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Essays in Financial Economics

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by

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Jinji Hao

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Dedicated to my family.

ABSTRACT OF THE DISSERTATION

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Professor Werner Ploberger, Chair

In the first chapter of my dissertation, I provide a novel framework – the cumulant generating function (cgf) of the market risk on the positive half real line – for studying the market risk which can be replicated by cross sections of index option prices in a model-free manner. Within this unifying framework, independent of the underlying price process, the VIX index measures the height of the cgf at one while the SVIX index proposed by Martin (2016) measures the convexity of the cgf over the interval $[0, 2]$. A tail index of the market risk, TIX, is proposed based on this framework which measures the tail decay rate of the distribution of market risk revealing the market perceptions of extreme risk. The change in TIX strongly predicts market returns both in and out of sample, with monthly R^2 statistics of 3.33% and 6.15%, respectively, outperforming the popular return predictors in the literature.

In the second chapter of my dissertation, I study a shadow banking system featuring collateral constraints to investigate the joint determination of haircut and interest rate, as well as its interaction with collateral asset pricing. The banks with limited commitment serve the households' need for consumption smoothing by taking deposits with a risky asset used as collateral and pursue the maximal leverage returns. In a collateral equilibrium as in Geanakoplos (1997, 2003), agents' marginal rates of substitution are equalized only in

non-default states, only the deposit contract with the highest liquidity value per unit of collateral is traded, and the risky asset price is boosted such that banks earn zero profit. Relative to the traditional banking with full commitment, banks are better off if they are endowed with the collateral asset while households are strictly worse off. I also find (i) higher households' risky asset endowment leads to a higher asset price because a stronger saving motive creates a scarcity of collateral, while higher banks' collateral endowment has the opposite effects; (ii) for the quality of collateral, the higher asset price resulting from an upside improvement simply leads to a higher haircut with the interest rate unchanged since lenders do not care about upside risk; on the contrary, for lenders with a high risk aversion, a downside improvement of quality decreases the asset price because it alleviates the tension of imperfect risk sharing and, therefore, reduces the collateral value, but everything goes in opposite directions for a low risk aversion; (iii) collateral use exhibits a diminishing return to scale in the amount of borrowing supported.

In the third chapter of my dissertation, I study specifically the implications of disaster concerns about financial intermediaries for stock market returns. Manela and Moreira (2017) develop a text-base measure of disaster concerns using phrase counts of front-page articles of the *Wall Street Journal*. While they do not find evidence for return predictability at the monthly horizon and, in particular, at any horizon for the financial intermediation category of this measure, I document that an *increase* in the news coverage of intermediation is followed by lower stock market returns next month in the sample since the World War II. The effect is economically large with a one-standard-deviation increase in the coverage associated with an 44 basis point decrease in next month's stock market excess return.

Chapter 1

A Model-Free Tail Risk Index and Its Return Predictability

This paper provides a novel framework–the cumulant generating function (cgf) of the market risk on the positive half real line–for studying the market risk which can be replicated by cross sections of index option prices in a model-free manner. Within this unifying framework, independent of the underlying price process, the VIX index measures the height of the cgf at one while the SVIX index proposed by Martin (2016) measures the convexity of the cgf over the interval $[0, 2]$. A tail index of the market risk, TIX, is proposed based on this framework which measures the tail decay rate of the distribution of market risk revealing the market perceptions of extreme risk. The innovation in TIX strongly predicts market returns both in and out of sample, with monthly R^2 statistics of 3.33% and 6.15%, respectively, outperforming the popular return predictors in the literature.

1.1 Introduction

Since the final payoffs of European options depend on the future realized price of the underlying, the current option prices reflect the market expectations of the future price movements. This forward-looking nature of option prices has inspired market participants to extract from them the possible information about the future market prices. The earliest and probably the most famous attempt was the Black-Scholes based option implied volatility. Moreover, the Breeden and Litzenberger (1978) formula shows that the density of final prices is just the second order derivative of European option prices with respect to the strike.

In this paper, a novel framework is provided to extract the information contained in option prices. Different from the implied volatility which relies on the Black-Scholes model, this framework is based on an option replication strategy and, therefore, is model-free. As with Breeden and Litzenberger (1978), I skip the price process and focus on the distribution of final prices. However, instead of recovering the distribution function of final prices directly, I extract the cumulant generating function (cgf) of the market risk, a mean zero random variable uniquely determining the outcome of stochastic final prices, with the same horizon as the maturity of options. Because of the possible non-existence of higher moments of log returns due to fat tails in their distributions in general, only the cgf on the positive half real line is extracted which is the object investigated further in this paper. It is a positive, increase, and convex curve starting with zero at the origin.

One advantage of working with the cgf of the market risk instead of the distribution of final prices comes from the empirical implementation. When adopting the approach of Breeden and Litzenberger (1978) to recover the distribution function of final prices, we either face the numerical instability involved in taking derivatives of the market option prices with respect to discrete strikes or have to rely on some parametric assumptions on the distribution function. However, the cgf of the market risk with a particular horizon can be replicated by the cross section of index option prices with the corresponding maturity.

I first show that this cgf of the market risk provides a unifying framework for interpreting the two leading forward-looking measures of market risk in the literature, the Chicago Board Options Exchange (CBOE) volatility index (VIX) and the SVIX proposed by Martin (2016). It turns out that VIX just measures the height of the cgf of the 30-day market risk at 1, or alternatively the generalized Itô type correction term, while SVIX measures the convexity of the function over the interval [0, 2]. The sum of SVIX and VIX measures exactly the height of the function at 2. These graphical interpretations are independent of any assumption regarding the price process. Nevertheless, for alternative processes assumed for the index price such as the finite moment log stable process in Carr and Wu (2003a) and the jump diffusion process in Merton (1976), their corresponding VIX and SVIX formulas can be easily obtained with these interpretations.

I then propose a new tail index based on the cgf of the market risk. It is defined as (the logarithm to base 2 of) the ratio of the cgf of the market risk evaluated at 2 over that at 1, which is called TIX. It measures the tail decay rate of the distribution of market risk. When the market risk is normal, it equals 2. When the market risk is stable as in Carr and Wu (2003a) more generally, the leading example in the paper, TIX is the stability parameter $\alpha \in (1, 2)$ of the distribution. A lower TIX indicates fatter tails of the distribution of market risk and, therefore, a higher extreme risk. Economically, TIX essentially reveals the market perceptions of the chance of large drops in index prices. The information contained in TIX is then extracted by examining the changes in market expectations. Namely, I obtain the innovations in TIX by taking the log difference of the TIX time series.

Empirically, using the S&P 500 index option prices, I construct a daily time series of TIX for the 30-day market risk born by the index from 1996:01 to 2015:08. It experienced significant drops during the episodes of market crashes such as the mini crash on Oct 27, 1997, the Russian default and LTCM collapse in 1998, the financial tsunami in 2008, the "Flash Crash" on May 6, 2010, the U.S. debt ceiling crisis in 2011, and China's economic slowdown in late August 2015. The innovations of this TIX time series also display an asymmetry of more frequent and larger drops than rises. It is not surprising that the large drops took place amid these episodes of crisis while the large rises appeared during the economic recoveries.

To examine the information content of the innovation in TIX, its predictability of future monthly market returns in the sample period 1996:02 to 2014:12 is compared with 14 popular predictor variables from Goyal and Welch (2008) and the short interest variable from Rapach, Ringgenberg, and Zhou (2016), which is arguably the best predictor in the literature so far.[1] Among the in-sample predictive regressions, the innovation in TIX has the largest impact with a one-standard-deviation increase in the innovation associated with an 88 basis point decrease in next month's stock market excess return. Its predictive coefficient is also the most significant at the 5% level and its in-sample R^2 statistic is 3.33% much bigger than the second best 2.59% from stock variance and the 2.03% from the short interest variable. For the more challenging out-of-sample test, only the innovation in TIX and the short interest can reduce the mean squared forecast error (MSFE) compared to the mean forecast benchmark, which assumes non-predictability. However, the innovation in TIX reduces the MSFE by 6.15% which is significant at the 5% level for a one-sided test while the short interest reduces it by only 0.75% with a significance level of 10%. These tests show that the change in market perceptions of future extreme risk contains significant additional information about future market returns.

This paper contributes to three strands of literature. First, it enriches the literature on developing option implied model-free measures of market risk. Britten-Jones and Neuberger (2000) derive a model-free implied volatility under an assumption of a diffusion process.

[1] For their sample period from 1973:01 to 2014:12.

Jiang and Tian (2005) extend their model-free implied volatility to asset price processes with jumps. Bakshi, Kapadia, and Madan (2003) provide a model-free estimator for the risk-neutral variance of holding period log returns. This paper shows that we can extract the cgf of the market risk in a model-free manner which provides a framework for studying and developing alternative measures of market risk. Indeed, the tail index proposed in this paper, which has a strong predictive power of market returns, is constructed within this framework. The cgf also provides fresh insights on VIX (Demeterfi et al., 1999) and SVIX of Martin (2016).

Second, the TIX proposed in this paper is related to a few fear, rare disaster, or tail indices developed in the literature. Kelly and Jiang (2014) estimate a common tail risk measure from the cross-section of stock returns. Bollerslev and Todorov (2011) estimate an Investor Fears Index (FI) as the proportion of variance risk premium for the S&P 500 index that is due to special compensation for possible large price jumps.[2] Du and Kapadia (2012) construct a jump and tail index (JTIX) as the difference between the risk-neutral variance of holding period log return[3] and VIX which, in a jump diffusion model, is related to the jump intensity and the jump size distribution. Based on this idea, Gao, Gao, and Song (2016) develop a Rare Disaster Index (RIX) using only the out-of-money put options written on six economic sector indices to identify the hedge funds with better skills in exploiting the market's ex ante rare disaster concerns. Manela and Moreira (2016) construct a news implied volatility (NVIX) as a proxy for disaster concerns using phrase counts of front-page articles of the *Wall Street Journal*. Instead, TIX reflects the market risk-neural expectations of extreme risk independent of the underlying price process.

Finally, this paper adds to the literature on the information content of option-based

[2]FI is estimated as the difference between the variance risk premium due to large positive jumps and that due to large negative jumps. It relies on reduced-from time series modeling of realized measures from high-frequency data to estimate the physical expectations and the short maturity out-of-the-money options to estimate the risk-neutral jump densities (Carr and Wu, 2003b).

[3]In this paper, the existence of second moment of log returns is not assumed in general.

measures for predicting returns.[4] It has been documented that the variance risk premium (Bollerslev, Tauchen, and Zhou, 2009; Drechsler and Yaron, 2011), the risk-neutral moments (Conrad, Dittmar, and Ghysels, 2013; Rehman and Vilkov, 2012), the implied volatility smirk or skew (Xing, Zhang, and Zhao, 2010; Ratcliff, 2013), and forward variances (Bakshi, Panayotov, and Skoulakis, 2011) are able to predict stock market returns, individual equity returns or Treasury bill returns. This paper shows that the change in market expectations of extreme risk as implied by index option prices is a powerful predictor of market returns.

The paper is organized as following. In Section 2, the framework of cgf of the market risk is laid down first and illustrated with two examples. Then a model-free extraction of the cgf from index option prices is provided. Finally, I examine what VIX and SVIX measure in general using this framework. In Section 3, a new tail index, TIX, is defined. In Section 4, I first describe the data used and extract the empirical cgf's of the S&P 500 index. I then construct TIX for the S&P 500 index and a predictor variable, the innovation in TIX. The market return predictability of this predictor is tested in Section 5. Section 6 contains further discussions and Section 7 concludes.

1.2 A Generic Framework of Market Risk

In this paper, I investigate the information contained in the equity market index option prices. European option prices are risk-neutral expectations of their final payoffs which are functions of only the future realized price of the underlying. Under the risk-neutral probability measure $\mathbb{Q}$, the future spot price S_T of the market index in general can be

[4]There is also a literature examining the information content of option-based measures in predicting realized variance, see Canina and Figlewski (1993) on Black-Scholes implied volatility and Jiang and Tian (2005) on model-free implied volatility.

expressed in an exponential form at time t,

$$S_T = S_t e^{(r-q)\tau + \mu\tau + \sigma Z_\tau}, \tag{1.1}$$

where $\tau = T - t$, and r and q denote, respectively, the continuously compounded risk-free rate and dividend yield, both of which are assumed to be deterministic. The term σZ_τ represents the *market risk* with Z_τ being a generic mean zero random variable standardized by the *size of risk* σ. When the second moment of Z_τ exists, the size of risk σ can be defined such that Z_τ has a standard deviation $\tau^{1/2}$. Otherwise, σ may be defined to standardize the corresponding notion of "width" of the distribution of Z_τ. For example, for the stable distribution which we will discuss in detail later, σ can be chosen such that Z_τ has a scale parameter $\tau^{1/2}$. Moreover, in order to have $E_t^*[S_T] = S_t e^{(r-q)\tau}$ which is the forward price of the underlying,[5] we need to add a correction term μ such that[6]

$$E_t^*\left[e^{\mu\tau + \sigma Z_\tau}\right] = 1. \tag{1.2}$$

Note that the specification in equation (1.1) does not rely on any assumption on the stochastic process of the index price.[7] For the class of diffusion processes with or without jumps in either price or volatility, which is widely used in the option pricing literature, the resulting distribution of the future spot price S_T can be fully captured by a random variable Z_τ as well as its normalizing constant σ and correction term μ. The effects of stochastic volatility and jumps will be reflected in the shape of the distribution of Z_τ, including its moments and tail behavior, and the magnitude of σ and μ. When the price follows a finite moment log stable process as in Carr and Wu (2003a), Z_τ is also stable distributed with

[5] The superscript $*$ is used to indicate the risk-neutral expectation throughout the paper.

[6] Whenever without causing confusion, the potential dependence of r, q, σ, and μ on the horizon τ is suppressed.

[7] Except for the deterministic interest rate and dividend yield assumed.

the same stable and skew parameters as the innovations.

Equation (1.1) reveals that the moment generating function (mgf) of the market risk σZ_τ, denoted as $M(\lambda) \equiv E_t^*\left[e^{\lambda\sigma Z_\tau}\right]$, plays a direct role in describing the future variations of the index price S_T or its simple return $R_{t,T} \equiv S_T/S_t$. The moments of simple returns are connected to the mgf of σZ_τ evaluated at the power of the moments

$$E_t^*[R_{t,T}^n] = e^{n(r-q)\tau+n\mu\tau}E_t^*\left[e^{n\sigma Z_\tau}\right] = e^{n(r-q)\tau+n\mu\tau}M(n). \tag{1.3}$$

For example, the second moment of $R_{t,T}$ depends on $M(2)$. Since the mgf represents a continuous curve, the information contained therein goes beyond its levels. In particular, varieties of measures of its shape may also reveal meaningful information about the market risk. The SVIX proposed by Martin (2016), which measures the risk-neutral variance of simple returns, indeed explicitly measures the convexity of the log of the mgf[8] of the market risk σZ_τ over the interval $[0, 2]$. I will also propose an index based on the mgf at different points capturing the tail risk incorporated in the market risk. By equation (1.2), the correction term μ is also related to the mgf of σZ_τ evaluated at 1 by

$$M(1) = e^{-\mu\tau}. \tag{1.4}$$

I will show later that the correction term μ is indeed what the CBOE VIX measures.

One of the advantages of working with the mgf of σZ_τ is that it can be replicated by portfolios of European index options with a maturity τ. Therefore, we can study the mgf of σZ_τ in a model-free manner using the option prices. Given our knowledge of the mgf of σZ_τ, we can recover particular characteristics of the market risk σZ_τ. Furthermore, we are able to investigate their information content and see if they have any predictive power of

[8] The log of mgf is cgf which will be defined very soon.

future market returns.[9]

Having set the scene, let us formalize the idea by reviewing several properties of mgf that also have economic interpretations in our setting. The mgf is usually considered useful because *if* the mgf itself is finite in a neighborhood around 0, its nth derivative at 0 exists and is the nth order moment of the random variable for any $n \geq 1$. However, this does not apply to the problem we have. It seems reasonable to assume that the simple return $R_{t,T}$ has finite moments up to a certain order N, so we may expect the mgf $M(\lambda)$ of the market risk σZ_τ to be finite for $\lambda \in [0, N]$. But there is no economic reason to expect the same for $\lambda < 0$. It should be pointed out that the moments of Z_τ or log returns[10] can still exist even if the mgf of σZ_τ does not exist for $\lambda < 0$. The following properties are useful for dealing with our more general case.

Property 1. *(Moment Generating Function) Let* $M(t) = E\left[e^{tX}\right]$ *be the mgf of a random variable* X *with a cdf* F *and* $m(t) \equiv \log M(t)$ *be its cgf, we have*

1. *M(0)=1 and m(0)=0.*
2. *If for some* $t_1 < t_2$, $M(t_1) < \infty$ *and* $M(t_2) < \infty$, *then for any* $t \in [t_1, t_2]$, $M(t) < \infty$.
3. *If for some* $t_1 < 0 < t_2$, $M(t_1) < \infty$ *and* $M(t_2) < \infty$, *then the moments of* X *exist for any order, i.e.,* $E(X^n) < \infty$ *for any* $n > 0$.
4. *(Convexity) On any interval on the real line where* $M(t)$ *is finite, it is superconvex, i.e.,* $m(t)$ *is convex.*

[9] It would also be interesting to examine their information content in predicting realized measures of risks but it is out of the scope of this paper.

[10] In this framework, the existence of moments of log returns is a stronger requirement than the existence of moments of simple returns. This is because the problem with the existence of moments of log returns arises from the fat left tail of its distribution. This problem is alleviated for simple returns which are exponential of log returns.

5. *(Tail Decay Rates) If $M(t_0)$ is finite for some $t_0 > 0$ ($t_0 < 0$, respectively), then the right (left, respectively) tail of F is exponentially bounded,*

$$P(X > x) \leq m(t_0)e^{-t_0 x}, \quad (P(X < x) \leq m(t_0)e^{-t_0 x}, \textit{respectively}). \tag{1.5}$$

   Conversely, if the right (left, respectively) tail of F is exponentially bounded, i.e., there exists $C > 0$ and $b > 0$ such that

$$P(X > x) \leq Ce^{-bx}, \quad (P(X < x) \leq Ce^{bx}, \textit{respectively}). \tag{1.6}$$

   then for any $t \in (0, b)$ ($t \in (-b, 0)$, respectively), $M(t) < \infty$.

We want the mgf of σZ_τ to be finite for $\lambda > 0$ for two reasons. First, it will guarantee the forward price of underlying to be finite for non-degenerate size of risk. Equivalently, this will make sure the correction satisfying equation (1.2) feasible. More importantly, it is necessary for the European option prices, which we rely on, to be finite. Properties 1.1 and 1.2 imply that if we assume the mgf $M(\lambda)$ is finite for a $\bar{\lambda} > 0$, then it is finite for all $\lambda \in [0, \bar{\lambda}]$. It is enough for our purpose if we only consider a bounded range of size of risk σ and a bounded horizon τ. I choose not to make this restriction and thus assume $M(\lambda)$ is finite for all $\lambda > 0$. Property 1.3 tells us to be careful about assuming the finiteness of mgf for any $\lambda < 0$ since it immediately implies the finiteness of mgf in a neighbourhood around 0 and thus the existence of moments of Z_τ or log returns of any order. These conditions are too strong as even the existence of second moment of log returns might be doubtful for serious econometricians which I will come back to later. Therefore, I propose the following criterion for the market risk σZ_τ.

Assumption 1. *The mgf of σZ_τ is finite on the positive half real line, i.e., $E_t^*\left[e^{\lambda \sigma Z_\tau}\right] < 0$ for $\lambda \in [0, \infty)$.*

This assumption adds a restriction on the set of random variables Z_τ that can be used to model the market risk. Property 1.5 implies that the right tail of the distribution of Z_τ has to decay fast enough such that it is exponentially bounded at any rate. On the other hand, I allow the mgf of σZ_τ to be infinite on the negative half real line. This means that the left tail of the distribution of Z_τ can decay slowly and its moments do not have to exist.

Let $m(\lambda) \equiv \log M(\lambda)$ be the cgf of the market risk σZ_τ. It necessarily has the following properties.

Lemma 1. *The cgf $m(\lambda)$ of the market risk σZ_τ is positive, increasing and convex on $[0, \infty)$ with $m(0) = 0$.*

Proof. By the Jensen's inequality and with the zero mean assumption on Z_τ, $E_t^*\left[e^{\lambda\sigma Z_\tau}\right] > e^{\lambda\sigma E_t^*[Z_\tau]}=1$. So $M(\lambda) > 1$ and $m(\lambda) > 0$ for $\lambda > 0$. $m(0) = 0$ by Property 1.1, and it is convex by Property 1.4. Then $m(\lambda)$ being increasing follows. □

Property 1.4 says that the mgf of the market risk σZ_τ increases extremely fast. After taking logarithm, the cgf increases at a slower rate but it is still convex. Since the cgf behaves less wildly, it is the more appropriate object for investigation. The correction term μ depends on the cgf of σZ_τ as following

Lemma 2. *The correction term μ is the negative of the annualized cgf of the market risk σZ_τ evaluated at 1. It is negative.*

Proof. By equation (1.4),

$$\mu = -\frac{1}{\tau}\log M(1) = -\frac{1}{\tau}m(1). \tag{1.7}$$

By Lemma 1, $\mu < 0$ in general. □

It is worth pointing out that the cgf is connected to a dispersion measure of random variables, entropy, recently employed in the finance literature. The entropy of a random variable is defined as $L(X) \equiv \log E[X] - E \log[X]$. Alvarez and Jermann (2005), Bansal and Lehmann (1997), and Backus, Chernov, and Martin (2011) use it to measure the dispersion of pricing kernels. Martin (2016) shows that VIX measures the entropy of simple returns, which is subsumed as a special case in this lemma.

Lemma 3. *Under the risk-neutral measure, the cgf of the market risk σZ_τ evaluated at n is the entropy of the simple return raised to the power n, i.e.,*

$$m(n) = L^*(R_{t,T}^n). \tag{1.8}$$

Proof. Taking logarithm of both sides in equation (1.3), we have

$$m(n) = \log E_t^*[R_{t,T}^n] - n(r - q + \mu)\tau. \tag{1.9}$$

Note that $n(r - q + \mu)\tau = E_t^*[\log R_{t,T}^n]$, so we have the result. □

Example 1. *(Diffusion Process) When Z_t is a diffusion process, σ is chosen to be the volatility of Z_τ. So the correction term μ corresponds to the Itô correction term $-\sigma^2/2$.*

1.2.1 Illustrating Examples

I illustrate the framework laid down so far with two examples. The leading example is a finite moment log stable process for the index price as in Carr and Wu (2003a) where the market risk is stable distributed. It subsumes the diffusion process as a special case. The second example is the jump diffusion process of Merton (1976) for which, although we do not have an explicit expression for the distribution of market risk, we can obtain its cgf analytically. By embedding these two examples in this general framework, it is straightforward to understand what two leading measures of market risk in the literature mean later.

Finite Moment Log Stable Process for the Underlying Price

There are several reasons for choosing this example. Firstly, the stable distribution followed by index log returns within this process is quite general but still analytically tractable in our framework. The family of stable distributions is described by a stability parameter or tail index $\alpha \in (0, 2]$, a skewness parameter $\beta \in [-1, 1]$, a scale parameter and a location parameter.[11] The family of stable distributions subsumes the normal distribution as a special case when $\alpha = 2$. It features fat tails when $\alpha < 2$, which is a stylized fact for equity returns, so that its second moment does not even exist. To guarantee the option prices are finite, Carr and Wu (2003a) propose to use a maximal negative parameter $\beta = -1$ such that the right tail of the distribution of log returns decays sufficiently fast. The tail index governs the decay rates of the left tail of the distribution.[12] Secondly, modeling index returns by a stable distribution might be justified by the generalized central limit theorem[13] which states that the sum of any sequence of independent and identical random variables, a reasonable assumption for index returns driven by random arrival of news at high frequencies, converges to a stable distribution. Thirdly, Carr and Wu (2003a) document empirically that the option pricing with stable distributions delivers superior performance than other more complicated models in the option pricing literature. Finally,

[11]The characteristic function is given by

$$E\exp(itX) = \begin{cases} \exp\left[it\mu - |ct|^{\alpha}(1 - i\beta \text{sign}(t) \tan \frac{\pi\alpha}{2})\right] & \alpha \neq 1, \\ \exp\left[it\mu - |ct|(1 + i\beta \text{sign}(t) \frac{2}{\pi} \log|t|)\right] & \alpha = 1, \end{cases}$$

where c and μ are the scale and location parameters, respectively.

[12]The stability parameter governs the tail decay rate in the following sense: for $\alpha \in (0, 2)$

$$\begin{cases} \lim_{x\to\infty} x^{\alpha} P(X > x) = C_{\alpha} \frac{1+\beta}{2} c^{\alpha}, \\ \lim_{x\to\infty} x^{\alpha} P(X < -x) = C_{\alpha} \frac{1-\beta}{2} c^{\alpha}, \end{cases}$$

where C_{α} is a constant depending only on α (Samorodnitsky and Taqqu, 1994, Property 1.2.15). When $\beta = -1$ as in Carr and Wu (2003a), the right tail decays faster than a power law. Indeed, if $\alpha < 1$, the support of the distribution is only $(-\infty, \mu]$. But the relevant case for this paper is $\alpha \in (1, 2)$ so the distribution still has a full support on the real line.

[13]It does not require the finite variance assumption as in the classical central limit theorem where the limit is a normal distribution.

this example also motivates the tail index I will propose later.

Suppose the equity index price follows a finite moment log stable process under the risk neural measure,

$$dS_t/S_t = rdt + \sigma dL_t^{\alpha,-1}$$

where $dL_t^{\alpha,-1}$ has a stable distribution with tail index $\alpha \in (1,2)$, zero drift, scale of $dt^{1/\alpha}$, and a maximal negative skew parameter $\beta = -1$. $\sigma > 0$ is the scale per unit of time. We can then write the index price at time T in an exponential form

$$S_T = S_t e^{(r-q)\tau + \mu\tau + \sigma L_\tau^{\alpha,-1}} \tag{1.10}$$

where $\sigma L_\tau^{\alpha,-1}$ represents the market risk which is also stable distributed with a tail index α, due to the stability property of the distribution, and a skewness -1. The scale instead of volatility of the log return distribution, σ, serves as the normalizing constant in our framework. The correction term μ is added such that

$$E_t\left[e^{\mu\tau + \sigma L_\tau^{\alpha,-1}}\right] = 1.$$

By the property of stable distribution (Samorodnitsky and Taqqu, 1994, Property 1.2.4, Proposition 1.2.12)

$$E_t\left[e^{\sigma L_\tau^{\alpha,-1}}\right] = e^{-\tau\sigma^\alpha \sec(\pi\alpha/2)}.$$

So the cgf of the market risk is

$$m(\lambda) = -\tau(\lambda\sigma)^\alpha \sec(\pi\alpha/2). \tag{1.11}$$

Hence, we also have an explicit expression for the correction term

$$\mu = \sigma^\alpha \sec \frac{\pi\alpha}{2}, \tag{1.12}$$

which depends on both the normalizing constant σ and the tail index α. Note that $\alpha \in (1, 2)$, so μ is negative.

Remark 1. *When $\alpha = 2$, the stable process turns into a diffusion process. But due to the parameterization of the stable distribution adopted here, the scale parameter of the stable distribution is not the standard deviation of the normal distribution in this special case. Indeed, the latter is the former multiplied by $\sqrt{2}$. So equivalently, we can replace $L_\tau^{\alpha,-1}$ by a Brownian motion $\sqrt{2}B_\tau$. By the Itô's lemma, we would have the correction term $\mu = -\sigma^2$, where σ is the scale parameter.*

Jump Diffusion Process for the Underlying Price

When the underlying price follows a jump diffusion process (Merton, 1976)

$$dS_t/S_{t^-} = (r - q)dt + \delta dW_t + (e^x - 1)dN(t) - \phi E^*[e^x - 1]dt$$

where $N(t)$ is a Poisson process with intensity ϕ and x is the i.i.d. jump size, the final price S_T can be expressed as

$$S_T = S_t \exp\left[(r - q - \frac{1}{2}\delta^2 - k)\tau + \underbrace{\delta W_\tau + \sum_{n=1}^{N(\tau)} x - \phi E^*[x]\tau}_{\sigma Z_\tau}\right] \tag{1.13}$$

where $k = \phi E^*[e^x - 1 - x]$ and the term labeled as σZ_τ represents the mean zero market risk. Clearly the correction term is

$$\mu = -\frac{1}{2}\delta^2 - k. \tag{1.14}$$

Although we do not have an explicit expression for the distribution of market risk, its cgf can be obtained as

$$m(\lambda) = \tau\left[\frac{1}{2}(\lambda\delta)^2 + \phi E^*[e^{\lambda x} - 1 - \lambda x]\right], \tag{1.15}$$

which is the sum of two terms coming from the diffusive and jump components, respectively.

1.2.2 Model-free Cumulant Generating Function of the Market Risk

Having established the cgf of the market risk as the object to study, I now show that the whole curve of the cgf can be extracted from cross sections of market index option prices in a model-free manner. This gives us the flexibility to study any feature of the cgf and, therefore, build alternative meaningful and informative measures of market risk which will be the focus of next section.

The intuition behind this model-free extraction is that the power functions can be replicated by European option payoff functions.[14] So the moments of S_T or simple return $R_{t,T}$ can be replicated by option prices. Because of the connection between these moments and the mgf of the market risk as shown in equation (1.3), we are able to extract the mgf and cgf from option prices as well. To extract the correction term μ, I apply a similar replication strategy for the log function of the final price as the construction of the CBOE VIX.

[14]Namely, $x^n = \bar{x}^n + n\bar{x}^{n-1}(x-\bar{x}) + n(n-1)\left\{\int_0^{\bar{x}} k^{n-2}\max(k-x,0)dk + \int_{\bar{x}}^{\infty} k^{n-2}\max(x-k,0)dk\right\}$ for any $\bar{x} > 0$.

Proposition 1. *The cgf of the market risk for any maturity τ can be extracted from the option prices as*

$$m(n) = \log\left[e^{n(r-q)\tau} + \frac{n(n-1)e^{r\tau}}{S_t^n}\left\{\int_0^{F_{t,T}} K^{n-2} put_{t,T}(K)dK + \int_{F_{t,T}}^{\infty} K^{n-2} call_{t,T}(K)dK\right\}\right] - n(r-q+\mu)\tau, \tag{1.16}$$

where $F_{t,T}$ is the futures price of the underlying and $call_{t,T}(K)$ and $put_{t,T}(K)$ are the prices of European call and put options with a strike K, respectively, at time t for an expiry date T. Note that μ itself is replicated by European option prices as

$$\mu = -\frac{e^{r\tau}}{\tau}\left\{\int_0^{F_{t,T}} \frac{1}{K^2} put_{t,T}(K)dK + \int_{F_{t,T}}^{\infty} \frac{1}{K^2} call_{t,T}(K)dK\right\}. \tag{1.17}$$

Proof. See the Appendix. □

We have already obtained the model-free extractions of the cgf of the market risk σZ_τ and its correction term μ. There is one parameter remaining to be extracted from the option prices, the normalizing constant σ. If the second moment of market risk or log return exists and σ is chosen to be the standard deviation, then $\sigma^2 = Var^*[\log(S_T/S_t)]$. The estimator developed by Bakshi, Kapadia, and Madan (2003) can be used to obtain σ^2. If the second moment of market risk or log return does not exist, we need alternative estimators for the corresponding notions of σ, which is not the focus of this paper.

Now with the general framework of market risk built and implemented, it is interesting to bring into it the measures of market risk developed in the literature. I examine two of them, the CBOE VIX and the SVIX proposed by Martin (2016).

