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1 Introduction

In the recent years, laser ranger finders mounted on mobile robots have become very common for various robot navigation tasks. They are capable of providing accurate range measurements in large angular fields in real time, and enable robots to perform more confidently a wide class of navigation tasks by fusing image data from the camera mounted on robots. In order to effectively use the data from the camera and laser range finder, it is important to know the relative position and orientation from each other, which affects the geometric interpretation of its measurements.

The calibration of each of these geometric sensors can be decomposed into internal calibration parameters and external parameters. The external calibration parameters are the position and orientation of the sensor relative to some fiducial coordinate system. The internal parameters, such as the calibration matrix of a

Figure 1: A B21r^{TM} Mobile Robot from iRobot Corporation can be configured to have a video camera (at the top) and a planar laser range finder which measures distances to points lying on a plane about 18 inches off the floor. The goal of this paper is to study a calibration method that finds the rotation Φ and the translation Δ which transform points in the camera coordinate system to points in the laser coordinate system

camera, affect how the sensor samples the scene. This work assumes the internal sensor calibration is known, and focuses on the external calibration. Here we propose a method for extrinsic calibration of a camera and laser range finder, that is, identifying the rigid transformation from the camera coordinate system to the laser coordinate system. The method employs a planar calibration pattern viewed simultaneously by the camera and laser range finder. For each different pose of the planar pattern, the method constraints the extrinsic parameters by registering the laser scanline on the planar pattern with the estimated pattern plane from the camera image.

It is important also to differentiate this work from two other problems that at first may appear similar. There have been several proposed methods for auto-calibration of a stereo camera pair with points that are matched between both cameras in the pair, and between images from different positions of the camera pair [1, 3, 7]. This is a fundamentally different kind of constraint. There has also been a great deal of work on calibration for laser scanners, which are the parts of active vision systems that project a point or a stripe which is then viewed by the camera. Finding the geometric relationship between the laser scanner and the camera is vital to creating metric depth estimates from the camera images, and calibration methods exist for this problem as well [4]. In this paper we consider an extrinsic calibration of a camera with a laser range finder where the laser points are invisible to the camera.

The next section introduces the basic equations associated with the extrinsic calibration, formalizes the problem we are going to solve and derives the geometric constraints on the rigid transformation from a camera coordinate system to the laser coordinate system. Section 3 gives methods for solving for the extrinsic calibration, first showing a closed-form solution, followed by nonlinear optimization methods which use the results from previous steps as initial conditions. A global optimization is also proposed to refine both camera intrinsic and extrinsic parameters. We conclude by giving experimental results showing the success of the techniques presented.

2 Basic Equations

A camera can be described by a usual pinhole model. A projection from the world coordinates $P = [X, Y, Z]^T$ to the image coordinates $\mathrm{p} = [u, v]^T$ can be represented as follows:

$$\mathrm{p} \sim \mathrm{K}(R\mathrm{P} + \mathrm{t}) \tag{1}$$

where K is the camera intrinsic matrix, R a 3×3 orthonormal matrix representing the camera's orientation, and t a 3-vector representing its position. In real cases, the camera can exhibit significant lens distortion, which can be modelled as a 5-vector parameter $\mathbf{k}$ consisting of radial and tangent distortion coefficients. Let (x, y) be the ideal normalized image coordinates, and $(\tilde{x}, \tilde{y})$ the distorted normalized image coordinates. We have [5]:

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} (1 + k_1 r^2 + k_2 r^4 + k_5 r^6) + \begin{bmatrix} 2k_3 xy + k_4(r^2 + 2x^2) \\ k_3(r^2 + 2y^2) + 2k_4 xy \end{bmatrix} \tag{2}$$

where $r^2 = x^2 + y^2$, $k_1,\ k_2,\ k_5$ are the first 3 coefficients of radial distortion, and $k_3,\ k_4$ are responsible for the tangent distortion.

