

Washington University in St. Louis

Washington University Open Scholarship

Arts & Sciences Electronic Theses and
Dissertations

Arts & Sciences

Spring 5-2017

Market Risk Management for Financial Institutions Based on GARCH Family Models

Qiandi Chen

Washington University in St. Louis

Follow this and additional works at: https://openscholarship.wustl.edu/art_sci_etds



Part of the [Applied Statistics Commons](#), [Finance and Financial Management Commons](#), and the [Longitudinal Data Analysis and Time Series Commons](#)

Recommended Citation

Chen, Qiandi, "Market Risk Management for Financial Institutions Based on GARCH Family Models" (2017). *Arts & Sciences Electronic Theses and Dissertations*. 1059.
https://openscholarship.wustl.edu/art_sci_etds/1059

This Thesis is brought to you for free and open access by the Arts & Sciences at Washington University Open Scholarship. It has been accepted for inclusion in Arts & Sciences Electronic Theses and Dissertations by an authorized administrator of Washington University Open Scholarship. For more information, please contact digital@wumail.wustl.edu.

Washington University in St. Louis
Graduate School of Arts and Sciences
Department of Mathematics

Market Risk Management for Financial Institutions Based on GARCH Family Models

by

Qiandi Chen

A thesis presented to
The Graduate School
of Washington University in
partial fulfillment of the
requirements for the degree
of Master of Arts

May 2017
Saint Louis, Missouri

Contents

List of Tables	iii
List of Figures	iv
Acknowledgments	v
Abstract	viii
1 Introduction	1
1.1 Background	1
1.2 Outline	3
2 Methodology	5
2.1 VAR	5
2.1.1 Calculation	5
2.1.2 Element Selection	7
2.2 Time Series and ARIMA Model	8
2.3 GARCH Family Model	10
2.3.1 ARCH	10
2.3.2 ARCH Effect Hypothesis-Testing	11
2.3.3 GARCH	12
2.3.4 E-GARCH	13
3 Data Analysis	15
3.1 Data Collection	15
3.2 Descriptive Statistics and Time Series Tests	16
3.3 Model Setup	20
3.3.1 ARMA Model	20
3.3.2 GARCH Family Model	22
3.4 VAR Calculation and Early Warning	29
3.5 Leverage Effect and News Impact Curve	32
4 Conclusion	35

List of Tables

3.1	S&P 500 Financials Sector Log Yield Descriptive Statistics	17
3.2	ARCH(q) Log Likelihood and AIC	22
3.3	Box-Ljung Test and ARCH-LM Test	25
3.4	GARCH Family Models' AIC and Fitted MSE	29
3.5	Future 10 days' Relative VAR at 99% level in 1-Day Horizon	31

List of Figures

3.1	S&P 500 Financials Sector Log Yield Histogram	17
3.2	S&P 500 Financials Sector Logarithmic Yield Time Series Plot	18
3.3	S&P 500 Financials Sector Log Yield ACF and PACF	20
3.4	ARMA Forecast Plot	21
3.5	Standardized Residuals of GARCH Family Models	26
3.6	VAR Historical Plot with Early Warning	32
3.7	News Impact Curve	34

Acknowledgments

This thesis took me a lot of time and energy this year, so that I feel that to complete such a paper is not an easy thing. However, I can feel the sense of accomplishment from the heart and deep gratitude when I was working on it. And there are several people gave me help from the early preparation to the idea construction, and from the late writing to the final revision.

Firstly, I would like to thank Washington University in St. Louis for providing me with a platform to write my thesis, so that I can have an opportunity to study the situation of US stock market. This is a valuable experience for me. Secondly, I would like to thank my thesis advisor Todd Kuffner, who helped me to determine the title and scope of my thesis, gave me a lot of advice on how to write a academic thesis, pointed out the insufficiency of my contents from the aspect of statistics, did the proofreading to supplement my English expression's weakness, and instructed me to correct the format. Thirdly, I would like to thank my other thesis committees, José E. Figueroa-López and Renato Feres, for their attendance on my oral defence to give me more suggestion. Finally, I would also like to thank my parents and friends who gave me so much passion and support to complete the thesis writing, even in the time when I was so upset about some failures.

Thank you all so much! I would continue to improve myself and make more efforts to write better theses.

Qiandi Chen

Washington University in Saint Louis

May 2017

Dedicated to my parents.

ABSTRACT OF THE THESIS

Market Risk Management for Financial Institutions Based on GARCH Family Models

by

Qiandi Chen

Master of Arts in Statistics

Washington University in St. Louis, May 2017

Research Advisor: Professor Todd Kuffner

The financial stock market turned out to rise and fall suddenly and sharply in recent years, which means that volatility and uncertainty is very significant in market and measuring the market risk accurately is of great importance. I collect the historical close price of S&P 500 Financials Sector Index from January 19th 2011 to January 31st 2017, and use the daily logarithm yield as time series data to build 2 ARMA models and 5 GARCH family models using t-distribution. Then I calculate future 10 days' relative VAR in 1-day horizon under 99% confidence level based on the selected model. E-GARCH model also shows the leverage effect of the time series, thus we know that the stock price is more sensitive to bad news than good news.

Keywords: Financial Market Risk Management, GARCH Family Models, VAR(Value at Risk), Early Warning, Leverage Effect

Chapter 1

Introduction

1.1 Background

The price of securities always exhibits uncertainty and unpredictability due to its substantial volatility and high speculation. Since volatility and uncertainty are correlated to risk directly, it is particularly important to describe and control risk in the market. Financial markets changed rapidly from several aspects with the increasing globalization of the economy, particularly for those financial institutions. Financial derivatives came into being and risk management and supervision have also been greatly strengthened. In order to measure and manage risk more effectively, quantitative risk must be understood.

Risk includes several types: market risk, credit risk, liquidity risk, operational risk, legal risk, accounting risk, information risk, and strategy risk. Market risk refers to the possible impact of the unexpected changes of price and rate in the market on the investments or operations. Markowitz (1952) proposed risk measurement model and use variance as an

important measurement of risk tools. Sharpe (1964) and Lintner (1965)'s CAPM also treat variance of yield rate as risk, but also found the limitations of this method. In the 1970s, financial market emerged many new models and methodologies that make risk measurement more accurate to numerical values. For instance, sensitivity methods and volatility methods are introduced to supplement the traditional risk management and the insufficiency of the normal distribution assumptions.

Among those various methods, VAR method is the mainstream one for financial institutions and regulators as the risk monitoring tool, which mainly examines the maximum probable loss (MPL). Morgan (1995) first used the VAR method, which established risk metrics on the RiskMetrics System and announced its VAR values.

The problem of VAR primarily refers to the variance of time series on price and rate, and this then is actually a problem about heteroskedasticity. We can have many methods like WLS to solve the problem of incremental heteroskedasticity, that is, the variance of the random error term changes with the change of explanatory variable. However, the interest rate, exchange rate, stock returns and some other financial time series exists the heteroskedasticity that does not belong to the incremental hteroskedasticity.

This kind of time series has some properties. The variance of the process not only changes with time, but also changes dramatically in some periods. It demonstrates volatility clustering characteristics by time, that is, variance is relatively small in a certain period, while it turns to be relatively large in another period. The value distribution exhibits leptokurtosis

characteristics, that is, the probability near mean and tail is larger than normal distribution, while the probability of the rest is smaller than normal distribution. Obviously the current variance is related to the volatility of the previous period.