1.2.3 What Does VIX Measure?

We first look at the CBOE VIX which is often referred to as the "fear index" in the market. The motivation of VIX comes from a variance swap contract which is an agreement at time t on the exchange of the sum of squared log returns

$$\left(\log\frac{S_{t+\Delta}}{S_t}\right)^2 + \left(\log\frac{S_{t+2\Delta}}{S_{t+\Delta}}\right)^2 + \cdots + \left(\log\frac{S_T}{S_{T-\Delta}}\right)^2$$

for a fixed strike price V. Based on Neuberger (1994), under a relatively strong assumption of a diffusion process $d\log S_t = r - q - \frac{1}{2}\sigma_t^2 + \sigma_t dZ_t$, in the continuous time limit as Δ shrinks to zero, the strike price can be heuristically expressed as

$$\begin{aligned}
V &= E_t^*\left[\int_t^T \sigma_t^2 dt\right] \\
&= 2E^*\left[\int_t^T \frac{1}{S_t}dS_t - \int_t^T d\log S_t\right] \\
&= 2(r-q)\tau - 2E_t^*\log\frac{S_T}{S_t} \\
&= 2e^{r\tau}\left\{\int_0^{F_{t,T}} \frac{1}{K^2}\mathrm{put}_{t,T}(K)dK + \int_{F_{t,T}}^{\infty}\frac{1}{K^2}\mathrm{call}_{t,T}(K)dK\right\}.
\end{aligned} \tag{1.18}$$

The last step follows by plugging in the replication of the log payoff in equation (3) in the proof of Proposition 1. Note in particular that this step is true in general independent of the assumption of a diffusion process.

Based on this result, the CBOE VIX is proposed (see Demeterfi et al., 1999) as the annualized strike price of the variance swap on the S&P 500 index

$$\mathrm{VIX}_t^2 = \frac{2e^{r\tau}}{\tau}\left\{\int_0^{F_{t,T}} \frac{1}{K^2}\mathrm{put}_{t,T}(K)dK + \int_{F_{t,T}}^{\infty}\frac{1}{K^2}\mathrm{call}_{t,T}(K)dK\right\}. \tag{1.19}$$

Although VIX is designed to be a measure of risk-neutral expectation of future realized variance as in equation (1.18) when initially launched, I will not take this for granted since a diffusion process for the underlying is assumed in the construction of VIX formula. From now on I will treat equation (1.19) as the definition of VIX and see what it really measures in more general settings.

Combining the definition of VIX in equation (1.19) and the replication of μ in equation (1.17), we can immediately see that VIX measures the correction term of the market risk, i.e., $\text{VIX}_t^2 = -2\mu$. As Martin (2016) points out, the last two lines in equation (1.18) also imply that VIX as defined by equation (1.19) is a measure of the entropy of simple return of the S&P 500 index,[15] i.e., $\text{VIX}_t^2 = \frac{2}{\tau} L^*(R_{t,T})$.

In our general framework of market risk, these results can be summarized as

Proposition 2. *Up to a constant scale, the CBOE VIX as defined by (1.19) is equivalent to the following measures*

1. *the cgf of the market risk σZ_τ at 1;*

2. *the correction term of the market risk;*

3. *the entropy of the market simple return.*

To be specific,

$$VIX_t^2 = \frac{2}{\tau} m(1) = -2\mu = \frac{2}{\tau} L^*(R_{t,T}). \tag{1.20}$$

Proof. With the discussions above, the results follow by taking into account either Lemma 3 for the case $n = 1$ or Lemma 2. □

It should be emphasized that these results do not rely on any assumption on the process followed by the index price. As depicted in Figure 1.1, half of the (unannualized squared) VIX measures exactly the height AB of the convex cgf of the market risk at 1. As the market

[15] Note that $\log E_t^*(S_T/S_t) = (r - q)\tau$. So the second last line in equation (1.18) equals $2L^*(R_{t,T})$.

risk evolves over time, the curve of its cgf shifts upwards or downwards, in particular, at 1 on the horizontal axis, and VIX fluctuates as well. Therefore, VIX reveals only a piece of information about the market risk that is contained locally by $m(1)$. We might potentially extract more information about the market risk by looking at the global shape of the curve.

Perhaps the simplest interpretation of VIX is the correction term μ. The correction term essentially gauges the gap, on a log scale, in the Jensen's inequality when we take expectation of the convex exponential function of a mean zero market risk. When the market risk is normal, VIX would reveal its standard deviation. Otherwise, VIX would contain information about the higher moments, if any, of market risk (Martin, 2013, Result 2). But the correction term interpretation is more general and it holds even if these moments do not exist. Using this interpretation of VIX, it is straightforward to obtain what VIX measures when the underlying price follows alternative processes in the examples below.

Example 2. *(Diffusion Process Continued) In the case of a diffusion process, since the correction term is $\mu = -\sigma^2/2$, we have $VIX_t^2 = \sigma^2$, where σ is the volatility.*

Example 3. *(Finite Moment Log Stable Process Continued) For this class of processes, we have the correction term in equation (1.12). By Proposition 2, we have*

$$VIX_t^2 = -2\sigma^\alpha \sec \frac{\pi\alpha}{2}. \tag{1.21}$$

It can be seen that when the underlying price is logstable VIX combines the tail risk α and the scale risk σ. It is not clear why this specific functional form of combination is meaningful. So it is worth measuring the two types of risk separately.

Example 4. *(Jump Diffusion Process Continued) When the underlying price follows a jump diffusion process, we have the correction term in equation (1.14) and, hence, VIX is* *given by*

$$VIX_t^2 = \delta^2 + 2\phi E^*[e^x - 1 - x].$$

Therefore, for a jump diffusion process, VIX is a biased estimator for the return variance contributed by diffusive innovations. The error term depends on the second and higher order moments of the jump size distribution as well as the jump intensity.

1.2.4 What Does SVIX Measure?

Due to the unlimited risk exposure to extreme price movements, the variance swap market experienced turmoil during the 2008 financial crisis. Martin (2016) proposes a novel simple variance swap contract which is an agreement on exchange of the sum of squared simple returns instead of log returns

$$\left(\frac{S_{t+\Delta} - S_t}{F_{t,t}}\right)^2 + \left(\frac{S_{t+2\Delta} - S_{t+\Delta}}{F_{t,t+\Delta}}\right)^2 + \cdots + \left(\frac{S_T - S_{T-\Delta}}{F_{t,T-\Delta}}\right)^2$$

in which $F_{t,s}$ is the forward price at time t of the underlying asset to time s. Under some mild assumptions and, in particular, allowing for jumps in the underlying process, Martin (2016) shows that in the continuous time limit as Δ shrinks to 0, the strike price of the simple variance swap contract can be replicated by options

$$V = \frac{2e^{r\tau}}{F_{t,T}^2}\left\{\int_0^{F_{t,T}} \mathrm{put}_{t,T}(K)dK + \int_{F_{t,T}}^{\infty} \mathrm{call}_{t,T}(K)dK\right\}.$$

Analogous to VIX, Martin (2016) proposes an index, SVIX, as the annualized strike of a simple variance swap for the S&P 500 index,

$$\mathrm{SVIX}_t^2 = \frac{2e^{r\tau}}{\tau \cdot F_{t,T}^2}\left\{\int_0^{F_{t,T}} \mathrm{put}_{t,T}(K)dK + \int_{F_{t,T}}^{\infty} \mathrm{call}_{t,T}(K)dK\right\}. \tag{1.22}$$

From now on equation (1.22) is treated as the definition of SVIX. Martin (2016) further shows that it measures the risk-neutral variance of simple (excess) return of the S&P 500 index

$$\mathrm{SVIX}_t^2 = \frac{1}{\tau}\mathrm{var}_t^*(R_{t,T}/\bar{R}), \tag{1.23}$$

where $\bar{R} \equiv e^{(r-q)\tau}$.

Within our general framework of market risk, we have the following interpretation of SVIX.

Proposition 3. *The SVIX measures the change in slope or the convexity of the cgf of the market risk σZ_τ over the interval* $[0, 2]$,

$$\log(1 + \tau \cdot SVIX_t^2) = m(2) - 2m(1) + m(0). \tag{1.24}$$

Proof. Note that $\mathrm{var}_t^*(R_{t,T}/\bar{R}) = E_t^*[R_{t,T}/\bar{R}]^2 - \big(E_t^*[R_{t,T}/\bar{R}]\big)^2$ and by the specification of price in equation (1.1) and the correction condition in equation (1.2)

$$E_t^*[R_{t,T}/\bar{R}] = 1,$$

and

$$E_t^*[R_{t,T}/\bar{R}]^2 = e^{2\mu\tau} E_t^*\left[e^{2\sigma Z_\tau}\right] = e^{2\mu\tau} M(2).$$

Then by the expression of the correction term in equation (1.7)

$$e^{2\mu\tau} = \frac{1}{M^2(1)}.$$

So

$$\mathrm{SVIX}_t^2 = \frac{1}{\tau}\left(\frac{M(2)}{M^2(1)} - 1\right).$$

Taking logarithm of an affine transformation of SVIX and with $m(0) = 0$, we obtain the result. □

As shown graphically in Figure 1.2, $m(1) - m(0)$ represents the slope of OB or BD. $m(2) - m(1)$ represents the slope of BE. So $m(2) - 2m(1) + m(0)$ represents the change in slope when we move along the curve of the mgf from $\lambda = 0$ to $\lambda = 2$. In this sense, SVIX measures the convexity of the cfg of the market risk over the interval $[0, 2]$. Alternatively, the difference in the slope between BD and BE or SVIX can be represented by DE in Figure 1.2. Interestingly, in the same Figure VIX is represented by CD. So the sum of VIX and SVIX measures the height, CE, of the cgf of the market risk at 2,

$$\tau \cdot \mathrm{VIX}_t^2 + \log(1 + \tau \cdot \mathrm{SVIX}_t^2) = m(2). \tag{1.25}$$

These observations illustrate the benefit of investigating the cgf of the market risk. It provides a unifying framework for intuitively interpreting these two leading forward-looking measures of market risk in the literature. Moreover, it offers the chance to build the connection between these two measures. Note that to compare VIX and SVIX in a meaningful way, we should work with $\tau \cdot \mathrm{VIX}_t^2$ and $\log(1 + \tau \cdot \mathrm{SVIX}_t^2)$ instead of directly with VIX_t and SVIX_t.[16] In the following example of finite moment log stable process we discussed previously, we can see analytically the relation between VIX and SVIX.

Example 5. *(Finite Moment Log Stable Process Continued) Based on the cgf of the market risk in equation (1.11) and by Proposition 3, the SVIX under a finite moment log stable process is given by*

$$\log(1 + \tau \cdot SVIX_t^2) = \tau(2 - 2^\alpha)\sigma^\alpha \sec(\pi\alpha/2). \tag{1.26}$$

[16] Although Martin (2016) works primarily with VIX_t and SVIX_t, he does find expressions for $\log(1+\tau \cdot \mathrm{SVIX}_t^2)$ and VIX_t^2 in Barro's consumption-based model with an Epstein-Zin representative agent and i.i.d. consumption growth. See his footnote 25 in Appendix.

Note that the VIX for a finite moment log stable process is given by equation (1.21). So in this example VIX and SVIX are related in the following way

$$\log(1 + \tau \cdot SVIX_t^2) = (2^{\alpha-1} - 1) \cdot \tau \cdot VIX_t^2. \tag{1.27}$$

In particular, when $\alpha = 2$, i.e., the price S_T is lognormal, we have

$$\log(1 + \tau \cdot SVIX_t^2) = \tau \cdot VIX_t^2. \tag{1.28}$$

This is the Result 4 obtained in Martin (2016) under the assumption that the stochastic discount factor (SDF) and S_T are conditionally jointly lognormal. Here we obtain the same result even without any assumption on the SDF.

When the market risk features fat left tails, i.e., $\alpha \in (1, 2)$, VIX is higher, $\tau \cdot VIX_t^2 > \log(1 + \tau \cdot SVIX_t^2)$.

Example 6. *(Jump Diffusion Process Continued) Based on the cgf of the market risk in equation (1.15) and by Proposition 3, the SVIX under a jump diffusion process can be obtained as*

$$\log(1 + \tau \cdot SVIX_t^2) = \tau\big(\delta^2 + \phi E^*[e^x - 1]^2\big). \tag{1.29}$$

As for VIX, the SVIX depends on both diffusive risk and jump risk. The difference between them is

$$\tau \cdot VIX_t^2 - \log(1 + \tau \cdot SVIX_t^2) = \tau\phi E^*\big[2(e^x - 1 - x) - (e^x - 1)^2\big] = \tau\phi E^*\left[-\frac{2}{3}x^3 + o(x^3)\right],$$

which depends mainly on the third moment of the jump size distribution. In particular, if the magnitude of negative jumps is larger than that of positive jumps, which is consistent with the empirical evidence, VIX is higher than SVIX, $\tau \cdot VIX_t^2 > \log(1 + \tau \cdot SVIX_t^2)$.

1.3 A Tail Index of Market Risk

As discussed above, both VIX and SVIX have their limitations as measures of market risk. They are motivated as the strike price for a swap on the second moment of either log or simple returns of the S&P 500 index. But the second moment is a rough description of the return distribution. It does not identify distinct features of the distribution separately such as the tails and the scale, which may all contribute to the second moment. As a result, VIX and SVIX also incorporate different types of market risk in their own ways. In the context of a finite moment log stable process for the underlying price, we have seen both VIX and SVIX mix the ordinary risk captured by the scale parameter and the extreme risk captured by the stability parameter. In the context of a jump diffusion process for the underlying price, both VIX and SVIX add up the risks from the diffusive and jump components. VIX is a bit more problematic because it theoretically does not necessarily measure the risk-neutral expectation of future realized variance without the assumption of a diffusion process for the underlying price.

Within the framework of the cgf of the market risk proposed in this paper, VIX reveals the information contained locally by the cgf at 1 while SVIX focuses on its convexity over the region $[0, 2]$. It seems that we are left with enough room for developing alternative measures of market risk from the shape of the cgf. Ideally, these measures have economic interpretations by targeting distinct features of market risk. In this section, I propose a tail index measure of the market risk, which is called “TIX".

Disaster risk has been documented in the literature to have significant implications for risk premia. Gabaix (2012), Wachter (2013) and Gourio (2008, 2012) show that variation in investors' concerns regarding rare disasters drives the time-varying equity risk premium. Bollerslev and Todorov (2011) find that fears of rare events account for roughly two-thirds of the total expected excess return and the left tail risks account for 88.4% of the total variance risk premium. Although varieties of indices have been developed in the literature to capture the disaster risk, an option-implied model-free one with the flavor of VIX and SVIX is absent. The TIX proposed here can fill this gap.

Note that the equation (1.27) in the example of finite moment log stable process can be rearranged to

$$\frac{\log(1+\tau\cdot\mathrm{SVIX}_t^2)+\tau\cdot\mathrm{VIX}_t^2}{\tau\cdot\mathrm{VIX}_t^2/2}=2^{\alpha}. \tag{1.30}$$

The right hand side now depends simply on the stability parameter capturing the tail risk. On the left hand side, by equation (1.20) the denominator is simply $m(1)$ while by equation (1.24) the nominator is simply $m(2)$. Since the cgf of the market risk $m(\lambda)$ can be extracted from the option prices, we now have a chance to obtain α from them as well. This insight motivates me to define a tail index in the following way.

Definition 1. *The tail index, or "TIX", is defined as logarithm to base 2 of the ratio of the cgf of the market risk σZ_τ evaluated at 2 over that evaluated at 1, namely*

$$TIX_t=\log_2\left(\frac{m(2)}{m(1)}\right). \tag{1.31}$$

Given the equivalence between the cgf of the market risk and the entropy of the simple returns raised to appropriate powers in Lemma 3, this definition can also be expressed as $\mathrm{TIX}_t=\log_2\big(L^*(R_{t,T}^2)/L^*(R_{t,T})\big)$.

In general, with the cgf of the market risk extracted from the option prices, TIX can be computed directly using the option prices in a model-free manner. Since the CBOE VIX is publicly listed, if the investors have SVIX in hand, the most convenient way of computing TIX is

$$\mathrm{TIX}_t=\log_2\left(\frac{\log(1+\tau\cdot\mathrm{SVIX}_t^2)+\tau\cdot\mathrm{VIX}_t^2}{\tau\cdot\mathrm{VIX}_t^2/2}\right).$$

An immediate observation is that TIX equals the stability parameter $\alpha\in(1,2)$ when the price S_T is logstable and, in particular, equals 2 when it is lognormal. For the finite moment log stable process, α governs only the left tail decay rate. So this paper mainly focuses on the left tail of the distribution of market risk corresponding to the large downside risk. This is consistent with the evidence documented in the literature that the downside jumps play a dominant role in asset pricing than the upside jumps; see, for example, Bakshi, Carr, and Wu (2008) and Du and Kapadia (2012).

Graphically, TIX is represented in Figure 1.2 by the ratio of CE over AB. In some sense, it also measures the convexity of the cgf of the market risk as SVIX. But they are quite different in comparing the heights of the cgf at 1 and 2 with the former in relative terms while the latter in absolute terms. For two market conditions with the same level of SVIX, the TIX can be very different. This is because TIX reflects the tail risk while SVIX reflects the second moment of simple returns. Similarly, TIX can be different for two market conditions with the same level of VIX.

1.4 Data Description

The general framework of market risk proposed in this paper is empirically implemented for the S&P 500 index in the U.S. equity market. The S&P 500 index option data for 1996:01 to 2015:08 used to extract the cgf's of the market risk is from the OptionMetrics. The zero couple yield data provided in the OptionMetrics is also used as the risk-free rate for extracting the cgf's. The dividend yield is backed out from the at-the-money option prices using put-call parity. I follow the CBOE's practice in constructing VIX as closely as possible when recovering the cgf's and computing TIX.[17] The detailed procedures are described in Appendix B. To clean the option data, I first apply the filters used by the CBOE. I exclude options that have a zero bid price. Then for call options as we move

[17]See the white paper for VIX on the CBOE's website.

to successively higher strike prices, once two consecutive call options are found to have zero bid prices, no calls with higher strikes are considered. This rule is applied for put options similarly. Secondly, I delete all replicated entries as in Martin (2016). Since only the highest bid and the lowest ask are available for each option in OptionMetrics, the mid of bid and ask is used as the option price as in the literature.

To test the market return predictability of the predictor to be constructed in this paper, 15 other popular predictor variables are considered for comparison. 14 of them are studied in Goyal and Welch (2008) and can be obtained from Amit Goyal's website.[18] The data on the short interest variable SII proposed in Rapach, Ringgenberg, and Zhou (2016), which is arguably the strongest known predictor of aggregate stock returns, is available from David Rapach's website.[19]

1.4.1 Empirical Cumulant Generating Functions

In this paper, I focus on the market risk with a horizon of 30 calendar days. This is consistent with the horizon targeted by the CBOE VIX and is also in align with the attempt to examine the return predictability at the monthly horizon later. At the end of each trading day, two cross sections of index option prices, the near-term and the next-term, are obtained with their maturities being the closest to and also bracketing the target of 30 calendar days. Proposition 1 is then applied for the two terms respectively to get their cgf's. Finally, a linear interpolation is employed to obtain the cgf for the market risk with a horizon of 30 calendar days.[20] To obtain a continuous curve, I extract the cgf's for every 0.2 along the horizontal axis starting from the origin. Figure 1.3 displays the monthly cgf's from Jan 1996 to August 2015 with each subfigure containing the twelve curves for the year labeled. To save space, the support of these cgf's is restricted to the interval $[0, 4]$. To

[18] http://www.hec.unil.ch/agoyal/.

[19] http://sites.slu.edu/rapachde/home/research.

[20] Occasionally, we need to outerpolate the 30 calendar days with two next-terms.

visualize the evolution of these cgf's, the same vertical scale is used across these subfigures.

There are several immediate observations from this figure. First, these curves are all positive, increasing and convex which is consistent with the theoretical properties of the cgf of the market risk established in Lemma 1. Second, there are salient time variations in the shape of these curves. The cgf spikes during the periods of the 1998 Russian default and LTCM collapse, the 2002 burst of dot-com"bubble", the 2008-2009 financial crisis, and the 2011 Euro sovereign debt crisis.

1.4.2 TIX

Given the time series of the cgf of the 30-day market risk, we can easily obtain the time series of TIX by computing the logarithm (to base 2) of the ratio of the height of each curve at 2 over that at 1. Panel A in Figure 1.4 plots the daily time series of TIX from Jan 4, 1996 to August 31, 2015. The TIX has been below 2 all the time during this period indicating fatter tails of the distributions of market risk than a normal distribution. Note that a lower TIX implies a slower tail decay rate of the distribution of market risk and, hence, a higher extreme risk.

Within this sample period, the first significant drop in TIX followed the mini crash in the global stock market on October 27, 1997 caused by the asian financial crisis. The second notable decline in TIX in the second half of 1998 was due to the Russian default and the LTCM collapse. The third and the largest slump in TIX was triggered by the bailout of Fannie and Freddie on September 7, 2008 as well as the takeover, bankruptcy, and bailout of Merrill Lynch, Lehman Brothers, and AIG on September 14, 15, and 16, 2008, respectively. The fourth bottom in TIX was recorded on May 7, 2010 immediately following the "Flash Crash" in the stock market on May 6, 2010 amid the concerns about the euro sovereign debt crisis. The fifth remarkable decline in TIX happened in the summer of 2011 due to the U.S. debt ceiling crisis and later that year as eurozone fears intensified. Another plunge in TIX was witnessed at the end of sample when the equity markets fell sharply in late August, 2015 propelled by fears that China's economic slowdown was turning out to be worse than feared. For all these episodes, the TIX fell below 1.9 or even 1.85.

There are also periods of milder declines in TIX. It fell to around 1.92 after September 11 attacks in 2011, after the Internet bubble bursting in 2002, in October, 2014 on concerns about global growth and worries about the spread of the Ebola virus, and at the end of the year due to mounting concerns about the currency crisis in Russia, the turmoil in oil markets and Greek elections.

1.4.3 Innovations in TIX

To recover the information contained in TIX, I obtain its innovations by taking the log difference of the time series of TIX. To avoid the noise in data at higher frequencies, I construct the monthly innovations in TIX. To be specific, I take the log difference of TIX at the end of two consecutive months. Since TIX is forward-looking with a 30 calendar day horizon, each innovation then tells us how the market updates its beliefs about the following 30-day extreme market risk. The way the market updates its beliefs could potentially reveal the fresh information flows in the market and, therefore, give rise to the return predictability. As TIX increases, the tail of the distribution of market risk gets thinner. So it indicates that the market adjusted downwards the expected extreme risk and the market conditions are improving. Therefore, I conjecture that the innovation in TIX is positively related to the future market returns. Figure 1.5 plots the non-overlapping monthly time series of innovations in TIX, $d\log \text{TIX}$. The series has a mean of -0.02%, a median of 0.05%, a standard deviation of 0.73%, a skewness of -1.22, and a kurtosis of 7.57.

It can be seen that there is an asymmetry in the innovations in the sense that negative innovations of large magnitude are more frequent than positive ones. The largest negative shocks occurred in October 1997 (-2.35%), August 1998 (-2.78%), September and October 2008 (-1.91% and -3.43%), and August 2015 (-3.64%). The largest positive shocks were recorded in November 1998 (2.19%) and November 2008 (1.59%).

To relate the information content of innovation in TIX to the large literature on market return predictability, I compare its predictive ability to that of 14 monthly predictor variables from Goyal and Welch (2008), which constitute a set of popular predictors in the literature, and that of the short interest variable SII from Rapach, Ringgenberg, and Zhou (2016), which is arguably the strongest predictor so far. Specifically, I include the following predictors:

1. Log dividend-price ratio (DP): log of a 12-month moving sum of dividends paid on the S&P 500 index minus the log of stock prices (S&P 500 index).

2. Log dividend yield (DY): log of a 12-month moving sum of dividends minus the log of lagged stock prices.

3. Log earnings-price ratio (EP): log of a 12-month moving sum of earnings on the S&P 500 index minus the log of stock prices.

4. Log dividend-payout ratio (DE): log of a 12-month moving sum of dividends minus the log of a 12-month moving sum of earnings.

5. Stock Variance (SVAR): computed as sum of squared daily returns on the S&P 500 index.

6. Book-to-market ratio (BM): book-to-market value ratio for the Dow Jones Industrial Average.

7. Net equity expansion (NTIS): ratio of a 12-month moving sum of net equity issues by NYSE-listed stocks to the total end-of-year market capitalization of NYSE stocks.

8. Treasury bill rate (TBL): interest rate on a three-month Treasury bill (secondary market).

9. Long-term yield (LTY): long-term government bond yield.

10. Long-term return (LTR): return on long-term government bonds.

11. Term spread (TMS): long-term yield minus the Treasury bill rate.

12. Default yield spread (DFY): difference between Moody's BAA- and AAA-rated corporate bond yields.

13. Default return spread (DFR): long-term corporate bond return minus the long-term government bond return.

14. Inflation (INFL): calculated from the Consumer Price Index (CPI) for all urban consumers.

15. SII: (standardized) detrended log of equal-weighted short interest.

Following the practice in the literature on predicting market returns, I focus on predicting the excess return on a value-weighted market portfolio. I measure the market excess return as the log return on the S&P 500 index minus the log return on a one-month Treasury bill.

1.4.4 Sample Properties

Table 3.1 shows the summary statistics for $d \log \text{TIX}$, 14 popular predictor variables from Goyal and Welch (2008), and the short interest variable SII from Rapach, Ringgenberg, and Zhou (2016) over the 1996:02 to 2014:12 sample period.[21]

Table 3.2 displays correlation coefficients for $d\log\text{TIX}$ and the other 15 popular predictor variables in the literature. It can be seen that many of the popular predictors from the literature exhibit strong correlations with each other. The correlation between short interest variable SII and NTIS is -0.51 in this sample. However, the newly proposed predictor $d\log\text{TIX}$ appears largely unrelated to these predictors. The strongest correlations (in magnitude) between $d\log\text{TIX}$ and these popular predictors occur with SVAR and DFR, which have correlations of only -0.25 and 0.25, respectively. The correlation between $d\log\text{TIX}$ and the short interest variable SII is merely -0.01. In other words, the predictor $d\log\text{TIX}$ seems to contain substantially different information from many of the stock return predictors used in the existing literature.

1.5 Predictability of Market Returns

In this section, I conduct both in-sample and out-of-sample tests of the return predictability of the innovation in TIX. During the sample period 1996:02-2014:12, the annualized equity risk premium is 5.78%, the volatility is 15.57%, and the Sharpe ratio is 0.37.

1.5.1 In-sample Test

To examine the return predictability of innovation in TIX and to compare it with other popular predictors in the literature, I first run the following in-sample predictive regression

[21] The data on the short interest variable SII in Rapach, Ringgenberg, and Zhou (2016) is available until 2014:12 on David Rapach's website. Since it is arguably the best predictor in the literature so far, I want to mainly compare the predictor variable $d\log\text{TIX}$ with SII. So I run the predictive regression using the sample until 2014:12 although the data for $d\log\text{TIX}$ is available until 2015:08 and the data on other variables in Goyal and Welch (2008) is available until 2015:12.

for the sample from 1996:02 to 2014:12

$$r_{t+1} = \alpha + \beta x_t + \epsilon_{t+1}, \tag{1.32}$$

where r_{t+1} is the market excess return in month $t+1$, x_t is the value of a particular predictor in month t, and ϵ_{t+1} is the residual.

To have the comparison of coefficient estimates meaningful, I standardize each predictor to have a standard deviation of one. I also take the negative of SVAR, TBL, LTY, DFY, and SII before running the regressions for these predictors so that their coefficient estimates are of a positive sign as the other predictors. I use a heteroskedasticity- and autocorrelation-robust t-statistic and compute the a wild bootstrapped p value to test the null hypothesis $\beta = 0$ against the alternative $\beta > 0$ as in Rapach, Ringgenberg, and Zhou (2016). For this sample period, after taking into account the lags, I have 227 observations for estimating equation (3.1).

Table 1.3 displays the predictive regression results. The second column shows the coefficient estimates $\hat{\beta}$ for predictors. In brackets under the coefficient estimates are their t-statistics as well as their significant levels. The third column presents the in-sample R^2 statistics of the OLS regressions. The new predictor $d \log \text{TIX}$ outperforms the others in all dimensions. First, its predicting coefficient estimate is the largest, meaning that the innovation in TIX has the largest impact on the predicted market returns. For a one-standard-deviation increase in the innovation in TIX, the predicted monthly market return decreases by 0.83%. It is followed by SVAR with a coefficient estimate of 0.72, DY with a coefficient estimate of 0.65, and SII with a coefficient estimate of 0.64. Second, its t-statistic is the highest with a significance level of 5%. Finally, its R^2 statistic, 3.33%, is much bigger than the other predictors which achieve at most an R^2 statistic of 2.59% by SVAR. The short interest variable SII has an R^2 statistic of 2.03%.

1.5.2 Out-of-sample Test

To check the robustness of the in-sample results and exclude the possibility of overfitting by the innovation in TIX, I look at the out-of-sample predictability of $d\log\text{TIX}$ as well as other predictors. To be specific, at the end of month t, I predict the market return next month $t+1$ by

$$\hat{r}_{t+1} = \hat{\alpha}_t + \hat{\beta}_t x_t \tag{1.33}$$

where $\hat{\alpha}_t$ and $\hat{\beta}_t$ are OLS estimates of α and β in equation (3.1), respectively, based on the data from the beginning of the sample to month t. I use the sample period 1996:02 to 2008:07 for the initial in-sample estimation of the predictive regression. The sample period 2008:08 to 2015:08 is used for out-of-sample performance evaluation. In particular, the mean forecast, the average excess return from the beginning of the sample to month t, serves as the benchmark. This forecast corresponds to the case of $\beta = 0$ in the predictive regression so that it assumes no predictability of market returns. The out-of-sample R^2 statistic, as defined in Campbell and Thompson (2008) as the proportional reduction in the mean squared forecast error (MSFE) of each predicting variable compared to the benchmark of mean forecast, is employed as a measure of out-of-sample predictability. To test if the out-of-sample R^2 statistic of a particular predictor variable is significant, I follow Rapach, Ringgenberg, and Zhou (2016) to use the Clark and West (2007) statistic to test the null hypothesis that the MSFE of mean forecast is less than or equal to that of the predictive forecast against the alternative that the MSFE of mean forecast is greater than that of the predictive forecast.

The out-of-sample R^2 statistics for the innovation in TIX as well as other popular predictor variables are shown in Table 1.4. It can be seen that the R^2 statistics are negative for all the 14 variables from Goyal and Welch (2008), meaning they are outperformed by the mean forecast. The short interest variable SII performs slightly better than the mean forecast with an R^2 statistic of 0.75% which is significant at the 10% level. However, the R^2 statistic is 6.15% for the innovation in TIX which is much larger and also more significant at the 5% level. Therefore, the innovation in TIX outperforms the mean forecast as well as other popular predictor variables by a large margin.

1.6 Discussions

1.6.1 A Robustness Check

In this paper, the construction of TIX is motivated by the observation that for a stable market risk $\log_2(m(2)/m(1))$ equals the stable parameter α which governs the tail decay rate. Indeed, in this case we have in general

$$\log_{\lambda_2/\lambda_1} \frac{m(\lambda_2)}{m(\lambda_1)} = \alpha,$$

for any positive $\lambda_1 \neq \lambda_2$. Then if the TIX constructed with $\lambda_1 = 1$ and $\lambda_2 = 2$ captures the market perceptions of extreme risk in the future, so should the TIX constructed with other pairs of $\lambda_1 \neq \lambda_2$. Given that the innovation in TIX constructed so far is able to predict market returns, we should expect the innovation in TIX constructed with alternative pairs of $\lambda_1 \neq \lambda_2$ contains the same information about future market returns. Figure 1.6 exhibits times series of monthly innovations in TIX constructed with six alternative pairs of (λ_1, λ_2): $(1, 2), (1, 3), (1, 4), (2, 3), (2, 4)$, and $(3, 4)$. It can be seen that they follow each other very closely. Indeed, their pairwise correlation coefficients are at least 0.98.

This exercise indicates one aspect in which the stable distribution might provide a good approximation to the distribution of market risk. Therefore, the construction of TIX as a tail index might hold more generally although it is motivated by a stable distributed market risk.

