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Essays in Heterogeneity, Irreversibility and Aggregate Fluctuations

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by

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Introduction

"**Essays on Heterogeneity, Irreversibility and Aggregate Fluctuations**" explores the connections between micro structure and technologies available to the agents operating in the economy and the dynamic of aggregate output and productivity. The thesis aims at further understanding the linkages between investment decisions of heterogeneous firms, the industry structure, and the aggregate dynamic of the economy.

The hypothesis explored in this dissertation is that the dynamic of the industry structure, the patterns of selection of firms and investment within an industry bear information as of the efficiency with which the economy operates.

The thesis consist of three essays organized in chapters.

Chapter I, **"Efficiency with Equilibrium Marginal Product Dispersion and Firm Selection"** investigates conditions under which reductions in marginal product of capital dispersion induce Pareto improving allocations. The economy is one in which dispersion in marginal products arise endogenously due to uncertainty and irreversible capital investment. The main result is that it is possible for allocations that display higher marginal product dispersion to be closer to the efficient one than allocations with lower marginal product dispersion. The intuition for such result is that the relevant statistic to assess efficiency is the covariance between the contributions of the firms to aggregate output and their shadow value of capital. Hence, allocations where firms with disparate marginal product contribute little to aggregate output, can be closer to the efficient allocation than allocations with lower dispersion, but where those with disparate marginal product contribute disproportionately more to output.

This essay contributes broadly to the study of optimal policy in economies with firm selection and heterogeneity. Existence of the competitive equilibrium is shown indirectly by providing a decentralization result of the efficient allocation. This result is of interest for the growing literature analyzing productivity gains from reallocation of factors across production units, and firm selection; as well as for the design of productivity enhancing policies.

Chapter II, **"Industry Dynamics, Investment and Business Cycles"** investigates the quantitative implications of irreversibilities in investment for aggregate productivity. Irreversibilities in investment induce marginal product dispersion in equilibrium when there is uncertainty in the economy. Through it, it affects aggregate productivity but also firm selection, which feeds back to the former through general equilibrium effects. I study a general equilibrium model with heterogeneous firms and aggregate uncertainty only. Investment, entry and exit decisions are modeled as real options and in each period, firms compete monopolistically. The main result of the essay is that efficiency losses associated to firm selection are quantitatively more important than those associated to lower equilibrium dispersion in marginal products, i.e. capital reallocation. This result supports other studies which have empirically documented productivity gains from changes in the pattern of firm churning in an industry.

I show also that the inefficiency induced by imperfect competition in the intermediate goods market interacts with the market incompleteness described in the first chapter. Which inefficiency is more important dictates the directions of the optimal policy. I calibrated the model economy to the US manufacturing sector and compute the implied optimal policy to implement the efficient allocation. The optimal policy implies subsidies to entry, the size of the subsidy is predicted higher in good times. In equilibrium there are more firms operating in the market under the efficient allocation. Upgrade costs are subsidized to induce better selection of firms in the market. The policy as of scrap values varies with the aggregate state and the technology operated by the firm. In good times, scrap values are lower for all capacities except for the bottom ones, to generate exit of the least productive units. In bad times, scrap values for the bottom capacities are predicted lower, and the scrap value of the firms at the top of the productivity/size distribution is higher. The latter induces exit by firms that are possibly capacity constrained.

Finally, this essay also shows that the relationship between aggregate productivity and dispersion in marginal products is not monotonous, and in particular, is not independent of the degree of uncertainty that firms face when investing. This result is important in view of the growing literature with cross country comparisons with measures of marginal and average product dispersion. In particular, it highlights that the efficient level of dispersion observed in an economy need not equalize the level of dispersion is another, and that their relationship will depend on the characteristics of the environment in which firms operate.

Chapter III, "Aggregate Fluctuations and the Industry Structure of the US Economy" documents changes in the input matrix of the US economy, and analyzes its implications for the relevance of sector specific and neutral shocks in aggregate fluctuations. The paper contributes to two strands of literature. The first one is the one characterizing linkages between sectors in the economy, and its relevance for the response of aggregate output to shocks. In this paper the focus is put on fairly aggregated sectors (Equipment and Consumption), but unlike the previous literature the intensity of trade is allowed to change as observed in the data. The essay is also related to the literature that studies the implications of investment specific and neutral shocks to aggregate volatility of output in economies that display investment specific technical change. The model economy analyzed in this paper is consistent with a balanced growth path in which investment specific technical change can be accommodated and intermediate good linkages across sectors do not vanish.

The main finding is that an economy where the input output entries are allowed to fluctuate as in the data generates larger amplification of shocks and a stronger role for neutral shocks than a comparable economy with a fixed input output structure. This result highlights the importance of modeling input output linkages in the now plain vanilla model of real business cycles with investment specific and neutral shocks.

Chapter I: Efficiency with Equilibrium Marginal Product Dispersion and Firm Selection

1 Introduction

The increased availability of firm level data has risen interest on the implications of firm heterogeneity in productivity, employment and capital allocations, for aggregate productivity. Recent work by Hsieh and Klenow (2009), has spawn off a growing literature that argues that measures of dispersion in revenue total factor productivity[1] for narrowly defined industries, can explain cross country disparities in aggregate total factor productivity (TFP). Part of the dispersion in revenue total factor productivity can be attributed to dispersion in marginal product of inputs, which is ubiquitous in industrial data[2]. From a static point of view, differences in marginal products can be associated to loses in aggregate productivity and welfare. If such disparities are generated through financial frictions[3] or policy distortions[4], welfare and aggregate productivity improves whenever dispersion is reduced. Dispersion can also be generated through features of the technology that firms operate (i.e. adjustment costs as in Asker *et al.* (2013) and Midrigan and Xu (2009)). In this case, dispersion can be consistent with dynamically optimal investment decisions, and it is not clear whether lower dispersion in marginal products would be productivity or welfare improving.

Little theoretical work has been done on the implications for efficiency of dispersion in marginal products that arise as the outcome of dynamically optimal investment decisions in economies with endogenous firm selection. In this paper, I address this question by focusing in an economy with dispersion in marginal product of capital generated through irreversible and indivisible investment. I consider the problem of a planner that faces the same technological restrictions that firms in the market face, and asks under which conditions a reduction in dispersion in marginal products is Pareto improving. The main result is that it is possible for an economy with higher marginal product dispersion to be closer to the efficient allocation than a comparable economy with lower dispersion. This result is relevant for the assessment of

[1]For a discussion on measures of revenue and quantity TFP see (Foster *et al.* (2008))

[2]For cross country evidence refer to Asker *et al.* (2013). For evidence for Korea, refer to Midrigan and Xu (2009). Also, Hsieh and Klenow (2009) provide evidence for the US, India and China. For evidence in Latin America, see Buso *et al.* (2013).

[3]See Buera and Shin (2011), Moll (2013), Midrigan and Xu (2009) and the extensive literature thereafter.

[4]Restuccia and Rogerson (2008) analyze a broad range of policy distorsions.Barstelman *et al.* (2013) document and study the impact of distorsions that are correlated with the size of firms.

potential productivity gains from reallocation of factors that induce less dispersion in marginal products. In our economy, such reallocation need not be efficiency improving. The reason is that the relevant statistic to assess efficiency gains is the correlation between the distribution of marginal products and the contribution of the firms to aggregate productivity. I also show that a market arrangement that would induce the efficient allocation when there is no equilibrium dispersion in marginal product of capital, fails to generate the efficient outcome when the allocation displays dispersion in marginal products. However, the efficient allocation can be decentralized under the same market structure, provided state contingent taxes and subsidies available.

The paper develops an infinite horizon model of investment by heterogeneous firms, with endogenous firm selection and idiosyncratic and aggregate uncertainty (Section 2). Firms produce goods out of capital and labor, and a Hicks neutral productivity level. Firms are entirely equity owned and rent capital and labor in competitive markets. The firm is identified by the realization of an exogenous idiosyncratic shock and an endogenous component of idiosyncratic productivity, i.e. a process. A process is defined as a productivity shifter and an associated minimum operating capacity in terms of capital. At the beginning of the period, after shocks are realized, firms decide whether to exit or not the market and if so, which process to operate. Entry, exit and process investment decisions are modeled as real options. The exercise of any of these options entails a one time fixed cost. Investment in processes is indivisible, because only a finite set of technologies (and associated minimum capacities) is available. It is also irreversible, in the sense that disinvestment in technology entails the firm liquidation in the current period, and a new draw of productivity in the next one.

Due to the real options feature of the model there are states of the world where firms hold a particular process while being constrained by its minimum capacity (holding excess capacity). The marginal product of capital for a constrained firm is lower than that of a comparable unconstrained firm. The identity of the firms that are constrained depends on the realization of current shocks in view of the history of shocks that the firm has experienced. For example, if a firm experiences a sequence of positive shocks, it is more likely to invest in better processes. But better processes have higher minimum running capacities. So when a negative shock hits, the firm is more likely to be running at overcapacity. The result resembles earlier intuitions drawn in Caballero and Hammour (1998) when analyzing factor specificity. The distribution of productivities and marginal products depends on the vintage structure of the firms operating in the market, and are endogenously related to each other. Endogenous selection of firms is therefore key in assessing the efficiency with which an economy operates [5].

In generating marginal product dispersion, I focus on a mechanism that relates to the literature on capital adjustment costs and Ss adjustment policies[6]. However, the adjustment policy in the model is asymmetric because the minimum capacity constraint only generates marginal product of capital below the interest rate in the market [7]. The empirical evidence supports the existence of minimum running capacities at the plant level. They had been documented in the energy industry, and the chemical industry among others[8]. In industries where output is produced on production lines, such as the car industry, minimum scales are also relevant[9].

When firms are capacity constrained, the price of capital in the market does not reflect their opportunity cost of capital nor does it reflect the heterogeneity in its shadow value for more and less constrained firms. Profit maximization is not enough to generate the efficient allocation of firms across technologies. Suppose first that there is a unique process that firms operate and that we allow for entry and exit. At which cost should the planner price the new activity generated upon entry? If marginal products are equalized, the entry of a new firm does not expand or reduce the space spanned in this economy[10]. When marginal products are different the space of possible tradable activities and transfers of capital across them gets possibly enlarged by a new dimension. I show that the planner prices the new activity at the average cost of capital in the market. In the market allocation, I assumed that firms pay for capital in a spot market at a cost that in equilibrium equals the marginal product of capital of firms that are unconstrained. This is the standard assumption in a plain vanilla model of industry dynamics. Notice that if we assume away entry and exit, these problem disappears when the space spanned is fixed. When in addition we assume, as in this paper, that firms can choose across technologies, the problem exacerbates as for each technology the space spanned gets larger or smaller depending on the adoption decisions of the firms in the market.

The fundamental source of inefficiency in the economy is a form of market incompleteness. The same market structure is enough for the first welfare theorem to hold in the comparable

[5] Theoretical work by Hopenhayn (1992) and Melitz (2003) point out the relevance of endogeneous selection for aggregate productivity. Empirical work by Eslava *et al.* (2004) also highlights the important of selection in shifiting aggregate productivity.

[6] Early work include Dixit and Pyndick (1994), Mariotti *et al.* (2006) and Caballero and Engel (1999).

[7] An Ss adjustment policy has the potential to generating marginal product above or below the interest rate.

[8] See Fuss (1981), Lyons (1980) and Tybout (2000)

[9] See Rodrik (1988) and Rhys (1977)

[10] One could think of firms, as assets with returns proportional to their idyosincratic productivity as in Diamond (1967)

economy with heterogeneity, firm selection and equalization of marginal products (for example Hopenhayn (1992) or Bilbiie *et al.* (2012)). The literature that studies efficiency in economies with selection and firm heterogeneity is sparse. A recent paper that studies the characteristic of the constrained optima in an economy with distortions in revenue product and endogenous entry and exit is Fattal Jaef and Hopenhayn (July 2012). They find that while the competitive allocation generates the efficient allocation of resources across a given set of technologies, it fails to generate the efficient level of entry and exit, and hence the efficient measure of active firms. The model analyzed here departs from their environment in several ways. First, this model studies allocations where marginal product dispersion occurs in equilibrium, in their economy the marginal product of labor is equated across firms. Second, the allocation of technologies run by the firm is endogenous, the allocation of employment and capital need not efficient in the competitive equilibrium. Third, the patterns of entry and exit in the competitive equilibrium can be below or above the efficient one depending on the endogenous joint distribution of marginal products and productivity[11]. I am explicit about this feature by constructing market allocations that while displaying higher marginal product dispersion, are closer to the efficient allocation (Section 5). This feature of the model highlights the importance of assessing the impact of dispersion in marginal products on aggregate productivity within a general equilibrium framework, where both selection and investment are intertwined and endogenously determined.

This paper contributes to the work initiated by Lucas and Prescott (1971). They showed that a competitive equilibrium can be decentralized as an industry equilibrium in which the planner maximizes overall surplus in the economy by allocating labor across firms. I show existence and uniqueness of the efficient allocation in an economy with irreversible and indivisible investment (Section 3). Furthermore, I show that there is a pseudo planner problem whose equilibrium allocation coincides with the decentralized solution as long as state contingent subsidies and taxes are available (Section 4). This taxes and subsidies are applied to the costs incurred by the firms when entering, upgrading process, or exiting the market[12]. The equivalence result follows closely the result described in Jones and Manuelli (1990) to study policy questions in convex economies with growth.

The study of optimal policy in economies with heterogeneous firms is not new. It has been

[11] In a recent paper, Cooper and Schott (2014) show that to correctly assess gains from reallocation in an economy with aggregate fluctuations the joint distribution of productivity and marginal products needs to be track.

[12] Hence, the decentralization mechanism bypass the absence of state contingent claims for every possible realization of the distribution of marginal product of capital.

done in models of international trade under oligopolistic competition in prices and quantities (Eaton and Grossman (1986)). For a model of industry dynamic without capital accumulation Lee and Mukoyama (2008) study the impact of alternative policies on labor regulations. However, their policies are ad hoc in the sense that there is no notion of efficiency associated to them. They consider i.i.d. policies and policies correlated with the productivity of the firms. Guner *et al.* (2008) study policies that target the size of the establishment, which in turn is correlated with their idiosyncratic productivity, and find a substantial role in shaping aggregate productivity. This paper contributes to the literature by providing an algorithm to solve for the optimal policy in economies where the efficient allocation displays marginal product dispersion.

2 Environment

This is an infinite horizon economy with time indexed by t. There is a final good which agents use for consumption and capital accumulation. The preferences of the planner in this economy are defined over consumption strems C_t of final output. Preferences are characterized by $U : \mathbb{R}^+ \to \mathbb{R}^+$.

Assumption 1: *U is concave, monotonically increasing and differentiable. Also, $U'(0) = +\infty$.*

The discount factor is β and the planner maximizes the present discounted value of the stream of consumption.

Final output Y_t is produced by means of a continuum of intermediate goods through technology $h : \mathbb{R}^+ \to \mathbb{R}^+$.

$$Y_t \leq h\left(\int y_{it} di\right)$$

Assumption 2: *The production function $h : \mathbb{R}^+ \to \mathbb{R}^+$ is differentiable and satisfies $h(0) = 0$, $h'(0) = +\infty$ and $h'(y) > 0$ for all $y \in \mathbb{R}^+$.*

Intermediate goods are substitutes in the production of final goods, as only the aggregate amount of intermediates determines final output.

Intermediate goods, y_t are produced by combining capital (k_t) and labor (l_t) and the technology operated by the firm. Productivity is assumed Hicks neutral.

$$y_{it} \leq s_t z_{it} \psi_i^n f(l_t, k_t)$$

Productivity entails three components: an exogenous and an endogenous idiosyncratic one, and an exogenous aggregate shock.

The exogenous component of idiosyncratic productivity follows a Markov process with transition probabilities $\mathbf{P}_z(z_{t+1}, z_t)$ for all $z_{t+1}, z_t \in \mathcal{Z} \equiv [\underline{z}, \overline{z}]$. The exogenous aggregate shock is denoted s_t and follows a Markov process with transition probabilities $\mathbf{P}_s(s_{t+1}, s_t)$ for all $s_{t+1}, s_t \in \mathcal{S} \equiv [\underline{s}, \overline{s}]$ .S is finite.

The endogenous component of productivity can be interpreted as a process for production. Each process is characterized by a productivity shifter, ψ^n and a minimum capacity constraint, $\underline{k}^n$. Processes are ordered so that $(\psi^n < \psi^{n+1})$ and $(\underline{k}^n < \underline{k}^{n+1})$ for all $n \leq N - 1$. In other words, more productive processes entail a higher minimum capacity constraint. Processes are chosen from the set $\mathcal{N} \equiv [0, N]$, where 0 indicates the firm is not operating and has exit the market. The adoption of a process is costly.

Assumption 3: *The function f is differentiable, increasing in both arguments and satisfies $f(0,0) = 0$, $f_l(0,0) = f_k(0,0) = +\infty$. Also, it displays decreasing returns to scale in capital and labor, such that $s_t z_t \psi^n f(l_t, k_t)(1-\rho) = kf_k + lf_l$*

The aggregate state of the economy is described by $\Xi_t = \left(s_t, \mathbf{z}_t, \left\{v_{t-1}^n\right\}_{n=1}^N, K_t\right)$ and includes the realization of exogenous aggregate and idiosyncratic shocks, s_t and the vector $\mathbf{z}_t$ respectively; the distribution of firms per process inherited from the previous period, v_{t-1}^n for $n \leq N$; and the available aggregate stock of capital. I denote $v_t^n(z, s_t) \equiv v_t^n([\underline{z}, z), s_t)$ the measure of firms with productivity at most z and technology n when the aggregate state is s_t. The law of motion of the distributions is denoted by T^{*S}.

The timing of decisions is as follows. The planner observes the realization of the aggregate and idiosyncratic shocks, and takes the aggregate stock of capital and the distribution of firms in the market as given. At that point it can decide whether or not to add production units to the market (entry). If entry is positive it pays $I(s_t)$ units of the final good. The idiosyncratic productivity of the firm entering the market is unknown before entry and all firms enter with the worse process. Immediate upgrades are allowed once the idiosyncratic state is revealed. The productivity of a firm entering the market is drawn from $G(z)$.

Assumption 4: *The function $G(z)$ is absolutely continuous $\mathcal{Z}$.*

The planner also decides which firms to liquidate, for which he receives $\Pi_e^n(s_t)$ per firm of type n; and finally which firms to upgrade in process at cost $I^{n+1}(s_t)$ (if upgrading a firm from process n to $n+1$). Downgrades in processes are not allowed. To induce it, the planner has to liquidate the firm and enter a new one. After all decisions on processes, entry and exit are completed, the planner chooses the allocation of capital and labor. After production takes place, the aggregate stock of capital depreciates at rate $\widehat{\delta}$. Also, some production units are liquidated exogenously with probability δ, for which the planner gets a scrap value of Π_e^f.

Assumption 5: *The scrap value if forced to exit, is less than or equal the scrap value when choosing to exit $\Pi_e^f \leq \Pi_e^n(s_t)$. Without loss of generality, $\Pi_e^f = 0$. Also, the cost of upgrade is higher or equal to the difference in scrap values, $I^{n+1} \geq \Pi_e^{n+1} - \Pi_e^n$.*

Hence, no resources can be generated simply by upgrading firms in the market.

Assumption 6: *The cost of entry is higher than the scrap value of the less productive process $I \geq \Pi_e^1$. Also, for any $s_1, s_2 \in S$ with $s_1 > s_2$, the entry cost satisfies $I(s_1) < I(s_2) + \int \Pi_e^1(s_1)dG(z_{it}) - \int \Pi_e^1(s_2)dG(z_{it})$.*

The first part of assumption 6 prevents generating resources by entering firms in the market, irrespective of whether they produce or not. The second part of the assumption is used later to assure that the measure of entrants is procyclical. The condition requires that entry cost do no "raise" too much during upturns, potentially desincentivizing entry.

As much as possibly I will refer to the cost structure as $\Upsilon_p(s_t) = \left[\{\Pi_e^n\}_{n=1}^N ; \{I^n\}_{n=1}^{N-1} ; I \right]$

3 Efficient Allocation

Define the problem of the planner as follows

$$V(\Xi_t) = \max_{C_t,\, K_{t+1},\, Y_t,\, \{z_t^e(\psi^n)\}_{e=1}^N,\, \{z_t^u(\psi^n)\}_{n=2}^N, M_t^{ent}, l_{it}, k_{it}} U(C_t) + \beta EV(\Xi_{t+1}) \tag{EA}$$

subject to

$$C_t + K_{t+1} - (1-\widehat{\delta})K_t + IM_t^{ent} + \text{Upgrade Costs} = Y_t + \sum_{n=1}^N \Pi_e^n M_{et}^n$$

$$h\left(\sum_j \int s_t z_{it} \psi^j f(l_{it}, k_{it}) di \right) = Y_t$$

$$\int l_i di = 1, \text{ and } \int k_i di = K_t$$

$$k_i \geq \underline{k} \text{ if } \psi_i = \psi^n$$

$$v_t = T^{*S}(v_{t-1}) \quad \text{and the exogenous transitions} \quad P_s \text{ and } P_z$$

where "Upgrade Costs" equals,

$\sum_{n=1}^{N-1} I^n \left[M_{ut}^n \; + M_t^{ent} \left(G(z^u(\psi_t^{n+1})) - G(z^u(\psi_t^n))\right)\right] + I^N \left[M_{ut}^n \; + M_t^{ent} \left(1 - G(z^u(\psi_t^n))\right)\right];$

$M_{et}^n(\Xi_{t-1}, \Xi_t)$ is the measure of exits for firms running process n, $M_{ut}^n(X_{t-1}, X_t)$ is the measure of incumbent upgrades in state Ξ_t to technology n; $M_t^{ent}(\Xi_t)$ is the corresponding measure of entrants. (See the Appendix for a detailed description).

Theorem 1 *The efficient allocation exists and it is unique.*

For expositional purposes the full proof can be found in the Appendix. Heuristically it goes as follows. The problem would be a standard concave problem if there were no sunk costs to technology adoption and no minimum capacity constraint that may bind in equilibrium. The presence of a continuum of heterogenous firms mitigates potential non-convexities in the aggregate set as in Mas-Colell (1977). The operator that describes the planner's problem is defined in the set of bounded absolutely continuous measures, and a unique fixed point is shown to exist. Among others, the existence of this equilibrium relies on the characteristics of the law of motion for the distribution of firms. For example, we need to make sure that the law of motion per process maps from and into the set of bounded absolutely continuous measures.

3.1 Law of Motion for the distribution of firms

I first show that in an economy without aggregate shocks, the economy has an invariant measure of firms across productivities.

Define the state space for the distribution as a Cartesian product $\mathcal{A} \equiv \mathcal{Z}\text{x}\mathcal{N}$ with a typical subset characterized by $A \equiv Z\text{x}N$.Let $\mathcal{B}$ be the sigma algebra of $\mathcal{A}$. The space $(\mathcal{A}, A)$ is a measurable space. Let $v(A)$ be the measure of agents in set A. Let $\Lambda^z((z,n), ZxN)$ be the probability that a firm with current state (z,n) transits to the set A next period. Hence $\Lambda^z(\mathcal{A}, A) \to [0,1]$ describes the law of motion of the system with idiosyncratic shocks only.

$$\Lambda^z((z,n), ZxN) = \int_{z' \in Z} \chi\left\{n'(z',n) \in N\right\} P_z(z',z)$$

where χ is an indicator function, and $n'(z',n)$ the optimal technology selection policy.

Define the operator T^* as

$$v_{t+1}^{z}(ZxN) = T^*(v_{t+1}) \equiv \int_{z\in Z} \Lambda^{z}((z,n), ZxN)d(v_t^z(z,n))$$

Assumption 7: *The cost structure for upgrade across technologies is such that* $\psi_T = \psi^N$ *if the firm experiences an arbitrary long sequence of good realizations of the idiosyncratic shock,* $\{\overline{z}\}_{t=1}^{T}$*for* $T > \widehat{T}(\{I^n\}_{n=1}^{N})$ *finite.*

Assumption 7 would be violated for example if the cost of upgrading to a particular process $n \leq N$ goes to infinity, i.e. $I^n \to \infty$. In this case, even under the best realizations of the shock the firm never finds optimal to upgrade to process n or better. This would in turn violate the monotone mixing condition needed to proof existence and uniqueness of the invariant measure.

Proposition 1 *The operator* T^* *has a unique fixed point in the space of measures defined over the measurable space* $(\mathcal{A}, A)$*.*

Now, augment the set A to include the state space for the realizations of the aggregate shock, i.e $\mathcal{A}\equiv\mathcal{S}\text{x}\mathcal{Z}\text{x}\mathcal{N}$ with typical subset characterized by $A^s \equiv S\text{x}Z\text{x}N$. Let $\Lambda^s((s,z,n), SxZxN)$, the probability that a firm with current state (s,z,n) transits to the set A next period.

$$\Lambda^s((s,z,n), SxZxN) = \sum_{s'\in S} \int_{z'\in Z} \chi\left\{n'(s',z',n) \in N\right\} P_z(z',z) P_s(s',s)$$

In general the law of motion of the distribution of firms in the market is described by an operator T^{*S}

$$v_{t+1}(SxZxN) = T^{*S}(v_t) \equiv \sum_{s\in S} \int_{z\in Z} \Lambda^s((s,z,n), SxZxN)d(v_t(s,z,n))$$

The operator T^{*S} is described in detail in the appendix.

If we consider the projections of v_t on the space N, v_t^n, it is possible to describe properties of the probability measure per technology.

Lemma 1 *The measure of firms per process* v_t^n*,belongs to the space of bounded and continuous measures on* $\mathcal{S}x\mathcal{Z}$*.*

3.2 Allocation

3.2.1 Capital Labor Ratios

In this section I describe the characteristics of the efficient allocation. Let λ_t^k (λ_t^l) be the lagrange multiplier associated to the feasibility constraint on aggregate capital (labor), λ_{it} the lagrange multiplies associated to each of the minimum capacity constraints of the production units operating in the market. Let μ_t the shadow value of consumption, i.e. the lagrange multiplies associated to the final goods feasibility constraint, and γ_t^n the shadow value of a firm operating process n.

The optimality conditions for labor and capital yield

$$\frac{k_{it}}{l_{it}} = \frac{\lambda_t^l}{\lambda_t^k - \lambda_{it}} \frac{f_k k_{it}}{f_l l_{it}}$$

where $\frac{f_k k}{f_l l}$ corresponds to the ratio of factor shares of total output under Assumption 3.

If the minimum capacity requirement is binding, the firm adjusts its resource allocation through the flexible factor, in this case labor. However, capital labor ratios are not equalized across production units [13]. The capital labor ratio of constrained firms is higher than that of unconstrained firms. In a static model with complete markets, disparate capital labor ratios are a sign of inefficiencies in the allocation. In the current set up however, these gaps are consistent with optimality.

Assumption 8: *f is separable in labor and capital when in logs, i.e* $\log(f(l,k)) = \log(\widehat{f}(l)) + \log(\widehat{f}(k))$

Under Assumption 8 labor and capital allocations can be described as a function of the productivity of the firm $x_t^{n_i} = z_{it}\psi_t^{n_i}$, and its marginal product of capital $\lambda_t^k - \lambda_{it}$.

$$l(x_t^{n_i}, X_t) = \frac{f_l^{-1}(x_t^{n_i}, \lambda_t^k - \lambda_{it})}{\int f_l^{-1}(x_t^{n_j}, \lambda_t^k - \lambda_{it})dj}$$

$$k(x_t^{n_i}, X_t) = K_t \frac{f_k^{-1}(x_t^{n_i}, \lambda_t^k - \lambda_{it})}{\int f_k^{-1}(x_t^{n_j}, \lambda_t^k - \lambda_{it})dj}$$

where f_l^{-1}indicates the inverse of the marginal product of labor, and f_k^{-1} is defined likewise.

If there is no dispersion in marginal product of labor (i.e. no firm is constrained), labor and

[13] In models where firms are financially constrained, the capital labor ratios of constrained firms is usually lower than that of unconstrained firms. Constrained firms hold less capital than they would if unconstrained, and have a higher marginal product of capital than the equilibrium cost of capital. In this model, constrained firms hold more capital than otherwise, and their MPK is lower than the interest rate.

capital demands are proportional to the relative productivity of the firm versus the rest of the economy[14].

In the analysis that follows it is useful to define two statistics, namely $Z^l = \int f_l^{-1}(x_t^{n_j}, \lambda_t^k - \lambda_{jt})dj$ and $Z^k = \int f_k^{-1}(x_t^{n_j}, \lambda_t^k - \lambda_{jt})dj$. Both are statistics of productivity adjusted by the marginal product of capital across all the firms in the economy. Capital labor ratios can alternatively be characterized in terms these statistics Z^l and Z^k.

$$\frac{k}{l} = K_t \frac{f_k^{-1}/f_l^{-1}}{Z_k/Z_l}$$

When there is no dispersion in marginal products, $\frac{f_k^{-1}/f_l^{-1}}{Z_k/Z_l} = 1$ and capital labor ratios are equalized.

3.2.2 Aggregates

All static decisions of the planner are summarized in the equilibrium capital and labor allocations. In studying the dynamic decisions, i.e. capital accumulation and firm allocation across processes, it is useful to rewrite aggregate output in terms of the static allocation.

Let the measure of firms operating in the market $M_t = \sum_{n=1}^{N-1} v_t^n(z^u(\psi_t^{n+1}, \Xi_t), s_t) + v_t^N(\overline{z}, s_t)$ where $z^u(\psi_t^{n+1}, \Xi_t)$ is the upgrade threshold from process n to $n+1$, and define a scaled measure $\widehat{v}_t^n = \frac{v_t^n}{M_t}$. Replacing capital and labor allocations in the aggregate production function, we obtain

$$Y(\Xi_t) = h\left(\sum_{n=1}^{N} \int s_t z_{it} \psi^n f(\frac{f_l^{-1}}{Z_l}, \frac{f_k^{-1}}{Z_k} K_t) dv_t^n(z_{it}, s_t)\right)$$

Under Assumption 8 one can rewrite it as

$$Y(\Xi_t) = h(TFP_t f(1, K_t))$$

Define total factor productivity as

[14] These are the demands for capital and labor in a plain vanilla industry dynamic model alla Hopenhayn (1989).

$$l(x_t^{n_i}, X_t) = \frac{f_l^{-1}(x_t^{n_i})}{\int f_l^{-1}(x_t^{n_j})dj}$$
$$k(x_t^{n_i}, X_t) = K_t \frac{f_k^{-1}(x_t^{n_i})}{\int f_k^{-1}(x_t^{n_j})dj}$$

$$TFP_t = M_t \sum_n \int s_t z_{it} \psi_t^n f(\frac{f_l^{-1}(x_t^{n_i}, \lambda_t^k - \lambda_{it})}{Z_l}, \frac{f_k^{-1}(x_t^{n_i}, \lambda_t^k - \lambda_{it})}{Z_k}) d\widehat{v}_t^n(z_{it}, s_t)^{15}$$

In other words, aggregate efficiency is determined by the realization of the exogenous shock, the measure of active firms in the market M_t (as usual in models with curvature in the profit function), and moments of the distribution of realized productivities for those firms. To better understand the impact of marginal product dispersion on the measures of efficiency in the economy, describe TFP as

$$TFP_t = M_t \sum_n \int s_t z_{it} \psi_t^n f(\frac{f_l^{-1}}{Z_l}, 1) f(1, \frac{Z_l}{Z_k} \frac{f_k^{-1}}{f_l^{-1}}) d\widehat{v}_t^n(z_{it})$$

If there are no firms capacity constrained, every firm has the same capital labor ratio, $\frac{Z_l}{Z_k}\frac{f_k^{-1}}{f_l^{-1}} = 1$, and the model boils down to the canonical firm dynamic one where

$$TFP_t = M_t \sum_n \int s_t z_{it} \psi_t^n f(\frac{f_l^{-1}}{Z_l}, 1) f(1, 1) d\widehat{v}_t^n(z_{it})$$

3.2.3 Industry Structure

With this characterization of the production possibility frontier of this economy, we can now describe the allocations of firms across processes, entry and exit.

If one computes the value for the planner of a change in the measure of firms operating technology n, one obtains

$$\frac{\gamma_t^n}{\mu_t} = \frac{\partial Y_t}{\partial M_t^n} + E_t \left[\widetilde{\beta}_{t+1} \frac{\gamma_{t+1}^n}{\mu_{t+1}} \right] \tag{1}$$

where the expectation is taken over the realizations of the aggregate state. From the optimality condition, $\frac{\gamma_t^n}{\mu_t}$ can be interpreted as the average contribution to aggregate output of a firm of type n at time t plus the discounted value of its average contribution tomorrow (valued at today's final goods).

Exit. In the efficient allocation the exit condition reads

$$\Pi_e^n(s_t) = \frac{\partial Y_t(\Xi_t)}{\partial z_t^e(\psi^n, \Xi_t)} \frac{1}{dv^n(z_t^e)} + E_t \left[\widetilde{\beta}_{t+1} VF_{t+1}^n(z_{t+1}, z_t^e(\psi^n, \Xi_t); \Xi_{t+1}) \right] \tag{2}$$

where $\widetilde{\beta}_{t+1} = \beta(1-\delta)\frac{\mu_{t+1}}{\mu_t}$ is the pricing kernel, $VF_{t+1}^n(z_{t+1}, z_t^e(\psi^n, \Xi_t); \Xi_{t+1})$ is the expected

[15] If we were to do a standard accounting exercise on aggregate output in this economy, total factor productivity would equal $h(TFP)$ under the assumption of additivity in intermediate inputs.

value of the firm with current productivity $z_t^e(\psi^n, \Xi_t)$ for the planner. It equals the value of the firm if it retains its process n, times the probability that the firm finds optimal to do so; plus the value of an upgraded firms minus the cost of upgrade, times the probability that it find optimal to upgrade, plus the scrap value of the firm adjusted by the probability of observing a low enough shock (see the appendix for an explicit expression). The expectation in 2 is computed over the aggregate shock. Aggregate shocks affect the thresholds for upgrade and exit tomorrow and hence the probability of each of those events occurring.

The contribution to output of the marginal firm is

$$\frac{\partial Y_t(\Xi_t)}{\partial z_t^e(\psi^n, \Xi_t)} \frac{1}{dv^n(z_t^e)} = h' \left(\int y_{it} di \right) s_t z_t^e \psi^n f(\frac{f_l^{-1}}{Z_l}, \frac{f_k^{-1}}{Z_k} K_t) - \alpha_t \rho \frac{Y_t}{K_t} k_{et}$$

In other words, it equals the output of the firm this period, minus the opportunity cost of capital allocated to that production unit. The planner values that cost as proportional to the **average productivity** of capital in the market. Under assumption 8, the expression $\alpha_t \frac{Y_t}{K_t}$ equals the marginal cost of capital λ_{kt} only when no firm in the market is constrained[16].

This a key object in the characterization of the allocation.

Upgrade. Optimality in upgrade thresholds is obtained when the cost of upgrade equalizes the gains in output from the upgrade, plus the discounted value of any future gains.

$$I_u^{n+1}(s_t) = \frac{\partial Y_t(\Xi_t)}{\partial z_t^u(\psi^{n+1}, \Xi_t)} \frac{1}{dv_t^n(z_t^u)} + E_t \left[\widetilde{\beta}_{t+1} \left(VF_{t+1}^{n+1}(z, z_t^u; \Xi_{t+1}) - VF_{t+1}^n(z, z_t^u; \Xi_{t+1}) \right) \right] \quad (3)$$

The derivative of aggregate output with respect to the upgrade threshold is the difference in the contribution to output of the marginal firm when operating technology n or $n+1$. Notice that the second term in the contribution of the firm to output is independent of the process it is running as long as the share of capital in total revenue is the same. Hence, when computing the difference in contribution the second term cancels out.

Entry. Finally, the efficient level of entry is obtained when the cost of entry equals the

[16] This fact becomes evident when we rewrite the contribution to output as

$$\frac{\partial Y_t(\Xi_t)}{\partial z_t^e(\psi^n, \Xi_t)} \frac{1}{dv^n(z_t^e)} = h' \left(\int y_{it} di \right) s_t z_t^e \psi^n f(\frac{f_l^{-1}}{Z_l}, \frac{f_k^{-1}}{Z_k} K_t) \{(1-\rho) - \alpha_t \rho \left(\frac{1}{K_t} \sum_n \int \frac{\lambda_t^k - \lambda_{it}}{\lambda_t^k - \lambda_{et}} k_{it} di \ - 1 \right) \}$$

When no firm is constrained, $\lambda_{it} = 0$ for all firms in the market, so that the contribution to output reduces to $h' \left(\int y_{it} di \right) s_t z_t^e \psi^n f(\frac{f_l^{-1}}{Z_l}, \frac{f_k^{-1}}{Z_k} K_t) (1-\rho)$.

expected value for the planner of an arbitrary firm.

$$\mu_t I(s_t) = \gamma_t^1 (G(z_{ut}^2) - G(z_{et}^1)) + \sum_{n=2}^{N-1} (\gamma_t^n - \mu_t I^n(s_t)) (G(z_{ut}^{n+1}) - G(z_{ut}^n)) + \left(\gamma_t^N - \mu_t I^N(s_t)\right) (1 - G(z_{ut}^N)) \tag{4}$$

The term $\frac{\gamma_t^n}{\mu_t}$ can be interpreted as the expected value for the planner of an arbitrary firm running process n. Hence, the optimality condition for entry equalizes the cost of entry, I_t to the expected social value of the firm, net of any cost it might incur in adopting a process.

4 Market Allocation

Before showing a mechanism that decentralized the efficient outcome as a market allocation, it is useful to understand why a market arrangement that is usually assumed in economies with heterogeneity and firm selection would fail to induce the efficient outcome.

The full description of the market structure can be found in the appendix. In this section I highlight the key features needed to understand the source of the inefficiency.

There is a representative consumer that rents capital and labor to the firms operating in the market at cost w_t and r_t respectively. The household trades shares of the firms operating in the market, and receives dividends from them at the end of each period (this dividends include any cost incurred in process adoption, entry or the liquidation value of the firms[17]). There is a representative firm producing in the final goods sector who purchases goods from the intermediate good producers.

Given the relevant aggregate state of the economy, $X_t = \left(s_t, \{v_t^n\}_{n=1}^N, K_t\right)$, intermediate good producers maximize the value of the firm. They choose capital and labor, given the restrictions on minimum capacity imposed by the technology, and decide which process to operate and when to exit. The free entry condition of the planner's allocation is imposed to pin down the level of entry.

The value of the firm is $W_t(x_t^n, X_t)$ when the aggregate state is X_t, the firm is operating process n, and its idiosyncratic state is $x_t^n = z_t \psi_t^n$.

If $n < N$, the value of the firm is

$$W_t(x_t^n, X_t) = Max\{\Pi_e^n(s_t), W_t(x_t^{n+1}, X_t) - I^{n+1}(s_t), \widetilde{W}_t(x_t^n, X_t)\} \tag{5}$$

[17] This market structure is analogous to the one chosen by Bilbiie *et al.* (2012)

subject to

$$X_{t+1} = \Gamma_f(X_t)$$

where $\widetilde{W}_t(x_t^n, X_t)$ is the continuation value of the firm when it decides to operate without upgrade in process and Γ_f is the perceived law of motion of the aggregate state for the firm. The continuation value is

$$\widetilde{W}_t(x_t^n, X_t) \ = \pi(x_t^n, X_t) + E_t\left[\widetilde{\beta}_{t+1} W_{t+1}(x_t^n, X_{t+1})\right]^{18}$$

where $\pi(x_t^n, X_t)$ are equilibrium profits and $\widetilde{\beta}_{t+1}(X_t, X_{t+1}) \equiv \beta\,(1-\delta)\,\frac{U'(C(X_{t+1}))}{U'(C(X_t))}$ is the stochastic discount factor of the household adjusted for the probability of survival of the firm, $\widetilde{\beta}_{t+1}$ to save notation. If the firm is operating the best technology, there is no possibility of upgrade so the second term in the value function W disappears.

Proposition 2 *$\widetilde{W}_t(x_t^n, X_t)$ is monotonic increasing in idiosyncratic productivity, z for any $n \leq N$. Hence, the optimal exit strategy of the firm is a trigger strategy such that if $z < z^e(\psi_t^n, X_t)$ the firm exits the market; if $z \geq z^u(\psi_t^{n+1}, X_t)$ the firm upgrades technology; if $z^e(\psi^n, X_t) \leq z < z^u(\psi^{n+1}, X_t)$ the firm produces using the n-th process. If the firm operates $n = N$, there is no further upgrade.*

Hence, as in the efficient allocation, the allocation of firms across technologies and the exit decisions are characterized by thresholds, $\left\{z^e(\psi_t^n, X_t), z^u(\psi_t^{n+1}, X_t)\right\}$ for $n \leq N$ and $t = 1, 2, .$Before comparing this policy to the efficient one, let me define a competitive equilibrium.

4.1 Competitive Equilibrium

Definition 1 *A competitive equilibrium is a system of thresholds $\left\{z^e(\psi_t^n, X_t), z^u(\psi_t^{n+1}, X_t)\right\}$ for $n \leq N$ and $t = 1, 2, \ldots$, distribution of firms $\{v_t\}_{t=0}^{\infty}$, a law of motion Γ for the aggregate state of the economy, $X_t = \left(s_t, \{v_t^n\}_{n=1}^{N}, K_t\right)$, a measure of entrants $\left\{M_t^{ent}\right\}_{t=0}^{\infty}$ with productivities drawn from $G(z)$, and consumption, aggregate capital and share holdings functions,*

$\left\{C(X_t), K_{t+1}(X_t), \{a^n(X_t)\}_{n=1}^{N}\right\}_{t=0}^{\infty}$ such that given the rental rates and the price of shares $\{r(X_t), w(X_t), P^n(X_t)\}_{t=0}^{\infty}$, the cost structure $\Upsilon_c(s_t) = \left[\{\Pi_e^n\}_{n=1}^{N}; \{I^n\}_{n=1}^{N-1}; I\right]$, the exogenous

[18] The expectation is computed over the realization of the aggregate and the idyosincratic shock $E_t\left(\widetilde{\beta}_{t+1} W_{t+1}(x_t^n, X_{t+1})\right) = \sum_{s_{t+1}\in S} P_s(s_{t+1}/s_t)\widetilde{\beta}_{t+1}(X_t, X_{t+1}) \int P_z(z', z) W_{t+1}(x_t^H, X_{t+1})dz'$.

laws of motion for aggregate shocks, $\mathbf{P}_s$*, and idiosyncratic shocks,*$\mathbf{P}_z$*; and the initial stock of capital in the economy* K_0 *and share holdings,* $a_0^n = 1\ \forall n \leq N$*,*

i) The representative consumer maximizes utility (as in (14)*)*

ii) Firms in the intermediate goods sector maximize their value (as described by 5)

iii) Firms in the final good sector maximize profits.

iv) Free entry holds, as in (25)

v) $M_t = M_t^{ent} + \sum_{n=1}^{N} (1-\delta)\, M_{t-1}^n - M_{nt}^n$ *where* $M_t = \sum_{n=1}^{N-1} v_t^n(z^u(\psi_t^n, X_t)) + v_t^N(\overline{z})$*.*

vi) Markets clear

(a) $\sum_{n=1}^{N} \int l(x_t, X_t) dv_t^n(z_{it}) = 1$

(b) $\sum_{n=1}^{N} \int k(x_t, X_t) dv_t^n(z_{it}) = K_t$

(c) $a_t^n = 1$*,* $\forall n \leq N$ *and* $t = 1, 2, ...$

(d) Feasibility in the goods market.

vii) Consistency for the law of motion of the aggregate state: $\Gamma = \Gamma_f = \Gamma_c$*.*

4.2 Market Allocation versus Efficient Allocation

The main difference between the efficient allocation vis a vis the market allocation stems from the allocation of firms across processes, exit and entry patterns. The reason is that the opportunity cost of capital in the efficient allocation is equalized to the average product of capital, and not to the marginal product of capital, the opportunity cost that firms account for in the market.

