}_t$ can differ, especially when the gross rate of return on computer capital and traditional capital are different. By inspection of (2.17), we see that this happens when either the investment prices and/or the depreciation rates of the two types of capital differ. In particular, the gross rate of return on computer capital is generally higher than traditional capital because the former depreciates more quickly. This implies that the standard way of computing TFP without separating out computer capital will *overestimate* the growth rate of aggregate productivity.

In Figure 2.16, we compare aggregate productivity from our benchmark calibration (A) against the TFP (standard Solow residual, $\hat{A}$), both according to our model (panel a) and in the data (panel b). For panel (a), we plug in our model-simulated data into (2.18). For panel (b), we impute all variables needed in (2.18) directly from the data. The figure confirms that the aggregate productivity growth is overestimated by about 10 percentage points over the past 30 years if computer capital is not explicitly separated, both in the data and also according to our model.

Summary of quantitative analysis There are two main findings from our quantitative analysis. First, constant occupation- and sector-specific technological progress necessarily slows down aggregate productivity growth over time, given complementarity across jobs and industries. Second, it was the dispersion in the growth rates across occupations (i.e., routinization) that was most responsible for the aggregate productivity slowdown. This



(a) Model (b) Data

Figure 2.16: Comparing different measures of TFP's

In panel (a), we plug in model-simulated income shares and quantities into the accounting equations in (2.18). In panel (b), we plug in data from NIPA and FAT directly.

negative impact of routinization on the growth rate of aggregate productivity was more or less perfectly counterbalanced by the impressive technological progress specific to the computer industry and its spillover through inter-industry linkages during the 1980s and the 1990s. The slower pace of the computer sector's productivity growth in recent years—and the associated deceleration of computer usage by other industries since 2000—is finally revealing the negative impact that decades of routinization has had on aggregate productivity growth.

2.5 Concluding Remarks

We presented a model in which productivities grow at heterogeneous rates across occupations (routinization), and also across industries. In particular, to understand the effect of the rise of the computer industry on aggregate productivity, we let its output be used in the production of all industries as a distinct type of capital.

We showed that when occupations and industries are complementary to one another and occupation- and sector-specific productivities grow at different rates, routinization in particular causes a slowdown in aggregate productivity. But such a slowdown was averted prior to the 2000s in the U.S., thanks to the rapid rise of the computer industry's productivity. It was only after the productivity of this sector slowed down that routinization began to reveal its negative impact on aggregate productivity growth.

The main message of our model is that multiple layers of the economy (i.e., occupations and sectors) can interact to generate interesting time trends that can help us reconcile evidence at the occupation and sector levels with aggregate trends. Moreover, we have also highlighted the importance of inter-industry linkages by showcasing that a single industry—in our case the computer industry—can have large effects on aggregate variables once such a propagation mechanism is taken into account.

In reality, all industries are interlinked, not only by providing intermediate inputs to one another as emphasized in some recent models (Acemoglu et al., 2012; Atalay, 2017) but also by serving different types of capital in which all industries need to invest (as we have modeled here). Modeling such additional layers of complexity is left for future research.

Chapter 3

Growth Facts with Intellectual Property Products: An Exploration of 31 OECD New National Accounts

This chapter was coauthored with Dongya Koh and Raül Santaeulália-Llopis.

3.1 Introduction

In 2009, the United Nation Statistical Commission adopted the new System of National Accounts (SNA) from 2008.[30] The most notable update in the new system is the capitalization of (some) intangibles in national accounts which recognizes the growting importance of intangibles in the economy. In SNA08, the intangible capital measured by the national accounts is labeled as intellectual property products (IPP). To be precise, the set of IPP

[30] *European Commission, International Monetary Fund, Organisation for Economic Co-operation and Development, United Nations, and World Bank,* System of National Accounts 2008 (New York: 2009)

measured by national accounts includes R&D and artistic originals, in addition to computer software introduced since SNA 1993. By 2016, most OECD countries have implemented the new system.[31]

We construct a new data set using new national accounts for 31 OECD countries that have implemented SNA08. We then use these database to document the secular behavior of economic growth and the big ratios (à la Kaldor (1957) and Jones (2016)) for these countries. We find 1) a decline of labor income share, 2) a rise of capital-output ratio, and 3) a rise of the rate of return to capital. We show that the new secular behavior of the big ratios that we document is entirely driven by the reclassification of IPP from expense to capital. In particular we show that treating IPP as expense, as in the pre-SNA93 accounting framework, we would find a relatively trendelss labor income share, capital-to-output ratio, and rate of return.

The main accounting assumption behind the capitalization of IPP implemented by national accounts is that all IPP rents are attributed to capital.[32] Specifically, the increase in IPP investment on the national products accounts is moved to gross operating surplus (hence, capital income) on the national income accounts. We argue that this accounting assumption that follows SNA08 guidelines is arbitrary and extreme. Indeed, we show that the assumption that all IPP rents are capital income is crucial in generating the new facts. Once we relax this assumption based, for example, on the cost structure of R&D (as in Koh et al. (2018)), we go back to the familar secular behavior of the big ratios in the pre-SNA93 accounting framework.

[31] A few exceptions are Turkey, Chile, and Japan.

[32] See a detailed description of the capitalization IPP implemented by the Bureau of Economic analysis (BEA) in the U.S in Koh et al. (2018).

The introduction of IPP as capital in national accounts poses some challenges for measurement that are not present for tangible capital (i.e., structures and equipment) (Corrado et al., 2005; McGrattan and Prescott, 2005). Indeed, although it is loable that IPP is treated as capital in national accounts given the long-run nature of its provided services, it is unclear what are the best assumptions behind the capitalization. First, most IPP is simply unobserved. Even within the context of the IPP items incorporated in national accounts (which are arguably better measured), a large part of their production (such as software or R&D) is conducted in-house without observable transactions for their valuation and pricing. Currently the national accounts measure this own account production based on costs (plus made-up nonmarket markups). Second, it is not obvious how to preserve the product-income identity in the presence of intangibles. Currently the national accounts equate rents generted from IPP to IPP invesment expenditure and then attribute all these rents to GOS. This is not justfied empirically. Indeed, many workers directly related to the production of intangibles (e.g., R&D lab managers) are paid a wage below their marginal value product in exchange of future equity in the firm (McGrattan and Prescott, 2010, 2014). We show that once we relax the SNA08 assumption on attributing all IPP rents to capital we find that the labor share of income, the capital-to-output ratio, and the rate of return are relatively trendless.

That the factor income share is sensitive to the distribution of IPP income has important implications on the quantitative importance of IPP capital as well, even though it does not alter the amount of IPP capital. For example, when IPP rents go more to labor, the contribution of IPP would work more through labor and less through capital. With the labor share observed in data, the contribution of IPP capital accounts for about a quarter of Solow residual in level and growth accounting. The additional explanatory power from IPP capital goes down to half, however, once relaxing the extreme assumption on the rent allocation based on the cost structure of R&D activities.

The paper is structured as follows. In Section 3.2 we describe the capitalization of IPP in the national accounts. In Section 3.3 we show the effects of IPP capitalization on economic growth and the big ratios including the labor share of income, the capital-to-output ratio, and the rate of return on capital. We conduct a development accounting exercise in Section 3.4 and a growth accounting exercise in Section 3.5. Section 3.6 concludes.

3.2 IPP Capitalization in the National Accounts

In 2009, the United Nation Statistical Commission adopted the new System of National Accounts, SNA 2008. The most notable update in the new system is an attempt to better measure the intangible capital in a national economy. In SNA 2008, the intangible capital measured by the national accounts of OECD countries is labeled as intellectual property products (IPP). IPP accounts include include R&D and artistic originals, in addition to computer software introduced since SNA 1993. By 2016, most OECD countries have implemented the new system.[33] Koh et al. (2018) explain in detail this accounting change using the US national income and product accounts.

Since most countries have implemented SNA 2008 very recently, and are still updating data figures, we build a new dataset that combines data from individual national sources with the OECD stats database. We construct capital series by type (i.e. tangible, IPP, and aggregate) using the perpetual inventory method with type specific depreciation rates obtained from the consumption of fixed capital data whenever available. For countries with no information on the consumption of fixed capital (either directly or indirectly from capital stock data), we use estimated depreciation rates corresponding to the level of log GDP per capita.[34] The labor share is also adjusted for self employed income using data for mixed

[33] A few exceptions are Turkey, Chile, and Japan.
[34] These include Spain, Mexico, and Portugal.

income or number of self employment, whichever provides longer data. The resulting dataset has 907 country-year observations covering 31 OECD countries for various time periods (see our Appendix for details). In documenting the growth facts, we exclude sample with GDP per capita less than 10,000 USD (in 2005), which is near 1940 in US, to focus on economies that are near balanced growth path in the sense of Kaldor (1957) and Jones (2016). This drops 37 out of 907 observations and makes no difference in our results.

Three major differences between our dataset and the Penn World Table (PWT) are noteworthy. First and most importantly, ours has IPP capital separately whereas PWT does not. This separation is essential for our study of the effects of IPP capitalization on growth and the big ratios across time and space. Second, we used longer series of mixed income or self employment data in general compared to PWT in the adjustment of labor share. Third, we used information of time varying depreciation rates for the construction of capital stock while PWT assumes constant depreciation rates for each capital type. These depreciation rates have implications for the measures of the stock of capital and hence growth and development accounting decompositions.

What does the IPP capitalization entails for the national product and income accounts? After the revision, expenditures on IPP capital (X_I) are treated as investment, and so the identity between the national product and national income is,

$$Y = C + X_T + X_I = \underbrace{RK}_{\text{gross operating surplus}} + \underbrace{WL}_{\text{compensation of employees}} . \tag{3.1}$$

Instead, before the revision, IPP investment was treated as an expense. Because the revision has the key accounting assumption that all IPP investment, X_I, is moved to gross operating surplus, GOS, we can summarize the result of the revision in the SNA as following. From the national income identity, production, expenditure and income side before the revision can be expressed as

$$Y_{Pre-SNA2008} = C + X_T = \underbrace{(RK - \chi X_I)}_{\text{gross operating surplus}} + \underbrace{(WL - (1-\chi)X_I)}_{\text{compensation of employees}} . \tag{3.2}$$

where χ refers to the fraction of IPP expenses coming from capital owners, whereas $1-\chi$ is the fraction of IPP expenses from workers. That is, χ captures the distribution of IPP rents across factors of production. The main accounting assumption behind the IPP capitalization implemented by national statistical offices—following the SNA2008 guidelines—is that $\chi = 1$. McGrattan and Prescott (2010) refer to χX_I and $(1-\chi)X_I$ as expensed and sweat investment, respectively. The current accounting practice under the SNA 2008 adds the entire X_I to the gross operating surplus, which implicitly assumes $\chi = 1$. In reality, χ is not neccessarily one as workers in R&D activities often get paid less than their marginal productivity with a promise of future equity compensation (McGrattan and Prescott, 2010). This is relevant as Koh et al. (2018) show that setting $\chi = 1$ has quantitative implications for the secular behavior of the labor share in the U.S.

3.3 The Effects of IPP Capitalization on Growth and the Big Ratios

First discuss the effects of IPP capitalizaiton on output growth and dispersion (Section 3.3.1). Second, we show that the decline of the accounting labor share observed in OECD countries can be explained by the capitalization of IPP (Secction 3.3.2). The capitalizaiton of IPP is also behind an increase in the capital-to-output ratio (Section 3.3.3) and in the rate of return to capital (Section 3.3.4).

Table 3.1: IPP investment at current PPP rates (Billions) in 2011

	IPP inv			IPP inv			IPP inv			IPP inv	
AUS	29.8	(3.4)	ESP	41.3	(3.0)	ISL	0.2	(2.3)	NZL	4.2	(3.3)
AUT	16.2	(5.1)	EST	0.7	(2.6)	ISR	9.8	(4.9)	POL	10.2	(1.4)
BEL	17.1	(4.4)	FIN	10.7	(6.0)	ITA	53.8	(2.9)	PRT	7.6	(3.2)
CAN	44.1	(3.4)	FRA	117.9	(5.7)	KOR	81.5	(6.1)	SVK	2.5	(2.0)
CHE	24.6	(6.3)	GBR	85.3	(4.3)	LUX	1.2	(3.1)	SVN	1.8	(3.7)
CZE	9.8	(3.8)	GRC	4.8	(1.9)	MEX	7.5	(0.4)	SWE	25.9	(7.6)
DEU	121.9	(4.1)	HUN	5.7	(3.0)	NLD	33.0	(5.0)	USA	783.8	(5.7)
DNK	11.7	(6.0)	IRL	11.7	(6.4)	NOR	9.8	(3.7)			

Notes: We write in parenthesis the proportion (%) of IPP investment in value added.

3.3.1 Effects of IPP Capitalization on Output Growth and Dispersion

Under the new SNA (2008) the production of IPP, x_I, is added to the pre-accounting measures of value added. This procedure has been gradually implemented by OECD countries. Precisely, the accouting change implies an increase in value added in the OECD output by 4% on average in 2011. Table 3.1 summarizes the effects of the IPP capitalization on value added for all our OECD countries in year 2011. The largest change occurs in the US with a value added that increases by 783.8 billions, the lowest change is by 0.2 billions in Iceland.

The accounting increase in value added due to the capitalization of IPP in percentage terms, γ_y, is captured by this ratio,

$$\gamma_y = \log \frac{y}{y - x_I}, \tag{3.3}$$

where y is value added x_I is IPP investment, and the denominator, $y - x_I$, captures value added before the capitalization of IPP. We plot γ_y for the OECD across time (panel (a1), Figure 3.1) and across space (panel (a2), Figure 3.1). The increasing importance of IPP investment across time and space is clear. Precisely, we find that γ_y increases from 0.9% in 1930 to 5.8% in 2014 on average in OECD countries. Across space, when a country's GDP per capita is near 8,000 USD (in 2005 PPP), γ_y is 0.7% on average. The γ_y increases to 5.7% on average when the GDP per capita attains near 70,000 USD (in 2005 PPP).

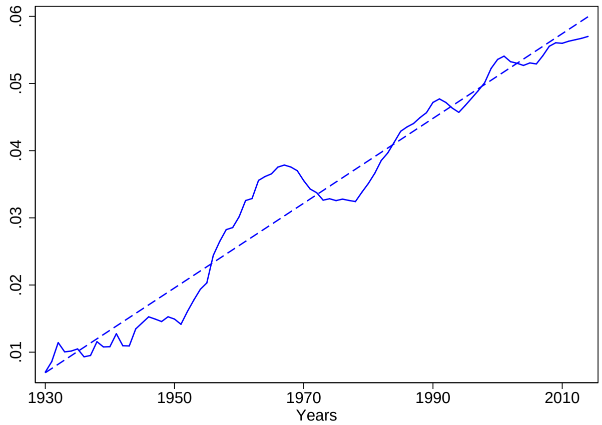
Naturally, the growth rate of value added also changes with the capitalization of IPP. The OECD value added growth rate currently averages 3.20% from 1950 to 2011, while this figure is 3.13 with the pre-SNA93 that expenses IPP. To be precise, we plot the changes over time for γ_y ($:= d\gamma_y$) which is the difference between the growth rate of value added corresponding to the current accounting and the growth rate of the pre-SNA93 accounting value added for the OECD across time (panel (b1), Figure 3.1) and across space (panel (b2), Figure 3.1). The difference between the growth rates has no clear trend over time and space, remaining at around 0.07% on average across time and space.

An interesting aspect of the IPP capitalization is that it increases value added proportionally more for countries with larger IPP investment. If countries that have large IPP investments are income-rich countries before the accounting change, then IPP capitalization can increase the dispersion of cross-country incomes. If countries that have large IPP investments are poor countries before the accounting chnage, then IPP capitaliation can decrease the dispersion of cross-country incomes. In Figure 3.2, we show the difference between cross-country standard deviation of log value added per capita before and after IPP capitalization across time. The cross-country standard deviation of value added per capita increases for all years with the capitalization of IPP (+ .77% on average between 1995 and 2011).

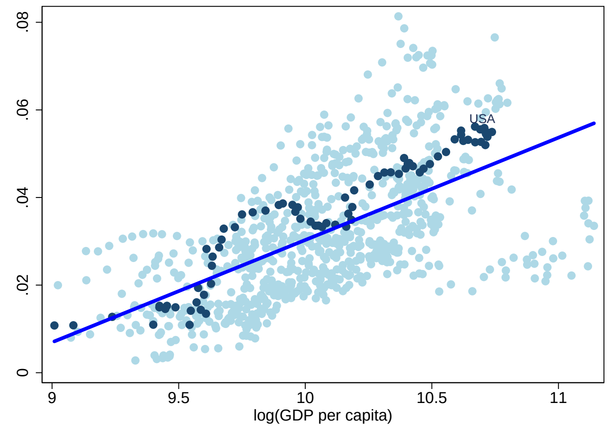
Figure 3.1: The Effects of IPP Capitalization on Value Added, 31 OECD countries

(a) Percentage Increase in Value Added due to IPP Investment (γ_y)

(a1) Across Time

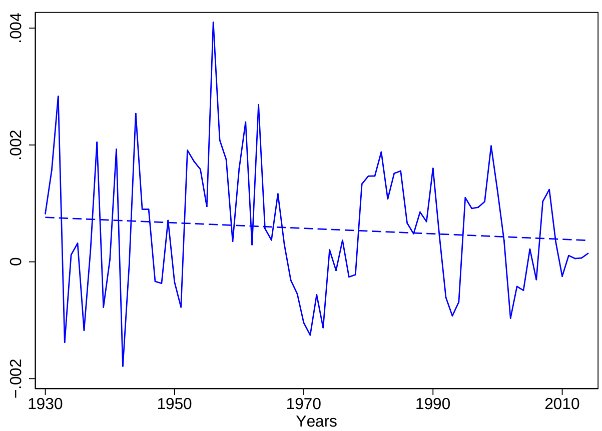


(a2) Across Space

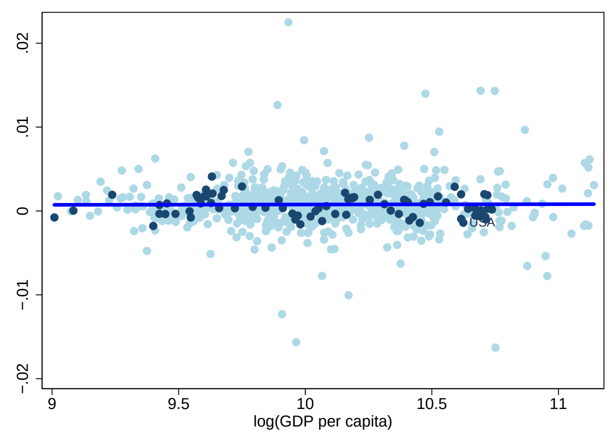


(b) Increase in Value Added Growth due to IPP Investment ($d\gamma_y$)

(b1) Across Time

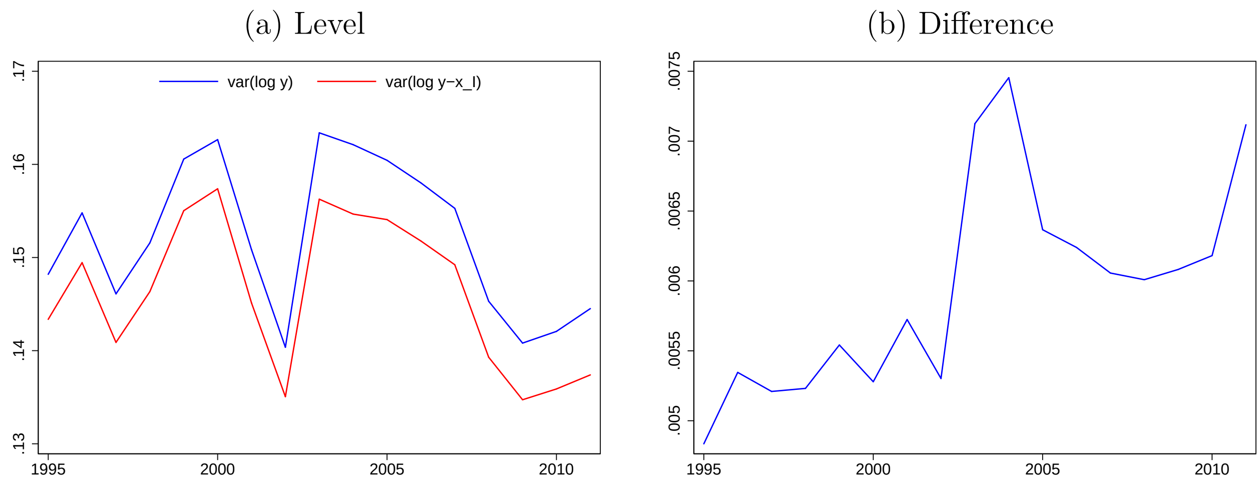


(b2) Across Space



Notes: Where γ_y is constructed as in equation (3.3). The average time series are based on the estimated time fixed effects using GDP (PPP) as weight.