1.6.2 Shape Variables of cgf Curves

TIX has been constructed as (logarithm to base 2 of) the ratio of the height of the cgf of the market risk at 2 over that at 1. The innovation in TIX has been shown to have strong predictive power of market returns. Since we essentially have a time series of functionals, it is reasonable to investigate the information content of alternative measures of the shape of cgf's. This is analogous to the attempts to recover information from implied volatility smirk or skew (Xing, Zhang, and Zhao, 2010; Ratcliff, 2013). Three sets of plausible shape variables are proposed here. The first set includes the height of the cgf, $m(\lambda)$, at $\lambda = 1, 2, 3, 4$. Note that $m(1)$ contains the same information as VIX^2. The second one includes the convexity of the cgf, $m''(\lambda) \equiv m(\lambda+1) - 2m(\lambda) + m(\lambda-1)$, at $\lambda = 1, 2, 3$. Note that $m''(1)$ contains the same information as $\log(1 + \tau \cdot \text{SVIX}^2)$. The last set includes the fourth order derivative of the cgf, $m^{(4)}(\lambda) \equiv m''(\lambda+1) - 2m''(\lambda) + m''(\lambda-1)$, at $\lambda = 2$.

Panel A in Table 1.5 presents the in-sample OLS regression results for height variables. Both of the coefficient estimates and R^2 statistics are close to zero when these variables are used as predictors directly, as shown in the second and third columns. The results for $m(1)$ are consistent with the evidence for VIX^2 in Bollerslev, Tauchen, and Zhou (2009). When their log differences are used as predictors, the coefficient estimates are still insignificant and the R^2 statistics are less than 0.70% compared to 3.33% for the innovation in TIX, as shown in the fourth and fifth columns. The results are very similar for the convexity variables as shown in Panel B of Table 1.5. Panel A in Table 1.6 exhibits the out-of-sample test results for height variables. None of them are able to beat the mean forecast benchmark although their changes do a better job. The same conclusions hold for the convexity variables as shown in Panel B of Table 1.6.

The exception is the 4th order derivative $m^{(4)}(2)$. As reported in Panel C in both Table 1.5 and Table 1.6, although the derivative itself does not predict the market returns in either in-sample or out-of-sample tests, its change achieves a coefficient of 0.64, significant at the 10% level, and an R^2 statistic of 2.04% in the in-sample regression test which are comparable to the short interest variable SII. It also reduces the MSFE compared to the mean forecast benchmark with an out-of-sample R^2 statistic of 2.10% although it is not significant. Overall, these shape variables are not as effective as the innovation in TIX in revealing the information about market returns in the future.

1.6.3 Predictability of VIX and SVIX

Note that $\tau \cdot \mathrm{VIX}^2$ is equivalent to $2m(1)$ and $\log(1 + \tau \cdot \mathrm{SVIX}^2)$ is equivalent to $m''(1)$. Although no evidence of return predictability was found for $m(1)$ and $m''(1)$ in either in-sample or out-of-sample tests, it may be worth examining the return predictability of VIX and SVIX themselves or their innovations. Table 1.7 reports the results of in-sample predictive regression tests. For both VIX and SVIX themselves, the coefficient estimates are close to zero and insignificant, and the R^2 statistics are literally zero. For the log changes in VIX and SVIX, the coefficient estimates (0.37 and 0.34, respectively) are still insignificant. The R^2 statistics (0.67% and 0.55%, respectively) are much lower than that for innovation in TIX. Both VIX and SVIX are also outperformed by the mean forecast benchmark in the out-of-sample tests as shown in Table 1.8. The results for SVIX^2 (unreported) are similar to these for SVIX and, in particular, their log differences are equivalent. Therefore, both VIX and SVIX are not able to predict market returns. It turns out that the change in TIX which is a function of VIX and SVIX displays superior performance in predicting market returns.

Martin (2016) comes to the opposite conclusion[22] for SVIX^2 but our tests are different. His in-sample regression tests for the null hypothesis that $\alpha = 0$ and $\beta = 1$ in equation (3.1). He reports an out-of-sample R^2 statistic of 0.42% but he uses $\tau \cdot \mathrm{SVIX}_t^2$ as the

[22]For the sample period 1996:01 to 2012:01.

predicted return directly without running a predictive forecast as in equation (3.2). These test results are regarded as supporting evidences for $\tau \cdot \mathrm{SVIX}_t^2$ being a tight lower bound for expected market risk premia.

1.6.4 Predictability of TIX

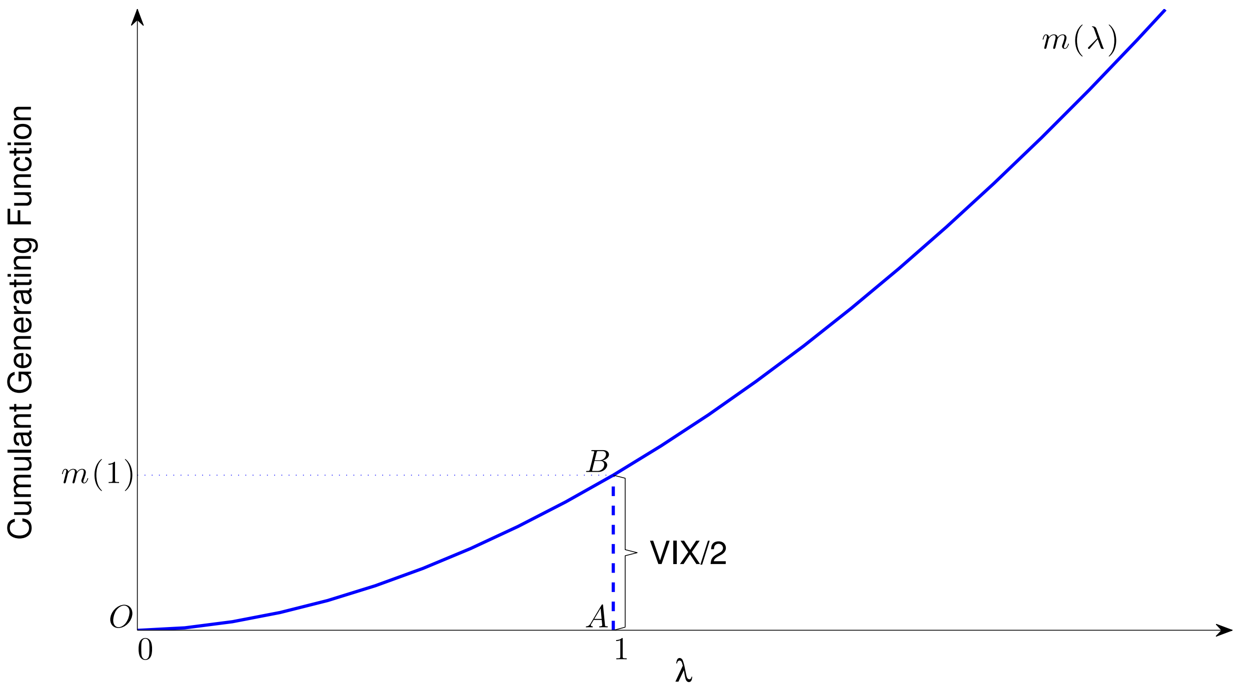
TIX measures the tail decay rate of the distribution of market risk which reflects the market perceptions of possible occurrence of large price drops. It is interesting to see if this forward-looking risk-neutral measure of extreme risk is related to the ex post market returns. The bottom line in Table 1.7 reports its in-sample predictive regression coefficient estimate of 0.07, which is close to zero and not significant, and negligible R^2 statistic of 0.03%. The out-of-sample R^2 statistic, as shown in the bottom line in Table 1.8, is -4.16% meaning it performs much worse than a forecast assuming non-predictability. This implies that the market price has incorporated the information contained in the level of TIX. When TIX is low as the market is concerned, we cannot make inference on whether the price will more likely go up or down. However, the change in TIX is informative about the future price movements. As TIX increases, the market becomes less concerned about price crashes in the future which turns out to be more likely followed by a rise in price. The importance of taking the difference in market expectations for forecasting returns is also reflected in Bakshi, Panayotov and Skoulakis (2011). They find that the forward variances, which are essentially the differences in the market expectation of the future variance at different horizons, are able to predictor returns while the expected variances themselves are not.

1.7 Conclusion

This paper provides a novel framework for extracting the information contained in option prices. I show that the cgf of the market risk on the positive half real line can be replicated by cross sections of index option prices in a model-free manner. It provides a unifying framework for interpreting the two leading forward-looking measures of market risk in the literature, the CBOE VIX and the SVIX proposed by Martin (2016). Independent of the underlying price process, the VIX measures the height of the cgf at 1 while the SVIX measures the convexity of the cgf over the range $[0, 2]$. I propose a tail index of market risk, TIX, as the ratio of the cgf of the market risk at 2 over that at 1 which measures the tail decay rate of the distribution of market risk. It reveals the market perceptions of extreme risk. I construct a daily time series of TIX for the 30-day market risk born by the S&P 500 index. To examine the information content of innovation in TIX, I test its predictability for future monthly market returns. The innovation in TIX strongly predicts market returns both in and out of sample, with monthly R^2 statistics of 3.33% and 6.15%, respectively, outperforming the popular return predictors in Goyal and Welch (2008) and the short interest variable in Rapach, Ringgenberg, and Zhou (2016).

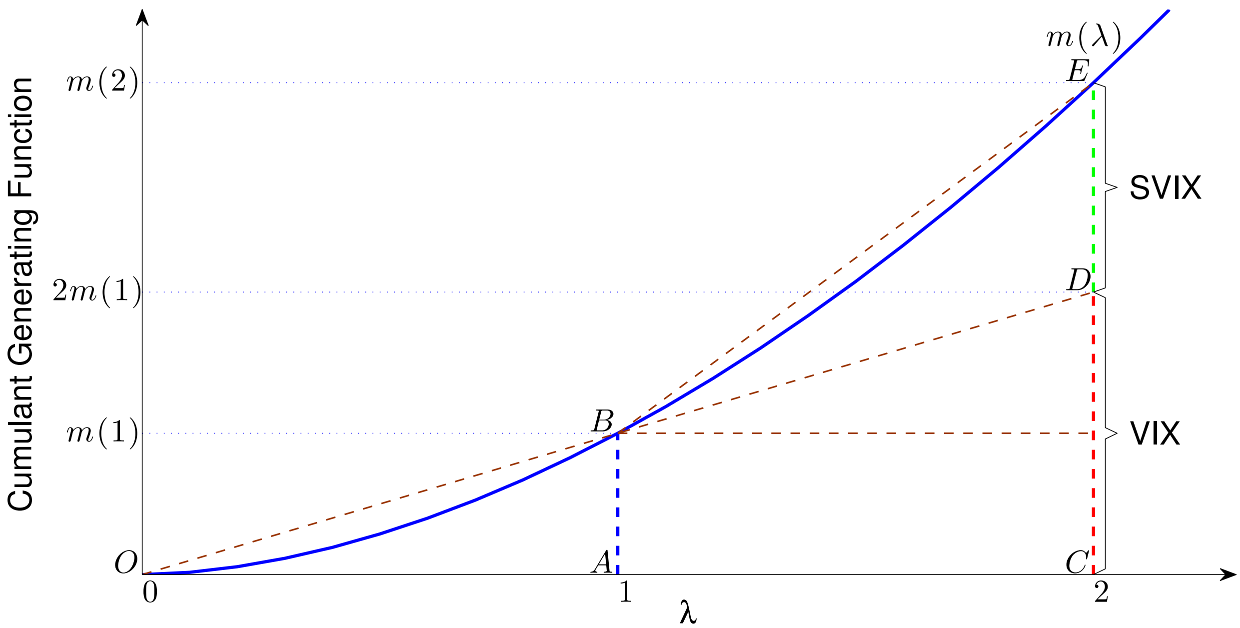
The future work would be to examine the predictability of the innovation in TIX at horizons beyond one month. As in Martin (2016) for SVIX, we need to construct a term structure for this tail index. The potential problem is that when we construct a TIX index for the one year horizon, for example, and take its difference between two consecutive months, the amount of information contained therein would be relatively small compared to the uncertainty involved at the one year horizon.

Figure 1.1: Cumulant Generating Function and VIX



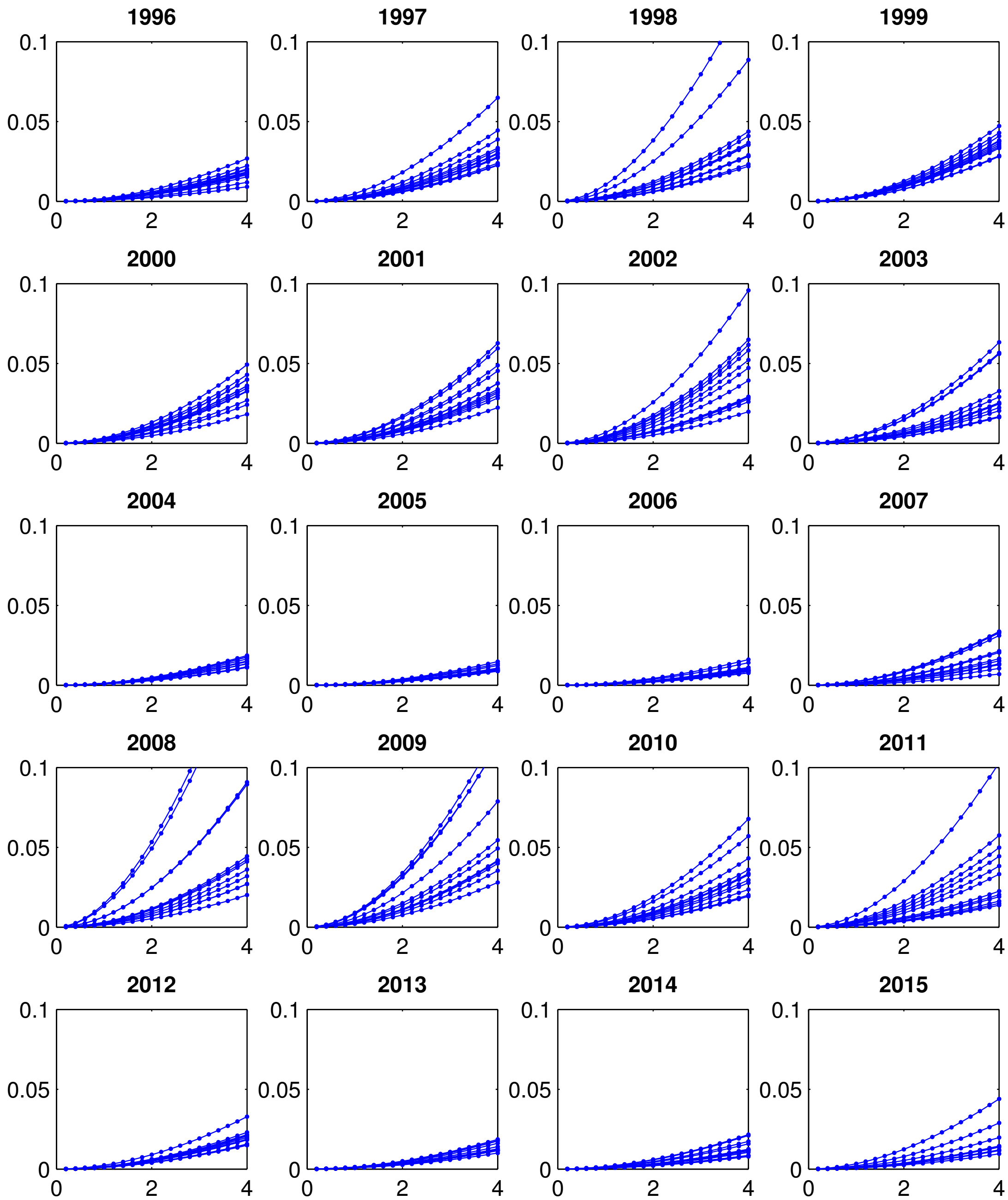
This figure draws the cumulant generating function $m(\lambda)$ of the market risk σZ_τ. The unannualized VIX, $\tau \cdot \text{VIX}^2$, divided by 2, measures the height of $m(\cdot)$ at 1 which is represented by AB in the figure.

Figure 1.2: Cumulant Generating Function and SVIX



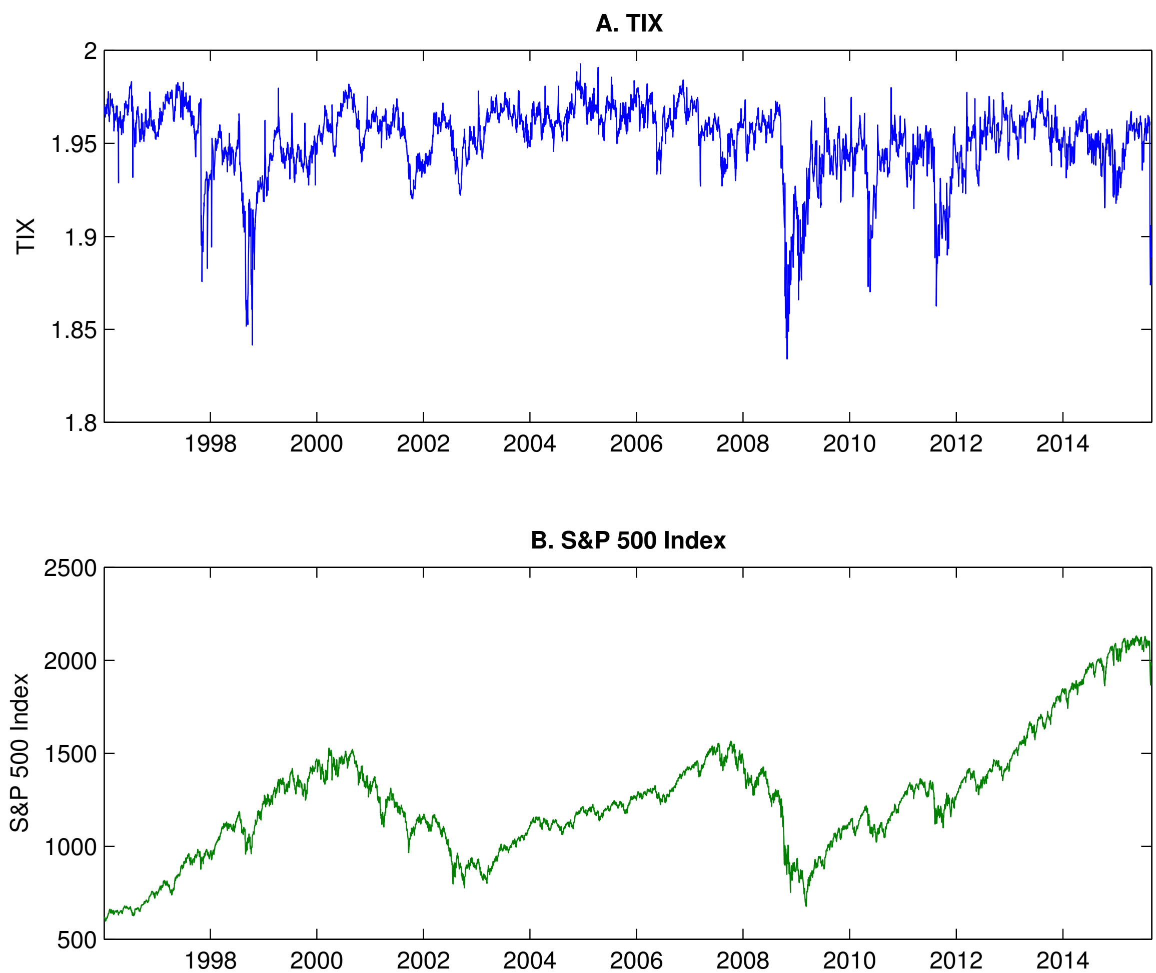
This figure draws the cumulant generating function $m(\lambda)$ of the market risk σZ_τ. The unannualized VIX, $\tau \cdot \mathrm{VIX}^2$, measures twice the height of $m(\cdot)$ at 1 which is represented by CD in the figure. The transformed unannualized SVIX, $\log(1 + \tau \cdot \mathrm{SVIX}^2)$, measures the difference between the height of $m(\cdot)$ at 2 and two times the height of $m(\cdot)$ at 1 which is represented by DE in the figure. Alternatively, it measures the convexity of the curve, i.e., the change in the slope between OB (or BD) and BE.

Figure 1.3: Cumulant Generating Functions of the 30-day Market Risk: 1996:01-2015:08



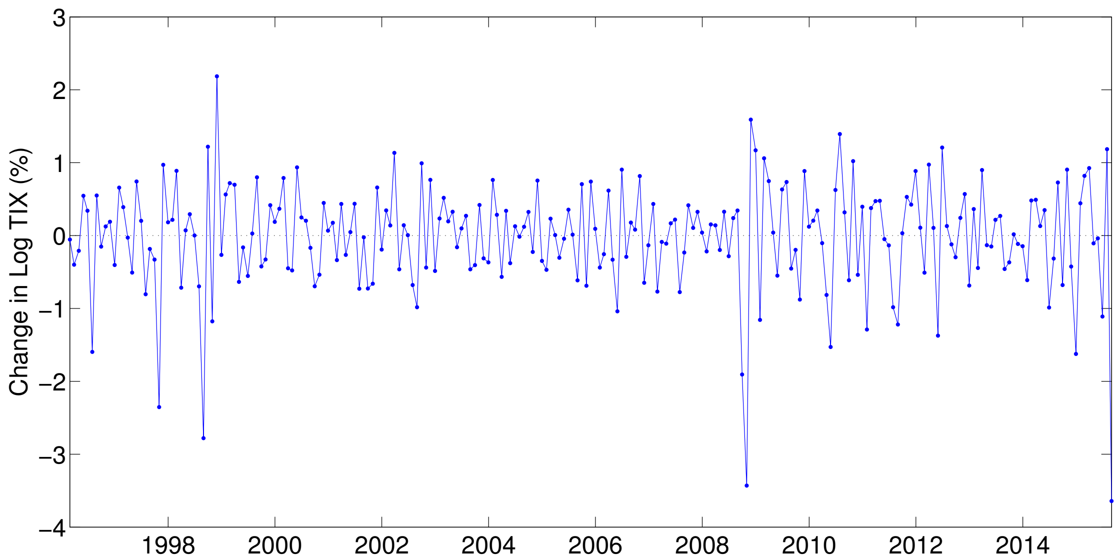
This figure plots the cgf's of the 30-day market risk in the S&P 500 index which are extracted from its option prices. The cgf curves are grouped by year. Each subfigure displays the 12 curves representing the cgf's at the end of each month within the year.

Figure 1.4: Daily Time Series of TIX and S&P 500 Index: 1996:01-2015:08



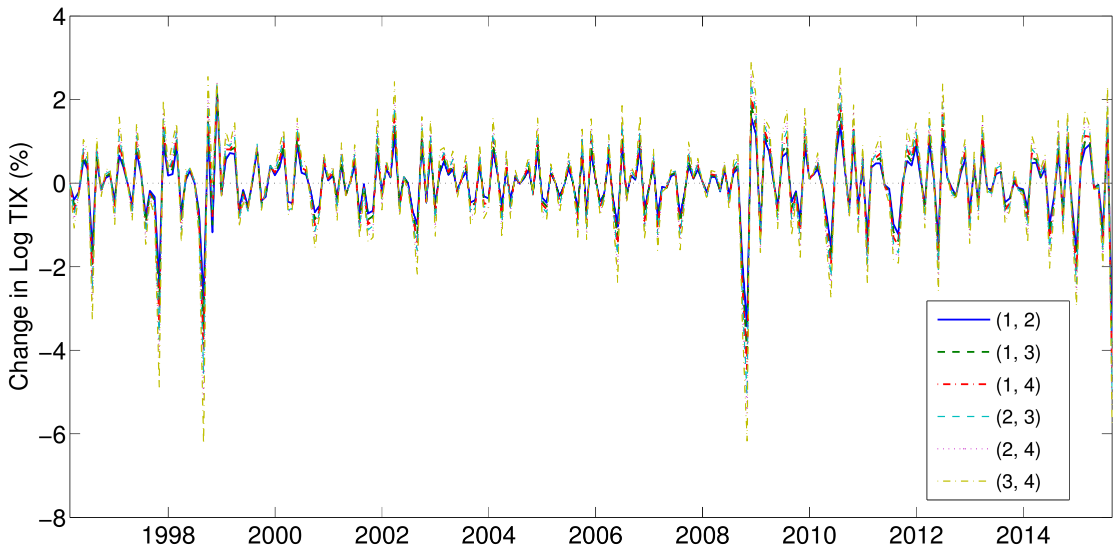
Panel A plots the time series of daily TIX. TIX measures the extreme risk of the market risk for next 30 calendar days. It is calculated as $\log_2(m(2/m(1)))$ where $m(\cdot)$ is the cgf of the market risk and is extracted from the option prices of the S&P 500 index. Panel B plots the time series of daily close price of the S&P 500 index.

Figure 1.5: Monthly Innovations in TIX: 1996:02:2015:08



This figure displays the monthly time series of innovations in TIX which are computed as the log difference in TIX at the end of each month.

Figure 1.6: Robustness Check of Innovations in TIX: 1996:02:2015:08



This figure displays the monthly time series of innovations in TIX with the TIX constructed as $\log_{\lambda_2/\lambda_1}(m(\lambda_2)/m(\lambda_1))$ for six alternative pairs of (λ_1, λ_2) which I use to label the corresponding time series.

Table 1.1: **Sample statistics, 1996:02-2014:12**

Predictor	mean	median	1st percentile	99th percentile	std. dev.
DP	-4.03	-4.03	-4.48	-3.38	0.22
DY	-4.03	-4.02	-4.48	-3.40	0.22
EP	-3.16	-3.01	-4.81	-2.66	0.41
DE	-0.87	-1.01	-1.24	1.29	0.47
SVAR	0.00	0.00	0.00	0.03	0.01
BM	0.26	0.27	0.12	0.42	0.08
NTIS	0.00	0.01	-0.05	0.03	0.02
TBL (%)	2.45	1.73	0.01	6.09	2.14
LTY (%)	4.81	4.84	2.19	7.18	1.27
LTR (%)	0.65	0.85	-7.26	7.77	3.05
TMS (%)	2.36	2.41	-0.26	4.45	1.34
DFY (%)	1.00	0.90	0.55	3.09	0.45
DFR (%)	-0.01	0.05	-6.20	5.99	1.81
INFL (%)	0.19	0.19	-0.83	0.88	0.29
SII	0.00	-0.06	-1.55	2.38	1.00
$d\log$ TIX (%)	-0.01	0.05	-2.45	1.44	0.69

The database contains 227 monthly observations for February 1996 to December 2014. The table displays summary statistics for 14 predictor variables from Goyal and Welch (2008), the short interest variable SII from Rapach, Ringgenberg, and Zhou (2016), and the innovation in TIX. DP is the log dividend-price ratio, DY is the log dividend yield, EP is the log earnings-price ratio, DE is the log dividend-payout ratio, SVAR is stock variance, BM is the book-to-market value ratio for the Dow Jones Industrial Average, NTIS is net equity expansion, TBL is the interest rate on a three-month Treasury bill, LTY is the long-term government bond yield, LTR is the return on long- term government bonds, TMS is the long-term government bond yield minus the Treasury bill rate, DFY is the difference between Moody's BAA- and AAA-rated corporate bond yields, DFR is the long-term corporate bond return minus the long-term government bond return, and INFL is inflation calculated from the CPI for all urban consumers. SII is the (standardized) detrended log of equal-weighted short interest. $d\log$ TIX is the log difference in TIX at the end of each month.

Table 1.2: **Correlation matrix, 1996:02-2014:12**

Predictor	DP	DY	EP	DE	SVAR	BM	NTIS	TBL	LTY	LTR	TMS	DFY	DFR	INFL	SII	$d\log$ TIX
DP	1.00															
DY	0.98	1.00														
EP	-0.01	-0.02	1.00													
DE	0.48	0.47	-0.88	1.00												
SVAR	0.30	0.22	-0.28	0.39	1.00											
BM	0.69	0.67	0.42	-0.04	0.08	1.00										
NTIS	-0.46	-0.43	0.10	-0.30	-0.33	-0.23	1.00									
TBL	-0.57	-0.57	-0.02	-0.25	-0.11	-0.69	0.22	1.00								
LTY	-0.53	-0.53	-0.25	-0.03	-0.04	-0.77	0.42	0.81	1.00							
LTR	0.03	-0.01	0.04	-0.03	0.18	0.05	0.02	0.00	-0.11	1.00						
TMS	0.41	0.41	-0.21	0.38	0.14	0.36	0.04	-0.83	-0.34	-0.09	1.00					
DFY	0.61	0.58	-0.52	0.74	0.59	0.33	-0.58	-0.44	-0.35	0.01	0.37	1.00				
DFR	0.01	0.10	-0.20	0.18	-0.24	-0.04	0.05	-0.08	-0.00	-0.46	0.12	0.13	1.00			
INFL	-0.17	-0.17	0.13	-0.19	-0.24	-0.10	0.04	0.16	0.16	-0.09	-0.10	-0.32	-0.11	1.00		
SII	0.19	0.17	-0.05	0.13	0.13	0.07	-0.51	0.07	0.07	0.02	-0.04	0.27	-0.10	0.13	1.00	
$d\log$ TIX	-0.04	0.05	-0.06	0.04	-0.25	-0.01	0.01	-0.00	-0.02	-0.02	0.00	0.06	0.25	-0.02	-0.01	1.00

The table displays correlation coefficients for 14 predictor variables from Goyal and Welch (2008), the short interest variable SII from Rapach, Ringgenberg, and Zhou (2016), and the innovation in TIX. 0.00 indicates less than 0.005 in absolute value.

Table 1.3: **In-sample predictive regression estimation results, 1996:02-2014:12**

Predictor	$\hat{\beta}$	$R^2(\%)$
DP	0.57	1.59
	[1.30]	
DY	0.65	2.09
	[1.65]*	
EP	0.26	0.34
	[0.61]	
DE	0.04	0.01
	[0.08]	
SVAR (-)	0.72	2.59
	[1.85]**	
BM	0.32	0.52
	[1.02]	
NTIS	0.59	1.69
	[1.28]	
TBL (-)	0.20	0.19
	[0.67]	
LTY (-)	0.32	0.51
	[1.21]	
LTR	0.09	0.04
	[0.30]	
TMS	0.01	0.00
	[0.03]	
DFY (-)	0.32	0.52
	[0.62]	
DFR	0.41	0.85
	[0.78]	
INFL	0.32	0.49
	[0.83]	
SII(-)	0.64	2.03
	[2.01]**	
$d \log$ TIX	0.83	3.33
	[2.10]**	

The table reports the ordinary least squares estimate of β and R^2 statistic for the predictive regression model,

$$r_{t+1} = \alpha + \beta x_t + \epsilon_{t+1} \quad \text{for} \quad t = 1, \cdots, T-1,$$

where r_{t+1} is the S&P 500 log excess return, x_t is the predictor variable in the first column, and $(-)$ indicates that I take the negative of the predictor variable. Each predictor variable is standardized to have a standard deviation of one. In brackets are heteroskedasticity- and autocorrelation-robust t-statistics for testing $H_0 : \beta = 0$ against $H_A : \beta > 0$; *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively, according to wild bootstrapped p-values; 0.00 indicates less than 0.005 in absolute value.

