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Communication in Collective Choice Environments

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by

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Dedicated to Erin, for being exceedingly loving, supportive, and patient.

ABSTRACT OF THE DISSERTATION

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In this dissertation, I present three models of communication in collective choice environments. The first two models demonstrate how collective choice procedures provide opportunities for informed communicators to manipulate outcomes by strategically obfuscating information to appeal to different coalitions at different times. Paradoxically, the members of the collective choice institution are often better off gathering no information at all rather than relying on an expert who manipulates outcomes in this way. The final chapter characterizes the incentives of candidates to reveal information about their preferences to voters in an election when multiple policy issues are at stake. I show that candidates can credibly reveal directional information about their preferences but will leave the voters uncertain about which candidate is more extreme.

Chapter 1

Preface

Communication and collective choice are central problems faced in political environments. Communication is necessary when some individuals possess information that is relevant to a choice that must be made by another individual or set of individuals. Since the seminal model of strategic information transmission by Crawford and Sobel (1982), social scientists have understood that successful communication is difficult when the parties involved have conflicting preferences about the choices to be made. Conflicting preferences are a distinguishing feature of politics, so communication problems abound in political environments. Collective choice – that is, the process of combining individuals' preferences or actions into a single choice for all of society – is another key feature of political institutions. Like communication, collective choice is difficult in the face of diverse preferences. The theorem of Schofield (1983) showed that, when there are multiple divisible issues at stake, there is generically no policy that would beat every other policy in a majority vote. Thus, the collective choice problem under majority rule is difficult in the sense that the rule does not prescribe any unambiguous "best" choice. This is known as the problem of collective choice cycles and it extends to many voting rules other than majority rule (Nakamura, 1979). Though both problems are well established and central to theories of politics, communication and collective choice have rarely been analyzed together.

In this dissertation, I present three models of communication in collective choice settings. In the first, I provide a general theory of information transmission by a single expert to a collective choice body. In the model, the policy-motivated expert has private information on the effect of a policy proposal and communicates to a set of voters prior to a vote over whether or not to implement the proposal. In contrast to previous game-theoretic models of information transmission, the results apply to situations involving multiple voters, multidimensional policy spaces and a broad class of voting rules. The results highlight how experts can use information to manipulate collective choices to the detriment of all of the voters. Opportunities for expert manipulation are the result of collective choice instability: all voting rules that allow collective preference cycles also allow welfare-reducing manipulative persuasion by an expert.

The second model in this dissertation applies similar insights about manipulative persuasion to a model of lobbying. Lobbyists play the role of the experts in this model, but the main distinguishing feature of the model compared to the first is the fact that two lobbyists compete for the votes of the legislators. Though interest groups gain influence by building coalitions between legislators with diverse goals and interests, political scientists' theories of interest group influence were previously built around models of communication to a single legislator. In contrast, I account for the special incentives that voting institutions provide for lobbyists to manipulate voting coalitions to their advantage. Even more than in the first model, this form of lobbyist manipulation has stark implications for legislator welfare: if interest group influence exists then it always has a negative effect on welfare. Competition between interest groups may help eliminate this negative effect but only for certain policy areas. In particular, interest group competition blocks the influence of interest groups advocating very targeted policies but has no effect if the policy area is sufficiently broad. The results suggests that interest group scholars should pay attention to the ways in which lobbyists use collective choice institutions to their advantage.

The final model is one of campaign communication in a two-candidate majority rule election with multidimensional policies. Candidate and voter preferences are private information and campaigns consist of both candidates sending cheap talk messages in order to communication information about their preferences. The game always possesses equilibria involving informative campaign messages. Campaign communications reveal information about the directions of the candidates' ideal points from the center of the policy space but leave the voters uncertain about which candidate is more extreme. There is always an equilibrium in which both candidates reveal whether they are left or right of the median on issue dimensions corresponding to the principal components of the preference distribution. Furthermore, if the distribution of candidates' ideal points is spherically symmetric then there exists an equilibrium in which the candidates fully reveal the direction of their ideal point from the center.

Together, these three models demonstrate that communicators have very different incentives when transmitting information to collective choice bodies rather than single decision-makers. The fact that many different coalitions may form in order to determine a collective choice increases the power of a communicator by expanding the set of persuasive appeals that help achieve the communicator's preferred outcome. In some cases this additional power works to the disadvantage of the members of the collective choice body.

Chapter 2

Expert Advice to a Voting Body

Expertise is at least as important as formal decision-making authority as a source of political power in many institutions. As a result, formal models of communication under asymmetric information have improved our understanding of topics such as legislative committees (Gilligan and Krehbiel, 1987, 1989, 1990), lobbying (Grossman and Helpman, 2001), and information acquisition by voters (Lupia and McCubbins, 1998). These models feature a key insight from the seminal Crawford and Sobel (1982) model: though more communication would lead to better policies, the expert faces incentives to obfuscate when her preferences diverge from the policy-maker's. Institutions, therefore, are posed as solutions to the problem of extracting the maximum amount of information from the expert. As Hirsch and Shotts (2012) noted, "uncertainty reduction, expertise, and the common good have become essentially synonymous, regardless of whether the empirical domain is institutional design, lobbying, or delegation."

Though the role of information is most pronounced in complex and multifaceted policy domains, the prevailing wisdom about communication in political institutions is built on models in which policy-making is one-dimensional. One-dimensional models excel at parsimony because they reduce the problem of communicating with a voting body to one of communicating with a single representative voter or legislator. This parsimony comes at the cost of generality because representative voters are unlikely to exist in multidimensional policy domains. In two or more dimensions, majority rule is known to be unstable in the sense that all policies can be beaten by some other policy (Schofield, 1983). Furthermore, this instability property extends beyond majority rule to a wide variety of voting rules (Nakamura, 1979). Though it is known that instability makes voting bodies subject to manipulation in other contexts (McKelvey, 1976; Riker, 1986), the implications of collective choice instability for information transmission are not well established.

In this paper, I present a model in which a policy-motivated expert has private information concerning the substantive effect of a multidimensional policy proposal under consideration by a set of voters. The expert sends a costless ("cheap talk") public message concerning the effect of the policy proposal. After seeing the expert's message, the voters take an up-or-down vote over whether to implement the new policy or retain the status quo. In contrast to standard accounts of communication in political institutions, information transmission from the expert to the voters may be *ex ante* detrimental to voter welfare. Welfare losses from information transmission result from a form of political manipulation by the expert in which information is presented in a way that exploits collective preference cycles. In fact, the phenomenon of manipulative persuasion is connected to collective preference instability in a precise way: all voting rules that permit collective choice instability also permit manipulative persuasion.

Example 1. To illustrate the logic of manipulative persuasion, consider the following example. A lobbyist wants to persuade a majority of a three-member legislative committee to approve a public project. The legislators' districts are labeled A, B, and C. The project will cost each legislator's district \$100 and provide a benefit b_i of uncertain size to each district. There are four possible effects of the policy, which the legislators initially believe are equally likely. The project may have an *equitable* effect, in which case it provides a good valued at \$105 to all districts. Alternatively, the policy may have a *provincial* effect, and provide a good valued at \$250 to one of the three districts. These four states and their resulting benefits are below:

Policy state	b_A	b_B	b_C
Equitable	105	105	105
Provincial A	250	0	0
Provincial B	0	250	0
Provincial C	0	0	250

Each legislator prefers to approve the proposal if the expected benefit to that legislator's district is greater than the \$100 cost. The lobbyist receives a payoff of zero if the proposal fails and some positive payoff if the proposal is passed. The lobbyist, who knows the effect of the policy, can advise the legislators in the form of a public speech revealing some information about the policy state.

What type of speech should the lobbyist make in each situation? If the legislators held a vote prior to listening to the lobbyists' speech, each legislator's expected payoff from passing the proposal would be \$355/4, which is less than the cost of \$100, so the proposal would fail unanimously. In fact, there is an equilibrium with no information transmission since, if legislators assume that all speeches are uninformative and vote in this fashion, the lobbyist cannot affect the chosen policy and therefore receives the same payoff from any message.

Next, suppose for the sake of argument that the lobbyist revealed all information to the legislators. Then the proposal would pass unanimously in the equitable state and fail in a 2-1 vote in every other state. However, if the legislators are persuaded to pass the proposal

Policy state	Message	Frequency
Equitable	$\{e\}$	100%
Provincial A	$\{a, b\}$	50%
	$\{a, c\}$	50%
Provincial B	$\{a, b\}$	50%
	$\{b, c\}$	50%
Provincial C	$\{a, c\}$	50%
	$\{b, c\}$	50%

Table 2.1: An equilibrium communication strategy for the game in Example 1.

when the lobbyist makes a speech implying that the policy is equitable, then the lobbyist has an incentive to make that same speech for other policy states, so it is not an equilibrium for the lobbyist to reveal all information.

Instead, suppose that the lobbyist gives a series of speeches with slightly less information content. Let the set of possible speeches be represented by subsets of the set $\{e, a, b, c\}$, so that the speech $\{e\}$ is interpreted as "The policy is equitable", $\{b\}$ is interpreted as "District B is the sole beneficiary of the policy", $\{a, c\}$ is interpreted as "The sole beneficiary of the project is either District A or District C", and so on. Consider the communication strategy in Table 2.1. Here, the lobbyist always reveals when the policy effect is equitable. The lobbyist uses mixed strategies for the other policy states. For instance, half of the time that the lobbyist learns that District A is the sole beneficiary of the project, she gives a speech meaning "the sole beneficiary of the project is either District A or District B" and the other half of the time she gives the same speech, where "B" is replaced with "C." A similar strategy is used for the other provincial policy states. When the lobbyist uses this strategy, the legislators are always persuaded by a speech claiming that the policy is equitable. Furthermore, when the lobbyist sends the message $\{a, b\}$, the posterior probability that the policy state is "Provincial A" is

$$\frac{\frac{1}{4} \cdot \frac{1}{2}}{\frac{1}{4} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{2}} = \frac{1}{2},$$

with the other $\frac{1}{2}$ probability concentrated on "Provincial B." Thus, legislators A and B each have an expected payoff of $\frac{\$250}{2} > \$100$ from the policy, so both legislators vote "Yes" and the proposal passes. The same calculation works for any of the ambiguous speeches sent by the lobbyist, so two legislators are willing to support the policy proposal following any speech given by the lobbyist as part of this strategy. Furthermore, since the policy always passes, the lobbyist never prefers to deviate from this speech strategy so this is an equilibrium.

Though the information provided by the expert is persuasive and accurate, the legislators are *ex ante* unanimously worse off in this equilibrium than if they had not consulted an expert. The *ex ante* expected payoff to each legislator when the lobbyist is persuasive is

$$\frac{1}{4} \cdot \$250 + \frac{1}{4} \cdot \$105 - \$100 = -\frac{45}{4}.$$

In comparison, the legislators receive an expected payoff of zero from automatically rejecting the policy without listening to any speech. Ex ante, the legislators would therefore unanimously prefer to commit to an uninformed decision, but a strategic expert can package information in a way that leads to sub-optimal outcomes for the legislature. □

In what follows, I demonstrate that this phenomenon is not peculiar to Example 1 but is applicable to a significant set of games resulting from a broad class of voting rules. I also show that manipulative (*ex ante* welfare-reducing) persuasion is tied to collective preference instability; regarding characteristics of voting rules, the possibility of collective choice cycles is equivalent to the possibility of manipulative persuasion. Furthermore, equilibria with manipulative persuasion are stable in the sense that they are not eliminated by small changes to the game. The results challenge the prevailing theories of institutions in which procedures are chosen in order to maximize information transmission. Instead, voters may unanimously prefer institutional arrangements that reduce opportunities for persuasion.

2.1 The model

Let $N = \{1, \ldots, n\}$ denote the set of voters and let E denote a single Expert. the voters must decide whether to implement a proposed policy ($x = 1$) or reject the proposal in favor of the status quo ($x = 0$). The utility to each agent from implementing the proposed policy depends on the state of the world, denoted $\omega = (\omega_E, \omega_1, \omega_2, \ldots, \omega_n)$. Specifically, the utility of agent i is represented by the function $u_i(x, \omega) = \omega_i x$ for all $i \in \{E\} \cup N$. Let $\Omega = [-a, a]^n \subset \mathbb{R}^n$ denote the set of feasible states of the world.[1]

Collective decision-making proceeds as follows. First, Nature determines the value of ω and reveals it to the Expert. Second, the Expert sends a payoff-irrelevant message $s \in \Omega$ to the voters. Finally, all voters observe the Expert's message and take an up-or-down vote $v_i \in \{\text{No}, \text{Yes}\}$. Let $\mathcal{D} \subseteq 2^N \backslash \{\emptyset\}$ denote a set of decisive coalitions. If $\{i \in N : v_i = \text{Yes}\} \in \mathcal{D}$, then the proposed policy is implemented ($x = 1$). Otherwise, the status quo persists ($x = 0$). Assume $\mathcal{D}$ is monotonic ($C \in \mathcal{D}$ and $C \subseteq C'$ imply $C' \in \mathcal{D}$) and proper ($C \in \mathcal{D}$ implies $N \backslash C \notin \mathcal{D}$). The class of voting rules permitted by these assumptions is related to that in Banks and Duggan (2000) and Kalandrakis (2006) and subsume all supermajority rules, weighted supermajority rules, certain bicameral voting rules, and many other institutional arrangements.

[1] Ω is restricted in this way primarily for expositional ease. The results rely only on the fact that Ω is convex and that $(0, 0, \ldots, 0) \in \text{int}(\Omega)$.

The value of ω is private information for the Expert. The distribution of ω is represented by a common prior $\mu_0 \in \Delta(\Omega)$, where $\Delta(\Omega)$ is the set of probability measures on Ω. [2] Voters' beliefs following the observation of a message s are denoted $\mu_s \in \Delta(\Omega)$. For any $\mu \in \Delta(\Omega)$, let $\mathbb{E}_\mu[\omega_i]$ denote the expected value of ω_i under the probability measure μ. I restrict my attention to prior distributions for which, absent any additional information, the voters would choose to reject the proposal in favor of the status quo (i.e. for some $C \in D$, we have $\mathbb{E}_\mu[\omega_i] < 0$ for all $i \in C$).

A strategy profile consists of a messaging strategy for the Expert and a voting strategy for each voter. The Expert's strategy is a function $\sigma : \Omega \to \Delta(\Omega)$, where $\sigma(s|\omega)$ is the probability distribution over the Expert's messages when the state of the world is ω. A voting strategy is a function $v_i^* : \Omega \to \{\text{No}, \text{Yes}\}$ mapping messages into voting decisions. The definition of v_i^* is potentially restrictive in that it rules out mixed voting strategies, but Theorem 10 in the Appendix demonstrates that this restriction has no effect on the communication outcomes that can be supported. Let $v^* = (v_1^*, \ldots, v_n^*)$ be the profile of all voting strategies. Let $x^*(s|v^*, \mathcal{D}) = 1$ if $\{i \in N : v_i^*(s) = \text{Yes}\} \in \mathcal{D}$ and 0 otherwise. Thus, x^* is a function mapping messages into voting outcomes given a profile of voting strategies.

I characterize perfect Bayesian equilibria in weakly undominated strategies. Thus, an equilibrium is a pair (σ, v^*) such that:

1. $s \in \text{supp}\sigma(\cdot|\omega)$ implies that $s \in \arg\max_{s' \in \Omega} u_E(x^*(s|v^*, \mathcal{D}), \omega)$; and
2. For all $i \in N$ and all $s \in \Omega$, $v_i^*(s) = \text{Yes}$ if $\mathbb{E}_{\mu_s}[\omega_i] > 0$ and $v_i^*(s) = \text{No}$ if $\mathbb{E}_{\mu_s}[\omega_i] < 0$; where μ_s is consistent with Bayes' rule for all $s \in \text{supp}(\sigma)$.

[2] More precisely, let $\mathcal{B}(\Omega)$ be the Borel σ-algebra on Ω. $\Delta(\Omega)$ is the set of all functions $\mu : \mathcal{B}(\Omega) \to [0,1]$ satisfying $\mu(\Omega) = 1$ and $\mu(\cup_t W_t) = \sum_t W_t$ for all countable sets of subsets $\{W_t\}$ in $\mathcal{B}(\Omega)$.

Let $S(\mu_0, \mathcal{D})$ be the set of all equilibria to the game induced by the prior distribution μ_0 and the voting rule $\mathcal{D}$. The ex ante expected utility to voter i from each strategy profile (σ, v^*) is $U_i^*(\sigma, v^*) = \mathbb{E}_{\mu_0}[\mathbb{E}_\sigma[\omega_i x^*(s|\sigma, \mathcal{D})]]$.

2.2 Babbling, Persuasion, and Expert Discretion

For any prior distribution, the game possesses babbling equilibria, defined as equilibria in which no information is transmitted ($\mu_s = \mu_0$ for all $s \in \Omega$). To see this, suppose the Expert mixes uniformly over all messages following any state of the world. Since $\sigma(W|\omega)$ is constant for all W and ω, $\mu_s = \mu_0$ for all $s \in \Omega$ and each $v_i^*(s) = 1$ only if $\mathbb{E}_{\mu_0}[\omega_i] \geq 0$. Furthermore, since $x^*(s|v^*, \mathcal{D})$ is constant with respect to s, $\arg\max_{s' \in \Omega} u_E(x^*(s|v^*, \mathcal{D}), \omega) = \Omega$ for all $\omega \in \Omega$. Thus, this strategy profile is an equilibrium to the game.

I focus on equilibria in which communication affects the policy outcome (i.e. $x^*(s|v^*, \mathcal{D}) \neq 0$ for some s). These are called *persuasive* equilibria. The game in Example 1 has a persuasive equilibrium in which all voting outcomes match the Expert's preferences toward the proposal. The next result establishes this a general property of persuasive equilibria to the game.

Lemma 1. *In any equilibrium, either $x^*(s|v^*, \mathcal{D})$ is constant or $x^*(s|v^*, \mathcal{D}) = 1$ only if $\omega_E \geq 0$ and $x^*(s|v^*, \mathcal{D}) = 0$ only if $\omega_E \leq 0$.*

Lemma 1 paints a pessimistic picture of communication in this environment. Normatively, one might hope that a voting body could benefit from the advice of the Expert while retaining some independent decision-making authority. However, in this model, all equilibria involve the voters either remaining uninformed or surrendering all policy discretion to the Expert.

In many situations, one may expect that the preferences of the Expert do not depend on her private information. For instance, in a judicial nomination hearing, the nominee has private information about her political preferences but likely prefers to be confirmed regardless of those political preferences. Similarly, a lobbyist who contracts with a client to advocate a proposal may gather information that, while relevant to the voters, will not change her own advocacy. In such circumstances, consultation with the Expert cannot be strictly beneficial to the voters.

Lemma 2. *If* $\mu_0(\{\omega : \omega_E > 0\}) = 1$ *or* $\mu_0(\{\omega : \omega_E < 0\}) = 1$ *then there does not exist a* $\mathcal{D}$*-preferred persuasive equilibrium.*

These results do not necessarily imply that persuasive equilibria are bad for voters. In fact, for some prior distributions there may be persuasive equilibria that improve voter welfare. However, if voters benefit from consultation with the Expert then they would also have benefited from delegating their authority direction to the Expert. A persuasive equilibrium (σ, v^*) is Pareto-preferred by voters if $U_i^*(\sigma, v^*) \geq 0$ for all $i \in N$ and $U_i^*(\sigma, v^*) > 0$ for some $i \in N$. A persuasive equilibrium is $\mathcal{D}$-preferred if there exists $C \in \mathcal{D}$ such that $U_i^*(\sigma, v^*) \geq 0$ for all $i \in C$ and $U_i^*(\sigma, v^*) > 0$ for some $i \in C$. As Lemma 3 states, there is a $\mathcal{D}$-preferred equilibrium only if all members of some decisive coalition would prefer to switch from the default (no communication) outcome to the opposite outcome if they knew that switching would make the Expert better off.

Lemma 3. *Assume* $\mu_0(\{\omega : \omega_E = 0\}) = 0$. *If a* $\mathcal{D}$*-preferred equilibrium exists then there exists* $C \in \mathcal{D}$ *such that*

$$\mathbb{E}_{\mu_0}[u_i(1,\omega)|u_E(1,\omega) > u_E(0,\omega)] \geq \mathbb{E}_{\mu_0}[u_i(0,\omega)|u_E(1,\omega) > u_E(0,\omega)]$$

for all $i \in C$ and this inequality is strict for some $i \in C$.

A direct implication of Lemma 3 is that any Pareto-preferred persuasive equilibria can be supported using a binary messaging strategy: the Expert accurately reveals whether or not she supports passage and a decisive coalition of voters follows the Expert's recommendation. Since Lemma 3 restricts the existence of $\mathcal{D}$-preferred equilibria, it is tempting to reach the stronger conclusion that $\mathcal{D}$-preferred equilibria must also have this form. This stronger conclusion is false, since there are circumstances under which more sophisticated messaging strategies are required to persuade voters to maintain the default action when the Expert prefers no change.[3]

Corollary 1. *If a Pareto-preferred equilibrium exists then the following strategy profile is an equilibrium. (1) For some $s, s' \in \Omega$ with $s \neq s'$, $\sigma(s|\omega) = 1$ for all ω such that $\omega_E \geq 0$, $\sigma(s'|\omega) = 1$ for all ω such that $\omega_E \leq 0$, and $\sigma(s''|\omega) = 0$ for all $s'' \in \Omega\backslash\{s, s'\}$ and all $\omega \in \Omega$. (2) For all $i \in N$, $v_i^*(s) = 1$ if and only if $\mathbb{E}_{\mu_0}[\omega_i|\omega_E \geq 0] \geq 0$ and $v_i^*(s') = 0$ if and only if $\mathbb{E}_{\mu_0}[\omega_i|\omega_E < 0] < 0$.*

Given Corollary 1, it is straightforward to verify the existence or non-existence of Pareto-preferred persuasive equilibria in a given game. However, as Example 1 illustrates, there are many possibilities for persuasive equilibria that are not *ex ante* preferred by voters. The following section characterizes the existence of such equilibria.

[3]For instance, suppose there exists $C \in \mathcal{D}$ such that $\mathbb{E}_{\mu_0}[u_i(1,\omega)|u_E(0,\omega) > u_E(0,\omega)] \geq \mathbb{E}_{\mu_0}[u_i(0,\omega)|u_E(1,\omega) > u_E(0,\omega)]$ for all $i \in C$ and this inequality is strict for some $i \in C$. Suppose there exists another $C' \in \mathcal{D}$ such that $\mathbb{E}_{\mu_0}[u_i(1,\omega)|u_E(1,\omega) < u_E(0,\omega)] \geq \mathbb{E}_{\mu_0}[u_i(0,\omega)|u_E(1,\omega) < u_E(0,\omega)]$ for all $i \in C'$. Then all members of C benefit from a persuasive equilibrium, though a persuasive equilibrium cannot be supported by a binary messaging strategy since members of C' would pass the proposal when the Expert recommends the status quo.

2.3 Manipulative Persuasion

The previous results suggest that it can be difficult to improve voter welfare through consultation with the Expert. Example 1 suggests a stronger result which is that the voters can be made strictly worse off by communicating with the expert. I call this outcome *manipulative persuasion*. In this section, I show that many voting bodies can be subject to manipulative persuasion. Furthermore, the possibility of manipulative persuasion is tied directly to collective preference instability: a voting rule is subject to manipulative persuasion if and only if it allows collective preference cycles.

A persuasive equilibrium involves manipulative persuasion if $U_i^*(\sigma, v^*) < 0$ for all $i \in N$. Since the existence of such equilibria depends on the prior distribution, I characterize the set of prior distributions that allow manipulative persuasion for a given voting rule. For each $\mathcal{D} \in 2^N \backslash \{\emptyset\}$, the *manipulative persuasion set* is

$$\begin{aligned} M(\mathcal{D}) = &\{\mu \in \Delta(\Omega) : \exists C \in D \text{ such that } \mathbb{E}_\mu[\omega_i] < 0 \forall i \in C \\ &\text{and } \exists (\sigma, v^*) \in S(\mu, \mathcal{D}) \text{ and } U_i^*(\sigma, v^*) < 0 \forall i \in N\}, \end{aligned} \tag{2.1}$$

or the set of prior distributions for which the proposal would fail with no communication and there exists an equilibrium with manipulative persuasion when the voting rule is $\mathcal{D}$. Though there are many welfare criteria under which one could say that persuasion is bad for voters, I focus on unanimously bad persuasion because it is the strongest criterion. If a persuasive equilibrium is dominated under this criterion, it is also dominated for some decisive coalition and dominated according to total welfare.