The laser range finder reports laser readings which are distance measurements to the points on a plane parallel to the floor. A laser coordinate system is defined with an origin at the the laser ranger finder, and the laser scan plane is the plane $Y = 0$. Suppose a point P in the camera coordinate system is located at a point P^f in the laser coordinate system, and the rigid transformation from the camera coordinate system to laser coordinate system can be described by :

$$P^f = \Phi P + \Delta \tag{3}$$

where Φ is a 3x3 orthonormal matrix representing the camera's orientation relative to the laser ranger finder and Δ is an 3-vector corresponding to its relative position.

Our goal in this paper is to develop ways to solve for these camera extrinsic parameters Φ and Δ which define the position and orientation of the camera with respect to the laser coordinate system.

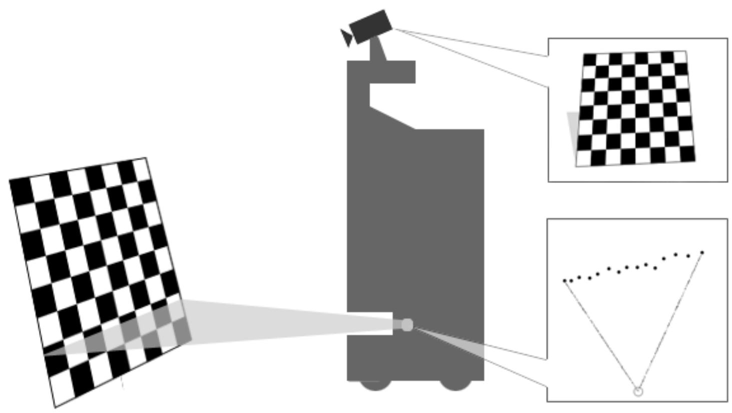

Figure 2: A planar calibration pattern is posed in the both views of the camera and the laser range finder.

Our proposed calibration method is to place in front of our system a planar pattern, say a checkerboard, which is visible to both the camera and the laser ranger finder. Figure 2 provides a general setup of this calibration method. For simplicity, when we talk about a *calibration plane*, we refer to the plane surface defined by the ckeckerboard. And we use *laser points* to refer to the laser measurements on the checkerboard, which is a portion of the whole laser reading.

Without loss of generality, we assume that the calibration plane is the plane $Z = 0$ in the world coordinate system. And in the camera coordinate system, the calibration plane can be parameterized by 3-vector N such that N is parallel to the normal of the calibration plane, and its magnitude $\|N\|$ equals the distance from camera to the calibration plane. With Equation (1) we can derive that

$$N = -R_3(R_3^T \cdot \mathrm{t}) \tag{4}$$

where R_3 is the 3rd column of rotation matrix R.

Since the laser points must lie on the calibration plane estimated from the camera, we get a geometric constraint on the rigid transformation between the camera coordinate system and the laser coordinate system. Given a laser point P^f in the laser coordinate system, from Equation (3), we can determine its coordinate P in the camera reference frame as $P = \Phi^{-1}(P^f - \Delta)$. Since the point P is on the calibration plane defined by N, it satisfies that $N \cdot P = \|N\|^2$. Then we have

$$N \cdot \Phi^{-1}(P^f - \Delta) = \|N\|^2 \tag{5}$$

For a measured calibration plane parameters N and laser point P^f, this gives a constraint on Φ and Δ.

3 Solving Extrinsic Calibration

This section provides the details how to effectively solve the extrinsic calibration problem for the system of a camera and a laser range finder. We will first propose a closed-form solution, followed by a nonlinear optimization with outlier detection. Finally, a global optimization is performed to refine the camera extrinsic parameters. This can be extended to refine the intrinsic parameters more accurately than standard singe camera calibration method.

3.1 Closed-from Solution

In this paper, we assume the camera is calibrated and what remains is to determine the calibration plane parameters by solving the orientation and position of the camera with respect to the checkerboard. Since the camera intrinsic parameters are known, it is easy to estimate R and t, which is discussed in [6]. Once the camera's extrinsic parameters are determined, the calibration plane parameter N can be obtained by Equation (4).