When there is marginal product dispersion, the entry of a new firm in the market possibly spans a whole new dimension of transfers across production units. The planner accounts for this by using the average cost of capital as the relevant opportunity cost of capital. When there is no dispersion, the opportunity cost of capital is identical for all the firms in the market, the average and marginal products coincide and hence the market allocation is efficient.

Disparities in the industry structure are important because they determine the equilibrium distribution of firms observed in the market, and the through it, affect the allocation of capital and labor across firms, equilibrium factor prices and the incentives for aggregate capital accumulation. All of these are described in the appendix.

In a previous section we showed that the exit condition in the efficient allocation is

$$\Pi_e^n(s_t) = \frac{\partial Y_t(\Xi_t)}{\partial z_t^e(\psi^n, \Xi_t)} \frac{1}{dv^n(z_t^e)} + E_t\left[\widetilde{\beta}_{t+1} VF_{t+1}^n(z_{t+1}, z_t^e(\psi^n, \Xi_t); \Xi_{t+1})\right]$$

The exit condition in the market allocation is

$$\Pi_t^n(s_t) = \pi(x_t^n, X_t) + E_t[\widetilde{\beta}_{t+1} W_{t+1}(x_t^n, X_{t+1})]$$

where the second term is the discounted value of the firm for every realization of the aggregate and idiosyncratic shocks.

If one computes the value for the planner of a change in the measure of firms operating technology n, one obtains

$$\frac{\gamma_t^n}{\mu_t} = \frac{\partial Y_t}{\partial M_t^n} + E_t\left[\widetilde{\beta}_{t+1}\frac{\gamma_{t+1}^n}{\mu_{t+1}}\right] \tag{6}$$

where the expectation is taken over the realizations of the aggregate state. Hence, $\frac{\gamma_t^n}{\mu_t}$ can be interpreted as the average contribution to aggregate output of a firm of type n at time t plus the discounted value of its average contribution tomorrow (valued at today's final goods). Given the definition of the expected value of the firm for the planner, VF_{t+1} and the recursion on the shadow value of a firm with technology n (6), disparities in the exit threshold between the market and the efficient allocations stem from differences in profits vis a vis the contribution of the firm to total output.

The equilibrium profit function dictates

$$\pi(z_t^e\psi^n, X_t) = h'\left(\int y_{it}di\right) s_t z_t^e \psi^n f\left(\frac{f_l^{-1}}{Z_l}, \frac{f_k^{-1}}{Z_k}K_t\right)\{(1-\rho) - \alpha_t\rho\left(\frac{r_t}{MPK_t^e} - 1\right)\} \tag{7}$$

In the efficient allocation, the contribution to output of the marginal firm is

$$\frac{\partial Y_t(\Xi_t)}{\partial z_t^e(\psi^n, \Xi_t)} \frac{1}{dv^n(z_t^e)} = h'(\int y_{it}di) s_t z_t^e \psi^n f(\frac{f_l^{-1}}{Z_l}, \frac{f_k^{-1}}{Z_k}K_t)\{(1-\rho) - \alpha_t\rho(\frac{1}{K_t}\sum_n \int \frac{\lambda_t^k - \lambda_{it}}{\lambda_t^k - \lambda_{et}} k_{it}di \; -1)\}$$

When the marginal product of capital of all firms equals the cost of capital in the market, the second term drops from both expressions and the profits of the firm coincide with the firm contribution to aggregate output. Hence, the optimal exit threshold is the same across allocations as the optimality conditions coincide. The optimality conditions are the same because given the recursion in 6, the expected value of a firm for the planner is the discounted expected value of profits. When there is a least one firm constrained by the minimum capacity requirement, the term in curly brackets differ. Proposition 4 describes the implications for the behavior of exit thresholds.

Proposition 3 *Suppose that there is at least one firm constrained by the minimum capacity. If the marginal firm exiting the market, given a particular process, is not constrained, the planner has an additional incentive to keep the firm active vis a vis the firm. Ceteris paribus, the exit threshold for the planner is lower than the one in the market allocation. If the marginal firm is constrained by the minimum capacity, and the marginal product of capital of the constrained firm is the same in the efficient and market allocation, the exit threshold differs. Ceteris paribus, it is lower (higher) in the efficient allocation than it is in the market allocation if* $\frac{1}{K_t}\sum_n \int \left(\lambda_t^k - \lambda_{it}\right) k_{it} di < (>) r_t$.

Upgrade. Optimality in upgrade thresholds dictates

$$I_u^{n+1}(s_t) = \frac{\partial Y_t(\Xi_t)}{\partial z_t^u(\psi^{n+1}, \Xi_t)} \frac{1}{dv_t^n(z_t^u)} + E_t\left[\widetilde{\beta}_{t+1}\left(VF_{t+1}^{n+1}(z, z_t^u; \Xi_{t+1}) - VF_{t+1}^{n}(z, z_t^u; \Xi_{t+1})\right)\right] \quad (8)$$

The derivative of aggregate output with respect to the upgrade threshold is the difference in the contribution to output of the marginal firm when operating technology n or $n+1$. If this difference coincides with the difference in profits, the optimality conditions in the efficient and the market allocation coincide. In general, when the equilibrium displays marginal product dispersion they do not.

Proposition 4 *Suppose that there is at least one firm constrained by the minimum capacity. Ceteris Paribus, if the marginal firm upgrading process is not constrained and the share of capital in total revenue is the same across processes, the decision for upgrade is the same for the planner and the firm. Ceteris paribus, if the firm upgrading technology is constrained by the minimum capacity after the upgrade, and the marginal product of capital is the same across allocations, the threshold for upgrade is lower (higher) in the efficient allocation than it is in the market allocation if* $\frac{1}{K_t}\sum_n \int \left(\lambda_t^k - \lambda_{it}\right) k_{it} di < (>) r_t$.

Entry. When the value of an arbitrary firm is the same in the market and efficient allocation, the free entry condition induces efficiency in entry. Otherwise,

Proposition 5 *Suppose that there is at least one firm constrained by the minimum capacity, and that firms that have upgraded are unconstrained. If the marginal firm exiting the market for the less productive process is not constrained, ceteris paribus, the planner generates more entry than the market allocation. If the marginal firm is constrained by the minimum capacity, and its marginal product of capital is the same in the efficient and market allocation, ceteris paribus, the planner generates more (less) entry than the market allocation if* $\frac{1}{K_t}\sum_n \int \left(\lambda_t^k - \lambda_{it}\right) k_{it} di < (>) r_t$.

The second assumption at the beginning of the proposition is key. For example, if the shadow value of capital for firms that upgrade to a given process is higher (lower) in the market allocation than in the planner's one, the upgrade threshold is lower (higher) in the market allocation. The expected value of the firm in the market is higher (lower) in the market allocation inducing more (less) entry than the efficient one. The phenomenon can occur for any of the process available, possibly with different directions. To assess whether entry levels are lower or higher than the efficient level a general equilibrium assessment is necessary.

5 Decentralization

The failure of the first welfare theorem under the current market structure, can be solved in several ways. One way to complete markets would be to allow for vertical integration of the final and intermediate producer. Whereas possible, this decentralization is bind to generate the centralized solution almost by assumption. Furthermore, it requires the final good producer to gather a lot of information as of the proceeds of each single production unit in the market. It needs to observe its capital demand, but also the realization of the productivity shock of the firm. Notice that when there is marginal product dispersion it is possible for two firms with different levels of productivity to generate the same profits or dividends, i.e. a high productivity constrained firm, and a low productivity unconstrained one. With marginal product equalization in this economy, dividends map one to one to the realization of the shock.

In this section I propose a simpler decentralization mechanism. To implement it, it is not necessary to know the idiosyncratic productivity of the firm at any point in time. It is however necessary to know the process they are operating and the aggregate state of the economy. Augment the cost structure to account for transfers, i.e. $\widehat{\Upsilon}_p(s_t) = \left[\{\Pi_e^n\}_{n=1}^N; \{I^n\}_{n=1}^{N-1}; I; T\right]$

The efficient allocation was solved for a cost structure $\widehat{\Upsilon}_p = \left[\{\Pi_e^n\}_{n=1}^N; \{I^n\}_{n=1}^{N-1}; I; 0\right] = \Upsilon_c$.

The idea of this decentralization is to change such cost structure in the market allocation $\Upsilon_c \neq \Upsilon_p$, so that the market and efficient allocations produce the same distribution of firms across technologies, and the same number of firms operating in the market. I do this indirectly. First, I solve a modified centralized problem whose allocation coincides with the market allocation. Then I show how to choose Υ_c to generate the efficient outcome.

Define an alternative centralized problem as follows

$$V(\Xi_t) = \max_{C_t,\ K_{t+1},\ Y_t,\ \{z_t^e(\psi^n)\}_{e=1}^N,\ \{z_t^u(\psi^n)\}_{n=2}^N, M_t^{ent}, l_{it}, k_{it}} U(C_t) + \beta E V(\Xi_{t+1}) \tag{Pseudo-Planner Problem}$$

subject to

$$C_t + K_{t+1} - (1-\widehat{\delta})K_t + IM_t^{ent} + \text{Upgrade Costs} = Y_t + T_t + \sum_{n=1}^{N} \Pi_e^n M_{et}^n$$

$$h\left(\sum_j \int s_t z_{it} \psi^j f(l_{it}, k_{it}) di\right) = Y_t$$

$$\int l_i di = 1, \text{ and } \int k_i di = K_t$$

$$k_i \geq \underline{k} \text{ if } \psi_i = \psi^n$$

$$v_{t+1} = T^{*S}(v_t) \text{ and the transitions } P_z \text{ and } P_s$$

where the main difference with the problem of efficiency is a transfer T_t that a planner takes a given.

Theorem 2 *a) For a given transfer scheme $\widehat{\Upsilon}_p$, the solution to this centralized problem exists and it is unique.*

b) There exist a cost structure $\left\{\widehat{\Upsilon}_p(s_t)\right\}_{t=0}^{\infty}$ such that the allocation of firms that solves this modified planner's problem coincides with the competitive allocation.

The argument for part (a) is the same as in the Theorem 1 and hence omitted. For part b), the proof has two steps. Analogous to Jones and Manuelli (1990), first I define an operator on the transfers, $\Omega(T(\Xi_t))$ and prove that it has a fixed point. At the fixed point, the feasibility constraint of the planner and competitive equilibrium are the same. Second, I need to define prices and a cost structure such that the optimality conditions hold in both cases. The price of capital and salaries are defined such that the optimal consumption and capital accumulation paths for the representative consumer coincide with those predicted by planner. To assure that the allocation of firms coincides, I use the linearity of the optimality conditions in both the market and the centralized problem. I show that one can define a unique set of subsidies/taxes, $\widehat{\tau}(\Xi_t)$ such that the thresholds of the decentralized problem satisfy the necessary conditions of the planner. I show that the transfer generated by $\widehat{\tau}\ (\Xi_t)$, $\mathbf{T}(\widehat{\tau}\ (\Xi_t))$ is a fixed point of Ω. Hence, the equivalence is proven. Note that if the equilibrium was Pareto optimal, then $\widehat{\tau}(\Xi_t)$ should be equal to zero across all states.

Corollary 1 *The solution to the competitive equilibrium exists*

Corollary 2 *The efficient allocation can be decentralized as a competitive allocation whenever state contingent subsidies/taxes are available.*$\widehat{\tau}^{\ c}(\Xi_t)$

The linearity in the optimality conditions of the firm allows me to recover the policy that would generate the efficient outcome as a market allocation.

6 Application

As explained when comparing the market and efficient allocation, the private and social value of a firm may in general differ when the equilibrium displays marginal product dispersion.

This section analyzes the contribution of a firm to output vis as vis its profits, for alternative distributions of exogenous idiosyncratic productivity, z_{it} and shadow value of capital, λ_{it}. To simplify the analysis I assume there are only two processes available, and technologies are Cobb-Douglas, being α the share of capital in value added. Under this assumption, the profits of the marginal firm in the market operating process n are

$$\pi(x_t, X_t) = \frac{Y_t}{Z^l}\left(\frac{z\psi^n}{MPK_t^{\alpha\rho}}\right)^{\frac{1}{1-\rho}}\left\{(1-\rho) - \alpha[\frac{r_t}{MPK_t} - 1]\right\} \tag{PV}$$

The contribution of the firm to aggregate output is

$$\frac{\partial Y_t(\Xi_t)}{\partial z_t^e(\psi^n, \Xi_t)}\frac{1}{dv^n(z_t^e)} = \frac{Y_t}{Z^l}(\frac{z\psi^n}{(MPK_t)^{\alpha\rho}})^{\frac{1}{1-\rho}}\left\{(1-\rho) - \alpha[\frac{Z^l}{Z^k}\frac{1}{MPK_t} - 1]\right\} \tag{SV}$$

The disparities that we have described generally in the previous section hold here. I first compare SV to PV for a given a distribution of firms productivity in the market and a distribution of shadows values of capital. I construct shadow values so that only firm using the worse technology are constrained. The distribution of productivities and shadows values are depicted in the top three panels of Figure 1.

The bottom right panels in Figure 1 are constructed such that as we move along the horizontal axis to the right, less firms are constrained. The blue line is the ratio of SV to PV for a given marginal product of capital of the marginal exiting firm. It is lower than one for all realizations indicating that the foregone output in the planner's allocation is higher than that accounted in the market allocation. Hence, if the scrap values are the same, the threshold for exit has to be higher in the market allocation than it is in the efficient allocation. When there are no constrained firms in the market both values coincide. The measure of dispersion in dispersion in marginal product of capital is lower as we move to right of the panel.

Next I allow for lower shadow value of capital for firms that have already upgraded. I start with an economy in which firms that have upgraded have no low marginal product of capital. I simulate alternative distributions for marginal product of capital such that I replace the marginal product of capital of the most productive firms running the worse technology (I set it equal to the interest rate in the market), and let firms that have upgraded have lower marginal product of capital (be constrained). I generate the replacement such that the dispersion in marginal products of capital is the same across allocations. In the last two realizations of the distributions of marginal products I drops its dispersion by allowing more firms operating the worse technology to be unconstrained.

Figure 2 depict the results of the simulations. Although the first 5 simulations have distributions of marginal product of capital with the same dispersion, the market and the efficient allocation depart from each other. As more firms operating the better technology are constrained SV gets relatively larger than PV, indicating that the losses in efficiency do not depend only on the observed dispersion but on the identity of the firms that have lower marginal product than the interest rate. In the last 2 simulations, the dispersion is lower than before. While the gap between SV to PV closes initially, the market value of the firm is further away from the social value, than in the simulation with higher dispersion in marginal product.

In these exercises, I have taken the distribution of shadow values of capital exogenously. It is expected that the disparities between SV and PV are reinforced or smoothed as the distribution in marginal products is allowed to vary endogenously in general equilibrium.

7 Conclusion

This paper contributes to the extensive literature linking disparities in marginal product of capital to differences in aggregate productivity across economies. It builds a formal framework for the study of efficiency in economies where dynamically optimal investment decisions of firms operating under uncertainty and endogenous firm selection, can generate dispersion in marginal products as an equilibrium outcome.

First, I show that a market arrangement that would induce the efficient allocation in an economy with no equilibrium dispersion in marginal product of capital, fails to generate the efficient outcome when the allocation displays dispersion in marginal products. The distribution of marginal products is a state of the economy and agents should be allowed to trade upon them for markets to be complete. I sidestep the absence of those assets, by providing a decentralization result that relies on changing entry and upgrade costs, as well as scrap values of firms, to generate the efficient allocation of firms across technologies.

Second, I show that it is possible to construct economies with higher marginal product dispersion, that are closer to the efficient allocation than comparable economies with lower dispersion in marginal products. This feature highlights the importance of studying the connection between marginal product dispersion and aggregate productivity within a general equilibrium framework, where the efficient allocation can be characterized.

While the focus of this paper is solely on marginal product of capital dispersion, it is known that similar indivisibilities and indivisibilities in investment are present in labor markets. Example of those are overhead labor costs, and firing costs. It is likely that dispersion in marginal product of labor and capital interact with each other, possibly to compensate one another. Whether higher joint dispersion in marginal product of labor and capital is detrimental for aggregate productivity and welfare remains to be shown. Likewise, the interactions between this source of marginal product dispersion (a technological one) with others such as financial frictions, remains to be studied.

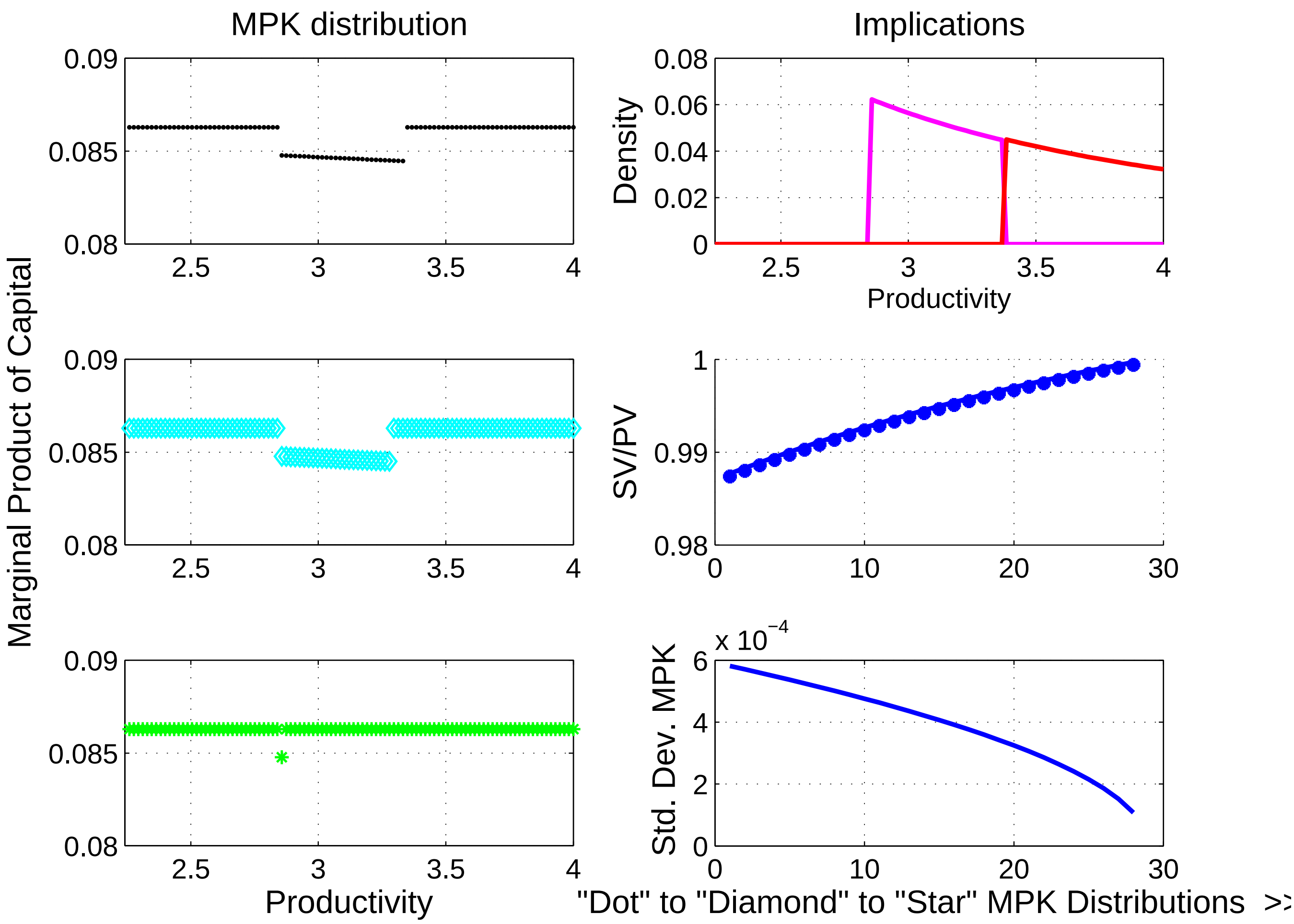


Figure 1: Social value versus Private value of the firm

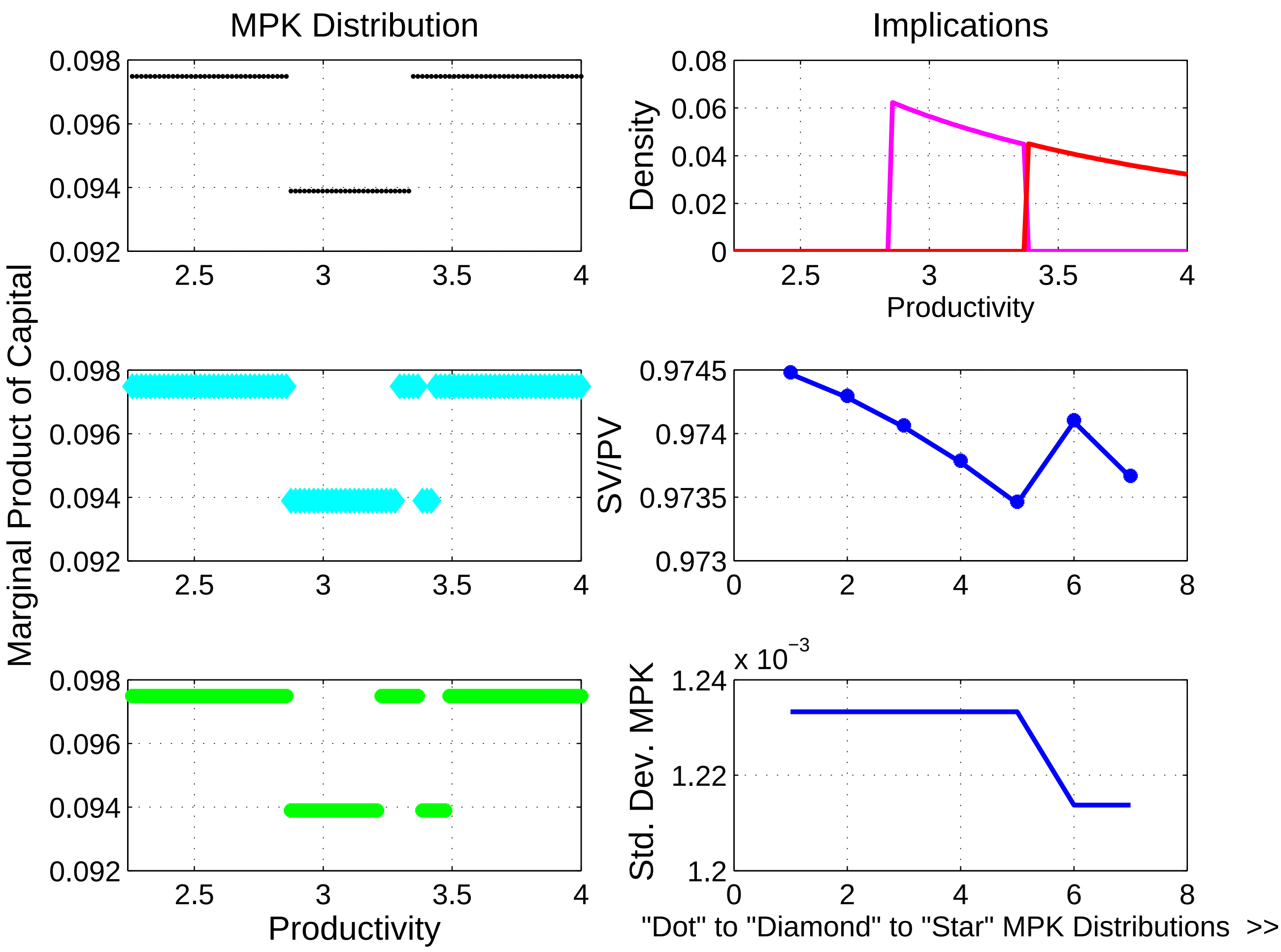


Figure 2: Social value versus Private value of the firm: Alternative Dispersion in MPK

Technical Appendix

7.1 Probability Measures

7.1.1 Existence and Uniqueness, $v^* : \mathcal{A} \rightarrow \mathcal{A}$

Proof of Proposition 2. To prove it, I use Theorem 2 in Hopenhayn and Prescott (1992).

To do use the theorem I need to add an order to the space $\mathcal{A}$. Define the order "$\geq$ " such that $a' \geq a$ if and only if $a' = \overline{a} \equiv \{\overline{z}, N\}$ or $a = \underline{a} \equiv \{\underline{z}, 0\}$ or $z' = z$ and $n' > n$. Then,($\mathcal{A}$,$\geq$) is an ordered space. If we add the Euclidean measure, $\mathcal{A}$ is a complete metric space.

First, Λ is a transition function, as it is a probability measure on ($\mathcal{A}$,$\mathcal{B}$) and is a measurable function. This is true because both P_z and P_s are measurable and the composition of measurable functions is measurable, as well as because of the continuity of $n'(n, z')$. Continuity of n' implies that for all $SxZxN \in \mathcal{B}(\mathcal{A})$, $\{(z, n) \in A : \Lambda((z, n)) \in ZxN\} \in \mathcal{B}(\mathcal{A})$.

Second, Λ satisfies monotonicity. In other words for any increasing, measurable and bounded function $f : \mathcal{A} \rightarrow \mathbb{R}$

$$\text{if } a_1 \geq a_2 \text{ then } (Tf)(a_1) = \int_{a' \in A} f(a')\Lambda(a_1, da') \geq (Tf)(a_2)$$

is increasing.

Under Assumption 9, z are not serially negatively correlated. The optimal policy dictates that firms using better processes (higher n) exit "after" firms with worse processes. Hence, if $a_1 > a_2$, Λ assigns measure zero to realizations of z' that have positive measure under n_1. Upgrades are costly, so conditional on the productivity level z', if a firm with technology n upgrades to technology $n + 2$ it is also optimal for the firm with process $n + 1$ to upgrade to $n + 2$. The latter implies that the policy function $n'(n, z')$ is non-decreasing, which completes the proof of monotonicity of Λ.

Finally we need to check that the monotone mixing condition is satisfied. Stated formally, there exist an element $a^* \in A$ and integer t such that

$$\Lambda^t(\overline{a}, [\underline{a}, a^*]) > 0 \text{ and } \Lambda^t(\underline{a}, [a^*, \overline{a}]) > 0$$

To show this property, assume $a = \underline{a}$ and take a sequence of "good shocks",$\{\overline{z}\}_{\tau=1}^{t}$The optimal policy and the cost structure (Assumption 7) dictates that the firm upgrades technologies to reach the best technology available. Hence, $a \in [a^*, \overline{a}]$. Suppose instead that we start with $a = \overline{a}$ and take a sequence of 'bad shocks" $\{\underline{z}\}_{\tau=1}^{t}$ While there is no downgrade in the process operated by the firm, the optimal policy dictates that the firm exits. Hence, $a \in [\underline{a}, a^*]$.

Under this three assumption, T^* has a unique fixed point in the space of measures. ■

7.1.2 Projections on the Space N (Law of Motion,Λ^n)

Let the measure of exits at any point in time

$$M_{et}^{1}(X_{t-1}, X_t) = (1-\delta)\int_{z^e(\psi_t^n, X_t)\geq z}\int v_{t-1}^{1}(z_{-1})P_z(z, z_{-1})dz_{-1}dz + M_t^{ent}G(z^e(\psi^1, X_t))$$
$$M_{et}^{n}(X_{t-1}, X_t) = (1-\delta)\int_{z^e(\psi_t^n, X_t)\geq z}\int v_{t-1}^{n}(z_{-1})P_z(z, z_{-1})dz_{-1}dz$$

In other words, it equals the measure of firms whose current idiosyncratic productivity component is below the current exit threshold plus entrants whose productivity draw is lower than the exit threshold for the lowest technology available.

The measure of incumbent upgrades equals

$$M_{ut}^{n}(X_t) = (1-\delta)\left[\int_{z\geq z^u(\psi_t^n, X_{t-1})}\int v_{t-1}^{n-1}(z_{-1})P(z, z_{-1})dz_{-1}dz\right] \quad \forall n > 1$$

the measure of firms running technology $n-1$ whose current realization of idiosyncratic productivity is above the upgrade threshold for process n.

With these definitions we can characterize the law of motion of the distribution of firms per process.

If $1 < n < N$ the law of motion is characterized by

$$\begin{aligned}
&v_t^n(\widehat{z}) = (1-\delta)\int_{z^e(\psi_t^n, X_t)}^{\widehat{z}}\int_{z^e(\psi_{t-1}^n, X_{t-1})}^{z^u(\psi_{t-1}^{n+1}, X_{t-1})} P(z, z_{-1})dv_{t-1}^{n}(z_{-1})dz && z^u(\psi_t^n, X_t) > \widehat{z} > z^e(\psi_t^n, X_t)\\
&v_t^n(\widehat{z}) = (1-\delta)\int_{z^u(\psi_t^n, X_t)}^{\widehat{z}}\int_{z^e(\psi_{t-1}^n, X_{t-1})}^{z^u(\psi_{t-1}^{n+1}, X_{t-1})} P(z, z_{-1})dv_{t-1}^{n}(z_{-1})dz + && z^u(\psi_t^{n+1}, X_t) > \widehat{z} \geq z^u(\psi_t^n, X_t)\\
&\quad + M_{ut}^n + M_t^{ent}\left(G(\widehat{z}) - z^u(\psi_t^n, X_t)\right) && \\
&v_t^n(\widehat{z}) = 0 && o/w
\end{aligned} \tag{9}$$

If $n=1$ the law of motion is

$$
\begin{aligned}
v_t^n(\widehat{z}) &= (1-\delta)\int_{z^e(\psi_t^n,X_t)}^{\widehat{z}}\int_{z^e(\psi_{t-1}^n,X_{t-1})}^{z^u(\psi_{t-1}^{n+1},X_{t-1})} P(z,z_{-1})dv_{t-1}^n(z_{-1})dz \\
&\quad +M_t^{ent}\left(G(\widehat{z})-G(z^e(\psi^n,X_t))\right) && z^u(\psi_t^{n+1},X_t) > \widehat{z} > z^e(\psi_t^n,X_t) \\
v_t^n(\widehat{z}) &= 0 && o/w
\end{aligned}
$$

For $n=N$, the law of motion is

$$
\begin{aligned}
v_t^n(\widehat{z}) &= (1-\delta)\int_{z^e(\psi_t^n,X_t)}^{\widehat{z}}\int_{z^e(\psi_{t-1}^n,X_{t-1})}^{z^u(\psi_{t-1}^{n+1},X_{t-1})} P(z,z_{-1})dv_{t-1}^n(z_{-1})dz && z^u(\psi_t^n,X_t) > \widehat{z} > z^e(\psi_t^n,X_t) \\
v_t^n(\widehat{z}) &= (1-\delta)\int_{z^u(\psi_t^n,X_t)}^{\widehat{z}}\int_{z^e(\psi_{t-1}^n,X_{t-1})}^{z^u(\psi_{t-1}^{n+1},X_{t-1})} P(z,z_{-1})dv_{t-1}^n(z_{-1})dz \\
&\quad +M_{ut}^n+M_t^{ent}\left(G(\widehat{z})-z^u(\psi_t^n,X_t)\right) && \overline{z} > \widehat{z} \geq z^u(\psi_t^n,X_t) \\
v_t^n(\widehat{z}) &= 0 && o/w
\end{aligned}
\tag{10}
$$

In other words, the measure of firms running process n with productivity at most $\widehat{z}$, equals the measure of firms operating in the previous period whose current idiosyncratic productivity is larger than the current exit threshold and at most $\widehat{z}$ minus the measure of exogenous liquidations, plus the measure of entrants with productivity up to $\widehat{z}$ if $\widehat{z}$ is larger than the upgrade threshold, and the measure of upgrades. If $n=1$ there are no upgrades for incumbents, and if $n=N$ there is no option for further upgrade.

Proof of Lemma 1. I split the proof in two. First, I show that the measure of absolutely continuous with respect to the lebesque measure on the real line, hence continuous. Then I show that the measure is bounded. ■

Lemma 2 (AC) *The measure associated to the distribution of types is absolutely continuous(AC) with respect to the lebesque measure on the real line*

Proof. The claim follows from the absolute continuity of the exogenous distribution of types. We prove by induction.

By definition

$$v_0^1(z) = \left[\frac{G(z)-G(z^e(\psi_0^1,X_0))}{1-G(z^e(\psi_0^1,X_0))}\right]$$

$$v_0^n(z) = \left[\frac{G(z)-G(z^u(\psi_0^n,X_0))}{1-G(z^e(\psi_0^1,X_0))}\right]\ \forall n>1$$

Take a sequence of of intervals $(a_k, b_k)_{k=1}^K$ and let

$$\sum_{k=1}^{K} |v_0^n(b_k) - v_0^n(a_k)| \leq \varepsilon$$

Replacing by the definition

$$\sum_{k=1}^{K} \left| \frac{1}{1 - G(z^e(\psi_0^1, X_0))} (G(b_k) - G(a_k)) \right| \leq \varepsilon$$

Let $\widehat{\varepsilon} = \varepsilon \left[1 - G(z^e(\psi_0^1, X_0))\right]$ .By absolute continuity of G, there exist $\widehat{\delta}$ such that

$$\sum_{k=1}^{K} |b_k - a_k| \leq \widehat{\delta}$$

Because ε was arbitrary, and $(a_k, b_k)_{k=1}^K$ too, v_0^n is absolutely continuous.

Suppose v_T^n is absolutely continuous. By definition,$v_{T+1}^n(z)$ follows either 9, **??** or 10. Hence, it is the sum of absolutely continuous functions which process that v_{T+1}^n is absolutely continuous. By induction, v_t^n is absolutely continuous for arbitrary t. ■

Lemma 3 (M) *The feasible measure of firms in the market is bounded*

Proof. By definition, the total measure of firms in the market is $M_t = \sum_{n=1}^{N-1} v_t^n(z^u(\psi_t^{n+1}, X_t), s_t) + v_t^N(\overline{z}, s_t)$. Using the aggregation results, one could right the feasibility constraint of the economy as

$$\begin{aligned} C_t = h(M_t \widetilde{TFP}_t f(1, K_t))) - K_{t+1} + (1 - \widehat{\delta})K_t \\ + \sum_{n=1}^{N} \Pi_e^n M_{et}^n \ - (IM_t^{ent} + \text{Upgrade Cost}) \\ M_t = (1 - \delta) M_{t-1} + M_t^{ent} - \sum_{n=1}^{N} M_{et}^n \end{aligned}$$

where $\widetilde{TFP}_t = TFP_t M_t^{-1}$.

A strategy to make the measure of firms grow without bound would be to never exit firms and enter as much as possible. Now, because entry is costly, optimality dictates that the marginal cost of an entrant equalizes the marginal return,

$$h'(((1 - \delta) M_{t-1} + M_t^{ent})\widetilde{TFP}_t f(1, K_t))) = I_t$$

which pins down a finite level of entry M_t^{ent} at each t. Replacing the entry level into the dynamic equation for the measure of firms we obtain

$$M_t = \frac{(h')^{-1}(I_t)}{\widetilde{TFP}_t f(1, K_t)}$$

which is bounded as f displays decreasing returns to scale in capital.

Alternatively, a strategy to make the measure of firms shrink without bound would be to never enter firms and exit as many as possible. Such strategy implies that the number of firms equals zero in finite time. Under Assumption 2 $h\prime(0) \to \infty$ and $h(0) = 0$. Hence, such strategy is not feasible.

Existence and Uniqueness of the centralized allocation ■

Before moving to the next result define Θ as the set of bounded absolutely continuous functions from $A \equiv \mathcal{S}\text{x}\mathcal{Z}\text{x}\mathcal{N} \rightarrow R^+$. Hence, $v_t \in \Theta\ \forall t$ as shown in Lemma 1. Let, $\overline{K} \subset R$ the feasible set for capital. Because there are decreasing returns to capital in the aggregate and there is no growth in the economy, it is without loss of generality to assume $\overline{K}$ is compact.

Lemma 4 (U) *$U : R^+ \rightarrow R^+$ is bounded.*

Proof. $U(C_t)$ can potentially be unbounded above or below. However, the feasible measure of firms in the market is always bounded above and away from zero (Lemma (M)). Also, due to decreasing returns in capital, the aggregate level of capital is bounded. Finally, the sets $\mathcal{S}$ and Z and $n' : \mathcal{S}\text{x}\mathcal{Z}\text{x}\mathcal{N} \rightarrow \mathcal{N}$ is continuous, hence bounded too. From the feasibility condition in the economy, aggregate consumption is bounded and under assumption 1 (continuity) $U(C_t)$ too. ■

7.2 Efficient Allocation (existence)

Proof of Theorem 1. We can write the planner's problem in terms of the operator $\digamma$ as

$$\digamma V(\Xi_t) = \underset{(v_t, K_{t+1}) \in \Gamma^*(s_t, v_{t-1}, K_t)}{Max} U(C(s_t, v_{t-1}, K_t, v_t, K_{t+1})) + \beta E_t[V(\Xi_{t+1})]$$

Let $H(\mathcal{S}x\Theta x\overline{K})$ be the set of functions (functional) $f : \mathcal{S}x\Theta x\overline{K} \rightarrow R$ continuous except potentially at the origin and bounded in the norm

$$\|f\| = \sup_{\|\Xi_t\|=1, \Xi_t \in \mathcal{S}x\Theta x\overline{K}} \|f(\Xi_t)\|$$

$\digamma : H(\mathcal{S}x\Theta x\overline{K}) \to H(\mathcal{S}x\Theta x\overline{K})$.

From Lemma (U) and Assumption 1 we know that U is bounded and continuous.

The state space $\mathcal{S}x\Theta x\overline{K}$ is compact. We have shown that Λ^s is a probability measure and hence satisfies the Feller property. We know that T^{*S} (the transition function) maps a convex compact set into itself (Lemma AC). $T^{*S} : \Theta \to \Theta$.

Hence $\digamma$ is a contraction, with a unique fixed point in $\mathcal{S}x\Theta x\overline{K}$. ■

7.3 Market Structure

7.3.1 Households

The representative household derives utility from consumption of the final good C_t.

The household is endowed with a unit of labor that for simplicity, is assumed to be supplied inelastically to the firms in the economy. She receives a wage w_t for those services. She can also accumulate capital K_t, priced in terms of the final good (the numeraire) and rent it at price r_t. The aggregate stock depreciates at rate $\widehat{\delta}$. Finally, the household can buy shares of N different mutual funds that entitle it to the dividends generated by the firms operating alternative processes in the economy. After dividends are paid, mutual funds shares a_t^n can be traded.

Her problem reads

$$\max_{C_t, n_t^L, n_t^H, K_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{11}$$

subject to

$$C_t + K_{t+1} - (1-\widehat{\delta})K_t + \sum_{n \leq N} P_t^n a_t^n = w_t + r_t K_t + \sum_{n \leq N} \left(d_t^n + P_t^n\right) a_{t-1}^n$$

$$X_{t+1} = \Gamma_c(X_t)$$

where P_t^n is the price of shares a_t^n of a mutual fund of firms operating process n at period t+1, which pay dividends d_{t+1}^n and can be sold tomorrow at price P_{t+1}^n. The discount factor is $\beta \in (0,1)$. In computing the return to the share holdings, the agent needs to forecast the law of motion of the distribution of firms in the market for each possible realization for the exogenous aggregate shock, s_t. The aggregate state of the economy $X_t = \left(s_t, \{v_t^n\}_{n=1}^N, K_t\right)$ entails the exogenous shock, s_t; the distribution of firms per process, v_t^n for $n \leq N$; and the available aggregate stock of capital. I denote $v_t^n(z, s_t) \equiv v_t^n([\underline{z}, z), s_t)$ the measure of firms with productivity at most z and technology n when the aggregate state is s_t. The subjective law of motion of the aggregate state for the representative consumer is denoted by Γ_c.

The optimality conditions of the problem are standard. The price of shares is the present discounted value of all future dividends of the portfolio of firms in period $t+1$with technology n, adjusted by the corresponding pricing kernel

$$P_t^n = E_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \frac{U'(C_\tau)}{U'(C_t)} d_\tau^n$$

7.3.2 Final Goods Sector

There is a representative competitive firm with a technology, $h : \mathbb{R}^+ \to \mathbb{R}^+$ that transforms intermediate inputs y_{it} into a final goods Y_t. Each intermediate good producer is identified with i. The final good producer takes the set of producers as given, and maximizes profits.

$$Max_{y_{it}}\ Y_t - \int p_{it} y_{it}\ di$$

subject to

$$Y_t \leq h\left(\int y_{it} di\right)$$

where p_{it} is the cost of good y_{it}. Intermediate goods are perfect substitutes in production of the final good, in the sense that only the total number of intermediate good produced matters for final good production[19].

The corresponding input demand for each variety i is determined by the FOC of the problem, i.e.

$$h'\left(\int y_{it} di\right) = p_t$$

Therefore the marginal cost of each intermediate good should be equalized.

7.3.3 Intermediate Goods

Firms use capital and labor to produce a homogeneous intermediate good y_t. The technology for production combines capital and labor according to f and productivity is Hicks neutral.

[19]With a continuum of intermediate producers this assumption allows me to avoid defining the production function h over a continuum of types. One could have assumed an arbitrary substitution pattern and a CES aggregator or alternatively, assumed a finite number of intermediate goods, and a very general substitution pattern across them.

$$y_t \leq s_t z_t \psi^n f(l_t, k_t)$$

Under Assumption 3, the owners of the firms (the household) receives a fraction of the production ρy_t as profits. Firms are assumed to be entirely equity owned.

Define $x_t^{n_i}$ as the vector of idiosyncratic state variables of firm i, i.e. $x_t^{n_i} = (z_t, \psi_t^{n_i})$. Let X_t be defined as before and define Γ_f as the law of motion for the aggregate state as perceived by any arbitrary firm; i.e. $X_{t+1} = \Gamma_f(X_t)$. The static problem of a firm i producing intermediate goods in any period t is

$$\pi(x_t^{n_i}, X_t) = Max_{p_t, l_t, k_t}\ (p_t y_{it} - w_t l_{it} - r_t k_{it})$$

subject to

$$y_{it} \leq s_t x_t^{n_i} f(l_{it}, k_{it})$$

$$k_{it} = [\underline{k}^{n_i}, \infty) \qquad (\lambda_i t)$$

The optimality conditions yield

$$\frac{k_{it}}{l_{it}} = \frac{w_t}{r_t - \lambda_{it}} \frac{f_k k_{it}}{f_l l_{it}}$$

where λ_{it} is the shadow value of capital when the firm is constrained by the minimum capacity, and $\frac{f_k k}{f_l l}$ corresponds to the ratio of factor shares of total output under Assumption 3.