Figure 3.2: The Effects of IPP Capitalization on Cross-Country Income Variation



3.3.2 Effects of IPP Capitalization on the Accounting Labor Share

The accounting labor share is experiencing a global decline that has attracted lots of attention (?). Figure 3.3 shows this decline across time (panel (a)) and space (panel (b)) for OECD countries. The accounting labor share is defined as

$$LS = 1 - \frac{GOS}{Y}.$$

where GOS is gross operating surplus and Y is gross domestic income.

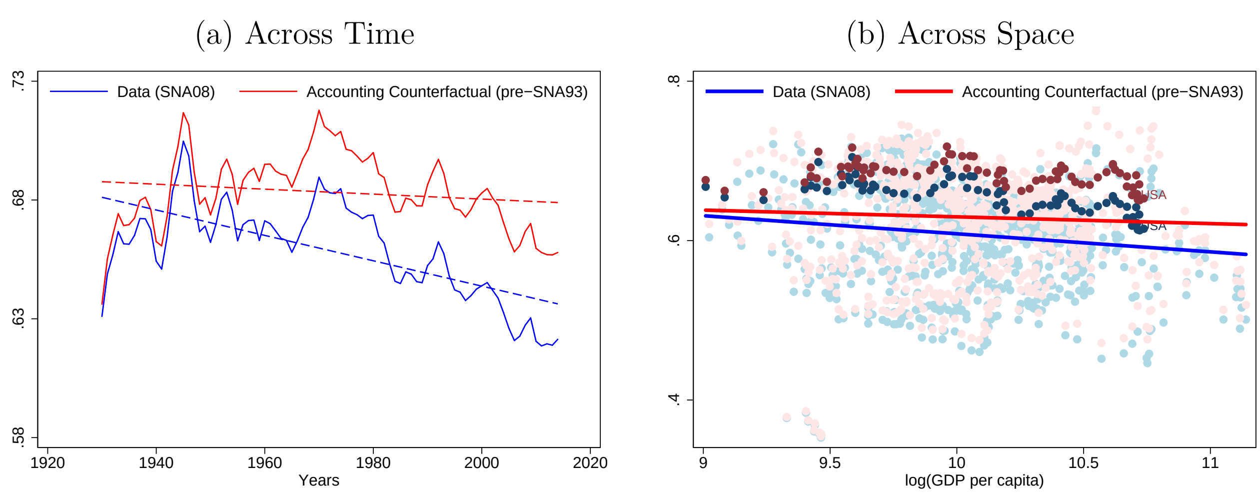
To measure the effects of the capitalization of IPP on the accounting LS, we follow the strategy in Koh et al. (2018) by constructing a counterfactual pre-SNA93 accounting LS in which IPP items are expensed as opposed to capitalized,

$$LS_{Pre-SNA93} = 1 - \frac{GOS - X_I}{Y - X_I},$$

where X_I is investment in IPP. Because $Y > GOS$, IPP capitalization unambiguously reduces labor share. Moreover, the revision can generate a declining trend for the labor share if the IPP investment is growing faster than value added which it does.

Figure 3.3 depicts accounting LS under the current SNA2008 scenario where IPP is capitalized and the pre-SNA1993 scenario where IPP is expensed. The time path of OECD labor share is obtained by the year fixed effects weighted by the dollor output as time coverages are different by countries[35] Both graphs show that the accounting LS declines in OECD countries across time and space under the current SNA2008, but the trend vanishes when IPP is expensed, i.e., under the pre-SNA2008 scenario. That is, the decline of the accounting LS is fully explained by the capitalization of IPP.

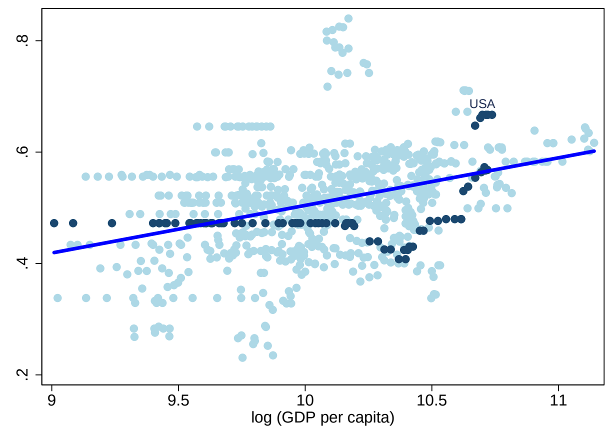
Figure 3.3: Effects of IPP Capitalization on Labor Share, 31 OECD Countries



The current accouting assumes that the IPP investment from the national product side is entirely attributed to GOS in national income (i.e. $\chi = 1$), see Section 3.2. This assumes

[35] We estimate $LS_{i,t} = c_i + \beta_t t + \varepsilon_{i,t}$ and then plot $\hat{\beta}_t$ where its 1950 value is normalized to the weighted average of 1950 labor share.

Figure 3.4: Labor Share in R&D Based on Cost Structure, 31 OECD Countries

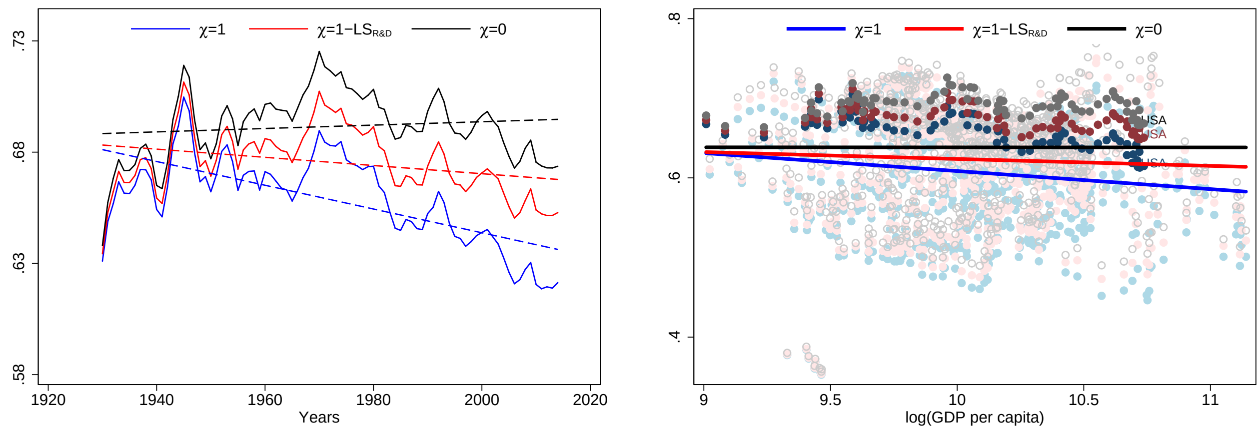


that the workers do not fund the R&D activities. However, it is widely happening in the R&D activities that workers get paid less than their contribution (marginal productivity) for a promise of future compensation such as stock options. We argue that this should be understood as evidence of $\chi < 1$. That is, workers also fund R&D investment, and their contribution should be understood as labor income, not capital income.

However, estimating χ is not a trivial matter as it requires a detailed micro-level information on the R&D activities. For now, we use the information based on the cost structure of R&D to examine the value of χ different from one. Specifically, we set $\chi = 1 - LS_{R\&D}$, where $1 - LS_{R\&D}$ is a fraction of capital expenses in total cost of R&D, obtained from OECD statistics database. Figure 3.4 confirms that $LS_{R\&D}$ is clearly different from 0, and has a slightly increasing trend over the development path (log GDP per capita). For example, for the US it raises from roughly 45% to 65% over the past 20 years.

Figure 3.5: Labor Share with alternative distributions of IPP rents, χ's

(a) Across Time (b) Across Space



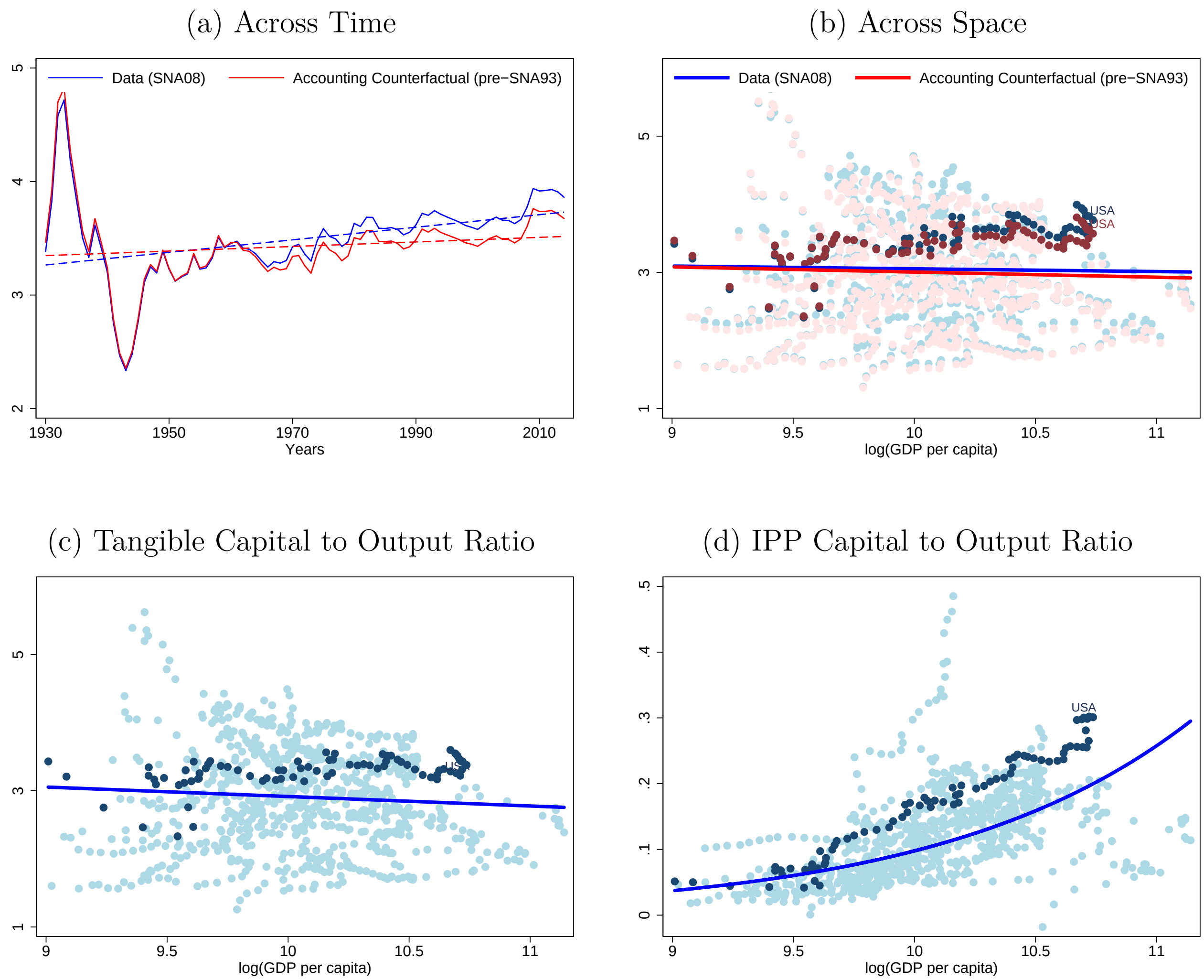
With our proxy for χ based on the cost structure of R&D, we compute an alternative labor share as following.[36]

$$LS_{\chi=1-LS_{R\&D}} = 1 - \frac{GOS - (1-\chi)X_I}{Y}.$$

We find that the role of χ is critical in understanding labor share decline. In particular, the decline of the labor share vanishes when relaxing the assumption that all the rents on IPP investment go to capital ($\chi = 1$). For our estimate of χ based on the cost structure of R&D activities labor share is trendless across time (panel (a), Figure 3.5) and space (panel (a), Figure 3.5). These findings extend to the OECD countries the accounting results in Koh et al. (2018) for the U.S.

[36] More precisely, we also adjust for the mixed income in computing labor share with any values for χ. That is, the labor share is $LS = [CE + (1-\chi)X_I \times (Y - MI)/Y]/(Y - MI)$ where MI is mixed income (mainly proprietors' income).

Figure 3.6: Effects of IPP Capitalization on the Capital to Output Ratio, 31 OECD Countries



3.3.3 Effects of IPP Capitalization on the Capital-to-Output Ratio

We plot the aggregate capital to output ratio with the current SNA08 and pre-SNA93 where all IPP was expensed. To replicate the pre-SNA93 scenario we compute the capital to output ratio as $\frac{K_T}{Y - X_I}$ where K_T is tangible capital and we remove investment in IPP from output in the denominator. It is clear that the capital to output ratio that incorporates IPP capital grows over time, while the capital to output ratio of the pre-IPP capitalization accounting is relatively trendless and consistent with the Kaldor facts (panel (a), Figure 3.6). Similarly, the capital to output ratio across space is larger when IPP is capitalized (panel (b), Figure 3.6). Although in this case we find relatively trendless capital-to-ouptut ratios across space in both scenarios, with and wihout IPP capitalization.

We decompose the sources behind the increase in the aggregate capital to output ratio. We compare the ratio of tangible capital K_T to output Y (panel (c), Figure 3.6) and the ratio of IPP capital K_I to output Y (panel (d), Figure 3.6). It is clear that it is the increase in the ratio of IPP capital to output over time that generates the increase in the aggregate capital to output ratio. Instead, the ratio of tangible capital to output decreases over time.

But is IPP capital accurately measured? A very important caveat of these findings is that the construction of the series of capital is based on the perpetuary inventory method (consistent with the procedure followed in the fixed asset tables of the national accounts) and this requires measures of unobserved IPP prices and unobserved IPP depreciation rates. National accounts capitalize structures and equipment, as well as IPP, using separate laws of motion for capital to obtain the series for K_T and K_I (see the appendix for the details). Therefore, the construction of the capital stock series implies that we need to use data on IPP prices and IPP depreciation rates which are unobservable and, we argue, subject to questionable assumptions in their construction. Precisely, in the US, the BEA does not provide an accounting measure of IPP depreciation but an economic one (Koh et al., 2018). To estimate R&D depreciation—aimed at capturing obsolescence and competion which are not directly observable—the BEA uses an economic model that maximizes profits over R&D choices with ad-hoc assumptions on the effect of R&D on profits (Li and Hall, 2016). Hence, treating the BEA IPP depreciation as measurement is only logically consistent with theory that complies with the BEA economic model that estimates IPP depreciation. In addition, the estimation of IPP depreciation requires IPP prices that we do not observe because there are no transactions of in-house production of intangibles and because R&D projects are heterogeneous in nature. Because we simply do not observe transactions of in-house production, the estimates of IPP prices for in-house production are hard to measure. A useful approach to estimate intangible capital that is unobservable is introduced in McGrattan and Prescott (2010). Instead, the BEA uses an input cost index as a proxy for the R&D output price change. However, an input cost index does not capture the impact of productivity change on real R&D output. Argumenting that R&D increases aggregate productivity, the BEA uses the economy-wide measure of multifactor productivity (MFP) from the BLS to proxy for unobserved R&D productivity and subtracts the growth rate of MFP from the input cost index (Crawford et al., 2014). Again, this is breeding ground for logical inconsistencies between theory and measurement if theory does not comply with the MFP from the BLS.

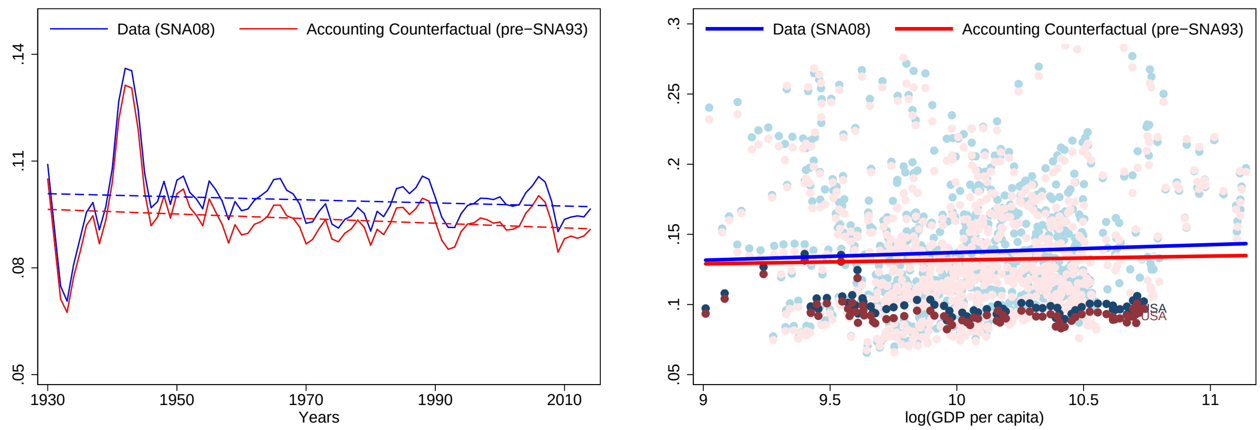
3.3.4 Effects of IPP Capitalization on the Rate of Return

The rate of return under the current system of national accounts is plotted across time and space in Figure 3.7. We find an increasing pattern for the rate of return in both cases. Instead, using the pre-SNA1993 accounting we go back to the standard Kaldor facts that deliver a rate of return that is relatively constant across time and space, see Figure 3.7.

Now we turn to an investigation of the quantitative importance of the IPP capital by level and growth accounting exercises in the following sections.

Figure 3.7: Effects of IPP Capitalization on the Rate of Return to Capital, 31 OECD Countries

(a) Across Time (b) Across Space



3.4 Development Accounting with IPP Capital

We first focus on the standard production function approach to level (or development) accounting. Second, we look at the product side (i.e., expenditures) of the national product.