Engle (1982) proposed the ARCH (Auto Regressive Conditional Heteroskedasticity) model to analyze the heteroscedasticity of the time series with these properties. In practice, the ARCH model might have high moving average order, thereby increasing the difficulty of parameter estimation and affecting the fitting accuracy. To solve this problem, Bollerslev (1986) proposed the GARCH (Generalized ARCH) model, which adjusts the variance of the error, and provides further analysis on volatility measurement and even prediction. Furthermore, E-GARCH model proposed by Nelson (1991) describes the asymmetric effect from different kind of news.

1.2 Outline

In this paper, I make an application to daily returns on the S&P 500 Financials Sector Index as the data set to find the VAR among financial institutions in the market. The time series includes the historical price and furthermore logarithmic yield from January 19th 2011 to January 31st 2017. This is a seven-year daily data, so we can analyze the recent seven year stock market on the aspects of only financial institutions.

Here we can build some useful volatility models based on these seven-year historical data and find a better one by comparisons. The GARCH models for financial stock return

in this paper would be based on the t-distribution hypothesis to estimate the VAR values, since the normal distribution is not sufficient for these kinds of time series.

Then we are to estimate the variance and calculate the VAR value. We can also make prediction on stock daily stock returns using mean value model and on VAR using volatility model for a period in the future. And thus early warning can be made according to VAR afterwards to help people reduce and control risk.

Chapter 2

Methodology

2.1 VAR

2.1.1 Calculation

VAR, value at risk, is the maximum probable loss (MPL) of financial asset portfolio given time horizon t and confidence level a when market is fluctuated normally. It can be expressed as

$$Prob(\Delta P < -VaR) = 1 - a.$$

Prob refers to the probability. $P(t)$ is the price of an investment or asset in time t . ΔP is the loss in the market during asset holding period t : $\Delta P = P(t + \Delta t) - P(t)$. VAR is the value in the risk condition with confidence level a . Thus the probability that the loss is more than the value at risk is $1 - a$. No matter whether we have positive or negative return, we treat VAR as a positive number for convenience expression here.

In the general distribution, assuming that P_0 is the initial value of the portfolio and the R is the return on investment during the holding period t . Then the value of the portfolio in the end period can be expressed as

$$P = P_0(1 + R) = P_0 + P_0R.$$

Assuming that the expected return and volatility of the return rate R is μ and σ . If the lowest value of the asset portfolio is

$$P^* = P_0(1 + R^*) = P_0 + P_0R^*,$$

then according to the definition of VAR, the maximum probable loss of asset portfolio over period of time t under confidence level α can be defined as VAR relative to the mean value of asset portfolio value (expected return), that is, the relative VAR is defined as

$$VAR_R = E(P) - P^* = P_0(1 + E(R)) - P_0(1 + R^*) = -P_0(R^* - \mu).$$

The minimum return here can be expressed as

$$R^* = -\alpha\sigma + \mu,$$

where α is the quantile value for confidence level a in standard normal distribution. And the area on the left showing in the graph is equal to $1 - a$.

Plug this equation into the relative VAR equation above, and we have that

$$VAR_R = -P_0(R^* - \mu) = P_0\alpha\sigma\sqrt{\Delta t}.$$

2.1.2 Element Selection

By definition we can see that the three elements of VAR are the choice of confidence level, the distribution of assets and the choice of asset holding period of time. Since the distribution of the asset is fixed when figuring the VAR of a certain asset, it is important to select the left two elements to measuring the VAR: basic time interval and confidence level.

The longer the holding period, and the higher confidence level, the larger is the VAR value. By contrast, the VAR value would be small if the time period is short and the confidence level is low. The length of the time period is selected by comprehensively considering the cost of frequent supervision. Some of the major commercial banks in the world select the interval of 1 day, which means that they publish the daily VAR value of the assets. The internal model of the Basel Committee selects the interval of 10 days and confidence level of 99%, and then the minimum capital requirement to ensure regulatory purpose can be obtained by multiplying the calculated VAR with a safety factor 3.

When it comes to confidence level, we should consider the availability and adequacy of historical data. The confidence level is set at the same time to balance the effectiveness and reliability of the VAR value. If the confidence level is high, the probability of events for which the loss exceeds the VAR value is reduced, and the demand for the number of data observations will be high. The confidence level should not to be set too high if the number of data observations is not enough. And if the confidence level is set too low, the effectiveness and reliability of the VAR value would be reduced.

2.2 Time Series and ARIMA Model

The mean function of time series r_t is defined as

$$\mu_t = E(r_t) = \int_{-\infty}^{\infty} r f_t(r) dr.$$

The auto-covariance function is defined as the second moment product

$$\gamma(s, t) = cov(r_s, r_t) = E[(r_s - \mu_s)(r_t - \mu_t)],$$

for all s and t . Then a time series r_t is said to be weakly stationary if (i) the mean value function μ_t is constant and does not depend on time t , and (ii) the auto-covariance function $\gamma_r(s, t)$ depends on s and t only through their difference $|s-t|$, that is, $\gamma(s, t) = \gamma(k, k+s-t)$.

The auto-correlation function (ACF) is defined as

$$\rho(s, t) = \frac{\gamma(s, t)}{\sqrt{\gamma(s, s)\gamma(t, t)}}.$$

The partial auto-correlation function (PACF) of a stationary process r_t , denoted ϕ_{hh} , for $h = 1, 2, \dots$, is

$$\phi_{11} = \text{corr}(r_{t+1}, r_t)$$

$$\phi_{hh} = \text{corr}(r_{t+h} - \hat{r}_{t+h}, r_t - \hat{r}_t), \quad h \geq 2.$$

And a time series z_t is white noise series if it is a collection of uncorrelated random variable z_t with mean 0 and finite variance σ_z^2 , and $\gamma(s, t) = \text{cov}(z_s, z_t) = 0, \text{ if } s \neq t$.

The Autoregressive Integrated Moving Average Model (ARIMA) is proposed by Box and Jenkins (1970). In ARIMA (p, d, q), p is the autoregressive term, d is the number of times of difference that makes the time series become stationary, and q is the moving average term.

After a certain times of difference, a time series would become stationary and we can only fit ARMA model here. This is an ARMA(p,q) equation

$$r_t = \mu + \sum_{i=1}^p \alpha_i r_{t-i} + \sum_{i=1}^q \beta_i \varepsilon_{t-i} + \varepsilon_t,$$

where r_t is the time series variable and ε_t is the error term.

2.3 GARCH Family Model

2.3.1 ARCH

If a stationary random variable r_t can be expressed as an AR(p) form

$$r_t = \mu + \sum_{i=1}^p \alpha_i r_{t-i} + \varepsilon_t$$

and the time series of its random error term ε_t can be given as

$$\varepsilon_t = \sigma_t z_t$$

where z_t is white noise series with zero mean and unit variance, the variance of the random error term is the expectation of the square error term

$$Var(\varepsilon_t) = E[\varepsilon_t^2] - (E[\varepsilon_t])^2 = E[\varepsilon_t^2]$$

and the variance can be described by the q-order lags of the square error term

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_q \varepsilon_{t-q}^2,$$

where $\alpha_0 > 0, \alpha_i \geq 0, i = 1, 2, \dots, q$ and $0 \leq \alpha_1 + \alpha_2 + \dots + \alpha_q < 1$ for stationary condition of time series. We say ε_t follows q-order ARCH process and is denoted as $\varepsilon_t \sim \text{ARCH}(q)$. We

can use this ARCH model to estimate the variance of r_t by

$$Var(r_t) = Var(\varepsilon_t) = \sigma_t^2.$$

There are several reasons to use ARCH family models to estimate the variance for risk management. We can assess how much the risk of the assets' holdings or transactions is brought to the revenue by predicting the amount of change in r_t or ε_t . The confidence interval of r_t , which changes over time, can be predicted. The correct estimation of the conditional heteroskedasticity can make the estimation of the regression parameter more accurate.