If the minimum capacity requirement is binding, the firm adjusts its resource allocation through the flexible factor, in this case labor. However, capital labor ratios are not equalized across production units [20]. The capital labor ratio of constrained firms is higher than that of unconstrained firms. In a static model with complete markets, disparate capital labor ratios are a sign of inefficiencies in the allocation. In the current set up however, these gaps might be consistent with optimality.

Under Assumption 8 one can describe labor and capital demands as a function of the pro-

[20] In models where firms are financially constrained, the capital labor ratios of constrained firms is usually lower than that of unconstrained firms. Constrained firms hold less capital than they would if unconstrained, and have a higher marginal product of capital than the equilibrium cost of capital. In this model, constrained firms hold more capital than otherwise, and their MPK is lower than the interest rate.

ductivity of the firm, and its marginal product of capital.

$$l(x_t^{n_i}, X_t) = \frac{f_l^{-1}(x_t^{n_i}, r_t - \lambda_{it})}{\int f_l^{-1}(x_t^{n_j}, r_t - \lambda_{jt})dj}$$

$$k(x_t^{n_i}, X_t) = K_t \frac{f_k^{-1}(x_t^{n_i}, r_t - \lambda_{it})}{\int f_k^{-1}(x_t^{n_j}, r_t - \lambda_{jt})dj}$$

where f_l^{-1}indicates the inverse of the marginal product of labor, and f_k^{-1} is defined likewise.

In the analysis that follows it is useful to define two statistics, namely $Z^l = \int f_l^{-1}(x_t^{n_j}, r_t - \lambda_{jt})dj$ and $Z^k = \int f_k^{-1}(x_t^{n_j}, r_t - \lambda_{jt})dj$. Both are statistics of productivity adjusted by the marginal product of capital across all the firms in the economy.

7.3.4 Exit and Upgrade

An active firm using process n get profits according to the state of the aggregate demand through its impact on the price of intermediate goods; a measure of productivity (adjusted by marginal product dispersion) as summarized by Z^k and Z^l; the productivity of the firm, $x_t^{n_i}$; and the share of capital expenses in total revenue. Under Assumption 3, the latter can be described as $\frac{f_k k_{it}}{f} = \alpha_t \rho$. Profits read

$$\pi(x_t^{n_i}, X_t) = h'\left(\int y_{it} di\right) s_t x_t^{n_i} f\left(\frac{f_l^{-1}}{Z_l}, \frac{f_k^{-1}}{Z_k} K_t\right)[(1-\rho) - \alpha_t \rho(\frac{r_t}{MPK_{it}} - 1)]$$

Whenever the minimum capacity constraint is binding the marginal product of capital of the firm is lower than the cost of capital in the market, and profits drop below those of an unconstrained firm with the same productivity. The drop in profits equals the gap between the cost of capital in the market and the firm's marginal product of capital times the firm's capital demand.

The value of the firm is $W_t(x_t^n, X_t)$ when the aggregate state is X_t,is as described in the body of the paper.

If $n = N$ the firm is already operating the best process in the economy. Hence, there is no upgrade in technology available and the value of the firm reads

$$W_t(x_t^N, X_t) = \max\{\Pi_e^N(s_t), \widetilde{W_t}(x_t^N, X_t)\}$$

$$\text{subject to } X_{t+1} = \Gamma_f(X_t)$$

7.3.5 Entry

A fraction δM_t of the total mass of firms operating in the market M_t, are forced out of the market at the end of each period, after production took place. At the beginning of next period, after the shocks have been realized some firms select themselves out of the market. There is a continuum of firms ready to enter the market at any period t. They invest $I(s_t)$ units of the numeraire and get a draw of productivity z_{it} from an exogenous distribution $G(z)$ with support $[\underline{z}, \overline{z}]$. At that point, they can decide whether to exit or operate in the market, and if operating, which process to use. They may choose to upgrade technology immediately at cost $\sum_{n=2}^{m} I_t^n(s_t)$ if choosing the m-th process available.

The first part of Assumption 6 prevents entrepreneurs from creating resources by entering and exiting immediately from the market. The second part bounds the difference in cost of entry across aggregate states, and will be used to assure procyclicality of the measure of entrants. Intuitively, if the cost of entry increases "too much" in good times, it is possible for entry to be discouraged altogether.

The mass of entrants M_t^{ent} is determined by the free entry condition,

$$I(s_t) \geq \int W(z_{it}, \psi^1, X_t) dG(z_{it}) \tag{12}$$

with equality if $M_t^{ent} > 0$.

7.3.6 Dividends

Dividends in the economy are

$$\begin{aligned}
d_t^n(X_t) &= \int \pi(x_t^{n_i}, X_t)\ dv_t^n(z_{it}) + \Pi_e^n(s_t) M_{et}^n(X_{t-1}, X_t) - M_t^{ent}(X_t) I(s_t) \qquad \text{if } \ n = 1 \\
d_t^n(X_t) &= \int \pi(x_t^{n_i}, X_t)\ dv_t^n(z_{it}) + \Pi_e^n(s_t) M_{et}^n(X_{t-1}, X_t) \qquad \text{if } 1 < n < N \\
&\quad - I^n(s_t) \left[M_{ut}^n\ (X_t) + M_t^{ent}(X_t) \left(G(z^u(\psi_t^{n+1}, X_t)) - G(z^u(\psi_t^n, X_t)) \right) \right] \\
d_t^n(X_t) &= \int \pi(x_t^{n_i}, X_t)\ dv_t^n(z_{it}) + \Pi_e^n(s_t) M_{et}^n(X_{t-1}, X_t) \qquad \text{if } n = N \\
&\quad - I^n(s_t) \left[M_{ut}^n\ (X_t) + M_t^{ent}(X_t) \left(1 - G(z^u(\psi_t^n, X_t)) \right) \right]
\end{aligned}$$

Hence, they equal the profit of active firm plus the scrap values of the ones that get liquidated, minus entry and upgrade costs. If we replace them in the budget constraint of the household we obtain the feasibility condition of this economy in terms of final goods.

7.3.7 Properties of the Value Function

Proof of Proposition 1. First notice that $\pi(x_t, X_t)$ is bounded and continuous in $z \in Z$ which follows from the boundness of the support of z and the continuity of f.

$$\pi(x_t, X_t) = h'\left(\int y_{it} di\right) s_t z_t \psi^n f\left(\frac{f_l^{-1}}{Z_l}, \frac{f_k^{-1}}{Z_k} K_t\right) [(1-\rho) - \alpha_t \rho(\frac{r_t}{MPK_{it}} - 1)]$$

Claim 3 *Profits are increasing in z.*

Proof. *By definition, $MPK_{it} = \min\{r_t, s_t z_{it} \psi_t^n f_k\}$. Hence, the term in brackets in the profit function is either zero or negative. MPK_{it} is increasing in z so that the term in brackets also increases in z. The claim follows.* ■

Second, let $W^*(x_t, X_t)$ be the unique fixed point to the operator T,

$$T(W(x_t, X_t)) = Max\left\{\Pi_e, \pi(x, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})(1-\delta) W(x_{t+1}, X_{t+1})\right), W(x_{t+1}, X_t) - I^{n+1}\right\}$$

We first show first that $W^*(\ x, X)$ is non-decreasing in z.

Let $C(Z)$ be the set of continuous bounded functions in z, and let $C'(Z)$ a closed subspace of non-decreasing functions. Take $W \in C(Z)$ and $z_1 < z_2$. then

$$\begin{aligned} T(W(z_1, \psi^n, X_t)) &= Max\left\{\begin{array}{c} \Pi_e, \pi(z_1, \psi^n, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})(1-\delta) W(x_{t+1}, X_{t+1})\right), \\ W(z_1, \psi^{n+1}, X_t) - I^{n+1} \end{array}\right\} \\ &\leq Max\left\{\begin{array}{c} \Pi_e, \pi(z_2, \psi^n, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})(1-\delta) W(x_{t+1}, X_{t+1})\right), \\ W(z_2, \psi^{n+1}, X_t) - I^{n+1} \end{array}\right\} \\ &= T(W(z_2, \psi^j, X_t)) \end{aligned}$$

When $n = N$ the last term in the operator disappears because there is no possibility of upgrade. Hence, the inequality in the second line follows from the monotonicity of profits in z; Assumption 9 and the definition of Λ^s (it satisfies the Feller condition), which implies that the expectation is increasing in z (Lemma 9.5 in Stokey *et al.* (1989)). This implies that $W(z_1, \psi^N, X_t)$ is non-decreasing. The same can be shown for $n < N$ by backward induction. All in all, $T(C'(Z)) \subseteq C'(Z)$ and using the Contraction Mapping Theorem $W^* \in C'(Z)$.

Now, we want to prove that for each (ψ^n, X) the function $\widetilde{W}\,(z, \psi^n, X_t)$ is strictly increasing in z. Take $z_1 < z_2$

$$\begin{aligned}\widetilde{W}(z_1, \psi^n, X_t) &= \pi(z_1, \psi^n, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W^*(z, \psi^n, X_{t+1})\right)\\ &< \pi(z_2, \psi^n, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W^*(z, \psi^n, X_{t+1})\right)\\ &= \widetilde{W}(z_2, \psi^n, X_t)\end{aligned}$$

which proves the claim. The inequality in the second term follows from the monotonicity of profits and as before, Lemma 9.5 in Stokey *et al.* (1989).

Given the monotonicity of the continuation values, the optimality of the trigger strategy follows. Suppose not. Hence, there is a firm with productivity z, such that $z < z^e(\psi^n, X_t)$ and the firm does not exit the market. But the firm with productivity $z + \Delta < z^e(\psi^n, X_t)$ did, so $Max\left\{\Pi_e, \widetilde{W}(z+\Delta, \psi^n, X_t)\right\} = \Pi_e$. From the monotonicity of $\widetilde{W}$, it follows that $\widetilde{W}(z+\Delta, \psi^n, X_t) > \widetilde{W}(z, \psi^n, X_t)$. In other words, $\Pi_e > \widetilde{W}(z, \psi^n, X_t)$ so that remaining in the market cannot be optimal. Analogous arguments hold for the upgrade threshold. ■

Proof of Lemma 2. The value of a firm operating technology $n < N$ is

$$W_t(x_t^n, X_t) = \max\left\{W_t(x_t^{n+1}, X_t) - I^{n+1}(X_t), \widetilde{W}_t(x_t^n, X_t), \Pi_e^n(X_t)\ \right\}$$

with continuation value given by

$$\begin{aligned}\widetilde{W}_t(x_t^n, X_t) = h'\left(\int y_{it}di\right)&[(1-\rho)\,s_t x_t f\left(\frac{f_l^{-1}(x_t^{n_i}, r_t - \lambda_{it})}{Z_l}, \frac{f_k^{-1}(x_t^{n_i}, r_t - \lambda_{it})}{Z_k}K_t\right)\\ &+ E_t\left[\widetilde{\beta}_{t+1}W_{t+1}(x_t^n, X_{t+1})\right]\end{aligned}$$

Using Assumption 4, rewrite the continuation value as

$$\widetilde{W}_t(x_t^n, X_t) = x_t f\left(f_l^{-1}(x_t^{n_i}, r_t - \lambda_{it}), f_k^{-1}(x_t^{n_i}, r_t - \lambda_{it})\right) h'\left(\int y_{it}di\right)[(1-\rho)\,s_t f\left(\frac{1}{Z_l}, \frac{K_t}{Z_k}\right)$$

$$\widetilde{W}_t(x_t^n, X_t) = z_t\varpi(\psi^n) f\left(f_l^{-1}(z_t, r_t - \lambda_{it}), f_k^{-1}(z_t, r_t - \lambda_{it})\right) h'\left(\int y_{it}di\right)[(1-\rho)\,s_t f\left(\frac{1}{Z_l}, \frac{K_t}{Z_k}\right)$$

where, $\varpi(\psi^n) \equiv \psi_t^n f\left(f_l^{-1}(\psi_t^n, 1), f_k^{-1}(\psi_t^n, 1)\right)$. The Value of a firm with the top technology can be described in terms of $\varpi(\psi^N)$ as

$$W_t(x_t^N, X_t) = \max\left\{\varpi(\psi^N)\widetilde{W}_t(z_t, 1, X_t), \Pi_e^N(s_t)\right\} = \varpi(\psi^N)\max\left\{\widetilde{W}_t(z_t, 1, X_t), \frac{\Pi_e^N(s_t)}{\varpi(\psi^N)}\right\}$$

$$W_t(x_t^N, X_t) = \varpi(\psi^N)\max\left\{\widetilde{W}_t(z_t, X_t), \widehat{\Pi}_e^N(s_t)\right\}$$

where $\widehat{\Pi}_e^N(X_t) = \frac{\Pi_e^N(X_t)}{\varpi(\psi^N)}$. Notice that z_t that solves for the exit threshold is the same in the "normalized" problem, versus the original one as $\widetilde{W}_t$ is also homogeneous in a function of idiosyncratic productivity. The previous equations indicate that $W_t(x_t^N, X_t)$ is homogenous in ψ^N.

The value of a firm of an arbitrary firm with technology $n = N - 1$ is

$$W_t(x_t^n, X_t) = \max\left\{\varpi(\psi^n)W_t(z_t, X_t) - I^{n+1}(s_t), \varpi(\psi^n)\widetilde{W}_t(z_t1, X_t), \Pi_e^n(s_t)\right\}$$

$$W_t(x_t^n, X_t) = \varpi(\psi^n)\max\left\{W_t(z_t, X_t) - \widehat{I}^{n+1}(s_t), \widetilde{W}_t(z_t1, X_t), \widehat{\Pi}_e^n(s_t)\right\}$$

where, $\widehat{I}^{n+1} = \frac{I^{n+1}(X_t)}{\varpi(\psi^n)}$Hence, homogeneous in ψ^n. The same argument holds for any $n < N$ which proves the statement. ■

7.3.8 Properties of the Allocation

The properties of the allocation depend on the characteristics of the value of the firms.

Lemma 5 (Homogeneity) *If the firm is unconstrained, its value is homogenous in productivity. Hence,* $W(x_t^n, X_t) \equiv W(z_t, \psi_t^n, X_t) = \varpi(\psi^n)W(z_t, 1, X_t)$ *with* $\varpi(\psi_t^n) \equiv \psi_t^n f\left(f_l^{-1}(\psi_t^n, 1), f_k^{-1}(\psi_t^n,\right.$

The homogeneity allows us to order optimal thresholds for upgrade and exit in terms of the process operated by the firm.

Assumption 8: *The scrap values are such that* $\frac{\Pi^e(\psi^{n+1})}{\varpi(\psi^{n+1})} < \frac{\Pi^e(\psi^n)}{\varpi(\psi^n)}$ $\forall n < N$

The assumption implies that the scrap value relative to a measure of productivity of the firm drops in the productivity of the process it operates. Hence there are less incentives to exit for larger more productive firms.

Assumption 9: *The Markov Chains describing the paths of z and s do not display negative serial correlation.*

Proposition 6 *Under Assumptions 1-9, the equilibrium allocation is such that*

1. *If firms are not minimum capacity constrained, exit thresholds for firms operating worse processes are higher than for firms running better processes, i.e.* $z^e(\psi_t^n, X_t) > z^e(\psi_t^{n+1}, X_t)$*.*

2. *The exit thresholds are increasing in the cost of capital, i.e.* $\frac{\partial z^e(\psi^n, X_t)}{\partial r_t} \geq 0 \ \forall n$*.*

3. *The upgrade threshold is higher than the exit threshold for a given technology, i.e.* $z^u(\psi_t^n, X_t) > z^e(\psi_t^n, X_t)$*.*

4. *The measure of entrants is procyclical.*

Proof.

1. It been shown in the previous section that $\pi(x_t, X_t)$ is monotonic in the firm exogenous idiosyncratic productivity. Analogous arguments hold for the technology shifter, ψ^n.

   It was also shown that $\widetilde{W}_t$ is increasing in idiosyncratic productivity. The optimality condition for the exit thresholds equalizes the firm value to its scrap value. Under **??**, it can be written as
   $$\frac{\Pi_e(\psi^n, X_t)}{\varpi(\psi^n)} = \widetilde{W}_t(z_t, 1, X_t)$$
   Assumption 7 then assures that $z^e(\psi_t^n, X_t) > z^e(\psi_t^{n+1}, X_t)$.

2. The profit function is such that $\frac{\partial \pi(x_t, X_t)}{\partial r_t} \leq 0$. The continuation value of the firm satisfies
   $$\widetilde{W}(z, \psi^n, X_t) = \pi(z, \psi^n, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W^*(z, \psi^n, X_{t+1})\right)$$
   and W^* is independent of the current interest rate (except possibly through its impact on the equilibrium distribution). Hence, the continuation value of the firm is non-increasing in the interest rate. The optimality condition for the exit threshold yields the result.

3. The result follows from Assumption 5.

4. The free entry condition dictates
   $$I(s_t) \geq \int W^*(z_{it}, \psi^1, X_t) dG(z_{it})$$
   By definition of W^*,
   $$I(s) \geq \int W^*(z_{it}, \psi^1, X_t) dG(z_{it}) \geq \int \Pi_e^1(s) dG(z_{it})$$

Pick $s' < s$. By Assumption 6

$$I(s') + \int \Pi_e^1(s)dG(z_{it}) - \int \Pi_e^1(s')dG(z_{it}) > \int \Pi_e^1(s)dG(z_{it})$$

$$I(s') > \int \Pi_e^1(s')dG(z_{it})$$

Which implies that $M^{ent}(s) > M^{ent}(s')$.

■

7.4 Market Allocation versus Efficient Allocation

7.4.1 Capital and Labor Allocation.

Using the equation for the shadow value of labor and capital $(\lambda_t^l, \lambda_t^k)$, and the optimality conditions of the firms,

$$\frac{k_{it}}{l_{it}} \frac{f_l l_{it}}{f_k k_{it}} = \frac{\lambda_t^l}{\lambda_t^k}$$

$$\frac{k_{it}}{l_{it}} \frac{f_l l_{it}}{f_k k_{it}} = \frac{w_t}{r_t}$$

Hence, the relative price of capital to labor in the market allocation coincides with the relative shadow values, only if the capital labor ratio of unconstrained firms is the same across allocations.

7.4.2 Aggregate Capital Accumulation.

The optimal path for aggregate capital in the efficient allocation is dictated by

$$U'\left(C_t^*\right) = E_t\left[U'\left(C_{t+1}^*\right)\beta\left(\lambda_{t+1}^k + 1 - \widehat{\delta}\right)\right]$$

In the market allocation the optimal capital accumulation decision of the household is characterized by

$$U'\left(C_t\right) = E_t\left[U'\left(C_{t+1}\right)\beta\left(r_{t+1} + 1 - \widehat{\delta}\right)\right]$$

If the pricing kernels are the same and the shadow value of capital is the same, both allocations yield identical paths for aggregate capital.

The shadow value of capital for the planner is

$$\lambda_t^k = f_k\left(\frac{K_t^{\lambda=0}}{\sum_n \int_{(\lambda=0)} f_k^{-1}(z_{it}, \psi^n, s_t)di}\right)$$

where $K_t^{\lambda=0}$ is the total capital intake of firms that are not constrained by the minimum capacity constrain, and $(\lambda = 0)$ is the set of those firms operating in the market

In the Market allocation, the interest rate solves

$$r_t = f_k\left(\frac{K_t^{\lambda=0}}{\sum_n \int_{(\lambda=0)} f_k^{-1}(z_{it}, \psi^n, s_t)di}\right)$$

Hence, the shadow value of capital in the planner's allocation and in the market allocation can differ because the set of firms that are currently unconstrained is different set $(\lambda = 0)$ across allocations. Those differences may in turn imply disparities in capital intake, as summarized by $K_t^{\lambda=0}$. If the allocation of firms across technologies differ, and the distribution of constrained firms does too, the induced differences in the cost of capital will affect also the pattern of aggregate capital accumulation.

7.4.3 Process Selection

Exit. The exit condition for a firm operating technology n in the planners' problem reads

$$\mu_t\left[\frac{\partial Y_t}{\partial z_t^e(\psi^n, X_t)} + \Pi_e^n(s_{t+1})\frac{\partial M_{et}^n}{\partial z_t^e(\psi^n, X_t)}\right] + \beta E\frac{\partial V(\Xi_{t+1})}{\partial z_t^e(\psi^n, X_t)} = 0$$

The envelope condition can be written in terms of the expected value of the firm which

$$\begin{aligned}
VF_{t+1}^n(z_{t+1}, z_{et}; \Xi_{t+1}) = {} & \frac{\gamma_{t+1}^n}{\mu_{t+1}}\int_{z_{t+1}^e}^{z_{t+1}^u} P(z_{t+1}, z_{et})dz_{t+1} \\
& + \sum_{m=n+1}^{N-1}\left(\frac{\gamma_{t+1}^m}{\mu_{t+1}} - \sum_{j=n+1}^{m} I_{t+1}^j\right)\int_{z_{t+1}^{u,j}}^{z_{t+1}^{u,j+1}} P(z_{t+1}, z_{et})dz_{t+1} \\
& + \left(\frac{\gamma_{t+1}^N}{\mu_{t+1}} - \sum_{j=n+1}^{N} I_{t+1}^j\right)\int_{z_{t+1}^{u,N}}^{\overline{z}} P(z_{t+1}, z_{et})dz_{t+1} \\
& + \Pi_e^n(s_{t+1})\int^{z_{t+1}^e} P(z_{t+1}, z_{et})dz_{t+1}
\end{aligned} \tag{13}$$

for $n < N$.

If the firm is operating the best technology,

$$VF^N_{t+1}(z_{t+1}, z_{et}; \Xi_{t+1}) = \frac{\gamma^N_{t+1}}{\mu_{t+1}} \int_{z^e_{t+1}}^{\overline{z}} P(z_{t+1}, z_{et}) dz_{t+1} + \Pi^N_e(s_{t+1}) \int^{z^e_{t+1}} P(z_{t+1}, z_{et}) dz_{t+1}$$

The derivative of output reads

$$\begin{aligned}\frac{\partial Y^*_t(X_t)}{\partial z^e_t(\psi^n, X_t)} \frac{1}{dv^n(z^e_t)} &= h'\left(\int y_{it} di\right) \{(1-\rho)\, s_t z_t \psi^n f(\frac{f_l^{-1}}{Z_l}, \frac{f_k^{-1}}{Z_k} K_t) - \\ &- \frac{f_l^{-1}(z^e_t)}{Z_l} f_l(z^e_t) s_t z^e_t \psi^n [\sum_n \int \frac{s_t z_{it} \psi^n}{s_t z^e_t \psi^{n_e}} \frac{f_l(z_{it}, r - \lambda_{it})}{f_l(z^e_t, r - \lambda_{et})} \frac{f_l^{-1}}{Z_l} di - 1] - \\ &- \frac{f_k^{-1}(z^e_t)}{Z_k} K_t f_k(z^e_t) s_t z^e_t \psi^n [\sum_j \int \frac{s_t z_{it} \psi^n}{s_t z^e_t \psi^{n_e}} \frac{f_k(z_{it}, r - \lambda_{it})}{f_k(z^e_t, r - \lambda_{et})} \frac{f_k^{-1}}{Z_k} di \ - 1]\}\end{aligned}$$

The second term cancels out because there is no dispersion in marginal products, $\frac{s_t z_{it} \psi^n}{s_t z^e_t \psi^{n_e}} \frac{f_l(z_{it}, r - \lambda_{it})}{f_l(z^e_t, r - \lambda_{et})} = 1 \forall i$ and feasibility in the labor market yields $\sum_j \int \frac{f_l^{-1}}{Z_l} di = 1$.

Upgrades. The optimality condition is

$$I^{n+1}_u(s_t) dv^n_t(z^u_t) = \frac{\partial Y_t(\Xi_t)}{\partial z^u_t(\psi^{n+1}, \Xi_t)} + E_t\left[\widetilde{\beta}_{t+1} \frac{\partial V\,(\Xi_{t+1})}{\partial z^u_t(\psi^{n+1}, \Xi_t)}\right]$$

The envelope condition can be written in terms of the difference in the value of the firms. The derivative with respect to output is

$$\begin{aligned}\frac{\partial Y_t}{\partial z^u_t} &= (1-\rho) h'\left(\int y_{it} di\right) s_t z^u_t \left(\psi^{n+1} f(\frac{f_l^{-1}}{Z_l}, \frac{f_k^{-1}}{Z_k} K_t) - \psi^n f(\frac{f_l^{-1}}{Z_l}, \frac{f_k^{-1}}{Z_k} K_t)\right) dv^n_t(z^u_t) \\ &- k^{n+1}_{ut} f_k(z^u_t) s_t z^u_t \psi^{n+1} [\frac{1}{K_t} \sum_j \int \frac{\lambda^k_t - \lambda_{it}}{\lambda^k_t - \lambda_{ut}} k_{it} di \ - 1] \\ &+ k^n_{ut} f_k(z^u_t) s_t z^u_t \psi^n [\frac{1}{K_t} \sum_j \int \frac{\lambda^k_t - \lambda_{it}}{\lambda^k_t - \lambda_{ut}} k_{it} di \ - 1]\end{aligned}$$

I now prove the propositions in section 3.

Proof of Proposition 4. The first part of the proposition follows from comparing the contribution to output and the profits of the marginal firm, if that marginal firm were to be the same in both allocations. Because the firm is not constrained, the relevant terms for comparison read

$$(1-\rho) - \alpha_t \rho \left(\frac{1}{K_t} \sum_n \int \frac{\lambda^k_t - \lambda_{it}}{\lambda^k_t} k_{it} di \ - 1\right) > (1-\rho)$$

As $\frac{1}{K_t}\sum_n \int \frac{\lambda_t^k-\lambda_{it}}{\lambda_t^k} k_{it}di < 1$. The optimality condition for exit implies then that the exit threshold for the planner is lower than the one in the market allocation (as the foregone output is higher than firms profits for ex ante identical firms).

To prove the second part of the proposition, compare$(1-\rho)-\alpha_t\rho\left(\frac{1}{K_t}\sum_n \int \frac{\lambda_t^k-\lambda_{it}}{\lambda_t^k-\lambda_{et}} k_{it}di \ -1\right)$ and $(1-\rho)-\alpha_t\rho\left(\frac{r_t}{MPK_t^e}-1\right)$.The proposition assumes $\lambda_t^k-\lambda_{et}=MPK_t^e$. Hence, if

$$\frac{1}{K_t}\sum_n \int \left(\lambda_t^k-\lambda_{it}\right) k_{it}di < r_t$$

the foregone output of liquidating the firm is higher in the planner's problem, than the profits accounted by the firm. Hence, the threshold for exit is lower in the efficient allocation than in the market one. ■

Proof of Proposition 5. The first part of the proposition follows from comparing the contribution to output and the profits of the marginal firm, if that marginal firm were to be the same in both allocations. Because the firm is not constrained, and the second term in both the output contribution of a firm, and the profits do not depend on the firm idiosyncratic type, they cancel out when computing the differences in the output contribution and profits across processes. Hence, both the planner and the firm in the market account for an increase in current value of $h'\left(\int y_{it}di\right)s_tz_tf\left(\frac{f_l^{-1}}{Z_l},\frac{f_k^{-1}}{Z_k}K_t\right)(1-\rho)\left(\psi^{n+1}-\psi^n\right)$.This difference is also accounted for in the expected value of the firm, which because firms are unconstrained, is homogeneous in ψ. The optimality conditions coincide.

For the second part, it is assumed that while the firm is currently unconstrained it might be constrained after upgrade. Hence the current value of the upgrade in terms of additional output is

$$h'\left(\int y_{it}di\right)s_tz_tf\left(\frac{f_l^{-1}}{Z_l},\frac{f_k^{-1}}{Z_k}K_t\right)(1-\rho)\left(\psi^{n+1}(1-\alpha_t\rho\left(\frac{1}{K_t}\sum_n \int \frac{\lambda_t^k-\lambda_{it}}{\lambda_t^k-\lambda_{ut}} k_{it}di \ -1\right))-\psi^n\right)$$

while expected profits for the firm read

$$h'\left(\int y_{it}di\right)s_tz_tf\left(\frac{f_l^{-1}}{Z_l},\frac{f_k^{-1}}{Z_k}K_t\right)(1-\rho)\left(\psi^{n+1}(1-\alpha_t\rho\left(\frac{r_t}{MPK_t^u}-1\right))-\psi^n\right)$$

. As in the exit condition, it is assumed that the shadow value of capital is the same after upgrade, i.e. $\lambda_t^k - \lambda_{ut} = MPK_t^u$. Hence, if

$$\frac{1}{K_t}\sum_n \int \left(\lambda_t^k - \lambda_{it}\right) k_{it} di < r_t$$

the output gain is higher in the planner's problem, than the gains in profits accounted for by the firm. Hence, the threshold for upgrade in lower in the efficient allocation. ■

Proof of Proposition 6. The result follows from the free entry condition (which by construction is the same in the efficient allocation and the market allocation) and the results obtained for upgrade and exit thresholds. Whenever firms that upgrade are unconstrained, the upgrade policy is efficient, ceteris paribus. Hence, disparities in the free entry condition between the efficient and market allocation stem only from differences in the marginal exit threshold and the value of the marginal exiting firm. If the marginal firm exiting the market for ψ^1 is not constrained, proposition **??** shows that the value of a firm for the planner is higher than for the firm in the market. Hence, for the same marginal firm, the free entry condition is not satisfied in the efficient allocation.

$$\mu_t I(s_t) < \gamma_t^1 (G(z_{ut}^2) - G(z_{et}^1)) + \sum_{n=2}^{N-1} \left(\gamma_t^n - \mu_t I^n(s_t)\right) \left(G(z_{ut}^{n+1}) - G(z_{ut}^n)\right) + \left(\gamma_t^N - \mu_t I^N(s_t)\right) \left(1 - G(z_{ut}^N)\right)$$

Entry is higher in the efficient allocation.

When the marginal firm is constrained, proposition 4 shows that whenever $\frac{1}{K_t}\sum_n \int \left(\lambda_t^k - \lambda_{it}\right) k_{it} di < r_t$ the value of the marginal firm is higher in the planner problem and hence there is more entry than in the market allocation. ■

7.5 Equivalence with the decentralized solution

To prove the equivalence between the centralized and decentralized solution define

$$\begin{aligned} &\Omega(\left\{z^e(\psi_t^n, \Xi_t), z^u(\psi_t^{n+1}, \Xi_t)\right\}_{n=1}^N, M^{ent}(\Xi_t); \Xi_t) \\ &\equiv \sum_{n=1}^N \tau_e^n \Pi_e^n(s_t) M_e^n(\Xi_t) + \sum_{n=1}^{N-1} \tau_u^{n+1} I^{n+1}(s_t) M_u^{n+1}(\Xi_t) + \tau(s_t) I(s_t) M^{ent}(\Xi_t) + Y - Y^* \end{aligned}$$

Lemma 6 $\Omega(\left\{z^e(\psi_t^n, \Xi_t), z^u(\psi_t^{n+1}, \Xi_t)\right\}_{n=1}^N, M^{ent}(\Xi_t); \Xi_t)$ *is continuous in the exit and upgrade thresholds as well as in the measure of entrants.*

Proof. Continuity in the measure of entrants is straightforward from the definition. Continuity in the thresholds follows from the definition of aggregate output and the measure of upgrades and exits in terms of the distribution of firms, jointly with the absolute continuity of v_t^n proved in Lemma (AC). ■

Lemma 7 *There exist a transfer scheme* $\mathbf{T}(\Xi_t)$ *such that*

$$\Omega(\left\{z^e(\psi_t^n, \mathbf{T}, \Xi_t), z^u(\psi_t^{n+1}, \mathbf{T}, \Xi_t)\right\}_{n=1}^N, M^{ent}(\mathbf{T}, \Xi_t); \Xi_t) = \mathbf{T}$$

Proof. Lemma (M) shows that the measure of firms operating in the market is bounded. Hence, there exist B such that $\Omega(\left\{z^e(\psi_t^n, \Xi_t), z^u(\psi_t^{n+1}, \Xi_t)\right\}_{n=1}^N, M^{ent}(\Xi_t); \Xi_t) < B$. The feasible measure of entrants is also bounded by Lemma (M). Let $\Phi \equiv [0, B]$, which is convex and compact by construction. The optimal thresholds are the maximizers of Pseudo-Planner Problem. By the theorem of the maximum they are u.h.c. in $\mathbf{T}(\Xi_t)$. Hence, Ω is an upper hemicontinuous convex valued correspondence and $\Omega \neq \varnothing$ for any $T \in \Phi$. Thus, Ω has a fixed point (Kakutani). ■

Note that there might be different combination of thresholds that generate the same transfer, and hence the fixed point is not unique. In other words, the decentralization need not be unique.

Proof of Theorem 2. Define, $\Upsilon^p(\Xi_t) = \Upsilon^c \widehat{\tau}(\Xi_t)$ where $\widehat{\tau}(\Xi_t)$ generates $\mathbf{T}(\Xi_t)$ (the fixed point of Ω)

When the Pseudo-Planner Problem is solved at $T_t = \mathbf{T}(\Xi_t)$ the budget constraint reads

$$C_t + K_{t+1} - (1-\widehat{\delta})K_t + IM_t^{ent} + \text{Upgrade Cost} \leq Y_t + \sum_{n=1}^{N} \Pi_e^n M_{et}^n$$

which is the market clearing condition in the decentralized allocation. Hence, for this cost structure the feasibility constraint of the planner coincides with that of the competitive equilibrium.

The dynamic optimality conditions for the firms need to hold at $\left\{z^e(\psi_t^n, \mathbf{T}, \Xi_t), z^u(\psi_t^{n+1}, \mathbf{T}, \Xi_t)\right\}$.

Claim 4 *There exist an industrial policy* $\widehat{\tau}(\Xi_t)$ *such that at the thresholds of the competitive equilibrium, the generated transfer* T_t *is a fixed point of* $\Omega; T\ (\widehat{\tau}(\Xi_t)) = \Omega(T)$.

Proof. *Note that the pseudo-planner's optimality conditions in terms of the allocation of firms across technologies and entry levels are linear in the cost of entry, upgrade and the scrap value*

(Equations 2 to 4). For notational convenience I collapse them to the following equation

$$\Upsilon^p = \Delta^p \left(\left\{ z^e(\psi_t^n, \Xi_t), z^u(\psi_t^{n+1}, \Xi_t) \right\}_{n=1}^N, M^{ent}(\Xi_t) \right)$$

Define $\widehat{\tau}(\Xi_t)$ to solve this system of equations at the equilibrium threshold and entry level of the competitive allocations, i.e.

$$\Upsilon^c \widehat{\tau}(\Xi_t) = \Delta^p \left(\left\{ z^e(\psi_t^n, X_t), z^u(\psi_t^{n+1}, X_t) \right\}_{n=1}^N, M^{ent}(X_t) \right)$$

The wedges $\widehat{\tau}(\Xi_t)$ are well defined because they solve a perfectly identified system of equations.

Suppose that $\mathbf{T}(\widehat{\tau}(\Xi_t))$ is not a fixed point of Ω. The level of output generated in by the centralized allocation is the same as in the decentralized allocation because the thresholds and measure of entries are the same. If $\mathbf{T}(\widehat{\tau}(\Xi_t))$ is not a fixed point of Ω, the budget constraint of the planner reads

$$C_t + K_{t+1} - (1-\widehat{\delta})K_t + IM_t^{ent} + \textit{Upgrade Cost} \leq Y_t + \sum_{n=1}^N \Pi_e^n M_{et}^n \; + \Upsilon^c \widehat{\tau}(\Xi_t)$$

which implies that the market clearing condition in the goods market in the competitive allocation is violated, which yields a contradiction. ■

Using the definition of the cost of capital in the market allocation, it is possible to show that as long as the allocation of firms is the same in the decentralized and centralized problem, the shadow value of capital coincides with the interest rate. Hence incentives for capital accumulation are the same in the market and planner's problem.

$$r_t = f_k \left(\frac{K_t^{\lambda=0}}{\sum_n \int_{(\lambda=0)} f_k^{-1}(z_{it}, \psi^n, s_t) di} \right) = \lambda_t^k$$

■

Proof of Corollary 2. The indifference conditions for the firms in the decentralized problem are linear in the costs too as seen from 25. To simplify notation, define the system of equations as

$$\Upsilon^c = \Delta^c \left(\left\{ z^e(\psi_t^n, X_t), z^u(\psi_t^{n+1}, X_t) \right\}_{n=1}^N, M^{ent}(X_t) \right)$$

Given the cost structure, the efficient allocation solves,

$$\Upsilon^c = \Delta^p \left(\left\{ z^e(\psi_t^n, \Xi_t), z^u(\psi_t^{n+1}, \Xi_t) \right\}_{n=1}^{N}, M^{ent}(\Xi_t) \right)$$

Define, $\widehat{\tau}^{\,c}(\Xi_t)$ to solve the system of equations Δ^c at the efficient threshold and entry level, i.e.

$$\Upsilon^c \widehat{\tau}^c(\Xi_t) = \Delta^c \left(\left\{ z^e(\psi_t^n, \Xi_t), z^u(\psi_t^{n+1}, \Xi_t) \right\}_{n=1}^{N}, M^{ent}(\Xi_t) \right)$$

The wedges $\widehat{\tau}^{\,c}(\Xi_t)$ are well defined because they solve a perfectly identified system of equations.

■

Chapter II: Industry Dynamics, Investment and Business Cycles

1 Introduction

Dispersion in marginal products within narrowly defined industries is a stylized fact of modern economies[21]. There are many reasons for which marginal productivity of inputs may differ across firms. Some of the most extensively analyzed mechanisms in the literature are size dependent policies [22], subsidies or taxes for particular firms[23] and market incompleteness (i.e. financial frictions [24]). These mechanisms can explain a large portion of the documented dispersion. They also imply that if this dispersion is eliminated, efficiency can be improved. In Caunedo (2014), I argue that dispersion in marginal products may arise as the outcome of an efficient allocation and that allocations that display lower equilibrium marginal product dispersion can be further away from the efficient allocation than those displaying lower marginal product dispersion. In this paper, I quantify aggregate productivity losses associated to dispersion in marginal product and inefficient firm selection in the US manufacturing sector. The latter has been extensively used as a benchmark for efficiency in many quantitative analysis comparing resource allocations across countries and hence a relevant starting point for our analysis. Additionally, I describe the policies that would induce the efficient allocation as a market outcome.

I extend the framework studied in Caunedo (2014) where firms face irreversibilities and indivisibilities in investment and operate under uncertainty, to allow for imperfect competition. The decisions to enter and exit the market, as well as the selection of technologies are costly and modelled as real options. A technology is a productivity level and an associated minimum capacity in terms of capital. More productive technologies have a higher minimum capacity associated to them. I assume away idiosyncratic shocks, so that at the moment of entry, each investor is assigned a blueprint (a technique to produce a good), the quality of which varies over a continuum of types and is constant in time. I solve for the industry equilibrium by

[21] For cross country evidence refer to Asker *et al.* (2013). For evidence for Korea, refer to Midrigan and Xu (2009). Also, Hsieh and Klenow (2009) provide evidence for the US, India and China. For evidence in Latin America, see Buso *et al.* (2013).

[22] Barstelman *et al.* (2013) document and study the impact of distorsions that are correlated with the size of firms.

[23] Restuccia and Rogerson (2008) analyze a broad range of policy distorsions.

[24] See Buera and Shin (2011), Moll (2013), Midrigan and Xu (2009) and the extensive literature thereafter.

means of a centralized problem whose allocation coincides with the market allocation for a given cost structure. I calibrate this stylized economy to the US manufacturing sector, and ask whether the allocation can be pareto improved either by narrowing differences in marginal product or by changing the industry dynamic. I show that shifts in the patterns of entry, exit and investment have a larger contribution to productivity gains than those associated to drops in marginal product dispersion. This finding is consistent with micro empirical analysis that documents substantial productivity improvements associated to shifts in the patterns of firm churning (Haltinwanger (2011), Davis *et al.* (2007) and Eslava *et al.* (2004)).

In terms of the macro implications of the asymmetry in capacity constraints there are two pieces of empirical evidence, that put together, suggest that this model can be in line with the data. First, measures of dispersion in the marginal product of capital fluctuate with the cycle (Eisfeldt and Rampini (2006)), they are countercyclical. In the model the aggregate state of the economy dictates fluctuations in the distribution of marginal products along this line. Second, there is also evidence that dispersion in revenue TFP at the plant level is countercyclical, and that the increase in dispersion is explained mostly by a larger right tail, i.e. more firms with lower revenue productivity (Kehrig (2011)). With constant return technologies, revenue productivity is proportional to marginal product of inputs. It is possible to argue that part of the increase in the right tail observed in the data is accounted for by an increase in dispersion in marginal product of capital, mostly through the right tail of its distribution. The model economy implies that recessions are periods where more firms operate with lower marginal product of capital which supports the empirical evidence[25].

In this economy, the equilibrium allocation is inefficient. On the one hand, monopolistic competition generates a gap between the social and private of the firm, equal to the constant markup charged by the firms in the decentralized market. On the other hand, markets are not complete in the arrow debreu sense. As explained in Caunedo (2014) when there is marginal product dispersion, an additional firm in the market (a firm can be interepreted as a new asset) may span a whole new dimesion of transfers across production units. The planner uses the average product of capital to account for the opportunity cost of capital, while the firms in the market allocation use the marginal product of capital of unconstrained firms,as reflected in the equilibrium interest rate. The gap between the private and social value of a firm varies

[25] The financial frictions story predicts more firms with higher marginal product of capital. Hence, dispersion should increase because the upper tail of the distribution of marginal products is getting larger.

endogenously with the conditions in the market. During downturns, it is more likely that firms hold excess capacity and hence, the decentralized allocation might be further away from the efficient one. It is worth to point out that if marginal products are equalized in equilibrium, the gap between decentralized and centralized allocations disappears and the allocation is efficient (the average and marginal product of capital are the same)

I characterize the policies that would bring the market and the efficient allocation with equilibrium marginal product dispersion. For the economy calibrated to the US manufacturing

sector, the planner's allocation dictates higher equilibrium investment, and a shift in output production towards larger, more productive firms. Improvements in aggregate productivity are 11% under the optimal policy. In the model, efficiency gains from the implementation of the optimal policy are accounted mostly by a change in firms entry, exit and investment patterns. Only a third of the gains in productivity are explained by reallocation of labor and capital across incumbent firms. The employment distribution varies slightly between the decentralized and planner's allocation. The optimal policy implies subsidies to entry, and the size of the subsidy is predicted higher in good times. In equilibrium there are more firms operating in the market under the efficient allocation. Upgrade costs are subsidized to induce better selection of firms in the market. The policy as of scrap values varies with the aggregate state and the technology operated by the firm. In good times, scrap values are lower for all capacities except for the bottom ones, to generate exit of the least productive units. In bad times, scrap values for the bottom capacities are predicted lower, and the scrap value of the firms at the top of the productivity/size distribution is higher. The latter induces exit by firms that are possibly capacity constrained.