Table 1.4: **Out-of-sample test results, 2008:09-2014:12**

Predictor	Out-of-sample R^2 statistics (%)
DP	-2.63
DY	-0.65
EP	-9.71
DE	-10.57
SVAR	-3.80
BM	-1.06
NTIS	-1.24
TBL	-1.44
LTY	-1.06
LTR	-1.85
TMS	-0.81
DFY	-8.02
DFR	-6.34
INFL	-1.78
SII	0.75^{*}
$d\log$ TIX	6.15^{**}

The second column reports the proportional reduction in mean squared forecast error (MSFE) at the monthly horizon for a predictive regression forecast of the S&P 500 log excess return based on the predictor variable in the first column vis-á-vis the prevailing mean benchmark forecast, where statistical significance is based on the Clark and West (2007) statistic for testing the null hypothesis that the prevailing mean MSFE is less than or equal to the predictive regression MSFE against the alternative hypothesis that the prevailing mean MSFE is greater than the predictive regression MSFE. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 1.5: **In-sample predictive regression estimation results for shape variables, 1996:02-2014:12**

Panel A: Height Variables

	Height: $m(\lambda)$		Change in Height: $d\log m(\lambda)$	
Predictor	$\hat{\beta}$	$R^2(\%)$	$\hat{\beta}$	$R^2(\%)$
$\lambda = 1(--)$	0.00	0.00	0.37	0.67
	[0.01]		[0.90]	
$\lambda = 2(+-)$	0.00	0.00	0.35	0.61
	[0.00]		[0.87]	
$\lambda = 3(+-)$	0.01	0.00	0.34	0.57
	[0.01]		[0.84]	
$\lambda = 4(+-)$	0.01	0.00	0.33	0.54
	[0.02]		[0.82]	

Panel B: Convexity Variables

	Convexity: $m''(\lambda)$		Change in Convexity: $d\log m''(\lambda)$	
Predictor	$\hat{\beta}$	$R^2(\%)$	$\hat{\beta}$	$R^2(\%)$
$\lambda = 1(+-)$	0.01	0.00	0.34	0.55
	[0.02]		[0.82]	
$\lambda = 2(+-)$	0.02	0.00	0.30	0.43
	[0.04]		[0.73]	
$\lambda = 3(+-)$	0.02	0.00	0.26	0.34
	[0.04]		[0.66]	

Panel C: 4th Order Derivative

	Derivative: $m^{(4)}(\lambda)$		Change in Derivative: $dm^{(4)}(\lambda)$	
Predictor	$\hat{\beta}$	$R^2(\%)$	$\hat{\beta}$	$R^2(\%)$
$\lambda = 2(--)$	0.17	0.14	0.64	2.04
	[0.37]		[1.61]*	

The table reports the ordinary least squares estimate of β and R^2 statistic for the predictive regression model,

$$r_{t+1} = \alpha + \beta x_t + \epsilon_{t+1} \quad \text{for} \quad t = 1, \cdots, T-1,$$

where r_{t+1} is the S&P 500 log excess return and x_t is the predictor variable which is either the shape variable itself (columns 2-3) or its change (columns 4-5) at a particular location in the first column. $(+-)$, for example, indicate that I keep the sign of the shape variable but take the negative of its change, respectively. Each predictor variable is standardized to have a standard deviation of one. Brackets below the $\hat{\beta}$ estimates report heteroskedasticity- and autocorrelation-robust t-statistics for testing $H_0 : \beta = 0$ against $H_A : \beta > 0$; *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively, according to wild bootstrapped p-values; 0.00 indicates less than 0.005 in absolute value.

Table 1.6: **Out-of-sample test results for shape variables, 2008:09-2014:12**

Panel A: Height Variables

	Out-of-sample R^2 statistics (%)	
Predictor	Height: $m(\lambda)$	Change in Height: $d\log m(\lambda)$
$\lambda = 1$	-8.09	-0.75
$\lambda = 2$	-7.86	-0.86
$\lambda = 3$	-7.67	-0.96
$\lambda = 4$	-7.52	-1.04

Panel B: Convexity Variables

	Out-of-sample R^2 statistics (%)	
Predictor	Convexity: $m''(\lambda)$	Change in Convexity: $d\log m''(\lambda)$
$\lambda = 1$	-2.63	-0.99
$\lambda = 2$	-7.01	-1.27
$\lambda = 3$	-6.56	-1.49

Panel C: 4th Order Derivative

	Out-of-sample R^2 statistics (%)	
Predictor	Derivative: $m^{(4)}(\lambda)$	Change in Derivative: $dm^{(4)}(\lambda)$
$\lambda = 2$	-13.54	2.10

This table reports the proportional reduction in mean squared forecast error (MSFE) at the monthly horizon for a predictive regression forecast of the S&P 500 log excess return based on the predictor variable, which is either the shape variable itself (column 2) or its change (column 3) at a particular location in the first column, vis-á-vis the prevailing mean benchmark forecast, where statistical significance is based on the Clark and West (2007) statistic for testing the null hypothesis that the prevailing mean MSFE is less than or equal to the predictive regression MSFE against the alternative hypothesis that the prevailing mean MSFE is greater than the predictive regression MSFE. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 1.7: **In-sample predictive regression estimation results for VIX, SVIX and TIX, 1996:02-2014:12**

	Index		Change in Log Index	
Predictor	$\hat{\beta}$	$R^2(\%)$	$\hat{\beta}$	$R^2(\%)$
VIX(+−)	0.01	0.00	0.37	0.67
	[0.03]		[0.90]	
SVIX(+−)	0.02	0.00	0.34	0.55
	[0.04]		[0.83]	
TIX(−+)	0.07	0.03	0.83	3.33
	[0.18]		[2.10]**	

The table reports the ordinary least squares estimate of β and R^2 statistic for the predictive regression model,

$$r_{t+1} = \alpha + \beta x_t + \epsilon_{t+1} \quad \text{for} \quad t = 1, \cdots, T-1,$$

where r_{t+1} is the S&P 500 log excess return and x_t is the predictor variable which is either the level (columns 2-3) or change (columns 4-5) of an index in the first column. (+−), for example, indicate that I keep the sign of the index level but take the negative of its change, respectively. Each predictor variable is standardized to have a standard deviation of one. Brackets below the $\hat{\beta}$ estimates report heteroskedasticity- and autocorrelation-robust t-statistics for testing $H_0 : \beta = 0$ against $H_A : \beta > 0$; *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively, according to wild bootstrapped p-values; 0.00 indicates less than 0.005 in absolute value.

Table 1.8: **Out-of-sample test results for VIX, SVIX and TIX, 2008:09-2014:12**

	Out-of-sample R^2 statistics (%)	
Predictor	Index	Change in Log Index
VIX	−4.84	−0.75
SVIX	−4.60	−0.99
TIX	−4.16	6.15**

This table reports the proportional reduction in mean squared forecast error (MSFE) at the monthly horizon for a predictive regression forecast of the S&P 500 log excess return based on the predictor variable, which is either the level (column 2) or change (column 3) of an index in the first column, vis-á-vis the prevailing mean benchmark forecast, where statistical significance is based on the Clark and West (2007) statistic for testing the null hypothesis that the prevailing mean MSFE is less than or equal to the predictive regression MSFE against the alternative hypothesis that the prevailing mean MSFE is greater than the predictive regression MSFE. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

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Chapter 2

Shadow Banking and Asset Pricing

A shadow banking system featuring collateral constraints is studied to investigate the joint determination of haircut and interest rate, as well as its interaction with collateral asset pricing. The banks with limited commitment serve the households' need for consumption smoothing by taking deposits with a risky asset used as collateral and pursue the maximal leverage returns. In a collateral equilibrium as in Geanakoplos (1997, 2003), agents' marginal rates of substitution are equalized only in non-default states, only the deposit contract with the highest liquidity value per unit of collateral is traded, and the risky asset price is boosted such that banks earn zero profit. Relative to the traditional banking with full commitment, banks are better off if they are endowed with the collateral asset while households are strictly worse off. I also find (i) higher households' risky asset endowment leads to a higher asset price because a stronger saving motive creates a scarcity of collateral; (ii) for lenders with a high risk aversion, an improvement of downside collateral quality decreases its price because it alleviates the tension of imperfect risk sharing and, therefore, reduces the collateral value, but everything goes in opposite directions for a low risk aversion.

2.1 Introduction

Financial institutions have been intensively engaged in shadow banking activities such as repo. A common feature of these lines of business is the essential use of collateral. A repo transaction between a repo investor and a shadow bank is analogous to the demand deposit for the traditional banking, with the collateral serving as kind of deposit insurance (Gorton and Metrick, 2010). For this type of collateralized deposit, there are two dimensions in the pricing of a promised but potentially risky repayment: the interest rate and the haircut[1]. A bank can offer more favorable terms to a depositor, who are concerned about the risk of a fixed payment in the future, by either putting up more collateral, which means a higher haircut, or by reducing the initial deposit, which means a higher interest rate.[2] The main questions I want to ask here are: how the interest rate and haircut are jointly determined? What are the factors that may play a role in the mechanism and how? Unfortunately, these questions are not well answered in the literature.[3]

A companion question is on the pricing of the asset used as collateral. It is expected that its price would interact with the collateral constraint (Kiyotaki and Moore, 1997) and a collateral value might be incorporated when the constraint is binding. But the knowledge is limited on how the price moves together with its haircut and the interest rate of loans backed by it. This is particularly relevant during the financial crisis because when the cash

[1]The term haircut is commonly used in the repo market. If a repo investor lends \$80 to a bank who puts up \$100 collateral, there is a haircut of 20% on the collateral. But haircut can be interpreted more broadly. If you buy a house of value \$1 million by a mortgage in which you pay a downpayment \$0.2 million and finance the rest with a bank, the bank indeed requires a haircut of 20% of the value of your house. If you start a business with a current value of \$1 million and finance the initial investment by a debt issuance of \$0.4 million, the bond holders are requiring a haircut of 60% from you, the equity holder.

[2]For example, in the programs of providing three-year loans to financial institutions as a Lender of Last Resort in December 2011 and February 2012, the European Central Bank charges a higher interest rate but a lower haircut on collateral than the primary market (Drechsler and Schnabl, 2014).

[3]Here is a quote from Gorton and Metrick (2012) who empirically investigate the behavior of the repo rate and haircut during the recent financial crisis (repo run in their terminology): "It could seem natural that repo spreads and repo haircuts should be jointly determined. Unfortunately, the theory is not sufficiently developed to provide much guidance here".

flows of securitized assets deteriorated with increased uncertainty, their prices and haircuts and the interest rates of loans might change dramatically, although differential patterns were present depending on the asset quality (Gorton and Metrick, 2012; Copeland, Martin, and Walker, 2014; Krishnamurthy, Nagel, and Orlov, 2014). So a framework for analyzing the relation between asset price, haircut and interest rate and the role of quality of collateral is desirable.

In this paper, I address these questions by investigating an exchange economy in which the risk averse households have consumption smoothing needs which could be met by the risk neutral banks who pursue the maximal leveraged return. But banks face a limited commitment problem in the sense that they cannot commit to pay back the deposits at the maturities.[4] However, there is a fixed supply of a risky asset in the economy which banks can put up as collateral to secure the deposits from households.

To be specific, banks and households are at the beginning endowed with a perishable consumption good and the risky asset which delivers a stochastic dividend next period. The choice of terms of deposit is modeled as buying bonds with face value 1 of a continuum of types indexed by the unit of risky asset, s, used as collateral in a competitive bond market in the spirit of Geanakoplos (1997, 2003). Agents choose optimally to hold the risky asset, buy bonds, or issue bonds (this requires collateral). In equilibrium, households find it optimal to only buy bonds, not holding the risky asset or issuing bonds because they are too risky. Moreover, banks only issue the single type of bond with the largest valuation difference, the liquidity value of a bond, per unit of collateral between households and banks. The risky asset price is boosted up from its fundamental value with banks as the marginal investor to reflect a collateral value which makes banks indifferent in issuing that bond. For any other

[4] This limited commitment assumption, the key friction considered in this paper, is motivated by the empirical observation in Pozsar (2011) that prior to the recent financial crisis, the global non-financial corporations and institutional investors were holding large cash pools exceeding the insufficient supply of safe assets such as the partially insured deposits and short-term government securities. Then the secured lending featuring the shadow banking system became the natural alternative investment.

type of bond not traded, households' valuation is less than the cost of the obligation for banks plus the collateral value paid for setting up the collateral required. There is always a possibility of default and agents' marginal rate of substitutions are equalized only for non-default states.

With the traditional banking with full commitment as a benchmark, although banks make the same zero profit in a shadow banking system, their welfare is enhanced if they are endowed with the collateral asset which earns a collateral value. Banks are taking advantage of the low deposit financing cost from households who are willing to pay more for the contract than what banks would ask for because of risk aversion. But it is costly to hold collateral because its price is boosted up offering a lower expected return than banks' time preference. In a completive equilibrium, these two factors offset exactly and banks earn an expected leveraged return equal to their time preference. Households' saving rate is adversely distorted except for a log-utility since there is a probability of default and the interest rate is risky. They save too little when their degree of risk aversion is low but too much when it is high. Even households' initial wealth is higher if endowed with the collateral asset, they are strictly worse off because of imperfect risk sharing.

The uniqueness of the type of bond actively traded, denoted as $\bar{s}$, gives rise to the joint determination of the interest rate and haircut in the shadow banking system. Factors that are relevant in the mechanism include the strengths of household and bank sectors in term of their endowments, agents' time preferences, households' risk aversion, and the quality of collateral. Table 2.1 lists all the factors and the predicted responses of variables of interest to shocks on them. The first several counterintuitive results illustrate the distinctive way this shadow banking system operates.

First, as households' endowment in the risky asset increases, the risky asset price turns out to increase as well. In this economy, as households become richer and thus are willing to save more to smooth consumption, it creates a scarcity of collateral with less units of asset backing each unit of promised repayment even though the total supply of asset is also higher which is of second order. This conservatism in using collateral is consistent with a higher collateral value and hence a higher risky asset price. So we have a rare case in which an excess supply generates an excess demand *outweighing* the excess supply.

Second, when households' initial endowment in either the consumption good or risky asset increases, even though households want to save more the interest rate goes up when households' relative risk aversion is less than 1. As discussed above, households' greater saving motive leads to less units of asset backing each unit of promised repayment. This implies a greater credit risk of the deposit contract. Although households' endogenous pricing kernel shifts upward because of risk aversion and a higher chance of default, the greater credit risk dominates when their relative risk aversion is less than 1. Households value the deposit contract less and a higher interest rate prevails. On the other hand, although the interest rate indeed decreases when the relative risk aversion is greater than 1, it is because households' increased valuation of payments in bad states dominates the greater credit risk.

Third, when banks are endowed with more risky asset, the effects are completely the opposite of the previous case. The reason is that when the household sector is held the same, more risky asset in the bank sector, where banks pursue leveraged returns, implies an affluence of collateral leading to more units of asset backing each unit of promised repayment. The collateral value of risky asset and hence its price decrease. The credit risk of the deposit contract declines driving the change of interest rate: decreases for low risk aversion while increases for high risk aversion.

Finally, although the households' risk aversion parameter plays both roles of risk aversion and elasticity of intertemporal substitution here, if we control for the effect of intertemporal substitution by assuming the same time preference for households and banks and let the risk aversion increase, one might expect households to be compensated by at least a higher interest rate or a higher haircut. However, both of them decrease if, for example, households are endowed with all the risky asset. This is because when households become more risk averse, they are more desperate in smoothing consumption and hence are less favored in the terms of deposit contract.

This paper also provides novel insights on the nature of haircut. Haircut can be interpreted as the equity-asset ratio if each collateralized deposit contract is considered as an establishment of a firm with the collateral, $\bar{s}$ units of risky asset, being the asset and the deposit being the debt. Hence, haircut is positively linked to the equity-debt ratio in which the equity is the banks' valuation of $\bar{s}$ call options on the risky asset with the $1/\bar{s}$ playing the role of a strike price, while the debt is the households' valuation of the standard debt-type payoff at the same strike price but reflecting their risk aversion. There are two channels for a shock to have an impact on the haircut. The first one is through the direct impact on the valuation of options contained in equity and debt which is relevant for shocks on the quality of collateral and households' risk aversion. The second one is through the indirect impact on the strike price, $1/\bar{s}$. In particular, we document the property that when the first channel is absent, the collateral use exhibits a diminishing return to scale in the amount of borrowing supported. That is, haircut is increasing in the unit of risky asset, $\bar{s}$, used as collateral, and therefore any shock, except the previous two, that increases $\bar{s}$ also increases haircut such as a negative shock on households' initial endowments, a positive shock on banks' risky asset endowment, a less patient household sector or a more patient bank sector.

The upside and downside quality of collateral[5] is shown to have asymmetric effects because households' valuation of the collateralized bonds depends only on its quality in states of default. When the upside quality improves, it does not change the type of deposit

[5]In the sense of a first order or a second order stochastic improvement over the right or left tail of the asset future dividend distributions.

contract traded and hence the interest rate and collateral value remain. The asset price increases simply because of the improvement of quality and the haircut adjusts upward accordingly. But when the downside quality improves, the equilibrium contract traded changes. It is interesting that households with low and high risk aversion respond differently because the contingent valuation of bond payment is increasing and concave (decreasing and convex, respectively) in the risky asset dividend when the risk aversion is low (high, respectively), leading to complete opposite predictions on changes of variables of interest. In particular, when the risk aversion is high, the asset price drops as its downside quality improves. This is because the collateral value falls as households' desire for consumption smoothing is dampened by the improvement in the downside asset quality. Also, although a first order and a second order stochastic dominance improvements of the quality have similar effects when they are local, asymmetry arises when they are global since the equity is always increasing and convex in the risky asset dividend.

Related Literature. This paper is closely related to Simsek (2013) which also employs the collateral equilibrium concept of Geanakoplos (1997, 2003). Namely, we both treat contracts with different features as commodities in a competitive market and let the market select and price, fortunately, the unique contract traded. This helps to transform the non-price variables such as collateral level and haircut involved in collateralized borrowing, difficulties faced by other approaches, into market prices. But the questions we address are different. Simsek (2013) considers the speculative bet between risk-neutral optimists and pessimists of a risky asset and focuses on the asymmetric disciplining of the collateral constraint on the excess leverage taken by optimistic borrowers because of the lenders' pessimistic view of the probabilities of default. However, this paper considers trades driven by the consumption smoothing needs from risk averse households and the motive of maximizing expected return from risk-neutral banks and focuses on factors relevant for determining the haircut and interest rate as well as the asset price. This paper is also connected to Fostel and Geanakoplos (2015) who shows that in a binomial economy any collateral equilibrium is equivalent (in real allocations and prices) to another one involving no default. This multiplicity and the no-default result come from the fact that all debt contracts involving partial default bear the same liquidity value as the safe contract with the maximum promise and, therefore, borrowers are indifferent between issuing any of them. If there is a continuum of states as in this paper, borrowers would optimally issue a unique contract with the maximum liquidity value which may involve default. Phelan (2015) also studies the asset pricing applications of collateral constraint. But the collateral asset price is determined by a budget constraint. Instead, it is priced as the fundamental value plus the collateral value in this paper. Moreover, Phelan (2015) focuses on the risk about future payoffs and endowments while this paper focuses on agents' initial endowments and preferences and the quality of collateral.

There is also a recent literature studying different aspects of the determination of repo haircut and interest rate. Eren (2014) focuses on the role of dealer banks as intermediaries of repo transactions in determining the repo haircut and repo rate. Although haircut and interest rate are both pinned down in his model[6], they are through separated mechanisms: the haircut is determined by the volume of lending by cash investors and the liquidity needs from dealer banks, and the interest rate is determined by the hedge funds' participation constraint. Moreover, collateral is assumed to be riskless there while in my model its quality is an important factor and its price is also endogenously determined. Dang, Gorton and Holmsrtöm (2013) also study the determination of haircuts. The tension in their model is that there is a chance that the borrower will be unable to pay off the debt in the future

[6]In his model, the hedge funds have investment projects as well as collateral assets while the cash investors want to store cash but they cannot meet each other. The deal banks, who have current liquidity needs, do a repo (collateralized borrowing) with cash investors and a reverse repo (collateralized lending) with hedge funds simultaneously and charge a higher haircut on the collateral from the latter than they are charged by the former. Since the haircut and interest rate charged by cash investors are assumed to be zero, only the haircut and interest rate charged to the hedge funds need to be determined. The haircut is simply the ratio of dealer banks' liquidity needs over cash investors' lending volume.

and, at the same time, the lender encounters a liquidity shock. Then the lender would like to sell the collateral to a third party at which stage he may face an adverse selection problem since the potential buyer may doubt the quality of the collateral. Therefore, the lender would suffer a loss which he takes into account up-front and requires a haircut.[7] Different from these models, this paper considers the determination of haircut and interest rate simply on the basis of endowments and preferences of borrowers and lenders and the quality of collateral without assuming any friction introduced by a third party. Hu, Pan, and Wang (2014) empirically study the behavior of haircut and repo rate in the tri-party repo market between dealer banks and money market funds.

This paper also relates to the literature on the role of financial intermediaries in transforming risky/illiquid assets into safe/liquid assets (Gorton, Lewellen, and Metrick, 2012). Here the shadow banking system essentially transforms the risky asset into safer collateralized bonds to facilitate the consumption smoothing of risk averse households. Krishnamurthy and Vissing-Jorgensen (2013) provide empirical evidence showing that the prevalence of short-term debt issued by the financial sector is driven by a large demand for safe and liquid asset for investment from the non-financial sector; that the financial sector supplies such debt by holding positions in other risky assets (loans, securities, etc.) that are funded by short-term debt. This evidence is consistent with the framework of this paper. Also, their evidence that investors are willing to pay a premium for holding such safe and liquid assets which incentivizes the financial sector is in line with the equilibrium feature of my model that the financing cost is so low that depositors are subsidizing banks[8]. From a theoretical point of view, Hébert (2015) also tries to answer the question why debt type contracts are so common in practice. In a securitized lending context, when the lender faces

[7]The determinants of repo haircuts identified in their model include: (i) the information sensitivity of collateral, (ii) the default probability of borrower, (iii) the intermediate liquidity needs of lender, and (iv) his default probability in a subsequent repo transaction.

[8]This paper focuses on the private supply of safe assets. For theoretical treatments of the private supply of liquid assets and the liquidity premium, see Holmsrtöm and Tirole (1998, 2011).

an agency problem from the borrower who can privately modify the quality of underlying assets, he shows that a debt type contract is optimal since it trades off the moral hazards of excessive risk-taking and lax effort from the borrower. This result provides a justification for limiting the contract space to collateralized bonds in this paper when dealing with the limited commitment problem of banks.

The paper is organized as following. The next section describes the model and considers the equilibrium in the benchmark of a traditional banking with full commitment. Section 3 characterizes the equilibrium and analyzes the welfare implications. Section 4 conducts the comparative static analysis of how the haircut and interest rate as well as the risky asset price respond to shocks in the endowments of the household sector and bank sector. Section 5 investigates how a change in the quality of collateral impacts its haircut and the interest rate of collateralized debt and how these effects feed back into its price. Section 6 documents the effects of agents' time preferences and households' degree of risk aversion. Section 7 concludes.

2.2 The Model

Consider an economy with two dates: time 0 and time 1. There is a single perfectly divisible risky asset (a Lucas tree) circulated in the economy on both dates. The total supply of the asset is fixed at K. There is another perishable consumption good which is the dividend (the fruit) of the risky asset. At time 0 the dividend has already realized and the ex-dividend price of the asset is p_0. Each unit of asset has a claim on the dividend $D_1 \in (0, \infty)$ at time 1 which is stochastic with a common known distribution function $F(D_1)$ and a density function $f(D_1)$.

There is a continuum of agents of two types with equal mass one. The type 1 agents are households who consume at time 0 and time 1 with a time-0 expected utility

$$u(c_0^h) + \beta^h E(u(c_1^h)),$$

where c_0^h and c_1^h are households' consumption at time 0 and 1, respectively, $\beta^h \in (0,1)$ is the households' time discount factor, and $u(c) = c^{1-\eta}/(1-\eta)$ is a CRRA utility function with a constant relative risk aversion coefficient $\eta > 0$. The type 2 agents are banks who are risk neutral and also consume at time 0 and time 1 with a time-0 expected utility[9]

$$c_0^b + \beta^b E(c_1^b),$$

where c_0^b and c_1^b are banks' consumption at time 0 and time 1, respectively, and $\beta^b \in (0,1)$ is the banks' time discount factor. At time 0, the households and banks are endowed with s_0^h and s_0^b units of risky asset, respectively, with $s_0^h + s_0^b = K$. The households and banks are also endowed with e_0^h and e_0^b units of consumption good, respectively.

At time 0, agents can borrow and lend between each other but they have no commitment to pay off the debt at time 1. However, agents can borrow by setting the risky asset as collateral. Since for each unit of promised repayment we do not know what the collateral value, or haircut, should be for a given interest rate or vice versa, we choose to model this borrowing and lending with a competitive market for collateralized bonds. Thus we are able to transform the non-price variable, haircut, into the type of a commodity and let the market make a choice of it and determine the corresponding price at the same time. We normalize the face value of all bonds to 1.[10] There are potentially a continuum of bonds

[9]We assume for convenience, as usual in the literature, that banks' time-0 consumption can be negative which can be interpreted as production of consumption good with a one-to-one labor cost. Alternatively, we can assume it is nonnegative but banks' endowment in consumption good is large relative to households' endowment in the risky asset such that this nonnegative constraint does not bind.

[10]For a simply debt contract in Simsek (2013), the unit of collateral asset is normalized to 1 and the contracts are indexed by their face values. For the convenience of working with bonds, I choose to normalize

with different units of risky asset set as collateral. In particular, we index the collateralized bond backed by s units of risky asset by s and call it bond-s. Therefore, the set of bonds is $\{\text{bond-}s : s \in S \equiv (0, \infty)\}$. At time 1, for each unit of bond-s, if it does not default, the holder gets the face value 1; if it defaults, the holder seizes its collateral obtaining the dividend of the risky asset. So the payoff of bond-s is $min(1, sD_1)$. Let q^s be the price of bond-s in terms of time-0 consumption good. There is also a competitive market for the risky asset at time 0, which is a claim for D_1 units of consumption good at time 1, with a price p_0. Agents are free in trading the risky asset and buying collateralized bonds, but specific amount of collateral is required for issuing any bond.

From the perspective of corporate finance, each bond-s issuance is equivalent to establishing a firm with a capital structure interpreted as following: the firm's total asset is s shares of the risky asset with a value sp_0, the debt holder or the lender has a claim for \$1 next period with a current value q^s, and the equity holder or the borrower keeps the residual next period with a current value $sp_0 - q^s$. The leverage ratio (debt/equity) of the firm is $q^s/(sp_0 - q^s)$, which is negatively related to the equity/asset ratio, or the haircut in a repo contract. It is also helpful to keep in mind that the expected return on equity $r_e = (1 + leverage) * r_a - leverage * r_d$, where the expected return on asset is $r_a = E(D_1)/p_0$ and the expected return on debt is $r_d = E[min(1, sD_1)]/q^s$.

The households maximize their expected discounted utility. Although consumption smoothing is feasible by simply holding the risky asset, they are willing to hold some less risky assets based on a risk-return tradeoff because of risk aversion. However, given that the banks are risk neutral, the risky asset would have the same expected return as the risk-free rate if any and, therefore, be dominated. In the absence of a risk-free asset in the economy, it turns out that a preferred way of deferring the current consumption good to time 1 is to buy collateralized bonds at time 0 in the market, which are "hybrid" assets in

the face value to 1. Nevertheless, they are equivalent.

the sense that they inherit the downside risk of the risky asset but are free of the upside risk. Therefore, households are the natural buyers of bonds and, hence, borrowers in the economy. They have no interest in doing the reverse, i.e., buying more risky asset and issuing bonds, which means a leverage. We will analyze this more formally in the model. So far no assumption is made to restrict their portfolios.

At this point, we have no clue about which types of bonds would be traded. We model the bond portfolio of the household in the most general way as two non-negative measures over the bond space $\mu^h, \mu^h_- : S \to \mathbb{R}$, corresponding to the long and short portfolios respectively.[11] Hence, for any Borel subset $S' \subseteq S$, $\mu^h(S')$ ($\mu^h_-(S')$) describes in the household's bond portfolio the mass of bonds with collateral level belonging to S' that the household bought (issued). Therefore, given the competitive bond prices $\{q^s\}_{s\in S}$ and the risky asset price p_0, the household's problem is[12]

$$\max_{\{c_0^h, c_1^h, s_+^h, \mu^h, \mu_-^h\}} u(c_0^h) + \beta^h E(u(c_1^h))$$

$$s.t. \quad c_0^h + s_+^h p_0 + \int_S q^s d\mu^h \leq e_0^h + s_0^h p_0 + \int_S q^s d\mu_-^h, \tag{2.1}$$

$$c_1^h + \int_S min(1, sD_1) d\mu_-^h \leq s_+^h D_1 + \int_S min(1, sD_1) d\mu^h, \tag{2.2}$$

$$\int_S s d\mu_-^h \leq s_+^h, \tag{2.3}$$

where s_+^h is the number of shares of risky asset the household purchases at time 0. Condition (2.1) says with the endowment at time 0 and the proceeds from issuing bonds, the household can either consume, buy the risky asset or bonds. Condition (2.2) says the household's

[11]Mathematically, μ^h and μ^h_- are Borel measures on the set S. That is, they are weakly increasing and right continuous with left limits. $\mu^h(s) - \lim_{s'\to s^-} \mu^h(s') \geq 0$ and $\mu^h_-(s) - \lim_{s'\to s^-} \mu^h_-(s') \geq 0$. Note that they may not be absolutely continuous with respect to the Lebesgue measure and, therefore, do not have a density function. They are also mutually exclusive.

[12]Variables with subscripts + or -, which indicate long and short positions respectively, are irrelevant in equilibrium, as will be seen later. Namely, in the equilibrium the households do not hold trees or issue bonds. Similarly, the banks do not buy bonds in their problem described below.

consumption at time 1 comes from the realized payoffs from the risky asset and long positions of bonds net of payments for short positions of bonds. The condition (2.3) is a collateral constraint which requires the household to hold enough risky asset as collateral for all bonds issued.

The banks also maximize the expected discounted utility. However, being risk neutral, they try to hold the asset with maximum expected return. In particular, they are in favor of leverage which is what collateralized borrowing achieves. Put in another way, they can finance their purchase of risky asset by issuing bonds and putting up the risky asset as collateral for the borrowing. Analogous to the household's problem, let the bank's short and long positions of bond portfolio be non-negative measures over the bond space $\mu^b, \mu^b_+ : S \to \mathbb{R}$, respectively.[13] Hence, for any Borel subset $S' \subseteq S$, $\mu^b(S')$ $(\mu^b_+(S'))$ describes among the bank's bond issuances (purchases) the mass of bonds with collateral level belonging to S'. Given the competitive bond prices $\{q^s\}_{s \in S}$ and the risky asset price p_0, the bank's problem is

$$\max_{\{c_0^b, c_1^b, s^b, \mu^b, \mu^b_+\}} c_0^b + \beta^b E(c_1^b)$$

$$s.t. \quad c_0^b + s^b p_0 + \int_S q^s d\mu^b_+ \leq e_0^b + s_0^b p_0 + \int_S q^s d\mu^b, \tag{2.4}$$

$$c_1^b \leq s^b D_1 + \int_S min(1, sD_1) d\mu^b_+ - \int_S min(1, sD_1) d\mu^b, \tag{2.5}$$

$$\int_S s d\mu^b \leq s^b, \tag{2.6}$$

where s^b is the number of shares of risky asset the bank purchases at time 0. Condition (2.4) says with the endowment at time 0 and the proceeds from issuing different types of bonds, the bank can either consume, buy the risky asset or bonds. Condition (2.5) says the bank's consumption at time 1 comes from the realized payoffs from the risky asset and

[13] $\mu^b(s) - \lim_{s' \to s^-} \mu^b(s') \geq 0$ and $\mu^b_+(s) - \lim_{s' \to s^-} \mu^b_+(s') \geq 0$.

long positions of bonds net of the payoffs of bonds to the lenders. The condition (2.6) is a collateral constraint which requires the bank to hold enough risky asset as collateral for all bonds issued.

Definition 2. *(Competitive Equilibrium with Homogenous Agents) A competitive equilibrium of the economy is a price system* $\{p_0, \{q^s\}_{s\in S}\}$ *and an allocation* $\{c_0^h, c_1^h, c_0^b, c_1^b\}$ *and asset holdings* $\{\mu^h, \mu^h_-, s^h_+, \mu^b_+, \mu^b, s^b\}$ *such that*

1. *Given the price system,* $\{c_0^h, c_1^h, \mu^h, \mu^h_-, s^h_+\}$ *solve the household's problem, and* $\{c_0^b, c_1^b, \mu^b_+, \mu^b, s^b\}$ *solve the bank's problem;*

2. *The bond market clears:* $\mu^h = \mu^b$, $\mu^h_- = \mu^b_+$ *;*

3. *The risky asset market clears:* $s^h_+ + s^b = K$*;*

4. *The consumption good markets clear:* $c_0^h + c_0^b = e_0^h + e_0^b$*, and* $c_1^h + c_1^b = KD_1$*.*

2.2.1 Benchmark: Traditional Banking with Full Commitment

When agents have full commitment, banks are taking deposit as in traditional banking with enough insurance, although the insurance is limited in reality.[14] A risk-free asset is available in the economy and all assets earn the same expected return, the risk-free rate, because of the risk neutrality of banks. For risk averse households, the risk-free asset dominates other assets and they smooth consumption by saving at a market risk-free rate. From the bank's preference, the market gross risk-free rate would be $R^* = 1/\beta^b$ and the price for the risky asset would be $p_0^* = \beta^b E(D_1)$.