2.3.2 ARCH Effect Hypothesis-Testing

Whether or not an autoregressive conditional heteroskedasticity (ARCH effect) exists in the error term of the mean equation should be tested. We can use ARCH-LM test here. Its null hypothesis is that the squared residuals series are not auto-correlated and there is no ARCH effect, while the alternative hypothesis is that the variance of the error term is not a constant and there exists an ARCH effect:

$$H_0 : \alpha_1 = \alpha_2 = \dots = \alpha_q = 0$$

$$H_1 : \alpha_1, \alpha_2, \dots, \alpha_q \text{ not all zero.}$$

Under null hypothesis, the OLS estimators are efficient, while the OLS estimators become inefficient in the case of the alternative hypothesis.

This are the steps to do this LM test: $\hat{\varepsilon}_t^2$ can be obtained by regressing r_t . Then regress $\hat{\varepsilon}_t^2$ with its q-order lags. Now R^2 is obtained from this new regression and use this one to construct the statistic $LM = TR^2$, where T is the sample size of the new regression.

Under the null hypothesis, $LM = TR^2 \sim \chi_{(q)}^2$. If $LM < \chi_{\alpha(q)}^2$, the null hypothesis would not be rejected. And if $LM > \chi_{\alpha(q)}^2$, it should be rejected.

2.3.3 GARCH

The ARCH(q) model is a distributed lag model for σ_t^2 . To avoid too many lag terms, we can use the methods of adding the lag terms of σ_t^2 , which utilizes the concept of Recall Reversibility.

The model is called generalized autoregressive conditional heteroskedasticity (GARCH)

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2,$$

denoted as GARCH(p,q), where $\omega > 0$, $\alpha_i \geq 0, i = 1, 2, \dots, q$, $\beta_j \geq 0, j = 1, 2, \dots, p$, and $0 \leq \sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j < 1$. Here p and q are orders of the lag and α_i and β_j are parameters of the model. ε_{t-i} are ARCH terms and ε_{t-j} are GARCH terms.

2.3.4 E-GARCH

Since the historical data is in the form of a square to affect the future volatility in the GARCH model, the rise and fall of the asset have same impact on future volatility. However, empirical analysis shows that the good news or bad news for financial asset has different qualitative effect on volatility result. This asymmetric impact from different kind of news on stock volatility is called the leverage effect.

E-GARCH, derived from the GARCH model, is to explain the leverage effect. In order to explain it, the symmetry function “square” should be abandoned, and the historical data could affect future volatility via an asymmetric function. However, the asymmetric function may break the rationality of the GARCH model, that is, the future volatility may be negative. The solution is to use the logarithm of volatility instead of simple volatility, and then the variance could be guaranteed as positive in the E-GARCH model.

The form of the E-GARCH model is

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^q \alpha_i \frac{|\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^p \beta_j \ln(\sigma_{t-j}^2)$$

or in Nelson form

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^q \alpha_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{i=1}^q \gamma_i \left(\left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| - \mu \right) + \sum_{j=1}^p \lambda_j \ln(\sigma_{t-j}^2),$$

where $\sum_{j=1}^{\infty} \alpha_j^2 < \infty$. μ is expectation of $|\frac{\varepsilon_t}{\sigma_t}|$, $\ln(\sigma_{t-j}^2)$ are log GARCH terms, $|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}| - \mu$ are ARCH terms, and $\frac{\varepsilon_{t-i}}{\sigma_{t-i}}$ are terms to describe the difference of good or bad information.

Since the right hand side is the logarithm of σ_t^2 , σ_t^2 would hold positive no matter whether the right hand side is positive or negative.

The term of $\frac{\varepsilon_{t-i}}{\sigma_{t-i}}$ is standardized new information. When there is good or bad news for asset, the term of $\frac{\varepsilon_{t-i}}{\sigma_{t-i}}$ shows as positive or negative respectively. Although the absolute value of positive and negative news is the same, the EGARCH model can distinguish the different effects of positive and negative news on the volatility.

Chapter 3

Data Analysis

3.1 Data Collection

I chose the historical close price of S&P 500 Financials Sector index with an investment horizon of 1 day as the origin data to apply for the risk testing methodology.

The data are collected from S&P Dow Jones Indices website¹. This index contains the companies included in the S&P 500 that are classified as members of the global industry classification system financial sector, which includes 3 industry groups: Banks, Diversified Financials and Insurance.

The historical time series data of stock price covers a period of 7 years from January 19th 2011 to January 31st 2017, which has 1519 observations. Obviously the time series is not stationary. First-order differenced data can be derived from this original data by using

¹<https://us.spindices.com/indices/equity/sp-500-financials-sector>

the natural logarithmic yield to be the variable of the volatility models:

$$r_t = \ln P_t - \ln P_{t-1},$$

where P_t is the stock price in time t . r_t is the stock return rate, which can be simply denoted as log yield.

3.2 Descriptive Statistics and Time Series Tests

The mean of the log yield is 0.0003609178. Its median is 0.000845984. The standard deviation is 0.01292466. Its skewness is -0.4055435, less than zero, illustrating a long left tail. The minimum value -0.1051823 is larger than the maximum 0.07889818 in absolute value. This roughly gives an information that the number of positive profit is more than the loss one, but the extent of loss might be greater than that of profit.

The kurtosis is 6.765386, larger than 3 in normal distribution, which demonstrates that the log yield exhibits leptokurtosis characteristic. We can notice this in the histogram in Figure 3.1. Jarque-Bera statistic ($JB = \frac{n-df}{6}(S^2 + \frac{(K-3)^2}{4})$) is 2938.5 with p-value 0.0000. Thus we can reject the null hypothesis of its normal distribution under 95% confidence level. We may consider to build the model using a t-distribution later.

Based on the time series plot of S&P 500 Financials Sector index's logarithmic yield in Figure 3.2, we can observe that the variance of the log yield not only changes through

Table 3.1: S&P 500 Financials Sector Log Yield Descriptive Statistics

Mean	Std. Dev.	Kurtosis	Skewness	Jarque-Bera
0.0003609178	0.01292466	6.765386	-0.4055435	2938.5