3.4.1 Production Function Approach

We conduct a standard development accounting exercise with the introduction of IPP capital in national accounts. Consider the following constant returns to scale (CRS) production function,

$$y_{j,t} = a_{j,t} k_{I,j,t}^{\theta_{I,j,t}} k_{T,j,t}^{\theta_{T,j,t}} h_{j,t}^{\theta_{h,j,t}} \tag{3.4}$$

where $y_{j,t}$ is output for country j in period t. The factor inputs of production are tangible capital, $k_{T,j,t}$, IPP capital, $k_{I,j,t}$, and labor in efficiency units, $h_{j,t}$. Each of these factors of

Table 3.2: Cross-Country Differences in Output per Capita: Value Added and the Importance of IPP Measured by National Accounts

(a) With $\chi = 1$ (SNA, 2008)

	1996	1999	2002	2005	2008	2011
Measure (A):						
Success	0.42	0.45	0.44	0.48	0.54	0.53
Success without IPP	0.32	0.34	0.32	0.36	0.41	0.40
Difference	0.10	0.11	0.12	0.12	0.13	0.13
Measure (B):						
Success	0.54	0.54	0.53	0.59	0.63	0.61
Success without IPP	0.41	0.41	0.38	0.45	0.48	0.45
Difference	0.13	0.13	0.15	0.14	0.15	0.16

(b) With $\chi = 1 - LS_{R\&D}$

	1996	1999	2002	2005	2008	2011
Measure (A):						
Success	0.37	0.39	0.38	0.42	0.47	0.46
Success without IPP	0.33	0.34	0.32	0.37	0.41	0.40
Difference	0.04	0.05	0.06	0.05	0.06	0.06
Measure (B):						
Success	0.50	0.50	0.48	0.55	0.58	0.56
Success without IPP	0.43	0.43	0.41	0.48	0.51	0.49
Difference	0.06	0.07	0.07	0.07	0.07	0.07

Notes: Success measure is fraction of variance explained by factor inputs. IPP explanation refers difference between success measure with IPP and without IPP out of output variation unexplained by traditional factors.

production contribute to output according to their respective coefficients θ, where $\theta_{I,j,t} + \theta_{T,j,t} + \theta_{h,j,t} = 1$.

We assume competitive markets which together with CRS technology implies that the coefficients θ are the factor shares of income. In terms of measurement, we compute each of these shares as:

$$\theta_{h,j,t} = \frac{wh_{j,t} + (1-\chi_{j,t})x_{I,j,t}}{y_{j,t}}, \tag{3.5}$$

$$\theta_{I,j,t} = \frac{\chi_{j,t}x_{I,j,t}}{Y_{k,t}}, \tag{3.6}$$

$$\theta_{T,j,t} = \frac{y_{j,t} - wh_{j,t} - x_{I,j,t}}{y_{j,t}} = 1 - \theta_{I,j,t} - \theta_{h,j,t}, \tag{3.7}$$

Again, consistently with the current system of national accounts (SNA, 2008) we use the accounting assumption that $\chi_{j,t} = 1 \; \forall j, t$. We parallelly examine the implications of this assumption by using the cost structure of R&D ($\chi_{j,t} = 1 - LS_{R\&D,j,t}$). Detailed data construction procedure for the level accounting is described in the appendix. Note that if $x_I = 0$, then we are back to the previous accounting (SNA 1993 where IPP capital was not capitalized). If $x_I > 0$ and $\chi = 1$, then we are in the current system of national accounts (SNA, 2008).

The quantitative assessment of the importance of the IPP capital in accounting for the cross-country differences in output, we need measures of IPP capital. National accounts for each country provides these measures constructed using the perpetual inventory method given series for IPP investment, IPP prices and IPP depreciation rates. As we discussed earlier, these series of capital are subject to substantial mismeasurement and are in large part of the result of accounting assumptions behind the series of IPP investment, IPP prices and IPP depreciation rates. For now, we take these series as given.

To see the impact of IPP on the cross-country per capita income differences we write production (3.4) for each period t in logs as,

$$\log(y_{j,t}) = \log(a_{j,t} + \underbrace{\theta_{I,j,t}\log(k_{I,j,t}) + \underbrace{\theta_{T,j,t}\log(k_{T,j,t}) + \theta_{h,j,t}\log(h_{j,t})}_{\log(q_{-I,j,t})}}_{\log(q_{j,t})}.$$

where $q_{j,t} = k_{I,j,t}^{\theta_{I,j,t}} k_{T,j,t}^{\theta_{T,j,t}} h_{j,t}^{\theta_{h,j,t}}$ captures the set of observable factor inputs in the national accounts and $q_{-I,j,t} = k_{T,j,t}^{\theta_{T,j,t}} h_{j,t}^{\theta_{h,j,t}}$ excludes IPP capital. To see the impact of IPP on the cross-country per capita income differences we compare measures of accounting success with and without IPP capital. Our two measures of accounting success follow Caselli (2005). First, we define,

$$\begin{aligned}
\text{Success A} \quad &= \frac{\text{var}(\log q_{j,t})}{\text{var}(\log y_{i,t})} \\
\text{Success A, without IPP} \quad &= \frac{\text{var}(\log q_{-I,j,t})}{\text{var}(\log y_{i,t})}
\end{aligned}$$

Because total factor productivity potentially comoves with the observable factor inputs we also use the following alternative measure of success:

$$\begin{aligned}
\text{Success B} \quad &= \frac{\text{var}(\log q_{j,t}) + \text{cov}(\log q_{j,t}, \log a_{j,t})}{\text{var}(\log y_{i,t})} \\
\text{Success B, without IPP} \quad &= \frac{\text{var}(\log q_{-I,j,t}) + \text{cov}(\log q_{-I,j,t}, \log a_{j,t})}{\text{var}(\log y_{i,t})}
\end{aligned}$$

Table 3.2 shows the results. With IPP capital, the success measure (A) increases by 10% in 1996, from 32 to 42%, and by 13% in 2011, from 40 to 53% (see panel (a) in Table 3.2). We find a similar increasing pattern of the contribution of IPP capital to cross-country per capita income differences over time using success measure (B). Precisely, we find that IPP capital increases success measure (B) by 13% in 1996, from 41 to 54%, and by 16% in 2011, from 45 to 61%. However, the results change with the value of χ, which can be easily seen

Table 3.3: Decomposition of the output variance in 2011

	IPP related			h related		k related
	var(k_I)	cov(k_I, h)	cov(k_I, k)	var(h)	cov(h, k_T)	var(k_T)
$\chi = 1$ (SNA08)	9.5	6.6	35.5	37.3	6.6	233.9
$\chi = 1 - LS_{R\&D}$	2.0	3.1	16.2	40.1	6.9	233.9
Difference	-7.5	-3.5	-19.3	+2.7	+0.2	+0.0

Table 3.4: Additional fraction explained by IPP capital in 2011

	δ_I	$\delta_I = 0$	$\delta_I = \delta_T$
$\chi = 1$ (SNA08)	0.22	0.20	0.23
$\chi = 1 - LS_{R\&D}$	0.10	0.08	0.10

from equation (3.5) to (3.7). Even though χ does not alter the level of IPP capital, it changes factor shares. Indeed, the additional explanation from the IPP capitalization goes down to less than a hal, when relaxing the extreme assumption that $\chi = 1$ to $\chi_{j,t} = 1 - LS_{R\&D,j,t}$, see panel (b) in Table 3.2.

Why does the explanation decrease with $\chi < 1$? The reduction in χ essentially lowers the IPP capital share with higher labor share. When the labor's contribution becomes more important, our understanding on cross country income differences gets smaller. This is because the variation in human capital is less useful for the understanding of the cross country income disparities than that of capital, at least for the human capital measured by the average years of schooling a la Barro-Lee in the PWT. Yet the precise measurement on human capital has not reached consensus (Schoellman and Hendricks, 2017, and references therein).

Note that the IPP rents going to labor makes workers' compensation higher but not their human capital better. When a country increases IPP investment, its additional contribution works through both labor and capital. The indirect channel working through additional compensation to labor explains less for the increase in output, as long as the level of human capital remains same. Therefore, it would be important to distinguish wage and human capital in the level accounting exercise, which is consistent with the point made in Caselli and Ciccone (2017) or Schoellman and Hendricks (2017).

To see how much the explanation coming from the variation in human capital is small using the Barro-Lee measures of human capital from the PWT, we decompose the variance of output as following:

$$
\begin{aligned}
\mathrm{var}(\log y) = & \underbrace{\mathrm{var}(\log a) + 2[\mathrm{cov}(a, \theta_h \log h) + \mathrm{cov}(a, \theta_I \log k_I) + \mathrm{cov}(a, \theta_T \log k_T)]}_{\text{TFP related}} \\
& + \underbrace{\mathrm{var}(\theta_h \log h) + \mathrm{cov}(\theta_h \log h, \theta_I \log k_I) + \mathrm{cov}(\theta_h \log h, \theta_T \log k_T)}_{\text{Human capital related}} \\
& + \underbrace{\mathrm{var}(\theta_I \log k_I) + \mathrm{cov}(\theta_h \log h, \theta_I \log k_I) + \mathrm{cov}(\theta_I \log k_I, \theta_T \log k_T)]}_{\text{IPP capital related}} \\
& + \underbrace{\mathrm{var}(\theta_T \log k_T) + \mathrm{cov}(\theta_h \log h, \theta_T \log k_T) + \mathrm{cov}(\theta_I \log k_I, \theta_T \log k_T)]}_{\text{Tangible capital related}}
\end{aligned}
$$

Table 3.3 shows the decomposition result, which confirms that the larger success from IPP and smaller success from human capital under the current accounting ($\chi = 1$). The magnitude of additional explanation from IPP capital is larger than that of human capital, resulting in larger overall success in the case with $\chi = 1$.

Another parameter to consider a variation is the depreciation of IPP capital (δ_I). In the accounting of IPP capital, the depreciation of IPP capital is higher than that of traditional capital. Different values for the depreciation may also change the level accounting results, as it alters the amount of IPP capital accumulated in the national economy. We examine the sensitivity of the level accounting to various depreciation rates; benchmark depreciation rate ($\delta_I = \hat{\delta}_I$), no depreciation ($\delta_I = 0$), and depreciation rate of traditional capital ($\delta_I = \hat{\delta}_T$). Contrary to the case of χ, changes in δ_I do not affect the results of level accounting much (see table 3.4).

3.4.2 Level Accounting from the Product Side of National Accounts

Because the production of IPP is equated to IPP investment in national accounts, an alternative approach to document the role of IPP in explaining cross-country differences in output per capita is conducting the analysis from the product side of the accounts, i.e.,

$$y = c + x_I + x_T + g \tag{3.8}$$

where c is private consumption, x_I is investment in IPP, x_T is investment in tangible assets, and g is government expenditure.

The level accounting from the product side of the accounts is interesting because it does not rely on measures of IPP capital (which requires measures of IPP prices and depreciation). At the same time it does not require mesaures of the factor share. Because the product side is additive (3.8), we can directly measure the contribution of its components to the cross-country variation in the level output. For example, to study the role of IPP investment we compute,

$$\frac{\mathrm{var}(x_{I,j,t}) + \mathrm{cov}(x_{I,j,t}, y_{j,t} - x_{I,j,t})}{\mathrm{var}(y_{j,t})}. \tag{3.9}$$

Note that because we do not take logs the variance of output across countries depends on its average. This is however irrelevant for our analysis as we are interested in the percentual

Table 3.5: Cross-Country Differences in Output per Worker: Contribution of IPP from the Product Side of National Accounts (%)

	1996	1999	2002	2005	2008	2011
Eq (3.9) (investment PPP)	4.0	4.5	4.7	4.6	5.2	6.4
Eq (3.9) (GDP PPP)	3.7	4.4	4.8	4.5	4.9	5.6

Notes: We use the decomposition in (3.9)

contribution of each of the components in the product side of the accounts (3.8) to output variation.

A difficulty is that the product components do not add up to the value added (y) when considering the price dispersion across countries. For example, we convert the unit of each product component into USD using the PPP rates that are different across items in the previous subsection. Therefore, we consider two different cases, one with x_I converted by investment PPP rate (as in the previous subsection) and another with x_I converted by GDP PPP rate.

Our results are in Table 3.5. We find that IPP investment explains about 4.8% of GDP variation on average. When using investment PPP, its explanation increases, but the difference is not significant. Because national accounts equate IPP investment to IPP income, we can use this result from the product side of the national accounts to validate the value of χ on the income side of the accounts. In this direction, we note that our results for the IPP contribution to cross-country income per capita differences is much more similar to the our preferred case with $\chi = 1 - LS_{R\&D}$ (4.3% on average) than the SNA08 assumption of $\chi = 1$ (8.9% on average). This suggests a value close to our choice $\chi = 1 - LS_{R\&D}$ is preferred.

3.5 Growth Accounting with IPP capital

In this section, we do growth accounting exercise. Many studies have attempted to account for the importance of innovational activities in economic growth. Related, Corrado et al. (2005, 2009) extend the standard growth accounting to incorporate a precisely measured innovation-related capital. The main difference with respect to Corrado et al. (2005, 2009) is that we focus on the implication of income allocation of the R&D activities (i.e. χ) in the growth accounting.

Note that under the assumption of constant return to scale and competitive market, growth accounting exercise is insensitive to exact form of production function. Specifically, given any constant return to scale production function $y = af(k_I, k_T, h)$, we have

$$\frac{dy}{y} = \theta_T \frac{dk_T}{k_T} + \theta_I \frac{dk_I}{k_I} + \theta_h \frac{dh}{h} + \frac{da}{a},$$

where y, k_T, k_I, h, and a are output per employment, traditional capital per employment, IPP capital per employment, average human capital, and total factor productivity, respectively. The θ_f's are the income share of factor input f.

Hence, the TFP growth from year s to year u can be approximated by

$$\log(a_u/a_s) = \log(y_u/y_s) - \bar{\theta}_T \log(k_{T,u}/k_{T,s}) - \bar{\theta}^I \log(k_{I,u}/k_{I,s}) - \bar{\theta}_h \log(h_u/h_s), \qquad (3.10)$$

where $\bar{\theta}$ is average factor share between s and u, y is GDP per worker, k_T is traditional capital per worker, k_I is IPP capital per worker, and h is average human capital (measured by the years of schooling). Decomposition of growth of output from a specific time period s to u is straightforwad by summing up each of four components. Similar to section 3.4, we

Table 3.6: IPP explanation for growth $(\log(z_u/z_s)/[\log(z_u/z_s)+\log(a_u/a_s)])$: OECD average

	δ_I	$\delta_I = 0$	$\delta_I = \delta_T$
$\chi = 1$ (SNA08)	0.25	0.55	0.49
$\chi = 1 - LS_{R\&D}$	0.12	0.26	0.23

define the additional IPP explanation as $\log(k_{I,u}/k_{I,s})/[\log(k_{I,u}/k_{I,s}) + \log(a_u/a_s)]$ and do the growth accounting exercise with various χ's and δ_I's.

Table 3.6 shows the summarized results with detailed results in table C4 and C5 in appendix C.2. On average, IPP capital contributes around 9% of output growth, which is slightly less than one half of the TFP (22%). This means that the additional IPP explanation is 25% under the benchmark χ (= 1) and δ_I. Again, the additional explanation from the IPP capital goes down to 12% with $\chi = 1 - LS_{R\&D} < 1$, which is less than half of the case with $\chi = 1$. The reason is similar to the case with level accounting: average human capital grows less than IPP capital itself.

Of course, the IPP growth increases when the depreciation rate is lower. For example, with no depreciation in IPP capital ($\delta_I = 0$), the IPP's explanation goes up to even higher than that of TFP (IPP explanation of 55%). But the fraction explained by IPP capital again is reduced to a half when relaxing the assumption on the distribution of IPP rent (i.e., $\chi = 1 \to \hat{\chi}$).

3.6 Conclusion

We document the rise of intellectual property products (IPP) captured by up-to-date national accounts in 31 OECD countries. These countries gradually adopt the new system of national accounts (SNA2008) that capitalizes IPP—which was previously treated as an intermediate expense in the pre-SNA1993 accounting framework. We examine how the capitalization of

IPP affects stylzed growth facts and the big ratios (Kaldor, 1957; Jones, 2016). We find that the capitalization of IPP generates (a) a decline of the accounting labor share, (b) an increase in the capital-to-output ratio across time, and (c) an increase in the rate of return to capital across time. The key accounting assumption behind the IPP capitalization implemented by national accounts is that the share of IPP rents that are attributed to capital, χ, is equal to one. That is, national accounts assume that IPP rents are entirely owed to capital. We argue that this assumption is arbitrary and extreme. More reasonable assumptions about the split of IPP rents between capital and labor—for example, based on the cost structure of R&D—generate a secularly trendless labor share, a constant capital-to-output ratio, and a constant rate of return across time. We discuss the implications of these new measures of IPP capital—conditional on χ—for cross-country income per capita differences using standard development and growth accounting exercises.

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Appendix A

Appendix to Chapter 1

A.1 Use of Equipment and Software by Occupation

The capital use by occupation data is constructed by combining BEA NIPA and O*NET Tools and Technology Database. In NIPA table 5.5, the investment on non-residential equipment are categorized by 25 types. In UNSPSC, the classification system used in O*NET Tools and Technology database, there are 4,300 commodities, which are in 825 classes, in 173 families, and in 36 segments.

To construct a mapping between two, we firstly assign one of NIPA investment types to the relevant segment in UNSPSC. Often, it is apparent that a segment includes several types of equipment investment in NIPA. In this case, we use the family categories in the assignment procedure. Again, if a family apparently includes several types in NIPA, we use classes. Through this procedure, we could make a rough concordance between a subset of UNSPSC and the types of equipment investment in NIPA. The constructed concordance is shown in table A1.

Next, we assume that two tools have same price if they are classified in the same category. For example, the "metal cutting machines" in UNSPSC is assigned to "metalworking machinery" in NIPA investment type. The value of using the metal cutting machines are then the amount of investment in metalworking machinery divided by total use of all the commodities in the metalworking machinery category, where the total use of all the tools in the metalworking machinery is defined as sum of a number of total employment of each occupation times a number of UNSPSC commodities assigned to the metalworking machinery that each occupation uses.

The method is assuming that the number of tools above well represent the value of them, only within the NIPA investment category. Across the NIPA investment categories, each number of tools used would get different weights, according to the average amount of investment given to each tool. The procedure may make a big difference from average number of tools if a category with many commodities had small values compared to a category with few commodities. However, as more differentiated categories are usually advanced (and hence have expensive items), we expect not much difference from the adjustment.