A notion of distance between probability distributions is helpful for characterizing the manipulative persuasion set of a game. If μ and μ' are two probability measures on Ω, the distance between μ and μ' is $\delta(\mu, \mu') = \sup_{A \in \mathcal{B}(\Omega)} |\mu(A) - \mu'(A)|$. When it is relevant, the metric space for probability distributions on Ω is assumed to be $(\Delta(\Omega), \delta)$.

The *collegium* of a voting rule is $\bigcap_{C \in \mathcal{D}} C$. A voting rule is *collegial* if it has a non-empty collegium and *non-collegial* otherwise (Austen-Smith and Banks, 2000). To illustrate the concept of collegial and non-collegial rules, consider the following examples:

- Three-person majority rule: The set of decisive coalitions is $\mathcal{D} = \{\{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$. Since $\{1, 2\} \cap \{1, 3\} \cap \{2, 3\} = \emptyset$, majority rule is non-collegial.
- Majority-rule with voter 1 as a veto player: $\mathcal{D} = \{C \subset N : |C| \geq \frac{n}{2}$ and $1 \in C\}$. Since $\bigcap_{C \in \mathcal{D}} C = \{1\}$, majority rule is collegial.
- Unanimity rule: The set of decisive coalitions is $\mathcal{D} = \{N\}$. Since the entire set of voters is in every decisive coalition, unanimity rule is collegial.

In other words, the collegium of a voting rule is the set of voters whose approval is always required to pass a proposal.

Define a binary relation $\succeq_{\mathcal{D}}$ over elements of Ω such that $\omega \succeq_{\mathcal{D}} \omega'$ if and only if $\{i : \omega_i \geq \omega'_i\} \in \mathcal{D}$. The voting rule $\mathcal{D}$ has a cycle in Ω if there exist $\omega, \omega', \omega'' \in \Omega$ such that $\omega \succeq_{\mathcal{D}} \omega'$ and $\omega' \succeq_{\mathcal{D}} \omega''$ but not $\omega \succeq_{\mathcal{D}} \omega''$. Voting rules that allow cycles are unstable in the sense that the existence of an unbeaten alternative cannot be guaranteed.

Theorem 1. *The following statements are equivalent:*

1. *$\mathcal{D}$ is non-collegial.*

2. *$\mathcal{D}$ has a cycle in Ω.*

3. *There exists a non-empty open set $\mathcal{O}(\mathcal{D}) \in \Delta(\Omega)$ such that $\mathcal{O} \subset M(\mathcal{D})$.*

The equivalence of conditions (1) and (2) in Theorem 1 is similar to standard results in social choice theory that is related, for example, to a common result on the acyclicity of voting rules by Nakamura (1979). The equivalence of conditions (1) and (3) is a new result and the proof is accomplished by constructing the set $\mathcal{O}(\mathcal{D})$. The fact that $\mathcal{O}(\mathcal{D})$ is an open set implies a form of stability: if prior distribution is in this set, the game resulting from a small perturbation of the prior distribution also possesses a manipulative persuasive equilibrium.

Theorem 1 shows that the distinction between collegial and non-collegial voting rules determines whether or not manipulative persuasion is possible. The next comparative statics result also provides insights into which rules are more or less manipulative within the class of non-collegial rules. Let $\mathcal{D}$ and $\mathcal{D}'$ be two distinct voting rules. $\mathcal{D}$ is *more resolute* than $\mathcal{D}'$ if and only if $\mathcal{D}' \subset \mathcal{D}$[4]. The concept of resoluteness provides a partial order over all voting rules considered in this model. Theorem 2 establishes that resoluteness also provides a partial ranking of the manipulability of voting rules.

Theorem 2. *If $\mathcal{D}$ is more resolute than $\mathcal{D}'$, then $M(\mathcal{D}') \subseteq M(\mathcal{D})$.*

Resoluteness facilitates comparisons between many commonly observed voting rules. For example, majority rule is more resolute than a supermajority requirement and the resoluteness of a supermajority rule decreases with the size of the required supermajority. By Theorem 2, it follows that manipulative persuasion should be less common for higher supermajority

[4]The definition of resoluteness provided in Austen-Smith and Banks (2000) is more general in that it applies to rules that cannot be fully represented by a set of decisive coalitions (for example, Borda count). That definition states that that Rule A is more resolute than Rule B if, any time an alternative strictly is socially preferred to another under Rule B, that alternative is also strictly preferred under Rule A. For the class of rules considered here, the two definitions are equivalent.

rules.[5] Though resoluteness is primarily useful for comparing non-collegial rules, it is also consistent with Theorem 1 in the sense that no collegial rule is more resolute than any non-collegial rule.

In the context of supermajority rules, Theorem 2 also provides a stronger connection between collective choice instability and a manipulability by an expert. Within the class of supermajority rules, simple majority rules are the most prone to cycles and stricter supermajority requirements reduce the propensity for cycles. Thus, a supermajority rule that is more resolute is more prone to instability and to expert manipulation.

Theorem 2 suggests that voters face a trade-off with respect to choosing voting rules. Less resolute rules increase the difficulty of passing a proposal by requiring greater consensus. Therefore, these rules may lead to excessive rejection of proposals that most voters favor. However, the increased difficulty of passing proposals may benefit the voters by preventing expert manipulation.

2.4 Potential Remedies to Manipulative Persuasion

In this section I explore several potential remedies to manipulative persuasion. I show that restricting the domain of voter utilities to those that can be represented by one-dimensional quadratic preferences eliminates the possibility of manipulation by an expert. Restricting the domain to quadratic spatial preferences in multiple dimensions does not rule out manipulative persuasion, so the effectiveness of this remedy is due to the one-dimensional aspect of the

[5] In a model of expert communication to committees in one-dimensional policy spaces, Jackson and Tan (2012) also find that supermajority rules can provide informational advantages. In their model, where information transmission is always beneficial to the voters, the advantage of supermajority rules comes from promoting more communication as opposed to preventing manipulation.

restriction. Restricting *communication* to a single dimension does not eliminate manipulative persuasion nor does allowing communication by multiple experts.

2.4.1 Domain restrictions

Though Theorems 1 and 2 are powerful results, they are limited in that they demonstrate what is possible rather than what is inevitable or even probable. In fact, my argument for the existence of a non-empty manipulative persuasion set relies on the assumption that all probability distributions over expected utility are feasible. This is a limitation shared by the classic results in social choice theory. Arrow's impossibility theorem[6], which shows that all non-dictatorial preference aggregation rules that are Pareto efficient must allow violations of Independence of Irrelevant Alternatives or allow preference cycles, relies on the assumption of an unrestricted domain of preference profiles. The Gibbard-Satterthwaite theorem[7], which demonstrates that if there are more than two alternatives to choose from then any non-dictatorial choice rule may provide incentives for voters to misreport their preferences, also relies on unrestricted domain. In the case of Arrow's theorem, restrictions on the domain of preferences such as single-peakedness can alleviate the problems raised by these results, though the negative result of the Gibbard-Satterthwaite theorem is not eliminated by restriction to single-peaked preferences (Penn, Patty and Gailmard, 2011).

In this section, I consider domain restrictions on distributions of utility. In the case of majority rule with an odd number of voters, if the state space is limited to distributions of utility that can be represented by quadratic preferences on a single dimension, the possibility

[6] See Arrow (1951).

[7] See Gibbard (1973) and Satterthwaite (1975)

of manipulative persuasion is eliminated.[8] Thus, returning to the one-dimensional world of most models of communication (for example, Gilligan and Krehbiel (1987, 1989, 1987)) in political science returns the conventional wisdom about the value of information. Theorem 3 states this result.

Theorem 3. *Assume that n is odd and let $\mathcal{D}^m = \{C \subset N : |C| > \frac{n}{2}\}$. Fix a vector $z = (z_1, \ldots, z_n) \in \mathbb{R}^n$ and for some $q \in \mathbb{R}$ let*

$$\Theta(q) = \{\omega \in \Omega : \exists \theta \in \mathbb{R} \text{ } s.t. \text{ } \omega_i = (z_i - q)^2 - (z_i - \theta)^2 \forall i \in N\}.$$

If $\mu_0(\Theta(q)) = 1$, then all persuasive equilibria are majority preferred.

Restricting the domain of voter utilities to those that can be represented in one dimension eliminates the possibility of expert manipulation precisely because it eliminates the instability associated with majority rule. However, since Ω includes distributions of utility that are not necessarily represented by any spatial preferences, one might be concerned that Theorem 1 results primarily from allowing non-standard distributions of utility. Unfortunately, results for spatial preferences are difficult to prove and straightforward examples are elusive. However, Example 2 demonstrates that manipulative persuasion is possible for two-dimensional quadratic preferences and continuous prior distributions.

Example 2. Consider a three-member majority-rule voting body. The preferences of the voters are represented by quadratic preferences in a two-dimensional policy space with a status quo located at the origin. That is, letting $z_i = (z_{i1}, z_{i2}) \in \mathbb{R}^2$ denote the ideal point of voter i and a state $\theta_i \in \mathbb{R}^2$, we have $\omega_i = z_{i1}^2 + z_{i2}^2 - (z_{i1} - \theta_1)^2 - (z_{i2} - \theta_2)^2$. Assume

[8] Since this model involves decision-making over lotteries, single-peakedness alone is not sufficient to guarantee the result. In addition, it is required that there is a voter who is decisive over lotteries, which is guaranteed in the case of an odd number of voters with quadratic preferences (Banks and Duggan, 2006).

Voter 1	$\left(-\frac{1}{2}, \frac{9}{10}\right)$
Voter 2	$\left(\frac{9}{10}, -\frac{1}{30}\right)$
Voter 3	$\left(-\frac{3}{5}, -\frac{9}{10}\right)$

Table 2.2: Ideal points for voters in Example 2.

that θ is uniformly distributed on the unit disk (i.e. the probability density is $f_0(\theta) = \frac{1}{2\pi}$ if $||\theta|| < 1$ and 0 otherwise). The payoff of the expert is equal to 1 if the proposal is passed an 0 otherwise. The ideal points of the voters are provided in Table 2.2.

The expectation of f_0 is equal to $(0, 0)$. Since the status quo is also located at $(0, 0)$ and the voters are risk-averse, the voters unanimously prefer to reject the proposal in favor of the status quo. However, there is a (manipulative) persuasive equilibrium to this game, as will be demonstrated below.

To state the equilibrium messaging strategy, it is helpful to re-express points in the policy space in polar coordinates. For any $\theta = (\theta_1, \theta_2)$, the polar coordinates (r, ϕ) are characterized by $\theta_1 = r\cos\phi$ and $\theta_2 = r\sin\phi$. Here r is the Euclidean distance of θ from the status quo and ϕ is the angle enclosed by the θ vector and positive horizontal axis (measured in radians and ranging from 0 to 2π). Consider the following pure messaging strategy:

$$\begin{aligned}
\text{supp}\sigma &= \{s_1, s_2, s_3\} \\
\sigma(s_1|\theta) &= 1 \text{ if } 0 \leq \phi \leq \frac{2}{3}\pi \\
\sigma(s_2|\theta) &= 1 \text{ if } \frac{2}{3}\pi \leq \phi \leq \frac{4}{3}\pi \\
\sigma(s_3|\theta) &= 1 \text{ if } \frac{4}{3}\pi \leq \phi \leq 2\pi.
\end{aligned}$$

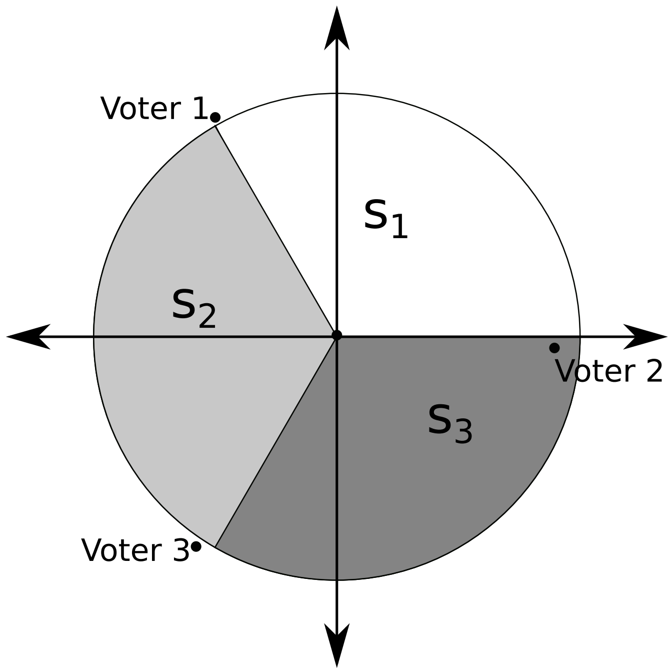

Figure 2.1: A visual depiction of the equilibrium in Example 2. The shaded regions represent a partition of the policy space which is bounded in the unit disk. The expert accurately reveals which partition contains the true state.

In other words, the expert divides the space into three equally sized circle segments and accurately reveals which of these segments contains the true state of the world. Figure 2.1 displays this strategy visually. The expected utilities to voters from passing the proposal following each signal are calculated in Appendix B and displayed numerically in Table 2.3.

	s_1	s_2	s_3
Voter 1	.105	.081	-1.180
Voter 2	0.063	-1.07	.015
Voter 3	-1.22	.16	.07

Table 2.3: Expected payoffs to each voter from passing a proposal following each signal in the persuasive equilibrium in Example 2. All payoffs are relative to the status quo.

If voters play best-responses to this messaging strategy, voters 1 and 2 approve the proposal following s_1, voters 1 and 3 approve following s_2, and voters 2 and 3 approve the proposal following s_3. Thus, this strategy profile constitutes a persuasive equilibrium in which the proposal passes following all messages. $\square$

Example 2 demonstrates that manipulative persuasion is possible in spatial settings with continuous probability distributions. Hence, the phenomenon described in this paper is not a result of non-standard distributions of preferences. The reliance on a uniform probability distribution is convenient but not necessary to support the outcome. In fact, the openness property proven in Theorem 1 implies that many nearby probability distributions could support the same outcome.

2.4.2 Limiting Communication

Since manipulative persuasion relies on the ability of the expert to induce collective preference cycles over potential lotteries faced by voters, it may be possible given some prior distributions to avoid manipulation by only allowing the expert to communicate about one policy dimension. This solution would mirror one of the insights of Shepsle and Weingast's (1981) concept of structure-induced equilibrium, that the well-known problems of manipulation by an agenda-setter can be avoided by requiring agenda-setting on only one dimension at a time. Unfortunately, limiting communication to a single dimension does not provide a general solution to the problem of manipulative persuasion, as Example 3 demonstrates.

Example 3. This example is constructed by embedding the game from Example 2 in a three dimensional space. Consider a three-member voting body operating by majority rule. Let $u_i(x,\theta) = -\sum_{j=1}^{3}(z_{ij} - \theta_i)^2$ and $z_1 = \left(-\frac{1}{2}, \frac{9}{10}, z_{13}\right)$, $z_2 = \left(\frac{9}{10}, -\frac{1}{30}, z_{23}\right)$, and $z_3 =$

$(-\frac{3}{5}, -\frac{9}{10}, z_{33})$. Thus, the ideal point of the voters on the first two policy dimensions are identical to those in Example 2. Furthermore, let $\theta \in \mathbb{R}^3$, let (θ_1, θ_2) be distributed uniformly on the set $\{(x, y) \in \mathbb{R}^2 : \sqrt{x^2 + y^2} \leq 1\}$ so that the marginal distributions of θ_1 and θ_2 are represented by the density f_0 from Example 2. Finally, if ϕ is the angular coordinate of (θ_1, θ_2) in two-dimensional polar coordinates and $\epsilon > 0$, assume that the conditional distribution of θ_3 is uniform on $(-\epsilon, 0)$ if $\phi \in [0, \frac{2}{3}\pi)$, uniform on $[0, \epsilon/2)$ if $[\frac{2}{3}\pi, \frac{4}{3}\pi)$, and uniform on $[\epsilon/2, \epsilon)$. If the expert perfectly reveals θ_3, conditional beliefs about θ_1 and θ_2 are equal to the posterior beliefs from Example 2. Furthermore, for any $\eta > 0$, if ϵ is sufficiently small then voters' expected utilities from each lottery are within η of the payoffs from Table 2.3. Thus, there exists a manipulative persuasive equilibrium to this game when ϵ is small enough. $\square$

Example 3 has an unusual structure but illustrates the relevant point: unidimensional communication can harm welfare if the dimension on which communication occurs is correlated with the other dimensions in a way that can induce cycles over conditional lotteries. Furthermore, the strategy profile suggested by Example 3 would still be an equilibrium if the expert did not know θ_1 or θ_2, which suggests that restricting the expert's information to a single dimension also does not eliminate the problem. Even requiring the expert to communicate only about the payoff of only one voter would not rule out manipulative persuasion for non-collegial rules, since that voter need not be in any of the decisive coalitions that are relevant to the final outcome.

It is possible, however, to eliminate the possibility of manipulative persuasion by limiting communication. In fact, as Lemma 3 suggests, manipulation can be eliminated in all cases by limiting communication to binary endorsements. In fact, the common practice of demanding yes-or-no answers from officials and experts in Congressional hearings suggests that legislators may have an intuitive understanding of this fact. Overall, restricting experts to coarse messaging strategies is a more promising approach to preventing manipulation than allowing unlimited communication on only one dimension.

2.4.3 Multiple experts

Though the model considers communication by a single expert, voting bodies are often able to consult with many experts on the same topic. In models of multidimensional cheap talk to a single decision-maker, the presence of multiple experts is known to dramatically improve the prospects for beneficial advice to the decision-maker (Battaglini, 2002). However, the main results of this paper are robust to the inclusion of multiple experts. In fact, including additional experts never eliminates a manipulative persuasive equilibrium and may expand the possibilities for manipulation.

To see that the inclusion of an additional expert does not eliminate the possibility of manipulation, fix a single-expert game $(N \cup E, \mathcal{D}, \mu_0)$. Consider a new game that is identical except for the inclusion of a new expert E'. The new game is $(N \cup E \cup E', \mathcal{D}, \mu_0')$ where $\mu_0' \in \Delta(\Omega \times [-a, a])$, $m(\omega_{E'})$ is the new marginal probability measure over the new expert's payoff, and $\mathbb{E}_m(W_{-E'}) = \mu_0(W)$ for all $W_{-E'} \in \mathcal{B}(\Omega)$.

If (σ, v^*) is a manipulative persuasive equilibrium to $(N \cup E, \mathcal{D}, \mu_0)$ then there is an equilibrium to $(N \cup E \cup E', \mathcal{D}, \mu_0')$ in which E plays the original strategy σ, voters use the original voting strategy profile v^* and the new expert E' babbles. Since E' transmits no new information, the strategies of the original players remain optimal. Furthermore, since voters' actions do not depend on messages from E', the new expert faces no incentive to deviate from this strategy. This demonstrates that including a new expert does not reduce the manipulative persuasion set $M(\mathcal{D})$ associated with any voting rule. In addition, if the distributions over

ω_E and $\omega_{E'}$ are fixed, there may be distributions over voter utility for which E' can manipulate but E cannot. Thus, instead of eliminating the potential for manipulation, adding more experts may reduce *ex ante* voter welfare.

2.5 A note on equilibrium selection

In the spirit of the canonical results in social choice theory, the focus of this study was on the set of outcomes that are possible for a given voting rule. As a consequence, I have not explored issues related to equilibrium selection. This might lead some to conclude that expert manipulation, while possible, is not a practical concern faced by voting bodies. After all, if the persuasive outcomes are all bad for the voters, they can always fall back on the babbling equilibrium outcome. In this section, I argue that if manipulative persuasion equilibria exist then they are the best prediction for the outcome of the game.

Blume and Sobel (1995) proposed a criterion for selecting equilibria in cheap talk games based on the idea the equilibrium should not be affected if new opportunities for communication arose. This equilibrium selection criterion is based on the notion of stable sets [9] with respect to a dominance relation. Assume that σ has finite support.[10] In the language of Blume and Sobel (1995) an *agreement* is a triple $(\sigma, \{v_i\}_{i\in N}, \mu)$ where μ gives the probability distribution over states and $(\sigma, \{v_i\}_{i\in N})$ is an equilibrium to the game given μ. The agreement $(\sigma, \{v_i\}_{i\in N}, \mu)$ *CP trumps* $(\sigma', \{v_i'\}_{i\in N}, \mu')$ if there exists a message s such that: (1) there is some ω such that $\sigma(s|\omega) > 0$ and $\mu(W) = \mu_s'(W)$ for all $W \in \mathcal{B}(\Omega)$ where μ_s' is the posterior probability measure on states after conditioning on the signal s; and (2) if

[9] The notion of stable sets was first defined by Neumann and Morgenstern (1944).

[10] The assumption that σ has finite support is without loss of generality for babbling equilibria and holds for any of the manipulative persuasion equilibria in the set characterized by Theorem 1.

$\sigma(s|\omega) > 0$ then $\mathbb{E}[u_E(x^*(s'|v, \mathcal{D}))|\sigma] > \mathbb{E}[u_E(x^*(s'|v', \mathcal{D}))|\sigma']$. Intuitively, $(\sigma, \{v_i\}_{i\in N}, \mu)$ CP trumps $(\sigma', \{v_i'\}_{i\in N}, \mu')$ if for some message sent in the equilibrium $(\sigma', \{v_i'\}_{i\in N})$, $(\sigma, \{v_i\}_{i\in N})$ is an equilibrium to the game that would be induced by allowing a second round of communication and the Expert strictly prefers this new equilibrium to the old one.

The stable set[11] of the "CP trumps" relation is a set of equilibria such that: (1) no equilibria inside the set is trumped by any other equilibrium inside the set, and (2) any equilibrium outside the set is trumped by some equilibrium inside the set. An equilibrium is called *communication proof* if it is an element of this stable set. Consider a game in which the equilibria consist of a set of persuasive equilibria and a set of babbling equilibria. Since persuasive equilibria yield the Expert the best feasible payoff (by Lemma 1), it is immediate that no equilibrium trumps any persuasive equilibrium. Furthermore, since babbling equilibria yield voter beliefs that are equal to the prior and give the Expert a lower payoff than the persuasive equilibrium in every state, every persuasive equilibrium trumps every babbling equilibrium. Thus, in such a game, only the persuasive equilibria are communication proof. This argument holds regardless of the welfare effects of the equilibrium for the voters, so this provides a rationale for selecting manipulative persuasion equilibria.

There is one situation in which the argument from above fails to select only persuasive equilibria. Suppose that there are no states for which the voters would pass the proposal if they had complete information. There may be a manipulative persuasive equilibrium to such a game.[12] However, there is also an equilibrium in which the expert fully reveals the state of the world and the voters reject the proposal following any message on or off the equilibrium path of play. This equilibrium is not CP trumped by any other equilibrium since the Expert

[11] Proposition One of Blume and Sobel (1995) shows that this stable set is unique.

[12] For instance, eliminating the "equitable" state from Example 1 and leaving the lobbyist's strategy the same but limited to the three remaining states yields a persuasive equilibrium to that game.

cannot un-reveal information that has been fully transmitted to the voters. Thus, both the persuasive equilibrium and the fully revealing equilibrium are communication proof even though the latter is payoff equivalent to the babbling equilibrium. However, the idea that the players believe that opportunities for further communication could arise suggests that the Expert would be unlikely to play this equilibrium, since the babbling equilibrium gives every player the same payoffs without foreclosing the idea of profitable communication at a later time.

2.6 Discussion

I have analyzed a model of communication from an expert to a voting body in a multidimensional policy space. In contrast to existing models of communication in political institutions, I show that persuasion by an expert can reduce the welfare of all voters. In fact, the possibility of expert manipulation is a feature of all cyclic voting rules. This result challenges prevailing informational theories of legislative and electoral institutions, which posit that uncertainty reduction inevitably leads to better policy decisions. For instance, the informational approach to legislative organization views legislative institutions as chosen for the purpose of providing "incentives for individuals to develop policy expertise and share policy-relevant information with fellow legislators" (Krehbiel, 1991). My argument implies that this theoretical approach requires revision in situations where policies are not unidimensional. In many situations, legislators would unanimously prefer institutions that reduce information transmission.