1.1 Literature Review

Models of industry equilibrium with complete markets (for example Hopenhayn (1992)) display aggregation. Hence, there is very little effect of heterogeneity in equilibrium allocations except possibly through selection. As marginal product and capital labor ratios are equalized, the model boils down to one of a representative firm with average productivity. Firm selection determines the equilibrium mean productivity in the market. When the relationship between productivity, size (employment or assets) and output is non-monotonic, heterogeneity matters in a non-trivial way.

Economies where heterogeneity cannot be reduced away are for example those of Lee and Mukoyama (2008), Clementi and Palazzo (2010) and Khan and Thomas (2008) (in incomplete markets). Lee and Mukoyama (2008) provide evidence of differential entry and exit behavior along the business cycle and propose a model to quantitatively explain those facts. They analyze the effect of fluctuations in fixed production costs and labor adjustment costs on the industry dynamic in a model with no capital. Clementi and Palazzo (2010) analyze the propagation of aggregate shocks due to entry and exit of firms when firms are allowed to accumulate capital. Khan and Thomas (2008) study the effect of irreversibilities and collateral constraints in equilibrium allocations in an economy with idiosyncratic shocks and without exit and entry. They find that both frictions reinforce each other in slowing down reallocation.

Khan and Thomas (2008) show that quantitatively the interaction of irreversibilities with financial frictions may explain large drops in aggregate efficiency and slow recoveries. As described by Caballero (1999) irreversibilities might have important consequences for the aggregate behavior of the economy when interacted with market incompleteness or informational asymmetries. In this paper, when irreversibilities are interacted with uncertainty in a fully fledged industry dynamic model, they generate a disparity between investment decisions of entrants and incumbents in the market. The vintage of the firm becomes relevant in explaining their investment behavior.

The most salient difference between this paper and previous work by Veracierto (2002), is that he abstracts from the entry and exit problem while it is determined endogenously in this paper[26]. Also, the nature of noncovexities in production is different from the one exploited in Veracierto (2002): while there partial irreversibility is allowed, here there is full irreversibility and invisibilities in technology adoption. The mechanism generating marginal product dispersion is close to that explored in Asker *et al.* (2013). In their model however, endogenous entry and exit is abstracted away. The results of the paper in terms of efficiency gains from the implementation of the optimal policy rely not only on the static gains from reallocation, which they explore, but also from the endogenous selection mechanism. This paper is also related to the work of Cooper and Schott (2014), who study productivity gains in the US manufacturing sector in response to cyclical factor reallocation. In their environment aggregate shocks do not generate cyclical losses in productivity, but shocks to the shadow value of capital or the

[26] As can be seen from table 4 in Veracierto (2002), when there is full irreversibility, the change in the exogenous death rate has considerable effect on investment dispersion across production units.

dispersion in idiosyncratic shocks do. In their model, entry and exit is abstracted away.

While I allow firms to endogenously determine efficiency through investments in distinct technologies, I assume that the idiosyncratic productivities of the firms are constant. I assume that the log productivity is drawn from an exponential distribution, so that the model can be interpreted as the limiting case of a model in which firms idiosyncratic productivity is stochastic and follows a Brownian Motion (See Luttmer (2010) for an example). The mechanism of the model does not vanish when idiosyncratic productivity is allowed to change in time. It can rather be reinforced, as negative idiosyncratic shocks may render previous investment decisions statically inefficient. Assuming idiosyncratic risk away allows me to separate the impact of technological restrictions versus market incompleteness.

Finally, as mentioned in the introduction, one of the main contributions is the characterization of the optimal industrial policy. This has been done for models of international trade under oligopolistic competition in prices and quantities (Eaton and Grossman (1986)). For a model of industry dynamic without capital accumulation Lee and Mukoyama (2008) study the impact of alternative policies on labor regulations. However, their policies are ad hoc in the sense that there is no notion of efficiency associated to them. Lee and Mukoyama (2008) study the impact of taxes to output and inputs in production over aggregate TFP, for both i.i.d. policies and policies correlated with the productivity of the firms. Guner *et al.* (2008) study policies that target the size of the establishment, which in turn is correlated with their idiosyncratic productivity, and find substantial role in shaping aggregate productivity. Distinctively, this paper characterizes the optimal policy in an environment in which the efficient allocation does not dictate equalization of marginal products across all firms in the economy.

2 Model

This is an infinite horizon economy with time indexed by t. There is a final good which agents use for consumption and capital accumulation. It is produced by means of a continuum of intermediate goods. Intermediate goods are produced by combining capital and labor. Each intermediate good is perfectly differentiated and each firm producing it faces a constant elasticity demand. Final goods are traded competitively while there is monopolistic competition in intermediate goods. The technology for production of intermediate goods is endogenously chosen, and each one is associated to a minimum running capacity in terms of capital goods.

There is aggregate uncertainty in the economy. The exogenous shock is denoted s_t that takes two values, i.e. $\{g, b\}$, $g > b$ associated to the "good" and "bad" state, respectively. The transition probabilities are given by the matrix $\mathbf{P} \equiv \begin{bmatrix} \gamma_g & 1-\gamma_g \\ 1-\gamma_b & \gamma_b \end{bmatrix}$ where $P(s_{t+1} = g/s_t = g) = \gamma_g$ and $P(s_{t+1} = b/s_t = b) = \gamma_b$.

2.1 Households

The representative household derives utility from consumption of the final good C_t.

The household is endowed with a unit of labor that for simplicity is supplied inelastically to the firms. She receives a wage w_t for the services. She can also accumulate capital K_t, priced in terms of the final good (the numeraire) and rent it at price r_t to the firms. The aggregate stock depreciates at rate $\widehat{\delta}$.The household can buy shares of two different mutual funds that entitled it to the dividends generated by the firms operating in the economy. The first mutual fund consist of all the firms running with low minimum capacity technology, and the second is build with all firms using a technology with higher minimum capacity. After dividends are paid, assets can be traded.

Her problem reads

$$Max_{\ C_t, n_t^L, n_t^H, K_{t+1}} E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t) \right] \tag{14}$$

subject to

$$C_t + K_{t+1} - (1-\widehat{\delta})K_t + \sum_{j=L,H} P_t^j n_t^j = w_t + r_t K_t + \sum_{j=L,H} \left(d_t^j + P_t^j \right) n_{t-1}^j \tag{15}$$

$$X_{t+1} = \Gamma_c(X_t) \tag{16}$$

where P_t^j is the price of shares n_t^j of a mutual fund of firms of technology with minimum capacity $j = L, H$ at period t+1, which pay dividends d_{t+1}^j and can be sold tomorrow at price P_{t+1}^j. In computing the return to the share holdings, the agent needs to forecast the law of motion of the distribution of firms in the market for each possible realization for the exogenous aggregate shock, s_t. The aggregate state $X_t = \left(s_t, v_t^L, v_t^H, K_t \right)$ includes the exogenous shock, s_t ; the distribution of firms per technology, v_t^j for $j = L, H$;and the available aggregate stock of capital. To save on notation I denote $v_t^j(z) \equiv v_t^j([0, z))$ the measure of firms with productivity at most z and technology j. The subjective law of motion for the representative consumer is denoted by Γ_c.

U fulfills the standard assumptions of concavity, monotonicity and differentiability. $\beta \in (0,1)$ is the discount factor. The optimality conditions of the problem are standard. Dynamic optimality yields

$$U'(C_t)P_t^k = \beta E_t \left[U'(C_{t+1})\left(d_{t+1}^k + P_{t+1}^k\right)\right] \tag{17}$$

For a standard CES specification $U(C_t) = \frac{C_t^{1-\theta}}{1-\theta}$ one can rewrite the price of shares as the present discounted value of all future dividends of firms that are active in period $t+1$with technology j, adjusted by the corresponding pricing kernel

$$P_t^j = E_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \frac{C_\tau^{-\theta}}{C_t^{-\theta}} d_\tau^j \tag{18}$$

2.2 Final Goods Sector

There is a a representative competitive firm with a CES technology that produces final goods Y_t out of intermediate inputs y_{it}. The firm maximizes profits as

$$Max_{y_{it}}\ Y_t - \int p_{it} y_{it}\ di$$

subject to

$$Y_t \leq \left(\int y_{it}^{\rho} di\right)^{\frac{1}{\rho}}$$

where p_{it} is the cost of good y_{it}. It is assumed $\rho \in (0,1)$ so that goods are substitutes in production.

The corresponding input demand for each variety i emerges from the FOC of the problem, i.e.

$$Y_t^{1-\rho} y_{it}^{\rho-1} = p_{it}$$

2.3 Intermediate Goods Sector

2.3.1 Capital and Labor Allocation

To produce each differentiated good, firms use capital and labor which are available at cost w_t and r_t, respectively, in units of the composite good. The technology is Cobb-Douglas,

$$y_t \leq s_t z \psi^j l_t^{1-\alpha} k_t^{\alpha}$$

There are two alternative technologies associated to a minimum capacity and a productivity shifter, $\{k_j, \psi^j\}$ for $j = L, H$. For simplicity we assume, $\psi^L = 1$ and $\psi^H > 1$. The capital choice sets are $[k_L, \infty)$ and $[k_H, \infty)$ for each technology, respectively. We interpret this indivisibility as the construction of a plant, or the set up of machinery which entails a particular capacity. The adoption of technology is costly. The problem of adoption, entry and exit into the market will be analyzed later. In this section, I study the allocation of capital and labor only.

Define x_t as the vector of idiosyncratic state variables to the firm, i.e. $x_t = (z, \psi^j)$. Let X_t be defined as before and define Γ_f as the law of motion for the aggregate state as perceived by any arbitrary firm; i.e. $X_{t+1} = \Gamma_f(X_t)$. The problem of a firm producing an intermediate good i in any period t is

$$\pi(x_t, X_t) = Max_{p_t, l_t, k_t} \ (p_t y_t - w_t l_t - r_t k_t)$$

subject to

$$y_t \leq s_t z \psi^j l_t^{1-\alpha} k_t^{\alpha}$$

$$\left(\frac{Y(X_t)}{y_t} \right)^{1-\rho} = p_t$$

$$k_t = [k_j, \infty)$$

Firms are assumed to be entirely equity owned. Because the elasticity of the demand is constant, the optimal price set by a firm is a constant markup over marginal cost. In particular,

$$p_t = \frac{(r_t - \lambda_t)^{\alpha} w_t^{1-\alpha}}{\rho \alpha^{\alpha} (1-\alpha)^{1-\alpha} s_t z}$$

where $\lambda_t \geq 0$ is the lagrange multiplier associated to the feasible set for capital. If the minimum capacity requirement is not binding then, $\lambda_t = 0$, otherwise $\lambda_t > 0$ and the markup for this firm is lower than otherwise.

From the FOC of the firms we can compute the labor and capital demand as follows

$$l_t = (s_t z \psi^j)^{\frac{\rho}{1-\rho}} \left[\left(\frac{1-\alpha}{w_t} \right)^{\frac{1}{\rho}-\alpha} \left(\frac{\alpha}{r_t - \lambda_t} \right)^{\alpha} \right]^{\frac{\rho}{1-\rho}} \rho^{\frac{1}{1-\rho}} Y(X_t) \tag{19}$$

$$k_t = \max \left\{ k_j, \left[s_t z \psi^j \left(\frac{1-\alpha}{w_t} \right)^{(1-\alpha)} \left(\frac{\alpha}{r_t} \right)^{\frac{1}{\rho}-(1-\alpha)} \right]^{\frac{\rho}{1-\rho}} \rho^{\frac{1}{1-\rho}} Y(X_t) \right\} \tag{20}$$

The higher the relative efficiency in production the higher the demand of labor when intermediate goods are substitutes in production. Labor and capital demands are non-creasing in their costs, and they are increasing in the demand level as summarized by $Y(X_t)$.

Importantly, capital labor ratios need not be equal across all firms in the economy as the shadow value of capital depends on whether firms are constrained or not

$$\frac{k_t}{l_t} = \frac{w_t}{r_t - \lambda_t} \frac{\alpha}{1-\alpha}$$

If the minimum capacity requirement is binding, the firm adjusts its resource allocation through the flexible factor, in this case labor. However, the last condition indicates that constrained firms' labor demand does not increase enough to equalize the firms capital labor ratios across all firms [27]. This disparity is at the heart of the dynamics studied in this paper. In a static model with complete markets, disparate capital labor ratios are a sign of inefficiencies in the allocation. In the current set up however, these gaps might be consistent with an efficient allocation as described later in the paper.

Define $Z^l = \int (\frac{\psi_i^j z_i}{(r_t(X_t)-\lambda_t^i)^{\alpha}})^{\frac{\rho}{1-\rho}} di$ and $Z^k = \int (\frac{\psi_i^j z_i}{(r_t(X_t)-\lambda_t^i)^{\frac{1-(1-\alpha)\rho}{\rho}}})^{\frac{\rho}{1-\rho}} di$, both statistics of productivity adjusted by the shadow value of capital across firms in the economy. Labor and capital demand are proportional to these statistics

$$l(x_t, X_t) = \frac{1}{Z^l} (\frac{\psi^j z}{(r_t(X_t) - \lambda_t)^{\alpha}})^{\frac{\rho}{1-\rho}} \tag{21}$$

$$k(x_t, X_t) = \frac{K_t}{Z^k} (\frac{\psi^j z}{(r_t(X_t) - \lambda_t)^{\frac{1-(1-\alpha)\rho}{\rho}}})^{\frac{\rho}{1-\rho}} \tag{22}$$

If no firm is constrained, shadow values of capital equalize across firms, and capital and labor demand are only a function of the relative productivity of the firms versus the average in

[27] In models where firms can be financially constrained, the capital labor ratios of constrained firms is usually lower than that of unconstrained firms. Constrained firms hold lower capital than they would if unconstrained, while in our model, constrained firms hold more capital.

the economy. When some firms are capacity constrained, the allocation of labor and capital is adjusted so that constrained firms can indeed retain more capital and labor inputs than if they were unconstrained.

2.3.2 Exit and Upgrade

Firms are exogenously liquidated with probability δ, getting a scrap value of Π_e^f.They can select out voluntarily, getting a scrap value of Π_e, net of exit costs. Assume $\Pi_e^f = 0$, $\Pi_e > 0$, so that the option to exit is meaningful. For simplicity, I assume the latter is constant along the cycle and across sizes, but the model can accommodate richer structures in which the value depends on the technology operated by the firm and potentially different across states. This is depicted in the quantitative section. Finally, an incumbent firm in the market may choose to upgrade its process at any point in time at cost I_H.

A firm using a high minimum capacity technology may choose to operate or exit in the current period. If it operates it will get profits according to

$$\pi(x_t, X_t) = (1-\rho)Y_t^{1-\rho}\left[\frac{s_t K_t^{\alpha}}{\left(Z^k\right)^{\alpha}\left(Z^l\right)^{1-\alpha}}\right]^{\rho}\left(\frac{z\psi^j}{MPK_t^{\alpha}}\right)^{\frac{\rho}{1-\rho}} - \frac{r_t - MPK_t}{MPK_t}\frac{K_t}{Z^k}\left(\frac{z\psi^j}{MPK_t^{\alpha}}\right)^{\frac{\rho}{1-\rho}}$$

Profits depend on the aggregate demand, a measure of productivity in the economy summarized by $\left(Z^k\right)^{\alpha}\left(Z^l\right)^{1-\alpha}$ and the productivity of the firm, adjusted for the value of its marginal product of capital. Whenever the minimum capacity constraint is binding the marginal product of capital of the firm is lower than the cost of capital in the market, and profits drop according to their gap and the demand of capital.

Before any production and endogenous technology, exit and entry decision take place, the firms can be exogenously liquidated with probability δ or continue operating. If it continues, it can exercise the option to exit irrespective of which state of the world s_t is realized. To save notation, let $x_t^j = z\psi^j$ for $j = L, H$.The value of the firm W_t follows

$$W_t(x_t^H, X_t) = Max\left\{\Pi_e, \pi(x_t^H, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})W_{t+1}(x_t^H, X_{t+1})\right)\right\} \tag{23}$$

subject to

$$X_{t+1} = \Gamma_f(X_t)$$

where $\widetilde{\beta}_{t+1}(X_t, X_{t+1}) \equiv \beta\left(1-\delta\right)\frac{U'(C(X_{t+1}))}{U'(C(X_t))}$ is the stochastic discount factor of the household adjusted for the probability of survival of the firm, $\widetilde{\beta}_{t+1}$ to save notation.

On the continuation region, when the option to exit is not exercised, the value of the firms is the present discounted value of all future expected profits. We call it $\widetilde{W}_t$ and it reads

$$\widetilde{W}_t(x_t^H, X_t) = \pi(x_t^H, X_t) + E_t\left(\widetilde{\beta}_{t+1} W_{t+1}(x_t^H, X_{t+1})\right)$$

Next, we move to the problem of firms currently holding a low minimum capacity requirement technology. After observing the aggregate state, they may decide to exit the market, to upgrade capacity or to operate at the current one. The cost of upgrade in technology is I_H units of the composite good that should be paid in the period of upgrade, after the aggregate shock is realized.

$$W_t(x_t^L, X_t) = Max\{\Pi_e, W_t(x_t^H, X_t) - I^H, \widetilde{W}_t(x_t^L, X_t)\} \tag{24}$$

subject to

$$X_{t+1} = \Gamma_f(X_t)$$

Their continuation value is

$$\widetilde{W}_t(x_t^L, X_t) \ = \pi(x_t^L, X_t) + E_t\left(\widetilde{\beta}_{t+1} W_{t+1}(x_t^L, X_{t+1})\right)$$

Let the function $z^e(\psi^j, X_t)$ determine the threshold for exit of j technology firms when the aggregate state of the economy is X_t. Let the function $z^u(\psi^H, X_t)$ determine the threshold for upgrade.

Proposition 7 *$\widetilde{W}_t(x_t^j, X_t)$ is monotonic increasing in idiosyncratic productivity, z. The optimal exit and upgrade strategy for the firm is such that if $z < z^e(\psi^j, X_t)$, the firm exits the market; if $z \geq z^u(\psi^H, X_t)$ the low minimum capacity technology upgrades; otherwise the firm holds a low minimum capacity requirement technology.*

2.3.3 Entry

A fraction δM_t of the total mass of firms operating in the market M_t, are forced out of the market each period. There is a continuum of firms ready to enter the market at any period t. They observe their productivity before investing I_L units to buy a low minimum capacity technology. Their productivity z is drawn from an exogenous distribution $G(z)$ with finite support $[\underline{z}, \overline{z}]$. For the problem to be well defined we need to assume $I_L \geq \Pi_e$. Otherwise, entrepreneurs could create resources just by entering and exiting immediately from the market. After entry, they may choose to upgrade technology immediately at cost I_H.

The mass of entrants M_t^{ent} is determined by the free entry condition,

$$I_L \geq \int\limits_{z^e(\psi^L, X_t)} W(z, \psi^L, X_t) dG(z/z \geq z^e(\psi^L, X_t)) \tag{25}$$

with equality if $M_t^{ent} > 0$.

It is worth noting that the equilibrium distribution of firms across productivity and technologies, which is used in the computation of the expected value of the firms (summarized in X_t), is indeed endogenously determined by the choice of exit and upgrade thresholds of firms in the market. Entrants correctly anticipate their future expected profits, so that pre-entry expected profit equalize the post entry value.

3 Aggregates

Let the measure of firms operating in the market $M_t = v_t^L(z^u(\psi^L, X_t)) + v_t^H(\overline{z})$ and define a scaled measure $\widehat{v}_t^j = \frac{v_t^j}{M_t}$. Replacing capital and labor demands in the aggregate production function, we obtain

$$Y(X_t) = TFP_t K_t^\alpha$$

where

$$TFP_t = s_t M_t^{\frac{1-\rho}{\rho}} \left(Z^l\right)^{\frac{1-\rho}{\rho}} \left(\frac{Z^l}{Z^k}\right)^\alpha \tag{TFP}$$

In other words, aggregate efficiency is determined by the realization of the exogenous shock, the measure of firms operating in the market (as usual in models of monopolistic competition), and a moment of the productivity of the firms operating in the market. If there are no firms capacity constrained, $\frac{Z^l}{Z^k} = r$, and the model boils down to the canonical firm dynamic one where $TFP_t = s_t M_t^{\frac{1-\rho}{\rho}} \left(\sum_j \int (\psi_i^j z_i)^{\frac{\rho}{1-\rho}} d\widehat{v}_t^j(z_i)\right)^{\frac{1-\rho}{\rho}}$. Also, as alpha goes to zero, disparity in marginal products becomes irrelevant for aggregate productivity, because the share of the factor for which the minimum constraint may bind becomes negligible. In general none of those is the case. It is important to note also that there might be multiple allocations (distributions across technologies) that yield the same TFP_t conditional on the aggregate state and the measure of operating firms.

Before moving to the definition of the equilibrium, let me close the model description by computing the dividends received by the household. They correspond to the sum of the profits of operating firms, plus the scrap value of the liquidated ones minus the costs of entry and upgrade.

$$\begin{aligned} d_t^L(X_t) &= \int \pi(z, \psi^L, X_t)\ dv_t^L(z) + \Pi_e M_t^{eL}(X_{t-1}, X_t) - I_L M_t^{ent}(X_t) \\ d_t^H(X_t) &= \int \pi(z, \psi^H, X_t)\ dv_t^H(z) + \Pi_e M_t^{eH}(X_{t-1}, X_t) - \\ &- I_H \left[M_t^u\ (X_t) + M_t^{ent}(X_t) \frac{1 - G(z_t^u(\psi^H))}{1 - G(z_t^e(\psi^L))} \right] \end{aligned}$$

where $M_t^{ej}(X_{t-1}, X_t)$ the measure of exits for firms running technology j, $M_t^u\ (X_t)$ is the measure of incumbent upgrades in state X_t; $M_t^{ent}(X_t)$ the corresponding measure of entrants, and $M_t^{ent}(X_t)\frac{1-G(z_t^u(\psi^H))}{1-G(z_t^e(\psi^L))}$ is the measure of entrants that upgrade immediately. The measure of exits is zero if $z^e(\psi^j, X_t) \leq z^e(\psi^j, X_{t-1})$ and positive otherwise, i.e. $M_t^{ej}(X_{t-1}, X_t) = (1-\delta)\, v_{t-1}^j(z^e(\psi^j, X_t))$ if $z^e(\psi^j, X_t) > z^e(\psi^j, X_{t-1})$.

Also, $M_t^u\ (X_t) = (1-\delta) \left[v_{t-1}^L(z^u(\psi^H, X_{t-1})) - v_{t-1}^L(z^u(\psi^H, X_t)) \right]$ whenever $z^u(\psi^H, X_{t-1}) > z^u(\psi^H, X_t)$ and zero otherwise.

4 Equilibrium

Definition 2 *A competitive equilibrium is a system of thresholds* $\left\{z^e(\psi^j, X_t), z^u(X_t)\right\}_{t=0}^{\infty}$, *distribution of firms* $\left\{v_t^L(z), v_t^H(z)\right\}_{t=0}^{\infty}$, *a law of motion for the dynamic of the distributions of firms,* Γ, *entrants* $\left\{M_t^{ent}\right\}_{t=0}^{\infty}$ *with productivities drawn from* $G(z)$, *and consumption, aggregate capital and share holdings functions* ,$\left\{C(X_t), K_{t+1}(X_t), n^H(X_t), n^L(X_t)\right\}_{t=0}^{\infty}$ *such that given prices* $\left\{r(X_t), w(X_t), P^L(X_t), P^H(X_t)\right\}_{t=0}^{\infty}$, *the cost structure* $\Upsilon_c = \left[\Pi^e, I^H, I^L\right]$, *the initial stock of capital in the economy* K_0, *share holdings,* $n_0^H = n_0^L = 1$, *and the exogenous law of motion for aggregate shocks* s_t *as characterized by* $\mathbf{P}$,

i) The representative consumer maximizes utility (as in (14)*)* *ii) Firms in the intermediate goods sector maximize their value as described by (23) and (24) given their residual demand and productivity z.*

iii) Firms in the final good sector maximize profits.

iv) $\int_{z^e(\psi^L,X_t)} W(z,\psi^L,X_t)dG(z/z \geq z^e(\psi^L,X_t)) \leq I_L$ *with equality if* $M_t^{ent} > 0$

v) $M_t = M_t^{ent} + (1-\delta)M_{t-1} - \left(M_t^{eL} + M_t^{eH}\right)$ *where* $M_t = v_t^L(z^u(\psi^H,X_t)) + v_t^H(\overline{z})$.

vi) Markets clear

(a) $\int l(x_t,X_t)dv_t^L(z) + \int l(x_t,X_t)dv_t^H(z) = 1$

(b) $\int k(x_t,X_t)dv_t^L(z) + \int k(x_t,X_t)dv_t^H(z) = K_t$

(c) $n_t^j = 1, j = L, H$

(d) $C_t + K_{t+1} - (1-\widehat{\delta})K_t + I_L M_t^{ent} + I_H \left[M_t^u + M_t^{ent}\frac{1-G(z_t^u(\psi^H))}{1-G(z_t^e(\psi^L))}\right] = Y_t + \Pi_e\left(M_t^{eL} + M_t^{eH}\right)$

vii) Consistency for the law of motion of the aggregate state: $\Gamma = \Gamma_f = \Gamma_c$.

Existence of the equilibrium is shown in Chapter I of this dissertation.

4.1 Properties of the allocation

Proposition 8 *The optimal allocation is such that*

1. *Assume costs and technologies are such that* $\frac{\Pi^e(\psi^H)}{\psi^{H\frac{1-\rho}{\rho}}} < \frac{\Pi^e(\psi^L)}{\psi^{L\frac{1-\rho}{\rho}}}$. *Then, exit thresholds for firms running the low minimum capacity technology are higher than for firms running the high minimum capacity one, i.e.* $z^e(\psi_L,X_t) > z^e(\psi_H,X_t)$ *if neither firm is constrained by the minimum capacity or both are.*

2. *Exit thresholds are increasing in the cost of capital, i.e.* $\frac{\partial z^e(\psi^j,X_t)}{\partial r_t} \geq 0$.

3. *The upgrade threshold across technology is higher than the exit threshold for high minimum capacity firms, i.e.* $z^u(\psi^H,X_t) > z^e(\psi_H,X_t)$.

4. *The measure of entrants is procyclical.*

5 Quantitative analysis

The first result indicates that firms running the low minimum capacity technology find optimal to exit before firms of the same idiosyncratic productivity running the high capacity technology . The second, that increases in the cost of capital, increase the likelihood of voluntary exit as equilibrium profits drop. The third result is important as it assures that costs are such that there is no upgrade in technology and immediate exit. Finally, the levels of entry are procyclical as they are in the data .Quantitative Analysis

In this section I assume there is a finite level (N) of minimum capacities/technologies, and there is no further investment in capacity conditional on a particular technology. I assume that there is a stock of capital ready to be used in any particular company. The stock is large enough so that any firm that decides to invest in capacity or enter the market can be supplied with the corresponding stock. The dynamic of the aggregate stock of capital will be pinned down by the consumption decisions of the planner, which in turn will pin down the dynamic of the measure of firms in the economy.

Production under each alternative technology is given by

$$y_t = s_t z k_j^{\alpha} l(x_t, X_t)^{1-\alpha} \text{ for } j = 1, ..., N$$

where $k_j < k_{j+1}$ for any j. A detailed explanation of the algorithm for computing the equilibrium is provided in Appendix A.

5.1 Calibration

The model is calibrated to the USA economy[28]. Although business cycle statistics are typically presented at quarterly frequency, industry dynamics statistics are only available on a yearly basis. Hence, the time unit of the model is a year. Some of the calibrated parameters are standard in the RBC literature. The persistence of expansions and recession periods were set to match the average duration of the phase of the business cycle in the USA. In particular, $\gamma_s = 1 - 1/t_s$ where t_s is the average length of a particular phase of the business cycle $s = g, b$. The average duration of an expansion was set to 3.175 years (or 12.7 quarters), and that of a recession to 1.425 years (or 5.7 quarters). The discount factor was set to match a steady state

[28] There are substantial differences in the firm size distribution of the USA versus other OECD countries (see Barstelman *et al.* (2009)). In particular, the right tail of the distribution is "fatter" in the USA than in other developed economies. Alternative calibrations can be accomodated.

interest rate of 2%, $1 + r = \beta^{-1}$. Log utility was assumed.

The substitutability across intermediate goods in the final good aggregator was set to match returns to entrepreneurship (ρ shapes the curvature of the profit function). Atkenson and Kehoe (2005) set a value of 15% to the returns to entrepreneurship, whose analogous in the model is $1 - \rho$ ($\rho = 0.85$). The share of capital in value added is set to 1/3 as standard in the literature. The hazard rate for exogenous exit, δ was set to 5,5%. It corresponds to the mean exit rate reported in Lee and Mukoyama (2008) based on statistics from the Annual Survey of Manufactures. Finally, the number of technologies is set arbitrarily to 4 and the lower bound of possible productivities equal to 0.01[29].

The remaining parameters of the model were calibrated jointly to match moments of the firm size distribution, as well as features of the industry dynamic and the aggregate volatility of the economy. To calibrate them I simulate the model economy via Montecarlo: I run the optimal policies for 1000 periods (discard the first 200) over 100 alternative paths for a variety of parameter specifications. The list of parameters calibrated jointly is presented in Table 2

While some parameters have closer tights to certain moments, they are not independent of the remaining variables of the economy. Let me describe their roles briefly. First, the size of aggregate shocks measured by $s_g - s_b$ is closely related to the volatility of the cyclical component of log GDP. The target in the data corresponds to the standard deviation of the hp-filtered series of log GDP from 1930 to 2011, equal to 2.1%. Positive shocks take a value of 1.027 and negative shocks of 0.97 (shocks are assumed symmetric around one). The observed variation in aggregate output is not independent however of the cost structure of the economy, as the latter determines how much investment or exit is observed in equilibrium, which in turn affects aggregate output.

The set of capacities as well as the range for idiosyncratic productivities, are related to the levels of log employment produced by the model[30]. The upper bound on capacities was set to 4 while the upper bound on productivities was set to 4.25. The firms at the top of the employment have a level of employment slightly above 10000 employees, consistent with the data. The distribution of sizes in the economy inherits also some of the properties of the exogenous distribution of idiosyncratic productivity, $G(z)$. The distribution of entrants is calibrated such that the $log(z)$ is exponential with parameter $\zeta_G = 1.9$. In other words, $G(z)$ is Pareto with parameter ζ_G.

[29] The minimum effective productivity operating in the market is determined endogenously.

[30] The finite level of capacities model predicts that relative labor demands are described by $\frac{l_i}{l_j} = \frac{\left(z_i k_i^{\alpha}\right)^{\frac{\rho}{1-\rho(1-\alpha)}}}{\left(z_i k_j^{\alpha}\right)^{\frac{\rho}{1-\rho(1-\alpha)}}}$

The generated firm size distribution is also related to the entry and upgrade costs per capacity, through the equilibrium allocations. To calibrate the cost structure, I assumed state independent costs for the pseudo planner problem. Once the allocations generated by the economy matched the targets for the US, I backed out the cost structure in the decentralized allocation. In other words, I computed the costs that would make the exit and upgrade threshold of the decentralized allocation coincide with the ones in the calibrated economy.

The total number of parameters for calibration is thirteen. The complete list of moments that were targeted to calibrate them are found in Table 3.The identified costs indicate slightly higher entry costs during expansions, fairly constant scrap values across states, but increasing in the capacity of the firms as expected. Upgrade costs are identified higher during expansions. In the ergodic distribution of the model, upgrades in capacity for incumbent firms average 2.8% of the total population of active firms, costs of upgrade should raise when incentives to upgrade increase to avoid shifts in the firm size distribution that will make it inconsistent with its fairly constant shape in the data. The establishment and employment shares are as reported by Lee and Mukoyama (2008), as well as the average exit and entry rates. Overall, the model predicts well the behavior of the establishment and employment distribution. The share of employment for firms at the top of the log employment distribution is slightly underpredicted. The model predicted share of establishments with less than 19 employees is below the observed number in the data. The firms at the top of the distribution reported by the BDS have 10.000 or more employees. They correspond to 6% of the total population of establishments in the economy. The model is conservative in this sense as the largest firm in the economy employs 10.829 employees.

In terms of firm entry and exit rates the model overpredicts exit rates by 0.7%, and underpredicts entry rates 0.6%. For the measure of firms to be stable in the ergodic distribution, these flows should be roughly the same, the model is calibrated to go half the way the difference in entry and exit rates reported in the data.I also targeted the percentage of firms with positive investment spikes as reported by Dums and Dunne (1998). A spike is defined as firm that reports an investment rate of 30% or higher in any given year. Given the capacity grid, any upgrade in capacity will be considered an investment spike, as well as any entry decision. The model produces a measure of spikes of about 1% higher than in the data once we account for investment of entrants. In the model, 40% of the measure of firms with investment spikes corresponds to incumbent firms. The contribution is rather small as for the calibrated aggregate shocks, investment thresholds move mildly. The introduction of firm specific shocks will increase fluctuations in the thresholds, potentially inducing more equilibrium investment for incumbents.

5.2 Results

5.2.1 Productivity

We first describe the predictions of the model for aggregate Total Factor Productivity (TFP). To express the results as close as those in the literature, note that when technology is Cobb[Douglas, total factor physical productivity ($TFPQ_i$) per firm is proportional to a geometric average of capital and labor productivity

$$TFPQ_i \triangleq MPK_i^{\alpha} MPL_i^{1-\alpha}$$

where the marginal product of capital and marginal product of labor are defined as $MPK_i = \rho\alpha \frac{y_i}{k_i}$ and $MPL_i = \rho(1-\alpha)\frac{y_i}{l_i}$respectively. Aggregating up, we obtain

$$TFP_t = \left(\sum_{j=L,H} \int (TFPQ_{z_i})^{\rho} \, dv_t^j(z_i) \right)^{\frac{1}{\rho}} \tag{26}$$

This expression is analogous to (TFP) presented in the aggregates section and is our baseline measure.

If there is no dispersion in marginal product across firms, aggregate total factor productivity simplifies to

$$TFP_t^{MC} = s_t \left[\sum_{j=L,H} \int (z_i)^{\frac{\rho}{1-\rho}} \, dv_t^j(z_i) \right]^{\frac{1-\rho}{\rho}} \tag{27}$$

Although in this case there are no losses in efficiency stemming from the technological friction, the presence of monopolistic competition might still affect productivity through the equilibrium number of operating firms in the market. We use this measure to test the properties of the baseline model against.

Table 4 shows the effect of irreversibilities and indivisibilities in production on computed aggregate TFP. All values are reported in log points. The first column reports the statistic described in (26) .The second column reports the same statistic for the optimal allocation of firms which is computed imposing the decentralized cost structure into the pseudo-planner problem absent of transfers. The first row reports aggregate productivity and the second row the standard deviation of the time series. The third row reports a measure of dispersion in computed TFPQ across firms. I report the coefficient of variation across economies.

Aggregate productivity under the optimal allocation is 11% higher than in the Baseline economy. While the optimal policy induces a drop in the coefficient of variation of TFPQ across firms, it induces higher volatility of productivity in the time series. From the definition of TFP one can see that the gains in efficiency in the constrained optima may stem from disparities in the allocation of firms across technologies and productivity, or from differences in the equilibrium measure of firms operating in the market. Further analysis on the sources of gains is included when describing the optimal policy.

To isolate the effect of irreversibilities and indivisibilities from the changes in the equilibrium measure of firms due to the monopolistic competition, I normalize the measure of active firms to one. Table 5 reports the statistics described in the previous table for the baseline economy, the optimal policy, and an economy in which marginal product of inputs in production equalizes across firms, i.e.(27). The allocation in which marginal products are equalized across firms yields the highest aggregate productivity and the lowest coefficient of variation for TFPQ. This is not surprising since the constrained optima cannot completely undo the impact of indivisibilities and irreversibilities on marginal product dispersion. The differences between them are large, while aggregate productivity almost double, the cross sectional dispersion drops to a third. Also time series productivity volatility raises even more when marginal products are equalized. Fluctuations in productivity in such economy stem from changes in the productivity of the marginal firm operating in the market. The irreversibilities and indivisibilities in the model induce lower adjustment, and less volatile aggregate productivity.

The measure of dispersion in TFPQ potentially hides distributional issues, i.e. the distortion generated by the irreversibility and the indivisibility is disparate across capacities/technologies. I compute the ratio of mean productivity per capacity in the model and under the assumption that firms equalize marginal products. An entry equal to 1 in Table 6 indicates the same mean productivity. The results suggest that the friction in the model generates firms with low capacity to held few resources (hence high marginal products), and productive firms running high capacity technologies, with too many resources compared to what they would held if marginal products were equalized. The friction in the model generates selection towards bigger more productivity firms. In the economy with equalization of marginal products, labor is shifted from the high capacity, low marginal productivity firms to low capacity higher marginal productivity ones. It is worth noting that the improvement in aggregate productivity induced for the optimal policy, is attained for a distribution of employment that resembles largely the one in the baseline economy.

5.2.2 Optimal Policy

As mentioned in the previous section efficiency gains may stem from improvements in the allocation of firms across technologies and productivity, or from differences in the equilibrium measure of firms operating in the market. For the calibrated economy, while total efficiency gains associated to the optimal policy are 11%, only a third of them stem from pure reallocation of resources. The rest, is induced by a larger measure of firms operating in the market in equilibrium: 17% more firms than in the baseline economy.

Accordingly, the industry dynamic is different. While entry and exit rates are lower under the optimal policy, the upgrade rate increases. Both combined indicate that there is a shift toward more productive larger firms. Upgrade rates of incumbent firms raises by 1% if compared to the baseline economy. Table 10 reports the firm dynamic. These patterns are consistent with the planner assigning a higher value to holding an additional large capacity firm than the private value of the firm in the decentralized equilibrium. The thresholds for upgrade and exit move accordingly. While in the baseline economy the exit thresholds are lower, the upgrade threshold are above the optimal levels as dictated by the efficient allocation. Average output per firm increases in the optimal allocation by 24.7% and average consumption increases 27%. The consumption equivalent compensation that would make an agent indifferent between living in the efficient or in the baseline economy should be 44% of the consumption in the baseline economy. Note that in this economy consumption equals output minus the good cost of entries and upgrades, plus the scrap value of the firms in the economy. Differences in the firm dynamic across allocations will be reflected in differences consumption equivalent measures even if the yield the same levels of output.

The optimal policy induces shifts in the contribution to output across firm sizes. It predicts a slightly larger share of output to be accounted for firms with more than 500 employees, as well as a larger contribution in employment. Capital however is allocated in the opposite direction, with a slightly higher share of the total used by the firms at the bottom of the distribution. This is not surprising since the marginal products at the bottom tend to be higher than those predicted by an economy with equalization of marginal products. Table 9 compares the predictions of the model and the optimal policy for the distribution of output, capital and employment.

One of the advantages of having the second welfare theorem to hold, is that we can study the characteristics of the optimal industrial policy, i.e. the cost structure that would induce a decentralized allocation that is efficient. Table 11 reports such cost structure and the one from the calibrated economy. The optimal policy dictates subsidies to the cost of entry in recessions

and higher entry costs during booms. Both policies combined induce less fluctuations in the measure of entrants to the market. Upgrade costs are subsidized across all aggregate states. Less costly upgrades induce shifts in the productivity distribution of the firms operating in the market to the right. Scrap values are identified lower than in the calibrated economy for all capacities except at the very bottom. Lower scrap values are consistent with lower exit rates predicted in by the optimal policy.

Note that I only describe differences across stationary equilibria. The exercises are silent as of the gains/losses that the economy may incur along the transition. Studying the path across equilibria is particularly challenging in economies like this one, where not only a statistic of the distribution needs to be carried along in the state space, but potentially full histories of a continuum of firms need to be considered. In the case where only two capacities are operated and there is no aggregate uncertainty the transition can be computed. In that case, the gains across stationary equilibrium are a lower bound to total gains whenever the transition occurs from an economy with a relatively low measure of active firms, to one with higher level of operating firms. For an increase in the measure of firms comparable to the one observed across steady states in the full model (17%), predicted transition gains are 60% larger than the steady state gains. Steady state gains in the simplified economy are 1%. This number is not readily comparable to the ones in the full economy because the cost structure and investment strategies do not map to each other. However, the exercise is useful to gain intuition. Gains are larger accounting for the transition because consumption convergence occurs from "above". By doing so, the planner avoids entering firms in the transition that will later on find themselves holding more capital that what they would need at the new steady state. In the transition the upgrade threshold jumps an overshoots the new steady state upgrade threshold. Any entrant that finds optimal to upgrade in the beginning of the transition will find optimal to do so all along it.

Also, induced entry decreases the relative measure of firms that are holding more capacity that what they would have chosen if entering the market this period. Hence, if the measure of firms is increasing in the market, the effect of the irreversibility on firms holding high capacity in the initial steady state vanishes in the aggregate.

5.2.3 Volatility and Aggregate TFP

In this section I investigate how features of the business cycle impact the entry and exit behavior of firms as well as our measures of aggregate productivity. The spirit of the exercise is to understand how the level of uncertainty that firms face affects aggregate productivity and equilibrium dispersion in marginal products.

In particular, I focus on changes in the unconditional variance of the shock. Suppose the aggregate shock s_t follows an AR(1)

$$s_t = \phi s_{t-1} + e_s$$

where ϕ is the persistence of the shock and e_s an i.i.d. shock with mean zero and standard deviation σ_e. The unconditional volatility of the aggregate shock is

$$\sigma_s^2 = \frac{\sigma_e^2}{1-\phi^2}$$

Hence, changes in unconditional volatility can be brought about by changes in the persistence or in the variance of the e_s shock. If the AR(1) process is approximated by a two state Markov chain, a la Rouwenhorst (1995), then

$$\left(\frac{s_g - s_b}{2}\right)^2 = \sigma_e^2$$

and

$$\gamma_g + \gamma_b - 1 = \phi$$

I first study whether changes in the persistence and the variance of e_s (for a given unconditional volatility) have different impact in entry and exit patterns as well as in aggregate efficiency. Second, I vary the unconditional variance by changing the variance of e_s only, and assess the implications for aggregate efficiency.

I assume that expansions are shorter than in the calibrated economy ($\gamma_g = .237$), about 1.1 years on average. I will call this Case G, for change in gamma. Alternatively, I set γ_g back to its calibration value, and increase $s_g - s_b$ to generate the same unconditional volatility. I will call this Case S, for change in the size of the shock.