Table A1: Concordance between NIPA equipment investment types and UNSPSC

NIPA		UNSPSC	
Line	Title	Code	Title
3	**Information processing equipment**		
4	Computers and peripheral equipement	43210000	Computer Equipment and Accessories
5	Communication equipment	43190000, 45110000	Communications Devices and Accessories, Audio and visual presentation and composing equipment
6	Medical equipment and instruments	42000000	Medical Equipment and Accessories and Supplies
9	Nonmedical instruments	41000000	Laboratory and Measuring and Observing and Testing Equipment
10	Photocopy and related equipment	45100000, 45120000	Printing and publishing equipment, Photographic or filming or video equipment
11	Office and accounting equipment	44100000, 31240000	Office machines and their supplies and accessories, Industrial optics
12	**Industrial equipment**		
13	Fabricated metal products	27000000, 31150000, 31160000, 31170000, 40140000, 40170000	Tools and General Machinery, Rope and chain and cable and wire and strap, Hardware, Bearings and bushings and wheels and gears, Fluid and gas distribution, Pipe piping and pipe fittings
14	Engines and turbines	26101500, 26101700	Engines, Engine components and accessories
17	Metalworking machinery	23240000, 23250000, 23260000, 23270000, 23280000	Metal cutting machinery and accessories, Metal forming machinery and accessories, Rapid prototyping machinery and accessories, Welding and soldering and brazing machinery and accessories and supplies, Metal treatment machinery
18 + 19	Special industry machinery, n.e.c. + General industrial, including materials handling, equipment	23100000, 23110000, 23120000, 23130000, 23140000, 23150000, 23160000, 23180000, 23190000, 23200000, 23210000, 23220000, 23230000, 23290000, 24100000, 24110000, 31140000, 40000000	Raw materials processing machinery, Petroleum processing machinery, Textile and fabric machinery and accessories, Lapidary machinery and equipment, Leatherworking repairing machinery and equipment, Industrial process machinery and equipment and supplies, Foundry machines and equipment and supplies, Industrial food and beverage equipment, Mixers and their parts and accessories, Mass transfer equipment, Electronic manufacturing machinery and equipment and accessories, Chicken processing machinery and equipment, Sawmilling and lumber processing machinery and equipment, Industrial machine tools, Material handling machinery and equipment, Containers and storage, Moldings, Distribution and Conditioning Systems and Equipment and Components
20 + 41	Electrical transmission, distribution, and industrial apparatus + Electrical equipment, n.e.c.	26101100, 26101200, 26101300, 26110000, 26120000, 26130000, 26140000, 39000000	Electric alternating current AC motors, Electric direct current DC motors, Non electric motors, Batteries and generators and kinetic power transmission, Electrical wire and cable and harness, Power generation, Atomic and nuclear energy machinery and equipment, Electrical Systems and Lighting and Components and Accessories and Supplies
21	**Transportation equipment**		
22 + 25	Trucks, buses, and truck trailers + Autos	25100000	Motor vehicles
26	Aircraft	25130000	Aircraft
27	Ships and boats	25110000	Marine transport
28	Railroad equipment	25120000	Railway and tramway machinery and equipment
29	**Other equipment**		
30	Furniture and fixtures	56000000	Furniture and Furnishings
33	Agricultural machinery	21000000	Farming and Fishing and Forestry and Wildlife Machinery and Accessories
36	Construction machinery	22000000	Building and Construction Machinery and Accessories
39	Mining and oilfield machinery	20000000	Mining and Well Drilling Machinery and Accessories
40	Service industry machinery	48000000	Service Industry Machinery and Equipment and Supplies

A.2 Discrete Approximation of the Model

This section discusses equilibrium conditions with discrete approximation of the model. For the approximation, assumption 1 and 4 are replaced by assumption 5 and 6 in section 1.3 and 1.4.

The task production is given by equation (1.19) with tasks discretized into $j = 0, 1, \cdots, J$. Now the tasks are discrete, so workers are sorted into each task according to cutoff level of human capital $\hat{h}_j$. More preciesely, we have a sequence of human capital $\{\hat{h}_j\}_{j=0,\cdots,J+1}$ such that a worker with $h \in [\hat{h}_j, \hat{h}_{j+1})$ are sorted into task j with $\hat{h}_0 = \underline{h}$ and $\hat{h}_{J+1} = \bar{h}$.

For a worker with exactly the threshold level of human capital should be indifferent between tasks so that

$$\omega_j b(\hat{h}_j, j) = \omega_{j-1} b(\hat{h}_j, j-1), \text{ for all } j, \text{ for } , j = 1, \cdots, J \tag{A.2.1}$$

replacing the original equilibrium condition (1.9).

The task production is solving

$$\max p_j T_j - \int_h w(h) l(h) dh - \int_{k=0}^{N_e} p_e(k) e(k) dk - \int_{k=0}^{N_s} p_s(k) s(k) dk,$$

which gives the FOCs,

$$w(h) = \omega_j b(h,j) = p_j T_j^{\frac{1}{\sigma_s}} H_j^{\frac{1}{\sigma_e} - \frac{1}{\sigma_s}} \left(\int_{\hat{h}_j}^{\hat{h}_{j+1}} b(h,j) \mu(h) dh \right)^{-\frac{1}{\sigma_e}} b(h,j),$$
$$\frac{1}{A_e \nu_e} = p_j T_j^{\frac{1}{\sigma_s}} H_j^{\frac{1}{\sigma_e} - \frac{1}{\sigma_s}} (N_e)^{\frac{\sigma_e - 1}{\sigma_e \nu_e} - 1} e_j^{-\frac{1}{\sigma_e}},$$
$$\frac{1}{A_s \nu_s} = p_j T_j^{\frac{1}{\sigma_s}} (N_s)^{\frac{\sigma_s - 1}{\sigma_s \nu_s} - 1} s_j^{-\frac{1}{\sigma_s}},$$

using the fact that $p_e = 1/(A_e\nu_e)$, $p_s = 1/(A_s\nu_s)$, $e_j(k) = e_j$, and $s_j(k) = s_j$ in equilibrium, and $H_j := \left[\alpha_{h,j}(\int_{\hat{h}_j}^{\hat{h}_{j+1}} b(h,j)\mu(h)dh)^{\frac{\sigma_e-1}{\sigma_e}} + \alpha_{e,j}(\int_{k=0}^{N_e} e(k)^{\nu_e}dk)^{\frac{\sigma_e-1}{\sigma_e\nu_e}}\right]^{\frac{\sigma_e}{\sigma_e-1}}$.

Combining the FOCs, we get

$$p_j = \left[\left(\alpha_{h,j}^{\sigma_e}\omega_j^{1-\sigma_e} + \alpha_{e,j}^{\sigma_e}\left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e-1}\right)^{\frac{1-\sigma_s}{1-\sigma_e}} + \alpha_{s,j}^{\sigma_s}\left(\nu_s A_s N_s^{\varphi_s}\right)^{\sigma_s-1}\right]^{\frac{1}{1-\sigma_s}}, \text{ for } j = 0, \cdots, J \tag{A.2.2}$$

which replaces equation (1.11).

The demand for each task is from

$$\max Y - \sum_j p_j T_j,$$

which gives

$$p_j = \left(\frac{\gamma_j Y}{T_j}\right)^{\frac{1}{\epsilon}}.$$

Combining this with FOCs, we obtain

$$p_j^{\epsilon-\sigma_s} = \frac{\gamma_j \alpha_{h,j}^{\sigma_s}\left(\alpha_{h,j}^{\sigma_e}\omega_j^{1-\sigma_e} + \alpha_{e,j}^{\sigma_e}\left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e-1}\right)^{\frac{\sigma_e-\sigma_s}{1-\sigma_e}} Y}{\omega_j^{\sigma_e}\int_{\hat{h}_j}^{\hat{h}_{j+1}} b(h,j)\mu(h)dh}, \text{ for } j = 0, \cdots, J, \tag{A.2.3}$$

replacing equation (1.10).

Now the equilibrium thresholds $\hat{h}_j$'s, wage rate ω_j's and prices p_j's are obtained by solving equation (A.2.1) to (A.2.3), which are $3J+1$ equations with the same number of unknowns.

A.3 Proof

Proof of lemma 2 *Since assignment function $\hat{h}(\tau)$ is strictly increasing, its inverse $\hat{\tau}(h)$ is well-defined. From the demand for task, equation (1.7), we know that there will be strictly positive task output $T(\tau) > 0$ (and hence $l(h, \hat{\tau}(h)) > 0$) for all $\tau \in [0, \bar{\tau}]$. The equation (1.8) and lemma 1 then implies*

$$w(h) = \omega(\hat{\tau}(h))b(h, \hat{\tau}(h)) \geq \omega(\hat{\tau}(h'))b(h, \hat{\tau}(h')), \text{ and}$$
$$w(h') = \omega(\hat{\tau}(h'))b(h', \hat{\tau}(h')) \geq \omega(\hat{\tau}(h))b(h', \hat{\tau}(h)).$$

Combining these two inequalities, we have

$$\frac{b(h, \hat{\tau}(h'))}{b(h, \hat{\tau}(h))} \leq \frac{\omega(\hat{\tau}(h))}{\omega(\hat{\tau}(h'))} \leq \frac{b(h', \hat{\tau}(h'))}{b(h', \hat{\tau}(h))}$$

Let $\tau = \hat{\tau}(h)$ and $\tau' = \hat{\tau}(h')$. Since $\hat{\tau}$ has an inverse function $\hat{h}$, above inequality is equivalent to

$$\frac{b(\hat{h}(\tau), \tau')}{b(\hat{h}(\tau), \tau)} \leq \frac{\omega(\tau)}{\omega(\tau')} \leq \frac{b(\hat{h}(\tau'), \tau')}{b(\hat{h}(\tau'), \tau)}$$

By taking log on both sides and dividing by $\tau' - \tau$,

$$\frac{\log b(\hat{h}(\tau), \tau') - \log b(\hat{h}(\tau), \tau)}{\tau' - \tau} \leq \frac{-(\log \omega(\tau') - \log \omega(\tau))}{\tau' - \tau} \leq \frac{\log b(\hat{h}(\tau'), \tau') - \log b(\hat{h}(\tau'), \tau)}{\tau' - \tau}$$

As $\tau' - \tau \to 0$, we have

$$\frac{d \log \omega(\tau)}{d\tau} = -\frac{\partial \log b(\hat{h}(\tau), \tau)}{\partial \tau},$$

which is the equation (1.9).

Now consider the task production. For notational convenience, we introduce

$$H(h,\tau) = \left[\alpha_h(\tau)(b(h,\tau)l(h))^{\frac{\sigma_e-1}{\sigma_e}} + \alpha_e(\tau)\left(\int_0^{N_e} e(k,\tau)^{\nu_e}dk\right)^{\frac{\sigma_e-1}{\sigma_e\nu_e}}\right]^{\frac{\sigma_e}{\sigma_e-1}}$$

From

$$\max\ p(\tau)T(\tau) - \int_h w(h)l(h,\tau)dh - \int_0^{N_s} p_s(k)s(k,\tau)dk - \int_0^{N_e} p_e(k)e(k,\tau)dk,$$

we have the following first order conditions:

$$w(h) \geq \alpha_h(\tau)p(\tau)T(\tau)^{\frac{1}{\sigma_s}}H(h,\tau)^{\frac{\sigma_s-\sigma_e}{\sigma_e\sigma_s}}l(h)^{-\frac{1}{\sigma_e}}b(h,\tau), \tag{A.3.1}$$

$$p_e(k) = \alpha_e(\tau)p(\tau)T(\tau)^{\frac{1}{\sigma_s}}H(h,\tau)^{\frac{\sigma_s-\sigma_e}{\sigma_e\sigma_s}}\left(\int_0^{N_e} e(k,\tau)^{\nu_e}\right)^{\frac{\sigma_e-1-\nu_e\sigma_e}{\nu_e\sigma_e}} e(k,\tau)^{\nu_e-1}, \tag{A.3.2}$$

$$p_s(k) = \alpha_s(\tau)p(\tau)T(\tau)^{\frac{1}{\sigma_s}}\int_0^{N_s} s(k,\tau)^{\nu_s}dk^{\frac{\sigma_s-1-\nu_s\sigma_s}{\nu_s\sigma_s}} s(k,\tau)^{\nu_s-1}, \tag{A.3.3}$$

In equipment- and software-producing sector, we solve

$$\max p_e(k)e(k) - e(k)/A_e, \quad \max p_s(k)s(k) - s(k)/A_s$$

subject to (A.3.2) *and* (A.3.3). *The solution gives*

$$p_e = 1/(\nu_e A_e), \quad p_s = 1/(\nu_s A_s) \quad \textit{for all } k. \tag{A.3.4}$$

Substituting (A.3.4) *into the FOCs, we get*

$$p(\tau) = \left[\left\{\alpha_h(\tau)^{\sigma_e}\omega(\tau)^{1-\sigma_e} + \alpha_e(\tau)^{\sigma_e}\left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e-1}\right\}^{\frac{1-\sigma_s}{1-\sigma_e}} + \alpha_s(\tau)^{\sigma_s}\left(\nu_s A_s N_s^{\varphi_s}\right)^{\sigma_s-1}\right]^{\frac{1}{1-\sigma_s}},$$

by combining the FOCs, which is the equation (1.11)*.*

Again from equation (A.3.1) *to* (A.3.3)*, the task production* $T(\tau)$ *can be expressed by*

$$T(\tau) = p(\tau)^{-\sigma_s}\omega(\tau)^{\sigma_e}\alpha_h(\tau)^{-\sigma_e}\left(\alpha_h(\tau)^{\sigma_e}\omega(\tau)^{1-\sigma_e} + \alpha_e(\tau)^{\sigma_e}\left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e-1}\right)^{\frac{\sigma_s-\sigma_e}{1-\sigma_e}}\int_h b(h,\tau)l(h,\tau)dh \tag{A.3.5}$$

From the labor market clearing condition and lemma 1, we have

$$l(h,\tau) = \mu(h)\delta[\tau - \hat{\tau}(h)],$$

where δ *is a Dirac delta function. Then we have*

$$\int_h b(h,\tau)l(h,\tau)dh = \int_{\tau'} b(\hat{h}(\tau'),\tau)\mu(\hat{h}(\tau))\delta[\tau-\tau']\hat{h}'(\tau')d\tau' = b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau).$$

Combining this with equation (1.7) *and* (A.3.5)*, we have*

$$\hat{h}'(\tau) = \frac{\gamma(\tau)p(\tau)^{\sigma_s-\epsilon}\alpha_h(\tau)^{\sigma_s}\left(\alpha_h(\tau)^{\sigma_e}\omega(\tau)^{1-\sigma_e} + \alpha_e(\tau)^{\sigma_e}\left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e-1}\right)^{\frac{\sigma_e-\sigma_s}{1-\sigma_e}} Y}{\omega(\tau)^{\sigma_e} b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))},$$

which is the equation (1.10)*.* ∎

Proof of lemma 3 *In steady state, if it exists,* $r = \pi_s/\eta_s = \pi_e/\eta_e = \rho$ *from the Euler equation* (1.17)*. Then* $\dot{X}/X = 0$ *for* $X = C$*,* E*,* S*,* N_e*, and* N_s *follow from usual argument. What we need to show is that there exist* N_s *and* N_e *that satisfy* $\pi_s/\eta_s = \pi_e/\eta_e = \rho$*.*

We start with the following lemma.

Lemma 4 *Fix $p(\tau)$ and $\hat{h}(\tau)$. There exists a pair $(\nu_s, \nu_e) \in (0,1) \times (0,1)$ such that $s(\tau)$ is strictly decreasing in N_s and $e(\tau)$ is strictly decreasing in N_e.*

Proof Combining equation (A.3.1) to (A.3.3) (FOCs), we have

$$\begin{aligned} s(\tau) = {} & N_s^{-1} N_s^{\varphi_s(\sigma_s-1)} \left(\nu_s A_s\right)^{\sigma_s} \alpha_s(\tau)^{\sigma_s} \alpha_h(\tau)^{-\frac{\sigma_e}{1-\sigma_e}} b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau) \\ & \times \left[\left(p(\tau)^{1-\sigma_s} - \alpha_s(\tau)^{\sigma_s} \left(\nu_s A_s N_s^{\varphi_s}\right)^{\sigma_s-1}\right)^{\frac{1-\sigma_e}{1-\sigma_s}} - \alpha_e(\tau)^{\sigma_e} \left(\nu_e A_e (N_s n_{es})^{\varphi_e}\right)^{\sigma_e-1} \right]^{\frac{\sigma_e}{1-\sigma_e}} \\ & \times \left(p(\tau)^{1-\sigma_s} - \alpha_s(\tau)^{\sigma_s} \left(\nu_s A_s N_s^{\varphi_s}\right)^{\sigma_s-1}\right)^{\frac{\sigma_s-\sigma_e}{1-\sigma_s}}, \end{aligned} \tag{A.3.6}$$

and

$$\begin{aligned} e(\tau) = {} & N_e^{-1} N_e^{\varphi_e(\sigma_e-1)} \left(\nu_e A_e\right)^{\sigma_e} \alpha_e(\tau)^{\sigma_e} \alpha_h(\tau)^{-\frac{\sigma_e}{1-\sigma_e}} b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau) \\ & \times \left[\left(p(\tau)^{1-\sigma_s} - \alpha_s(\tau)^{\sigma_s} \left(\nu_s A_s (N_e/n_{es})^{\varphi_s}\right)^{\sigma_s-1}\right)^{\frac{1-\sigma_e}{1-\sigma_s}} - \alpha_e(\tau)^{\sigma_e} \left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e-1} \right]^{\frac{\sigma_e}{1-\sigma_e}}, \end{aligned} \tag{A.3.7}$$

where $n_{es} := N_e/N_s$.

From equation (A.3.6) and (A.3.7), we can express

$$\frac{\partial \log s(\tau)}{\partial N_s} = -\frac{1}{N_s} + s_1(\tau;\varphi_s), \tag{A.3.8}$$

$$\frac{\partial \log e(\tau)}{\partial N_e} = -\frac{1}{N_e} + e_1(\tau;\varphi_e), \tag{A.3.9}$$

and it's straightforward to check that $\lim_{\varphi_s \downarrow 0} |s_1(\tau;\varphi_s)| = 0$, $\lim_{\varphi_e \downarrow 0} |e_1(\tau;\varphi_e)| = 0$, and $\partial s_1/\partial\varphi_s > 0$, $\partial e_1/\partial\varphi_e > 0$. This implies that there should be $0 < \nu_s < 1$ and $0 < \nu_e < 1$ which make $s(\tau)$ strictly decreasing in N_s and $e(\tau)$ strictly decreasing in N_e. ∎

Lemma 5 *Fix $p(\tau)$ and $\hat{h}(\tau)$. With ν_e and ν_s close to one, we have the following:*

$$\lim_{N_s \to 0} s(\tau) = \infty, \quad \lim_{N_e \to 0} e(\tau) = \infty, \quad \lim_{N_s \to \infty} s(\tau) = 0, \quad \lim_{N_e \to \infty} e(\tau) = 0.$$

Proof By substituting $\nu_e = 1$ and $\nu_s = 1$ (and hence $\varphi_e = \frac{1-\nu_e}{\nu_e} = 0$ and $\varphi_s = \frac{1-\nu_s}{\nu_s} = 0$) into equation (A.3.6) and (A.3.7), we have

$$\begin{aligned} s(\tau) &= N_s^{-1} (\nu_s A_s)^{\sigma_s} \alpha_s(\tau)^{\sigma_s} \alpha_h(\tau)^{-\frac{\sigma_e}{1-\sigma_e}} b(\hat{h}(\tau), \tau) \mu(\hat{h}(\tau)) \hat{h}'(\tau) \\ &\times \left[\left(p(\tau)^{1-\sigma_s} - \alpha_s(\tau)^{\sigma_s} (\nu_s A_s)^{\sigma_s - 1} \right)^{\frac{1-\sigma_e}{1-\sigma_s}} - \alpha_e(\tau)^{\sigma_e} (\nu_e A_e)^{\sigma_e - 1} \right]^{\frac{\sigma_e}{1-\sigma_e}} \\ &\times \left(p(\tau)^{1-\sigma_s} - \alpha_s(\tau)^{\sigma_s} (\nu_s A_s)^{\sigma_s - 1} \right)^{\frac{\sigma_s - \sigma_e}{1-\sigma_s}}, \end{aligned} \tag{A.3.10}$$

and

$$\begin{aligned} e(\tau) &= N_e^{-1} (\nu_e A_e)^{\sigma_e} \alpha_e(\tau)^{\sigma_e} \alpha_h(\tau)^{-\frac{\sigma_e}{1-\sigma_e}} b(\hat{h}(\tau), \tau) \mu(\hat{h}(\tau)) \hat{h}'(\tau) \\ &\times \left[\left(p(\tau)^{1-\sigma_s} - \alpha_s(\tau)^{\sigma_s} (\nu_s A_s)^{\sigma_s - 1} \right)^{\frac{1-\sigma_e}{\cdot}} 1 - \sigma_s - \alpha_e(\tau)^{\sigma_e} (\nu_e A_e)^{1-\sigma_e} \right]^{\frac{\sigma_e}{1-\sigma_e}} \end{aligned} \tag{A.3.11}$$

The result is straightforward from equation (A.3.10) and (A.3.11). ∎

Since π_e and π_s are proportional to integration of $s(\tau)$ and $e(\tau)$, lemma 4 and 5 imply the existence of unique steady state under some ν_e and ν_s large enough, fixing static equilibrium.