My main results are related to previous theoretical results in the literature. Fernandez and Rodrik (1991) showed that when there is ex ante uncertainty about who gains or loses from reforms and policies are chosen through a majority vote, reforms may be rejected ex ante even though a majority of voters benefit from the reform ex post. Chakraborty and Harbaugh (2010) show by example that communication from an expert can reduce voter welfare. Their example features a jury operating by unanimity rule. Jurors are privately informed about their preferences and a defense attorney is privately informed about the case facts. Communication by the defense attorney lowers expected voter welfare by increasing the probability of acquital to suboptimal levels.[13] Finally, Alonzo and Câmara (2014) develop a model of information and voting in which the expert has no private information but can design the information content of a public signal. These public signals, like the expert communication in my model, can harm public welfare. Furthermore, as in this study, stricter supermajority rules reduce the harm from informational manipulation.

The current model assumes that experts are not voters. A natural extension is to model situations in which voters themselves have expert information, which introduces the additional complication that the expert's vote conveys some information and voters can condition on being pivotal (Austen-Smith and Feddersen, 2006*a*). The question of whether deliberation among voters with private information is subject to welfare-reducing manipulation is still open. Extending the model to incorporate multiple voting experts would be a first step toward a multidimensional model of deliberation and debate in the spirit of Austen-Smith and Riker (1987) or Austen-Smith (1990).

The model also assumes that policy proposals are generated exogenously and cannot be altered once introduced. The exogenous proposals distinguishes this paper from models of

[13]The voting example in Chakraborty and Harbaugh (2010) differs from this study because welfare effects depend on random components of voter preferences rather than on instability of collective preferences. In fact, welfare-reducing persuasion is not possible for unanimity rule in my model. Futhermore, in the Chakraborty and Harbaugh (2010) example, if voter welfare is measured after voters know their own preferences (as in this study, where voter preferences for a given state are known), expert communication improves voter welfare.

legislative bargaining, which have been extended to incomplete information environments by Meirowitz (2007*a*). The fact that proposals cannot be altered amounts to an assumption that the voting body operates under a closed rule. Though this assumption is appropriate in popular elections and some legislatures, it contrasts with models of the United States Congress in which bills can be considered under open or closed rules . Unfortunately, though one-dimensional models rely on the assumption that bills are amended to reflect the preferences of the chamber median under open rules, there is no similarly justifiable assumption in the absense of a core. However, the fact that information transmission can be problematic under closed rules is significant in light of the common theoretical claim that closed amendment rules are justified on informational grounds (Gilligan and Krehbiel, 1987).

Overall, this paper provides novel results regarding information transmission by experts and its substantive impact on voting. It also provides a general, flexible formal framework that can be easily extended to study a number of legislative and electoral institutions. This study also suggests that exploring the connections between signaling models and social choice thoeretic concepts is a fruitful path for future research.

Chapter 3

Informational Lobbying and Legislative Voting

The central importance of lobbying in American politics is often viewed as part of a mutually beneficial arrangement between legislators and interest groups. Legislators lack information required to make good policies and get reelected and interest groups supply this information and in so doing gain valuable influence on policy-making. As Wright (1996) phrased the argument, "The informational relationship between legislators and lobbyists has a marketlike quality to it: legislators demand information to reduce uncertainty, and lobbyists supply it" (p. 88). However, this reliance on lobbyists also makes legislators vulnerable to manipulation since lobbyists may exaggerate or misprepresent facts to suit their needs (Schattschneider, 1947, p. 199). Do the gains in information from interest group influence make up for the losses in terms of representation?

In this paper, I analyze the positive and normative implications of lobbying in an environment with multiple lobbyists and multiple legislators. A critical feature of the model is the incorporation of voting with multiple legislators. This is a departure from the literature on informational lobbying which is based on one-legislator models. However, incorporating voting institutions into theories of lobbying is realistic and substantively critical: at its heart, lobbying is about building *coalitions* in favor of one's preferred policy.[14] Lobbyists are unlikely to expend resources persuading a particular legislator to support a policy if they will fall short of a majority with or without that legislator's support. Furthermore, lobbyists can emphasize different aspects of a policy to appeal to different groups: an opponent of the Affordable Care Act may have appealed to libertarians on the basis of federal spending and to social conservatives on the basis of abortion policy. The presence of multiple winning coalitions also affects the welfare implications of information transmission since providing information to a single decision-maker always improves welfare but information transmission to a voting body can reduce *ex ante* welfare (Schnakenberg, 2014).

I also depart from previous models by analyzing interest groups that are purely vote-oriented in the sense that their preferences do not depend on the aspects of the policy that affect legislative preferences. This is a realistic focus for many policy settings. For instance, representatives from lobbying firms may contract with clients to advocate certain policies. In these cases, the firms gather information relevant to the legislators' support for that policy, but their own advocacy does not depend on that information. Similarly, a manufacturer of textiles may lobby in favor of higher tariffs on foreign textiles on the basis of domestic unemployment or foreign labor rights even though their support for the policy depends only on its effect on their company's profits. The focus on vote-oriented lobbyists is useful in part because traditional one-legislator models would predict that these groups could not be effective[15] yet they often influence policy-making in the multiple legislator model because

[14]Baron and Hirsch (2012) analyze a model in which lobbyists attempt to influence coalitions in addition to policy in a parliamentary government formation setting. In this model, the means of lobbying is through direct contributions rather than persuasion.

[15]To see why vote-oriented lobbyists cannot be effective in one legislator models, suppose that when the lobbyist's information suggests that the legislator's preferences would align with the lobbyist's, the lobbyist

they can inform the legislators about *which* coalition is likely to benefit from the proposed policy. Thus, even purely vote-oriented groups can gain influence by appealing to different coalitions at different times.

The normative implications of the model are straightforward but differ from traditional models: influential lobbying by vote-oriented interest groups is always bad for welfare. In contrast, lobbyist influence in one legislator models always weakly benefits the legislator since rationality of the legislator provides a check on the lobbyist's incentive to lie. If the legislator is rational, the worst-case scenario is the new information is useless, but then the legislator is free to ignore it and make whatever decision she would have made in the absence of that information. In voting settings, however, legislators can lose in expectation from the introduction of new information since they may be left out of the winning coalition as often as they are included. For majority voting, this result is related to Anscombe's (1976) paradox, which states that if several issues are decided by a yes-or-no vote, the majority might vote in the minority on the majority of issues. In probabilistic terms, a related fact is that majority preferences over a set of lotteries over policies may conflict with majority preferences over a compound lottery made up of those lotteries.

The positive implications of the model are more nuanced. The prediction of the model depends on the expected distribution of benefits from the policy in the following way. If the relative potential gains to losses from the policy are sufficiently high, an interest group promoting that policy can always influence the legislature to pass that policy when there is no competition from opposing interest groups. In the presence of competition from opposing

gives a speech interpreted as "Trust me, you will like what happens if you vote in favor of this policy." If the legislator believes this speech and votes in favor of the policy, then the lobbyist will be tempted to send give this speech even when it is not true. Therefore, the legislator, knowing that the lobbyist faces this temptation, expects that the lobbyist would make the same speech whether or not she would truly like the effect of the policy. Therefore, the legislator assumes that the lobbyist's speech is uninformative about her true preferences regarding the policy, and disregard the speech.

interest groups, an interest group promoting a policy can still influence the legislature to pass the policy if the distribution of expected benefits from the policy are not too targeted to a minority of legislators. Otherwise, the interest group promoting the status quo can block the other interest group from influencing the legislators.

3.1 Related literature

The model closely relates to a large body of theory on informational lobbying.[16] The seminal work on informational lobbying came in the early 1990's. Potters and Winden (1990) formalize idea of political pressure as information transmission and prove the existence of political pressure in a dynamic game between an interest group and a legislator. Another single interest-group model by Potters and Winden (1992) shows that conflicts of interest between interest groups and legislators can prevent credible informational lobbying and that the imposition of lobbying costs restores credibility when conflicts of interest are not too severe.[17] Austen-Smith and Wright (1992) analyze a model in which two opposing interest groups transmit information to a single legislator in order to influence her vote. In that model, legislators make better decisions on average with lobbying than without and the informational gains from lobbying are more pronounced when the issue is very salient to the interest group. Ainsworth (1993) shows how legislators have incentives to regulate lobbying in ways that make their signals costly and limit their temptation to exaggerate. More recent theoretical studies focus on the implications of the fact that financial contributions to

[16]See Grossman and Helpman (2001, Chapter 4) for a thorough review of the early work in this literature.

[17]This relates to a more general point, due to Crawford and Sobel (1982), that "cheap talk" information transmission is less informative the more the preferences of the sender and receiver diverge.

campaigns grants lobbyists access to legislators so that they can attempt to persuade them to support their preferred policies (Cotton, 2012).[18]

The most significant challenge to informational theories was raised by Hall and Deardorff (2006). Though existing informational theories suggest that interest groups should focus their lobbying efforts on legislators that are undecided or inclined to vote against their position, the empirical patterns are exactly the opposite: interest groups spend most of their time lobbying their natural allies. Though Austen-Smith and Wright (1994) addressed this inconsistency with a model showing that lobbyists may target allies in order to prevent them from being influenced by opposing interest groups, their model still predicts that interest groups only lobby allies that are close to the fence and will do so less often than non-allies, so informational theories are not fully consistent with empirical patterns of lobbying. Thus, Hall and Deardorff conclude that the informational model does not capture the nature of lobbying activity. Instead, they claim that lobbying is a form of legislative subsidy: lobbyists provide labor and expertise to like-minded legislators who are strapped for resources in order to make them more effective at advancing their interests.

As I discuss later in this paper, the criticism raised by Hall and Deardorff illustrates the theoretical confusion that results from making predictions about *which* legislators groups should lobby using theories with only *one* legislator to be lobbied. Furthermore, though informational lobbying and legislative subsidy are usually posed as competing theories, they are not mutually exclusive and in fact complement one another. As Hall and Deardorff note, expertise is one of the resources that lobbyists make available to legislators. Certainly the information provided to legislators by lobbyists is used to build coalitions in favor of their

[18] Lohmann (1995) also addresses this topic and shows that campaign contributions can increase the credibility of communication from lobbyists. Cotton (2012) examines the implications of these access costs for the welfare effects of limits on campaign contributions.

policy positions and lobbyists provide information along with advice about how it should be disseminated. Thus, the expertise provided by lobbyists as a subsidy to like-minded legislators is only one step removed from standard accounts of informational lobbying.[19]

One exception to the tendency of the informational lobbying literature to focus on single legislator models is a study by Bennedsen and Feldmann (2002) that complements this study. In that model, a single lobbying group communicates with legislators about demand for a public good and then a randomly chosen agenda-setter proposes a distribution of the public good across the districts. The proposal is then subject to a majority vote. Like this study, the model in Bennedsen and Feldmann (2002) shows that lobbying a voting body can provide greater opportunity for information transmission than lobbying a single individual because the information transmitted can affect the composition of the winning coalition that comes together to pass or block a policy proposal. Unlike this study, Bennedsen and Feldmann's (2002) model has endogenous proposals and communication occurs at the agenda-setting stage. Furthermore, in their model, the lobbying group's utility is increasing in the total amount of the public good that is provided. Their model does not produce the negative welfare results in this study, which suggests that interest group influence is more productive at the agenda-setting stage of policy-making and when the lobbying group has state-dependent preferences.

A model by Caillaud and Tirole (2007) also explores strategies that can be used to persuade a voting body to pass a bill. In that model, the lobbyist[20] does not have private information about the effect of the bill on a legislator's payoff but can provide legislators with a report

[19] For instance, if a lobbyist provides information to a legislator with the same preferences as herself it is an equilibrium for that lobbyist to provide instructions to allow that legislator to implement the strategy she would have used to persuade the entire legislature.

[20] In Caillaud and Tirole (2007) this player is called a "sponsor" and the voting players are called "group members." I am referring to them as lobbyists and legislators to make clear the relationship of their paper to the current application.

that allows them to determine this information for themselves. They show that by targeting key legislators the lobbyist can sometimes engineer "persuasion cascades" in which bringing key members on board sways the opinions of others.

3.2 Model

The set of players is $G_1 \cup G_2 \cup N$ where $N = \{1, \ldots, n\}$ is a set of legislators with $n \geq 3$, G_1 is an interest group in favor of some new policy, and G_2 is an interest group in favor of the status quo. The legislators must decide whether to implement a new policy ($x = 1$) or reject it in favor of the status quo ($x = 0$). The state of the world $W \in 2^N$ is a set of beneficiaries of the policy. Thus, if $W = \emptyset$ then no legislators would benefit from implementing the policy, if $W = N$ then all legislators would benefit from the policy, and if $W = C \subset N$ then only legislators in the set C would benefit from implementing the policy.

The set of beneficiaries W is private information to the interest groups and is not observed by the legislators. The legislators' beliefs about the distribution of W are represented by a common prior p that assigns a probability to each subset of N. I assume that p has full support on 2^N, so that $p(W) > 0$ for all $W \subset N$. For each $k \in \{0, \ldots, n\}$, let $p_k = \sum_{W:|W|=k} p(W)$ denote the probability that the number of beneficiaries is equal to k.

The sequence of play is as follows. First, G_1 chooses a set of legislators $M_1 \subset N$ to lobby. The role of lobbying in this model is purely communicative – to lobby a particular legislator is to approach that legislator and make a speech having the interpretation "I recommend that you vote in favor of my preferred policy." $M_1 = \emptyset$ is interpreted as a decision not to engage in lobbying. Next, G_2 observes M_1 and chooses a set of legislators $M_2 \subset N$ to lobby.

Finally, the legislators observe M_1 and M_2, update their beliefs about the set of beneficiaries, and take an up-or-down vote over whether or not to implement the new policy. Legislators' votes are translated into outcomes according to a q-majority rule: the policy is implemented if and only if the number of legislators who vote in favor is at least q, where $\frac{n}{2} < q \leq n$.

The interests groups have opposing preferences: G_1 is in favor of the new policy and G_2 is in favor of the status quo. In addition to the utility they receive from policy, the groups pay a fixed cost for lobbying. Thus, letting $m_1 = 1$ if $M_1 \neq \emptyset$ and $m_1 = 0$ otherwise, the preferences of G_1 are represented by the utility function

$$u_{G_1}(x, m_1) = x - cm_1 \tag{3.1}$$

where $0 < c < 1$. Likewise, the utility function for G_2 is

$$u_{G_2}(x, m_2) = -x - cm_2 \tag{3.2}$$

where $m_2 = 1$ if $M_2 \neq \emptyset$ and 0 otherwise. The lobbying cost c may represent direct labor and expenses from lobbying, access costs such as campaign donations or costs of acquiring expertise.[21]

The legislators' utility functions are

$$u_i(x, W) = \begin{cases} xH & \text{if } i \in W \\ -xL & \text{if } i \notin W \end{cases} \tag{3.3}$$

[21] Interpreting c as the cost of acquiring expertise would require modifying the game so that the the set of beneficiaries is only revealed to each group after they decide in favor of lobbying. This modification would not substantively affect the results, since the decision of whether or not to lobby does not depend on the group's private information in equilibrium.

for all $i \in N$, where $H > 0$ and $L > 0$. In other words, each legislator receives a payoff of H if the policy is implemented and she is a beneficiary, $-L$ if the policy is implemented and she is not a beneficiary, and 0 if the policy is rejected.

To simplify the presentation of results, I assume that the policy proposal would fail in the absence of lobbying. This requires that at least q legislators' prior expected utilities are negative. Thus, the focus of the analysis is on cases in which G_1 attempts to influence the legislators to vote in favor of the policy, though the results would extend naturally to the opposite case.

3.3 Strategies and Equilibrium

The analysis allows interest groups to use mixed lobbying strategies. A strategy for G_1 is a function $\sigma_1 : 2^N \to \Delta(2^N)$, where $\Delta(2^N)$ is the set of probability distributions over 2^N. The function σ_1 specifies a probability of lobbying each subset of legislators for each possible set of beneficiaries. Thus, $\sigma_1(M|W)$ denotes the probability that $M_1 = M$ when the set of beneficiaries is W.

A strategy for G_2 is a function $\sigma_2 : 2^N \times 2^N \to \Delta(2^N)$ specifying a probability of lobbying each subset of legislators given the set of beneficiaries and the set of legislators lobbied by G_1. Thus, $\sigma_2(M|W, M_1)$ is the probability that $M_2 = M$ given that the set of beneficiaries is W and the set of legislators lobbied by G_1 is M_1.

Without loss of generality, I consider only pure voting strategies.[22] The voting strategy of legislator $i \in N$, denoted $v_i(M_1, M_2)$, is equal to 1 if that legislator votes in favor of

[22] Theorem 4 in Schnakenberg (2014) implies that, for any equilibrium in mixed voting strategies, there is some equilibrium in pure voting strategies where G_1 and G_2 use exactly the same lobbying strategies.

the policy following M_1 and M_2 and zero otherwise. Given the q-majority rule, the policy outcome $x^*(M_1, M_2)$ is equal to 1 if $\sum_{i \in N} v_i(M_1, M_2) \geq q$ and zero otherwise.

The beliefs of the legislators about the state of the world following M_1 and M_2 are denoted $\pi(W|M_1, M_2)$. Given these beliefs, the expected utility of implementing the policy for each legislator is

$$U_i^\pi(M_1, M_2) = \sum_{W \in 2^N} \pi(W|M_1, M_2) u_i(1, W). \tag{3.4}$$

The analysis characterizes perfect Bayesian equilibria in weakly undominated strategies. Specifically, an equilibrium to the game is a profile of strategies such that:

- M_1 maximizes $x^*(M_1, M_2)$ given σ_2 and $\{v_i\}_{i \in N}$;
- M_2 minimizes $x^*(M_1, M_2)$ given M_1 and $\{v_i\}_{i \in N}$;
- Legislators vote in favor of the policy if and only if[23] $U_i^\pi(M_1, M_2) \geq 0$; where
- $\pi(W|M_1, M_2)$ is consistent with Bayes rule when possible.

The decision rule for legislators reduces to a simple comparison of the relative value of potential policy gains and losses to the odds of being a non-beneficiary of the policy. Let $\Pi_i(M_1, M_2) = \sum_{W: i \in W} \pi(W|M_1, M_2)$ denote i's probability of being a beneficiary following the lobbying choices M_1 and M_2. Legislator i's expected payoff from passing the policy is $U_i^\pi(M_1, M_2) = \Pi_i(M_1, M_2)H - (1 - \Pi_i(M_1, M_2))L$. Thus, $v_i(M_1, M_2) = 1$ is optimal if and only if

$$U_i^\pi(M_1, M_2) = \Pi_i(M_1, M_2)H - (1 - \Pi_i(M_1, M_2))L \geq 0$$

[23] I assume for the sake of convenience that legislators vote in favor of the policy when they are indifferent. This assumption does not substantively affect the results.

which is true if

$$\frac{H}{L} \geq \frac{1 - \Pi_i(M_1, M_2)}{\Pi_i(M_1, M_2)}. \tag{3.5}$$

This formulation of the legislators' decision-rules is used in all of the main results.

Since the costs of lobbying do not depend on the true set of policy beneficiaries,there are uninformative equilibria that correspond to the "babbling" equilibria common to cheap talk games: if the legislators believe that both interest groups will lobby uninformatively, neither interest group has an incentive to be informative and therefore both groups will choose not to lobby. When more informative equilibria exist, these equilibria are implausible: a lobbyist who chose to babble when she could be influential would not remain a lobbyist for very long. I therefore focus on equilibria that are optimal in the sense that the legislators believe that each interest group will play the strategy that maximizes that group's equilibrium payoff given the strategy of the other interest group. For the rest of this paper, I will simply use the word *equilibrium* to refer to perfect Bayesian equilibria that are optimal in this sense.

3.4 Analysis

In the analysis I characterize the equilibria to the game and demonstrate that interest group influence in this environment is always bad for the *ex ante* welfare of the legislators. The game has three categories of equilibria which depend on the prior probability distribution over beneficiaries and on the relative values of the potential gains and losses from policy. In a *no influence* equilibrium, G_1 cannot persuade the legislators to implement the policy even in the absence of competition from G_2. In an *influential* equilibrium, G_1 persuades the legislators to implement the policy and G_2 cannot credibly prevent that influence. Finally, in a

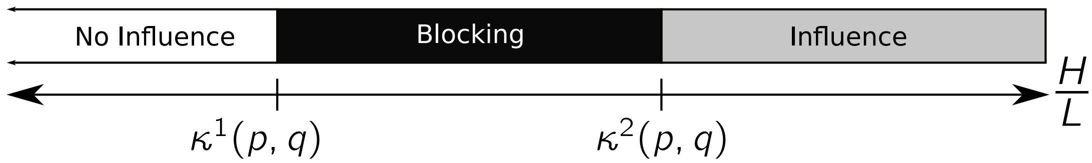

Figure 3.1: The figure displays equilibria of the lobbying game as a function of the ratio of potential benefits of the proposed policy to the potential costs.

blocking equilibrium, competition between the interest groups prevents G_1 from being influential. After characterizing each category of equilibrium I show how stricter supermajority requirements may prevent interest group influence.

Figure 3.1 provides an overview of the results regarding the types of equilibria that can be supported for some policy areas. The equilibria are defined by the ratio of the potential benefits of the policy to the potential costs. When H/L is very small, the beneficiaries of the project receive only a small payoff relative to the amount lost by non-beneficiaries. In those cases, G_1 is never influential. When H/L is large, the beneficiaries receive more relative to the losses of the non-beneficiaries and G_1 is influential even in the face of competition from G_2. Finally, when H/L is at an intermediate level, the presence of lobbying competition blocks G_1 from being influential when it would otherwise persuade the legislators to implement the proposal. These three regimes are defined by two cutpoints which are labeled $\kappa^1(p, q)$ and $\kappa^2(p, q)$ and are fully defined in the following sections.

3.4.1 Legislator welfare

Before getting into the details of the equilibria I present the main normative result which is that the welfare effects of interest group influence must be negative. Since the analysis is limited to interest groups that are purely vote-oriented in the sense that they are not moved by the same information that moves legislators, the result should not be interpreted to mean that *all* interest group influence must harm welfare. However, the result raises normative concerns about the influence of for-profit lobbying firms and certain narrow business interests in American politics.

The key fact that leads vote-oriented interest groups to harm welfare when they are influential is that it is impossible for such a group to be only partially influential. If there is any lobbying choice that leads the legislators to select such an interest group's preferred policy, then that interest group will always make that lobbying choice rather than one leading to an unfavorable outcome, regardless of the nature of the group's information. Lemma 4 shows the consequence of this reasoning: all equilibria to the game induce the same policy choice in every situation. Recall that, since I have assumed that the legislators initially do not wish to pass the policy, an influential equilibrium to this game is one in which the policy sometimes passes. Lemma 4 tells us that, in an influential equilibrium, the policy must always pass.

Lemma 4. *In any equilibrium to the game, $x^*(M_1, M_2)$ is constant.*

In a game with only one legislator, Lemma 4 would imply that interests groups are never influential: since the legislator knows that an interest group lobbyist will say whatever it takes to sway the legislator to her side, the legislator would interpret lobbying choices as being completely uninformative. However, in a collective choice setting lobbying can still be interpreted as being informative because the lobbyist can choose to appeal to one coalition rather than another.

As an example, suppose an environmentalist legislator is approached by an auto industry lobbyist, who tells her "You should support increasing tariffs on Chinese cars to prevent an influx of vehicles with inferior emissions standards." If that legislator were the only decision-maker that could affect tariffs, she should dismiss the message on the basis that the lobbyist would say anything to get her to support higher tariffs. However, in a legislative setting the message may be credible. The legislator's thought process might be: *I know that the industry lobbyist would say anything to get the legislature to support higher tariffs. However, if the lobbyist's information did not suggest that the Chinese cars adhered inferior emissions standards, she most likely would not have lobbied me and would instead focus her efforts on winning over labor and business oriented legislators.* Thus, the legislator should interpret the lobbyist's message as credible.