[14] The current FDIC standard deposit insurance amount in 2015 is $250,000 per depositor, per insured bank, for each account ownership category.

The household's problem is

$$\max_{\{c_0^h, c_1^h\}} u(c_0^h) + \beta^h E(u(c_1^h))$$
$$s.t. \quad c_0^h + \beta^b c_1^h \leq e_0^h + s_0^h \beta^b E(D_1),$$

which implies the Euler equation

$$\beta^b = \beta^h \left(\frac{c_1^h}{e_0^h + s_0^h \beta^b E(D_1) - \beta^b c_1^h} \right)^{-\eta}, \tag{2.7}$$

i.e., the bank and household have the same intertemporal marginal rate of substitution, the inverse of risk-free rate. Let $w^* = e_0^h + s_0^h p_0^*$ be households' initial wealth. So households' optimal saving rate is

$$\kappa^* = \frac{\beta^b}{\beta^b + (\beta^b/\beta^h)^{1/\eta}},$$

and consumption plan is $((1-\kappa^*)w^*, \kappa^* w^*/\beta^b)$. For $\eta \neq 1$, their expected discounted utility can be expressed as[15]

$$U_h^* = \frac{w^{*1-\eta}}{1-\eta}(1-\kappa^*)^{-\eta}.$$

The banks' expected utility is their initial wealth

$$U_b^* = e_0^b + s_0^b p_0^*$$

with a consumption plan $(e_0^b - s_0^h p_0^* + \kappa^* w^*, K D_1 - \kappa^* w^*/\beta^b)$.

Although agents can buy or issue collateralized bonds and the households can also

[15]This is a feature specific to CRRA utility. To see this for $\eta \neq 1$

$$U_h^* = \frac{(c_0^h)^{-\eta}}{1-\eta}\left[c_0^h + \beta^h E\left(\left(\frac{c_1^h}{c_0^h}\right)^{-\eta} c_1^h\right)\right] = \frac{(c_0^h)^{-\eta}}{1-\eta}[c_0^h + \beta^b E(c_1^h)] = \frac{(c_0^h)^{-\eta}}{1-\eta} w^*,$$

which can be interpreted as the product of time-0 marginal utility and initial wealth adjusted by $1-\eta$, or the one time utility from the initial wealth adjusted by the saving rate.

hold the risky asset, they have no incentive to do so. Moreover, the deposit opportunity improves the welfare of the households, while the banks' expected utility remains the same. Note that the equilibrium outcome is unchanged if the arrow securities are traded and the market gets complete. So this allocation is Pareto efficient.

2.3 Characterization of the Equilibrium

In this section, we characterize the equilibrium by first looking at the household's and bank's problems, respectively. As the previous analysis suggests and to save notation, we conjecture an equilibrium where only the banks hold the risky asset and issue bonds, which will be verified later. So currently we suppress the variables s_+^h, μ_-^h and μ_+^b.

For the household's problem, the budget constraints (2.1) and (2.2) must bind in equilibrium. Hence, $c_0^h = e_0^h + s_0^h p_0 - \int_S q^s d\mu^h$ and $c_1^h = \int_S min(1, sD_1) d\mu^h$. Let $\{\gamma^s \geq 0\}_{s \in S}$ be the Lagrangian multipliers associated with the nonnegative constraints (**??**) of bond positions. The first order conditions for the household's problem are bond pricing conditions

$$q^s = E\Big[\underbrace{\beta^h \frac{u'(c_1^h)}{u'(c_0^h)}}_{\text{pricing kernel } M^h(D_1)} min(1, sD_1)\Big] + \frac{\gamma^s}{u'(c_0^h)}, \forall s \in S,$$

and slackness conditions

$$\gamma^s \begin{cases} = 0, & \text{if bond-}s \text{ is actively traded;} \\ \geq 0, & \text{if bond-}s \text{ is not actively traded.} \end{cases}$$

In particular, for the actively traded bonds, their market prices equal the household's valuations $E\big[M^h(D_1) min(1, sD_1)\big]$ which can be considered as their bid prices. For any non-traded bond, it is too expensive relative to the household's valuation which is captured by the multiplier $\frac{\gamma^s}{u'(c_0^h)} \geq 0$.

For the bank's problem, the budget constraints (2.4) and (2.5) must bind in equilibrium too. Hence, $c_0^b = e_0^b + (s_0^b - s^b)p_0 + \int_S q^s d\mu^b$ and $c_1^b = s^b D_1 - \int_S min(1, sD_1)d\mu^b$. Let $\lambda^b \geq 0$ be the Lagrangian multiplier associated with the collateral constraint (2.6) and $\{\lambda^s \geq 0\}_{s \in S}$ be the Lagrangian multipliers associated with the nonnegative constraints (**??**) of bond positions. The first order conditions for the bank's problem are bond pricing conditions

$$q^s = E\big[\beta^b min(1, sD_1)\big] + \lambda^b s - \lambda^s, \forall s \in S,$$

the risky asset pricing condition

$$p_0 = E(\beta^b D_1) + \lambda^b, \tag{2.8}$$

and slackness conditions

$$\lambda^b \left(s^b - \int_S s d\mu^b \right) = 0,$$

$$\lambda^s \begin{cases} = 0, & \text{if bond-}s\text{ is actively traded;} \\ \geq 0, & \text{if bond-}s\text{ is not actively traded.} \end{cases}$$

λ^b reflects the scarcity of collateral. If the collateral constraint binds, the market price of the risky asset is its fundamental value $E(\beta^b D_1)$ for the bank plus a *collateral value* λ^b. For any actively traded bond, the bond market price equals the bank's cost of issuing that bond which we can think of as an ask price consisting of the bank's valuation of the obligation, $E\big[\beta^b min(1, sD_1)\big]$, and a premium, $\lambda^b s$, paid for the collateral required. The higher the collateral requirement s, the higher the premium is. For any non-traded bond, the bond market price is not enough to compensate the bank's cost of issuing the bond which is captured by the multiplier $\lambda^s \geq 0$. If the collateral constraint does not bind, the bank's valuation of the risky asset and the cost of issuing any bond are equal to their fundamental values.

If an equilibrium exists, then for any actively traded bond-s, the household's valuation of the bond must be equal to the bank's cost of issuing the bond, and they are both consistent with the bond market price q^s. For any non-traded bond-s, the household's valuation of the bond must be lower than or equal to the bank's cost of issuing the bond, and bond market price q^s is somewhere in between. Define $L(s)$ as the *liquidity value*[16] of bond-s, which is the difference between the bond fundamental values for the household and bank

$$L(s) = E[\quad \underbrace{(M^h(D_1) - \beta^b)}_{\text{Difference in pricing kernels}} \quad min(1, sD_1)]. \tag{2.9}$$

Let $S^* \subseteq S$ be the set of actively traded bonds in equilibrium. Then as analyzed above, we must have

$$L(s) = \lambda^b s, \forall s \in S^*; \quad L(s) \leq \lambda^b s, \forall s \notin S^*. \tag{2.10}$$

The liquidity value $L(s)$ depends on the household's bond portfolio $\mu^h|_{S^*}$, which fully pins down the household's consumption in both periods, as well as the distribution of dividend D_1. In equilibrium, given the bond holdings $\mu^h|_{S^*}$, the household's time-0 consumption is fixed and time-1 consumption $c_1^h(D_1)$ is a continuous, (weakly) increasing and (weakly) concave function of D_1 with $\lim_{D_1 \to 0} c_1^h(D_1) = 0$ and $\lim_{D_1 \to \infty} c_1^h(D_1) = \mu^h(S^*)$.[17] Hence the household's pricing kernel $M^h(D_1)$ is continuous and (weakly) decreasing with $\lim_{D_1 \to 0} M^h(D_1) = \infty$ and $\lim_{D_1 \to \infty} M^h(D_1) = M^h(\mu^h(S^*))$. The following properties for

[16]The terminology introduced by Fostel and Geanakoplos (2008).

[17]It is possible that $\lim_{D_1 \to \infty} c_1^h(D_1) = \mu^b(S^*) = \infty$ and $\lim_{D_1 \to \infty} M^h(D_1) = M^h(\mu^h(S^*)) = 0$.

the function $L(s)$ are intuitive:

$$\lim_{s \to 0} L(s) = 0, \tag{2.11}$$

$$L'(s) = \int_0^{1/s} (M^h(D_1) - \beta^b) D_1 dF(D_1), \tag{2.12}$$

$$L''(s) = -\frac{1}{s^3}\left(M^h\left(\frac{1}{s}\right) - \beta^b\right) f\left(\frac{1}{s}\right). \tag{2.13}$$

If the level of collateral is low enough, the bank and household agree that the bond value is close to zero. When the level of collateral increases, the marginal increase in the bond value for the bank is the expected value of losing a tree upon a default, while it is the expected value of obtaining a tree upon a default for the household. With different pricing kernels, they have different valuations of a marginal increase in collateral, which drives the shape of the liquidity value function. If the household's pricing kernel is higher at the default threshold of dividend, the difference in the valuation of a marginal increase in collateral decreases since the bond gets safer at the margin.

We can go one step further to have the following insights on the equilibrium. The household's pricing kernel is decreasing but in equilibrium it cannot be greater than the bank's for all levels of dividend. Otherwise, an equilibrium satisfying (2.10) cannot be obtained even with a binding collateral constraint which adds a linear issuance cost for the bank.[18] See Case I in Figure 2.1. Hence, we are left with possibilities in equilibrium: (i) the household's pricing kernel intersects the bank's at some dividend level $\hat{D}_1 = 1/\hat{s}$,[19] see Case II in Figure 2.1; or (ii) the household's pricing kernel touches the bank's at some

[18] Note that the bank's valuation is then lower than the household's for any bond and the bank's valuation of a marginal increase in collateral is also always lower (the function $L(s)$ is strictly increasing) and, in particular, the marginal difference is strictly decreasing to zero as the level of collateral approaches infinity (the function $L(s)$ is strictly concave).

[19] In this case, $M^h(\hat{D}_1) = \beta^h$, $M^h(D_1) > \beta^h, \forall D_1 \in (0, \hat{D}_1)$, and $M^h(D_1) < \beta^h, \forall D_1 \in (\hat{D}_1, \infty)$. Moreover, $L'(\hat{s}) > 0$ and $L''(\hat{s}) = 0$; $\forall s \in (0, \hat{s})$, $L''(s) > 0$, and $\lim_{s \to 0} L'(s) = E[(M^h(D_1) - \beta^b)D_1]$; $\forall s \in (\hat{s}, \infty)$, $L''(s) < 0$, and $\lim_{s \to \infty} L'(s) = 0$.

dividend level $\tilde{D}_1 = 1/\tilde{s}$ and remains the same beyond that level,[20] see Case III in Figure 2.1.

Based on these properties of the liquidity value function, we have the following result regarding the set of actively traded bonds in equilibrium.

Lemma 4. *If an equilibrium exists, the set of actively traded bonds $S^* \subseteq S$ satisfying (2.10) is a singleton, i.e., $S^* = \{\bar{s}\}$. Moreover, the collateral constraint binds.*

Proof. See Appendix. □

Now we are ready to show the existence and characterize an equilibrium. Suppose in equilibrium the bond-$\bar{s}$ is traded and the collateral constraint binds, then the trading volume is $\frac{K}{\bar{s}}$. From the household's problem, the price for bond-$\bar{s}$ is

$$q^{\bar{s}} = E\left[\beta^h \left(\frac{min(1, \bar{s}D_1)K/\bar{s}}{e_0^h + s_0^h p_0 - q^{\bar{s}} K/\bar{s}}\right)^{-\eta} min(1, \bar{s}D_1)\right]. \tag{2.14}$$

The households and the banks agree on this price, that is, $L(\bar{s}) - \lambda^b \bar{s} = 0$, which implies that the risky asset's collateral value is given by

$$\lambda^b = \frac{q^{\bar{s}} - \beta^b E[min(1, \bar{s}D_1)]}{\bar{s}}. \tag{2.15}$$

So the price of collateral asset is boosted up by a collateral value such that banks are indifferent in issuing the type of bond traded. The first order condition $L'(\bar{s}) - \lambda^b = 0$

[20]In this case, $M^h(D_1) > \beta^h, \forall D_1 \in (0, \tilde{D}_1)$ and $M^h(D_1) = \beta^h, \forall D_1 \in [\tilde{D}_1, \infty)$, and any bond-$s$ with $s < \tilde{s}$ is not traded and at least bond-$\tilde{s}$ is traded.

implies[21]

$$\beta^b = \beta^h \left(\frac{K/\bar{s}}{e_0^h + s_0^h p_0 - q^{\bar{s}} K/\bar{s}} \right)^{-\eta}, \tag{2.16}$$

which says the household's and bank's pricing kernels are the same when the dividend exceeds the default threshold and the household gets the maximum time-1 consumption. So the Case III in Figure 2.1 turns out to be true in equilibrium. Substituting (2.16) into (2.14), we get the bond price as a function of collateral level $\bar{s}$

$$q^{\bar{s}} = \beta^b E[min(1, \bar{s}D_1)^{1-\eta}], \tag{2.17}$$

and substituting it into (2.15)

$$\lambda^b = \frac{\beta^b E[min(1, \bar{s}D_1)^{1-\eta}] - \beta^b E[min(1, \bar{s}D_1)]}{\bar{s}} > 0. \tag{2.18}$$

For actively traded bond-$\bar{s}$, the household's valuation is higher than the bank's. Hence the banks would like to issue as many bonds-$\bar{s}$ as possible. But this drives up the price of the risky asset which is required for issuing any amount of bond. In equilibrium, the price of risky asset is such that the banks are indifferent between issuing or not issuing the actively traded bond, which is unique, and prefer not issuing any other bond.

The risky asset pricing condition (2.8) together with (2.16), (2.17) and (2.18) form a system of equations for $\bar{s}$, $q^{\bar{s}}$, λ^b and p_0. Indeed, there exists a unique solution.

[21] This can be seen from

$$\begin{aligned} L'(s)|_{s=\bar{s}} - \lambda^b &= \int_0^{1/\bar{s}} \left[\beta^h \left(\frac{min(1, \bar{s}D_1)K/\bar{s}}{e_0^h + s_0^h p_0 - q^{\bar{s}} K/\bar{s}} \right)^{-\eta} - \beta^b \right] D_1 dF(D_1) - \frac{q^{\bar{s}} - \beta^b E[min(1, \bar{s}D_1)]}{\bar{s}} \\ &= \frac{1}{\bar{s}} \left[\int_{1/\bar{s}}^{\infty} \beta^b dF(D_1) - \int_{1/\bar{s}}^{\infty} \beta^h \left(\frac{min(1, \bar{s}D_1)K/\bar{s}}{e_0^h + s_0^h p_0 - q^{\bar{s}} K/\bar{s}} \right)^{-\eta} dF(D_1) \right] \\ &= \frac{1}{\bar{s}} \left(1 - F\left(\frac{1}{\bar{s}}\right) \right) \left[\beta^b - \beta^h \left(\frac{K/\bar{s}}{e_0^h + s_0^h p_0 - q^{\bar{s}} K/\bar{s}} \right)^{-\eta} \right] \\ &= 0. \end{aligned}$$

Theorem 1. *There exists a unique equilibrium in which the collateral constraint binds and a single bond-$\bar{s}$ is actively traded, i.e., $S^* = \{\bar{s}\}$, with $\bar{s} \in S$ determined by*

$$\beta^b = \beta^h \left(\frac{K/\bar{s}}{e_0^h + s_0^h E(\beta^b D_1) - \{(K - s_0^h)\beta^b E[min(1, \bar{s}D_1)^{1-\eta}] + s_0^h \beta^b E[min(1, \bar{s}D_1)]\}/\bar{s}} \right)^{-\eta}. \tag{2.19}$$

The collateral value for the risky asset is given by (2.18) and the price for the risky asset is given by (2.8). The price for bond-$\bar{s}$ is given by (2.17). For any bond-s, $s \in S \setminus \{\bar{s}\}$, the price q^s is indeterminate and can be anything in the bid-ask interval

$$\left[\beta^b E[min(1, \bar{s}D_1)^{-\eta} min(1, sD_1)], \beta^b E[min(1, sD_1)] + s\lambda^b\right]. \tag{2.20}$$

The allocations are

$$\begin{aligned}
(c_0^h, c_1^h) &= \left(e_0^h + s_0^h p_0 - \frac{K}{\bar{s}} q^{\bar{s}}, \frac{K}{\bar{s}} min(1, \bar{s}D_1) \right), \\
(c_0^b, c_1^b) &= \left(e_0^b - s_0^h p_0 + \frac{K}{\bar{s}} q^{\bar{s}}, \frac{K}{\bar{s}} max(\bar{s}D_1 - 1, 0) \right),
\end{aligned}$$

with bond portfolio $\mu^h = \mu^b$ being a Dirac measure with mass $\frac{K}{\bar{s}}$ at $\bar{s} \in S$.

Proof. See Appendix. □

Recall that we conjectured the equilibrium in which only banks hold the risky asset and issue bonds. It is the time to verify the households indeed have no incentive to do the same thing in the equilibrium characterized above. It is clear that it is not in the household's interest to issue collateralized bonds because the fundamental values of bonds for them are already higher than those for the banks, $L(s) > 0$, let alone the issuer of bonds needs to pay a premium due to the collateral required. For the risky asset, its value for the households turns out to be the same as the market price, making them indifferent in holding a marginal share of it. To see this,

$$E[M^h(D_1)D_1] - p_0 = \int_0^{1/\bar{s}} (M^h(D_1) - \beta^b)D_1 dF(D_1) - \lambda^b = L'(\bar{s}) - \lambda^b = 0. \tag{2.21}$$

A dividend payment higher than $1/\bar{s}$ has the same value for the household and bank. However, the household's excess valuation of dividend payments lower than $1/\bar{s}$ over the bank's exactly matches the collateral value of the risky asset. This completes the characterization. In Part B of the Appendix, it is shown that this equilibrium is robust to a heterogeneous wealth distribution.

Several remarks are helpful to clarify the equilibrium obtained. First, one of the key insights in the characterization is that market selects the type of bond $\bar{s}$ with the maximum liquidity value per unit of collateral *given* the households' pricing kernel in equilibrium. This does not mean $\bar{s}$ maximizes the unconditional liquidity value per unit of collateral or the collateral value in (2.18). Indeed, we will see later that $\partial\lambda^b/\partial\bar{s} < 0$ so a maximum does not exist.

Second, banks choose to issue bond-$\bar{s}$ because prices of other bonds are too low. This can certainly be interpreted as maximizing the expected leveraged return given the equilibrium prices $\{q^s\}_{s\in S}$, for example, at the lower bound of the bid-ask interval (2.20). Simsek (2013) shows that the collateral constraint provides a discipline against the excess leverage taken by the borrower since the financing cost is increasing in the leverage.[22] But here the

[22] In the model of collateralized lending between the pessimist and the optimist in Simsek (2013), the market selection of the contract traded is equivalent to a principal-agent problem in which, subject to lender's participation constraint, the optimist (borrower) maximizes an expected return analogous to (no discounting)

$$r'_e = \frac{E_o[\bar{s}D_1] - E_o[min(1, \bar{s}D_1)]}{\bar{s}p_0 - E_p[min(1, \bar{s}D_1)]} \tag{2.22}$$

with E_o and E_p denoting the expectation operators by the risk-neutral optimist and pessimist (lender), respectively. The optimist wants to leverage the asset return $r'_a = E_o(D_1)/p_0$ by financing with a perceived gross interest rate cost of

$$r'_d = \frac{E_o[min(1, \bar{s}D_1)]}{E_p[min(1, \bar{s}D_1)]} \geq 1, \tag{2.23}$$

expected cost of deposit financing in equilibrium is

$$r_d = \frac{E[min(1, \bar{s}D_1)]}{q^{\bar{s}}} = \frac{E[min(1, \bar{s}D_1)]}{\beta^b E[min(1, \bar{s}D_1)^{1-\eta}]}, \tag{2.24}$$

which is not necessarily decreasing in $\bar{s}$ so as to provide a force against leverage.[23]

Third, the market's *unconditional* selection of the type of bond traded is based on an equality of marginal rate of substitution (2.16) between banks and households in non-default states. Analogous to the Euler equation (2.7) in the benchmark, it is as if households treat the nominal interest rate $1/q^{\bar{s}}$ as risk free and save the amount of $K/\bar{s}$. This condition comes from the observation that only one type of bond would be traded, the characterization of the set of bonds actively traded (2.10) and the implied first order condition.

2.3.1 Welfare Analysis

Let us first look at the expected returns on the contingent claims from the banks' perspective. The expected cost of deposit financing is given in (2.24) with $r_d < 1/\beta^b$ which is cheap relative to their time preference. But they cannot make profit directly from it because of the collateral requirement. The holding of collateral indeed incurs a cost for banks as can be seen by the expected return on the risky asset

$$r_a = \frac{E[\bar{s}D_1]}{\bar{s}p_0} = \frac{E(D_1)}{\beta^b E(D_1) + \lambda^b} < \frac{1}{\beta^b}, \tag{2.25}$$

which would be too low for banks to hold it in the absence of the purpose of using it for collateral. Although banks are the marginal investors of the risky asset in both settings with or without commitment, in this shadow banking system, the price is more than the simple expected discounted dividend because of a collateral value on top of it. Put in

which is decreasing in the collateral level $\bar{s}$ providing a counterforce against the optimist's desire for leverage.

[23] For example, when $\eta \in (1, \infty)$.

another way, we can say the banks try to leverage the low expected return on holdings of the risky asset by financing at an even lower cost of the deposit since $r_a > r_d$. In either way, the banks' true return on capital is the equity return

$$r_e = \frac{E[max(\bar{s}D_1 - 1, 0)]}{\bar{s}p_0 - q^{\bar{s}}} = \frac{1}{\beta^b}. \tag{2.26}$$

So the low return on the collateral asset and the low cost of deposit financing exactly cancel out and banks actually do not make any profit from this business. This is not surprising since banks are indifferent in issuing bond-$\bar{s}$ in equilibrium. Therefore, banks' expected utility is their initial wealth $U_b = e_0^b + s_0^b p_0$. If banks are not endowed with any risky asset, they would have the same expected utility as in the benchmark of traditional banking. However, if banks are endowed with an amount of the risky asset, they would enjoy welfare gains relative to the traditional banking because the risky asset endowment is earning a collateral value, $p_0 > p_0^*$.

Now we look at the welfare of households. Their initial wealth is $w = e_0^h + s_0^h p_0$ and saving rate is

$$\kappa = \frac{q^{\bar{s}}}{q^{\bar{s}} + (\beta^b/\beta^h)^{1/\eta}}.$$

It turns out that for $\eta \neq 1$ their expected discounted utility can again be expressed as[24]

$$U_h = \frac{w^{1-\eta}}{1-\eta}(1-\kappa)^{-\eta} \tag{2.28}$$

[24]Note that when $\eta \neq 1$

$$\begin{aligned} U_h &= \frac{(c_0^h)^{-\eta}}{1-\eta}\left[c_0^h + \frac{K}{\bar{s}}\beta^h E\left(\left(\frac{K/\bar{s}}{c_0^h}\right)^{-\eta} min(1, \bar{s}D_1)^{1-\eta}\right)\right] \\ &= \frac{(c_0^h)^{-\eta}}{1-\eta}\left[w - \frac{K}{\bar{s}}q^{\bar{s}} + \frac{K}{\bar{s}}\beta^b E[min(1, \bar{s}D_1)^{1-\eta}]\right] \\ &= \frac{(c_0^h)^{-\eta}}{1-\eta}w. \end{aligned} \tag{2.27}$$

i.e., the one time utility from the wealth adjusted by the saving rate.

If households are not endowed with the risky asset, then $w = w^*$. For $\eta \in (0,1)$, the nominal interest rate in shadow banking is greater than the risk-free rate in traditional banking $1/q^{\bar{s}} > 1/\beta^b$ and the saving rate is distorted downward $\kappa < \kappa^*$. So the marginal utility at time 0 is lower and the expected utility is lower. For $\eta \in (1,\infty)$, the nominal interest rate in shadow banking is less than the risk-free rate in traditional banking $1/q^{\bar{s}} < 1/\beta^b$ and the saving rate is distorted upward $\kappa > \kappa^*$. So the marginal utility at time 0 is larger and the expected utility (negative) is again lower. Households save too little when their risk aversion is low and too much when it is high.

For the special case of a log-utility with $\eta = 1$, the nominal interest rate in the shadow banking is the same as the risk-free rate in the traditional banking $1/q^{\bar{s}} = 1/\beta^b$, so is the saving rate $\kappa = \kappa^*$. Although the nominal interest rate in the shadow banking is risky, the low payment in a bad future state is offset by households' high valuation of it at the same time. Nevertheless, their expected utility is still lower since their consumption at time 1 is the same as that in the benchmark when banks do not default but is lower otherwise.

When households are endowed with some risky asset, $s_0^h > 0$, they would experience a positive wealth effect of a higher risky asset price, $w = w^* + s_0^h\lambda^b$. But this is not enough to reverse the fact they are choosing an affordable but not optimal consumption plan in the benchmark.[25] The analysis above can be summarized as following:

Theorem 2. *The banks' expected utility in the shadow banking is no less than that in the traditional banking and is strictly higher if their endowment in the risky asset is positive, $s_0^b > 0$. The households are strictly worse off in the shadow banking compared to in the traditional banking.*

In this shadow banking, the collateral constraint resulting from the limited commitment

[25]Note that the cost of households' consumption plan here at the prices in the benchmark is $w^* - s_0^b\lambda^b$.

problem prevents agents from perfect risk sharing. The trading of collateralized bonds only facilitates the upside risk sharing leaving households to bear the downside risk which creates the inefficiency relative to the traditional banking. Because of the concern of downside risk, households are willing to pay an extra liquidity value to banks when buying bonds. Although agents are not profiting directly from deposits with banks being indifferent in issuing bond-$\bar{s}$ and households paying the fundamental value for bonds,[26] the excess valuation of the bond by households over banks generates a collateral value for the initial holders of the risky asset. Although banks and households can both benefit from this positive wealth effect, households are still worse off overall. Note that agents have homogenous beliefs and this collateral value are not from a speculative bet between optimists and pessimists.[27] Instead, it comes from the difference in the endogenously determined pricing kernels.

2.4 Scarcity of Collateral

The uniqueness of the type of bond traded in the equilibrium characterized above pins down the interest rate and haircut simultaneously in the shadow banking system. The framework also makes it feasible to analyze the response of these two variables of interest to varieties of shocks to the economy. Given the equilibrium prices, define the haircut H as

$$H = 1 - \frac{q^{\bar{s}}}{\bar{s}p_0} \propto \frac{\bar{s}p_0}{q^{\bar{s}}} = 1 + \frac{E[max(\bar{s}D_1 - 1, 0)]}{E[min(1, \bar{s}D_1)^{1-\eta}]}, \tag{2.29}$$

which is as a percentage of the collateral value, the difference between the collateral value and the money raised (bond price). For example, if a borrower puts up 2 shares of a stock with price \$0.5 and issues one unit of bond with face value \$1 at a price \$0.8, the haircut

[26] This is in contrast to Simsek (2013) in which the optimist/borrower has all the bargaining power and extracts all the surplus while the lender earns the same expected return as the storage technology.

[27] Speculative bet driven by heterogenous beliefs is the case in Simsek (2013) and Brunnermeier, Simsek and Xiong (2014).

is 20%, i.e., \$0.2. If we interpret the collateral value as the asset of a "firm" and the bond price as its debt, haircut is positively related to the asset/debt ratio or the equity/debt ratio at the most right hand side of the equation. Clearly, equity is the value of a call option with $1/\bar{s}$ playing the role of a strike price, and debt is households' valuation of a standard debt-type payoff. Define the interest rate R as

$$R = \frac{1}{q^{\bar{s}}} - 1,$$

so in the example above, the interest rate would be 25%.

We first look at the comparative statics with respect to the strength of the household sector, i.e., their initial endowments in consumption good and risky asset. Since the bank sector endowment in consumption good is irrelevant in determining the equilibrium prices (only matters for bank's time-0 consumption) in this economy, the shock to the household sector endowment in consumption good can be simply interpreted as a shock to the dividend per share of asset at time 0, keeping the asset endowments fixed. But for a positive shock to the household sector endowment in asset, that could be due to an increase in the total supply, keeping the bank sector strength fixed, or due to a reallocation from the bank to the household sector, keeping the total supply fixed. In either way, however, we have the following unambiguous predictions on the changes in the asset price, the interest rate and haircut.

Proposition 4. *When the household sector time-0 dividend endowment e_0^h or asset endowment s_0^h increases, keeping either the total supply of the asset K fixed or the bank's asset endowment s_0^b fixed, the collateral level $\bar{s}$ drops, the collateral value λ^b and the price of risky asset p_0 increase; the haircut H decreases. For $\eta \in (0, 1)$, the bond price $q^{\bar{s}}$ decreases and the interest rate R increases; for $\eta \in (1, \infty)$, the bond price $q^{\bar{s}}$ increases and the interest rate R decreases.*

Proof. See Appendix. □

The possible increase in interest rate is quite surprising since when the households become rich, they would like to smooth consumption and save more and, therefore, buy more bonds, which would typically push the interest rate down for traditional banking based on a simple demand/supply analysis. However, for the shadow banking, given the limited supply of risky asset[28], the higher demand for bonds first drives down the collateral level $\bar{s}$ per unit of bond. The bond becomes riskier but households' valuations of payments in bad states also increase. For $\eta \in (0, 1)$, the effect of an increased risk dominates, bond-$\bar{s}$ becomes cheaper implying a higher interest rate; for $\eta \in (1, \infty)$, the effect of higher valuations in bad states dominates, bond-$\bar{s}$ becomes more expensive implying a lower interest rate. The increase in the risky asset price is also counterintuitive when its total supply becomes higher in the economy. The reason is that in this economy, a low level of collateral coincides with a scarcity of collateral which implies a higher collateral value λ^b and price p_0 for the risky asset. Namely, we have $\partial \lambda^b / \partial \bar{s} < 0$ in general[29]. Furthermore, everything else being equal[30], the haircut is increasing in $\bar{s}$, that is, the number of shares put up as collateral exhibits a diminishing return to scale in terms of the amount of borrowing they can support. To see this, note that $\partial(\bar{s}p_0/q^{\bar{s}})/\partial \bar{s} > 0$. So the haircut decreases in the economy which, in together with a greater risk of the bond, facilitates the higher demand of saving. Note that, for $\eta \in (0, 1)$, an increase in the risky asset price is outweighed by a lower collateral level, so the collateral value $\bar{s}p_0$ declines.

On the other hand, if the bank sector gets stronger in a way of shifting the risky asset from the household to the bank sector, we simply have the results above reversed. The more interesting case is what would happen if the bank sector has more collateral available

[28]The increase in the total supply of asset in the second case is of second order.

[29]After controlling for banks' time preference β^b, the cdf F and the relative risk aversion coefficient η.

[30]After controlling for the cdf F and the relative risk aversion coefficient η.

while the strength of the household sector remains the same.

Proposition 5. *When banks' risky asset endowment s_0^b increases (K increases accordingly), the collateral level $\bar{s}$ increases, the collateral value λ^b and the price of risky asset p_0 decrease; the haircut H increases. For $\eta \in (0,1)$, the bond price $q^{\bar{s}}$ increases and the interest rate R decreases; for $\eta \in (1,\infty)$, the bond price $q^{\bar{s}}$ decreases and the interest rate R increases.*

Proof. See Appendix. □

The banks in this economy always pursue an expected return as high as possible. With more assets in hand, they want to leverage up. This creates an excess supply of bonds relative to households' demand at the equilibrium prices. This tension is solved by banks' increasing the collateral level, $\bar{s}$, for each unit of bond. The interest rate declines for $\eta \in (0,1)$ because the bond is safer, while it increases for $\eta \in (1,\infty)$ because households value payments in bad states less. The property of a diminishing return to scale in the use of collateral implies a higher haircut. Remember the haircut is the equity/asset ratio of a firm in the language of corporate finance, so a higher haircut means a lower leverage. Different from an increase in the supply of risky asset in the household sector, an increase in the bank sector is now of first order since the strength of the household sector is unchanged. The affluence of collateral implies a lower collateral value and, therefore, a lower asset price. We might expect banks to earn a lower expected return in a competitive market, but the return on asset increases while the cost of deposit financing may either increase or decrease, and the lower leverage is such that banks earn the same expected return $1/\beta^b$ as before.