Since all laser points are on the plane Y=0 in the laser coordinate system, a laser point P^f can be represented by $\hat{P}^f = [X, Z, 1]^T$. Then we rewrite Equation (5) as:

$$N \cdot H\hat{P}^f = \|N\|^2 \tag{6}$$

where $H = \Phi^{-1}\begin{pmatrix} 1 & 0 & \\ 0 & 0 & -\Delta \\ 0 & 1 & \end{pmatrix}$, a 3x3 transform matrix from the laser coordinate system to the camera coordinate system. and let $H = [\ h_{ij}\ ]_{3\times 3}$, and $N = [\ n_i\ ]_{3\times 1}$. Then we have a linear constraint on H for each measurement:

$$a \cdot h = \|N\|^2 \tag{7}$$

with $a = [n_1 x, n_1 z, n_1, n_2 x, n_2 z, n_2, n_3 x, n_3 z, n_3]$ and $h = [h_{11}, h_{12}, h_{13}, h_{21}, h_{22}, h_{23}, h_{31}, h_{32}, h_{33}]^T$. If total n laser points we observed from the poses, by stacking n such equations we obtain:

$$Ah = B \tag{8}$$

where A is $n \times 9$ matrix and B is $n \times 1$ vector. If $n \geq 9$, we can obtain a unique solution h by $h = A^T(AA^T)^{-1}B$. Once H is determined by solving for h, we can estimate camera relative orientation and position as follows:

$$\begin{aligned} \Phi &= [H_1, -H_1 \times H_2, H_2]^T \\ \Delta &= -[H_1, -H_1 \times H_2, H_2]^T H_3 \end{aligned}$$

where H_i is the ith column of matrix H.

Because of noise in data, computed matrix Φ doesn't always meet the properties of a rotation matrix. We can compute a rotation matrix $\hat{\Phi}$ to approximate the computed matrix Φ by minimizing Frobenius norm of the difference $\hat{\Phi} - \Phi$, subject to $\hat{\Phi}\hat{\Phi}^T = I$. The details about this matrix computation can be referred in [2]. These estimates of $\hat{\Phi}$ and Δ are used as initial conditions for the nonlinear optimization in the next section.

3.2 Nonlinear Optimization

The above solution is obtained by minimizing an algebraic distance which does not have direct interpretation in the measurement space. We can refine it by a nonlinear minimization on the Euclidean distances from laser points to the checkerboard planes, and it is more physically meaningful.

Given different poses of the checkerboard, the camera extrinsic parameters Φ and Δ can refined by minimizing the below functional:

$$\sum_i \sum_j \left(\frac{N_i}{\|N_i\|} \cdot (\Phi^{-1}(P_{ij}^f - \Delta)) - \|N_i\| \right)^2 \tag{9}$$

where N_i defines the checkerboard plane in the ith pose. A rotation Φ is parameterized by the Rodrigues formula as a 3-vector parameter, which is in the direction the rotation axis and has a magnitude equal to the rotation angle. We minimize Equation (9) as a nonlinear optimization problem with Lavenberg-Marquardt Algorithm in Matlab. This requires an initial guess of Φ and Δ, which is obtained using the method described in the previous section.

Both camera and laser range finder have some noise in their outputs, and we found the wrong estimates of calibration planes will badly effect the final results. We develop an outlier-removal approach that iteratively throws away some data sets which have too large residuals from Equation (9). A brief description is given as follows:

1. for each checkerboard view, the calibration plane parameter $\vec{N}$ is estimated in the camera coordinate system, and the laser points are extracted in the laser coordinate system. Construct a valid pose set with all poses.

2. Based on current valid pose set, estimate camera orientation Φ and position Δ by minimizing the residual function from the geometric constraints in Equation (9). Then empty the valid pose set.

3. for each pose, compute the average 3D projection error ϵ of laser points to the checkerboard plane with current estimated camera orientation and position. If the projection error $\epsilon < \delta$, then we add this pose into the valid pose set, where δ is threshold for maximum average error.