The influence of interest groups in this environment therefore comes from manipulating legislative coalitions. The harm to legislator welfare from this manipulation comes from the conflict between the *ex ante* welfare of the legislators before the lobbyists walk through the door and the *ex post* welfare of the legislators who are influenced by the lobbyist. Though each legislator who is influenced has every incentive to vote in a way that is consistent with the lobbyist's recommendation, a legislator does not know at the beginning of the game whether or not she will be one of the people lobbied. Furthermore, since the legislature did not intially wish to pass the policy and the policy always passes in an influential equilibrium, at least a winning coalition of legislators are worse off on average with interest group influence than without it.

Theorem 4. *Any influential equilibrium gives all legislators a lower ex ante expected utility relative to the outcome when lobbying is not allowed.*

The intuition driving the negative welfare result in Theorem 4 relates to Anscombe's paradox which states that, when a binary majority vote is taken over a series of issues, a majority of the voters may be on the losing side on the majority of issues. This is one of many voting paradoxes that result from the cyclic nature collective preferences[24] and the problem extends to supermajority rules. Anscombe's paradox illustrates just one way that a lobbyist might manipulate a legislature. Each different coalition that the interest group may lobby presents the legislature with a different "issue" or, more directly, a different lottery over possible effects of the policy in question. Moreover, the lobbyist may present these issues in such a way that most legislators lose out most of the time even though a majority (or supermajority) prefers to follow the lobbyist's advice on any given issue.

3.4.2 Interest group influence without competition

I now turn my attention to the positive implications of the model. I begin by characterizing the conditions required for an interest group to be influential in the absence of any competition. This is a useful benchmark against the results with competition but it is also a substantively important case because one common path to lobbyist influence is to gravitate toward narrow issue niches without substantial competition (Browne, 1990; Baumgartner and Leech, 2001; Gray and Lowery, 1997).

Since a large number of distinct lobbying strategies can achieve the same effect in any given situation, explicitly characterizing every equilibrium lobbying strategy is difficult and uninformative. Instead, my approach is to establish necessary and sufficient conditions for interest group influence by first establishing an upper bound on the expected utility that a lobbyist can credibly offer a winning coalition of legislators and then by demonstrating that there exists a lobbying strategy that achieves that upper bound. Thus, there is an influential equilibrium if and only if, when the lobbyist promises the maximum credible expected utility

[24] See Nurmi (1999) for a thorough treatment of the topic of voting paradoxes.

to the members of some winning coalition, the legislators in that winning coalition are willing to approve the proposed policy. Lemma 5 establishes the relevant upper bound.

Lemma 5. *In any equilibrium to the game with or without competition, following some* $M_1, M_2 \in 2^N$*, we have*

$$U_i^\pi(M_1, M_2) \leq H \sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\} - L(1 - \sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\})$$

for at least $n - q$ *legislators.*

In other words, Lemma 5 says that the maximum probability that all members of a winning coalition can assign to being a beneficiary of the policy is $\sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\}$. Recall that p_k is the probability that the number of beneficiaries is k. Thus, in the best case scenario for the lobbyist, a winning coalition of legislators are persuaded to compute their probability of being a beneficiary according to the following reasoning after they are lobbied: *Given that I was lobbied, I believe that if there is a winning coalition that will benefit from the policy then I am a beneficiary for sure. Otherwise, if the set of beneficiaries falls short of a winning coalition, the probability of benefiting is evenly distributed among a minimal winning coalition that includes myself.*

The lobbying strategy defined below induces this reasoning among exactly q legislators following any lobbying choice. The strategy is called "minimal" because the lobbyist always lobbies exactly a minimal winning coalition of legislators. The group lobbies only a minimal winning coalition even if the set of true beneficiaries is much larger. At first glance it seems counterintuitive that a group who always wants a policy to pass would ignore legislators who would truly benefit from the policy even when it means they will vote against the policy. However, there is no extra gain to passing a policy with an oversized coalition and if the group sometimes lobbied large coalitions then the legislators could infer that the set of beneficiaries is small when the group only lobbies a minimal coalition. In contrast, when a minimal strategy is used, the lobbying choices do not inform the legislators about the number of beneficiaries. The minimal strategy is defined below.

Definition 1 (Minimal strategy for G_1)**.** *A minimal strategy for G_1 is a mixed strategy $\sigma_1^*: 2^N \to \Delta(2^N)$ defined as follows:*

$$\sigma_1^{\min}(M_1|W) = \begin{cases} \left(p(W)\binom{n-|W|}{q-|W|}\right)^{-1} & \textit{if } |M_1| = q \textit{ and } W \subseteq M_1 \\ \left(p(W)\binom{|W|}{q}\right)^{-1} & \textit{if } |M_1| = q \textit{ and } W \supset M_1 \\ 0 & \textit{otherwise.} \end{cases} \tag{3.6}$$

Though the minimal strategy provides some intuition about the incentives of the lobbyist in this legislative setting, it is not a prediction about the lobbying strategy that in interest group will use in any given situation. The minimal strategy is the most robust in the sense that it allows interest group influence any time influence is possible. However, in a given situation, it is likely that there are many other lobbying strategies lead to influential equilibria. Thus, the strategies should not be interpreted as providing a point prediction about which legislators should be lobbied by a given interest group. Notably, if the lobbyist uses the minimal strategy, all of the legislators are equally likely to be lobbied.[25] Thus, there

[25]To see this, note that the probability that i is lobbied is

$$\sum_{W:i\in W}\sum_{M:i\in M} p(W)\sigma(M|W) = \sum_{W:i\in W\wedge|W|<q} p(W)(p(W)\binom{n-|W|}{q-|W|})^{-1} + \sum_{W:i\in W\wedge|W|\geq q} p(W)(p(W)\binom{n-|W|}{q-|W|})^{-1}$$

which reduces to

$$\sum_{W:i\in W\wedge|W|<q}\binom{n-|W|}{q-|W|}^{-1} + \sum_{W:i\in W\wedge|W|\geq q}\binom{n-|W|}{q-|W|}^{-1}$$

which is the same for all i.

is at least one equilibrium strategy for which the criticism of Hall and Deardorff does not apply. In fact, when the environment does not require the lobbyist to use the most robust strategy available, there will be equilibria in which the interest group more frequently lobbies allies. Lemma 6 verifies that the minimal strategy described above implements the lobbyist's best case scenario by promising the maximum possible expected utility to a minimal winning coalition of legislators.

Lemma 6. *If G_2 never lobbies and G_1 uses the strategy $\sigma_1^{\min}$, for each M_1 such that $|M_1| = q$, we have*

$$U_i^{\pi}(M_1, \emptyset) = H \sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\} - L\left(1 - \sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\}\right)$$

for all q of the legislators in M_1.

To relate the expected utilities of the legislators from Lemma 6 to the decision rule described in Equation 3.5, let

$$\kappa^1(p, q) = \frac{1 - \sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\}}{\sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\}}. \tag{3.7}$$

The condition required for influence without competition is stated in Theorem 5.

Theorem 5. *When there is no competition, G_1 is influential if and only if*

$$\frac{H}{L} \geq \kappa^1(p, q).$$

Though Theorem 5 does not make a prediction about who an interest group will lobby, it does provide a prediction about when an interest group will lobby by defining which policy issues are winnable. The legislature as a whole must be close enough to indifference in the sense that, though the legislature would not approve the policy without lobbying, there are enough expected benefits to go around that the lobbyist can make a good case.

3.4.3 Blocking influence through competitive lobbying

Next, I focus on how competitive lobbying can alter the conditions under which an interest group is influential. In some cases, the presence of competition between interest groups eliminates the possibility of influence. However, if the distribution of expected benefits from a policy are not too narrowly targeted and the expected loss from the policy is not too large, an interest group can successfully influence the legislature to implement a new policy in the presence of competition.

Assume that σ_1 is a lobbying strategy in which G_1 lobbies with positive probability. I will say that G_2 *blocks* the strategy σ_1 if:

1. there exists a strategy σ_2 such that σ_2 is a best response to σ_1 for some legislator strategies that are consistent with Bayesian beliefs, and
2. given the same legislator strategies, σ_1 is not a best response to σ_2.

In other words, G_2 blocks the strategy σ_1 if there is a strategy that prevents σ_1 from being influential.

Clearly G_2 blocks σ_1 if there is a strategy by which G_2 can always prevent the policy from being implemented following every lobbying choice by G_1 under σ_1. Lemma 7 shows that G_2 also blocks σ_1 if there is a strategy that prevents the policy from being implemented following even one of the lobbying choices made by G_1. The reasoning behind Lemma 7 is as follows. Suppose that if G_1 uses the strategy σ_1 and that G_2 has a strategy that could prevent passage of the policy following some $\tilde{M}_1$ in the support of σ_1 but not necessarily following any other lobbying choice by G_1. Then G_2 should play the strategy that prevents passage whenever possible and choose not to lobby the rest of the time. However, this implies that the policy never passes following $\tilde{M}_1$, which means that G_1 strictly prefers to deviate to a strategy the places zero probability on $\tilde{M}_1$.

Lemma 7. *G_2 blocks the strategy σ_1 if there exists some M_1 and σ_2 such that $\sum_{W \in 2^N} \sigma_1(M_1|W) > 0$ and $x^*(M_1, M_2) = 0$ for all M_2 such that $\sum_{W \in 2^N} \sigma_2(M_2|M_1, W) > 0$.*

Lemma 7 establishes the path to verifying the existence of a blocking equilibrium to the game. If G_2 can prevent passage of the policy following at least one lobbying choice given any strategy by G_1, then G_2 blocks every possible strategy for σ_1 and the prediction of the game is a blocking equilibrium. Lemma 8 establishes an upper bound on the expected policy payoff that G_1 can always offer a winning coalition of legislators in the presence of competition.

Lemma 8. *For any lobbying strategy by G_1, t here is a strategy for G_2 such that for any (M_1, M_2) on the equilibrium path $U_i^\pi(M_1, M_2) \leq H \sum_{k=q}^{n} p_k - L \sum_{k=0}^{q-1} p_k$ for at least $n - q$ legislators.*

Thus, if G_2 lobbies effectively it cannot be the case that there are always q or more legislators who believe that they are beneficiaries of the policy with a probability higher than the total probability that there are q or more beneficiaries. Given this result, it is now possible to state a sufficient condition for a blocking equilibrium. Let

$$\kappa^2(p, q) = \frac{\sum_{k=0}^{q-1} p_k}{\sum_{k=q}^{n} p_k}.$$

be the odds that the set of beneficiaries is smaller than q. I interpret $\kappa^2(p, q)$ as a measure of the extent to which a policy area is targeted or broad in relation to the voting rule. If $\kappa^2(p, q)$ is very high, the legislators believe that the benefits of the proposed policy are likely to be targeted to a small number of districts. This may represent lobbying over pork projects or policies designed to satisfy a small ideological niche. In contrast, a low value of $\kappa^2(p, q)$ is associated with a policy that is expected to be broad in the sense of appealing to a broad coalition. As Theorem 6 establishes, competition between interest groups tends to eliminate the possibility of influence in targeted policy areas but not necessarily in broad ones.

Theorem 6. *There is a blocking equilibrium if*

$$\kappa^1(p, q) \leq \frac{H}{L} < \kappa^2(p, q).$$

Theorem 6 establishes a necessary condition for G_1 to be influential – the policy area must be sufficiently broad that G_2 cannot block every strategy. Theorem 7 establishes that the same condition is also sufficient. When the expected distribution of benefits is sufficiently broad compared to the ratio of potential gains and losses from the policy, the interest group promoting the policy can influence the legislature even the presence of competition.

The blocking equilibria have a similar quality to the "jamming" that occurs in Minozzi's (2011) model of communication from two senders to a single decision-maker. In that model, one sender employs a communication strategy designed to leave the decision-maker uncertain about who has sent a truthful message. Here, competitive lobbying has a similar effect on the legislators who are lobbied by both groups. These legislators believe that G_1 is truthful about the identity of the beneficiaries when the number of beneficiaries is large and that G_2 is truthful when the number of beneficiaries is small. However, since they are not informed about the size of the set of beneficiaries, they remain uncertain about which message should be believed.

Theorem 7. *There exists an influential equilibrium in the presence of interest group competition if*

$$\frac{H}{L} \geq \kappa^2(p, q).$$

When interpreting Theorem 7, it is important to recognize that the condition depends on three parameters of the game: the ratio of potential gains and losses from the policy, the prior distribution over beneficiaries, and the voting rule. All three are important for the result. If the potential gains to a policy are extremely small and the potential losses extremely large, G_1 may be unable to influence the legislature even if the policy is very broad as measured by $\kappa^2(p, q)$. Furthermore, there exist prior distributions for which the policy would pass with no lobbying any time $H/L \geq \kappa^2(p, q)$, for instance if the policy is extremely targeted in the sense that it almost certainly benefits only a minority of legislators. Finally, as Corollary 2 states, stricter supermajority rules make influence more difficult. This is true without competition, since $\kappa^1(p, q)$ increases with q, and with competition, since $\kappa^2(p, q)$ also increases in q.

Corollary 2. *Increasing the supermajority rule reduces the set of circumstances leading to influential equilibria. Influence is never possible under a unanimity rule ($q = n$).*

Considering the welfare result of Theorem 4, this conclusion about voting rules provides a rationale for stricter supermajority rules in some legislatures. Increasing the supermajority rule can limit the welfare losses experienced by legislators as a result of manipulation by interest groups. In fact, a common argument in favor of supermajority rules is that they help prevent capture by special interests (McGinnis and Rappaport, 1998).

3.5 Discussion

The model has many implications for interest group scholars and more generally for the understanding of policy-making. In this section, I summarize some of the main insights from the model regarding the nature of interest group influence.

Informational lobbying can be bad for legislators. The informational view of lobbying is often taken to mean that lobbyists provide a service to legislators that increases the quality of representation. For instance, one of the seminal papers on informational lobbying concludes that "a legislator will on average make 'better' decisions with lobbying than without" (Austen-Smith and Wright, 1992, p. 229). Similarly, before applying the informational lobbying framework to the analysis of the role of *amici curiae* briefs in the Supreme court, Epstein and Knight (1999) summarize the argument for informational lobbying as follows:

> Lobbyists provide information to MCs [Member of Congress] about consequences of alternative courses of action (such as voting for or against a bill). With this information in hand, MCs can then make rational choices, that is, choices designed to maximize their preference for reelection as opposed to electoral ouster. This is one reason why reelection rates for MCs remain so high. Or so the argument goes (p. 215).

My model suggests that the optimistic conclusions in the literature about interest group influence are tied to the one-legislator models on which most of the literature is based. When the models are adjusted to account for multiple legislators interacting in a collective choice environment, the welfare gains from informational lobbying can be reversed. In fact, as I demonstrate, if interest groups are purely vote-oriented then the effect of interest group influence on legislator welfare is always negative.

Lobbying at the voting stage is most manipulative. I have focused on lobbying that occurs at the voting stage of policy-making. This is a unique stage of the policy-making process because it is the stage at which legislators are the least flexible. In fact, it is likely that interest group influence is much more socially productive at the agenda-setting stage. Austen-Smith (1993) presents a model in which interest groups can lobby at the agenda-setting stage of policy-making in addition to the voting stage and shows that the prospects for influence are generally greater at the agenda-setting stage.[26] Furthermore, Bennedsen and Feldmann (2002) show that majority voting creates opportunities for socially productive informational lobbying prior to agenda-setting. Therefore, it may be most beneficial for legislators to grant access to interest groups when policy proposals are being drafted and legislators are deciding how to allocate their time and resources but deny them access during floor votes.

Unfortunately, in some policy environments, most interest group influence is bound to occur at the voting stage. For instance, Caldeira and Wright (1998) find evidence of lobbyist influence on votes over confirmation of Supreme Court nominees. Interest groups are likely less capable of influencing decisions over who to nominate given that access to the President is more limited and potential nominees are vetted behind closed doors. Furthermore, agenda-setting is often a function of external events that interest groups do not control. For instance, gun control legislation is placed on the legislative agenda in response to mass shootings, and changes in import tariffs are placed on the agenda in response to international negotiations that happen outside of Congress.

The absence of lobbying does not imply the absence of impact. In some circumstances an interest group would never be influential with or without competition. In other

[26] In Austen-Smith's (1993) model, interest group influence always improves the legislator's welfare.

circumstances, the presence of competition between interest groups is the force that blocks a particular interest group from becoming influential. These two situations are substantively very different since the presence of an opposing interest group changes the final policy in the latter case. Thus, the fact that an interest is represented by a group with the capability to lobby improves the quality of representation for that group even when that group is not actively lobbying. Unfortunately for empirical work on interest group influence, non-influential equilibria and blocking equilibria are observationally equivalent. Thus, empirical studies that look at policy outcomes as a function of lobbying activity may only provide a lower bound on the extent of interest group influence in legislatures.

In the presence of interest group competition, targeted policies are more difficult to advocate than broad-based policies. My model predicts that, holding the magnitude of the benefits and losses of the policy constant, competition between interest groups should reduce the influence of interest group advocates of targeted policies but not broad-based policies.

This prediction expands the insights of collective action models of interest group formation which predict that targeted interests are more likely to gain interest group representation (Olson, 1965). In international political economy, this argument is often cited as a reason that protectionist interests gain lobbying representation at higher rates than free trade interests (Ehrlich, 2008, 2007). Note that the definitions of broad and narrow policy areas in this model are reciprocal: if the advocates of a policy represent a broad-based interest then the opponents represent a targeted interest, and vice-versa. Thus, collective action models suggest that interest groups are least likely to face competition in exactly the circumstances where my model suggests competition would be most effective in blocking the influence of special interests.

3.6 Conclusion

I have analyzed a model of interest group influence on legislative voting through information transmission. Unlike most models of informational lobbying, mine accounts for the special incentives that collective choice institutions provide for lobbyists to manipulate voting coalitions to their advantage. This form of lobbyist manipulation has stark implications for legislator welfare: if interest group influence exists then it always has a negative effect on welfare. Competition between interest groups may help eliminate this negative effect but only for certain policy areas. In particular, interest group competition blocks the influence of interest groups advocating very targeted policies but has no effect if the policy area is sufficiently broad.

The model could be extended in several directions to increase the breadth of its empirical applications. The model considers only one tool at lobbyists' disposal: lobbying for support once the bill is ready for a floor vote. Of course, lobbyists attempt to influence the policy-making process at every stage of the process, including agenda-setting, drafting bills, and affecting implementation by agencies. The negative welfare implications of interest group influence in this model are driven in part by the fact that the legislature faces a take-it-or-leave-it choice regarding whether to pass the bill. Furthermore, I have considered interest groups that are purely vote-oriented in the sense that they are not moved by the same policy information that moves legislators. Though this is a realistic assumption for many interest groups, others are clearly more directly policy-motivated and this should increase the chances for welfare-enhancing interest group influence.

Chapter 4

Directional Cheap Talk in Political Campaigns

In order to make good choices, voters must rely at some level on the willingness of candidates to transmit accurate information about their policy positions. Unfortunately, communication from candidates to voters is limited by the problem of credibility: because elections incentivize candidates to say whatever it takes to win, voters ought to be skeptical of any information provided by candidates. This insight corresponds to the popular intuition that talk is cheap in politics and has lead to pessimism about the chances for meaningful voter learning during electoral campaigns.

How does the credibility problem shape the qualitative nature of policy discussions during the course of a campaign? Can candidates credibly communicate meaningful information about their policy preferences to voters? Here, I develop a model of electoral campaigns in multidimensional policy environments in which candidates and voters are each uncertain about the preferences of the other players. In the model, two candidates send cheap talk messages to voters prior to a majority election and, if elected, implement their most preferred policy. In particular, the winning candidate is unconstrained by her campaign promises. I find that candidates can credibly reveal information about the direction (but not the distance) of their ideal points from the center of the space. Credibility problem constrains the content of campaigns by causing voters to choose between candidates on the basis of directional information instead of immediate information about the level of preference divergence between themselves and the candidates.

The analysis produces two main contributions. First, I show that campaigns are informative even when candidates only engage in cheap talk. Since the seminal study of Crawford and Sobel (1982) social scientists have understood that information transmission is only possible when the speaker and the audience have shared interests. Candidates lack shared interests with voters since they want to win the election even if informed voters would surely choose their opponents, so early applications of cheap talk models to elections in simple policy environments produced negative results on the impossibility of information transmission in campaigns (Harrington, 1992). The observation that candidates lack credibility when talk is cheap motivates theoretical studies in which institutions such as political parties (Snyder and Ting, 2002) or repeated elections (Banks, 1990; Harrington, 1993) are posed as mechanisms to make candidate communications believable by allowing costly signals that induce greater shared interests between candidates and voters.[27] This intuition also influences empirical work on elections where the observation that politicians' policies coincide with their campaign messages is contrasted with the counterargument that campaigns are "just cheap talk."[28]

[27]Banks (1990) uses a costly signaling model and uses retrospective voting in repeated elections as the rationale for this choice. Callander and Wilkie (2007) allow candidates to have heterogeneous lying abilities and find that the presence of liars (i.e. candidates for whom talk is cheap) affects the communication strategies of all the candidates since honest candidates know that liars are tempted to imitate them. In that game, the electoral process favors liars, though honest candidates win a significant proportion of elections.

[28]To cite two recent examples, the frontmatter of Sulkin's (2011) book summarizes the main results as showing that "contrary to the conventional wisdom that candidates' appeals are just 'cheap talk,' campaigns actually have a lasting legacy in the content of representatives' and senators' behavior in office." And Claibourn (2012) state that "empirical work on party platform fulfillment and presidential promise keeping shows that these agenda priority signals are not cheap talk."

Though credibility imposes qualitative limitations on the ways that political issues are discussed in campaigns, credible communication is often not very limiting in terms of the amount of information that candidates convey and the effect of campaigns on the quality of voters' decision-making. For example, if voters believe that candidates' preferences on each policy issue are equally variable and are uncorrelated with their positions on other policy issues, credible directional communication in campaigns comes close, in a precise sense, to full revelation of candidate preferences. More generally, the candidates can convey binary information (e.g. pro or con, yes or no, left or right) on each issue dimension in every situation described by the model.

A second contribution of this study is to show how strategic communication structures the selection and presentation of policy issues in campaigns. The model predicts that candidates will reveal information about the *direction* in which they would change policy but not the magnitude of those changes. For instance, a statement such as "I will reduce entitlements and increase military spending" can be credible but not statements such as "I will cut entitlements by 30% and increase military expenditures by 10%." These directional campaign messages lend a predictable structure to electoral cleavages – the voters supporting each candidate are separated into two roughly equal sets by a plane passing through the center of the policy space. Thus, both candidates appeal equally to a voter whose ideal point is exactly at the center of the policy space and the candidates' supporters form two cohesive voting groups divided by a single plane.

The paper proceeds as follows. Section 4.1 presents an overview of the argument using three illustrative examples. Section 4.2 explains the general model and Section 4.3 presents the results. Section 4.4 discusses some normative and positive implications of the model and concludes.

4.1 Overview of the argument

In this section, I summarize the intuition behind the main results before presenting the model in full detail. The players have standard spatial preferences: a policy is represented by a point in Euclidean space and each players' policy preferences are maximized at her ideal point. Each candidate's ideal point is private information – unknown to the voters and to the other candidate – and both candidates are also uncertain about the ideal points of the voters. Unlike in the standard Downsian model, candidates cannot commit to policy platforms – if elected, a candidate will simply implement her ideal point. However, prior to the election, both candidates have the opportunity to send cheap talk messages to the voters about their ideal points. Assume these message take the form "My ideal point is somewhere in the set S" where S is a subset of policies.

Example 4. The main results can be seen as a generalization of the following one-dimensional result. When the policy space is one-dimensional, suppose that the players believe that all other players' (both candidates' and voters') ideal points are drawn from a probability distribution that is symmetric around zero (for example, ideal points may be drawn from a standard normal distribution or a uniform distribution on [-1, 1]).[29] In this setting, there is an equilibrium in which each candidate accurate reveals whether her ideal point lies to the left or the right of zero. Thus, the candidates communicate directional information about their preferences without revealing the intensity of their preferences in that direction.

To see why this is an equilibrium, consider what happens when one candidate announces "My ideal point is left of center" and the other announces "My ideal point is right of center" as displayed in Figure 4.1. Since the distribution of candidates' ideal points is symmetric,

[29] A distribution with a density function f is symmetric in one-dimension if $f(x) = f(-x)$ for all $x \in \mathbb{R}$.