Figure 3.1: S&P 500 Financials Sector Log Yield Histogram

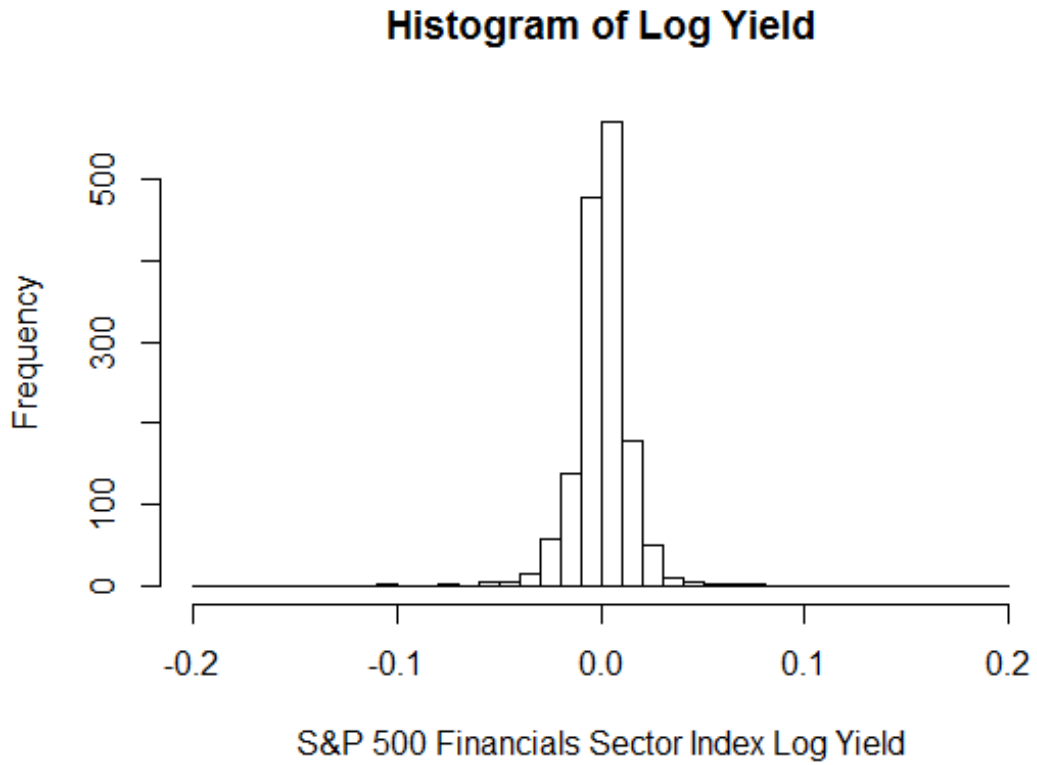
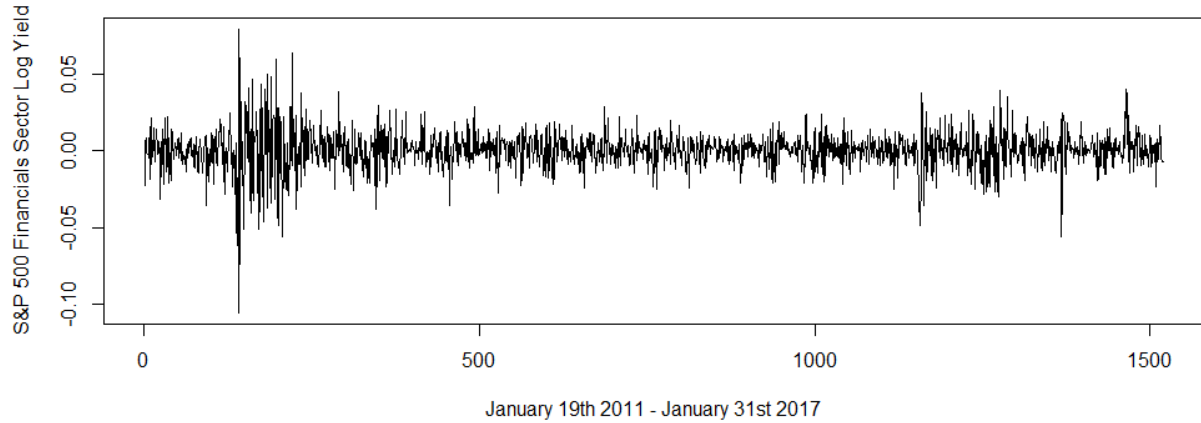


Figure 3.2: S&P 500 Financials Sector Logarithmic Yield Time Series Plot



the time, but also changes very dramatically through the time. There exists a volatility clustering feature in this plot: the variances are small in some period (e.g. observations 500-1100), while the variances are sometimes quite large (e.g. observations 100-300 and 1100-1400).

Thus the log yield time series might have ARCH effect and some further statistical tests on the data need to be taken before we build models.

The first test for time series is to see whether the data are stationary or not. Here I use the Augmented-Dickey-Fuller Unit Root Test. The null hypothesis is that the time series has a unit root and is not stationary, while the alternative hypothesis is that it does not have a unit root and is stationary. Here the ADF statistic of log yield series is -10.977 with its p-value smaller than 0.01. Thus the null hypothesis that the time series is non-stationary would be rejected under 95% confidence level.

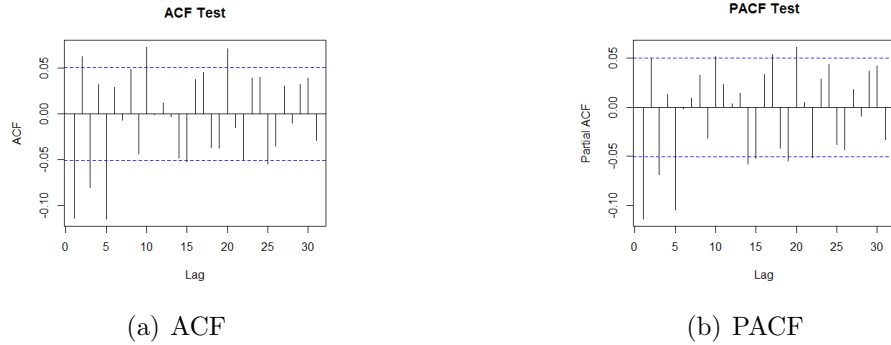
The second test is to see whether it is a white noise process. We can do Box-Ljung test, which is a test about overall randomness. The Box-Ljung test statistic is

$$Q = n(n+2) \sum_{k=1}^h \frac{\hat{p}_k^2}{n-k},$$

which follows $\chi^2(h)$ in significance level α . We can see whether the autocorrelations of time series are different from zero via Box-Ljung test. The null hypothesis is that the series is independently distributed, while the alternative hypothesis is that it is not independently distributed and has serial correlation. Here the Box-Ljung test result of log yield is $\chi^2 = 57.038$, $df = 7.3258$, $p\text{-value} = 8.485e-10$. Thus the null hypothesis that $\hat{p}_1^2 = \hat{p}_2^2 = \dots = \hat{p}_n^2 = 0$ would be rejected under 95% confidence level. The variables might not be independent.

Autocorrelation also can be seen in the ACF and PACF chart in Figure 3.3. It is shown in the chart that several coefficients of autocorrelation and partial autocorrelation are larger than confidence interval. So there exists significant autocorrelations. Therefore, we need to consider ARMA model to build the mean equation later.

Figure 3.3: S&P 500 Financials Sector Log Yield ACF and PACF



3.3 Model Setup

3.3.1 ARMA Model

Since both ACF and PACF have tails and do not actually truncated after a certain lag, I consider a mixed ARMA model would be better to fit the mean model for stock return rate.

Using Akaike’s Information Criterion ($AIC = 2p - 2\log(L)$) to find the ARMA model in R automatically, we could get ARIMA(2,0,1) with minimum AIC -8929.25. Similarly we can find another ARIMA(1,0,1) model by using Bayes Information Criterion ($BIC = p\log(n) - 2\log(L)$) with minimum BIC -8914.49. The penalty for bigger parameters of BIC is more strict than AIC. Both of these two models could be considered to build GARCH model, and we could choose one according to the result of GARCH model fit.

ARMA Model 1 (AIC):

$$r_t = -0.9042r_{t-1} - 0.0268r_{t-2} + 0.8075\varepsilon_{t-1} + \varepsilon_t$$

ARMA Model 2 (BIC):

$$r_t = -0.8536r_{t-1} + 0.7726\varepsilon_{t-1} + \varepsilon_t$$

Figure 3.4 is the plot of ARMA(2,1) model with predicted future return plot for 30 days using ARMA(2,1) under 85% and 95% confidence level. It shows that the stock yield is around mean zero with variances, which is reasonable for economic meaning. However, obviously this forecast is too rough to provide any more valid information since the variance hold unchanged during the predicted time.