4. Repeat Step 2, 3 until convergence (empirically it takes 2 to 3 iterations to converge).

3.3 Global Optimization

In practice, the camera calibration is not precisely known, and the errors affect the performance of the extrinsic calibration. Given the relative orientation and position of a camera, the laser data gives extra constraints on the position of the planar pattern, which could be analyzed in the camera intrinsic calibration. So intuitively, the camera intrinsic and extrinsic parameters can be refined by performing a global optimization with the initial estimate of the camera extrinsic parameters Φ and Δ, and an estimate of the intrinsic parameter K.

Given different views of the checkerboard and grid points on the plane, We can obtain the projection error of grid points:

$$\sum_i \sum_j \|\mathbf{p}_{ij} - \tilde{\mathbf{p}}(\mathrm{K}, \mathrm{k}, R_i, \mathrm{t}_i, P_j)\| \tag{10}$$

where $\tilde{\mathbf{p}}(\mathrm{K}, \mathrm{k}, R_i, \mathrm{t}_i, P_j)$ is the projection of point P_j in image i according to Equation (1) and (2).

With Equation (9), we can do a global optimization by minimizing the combination of reprojection error and laser to calibration plane error as follows:

$$\sum_i \sum_j \mathrm{d}^2(P(\Phi, \Delta, P_{ij}^f), N(R_i, t_i)) + \alpha \sum_i \sum_j \|\mathbf{p}_{ij} - \tilde{\mathbf{p}}(\mathrm{K}, \mathrm{k}, R_i, \mathrm{t}_i, P_j)\|^2 \tag{11}$$

where α is a scalar weight, $N(R_i, \mathrm{t}_i)$ the calibration plane parameter in pose i according to Equation (4), $P(\Phi, \Delta, P_{ij}^f)$ the coordinate of laser point P_{ij}^f in the camera coordinate system according to Equation (3), and $\mathrm{d}^2(P, N)$ the squared Euclidean distance from point P to plane N.

This nonlinear minimization problem can be solved with the Levenberg-Marquardt Algorithm implemented in Matlab optimization toolkits. It demands the initial guesses of Φ, Δ, and $\{R_i, \mathrm{t}_i \mid i = 1 \ldots n\}$, which can be obtained with the methods discussed in the previous sections. An initial guess of camera intrinsic parameters K and distortion parameter k can be just the original inputs.

3.4 Algorithm Summary

The complete algorithm can be described as the following steps:

1. Build a big checkerboard and place it in front of the camera-laser range finder system in the different orientations.

2. For each checkerboard pose, extract the laser points in the laser reading, and detect the checkerboard grid points in the image. Estimate the camera orientation R_i and position t_i with respect to the checkerboard, and then compute the calibration plane parameter N_i.

3. Estimate the parameter Φ and Δ using the closed-form solution as described in Section 3.1.

4. Refine Φ and Δ using the techniques described in Section 3.2.

5. If necessary, refine all parameters by minimizing (11).

4 Experiments

The proposed algorithm has been implemented in Matlab and tested on both computer simulated data and real data. The closed-form solution provides initial conditions for the nonlinear refining. A global optimization on all camera calibration parameters is also tested on simulation data.

4.1 Computer Simulations

The placement of the camera relative to the laser range finder is described by the extrinsic parameters $\Phi = \begin{pmatrix} 0.9998 & 0.0124 & -0.0185 \\ -0.0074 & 0.9689 & 0.2475 \\ 0.0210 & -0.2473 & 0.9687 \end{pmatrix}$ and $\Delta = [\ 0 \quad -1.0 \quad 0.1\ ]^T$ meters. The camera is simulated as an ideal pinhole with focal length 750 and principal point $(320, 240)$. The calibration pattern plane is a checkerboard defined by 10×10 grids, and the size of the pattern square is $76mm \times 76mm$. The orientation of the checkerboard is generated as follows: the plane is initially parallel to the image plane; a rotation axis is randomly chosen on the plane and the plane is rotated around that axis with angle θ. The position of the plane is chosen properly such that the checkerboard grids can appear entirely on the image plane. Gaussian noise with mean 0 and standard deviation 0.5 pixel is added to the projected image points. The laser points are computed based on the placement of the camera and the setting of checkerboard. We also add uniform noise into the laser points to simulate their accuracy of 5cm.