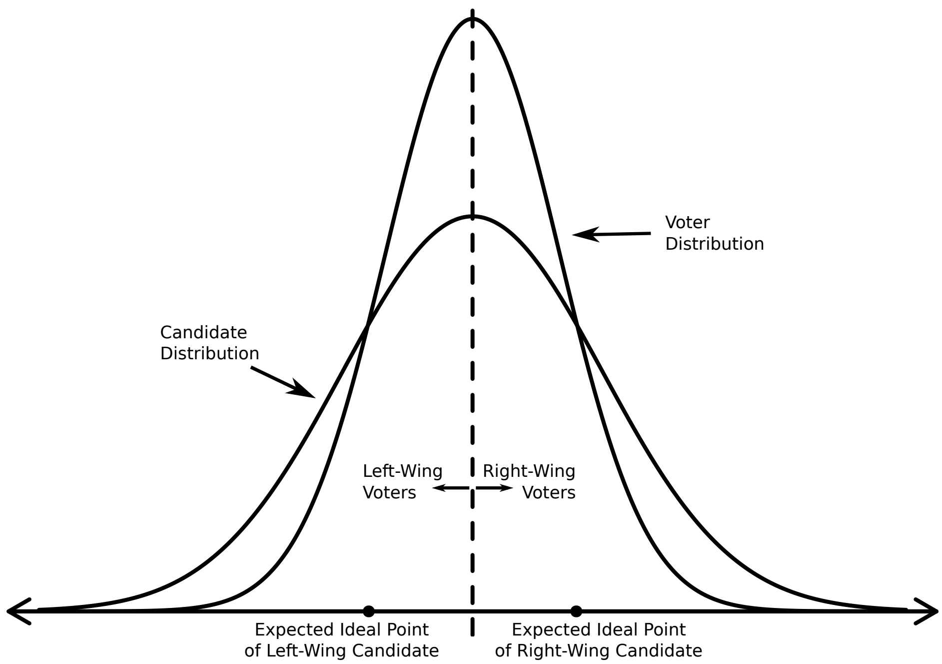

Figure 4.1: A one-dimensional directional equilibrium from Example 4.

the voters expect still expect the candidates to be equally extreme. As a result, left-of-center voters prefer the left-wing candidate and right-of-center voters prefer the right-wing candidate. Since the distribution of voters' ideal points is also symmetric and the candidates remain uncertain about voter preferences, this means that each candidate expects to win the election with a probability of one-half. Thus, neither candidate has an incentive to deviate to a dishonest campaign message. Furthermore, even though the candidates have the same expected probability of winning before they know the voters' ideal points, the voters gain from campaign speech in expectation since ex post winner of the election is the candidate that would move policy in the direction preferred by a majority of voters. $\square$

The remainder of this paper shows that the directional logic of strategic communication in the one-dimensional model generalizes to multidimensional policy spaces. The generalization of the result to multiple dimensions involves more subtlety for two reasons. First, though in a one-dimensional space there are only two directions away from the center (left or right), in two or more dimensions there are an infinite number of directions away from the center. Second, the results rely on symmetry in the distribution of ideal points and, while symmetry is an unambiguous concept for one-dimensional distributions, there are many different generalizations of one-dimensional symmetry to multiple dimensions and each have different implications for the directional information that candidates transmit. The following examples demonstrate both points.

Example 5. Consider a city council election in which the two issues at stake are levels of spending on parks and public transportation. Assume that all players' ideal points are distributed according to a multivariate normal distribution with a mean of $(0, 0)'$ and a covariance matrix equal to the 2×2 identity matrix.[30] That is, each player's ideal point in two dimensions consists of two numbers, each drawn independently from a standard normal distribution. The direction of a candidate's ideal point form the center in two dimensions is a ray starting from $(0, 0)$ and passing through the candidate's ideal point. This is represented by an angle, ranging from 0 to 2π when measured in radians and ranging from 0 to 360 when measured in degrees. There is an equilibrium to this game in which each candidate perfectly reveals this direction.

Two see why this is an equilibrium, suppose that the ideal points of Candidate A and Candidate B are as shown in Figure 4.2. The circles represent contours of the distribution of ideal points, or sets of points with the same density. Suppose that both candidates reveal directions from the center represented by rays passing through their ideal points. Candidate A's message can be interpreted roughly as "I will cut spending on both parks and transportation, but I will cut five times as much on parks as on transportation." Similarly, Candidate B's message is interpreted as "I will increase spending on parks and transportation, but I will only increase spending on transportation at two thirds the rate that I will increase spending on parks." Given this information, the voters expect the candidates to be equally extreme in their chosen directions and therefore choose the candidate that would move policy in the direction most similar to their own.[31] Thus, the voters for each candidate are divided by a line passing through the center which, because of the symmetry of the voter distribution, divides the voter distribution exactly in half. Hence, each candidate expects to win the election with a probability of one half. Furthermore, this holds for any two rays that the

[30] In this example and in Example 6, I assume that voters' and candidates' ideal points come from the same distribution. I relax this assumption in the full model and the most stringent requirements regarding distributional symmetry apply to the ideal points of the candidates rather than the voters.

[31] Specifically, they will choose the candidate for whom the angle enclosed by their ideal point and the ray revealed by the candidate has the largest cosine, as is shown in the proof of Theorem 8.

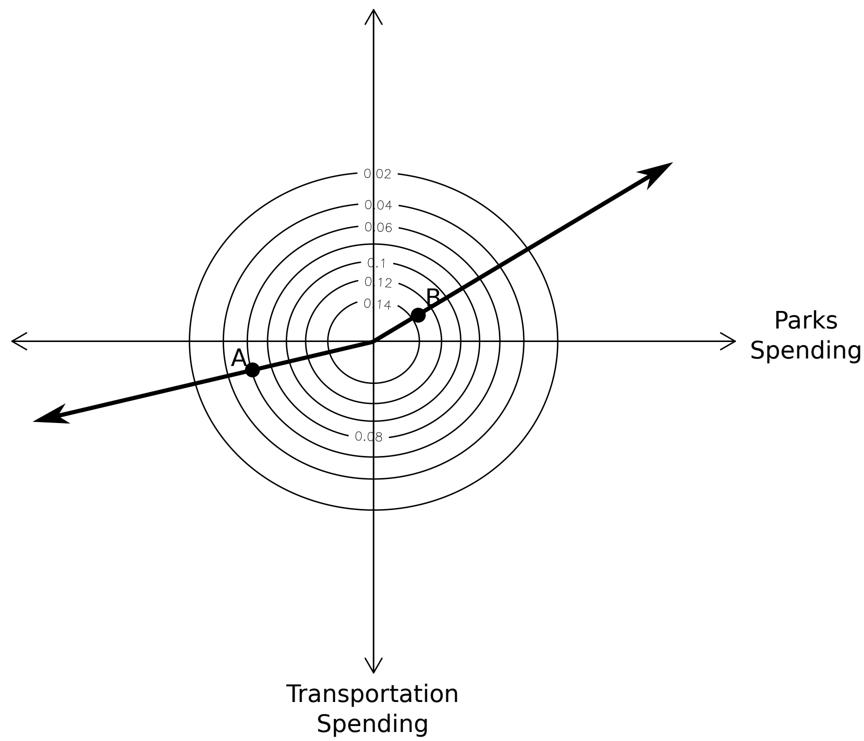

Figure 4.2: The directional equilibrium in Example 5.

candidates may reveal, so it is an equilibrium for all candidates to fully reveal their directions from the center. $\square$

The equilibrium in Example 5 involves a large degree of information transmission. In two or more dimensions, revealing candidates' directions from the center is close to full revelation in the sense that voters' beliefs about each candidate are concentrated on a set with zero prior density that contains the true ideal point. This high level of information transmission is made possible by the fact that the distribution of ideal points is neutral with respect to direction – the probability density at a particular point depends only on the distance of that point from the center and not on its direction. This property is known as spherical symmetry and is the strongest generalization of the notion of symmetry to multivariate distributions. As Example 6 demonstrates, when the distribution of ideal points violates this property but satisfies weaker notions of symmetry, the equilibria are qualitatively similar but less informative.

Example 6. Consider an election involving the same two issues as in Example 5 but in which players' preferences on the issues are correlated: people who support increasing spending on parks tend to also support increasing spending on transportation and people who prefer cuts in one program tend to prefer cuts in both. One interpretation of this correlation is that there is some principal dimension (e.g. preferences for more or less government spending) that explains preferences on both issues. To represent this idea, let ideal points be drawn from a multivariate normal distribution with a mean of $(0, 0)$ and a covariance matrix of

$$\begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}.$$

While the contours of the distribution in Example 5 were circles, the contours of the new distribution are ellipses which are longer in the direction of the principal preference dimension. In this environment, it can no longer be an equilibrium for candidates to fully reveal their directions from the center. The reason is that the directions of their ideal points now contain some information about the extremity of the candidates and candidates who reveal themselves to be more extreme are at a disadvantage.

However, there is an equilibrium in which the candidates imperfectly reveal directional information about their preferences. This equilibrium is illustrated in Figure 4.3. In this equilibrium, there are four regions (labelled using Roman numerals) and the candidates accurately reveal the regions containing their ideal points. These regions are defined by rotating the axes until one axis represents the principal dimension explaining the correlation in ideal points. In this new rotated space, the dimensions of the candidates' ideal points are uncorrelated, though they have different variances. The regions illustrated in Figure 4.3 represent the four quadrants of the space with respect to these rotated axes. Like the rays in Example 6, these sets give the candidates an equal expected distance to the center from the perspective of voters. As a result, the voters for each candidate are once again divided in half according to their preferences over the direction of the new policy from the center. Thus, for any pair of quadrants that the candidates reveal, both candidates expect to win the election with a probability of one half. $\square$

4.2 The model

I consider a model of an election in which candidates are unable to commit to policy platforms. Candidates send campaign messages to convey information to voters about their

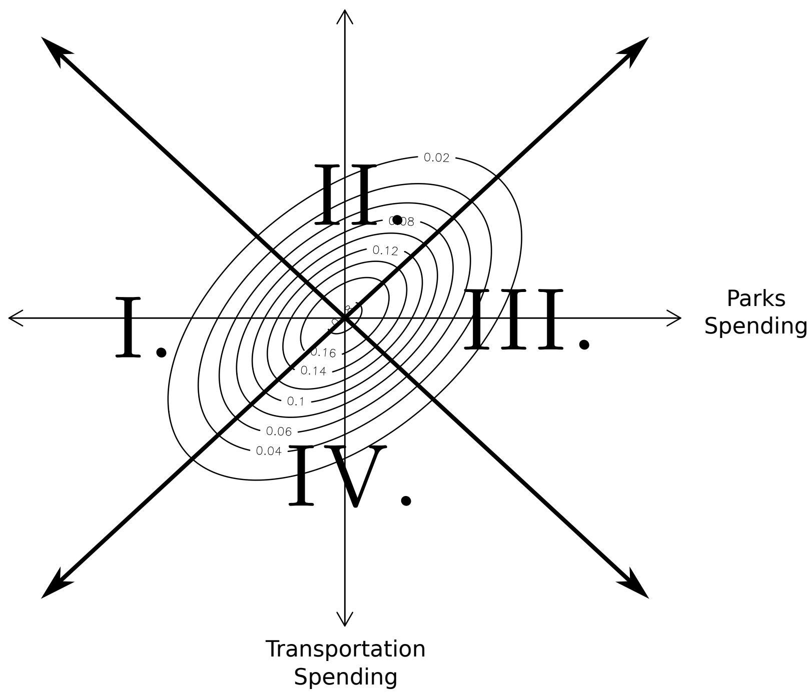

Figure 4.3: The partially revealing directional equilibrium in Example 6.

policy intentions. The policy space is $\mathbb{R}^d$, with $d \geq 1$. The set of players consists of two candidates, denoted A and B, and a finite set N with an odd number n of voters. The ideal policy of each player $i \in A \cup B \cup N$ is denoted $\mathbf{z}_i \in \mathbb{R}^d$. Each player's preferences are represented by a quadratic Euclidean utility function $u(\mathbf{x}, \mathbf{z}_i) = -||\mathbf{x} - \mathbf{z}_i||^2$ where $\mathbf{x}$ is the policy that is implemented and $||\cdot||$ is the Euclidean norm.

The election proceeds as follows. First, both candidates send costless and public campaign messages $\mathbf{m}_A, \mathbf{m}_B \in \mathbb{R}^d$ to the voters about their policy preferences.[32] Second, the voters observe the messages, update their beliefs about the ideal points of the candidates, and each voter votes in favor of one of the candidates. Finally, the candidate with the majority of votes wins the election and implements her ideal policy.

Each player's ideal point is private information. The players believe that the candidates' ideal points are independently and identically distributed from a continuous probability distribution with density $f(\mathbf{z}) = c \cdot f_0(\mathbf{z}'\mathbf{\Sigma}^{-1}\mathbf{z})$ where f_0 is continuous and non-negative and the scale parameter $\mathbf{\Sigma}$ is a real, symmetric, positive definite $d \times d$ matrix and c is a normalizing constant. This implies that f is *elliptically symmetric* around the point $(0, \ldots, 0)$ (Fang, Kotz and Ng, 1990). The players believe that the voters' ideal points are independently and identically distributed according to a probability measure G. I assume that G is *angularly symmetric* about $(0, \ldots, 0)$, which means that $\mathbf{z}_i/||\mathbf{z}_i||$ and $-\mathbf{z}_i/||\mathbf{z}_i||$ have the same distribution or, equivalently, that any hyperplane passing through $(0, \ldots, 0)$ will divide $\mathbb{R}^d$ into two half-spaces with equal probability under G.[33]

[32] The main existence results do not depend on whether the candidates' messages are simultaneous or sequential so I leave this aspect of the model open to interpretation.

[33] This definition is due to Liu (1988). The assumptions on the distribution of voters' ideal points are weaker than the assumptions on candidates' ideal points in that all elliptically symmetric distributions are also angularly symmetric, but both forms of symmetry are satisfied by any multivariate normal distribution and a variety of others. For an overview of these and other notions of multivariate symmetry in probability distributions, see Serfling (2004).

I consider symmetric perfect Bayesian equilibria in weakly undominated strategies. A strategy for the candidates is a function $\sigma : \mathbb{R}^d \to \mathbb{R}^d$ mapping each possible ideal point into a campaign message. Thus, if $\sigma(\mathbf{z}) = \mathbf{m}$ means that a candidate with an ideal point of $\mathbf{z}$ sends the message $\mathbf{m}$. The focus on symmetric equilibria means that candidates A and B would send identical messages if they had the same ideal point. A strategy for each voter is a function $v : (\mathbb{R}^d)^3 \to [0, 1]$ mapping the voter's ideal point and both candidates' messages into a probability of voting for candidate A. For example, if $v(\mathbf{z}_i, \mathbf{m}_A, \mathbf{m}_B) = 1$, then a voter with ideal point $\mathbf{z}_i$ votes for candidate A with probability 1 when the candidates send the message $\mathbf{m}_A$ and $\mathbf{m}_B$. The focus on weakly undominated strategies means that voters always choose the candidate that, if elected, would give them the highest expected utility. This rules outthe possibility that voters choose their least-favored candidate when they are not pivotal. Finally, let $\mu(\mathbf{m})$ denote the beliefs of the voters about the ideal point of a candidate who sends the message $\mathbf{m}$. Voters' beliefs about a candidate depend only on that candidate's message.

An equilibrium to the game is a strategy profile in which:

1. $\sigma(\mathbf{z})$ maximizes each candidate's probability of winning the election given that the voters' play v and the other candidate plays σ,

2. $v(\mathbf{z}_i, \mathbf{m}_A, \mathbf{m}_B) = 1$ if A gives a voter with ideal point $\mathbf{z}_i$ a higher expected utility under beliefs μ, $v(\mathbf{z}_i, \mathbf{m}_A, \mathbf{m}_B) = 0$ if B gives that voter a higher expected utility under beliefs μ, and $v(\mathbf{z}_i, \mathbf{m}_A, \mathbf{m}_B) = 1/2$ if the expected utilities for a voter with ideal point $\mathbf{z}_i$ are equal for both candidates under beliefs μ, and

3. μ is consistent with Bayes' rule: letting $S(\mathbf{m}) = \{\mathbf{z} \in \mathbb{R}^d : \sigma(\mathbf{z}) = \mathbf{m}\}$, we have $\mu(\mathbf{m}) = f(\mathbf{x})/[\int_{S(\mathbf{m})} f(\mathbf{x})]$ whenever $S(\mathbf{m})$ is non-empty.

The meanings of the candidates' messages are determined only by the sets of types who would send those messages in equilibrium. Therefore, the analysis is concerned with the the sets $S(\mathbf{m})$ induced by the messages rather than the messages themselves. If $S(\mathbf{m}) = S$, the message $\mathbf{m}$ is meant to convey the information "My ideal point is somewhere in the set S." In reality, the candidate would likely use more natural language such as "I am economically liberal and socially conservative" or "I prefer to increase spending on food stamps twice as much as on science funding." The effect of these messages is to inform the voter by reducing the set of preferences that the candidate might hold.

4.3 Results

The results mirror the examples in Section 4.1. Section 4.3.1 shows that if the distribution of candidates' ideal points is spherically symmetric (as in Example 5) then there is an equilibrium in which candidates fully reveal the direction of their ideal points from the center. Section 4.3.2 shows that there always exists an equilibrium in which candidates reveal the orthants containing an orthogonal rotation of their ideal points, as in Example 6.

4.3.1 Directional communication

The *direction* of an ideal point $\mathbf{z} = (z^1, z^2, \ldots, z^d) \in \mathbb{R}^d$ is a vector of numbers $\phi(\mathbf{z}) = (\phi^1(\mathbf{z}), \ldots, \phi^{d-1}(\mathbf{z}))$ such that

$$z^i = ||\mathbf{z}|| \prod_{j<i} \sin \phi^j(\mathbf{z}) \cos \phi^i(\mathbf{z})$$

for all $i \in \{1, \ldots, d-1\}$ and

$$z^d = ||\mathbf{z}|| \prod_{j<d} \sin \phi^j(\mathbf{z}).$$

These are the angular components of the hyperspherical coordinates of $\mathbf{z}$. The function $\phi(\mathbf{z})$ determines which point on the unit sphere lies on the same ray from the origin as the point $\mathbf{z}$.

A *directional communication equilibrium* is one in which the candidates fully reveal the direction of their ideal point and nothing more. Since the specific messages chosen by the candidate are not of interest, it is convenient to consider candidate strategies that place positive probability only on messages on the unit hypersphere. Thus, a directional communication equilibrium is one in which $S(\mathbf{m}) = \{\mathbf{z} \in \mathbb{R}^d : \phi(\mathbf{z}) = \phi(\mathbf{m})\}$ if $||m|| = 1$ and $S(\mathbf{m}) = \emptyset$ otherwise. In a directional communication equilibrium, voters' beliefs are concentrated on a ray starting from the origin and pointing in the direction of $\mathbf{m}$.

Theorem 8. *There exists a directional communication equilibrium if $\mathbf{\Sigma} = k\mathbf{I}_d$ where $k > 0$ is a constant scalar and $\mathbf{I}_d$ is the identity matrix.*

In a one-dimensional policy space, the equilibrium described in Theorem 8 corresponds to the equilibrium from Example 4 in which the candidates reveal whether their ideal point is to the left or to the right of the center. In multidimensional policy spaces, a directional communication equilibrium implies nearly complete information transmission in the sense that voters' beliefs are concentrated on very small sets. The interpretation of directional communication as being close to full revelation is supported by Corollary 3.

Corollary 3. *If $d > 1$ and $\mathbf{\Sigma} = k\mathbf{I}_d$ then there exists and equilibrium in which voters' beliefs about candidates are concentrated on sets with measure zero containing the the candidates' true ideal points.*

Directional equilibria clearly fall short of full revelation in other ways. Voters are unable to distinguish between moderate and extreme candidates. Therefore, there is a chance that voters will make mistakes. However, the winner of the election will be the candidate who would move policy in the direction preferred by the majority of voters.

4.3.2 Principal orthant communication

Let $\mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^{-1} = \mathbf{\Sigma}$ be an eigendecomposition of the matrix $\mathbf{\Sigma}$, where $\mathbf{\Lambda}$ is a diagonal matrix with the diagonal elements equal to the eigenvalues of $\mathbf{\Sigma}$, and the columns of $\mathbf{Q}$ are the corresponding eigenvectors. This decomposition is useful because the any two dimensions in the distribution of $\mathbf{Qz}$ are orthogonal. Furthermore, $\mathbf{Q}$ can be chosen to be a rotation matrix so that $\mathbf{Qz}$ can be interpreted as the coordinates of $\mathbf{z}$ when the axes of the space are rotated according to the eigenvalues of $\mathbf{\Sigma}$. These new axes represent, in statistical parlance, the principal components of the distribution f and, in geometric terms, the principal axes of the ellipsoids formed by the contours of f.

I define the *principal orthant* of $\mathbf{z}$ as the set of points $\tilde{\mathbf{z}} \in \mathbb{R}^d$ such that $\mathbf{Qz}$ and $\mathbf{Q}\tilde{\mathbf{z}}$ have the same sign on every dimension. There always exists an equilibrium in which the candidates reveal the principal orthant of their ideal points. Since the particular messages used by the candidates are unimportant, I restrict attention to messages consisting of a sequence numbers equal to -1 or 1. A principal orthant equilibrium is one in which $S(\mathbf{m}) = \{\mathbf{z} \in \mathbb{R}^d : \mathbf{m} \cdot \mathbf{z} >> 0\}$ if $\mathbf{m} \in \{-1, 1\}^d$ and $S(\mathbf{m}) = \emptyset$ otherwise.

Theorem 9. *There exists a principal orthant equilibrium.*

Theorem 9 implies that there is always an equilibrium that partitions the policy space into 2^d convex sets. Since all ideal points along the same ray from the origin belong to the same principal orthant, principal orthant equilibria partially reveal the direction of each candidate from the center. Thus, though less informative, principal orthant equilibria are qualitatively similar to directional communication.

Assuming the principal components of the ideal point distributions are well-understood by the voters, principal orthant equilibria are easily understood by the voters since they correspond to statements such as "I am socially conservative and economically liberal." In fact, since the principal components explain the most variance in political preferences, there is reason to believe that these dimensions are the most likely to correspond to how voters think about politics. In fact, this is the motivation behind principal components analysis, factor analysis, and related methods which have been used to measure voters' political ideology in empirical work for decades (Schofield, Gallego and Jeon, 2011; Enelow and Hinich, 1984; Aldrich and McKelvey, 1977).

4.4 Discussion

In this section I discuss the normative and positive implications of the model and situate the results in the existing literature.

4.4.1 Welfare implications

Though the candidates believe they have the same probability of victory, the actual winner of the election gives a higher expected utility to a majority of voters. Cheap talk can improve voters' welfare in this model because of the assumption that candidates are uncertain about the preferences of the voters. Thus, candidates can provide information that is useful to the voters and this information is credible because it does not change the candidates' beliefs about the probability that they win the election. Though candidates may be willing to say anything it takes to win, productive communication is still possible when candidates cannot be sure precisely what they should say to be elected.

4.4.2 Implications for existing theoretical work

Compared to previous work, this study is considerably more optimistic about the prospects for successful cheap talk in electoral campaigns. Early results suggested that information transmission does not occur when candidate messages are cheap talk. For instance, Harrington (1992) presented a model of cheap talk campaigns in a one-dimensional environment and showed that information transmission does not occur. Banks's (1990) study of campaign communication in a one-dimensional policy space focused on the case of costly signals, but showed that there is no information transmission when signaling costs are close to zero. In informal work on elections, "cheap talk" is basically a synonym for meaningless communication.

Since cheap talk communication was unsuccessful in these settings, political scientists have focused on electoral signaling models in which obfuscation is costly. For example, an informational rationale for political parties is that party screening mechanisms induce costly signals that allow candidates to communicate their preferences to voters (Snyder and Ting, 2002; Ashworth and de Mesquita, 2008). I do not dispute the empirical realism of these models. Certainly not all campaign communication is cheap talk. However, a the theoretical contribution of this study is to show that institutions that provide costly signals are not *necessary* for sustaining information transmission in elections. In the richer policy environment considered in this study, campaign talk is meaningful without being costly.