ARCH effect should be tested since we can roughly find there is significant heteroskedasticity for this time series in Figure 3.4. We use ARCH-LM test here, which is stated before in 2.3.2. The result is that the LM statistic is 316.05 with p-value less than 2.2e-16. Thus the null hypothesis of no ARCH effect is rejected under 95% confidence level. I would build GARCH family models to find more efficient estimators.

Figure 3.4: ARMA Forecast Plot

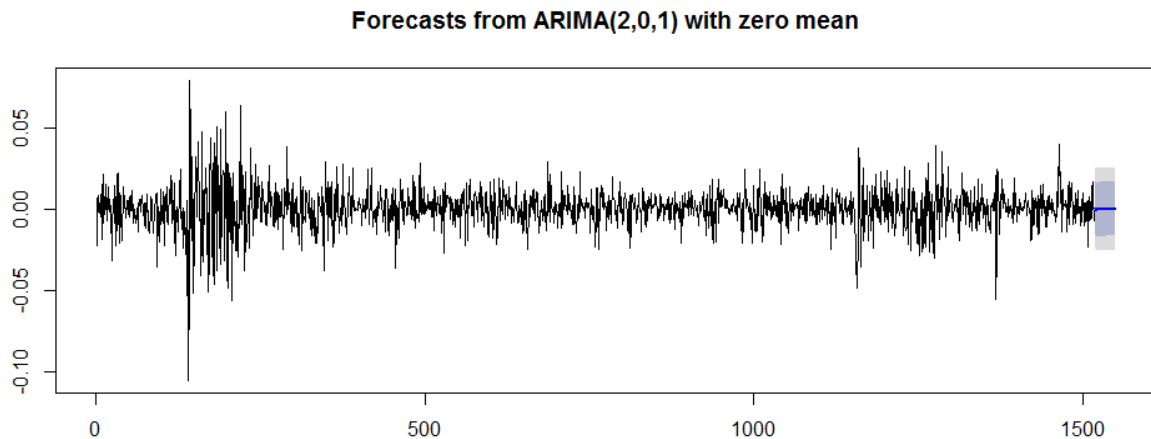


Table 3.2: ARCH(q) Log Likelihood and AIC

Model	q	df	Log Likelihood	AIC
ARCH(1)	1	2	4654.143	-6.118688
ARCH(2)	2	3	4706.392	-6.186165
ARCH(3)	3	4	4726.472	-6.211286
ARCH(4)	4	5	4733.214	-6.218847
ARCH(5)	5	6	4739.571	-6.225900
ARCH(6)	6	7	4745.285	-6.232106
ARCH(7)	7	8	4747.802	-6.234104
ARCH(8)	8	9	4753.363	-6.240110
ARCH(9)	9	10	4754.665	-6.240507
ARCH(10)	10	11	4755.007	-6.239641

3.3.2 GARCH Family Model

I use residuals of both ARMA(2,1) and ARMA(1,1) fit the GARCH family models. The results with ARMA(2,1) are mainly better than the ARMA(1,1) one. Thus I treat ARMA(2,1) as the default choice to fit GARCH family models here. And I will also mention ARMA(1,1) when it turns to be a better choice.

As for the ARCH(q) models, ARCH orders and parameters could be selected by AIC criterion. I fit 10 ARCH models to the residuals in total, and then calculate the log likelihood and AIC for each models. The result is shown in Table 3.2. ARCH(9) has minimum AIC -6.240507 during these 10 ARCH models and but the coefficient of the ninth lag term is significant under 95% confidence level. Therefore, ARCH(8) with second minimum AIC is the selected model for ARCH one. As for GARCH(p,q) models, 4 models may be considered at first: GARCH(1,1), GARCH(2,1), GARCH(1,2) and GARCH(2,2). Since the two coefficients

of ARCH terms in GARCH(1,2) and GARCH(2,2) are not significant, GARCH(1,1) and GARCH(2,1) are selected.

Similarly the coefficients in E-GARCH(2,1) are not all significant, thus it reduced to be E-GARCH(1,1). However, the coefficients in E-GARCH(1,2) are significant when it uses ARMA(1,1) model as mean equation.

Here are all the models discussed to be considered above using t-distribution.

Model 1 ARMA(2,1)+ARCH(8):

$$r_t = 8.614 \times 10^{-5} + 0.8511r_{t-1} + 0.03732r_{t-2} - 0.9187\varepsilon_{t-1} + \varepsilon_t$$

$$\sigma_t^2 = 3.62 \times 10^{-5} + 0.1032\varepsilon_{t-1}^2 + 0.2171\varepsilon_{t-2}^2 + 0.1228\varepsilon_{t-3}^2 + 0.0572\varepsilon_{t-4}^2 + 0.0752\varepsilon_{t-5}^2 + 0.1146\varepsilon_{t-6}^2 + 0.0247\varepsilon_{t-7}^2 + 0.0827\varepsilon_{t-8}^2$$

Model 2 ARMA(2,1)+GARCH(1,1):

$$r_t = 9.49 \times 10^{-5} + 0.8381r_{t-1} + 0.0389r_{t-2} - 0.913\varepsilon_{t-1} + \varepsilon_t$$

$$\sigma_t^2 = 5.799 \times 10^{-6} + 0.1536\varepsilon_{t-1}^2 + 0.8144\sigma_{t-1}^2$$

Model 3 ARMA(2,1)+GARCH(2,1):

$$r_t = 1.598 \times 10^{-3} - 0.9898r_{t-1} - 0.0551r_{t-2} + 0.9175\varepsilon_{t-1} + \varepsilon_t$$

$$\sigma_t^2 = 7.936 \times 10^{-6} + 0.09984\varepsilon_{t-1}^2 + 0.09845\varepsilon_{t-2}^2 + 0.7577\sigma_{t-1}^2$$

Model 4 ARMA(2,1)+E-GARCH(1,1):

$$r_t = 4.37 \times 10^{-4} - 0.9789r_{t-1} - 0.0426r_{t-2} + 0.9184\varepsilon_{t-1} + \varepsilon_t$$

$$\ln \sigma_t^2 = -0.4263 - 0.1563(|\varepsilon_{t-1}| + 0.2417\varepsilon_{t-1})/\sigma_{t-1} + 0.9533 \ln \sigma_{t-1}^2$$

Model 5 ARMA(1,1)+E-GARCH(1,2):

$$r_t = 0.000447 - 0.0256r_{t-1} - 0.0358\varepsilon_{t-1} + \varepsilon_t$$

$$\ln \sigma_t^2 = -0.4124 - 0.1488(|\varepsilon_{t-1}| + 0.23135\varepsilon_{t-1})/\sigma_{t-1} + \ln \sigma_{t-1}^2 - 0.0452 \ln \sigma_{t-2}^2$$

Except for coefficients of the fourth and seventh lag terms in model 1, all the coefficients in GARCH family models shown above are significant under 95% confidence level. This means that all the coefficients in conditional variance models are significant. And it demonstrates that the stock return rate time series r_t has significant volatility clustering, which is the same result as discussion in descriptive statistics part.