Note that both $\hat{h}$ and $\mu(h)dh$ are bounded above by assumtion and boundary conditions, and $p(\tau)$ is also bounded as $\int_\tau \gamma(\tau) p(\tau)^{1-\epsilon} d\tau = 1$. Hence, the existence follows when π_e and π_s are continuous in N_e and N_s even when considering changes in static equilibrium. Recall that $p(\tau)$ and $\hat{h}(\tau)$ could be obtained from the system of differential equations (1.9) *to* (1.11)*. Since all functions in equation* (1.9) *to* (1.11) *are differentiable, π_e and π_s are also continuous in N_e and N_s and the desired result follows.*

Intuitively, large ν_e and ν_s mean small returns to introducing additional variety, in turn, meaning decreasing rete of return. To see this intuition more clearly, recall that the task production function is given by

$$T(\tau) = \Bigg[\left\{ \alpha_h(\tau) \left(b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau) \right)^{\frac{\sigma_e-1}{\sigma_e}} + \alpha_e(\tau) N_e^{\frac{\sigma_e-1}{\sigma_e \nu_e}} e(\tau)^{\frac{\sigma_e-1}{\sigma_e}} \right\}^{\frac{\sigma_e(\sigma_s-1)}{(\sigma_e-1)\sigma_s}}$$

$$+\alpha_s(\tau) N_s^{\frac{\sigma_s-1}{\sigma_s \nu_s}} s(\tau)^{\frac{\sigma_s-1}{\sigma_s}} \Bigg]^{\frac{\sigma_s}{\sigma_s-1}}, \tag{A.3.12}$$

as $s(k,\tau) = s(\tau)$ and $e(k,\tau) = e(\tau)$ in equilibrium. The production is homogeneous of degree one in labor, N_e and N_s when $\nu_e \to 1$ and $\nu_s \to 1$. Since labor is fixed component, the production features strict concavity along N_e and N_s, meaning decreasing returns to scale in terms of total varieties.

The second part of lemma (3) is when $\sigma_e = \sigma_s = 1$. In this case,

$$p(\tau)T(\tau) = \frac{\omega(\tau)b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau)}{\alpha_h(\tau)}, \tag{A.3.13}$$

$$s(\tau) = \frac{\nu_s A_s \alpha_s(\tau) p(\tau) T(\tau)}{N_s}, \tag{A.3.14}$$

$$e(\tau) = \frac{\nu_e A_e \alpha_e(\tau) p(\tau) T(\tau)}{N_e}. \tag{A.3.15}$$

Combining the FOCs, $T(\tau)$ satisfies

$$p(\tau)T(\tau) = p(\tau)^{\frac{1}{\alpha_h(\tau)}} \kappa(\tau) N_s^{\Psi_{es}(\tau)} \left(\frac{N_e}{N_s} \right)^{\Psi_e(\tau)} B(\tau), \tag{A.3.16}$$

where $\kappa(\tau) := (\alpha_s(\tau)\nu_s A_s)^{\frac{\alpha_s(\tau)}{\alpha_h(\tau)}} (\alpha_e(\tau)\nu_e A_e)^{\frac{\alpha_e(\tau)}{\alpha_h(\tau)}}$, $\Psi_{es}(\tau) := \frac{1-\nu_s}{\nu_s}\frac{\alpha_s(\tau)}{\alpha_h(\tau)} + \frac{1-\nu_e}{\nu_e}\frac{\alpha_e(\tau)}{\alpha_h(\tau)}$, $\Psi_e(\tau) := \frac{1-\nu_e}{\nu_e}\frac{\alpha_e(\tau)}{\alpha_h(\tau)}$, and $B(\tau) := b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau)$ are introduced to simplify notation.

From equation (A.3.14) *and* (A.3.15)*, it is apparent that* $s(\tau)$ *and* $e(\tau)$ *are dcreasing in* N_s *and* N_e *respectivley when* $\Psi_{es}(\tau) < 1$*, which is a condition given in lemma 3.* ∎

Proof of proposition 1 (job polarization) *Substituting* $p(\tau)$ *out from equation* (1.9) *to* (1.11)*, we have*

$$\begin{aligned}
\hat{h}'(\tau) &= \frac{\gamma(\tau)\alpha_h(\tau)^{1-\alpha_h(\tau)(1-\epsilon)}Y}{b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\omega(\tau)^{1-\alpha_h(\tau)(1-\epsilon)}}\times \\
&\quad \left[\left(\alpha_s(\tau)\nu_s A_s N_s^{(1-\nu_s)/\nu_s}\right)^{\alpha_s(\tau)}\left(\alpha_e(\tau)\nu_e A_e N_e^{(1-\nu_e)/\nu_e}\right)^{\alpha_e(\tau)}\right]^{\epsilon-1} && \text{(A.3.17)} \\
\frac{d\log\omega(\tau)}{d\tau} &= -\frac{\partial \log b(\hat{h}(\tau),\tau)}{\partial\tau} && \text{(A.3.18)}
\end{aligned}$$

First, we show $\hat{h}_1$ *and* $\hat{h}_2$ *has to cross at least once. Suppose there is no crossing. Since* $\hat{h}_1(0) = \hat{h}_2(0)$ *and* $\hat{h}_1(\bar{\tau}) = \hat{h}_2(\bar{\tau})$*, we have*

$$\begin{aligned}
\left(\frac{\omega_1(0)}{\omega_2(0)}\right)^{1-\alpha_h(0)(1-\epsilon)} &= \frac{\hat{h}_2'(0)}{\hat{h}_1'(0)}\left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)\alpha_e(0)}, && \text{(A.3.19)} \\
\left(\frac{\omega_1(\bar{\tau})}{\omega_2(\bar{\tau})}\right)^{1-\alpha_h(\bar{\tau})(1-\epsilon)} &= \frac{\hat{h}_2'(\bar{\tau})}{\hat{h}_1'(\bar{\tau})}\left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)\alpha_e(\bar{\tau})}, && \text{(A.3.20)}
\end{aligned}$$

from equation (A.3.17)*. Combining,*

$$\left(\frac{\omega_1(\bar{\tau})/\omega_1(0)}{\omega_2(\bar{\tau})/\omega_2(0)}\right)^{1-\alpha_h(0)(1-\epsilon)}\left(\frac{\omega_1(\bar{\tau})}{\omega_2(\bar{\tau})}\right)^{(\alpha_h(0)-\alpha_h(\bar{\tau}))(1-\epsilon)} = \frac{\hat{h}_2'(\bar{\tau})/\hat{h}_2'(0)}{\hat{h}_1'(\bar{\tau})/\hat{h}1'(0)} \qquad \text{(A.3.21)}$$

Since $\hat{h}(\tau)$ *is strictly monotone and continous, with no crossing on entire* $(0,\bar{\tau})$*, we have to have either (i)* $\hat{h}_2'(\bar{\tau})/\hat{h}_2'(0) < \hat{h}_1'(\bar{\tau})/\hat{h}_1'(0)$ *and* $\hat{h}_1(\tau) < \hat{h}_2(\tau)$ *for* $\tau \in (0,\bar{\tau})$*, or (ii)* $\hat{h}_2'(\bar{\tau})/\hat{h}_2'(0) > \hat{h}_1'(\bar{\tau})/\hat{h}_1'(0)$ *and* $\hat{h}_1(\tau) > \hat{h}_2(\tau)$ *for* $\tau \in (0,\bar{\tau})$*. However, from equation* (A.3.18) *and log supermodularity of* $b(h,\tau)$*, we have* $\omega_1(\bar{\tau})/\omega_1(0) > \omega_2(\bar{\tau})/\omega_2(0)$ *with* $\hat{h}_1(\tau) < \hat{h}_2(\tau)$*.*

With small enough $\alpha_s(\bar{\tau})$, $(\omega_1(\bar{\tau})/\omega_2(\bar{\tau}))^{(\alpha_h(0)-\alpha_h(\bar{\tau}))(1-\epsilon)}$ *goes close to one, and hence equation* (A.3.21) *contradicts log supermodularity of* $b(h,\tau)$*.*

Second, we show that when $\hat{h}_1(\tau)$ *and* $\hat{h}_2(\tau)$ *cross at any three points* $\tau_a < \tau_b < \tau_c$*, we have* $\hat{h}_1'(\tau_a)/\hat{h}_1'(\tau_b) < \hat{h}_2'(\tau_a)/\hat{h}_2'(\tau_b)$ *with* $\hat{h}_2(\tau) > \hat{h}_1(\tau)$ *for* $\tau \in (\tau_a,\tau_b)$ *and* $\hat{h}_1'(\tau_c)/\hat{h}_1'(\tau_b) < \hat{h}_2'(\tau_c)/\hat{h}_2'(\tau_b)$ *with* $\hat{h}_1(\tau) > \hat{h}_2(\tau)$ *for* $\tau \in (\tau_b,\tau_c)$*.*

From equilibrium condition (A.3.17)*,*

$$\left(\frac{\omega_1(\tau_b)/\omega_1(\tau_a)}{\omega_2(\tau_b)/\omega_2(\tau_a)}\right)^{1-\alpha_h(\tau_a)(1-\epsilon)}\left(\frac{\omega_1(\tau_b)}{\omega_2(\tau_b)}\right)^{(\alpha_h(\tau_a)-\alpha_h(\tau_b))(1-\epsilon)} = \frac{\hat{h}_2'(\tau_b)/\hat{h}_2'(\tau_a)}{\hat{h}_1'(\tau_b)/\hat{h}_1'(\tau_a)}\left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)(\alpha_e(\tau_b)-\alpha_e(\tau_a))} \tag{A.3.22}$$

$$\left(\frac{\omega_1(\tau_c)/\omega_1(\tau_b)}{\omega_2(\tau_c)/\omega_2(\tau_b)}\right)^{1-\alpha_h(\tau_c)(1-\epsilon)}\left(\frac{\omega_1(\tau_b)}{\omega_2(\tau_b)}\right)^{(\alpha_h(\tau_b)-\alpha_h(\tau_c))(1-\epsilon)} = \frac{\hat{h}_2'(\tau_c)/\hat{h}_2'(\tau_b)}{\hat{h}_1'(\tau_c)/\hat{h}_1'(\tau_b)}\left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)(\alpha_e(\tau_c)-\alpha_e(\tau_b))} \tag{A.3.23}$$

With small enough $\alpha_h'(\tau)$*, these equations are approximated to*

$$\left(\frac{\omega_1(\tau_b)/\omega_1(\tau_a)}{\omega_2(\tau_b)/\omega_2(\tau_a)}\right)^{1-\alpha_h(\tau_a)(1-\epsilon)} \approx \frac{\hat{h}_2'(\tau_b)/\hat{h}_2'(\tau_a)}{\hat{h}_1'(\tau_b)/\hat{h}_1'(\tau_a)}\left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)(\alpha_e(\tau_b)-\alpha_e(\tau_a))} \tag{A.3.24}$$

$$\left(\frac{\omega_1(\tau_c)/\omega_1(\tau_b)}{\omega_2(\tau_c)/\omega_2(\tau_b)}\right)^{1-\alpha_h(\tau_c)(1-\epsilon)} \approx \frac{\hat{h}_2'(\tau_c)/\hat{h}_2'(\tau_b)}{\hat{h}_1'(\tau_c)/\hat{h}_1'(\tau_b)}\left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)(\alpha_e(\tau_c)-\alpha_e(\tau_b))} \tag{A.3.25}$$

The only possibility that this can hold at the same time is when $\alpha_e(\tau_b) > \alpha_e(\tau_a)$ *and* $\alpha_e(\tau_b) > \alpha_e(\tau_c)$ *so that the signs of exponent term with respect to* (A_{e2}/A_{e1}) *are different. Recall that* $\omega_1(\tau_b)/\omega_1(\tau_a) < \omega_2(\tau_b)/\omega_2(\tau_a)$ *implies* $\hat{h}_2'(\tau_b)/\hat{h}_2'(\tau_a) > \hat{h}_1'(\tau_b)/\hat{h}_1'(\tau_a)$ *from equilibrium condition* (A.3.18) *and log supermodularity of* $b(h,\tau)$*. Since* $q_{e1} > q_{e2}$, $\alpha_e(\tau_b) > \alpha_e(\tau_a)$*, and* $\alpha_e(\tau_b) > \alpha_e(\tau_c)$*, we must have* $\omega_1(\tau_b)/\omega_1(\tau_a) > \omega_2(\tau_b)/\omega_2(\tau_a)$ *and* $\omega_1(\tau_c)/\omega_1(\tau_b) < \omega_2(\tau_c)/\omega_2(\tau_b)$*, which implies* $\hat{h}_1(\tau) < \hat{h}_2(\tau)$ *for* $\tau \in (\tau_a,\tau_b)$ *and* $\hat{h}_1(\tau) > \hat{h}_2(\tau)$ *for* $\tau \in (\tau_b,\tau_c)$*.*

The proof in the first part rules out any even number of crossings and no crossing. The second part implies they have to cross only a single time on $\tau \in (0, \bar{\tau})$ as they already meet at 0 and $\bar{\tau}$. Then the result follows from the second part of proof. ∎

Proof of proposition 2 (the rise of software) *We firstly show that the production share of middle skill task (task 1) falls and that of high skill task (task 2) rises in response to the decline of price of equipment in a discretized model as well. To be specific, we prove the following lemma first.*

Lemma 6 *Fix N_e and N_s. Consider a decline of the price of equipment; $d \log A_e > 0$ and suppose $\epsilon < 1$ and assumption 2, 5, and 6. Then we have $d \log p_1 < 0$ and $d \log p_2 > 0$.*

Proof From the equilibrium conditions (A.2.1) to (A.2.3),

$$\sum_{j=0}^{2} \gamma_j p_j^{1-\epsilon} = 1$$

$$p_j = \left(\frac{\omega_j}{\alpha_{h,j}}\right)^{\alpha_{h,j}} \left(\frac{1}{\nu_e A_e \alpha_{e,j}}\right)^{\alpha_{e,j}} \left(\frac{1}{\nu_s A_s \alpha_{s,j}}\right)^{\alpha_{s,j}} N_e^{-\varphi_e \alpha_{e,j}} N_s^{-\varphi_s \alpha_{s,j}}, \text{ for } j = 0, 1, 2$$

$$w_{j-1} b(\hat{h}_j, j-1) = w_j b(\hat{h}_j, j), \text{ for } j = 1, 2$$

$$\frac{\omega_{j-1} \int_{\hat{h}_{j-1}}^{\hat{h}_j} b(h, j-1)\mu(h) dh}{\omega_j \int_{\hat{h}_j}^{\hat{h}_{j+1}} b(h, j)\mu(h) dh} = \frac{\alpha_{h,j-1}\gamma_{j-1}}{\alpha_{h,j}\gamma_j} \left(\frac{p_{j-1}}{p_j}\right)^{1-\epsilon}, \text{ for } j = 1, 2,$$

with $\sigma_s = \sigma_e = 1$.

Let $\Delta x = d\log(x)$. Then by differentiating above and using assumption 5,

$$\Delta p_j = \alpha_{h,j}\Delta\omega_j - \alpha_{e,j}\Delta A_e \tag{A.3.26}$$

$$\Delta\omega_{j-1} = \Delta\omega_j + \Delta b(\hat{h}_j, j) - \Delta b(\hat{h}_j, j-1) \tag{A.3.27}$$

$$\Delta\omega_{j-1} - \Delta\omega_j = (1-\epsilon)(\Delta p_{j-1} - \Delta p_j) \tag{A.3.28}$$

$$\sum_{j=0}^{2} \gamma_j p_j^{1-\epsilon}\Delta p_j = 0 \tag{A.3.29}$$

Eliminating ω_j's,

$$\left(\frac{1}{\alpha_{h,0}} - (1-\epsilon)\right)\Delta p_0 = \left(\frac{1}{\alpha_{h,1}} - (1-\epsilon)\right)\Delta p_1 + \left(\frac{\alpha_{e,1}}{\alpha_{h,1}} - \frac{\alpha_{e,0}}{\alpha_{h,0}}\right)\Delta A_e \tag{A.3.30}$$

$$\left(\frac{1}{\alpha_{h,2}} - (1-\epsilon)\right)\Delta p_2 = \left(\frac{1}{\alpha_{h,1}} - (1-\epsilon)\right)\Delta p_1 + \left(\frac{\alpha_{e,1}}{\alpha_{h,1}} - \frac{\alpha_{e,2}}{\alpha_{h,2}}\right)\Delta A_e \tag{A.3.31}$$

Since $1/\alpha_{h,j} > (1-\epsilon)$ for all j's and $\alpha_{e,1}/\alpha_{h,1} > \alpha_{e,j}/\alpha_{h,j}$ for $j = 0, 2$, it is easy to check that $\Delta p_1 < 0$ by substituting equation (A.3.30) and (A.3.31) into equation (A.3.29).