Methodologically, this study is related to recent work on multidimensional cheap talk, which has generally shown that the opportunities for information transmission can be greatly improved when information is multidimensional compared to it is unidimensional (Battaglini, 2002; Chakraborty and Harbaugh, 2007, 2010). In my model, the equilibrium of the multidimensional game is similar in character to the one-dimensional equilibrium, though the information revealed may appear more meaningful in many dimensions than in one.[34]

The contrast in my model between spherically symmetric (Theorem 8) and merely elliptically symmetric (Theorem 9) candidate distributions corresponds to the observation about multidimensional cheap talk made by Levy and Razin (2007):

> Generally, communication on one dimension may reveal information on others. Therefore, even when the two players have no conflict on a particular dimension, such informational spillovers may restrict their ability to communicate (885).

This argument describes why directional information is fully transmitted in the spherical case of Theorem 8 and not in other cases. On average, candidates have no conflict with voters when it comes to the direction of their ideal point since a voter at the center is indifferent about which direction policy is set from the center. However, if the distribution of candidates is not spherically symmetric, information about the direction of a candidate's ideal point spills over into information about the extremity of that candidate.

[34] For instance, perfectly revealing a candidate's direction divides the space in half in the one-dimensional model and into sets with measure zero in the multidimensional model. Similarly, revealing "pro versus con" information in one dimension corresponds to a modest level of information transmission while revealing the same level of information on 20 different issues conveys considerable information about a candidate.

4.4.3 Empirical implications

The model has several empirical implications. First, the model makes qualitative predictions about the ways that candidates are likely to discuss their positions on political issues. Specifically, the information that candidates convey should inform voters about the direction in which they would like to move policy while leaving voters uncertain about their distance from the center of the policy space.

The directional nature of information in my theory corresponds well to work on voting behavior which claims that voters appear to engage in directional rather than proximity (spatial) voting (Rabinowitz and Macdonald, 1989). According to this argument, citizens respond to political objects in terms of direction rather than proximity. Though proponents of directional voting theories interpreted these ideas as implying that "the traditional spatial theory of elections is seriously flawed" (p. 114), my theory shows why voting would appear directional even if voters have standard spatial preferences. Here, voting appears directional not because of psychological characteristics of voters but because only directional information can be credibly communicated by candidates.

Second, the model provides some microfoundations for the way empirical researchers operationalize ideology in the spatial model of voting. For many years, political scientists have employed psychometric methods related to principal components analysis and factor analysis to measure political preferences. These methods involve performing a singular value decomposition on the correlation matrix of the data (e.g. answers to issue questions on a survey) and describing preferences along the dimensions corresponding to some number of eigenvectors, called factors in factor analysis. The justification for this approach is that these eigenvectors correspond to orthogonal dimensions that are ordered according to the amount of variance each explains in the data. In my theory, the candidates transform the space in exactly the same way in order to reveal information to voters about their preferences. The reason is that candidates can more easily communicate directional information about their preferences along orthogonal dimensions, because voters expect candidates in each orthant to be equally extreme.

The various normative and positive implications arising from the model indicate the advantages of analyzing the incentives for candidates to communicate policy information to voters in multidimensional policy spaces. The formal theory literature on incomplete information in elections has neglected this task even though most elections involve the consideration of multiple policy issues. My model is purposely simple, involving very little institutional detail other than majority rule. Inclusion of institutions such as repeated elections to induce reputation concerns would likely enhance the opportunities for productive communication. Similarly, the inclusion of (costly) informative party labels would improve prospects for information transmission in multiple dimensions as it does in one-dimensional models.

Chapter 5

Conclusions

I have analyzed three models of communication in collective choice environments. In all of the models, the communicators possess private information that determines the effects of different choices available to the members of a collective choice body. Furthermore, the effects of the choices in each model are multidimensional, so that collective preferences over the effects are not necessarily transitive. In each situation, the opportunities for communication are expanded relative to settings with a single recipient of information. In my model of political campaigns, these new opportunities for communication work to the benefit of the voters. In the other models, additional opportunities for communication come in the form of manipulative persuasion that harms voters' welfare. The results show that strategic information transmission is substantially different when the audience is a collective choice body rather than a single decision-maker.

The models leave many questions open for future research. In all of the models in this dissertation, the voters face a choice between only two alternatives. I neglect the issue of agenda-setting and assume that the players have no proposal power. It is possible that some of the negative welfare implications of expert manipulation would be eliminated or reversed if voters could modify proposals prior to voting. In a dynamic model with multidimensional policies involving communication and proposal power, Meirowitz (2007*a*) proves that outcomes converge to the core of the voting rule if one exists. However, as Chapter 1 suggests, communication strategies are very different in the absence of a core. The effects of communication on voter welfare in the absence of a core when proposals can be modified is still unknown.

The voters and legislators in my models possess no private information of their own. In that way, my models differ from models of deliberation and debate in which all voters have opportunities to transmit information (Meirowitz, 2007*b*; Austen-Smith, 1990; Austen-Smith and Feddersen, 2006*b*). Future research may reveal how similar deliberative processes operate in multidimensional environments like those considered in this dissertation.

A broader normative problem related to my theoretical framework is one of institutional design: How should institutions be designed to efficiently aggregate preferences and take advantage of agents' varying expertise? As my research suggests, such institutions must account for the special difficulties that arise when communication and voting are combined in complex policy-making environments.

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Appendix A

Appendix to Chapter 2

A.1 Proofs of results

For the first few results in this section I allow for mixed voting strategies. Let $V_i : \Omega \to [0,1]$ be a mixed voting strategy that maps messages into probabilities of voting "Yes." Let V be a profile of such strategies and $\mathcal{D} - \{C_1, \ldots, C_K\}$. The probability that the proposal passes following a message $s \in \Omega$ given the profile V of voting strategies is

$$\chi(s,V) = \sum_{h=1}^{K} \left((-1)^{h-1} \sum_{D^* \subset \mathcal{D} : |D^*| = h} \prod_{i \in D^*} V_i(s) \right). \tag{A.1}$$

Lemma 9. *Assume (σ, V) is a perfect Bayesian equilibrium in weakly undominated strategies. For any $s \in \Omega$, if $\chi(s,V) \in (0,1)$, there is some $C \in \mathcal{D}$ such that $\mathbb{E}_{\mu_s}[\omega_i] \geq 0$ for all $i \in C$.*

Proof. For any $i \in N$, in weakly undominated strategies $V_i(s) \in (0,1)$ implies that $\mathbb{E}_{\mu_s}[\omega_i] = 0$ and $V_i(s) = 1$ implies that $\mathbb{E}_{\mu_s}[\omega_i] \geq 0$. Since $\chi(s,V) > 0$, we must have some $C \in \mathcal{D}$ such that $V_i(s) > 0$ for all $i \in C$, which proves that $\mathbb{E}_{\mu_s}[\omega_i] \geq 0$ for all $i \in C$. $\square$

Lemma 10. *Assume* (σ, V) *is perfect Bayesian equilibrium in weakly undominated strategies and let* $P^\chi = \{p \in [0,1] : \exists s \in \Omega$ *such that* $\chi(s, V) = p\}$*, we have: (a)* $\omega_E > 0 \Rightarrow \chi(s, V) = \max P^\chi$ *for all* s *such that* $\sigma(s|\omega) > 0$ *and (b)* $\omega_E < 0 \Rightarrow \chi(s, V) = \min P^\chi$ *for all* s *such that* $\sigma(s|\omega) > 0$

Proof. To prove (a), assume $\omega_E > 0$ and $\chi(s, V) \neq \max P^\chi$. Then there exists $\overline{s} \in \Omega$ such that $\chi(\overline{s}, V) > \chi(s, V)$, which implies that E receives a strictly higher payoff from playing $\overline{s}$, so (σ, V) is not an equilibrium. To prove (b), assume $\omega_E < 0$ and $\chi(s, V) \neq \min P^\chi$. Then there exists $\underline{s} \in \Omega$ such that $\chi(\underline{s}, V) < \chi(s, V)$ which implies that E strictly prefers to deviate to $\underline{s}$, so (σ, V) is not an equilibrium. $\square$

Lemma 11. *For all* (σ, v^*)*, if* $x^*(s|v^*, \mathcal{D})$ *is constant for all* $s \in \Omega$ *then* (σ, v^*) *is not* $\mathcal{D}$*-preferred.*

Proof. If $x^*(s|v^*, \mathcal{D}) = 0$ then $U_i^*(\sigma, v^*) = 0$ for all $i \in N$ and (σ, v^*) is not $\mathcal{D}$-preferred. Let $x^*(s|v^*, \mathcal{D}) = 1$. Then $U_i^*(\sigma, v^*) = \mathbb{E}_{\mu_0}[\omega_i]x^*(s|v^*, \mathcal{D})$. Since $x = 0$ is chosen in the babbling equilibrium, there exists $C \in \mathcal{D}$ such that $0 \geq \mathbb{E}_{\mu_0}[\omega_i]x^*(s|v^*, \mathcal{D})$ for all $i \in C$. Thus, (σ, v^*) is not $\mathcal{D}$-preferred. $\square$

Theorem 10. *If* (σ, V) *is a perfect Bayesian equilibrium to the game then there exists a perfect Bayesian equilibrium* (σ, v^*) *where* v^* *is in pure strategies.*

Proof. Let (σ, V) be an equilibrium. To construct a new strategy profile (μ, v^*), let

$$\overline{X}(V) = \{s \in \Omega : \chi(s, V) = \max P^\chi\} \tag{A.2}$$

$$\underline{X}(V) = \{S \in \Omega : \chi(s, V) = \min P^\chi\} \text{ and} \tag{A.3}$$

$$I(V, s) = \{i \in N : V_i(s) \in (0, 1)\} \tag{A.4}$$

where P^{χ} is defined as in Lemma 10.Construct (σ, v^*) as follows. For each $i \in N$ and $s \in \Omega$:

$$v_i^*(s) = \begin{cases} V_i(s) & \text{if } i \notin I(V,s) \\ 1 & \text{if } i \in I(V,s) \text{ and } s \in \overline{X}(V) \\ 0 & \text{if } i \in I(V,s) \text{ and } s \in \underline{X}(v) \\ 1 & \text{if } i \in I(V,s) \text{ and } s \in \Omega \backslash (\overline{X}(V) \cup \underline{X}(V)) \end{cases} \tag{A.5}$$

Thus, v^* is in pure strategies. Since $\mathbb{E}_{\mu_s}[\omega_i] = 0$ for all $i \in I(V,s)$ as demonstrated in the proof of Lemma 9, no voter strictly prefers to deviate from v^*. Furthermore, by Lemma 10, $\sigma(s|\omega) > 0$ implies that $\omega_E \geq 0$ for all $s \in \overline{X}(V)$, $\sigma(s'|\omega) > 0$ implies that $\omega_E \leq 0$ for all $s' \in \underline{X}(V)$, and $\omega_E = 0$ for all ω such that $\sigma(s''|\omega) > 0$ and $s'' \in \Omega \backslash (\overline{X}(V) \cup \underline{X}(V))$. Thus, if (σ, V) is an equilibrium, so is (σ, v^*). □

Lemma 1 *In any equilibrium, either $x^*(s|v^*, \mathcal{D})$ is constant or $x^*(s|v^*, \mathcal{D}) = 1$ only if $\omega_E \geq 0$ and $x^*(s|v^*, \mathcal{D}) = 0$ only if $\omega_E \leq 0$.*

Proof. Consider the strategy profile (σ, v^*) and assume that $x^*(s|v^*, \mathcal{D})$ is not constant. Then $\exists s, s' \in \Omega$ such that $x^*(s|v^*, \mathcal{D}) = 0$ and $x^*(s'|v^*, \mathcal{D}) = 1)$. Suppose there exists $\hat{s}$ such that $\sigma(\hat{s}|\omega) > 0$ for some ω such that $\omega_E < 0$ and $x^*(\hat{s}|v^*, \mathcal{D}) = 0$. Then $u_E(x^*(s|v^*, \mathcal{D}), \omega) > u_E(x^*(\hat{s}|v^*, \mathcal{D}), \omega)$ which shows that (σ, v^*) is not an equilbrium. Similarly, suppose there exists $\tilde{s}$ such that $\sigma(\tilde{s}|\omega') > 0$ for some ω' such that $\omega_E > 0$ and $x^*(\hat{s}|v^*, \mathcal{D}) = 1$. Then $u_E(x^*(s'|v^*, \mathcal{D}), \omega') > u_E(x^*(\tilde{s}|v^*, \mathcal{D}), \omega')$ which shows that (σ, v^*) is not an equilbrium. □

Lemma 2 *If $\mu_0(\{\omega : \omega_E > 0\}) = 1$ or $\mu_0(\{\omega : \omega_E < 0\}) = 1$ then there does not exist a $\mathcal{D}$-preferred persuasive equilibrium.*

Proof. By Lemma 11, if $x^*(s|v^*, \mathcal{D})$ is constant for (σ, v^*) then (σ, v^*) is not $\mathcal{D}$-preferred. Thus, I need to show that if $x^*(s|v^*, \mathcal{D})$ is not constant for (σ, v^*) and $\mu_0(\{\omega : \omega_E > 0\}) = 1$ or $\mu_0(\{\omega : \omega_E < 0\}) = 1$, then (σ, v^*) is not an equilibrium. Let $x^*(s|v^*, \mathcal{D}) = 0$ and $x^*(s'|v^*, \mathcal{D}) = 1$ for some $s, s' \in \Omega$ such that $\sigma(s|\omega)\mu_0(s) > 0$ and $\sigma(s'|\omega')\mu_0(s') > 0$ for some $\omega, \omega' \in \Omega$. If $\mu_0(\{\omega : \omega_E > 0\}) = 1$ then $u_E(s', \omega) > u_E(s, \omega)$ which proves that (σ, v^*) is not an equilibrium. If $\mu_0(\{\omega : \omega_E < 0\}) = 1$ then $u_E(s, \omega') > u_E(s', \omega')$ which also proves that (σ, v^*) is not an equilibrium. □

Lemma 3 *Assume $\mu_0(\{\omega : \omega_E = 0\}) = 0$. If a $\mathcal{D}$-preferred equilibrium exists then there exists $C \in \mathcal{D}$ such that*

$$\mathbb{E}_{\mu_0}[u_i(1, \omega)|u_E(1, \omega) > u_E(0, \omega)] \geq \mathbb{E}_{\mu_0}[u_i(0, \omega)|u_E(1, \omega) > u_E(0, \omega)]$$

for all $i \in C$ and this inequality is strict for some $i \in C$.

Proof. By Lemma 2, if (σ, v^*) is $\mathcal{D}$-preferred then $\mu_0(\{\omega : \omega_E > 0\}) \in (0, 1)$. By Lemma 11, if $x^*(s|v^*, \mathcal{D})$ is constant for (σ, v^*) then (σ, v^*) is not $\mathcal{D}$-preferred. Thus, by Lemma 1, if (σ, v^*) is a $\mathcal{D}$-preferred equilibrium then $x^*(s|v^*, \mathcal{D}) = 1$ only if $\omega_E \geq 0$ and $x^*(s|v^*, \mathcal{D}) = 0$ only if $\omega_E \leq 0$. Thus,

$$U_i^*(\sigma, v^*) = \mu_0(\{\omega : \omega_E > 0\})\mathbb{E}_{mu_0}[\omega_i|\omega_E > 0]. \tag{A.6}$$

Then

$$U_i^*(\sigma, v^*) = \mu_0(\{\omega : \omega_E > 0\})\mathbb{E}_{mu_0}[\omega_i|\omega_E > 0] \tag{A.7}$$

$$= \mu_0(\{\omega : \omega_E > 0\})\mathbb{E}_{\mu_0}[u_i(1, \omega)|u_E(1, \omega) > u_E(x_0, \omega)]. \tag{A.8}$$

Since $\mu_0(\{\omega : \omega_E > 0\}) > 0$, $U_i^*(\sigma, v^*) - \mathbb{E}_{\mu_0}[\omega_i]x_0 \geq 0$ if and only if $\mathbb{E}_{\mu_0}[u_i(x_0^c, \omega)|u_E(x_0^c, \omega) > u_E(x_0, \omega)] \geq 0$ and $U_i^*(\sigma, v^*) - \mathbb{E}_{\mu_0}[\omega_i]x_0 > 0$ if and only if $\mathbb{E}_{\mu_0}[u_i(x_0^c, \omega)|u_E(x_0^c, \omega) > u_E(x_0, \omega)] > 0$. □

Lemma 12. *If $\mu_0 \in \Delta(\Omega)$, $\{\mu_1, \ldots, \mu_K\} \subset \Delta(\Omega)$ and $\sum_{j=1}^{K} \mu_j(W) = \mu_0(W)$ for all $W \in \mathcal{B}(\Omega)$, then there exists a mixed messaging strategy σ such that supp$(\sigma) = \{s_1, s_2, \ldots, s_K\}$ and $\mu_{s_j} = \mu_j$ for all $j \in \{1, \ldots, K\}$.*

Proof. The result and proof generalize Proposition 1 of Kamenica and Gentzkow (2011) but follow a similar logic. Let $\mu_0 \in \Delta(\Omega)$ and assume that $\sum_{j=1}^{K} \mu_j(W) = \mu_0(W)$ for all $W \in \mathcal{B}(\Omega)$ for $\{\mu_1, \ldots, \mu_K\} \subset \Delta(\Omega)$. Consider the Lebesgue decomposition of μ_0:

$$\mu_0 = \mu_0^{(ac)} + \mu_0^{(s)}.$$

where $\mu_0^{(ac)}$ is absolutely continuous with respect to the Lebesgue measure λ and $\mu_0^{(s)}$ is singular. By the Radon-Nikodym theorem, there exists a nonnegative measurable function f_0 on Ω such that for all $W \in \mathcal{B}(\Omega)$:

$$\mu_0(W) = \int_W f_0 d\lambda. \tag{A.9}$$

Similarly, for $j \in \{1, \ldots, K\}$, let $\mu_j = \mu_j^{(ac)} + \mu_j^{(s)}$ characterize the Lebesgue decomposition of μ_j. Since $\sum_{j=1}^{K} \mu_j(W) = \mu_0(W)$ for all $W \in \mathcal{B}(\Omega)$, each μ_j is absolutely continuous with respect to μ_0 and therefore there exists a nonnegative μ_0-measurable function such that $\mu_j(W) = \int_W f d\mu_0$ for all $W \in \mathcal{B}(\Omega)$.

For any $\omega \in \text{supp}(\mu_0)$, define the following mixed messaging strategy on $\{s_1, \ldots, s_K\}$:

$$\sigma(s_j|\omega) = \begin{cases} \frac{\mu_j(\omega)p_j}{\mu_0(\omega)} & \text{if } \mu_0(\omega) > 0 \\ \frac{f_j(\omega)p_j}{f_0(\omega)} & \text{otherwise.} \end{cases} \tag{A.10}$$

By Proposition 2 of Macci (1996), the Lebesque decomposition of the posterior measure with respect to μ_0 is characterized as follows:

$$\begin{aligned}
\int_W \frac{\sigma(s_j|\omega)f_0(\omega)}{\int_\Omega \sigma(s_j|\omega')d\mu_0(\omega')}d\mu_0(\omega) &= \int_W \frac{\frac{f_j(\omega)p_j}{f_0(\omega)}f_0(\omega)}{\int_\Omega \frac{f_j(\omega')p_j}{f_0(\omega')}d\mu_0(\omega')}d\mu_0(\omega) && \text{(A.11)} \\
&= \int_W \frac{f_j(\omega)p_j}{p_j \int_\Omega f_j(\omega')d\mu_0(\omega')}d\mu_0(\omega) && \text{(A.12)} \\
&= \int_W f_j(\omega)d\mu_0(\omega) && \text{(A.13)} \\
&= \mu_j^{(ac)}(W) && \text{(A.14)}
\end{aligned}$$

$$\begin{aligned}
\int_W \frac{\sigma(s_j|\omega)\mu_0(\omega)}{\int_\Omega \sigma(s_j|\omega')d\mu_0(\omega'}d\mu_0(\omega) &= \int_W \frac{\frac{\mu_j(\omega)p_j}{\mu_0(\omega)}\mu_0(\omega)}{\int_\Omega \frac{\mu_j(\omega')p_j}{\mu_0(\omega')}d\mu_0(\omega')}d\mu_0(\omega) && \text{(A.15)} \\
&= \int_W \frac{\mu_j(\omega)p_j}{p_j \int_\Omega \mu_j(\omega')d\mu_0(\omega')}d\mu_0(\omega) && \text{(A.16)} \\
&= \int_W \mu_j(\omega)d\mu_0(\omega) && \text{(A.17)} \\
&= \mu_j^{(s)}(W) && \text{(A.18)}
\end{aligned}$$

Thus, since $\mu_j = \mu_j^{(ac)} + \mu_j^{(s)}$, the strategy σ induces the posterior measures $\{\mu_j\}_{j=1}^K$ for each $\{s_j\}_{j=1}^K$. $\square$

Lemma 13. *Let $\mu, \mu' \in \Delta(\Omega)$ and let $A \subseteq B$ for $A, B \in \mathcal{B}(\Omega)$. If $\delta(\mu, \mu') < \frac{\mu(B)\eta}{\eta+2}$ then*

$$\left|\frac{\mu(A)}{\mu(B)} - \frac{\mu'(A)}{\mu'(B)}\right| < \eta.$$

Proof. Let $\mu'(A) = \mu(A) + \eta_A$ and $\mu'(B) = \mu(B) + \eta_B$. We have

$$\left|\frac{u(A)}{\mu(B)} - \frac{\mu'(A)}{\mu'(B)}\right| = \left|\frac{\mu(A)}{\mu(B)} - \frac{\mu(A) + \eta_A}{\mu(B) + \eta_B}\right| \tag{A.19}$$

$$= \left|\frac{\mu(A)\eta_B - \mu(B)\eta_A}{\mu(B)(\mu(B) + \eta_B)}\right|. \tag{A.20}$$

This absolute value is maximized when η_B is negative and $|\eta_A - \eta_B|$ is as large as possible. Let $\overline{\eta} = \delta(\mu, \mu')$. Thus, we have

$$\left|\frac{u(A)}{\mu(B)} - \frac{\mu'(A)}{\mu'(B)}\right| \leq \left|\frac{-\mu(A)\overline{\eta} - \mu(B)\overline{\eta}}{\mu(B)(\mu(B) - \overline{\eta})}\right| \tag{A.21}$$

$$= \frac{\overline{\eta}(\mu(A) + \mu(B))}{\mu(B)(\mu(B) - \overline{\eta})} \tag{A.22}$$

$$< \frac{2\mu(B)\overline{\eta}}{\mu(B)(\mu(B) - \overline{\eta})} \tag{A.23}$$

$$= \frac{2\overline{\eta}}{\mu(B) - \overline{\eta}}. \tag{A.24}$$

If $\overline{\eta} < \frac{\mu(B)\eta}{\eta+2}$, we have

$$\frac{2\overline{\eta}}{\mu(B) - \overline{\eta}} < \frac{2\overline{\eta}}{\mu(B) - \frac{\mu(B)\eta}{\eta+2}} \tag{A.25}$$

$$= \frac{\frac{\mu(B)\eta}{\eta+2}}{1 - \frac{\eta}{\eta+2}} \tag{A.26}$$

$$= \frac{2\eta}{\frac{\eta}{\eta+2}(\eta + 2)} \tag{A.27}$$

$$= \eta. \tag{A.28}$$

Thus,

$$\left|\frac{\mu(A)}{\mu(B)} - \frac{\mu'(A)}{\mu'(B)}\right| < \eta.$$

if $\delta(\mu, \mu') < \frac{\mu(B)\eta}{\eta+2}$. $\square$

Theorem 1 *The following statements are equivalent:*

1. *$\mathcal{D}$ is non-collegial.*
2. *$\mathcal{D}$ has a cycle in Ω.*
3. *There exists a non-empty open set $\mathcal{O}(\mathcal{D}) \in \Delta(\Omega)$ such that $\mathcal{O} \subset M(\mathcal{D})$.*

Proof. I will prove the equivalence of (1) and (2) and then of (1) and (3). The proof that (1) $\Rightarrow$ (2) closely follows the proof of Theorem 2.4 in Austen-Smith and Banks (2000). Assume $\mathcal{D}$ is non-collegial. Since Ω is a convex set, there exists as set $\{\omega^1, \dots, \omega^n\} \subset \Omega$ such that

$$\begin{aligned} \omega_1^1 > \omega_1^2 > \cdots > \omega_1^n \\ \omega_2^2 > \omega_2^3 > \cdots > \omega_2^n > \omega_2^1 \\ \omega_3^3 > \omega_3^4 > \cdots > \omega_3^n > \omega_3^1 > \omega_3^2 \\ \cdots \\ \omega_n^n > \omega_n^1 > \omega_n^2 \cdots > \omega_n^{n-2} > \omega_n^{n-1} \end{aligned}$$

Since $\mathcal{D}$ is non-collegial, for all $i \in N$ there exists $C \in \mathcal{D}$ such that $i \notin C$. For each element of $\{\omega^1, \dots, \omega^n\}$, we have $\{i : \omega_i^{j-1} > \omega_i^j\} = N \backslash \{j\}$ for all $j \in N \backslash \{1\}$ and $\{i : \omega_i^n > \omega_i^1\} = N \backslash \{1\}$. Since $\mathcal{D}$ is proper, we have $N \backslash \{j\} \in \mathcal{D}$ for all $j \in N$, which implies that $\omega^n \succeq_{\mathcal{D}} \omega^1 \succeq_{\mathcal{D}} \omega^2 \succeq_{\mathcal{D}} \cdots \succeq_{\mathcal{D}} \omega^n$. Thus, $\mathcal{D}$ has a cycle in Ω. To show that (2) $\Rightarrow$ (1), assume that $\mathcal{D}$ is collegial. Then there exists $i \in N$ such that $i \in C$ for all $C \in \mathcal{D}$ and transitivity of i's preferences implies that $\mathcal{D}$ does not cycle in Ω.