The variance of error term in stock yield mean model can be described by past squared error term and past squared variance itself with lags by parameters shown above. All the ARCH coefficients of ARCH and GARCH models are larger than 0, which illustrates that external effects would intensify system volatility. The GARCH coefficients (0.8144, 0.7577 and 0.9533) are all close to 1, reflecting the long-term memory of the system. The sum of ARCH coefficient and GARCH coefficient is smaller than 1 for each model, thereby satisfying the condition of GARCH family models stated in methodology part. Thus we know these models' processes are stationary, and the impact of past volatility on the future is gradually decaying. Further discussion about the coefficients of E-GARCH would be covered in 3.5 part.

Table 3.3: Box-Ljung Test and ARCH-LM Test

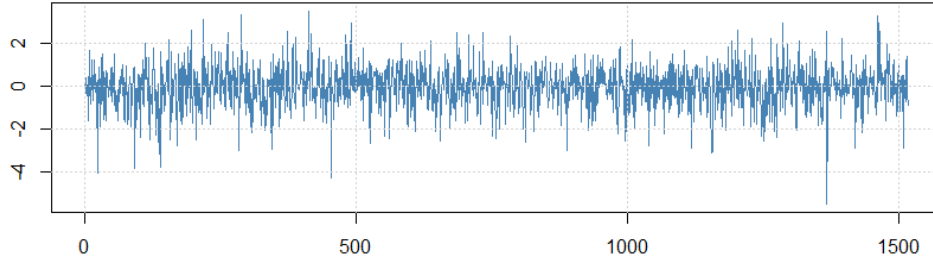
Test	Box-Ljung Test (lag12)		ARCH-LM Test (lag 24)	
	Q Statistic	p-value	LM Statistic	p-value
Model 1 ARCH(8)	13.662	0.3228	12.854	0.9685
Model 2 GARCH(1,1)	15.981	0.1921	25.665	0.3704
Model 3 GARCH(2,1)	13.045	0.3658	21.960	0.5817
Model 4 E-GARCH(1,1)	11.281	0.5050	23.567	0.4866
Model 4 E-GARCH(1,2)	12.832	0.3814	23.491	0.4910

Now we need to do the tests to check whether the standardized residuals of those models have ARCH effect right now. Figure 3.5 shows the standardized residuals time series plots, their QQ-plots, and their ACF figures.

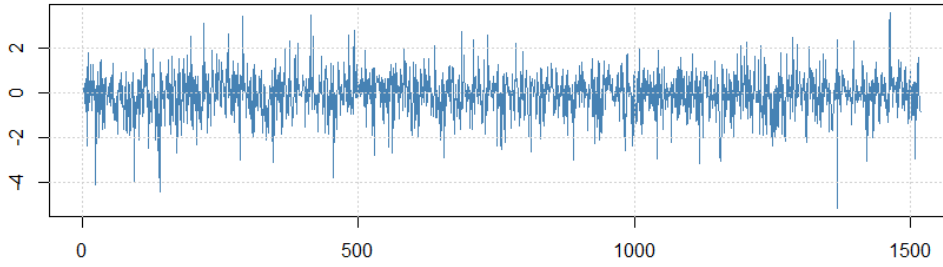
There is no obvious volatility clustering phenomenon in standard residuals plot from (a) to (e) in Figure 3.5. The points almost lie on the $x = y$ lines in the QQ-plots, thus it is reasonable to build model using t-distribution. Two E-GARCH models have better QQ-plot of t-distribution than that of other three models. Except for Model 5 E-GARCH(1,2), the coefficients of autocorrelation roughly stand inside the confidence interval in ACF figures from (k) to (o).

In order to check their independence and ARCH effect, Ljung-Box test and ARCH-LM test need to be taken. The results are shown in Table 3.3.

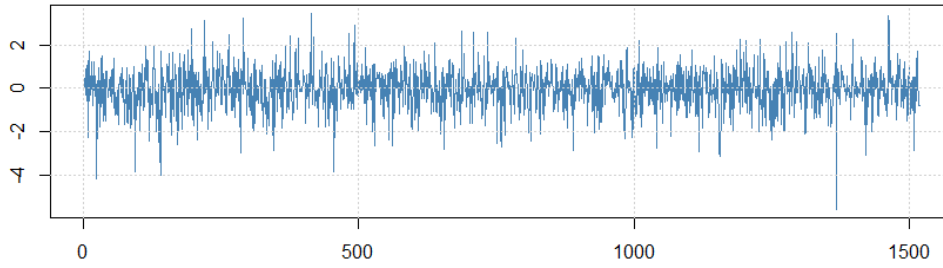
Figure 3.5: Standardized Residuals of GARCH Family Models