Substituting equation (A.3.30) and (A.3.31) into equation (A.3.29), we also have

$$\begin{aligned}
&\left[\gamma_0 p_0^{1-\epsilon}\left(\frac{\frac{1}{\alpha_{h,2}} - (1-\epsilon)}{\frac{1}{\alpha_{h,0}} - (1-\epsilon)}\right) + \gamma_2 p_2^{1-\epsilon} + \gamma_1 p_1^{1-\epsilon}\left(\frac{\frac{1}{\alpha_{h,2}} - (1-\epsilon)}{\frac{1}{\alpha_{h,1}} - (1-\epsilon)}\right)\right]\Delta p_2 \\
&+\gamma_0 p_0^{1-\epsilon}\left[\frac{\left(\frac{\alpha_{e,1}}{\alpha_{h,1}} - \frac{\alpha_{e,0}}{\alpha_{h,0}}\right) - \left(\frac{\alpha_{e,1}}{\alpha_{h,1}} - \frac{\alpha_{e,2}}{\alpha_{h,2}}\right)}{\frac{1}{\alpha_{h,0}} - (1-\epsilon)}\right]\Delta A_e \\
&-\gamma_1 p_1^{1-\epsilon}\frac{\frac{\alpha_{e,1}}{\alpha_{h,1}} - \frac{\alpha_{e,2}}{\alpha_{h,2}}}{\frac{1}{\alpha_{h,1}} - (1-\epsilon)}\Delta A_e = 0
\end{aligned} \tag{A.3.32}$$

By assumption 6 and $\epsilon < 1$, we have

$$\left[\gamma_0 p_0^{1-\epsilon}\left(\frac{\frac{1}{\alpha_{h,2}} - (1-\epsilon)}{\frac{1}{\alpha_{h,0}} - (1-\epsilon)}\right) + \gamma_2 p_2^{1-\epsilon} + \gamma_1 p_1^{1-\epsilon}\left(\frac{\frac{1}{\alpha_{h,2}} - (1-\epsilon)}{\frac{1}{\alpha_{h,1}} - (1-\epsilon)}\right)\right] > 0,$$

$$\gamma_0 p_0^{1-\epsilon}\left[\frac{\left(\frac{\alpha_{e,1}}{\alpha_{h,1}} - \frac{\alpha_{e,0}}{\alpha_{h,0}}\right) - \left(\frac{\alpha_{e,1}}{\alpha_{h,1}} - \frac{\alpha_{e,2}}{\alpha_{h,2}}\right)}{\frac{1}{\alpha_{h,0}} - (1-\epsilon)}\right] = 0,$$

$$\gamma_1 p_1^{1-\epsilon}\frac{\frac{\alpha_{e,1}}{\alpha_{h,1}} - \frac{\alpha_{e,2}}{\alpha_{h,2}}}{\frac{1}{\alpha_{h,1}} - (1-\epsilon)} > 0,$$

implying $\Delta p_2 > 0$ from equation (A.3.32). ∎

Now we show that lemma 6 implies a relative increase of software variety in the new steady state. Note that the profits from providing software and equipment variety are given by

$$\pi_s = \sum_j \frac{1-\nu}{\nu A_s} s_j \quad \text{and } \pi_e = \sum_j \frac{1-\nu}{\nu A_e} e_j.$$

From the FOC and using (A.3.4) *($p_e = 1/(\nu A_e)$ and $p_s = 1/(\nu A_s)$), demand for equipment and software for each task are $e_j = \nu_e A_e \alpha_{e,j} p_j T_j / N_e$ and $s_j = \nu_s A_s \alpha_{s,j} p_j T_j / N_s$.*

From lemma 3, we know $\pi_e/\eta = \pi_s/\eta = \rho$ in any steady state equilibrium, and hence,

$$d\pi_e = (1-\nu)\Bigg[(1-\epsilon)(\alpha_{e,0} p_0^{-\epsilon} dp_0 + \alpha_{e,1} p_1^{-\epsilon} dp_1 + \alpha_{e,2} p_2^{-\epsilon} dp_2) Y$$
$$+ (\sum_j \alpha_{e,j} p_j^{1-\epsilon}) dY - \frac{1}{N_e} \sum_j \alpha_{e,j} p_j^{1-\epsilon} Y dN_e\Bigg] = 0$$
$$d\pi_s = (1-\nu)\Bigg[(1-\epsilon)(\alpha_{s,0} p_0^{-\epsilon} dp_0 + \alpha_{s,1} p_1^{-\epsilon} dp_1 + \alpha_{s,2} p_2^{-\epsilon} dp_2) Y$$
$$+ (\sum_j \alpha_{s,j} p_j^{1-\epsilon}) dY - \frac{1}{N_s} \sum_j \alpha_{e,j} p_j^{1-\epsilon} Y dN_s\Bigg] = 0$$

Combining,

$$
\begin{aligned}
&(1-\epsilon)[(\alpha_{e,1}-\alpha_{s,1})p_1^{-\epsilon}dp_1+(\alpha_{e,2}-\alpha_{s,2})p_2^{-\epsilon}dp_2] \\
&=\sum_j \alpha_{e,j}p_j^{1-\epsilon}\left[\frac{dN_e}{N_e}-\frac{dY}{Y}\right]-\sum_j \alpha_{s,j}p_j^{1-\epsilon}\left[\frac{dN_s}{N_s}-\frac{dY}{Y}\right] \\
&=\sum_j \alpha_{s,j}p_j^{1-\epsilon}\left[\frac{dN_e-dN_s}{N_s}-\left(1-\frac{N_e}{N_s}\right)\frac{dY}{Y}\right]<0,
\end{aligned}
$$

where the last equality is from no arbitrage condition (1.16) *(*$\frac{N_s}{N_e}=\frac{\sum_j \alpha_{s,j}\gamma_j p_j^{1-\epsilon}}{\sum_j \alpha_{e,j}\gamma_j p_j^{1-\epsilon}}$*), and the inequality is from lemma 6 and assumption 6.*

Hence, we have

$$dN_s > dN_e + (N_e - N_s)\frac{dY}{Y}.$$

Since decrease in the price of equipment raise the level of production, we have $dY/Y>0$*. Hence, with the condition given in this proposition (*$N_e \geq N_s$*),* $(N_e-N_s)dY/Y \geq 0$ *and so* $dN_s > dN_e$*. Finally, since* $N_e \geq N_s$*, we have*

$$dN_s/N_s > dN_e/N_e,$$

which was to be shown. ∎

Proof of proposition 3 (skill demand reversal) *Suppose they cross at least once. It means that we have at least three points* $\tau_a<\tau_b<\tau_c$ *such that* $\hat{h}_1(\tau_a)=\hat{h}_2(\tau_a)$*,* $\hat{h}_1(\tau_b)=\hat{h}_2(\tau_b)$*,* *and* $\hat{h}_1(\tau_c) = \hat{h}_2(\tau_c)$*. Then, we have*

$$\left(\frac{\omega_1(\tau_b)/\omega_1(\tau_a)}{\omega_2(\tau_b)/\omega_2(\tau_a)}\right)^{1-\alpha_h(\tau_a)(1-\epsilon)} \left(\frac{\omega_1(\tau_b)}{\omega_2(\tau_b)}\right)^{(\alpha_h(\tau_a)-\alpha_h(\tau_b))(1-\epsilon)} = \frac{\hat{h}_2'(\tau_b)/\hat{h}_2'(\tau_a)}{\hat{h}_1'(\tau_b)/\hat{h}_1'(\tau_a)} \left(\frac{N_{s2}}{N_{s1}}\right)^{\varphi_s(1-\epsilon)(\alpha_s(\tau_b)-\alpha_s(\tau_a))} \tag{A.3.33}$$

$$\left(\frac{\omega_1(\tau_c)/\omega_1(\tau_b)}{\omega_2(\tau_c)/\omega_2(\tau_b)}\right)^{1-\alpha_h(\tau_c)(1-\epsilon)} \left(\frac{\omega_1(\tau_b)}{\omega_2(\tau_b)}\right)^{(\alpha_h(\tau_b)-\alpha_h(\tau_c))(1-\epsilon)} = \frac{\hat{h}_2'(\tau_c)/\hat{h}_2'(\tau_b)}{\hat{h}_1'(\tau_c)/\hat{h}_1'(\tau_b)} \left(\frac{N_{s2}}{N_{s1}}\right)^{\varphi_s(1-\epsilon)(\alpha_s(\tau_c)-\alpha_s(\tau_b))} \tag{A.3.34}$$

where $\varphi_s \equiv (1-\nu_s)/\nu_s$*.*

With small enough $\alpha_h'(\tau)$*, above equations can be approximated to*

$$\left(\frac{\omega_1(\tau_b)/\omega_1(\tau_a)}{\omega_2(\tau_b)/\omega_2(\tau_a)}\right)^{1-\alpha_h(\tau_a)(1-\epsilon)} \frac{\hat{h}_1'(\tau_b)/\hat{h}_1'(\tau_a)}{\hat{h}_2'(\tau_b)/\hat{h}_2'(\tau_a)} \approx \left(\frac{N_{s2}}{N_{s1}}\right)^{\varphi_s(1-\epsilon)(\alpha_s(\tau_b)-\alpha_s(\tau_a))} \tag{A.3.35}$$

$$\left(\frac{\omega_1(\tau_c)/\omega_1(\tau_b)}{\omega_2(\tau_c)/\omega_2(\tau_b)}\right)^{1-\alpha_h(\tau_c)(1-\epsilon)} \frac{\hat{h}_1'(\tau_c)/\hat{h}_1'(\tau_b)}{\hat{h}_2'(\tau_c)/\hat{h}_2'(\tau_b)} \approx \left(\frac{N_{s2}}{N_{s1}}\right)^{\varphi_s(1-\epsilon)(\alpha_s(\tau_c)-\alpha_s(\tau_b))} \tag{A.3.36}$$

Again, since matching function is continuous and monotone, and $b(h,\tau)$ *is log supermodular, signs of log of LHS in both equation* (A.3.35) *and* (A.3.36) *should be different. However, since* $\alpha_s(\tau)$ *is strictly increasing, signs of log of RHS in equation* (A.3.35) *and* (A.3.36) *are same, which is contradiction.*

Finally, to show $\hat{h}_2(\tau) < \hat{h}_1(\tau)$ *for* $\tau \in (0, \bar{\tau})$*, recall that equilibrium condition* (A.3.17) *implies*

$$\left(\frac{\omega_1(\bar{\tau})/\omega_1(0)}{\omega_2(\bar{\tau})/\omega_2(0)}\right)^{1-\alpha_h(\bar{\tau})(1-\epsilon)} \frac{\hat{h}_1'(\bar{\tau})/\hat{h}_1'(0)}{\hat{h}_2'(\bar{\tau})/\hat{h}_2'(0)} = \left(\left(\frac{N_{s2}}{N_{s1}}\right)^{\varphi_s} \frac{\omega_2(0)}{\omega_1(0)}\right)^{(1-\epsilon)(\alpha_s(\bar{\tau})-\alpha_s(0))} \tag{A.3.37}$$

Since $(1-\epsilon)(\alpha_s(\bar{\tau}) - \alpha_s(0)) > 0$ *and* $N_{s2} > N_{s1}$*, we have to have* $\omega_1(\bar{\tau})/\omega_1(0) > \omega_2(\bar{\tau})/\omega_2(0)$*, which implies* $\hat{h}_2(\tau) > \hat{h}_1(\tau)$*.* ∎

A.4 Numerical Examples: Continuous Tasks

To illustrate the comparative statics, we provide some numerical examples. For this example, we set:

$$b(h,\tau) = h - \tau,\ \mathcal{M}(h) = \frac{1-h^{-a}}{1-\bar{h}^{-a}},\ \gamma(\tau) = 1,$$
$$\alpha_e(\tau) = -2.5(\tau - .5)^2 + .6, \quad \text{and} \quad \alpha_s(\tau) = .3\tau + .025.$$

For the parameter values, we use $\bar{\tau} = 1$, $\bar{h} = 4$, $d\tau = .005$, $a = 2.5$, $\epsilon = 0.7$, $\nu_s = 0.65$, $\nu_s = .8$, $\eta_s = \eta_e = A_s = A_e = 1$, $\theta = 1$, and $\rho = .03$.

In the inner loop, we solve static equilibrium given N_s and N_e. The equilibrium assignment function is computed from equation (1.9) to (1.11). Specifically, we use $\hat{h}(0) = 1$ and guess $\hat{h}'(0)$ and $\omega(0)$. With the guess, differential equation is solved using finite difference method. We iterate until $\hat{h}(1) = 4$ and $\int p(\tau)^{1-\epsilon} d\tau = 1$ using Gauss-Newton method.

Then in the outer loop, we search for N_s and N_e that equate $\pi_s/\eta_s = \pi_e/\eta_e = \rho$, again using Gauss-Newton method.

Factor intensities and the equilibrium assignment function in this example are shown in figure A1. The equipment intensity $\alpha_e(\tau)$ is increasing on $\tau \in [0, 0.5]$ and decreasing on $\tau \in [0.5, 1]$, while the software intensity $\alpha_s(\tau)$ is increasing from 0 to 1. We can also see that the equilibrium assignment function $\hat{h}(\tau)$ is strictly increasing on τ.

Now we compare equilibrium with $A_e = 1$ and $A_e = 5$ in figure A2. The assignment function in the original equilibrium (with $A_e = 1$), in the static equilibrium (with $A_e = 5$) and the new steady state (with $A_e = 5$) are depicted in figure A2(a). As expected from proposition 1 through proposition 3, we see that the assignment function in the static equilibrium (blue line) cross with the original assignment function (black line) at the middle of τ. The assignment function in the steady state (red line) is generally located above the assignment function in static equilibrium (blue line).

To see the changes in the employment structure more clearly, we also plot changes in the employment share by skill percentile in figure A2(b), similar to the graph shown in figure 1.2(a). To be specific, the horizontal axis shows the tasks ($\hat{\tau}(h)$) corresponding to each percentile in the skill distribution $\mathcal{M}$, and the vertical axis shows the changes in the employment share of those tasks from the original equilibrium to new static equilibrium (blue line) and from the new static equilibrium to new steady state (red line). For example, the first two points on the horizontal axis is two task $\hat{\tau}_1(h_1)$ and $\hat{\tau}_1(h_2)$ where $\hat{\tau}_1$ represents the original (inverse) assignment function and $h_1 = 1$ and $h_2 = \mathcal{M}^{-1}(1)$. Then the first point on the blue line is difference between $\mathcal{M}(\hat{h}_2(\hat{\tau}_1(h_1))) - \mathcal{M}(\hat{h}_2(\hat{\tau}_1(h_2))$ and $\mathcal{M}(h_2) - \mathcal{M}(h_1)$, where $\hat{h}_2$ is the assignment function in the static equilibrium corresponding to $A_e = 5$.

Figure A1: Factor intensities and assignment function

(a) Intensities $\alpha(\tau)$ (b) Assignment function $\hat{h}(\tau)$

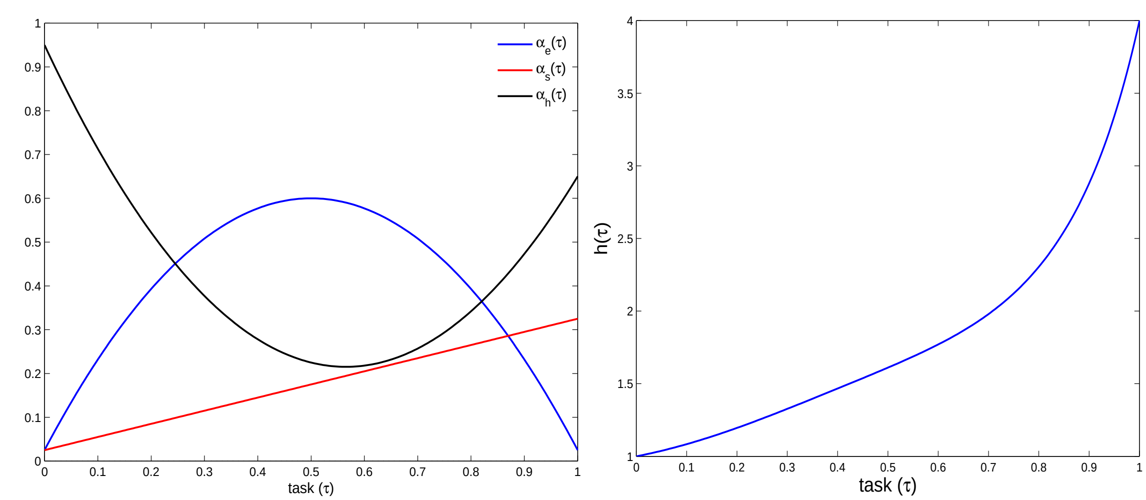
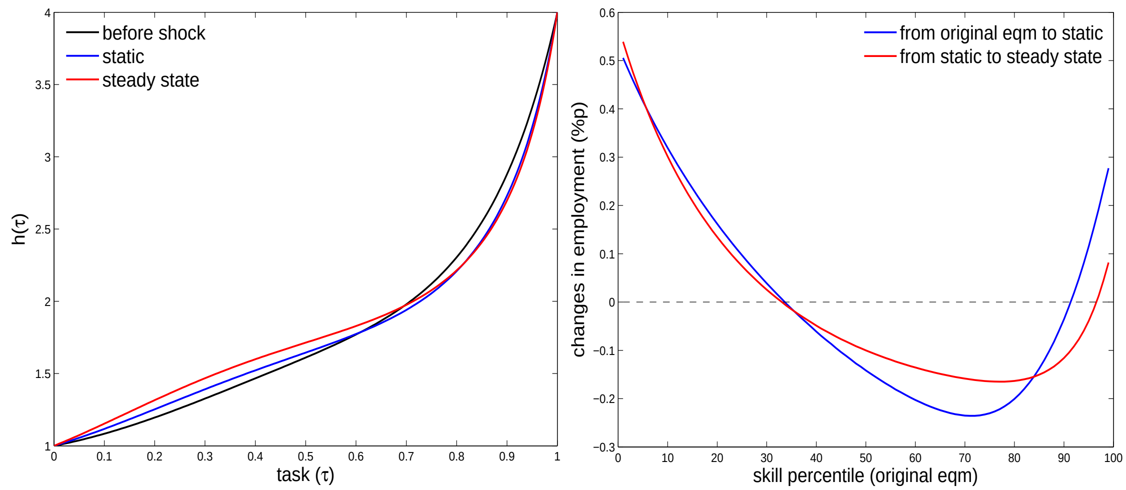


Figure A2: Equilibrium comparison with $A_e = 1$ and $A_e = 5$

(a) Assignment function $\hat{h}(\tau)$ (b) Changes in employment by skill percentile



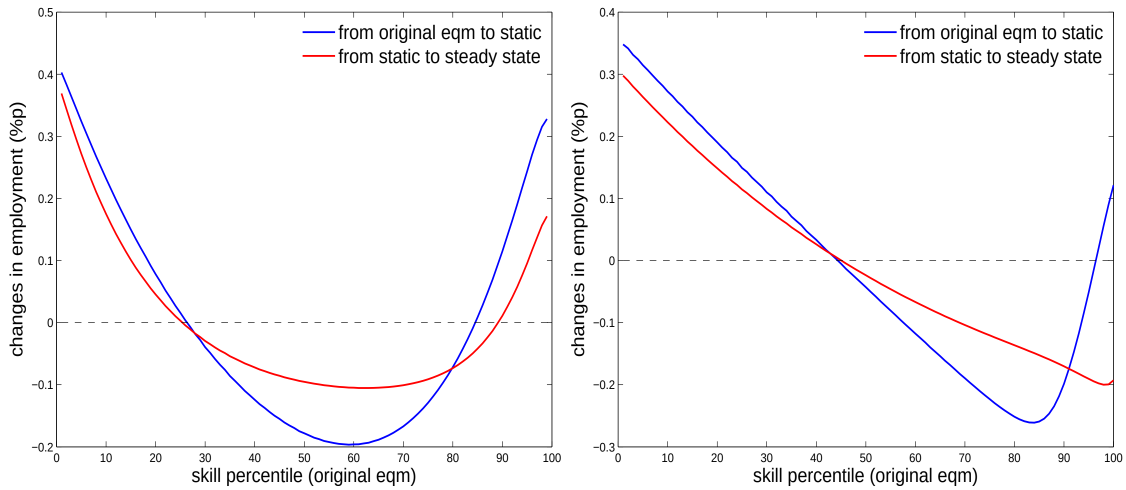
For the relative size of software variety to equipment variety, it was initially .74 in the original equilibrium, and increases to .77 in the new steady state, which is about 6% increase.

CES Task Production To characterize the analytical results, we assume unitary elasticity between labor and capital. However, the crucial characteristic is that the elasticity of substitution between labor and capital is greater than the elasticity of substitution between tasks (ϵ). Furthermore, we expect that task production need not be Cobb-Douglas in generating responses consistent with propositions 1 to 3, at least numerically.

What could we expect if the elasticity of substitution between labor and capital is different from one? We predict that the larger the elasticity of substitution between labor and equipment becomes, the higher the polarization effect would appear. We also expect that the skill demand reversal effect (decreasing high-skill demand) and the rise of software would be enhanced as the elasticity of substitution between labor and software increases.

Figure A3: Equilibrium comparison with $A_e = 1$ and $A_e = 5$: CES task production

(a) Changes in employment with $\sigma_e = 1.2$ (b) Changes in employment with $\sigma_s = 1.2$



Intuitively, when the elasticity of substitution between equipment and labor is greater than one, a decrease in the price of equipment lowers the demand for middle-skill tasks not only through the adjustment in the assignment but also through the adjustment between labor and equipment within a task. Additionally, when the elasticity of substitution between software and labor is greater than one, corresponding increases in software would substitute high-skill labor more than before.