The proof that (1) implies (3) is by construction. Assume that $\mathcal{D}$ is non-collegial. Let

$$\mathcal{F}(\mathcal{D}) = \{D \subseteq \mathcal{D} : \bigcap_{C \in D} C = \emptyset \text{ and } \forall C \in \mathcal{D}, C' \subset C \Rightarrow C' \notin \mathcal{D}\} \tag{A.29}$$

be the set of all sets of minimal decisive coalitions with an empty intersection. Since $\mathcal{D}$ is non-collegial, $\mathcal{F}(\mathcal{D})$ is non-empty. For each $D = \{C_1, \ldots, C_{|D|}\} \in \mathcal{F}(\mathcal{D})$, let

$$\begin{aligned}\mathcal{L}(D, u^+, u^-) = \Big\{ & L = (\mu_1, \ldots \mu_{|D|+1} \in \Delta(\Omega)^{|D|+1} : \mu_j(\{\omega : \omega_E \geq 0\}) = 1 \forall j \in \{1, \ldots, |D|\} \\ & \text{and } \mu_{|D|+1}(\{\omega : \omega_E < 0\}) = 1 \\ & \text{and } \forall j \in \{1, \ldots, |D|\}, \mathbb{E}_{\mu_j}[\omega_i] \in \big(0, u^+\big) \, \forall i \in C_j \text{ and } \mathbb{E}_{\mu_j}[\omega_i] < -u^- \forall i \notin C_j \\ & \text{and } \mathbb{E}_{\mu_{|D|+1}}[\omega_i] < 0 \forall i \in N \Big\}\end{aligned} \tag{A.30}$$

For each $\mathcal{L}(D)$ and $L = (\mu_1, \ldots, \mu_{|D|+1}) \in \mathcal{L}(D)$, let

$$\rho^*(D, u^+, u^-) = \left\{ p \in \mathbb{R}^{|D|+1} : \min_j p_j > 0, \sum_{j=1}^{|D|+1} p_j = 1, \text{ and } \min_{j \in \{1, \ldots, |D|\}} p_j > \frac{u^+}{u^+ + u^-} \right\}. \tag{A.31}$$

The set $\rho^*(D, u^+, u^-)$ is non-empty provided that $u^+ < \frac{a}{|D|+1}$ and $u^- > u^+(|D| - 1)$.

Let

$$\mathcal{O}^*(\mathcal{D}) = \bigcup_{D \in \mathcal{F}(\mathcal{D})} \bigcup_{\substack{u^+ \in (0, \frac{a}{|D|-1}) \\ u^- \in (u^+(|D|-1), a)}} \bigcup_{L \in \mathcal{L}(D, u^+, u^-)} \bigcup_{p \in \rho^*(D, u^+, u^-)} \left\{ \mu \in \Delta(\Omega) : \mu(\omega) = \sum_{j=1}^{|D|+1} p_j \mu_j(\omega) \forall \omega \in \Omega \right\}. \tag{A.32}$$

I will show that $\mathcal{O}^*(\mathcal{D}) \subseteq M(\mathcal{D})$ and that $\mathcal{O}^*(\mathcal{D})$ is open in $(\Delta(\Omega), \delta)$.

To show that $\mathcal{O}^*(\mathcal{D}) \subseteq M(\mathcal{D})$, let $L \in \mathcal{L}D$ for some $D \in \mathcal{F}(\mathcal{D})$ and let $p \in \rho^*(L)$. Let $\mu^*(W) = \sum_{j=1}^{|D|+1} p_j \mu_j(W)$ for all $W \in \mathcal{B}(\Omega)$, where $L = (\mu_1, \ldots, \mu_{|D|+1})$. By Lemma 12, there exists a mixed strategy σ with $\text{supp}(\sigma) = \{s_j\}_{j=1}^{|D|+1}$ such that $\mu_{s_j} = \mu_j$ for each $j \in \{1, \ldots, |D|+1\}$. By definition of $\mathcal{I}^*(\mathcal{D})$, there exists $C \in \mathcal{D}$ such that $\mathbb{E}_{\mu_{s_j}}[\omega_i]$ for all $i \in C$ for s_j such that $\sigma(s_j|\omega) > 0$ for some ω such that $\omega_E \geq 0$ (i.e. s_j for all $j \in \{1, \ldots, |D|\}$. Furthermore, $\sigma(s_{|D|+1}|\omega) = 1$ for all ω such that $\omega_E < 0$ and $\mathbb{E}_{\mu_{s_{|D|+1}}}[\omega_i] < 0$ for all i. Thus, this profile is an equilibrium.

Since $D \in \mathcal{F}(\mathcal{D})$, for all $i \in N$ there exists $C \in D$ such that $i \notin C$. Thus, since $\min_{j \in \{1,\ldots,|D|\}} p_j > \frac{u^+(L)}{u^+(L)+u^-(L)}$, for all $i \in N$ we have

$$\mathbb{E}_{\mu^*}[\omega_i] < \left(1 - \frac{u^+(L)}{u^+(L)+u^-(L)}\right) u^+(L) - \frac{u^+(L)}{u^+(L)+u^-(L)} u^-(L) + p_{|D|+1}\mathbb{E}_{|D|+1}[\omega)i] \tag{A.33}$$

$$< \left(1 - \frac{u^+(L)}{u^+(L)+u^-(L)}\right) u^+(L) - \frac{u^+(L)}{u^+(L)+u^-(L)} u^-(L) \tag{A.34}$$

$$= u^+(L) - \frac{u^+(L)}{u^+(L)+u^-(L)} u^+(L) - u^-(L) \frac{u^+(L)}{u^+(L)+u^-(L)} \tag{A.35}$$

$$= u^+(L) - \frac{u^+(L)}{u^+(L)+u^-(L)} \left[u^+(L) + u^-(L)\right] = 0. \tag{A.36}$$

Thus, we have $x_0 = 0$ for all $\mu \in \mathcal{O}^*(\mathcal{D})$ and

$$U_i^*(\sigma^*, v^*) < \left(1 - \frac{u^+(L)}{u^+(L)+u^-(L)}\right) u^+(L) - \frac{u^+(L)}{u^+(L)+u^-(L)} u^-(L) = 0 \tag{A.37}$$

which shows that $\mathcal{O}^*(\mathcal{D}) \subseteq M(\mathcal{D})$.

To show that $\mathcal{O}^*(\mathcal{D})$ is open, let $\mu \in \mathcal{O}(\mathcal{D})$. I will show that there exists $\epsilon > 0$ such that if $\mu' \in \Delta(\Omega)$ and $\delta(\mu, \mu') < \epsilon$ then $\mu' \in \mathcal{O}(\mathcal{D})$. For some $D \in \mathcal{F}(\mathcal{D})$, let $u^+ < \frac{a}{|D|+1}$,

$u^- > u^+(|D|-1)$, and let $L = (\mu_1, \ldots, \mu_{|D|+1}) \in \mathcal{L}(D, u^+, u^-)$ and $p = (p_1, \ldots, p_{|D|+1}) \in \rho^*(D, u^+, u^-)$ satisfy $\mu(\omega) = \sum_{j=1}^{|D|+1} \mu_j(\omega) p_j$.

Let $\Omega^+ = \{\omega \in \Omega : \omega_E \geq 0\}$ and $\Omega^- = \{\omega \in \Omega : \omega_E < 0\}$. Consider an arbitrary $\mu' \in \Delta(\Omega)$. Consider any $\mu' \in \Delta(\Omega)$. Let $p' = (p', \ldots, p'_{|D|+1}) = (p'_1, p_2, p_3, \ldots, p_{|D|}, p'_{|D|+1})$ where $p'_1 = p_1 + \mu'(\Omega^+) - \mu(\Omega^+)$ and $p'_{|D|+1} = p_{|D|+1} + \mu'(\Omega^-) - \mu(\Omega^-)$. Since $\mu_j(\Omega^+) = 1$ for all $j \in \{1, \ldots, |D|\}$ and $\mu(\Omega^+) = \sum_{j=1}^{|D|} p_j \mu_j(\Omega^+)$, it must be the case that $\sum_{j=1}^{|D|} p_j = \mu(\Omega^+)$ and $p_{|D|+1} = \mu(\Omega^-)$. It follows that

$$\sum_{j=1}^{|D|} p'_j = \sum_{j=1}^{|D|} p_j + \mu'(\Omega^+) - \mu(\Omega^+) = \mu'(\Omega^+) \tag{A.38}$$

and therefore that $p'_{|D|+1} = \mu'(\Omega^-)$.

Let $L' = (\mu'_1, \ldots, \mu'_{|D|+1}) = (\mu'_1, \mu_2, \ldots, \mu_{|D|}, \mu'_{|D|+1})$,, where

$$\mu'_1(W) = \frac{\frac{\mu'(W \cap \Omega^+)}{\mu'(\Omega^+)} - \sum_{j=2}^{D} p_j \mu_j(W \cap \Omega^+)}{p'_1} \tag{A.39}$$

and

$$\mu'_{|D|+1}(W) = \frac{\omega'(W \cap \Omega^-)}{\omega'(\Omega^-)} \tag{A.40}$$

for all $W \in \mathcal{B}(\Omega)$.

For any open interval $(\underline{b}, \overline{b})$ with $-a < \underline{b} < \overline{b} < a$ and for any $i \in N$, continuity of expected utility implies that

$$\{\mu \in \Delta(\Omega) : \mathbb{E}_\mu[\omega_i] \in (\underline{b}, \overline{b})\} \tag{A.41}$$

is an open set. Furthermore, for any $C \subseteq N$, the set

$$\{\mu \in \Delta(\Omega) : \mathbb{E}_\mu[\omega_i] \in (\underline{b}, \overline{b}) \forall i\}, \tag{A.42}$$

being a finite intersection of open sets, is also open. Thus, since $\mu'_j(\Omega^+) = 1$ for all $j \in \{1, \ldots, |D|\}$ and $\mu'_{|D|+1}(\Omega^-) = 1$, there exists $\eta > 0$ such that $\max\{\delta(\mu_1, \mu'_1), \delta(\mu_{|D|+1}, \mu'_{|D|+1})\} < \eta$ and $L \in \mathcal{L}(D, u^+, u^-)$ implies that $L' \in \mathcal{L}(D, u^+, u^-)$.

For such an $\eta > 0$, let $\epsilon' < \min\{\frac{\mu(\Omega^+)p_1\eta}{p_1\eta+2}, \frac{\mu(\Omega^-)\eta}{\eta+2}\}$. By Lemma 13, if $\delta(\mu, \mu') < \epsilon'$ then $\delta(\mu_{|D|+1}, \mu'_{|D|+1}) < \eta$. Furthermore, for all $W \in \mathcal{B}(\Omega^+)$ (the Borel σ-algebra on Ω^+), we have

$$p_1\mu_1(W) + \sum_{j=2}^{|D|} p_j\mu_j(W) = \frac{\mu(W)}{\mu(\Omega^+)} \text{ and} \tag{A.43}$$

$$p'_1\mu'_1(W) + \sum_{j=2}^{|D|} p_j\mu_j(W) = \frac{\mu'(W)}{\mu'(\Omega^+)}. \tag{A.44}$$

By Lemma 13, if $\delta(\mu, \mu') < \epsilon'$ then $\delta(\mu_1, \mu'_1) < p_1\eta$, so

$$\left|p_1\mu_1(W) - p'_1\mu'_1(W)\right| = \left|\frac{\mu(W)}{\mu(\Omega^+)} - \frac{\mu(W)}{\mu(\Omega^+)}\right| < p_1\eta. \tag{A.45}$$

Thus, we have

$$\begin{aligned}
\left|p_1\mu_1(W) - p'_1\mu'_1(W)\right| &= \left|p_1(\mu_1(W) - \mu'_1(W)) - \mu'(W)(\mu(\Omega^+) - \mu'(\Omega^+))\right| && \text{(A.46)}\\
&\leq p_1|\mu_1(W) - \mu'_1(W)| + \mu'(W)|\mu(\Omega^+) - \mu'(\Omega^+)| && \text{(A.47)}\\
&< p_1\eta && \text{(A.48)}
\end{aligned}$$

This implies that

$$|\mu_1(W) - \mu_1'(W)| < \frac{p_1\eta - \mu_1'(W)|\mu(\Omega^+) - \mu'(\Omega^+)|}{p_1} < \frac{p_1\eta}{p_1} = \eta. \tag{A.49}$$

Thus, $\delta(\mu, \mu') < \epsilon'$ implies that $\delta(\mu_1, \mu_1') < \eta$. Finally, since $p_1' - \frac{u^+}{u^+ + u^-}$ is a continuous function and $\left(0, \frac{u^+}{u^+ + u^-}\right)$ is an open interval, there exists ϵ'' such that if $p_1 \in \left(0, \frac{u^+}{u^+ + u^-}\right)$ then $(p_1 - \epsilon'', p_1 + \epsilon'') \subset \left(0, \frac{u^+}{u^+ + u^-}\right)$. Since $p_1' = p_1 + \mu'(\Omega^+) - \mu(\Omega^+)$, we have $p' \in \rho^*(D, u^+, u^-)$ if $p \in \rho^*(D, u^+, u^-)$ and $\delta(\mu, \mu') < \epsilon''$. Combining these facts, if $\delta(\mu, \mu') < \min\{\epsilon', \epsilon''\}$ and $L \in \mathcal{L}(D, u^+, u^-)$ and $p \in \rho^*(D, u^+, u^-)$, then $L' \in \mathcal{L}(D, u^+, u^-)$ and $p' \in \rho^*(D, u^+, u^-)$. Thus, if $\delta(\mu, \mu') < \min\{\epsilon', \epsilon''\}$ and $\mu \in \mathcal{O}(\mathcal{D})$ then $\mu' \in \mathcal{O}(\mathcal{D})$.

Finally, to show that (3) implies (1), assume that $\mathcal{D}$ is collegial. Then there exists $i \in N$ such that $i \in C$ for all $C \in \mathcal{D}$. Thus, $x^*(s|v^*, \mathcal{D}) = 1$ if $\mathbb{E}_{\mu_s}[\omega_i] > 0$. This implies that $U_i^*(\sigma, v*) \geq 0$ in any equilibrium. □

Theorem 2 *If $\mathcal{D}$ is more resolute than $\mathcal{D}'$, then $M(\mathcal{D}') \subseteq M(\mathcal{D})$.*

Proof. Assume that $\mathcal{D}$ is more resolute than $\mathcal{D}'$ and $\mu \in M(\mathcal{D})$. Thus, there exists (σ, v^*) such that $(\sigma, v^*) \in S(\mu, \mathcal{D})$ and $U_i^*(\sigma, v^*) < \mathbb{E}_\mu[\omega_i]x_0$ for all $i \in N$. Thus, for each $s \in \text{supp}(\sigma)$, $x^*(s|v^*, \mathcal{D}) = 1$ implies that there exists some $C \in \mathcal{D}$ such that $\mathbb{E}_{\mu_s}[\omega_i] \geq 0$ for all $i \in C$. Since $\mathcal{D}$ is more resolute than $\mathcal{D}'$, $C \in \mathcal{D}'$, so we have $x^*(s|\hat{v}, \mathcal{D}') = 1$ in some equilibrium $(\sigma, \hat{v}) \in S(\mu, \mathcal{D}')$. This proves that $M(\mathcal{D}') \subseteq M(\mathcal{D})$. □

Theorem 3 *Assume that n is odd and let $\mathcal{D}^m = \{C \subset N : |C| > \frac{n}{2}\}$. Fix a vector $z = (z_1, \ldots, z_n) \in \mathbb{R}^n$ and for some $q \in \mathbb{R}$ let*

$$\Theta(q) = \{\omega \in \Omega : \exists\theta \in \mathbb{R} \ s.t. \ \omega_i = (z_i - q)^2 - (z_i - \theta)^2 \forall i \in N\}.$$

If $\mu_0(\Theta(q)) = 1$, then all persuasive equilibria are majority preferred.

Proof. Let $\overline{z}$ be the median of $(z_1, \ldots, z_n)$ and let $\overline{i}$ be the index of the voter with $z_i = \overline{z}$. Since the number of voters is odd, $\overline{z}$ is preferred to all points in $\Theta(q)$. Banks and Duggan (2006) show that, if voters' utility functions are quadratic and there exists a core point at a voter's ideal point, then the core voter is decisive over lotteries. That is, for all $\mu \in \Delta(\Omega)$, $\{i \in N : \mathbb{E}_\mu[\omega_i] \geq 0\} \in \mathcal{D}^m$ if and only if $\mathbb{E}_\mu[\omega_{\overline{i}}] \geq 0\}$. Thus, since proposals only pass if $\mathbb{E}_{\mu_s}[\omega_{\overline{i}}] > 0$, it must be the case that $U^*_{\overline{i}}(\sigma, v^*) \geq x_0 \mathbb{E}_{\mu_0}[\omega_{\overline{i}}]$ if (σ, v^*) is a persuasive equilibrium. Furthermore, by the result of Banks and Duggan, this implies that $\{i \in N : U^*_i(\sigma, v^*) \geq x_0 \mathbb{E}_{\mu_0}[\omega_i]\} \in \mathcal{D}^m$. $\square$

A.2 Expected Utility Calculations for Example 2

Let $\upsilon_i(x, \theta) = (z_{i1} - \theta_1)^2 + (z_{i2} - \theta_2)^2$, so that $\omega_i = z_{i1}^2 + z_{i2}^2 - \upsilon_i(x, \theta)$. Since ω is a quadratic function of θ, we have

$$\mathbb{E}_\mu[\upsilon_i(x, \theta)] = (z_{i1} - \mathbb{E}_\mu[\theta_1])^2 + (z_{i2} - \mathbb{E}_\mu[\theta_2])^2 + \mathrm{Var}_\mu[\theta_1] + \mathrm{Var}_\mu[\theta_2]. \tag{A.50}$$

Let μ_0 be the prior distribution as usual, and let $\{\mu_j\}_{j=1}^3$ be the posterior beliefs resulting from each signal. The expectation and variance of these distributions are as follows.

$$\mathbb{E}_{\mu_0}[\theta_1] = \frac{1}{2\pi} \int_0^1 \int_0^{2\pi} r \cos \phi d\phi dr = 0 \tag{A.51}$$

$$\mathbb{E}_{\mu_0}[\theta_2] = \frac{1}{2\pi} \int_0^1 \int_0^{2\pi} r \sin \phi d\phi dr = 0 \tag{A.52}$$

$$\mathrm{Var}_{\mu_0}[\theta_1] = \frac{1}{2\pi}\int_0^1\int_0^{2\pi}(r\cos\phi)^2 d\phi dr = \frac{1}{6} \tag{A.53}$$

$$\mathrm{Var}_{\mu_0}[\theta_2] = \frac{1}{2\pi}\int_0^1\int_0^{2\pi}(r\sin\phi)^2 d\phi dr = \frac{1}{6} \tag{A.54}$$

$$\mathbb{E}_{\mu_1}[\theta_1] = \frac{3}{2\pi}\int_0^1\int_0^{\frac{2}{3}\pi} r\cos\phi d\phi dr = \frac{3\sqrt{3}}{8\pi} \tag{A.55}$$

$$\mathbb{E}_{\mu_1}[\theta_2] = \frac{3}{2\pi}\int_0^1\int_0^{\frac{2}{3}\pi} r\sin\phi d\phi dr = \frac{9}{8\pi} \tag{A.56}$$

$$\mathrm{Var}_{\mu_1}[\theta_1] = \frac{3}{2\pi}\int_0^1\int_0^{\frac{2}{3}\pi}(r\cos\phi - \frac{3\sqrt{3}}{8\pi})^2 d\phi dr = \frac{1}{6} - \frac{27}{64\pi^2} - \frac{\sqrt{3}}{16\pi} \tag{A.57}$$

$$\mathrm{Var}_{\mu_1}[\theta_2] = \frac{3}{2\pi}\int_0^1\int_0^{\frac{2}{3}\pi}(r\sin\phi - \frac{9}{8\pi})^2 d\phi dr = \frac{1}{6} - \frac{81}{64\pi^2} - \frac{\sqrt{3}}{16\pi} \tag{A.58}$$

$$\mathbb{E}_{\mu_2}[\theta_1] = \frac{3}{2\pi}\int_0^1\int_{\frac{2}{3}\pi}^{\frac{4}{3}\pi} r\cos\phi d\phi dr = -\frac{3\sqrt{3}}{4\pi} \tag{A.59}$$

$$\mathbb{E}_{\mu_2}[\theta_2] = \frac{3}{2\pi}\int_0^1\int_{\frac{2}{3}\pi}^{\frac{4}{3}\pi} r\sin\phi d\phi dr = 0 \tag{A.60}$$

$$\mathrm{Var}_{\mu_2}[\theta_1] = \frac{3}{2\pi}\int_0^1\int_{\frac{2}{3}\pi}^{\frac{4}{3}\pi}(r\cos\phi + \frac{3\sqrt{3}}{4\pi})^2 d\phi dr = \frac{1}{6} - \frac{27}{16\pi^2} - \frac{\sqrt{3}}{8\pi} \tag{A.61}$$

$$\mathrm{Var}_{\mu_2}[\theta_2] = \frac{3}{2\pi}\int_0^1\int_{\frac{2}{3}\pi}^{\frac{4}{3}\pi}(r\sin\phi)^2 d\phi dr = \frac{1}{6} - \frac{\sqrt{3}}{8\pi} \tag{A.62}$$

$$\mathbb{E}_{\mu_0}[\theta_1] = \frac{3}{2\pi}\int_0^1\int_{\frac{4}{3}\pi}^{2\pi} r\cos\phi d\phi dr = \frac{3\sqrt{3}}{8\pi} \tag{A.63}$$

$$\mathbb{E}_{\mu_0}[\theta_2] = \frac{3}{2\pi}\int_0^1\int_{\frac{4}{3}\pi}^{2\pi} r\sin\phi d\phi dr = -\frac{9}{8\pi} \tag{A.64}$$

$$\mathrm{Var}_{\mu_0}[\theta_1] = \frac{3}{2\pi}\int_0^1\int_{\frac{4}{3}\pi}^{2\pi}(r\cos\phi - \frac{3\sqrt{3}}{8\pi})^2 d\phi dr = \frac{1}{6} - \frac{27}{64\pi^2} - \frac{\sqrt{3}}{16\pi} \tag{A.65}$$

$$\mathrm{Var}_{\mu_0}[\theta_2] = \frac{3}{2\pi}\int_0^1\int_{\frac{4}{3}\pi}^{2\pi}(r\sin\phi + \frac{9}{8\pi})^2 d\phi dr = \frac{1}{6} - \frac{81}{64\pi^2} - \frac{\sqrt{3}}{16\pi} \tag{A.66}$$

Plugging these values into Equation A.50 gives us the expected values of $\upsilon_i(x, \theta)$ for each voter.

$$\mathbb{E}_{\mu_1}[\upsilon_i(x,\theta)] = \frac{1}{3} + \left(z_{i2} - \frac{9}{8\pi}\right)^2 + \left(z_{i1} - \frac{3\sqrt{3}}{8\pi}\right)^2 - \frac{27}{16\pi^2} \tag{A.67}$$

$$\mathbb{E}_{\mu_2}[\upsilon_i(x,\theta)] = \frac{1}{3} + z_{i2}^2 + \left(z_{i1} + \frac{3\sqrt{3}}{4\pi}\right)^2 - \frac{27}{16\pi^2} \tag{A.68}$$

$$\mathbb{E}_{\mu_3}[\upsilon_i(x,\theta)] = \frac{1}{3} + \left(z_{i2} + \frac{9}{8\pi}\right)^2 + \left(z_{i1} - \frac{3\sqrt{3}}{8\pi}\right)^2 - \frac{27}{16\pi^2} \tag{A.69}$$

Substituting the ideal points from Table 2.2 for z_{i1} and z_{i2} and subtracting the result from $z_{i1}^2 + z_{i2}^2$ gives the payoffs in Table 2.3.