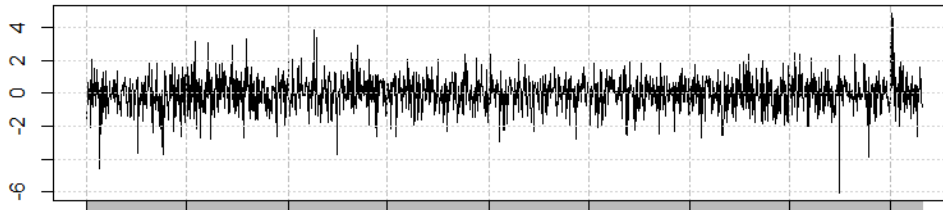
(a) Model 1 ARCH(6) Std Res Plot



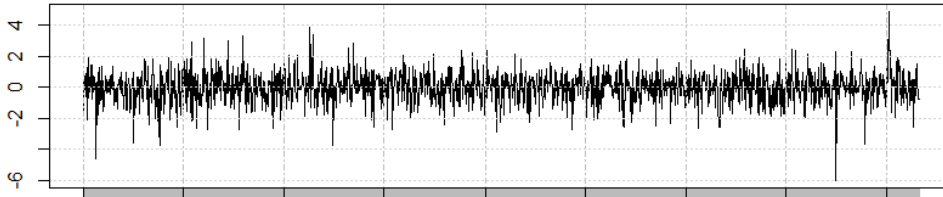
(b) Model 2 GARCH(1,1) Std Res Plot



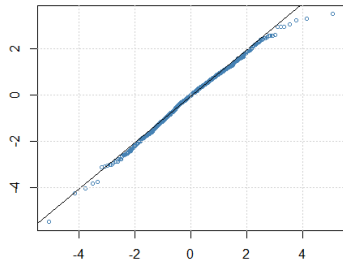
(c) Model 3 GARCH(2,1) Std Res Plot



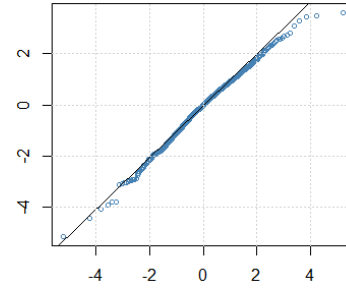
(d) Model 4 E-GARCH(1,1) Std Res Plot



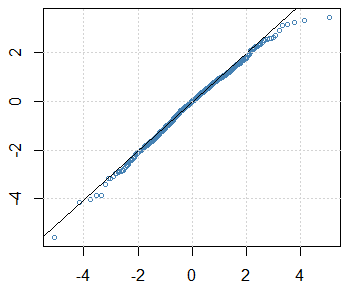
(e) Model 5 E-GARCH(1,2) Std Res Plot



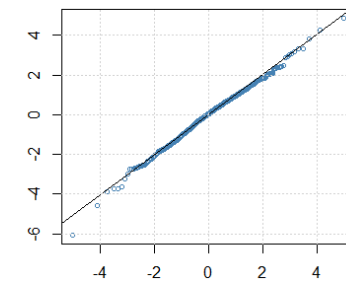
(f) Model 1 ARCH(8) Std Res QQ-Plot



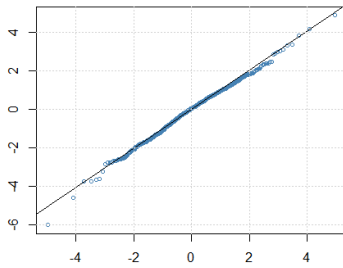
(g) Model 2 GARCH(1,1) Std Res QQ-Plot



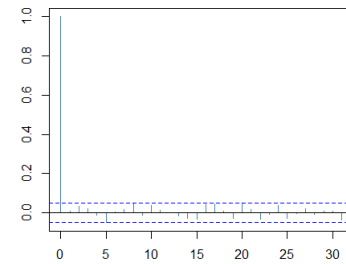
(h) Model 3 GARCH(2,1) Std Res QQ-Plot



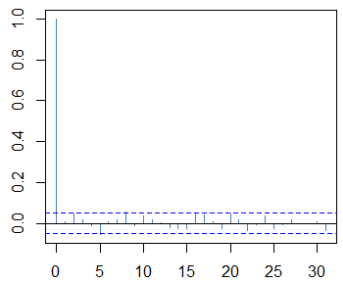
(i) Model 4 E-GARCH(1,1) Std Res QQ-Plot



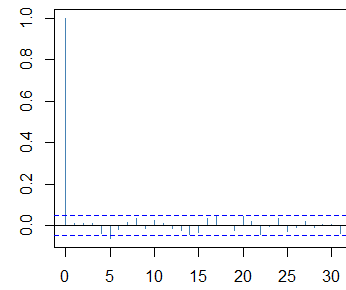
(j) Model 5 E-GARCH(1,2) Std Res QQ-Plot



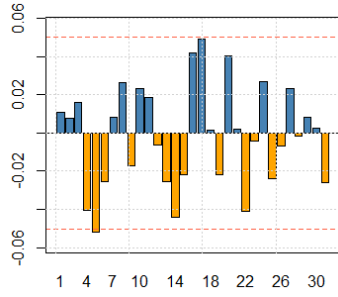
(k) Model 1 ARCH(8) Std Res ACF



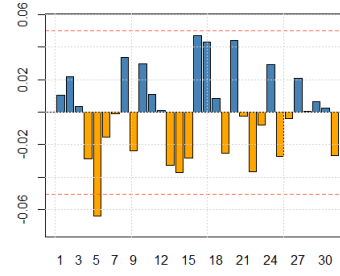
(l) Model 2 GARCH(1,1) Std Res ACF



(m) Model 3 GARCH(2,1) Std Res ACF



(n) Model 4 E-GARCH(1,1)
Std Res ACF



(o) Model 5 E-GARCH(1,2)
Std Res ACF

Since the p-values of Q statistics of Ljung-Box tests are all larger than 0.05, the null hypothesis that the series is independently distributed would not be rejected under 95% confidence level. The p-values of LM statistics are also larger than 0.05, and the null hypothesis that there is no autoregressive conditional heteroskedasticity would not be rejected under 95% confidence level.

Among those five models, the first ARCH model has 8 parameters that make the model still not simple enough. It's better to focus more on the latter four models. We could use Akaike Information Criterion to select a better model for further analysis. We could also select model using method of mean square error (MSE) by comparing the fitted value of the models with the original series data:

$$MSE = \frac{SSE}{n} = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2.$$

The results of AIC and MSE are shown in Table 3.4. Model 3 GARCH(2,1) has lowest fitted value mean square error, while Model 4 E-GARCH(1,1) has lowest AIC.

Table 3.4: GARCH Family Models' AIC and Fitted MSE

Model	AIC	Fitted MSE
Model 1 ARCH(8)	-6.2405	0.0001651355
Model 2 GARCH(1,1)	-6.2448	0.0001651969
Model 3 GARCH(2,1)	-6.2420	0.0001640247
Model 4 E-GARCH(1,1)	-6.2588	0.0001641403
Model 5 E-GARCH(1,2)	-6.2580	0.0001652685

Comprehensively considering the best QQ-plot, the lowest AIC, a relatively small MSE and also its economic leverage meaning, Model 4 E-GARCH(1,1) is a good choice. But also Model 3 GARCH(2,1) is a good one here, due to the good diagnose result and the lowest MSE. I would use GARCH(2,1) to make prediction and forecasts for stock market while using E-GARCH(1,1) to discuss about leverage and the effect of new information in stock market.

3.4 VAR Calculation and Early Warning

According to the definition of VAR, the minimum return rate is

$$R^* = -\alpha\sigma + \mu$$

and the relative VAR is

$$VAR_R = -P_0(R^* - \mu) = P_0\alpha\sigma\sqrt{\Delta t}.$$

Table 3.5 is the VAR calculation for 10 days head based on prediction from Model 2 GARCH(2,1) at 99% confidence level in 1-day horizon. I also provide the real world rate and price from testing sample for comparison, but the VAR is still calculated by the prediction one rather than the real one. The actual days would be from February 1st 2017(day 1) to February 14th 2017(day10) with totally 10 working days in financial market.

This means that 14.02 dollars as value at risk are predicted to be the maximum probable loss in day 1 with initial price 390.08 dollars for 1 volume in 1-day horizon at 99% confidence level. Similarly 14.25 dollars are the second day's value at risk with initial price 390.65 dollars for 1 volume in 1-day horizon at 99% confidence level, and so on as the other days left.

We can also use this method to predict more future VAR to provide quantitative information for risk management and company strategy. How this would produce real world economic significance and make sense for risk management? We see that there exists volatility clustering in the historical log yield time series. This means that the violent fluctuation and huge volatility did not turn up suddenly without any signals. They came into being gradually in the market. Thus we conclude that there would be any corresponding signals before huge volatility and ruined loss, for example, some extremely low rate of return. Thus

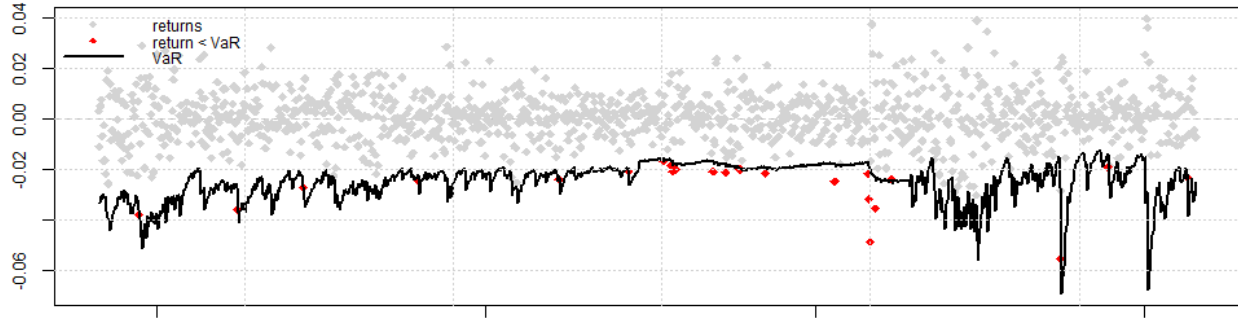
Table 3.5: Future 10 days' Relative VAR at 99% level in 1-Day Horizon

Day	Predict Rate	Predict Price	Predict Sigma	VAR
0	0.0010528	390.0802411	0.0099253	N/A
1	0.0014499	390.6458194	0.0096956	14.02172
2	0.0010438	391.0535576	0.0098384	14.24887
3	0.0010261	391.4548009	0.0099748	14.46149
4	0.0009954	391.8444682	0.010105	14.66529
5	0.0009691	392.2241966	0.0102294	14.86061
6	0.0009458	392.5951625	0.0103485	15.0482
7	0.0009253	392.9584175	0.0104625	15.22836
8	0.0009072	393.3148898	0.0105716	15.40139
9	0.0008912	393.6653997	0.0106762	15.56789
10	0.0008771	394.0106718	0.0107764	15.72801

institutions could use these signals to predict value at risk and make corresponding risk management afterwards.