In the experiment, the estimated extrinsic parameters are compared with the ground truth. We measure the error for camera orientation Φ by computing the angle between the estimate and the true orientation, and the error for camera position Δ by computing the distance between the estimate and the true camera position.

Performance w.r.t. the number of checkerboard poses. This experiment deals with how the number of plane poses effects the performance. We vary the number of poses from 6 to 24. For each experiment number, 100 trials of independent checkerboard plane orientations with $\theta = 60^\circ$ and independent checkerboard plane positions are conducted. The Gaussian noise added in the projected image points is also independent between trials, as well as the uniform noise in the laser points. The results is shown in Figure 3. The error decreases when more poses are used.

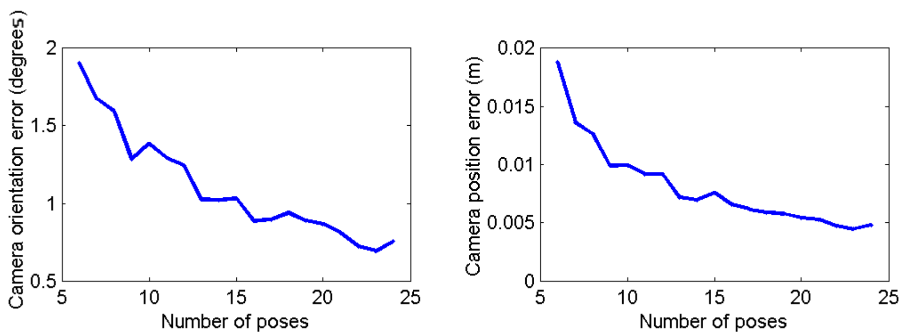

Figure 3: Errors vs. the number of poses of the checkerboard plane

Performance w.r.t. the orientation of checkerboard plane. This experiment is performed for different orientations of the checkerboard plane to examine its influence in the calibration performance. We compute the average errors by running 100 trials with 10 checkerboard poses. The orientation angle of checkerboard plane varies from 10° to 80°, and the result is shown in Figure 4. We found that the result improves when the orientation angle increases, due to more precise estimates of calibration planes with large angles with respective to the image plane. Best performance seems to be achieved around 70°. Note that in practice, when the angle increases, foreshortening makes the corner detection less accurate, and the number of laser points probably decreases, but these are not considered in this experiment.