Appendix B

Appendix to Chapter 3

B.1 Proofs of results

Lemma 4 *In any equilibrium to the game, $x^*(M_1, M_2)$ is constant.*

Proof. Assume there is some strategy profile (σ_1, σ_2) such that $x^*(M_1, M_2) = 0$ and $x^*(M_1', M_2') = 1$ for some pairs of lobbying choices. I must show that this strategy profile is not an equilibrium. If here exists M_2'' such that $x^*(M_1', M_2'') = 0$, then M_2'' is a strictly better response to M_1' than M_2' for any W, which implies that (σ_1, σ_2) is not an equilibrium. If there does not exist such a M_2'', then $x^*(M_1', \tilde{M}) = 1$ for all $\tilde{M} \in 2^N$. In particular, M_1' gives G_1 a strictly higher expected payoff than M_1, which also implies that (σ_1, σ_2) is not an equilibrium. $\square$

Lemma 5 *In any equilibrium to the game with or without competition, following some $M_1, M_2 \in 2^N$, we have*

$$U_i^\pi(M_1, M_2) \leq H \sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\} - L(1 - \sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\})$$

for at least $n - q$ legislators.

Proof. The proof is by contradiction. Assume there is an equilibrium in which $U_i^{\pi}(M_1, M_2) > H \sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\} - L(1 - \sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\})$ for at least q legislators following any pair of lobbying choices. Thus, there must be at least q legislators such that $\Pi_i(M_1, \emptyset) > \sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\}$ for each M_1 in the support of σ_1. Since $\Pr[|W| \geq q | M_1, M_2] \leq \sum_{k=q}^{n} p_k$ for some (M_1, M_2), this implies that, for some $k \in \{0, \ldots, q-1\}$, we have $\Pi_i(M_1, \emptyset || W| = k) > \frac{k}{q}$. Number these q legislators as $\{1, \ldots, q\}$. Since $\Pi_1(M_1, \emptyset || W| = k) > \frac{k}{q}$, we have

$$\Pr[W \subset \{2, \ldots, q\} | M_1, \emptyset, |W| = k] < \frac{q-k}{q}.$$

Similarly, since $\Pi_2(M_1, \emptyset || W| = k) > \frac{k}{q}$, we have

$$\Pr[W \subset \{1, 3, \ldots, q | M_1, \emptyset, |W| = k\}] < \frac{q-k}{q}.$$

In fact, for all $i \in \{1, \ldots, q-1\}$, we have

$$\Pr[W \subset \{1, \ldots, q\} \backslash \{i\} | M_1, \emptyset, |W| = k] < \frac{q-k}{q}.$$

Let

$$Q(i) = \left\{W \in 2^N : |W| = k, \{q\} \cup \{1, \ldots, i\} \subseteq W \ \wedge i \notin W\right\}.$$

We have

$$\Pr[q \in W | M_1, \emptyset, |W| = k] = \sum_{W : |W| = k \wedge q \in W} \pi(W | M_1, \emptyset, |W| = k) \tag{B.1}$$

$$= \sum_{i=1}^{k-1} \pi(Q(i) | M_1, \emptyset, |W| = k) \tag{B.2}$$

$$< \sum_{i=1}^{k-1} \frac{q-k}{q} \tag{B.3}$$

$$= \frac{(k-1)(q-k)}{q} \tag{B.4}$$

$$< \frac{k}{q}. \tag{B.5}$$

This contradicts the statement that $\Pi_i(M_1, \emptyset || W| = k) > \frac{k}{q}$ for all $i \in \{1, \ldots, q\}$. Thus, such an equilibrium cannot exist. $\square$

Lemma 6 *If G_2 never lobbies and G_1 uses the strategy $\sigma_1^{\min}$, for each M_1 such that $|M_1| = q$, we have*

$$U_i^{\pi}(M_1, \emptyset) = H \sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\} - L(1 - \sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\})$$

for all q of the legislators in M_1.

Proof. Let $\sigma_2(\emptyset | W, M_1) = 1$ for all W and M_1 and let $\sigma_1 = \sigma_1^{\min}$. For each M_1 such that $|M_1| = q$ and each $W \in 2^N$, we have

$$\pi(W|M_1) = \begin{cases} \binom{n-|W|}{q-|W|}^{-1} & \text{if } W \subseteq M_1 \\ \binom{|W|}{q}^{-1} & \text{if } W \supset M_1 \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, the posterior distribution conditional on the size of the set of beneficiaries is

$$\pi(W|M_1, |W| = k) = \begin{cases} \binom{q}{k}^{-1} & \text{if } k < q \text{ and } W \subset M_1 \\ 1 & \text{if } q \geq k \text{ and } W \supset M_1 \\ 0 & \text{otherwise.} \end{cases}$$

Hence, if $|W| = k$ for $k \in \{0, \ldots, q-1\}$, the probability that $i \in W$ given $i \in M_1$ is equal to

$$\Pi_i(M_1, \emptyset \| W| = k) = \sum_{W' : i \in W' \wedge |W'| = k} \binom{q}{k}^{-1} \tag{B.6}$$

$$= \binom{q-1}{k-1} \binom{q}{k}^{-1} \qquad \frac{k}{q}. \tag{B.7}$$

Thus, the total probability that $i \in W$ given $i \in M_1$ is equal to

$$\Pi_i(M_1, \emptyset) = \sum_{k=0}^{q-1} \frac{k}{q} + \sum_{k=q}^{n} 1 \tag{B.8}$$

$$= \sum_{k=0}^{n} \min\{\frac{k}{q}, 1\}. \tag{B.9}$$

Thus,

$$U_i^{\pi}(M_1, \emptyset) = H \sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\} - L(1 - \sum_{k=0}^{n} p_k \min\left\{\frac{k}{q}, 1\right\})$$

for all $i \in M_1$ for each M_1 such that $\sum_{W \in 2^N} \sigma_1(M_1|W) > 0$. Since $|M_1| = q$ for all such M_1 given $\sigma_1^{\min}$, this completes the proof. $\square$

Theorem 5 *When there is no competition, G_1 is influential if and only if*

$$\frac{H}{L} \geq \kappa^1(p, q).$$

Proof. The legislator decision rule in Equation 3.5 implies that each legislator will vote in favor of passage if and only if $\frac{H}{L} \geq \frac{1 - \Pi_i(M_1, \emptyset)}{\Pi_i(M_1, \emptyset)}$. Lemma 6 implies that $\frac{1 - \Pi_i(M_1, \emptyset)}{\Pi_i(M_1, \emptyset)} = \kappa^1(p, q)$ for all $i \in M_1$. Thus, if G_1 plays a minimal strategy then the policy is implemented if and only if $\frac{H}{L} \geq \kappa^1(p, q)$. G_1 has no incentive to deviate from this strategy since the policy is implemented following any message, so this is an equilibrium. Furthermore, Lemma 5 implies that there is no pair of messages for which $\frac{1-\Pi_i(M_1,\emptyset)}{\Pi_i(M_1,\emptyset)} < \kappa^1(p,q)$, which implies that there is no strategy to support an influential equilibrium if $\frac{H}{L} < \kappa^1(p,q)$. Thus, G_1 is influential if and only if $\frac{H}{L} \geq \kappa^1(p,q)$. □

Lemma 7. *G_2 blocks the strategy σ_1 if there exists some M_1 and σ_2 such that $\sum_{W\in 2^N} \sigma_1(M_1|W) > 0$ and $x^*(M_1,M_2) = 0$ for all M_2 such that $\sum_{W\in 2^N} \sigma_2(M_2|M_1,W) > 0$.*

Proof. Consider a particular σ_1 and let

$$\mathcal{M}(\sigma_1) = \left\{ M_1 : \sum_{W\in 2^N} \sigma_1(M_1|W) > 0 \text{ and } \exists \sigma_2 \text{ s.t. } x^*(M_1,M_2) = 0 \forall M_2 \text{ s.t. } \sum_{W\in 2^N} \sigma_2(M_2|M_1,W) > 0 \right\}.$$

Assume that $\mathcal{M}$ is non-empty and for each $\tilde{M} \in \mathcal{M}$ let $\sigma_2^{\tilde{M}}$ be some strategy that prevents passage of the policy when $M_1 = \tilde{M}$. If $\mathcal{M}(\sigma_1) = \text{supp}(\sigma_1)$ then the strategy $\sigma_2'(M_2|W,M_1) = \sigma_2^{M_1}(M_2|W,M_1)$ prevents passage following all messages, so G_2 blocks σ_1 and we are done. Otherwise, let $\mathcal{M} \subset \text{supp}(\sigma_1)$ and define the strategy σ_2'' as follows:

$$\sigma_2''(M_2|W,M_1) = \begin{cases} \sigma_2^{M_1}(M_2|W,M_1) & \text{if } M_1 \in \mathcal{M} \\ 1 & \text{if } M_2 = \emptyset \text{ and } M_1 \notin \mathcal{M} \\ 0 & \text{otherwise.} \end{cases}$$

This strategy is a best response when

$$\pi(W|M_1,M_2) = \frac{p(W)\sigma(M_1|W)}{\sum_{W'\in 2^N} p(W')\sigma(M_1|W')}$$

for all $M_1 \in \text{supp}(\sigma_1)\backslash\mathcal{M}$ and $M_2 \neq \emptyset$, which is consistent with Bayes rule given these lobbying strategies, and by definition of $\mathcal{M}$ and σ_2'' this implies that $x^*(M_1, M_2) = 0$ for all $M_1 \in \mathcal{M}$ and M_2 sent with positive probability following M_1 and $x^*(M_1, \emptyset) = 1$ for all $M_1 \in \text{supp}(\sigma_1)\backslash\mathcal{M}$. Thus, G_1 strictly prefers to deviate to a strategy that places positive probability only on lobbying choices not in $\mathcal{M}$. This shows that G_2 blocks the strategy σ_1. $\square$

Lemma 8. *For any lobbying strategy by G_1, there is a strategy for G_2 such that for any (M_1, M_2) on the equilibrium path $U_i^\pi(M_1, M_2) \leq H\sum_{k=q}^{n} p_k - L\sum_{k=0}^{q-1} p_k$ for at least $n - q$ legislators.*

Proof. Let

$$\tau(M_1, M_2|\sigma_1, \sigma_2) = \sum_{W \in 2^N} \sigma_1(M_1|W)\sigma_2(M_2|W, M_1)$$

denote the total probability of the lobbying choices M_1 and M_2. The laws of probability require that

$$\sum_{2^N \times 2^N} \tau(M_1, M_2|\sigma_1, \sigma_2)\pi(W|M_1, M_2) = p(W)$$

for all $W \in 2^N$. This implies that

$$\begin{aligned}\sum_{W:|W|\geq} \sum_{2^N \times 2^N} \tau(M_1, M_2|\sigma_1, \sigma_2)\pi(W|M_1, M_2) &= \sum_{2^N \times 2^N} \tau(M_1, M_2|\sigma_1, \sigma_2)\Pr[|W| \geq q|M_1, M_2] \\ &= \sum_{k=q}^{n} p_k. \qquad \text{(B.10)}\end{aligned}$$

Consider a strategy σ_2 such that $\sigma_2(M_2|W, M_1) = 0$ when $|W| < q$ and $M_2 \cap W \neq \emptyset$. Furthermore, assume σ_2 only places positive probability on sets of size $n - q$. For such a strategy, the probability that $i \in M_2$ is equal to zero if $i \in W$ and $|W| < q$. Thus, for all

$i \in M_2$:

$$\Pi_i(M_1, M_2) = \Pi_i(M_1, M_2||W| < q) \Pr[|W| < q|M_1, M_2] + \Pi_i(M_1, M_2||W| < q) \Pr[|W| < q|M_1, M_2] \tag{B.11}$$

$$= 0 + \Pi_i(M_1, M_2||W| < q) \Pr[|W| < q|M_1, M_2]. \tag{B.12}$$

If $\Pr[|W| < q|M_1, M_2] = \sum_{k=0}^{q-1}$ for all (M_1, M_2) such that $\tau(M_1, M_2|\sigma_1, \sigma_2) > 0$ then $U_i^\pi(M_1, M_2) = H \sum_{k=q}^{n} p_k - L \sum_{k=0}^{q-1} p_k$ for all $n - q$ legislators in M_2 and the proof is complete. Otherwise, Equation B.10 implies that $\Pr[|W| < q|M_1, M_2] < \sum_{k=0}^{q-1}$ for some (M_1, M_2) such that $\tau(M_1, M_2|\sigma_1, \sigma_2) > 0$. Therefore, $U_i^\pi(M_1, M_2) < H \sum_{k=q}^{n} p_k - L \sum_{k=0}^{q-1} p_k$ for all $n - q$ legislators in M_2. Thus, for any σ_1, there exists a σ_2 such that $U_i^\pi(M_1, M_2) \leq H \sum_{k=q}^{n} p_k - L \sum_{k=0}^{q-1} p_k$ for at least $n - q$ legislators for any (M_1, M_2) on the equilibrium path. □

Theorem 6. *There is a blocking equilibrium if*

$$\kappa^1(p, q) \leq \frac{H}{L} < \kappa^2(p, q).$$

Proof. Let $\kappa^1(p, q) \leq \frac{H}{L} < \kappa^2(p, q)$. By Theorem 5 there is an influential equilibrium when there is no competition. Furthermore, by Lemma 8 and the legislator decision rule in Equation 3.5, for every σ_1 there is some σ_2 such that $x * (M_1, M_2) = 0$ for some M_1 such that $\sum_{W \in 2^N} \sigma_1(M_1|W) > 0$ and all M_2 such that $\sum_{W \in 2^N} \sigma_2(M_2|W, M_1) > 0$. By Lemma 7, this implies that G_2 blocks every lobbying strategy by G_1, which shows that there is a blocking equilibrium. □

Theorem 7. *There exists an influential equilibrium in the presence of interest group competition if*

$$\frac{H}{L} \geq \kappa^2(p,q).$$

Proof. Let $\sigma_1 = \sigma_2^{\min}$. Then $\Pi_i(M_1, M_2||W| \geq q) = 1$ for all $i \in M_1$ given any M_2 for any σ_2. Thus,

$$\Pi_i(M_1, M_2) = \Pi_i(M_1, M_2||W| < q)\sum_{k=0}^{q-1} p_k + \Pi_i(M_1, M_2||W| \geq q)\sum_{k=q}^{n} p_k \geq H\sum_{k=q}^{n} p_k - L\sum_{k=0}^{q-1} p_k.$$

This implies that $x^*(M_1, M_2) = 1$ for all M_2 and all M_1 such that $\sum_{W\in 2^N} \sigma_1(M_1|W) > 0$. $\square$

Theorem 4. *Any influential equilibrium gives at least legislators a lower ex ante expected utility relative to the outcome when lobbying is not allowed.*

Proof. By Lemma 4, in an influential equilibrium $x^*(M_1, M_2) = 1$ for all M_1, M_2. Thus, in an influential equilibrium, the policy is passed with probability one. By assumption, $\sum_{W\in 2^N} p(W)u_i(1, W) < 0$ for at least q legislators since the policy, since the legislators would not pass the policy in the absence of lobbying. Therefore, at least q legislators get a negative ex ante expected utility from an influential equilibrium. $\square$

Appendix C

Appendix to Chapter 4

C.1 Proofs of Results

Theorem 8 *There exists a directional communication equilibrium if* $\mathbf{\Sigma} = k\mathbf{I}_d$ *where* $k > 0$ *is a constant scalar and* $\mathbf{I}_d$ *is the identity matrix.*

Proof. If $\mathbf{\Sigma} = k\mathbf{I}_d$ then $\mathbf{z}'\mathbf{\Sigma}\mathbf{z} = k||\mathbf{z}||^2$ for all $\mathbf{z} \in \mathbb{R}^d$. Thus, we can write $f(\mathbf{z}) = f^*(||\mathbf{z}||)$ where the domain of $f^*(||\mathbf{z}||)$ is $\mathbb{R}_+$. Let for any $\mathbf{z}$ and $\tilde{\mathbf{z}}$, let $\theta(\mathbf{z}, \tilde{\mathbf{z}})$ be the angle enclosed by the two vectors.

By the law of cosines,

$$||\mathbf{z} - \tilde{\mathbf{z}}||^2 = ||\mathbf{z}||^2 + ||\tilde{\mathbf{z}}||^2 - 2||\mathbf{z}||||\tilde{\mathbf{z}}|| \cos\theta(\mathbf{z}, \tilde{\mathbf{z}}). \tag{C.1}$$

Thus, the expected utility to voter i from electing a candidate $j \in \{A, B\}$ using the directional communication strategy who sends the message m is

$$-\mathbb{E}[||\mathbf{z}_i - \mathbf{z}||\ |m] = -\int_0^\infty \left[||\mathbf{z}||^2 + r^2 - 2||\mathbf{z}||r\cos\theta(\mathbf{z}_i, \mathbf{m})\right] dr \tag{C.2}$$

$$= -||\mathbf{z}_i||^2 - \int_0^\infty r^2 f^*(r)dr + 2||\mathbf{z}_i|| \cos\theta(\mathbf{z}_i, \mathbf{m}) \int_0^\infty r^2 f^*(r)dr. \qquad \text{(C.3)}$$

Therefore, if A and B send $\mathbf{m}_A$ and $\mathbf{m}_B$ respectively and both use directional communications strategies, i strictly prefers A to B if and only if

$$-||\mathbf{z}_i||^2 - \int_0^\infty r^2 f^*(r)dr + 2||\mathbf{z}_i|| \cos\theta(\mathbf{z}_i, \mathbf{m}_A) \int_0^\infty r^2 f^*(r)dr \qquad \text{(C.4)}$$
$$> -||\mathbf{z}_i||^2 - \int_0^\infty r^2 f^*(r)dr + 2||\mathbf{z}_i|| \cos\theta(\mathbf{z}_i, \mathbf{m}_B) \int_0^\infty r^2 f^*(r)dr$$

$$2||\mathbf{z}_i|| \cos\theta(\mathbf{z}_i, \mathbf{m}_A) > 2||\mathbf{z}_i|| \cos\theta(\mathbf{z}_i, \mathbf{m}_B) \qquad \text{(C.5)}$$

$$\cos\theta(\mathbf{z}_i, \mathbf{m}_A) > \cos\theta(\mathbf{z}_i, \mathbf{m}_B). \qquad \text{(C.6)}$$

Since $\cos\theta(\mathbf{z}_i, \mathbf{m}) = \frac{\mathbf{z}_i \cdot \mathbf{m}}{||\mathbf{z}_i|| ||\mathbf{m}||} = \frac{\mathbf{z}_i \cdot \mathbf{m}}{||\mathbf{z}_i||}$ by the definition of the dot product and the fact that $||m|| = 1$, we have $\cos\theta(\mathbf{z}_i, \mathbf{m}_A) > \cos\theta(\mathbf{z}_i, \mathbf{m}_B)$ if and only if $\mathbf{z}_i \cdot (\mathbf{m}_A - \mathbf{m}_B) > 0$. Thus, the set of $\mathbf{z}_i$ satisfying this condition defines an open halfspace with $(0, \ldots, 0)$ on the boundary. By the assumption that G is angularly symmetric, this implies that each voter strictly prefers A to B with probability $\frac{1}{2}$ and, by symmetry, strictly prefers B to A with the same probability. Furthermore, since this holds for any two messages, neither candidate has a strict incentive to deviate from the directional communication strategy. $\square$

Theorem 9 *There exists a principal orthant equilibrium.*

Proof. Let $\mathbf{y} = \mathbf{Q}\mathbf{z} = (y^1, \ldots, y^d)$. Since $\mathbf{\Sigma}$ is a symmetric real matrix, $\mathbf{Q}$ is an orthogonal matrix, meaning that $\mathbf{Q}'\mathbf{Q} = \mathbf{I}_d$. Some algebraic manipulation shows that

$$f(\mathbf{y}) = f(\mathbf{Q}\mathbf{z}) = c \cdot f_0(\mathbf{z}'\mathbf{Q}'\mathbf{Q}\mathbf{\Lambda}^{-1}\mathbf{Q}^{-1}\mathbf{Q}\mathbf{z}) = c \cdot f_0(\mathbf{z}'\mathbf{\Lambda}^{-1}\mathbf{z}). \tag{C.7}$$

Since $\mathbf{\Lambda}^{-1}$ is a diagonal matrix, this implies that the dimensions of $\mathbf{Q}\mathbf{z}$ are distributed independently and we can write

$$f(\mathbf{y}) = \prod_{j=1}^{d} f_j(y_j) \tag{C.8}$$

where f_j is the marginal density of y_j. Furthermore, since $f(\mathbf{y}) = f(-\mathbf{y})$, we have $\int_{-\infty}^{0} f_j(y_j)dy_j = \int_{0}^{\infty} f_j(y_j)dy_j = \frac{1}{2}$ and $\int_{-\infty}^{0} y_j^2 f_j(y_j)dy_j = \int_{0}^{\infty} y_j^2 f_j(y_j)dy_j$ for all j. The latter implies that $\mathbb{E}[||\mathbf{z}||^2|\mathbf{m}] = \mathbb{E}[||\mathbf{z}||^2|\tilde{\mathbf{m}}]$ and $\text{Var}[\mathbf{z}|\mathbf{m}] = \text{Var}[\mathbf{z}|\tilde{\mathbf{m}}]$ if $S(\mathbf{m})$ and $S(\tilde{m})$ are both principal orthants of $\mathbb{R}^d$ with respect to f.

Let $\mathbb{E}[||\mathbf{z}||^2|\mathbf{m}] = \overline{z}$ and $\text{Var}[\mathbf{z}|\mathbf{m}] = V$ and let $\overline{s}(\mathbf{m})$ and $\overline{s}(\tilde{m})$ be the (vector-valued) means of $S(\mathbf{m})$ and $S(\tilde{m})$, respectively. The mean-variance representation of quadratic expected utility implies that $\mathbb{E}[u(\mathbf{z}_A, \mathbf{z}_i)|\mathbf{m}_A = \mathbf{m}] > \mathbb{E}[u(\mathbf{z}_B, \mathbf{z}_i)|\mathbf{m}_B = \tilde{\mathbf{m}}]$ if and only if

$$-||\mathbf{z}_i - \overline{s}(\mathbf{m})|| - V > -||\mathbf{z}_i - \overline{s}(\tilde{\mathbf{m}})|| - V \tag{C.9}$$

$$-||\mathbf{z}_i|^2 - \overline{z}^2 + 2\mathbf{z}_i \cdot \overline{s}(\mathbf{m}) - V > -||\mathbf{z}_i|^2 - \overline{z}^2 + 2\mathbf{z}_i \cdot \overline{s}(\tilde{\mathbf{m}}) - V \tag{C.10}$$

$$2\mathbf{z}_i \cdot \overline{s}(\mathbf{m}) > 2\mathbf{z}_i \cdot \overline{s}(\tilde{\mathbf{m}}) \tag{C.11}$$

$$\mathbf{z}_i \cdot (\overline{s}(\mathbf{m}) - \overline{s}(\tilde{\mathbf{m}})) > 0. \tag{C.12}$$

The set of $\mathbf{z}_i$ satisfying this condition defines an open halfspace with $(0, \ldots, 0)$ on the boundary. Since G is angularly symmetric, this implies that each candidate expects to gain any voter's support with probability $\frac{1}{2}$. Since this holds for any pair of principal orthans, no candidate has a strict incentive to deviate from this strategy. $\square$