Table 7 reports the results. The first row reports aggregate TFP, the second its volatility. The third row reports the coefficient of variation of TFPQ across firms. The fourth, the ratio of aggregate TFP defined as (26)/(27) when the measure of firms is normalized to 1. The fourth row reports the implied volatility of output. The fifth and sixth columns report the cross sectional dispersion in productivity. As expected the predicted volatility of output is larger in the cases under study than under the calibrated model. In this particular example, the volatility of output is substantially higher when the size of shocks changes rather than when the persistence of the process does. On the one hand, lower persistence of the shock affects the discounting of future profits and hence the trade off between current and future consumption. While shocks are more frequent, firms are also less willing to respond to the aggregate fluctuations by investing or disinvesting On the other hand, the size of the shocks affects the actual payoffs of investment. Because the firms have an outside option given by their scrap value when exiting, increases in the size of the shock improve the payoffs of investment, inducing larger responses in output.

A feature to highlight is that the impact on aggregate TFP is not monotonous. While in Case G productivity raises about 10%, it drops one third in Case S. The cross sectional dispersion of TFPQ drops by similar magnitudes in both cases, yet aggregate efficiency is very different. The volatility of aggregate output raises substantially. In terms of allocations, the relative efficiency of these economies against their equal marginal products counterparts are fairly constant. Hence, much of the differences across economies stem from the equilibrium measure of firms in the market. The economy of Case G has 4 times more firms than the economy of Case S.

The underlying industry dynamic, i.e. patterns of entry, exit and investment, also differ. Table 8 depicts mean exit, entry and upgrade rates from montecarlo simulations. In both cases the increase in volatility induces higher upgrade rates. Although in Case S, upgrade rates augments almost 5 times with respect to the baseline, selection does not induce higher average productivity (in part because exit rates are also larger). In Case G instead, entry and exit rates drop with respect to the baseline, while upgrade increase and average productivity raises.

This example points out that different features of the underlying process of exogenous shocks, can produce substantially different responses of the economy even when the underlying measure of uncertainty (unconditional volatility) is the same. This is embedded in the non-convexities of the model. The disparity in the behavior of exit and entry rates as well as investment rates, may be a promising tool in identifying characteristics of the productivity process. A limitation however, is that the relationship between the industry dynamic and the nature of shock depends on the underlying friction in the economy.

Finally, I assess the impact of changes in the unconditional volatility of the shock from changes in the size of the shocks only. I simulate the economy for a grid of $s_g - s_b$ between 0.04 to 0.15 (equivalent to positive and negative shocks of sizes 0.02 and 0.07, respectively). The predicted relationship between the volatility of output (and hence the unconditional volatility of the aggregate shock) and the cross sectional dispersion in productivity is non-monotonic. Also, the relationship between dispersion in computed productivities at the firm level and aggregate productivity is not independent of aggregate uncertainty. Figure 5 displays a scatter plot of measures of dispersion and aggregate TFP under alternative shocks.

5.3 Sensitivity Analysis

I perform robustness check with respect to some of the parameters that characterize the size distribution of firms. In particular, the parameter ς_G that parametrizes the exponential distribution from which productivity draws for entrants are obtained. Second I compare the predictions of the calibrated Model to one in which the exogenous rate of exit is substantially lower.

I first set the parameter that characterizes the exponential distribution to 1.01. This number is not arbitrary as it correspond to the estimated parameter for the Pareto distribution that characterizes the firm size distribution in the data (See Axtell (2001)). The predicted distribution of establishment across log employment lies to the right of the calibrated one. Note that a lower parameter for an exponential distribution indicates a "fatter' tail. In other words, entrants in this alternative economy start too productive inducing selection at the bottom and a shift in the allocation towards larger firms.

As the parameter increases the average productivity of entrants gets lower. Entrants with lower productivity affect the average productivity in the market and the allocation of employment and capacity across productivities. Matching accurately the firm distribution by employment and establishment is important. The economy with $\varsigma_G = 1.2$ cannot match the employment distribution in the data. It generates a distribution highly skewed to the right.

I also test the predictions of the model when the exogenous exit rate drops to 1.1% per year. The equilibrium industry dynamic changes by construction generating lower entry and exit rates in equilibrium. The size distribution of firms gets skewed to the right, indicating reallocation towards high capacity more productive firms. The equilibrium number of firms operating in the market drops. Finally, the time of the transition to the stationary distribution of firms doubles. Although transitional dynamics is not the objective of this paper, this result indicates that the study of the impact of policies that changes the incentives to firm liquidation should account for longer or shorter transition paths.

6 Conclusion

This paper explores the implications of investment irreversibility and technology indivisibilities for aggregate efficiency in production. I find that observed dispersion in marginal products is not independent of other features of the economy, such as the business cycle or more broadly the degree of demand uncertainty that firms face. The paper highlights that dispersion in marginal products is an imperfect measure of the associated efficiency losses.

When the industry dynamic is incorporated in a general equilibrium framework, high aggregate productivity allocations are associated with relatively low dispersion in marginal products. But low aggregate productivity allocations can also be associated to low dispersion in marginal products and hence in measured productivity. For a calibrated economy to the US manufacturing sector, I show that most of the gains in productivity from shifting to the efficient allocation of resources stem from changes in the industry dynamic rather than static reallocation of resources.

Partial irreversibility and higher divisibility in capital allocations will lessen the model generated excess dispersion in marginal products, for a given volatility of the aggregate process. However, as long as the movements in investment thresholds are such that the measure of incumbents firms holding capital away from the one chosen by entrants with the same blueprint does not vanish, non-convexities at the micro level will induce dispersion in marginal products and computed productivity.

I have abstracted from idiosyncratic risk. If incorporated in the model, I expect higher induced dispersion in marginal products. Higher uncertainty at the firm level will move optimal investment thresholds at the firm level even more than in the economy with aggregate shocks only. Large regions of inaction for alternative realizations of the idiosyncratic productivity shock or demand shock, are consistent with sustained disparities in marginal products.

While this paper focuses on the US manufacturing sector, the relationship between uncertainty, investment, industry structure and disparities in marginal products across production units might be a promising line of research in the context of the study of cross country differences in aggregate TFP. In other words, are economies characterized by more instability (i.e. political instability that leads to uncertainty on tax schemes, or fluctuations in the terms of trade in economies with a highly concentrated production base) prone to higher and persistent disparities in marginal products? How does the industry structure and firm dynamics vary across these economies? Can those patterns help us identify features of the aggregate productivity process?

Suppose that one would like to compare alternative economies for which we observe some statistic of marginal product dispersion. Suppose that these economies differ in the process characterizing the aggregate shock. In the model, it is possible for these economies to have similar dispersion marginal products and substantial differences in aggregate productivity. At one extreme, when the volatility of the aggregate productivity process is low, the economy approximates a stationary one. There is exit and entry in equilibrium as well as upgrades in technology. However, because the size of the aggregate shock is small, the main determinant of investment decisions is the firm's idiosyncratic productivity (as it will be in an economy with no shocks). The mechanism discussed in the example at the beginning becomes irrelevant. At the other extreme, when the volatility of the process is very high, incumbent firms find it more valuable to wait and not upgrade. Hence, in equilibrium upgrades in technology are delayed. Exit rates increase so that firms holding capital away from the level that they would have chosen in the current period are selected out of the market whenever a bad shock hits the economy. The mechanism described above vanishes again. While both economies display low dispersion in marginal products, the one with higher volatility is on average less productive than the one with lower volatility. Hence, the link between aggregate productivity and dispersion in marginal products depends on features of the macroeconomy and the patterns of firms entry, exit and investment.

Finally, it is worth mentioning that the results presented in the paper correspond to the behavior of firm distributions in the long run. The properties of the transitions to the ergodic distributions remain to be studied. The presence of indivisibilities in technologies may slow down the transition, affecting not only the equilibrium technologies adopted but also the return to capital and the path of output and capital accumulation, as well as the implications for the design of optimal policy.

Appendix (A)

6.1 Numerical Solution

Given a cost structure, Υ,the solution to the pseudo-planner problem is a set of functions $z^{e*}(k_j, \Xi_t; \Upsilon)$, $z^{u*}(k_j, \Xi_t; \Upsilon)$ and a measure of entrants M^{ent*} that solves the corresponding optimality conditions. The algorithm to solve the equilibrium allocations is

1. Assume an arbitrary cost structure for the planner $\Upsilon = \left[\Pi^e, \Pi^e, I^H, I^L, 0\right]$ (with no transfers, T).
2. Compute the dynamic of the joint distribution of capital and productivity for an arbitrary initial distribution v_0.
3. Approximate the value function of the planner
4. For a given optimal policy for the planner, run montecarlo simulations over the predicted distribution of $\{v_t\}_{t=1}^{T_M}$.
5. Calibration: The moments of $v = v_{T_M}$ for T_M large enough, are used to matched moments of firms dynamic in the data.
6. Use the calibrated cost structure of the planner Υ, and the optimality conditions delivered from the decentralized problem to compute the cost structure of the decentralized allocation $\Upsilon^c = [\Pi_e^c(k_j, \Xi_t), I_H^c(k_j, \Xi_t), I_L^c, 0]$.
7. Use the decentralized cost structure to solve for the optimal policy (planner's allocation).

6.1.1 Dynamic of the Distribution

We need first to construct the grid of capacity levels in the economy, Ψ^k and that of idiosyncratic productivities Ψ^z. The grid for capacities is equally spaced, and the grid of idiosyncratic productivities is log spaced. Points in the Ψ^z will be concentrated in the left tail.

Let J be the number of capacity levels. Define the grid if exit thresholds Ψ_j^e for $j = 1, ..., J$; and three grids for upgrade threshold grids Ψ_j^u for $j = 1, ..., J-1$ where Ψ_j^u indexes the grid of upgrade thresholds from capacity j to $j+1$. Finally, we need a grid for entry levels, Ψ^{ent}.

To generate the grids we do it jointly via the Smolyak algorithm. The algorithm constructs a sparse multidimensional grid.

The grid and transition matrix for the aggregate exogenous state s is constructed following Tauchen (1986).

For given Λ_0, I compute Λ_1 using the law of motion described in the body of the paper, for each of the points in the sparse grid.

6.1.2 Approximation of the Value Function

I implement standard value function iteration over the centralized problem.

To interpolate the value function, I use tensor products using the sparse grid as interpolation points.

I solve for the coefficients of the interpolating function given an initial guess of the value function, θ_0 and the cost structure of the model, Υ.

Then update the guess by optimizing numerically

$$V_1(v,s,M_t) = \underset{\{z^x_{jt}\}_{j=1}^J,\{z^u_{jt}\}_{j=1}^J,M^e_t}{Max} U\left(C_t(\{z^e_{jt}\}_{j=1}^J,\{z^u_{jt}\}_{j=1}^J,M^e_t)\right)$$
$$+\beta[\Pr(s'=s_1/s)V_0\left(v',s_1,M_t(1-\delta)-M^{eL}_t-M^{eH}_t+M^{ent}_t\right)$$
$$+\Pr(s'=s_2/s)V_0\left(v',s_2,M_t(1-\delta)-M^{eL}_t-M^{eH}_t+M^{ent}_t\right)]$$

subject to

$$C_t + I^{L*}M^{ent}_t + I^{H*}M^{up}_t \leq Y_t + T_t + \Pi^{e*}_j M^e_j$$

$$v' = \Lambda(v,\{z^x_{jt}\}_{j=1}^J,\{z^u_{jt}\}_{j=1}^J,M^{ent}_t,M_t)$$

$$\left(\sum_{\Psi^k}\sum_{\Psi^z}\left(z_i l_i^{1-\alpha}k_j^{\alpha}\right)^{\rho}\eta^j(z_i)\right)^{\frac{1}{\rho}} = Y_t$$

$$\frac{v^j_t(z_i)-v^j_t(z_{i-1})}{z_i-z_{i-1}} = \eta^j(z_i)$$

$$\int l_i di = 1$$

Using the updated value function V_1 recompute θ.Iterate until convergence.

6.1.3 Montecarlo Simulations

From the calibrated transition probabilities of the aggregate shock, generate 100 paths of 1000 periods each and simulate the path of allocations given the optimal policy of the planner.

Compute statistics of interest characterizing the firm dynamics of the economy, i.e. entry rates, exit rates and investment rates per capacity, dispersion in productivity, etc.

6.1.4 Cost Structure in the Decentralized Allocation

The optimality conditions for the firms, as well as those of the centralized problem are linear in the adjustment costs. Hence, if we replace the allocation that solves the pseudo planner problem into the system of equations that solves the decentralized allocation, we can infer the cost structure that decentralizes the allocation.

At the centralized allocation, the optimality conditions from the decentralized problem would typically not hold. To bring the equilibrium about, we redefine the adjustment costs faced by firms as

$$\Pi_j^c(k_j, s_t, v_t) = \Pi_e(1 + \tau^e(k_j, s_t, v_t))$$

$$I_j^{Hc}(k_j, s_t, v_t) = I_H(1 + \tau^u(k_j, s_t, v_t))$$

$$I^{Lc}(s_t, v_t) = I_L(1 + \tau^{ent}(s_t, v_t))$$

and solve a system of nonlinear equations for the tax/subsidy scheme. The cost structure of the decentralized allocation is $\Upsilon^c = \left[\left\{ \Pi_j^c(k_j, s_t, v_t) \right\}_{j=1}^{J}, \left\{ I_j^{Hc}(k_j, s_t, v_t) \right\}_{j=1}^{J-1}, I^{Lc}(s_t, v_t), 0 \right]$.

Appendix (B)

6.2 Results

Figure 3: Establishment Distribution

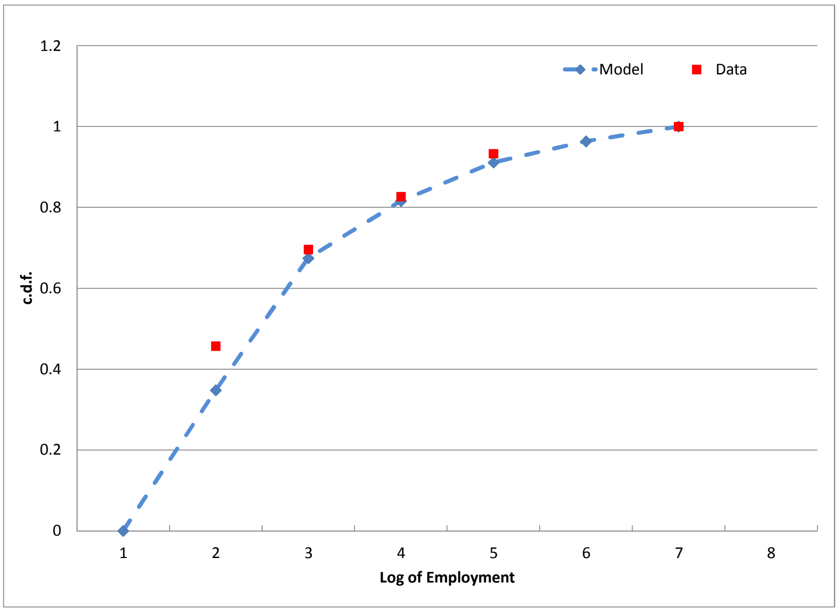
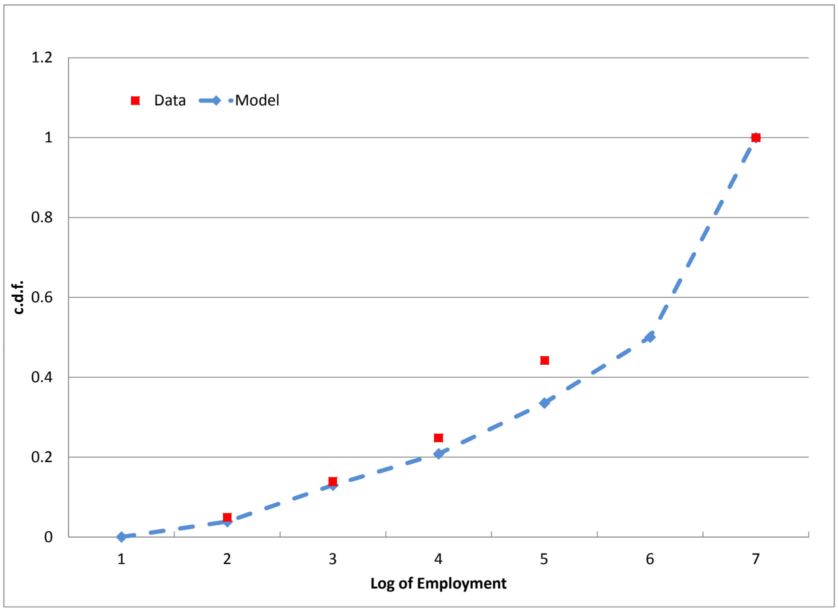


Figure 4: Employment Distribution



Parameter	Target	Value
γ_g	Persistence of Expansions	.685
γ_b	Persistence of Recessions	.298
β	Average Annual Interest Rate	.98
α	Share of Capital	33%
$\sigma(\rho)$	Returns to entrepreneurship	6.66 (0.85)
δ	Mean Exit Rate	0.055
θ	Intertemporal Elasticity of Substitution	1 (log utility)
$\underline{z}$	Lower Bound of Idyosincratic Productivity	0.01
N	Number of Technologies/Capacities	4

Table 1: Parametrization

Parameter	Definition	Value
$s_g - s_b$	Size of the Shocks (Symmetric)	$exp(0.0267) - exp(-0.0267)$
$[\underline{k}, \overline{k}]$	Range of Capacities	$[1, 4]$
$[\underline{z}, \overline{z}]$	Range for Idiosyncratic Productivity (Upper Bound)	$[0.01, 4.25]$
I^{L31}	Entry Costs	$\begin{bmatrix} 1.09 \\ 0.93 \end{bmatrix}$
I^H	Upgrade Costs	$\begin{bmatrix} 4.55 & 11.37 & 37.1 \\ 4.28 & 4.26 & 1.98 \end{bmatrix}$
Π^e	Scrap Values	$\begin{bmatrix} 0.85 & 2.47 & 9.27 & 9.1 \\ 0.86 & 2.46 & 9.2 & 9.13 \end{bmatrix}$
ς_G	Pareto Tail of the productivity distirbution at entry	1.9

Table 2: Jointly Calibrated Parameters

Moment	Data	Model	Moment	Data	Model
Emp. Share, 1-19	0.05	0.04	Estab. Share, 1-19	0.46	0.35
Emp. Share, 20-49	0.14	0.13	Estab. Share, 20-49	0.69	0.67
Emp. Share, 50-99	0.25	0.21	Estab., 50-99	0.83	0.82
Emp. Share, 100-249	0.44	0.36	Estab., 100-249	0.93	0.91
Entry Rate	6.9%	6.24%	Exit Rate	5.5%	6.23%
Investment Spikes[32]	8%	9.1%	Log Emp. (upper bound)	10000+	10829
Output Volatility	2.09%	2.1%			

Table 3: Targeted Moments

	Baseline	Optimal Allocation
Aggregate TFP	3.36	3.73
Standard Deviation TFP	7.9%	8.4%
Coefficient of Variation, TFPQ	3.01	2.66

Table 4: Productivity Statistics

	Baseline	Optimal Allocation	TFP^{mc}
Aggregate TFP	1.31	1.36	2.33
Standard Deviation TFP	2.6%	2.6%	3.3%
Coefficient of Variation, TFPQ	3.01	2.66	1.05

Table 5: Productivity Statistics: Normalized Measure

k	**Ratio mean** $TFPQ$
1	1.02
2	0.99
3	0.99
4	0.98

Table 6: Efficiency across capacities

	Baseline	**Case G**	**Case S**
	γ_g= **.685**	γ_g= **.237**	γ_g= **.685**
	$s_g - s_b$= **0.053**	$s_g - s_b$= **0.053**	$s_g - s_b$= **0.064**
TFP	3.36	3.72	2.19
Standard Deviation TFP	7.9%	9.1%	30.4%
Coefficient of Variation TFPQ	3.01	2.7	2.64
$\mathbf{TFP}_{M=1}/\mathbf{TFP}^{mc}$	0.56	0.58	0.56
Volatility of Output	2.1%	2.5%	8.6%

Table 7: Features of Aggregate Uncertainty

	Model	**Case G**	**Case S**
Entry Rate	6.24%	5.95%	20.4%
Exit Rate	6.23%	5.94%	12.6%
Upgrade Rate	9.1%	9.7%	45.1%

Table 8: Firm Dynamics

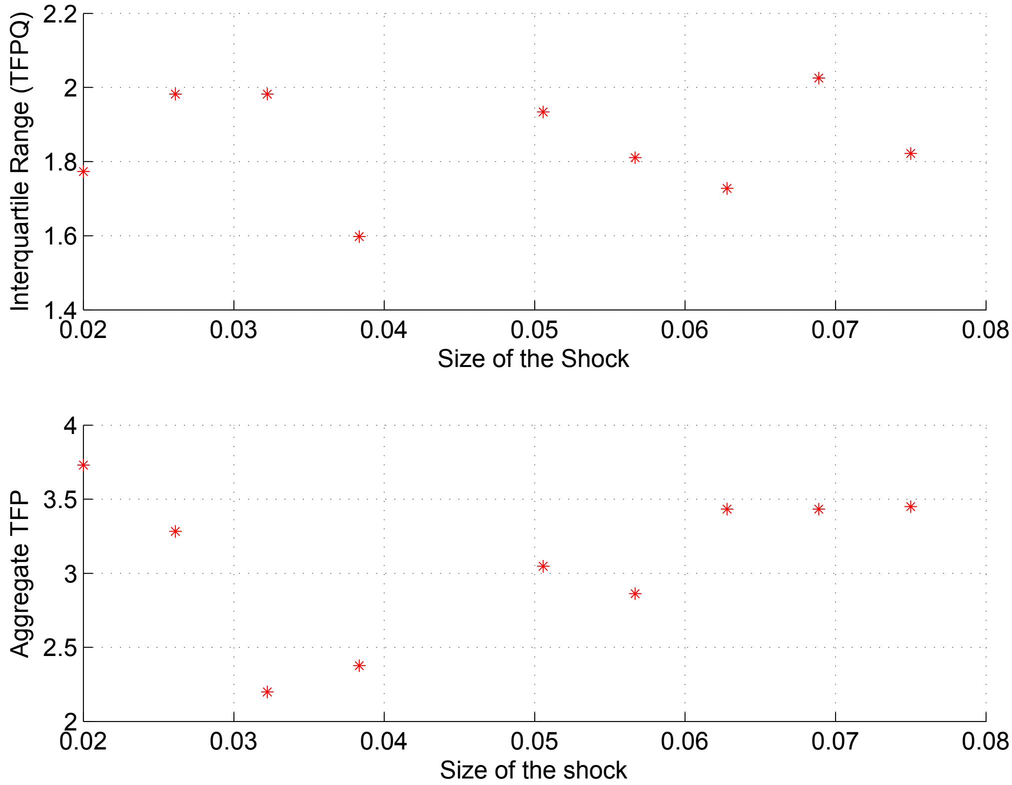


Figure 5: Dispersion in TFP, Aggregate TFP and the Cyclical Component of GDP

Employment	0-49	50-149	150-499	500+
Output Share	0.14	0.06	0.05	0.75
Opt. Policy	0.13	0.05	0.04	0.78
Capital Share	0.69	0.16	0.08	0.07
Opt. Policy	0.71	0.15	0.08	0.06
Employment Share	0.16	0.11	0.15	0.58
Opt. Policy	0.16	0.11	0.14	0.59

Table 9: Optimal Policy: Distributional Implications

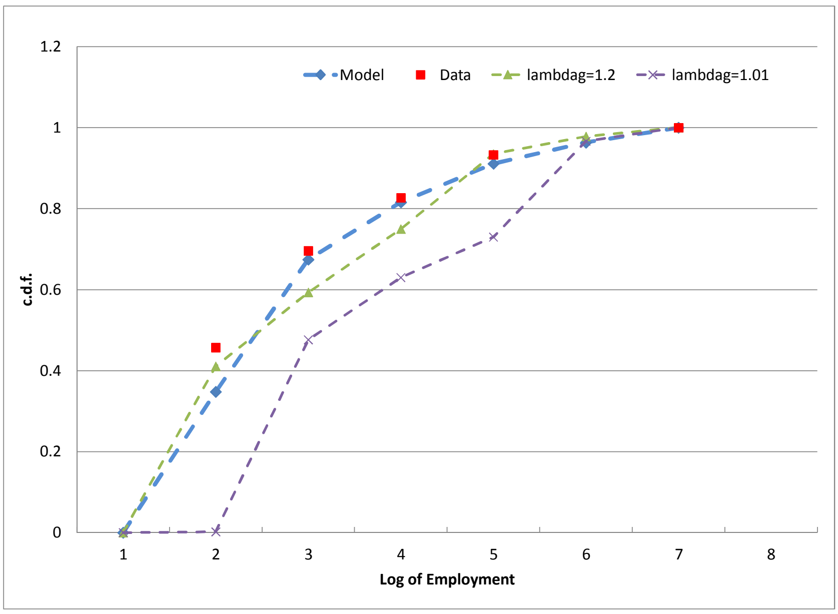
	Model	Optimal Policy
Entry Rate	6.24%	5.85%
Exit Rate	6.23%	5.84%
Upgrade Rate	9.1%	9.8%

Table 10: Optimal Policy: Firm Dynamics

	Good Times		Bad Times	
	Baseline	**Optimal Policy**	**Baseline**	**Optimal Policy**
I^L/Y	0.30	0.28	0.30	0.29
I^H/Y	[0.87 3.28 3.22]	[0.45 0.81 2.51]	[0.87 3.25 3.23]	[0.43 0.83 2.57]
Π^e/Y	[0.39 1.61 4.02 13.11]	[0.50 0.49 2.27 8.33]	[0.33 1.51 1.51 0.70]	[−0.02 0.29 0.79 1.84]

Table 11: Tax/subsidy Structure in terms of output per worker

Figure 6: Establishment Distribution, Sensitivity Analysis



Appendix (C)

6.3 Features of the Solution

Proposition 9 *Continuation Values* $\widetilde{W}$ *are monotonic increasing in idiosyncratic productivity,* z *and the optimal investment/disinvestment strategy of the firm is a set of thresholds such that if* $z < z^e(\psi^J, X_t)$ *they exit the market, and if* $J = L$ *whenever* $z \geq z^u(\psi^H, X_t)$ *the firm upgrades capacity.*

Proof. First notice that $\pi(x_t, X_t)$ is bounded and continuous in z. (Replace the optimal factor demands in the profit function).

Second, let $W^*(x, X)$ be the unique fixed point to the operator T,

$$T(W(x, X_t)) = Max\left\{\Pi_e, \pi(x, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W(x, X_{t+1})\right)\right\}$$

We first show first that $W^*(x, X)$ is non-decreasing in z.

Let $C(Z)$ be the set of continuous bounded functions in z, and let $C'(Z)$ a closed subspace of non-decreasing functions. Take $W \in C(Z)$ and $z_1 < z_2$. then

$$\begin{aligned} T(W(z_1, \psi^j, X_t)) &= Max\left\{\Pi_e, \pi(z_1, \psi^j, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W(x, X_{t+1})\right)\right\} \\ &\leq Max\left\{\Pi_e, \pi(z_2, \psi^j, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W(x, X_{t+1})\right)\right\} \\ &= T(W(z_2, \psi^j, X_t)) \end{aligned}$$

so that $T(C'(Z)) \subseteq C'(Z)$. Hence by the Contraction Mapping Theorem $W^* \in C'(Z)$.

Now, we want to prove that for each $\left(\psi^j, X\right)$ the function $\widetilde{W}\ (z,\psi^j, X_t)$ is strictly increasing in z. Note that the expectation operator in the last term of the previous equation defined over the aggregate of the economy and independent of the productivity of the firm except through the function W^*. Take $z_1 < z_2$

$$\begin{aligned} \widetilde{W}(z_1, \psi^j, X_t) &= \pi(z_1, \psi^j, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W^*(z_1, \psi^j, X_{t+1})\right) \\ &< \pi(z_2, \psi^j, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W^*(z_2, \psi^j, X_{t+1})\right) \\ &= \widetilde{W}(z_2, \psi^j, X_t) \end{aligned}$$

which proves the claim.

Given the monotonicity of the continuation values, the optimality of the trigger strategy follows. Suppose not. Hence, there is a firm with productivity z, such that $z < z^e(\psi^J, X_t)$ and the firm does not exit the market. But the firm with productivity $z + \Delta < z^e(\psi^J, X_t)$ did, so $Max\left\{\Pi_e, \widetilde{W}(z+\Delta, \psi^j, X_t)\right\} = \Pi_e$. From the monotonicity of $\widetilde{W}$, it holds $\widetilde{W}(z+\Delta, \psi^j, X_t) > \widetilde{W}(z, \psi^j, X_t)$ so that $\Pi_e > \widetilde{W}(z, \psi^j, X_t)$ and hence remaining in the market cannot be optimal. Analogous argument hold for the upgrade thresholds. ■

Proposition 10 *The optimal allocation satisfies*

1. *If the minimum capacity constraint is not binding,* $z^e(\psi_L, X_t) > z^e(\psi_H, X_t)$
2. *The exit thresholds are increasing in the cost of capital, i.e.* $\frac{\partial z^e(\psi^j, X_t)}{\partial r_t} \geq 0$.
3. *The upgrade threshold across technology is higher than the exit threshold for high minimum capacity firms, i.e.* $z^u(\psi^H, X_t) > z^e(\psi_H, X_t)$.
4. *The measure of entrants is procyclical.*

Before proving the results note that the instantaneous profits of a firm are

$$\pi(x_t, X_t) = [(1-\rho)Y_t^{1-\rho}\left[\frac{s_t K_t^\alpha}{(Z^k)^\alpha (Z^l)^{1-\alpha}}\right]^\rho - \frac{r_t - MPK_t}{MPK_t}\frac{K_t}{Z^k}]\left(\frac{z\psi^j}{MPK_t^\alpha}\right)^{\frac{\rho}{1-\rho}}$$

1. **Proof.** Note first that the profit function $\pi(x_t, X_t)$ is monotonic in the firm idiosyncratic productivity and the technology shifter. We have proved that firms' continuation values are also monotonic. Hence $W(z, \psi_H, X_t) > W(z, \psi_L, X_t)$ for all z whenever the minimum capacity constraint is not binding. The value of the firm is homogenous in the productivity of the process (follows from the form of the profit function). The optimality condition for the exit thresholds equalizes the firm to its scrap value. Hence, if $\frac{\Pi(\psi^H)}{(\psi^H)^{\frac{\rho}{1-\rho}}} < \frac{\Pi(\psi^L)}{(\psi^L)^{\frac{\rho}{1-\rho}}}$ then $z^e(\psi_L, X_t) > z^e(\psi_H, X_t)$. ■

   Proof. The profit function is such that $\frac{\partial \pi(x_t, X_t)}{\partial r_t} < 0$. Following the same strategy than for the monotonicity in idiosyncratic productivity one can show that $W(z, \psi^j, X_t)$ is non increasing in the cost of capital and the continuation value $\widetilde{W}(z, \psi^j, X_t)$ is decreasing in r_t. As in the previous proof, the result follows from the optimality condition for the exit threshold. ■

Proof. Given that upgrades in technology are costly and the scrap value at exit is independent of the technology operated by the firm. It cannot be optimal to upgrade and exit immediately. For this strategy, I_H units of goods are paid , while exiting while running the low minimum capacity technology yields the same scrap value and no associated cost. ■

Proof. $\widetilde{W}_t(z, \psi_L, X_t)$ is increasing in the aggregate state of technology s. From the free entry condition the result follows. ■

The fact that the scrap value of the firm is independent of the cost capital and the idiosyncratic characteristics of the firm is critical to prove the previous results.

6.4 Law of motion for the distribution of firms

For notational convenience I redefine any function $f(a, X_t)$ as $f_t(a)$. Let $\Lambda : C^v \times C^v \times \{s_g, s_b\} \to C^v \times C^v$ be the equilibrium law of motion for the distribution of firms of low and high minimum capacity technologies. The law of motion is characterized by

$$\begin{array}{ll} v_t^L(z) = (1-\delta)\, v_{t-1}^L(z) - M_t^{eL} + M_t^{ent} \frac{G(z) - G(z_t^e(\psi^L))}{1 - G(z_t^e(\psi^L))} & z_t^u(\psi^H) > z > z_t^e(\psi^L) \\ v_t^L(z) = 0 & \text{o/w} \end{array}$$

In other words, the measure of firms running the low minimum capacity technology equals the measure of firms from the previous period with productivity larger than the current exit threshold, net of exogenous liquidations, plus the measure of entrants with productivity up to the upgrade threshold.

The dynamic for the distribution of high minimum capacity technology is

$$\begin{array}{ll} v_t^H(z) = (1-\delta)\, v_{t-1}^H(z) - M_t^{eH} & z_t^u(\psi^H) > z > z_t^e(\psi^H) \\ v_t^H(z) = (1-\delta)\ v_{t-1}^H(z) + M_t^{ent} \frac{G(z) - G(z_t^u(\psi^H))}{1 - G(z_t^e(\psi^L))} & \overline{z} > z > z_t^u(\psi^H) \end{array}$$

whenever $z_{t-1}^u(\psi^H) \leq z_t^u(\psi^H)$.Otherwise

$$\begin{array}{ll} v_t^H(z) = (1-\delta)\ v_{t-1}^H(z) - M_t^{eH} & z_t^u(\psi^H) > z > z_t^e(\psi^H) \\ v_t^H(z) = (1-\delta)\, v_{t-1}^H(z) - M_t^{eH} + M_t^u(z) + M_t^{ent} \frac{G(z) - G(z_t^u)}{1 - G(z_t^e(\psi^L))} & z_{t-1}^u(\psi^H) > z > z_t^u(\psi^H) \\ v_t^H(z) = (1-\delta)\, v_{t-1}^H(z) - M_t^{eH} + M_t^u(z_{t-1}^u(\psi^H)) + M_t^{ent} \frac{G(z) - G(z_t^u)}{1 - G(z_t^e(\psi^L))} & \overline{z} > z > z_{t-1}^u(\psi^H) \end{array}$$

The measure of firms running at high minimum capacity equals the measure of firms that survived from the previous period, minus exits, plus entrants with productivity larger than the current upgrade threshold. If the current threshold is above the previous one, this measure also includes firms that upgraded technologies under the previous threshold rule and decide to remain in the market under the current exit rule.

Chapter III: Aggregate Fluctuations and the Industry Structure of the US Economy

1 Introduction

The input output structure (a summary of the trade in intermediate inputs across sectors) is usually assumed constant in time. However, recent input output data reported at annual frequencies, suggests that the structure changes in time and that those changes are correlated with aggregate activity in the economy.

1. The average absolute change of intermediate inputs' cost shares in the equipment production (consumption goods) sector is 1.9% (1.1%) annually.
2. The off diagonal terms of the input output matrix change more than the diagonal terms (intermediate inputs produced by the same sector).
3. Cost shares of intermediate inputs[33] produced by the equipment (consumption) sector correlate positively (negatively) with aggregate activity.

Changes in the entries of the input-output matrix are a reflection of the pattern of reallocation in the economy in response to changes in relative prices. Relative price change when the efficiency in production of certain sectors improve over others in the economy. Hence, cost share behavior bears information as of the reallocation of factors in response to changes in efficiency in production across sectors, and through it, of the propagation of shocks in the economy. In this paper, I revisit an old question in real business cycle theory: what is the role of sectoral and neutral shocks in aggregate fluctuations? To answer this question I study a two sector economy augmented to allow for intermediate input linkages and consistent with the movements in the input output structure observed in the data. This framework is homomorphic to the canonical model with two sectors, and investment specific and neutral shocks (Foerster *et al.* (2008) and Abel and Eberly (1997)). The main result is that the augmented economy predicts a stronger role for neutral shocks in the volatility of aggregate output vis a vis a comparable economy with a fixed input output structure. Also, the amplification of sectoral shocks is stronger in the flexible cost share economy than in the canonical one.

[33] Based on BEA, description of Annual Industry Accounts this include energy, raw materials, semi-finished goods, and purchased services

After the work of Greenwood *et al.* (1997) we have seen the development of a fruitful research agenda that studies the role of investment specific versus neutral shocks in long run growth and aggregate cyclical fluctuations (See Foerster *et al.* (2008) and Abel and Eberly (1997)). I augment that economy to allow for intermediate good linkages across sectors. The consumption sector produces final and intermediate goods out of capital and intermediate goods from the equipment and consumption sector. In the equipment sector, there are two subsectors. One produces investment goods out of capital and intermediate goods, and the second on produces intermediate equipment goods out of capital and intermediate consumption goods. Capital is sector specific and the stock of capital is fixed at the beginning of each period, before shocks are realized. Although the structure is richer than the canonical model, I show that under certain conditions on the share of inputs in production, the augmented economy reduces to a two sector economy indistinguishable from the standard economy studied in the literature. Among others, the economy is consistent balanced growth and investment specific technical change[34].

In a one sector model, the share of intermediate goods in production has a role in determining the level of GDP, Jones (2011). But GDP growth depends only on aggregate productivity growth, measured as the change in output not explained by a change in primary inputs (labor and capital). In other words, intermediate inputs are irrelevant in determining aggregate fluctuations. In a multisector neoclassical model instead, the production possibility frontier is a weighted measure of the Solow residuals in each sector, Hulten (1978). The computed solow residuals depend not only on the allocation of primary factors but also on the allocation of intermediate goods across sectors. Hence, aggregate output fluctuations are determined by changes in the allocation of intermediate goods across sectors in response to changes in relative prices.

If we assume that markets are competitive, cost shares of inputs are equal to the elasticity of inputs in production, i.e.

$$Csh^{iJ} = \frac{p^i M^i}{p^J Y^J} = \frac{\partial Y^J}{\partial M^i}\frac{M^i}{Y^J} = \left|\varepsilon^{iJ}\right|$$

where p index prices, Y gross output and M intermediate good consumption. In a frictionless economy, changes in relative prices reflect changes in relative productivity across sectors. When

[34] While unexplored in this paper, these characteristics are key if the framework is to be used in the empirical analysis of the role of neutral and investment specific technical change through long run restrictions as in Fisher (2006)

input productivity moves along relative prices one to one, cost shares are constant and the input output structure of the economy does not change. Substitution towards more productively produced inputs may generate output increases in the sector producing the intermediate good as well as in the one consuming it. Cost shares can go up or down. Hence, cost shares movements, jointly with the behavior of relative prices bring us direct evidence of the pattern of reallocation.

Why do these patterns imply different roles for sector specific and neutral shocks? To illustrate, let's think of an economy with three sectors. Two sectors produce intermediate goods out of a linear technology in sectoral productivity. The third sector combines these two inputs to generate the consumption good in the economy using a Leontief technology.

$$Y = \min\left\{aM^1, M^2\right\} \qquad M^1 = A^1,\, M^2 = A^2$$

where A^i is exogenous. The equilibrium price of output satisfies

$$p = p^2 + \frac{1}{a}p^1$$

Suppose that productivity improves in sector 2. Then $\Delta A^2 > 0$, and the cost share for input 2 drops as $\Delta p^2 < 0$. The cost share of input 2 is just the relative price of input to output. Total value added does not change because $Y = aA^1$, but aggregate productivity improves as $\Delta TFP = \frac{A^2}{aM^1}\Delta A^2$. Hence, a purely sectoral shock has no impact on aggregate output but improves productivity. A neutral shock (that raises both A^1 and A^2) improves both aggregate productivity and output. Furthermore, should the economist analyzing the economy had imposed a constant cost share structure, it would have predicted an increase in aggregate output after the shock. Substitution towards the now more efficiently produced input would have induced an increase in output of $\Delta Y = Csh_{2Y}\Delta A^2$.

While in the example the disparity in cost share behavior is fully characterized by the underlying production function describing output in each sector, there are many other mechanisms for which cost shares may change differently across sectors, even when operating the same technology. Input specificity is one of them. When looking at aggregate sectoral data, many goods are bundled together. Movements in cost shares may reflect the inability to easily switch across goods that are close together (belong to the same 3 digit NAICS code) but not necessarily the same. Another potential source of cost share fluctuations is the presence of inventories. While inventories should be accounted independently of intermediate goods, data measurements may include items that we would consider inventories. A similar argument follows for equipment parts, which should be accounted as part of investment goods. If a firm has stock up enough intermediate inputs for production within a year of production, changes in relative prices anytime during the year will not be reflected in its input intake. In this paper, I assume movements in costs shares are generated by differences in production technologies only. This allows me to assess the quantitative impact of changes in cost shares while keeping a structure that is very close to the canonical two sector model in the literature.

In the paper, conditions are provided for the existence of a balanced growth path in which all inputs are used in production, yet productivity growth rates in the equipment and consumption sector may differ. When the detrended economy is calibrated to predict the patterns of cost share movements observed in the data, the contribution of neutral shocks to output volatility increases relative to a comparable economy with constant cost shares. In other words, the variance decomposition of an economy calibrated to the same steady state in which constant cost shares are imposed (Cobb-Douglas technologies), indicates that neutral shocks contribute 8% less to aggregate output volatility than they do in a flexible cost share economy. Aggregate output impulse responses to persistent and fully temporary shocks depend on the underlying reallocation patterns embedded in the economy.

Finally, the impact of sectoral shocks is amplified in the flexible cost share economy versus the constant one. In other words, to generate the same volatility in aggregate GDP, and gross output in the consumption and equipment sector, the identified size of the shocks in the economy with a fixed input output structure is larger, than that in the flexible cost share economy.

The rest of the paper is organized as follows. Section 2 described the related literature, Section 3 documents the main finding in the data. Section 4 describes the model and the characterization of the BGP. Section 4 presents the calibration and quantitative results. Section 5 concludes.

1.1 Literature Review

The literature on the role of sectoral shocks is extensive. The seminal work by Hulten (1978) paved the way for the study of the role of input output linkages in the transmission of sectoral shocks. While the authors find a substantial role for sectoral shocks in shaping aggregate fluctuations in output, much discussion has been triggered since on the plausibility of transmission of idiosyncratic shocks to the aggregate economy. At the heart of the arguments is whether law of large arguments apply to the units that we define as sectors[35].

There are quantitative approaches that exploit the factor structure of a model with input output linkages as in Long and Plosser (1983). The work of Foerster *et al.* (2008) show that the role of sectoral shocks in explaining aggregate volatility has increased (in relative terms) after the great moderation. Key to the econometric strategy of the paper is the assumption that the input output structure is stable. This paper departs fundamentally from it by allowing trade intensities in intermediate goods to change across time. More recently, Abel and Eberly (1997) has used intermediate input purchases to identify the relative importance of industry-specific shocks. In his framework he estimates an elasticity of substitution between value added and intermediate goods different than one. In this paper, I assume the elasticity is between intermediates and value added unitary so that while sector productivity trends may differ across sectors, the economy is consistent with a balance growth path in which all intermediate goods are used in production.

After the work of Greenwood *et al.* (1997), the analysis of economies with neutral and investment specific shocks, is the preferred choice in the literature studying business cycles. Both the consistency with long run growth and a trend a in the relative price of equipment to consumption is key in a two sector economy like the one I study in this paper. While the papers described earlier allow for a large degree of heterogeneity across sectors in the economy, I keep the economy as close to the now plain vanilla business cycle model as possible. By doing this, I can uncover the role of the input output structure, while 1) providing a flexible framework that a) can be enriched to analyze a richer shocks structure as in Smets and Wouters (2007), b) can accommodate stochastic trends between the investment and consumption sector (as in Schmitt-Grohe and Uribe (2011)) 2) paving the way for future research on the its implications for long run identification strategies as in the seminal work of Gali (1999), augmented later to allow for investment specific shocks in Fisher (2006).