To confirm the intuition, we provide several numerical illustrations in figure A3 (details can be found in appendix A.4). As expected, the magnitude of the decreasing middle increases as σ_e increases, and the decrease in high-skill demand is enhanced as σ_s increases.

A.5 Data Construction for Section 1.5

For relative employment by industry, we use a ratio of employment of routine occupations and employment of cognitive occupations. Routine occupations include machine operators, office and sales, mechanics, construction and production, and transportation occupations. Cognitive occupations are management, professionals, and technicians. The level of employment is obtained from Census 1980, 1990, and 2000, and American Community Survey (ACS) 2010, received from IPUMS. We made a concordance between consistent industry code `ind1990` and `indnaics` using employment in Census 2000. Then employment by `indnaics` is merged into 61 BEA industry code based on a concordance between BEA industry code and NAICS.

The price of equipment and software by industry is from Section 2 of Fixed Asset Table from BEA. The price index is constructed by dividing nominal investment by real investment. We use private non-residential equipment investment by industry for the benchmark, although other series (e.g. industrial equipment) also give similar results.

For growth of software innovation, we use log difference of own account software investment by industry, which captures software investment made in-house by firms. We believe this as a good proxy for software innovation, as in-house software investment is made to develop new software for firm's production process.

It is not straightforward to measure R&D for equipment related innovation from industry level data, as BEA records R&D expenditures only by sources of funds. We think that R&D expenditures funded by equipment producing industries are likely to be used for equipment related innovation, but they should be only a subset of total equipment related innovations. It is also likely that most of these expenditures are used by equipment producing industries, not others, which makes it difficult to capture industry variation. Therefore, we use total R&D expenditures other than software as a benchmark series for N_e, and examine robustness using many different combinations of R&D data. All combinations, including a case with own-account software only, show similar positive relation against relative price.

A.6 Calibration Procedure

This section describes the detailed calibration procedure. We normalize exogenous variables M_j's, A_e, and A_s to one in 1980.

1. We start from $\hat{h}_j$'s that correspond to employment share of occupation j in 1980 and fix ϵ, σ_s and σ_e arbitrarily.

2. By indifference between tasks at the threshold level of skills, we have

$$\frac{w_j}{w_{j-1}} = \frac{\hat{h}_j - \chi_{j-1}}{\hat{h}_j - \chi_j},$$

   and so $w_j = w_0 \prod_{k=1}^{j} (\hat{h}_k - \chi_{k-1})/(\hat{h}_k - \chi_k)$. Therefore, payroll share of occupation j is given by

$$\frac{\prod_{k=1}^{j} (\hat{h}_k - \chi_{k-1})/(\hat{h}_k - \chi_k) \int_{\hat{h}_{j-1}}^{\hat{h}_j} (h - \chi_j) h^{-a-1} dh}{\sum_j \prod_{k=1}^{j} (\hat{h}_k - \chi_{k-1})/(\hat{h}_k - \chi_k) \int_{\hat{h}_{j-1}}^{\hat{h}_j} (h - \chi_j) h^{-a-1} dh}.$$

   We set 8 parameters χ_j's and 1 parameter a to minimize distance between payroll share in data and the model for 9 occupations.

3. Guess $\alpha_{j,e}$ and $\alpha_{j,s}$. We find γ_j's that match with $\hat{h}_j$'s in equilibrium.

4. We iterate over $\alpha_{j,e}$ and $\alpha_{j,s}$ untill aggregate labor share, E_j and S_j in the model match with aggregate labor share, equipment and software investment by occupation in data.

5. We solve for M_j's for routine occupations ($j = 2, 3, 4, 5, 7, 8$) to match employment share of routine occupations in data. Note that we already have different values of A_e and A_s for each period obtained from data.

6. Iterate over σ_s and σ_e so that labor share with and without software match with trend implied level in 2010.

7. Iterate over ϵ so as to minimize an average distance between changes in payroll share by occupation in the model and data.

The procedure gives all the parameters needed to be calibrated. For ν_e and ν_s, we use estimated value as described in the section 1.6.

Appendix B

Appendix to Chapter 2

B.1 Tables and Figures Not Included in Text

	Const	FIRE	Health	High serv.	Low serv.	Dur	Mine	Non-durable	Trade	Comp-uter
$\sigma = .1$	1.577	1.260	1.497	1.579	1.447	1.222	1.423	1.462	1.505	1.828
$\sigma = .5$	1.593	1.245	1.473	1.546	1.436	1.225	1.422	1.473	1.480	1.825
$\sigma = .7$	1.624	1.229	1.444	1.506	1.423	1.234	1.426	1.498	1.451	1.825
$\sigma = .815$	1.699	1.213	1.413	1.461	1.415	1.263	1.445	1.559	1.419	1.840

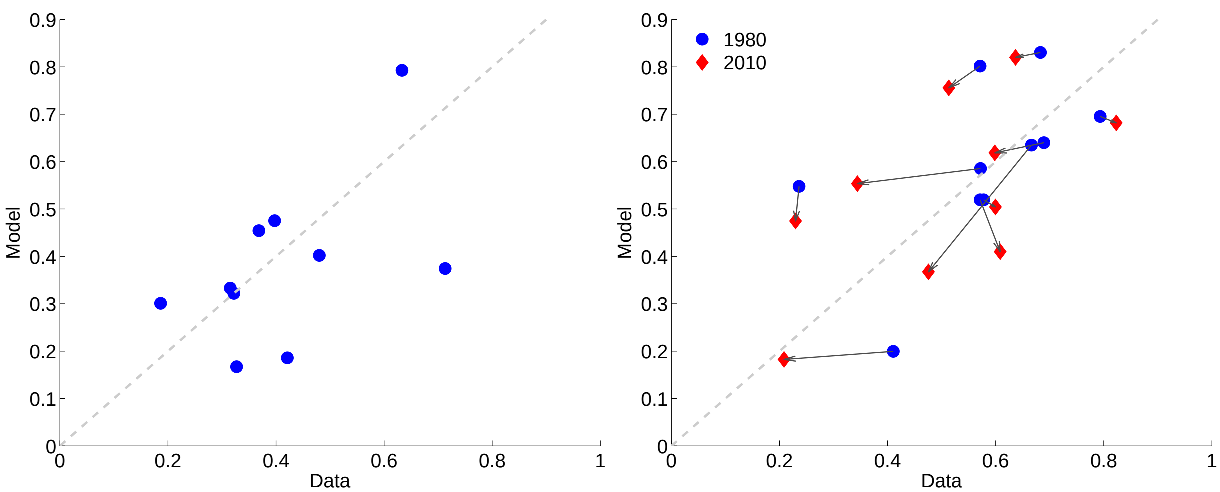
Table B1: Calibrated ρ_i's across various σ's

For each value of σ, the ρ_i's are calibrated as explained in Section 2.4.1 except that σ is fixed. We found that for values of σ above its benchmark value of 0.815, the model fit quickly becomes exponentially poor with no solution as it approaches 1. The reason is that occupation-specific productivities M_j become so large that it becomes impossible to simultaneously match employment share changes and measured TFP by industry.

	Output		Productivity	
	Data	Model	Data	Model
1980-	3.41	3.81	0.43*	0.85
1990-	3.75	3.28	1.26	0.73
2000-	**1.54**	**1.30**	**0.44**	**0.34**

Table B2: Model Fit to Aggregate Output and Productivity

Data source: BEA NIPA. *Although average productivity growth seems low in the data for the 1980s, this is more of a cyclical phenomena in the early 1980s that persisted from the late 1970s. For example, average productivity growth from 1982-1990 is 1.18%.



(a) Traditional Capital share (b) Labor share

Figure B1: Factor income shares by industry: model vs. data

In the model, traditional capital income shares are computed by first fixing that of the computer industry's to 1980 data as explained in Section 2.4.1, and then calibrating them for all other industries using a method of moments. Data traditional capital income shares are computed by applying the procedure in Section 2.4.1 too all industries in 1980. All data from BEA NIPA and FAT.

Appendix C

Appendix to Chapter 3

C.1 The Data

C.1.1 Data sources

We use National Accounts data of countries following SNA 08, which are AUS, AUT, BEL, CAN, CHE, CZE, DEU, DNK, ESP, EST, FRA, FIN, GBR, GRC, HUN, IRL, ISL, ISR, ITA, KOR, LUX, MEX, NLD, NOR, NZL, POL, PRT, SVK, SVN, SWE, and USA. Data are from either OECD statistics or National statistical offices, which gives longer or conceptually more accurate series. The data sources are summarized in table C1 and C2.

C.1.2 Investment

We classify type of investments by traditional and IPP. Traditional investment includes dwellings, other buildings and structures, and equipments & weapon systems. We exclude

Table C1: National sources

Country	Name of Institution	Name of Table
AUS	Australian Bureau of Statistics	Australian System of National Accounts
AUT	Statistics Austria	National Accounts
BEL	NBB statistics	National Accounts
CAN	Statistics Canada	System of macroeconomic accounts
CHE	Swiss Statistics	National Accounts
CZE	Czech Statistical Office	National Accounts
DNK	Statistics Denmark	National accounts and government finances
DEU	Statistisches Bundesamt	National Accounts
ESP	National Statistics Institute	National Accounts
EST	Statistics Estonia	National Accounts
FIN	Statistics Finland	National Accounts
FRA	National Institute of Statistics and Economic Studies	National Accounts
GBR	Office for National Statistics	National Accounts
GRC	Hellenic Statistical Authority	National Accounts
HUN	Hungarian Central Statistical Office	Integrated economic accounts
IRL	Central Statistical Office	National Accounts
ISL	Statistics Iceland	National Accounts
ISR	Bank of Israel	National Accounts
ITA	Italian National Institute of Statistics	National Accounts
KOR	Bank of Korea	National Accounts
LUX	Grand-Duchy of Luxembourg	National Accounts
NLD	Statistics Netherlands	Macroeconomics table
NOR	Statistics Norway	National Accounts
NZL	Statistics New Zealand	National Accounts
POL	Central Statistical Office of Poland	National Accounts
PRT	Statistics Portugal	National Accounts
SVK	Statistical Office of the Slovak Republic	Macroeconomic Statistics
SVN	Statistical Office RS	National Accounts
SWE	Statistics Sweden	National Accounts
USA	Bureau of Economic Analysis	National Income and Product Account
OECD	OECD Statistics	National Accounts

cultivated biological resources from both classification of which shares in total investments is less than 1% on average.

Since statistical office does not provide real value of traditional investment, we construct it from subitems – dwellings, other buildings and structures, and equilpments & weapon systems – using Törnqvist index. Specifically, price change of traditional investment (π_t^T) is

$$\pi_t^T = \omega_t^R \pi_t^R + \omega_t^S \pi_t^S + \omega_t^E \pi_t^E,$$

Table C2: Data sources by country

	Variables										
	CE	MI	GVA	SE[5)]	P^c	NI	RI	NK	RK	CFC	D
AUS	NS	NS	NS	–	NS	NS	NS	NS	NS	NS	NS
AUT	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	–
BEL	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	–
CAN	OECD	OECD	OECD	OECD	OECD	OECD	OECD	NS	–	OECD	–
CHE	OECD	OECD	OECD	–	OECD	OECD	OECD	NS	NS	OECD	–
CZE	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	–
DEU	OECD	OECD	OECD	OECD	OECD	OECD	OECD	NS	NS	OECD	–
DNK	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	–
ESP	OECD	OECD	OECD	OECD	OECD	OECD	OECD	–	–	OECD	–
EST	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	–
FRA	OECD	OECD	OECD	–	OECD	OECD	OECD	NS	OECD	OECD	NS
FIN	OECD	NS	OECD	OECD	OECD	OECD	OECD	OECD	OECD	NS	–
GBR	NS	NS	NS	OECD	NS	OECD	OECD	OECD	OECD	OECD	NS
GRC	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	–
HUN	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	–
IRL	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	–
ISL	OECD	–	OECD	–	OECD	OECD	OECD	–	–	OECD	–
ISR	OECD	OECD[4)]	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	–
ITA	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	–
KOR	OECD	OECD[4)]	OECD	OECD	OECD	OECD	OECD	NS	NS	NS	–
LUX	OECD	–	OECD	OECD	OECD	OECD	OECD	OECD	–	–	–
MEX	OECD	OECD	OECD	OECD	OECD	OECD	OECD	–	–	OECD	–
NLD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	–
NOR	OECD	NS	OECD	OECD	OECD	OECD	OECD	OECD	–	NS	–
NZL	OECD	OECD[4)]	OECD	OECD	OECD	OECD	NS	NS	OECD	NS	–
POL	OECD	NS	OECD	OECD	OECD	OECD	OECD	OECD	OECD	NS	–
PRT	OECD	OECD	OECD	–	OECD	OECD	OECD	OECD	OECD	OECD	–
SVK	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	OECD	–
SVN	OECD	OECD	OECD	–	OECD	OECD	OECD	OECD	OECD	OECD	–
SWE	NS	NS	NS	OECD	NS	NS	NS	OECD	OECD	NS	–
USA	NS	NS	NS	–	NS	NS	NS	NS	NS	NS	NS

Notes: 1) CE: compensation of employees, MI: gross mixed income, GVA: gross value added at basic price, SE: total employment / (total employment - # of self employee), P^c: price index of private consumption, NI: nominal investment by type, RI: real investment by type, NK: nominal net capital stock by type, RI: real net capital stock by type, CFC: consumption of fixed capital from income account, D: consumption of fixed capital by type.

2) NS refers to national source.

3) Marked as OECD when OECD series and NS series are same.

4) Gross operational surplus of households sector is used instead of MI for ISR, KOR, and NZL.

5) SE is used (and appeared here) only when it is longer available than mixed income.

where R, S, E refer to dwellings (R), other buildings and structures (S), and equipments & weapon systems (E), ω refer to two-year moving average of nominal share of each item in total investments, and π's refer to price changes. Then the price index of traditional investment is given by $P_t^T = \prod_{i=0}^{t}(1+\pi_i^T)$, with $\pi_0^T = 0$. Nominal investment is simply sum of subitems ($I_t^T = I_t^R + I_t^S + I_t^E$) and real investment is nominal investment divided by price index ($X_t^T = I_t^T / P_t^T$).

C.1.3 Depreciation rates

Depreciation rates is defined as consumption of fixed capital divided by capital stock at end of previous year. When both real value of consumption of fixed capital (CFC) and capital stock data are available, we use data (AUS, GBR, and USA) where real value of traditional capital and traditional consumption of fixed capital are constructed using Törnqvist index as above. When only nominal value of CFC is available (FRA), depreciation rate is obtained by

$$\delta_t^i = \frac{NCFC_t^i}{NK_{t-1}^i \times PK_t^i / PK_{t-1}^i},$$

where NCFC is nominal CFC in data, NK is nominal capital in data, and PK is price of capital in data for IPP and Törnqvist index for traditional, and $i \in \{T, IPP\}$.

However, most countries do not provide CFC data by asset type. For these countries, we consider two estimates of CFC from data. Firstly, we can estimate real value of CFC by asset type using

$$R\hat{C}FC_t^i = RK_{t-1}^i + X_t^i - RK_t^i,$$

where RK is real value of capital, X is real value of investments, and $i \in \{T, IPP\}$. It is worth noting that price of capital is different from price of investment in data since all subitems in each category differ in terms of both depreciation rates and price changes. When

RK is not available in data (e.g. CAN, ESP, ISL, LUX, MEX, and NOR), however, we use price of investment for price of capital.

Secondly, we can estimate nominal value of CFC by

$$N\hat{C}FC_t^i = NK_{t-1}^i \times \frac{PK_t^i}{PK_{t-1}^i} + I_t^i - NK_t^i,$$

where NK is nominal capital, I is nominal investments, and $i \in \{T, IPP\}$. Note that $N\hat{C}FC/R\hat{C}FC$ is not simply PK since price of investment and capital are different.

The prices of CFC, capital, and investment are all different since composition of subitems are different. In this sense, RCFC should be better measure for true depreciation rates than NCFC. However, we can use more information with NCFC, which is total CFC that can be obtained from income accounts. Specifically, we can obtain one of NCFC as residual from total CFC of income accounts, for example, $N\hat{C}FC_t^T = CFC_t - N\hat{C}FC_t^{IPP}$.

Note that dep rates are actually stable for countries with CFC data available, but CFC estimated above could fluctuate due to re-valuation and inventory adjustment. Hence, in practice, we plot depreciation rates from both $R\hat{C}FC$ and $N\hat{C}FC$, and then chooses dep rates that are more stable. If they are similar, we went with $RCFC$. The countries with $RCFC$ are AUT, CHE, DEU, FIN, GRC, HUN, ISR, ITA, LUX, NLD, and PRT. Those with $NCFC$ are BEL, CAN, CZE, EST, IRL, KOR, NOR, NZL, POL, SVK, SVN, and SWE.

C.1.4 Capital

Depending on methods of getting depreciation rates, it is possible that RK in data is not compatible with implied depreciation rates. Importantly, this includes cases where we get depreciation rates from CFC data. This is because in data, capital is adjusted for revaluation and inventories, where gross fixed capital formation does not include them. To make capital series to be compatible with investment data in a standard model sense, we construct real value of capital as following.

$$K_{t+1}^i = (1 - \delta_t^i) K_t^i + X_t^i, \tag{C.1.1}$$

with K_0^i being nominal capital data of base year.

Note that above methods require estimated δ which requires data for capital and investment. In many countries, however, we have longer investment series available than capital series. For these countries (AUT, CAN, CHE, CZE, ESP, EST, FIN, FRA, GBR, ISL, ITA, KOR, LUX, MEX, NLD, POL, PRT, SVK, SVN, and SWE), it could be useful to consider extension of capital series.

With X_t^i given as data, what we need is δ_t^i for those years without capital data. For the depreciation rates, we use fitted value obtained from the following regression.

$$\delta_{j,t}^i = \beta_j + \gamma \log(\text{GDP per capita}_{j,t}) + \varepsilon_{j,t},$$

where j refers each country. To make GDP per capita comparable across countries, we use constant PPP rates obtained from PWT 8.1.[37]

With estimated depreciation rates $\hat{\delta}_{j,t}$ at hands, we can get capital series by computing

$$K_t^i = \frac{K_{t+1}^i - X_t^i}{1 - \hat{\delta}_t^i}. \tag{C.1.2}$$

The problem with this method, however, is that it is very sensitive to even very small error in base year because errors are accumulated across the extension. To be precise, when NK_0 in

[37] To be specific, PPP rates (pppr) is obtained from pppr = q_gdp/rgdpo, where q_gdp is real GDP in national currency from NA data of PWT 8.1 and rgdpo is output-side real GDP at chained PPPs. We then multiply 1/pppr to our series of real GDP with SNA 08. We assume $\text{pppr}_t = \text{pppr}_{2011}$ for $t > 2011$.

data is a little bit different from K_0 that could have been obtained if we had data for K_{-10}, estimated $\hat{K}_{-10}$ from NK_0 can be very different from true K_{-10} because the small difference in time 0 is accumulated from $t = 0$ to $t = -10$. To see this more clearly, it is useful to see an example.