Remark 2. *The prices in the economy, including the collateral level, the bond price and interest rate, the collateral value and risky asset price, and the haircut, are homogenous of degree 0 with respect to the total supply of risky asset, keeping the dividend per share and the proportion allocation of risky asset fixed, while the quantities of consumption and bond trading volume are homogenous of degree 1.*

2.5 Quality of Collateral

The quality of collateral in this economy is described by the distribution of future dividend of the risky asset at time 1. During the recent financial crisis, the housing market went down which deteriorated the quality of the mortgage-related collateral assets and led further to the turmoil in the shadow banking system including markets such as repo and asset backed commercial paper. Now we consider the theoretical predictions of the model on how changes in the quality of collateral affect its price and haircut as well as the interest rate of deposits.

To simplify the analysis and to focus on the role of quality of collateral in the equilibrium deposit contract, we shut down the channel of a wealth effect of collateral quality through households' endowment by assuming they are endowed with zero risky asset but positive consumption good at time 0. Then note that the households care about the asset quality only because of the credit risk of their deposit and in an asymmetric way. If a default would not happen, no matter how promising the asset is, the households enjoy no benefit; but the downside risk is relevant because they would keep the collateral upon default. On the other hand, the banks care about the upward potential of the risky asset because the downside risk in the asset holdings is hedged by the short positions in bonds, or put differently, they are shareholders of "firms". If a default is triggered, how bad the situation is has nothing to do with them. However, there is a caveat here. The downside risk still matters for banks through the collateral constraint because the bond price reflects the households' valuation of the downside risk.

This role played by the collateral constraint asymmetrically for the downside risk is similar to its asymmetric disciplining of the downside optimism in the Simsek's (2013) setting with belief disagreements, but the mechanisms are different. In Simsek (2013), the optimist (borrower) maximizes an expected return analogous to (2.22) by leveraging the asset return with a perceived financing cost given by (2.23). The more pessimistic is the lender relative to the borrower over the downside risk, the higher the perceived interest cost which disciplines the leverage taken and limits the impact of belief disagreements on the asset price. In the current setting, agents have the same belief and the perceived deposit financing cost for banks as given by (2.24) is even lower than their time preference $r_d < 1/\beta^b$ because households value payments in bad future states more. So the deposit financing is even a subsidy without taking the collateral requirement into account and the asymmetric disciplining story is absent here. Nevertheless, the downside risk plays an asymmetric role here relative to the upside risk. That being said, it is clear to see the impact of an improvement of asset quality in the upper tail in a sense of first order stochastic dominance (FOSD) or second order stochastic dominance (SOSD). Suppose the collateral level is $\bar{s}_0$ before the shock.

Proposition 6. *Assume $s_0^h = 0$. If there is a FOSD improvement of the quality of collateral over the upper tail $(\bar{s}_0, \infty)$, keeping the quality unchanged in the lower tail $(0, \bar{s}_0)$, then the collateral level $\bar{s}$, the bond price $q^{\bar{s}}$, the interest rate R, and the collateral value λ^b remain the same; the price of risky asset p_0, the collateral asset value $\bar{s}p_0$, and the haircut H increase.*

However, a mean preserving (mean increasing, respectively) SOSD improvement of the quality of collateral over the upper tail $(\bar{s}_0, \infty)$, keeping the quality unchanged in the lower tail $(0, \bar{s}_0)$, has no effect (the same effect as FOSD, respectively) in the economy.

Proof. See Appendix. □

If the quality of collateral in the lower tail $(0, \bar{s}_0)$ does not change, for either a FOSD or a SOSD improvement in the upper tail, households's valuations of bonds and banks' costs of issuing bonds do not change. So market selects the same of type of bond traded $\bar{s}$ as before, and the same bond price $q^{\bar{s}}$, interest rate R and collateral value prevail. For a FOSD improvement in the upper tail, the asset price increases because its fundamental value $\beta^b E[D_1]$ is higher, so is the collateral value $\bar{s}p_0$. Then the haircut increases because the money raised $q^{\bar{s}}$ is the same as before. For a mean preserving SOSD improvement in the upper tail, the fundamental value of the asset does not change, so does the haircut. The message here is that households only care about the downside risk, and any shock to the upper tail of the quality of asset and, hence, asset price will be reflected in the haircut without changing the interest rate. For banks, although the expected return on asset may increase while the cost of deposit financing does not change, their expected return on equity remains because of deleveraging.

Now we consider a change in the quality of collateral in the lower tail.

Proposition 7. *Assume $s_0^h = 0$. If there is a FOSD improvement of the quality of collateral over the lower tail $(0, \bar{s}_0)$, keeping the quality unchanged in the upper tail $(\bar{s}_0, \infty)$, then for $\eta \in (1, \infty)$, the collateral level $\bar{s}$ and the bond price $q^{\bar{s}}$ decrease, and the interest rate R increases; the asset price p_0 decreases; the haircut increases if and only if*

$$E[D_1|D_1 > 1/\bar{s}_0] > (\beta^h/\beta^b)^{1/\eta} e_0^h/K. \tag{2.30}$$

For $\eta \in (0, 1)$, the collateral level $\bar{s}$ and the bond price $q^{\bar{s}}$ increase, and the interest rate R drops; the asset price p_0 increases; the haircut decreases if and only if (2.30) holds.

Moreover, these predictions are the same for a SOSD improvement of the quality of collateral over the lower tail $(0, \bar{s}_0)$, keeping the quality unchanged in the upper tail $(\bar{s}_0, \infty)$.

Proof. See Appendix. □

Take the case $\eta \in (1, \infty)$ for illustration. Since the value of contingent bond payment $\beta^b min(1, \bar{s}D_1)^{1-\eta}$ for households is a strictly decreasing and strictly convex function of D_1 in the lower tail, a FOSD or SOSD improvement of the quality of collateral in this region decreases the households' valuation of the bond. The increased interest rate has a positive income effect on households who will save more pushing the collateral level $\bar{s}$ down.[31]

Although the improvement in the downside quality increases the fundamental value (except for mean preserving SOSD) and the decrease in $\bar{s}$ tends to increase the collateral value, these two effects are dominated by the decline in households' valuation of the bond which reduces the collateral value and, therefore, the risky asset price. To see it more clearly, recall that, in the capital structure terminology, the risky asset price is the firm value per unit of collateral and the firm value can be interpreted as the sum of equity and debt. Then we have[32]

$$p_0 = \frac{\text{equity} + \text{debt}}{\bar{s}} = \beta^b E\left[max\left(D_1 - \frac{1}{\bar{s}}, 0\right)\right] + \frac{e_0^h}{K} - \frac{1}{\bar{s}}\left(\frac{\beta^b}{\beta^h}\right)^{1/\eta}, \tag{2.31}$$

in which the equity per unit of collateral decreases because the strike price increases. In this economy with a binding collateral constraint, a lower $\bar{s}$ implies a higher consumption for households in non-default states. To have agents' intertemporal substitution equalized in non-default states, households must have saved less which indicates a lower bond price per unit of collateral. So the price of risky asset decreases unambiguously as its downside quality improves. The intuition is that when the risk aversion is high, households are desperate in smoothing consumption and willing to pay a higher bond price. The improvement in the downside quality of asset alleviates this tension and the bond price falls accordingly, so does the collateral value.

Although a lower $\bar{s}$ tends to reduce the haircut by the diminishing return to scale property, the improvement of quality of collateral decreases the debt value and thus does

[31] Although a lower $\bar{s}$ tends to increase the bond price in this case, it is of second order.

[32] When $s_0^h = 0$, the bond price can be expressed as $\bar{s}e_0^h/K - (\beta^b/\beta^h)^{1/\eta}$ as shown in the proof.

the opposite. This tradeoff can be reduced to a comparative statics with respect to only $\bar{s}$

$$H \propto \frac{\text{equity}}{\text{debt}} = \frac{\beta^b E[max(\bar{s}D_1 - 1, 0)]}{\bar{s}e_0^h/K - (\beta^b/\beta^h)^{1/\eta}} \tag{2.32}$$

which is decreasing in $\bar{s}$ if and only if the inequality (2.30) holds. The intuition behind is that both equity and debt are increasing in $\bar{s}$ and locally have negative intercepts, the ratio is decreasing in $\bar{s}$ if and only if the local slope-intercept (absolute value) ratio is larger for the equity. That is, if the conditional expected value of the upper tail of the risky asset is large relative to households' consumption good endowment per unit of total supply of collateral, the haircut increases.

For $\eta \in (0, 1)$, since the value of contingent bond payment $\beta^b min(1, \bar{s}D_1)^{1-\eta}$ for households is a strictly increasing and strictly concave function of D_1 in the lower tail, it is interesting that everything above goes in the opposite direction.

If the quality of collateral changes involving both the lower and upper tails, some of the results above can be generalized. For the interest rate, since only the downside quality matters for the bond payoff and therefore for the strike price $1/\bar{s}$ in equilibrium, the argument in Proposition 7 goes through and for either a FOSD or SOSD over the full support, the interest rate decreases for $\eta \in (0, 1)$ but increases for $\eta \in (1, \infty)$.

For the price of risky asset, first consider a FOSD improvement for $\eta \in (0, 1)$, it increases $\bar{s}$ as in Proposition (7) as well as the equity per unit of collateral in (2.31), so the price of risky asset increases. However, for $\eta \in (1, \infty)$, it decreases $\bar{s}$ but still increases the equity per unit of collateral, so the prediction is not clear. Now consider a SOSD, it increases $\bar{s}$ for $\eta \in (0, 1)$ while decreases $\bar{s}$ for $\eta \in (1, \infty)$; however, since a call option is increasing in the volatility, it decreases the equity per unit of collateral. So the price of risky asset decreases for $\eta \in (1, \infty)$ but it depends for $\eta \in (0, 1)$. These asymmetries are due to the different responses of equity and bond (under different risk aversions) valuations to changes of future distribution of dividend.

Similarly for the haircut, in the expression (2.32), the downside quality affects the number of call option $\bar{s}$ and the strike price $1/\bar{s}$ as before through its impact on the bond price, while the upside quality affects the equity. Since the prediction of first effect depends on the inequality (2.30), for a FOSD, the second effect strengths (loosens) the inequality for a $\eta \in (0, 1)$ ($\eta \in (1, \infty)$). However, for a SOSD, the second effect loosens (strengths) the inequality for a $\eta \in (0, 1)$ ($\eta \in (1, \infty)$).

2.6 Time and Risk Preferences

We now look at the effects of agents' time preferences.

Proposition 8. *When the household time preference β^h increases, the collateral level $\bar{s}$ drops, the collateral value λ^b and the price of risky asset p_0 increase; the haircut H decreases. For $\eta \in (0, 1)$, the bond price $q^{\bar{s}}$ decreases and the interest rate R increases; for $\eta \in (1, \infty)$, the bond price $q^{\bar{s}}$ increases and the interest rate R decreases.*

Proof. See Appendix. □

When the households become more patient, they would like to transfer more consumption to time 1 and, therefore, buy more bonds. This will have the same effects as a stronger household sector as in Proposition 4.

On the other hand, if the banks are more patient, there are two direct effects. First, the interest rate for bond-$\bar{s}$ tends to be lower which has a substitution effect inducing households to consume more and save less. Second, the lower interest rate has a negative income effect for households, while the higher price of the risky asset at given $\bar{s}$ has a positive wealth effect. Nevertheless, if households are not endowed with any risky asset or for large elasticity of intertemporal substitution, $\eta \in (0, 1)$, the substitution effect dominates.

Proposition 9. *Assume banks are endowed with all the risky asset, $s_0^h = 0$. When the bank time preference β^b increases, the collateral level $\bar{s}$ increases, the bond price $q^{\bar{s}}$ increases, and the interest rate R drops; the price of risky asset p_0 increases; the collateral asset value $\bar{s}p_0$ and the haircut H increase.*

If $s_0^h > 0$, as long as the elasticity of intertemporal substitution is large $\eta \in (0, 1)$, then when the bank time preference β^b increases, the collateral level $\bar{s}$ increases, the bond price $q^{\bar{s}}$ increases, and the interest rate R drops; the collateral asset value $\bar{s}p_0$ and the haircut H increase.

Proof. See Appendix. $\square$

When banks get more patient, the current business bringing them an expected return of $1/\beta^b$ becomes profitable, and they would like to expand the leveraged investment until the expected return is pushed down to a new zero profit level. When the wealth effect through the risky asset endowment is absent, $s_0^h = 0$, the higher intertemporal rate of substitution of banks gives rise to a substitution effect inducing households to save less. This results in a higher collateral level $\bar{s}$ for each unit of bond and a higher haircut follows from the decreasing return to scale property. Although a higher collateral level $\bar{s}$ tends to decrease the collateral value, banks' expanded investment due to improved patience increases the asset price. Even a higher collateral level $\bar{s}$ tends to decrease the bond price for $\eta \in (1, \infty)$, the upward shift of households' intertemporal rate of substitution dominates leading to a higher bond price and a lower interest rate. Note that this lower interest rate has a negative income effect for households which also induces them to save less. During the adjustment, banks experience a lower expected return on asset and a process of deleveraging.

We now look at the comparative statics with respective to η which plays two roles here governing both risk aversion and elasticity of intertemporal substitution of households. The rise aversion is reflected in the decreasing marginal utility and the pricing of contingent claims. We decompose the analysis in these two respects. To consider the effect of a higher risk aversion, assume households and banks have the same time preference.

Proposition 10. *Assume* $\beta^h = \beta^b$. *If* $s_0^h \in [0, K)$, *when the household's risk aversion* η *increases, the collateral level* $\bar{s}$ *and the bond price* $q^{\bar{s}}$ *increase, and the interest rate* R *declines; the collateral value* λ^b *and the price of risky asset* p_0 *increase; the collateral asset value* $\bar{s}p_0$ *increases, and the haircut increases if and only if* $E[D_1 | D_1 > 1/\bar{s}] < e_0^h/K$.

If $s_0^h = K$, *when the household's risk aversion* η *increases, the collateral level* $\bar{s}$ *is unchanged, the bond price* $q^{\bar{s}}$ *increases, and the interest rate* R *declines; the collateral value* λ^b *and the price of risky asset* p_0 *increase; the collateral asset value* $\bar{s}p_0$ *increases and the haircut* H *decreases.*

Proof. See Appendix. □

An increase in households' risk aversion increases the market price of the bond. As long as the households do not own the entire supply of risky asset, it has a negative income effect and decreases their saving demand implying a higher collateral level $\bar{s}$ for each unit of bond. Even an increase in $\bar{s}$ tends to decrease the bond price for $\eta \in (1, \infty)$, the bond price increases overall and the interest rate declines. At the same time, the increase in the bond price lowers the cost of deposit financing for banks. Their pursuit of leveraged investment would push up the asset price. While an increase in $\bar{s}$ tends to increase the haircut, the improved willingness of households to lend money has the opposite effect. A critical condition for a higher haircut is that the conditional mean quality of collateral in non-default states is lower than households' consumption good endowment per unit of supply of collateral asset in the economy.

When the wealth effect due to the impact the risk averse has on the prices, as well as the substitution effect, is absent, the type of bond traded remains the same. The bond price increases and interest rate drops since a higher risk aversion increases the value of payments in bad states. The risky asset price increases as it has a greater collateral value for banks who want to take advantage of the lower financing cost. The haircut drops simply because the households are more willing to contribute debt capital.

To consider the effect of a lower elasticity of intertemporal substitution, we assume households are endowed with all the risky asset so that the effects of risk aversion on the bond price and collateral value are offset by each other.

Proposition 11. *Assume $s_0^h = K$. If $\beta^h < \beta^b$, when the household's elasticity of intertemporal substitution $1/\eta$ decreases, the collateral level $\bar{s}$ decreases, the collateral value λ^b and the price of risky asset p_0 increase; the haircut H decreases. For $\eta \in (1, \infty)$, the bond price $q^{\bar{s}}$ increases, and the interest rate R declines.*

Proof. See Appendix. □

Since households are less patient, they maintain a negative consumption growth pattern. When the elasticity of intertemporal substitution decreases, the optimal consumption pattern becomes flatter. So households save more which pushes down the collateral level $\bar{s}$. Furthermore, a higher liquidity value for any bond-s resulting from a higher risk aversion attracts more business from banks in issuing bonds. Overall, this creates a scarcity of collateral increasing the collateral value and price of risky asset. The haircut decreases because of both a diminishing return to scale in using collateral and an enhanced willingness of household to purchase bonds as a result of higher valuations of payments in bad states. Although the bond price tends to increase for a higher risk aversion, households' low consumption in bad states from a riskier bond traded makes the change of its price depend on the degree of risk aversion. For $\eta \in (1, \infty)$, a higher risk aversion and a higher risk both increase the price of bond traded.

2.7 Conclusion

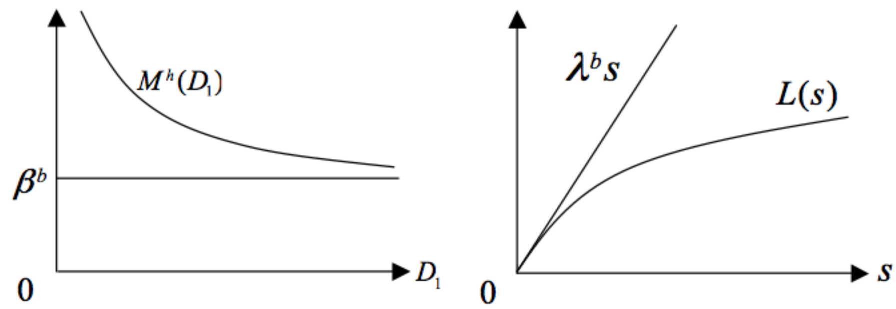
This paper provides a framework to analyze the joint determination of haircut and interest rate and the pricing of collateral assets in a shadow banking system. The key mechanism is that banks issue the type of bond with the highest liquidity value per unit of collateral, the asset price increases to bear a collateral value such that banks are indifferent in issuing that bond at the margin, and agents' marginal rates of substitution are equalized only in non-default states. In terms of welfare, banks are no worse than in the traditional banking but are strictly better off if endowed with the risky asset while households are strictly worse off.

The determinants of the variables of interest and their effects are fully documented. An increase in households' endowment increases the asset price and reduces the haircut with the change of interest rate depending on the tradeoff between state prices and credit risk governed by the level of risk aversion. An increase in banks' asset endowment has the opposite effects. The households' more patient time preference has the same effects as an increase in their endowment through the enhanced saving motive. The banks' more patient time preference (controlling for the wealth effect) reduces the interest rate and increases the haircut and asset price. Households' increased risk aversion (controlling for the substitution effect) reduces the interest rate and increases the asset price.

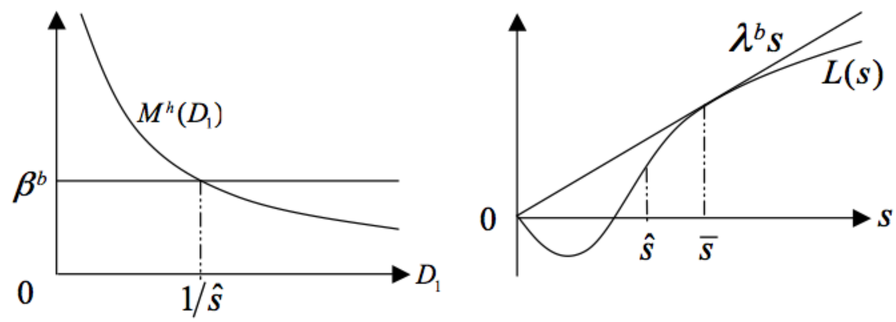
The upside and downside quality of collateral have asymmetric effects on the variables of interest. The improvement of upside quality only increases asset price with the haircut adjusting upward accordingly and the credit risk and interest rate unchanged. But the downside quality improvement has an impact on all variables. In particular, the effects are the opposite for households with a high versus a low risk aversion. This has empirical relevance since researchers typically use measures of asset overall quality such as variance of returns. This paper shows that the predictions regarding the effects of the overall quality on the haircut and interest rate are quite ambiguous and it is important to distinguish the upside and downside quality.

Several interesting properties in an economy with collateral are found including a diminishing return in scale in collateral use and a decreasing collateral value in the units of collateral, everything else being equal. Predictions on the changes in the asset price and interest rate may be far different from those made based on a typical demand/supply analysis. An increase in the supply of assets may raise their prices and an improvement of the quality of assets may bring their prices down. There is not necessarily a tradeoff between the haircut and interest rate and they can move in the same or the opposite directions depending on the type of shocks.

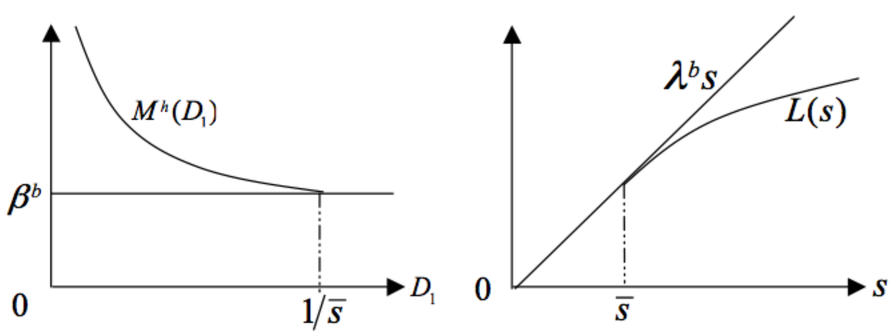
Figure 2.1: Pricing Kernels (left panel) and Liquidity Value of Bonds (right panel)



(a) Case I

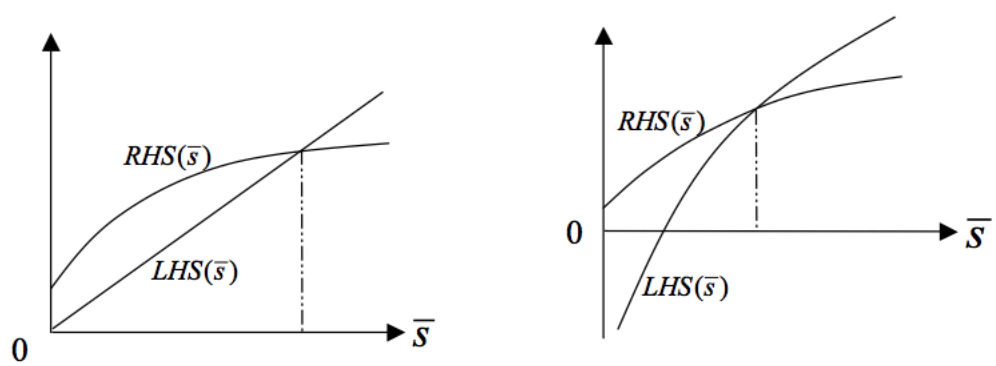


(b) Case II



(c) Case III

Figure 2.2: Existence and Uniqueness of Equilibrium



(a) $\eta \in (0, 1)$

(b) $\eta \in (1, \infty), s_0^h \in (0, K)$

Table 2.1: **Predictions on the Responses of Interest Rate, Haircut and Asset Price to Shocks**

		Predictions		
Shocks	Cases	Interest Rate R	Haircut H	Asset Price p_0
Household's endowments $e_0^h \uparrow$ or $s_0^h \uparrow$	-	$\uparrow$ for $\eta \in (0,1)$;$\downarrow$ for $\eta \in (1,\infty)$	$\downarrow$	$\uparrow$
Bank's endowment $s_0^b \uparrow$	-	$\downarrow$ for $\eta \in (0,1)$;$\uparrow$ for $\eta \in (1,\infty)$	$\uparrow$	$\downarrow$
Improving Collateral Quality ($s_0^h = 0$)				
FOSD/MI SOSD over upper tail $(1/\bar{s}_0, \infty)$	-	remains	$\uparrow$	$\uparrow$
MP SOSD over upper tail $(1/\bar{s}_0, \infty)$	-	remains	remains	remains
FOSD/SOSD over lower tail $(0, 1/\bar{s}_0)$	$\eta \in (0,1)$	$\downarrow$	$\downarrow$ iff $E[D_1 \mid D_1 > \frac{1}{\bar{s}_0}] > \left(\frac{\beta^h}{\beta^b}\right)^{\frac{1}{\eta}} \frac{e_0^h}{K}$	$\uparrow$
	$\eta \in (1,\infty)$	$\uparrow$	$\uparrow$ iff $E[D_1 \mid D_1 > \frac{1}{\bar{s}_0}] > \left(\frac{\beta^h}{\beta^b}\right)^{\frac{1}{\eta}} \frac{e_0^h}{K}$	$\downarrow$
Household's time preference $\beta^h \uparrow$	-	$\uparrow$ for $\eta \in (0,1)$;$\downarrow$ for $\eta \in (1,\infty)$	$\downarrow$	$\uparrow$
Bank's time preference $\beta^b \uparrow$	$s_0^h = 0$	$\downarrow$	$\uparrow$	$\uparrow$
	$s_0^h > 0, \eta \in (0,1)$	$\downarrow$	$\uparrow$	-
Household's risk aversion $\eta \uparrow (\beta^b = \beta^h)$	$s_0^h < K$	$\downarrow$	$\uparrow$ iff $E[D_1 \mid D_1 > \frac{1}{\bar{s}}] < \frac{e_0^h}{K}$	$\uparrow$
	$s_0^h = K$	$\downarrow$	$\downarrow$	$\uparrow$
Household's EIS $1/\eta \downarrow (s_0^h = K)$	$\beta^b > \beta^h$	$\downarrow$ for $\eta \in (1,\infty)$	$\downarrow$	$\uparrow$

Note: In this table, e_0^h and s_0^h are households' initial endowments in consumption good and risky asset, respectively. s_0^b is banks' initial endowment in risky asset. $K = s_0^h + s_0^b$ is the total supply of the risky asset. β^h and β^b are households' and banks' time preferences, respectively. η is households' degree of risk aversion. D_1 is the random dividend payment of the risky asset at time 1. $\bar{s}_0$ is the collateral level prior to shocks. FOSD and (MP, MI) SOSD stand for first and (mean preserving, mean increasing) second order stochastic dominance, respectively.

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Chapter 3

News Coverage of Intermediation and Stock Returns

This paper documents that an increase in the news coverage of intermediation, as measured by Manela and Moreira (2017) using phrase counts of front-page articles of the Wall Street Journal, is followed by lower stock market returns next month since the World War II. The effect is economically large with a one-standard-deviation increase in the coverage associated with an 44 basis point decrease in next month's stock market excess return. The in and out of sample R^2 statistics are 1.09% and 0.86%, respectively, outperforming the known popular predictors in the literature.

3.1 Introduction

This paper studies the asset pricing implications of the news coverage of financial intermediation. It is found that an increase in the intensity of news coverage of intermediation is followed by a significantly lower equity market return next month since the World War II.

The intensity of news coverage of intermediation is measured by the intermediation component of the news implied index of disaster concerns, NVIX, in Manela and Moreira (2017). This measure is based on the frequencies of intermediation related phrases appearing on the *Wall Street Journal* front-page titles and abstracts. These frequencies are weighted, together with phrases in other categories, to fit the option implied volatility index VIX reported by the Chicago Board Options Exchange (CBOE).

This study is motivated by two strands of literature. First, Hao (2017) documents that the *change* in an option implied tail risk index strongly predicts future monthly market excess returns while its *level* does not. But the evidence is limited to a relatively short sample period 1996-2014 due to the availability of the S&P 500 index option prices. However, this news implied index of disaster concerns in Manela and Moreira (2017) goes back to 1880s which allows us to examine the return predictability of change in disaster risk with a much longer sample. Moreover, this text based approach also makes it possible to speak to the sources of change in disaster concerns. Indeed, Manela and Moreira (2017) also study the return predictability of NVIX and its components but focus on their *levels* instead of *changes*.

Second, a recent asset pricing literature argues that financial intermediaries are more likely to be the marginal investors of many asset classes than a representative consumer because they are actively trading these assets using sophisticated models. Therefore, the marginal value of wealth of these financial intermediaries is more informative about the stochastic discount factor (SDF). The leverage of financial intermediaries has been used to measure the tightness of intermediary funding constraints and therefore their marginal value of wealth. Adrian, Etula, and Muir (2014) (henceforth AEM) use the leverage ratio of broker-dealers while He, Kelly, and Manela (forthcoming) (henceforth HKM) employ the capital ratio of holding companies of primary dealers. Interestingly, although these two variables are supposed to be negatively correlated since they are the inverse of each other by definition, they are indeed positively correlated empirically. That is, high leverage for broker-dealers coincides with low leverage for holding companies of primary dealers. HKM attribute this to the difference between holding companies, which subsume primary dealers as their subsidiaries, and broker-dealers, who are isolated from other parts of their larger institutions. However, the news coverage of intermediation, the measure of financial distress or disaster concerns adopted in this paper, turns out to be positively correlated with leverage of both broker-dealers and holding companies of primary dealers. Therefore, the news coverage of intermediation might capture the overall financial distress better than the state variables used in AEM and HKM.

The main findings of this paper are the following. While Manela and Moreira (2017) do not find any evidence for return predictability of NVIX and its components at the monthly horizon and, in particular, at any horizon for the financial intermediation component, I document that an increase in the news coverage of intermediation is followed by lower stock market returns next month in the sample since World War II. The effect is economically large with a one-standard-deviation increase in the coverage associated with an 44 basis point decrease in next month's stock market excess return. The in and out of sample R^2 statistics are 1.09% and 0.86%, respectively, outperforming the known popular predictors in the literature.

This return predictability of change in news coverage of intermediation is not due to market sentiment (Huang, Jiang, Tu, and Zhou, 2014) or momentum (Moskowitz, Ooi, and Pedersen, 2012). Although the market sentiment also predicts the monthly market returns, the predictability power is less than the change in news coverage of intermediation with both a smaller economics impact and a lower in-sample R^2 statistic. Importantly, the predictability of either of them is not affected by controlling for the other implying that they contain distinctive information content about future market returns. The predictability of change in news coverage of intermediation is not affected either by controlling for the equity market momentum. Indeed, the equity market momentum does not predict the monthly market returns in the sample 1985-2013.

This return predictability of change in news coverage of intermediation is not driven by fluctuations in the leverage of financial intermediaries. Although an intensive news coverage of intermediation is accompanied by high leverage taken by both broker-dealers and holding companies of primary dealers, the change in news of coverage of intermediation is only significantly positively correlated with the innovation in leverage of broker-dealers at the quarterly frequency. An in-sample predictive regression test shows that neither the capital ratio factor in HKM nor the leverage factor in AEM is able to predict future monthly market returns. The predictability of change in news coverage of intermediation is not affected by controlling for either of them.

3.2 Data Description

This paper measures the news coverage of intermediation, denoted as NCI, by the intermediation component of the news implied volatility index, NVIX, in Manela and Moreira (2017). The text based NVIX is constructed in the following way in Manela and Moreira (2017). First, they count the phrases of the *Wall Street Journal* front-page titles and abstracts each day and aggregate them to the monthly frequency. Second, by a machine learning technique of supported vector regression, the normalized phrase frequencies are weighted to fit the implied volatility indices VIX and VXO reported by the CBOE for the recent period 1995-2009. The fitted time series is called NVIX. Third, these weights are used to constructed NVIX back to 1880s. In their updated dataset, these weights are also used to extend NVIX to March 2016. In particular, the phrases are putted into the following categories: government, financial intermediation, natural disasters, stock markets, war, and unclassified. The corresponding components of NVIX are obtained by setting weights for phrases in other categories to zero. See Manela and Moreira (2017) for more details for the construction. The updated time series data of news implied volatility NVIX and its components is available from July 1889 to March 2016 on Asaf Manela's website.[1]

This paper proposes the monthly change in news coverage of intermediation, dNCI, as a predictor for market returns. To test its return predictability, 15 other predictor variables are considered for comparison. 14 of them are studied in Goyal and Welch (2008) and constitute a set of popular predictors in the literature. The monthly sample for the period from 1870 to 2015 can be obtained from Amit Goyal's website.[2] The short interest variable SII proposed in Rapach, Ringgenberg and Zhou (2016), which is arguably the strongest known predictor of aggregate stock returns, is available for the period from 1973 to 2014 on David Rapach's website.[3] Specifically, I include the following predictors:

1. Log dividend-price ratio (DP): log of a 12-month moving sum of dividends paid on the S&P 500 index minus the log of stock prices (S&P 500 index).