There is an extensive literature in business cycles studying the impact of investment specific

[35] Dupor (1999) shows that when the network that describes the input output structure is a balanced one, sectoral shocks indeed do not affect aggregates. We have learn much about the characteristics of the network structure since. Horvath (2000) shows that when the input output structure is sparse (as is the case in the data) sectoral shocks do not fade away in the aggregate. Alternatively, Carvalho and Gabaix (2013) show that when the role of sectors in the economy is unbalanced, in the sense that a few sectors account for most of the value added in the economy, the law of large numbers fails and sectoral shocks can have aggregate impact. Along the same line are the network results by Acemoglu *et al.* (2012) and Oberfield (2011). Hence, there is nowadays consensus that sectoral shocks can be transmitted to the aggregate economy and have quantitative impact.

and neutral shocks for aggregate fluctuations. Justiniano *et al.* (2010) show that in a full DSGE model with price markup shocks, neutral technology shocks, Calvo pricing, wage markups shocks, preference shocks, and investment shocks, most of the variability of output is explained by shocks to the marginal efficiency with which final goods are transformed into capital. The structure of intermediate goods trade across sectors is abstracted away. In this paper, the economy is strip out from the rich shock structure and augmented to allow for intersectoral trade in intermediates. This allows me to highlight the relevance of modeling the input output structure with endogenous fluctuations vis as vis an economy with a fixed input output structure. It is shown that the amplification of sectoral shocks is stronger in this economy, than in a comparable economy with a constant input output structure. The particular modelling strategy in which the equipment production sector is split into an investment good producing sector and an intermediate good producing sector allows me to identify the differential impact of shocks to each of these activities. Shocks to the production of investment and intermediates goods are in nature disparate and are shown to have distinct relative impact in output volatility.

Finally, there is an incipient literature applying notions of networks to understand the generation of trade linkages between firms (Oberfield (2013) and Carvalho and Voigtlander (2014)). While the focus of the analysis is different from the one in this paper, both are complementary to each other. Is through the coordinated behavior of all those firms that the decision on trade linkages matters for the aggregate dynamic of the economy. In this paper I show that cost shares of fairly aggregated sectors fluctuate in time and they are relevant in understanding the role of investment specific and neutral shocks in the economy. It remains to be shown that the fluctuations in aggregate cost shares are consistent with the firm behavior observed in the data.

2 Empirical Facts

2.1 Input-Output structure

I study make-use tables from 1993 to 2012 as reported by the Bureau of Labor Statistics based on BEA data. The series are presented for 195 sectors, and values are current US dollars. The data Appendix describes in detail the sectors that have been included in the analysis.

The objective of this analysis is to describe the changes in the input output structure vis a vis the aggregate level of activity in economy. The level of aggregation across sectors is key in producing the facts documented in this section. For the purpose of the analysis in an economy with two sectors as the one presented in the body of the paper, aggregation of consumption and equipment/investment sectors is enough. However, in the analysis of the empirical facts I present results where I aggregate sectors to build an Input Output matrix with 33 industries, consistent with the KLEMS sectoral data available at BEA. Then I further classify these 33 sectors as investment/equipment, consumption sector, agricultural and mining sector.

Independent of the level of aggregation, the investment sector is constructed to include equipment producing sectors consistent with the analysis in Cummins and Violante (2002). In other words, the aggregation rule is consistent with the construction of relative price indexes that are used to describe the path of investment specific technical change.

In the analysis I abstract from the behavior of agricultural and mining sectors for several reasons. First, the assumption of constant returns to scale in technology that I use later in the model economy is unlikely to hold in these sectors, where there are large fixed costs of operation. Second and most important, fluctuations in price of these commodities may not be tight to changes in relative productivity vis a vis other sectors in the economy, but rather to developments in international commodity markets. The government, except postal services, has been abstracted away from this analysis.

The cost share of input i in sector j is defined as

$$Csh^{iJ} = \frac{p^i M^{iJ}}{p^J Y^J}$$

where Y^J denotes gross output in sector j, M^{iJ} is the intermediate good i intake of sector j and p denote prices. Hence, cost shares fluctuate whenever changes in relative prices are not fully translated into changes in input productivity ($\frac{Y^J}{M^{iJ}}$).

Figures 7 to 12 display time series of cost shares of consumption and equipment intermediate goods for various sectorial aggregations. The period of analysis includes the Great Recession, where the input output structure experienced a substantial shake out. The nature of those changes are out of the scope of this particular analysis. However, the panels include a line identifying the date collapse in the US financial market. This panel are constructed singling out the behavior of particular sectors that might be driving the behavior of the aggregate cost shares of equipment and consumption. In the first two panels I present data with all sectors are described in the baseline classification in the appendix. In the next two panels, I single out the behavior of the construction sector, potentially important in the developments of the late 90's and 2000's up to the recession in 2007. I also single out Utilities. In the last two panels I show the behavior of cost shares of the finance sector and the real state sector. In each of the last 4 panels,the cost shares of consumption and equipment have the corresponding sector under analysis, substracted out.

In Figures 7 and 8 I show the behavior of cost shares of each type of intermediate goods in the consumption and equipment sector, and for completeness, in the agricultural and mining sector. Own cost shares fluctuate substantially (consumption in consumption, and equipment in equipment). These sectors aggregate up changes in relative prices across subsectors, so it is possible for the within sector cost share to fluctuate with movements in relative prices and not only with input productivity. The cost share of consumption intermediates in equipment displays a slight upward trend up to 2000s that reverts in the next decade. The cost share of equipment intermediates in equipment displays a mirror dynamic, with the cost share dropping up to the 2000s and increasing later on. The cost share of equipment in consumption displays a downward trend that may well be explained by the drop in the relative price of equipment to consumption goods. If we normalize cost shares to account only for those sectors accounted in the consumption and equipment sectors[36], the cost share of consumption in the equipment sector averages 43% in the sample period, while the cost share of equipment intermediates in consumption averages 7% and displays a declining trend. If only these two aggregate sectors are considered in the sample, the cost share structure is as depicted in Table (12) .In the sample period, gross output of the consumption sector is four times larger than gross output of the equipment sector. Also, in the consumption(equipment) sector 67% (48%) of gross output is used as intermediate input in other sectors. The rest is either consumed or adds to the capital stock.

In Figures 9 and 10 I disaggregate the behavior of the construction and utilities sector from the overall consumption sector. The exercise is designed to understand if some of the dynamic described before are explained by particular sectors. When construction and utilities are abstracted away the consumption sector, the cost share of equipment in consumption displays a less steep downward trend than for the full sector, in particular after 2000.The cost shares of i equipment intermediate inputs in construction is relatively stable. The cost share of consump-

[36]This normalization is consistent with the model presented in the next section, in which only this two sectors are accounted for.

tion intermediate drops substantially up to 2000 and raises thereafter. Utilities cost shares of equipment and consumption intermediate goods behave as mirror images. The dynamic of the cost share in of consumption in consumption is very similar to the one depicted in the previous panel. This indicates that neither construction nor utilities explain the decrease in cost share of consumption intermediates in consumption from the 2000s onwards.

Figures 11 and 12 display a disaggregation for the finance sector and real state sector. The share of consumption intermediates in the consumption sector displays a downward trend for the whole sample period, indicating that part of the raise in the early 2000 are explained by the consumption intermediate share in the finance sector, which remains relatively stable. Noticeably, the cost share of consumption in equipment is much more volatile when the finance and real state sector are abstracted from consumption.

In summary, cost shares fluctuate substantially. The cost share of equipment in consumption sectors displays a downward trend, while the remaining cost shares are relatively stable. Cost shares of consumption intermediates went up on average up to the beginning of the 2000s and then down to the end of the sample period. The cost share of equipment in equipment sectors behaved as mirror image of that pattern.

Next I would like to describe the year on year changes in cost shares. In other words, I would like to describe changes in (12). To study variation across inputs I compute average absolute changes year on year. Those are presented in table (13)

The off diagonal terms, are larger than the diagonal terms, and in particular, the largest movements are for equipment intermediates in the consumption sector. Relative price changes within a category, i.e. computers and transportation equipment, are aggregated out in the changes reported in the diagonal of the table. Changes in the relative price of consumption and equipment basket are reflected in the fluctuations in the off diagonal terms. To grasp the magnitude of these changes,one could compute the absolute average deviations from the mean share over the sample period, as in table(14)

Changes in the cost share of equipment in consumption are accounted large (10.1% on average), and those of consumption in the equipment sector average 3% of the mean. However, as depicted in figures 7 to 12, some of the share series contain longer term trends. To avoid imputing changes in cost shares as just shifts along the trend, I also report deviations from an hp-trend. These absolute deviations are reported in (15). Once we account for this trend, deviations in own intermediate inputs cost shares drop below 1% per year. and deviations in the cost share of consumption in equipment and equipment in consumption are 1.3% and 1.5% respectively.

To summarize, changes in cost shares are not negligible whether accounted as year on year changes or as deviations from trend. Changes in cost shares of intermediate inputs produced by other sectors (the off diagonal terms in the last three tables) are larger than those in the diagonal terms.

The asymmetry in the industry structure depicted in Table(12) and the contribution of each sector in value added and gross output are important features that the model economy needs to capture to assess: a) the elasticity of the cost shares to changes in relative prices, b) the aggregate impact of those changes. Elasticities of substitution across inputs (hence, cost shares) depend on the industry structure as summarized by (12) .However, the relevance of shocks in the aggregate depend on their contributions to value added and gross output.

Finally, let me describe the correlation of cost share changes with aggregate activity. I define value added as the sum of the dollar value of value added in the equipment and consumption sectors. I report correlations for three different time series. The full sample includes the Great Recession (GR), the second sample only focuses on the periods up to the GR. The third sample interpolates the pre and post 2008 values abstracting from the drop in activity in 2008. Table (16) shows that cost shares of consumption intermediate goods are countercyclical irrespective of the sample period. Cost shares of equipment intermediates in the equipment sector are procyclical in the full sample, but acyclical if we consider the period pre 2008 or the interpolated data. The correlation of the cost share of equipment intermediates in the consumption sector with aggregate value added displays the largest disparities across sample periods. While for the full sample the correlation is positive , in the pre 2008 period it is identified negative of about the same magnitude. When we interpolate to abstract from the GR the cost share appears acyclical.

For the calibrated exercise at the end of the paper I will use data from the full sample. The particular shifts in the input output structure that occur during the break out of the recession remain to be studied, possibly at a higher level of disaggregation.

The countercyclicality of the cost share of consumption in equipment is important in view of the extensively documented countercyclicality of the price of equipment. Negatively correlated cost shares indicate that input productivity drops less than the drop in the relative price of equipment to consumption. In good times the relative price goes down, inducing the cost share of consumption in equipment to increase if there are no changes in input productivity. For the cost share to drop, input productivity has to increase in the equipment sector.

If we abstract from the changes in relative prices that are certainly occurring within each of these fairly aggregate sectors, the correlations on the diagonal terms of the table indicate that: a) input productivity increases in the consumption sector when aggregate activity booms on average, and b) that input productivity in the equipment sector drops during activity booms on average. Note that if the relative price of one of the categories within a sector is dropping dramatically, say the price of computers relative to transportation equipment, that shift in prices might be reflected as a procyclical cost share for intermediates produced in the same sector.

As mentioned early, these results are not invariant to the degree of aggregation in the economy. In Figures 13 and 14 I compute the correlation of cost shares of equipment and consumption intermediate goods for 33 sectors, and for the full sample of 170 sectors (abstracting the government).

To conclude this section, let me summarize the main features observed in the data:

By studying an economy consistent with these features I will argue that these facts are key in understanding the role of sectoral and neutral shocks as well as the amplification of shocks in the economy.

1. Sectors have disparate roles as input suppliers of other sectors in the economy.

2. The average absolute change of intermediate inputs' cost shares in the equipment production (consumption goods) sector is 1.9% (1.1%) annually.

3. The off diagonal terms of the input output matrix change more than the diagonal terms(intermediate inputs produced by the same sector).

4. Cost shares of intermediate inputs produced by the equipment (consumption) sector correlate positively (negatively) with aggregate activity.

3 Model

3.1 Environment

This is a discrete time, infinite horizon economy.

There are two final goods produced in the economy, equipment and consumption goods. Additionally, intermediate equipment goods are produced.

There are three production sectors in the economy. I assume there is a representative firm in each sector. All markets are competitive and the technologies are constant returns to scale. The diagram 17 displays the input output structure of the economy under analysis. A cross indicates a positive entry in the matrix. To distinguish between goods produced by the equipment sector, I call X_2 the sector producing intermediate equipment and X_1 the sector producing investment goods (capital)[37].

Total value added in this economy equalizes the sum across entries in the last two columns (GDP). Gross output per sector corresponds to the row sum across all columns. Total cost corresponds to the column sum per sector. The cost share is the ratio between a particular entry in the intermediate demand section and gross output. The cost shares in the model are constructed as in the data, separating out expenses in capital services.

There is a representative household with standard preferences over final consumption goods. She maximizes lifetime utility by choosing a consumption stream as well as purchases of investment goods.

3.2 Representative Household

The representative household maximizes lifetime utility subject to its budget constraint. Her income stems from the rental of capital to the firms in the economy and from claims to the profits of those firms. Note that capital is specific to a sector and hence capital cannot be instantaneously reallocated from one sector to another. The non-negativity constraint in investment goods should hold for each capital type.

$$\max_{c_t^s, c_t^m, x_t} \sum_{t=0}^{\infty} \beta^t \; U(c_t)$$

[37] This distinction is useful for the analysis of the balance growth path in particular. I will show that under certain conditions, this economy reduces to a two sector economy where both sectors produce final and intermediate goods, and it is possible to observe investment specific technical change.

subject to

$$p_t^y c_t + p_t^{x1} x_1 \leq r_t\left(k_{x1} + k_{x2} + k_y\right) + \sum_{j=y,x_1,x_2} \pi_t^j$$

$$k_{x1}^{'} - k_{x1}(1-\delta_x) = i_{x1} \qquad (\kappa_x)$$

$$k_{x2}^{'} - k_{x2}(1-\delta_x) = i_{x2} \qquad (\kappa_x)$$

$$k_y^{'} - k_y(1-\delta_y) = i_y \qquad (\kappa_y)$$

$$i_y + i_{x1} + i_{x2} = x_1 \qquad \text{and } i_j \geq 0 \text{ for } j = y, x_1, x_2$$

where β is the discount factor; p^j indexes prices for alternative goods $j = y, x_1$, i.e. final consumption and investment goods, respectively; the capital stock is k_t^j with rental rate r_t and the profits of the firms in each sector are π_t^j. The depreciation rate is allowed to differ between equipment production sectors and consumption production sectors.

3.3 Consumption Goods Sector

The representative firm in the consumption sector maximizes profits each period. It has available a technology that uses intermediate goods (M) and capital goods. Once the productivity of all sectors is realized, it chooses its input purchases. The problem of the firm reads

$$\max_{M_t^{yy}, M_t^{xy}, k_t^y} \; p_t^y Y_t^y - p_t^y M_t^{yy} - p_t^{x_2} M_t^{xy} - r_t k_t^y$$

subject to

$$Y_t = \exp(A_t^g)\left(k_t^y\right)^{\alpha_{y1}}\left(\alpha_{y2}\left(M_t^{yy}\right)^{\rho_y} + (1-\alpha_{y2})\left(M_t^{xy}\right)^{\rho_y}\right)^{\frac{\alpha_{my}}{\rho_y}}$$

where A_t^g is a Hicks Neutral productivity shock. For simplicity we have assumed that shocks to the productivity of the consumption good sector correspond to aggregate shocks[38]. The intermediate good purchases from sector j are M_t^{jy}; k_t^y is the stock of capital used in production, α^{y1} is the share of the capital/value added in gross output; $(1-\rho_y)^{-1} \in (-\infty, 1)$ is the elasticity of substitution across intermediate goods in the equipment sector; α_{my} is the share of intermediates in value added; and α_{y2} corresponds to the share of consumption intermediate

[38] Shocks particular to this sector can be incorporated. However, we expect the predictions of that economy to be analogous to this one. The current set up correspond in which any change in idyosincratic productivity in this sector is reflected in changes in the relative productivity of the other two sectors in the economy. Quantitatively, the modeling strategy may make a difference in the variance decomposition exercise, so robustness checks will be run.

inputs in the production of consumption goods when $\rho_y = 0$ (Cobb-Douglas technology).

Labor is assumed away in this analysis. Or in other words, one could assume that capital and labor are one to one in production, with an elasticity of substitution equal to zero, i.e. a Leontieff technology. It might be potentially important to model the substitution patterns between labor and capital as in Koh and Santaeulalia-Llopis (2014). This substitution patterns can be complementary to the shifts in intermediate input intake generated by the model.

3.4 Equipment Sectors

3.4.1 Investment Goods

The representative firm in the equipment investment sector maximizes profits by choosing intermediate good purchases of both consumption and intermediate equipment goods. Its problem reads

$$\max_{M_t^{xx}, M_t^{yx_1}} \; p_t^{x_1} X_t^1 - p_t^{x_2} M_t^{xx_1} - p_t^y M_t^{yx_1}$$

subject to

$$X_t^1 = \exp(A_t^g) \exp(A_t^{x_1}) \, (k_t^{x_1})^{\alpha_{x1}} \left(\alpha_{x2} \, (M_t^{xx_1})^{\rho_x} + (1-\alpha_{x2}) \, (M_t^{yx_1})^{\rho_x}\right)^{\frac{\alpha_{mx}}{\rho_x}}$$

where $A_t^{x_1}$ is a Hicks Neutral sectoral productivity shock, M^{ix} are intermediate good i purchases in sector X_1; $(1-\rho_x)^{-1} \in (-\infty, 1)$ is the elasticity of substitution across intermediate goods in the equipment sector; α_{mx} is the share of intermediates in value added; and α_{x2} corresponds to the share of intermediate equipment inputs in the production of investment goods when $\rho_x = 0$.

3.4.2 Intermediate Goods

The representative firm in this sector maximizes profits by choosing capital and intermediate goods from the consumption sector. Its problem reads

$$\max_{M_t^{yx_2}, k_t^y} \; p_t^{x_2} X_t^2 - p_t^y M_t^{yx_2} - r_t k_t^{x_2}$$

subject to

$$X_t^2 = \exp(A_t^g) \exp(A_t^{x_2}) \left(k_t^{x2}\right)^{\zeta} (M_t^{yx_2})^{\alpha_{mx_2}}$$

where A_t^{x2} is a Hicks Neutral productivity shock, $M_t^{yx_2}$ is the purchase of intermediate consumption goods; k_t^{x2} is the stock of capital used in production; and ς corresponds to the share of capital in the production of intermediate equipment goods and α_{mx_2} is the share of intermediates in value added.

3.5 Productivity

Each sector takes the realization of the productivity process as given. Productivity has two elements, a deterministic trend and a noise term. Let $\mathbf{A}_t \equiv \{A_t^g, A_t^{x_1}, A_t^{x_2}\}$ be the current realization of the shocks in the economy. The dynamic of $\mathbf{A}$ is described as

$$\mathbf{A}_t = \Gamma(\mathbf{A}_{t-1})$$

$$\mathbf{A}_t = (1+\gamma_t)\mathbf{A}_0 + \Lambda_t$$

where γ_t is a vector collecting the time trends and Λ_t is the noise in the process, $E(\Lambda_t) = 0$.

The noise term has in turn two elements. One that is purely temporary and I call ϵ_t and a persistent component $\mathbf{z}_t$ with persistence θ and innovation η_t. In other words, the noise structure is

$$\Lambda_t = \mathbf{z}_t + \epsilon_t$$

$$\mathbf{z}_t = \theta \mathbf{z}_{t-1} + \eta_t$$

$$\Lambda_t = \theta\Lambda_{t-1} - \theta\epsilon_{t-1} + \eta_t + \epsilon_t$$

$\epsilon_t \backsim N(0, \Sigma^\epsilon)$ and $\eta_t \backsim N(0, \Sigma^\eta)$. The variance covariance matrix of the shocks are Σ^ϵ and Σ^η independent from each other. Whereas the ϵ_t shocks are purely temporary, the time series of gross output and value added at the sector level will display persistence, through the impact of these shocks on the accumulation of sector specific capital. Also, whereas both persistent and purely temporary shocks may be independent across sectors, the economy will display comovement due to the intermediate input linkages.

4 Equilibrium

Before defining the equilibrium let me introduce some additional notation. Let $\mathbf{p}_t \equiv \{p_t^y, p_t^{x_1}, p_t^{x_2}\}$ be the vector of prices in the economy, $\mathbf{M}_t^y \equiv \{M_t^{yx_1}, M_t^{yx_2}, M_t^{yy}\}$ be the vector of intermediate consumption goods, $\mathbf{M}_t^x \equiv \{M_t^{xx_1}, M_t^{xy}\}$ be the vector of intermediate equipment goods.

Definition 3 *A competitive equilibrium is an allocation of consumption,investment and capital* $\left\{c_t, \left\{i_t^j, k_{t+1}^j\right\}_{j=y,x_1,x_2}\right\}_{t=0}^{\infty}$*, as well as intermediate good consumption* $\{\mathbf{M}_t^y, \mathbf{M}_t^x\}_{t=0}^{\infty}$*, such that given a system of prices,* $\{\mathbf{p}(\mathbf{A}_t), r(\mathbf{A}_t)\}_{t=0}^{\infty}$*, the exogenous dynamic for sectoral productivity* $\mathbf{A}_{t+1} = \Gamma(\mathbf{A}_t)$ *and the initial stock of capital* k_0^j*,*

1. *The representative household maximizes utility*
2. *The representative firm in each sector maximizes profits*
3. *Markets clear:*
   - *(a)* $c_t + M_t^{yy} + M_t^{yx_1} + M_t^{yx_2} = Y_t$
   - *(b)* $i_t^y + i_t^{x_1} + i_t^{x2} = X_t^1$
   - *(c)* $M_t^{xx_1} + M_t^{xy} = X_t^2$

I now describe how the production possibility frontier changes in this multisector economy. As in Hulten (1978), the PPF is a weighted average of Solow residuals of different sectors in the economy. Let $\widetilde{x}_t$ be the log deviation of variable x from its steady state value. Deviations in aggregate efficiency in period t, can be described by

$$\widetilde{T}_t = \sum_J \frac{p_t^J Y_t^J}{\sum_j p_t^j \left(Y_t^j \gamma_t^j\right)} \widetilde{Z}_t^J \tag{28}$$

Fluctuations in the solow residual in each sector are characterized by

$$\widetilde{Z}_t^y = \widetilde{Y}_t - \sum_{i=1}^{S} Csh^{iy} \widetilde{M}_t^{iy} - Csh^{ky} \widetilde{k}_t^y \tag{29}$$

$$\widetilde{Z}_t^{x^j} = \widetilde{X}_t^j - \sum_{i=1}^{S} Csh^{ix} \widetilde{M}_t^{ix} - Csh^{kx} \widetilde{k}_t^{x^j} \tag{30}$$

Hence, the residual is the change in aggregate output not explained by changes in the input of production. Using the definition of cost shares, movements in intermediate input intake per sector can be characterized by

$$\widetilde{M}_t^{iJ} = \widetilde{Csh}_t^{iJ} + \widetilde{p}_t^J - \widetilde{p}_t^i + \widetilde{Y}_t^J$$

Hence, whether equilibrium prices adjust so that the log deviation in intermediate purchases is the same in the constant and flexible cost share economy should be assessed in the context of a general equilibrium model. There is no reason to believe this will be the case. If predicted log deviations in intermediate purchases differ across economies, so will productivity and through them, predicted aggregate productivity changes.

4.1 Balanced Growth Path

Before moving on to the quantitative assessment of the model, I describe the balance growth path of the economy and the conditions that reduce this economy to a plain vanilla two sector economy as in Greenwood *et al.* (1997).

A Balance Growth Path is a path of gross output in the consumption, aggregate consumption, intermediate inputs intake, and equipment intermediate sector gross output such that they all grow at a constant equal rate, and a path of aggregate investment, capital and investment at the sectoral level, such they also grow at a equal constant rate, possibly different from the one of aggregate consumption.

Theorem 5 *Given the technologies assumed for this economy, a Balanced Growth Path (BGP) where all intermediate goods are used in production exists iff technology is Cobb Douglas in capital and intermediates and either 1) the elasticity of substitution across intermediate goods, equals unity (Cobb-Douglas technology); or 2) there are no linkages through intermediate goods between the equipment and consumption sector; 3) productivity growth in the consumption sector and intermediate equipment sector are proportional by a factor* $(g^x)^{\alpha_{y1}-\zeta}(g^y)^{\alpha_{my}-\alpha_{mx_2}}$ *,where* g^x *is the growth rate of gross output in the investment sector and* g^y *is the one in the consumption sector.*

The first result is analogous to that in Ngai and Pissarides (2007) in an economy with structural change. The second result is well know as it reduces the economy to one like the one in Greenwood *et al.* (1997). The third one allows me to study the economy that has been described in the previous section. One with non-trivial heterogeneous productivity processes across different sectors while allowing for fluctuations in cost shares. It is worth mentioning that while cost shares are allowed to change, in equilibrium they will be constant along the BGP. In the third case, productivity growth in the intermediate equipment and consumption sector are allowed to differ iff the shares of capital and intermediates in value added are different across sectors. If productivity growth rates are proportional, then output in the intermediate equipment sector and in the output sector grow at the same rate.

Corollary 3 *For a given productivity growth rate in the investment sector, it is possible to find a set of parameters (share of capital and intermediates) such that productivity growth in the intermediate equipment sector equalizes the one in the investment sector and the BGP is preserved. Productivity gains should satisfy*

$$\gamma^x = \gamma^{x2} = (\gamma^y)^{\frac{1+\psi_{x2}\left(\alpha_{y1}-\zeta\right)+\psi_{y2}(\alpha_{my}-\alpha_{mx_2})}{1-\psi_{x1}\left(\alpha_{y1}-\zeta\right)+\psi_{y1}(\alpha_{my}-\alpha_{mx_2})}}$$

Hence, if the previous relationship is satisfied between productivity gains in the investment and the consumption sector, the economy resembles an economy with two sectors in which the only shocks are a neutral and an investment specific one.

It is worth describing equilibrium growth rates for the case that will be analyzed in the rest of the paper (3). Growth rates of gross output along the BGP are convex combinations of the productivity growth in the investment and consumption sector.

$$g^x = (\gamma^x)^{\psi_{x1}} (\gamma^y)^{\psi_{x2}}$$

$$g^y = (\gamma^x)^{\psi_{y1}} (\gamma^y)^{\psi_{y2}} = g^{x2}$$

$\psi_{x1} = \frac{1-\alpha_{my}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$ and $\psi_{x2} = \frac{a_{mx}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$. Also, $\psi_{y1} = \frac{a_{y_1}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$ and $\psi_{y2} = \frac{1-a_{x_1}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$.

In an economy with constant returns to scale and no labor, so that $a_{y_1} + a_{my} = 1$ and $a_{x_1} + a_{mx} = 1$, the growth rates of output are identical across sectors. In an economy where $a_{y_1} + a_{my} < 1$ and $a_{x_1} + a_{mx} = 1$, the consumption sector gross output will grow slower than the investment sector. If instead, $a_{y_1} + a_{my} = 1$ and $a_{x_1} + a_{mx} < 1$, the consumption sector will grow faster than the investment equipment sector. In other words, whenever the share of intermediates in production in the equipment (consumption) sector is relatively small, the consumption sector grows faster (slower) than the investment equipment sector.

5 Quantitative Exercises

The quantitative strategy is as follows. First the model is detrended using the BGP results from the previous section. In particular, the following transformation of variables generates a stationary economy

$$\frac{Y_t}{Q_t^y}, \frac{X_t^2}{Q_t^y}, \frac{M_t^{ij}}{Q_t^y}, \frac{C_t}{Q_t^y}, \frac{X_t^1}{Q_t^x}, \frac{k_t^i}{Q_t^x}, \frac{i_t^i}{Q_t^x}$$

where

$$Q_t^x = (A^{x_1})^{\psi_{x1}} (A^y)^{\psi_{x2}} \quad \text{and} \quad Q_t^y = (A^{x_1})^{\psi_{y1}} (A^y)^{\psi_{y2}}$$

Second, I calibrate the model to match the steady state behavior of the industry structure (i.e. the share of intermediate inputs in each sector), the volatility of gross output and value added. To match the cyclical behavior of cost shares observed in the data, I calibrate the variance covariance matrix of the shock structure as well as the elasticities of substitution across intermediate goods.

Third, I calibrate a comparable economy with Cobb-Douglas technologies (constant cost shares) to generate the same steady state of the baseline economy.

With these two economies I run alternative experiments. First, I compute impulse responses for identical shocks to test the propagation properties of each economy. Second, I simulate each economy and compute a variance decomposition of the generated path for output and aggregate TFP, for neutral and investment specific shocks.

I have used data from the Capital Flow Table of 1997 to compute investment levels across sectors. Capital stocks for the same year across sectors were obtained from the EUKLEMS database. Nominal shares of intermediate inputs were obtained from annual Input Output tables at chained dollars of 2005 as reported by BLS. The relative price of equipment to consumption good was obtained as averages of quarterly data as reported in DiCecio (2009), computed following Cummins and Violante (2002) methodology.

For these exercises, I assume that the share of intermediates in value added is the residual after deducting the share of capital to assure constant returns in each sector. Under such specification, growth rates of gross output are the same across all sectors (as explicit in the definition of ψ_{x_i}, ψ_{y_i} whenever $\alpha_{j1} = (1 - \alpha_{mj})$)

5.1 Calibration

The model is calibrated to annual frequencies mainly because the data on intermediate good cost shares is available at that frequency. Table 18 describes the set of parameters that were set independently of the model conditions. The discount factor was set consistent with a annual interest rate of 2%. The capital depreciation was set to 5% per year (as in Cooley and Prescott, 1995). I also need to calibrate the growth rate along the BGP. The trends are obtained as gross output weighted average sector growth rates by KLEMS. These are computed 1978 to 2007[39]. Because there is no labor in the economy, the model is bounded to generate the same growth trend in the equipment and consumption sector. In the baseline calibration I use the growth rate for the equipment sector equal to 3.15% per year. I later draw sensitivity analysis assuming instead the average growth rate observed in the consumption sectors, 1.5%.

From the optimality conditions for capital (x^* corresponds to the steady state value of variable x) we obtain,

$$\frac{1 + g^x - \beta(1-\delta)}{\beta} \frac{k^*_{x_1}}{X^*_1} = \alpha_{x1}$$

Hence, I need either a measure of capital output ratio in the investment sector, or a measure of capital capital services in gross output, α_{x1}. I use the latter. The feasibility condition that dictates that gross output in sector X_1 corresponds to total investment in the economy as reported in the Flow of Funds. Capital services are obtained from KLEMS data.

Following a similar strategy we can calibrate the share of capital in the consumption sector as

$$\frac{1 + g^x - \beta(1-\delta)}{\beta} \frac{\lambda^*_x k^*_y}{\lambda^*_y Y^*} = \alpha_{y1}$$

Consistent with the literature (Hornstein and Praschnik (1997) and Huffman and Wynne (1999)), the calibrated share of capital in the consumption sector is slightly higher than the one in the investment sector.

We are left to calibrate, two elasticities of substitution ($\frac{1}{1-\rho_x}, \frac{1}{1-\rho_y}$) and the shares of input in production, as well as the variance covariance matrix of the shocks. I calibrate them jointly matching moments of the data. The moments targeted are twelve, described in Table 20. They include the industry structure (the cost shares reported in Table (12)); the correlation of cost shares to aggregate value added (reported for the pre Great Recession period in Table (16));

[39] If the average is computed over a time frame comparable to the input output data, the growth rate in the equipment sector raises 1% and in the consumption sector raises 0.2%. Productivity growth for these period is 5.99% in the equipment sector and 1.5% in the consumption sector (value added measures of TFP).

the volatility of aggregate GDP, and the volatility of gross output in each sector as well as the persistence of the cyclical component (hp filtered) of each series.

Table 19 describes the set of calibrated parameters. The first two parameters are closely tight to the share of intermediates from each sector. The elasticities of substitution across inputs and the full structure of shocks are identified through the correlations of cost shares with aggregate activity, as well as the persistence of shocks and volatility of the aggregate series. The persistence parameters θ where taken as primitive in the simulations, and several sensitivity analysis done over the values of the primitive. The calibrated persistence and volatility of the aggregate series are particularly sensitive to the underlying persistence of the shock in the investment sector θ_x. I choose ς to assure that the steady state of the model is well defined, i.e. there is a set of non-negative prices that solve the allocation.

In the preferred calibration, the share of consumption intermediate across sectors is similar in the investment and consumption sectors. The elasticities of substitution across intermediate goods are identified less than one in both sectors. The elasticity of substitution is higher in the consumption sector. The shock structure is such that the volatility of neutral shocks, temporary and persistent are always lower than the volatility in the equipment sector. Among equipment producing sectors, we identify higher (lower) volatility for purely temporary(persistent) shocks in the production of intermediates, than in the investment production sector. Finally the covariance of temporary shocks in the equipment production sector is identified negative at -0.72. Intermediate equipment goods are typically parts and unfinished goods that would eventually contribute to the stock of capital in the economy. When the final equipment sector entails a positive shock capital goods turn cheaper vis a vis intermediate equipment goods. Output in the equipment sector raises and the relative price of final to intermediate equipment goods drops. In the data, we have identified that cost shares in the equipment sector are procyclical. Hence, in input productivity in the equipment final goods sector raises, it may possible offset the impact of the change in relative prices, inducing countercyclical cost shares which is at odds with the data (recall that the elasticity of substitution across intermediate inputs in the investment sector is close to one).

With this calibration the model is able to generate an industry structure where the cost shares of consumption and equipment in the equipment sector are very close to the ones observed in the data (the cost share of consumption goods in equipment is 57% and the model predicts it to be 53%). The model however generates a cost share of consumption goods in the consumption sector 10% lower than in the data(the cost share of consumption goods is 81% in the model and 93% in the data).

In terms of the correlations of cost share movements with aggregate value added, the model replicates the countercyclicality of consumption sector cost shares observed in the data. Almost by construction however, it is not able to generate the disparity in correlations across different types of intermediate inputs. The size of the correlation is slightly overestimated for the cost share of equipment and consumption intermediates in equipment. The model predicts a lower correlation of cost shares in the consumption sector. The predicted correlation is closer in magnitude to the one documented in the data for the cost share of equipment in consumption. The model underestimates the correlation of the cost share of consumption in consumption. Correlations between aggregate output and cost shares increase when the size of the shocks, in particular the neutral shock, increases. However, higher volatility of the neutral shocks implies a much larger volatility of output than observed in the data.

Regarding the volatility of output, the statistic in the data is slightly higher than predicted by the model(1.5% in the data versus 1.35% in the model). The model generates higher volatility for gross output in consumption and equipment than in aggregate value added, and higher volatility in the equipment sector. The standard deviations of gross output in the consumption sector is 1.8% in the data and 2% as predicted by the model; while the standard deviation in the equipment sector is 4.8% in the data and 5.3% in the model. Finally, the model predicts autocorrelations close in magnitude to the ones observed in the data. The correlation predicted in the equipment is lower than in the consumption sector (as observed in the data).

5.2 Results

5.2.1 Aggregate and Sectoral Shocks

Table 21 presents the variance decomposition across shocks for aggregate value added, aggregate productivity, investment and consumption. Shocks to the equipment sector combined explain about 20% of the volatility of output in out baseline calibration. Roughly 60% of those movements are accounted by the volatility of the transitory component of the intermediate equipment sector,and the remaining to the persistent and transitory component of the innovations in the investment equipment sector. Neutral shocks explain the remaining of the volatility of GDP, with two thirds of it accounted by the transitory component of shocks, and the rest by persistent shocks.

s shown in the body of the paper, the aggregate production possibility frontier shifts with changes in the relative intensity with which inputs are used in production across different sectors, and the relevance of each sector for gross output. For our baseline calibration, 47% of the volatility of aggregate productivity is explained by shocks to the equipment sector; 27% of the fluctuations in aggregate productivity originate in persistent shocks. Neutral shocks explains 53% of the induced variation in TFP, with roughly 60% of it contributed by the transitory component. It is important to highlight that shocks to the intermediate production of equipment goods account for most of the volatility induced by shocks to the equipment sector. This model implies that changes in the productivity with which intermediate goods are produced have a stronger impact on aggregate productivity than shocks to the investment good production. Quantitatively the introduction of intermediate goods production is important in explaining changes in aggregate TFP.

The contribution of shocks to the volatility of aggregate investment is very similar to the one found for aggregate productivity. The volatility of aggregate consumption in the model is mostly explained by transitory shocks (87%), and most of it contributed by shocks to the investment equipment sector (67%). Shocks to the equipment intermediate sector barely affect aggregate consumption volatility.

5.2.2 Constant versus Flexible Cost Shares

In the previous section I highlighted the importance of modelling intermediate input in a the standard two sector economy, in particular for the impact of shocks in TFP and aggregate output. Now, I would like to show that allowing for a flexible cost share structure is also relevant in assessing the impact of neutral and sectoral shocks for aggregate volatility. To do so, I compare the calibrated flexible cost share economy with a comparable economy assuming a constant input output structure. I calibrate an economy where the elasticity of substitution is equal to one (Cobb Douglas technology) to generate the same input structure as the benchmark economy in steady state. The steady state input structure is important because the elasticities of inputs to changes in relative prices (in the flexible cost share economy) depend on the initial shares of inputs in production. I do this in two steps, first I only focus on generating the same input structure in steady state given the shock structure. As it turns out, the constant cost share economy, generates consistently lower volatility for the aggregates in the economy (i.e.

value added, and gross output in each of the sectors), indicating that the amplification of shocks is weaker than in the flexible cost share economy. I hence recalibrate the shock structure to match the volatility and autocorrelation of GDP and gross output in the economy with flexible cost shares and compare results.

Table 22 reports the calibration of the model and compares it to the benchmark allocation. The shares of capital are identical across specifications, except in the intermediate equipment sector, where the share is computed to assure that the steady state is well defined. The elasticities of substitution across intermediate inputs are set to 1 ($\rho = 0$), so that the technology is Cobb Douglas in all inputs. The share of equipment (consumption) intermediates in the investment (consumption) sector is slightly higher than in the benchmark economy. As in the benchmark calibration, both models predict relatively well the input composition in the equipment sector, but do not account for all the disparity in intermediate input intake in the consumption sector.

Without adjustment of the cost structure the economy with constant cost shares generates much lower volatility of the series of GDP and gross output in the consumption and equipment sector. This is already a symptom of the differential impact of shocks in a flexible and constant cost share economy that I will describe through the predicted impulse response functions.

When I recalibrate the shocks to generate the moments of the flexible cost share economy I identify a standard deviation for the investment equipment sector shock twice as large as the one identified in the benchmark model. The standard deviation of the transitory component of the shock in the intermediate equipment sector is identified 50% as large as in the benchmark economy. Finally, while the transitory component of the neutral shock is as large as in the flexible cost share economy, the standard deviation of the persistent shock is identified almost twice as large and in the latter.

Variance Decomposition Table 24 displays the contribution to the variance of GDP, aggregate productivity, investment and aggregate consumption from neutral and sector specific shocks when we only match the industry structure across economies.

If we compare the variance decomposition of GDP, both the constant and flexible cost share economy predict similar contributions from temporary and persistent shocks. The benchmark economy predicts a slightly higher contribution of neutral shocks with value added volatility (2%) and a slightly lower contribution of temporary equipment sector shocks (3% in the investment sector and 1% in the equipment intermediate sector).

The contribution of neutral shocks to the variance in total factor productivity is lower in the constant cost share economy (5% in temporary shocks, and 2% for persistent shocks) and shocks to the intermediate equipment sector are predicted to explain relatively more of TFP variance. The variance decomposition of aggregate investment is very close across economies, and the variance of aggregate consumption is mostly explained by investment shocks as in the benchmark economy.

When the constant cost share economy is recalibrated to generate the volatility and autocorrelation of the aggregate series, the predictions of each of the models depart substantially. The economies that compare next generate the same steady state industry structure and the simulated series have the same statistical properties. Yet, the constant cost share economy predicts that 60% of the volatility of aggregate output is explained by shocks to the equipment sector while in the benchmark economy they explain 50%. The temporary (persistent) component of neutral shocks explains 19% (22%) of output volatility in the constant cost share economy and 35% (15%) in the benchmark one. Hence, the economy with a fixed input output structure predicts larger contribution for equipment sector shocks, but also if shifts the contribution of neutral shocks from temporary to persistent shocks.

This shift across type of shocks also occurs in the contributions to the volatility of aggregate TFP. More important, while investment equipment sector shocks are negligible in explaining TFP volatility in the benchmark economy, they are predicted to explain 27% of its volatility in the constant cost share economy. As described in the body of the paper the production possibility frontier of a multisector economy depends not only on the productivity of each sector, but also on the contribution of each of them to aggregate gross output. It is not surprising then that in a flexible cost share economy (where those contributions are responding to shifts in relative efficiency in production) the contribution of intermediate equipment sector shock to the volatility of TFP is larger than in the economy with a fixed input output structure (its transitory component contributes almost twice as much, and its persistent component 4 times more).

The comparison in the variance decomposition of aggregate investment is very close to the one described for TFP. In the case of consumption volatility, the role of transitory investment sector shocks is stronger in the constant cost share economy, reaching above 90% of consumption volatility.

Given that the predicted aggregate volatility in each economy differ, I recalibrate the constant cost share economy to generate volatility and persistence of the aggregate series closer to the ones predicted by the flexible cost share economy (See 26). Then, I compute a variance decomposition of the shocks and compare it to the baseline economy (as shown in 27).

The most salient feature in this comparison is that shocks to the equipment sector become substantially more relevant in explaining both aggregate output volatility and aggregate productivity volatility. While in the baseline case they only explain 20% of GDP volatility, they account for 45% of GDP volatility in the recalibrated economy. Also, while they explain roughly 45% of TFP volatility in the baseline economy, they are predicted to account for 60% of it in the recalibrated constant cost share economy. This shifts are substantial and depict the value in modelling carefully the input output structure, and in turn the variance covariance matrix of the underlying shocks.

Impulse Responses This section presents the predicted responses of key macroeconomic variables to shocks to productivity in alternative sectors. The focus in on a comparative analysis of responses in the flexible versus the constant cost share economy.

I study the behavior of aggregate output, TFP, aggregate consumption, gross output in the consumption sector, investment, and the relative price of new capital goods versus consumption goods. Figures 6.3 to 6.3 depict the responses of this variables to transitory neutral shocks, investment specific shocks, and shocks to the equipment intermediates sector. Figures 6.3 to 6.3 display the responses to shocks in the persistent component of productivity for the same three alternatives.