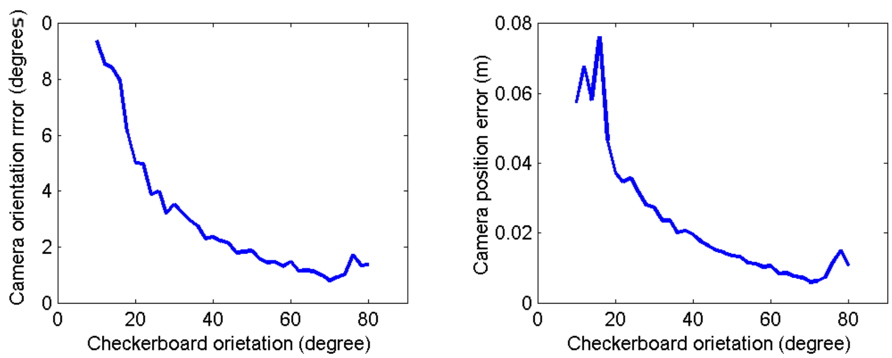

Figure 4: Errors vs. the orientation of the checkerboard plane

Refinement of camera intrinsic parameters. This experiment investigates how the laser data help the camera calibration, and examines its performance on refining camera intrinsic and extrinsic parameters. Here we run 100 independent trials and then compare the average result with the ground truth. For each trial, 10 checkerboard poses are used. The camera focal length are corrupted with Gaussian noise with mean 0 and standard deviation 10. For the camera principal point, Gaussian noise with mean 0 and deviation 5 pixels is added. The checkerboard is placed in the orientation ranging from 50° to 70°. Initially, we perform the extrinsic calibration with the corrupted camera intrinsic matric K. Then use the initial camera orientation Φ and position Δ to do global optimization to estimate final camera intrinsic and extrinsic parameters. The improvement of camera intrinsic matrix can be evaluated by the ratio of the Frobenius norm of the difference of the estimated K and the ground truth to the Frobenius norm of the difference of the corrupted K and the ground truth. The result is shown in Table 1. As we can see, when the global optimization is applied on both camera intrinsic and extrinsic parameters, the accuracy of the camera intrinsic matrix K is improved by about 30%. The error of estimates of camera orientation and position with respect to the laser coordinate system also decreases by around 30%.

	Orientation error(Φ)	Position error(Δ)	Fro. norm ratio w.r.t K
initial	2.33°	3.78cm	
final	1.95°	2.37cm	0.6969

Table 1: The result of global opitimization on the camera intrinsic and extrinsic parameters

4.2 Real Data

The proposed method has been tested on a robotic platform shown in Figure 1, equipped with a SICK-PLS laser range finder and a Fujifilm 4800 camera mounted on top of the robot. The laser range finder is capable of scanning 180 degrees of the environment parallel to the floor, with an angle resolution of one measurement per degree and a range measuring accuracy of 5cm. Experiments have been performed with real data, and the method operates well given reliable calibration parameters of the camera. Here we present the result with one example.

The camera resolution is set as 640×480. The calibration pattern is a 12×10 checkerboard, and the size of a checker square is $76mm \times 76mm$. 12 images of the checkerboard are taken along with 12 laser readings. The laser points can be manually selected among the whole laser measurements. Figure 5 demonstrates the results of the algorithm applied to the configuration. We map the laser points onto the calibration plane with estimated Φ and Δ, and the average distance error from the laser points to the calibration plane is around 2-3cm. Although we do not have the ground truth of the extrinsic parameters Φ and Δ, but the mapping results are quite reasonable.

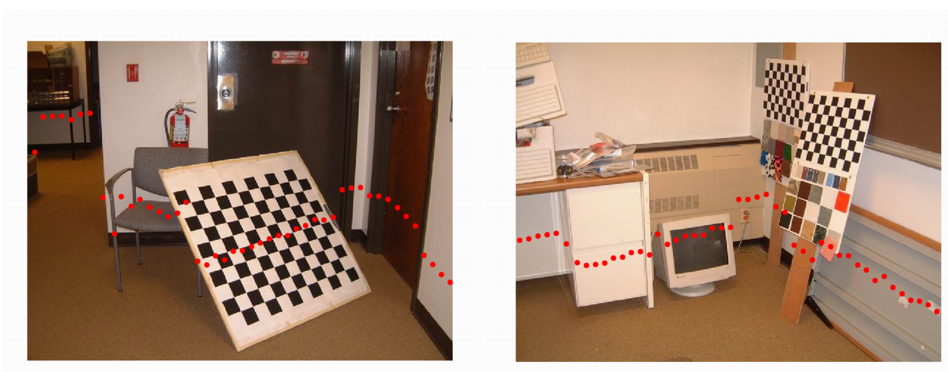

Figure 5: The left figure shows one checkerboard setting captured by the camera, and the red points overlaying on the image are the mapping laser reading with estimated extrinsic parameters Φ and Δ. The right figure shows one example of laser reading mapped onto the camera image after the extrinsic calibration

5 Conclusion

In this paper, we presented an extrinsic calibration method to estimate the orientation and position of a camera with respect to a laser ranger finder for the robot.

The proposed method requires a few poses of planar pattern which is visible for both the camera and the laser range finder, and then a geometric constraint on the camera extrinsic parameters is imposed. For a well calibrated camera and laser range finder, this method succeeds in calibrating camera extrinsic parameters with respect to the laser range finder. Moreover,the camera intrinsic calibration can be also refined by fusing laser data, which we believed is helpful for more robust robotic tasks.

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