2. Log dividend yield (DY): log of a 12-month moving sum of dividends minus the log of lagged stock prices.

3. Log earnings-price ratio (EP): log of a 12-month moving sum of earnings on the S&P 500 index minus the log of stock prices.

4. Log dividend-payout ratio (DE): log of a 12-month moving sum of dividends minus the log of a 12-month moving sum of earnings.

5. Excess stock return volatility (RVOL): computed using a 12-month moving standard deviation estimator, as in Mele (2007).

6. Book-to-market ratio (BM): book-to-market value ratio for the Dow Jones Industrial Average.

[1] http://apps.olin.wustl.edu/faculty/manela/data.html.
[2] http://www.hec.unil.ch/agoyal/.
[3] http://sites.slu.edu/rapachde/home/research.

7. Net equity expansion (NTIS): ratio of a 12-month moving sum of net equity issues by NYSE-listed stocks to the total end-of-year market capitalization of NYSE stocks.

8. Treasury bill rate (TBL): interest rate on a three-month Treasury bill (secondary market).

9. Long-term yield (LTY): long-term government bond yield.

10. Long-term return (LTR): return on long-term government bonds.

11. Term spread (TMS): long-term yield minus the Treasury bill rate.

12. Default yield spread (DFY): difference between Moody's BAA- and AAA-rated corporate bond yields.

13. Default return spread (DFR): long-term corporate bond return minus the long-term government bond return.

14. Inflation (INFL): calculated from the Consumer Price Index (CPI) for all urban consumers.

15. SII: (standardized) detrended log of equal-weighted short interest from Rapach, Ringgenberg and Zhou (2016).

Following the practice in literature on predicting market returns, I focus on predicting the excess return on a value-weighted market portfolio. I measure the market excess return as the log return on the S&P 500 index minus the log return on a one-month Treasury bill. To compute the log return on the S&P 500 index, I use the CRSP S&P 500 index value-weighted data.[4]

[4] I use the CRSP S&P 500 index value-weighted data after year 1926 and the log return based on S&P 500 index before year 1926.

3.2.1 Sample Properties

Table 3.1 shows the summary statistics for the change in news coverage of intermediation dNCI, 14 popular predictor variables from Goyal and Welch (2008), and the short interest variable SII from Rapach, Ringgenberg and Zhou (2016) over the 1980:01 to 2015:12 sample period.[5]

Table 3.2 displays correlation coefficients for dNCI and the other 15 popular predictor variables in the literature. While many of the popular predictors from the literature exhibit strong correlations with each other, the newly proposed predictor dNCI appears largely unrelated to these predictors. The strongest correlation (in magnitude) between dNCI and one of the popular predictors occur with DFR, which has correlation of only -0.18. The correlation between dNCI and the short interest variable SII is merely 0.05. In other words, the predictor dNCI appears to contain substantially different information from many of the stock return predictors used in the existing literature.

3.3 Predictability of Market Returns

In this section, I conduct both in-sample and out-of-sample tests of the return predictability of the innovation in news coverage of intermediation.

3.3.1 In-sample Test

To examine the return predictability of innovation in NCI and to compare it with other popular predictors in the literature, I first run the following in-sample predictive regression

$$r_{t+1} = \alpha + \beta x_t + \epsilon_{t+1}, \tag{3.1}$$

[5]The statistics for the short interest variable SII in Rapach, Ringgenberg and Zhou (2016) as well as its correlations with other predictors are based on the sample from 1980:01 to 2014:12.

where r_{t+1} is the market excess return in month $t+1$, x_t is the value of a particular predictor in month t, and ϵ_{t+1} is the residual. To have the comparison of coefficient estimates meaningful, I standardize each predictor to have a standard deviation of one. I use a heteroskedasticity- and autocorrelation-robust t-statistics and compute the a wild bootstrapped p value to test the null hypothesis $\beta = 0$ against the alternative $\beta > 0$ as in Rapach, Ringgenberg and Zhou (2016).

The dataset for the phrase counts of the *Wall Street Journal* front-page titles and abstracts goes back to 1880s. Manela and Moreira (2017) study the return predicability of NVIX and its components using the post-World War II sample 1945-2009 because of the high-quality stock market data for this period. Since this paper focuses specifically on the news coverage of intermediation, we might want to examine if there is a time variation in the information content of dNCI before running a single regression with a sample of more than a century. To do this, I run the in-sample predictive regression by decades.

I take the negative of NTIS, TBL, LTY, INFL, SII, and dNCI before running the regressions for these predictors so that their coefficient estimates are of a positive sign as the other predictors.

3.3.2 Out-of-sample Test

To check the robustness of the in-sample results and exclude the possibility of overfitting by dNCI, I look at the out-of-sample predictability of dNCI as well as other predictors. To be specific, at the end of month t, I predict the market return next month $t+1$ by

$$\hat{r}_{t+1} = \hat{\alpha}_t + \hat{\beta}_t x_t \tag{3.2}$$

where $\hat{\alpha}_t$ and $\hat{\beta}_t$ are OLS estimates of α and β in equation (3.1), respectively, based on the data from the beginning of the sample to month t.

In particular, the mean forecast, the average excess return from the beginning of the sample to month t, serves as the benchmark. This forecast corresponds to the case of $\beta = 0$ in the predictive regression so that it assumes no predictability of market returns. The out-of-sample R^2 statistic, as defined in Campbell and Thompson (2008) as the proportional reduction in the mean squared forecast error (MSFE) of each predicting variable compared to the benchmark of mean forecast, is employed as a measure of out-of-sample predictability. To test if the out-of-sample R^2 statistic of a particular predictor variable is significant, I follow Rapach, Ringgenberg and Zhou (2016) to use the Clark and West (2007) statistic to test the null hypothesis that the MSFE of mean forecast is less than or equal to that of the predictive forecast against the alternative that the MSFE of mean forecast is greater than that of the predictive forecast.

3.3.3 Empirical Results

Table 3.3 presents the in-sample regression coefficients, t-statistics and R^2 of 14 predictors from Goyal and Welch (2008), the short interest variable SII from Rapach, Ringgenberg and Zhou (2016), and the innovation in NCI for different sample periods. Table 3.4 presents the out-of-sample R^2 statistics of these predictors for the corresponding sample periods.

Post-World War II Sample

Because of the high quality of stock market data after the World War II, it is worth examining the return predictability of dNCI for the post-war sample 1945:01 to 2015:12.

The fifth and sixth columns of Table 3.3 present the in-sample regression coefficients, t-statistics and R^2 of 14 predictors from Goyal and Welch (2008) and dNCI for the post-war sample. Eight out of the 14 predictors from Goyal and Welch (2008) have significant $\hat{\beta}$ estimates: DP, DY, RVOL, TBL, LTY, LTR, TMS, and INFL. Their β estimates range from 0.26 to 0.37 with the maximum attained by LTR. However, the $\hat{\beta}$ estimates for dNCI is 0.44, which is significant at the 5% level. This means that for a one-standard-deviation increase in dNCI, the predicted monthly market excess return decreases by 0.44% (6.28% in annualized term). In terms of the proportion of variation in monthly market returns that can be explained by predictors, LTR has the highest R^2 of 0.78% among the 14 predictors from Goyal and Welch (2008). However, dNCI achieves the largest R^2 of 1.09%.

The third column of Table 3.4 presents the out-of-sample R^2 for the post-war sample with the first 20 years 1945:01 to1964:12 used for the initial estimation. Although for six out of the 14 predictors from Goyal and Welch (2008) the null hypothesis that the mean MSFE is less than or equal to the predictive regression MSFE is rejected, only the LTR obtains a positive R^2 of 0.19%.[6] However, dNCI attains the largest R^2 of 0.86%, which is significant at the 5% level.

Comparison with the Short Interest Variable

For the sample period 1973:01 to 2014:12, Rapach, Ringgenberg, and Zhou (2016) document that short interest, aggregated across securities, is arguably the strongest predictor of the equity risk premium identified to date. It outperforms the 14 popular predictors from Goyal and Welch (2008) in and out of sample and at horizons from one month to one year. Hao (2017) shows that the change in an option implied tail risk index, TIX, predicts market returns much better than short interest at the one month horizon for the sample period 1996:01 to 2014:12. TIX essentially measures market disaster concerns as the news implied index, NVIX, developed by Manela and Moreira (2015). However, given the long history of data available for the news coverage of intermediation, we now have the opportunity to compare the return predictability of change in disaster concerns with short interest for the same sample period as in Rapach, Ringgenberg, and Zhou (2016).

[6]RVOL also has a positive R^2 of 0.19% but it is not significant.

The third and forth columns of Table 3.3 present the in-sample regression coefficients, t-statistics and R^2 of 14 predictors from Goyal and Welch (2008), the short interest variable SII from Rapach, Ringgenberg and Zhou (2016), and dNCI for the sample 1973:01 to 2014:12. Four out of the 14 predictors from Goyal and Welch (2008) have significant $\hat{\beta}$ estimates: RVOL, LTR, TMS, and DFR. Among them the maximum estimate is from DFR, 0.48. SII has a slightly higher $\hat{\beta}$ estimate of 0.50 which is significant at the 1% level. However, the $\hat{\beta}$ estimate for dNCI is 0.72, significant at the 1% level, which is much higher than the estimates for other predictors. It is also economically important. For a one-standard-deviation increase in dNCI, the predicted monthly market excess return decreases by 0.72% (8.64% in annualized term). In terms of the proportion of variation in monthly market returns that can be explained by predictors, DFR has the highest R^2 of 1.14% among the 14 predictors from Goyal and Welch (2008). SII can explain slightly more with an R^2 of 1.24%. However, dNCI achieves an R^2 of 2.57% which is more than twice that for SII.

The second column of Table 3.4 presents the out-of-sample R^2 with the sample 1973:01 to 1989:12 used for the initial estimation as in Rapach, Ringgenberg and Zhou (2016). None of the 14 predictors from Goyal and Welch (2008) can reduce the MSFE of the mean benchmark forecast with R^2 all being negative. SII outperforms the mean benchmark forecast with an R^2 of 1.94%, significant at the 1% level. However, dNCI attains the largest R^2 of 3.36%, significant at the 5% level, which is much higher than that for SII.

Sample since 1980s

The second and third columns of Table 3.3 report the main in-sample regression results using the sample 1980:01 to 2015:12. Among the 14 predictors from Goyal and Welch (2008), only LTR and DFR are significant at the 10% confidence level. A one-standard-deviation increase in LTR and DFR increases the predicted monthly market excess return by 30 base points and 47 base points, respectively. Their R^2 statistics are 0.47% and 1.13%, respectively. In contrast, the new predictor dNCI is significant at the 1% level. The economic impact is also much bigger with a one-standard-deviation increase in dNCI associated with an 88 base point (10.56% in annualized term) decrease in the predicted monthly market excess return. The R^2 statistic is 3.97% which is more three times that for the DFR, the best one among other popular predictors.

The out-of-sample test results are reported in the first column of Table 3.4. I use the sample period 1980:01-1989:12 for the initial estimation and the sample period 1990:01-2015:12 for performance evaluation. It can seen that none of the 14 variables Goyal and Welch (2008) is able to reduce the MSFE of the benchmark mean forecast model with their out-of-sample R^2 statistics all being negative. In contrast, the new predictor dNCI achieves a proportional reduction of the MSFE of the benchmark model of 3.82%, which is significant at the 5% level.

Full Sample

The last two columns of Table 3.3 present the in-sample regression coefficients, t-statistics and R^2 for 5 predictors from Goyal and Welch (2008) and dNCI for the full sample 1889:08-2015:12. We can see none of them is significant. The last column of Table 3.3 presents the out-of-sample R^2 for the full sample with the first 20 years 1889:08-1909:07 used for the initial estimation. Again, none of them can outperform the mean benchmark forecast.

3.4 Discussions

3.4.1 News Coverage of Intermediation, Sentiment, and Momentum

The news coverage of intermediation reflects the attention paid by reporters to the the financial intermediaries. It is therefore tempting to think that it is related to the market sentiment. Since the return predictor proposed in this paper is the change in news coverage of intermediation, it is also worth examining if its return predictability is driven by momentum. In this section, I study the relation between the change in news coverage of intermediation, sentiment, and momentum. I use the sentiment series PLS from Huang, Jiang, Tu, and Zhou (2014) which is an improved sentiment index over that in Baker and Wurgler (2006, 2007). The time series equity market momentum index MOM is from Moskowitz, Ooi, and Pedersen (2012). The sample used for the comparison with sentiment is over 1980:01 to 2014:12. The sample used for comparison with the momentum is over 1985:01 to 2013:12.

Table 3.5 presents the in-sample predictive regression results for the change in news coverage of intermediation, sentiment, and momentum. For the comparison with sentiment (column 1-3), both the change in news coverage of intermediation and sentiment are significant predictors of future monthly market excess returns. But the change in news coverage of intermediation performs better with a predictive coefficient estimate of 0.89 versus 0.70 and an R^2 statistic of 4.13% versus 2.48%. Interestingly, in a bivariate predictive regression with these two predictors (column 3), the $\hat{\beta}$ estimates for both of them do not change from their univariate regressions and are still significant. The R^2 statistic 6.58% is also close the sum of those in their univariate regressions (6.61%). This implies that the change in news coverage of intermediation and sentiment contain completely distinctive information about future monthly market excess returns.

For the comparison with momentum (column 4-6), the change in news coverage of intermediation is still significant in this sample with a predictive coefficient of 0.96 and an R^2 statistic of 4.59%. But the momentum is not a significant predictor. These results still hold in a bivariate regression with these two predictors (column 6). The $\hat{\beta}$ estimate for the change in news coverage of intermediation and its t-statistic do not change. Therefore, the momentum does not reflect the information about future monthly market excess returns contained in the change in news coverage of intermediation.

3.4.2 News Coverage of Intermediation and Financial Sector Leverage

In this section, I examine the relation between the news coverage of intermediation and the financial intermediary leverage. AEM argue that the marginal value of wealth of financial intermediaries is more informative about the SDF than that of a representative consumer. They find that the innovation in the leverage ratio of broker-dealers is a priced factor across equity and bond portfolios. HKM argue that the financial intermediaries are more likely to be marginal investors and their pricing kernels are more relevant to asset returns. They find that the innovation in the capital ratio of holding companies of primary dealers is able price many classes of assets. I use the sample 1980:01 to 2012:12 for the leverage ratio and capital ratio which is available from Asaf Manela's website.

Table 3.6 displays the correlations between news coverage of intermediation, leverage ratio of broker-dealers, capital ratio of holding companies of primary dealers. Interestingly, although these leverage ratio and capital ratio are supposed to negatively correlated since they are the inverse of each other, HKM find that these two time series are indeed significantly positively correlated. This is confirmed in Panel B at the quarterly frequency

(36%).[7] So high leverage for broker-dealers is accompanied by low leverage for holding companies of primary dealers. HKM explain this by the different funding constraints of broker-dealers and holding companies of primary dealers. However, Panel B also hows that at the quarterly frequency the news coverage of intermediation is significantly negatively correlated (-25%) with the capital ratio of holding companies of primary dealers and, at the same time, significantly positively correlated (53%) with the leverage ratio of broker-dealers. The correlation between news coverage of intermediation and capital ratio of holding companies of primary dealers is also confirmed at the monthly frequency in Panel A. This implies that a high news coverage of intermediation is consistent with high leverage for both broker-dealers and holding companies of primary dealers.

When we turn to correlations between the innovations of these variables, which will be the relevant predictors in this paper and factors in AEM and HKM, Panel C and D of Table 3.6 show that the change in news coverage of intermediation is not significantly correlated with the capital risk factor at either monthly (-8%) or quarterly (-10%) frequency. The change in news coverage of intermediation is significantly positively correlated (40%) with the leverage factor at the quarterly frequency although this correlation disappears at the monthly frequency (1%). The correlations between capital risk factor and leverage factor is insignificant at the quarterly frequency (9%) although it is significantly at the monthly frequency (29%).[8] Since the leverage factor contains part of the information in the change in news coverage of intermediation, it is inspiring to examine if the leverage factor also predicts future monthly market excess returns.

[7]The leverage ratio of broker-dealers is not available at the monthly frequency.

[8]The raw data for leverage of financial intermediaries is only available at the quarterly frequency. The monthly data for leverage factor is constructed from a leverage mimicking portfolio at the quarterly frequency. The monthly data for capital factor is based on the most recently reported quarterly debt and ignores the within-quarter variation in the debt. So the correlations at the quarterly frequency are more reliable. The positive correlation between the capital risk factor and leverage factor at the monthly frequency may be simply driven by the correlation between the equity returns of holding companies of primary dealers and the return of the leverage mimicking portfolio.

Table 3.7 presents the in-sample predictive regression results for capital risk factor and leverage factor as well as the change in news coverage of intermediation. For the univariate regressions, the change in news coverage of intermediation (column 1) is still significant and even has a slightly higher predictive coefficient, 0.93, and an larger R^2, 4.24%, than those in the main sample. In contrast, both capital risk factor and leverage factor are not significant as market return predictors (column 2 and 4). The t-statistic for the capital risk factor is 1.00 and that for the leverage factor is merely 0.17. For the bivariate regressions with the change in news coverage of intermediation as one of the predictors (column 3 and 5), the addition of either capital risk factor or leverage factor hardly changes the predictive coefficient, R^2, or t-statistic of the change in news coverage of intermediation compared to those in its univariate regression. So capital risk factor and leverage factor are not able to predict future monthly market excess returns.[9]

The takeaway from this section is that although an intensive news coverage of intermediation is consistent with high leverage for both broker-dealers and holding companies of primary dealers and there is an overlap in the information contained in the change in news coverage of intermediation and the leverage factor, the change in news coverage of intermediation contains significant amount of information about future monthly market excess returns which is absent in either leverage factor or capital risk factor.

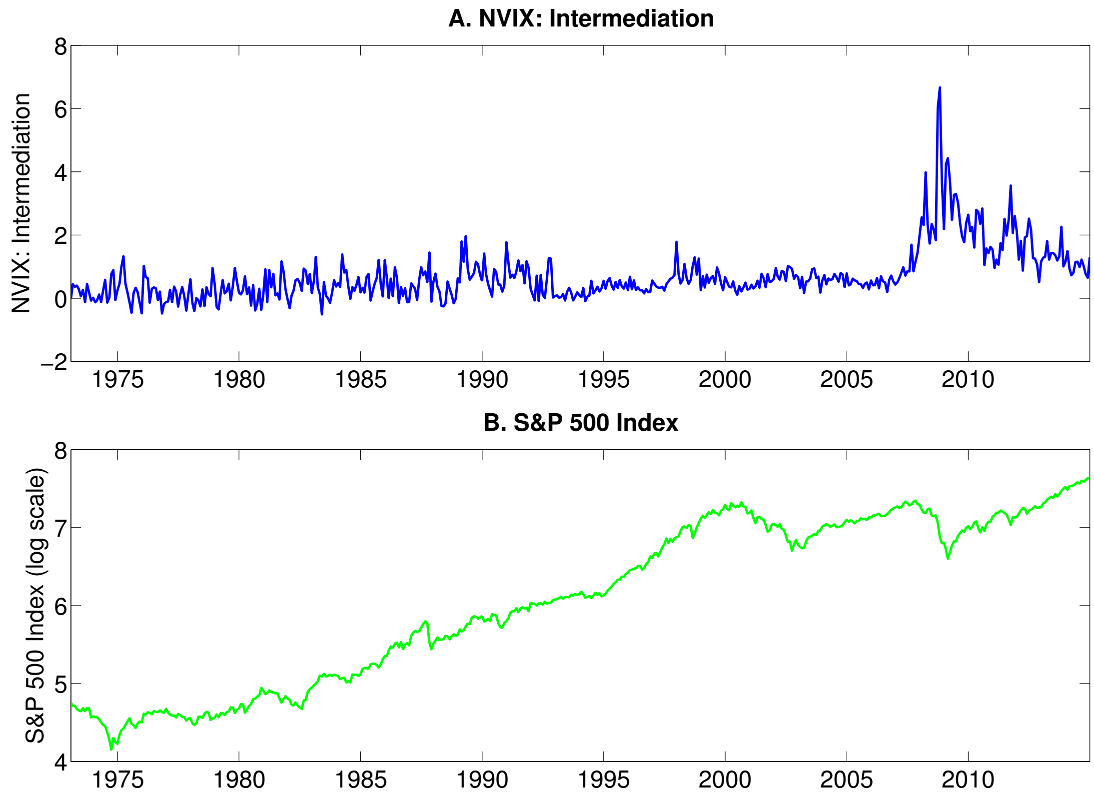
3.5 Conclusion

This paper documents that the change in news coverage of intermediation strongly predicts future monthly market excess returns. This return predictability is not driven by market sentiment, equity market momentum, or the fluctuations in leverage of financial intermediaries. It implies that the variation in the attention paid by reporters to financial

[9]The quarterly change in the intermediary component of NVIX or the Leverage Factor in Adrian, Etula, and Muir (2014) cannot predict the future monthly or quarterly excess returns.

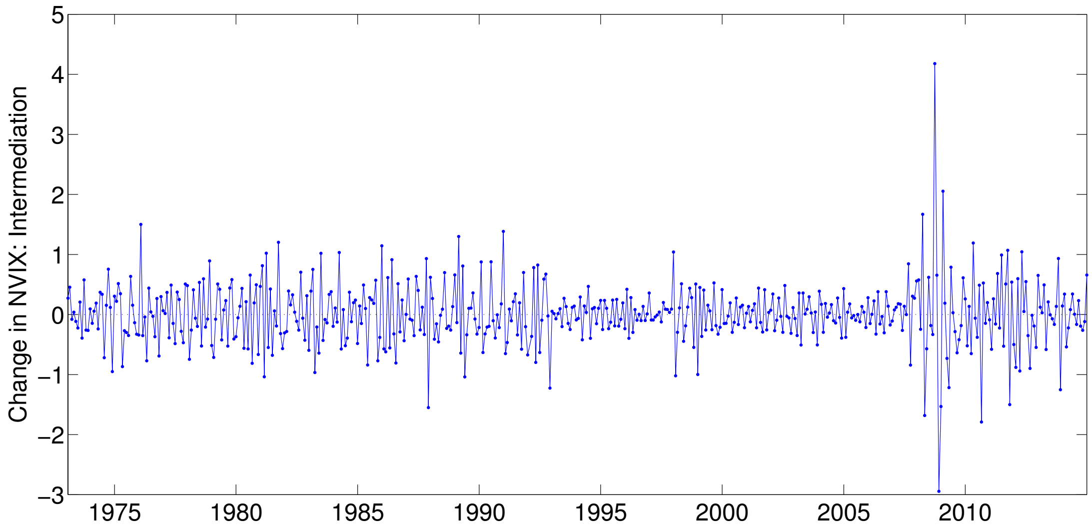
intermediates contains information about future market performance which has been fully incorporated in the current market prices.

Figure 3.1: Monthly Time Series of Intermediation Component of NVIX and S&P 500 Index: 1973:01-2014:12



Panel A plots the time series of monthly intermediation component of NVIX. Panel B plots the time series of daily close price of the S&P 500 index.

Figure 3.2: Monthly Innovations in the intermediation component of NVIX: 1973:01:2014:12



This figure displays the monthly time series of innovations in the intermediation component of NVIX which are computed as the difference at the end of each month.

Table 3.1: **Sample statistics, 1980:01-2015:12**

Predictor	mean	median	1st percentile	99th percentile	std. dev.
DP	-3.71	-3.82	-4.47	-2.83	0.42
DY	-3.71	-3.81	-4.47	-2.82	0.42
EP	-2.93	-2.92	-4.69	-2.02	0.46
DE	-0.79	-0.85	-1.24	1.09	0.37
RVOL	0.15	0.14	0.06	0.30	0.05
BM	0.41	0.33	0.13	1.16	0.24
NTIS	0.01	0.01	-0.05	0.04	0.02
TBL (%)	4.56	4.81	0.02	15.05	3.60
LTY (%)	6.85	6.34	2.18	14.04	2.99
LTR (%)	0.80	0.83	-6.61	9.99	3.28
TMS (%)	2.29	2.47	-2.51	4.38	1.44
DFY (%)	1.11	0.96	0.55	2.93	0.48
DFR (%)	-0.01	0.05	-5.31	4.26	1.53
INFL (%)	0.27	0.24	-0.56	1.20	0.30
SII	-0.00	-0.05	-2.13	2.51	1.00
dNCI	0.00	-0.00	-1.54	1.31	0.54

The database contains 432 monthly observations for January 1980 to December 2015 except that the statistics for SII are based on the sample until December 2014. The table displays summary statistics for 14 predictor variables from Goyal and Welch (2008), the short interest variable SII from Rapach, Ringgenberg, and Zhou (2016), and the innovation in the news coverage of intermediation. DP is the log dividend-price ratio, DY is the log dividend yield, EP is the log earnings-price ratio, DE is the log dividend-payout ratio, RVOL is the volatility of excess stock returns, BM is the book-to-market value ratio for the Dow Jones Industrial Average, NTIS is net equity expansion, TBL is the interest rate on a three-month Treasury bill, LTY is the long-term government bond yield, LTR is the return on long- term government bonds, TMS is the long-term government bond yield minus the Treasury bill rate, DFY is the difference between Moody's BAA- and AAA-rated corporate bond yields, DFR is the long-term corporate bond return minus the long-term government bond return, and INFL is inflation calculated from the CPI for all urban consumers. SII is the (standardized) detrended log of equal-weighted short interest. dNCI is the monthly change in the intermediation component of the news implied volatility from Manela and Moreira (2015).

Table 3.2: **Correlation matrix, 1980:01-2015:12**

Predictor	DP	DY	EP	DE	SVAR	BM	NTIS	TBL	LTY	LTR	TMS	DFY	DFR	INFL	SII	$\Delta NVIX_{Intermediation}$
DP	1.00															
DY	0.99	1.00														
EP	0.64	0.64	1.00													
DE	0.34	0.33	-0.50	1.00												
RVOL	-0.02	-0.01	-0.35	0.42	1.00											
BM	0.89	0.88	0.75	0.08	0.00	1.00										
NTIS	-0.08	-0.08	-0.00	-0.08	-0.07	-0.02	1.00									
TBL	0.66	0.66	0.64	-0.04	-0.04	0.72	0.07	1.00								
LTY	0.78	0.78	0.62	0.13	0.02	0.79	0.14	0.92	1.00							
LTR	0.07	0.07	0.06	-0.00	-0.01	0.07	-0.05	0.04	0.01	1.00						
TMS	-0.03	-0.03	-0.32	0.36	0.16	-0.16	0.11	-0.59	-0.23	-0.08	1.00					
DFY	0.59	0.59	0.16	0.48	0.38	0.65	-0.35	0.30	0.37	0.09	0.03	1.00				
DFR	0.02	0.04	-0.10	0.14	0.14	0.01	0.02	-0.03	0.02	-0.48	0.11	0.10	1.00			
INFL	0.32	0.31	0.39	-0.12	-0.03	0.39	0.09	0.48	0.42	-0.04	-0.32	0.05	-0.11	1.00		
SII	-0.08	-0.09	-0.20	0.15	-0.08	-0.23	-0.36	-0.04	-0.06	-0.02	-0.02	-0.04	-0.07	0.01	1.00	
dNCI	-0.01	-0.02	0.04	-0.05	-0.05	-0.00	-0.02	0.02	0.01	-0.08	-0.03	-0.06	-0.18	0.05	0.05	1.00

The table displays correlation coefficients for 14 predictor variables from Goyal and Welch (2008), the short interest variable SII from Rapach, Ringgenberg, and Zhou (2016), and the innovation in the intermediation component of the news implied volatility from Manela and Moreira (2017). 0.00 indicates less than 0.005 in absolute value. The statistics for SII are based on the sample until December 2014.

Table 3.3: **In-sample predictive regression estimation results**

Subsample	1980:01-2015:12		1973:01-2014:12		1945:01-2015:12		1889:08:2015:12	
Predictor	$\hat{\beta}$	$R^2(\%)$	$\hat{\beta}$	$R^2(\%)$	$\hat{\beta}$	$R^2(\%)$		
DP	0.25	0.33	0.15	0.12	0.32	0.60	0.08	0.03
	[1.14]		[0.74]		[2.25]**		[0.43]	
DY	0.28	0.40	0.17	0.15	0.34	0.66	0.14	0.08
	[1.27]		[0.85]		[2.37]**		[0.74]	
EP	0.20	0.20	0.09	0.04	0.25	0.36	0.19	0.15
	[0.69]		[0.36]		[1.33]		[1.32]	
DE	0.04	0.01	0.06	0.02	0.10	0.06	−0.13	0.07
	[0.14]		[0.23]		[0.53]		[−0.60]	
RVOL	0.23	0.28	0.35	0.62	0.27	0.41	0.15	0.09
	[1.23]		[1.90]**		[1.90]**		[0.45]	
BM	0.09	0.04	−0.01	0.00	0.15	0.13		
	[0.37]		[−0.05]		[0.94]			
NTIS(−)	−0.03	0.01	0.07	0.02	0.04	0.01		
	[−0.13]		[0.27]		[0.19]			
TBL(−)	0.20	0.21	0.26	0.34	0.36	0.74		
	[0.92]		[1.28]		[2.35]**			
LTY(−)	0.14	0.09	0.15	0.10	0.26	0.40		
	[0.61]		[0.71]		[1.74]**			
LTR	0.30	0.47	0.34	0.57	0.37	0.78		
	[1.44]*		[1.71]**		[2.45]***			
TMS	0.23	0.27	0.33	0.56	0.27	0.42		
	[1.06]		[1.66]*		[1.78]**			
DFY	−0.04	0.01	0.15	0.11	0.05	0.02		
	[−0.13]		[0.55]		[0.26]			
DFR	0.47	1.13	0.48	1.14	0.18	0.18		
	[1.36]*		[1.51]*		[0.77]			
INFL(−)	−0.04	0.01	0.05	0.01	0.35	0.71		
	[−0.15]		[0.20]		[1.83]*			
SII(−)			0.50	1.24				
			[2.50]***					
dNCI(−)	0.88	3.97	0.72	2.57	0.44	1.09	0.17	0.12
	[2.95]***		[2.60]***		[2.15]**		[1.08]	

The table reports the ordinary least squares estimate of β and R^2 statistic for the predictive regression model,

$$r_{t+1} = \alpha + \beta x_t + \epsilon_{t+1} \quad \text{for} \quad t = 1, \cdots, T-1,$$

where r_t is the S&P 500 log excess return for month t, x_t is the predictor variable in the first column, and (−) indicates that we take the negative of the predictor variable. Each predictor variable is standardized to have a standard deviation of one. Brackets below the $\hat{\beta}$ estimates report heteroskedasticity- and autocorrelation-robust t-statistics for testing $H_0 : \beta = 0$ against $H_A : \beta > 0$; *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively, according to wild bootstrapped p-values; 0.00 indicates less than 0.005 in absolute value.