In general terms, on impact the economy with flexible cost shares reacts more to a given shock than the constant cost share economy does. Purely transitory shocks can have persistent effect on aggregate variables through the effect on equilibrium capital accumulation in each of the sectors. The persistent effect is stronger in the benchmark economy, except for aggregate TFP, which in both economies returns to its steady state level after one period.

Let me describe the response to each of the possible shocks one at the time. If the shock is neutral, a one standard deviation shock induces a three fold raise in aggregate output. In the constant cost share economy, the predicted increase in activity is 3/4 of the one predicted under flexible cost shares. As expected, aggregate consumption reacts less than aggregate value added, and investment and gross output in the consumption sector almost one to one with the increase in GDP. The relative price of intermediate inputs (equipment to consumption) drops on impact but convergences above steady state after that. The drop in the relative price in response to the neutral shock is slightly above 10% of its steady state level. Although the shock is neutral in nature, the disparity in input intensity across sectors generates changes in relative prices, and asymmetric responses of gross output in different sectors.

When the shock originates in the investment sector and it is persistent, one standard deviation shock (0.01) generates a reaction on GDP upon impact of 7% of the size of the shock. In the long run, GDP augments to 10% of the size of the shock in the investment sector. Gross final output falls upon impact to then overshoot its steady state level. This is explained mostly by the increase in the capital stock. The relative price of intermediate equipment goods raises 15% on impact and drops to slightly below 13% in the long run. In the constant cost share economy, the predicted change in relative prices is only 11% and converges to 9% in the long run. The disparity in the behavior of relative prices after impact is related to the more pronounced response of aggregate investment on impact for the flexible cost share economy. If the shock is purely transitory instead, gross value added is predicted to drop upon impact and raise above steady state levels after that. This is expected in response to the raise in investment upon impact.

Finally, if the shock originates in the intermediate equipment sector and is transitory, a one standard deviation shock (0.014) generates a positive reaction of GDP upon impact of about one third of the size of the shock. Gross output in the consumption sector increases half as much as the size of the shock in the flexible cost share economy, but slightly about a quarter of the size of the shock in the constant cost share economy. Investment increases 60% of the size of the shock and 40% in the constant cost share economy. The disparity in the behavior of gross output is mostly explained by the differences in the predictions for aggregate total factor productivity. While in the benchmark economy it is predicted to increase three times the size of the shock (recall that the Domar weights in the computation of TFP shift with the composition of gross output and value added in the economy) in the constant cost share economy, productivity raises two thirds of that.

In summary, while the dynamic predicted in either economy are comparable. Quantitatively the amplification of shocks in the flexible cost share economy is larger than in the constant cost share economy.

6 Conclusion

This paper studies the effect of fluctuations in the cost shares of intermediate inputs for the volatility of output. The model economy is calibrated to match the industry structure of the USA economy and the cyclical cost share behavior documented in the data. When tested against a comparable constant cost share economy, neutral shocks account for 8% more of aggregate output volatility. Responses of aggregate output and productivity to sectoral shocks are magnified when patterns of reallocation of factors are consistent with constant cost shares.

The disparities in cost share behavior across sectors may provide identifying restrictions for the nature of shocks to the economy. The results stems not only from the disparate contribution of sectors to value added, but also from degree of substitutability in inputs of production. Additional empirical analysis on the latter might be a promising avenue for further work. Furthermore, the disparities in the predictions of the dynamic of key aggregate variables may provide additional identification restrictions for the nature of shocks in the economy.

The paper illustrate the quantitative implications of disciplining the model economy to generate the pattern of reallocation observed in the data. It is still an open questions which are the mechanisms that generate those patterns. Are they consistent with factor specificity? do inventories play a role? do these patterns change when credit conditions change? Analysis of the input output structure dynamic for more disaggregated sector can shed light to some of these questions.

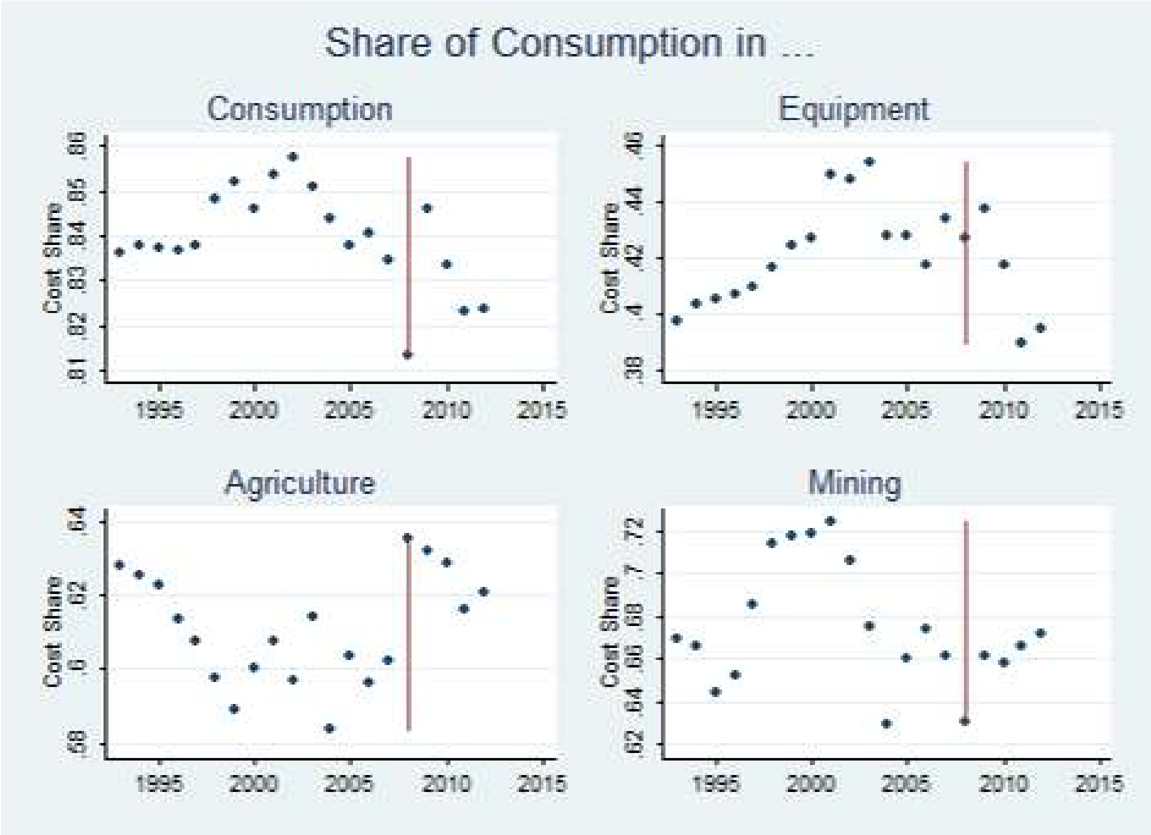


Figure 7: Consumption Cost Shares

Input/Sector	Equipment	Consumption
Equipment	43%	7%
Consumption	57%	93%

Table 12: Share of Service and Manufacturing Inputs (intermediate goods from the Agriculture and Mining sector have been factored out)

Input/Sector	Equipment	Consumption
Equipment	1.6%	2.9%
Consumption	2.2%	0.9%

Table 13: Yearly average absolute change

Input/Sector	Equipment	Consumption
Equipment	2.4%	10.1%
Consumption	3.5%	0.9%

Table 14: Average absolute deviation relative to mean share

Input/Sector	Equipment	Consumption
Equipment	0.9%	1.5%
Consumption	1.3%	0.4%

Table 15: Average absolute deviation relative to HP- trend (smoothing factor, 6.25)

	Full Sample		Pre-2008		without GR	
Input/Sector	Eq	Co	Eq	Co	Eq	Co
Equipment (Eq)	0.22	0.10	0.05	-0.14	0.07	-0.08
Consumption (Co)	-0.22	-0.27	-0.13	-0.33	-0.12	-0.23

Table 16: Correlation with Industrial Value Added

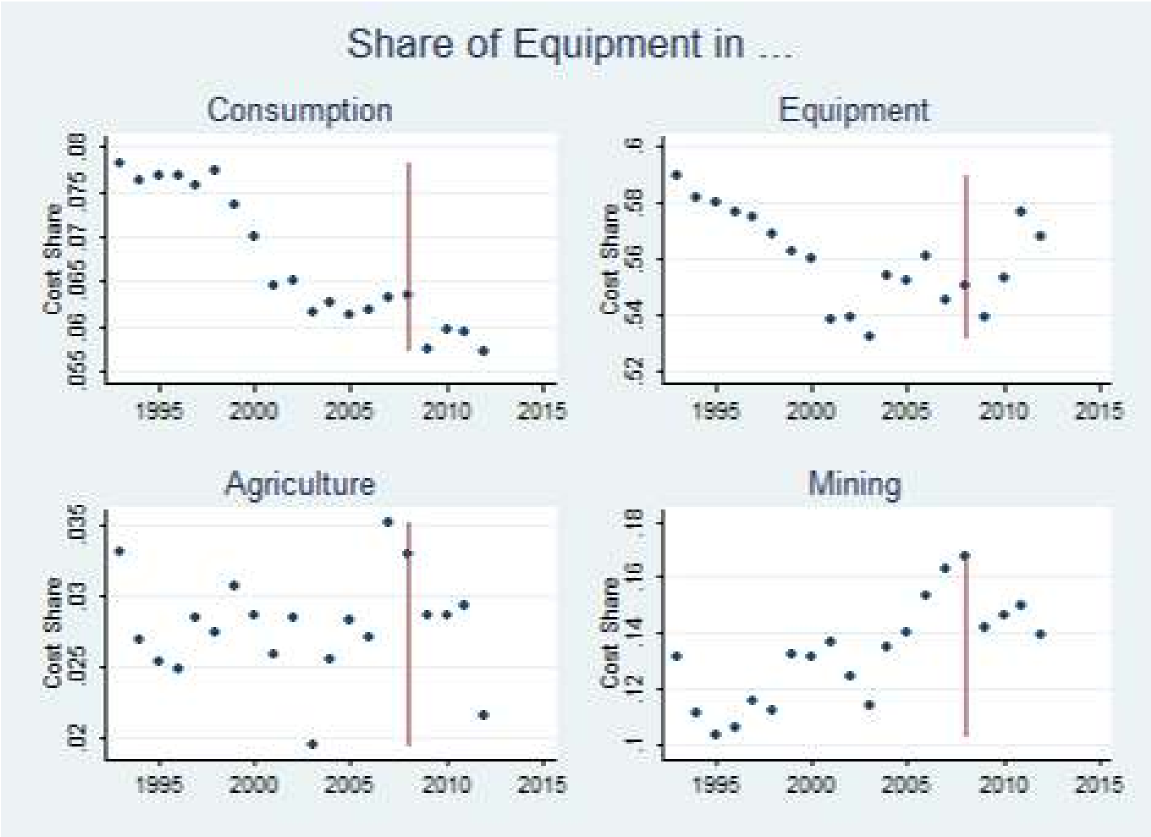


Figure 8: Equipment Cost Shares

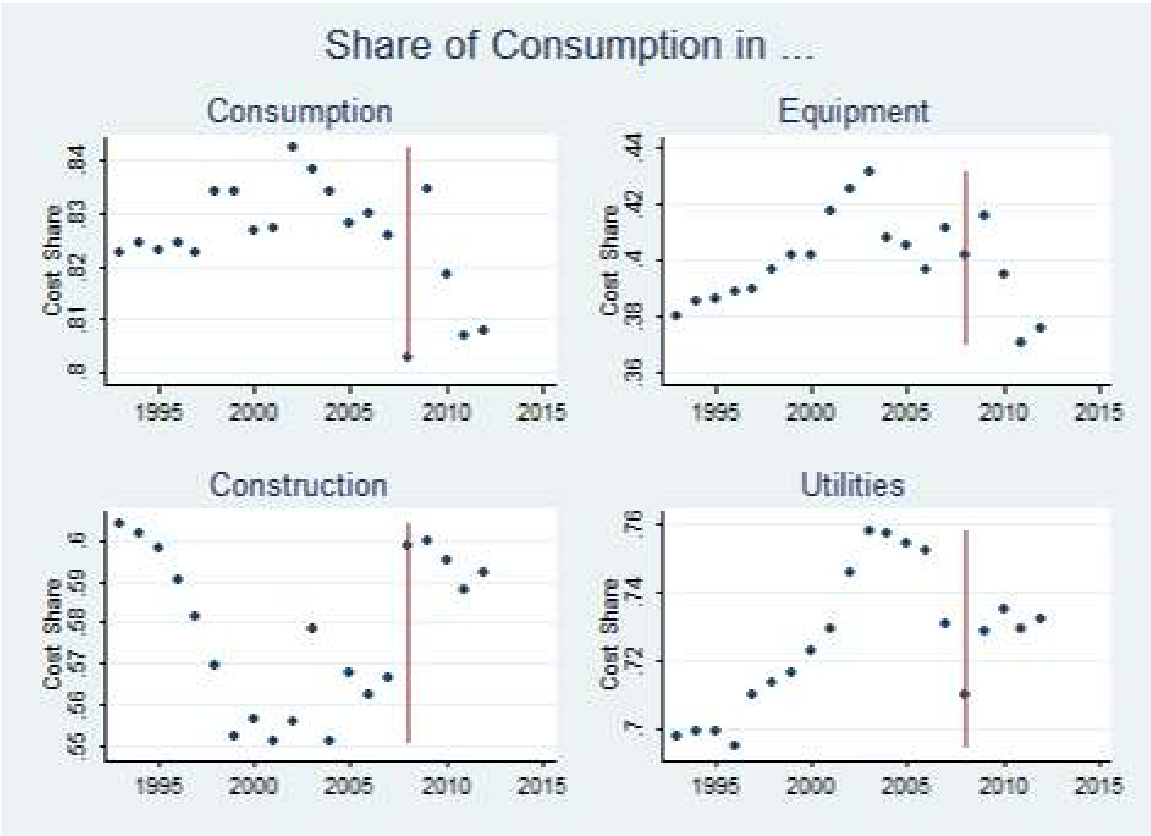


Figure 9: Consumption Cost Shares

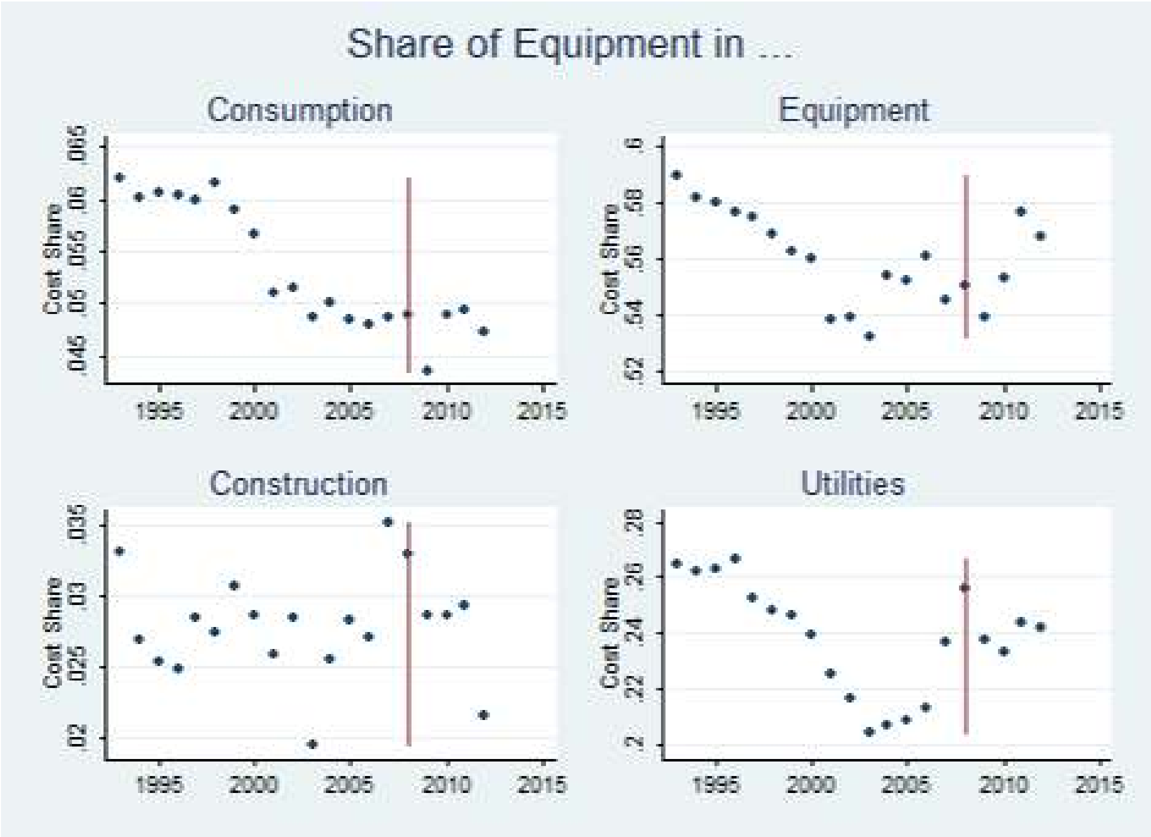


Figure 10: Equipment Cost Shares

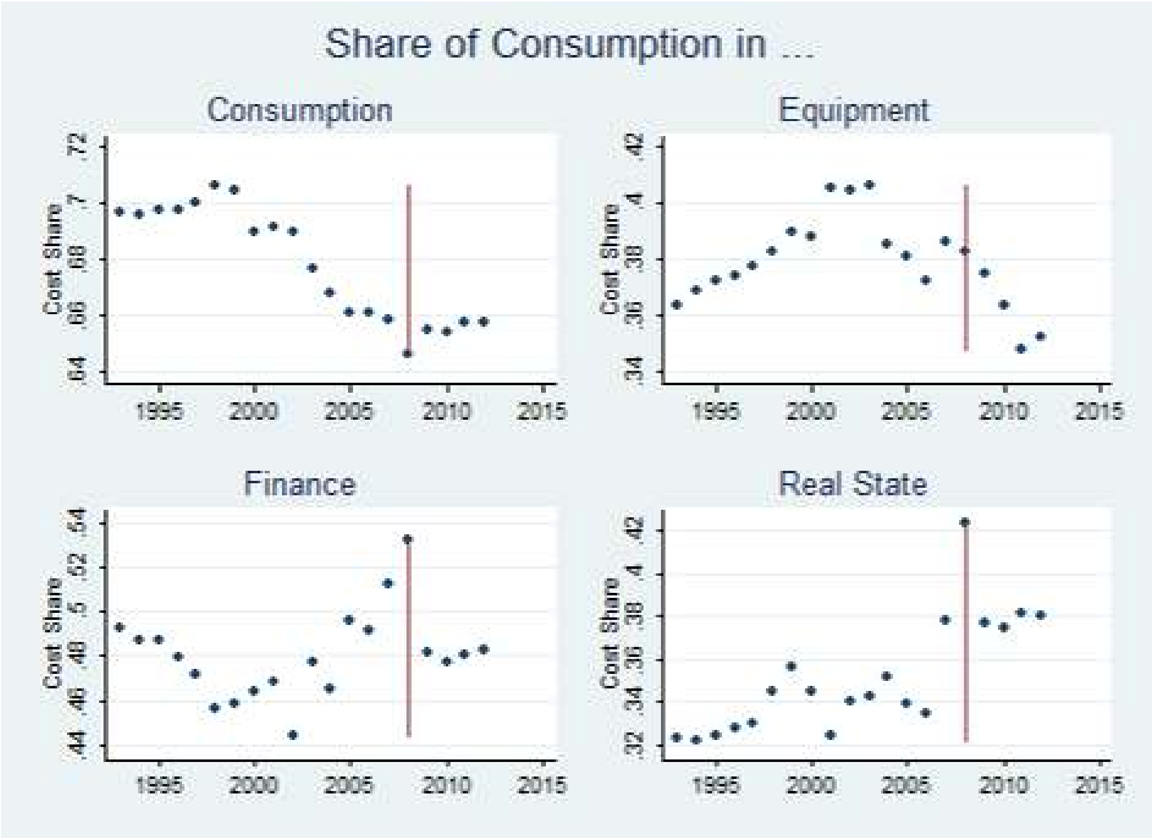


Figure 11: Consumption Cost Shares

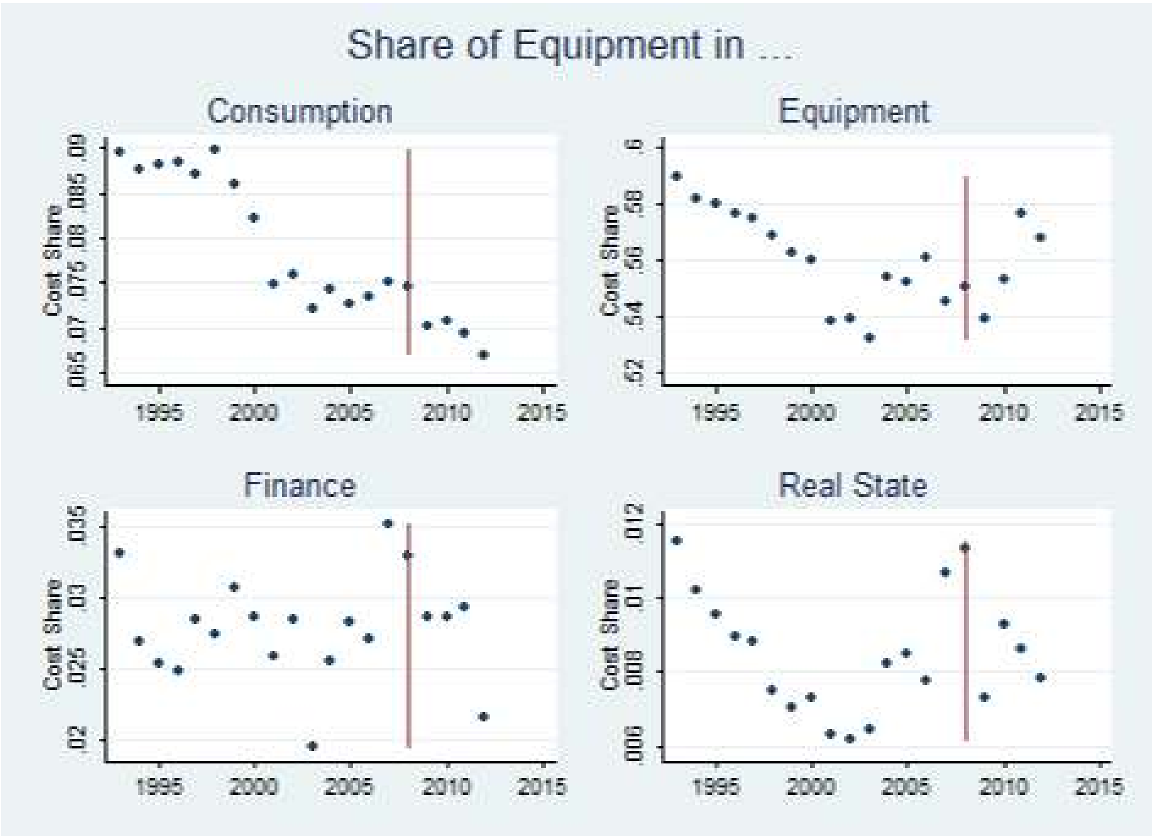


Figure 12: Equipment Cost Shares

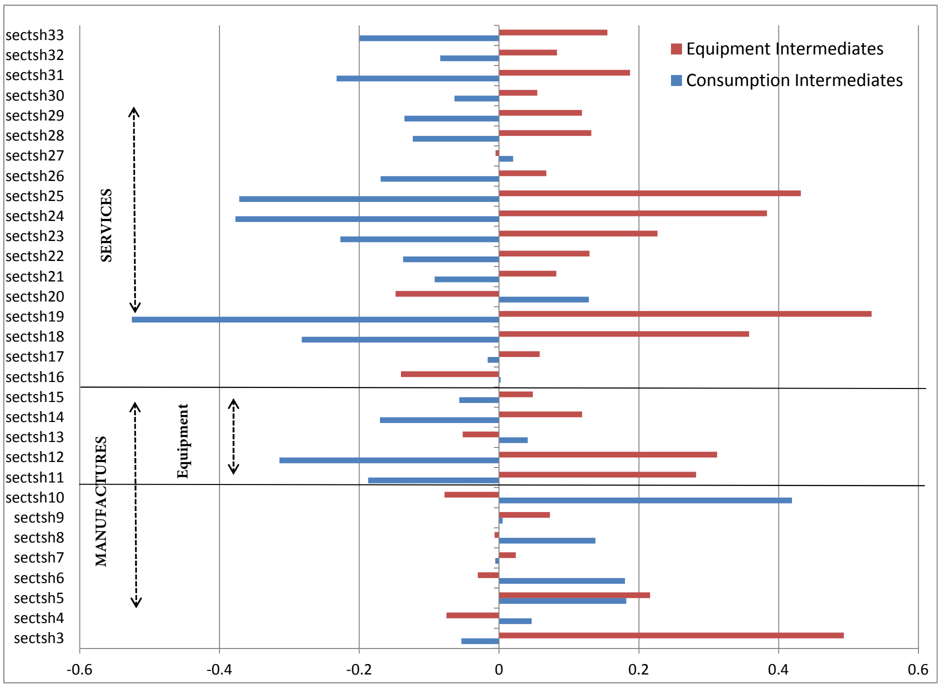


Figure 13: Correlation with Aggregate Output

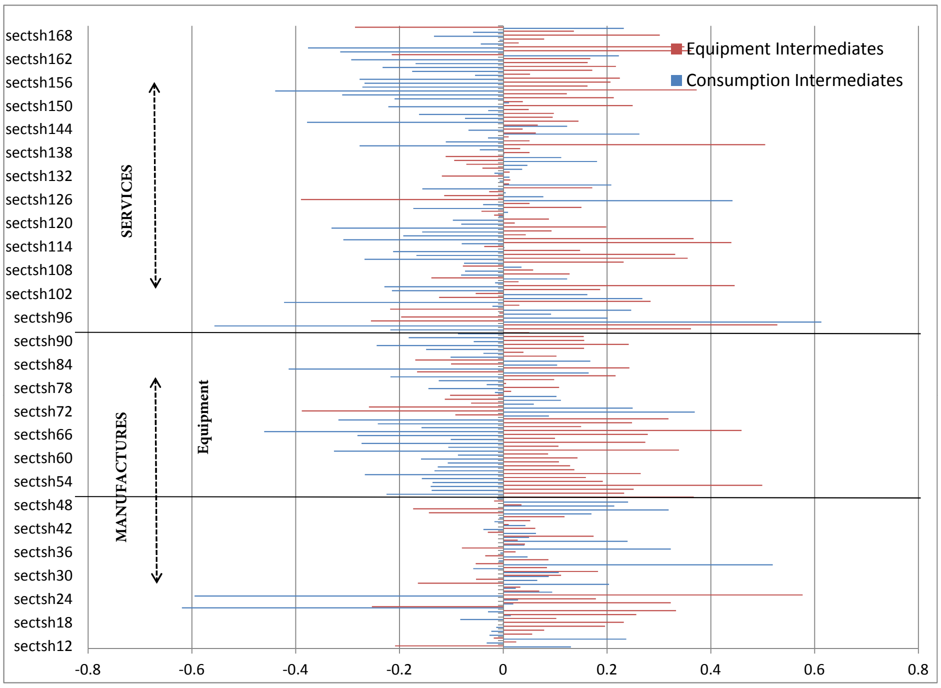


Figure 14: Correlation with Aggregate Output

Sector		Intermediate Demand E X_2	Intermediate Demand E X_1	Intermediate Demand C Y	Final Demand Equipment	Final Demand Consumption
E	X_2		x	x		
	X_1				x	
C	Y	x	x	x		x
Capital		x	x	x		

Table 17: Input Output matrix of the economy

Appendix(A)

6.1 Balance Growth Path

Theorem 6 *A BGP where all intermediate goods are used in production exists iff technology is Cobb Douglas in capital and intermediates and either 1) the elasticity of substitution across intermediate goods, equals unity (Cobb-Douglas technology); or 2) there are no linkages through intermediate goods between the investment and consumption sector; 3) productivity growth in the consumption sector and intermediate equipment sector are proportional by a factor* $(g^x)^{\alpha_{y1}-\zeta}(g^y)^{\alpha_{my}-\alpha_{mx2}}$ *,where* g^x *is the growth rate of output in the investment sector and* g^y *is the one in the consumption sector.*

1) and 2) are special cases of 3), hence I prove the latter first.

Proof. Suppose that productivity in the investment durable sector grows at rate γ^x and

productivity in the consumption and intermediate equipment sector grows at rate γ^y and γ^{x_2} respectively. Let g^j be the growth rate of output in sector $j = y, x, x_2$.

The feasibility restrictions in the economy imply

$$g^y = g^{M_{yy}} = g^{M_{yx}} = g^c$$

$$g^{x2} = g^{M_{xx}} = g^{M_{xy}}$$

$$g^x = g^{i_j} = g^{k_j} \text{ for any } j = y, x, x_2$$

Given production technologies, for intermediate goods from the consumption and equipment sector to be used in production along the BGP, the growth rate of output in the consumption and intermediate equipment should equalize. Such feature stems from the optimality conditions of the firms in intermediate inputs.

$$\alpha_{my}\alpha_{y2}\left(\frac{Y}{M_{yy}}\right)\frac{1}{\alpha_{y2}+(1-\alpha_{y2})\left(\frac{M_{xy}}{M_{yy}}\right)^{\rho_y}} = 1 \tag{31}$$

Unless the growth rates of input intake from the equipment and consumption sector are the same, the optimality condition would not be satisfied along the BGP.

$$g^{x_2} = g^y$$

From the production technology in the consumption sector and intermediate equipment sector we obtain

$$g^y = \gamma^y \left(g^x\right)^{\alpha_{y1}} \left(g^y\right)^{\alpha_{my}}$$

$$g^{x_2} = \gamma^{x_2} \left(g^x\right)^{\varsigma} \left(g^y\right)^{\alpha_{mx_2}}$$

Hence,

$$\gamma^{x_2} = \gamma^y \left(g^x\right)^{\alpha_{y1}-\zeta} \left(g^y\right)^{\alpha_{my}-\alpha_{mx_2}}$$

which depends on the relative capital intensity of the consumption sector and intermediate equipment sector.

Finally, from the investment sector technology, we have

$$g^x = \gamma^x \left(g^x\right)^{\alpha_{x1}} \left(g^y\right)^{1-\alpha_{x1}}$$

Combining this equation with the one describing growth rates in the consumption sector, we obtain the BGP of the economy, i.e.

$$g^x = \left(\gamma^x\right)^{\psi_{x1}} \left(\gamma^y\right)^{\psi_{x2}}$$

$$g^y = \left(\gamma^x\right)^{\psi_{y1}} \left(\gamma^y\right)^{\psi_{y2}}$$

$\psi_{x1} = \frac{1-\alpha_{my}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$ and $\psi_{x2} = \frac{a_{mx}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$. Also, $\psi_{y1} = \frac{a_{y_1}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$ and $\psi_{y2} = \frac{1-a_{x_1}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$

To show number 1), note that the problem that was pointed out in 31 is not present anymore, for $\rho = 0$. Hence, the condition $g^{x_2} = g^y$ need not hold. The algebra gets more cumbersome but it is possible to show that the BGP will solve

$$g^x = \gamma^x \left(g^x\right)^{\alpha_{x1}} \left(\left(g^{x_2}\right)^{\alpha_{x2}} \left(g^y\right)^{1-\alpha_{x2}}\right)^{\alpha_{mx}}$$

$$g^y = \gamma^y \left(g^x\right)^{\alpha_{y1}} \left(\left(g^{x_2}\right)^{1-\alpha_{y2}} \left(g^y\right)^{\alpha_{y2}}\right)^{\alpha_{my}}$$

$$g^{x_2} = \gamma^{x_2} \left(g^x\right)^{\varsigma} \left(g^{x_2}\right)^{\alpha_{mx_2}}$$

Number 2) is analogous to number 3) but now the system of equations to be solved is

$$g^x = \gamma^x \left(g^x\right)^{\alpha_{x1}} \left(\left(g^{x_2}\right)^{\alpha_{x2}}\right)^{\alpha_{mx}}$$

$$g^y = \gamma^y \left(g^x\right)^{\alpha_{y1}} \left(\left(g^y\right)^{\alpha_{y2}}\right)^{\alpha_{my}}$$

$$g^{x_2} = \gamma^{x_2} \left(g^x\right)^{\varsigma} \left(g^{x_2}\right)^{\alpha_{mx_2}}$$

■

6.2 Optimality and Steady State

Feasibility dictates

$$k_{x1}^{'}(1+g^{x}) - k_{x1}(1-\delta) = i_{x1} \tag{κ_x}$$

$$k_{x2}^{'}(1+g^{x}) - k_{x2}(1-\delta) = i_{x2}$$

$$k_{y}^{'}(1+g^{y}) - k_{y}(1-\delta) = i_{y} \tag{κ_y}$$

$$i_y + i_{x1} + i_{x2} = X_1 \tag{λ_x}$$

$$M_{xx} + M_{xy} = X_2 \tag{$\lambda_x 2$}$$

$$C + M_{yy} + M_{yx} = Y \tag{λ_y}$$

$$M_{yx} = M_{yx1} + M_{yx2}$$

The corresponding optimality conditions are

$$\lambda_x(1+g^x) = \beta\lambda_x' \left[\alpha_{x1}\left(\frac{X_1'}{k_{x_1}'}\right) + (1-\delta)\right]$$

$$\lambda_x(1+g^{x_2}) = \beta\lambda_x' \left[\frac{\lambda_{x2}'}{\lambda_x'}\zeta_x\left(\frac{X_2'}{k_{x_2}'}\right) + (1-\delta)\right]$$

$$\lambda_x(1+g^{y}) = \beta\lambda_x' \left[\frac{\lambda_{y}'}{\lambda_x'}\alpha_{y1}\left(\frac{Y'}{k_{y}'}\right) + (1-\delta)\right]$$

$$(1-\alpha_{y1})\,\alpha_{y2}\left(\frac{Y}{M_{yy}}\right)\frac{(M_{yy})^{\rho_y}}{\alpha_{y2}\,(M_{yy})^{\rho_y} + (1-\alpha_{y2})\,(M_{xy})^{\rho_y}} = 1 \tag{$M_y y$}$$

$$\lambda_y\,(1-\alpha_{y1})\,(1-\alpha_{y2})\left(\frac{Y}{M_{xy}}\right)\frac{(M_{xy})^{\rho_y}}{\alpha_{y2}\,(M_{yy})^{\rho_y} + (1-\alpha_{y2})\,(M_{xy})^{\rho_y}} = \lambda_{x2} \tag{$M_x y$}$$

$$\lambda_x\,(1-\alpha_{x1})\,\alpha_{x2}\left(\frac{X_1}{M_{xx}}\right)\frac{(M_{xx})^{\rho_x}}{\alpha_{x2}\,(M_{xx})^{\rho_x} + (1-\alpha_{x2})\,(M_{yx1})^{\rho_x}} = \lambda_{x2} \tag{$M_x x$}$$

$$\lambda_x\,(1-\alpha_{x1})\,(1-\alpha_{x2})\left(\frac{X_1}{M_{yx1}}\right)\frac{(M_{yx1})^{\rho_x}}{\alpha_{x2}\,(M_{xx})^{\rho_x} + (1-\alpha_{x2})\,(M_{yx1})^{\rho_x}} = \lambda_{y} \tag{$M_y x$}$$

$$\lambda_{x2}(1-\zeta_x)\left(\frac{X_2}{M_{yx2}}\right) = \lambda_y \tag{$M_y x2$}$$

This is a standard convex economy. Hence, the equilibrium exits and its unique. Also the welfare theorems hold.

6.3 Steady State

From the production function, we obtain

$$\frac{X_1}{M_{xx}} = \left(\frac{k_{x1}}{M_{xx}}\right)^{\alpha_{x1}} (\alpha_{x2} + (1-\alpha_{x2})\left(\frac{M_{yx1}}{M_{xx}}\right)^{\rho_x})^{\frac{1-\alpha_{x1}}{\rho_x}}$$

Using the optimality condition in intermediate goods and capital we can rewrite the equation as

$$\frac{\lambda_{x2}}{\lambda_x}\frac{1}{(1-\alpha_{x1})\,\alpha_{x2}} = \left(\frac{\alpha_{x1}}{(1-\alpha_{x1})\,\alpha_{x2}}\frac{\beta}{1+g^x-\beta(1-\delta)}\frac{\lambda_{x2}}{\lambda_x}\right)^{\alpha_{x1}}\left[\alpha_{x2} + (1-\alpha_{x2})\left(\frac{1-\alpha_{x2}}{\alpha_{x2}}\frac{\lambda_x}{\lambda_y}\right)^{\frac{\rho_x}{1-\rho_x}}\right]^{(1-\alpha_{x1})\frac{1-\rho_x}{\rho_x}} \tag{32}$$

which defines the equilibrium relative prices of investment goods versus consumption goods.

Using the production function in the final good sector we can solve for $\frac{\lambda_{x2}}{\lambda_y}$ as

$$\frac{Y}{M_{yy}} = \left(\frac{k_y}{M_{yy}}\right)^{\alpha_{y1}} (\alpha_{y2} + (1-\alpha_{y2})\left(\frac{M_{xy}}{M_{yy}}\right)^{\rho_y})^{\frac{1-\alpha_{y1}}{\rho_y}}$$

Following the same procedure as before, we can express this equation as a function of the relative price of investment and final goods.

$$\frac{1}{(1-\alpha_{y1})\,\alpha_{y2}} = \left(\frac{\alpha_{y1}}{(1-\alpha_{y1})\,\alpha_{y2}}\frac{\beta}{1+g^y-\beta(1-\delta)}\frac{\lambda_y}{\lambda_x}\right)^{\alpha_{y1}}\left[\alpha_{y2} + (1-\alpha_{y2})\left(\frac{1-\alpha_{y2}}{\alpha_{y2}}\frac{\lambda_y}{\lambda_{x2}}\right)^{\frac{\rho_y}{1-\rho_y}}\right]^{\left(1-\alpha_{y1}\right)\frac{1-\rho_y}{\rho_y}} \tag{33}$$

Finally, the production technology in the third sector dictates

$$\frac{X_2}{k_{x2}} = \left(\frac{M_{yx2}}{k_{x2}}\right)^{(1-\varsigma_x)}$$

$$\left(\frac{1+g^y-\beta(1-\delta)}{\beta}\frac{\lambda_x}{\lambda_{x2}}\frac{1}{\varsigma_x}\right) = \left(\frac{1+g^y-\beta(1-\delta)}{\beta}\frac{\lambda_x}{\lambda_y}\frac{1-\varsigma_x}{\varsigma_x}\right)^{(1-\varsigma_x)} \tag{34}$$

Hence, equations (32), (33) , (34) define a system of three equations and three unknowns. Given the calibrated parameters I impose conditions on the share of value added in the third sector so that the system is exactly determined.

From the feasibility condition in intermediate goods of the investment sector we obtain

$$\frac{M_{xx}}{k_{x2}}+\frac{M_{xy}}{k_{x2}}=\frac{X_2}{k_{x2}}$$

$$\frac{\lambda_x}{\lambda_{x2}}\frac{(1-\alpha_{x1})\,\alpha_{x2}}{\alpha_{x2}+(1-\alpha_{x2})\left(\frac{1-\alpha_{x2}}{\alpha_{x2}}\frac{\lambda_{x2}}{\lambda_y}\right)^{\frac{\rho_x}{1-\rho_x}}}\frac{X_1}{X_2}+\frac{\lambda_y}{\lambda_{x2}}\frac{(1-\alpha_{y1})\,(1-\alpha_{y2})}{\alpha_{y2}\left(\frac{\alpha_{y2}}{1-\alpha_{y2}}\frac{\lambda_{x2}}{\lambda_y}\right)^{\frac{\rho_y}{1-\rho_y}}+(1-\alpha_{y2})}\frac{Y}{X_2}=1$$

which determines the ratio of gross output in the production of equipment, as well as the ratio of consumption good production to intermediate investment goods, as a function of parameters and equilibrium prices.

If we now turn to the feasibility condition in the final production investment sector, we have

$$\delta(1+\frac{k_y}{k_x}+\frac{k_{x2}}{k_x})=\frac{X_1}{k_x}$$

$$\delta(1+\frac{\lambda_y}{\lambda_x}\frac{Y}{X_2}\frac{\alpha_{y1}}{\alpha_{x1}}\frac{X_2}{X_1}+\left(\frac{\lambda_{x2}}{\lambda_x}\right)\frac{\zeta_x}{\alpha_{x1}}\frac{X_2}{X_1})=\frac{1+g^x-\beta(1-\delta)}{\beta}\frac{1}{\alpha_{x1}}$$

If we put both feasibility conditions together we obtain a system of two equations in two unknowns, i.e. the ratios of gross output across sectors.