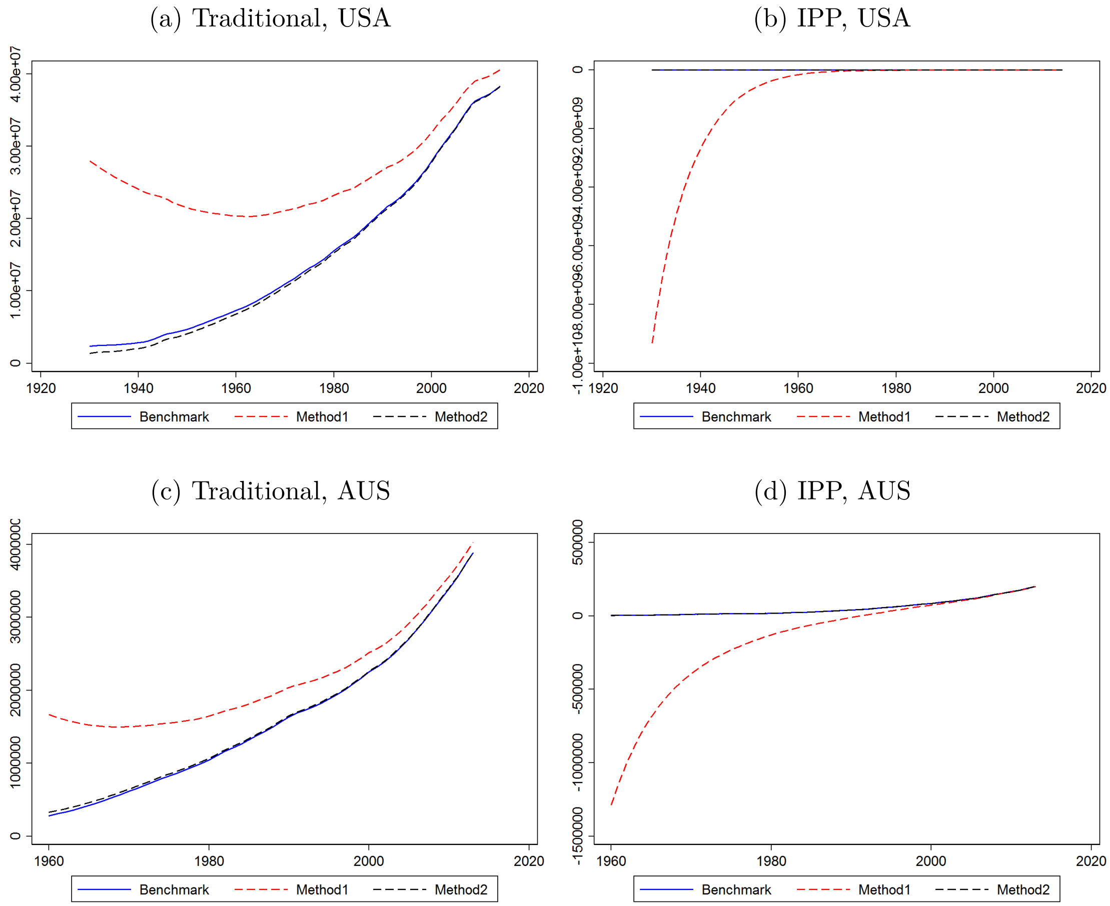
Figure C1 compares K from equation (C.1.1) with $K_0 = K_{1929}$ (call this $K1$, a blue line) and K from equation (C.1.2) with $K_0 = K_{2005}$ (call this $K2$, a red line). Because of reasons stated above, $K1$ is not exactly same with K_t in data. Since we use exactly same δ_t, $K1$ has to be equal to $K2$ if $K1_{2005} = K_{2005}$. However, $K1$ is a little bit different from K at 2005 and this makes $K2$ a lot different from $K1$ as time goes back.

One way to mitigate this problem is to set a restriction on the initial movement of capital. Since errors are accumulated, magnitude of K_1/K_0 becomes really big (either positive or negative as can be seen in graphs) if there was an error in base period. By restricting K_1/K_0 to be a reasonably small number (e.g. fitted growth rate of capital against log GDP per capita), we can mitigate the exploision problem as can be seen by a black line in figure C1. Precisely, the black line is obtaind by equation (C.1.1), with

$$K_1^i = \hat{g}_0^i K_0^i,\ K_1^i = (1 - \hat{\delta}_0^i) K_0^i + X_0^i \ \rightarrow \ K_0^i = \frac{X_0^i}{\hat{g}_0^i + \hat{\delta}_0^i}, \tag{C.1.3}$$

where $\hat{g}_0^i$ and $\hat{\delta}_0^i$ are fitted growth rate and depreciation rate of capital against log GDP per capita. Note that the assumption we use is not a steady state assumption because we use estimated depreciation rates that are fluctuating over time. Rather, our assumption is simply stating that the growth rate of capital from the initial period to next period is set to fitted growth rate. From then on, we use exactly same procedure of making capital series via equation (C.1.1) using freely moving depreciation rates, $\hat{\delta}_t^j$.

Figure C1: Extended capital by different methods



Notes: Benchmark: $K' = K(1-\delta) + X$ with K_0 =data, Method 1: $K = (K' - X)/(1-\delta)$ for $t < 2005$, Method 2: $K' = K(1-\delta) + X$ with $K_0 = X_0/(g+\delta)$.

In practice, we plot capital series obtained from equation (C.1.2) (method 1), and if capital series go up or become negative as time goes back, we use the restriction (C.1.3) (method 2). As a result, we apply method 2 to traditional capital of NLD, ITA, and PRT, and to IPP capital of AUT, CAN, CZE, EST, FRA, GBR, IRL, ITA, NLD, POL, SVK, SVN and SWE.

We have three countries in our sample with no capital stock available in data (ESP, ISL, and MEX). For these countries, we set initial level of capital as a fitted value from the following regression,

$$\log\left(\frac{K^i}{Y}\right) = \beta + \gamma \log(\text{GDP per capita}_{j,t}) + \varepsilon_{j,t},$$

and then apply equation (C.1.1). Since Mexico gives decreasing IPP capital near initial period, we apply method 2 (equation (C.1.3)) for IPP capital of Mexico.

C.1.5 Labor Share

We adjust for mixed income following Koh, Santaeulàlia-Llopis, and Yu (2015) in constructing our baseline labor share. To begin with, we classify Gross Domestic Income into unambiguous capital income (UCI), unambiguous income (UI), and ambiguous income (AI). Unambiguous capital income (UCI) is the gross operating surplus (GOS) which does not include gross mixed income (GMI) in the National Accounts. Note that both gross operating surplus and gross mixed income includes consumption of fixed capital. Adding compensation of employees (CE) to unambiguous capital income (UCI), we get unambiguous income (UI=UCI+CE). Ambiguous income is income other than UI, which is sum of gross mixed income and tax net of subsidy (AI=GMI+Tax-Sub). We assume gross capital income share in ambiguous income is same as gross capital income share of unambiguous income. Then the total capital income can be obtained by summing up unambiguous capital income and capital income in ambiguous income (KI=UCI+$\theta\times$AI, θ =UCI/UI). Finally, labor share is one minus capital share which is capital income divided by total income (LS=1-KI/GDI).

$$\begin{aligned}
&\text{Unambiguous Capital Income, UCI} = \text{GOS} \\
&\text{Unambiguous Income, UI} = \text{CE} + \text{UCI} \\
&\text{Ambiguous Income, AI} = \text{GMI} + \text{Tax} - \text{Sub} \\
&\text{Capital Income, KI} = \text{UCI} + \text{AI} \times \theta,\ \ \theta = \text{UCI}/\text{UI} \\
&\text{Labor Share, LS} = 1 - \frac{\text{KI}}{\text{UI} + \text{AI}} = 1 - \frac{\text{KI}}{\text{GDI}}
\end{aligned} \tag{C.1.4}$$

The differences between ours and Koh, Santaeulàlia-Llopis, and Yu (2015) are that we do not adjust for Business Current Transfer Payments in gross operating surplus due to limited data availability (table C3) and that we use gross operating surplus not net operating surplus. However, the Business Current Transfer Payments is only 0.5% of GDI on average and does not affect trend of labor share. The BEA only provides proprietor's income excluding consumption of fixed capital, i.e net mixed income. Hence we have to use net labor share to get accurate labor income of proprietors for US. Net capital income share of unambiguous income is $\tilde{\theta}$=NOS/(CE+NOS) and so total capital income becomes KI=NOS+$\tilde{\theta}\times$NMI+$\theta\times$(Tax-Sub)+DEP, where θ is gross capital share and $\tilde{\theta}$ is net labor share. Labor share is then computed by LS=1-KI/GDI.

To avoid confusion, we call net operating surplus excluding net proprietor's income as net operating surplus (NOS). Note, however, that Net operating surplus in NIPA table includes (net) proprietor's income so that net operating surplus in NIPA table is different from what we call NOS here (see table C3).

In cases where longer series of self employee are available (i.e. AUT, BEL, CAN, CHE, CZE, DEU, DNK, ESP, EST, FIN, GRC, IRL, ISR, ITA, KOR, MEX, NLD, NOR, NZL, POL, PRT, and SVK), we extend labor share in equation (C.1.4) with self employee adjusted labor share as

$$LS_{t-1} = LS_t \times (LS^{SE}_{t-1}/LS^{SE}_t),$$

where $LS^{SE} = \frac{\text{CE}}{\text{GDI-(Tax-Sub)}} \times \frac{\text{Total employment}}{\text{Total employment - \# of self employees}}$. In words, LS^{SE} is labor share adjusted with assumption that average wage of self employees is same with that of employees. Since average wage of self employees is usually less than that of employees, LS^{SE} is likely to overestimate the level of labor share. However, LS^{SE} gives similar pattern with our baseline labor share and we only reflect changes in labor share to extend our baseline labor share which we believe the best measure for labor share in the economy. The exceptions are LUX and ISL where only LS^{SE} is available (LUX) or neither MI nor SE is available (ISL).

An adjustment of IPP effects on labor share is as following. From the standard representative firm's profit maximizing problem, we have

$$R^i_{t+1} = (1 + r_{t+1})\frac{1}{V^i_t} - (1 - \delta^i_{t+1})\frac{1}{V^i_t},$$

Table C3: Structure of income account: BEA NIPA and OECD National Accounts

BEA NIPA (USA)	OECD NA
GDI	GDI
Compensation of employ (CE)	Compensation of employ (CE)
Taxes (Tax)	Taxes (Tax)
Subsidies (Sub)	Subsidies (Sub)
Net operating surplus (NOS+NMI)	
Net intersts	
Business current transfer payments	
Proprietor's income (NMI)	Gross operating surplus (GOS)
Rental income	Gross mixed income (GMI)
Corporate profits	
Current surplus of government enterprises	
Consumption of fixed capital (DEP)	

where R is gross return, r is net return, $V^i = P^c/P^i$, and $i \in \{T, IPP\}$. Also, labor share in data can be expressed as

$$LS = 1 - \frac{R^T K^T}{Y} - \frac{R^{IPP} K^{IPP}}{Y},$$

from any constant returns to scale production function.

Assuming common net return for T and IPP (i.e. no arbitrage), these constitute three equations for three unknowns R^T, R^{IPP}, and r. Then the labor share without IPP (LS^T) is obtained by

$$LS^T = 1 - \frac{R^T K^T}{Y - R^{IPP} K^{IPP}}.$$

Note that this adjustment is available only when our capital series are available. Since capital was extended up to a point with investment data available, we have LS^T whenever investment data are available. However, for some countries in our sample, labor share data covers longer periods than investments. To extend LS^T up to a point when LS data starts, we estimate following regression.

$$dif_{j,t} = \beta_j + \gamma \log(\text{GDP per capita}_{j,t}) + \varepsilon_{j,t},$$

where $dif_{j,t} = \frac{LS^T}{LS} - 1 = \frac{R^{IPP} K^{IPP}}{Y}$. Then extended LS^T is computed by

$$\hat{LS}^T_{j,t} = LS_{j,t} \times (1 + \hat{dif}_{j,t}).$$

C.2 Growth Accounting Results

Table C4: Growth Accounting with $\chi = 1$

	Growth rates					Percent explained				time	
country	y	h	k	z	tfp	h	k	z	tfp	s	t
AUS	1.51	0.10	0.70	0.14	0.57	6.4	46.2	9.4	38.0	1985	2012
AUT	1.39	0.34	0.41	0.17	0.47	24.3	29.6	12.3	33.8	1977	2014
BEL	0.66	0.24	0.20	0.14	0.09	36.2	30.2	20.6	13.0	1996	2014
CAN	1.14	0.36	0.63	0.11	0.04	31.2	55.7	9.4	3.7	1982	2010
CHE	0.91	0.17	0.00	0.16	0.57	19.1	0.3	17.5	63.2	1995	2013
CZE	2.47	0.25	1.06	0.04	1.12	10.0	42.9	1.8	45.4	1994	2014
DEU	0.57	0.17	0.25	0.08	0.06	30.2	44.7	14.9	10.2	1992	2014
DNK	1.74	0.38	0.64	0.20	0.53	21.6	36.9	11.4	30.2	1967	2013
ESP	0.47	0.39	1.08	0.13	-1.13	83.4	230.8	27.1	-241.3	1996	2011
EST	5.17	0.49	2.45	0.21	2.01	9.4	47.5	4.1	39.0	1996	2013
FIN	1.87	0.46	0.55	0.52	0.34	24.6	29.2	28.0	18.3	1976	2014
FRA	2.29	0.47	0.91	0.15	0.77	20.5	39.5	6.4	33.6	1961	2014
GBR	1.70	0.42	0.76	0.13	0.39	24.8	44.8	7.5	23.0	1981	2014
GRC	0.78	0.36	1.42	0.07	-1.08	46.4	181.3	9.5	-137.2	1996	2013
HUN	1.82	0.47	0.44	0.20	0.71	26.0	24.3	11.0	38.7	1996	2013
IRL	3.48	0.27	1.70	0.45	1.05	7.9	48.9	13.0	30.3	1996	2013
ISL	1.98	0.45	1.11	0.09	0.34	22.4	56.1	4.4	17.1	1998	2011
ISR	1.12	0.44	0.21	0.02	0.45	39.1	18.6	1.8	40.5	1996	2014
ITA	1.18	0.53	0.55	0.07	0.03	45.0	46.9	5.7	2.4	1971	2014
KOR	4.30	0.87	2.57	0.40	0.45	20.2	59.8	9.4	10.5	1970	2013
LUX	0.63	0.56	0.25	0.23	-0.41	89.3	39.9	36.7	-65.8	1997	2012
MEX	0.72	0.25	1.57	0.00	-1.10	35.0	219.0	-0.3	-153.6	2004	2011
NLD	0.69	0.29	0.45	0.15	-0.20	42.0	64.7	22.1	-28.8	1981	2014
NOR	2.46	0.38	0.60	0.17	1.30	15.5	24.4	7.1	53.0	1971	2013
NZL	0.41	0.04	0.91	0.15	-0.68	9.5	219.1	35.6	-164.2	1972	2011
POL	3.24	0.43	1.60	0.07	1.14	13.3	49.3	2.1	35.2	1996	2013
PRT	0.91	0.49	0.96	0.06	-0.60	53.9	105.3	6.7	-65.9	1996	2013
SVK	2.81	0.41	0.41	0.06	1.93	14.7	14.5	2.2	68.6	1996	2013
SVN	2.11	0.29	0.55	0.11	1.14	14.0	26.3	5.4	54.3	1996	2013
SWE	2.14	0.20	0.90	0.09	0.96	9.1	41.8	4.4	44.7	1993	2013
USA	1.63	0.36	0.56	0.13	0.57	22.3	34.5	8.1	35.0	1950	2014
OECD	1.75	0.37	0.85	0.15	0.38	20.9	48.6	8.7	21.8		

Notes: Growth rates are computed by $100 \times (\ln(x_t) - \ln(x_s))/(t-s)$, where t and s refers to final and initial point. OECD refers to average.

Table C5: Growth Accounting with $\chi = \hat{\chi}$

	Growth rates					Percent explained				time	
country	y	h	k	z	tfp	h	k	z	tfp	s	t
AUS	1.51	0.10	0.70	0.08	0.63	6.6	46.0	5.4	42.0	1985	2012
AUT	1.39	0.35	0.41	0.08	0.55	25.0	29.5	5.7	39.9	1977	2014
BEL	0.66	0.25	0.20	0.06	0.16	37.6	30.1	8.7	23.7	1996	2014
CAN	1.14	0.37	0.63	0.05	0.10	32.1	55.4	4.2	8.3	1982	2010
CHE	0.91	0.18	0.00	0.07	0.66	20.0	0.3	7.3	72.4	1995	2013
CZE	2.47	0.25	1.06	0.03	1.14	10.3	42.7	1.1	46.0	1994	2014
DEU	0.57	0.18	0.25	0.03	0.10	31.3	44.4	6.0	18.4	1992	2014
DNK	1.74	0.39	0.64	0.07	0.64	22.3	36.6	4.3	36.7	1967	2013
ESP	0.47	0.40	1.08	0.06	-1.06	85.4	229.5	12.0	-226.9	1996	2011
EST	5.17	0.49	2.45	0.14	2.09	9.5	47.4	2.7	40.4	1996	2013
FIN	1.87	0.47	0.54	0.27	0.58	25.2	29.1	14.6	31.2	1976	2014
FRA	2.29	0.49	0.90	0.06	0.84	21.4	39.1	2.7	36.8	1961	2014
GBR	1.70	0.43	0.76	0.07	0.44	25.4	44.6	3.9	26.1	1981	2014
GRC	0.78	0.37	1.41	0.04	-1.04	47.5	180.0	4.8	-132.3	1996	2013
HUN	1.82	0.48	0.44	0.12	0.78	26.4	24.2	6.6	42.8	1996	2013
IRL	3.48	0.29	1.69	0.20	1.30	8.3	48.7	5.7	37.4	1996	2013
ISL	1.98	0.46	1.11	0.03	0.38	23.0	56.1	1.8	19.1	1998	2011
ISR	1.12	0.47	0.21	0.00	0.45	41.5	18.3	0.4	39.8	1996	2014
ITA	1.18	0.54	0.55	0.03	0.06	46.0	46.6	2.5	4.9	1971	2014
KOR	4.30	0.89	2.55	0.23	0.63	20.8	59.3	5.3	14.6	1970	2013
LUX	0.63	0.58	0.25	0.09	-0.29	91.4	39.8	14.7	-45.8	1997	2012
MEX	0.72	0.25	1.57	0.00	-1.10	35.1	218.9	-0.2	-153.8	2004	2011
NLD	0.69	0.30	0.44	0.05	-0.11	43.7	64.2	7.3	-15.3	1981	2014
NOR	2.46	0.39	0.60	0.07	1.40	16.0	24.2	2.8	57.0	1971	2013
NZL	0.41	0.04	0.91	0.07	-0.60	9.7	218.9	17.3	-145.9	1972	2011
POL	3.24	0.44	1.59	0.04	1.17	13.5	49.1	1.3	36.1	1996	2013
PRT	0.91	0.50	0.96	0.03	-0.58	54.8	104.8	3.8	-63.5	1996	2013
SVK	2.81	0.42	0.40	0.04	1.94	14.9	14.4	1.5	69.1	1996	2013
SVN	2.11	0.30	0.55	0.06	1.20	14.3	26.1	2.7	56.8	1996	2013
SWE	2.14	0.21	0.89	0.05	1.00	9.6	41.6	2.3	46.5	1993	2013
USA	1.63	0.38	0.56	0.05	0.65	23.1	34.3	3.1	39.5	1950	2014
OECD	1.75	0.38	0.85	0.07	0.45	21.5	48.4	4.2	26.0		

Notes: Growth rates are computed by $100 \times (\ln(x_t) - \ln(x_s))/(t-s)$, where t and s refers to final and initial point. OECD refers to average.