Table 3.4: **Out-of-sample test results**

	Out-of-sample R^2 statistics (%)			
Predictor	1990:01-2015:12	1990:01-2014:12	1965:01-2015:12	1909:08-2015:12
DP	-0.57	-2.06	-0.08^{*}	-0.55
DY	-0.61	-2.20	-0.02^{*}	-0.60
EP	-0.71	-1.14	-0.33	-0.08
DE	-2.00	-2.27	-0.61	-0.38
RVOL	-0.16	-0.56	0.19	-1.69
BM	-0.32	-0.56	-1.02	
NTIS	-2.91	-3.22	-1.18	
TBL	-1.74	-0.38	-0.14^{**}	
LTY	-1.38	-0.31	-0.37^{**}	
LTR	-0.25	-0.47	0.19^{**}	
TMS	-0.83	-0.76	-0.54^{**}	
DFY	-0.98	-3.07	-0.73	
DFR	-2.29	-1.96	-0.59	
INFL	-0.83	-0.64	-0.20	
SII		1.94^{***}		
dNCI	3.82^{**}	3.36^{**}	0.86^{**}	-0.10

The second column reports the proportional reduction in mean squared forecast error (MSFE) at the one month horizon for a predictive regression forecast of the S&P 500 log excess return based on the predictor variable in the first column vis-á-vis the prevailing mean benchmark forecast, where statistical significance is based on the Clark and West (2007) statistic for testing the null hypothesis that the prevailing mean MSFE is less than or equal to the predictive regression MSFE against the alternative hypothesis that the prevailing mean MSFE is greater than the predictive regression MSFE. The sample 1973:01:1989:12 is used for the initial estimation.*, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 3.5: **Comparison with Sentiment and Momentum**

	Sentiment			Momentum		
	r_{t+1}	r_{t+1}	r_{t+1}	r_{t+1}	r_{t+1}	r_{t+1}
dNCI (−)	0.89		0.89	0.96		0.94
	[2.98]***		[3.11]***	[2.78]***		[2.78]***
PLS (−)		0.70	0.69			
		[3.52]***	[3.49]***			
MOM (−)					0.34	0.25
					[1.16]	[1.00]
R^2(%)	4.13	2.48	6.58	4.59	0.57	4.91
#Obs	420	420	420	348	348	348

The table reports the ordinary least squares estimate of β and R^2 statistic for the predictive regression model,

$$r_{t+1} = \alpha + \beta X_t + \epsilon_{t+1} \quad \text{for} \quad t = 1, \cdots, T-1,$$

where r_t is the S&P 500 log excess return for month t, X_t is a vector of predictor variables in the first column, and (−) indicates that I take the negative of the predictor variable. PLS is the sentiment index from Huang, Jiang, Tu, and Zhou (2014) and MOM is the time series equity market momentum index from Moskowitz, Ooi, and Pedersen (2012). Each predictor variable is standardized to have a standard deviation of one. Brackets below the $\hat{\beta}$ estimates report heteroskedasticity- and autocorrelation-robust t-statistics for testing $H_0 : \beta = 0$ against $H_A : \beta > 0$; *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively, according to wild bootstrapped p-values; 0.00 indicates less than 0.005 in absolute value. The sample used for comparison with the sentiment is over January 1980 to December 2014. The sample used for comparison with the momentum is over January 1985 to December 2013.

Table 3.6: **Correlation Matrix: Change in News Coverage, Capital Risk Factor, and Leverage Factor**

Panel A: Monthly Levels

	NCI	Capital Ratio	Leverage Ratio
NCI	1.00		
Capital Ratio	-0.25^{***}	1.00	
Leverage Ratio	n/a	n/a	1.00

Panel B: Quarterly Levels

	NCI	Capital Ratio	Leverage Ratio
NCI	1.00		
Capital Ratio	-0.25^{***}	1.00	
Leverage Ratio	0.53^{***}	0.36^{***}	1.00

Panel C: Monthly Innovations

	dNCI	Capital Risk Factor	Leverage Factor
dNCI	1.00		
Capital Risk Factor	-0.08	1.00	
Leverage Factor	0.01	0.29^{***}	1.00

Panel D: Quaterly Innovations

	dNCI	Capital Risk Factor	Leverage Factor
dNCI	1.00		
Capital Risk Factor	-0.10	1.00	
Leverage Factor	0.40^{***}	0.09	1.00

Capital Risk Factor and Leverage Factor are from He, Kelly, and Manela (forthcoming). The sample is over Jan 1980 to Dec 2012.

Table 3.7: **Comparison with Capital Risk Factor and Leverage Factor**

	r_{t+1}	r_{t+1}	r_{t+1}	r_{t+1}	r_{t+1}
dNCI (−)	0.93		0.91		0.93
	[2.96]***		[2.90]***		[2.95]***
Capital Risk Factor		0.24	0.17		
		[1.00]	[0.69]		
Leverage Factor (−)				0.05	0.04
				[0.17]	[0.15]
R^2(%)	4.24	0.29	4.37	0.01	4.24
#Obs	396	396	396	396	396

The table reports the ordinary least squares estimate of β and R^2 statistic for the predictive regression model,

$$r_{t+1} = \alpha + \beta X_t + \epsilon_{t+1} \quad \text{for} \quad t = 1, \cdots, T-1,$$

where r_t is the S&P 500 log excess return for month t, X_t is a vector of predictor variables in the first column, and (−) indicates that I take the negative of the predictor variable. Capital Risk Factor and Leverage Factor are from He, Kelly, and Manela (forthcoming). Each predictor variable is standardized to have a standard deviation of one. Brackets below the $\hat{\beta}$ estimates report heteroskedasticity- and autocorrelation-robust t-statistics for testing $H_0 : \beta = 0$ against $H_A : \beta > 0$; *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively, according to wild bootstrapped p-values; 0.00 indicates less than 0.005 in absolute value. The sample is over January 1980 to December 2012.

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Appendix

Appendix: Chapter 1

A: Proofs

Proof of Proposition 1

Proof. I first replicate the log function of the underlying price at time T by the payoff functions of a bond, a futures contract, and the out-of-the-money put and call options

$$\log \frac{S_T}{S_t} = \log \frac{F_{t,T}}{S_t} + \frac{S_T - F_{t,T}}{F_{t,T}} - \int_0^{F_{t,T}} \frac{1}{K^2} \max(K - S_T, 0) dK - \int_{F_{t,T}}^{\infty} \frac{1}{K^2} \max(S_T - K, 0) dK. \tag{3}$$

Note that $F_{t,T}/S_t = e^{(r-q)\tau}$ and $E_t^*(S_T - F_{t,T}) = 0$. Then equation (1.17) follows by taking the risk-neutral expectation of both sides and employing the risk-neutral pricing of options.

Next, for $n \neq 1$, I can also replicate the power function by the same set of payoff functions

$$\begin{aligned} S_T^n = F_{t,T}^n + nF_{t,T}^{n-1}(S_T - F_{t,T}) + n(n-1)\Bigg\{ \int_0^{F_{t,T}} K^{n-2} \max(K - S_T, 0) dK \\ + \int_{F_{t,T}}^{\infty} K^{n-2} \max(S_T - K, 0) dK \Bigg\}. \end{aligned}$$

Dividing both sides by S_t^n and taking risk-neutral expectation of them, we have

$$E_t^*[R_{t,T}^n] = e^{n(r-q)\tau} + \frac{n(n-1)e^{r\tau}}{S_t^n}\left\{\int_0^{F_{t,T}} K^{n-2}\text{put}_{t,T}(K)dK + \int_{F_{t,T}}^{\infty} K^{n-2}\text{call}_{t,T}(K)dK\right\}.$$

Then by equation (1.9), we have equation (1.16).

For $n = 1$, recall $m(1) = -\mu\tau$ in Lemma 2, so equation (1.16) is true in general. □

B: Constructions of the Cumulant Generating Function of the Generic Market Risk and TIX

I use the option quotes from the OptionMetrics. Although this dataset may be different from the one the CBOE uses to compute VIX, when constructing the cgf and TIX, I try to follow as closely as possible the practice for constructing the CBOE VIX as described in the VIX white paper.

I clean the data in two ways. First, as the CBOE does for VIX, I exclude options that have a bid price of zero. Moreover, for call options as we move to successively higher strike prices, once two consecutive call options are found to have zero bid prices, no calls with higher strikes are considered. I do the same cleaning for put options as we move to successively lower strike prices. Second, I delete all replicated entries as in Martin (2016).

B.1. Construction of μ_t

I first compute VIX_t^2 following exactly the same procedure as that for the CBOE VIX. Then we have $\mu_t = -\text{VIX}_t^2/2$.

A special note is that in OptionMetrics the strikes at which we have quotes for call options may not match those for put options. When determining the forward SPX level, F, I first obtain the strikes at which we have quotes for both call and put options. Across these strikes, I identify the one at which the absolute difference between the call and put prices is smallest. Then by put-call parity, $F = \text{Strike Price} + \exp(rT) \times (\text{Call Price} - \text{Put Price})$. I then determine K_0, the strike price immediately below the forward index level at which we at least have the quote for either the call or put option. If the price for one of the two options is missing, I apply the put-call parity to recover it from the other one. Then the procedure for the CBOE VIX can be followed exactly.

B.2. Construction of cgf for a given maturity T

To construct the cgf using option prices as in equation (1.16), we need to deal with the two terms on the right hand. For the second term, we have already obtained μ in B.1. We can also get the risk-free rate minus dividend yield implied by option prices as $r - q = \log(F/S_t)/\tau$.

For the first term, the integration

$$e^{r\tau}\left\{\int_0^{F_{t,T}} K^{n-2}\text{put}_{t,T}(K)dK + \int_{F_{t,T}}^{\infty} K^{n-2}\text{call}_{t,T}(K)dK\right\}$$

is similar to the integration in the VIX formula (1.19) except that the weights for options at strike K_i are now $1/K_i^{2-n}$ instead of $1/K_i^2$. So we can follow the same procedure as I did in B.1 for constructing VIX^2. In particular, the first term can be computed as

$$\log\left(\frac{n(n-1)}{S_t^n}\sum_i \frac{\Delta K_i}{K_i^{2-n}} e^{r\tau} Q(K_i) + \frac{K_0^{n-1}}{S_t^n}\big((1-n)K_0 + nF\big)\right). \tag{4}$$

B.3. Construction of cgf for the 30-day maturity T_{30}

As for the CBOE VIX, I use the near-term and the next-term options to target the 30-day maturity. Specifically, I compute the cgf implied by the near- and next- term options, respectively, following the steps in B.1 and B.2 above. This will give us the cgf $m_n(n)$ and

$m_f(n)$ for the maturities T_n and T_f, respectively.[10] Then I interpolate the cgf of the 30-day market risk by

$$m(n) = \frac{m_n(n) * (T_f - T_{30}) + m_f(n) * (T_{30} - T_n)}{T_f - T_n}. \tag{5}$$

B.4. Construction of TIX for the 30-day maturity T_{30}

Having the cgf of the 30-day market risk, I construct TIX as

$$\text{TIX} = \log_2\left(\frac{m(2)}{m(1)}\right). \tag{6}$$

[10] Subscript n stands for "near" and subscript f stands for "far".

Appendix: Chapter 2

A: Proofs.

Proof of Lemma 1: For any $s \in S^*$, s is a local maximum of $L(s) - \lambda^b s$. Hence it must be that $L''(s) \leq 0$ and $L'(s) - \lambda^b = 0$.

For the case there exists a unique $\hat{s}$ such that $M^h(1/\hat{s}) = \beta^h$, the second order condition implies $s \in [\hat{s}, \infty)$. Since $L'(s)$ is strictly decreasing in $(\hat{s}, \infty)$, there is at most one s satisfying the first order condition. Moreover, $L'(s) > 0$ in $(\hat{s}, \infty)$, hence $\lambda^b > 0$.

For the case $M^h(D_1) = \beta^h, \forall D_1 \in [1/\tilde{s}, \infty)$ for some maximum $\tilde{s}$, the liquidity value function is linear in $(0, \tilde{s}]$ and strictly concave in $[\tilde{s}, \infty)$. Then bond-$\tilde{s}$ is the only bond that could be traded and $\lambda^b > 0$.

Proof of Theorem 1: Substituting the bond price, the collateral value and the risky asset price into the pricing kernel equality (2.16), we have (2.19). To show the existence and uniqueness, rearrange (2.19)

$$\underbrace{\bar{s}[e_0^h + s_0^h E(\beta^b D_1)]}_{\text{LHS}(\bar{s})} = \underbrace{(K - s_0^h)\beta^b E[min(1, \bar{s}D_1)^{1-\eta}] + s_0^h \beta^b E[min(1, \bar{s}D_1)] + K\left(\frac{\beta^b}{\beta^h}\right)^{1/\eta}}_{\text{RHS}(\bar{s})}. \tag{7}$$

Note that $\lim_{\bar{s}\to 0} RHS(\bar{s}) > 0$ and $\lim_{\bar{s}\to\infty} RHS(\bar{s}) < \infty$. This guarantees the existence of an equilibrium. For the uniqueness, first consider the case $\eta \in (0, 1]$. Note that

$$LHS'(\bar{s}) = e_0^h + \frac{\beta^b}{\bar{s}} s_0^h \int_0^\infty \bar{s}D_1 dF(D_1),$$

$$RHS'(\bar{s}) = \frac{\beta^b}{\bar{s}}\left[(1-\eta)(K - s_0^h)\int_0^{1/\bar{s}} (\bar{s}D_1)^{1-\eta} dF(D_1) + s_0^h \int_0^{1/\bar{s}} \bar{s}D_1 dF(D_1)\right],$$

$$RHS''(\bar{s}) = -\frac{\beta^b}{\bar{s}^2}\left[\eta(1-\eta)(K - s_0^h)\int_0^{1/\bar{s}} (\bar{s}D_1)^{1-\eta} dF(D_1) + \frac{1}{\bar{s}} f\left(\frac{1}{\bar{s}}\right)[(1-\eta)(K - s_0^h) + s_0^h]\right].$$

Hence

$$RHS'(\bar{s}) > 0, RHS''(\bar{s}) < 0,$$

so the solution is unique. See the left panel in Figure 2.2.

For $\eta \in (1, \infty)$, consider

$$\underbrace{\bar{s}[e_0^h + s_0^h E(\beta^b D_1)] - (K - s_0^h)\beta^b E[min(1, \bar{s}D_1)^{1-\eta}]}_{\text{LHS}(\bar{s})} = \underbrace{s_0^h \beta^b E[min(1, \bar{s}D_1)] + K\left(\frac{\beta^b}{\beta^h}\right)^{1/\eta}}_{\text{RHS}(\bar{s})}. \tag{8}$$

Note that for $s_0^h \in (0, K)$, the $LHS(\bar{s})$ is strictly increasing and strictly concave with $\lim_{\bar{s}\to 0} LHS(\bar{s}) = -\infty$ and $\lim_{\bar{s}\to\infty} LHS(\bar{s}) = \infty$; the RHS is strictly increasing and strictly concave with $\lim_{\bar{s}\to 0} RHS(\bar{s}) = K(\beta^b/\beta^h)^{1/\eta}$ and $\lim_{\bar{s}\to\infty} RHS(\bar{s}) = s_0^h\beta^b + K(\beta^b/\beta^h)^{1/\eta}$. Moreover, $RHS'(\bar{s}) < LRH'(\bar{s})$ for any $\bar{s} \in S$. Hence there exists a unique solution. It is also true for $s_0^h = 0$ or $s_0^h = K$. See the right panel in Figure 2.2.

Proof of Proposition 1: When e_0^h increases, for either (7) with $\eta \in (0, 1)$ or (8) with $\eta \in (1, \infty)$, the LHS shifts upward, while the RHS is unchanged. This implies a drop in $\bar{s}$.

When s_0^h increases, total differentiating (7), we have in general

$$\frac{\partial \bar{s}}{\partial s_0^h} = \frac{\beta^b E[min(1, \bar{s}D_1)] - \beta^b E(\bar{s}D_1) + \frac{\partial(K - s_0^h)}{\partial s_0^h}\beta^b E[min(1, \bar{s}D_1)^{1-\eta}]}{LHS'(\bar{s}) - RHS'(\bar{s})},$$

where $\frac{\partial(K - s_0^h)}{\partial s_0^h}$ equals -1 if K is fixed or 0 if s_0^b is fixed. Note that the denominator is positive from the proof of Theorem 1. In the nominator, the first two terms already result in a negative value, so it is negative regardless of the assumption made on fixing K or s_0^b. Overall, $\bar{s}$ declines.

For the collateral value, note that

$$\frac{\partial \lambda^b}{\partial \bar{s}} = -\frac{\eta\beta^b}{\bar{s}^2}\int_0^{1/\bar{s}} (\bar{s}D_1)^{1-\eta} dF(D_1) < 0,$$

For the haircut, consider

$$\frac{\bar{s}p_0}{q^{\bar{s}}} = 1 + \frac{\int_{1/\bar{s}}^{\infty}(\bar{s}D_1 - 1)dF(D_1)}{E[min(1, \bar{s}D_1)^{1-\eta}]}$$

with

$$\frac{\partial(\bar{s}p_0/q^{\bar{s}})}{\partial\bar{s}} = \frac{\int_{1/\bar{s}}^{\infty} D_1 dF(D_1) E[min(1, \bar{s}D_1)^{1-\eta}] - (1-\eta)\int_{1/\bar{s}}^{\infty}(D_1 - \frac{1}{\bar{s}})dF(D_1)\int_0^{1/\bar{s}}(\bar{s}D_1)^{1-\eta}dF(D_1)}{E[min(1, \bar{s}D_1)^{1-\eta}]^2} > 0. \tag{9}$$

The results for $q^{\bar{s}}$ and R can be seen from (2.17) with $\partial q^{\bar{s}}/\partial\bar{s} > 0$ for $\eta \in (0,1)$ and $\partial q^{\bar{s}}/\partial\bar{s} < 0$ for $\eta \in (1,\infty)$. The result for p_0 can be seen from (2.8).

Proof of Proposition 2: For (7) with $\eta \in (0,1)$, when s_0^b increases, the LHS does not change while the RHS shifts upward. So $\bar{s}$ increases. For (8) with $\eta \in (1,\infty)$, when s_0^b increases, the LHS shifts downward while the RHS does not change. So $\bar{s}$ also increases. The changes in other variables follow from the proof in Proposition 1.

Proof of Proposition 3: For the case $s_0^h = 0$, (7) becomes

$$\bar{s}e_0^h = K\beta^b E[min(1, \bar{s}D_1)^{1-\eta}] + K\left(\frac{\beta^b}{\beta^h}\right)^{1/\eta}. \tag{10}$$

Since the quality of collateral in the lower tail $(0, \bar{s}_0)$ remains the same for either of the two shocks, the collateral level $\bar{s}$ is the same as before. So the bond price $q^{\bar{s}}$, by (2.17), and its interest rate R do not change. By (2.18), the collateral value λ^b is unaffected. By (2.8), the asset price increases for a FOSD improvement in the upper tail, so does the collateral value $\bar{s}p_0$. Then the haircut increases by (2.29). For a mean preserving SOSD improvement in the upper tail, the asset price and haircut do not change. A mean increasing SOSD improvement is a combination of the previous two cases.

Proof of Proposition 4: First consider $\eta \in (0,1)$. In (10), note that $min(1, \bar{s}_0 D_1)^{1-\eta}$ is strictly increasing in D_1 over $(0, 1/\bar{s}_0)$. So a FOSD improvement increases $E[min(1, \bar{s}D_1)^{1-\eta}]$, leading to a greater collateral level $\bar{s}$ which further increases the bond price $q^{\bar{s}}$ in (2.17), and a lower interest rate R follows. Substituting the expression of the bond price in (10) into the asset price (2.8), we have (2.31) which increases with $\bar{s}$. For the haircut, substituting the same expression into (2.29), we have (2.32) with $\partial(\text{equity/debt})/\partial\bar{s} < 0$ if and only if

$$\frac{\int_{1/\bar{s}}^{\infty} D_1 dF(D_1)}{\bar{s}\int_{1/\bar{s}}^{\infty} D_1 dF(D_1) - \int_{1/\bar{s}}^{\infty} dF(D_1)} < \frac{e_0^h/K}{\bar{s}e_0^h/K - (\beta^b/\beta^h)^{1/\eta}}. \tag{11}$$

Since the function $a/(\bar{s}a - 1)$ is decreasing in a in $(1/\bar{s}, \infty)$, the haircut decreases if and only if the inequality (2.30) holds.

For a SOSD improvement, note that $min(1, \bar{s}_0 D_1)^{1-\eta}$ is strictly concave over $D_1 \in (0, 1/\bar{s}_0)$. So $E[min(1, \bar{s}D_1)^{1-\eta}]$ increases and the rest of results are the same as above.

For $\eta \in (1, \infty)$, note that $min(1, \bar{s}_0 D_1)^{1-\eta}$ is now strictly decreasing and strictly convex over $D_1 \in (0, 1/\bar{s}_0)$ and, therefore, the bond price $\beta^b E[min(1, \bar{s}D_1)^{1-\eta}]$ tends to decrease for either a FOSD or a SOSD improvement. So $\bar{s}$ decreases. Although this tends to increase the bond price, it is of second order as can be seen from (10) in which the bond price changes in the same direction as $\bar{s}$. So the interest rate increases. The asset price decreases by (2.31) and haircut decreases if the opposite of (11) is true.

Proof of Proposition 5: When β^h increases, the LHS of either (7) or (8) is unchanged, while the RHS shifts downward. The rest of the proof is the same as above.

Proof of Proposition 6: When $s_0^h = 0$, total differentiating (7) with respect to β^b, we have

$$\frac{\partial\bar{s}}{\partial\beta^b} = \frac{KE[min(1, \bar{s}D_1)^{1-\eta}] + \frac{K}{\eta\beta^b}\left(\frac{\beta^b}{\beta^h}\right)^{1/\eta}}{LHS'(\bar{s}) - RHS'(\bar{s})} > 0,$$

so $\bar{s}$ increases. For the bond price $q^{\bar{s}}$,[11] we have

$$\frac{\partial q^{\bar{s}}}{\partial \beta^b} = E[min(1,\bar{s}D_1)^{1-\eta}] + \frac{\partial \beta^b E[min(1,\bar{s}D_1)^{1-\eta}]}{\partial \bar{s}} \frac{\partial \bar{s}}{\partial \beta^b}$$
$$= \frac{e_0^h E[min(1,\bar{s}D_1)^{1-\eta}] + \frac{\partial \beta^b E[min(1,\bar{s}D_1)^{1-\eta}]}{\partial \bar{s}} \frac{K}{\eta \beta^b}\left(\frac{\beta^b}{\beta^h}\right)^{1/\eta}}{LHS'(\bar{s}) - RHS'(\bar{s})}$$
$$= \frac{\bar{s}e_0^h E[min(1,\bar{s}D_1)^{1-\eta}] + (\frac{1}{\eta}-1)\int_0^{1/\bar{s}}(\bar{s}D_1)^{1-\eta}dF(D_1)K\left(\frac{\beta^b}{\beta^h}\right)^{1/\eta}}{\bar{s}[LHS'(\bar{s}) - RHS'(\bar{s})]}. \tag{12}$$

Since by (7) $\bar{s}e_0^h - K(\beta^b/\beta^h)^{1/\eta} > 0$, $\frac{\partial q^{\bar{s}}}{\partial \beta^b} > 0$, so $q^{\bar{s}}$ increases and R declines. For the asset price

$$\begin{aligned}
\frac{\partial p_0}{\partial \beta^b} &= E[D_1] + \frac{E[min(1,\bar{s}D_1)^{1-\eta}] - E[min(1,\bar{s}D_1)]}{\bar{s}} - \frac{\eta\beta^b}{\bar{s}^2}\int_0^{1/\bar{s}}(\bar{s}D_1)^{1-\eta}dF(D_1)\frac{\partial \bar{s}}{\partial \beta^b} \\
&= \frac{1}{\bar{s}[LHS'(\bar{s}) - RHS'(\bar{s})]}\Bigg[\Bigg(\int_{1/\bar{s}}^{\infty}(\bar{s}D_1 - 1)dF(D_1) + E[min(1,\bar{s}D_1)^{1-\eta}]\Bigg)\Bigg(e_0^h - \\
&\quad \frac{(1-\eta)K\beta^b}{\bar{s}}\int_0^{1/\bar{s}}(\bar{s}D_1)^{1-\eta}dF(D_1)\Bigg) - \\
&\quad \frac{\eta\beta^b}{\bar{s}}\int_0^{1/\bar{s}}(\bar{s}D_1)^{1-\eta}dF(D_1)\Bigg(KE[min(1,\bar{s}D_1)^{1-\eta}] + \frac{K}{\eta\beta^b}\left(\frac{\beta^b}{\beta^h}\right)^{1/\eta}\Bigg)\Bigg] \\
&> \frac{1}{\bar{s}[LHS'(\bar{s}) - RHS'(\bar{s})]}\Bigg[\Bigg(\int_{1/\bar{s}}^{\infty}\bar{s}D_1 dF(D_1) + \int_0^{1/\bar{s}}(\bar{s}D_1)^{1-\eta}dF(D_1)\Bigg)\Bigg(e_0^h - \\
&\quad \frac{K\beta^b}{\bar{s}}\int_0^{1/\bar{s}}(\bar{s}D_1)^{1-\eta}dF(D_1)\Bigg) - \frac{K}{\bar{s}}\left(\frac{\beta^b}{\beta^h}\right)^{1/\eta}\int_0^{1/\bar{s}}(\bar{s}D_1)^{1-\eta}dF(D_1)\Bigg] \\
&> \frac{\int_{1/\bar{s}}^{\infty}\bar{s}D_1 dF(D_1)\left(e_0^h - \frac{K\beta^b}{\bar{s}}\int_0^{1/\bar{s}}(\bar{s}D_1)^{1-\eta}dF(D_1)\right)}{\bar{s}[LHS'(\bar{s}) - RHS'(\bar{s})]} \\
&> 0.
\end{aligned} \tag{13}$$

The last two inequalities use the equilibrium condition (7), $\bar{s}e_0^h = K\beta^b E[min(1,\bar{s}D_1)^{1-\eta}] + K(\beta^b/\beta^h)^{1/\eta}$, when $s_0^h = 0$. So p_0 and $\bar{s}p_0$ increase. The haircut increases by (2.29).

[11]It is straightforward for $\eta \in (0,1)$ since $q^{\bar{s}}$ is increasing in both β^b and $\bar{s}$.

For $s_0^h \in (0, K]$ and $\eta \in (0, 1)$, total differentiating (7) and substituting it into the nominator, we have

$$\frac{\partial \bar{s}}{\partial \beta^b} = \frac{1}{\beta^b} \frac{\bar{s}e_0^h - K(\beta^b/\beta^h)^{1/\eta} + \frac{K}{\eta}(\beta^b/\beta^h)^{1/\eta}}{LHS'(\bar{s}) - RHS'(\bar{s})} > 0.$$

$q^{\bar{s}}$ increases since it is increasing in $\bar{s}$ for $\eta \in (0, 1)$, as well as β^b. The haircut increases by (2.29). Given that $q^{\bar{s}}$ and H increase, $\bar{s}p_0$ must increase too.

Proof of Proposition 7:

When $\beta^b = \beta^h$, for the case $s_0^h \in [0, K)$, as η increases, the LHS of (7) and the RHS of (8) do not change, while the RHS of (7) increases and the LHS of (8) decreases, so $\bar{s}$ increases for any η. Then note that both of the two equations can be written as

$$q^{\bar{s}} = \frac{\bar{s}e_0^h + s_0^h \beta^b E[max(\bar{s}D_1 - 1, 0)] - K}{K - s_0^h}, \tag{14}$$

so $q^{\bar{s}}$ increases as the right hand is increasing in $\bar{s}$, and R declines. For the asset price, substituting (14) into (2.8)

$$p_0 = \frac{K}{K - s_0^h}\left(\beta^b E[max(D_1 - 1/\bar{s}, 0)] - \frac{1}{\bar{s}} + \frac{e_0^h}{K}\right), \tag{15}$$

so p_0 increases as the right hand is also increasing in $\bar{s}$, which implies λ^b increases. For the haircut, substituting (14) into (2.29), we have

$$\frac{\bar{s}p_0}{q^{\bar{s}}} = 1 + \beta^b(K - s_0^h)\frac{E[max(\bar{s}D_1 - 1, 0)]}{\bar{s}e_0^h - K + \beta^b s_0^h E[max(\bar{s}D_1 - 1, 0)]}$$

with $\frac{\partial(\bar{s}p_0/q^{\bar{s}})}{\partial \bar{s}} > 0$ if and only if

$$K\int_{1/\bar{s}}^{\infty} D_1 dF(D_1) < e_0^h \int_{1/\bar{s}}^{\infty} dF(D_1)$$

or $E[D_1|D_1 > 1/\bar{s}] < e_0^h/K$. So the haircut increases if and only if this inequality holds.

For the case $s_0^h = K$, η has no effect on either (7) or (8). So $\bar{s}$ remains the same. The bond price $q^{\bar{s}}$ increases by (2.17) driving down R, which also leads to a higher collateral value λ^b, by (2.18), and p_0, and a lower haircut by (2.29).

Proof of Proposition 8: The RHS of either (7) or (8) shifts downward, while the LHS is unaffected. So $\bar{s}$ decreases. Since $\frac{\bar{s}p_0}{q^{\bar{s}}}$ is increasing in $\bar{s}$ but decreasing in η, the haircut decreases. Since λ^b is decreasing in $\bar{s}$ but increasing in η, the collateral value and asset price increase. When $\eta \in (1, \infty)$, the bond price $q^{\bar{s}}$ is decreasing in $\bar{s}$ but increasing in η, the interest rate drops.

B: Robustness of the Equilibrium to the Wealth Distribution.

In this section, we show that the equilibrium established in the main context is robust to a heterogenous endowment distribution. Since banks have linear utility function, it does not matter for the equilibrium if they have different endowments at time 0 except for the banks' consumption flow. So we assume the banks still have the same endowments. However, if the households have different endowments in either the dividend or the risky asset at time 0, we would like to know how this may affect the equilibrium obtained before with homogenous agents.

Assume there are I types of household and each type i , $i = 1, \cdots, I$, is of mass m_i with $\sum_i m_i = 1$. Each type i household is endowed with e_{i0}^h units of dividend and s_{i0}^h units of risky asset at time 0 with $e_0^h \equiv \sum_i m_i e_{i0}^h = KD_0 - e_0^b$ and $s_0^h \equiv \sum_i m_i s_{i0}^h = K - s_0^b$. [12] It turns out that the equilibrium is largely independent of the endowment distribution across the households. Each household with whatever endowments will trade only one type of bond by Lemma 1. Moreover, all types of household with different endowments will trade the same type of bond-$\bar{s}$ because the collateral value in (2.18) is consistent with only

[12] This can be extended to a continuum of types.

a single collateral level $\bar{s}$. Each type i household will buy quantity $K_i/\bar{s}$ of bond-$\bar{s}$ such that all households have the same pricing kernel. Households with higher wealth buy more bonds. On the bank side, the collateral is allocated for trading with different types of household such that $K = \sum_i m_i K_i$.

Remark 3. *There exists a unique equilibrium in the economy* $(I, \{m_i, e_{i0}^h, s_{i0}^h\}_{i=1}^I, e_0^b, s_0^b, K)$ *in which the collateral constraint binds and a single bond-*$\bar{s}$ *is actively traded, i.e.,* $S^* = \{\bar{s}\}$, *with* $\bar{s} \in S$ *determined by (2.19) where* e_0^h *and* s_0^h *are the aggregate dividend and risky asset endowments of households.*

The collateral value for the risky asset, the price for the risky asset, and price for bond-$\bar{s}$ *are still given by (2.18), (2.8), and (2.17), respectively. For any bond-*s, $s \in S \setminus \{\bar{s}\}$, *the price* q^s *is undetermined and can be anything in the interval given by (2.20).*

The allocations are

$$(c_{i0}^h, c_{i1}^h) = \left(e_{i0}^h + s_{i0}^h p_0 - \frac{K_i}{\bar{s}} q^{\bar{s}}, \frac{K_i}{\bar{s}} min(1, \bar{s} D_1)\right), \quad i = 1, \cdots, I$$
$$(c_0^b, c_1^b) = K(D_0, D_1) - \sum_i m_i (c_{i0}^h, c_{i1}^h),$$

where

$$\frac{K_i}{\bar{s}} = \frac{e_{i0}^h + s_{i0}^h p_0}{q^{\bar{s}} + (\beta^b/\beta^h)^{1/\eta}}.$$

The bond portfolio μ_i^h *is a Dirac measure with mass* $\frac{K_i}{\bar{s}}$ *at* $\bar{s} \in S$ *and* μ^b *is a Dirac measure with mass* $\frac{K}{\bar{s}}$ *at* $\bar{s} \in S$.