To pin down the levels of the variables use the feasibility constraint in the consumption good sector.

$$\frac{U^{-1}(\lambda_y^*)}{M_{yy}}+1+\frac{M_{yx}}{M_{yy}}=\frac{Y}{M_{yy}}$$

where $\frac{Y}{M_{yy}}=\frac{\alpha_{y2}+(1-\alpha_{y2})\left(\frac{M_{xy}}{M_{yy}}\right)^{\rho_y}}{(1-\alpha_{y1})\alpha_{y2}}$. As we have shown before, $\frac{M_{xy}}{M_{yy}}$ is a function of the prices in the economy. In other words, M_{yy} solves

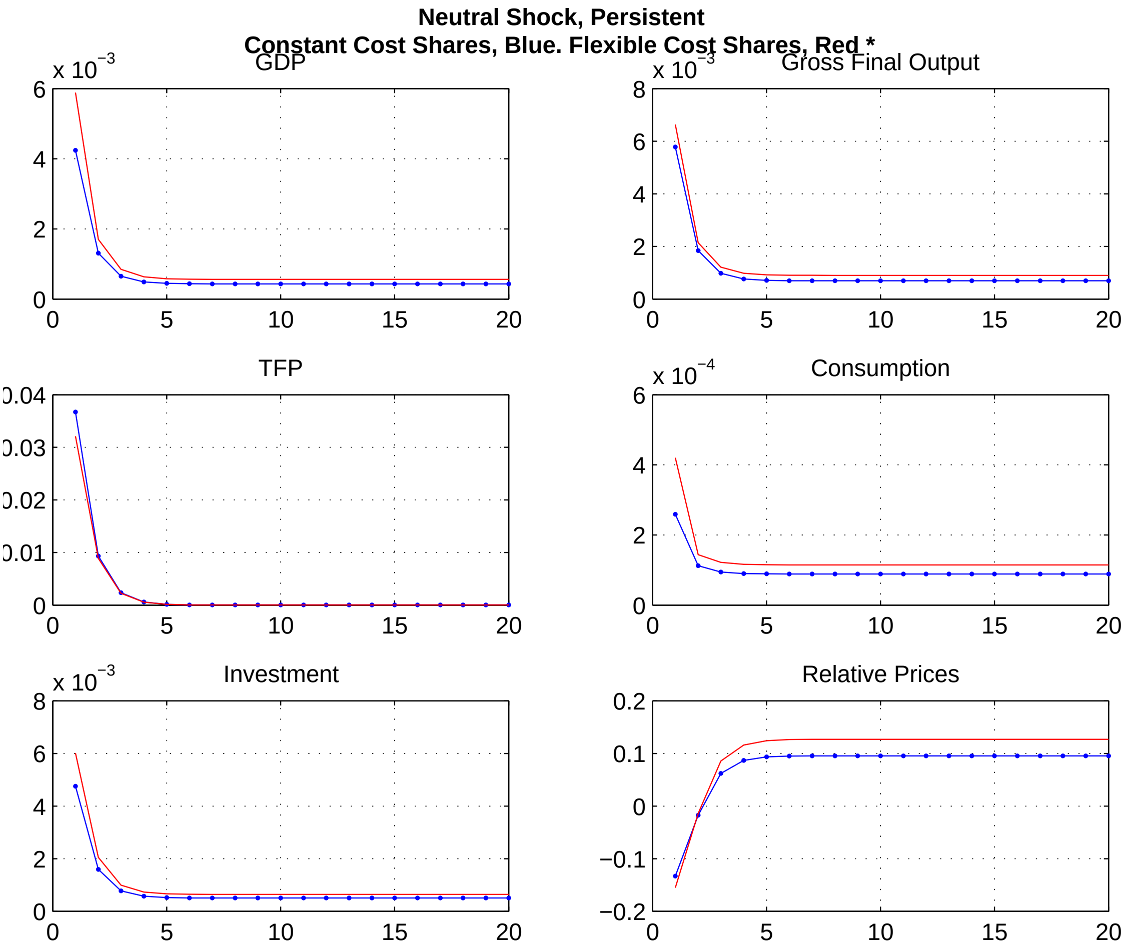
$$\frac{U^{-1\prime}(\lambda_y^*)}{M_{yy}}+1+\frac{\alpha_{y2}+(1-\alpha_{y2})\left(\frac{M_{xy}}{M_{yy}}\right)^{\rho_y}}{(1-\alpha_{y1})\,\alpha_{y2}}[\frac{X_1}{Y}\frac{\lambda_x}{\lambda_y}\frac{(1-\alpha_{x1})\,(1-\alpha_{x2})}{\alpha_{x2}\left(\frac{M_{xx}}{M_{yx}}\right)^{\rho_x}+(1-\alpha_{x2})}$$
$$+\left(\frac{\lambda_{x2}}{\lambda_y}\right)\frac{(1-\varsigma_x)}{(1-\alpha_{y1})\,\alpha_{y2}}\frac{X_2}{Y}]$$
$$=\frac{\alpha_{y2}+(1-\alpha_{y2})\left(\frac{M_{xy}}{M_{yy}}\right)^{\rho_y}}{(1-\alpha_{y1})\,\alpha_{y2}}$$

where $\frac{M_{xy}}{M_{yy}}=\left(\frac{\lambda_y}{\lambda_{x2}}\frac{1-\alpha_{y2}}{\alpha_{y2}}\right)^{\frac{1}{1-\rho_y}}$ and $\frac{M_{xx}}{M_{yx}}=\left(\frac{\alpha_{x2}}{1-\alpha_{x2}}\frac{\lambda_y}{\lambda_{x2}}\right)^{\frac{1}{1-\rho_x}}$.

Once M_{yy}^* is determined M_{yx}^* is too, as well as Y^*, k_y^*, M_{xy}^* from the optimality conditions.

k_x^* is determined using $\frac{k_y}{k_x}^*$ and then K^* can be computed. X^* is solved by the equilibrium ratio $\frac{X}{Y}^*$ and Y^*.

Appendix (B)



Neutral Shock, Persistent
Constant Cost Shares, Blue. Flexible Cost Shares, Red *
GDP
x 10^-3
Gross Final Output
x 10^-3
TFP
Consumption
x 10^-4
Investment
x 10^-3
Relative Prices

Parameter	Definition	Model
α_{x1}	Capital Share, Equipment	0.182
α_{y1}	Capital Share, Consumption	0.210
ς	Capital Share, Equipment Intermediates	0.901
θ_g	Consumption Shock Persistence	0.2
θ_{x_2}	Equipment Intermed. Shock Persistence	0.25
θ_{x_1}	Investment Shock Persistence	0.25
g^x	Gross Output Growth Rate, Equipment	3.15%
β	Discount Factor	0.98
δ	Capital Depreciation	0.05

Table 18: Parameters calibrated outside the model

Parameter	Definition	Value
ρ_x	Elasticity of Substitution M^{xx_1}, M^{yx_1}	-0.12
α_{x2}	Share Equipment Intermediates, Equipment	0.579
ρ_y	Elasticity of Substitution M^{yy}, M^{xy}	-1.36
α_{y2}	Share Consumption Intermediates, Consumption	0.658
	Volatility Transitory Shocks	
σ_ε^g	Neutral	0.0023
$\sigma_\varepsilon^{x_1}$	Investment	0.01
$\sigma_\varepsilon^{x_2}$	Intermediate Equipment	0.014
$Corr\left(\varepsilon^{x2}, \varepsilon^{x_1}\right)$	Covariance Equipment Sector	-0.728
	Volatility Persistent Shocks	
σ_η^g	Neutral	0.0016
σ_η^{x1}	Investment	0.0033
σ_η^{x2}	Intermediate Equipment	0.0001

Table 19: Jointly calibrated Parameters

Moment	Model	Data
Cost Shares		
Consumption Goods in Consumption	0.81	0.93
Consumption Goods in Equipment	0.52	0.57
Correlation, GDP Cycle and Cost Shares		
Consumption Goods in Consumption	-0.27	-0.13
Equipment Goods in Consumption	0.10	0.13
Equipment Goods in Equipment	0.22	0.26
Consumption Goods in Equipment	-0.22	-0.26
Standard Deviation		
GDP	0.013	0.015
Gross Output, Consumption	0.02	0.018
Gross Output, Equipment	0.053	0.0476
Autocorrelation (1)		
GDP	0.36	0.38
Gross Output, Consumption	0.39	0.36
Gross Output, Equipment	0.36	0.29

Table 20: Moments

	TRANSITORY, ε			PERSISTENT, η		
	A^g	A^{x_1}	A^{x_2}	A^g	A^{x_1}	A^{x_2}
GROSS DOMESTIC PRODUCT	52.8	8.12	12.1	26.8	0.08	0.00
TOTAL FACTOR PRODUCTIVITY	35.4	9.22	28.58	18.39	8.67	0.00
INVESTMENT	34	10.5	28.8	17.6	9.1	0.00
CONSUMPTION	14	66.8	1.73	6.99	10.5	0.00

Table 21: Variance Decomposition, Baseline

PARAMETER	DEFINITION	CES	COBB-DOUGLAS
ρ_x	Elasticity of substitution, Equipment	-0.12	0
α_{x2}	Share of equipment intermediates, Equipment	0.579	0.49
ρ_y	Elasticity of substitution, Consumption	-1.36	0
α_{y2}	Share of consumption intermediates, Consumption	0.658	0.81
α_{x1}	Share of capital, Equipment	0.182	0.182
ς	Share of capital, Equipment intermediates	0.901	0.973
α_{y1}	Share of capital, Consumption	0.210	0.210

Table 22: Parametrization, Baseline CES vs. Cobb Douglas technology

MOMENT	CES	CD	DATA
Cost Shares			
Consumption Goods in Consumption	0.81	0.81	0.93
Consumption Goods in Equipment	0.52	0.52	0.57
Correlation, GDP Cycle and Cost Shares			
Consumption Goods in Consumption	-0.27	.	-0.13
Equipment Goods in Consumption	0.10	.	0.13
Equipment Goods in Equipment	0.22	.	0.26
Consumption Goods in Equipment	-0.22	.	-0.26
Standard Deviation			
GDP	0.013	0.01	0.015
Gross Output, Consumption	0.02	0.015	0.018
Gross Output, Equipment	0.053	0.041	0.0476
Autocorrelation (1)			
GDP	0.36	0.26	0.38
Gross Output, Consumption	0.39	0.41	0.36
Gross Output, Equipment	0.36	0.37	0.29

Table 23: Moments

	Transitory, ε			Persistent, η		
GDP	A^g	A^{x_1}	A^{x_2}	A^g	A^{x_1}	A^{x_2}
CES	52.8	8.12	12.1	26.8	0.08	0.00
Cobb-Douglas	47.7	12.09	15.6	24.4	0.06	0.00
TPF						
CES	35.4	9.22	28.58	18.4	8.67	0.00
Cobb-Douglas	39	20.26	12.57	20.17	7.8	0.00
Investment						
CES	34	10.5	28.8	17.6	9.1	0.00
Cobb-Douglas	34.5	12.52	25.9	17.9	9.1	0.00
Consumption						
CES	14	66.8	1.73	6.99	10.5	0.00
Cobb-Douglas	8.06	78.1	0.12	4.03	9.71	0.00

Table 24: Variance Decomposition

Parameter	Definition	CES	CD
	Volatility Transitory Shocks		
σ_{ε}^{g}	Neutral	0.0023	0.0036
$\sigma_{\varepsilon}^{x_1}$	Investment	0.01	0.028
$\sigma_{\varepsilon}^{x_2}$	Intermediate Equipment	0.014	0.037
$Corr\left(\varepsilon^{x2}, \varepsilon^{x_1}\right)$	Covariance Equipment Sector	-0.72	-0.72
	Volatility Persistent Shocks		
σ_{η}^{g}	Neutral	0.0016	0.0036
σ_{η}^{x1}	Investment	0.0033	0.0068
σ_{η}^{x2}	Intermediate Equipment	0.0001	0.0009

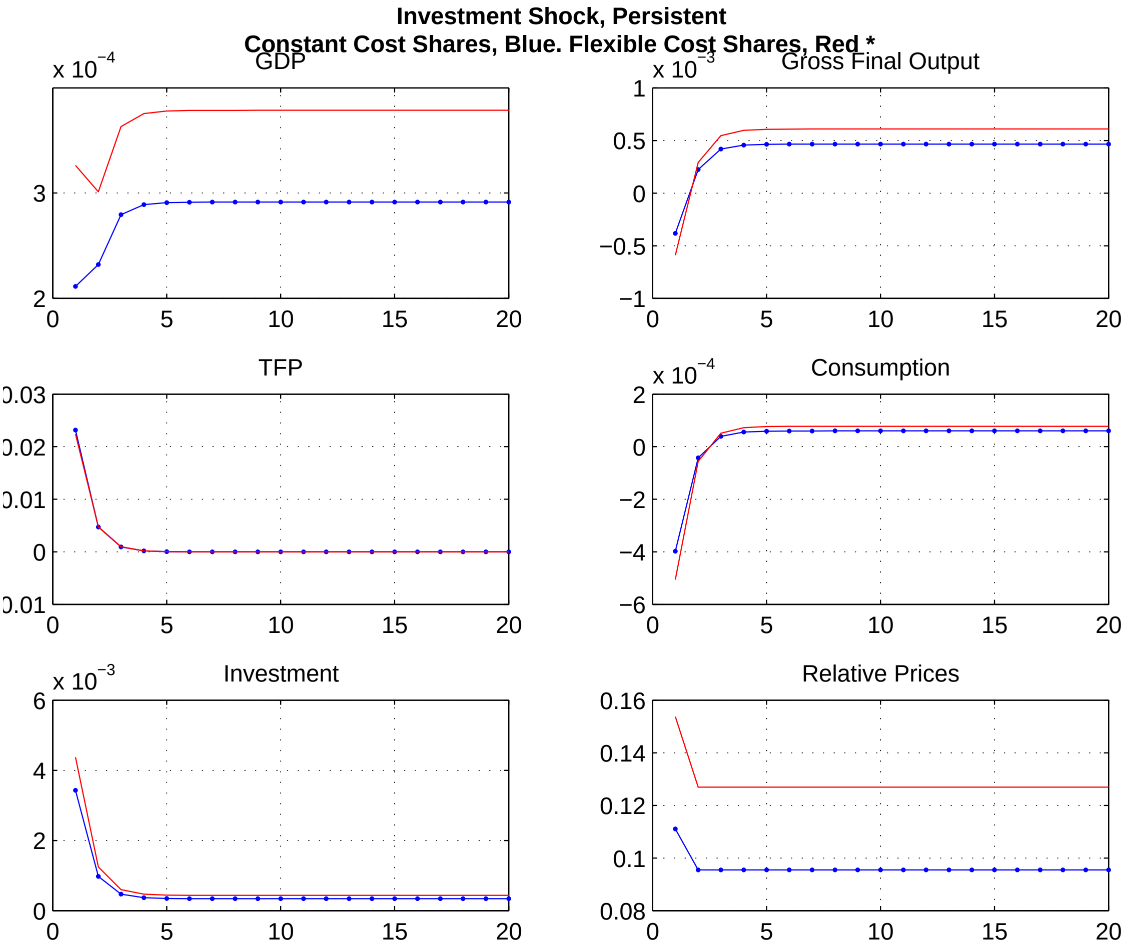
Table 25: Calibrated Shocks, Constant Cost Share Economy

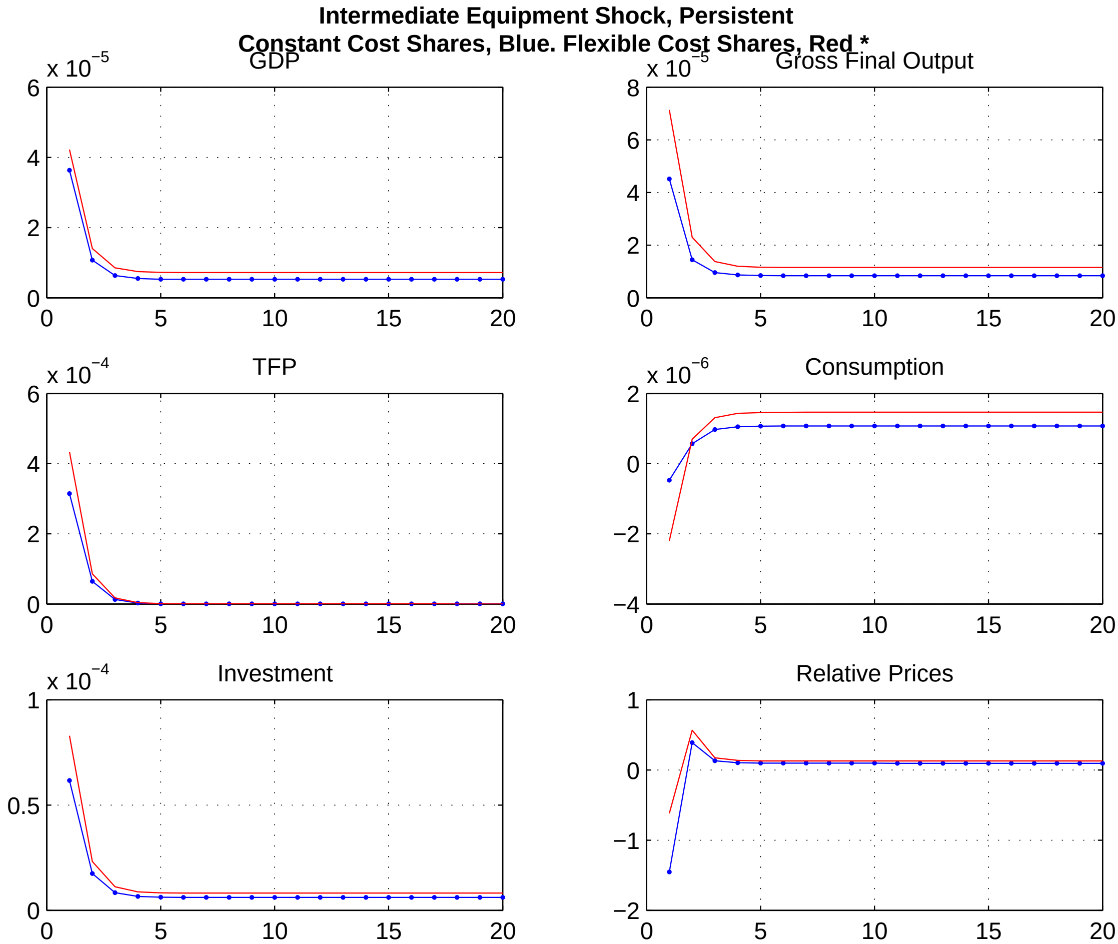
Moment	CES	CD	CD Recalibrated
Cost Shares			
Consumption Goods in Consumption	0.81	0.81	0.81
Consumption Goods in Equipment	0.52	0.52	0.52
Correlation, GDP Cycle and Cost Shares			
Consumption Goods in Consumption	-0.27	.	.
Equipment Goods in Consumption	0.10	.	.
Equipment Goods in Equipment	0.22	.	.
Consumption Goods in Equipment	-0.22	.	.
Standard Deviation			
GDP	0.013	0.01	0.023
Gross Output, Consumption	0.02	0.015	0.033
Gross Output, Equipment	0.053	0.041	0.032
Autocorrelation (1)			
GDP	0.36	0.26	0.37
Gross Output, Consumption	0.39	0.41	0.40
Gross Output, Equipment	0.36	0.37	0.37

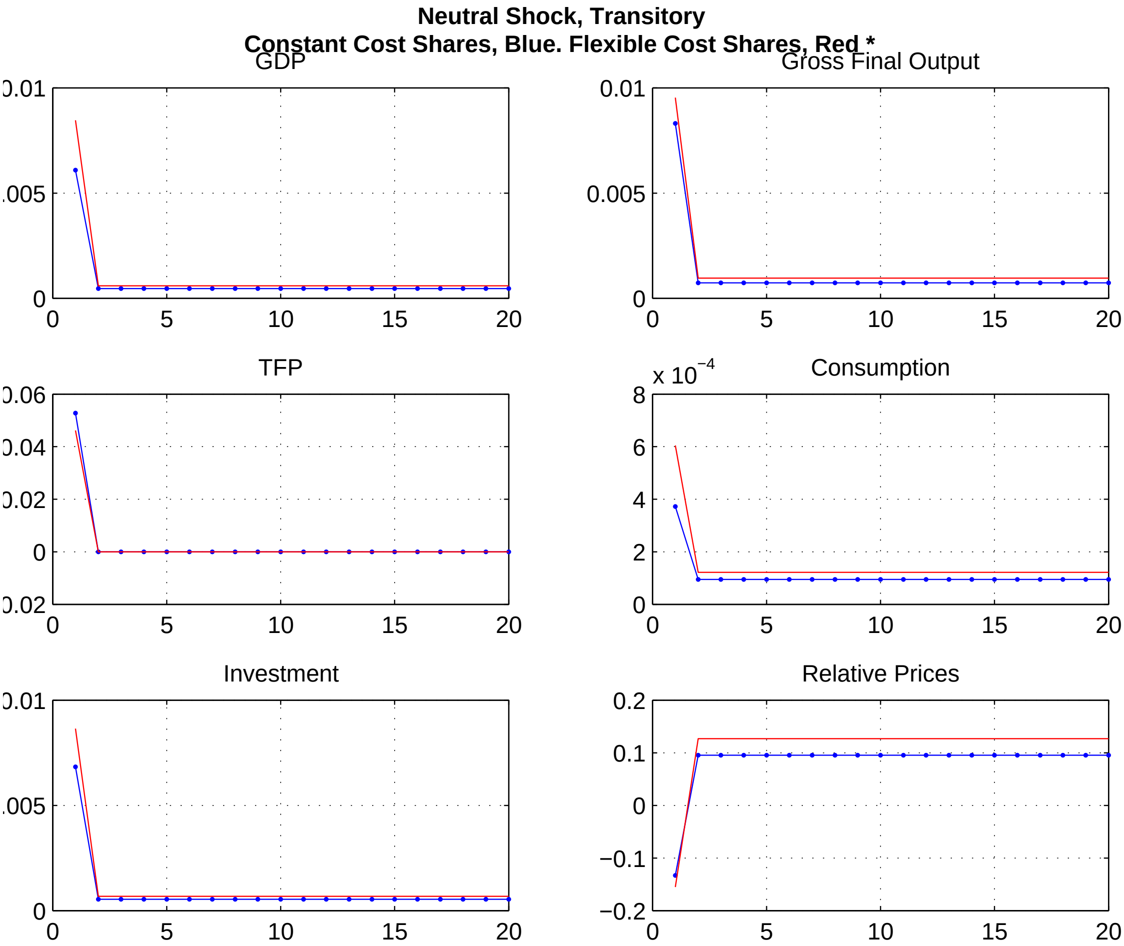
Table 26: Moments

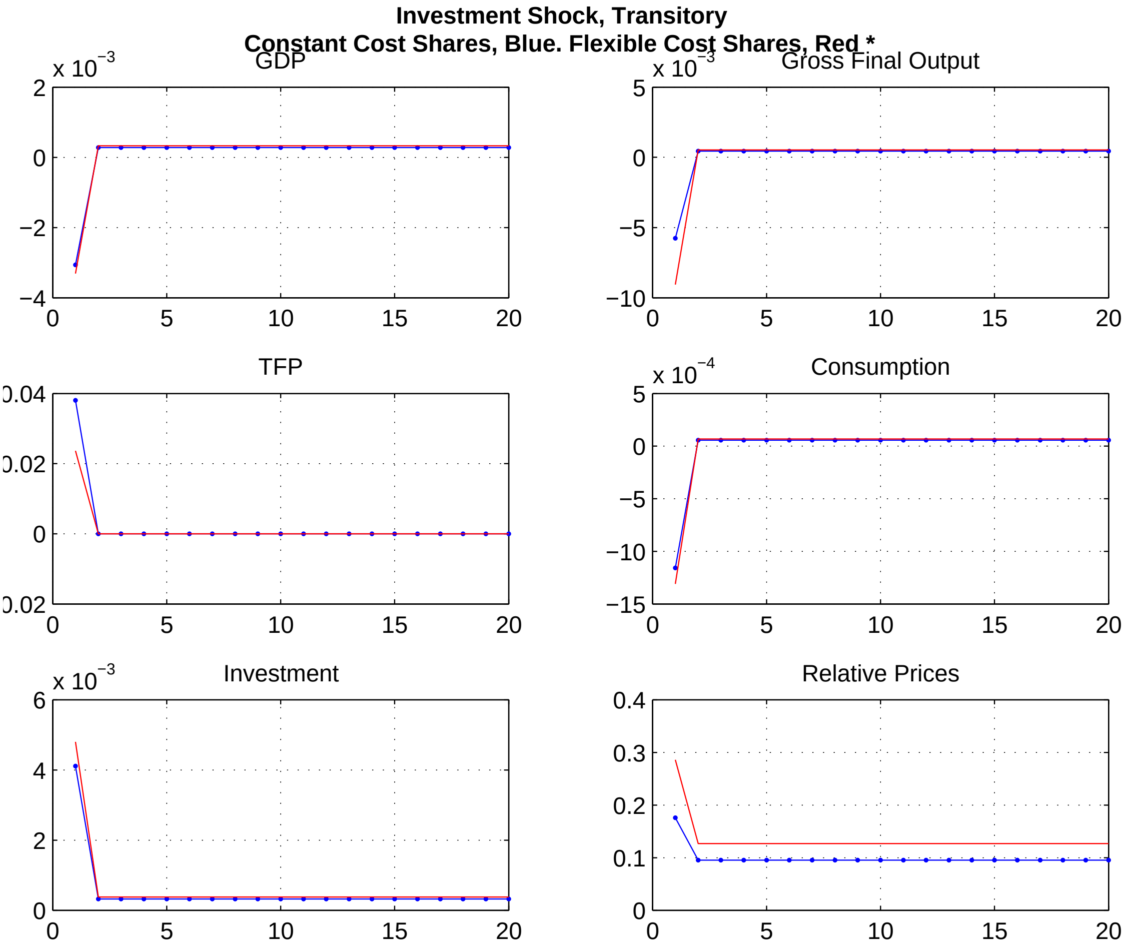
	Transitory, ε			Persistent, η		
GDP	A^{g}	A^{x_1}	A^{x_2}	A^{g}	A^{x_1}	A^{x_2}
CES	52.8	8.12	12.1	26.8	0.08	0.00
Cobb-Douglas*	26.7	19.4	25.3	28.2	0.06	0.03
TPF						
CES	35.4	9.22	28.58	18.4	8.67	0.00
Cobb-Douglas*	19.4	34.6	18.5	20.7	6.73	0.02
Investment						
CES	34	10.5	28.8	17.6	9.1	0.00
Cobb-Douglas*	16.6	21.8	36.3	17.7	7.5	0.05
Consumption						
CES	14	66.8	1.73	6.99	10.5	0.00
Cobb-Douglas*	2.8	88.2	0.12	2.93	5.92	0.00

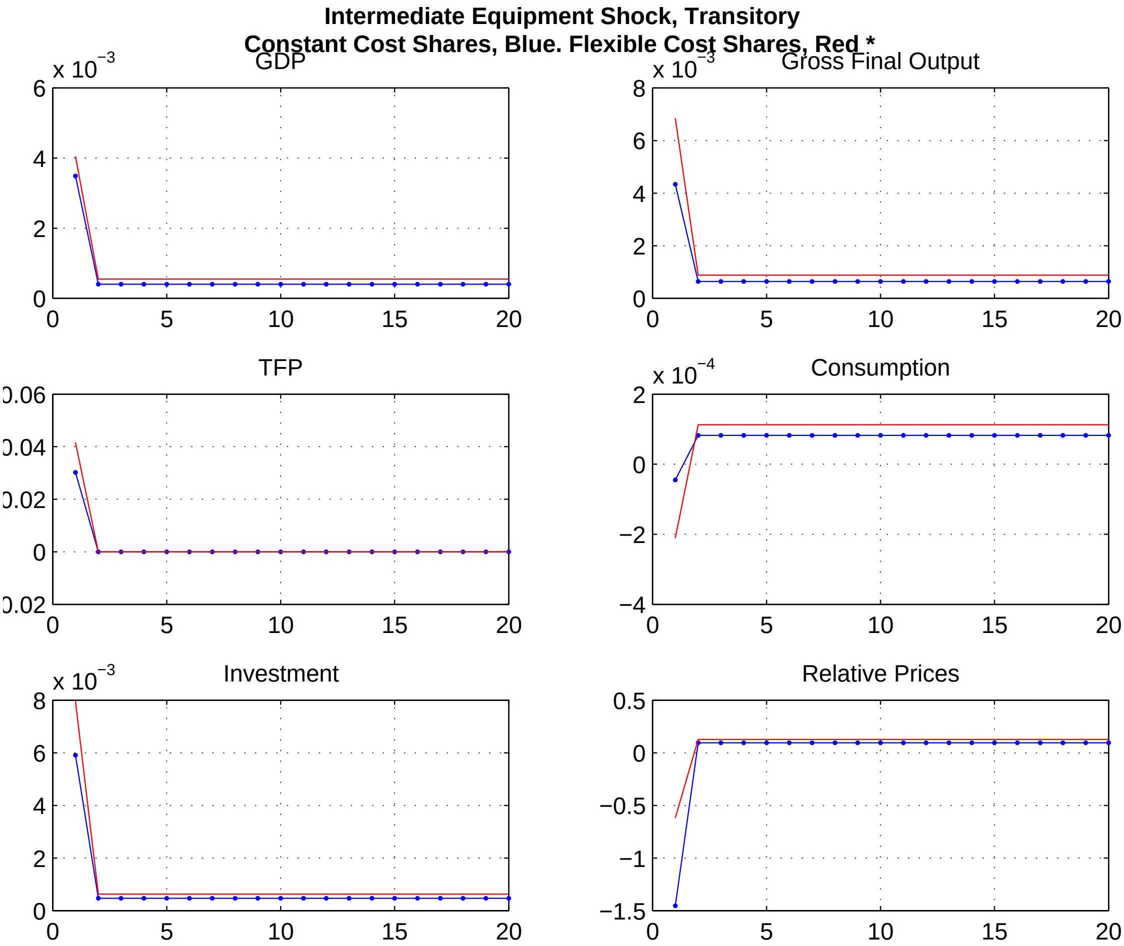
Table 27: Variance Decomposition: Cobb Douglas Economy, recalibrated


Investment Shock, Persistent
Constant Cost Shares, Blue. Flexible Cost Shares, Red *
GDP
x 10^-4
Gross Final Output
x 10^-3
TFP
Consumption
x 10^-4
Investment
x 10^-3
Relative Prices


Intermediate Equipment Shock, Persistent
Constant Cost Shares, Blue. Flexible Cost Shares, Red *
GDP
x 10^−5
Gross Final Output
x 10^−5
TFP
x 10^−4
Consumption
x 10^−6
Investment
x 10^−4
Relative Prices


Neutral Shock, Transitory
Constant Cost Shares, Blue. Flexible Cost Shares, Red *
GDP
Gross Final Output
TFP
Consumption
x 10^−4
Investment
Relative Prices


Investment Shock, Transitory
Constant Cost Shares, Blue. Flexible Cost Shares, Red *
GDP
x 10^-3
Gross Final Output
x 10^-3
TFP
Consumption
x 10^-4
Investment
x 10^-3
Relative Prices


Intermediate Equipment Shock, Transitory
Constant Cost Shares, Blue. Flexible Cost Shares, Red *
GDP
x 10^-3
Gross Final Output
x 10^-3
TFP
Consumption
x 10^-4
Investment
x 10^-3
Relative Prices

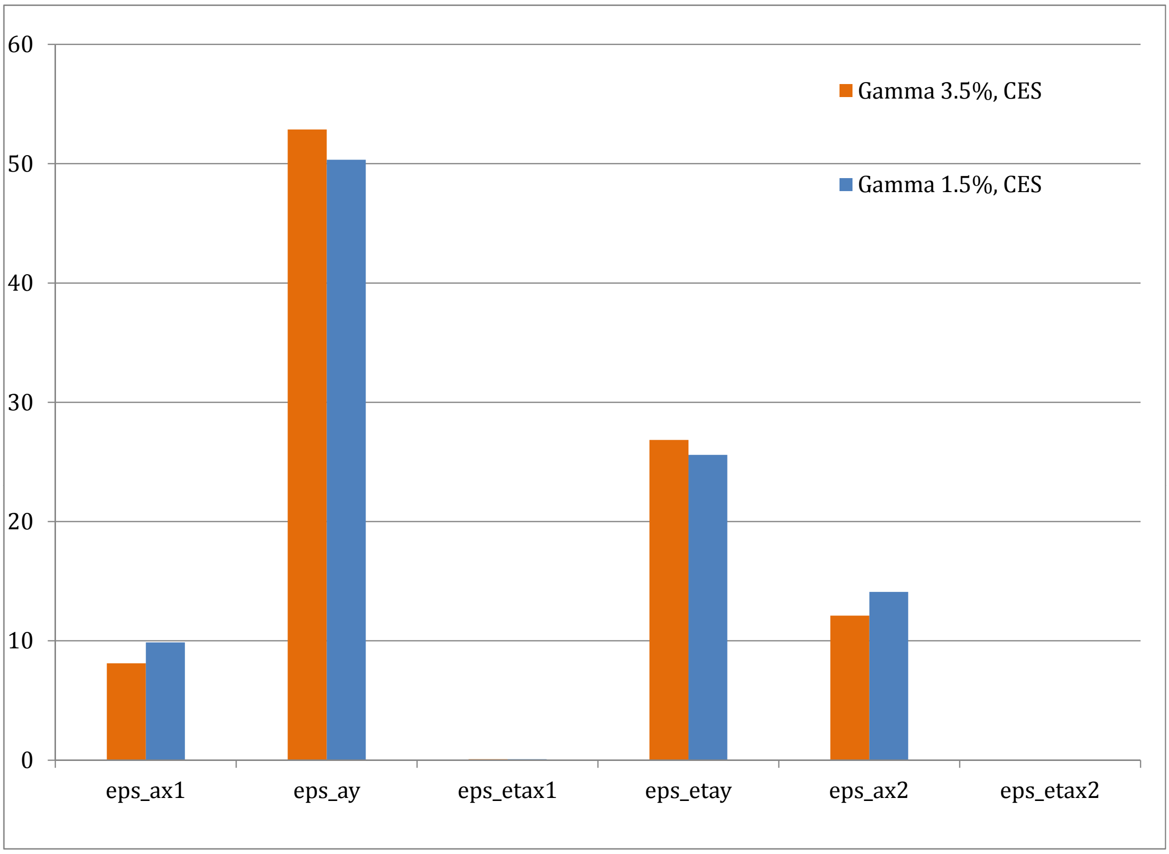


Figure 15: Alternative Aggregate Growth Rates

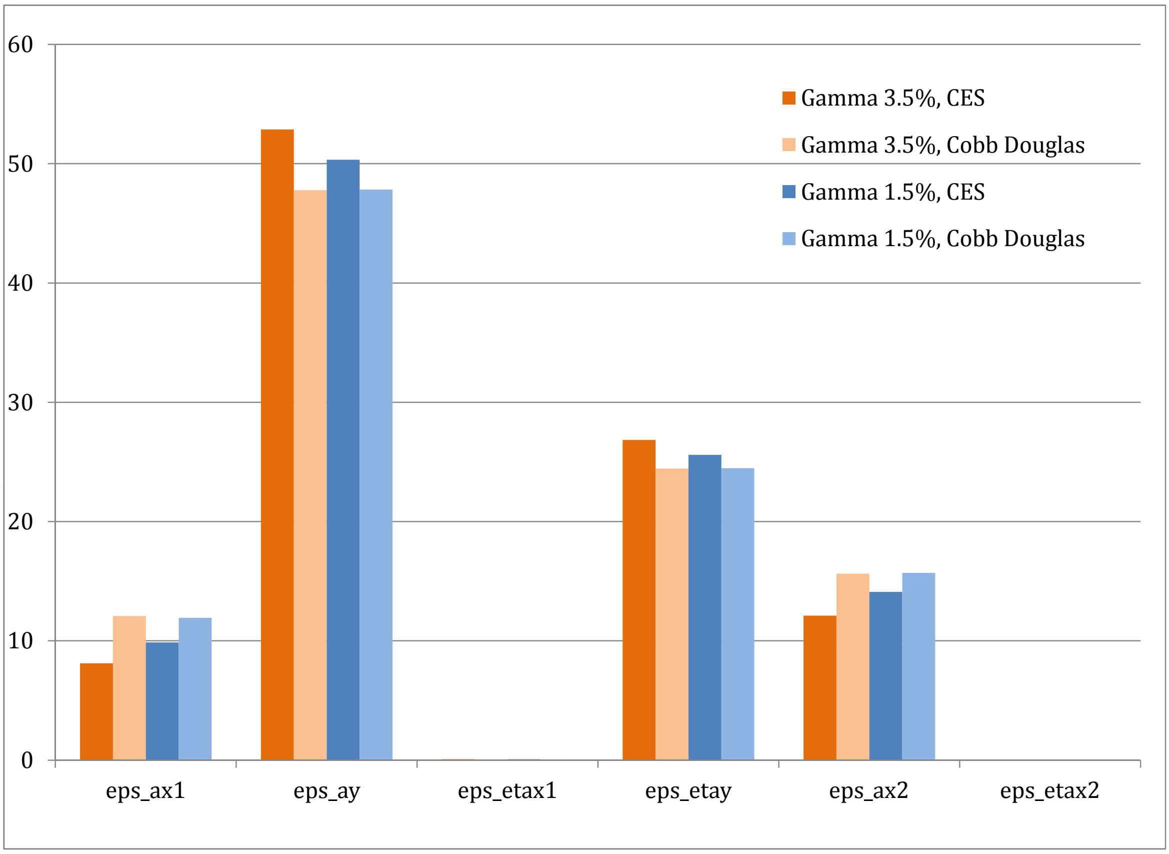


Figure 16: Alternative Aggregate Growth Rates, Constant and Flexible Cost Shares

7 Data Appendix

The following tables include a description of the sectors under analysis. Our definition of the equipment sector entails sectro 33 in the NAICS.

Industry/Commodity Description	NAICS
Agriculture, forestry, fishing and hunting	11
Crop production	111
Animal production	112
Forestry	1131, 1132
Logging	1133
Fishing, hunting and trapping	114
Support activities for agriculture and forestry	115
Mining	21
Oil and gas extraction	211
Coal mining	2121
Metal ore mining	2122
Nonmetallic mineral mining and quarrying	2123
Support activities for mining	213
Utilities	22
Electric power generation, transmission and distribution	2211
Natural gas distribution	2212
Water, sewage and other systems	2213
Construction	
Construction	23
Manufacturing	31, 32, 33
Animal food manufacturing	3111
Grain and oilseed milling	3112
Sugar and confectionery product manufacturing	3113
Fruit and vegetable preserving and specialty food manufacturing	3114
Dairy product manufacturing	3115
Animal slaughtering and processing	3116
Seafood product preparation and packaging	3117
Bakeries and tortilla manufacturing	3118
Other food manufacturing	3119
Beverage manufacturing	3121
Tobacco manufacturing	3122

Industry/Commodity Description	**NAICS**
Textile mills and textile product mills	313,314
Apparel manufacturing	315
Leather and allied product manufacturing, including footwear manufacturing	316
Sawmills and wood preservation	3211
Veneer, plywood, and engineered wood product manufacturing	3212
Other wood product manufacturing	3219
Pulp, paper, and paperboard mills	3221
Converted paper product manufacturing	3222
Printing and related support activities	323
Converted paper product manufacturing	3222
Printing and related support activities	323
Petroleum and coal products manufacturing	324
Basic chemical manufacturing	3251
Resin, synthetic rubber, and artificial synthetic fibers and filaments manufacturing	3252
Pesticide, fertilizer, and other agricultural chemical	3253
Pharmaceutical and medicine manufacturing	3254
Paint, coating, and adhesive manufacturing	3255
Soap, cleaning compound, and toilet preparation	3256
Other chemical product and preparation manufacturing	3259
Plastics product manufacturing	3261
Rubber product manufacturing	3262
Clay product and refractory manufacturing	3271
Glass and glass product manufacturing	3272
Cement and concrete product manufacturing	3273
Lime, gypsum and other nonmetallic mineral product	3274, 3279
Iron and steel mills and ferroalloy manufacturing	3311
Steel product manufacturing from purchased steel	3312
Alumina and aluminum production and processing	3313
Nonferrous metal (except aluminum) production	3314
Foundries	3315
Forging and stamping	3321
Cutlery and handtool manufacturing	3322
Architectural and structural metals manufacturing	3323
Boiler, tank, and shipping container manufacturing	3324
Hardware manufacturing	3325
Spring and wire product manufacturing	3326
Machine shops;	3327

Industry/Commodity Description	NAICS
Coating, engraving, heat treating, and allied activities	3328
Other fabricated metal product manufacturing	3329
Agriculture, construction, and mining machinery	3331
Industrial machinery manufacturing	3332
Commercial and service industry machinery	3333
Ventilation, heating, air-conditioning, equipment	3334
Metalworking machinery manufacturing	3335
Engine, turbine, and power transmission equipment	3336
Other general purpose machinery manufacturing	3339
Computer and peripheral equipment manufacturing	3341
Communications equipment manufacturing	3342
Audio and video equipment manufacturing	3343
Semiconductor and other electronic component	3344
Navigational, measuring, electromedica manufacturing	3345
Manufacturing and reproducing magnetic and optical media	3346
Electric lighting equipment manufacturing	3351
Household appliance manufacturing	3352
Electrical equipment manufacturing	3353
Other electrical equipment and component manufacturing	3359
Motor vehicle manufacturing	3361
Motor vehicle body and trailer manufacturing	3362
Motor vehicle parts manufacturing	3363
Aerospace product and parts manufacturing	3364
Railroad rolling stock manufacturing	3365
Ship and boat building	3366
Other transportation equipment manufacturing	3369
Household and institutional furniture and kitchen cabinet manufacturing	3371
Office furniture (including fixtures) manufacturing	3372
Other furniture related product manufacturing	3379
Medical equipment and supplies manufacturing	3391
Other miscellaneous manufacturing	3399
Wholesale trade	42
Wholesale trade	42

Industry/Commodity Description	NAICS
Retail trade	44, 45
Retail trade	44, 45
Transportation and warehousing	48, 49
Air transportation	481
Rail transportation	482
Water transportation	483
Truck transportation	484
Transit and ground passenger transportation	485
Pipeline transportation	486
Scenic and sightseeing transportation and support activities for transportation	487, 488
Couriers and messengers	492
Warehousing and storage	493
Information	
Newspaper, periodical, book, and directory publishers	5111
Software publishers	5112
Motion picture, video, and sound recording industries	512
Broadcasting (except internet)	515
Telecommunications	517
Data processing, hosting, related services, and other information services	518, 519
Finance and insurance	52
Monetary authorities, credit intermediation	521, 522
Securities, commodity contracts, and other invest	523
Insurance carriers	5241
Agencies, brokerages, and other insurance	5242
Funds, trusts, and other financial vehicles	525
Real estate and rental and leasing	53
Real estate	531
Automotive equipment rental and leasing	5321
Consumer goods rental and general rental centers	5322, 5323
Commercial and industrial machinery and equipment rental and leasing	5324
Lessors of nonfinancial intangible assets	533

Industry/Commodity Description	NAICS
Professional, scientific, and technical services	
Legal services	5411
Accounting, tax preparation, bookkeeping,	5412
Architectural, engineering, and related services	5413
Specialized design services	5414
Computer systems design and related services	5415
Management, scientific, and technical consulting	5416
Scientific research and development services	5417
Advertising and related services	5418
Other professional, scientific, and technical services	5419
Management of companies and enterprises	55
Management of companies and enterprises	55
Administrative and support	56
Office administrative services	5611
Facilities support services	5612
Employment services	5613
Business support services	5614
Travel arrangement and reservation services	5615
Investigation and security services	5616
Services to buildings and dwellings	5617
Other support services	5619
Waste management and remediation services	562
Educational services	61
Elementary and secondary schools	6111
Junior colleges, colleges, universities, prof schools	6112, 6113
Other educational services	6114-7
Health care and social assistance	62
Offices of health practitioners	6211, 6212, 6213
Home health care services	6216

Industry/Commodity Description	**NAICS**
Outpatient, laboratory, and other ambulatory care	6214, 6215, 6219
Hospitals	622
Nursing and residential care facilities	623
Individual and family services	6241
Community and vocational rehabilitation services	6242, 6243
Child day care services	6244
Arts, entertainment, and recreation	71
Performing arts companies	7111
Spectator sports	7112
Promoters of events, and agents and managers	7113, 7114
Independent artists, writers, and performers	7115
Museums, historical sites, and similar institutions	712
Amusement, gambling, and recreation industries	713
Accommodation and food services	72
Accommodation	721
Food services and drinking places	722
Other services (except public administration)	81
Automotive repair and maintenance	8111
Electronic and precision equipment repair	8112
Commercial and industrial machinery and equipment	8113
Personal and household goods repair	8114
Personal care services	8121
Death care services	8122
Drycleaning and laundry services	8123
Other personal services	8129
Religious organizations	8131
Grantmaking and giving services and social advocacy organizations	8132, 8133
Civic, social, professional, and similar organizations	8134, 8139
Private households	814
Postal Service	491

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