Figure 3.6 is the plot of early warning for low stock yield in history. We could see some points are under the VAR curve, shown as red points, before some huge volatility. They would be the signals and early warning for crisis. However, the accuracy of the early warning by model is not good in stationary period, for example from observation 700 to 1100. Thus the early warning by this model has more economic meaning in non-stationary volatility period.

Figure 3.6: VAR Historical Plot with Early Warning



3.5 Leverage Effect and News Impact Curve

As stated before, the impact of same unit of bad news on stock volatility is often greater than the impact of good news. When the bad news decrease the asset price in one period, thereby reducing the capital invested in the new business, the debt-to-capital ratio would rise, which leads to an increase in the variance risk of the company's expected return. In addition, the asymmetry also has a significant effect on the assets return and stock market return covariance-to-variance ratio. Therefore, we need to also consider the leverage effect using E-GARCH model to describe the asymmetry.

Particularly in the Model 4 E-GARCH(1,1) with ARMA(2,1), we have variance equation in Nelson form:

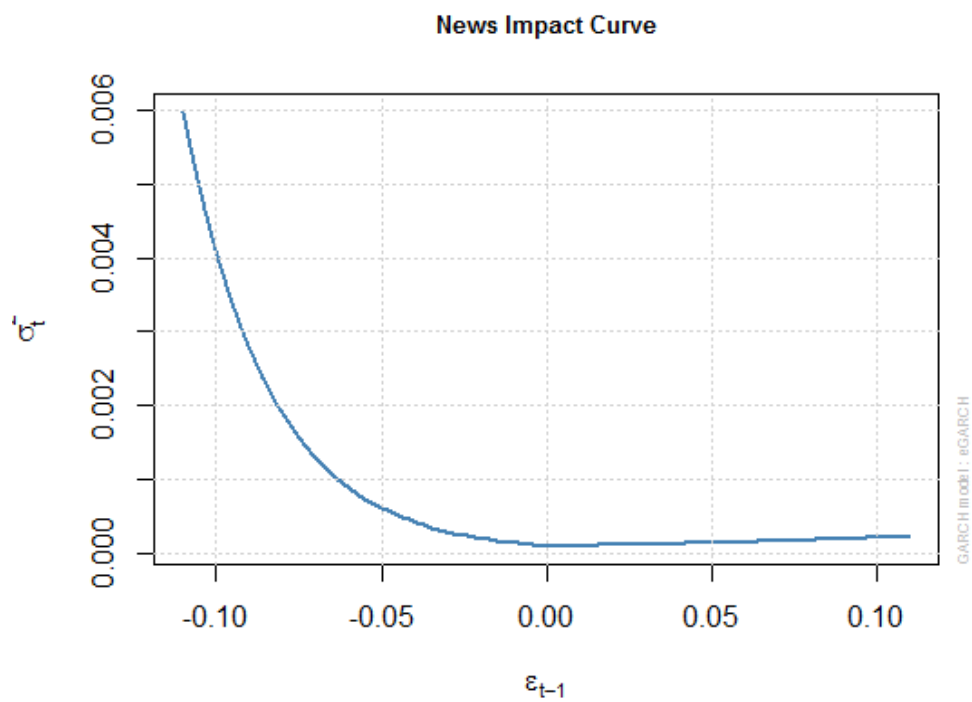
$$\ln(\sigma_t^2) = -0.55103 - 0.03778 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} - 0.1563 \left(\left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - 0.798 \right) + 0.9533 \ln(\sigma_{t-1}^2).$$

The coefficient of $\frac{\varepsilon_{t-1}}{\sigma_{t-1}}$ is -0.03778 and is significant. This term describe the difference between good and bad news. Thus it can be concluded that the model has significant leverage effect.

We take the observation 1519 (January 31st 2017) as an example. When the news is good and assuming $\varepsilon_{1518}/\sigma_{1518} = 1$, $\ln(\sigma_{1519}^2) = 0.000092$. When the news is bad and assuming $\varepsilon_{1518}/\sigma_{1518} = -1$, $\ln(\sigma_{1519}^2) = 0.0001356$. There is a large difference between the two variances, which shows that the bad news has more influence on volatility than the good news.

It is also very obvious to see the asymmetry on news impact curve in Figure 3.7. The stock yield is steep when information term is less than zero with bad news or negative shocks. The stock yield is relatively gentle and stationary when the information term is greater than zero with good news or positive shocks. This means stock price is more sensitive to bad news, and it also coincides with the phenomenon that stock price rises slowly but falls very quickly. It performs in the time series that the volatility in the falling direction is larger than that of rising direction. Thus the anti-risk ability in financial market should be strengthened for risk management.

Figure 3.7: News Impact Curve



Chapter 4

Conclusion

The financial stock market turned out to rise and fall suddenly and sharply in recent years, which means that the volatility and uncertainty is very significant in market and measuring the market risk accurately is of great importance. This paper uses the historical price of S&P 500 Financials Sector Index from January 19th 2011 to January 31st 2017 as the original time series data, and find the logarithm yield as daily returns to build 2 ARMA and 5 GARCH family models using t-distribution. Then I calculate future 10 days' relative VAR in 1-day horizon under 99% confidence level based on the selected model GARCH(2,1) with ARMA(2,1). According to the results, we could make conclusion as follows.

First, since the log yield time series reject the normality assumption and ARMA models have ARCH effect, we need to build the GARCH family models to find variance of the error term in mean equation for further analysis. It is obvious that there exists volatility clustering feature in log yield time series and they are auto-correlated. And ARMA models also have ARCH effect that the estimation would not consistent and efficient enough. Thus I build

5 GARCH family models using t-distribution to solve this problem. All the models remove ARCH effect of time series under 95% confidence level.

Second, I select GARCH(2,1) model to make prediction of stock yield and stock price among 5 GARCH family models according to significance of coefficients, tests on standardized residuals, and MSE between historical data and fitted value.

Third, I calculate future 10 days' relative VAR in 1-day horizon under 99% confidence level based on the selected model GARCH(2,1) with ARMA(2,1) as the quantitative risk measuring method. I also plot the historical VAR in time series and treat the returns that lower than the minimum return of VAR as the signal of early warning for crisis. The early warning is particularly effective in non-stationary volatility period, while the accuracy turn to be not good enough in stationary period. Institutions and Investors could make risk management and investment strategies with VAR signals during volatility period.

Finally, there is asymmetry on news impact curve derived through E-GARCH(1,1) with ARMA(1,1) model, which means that the log yield series have leverage effect. Thus it can be concluded that the impact of same unit of bad news on stock volatility is greater than the impact of good news in this financial market.

References

MARKOWITZ, H. (1952). Portfolio Selection. *The Journal of Finance* **7**(1), 77–91.

SHARPE, W.F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *Journal of Finance* **19**(3), 425–442.

LINTNER, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *The Review of Economics and Statistics* **47**(1), 13–37.

MORGAN, J.P. (1995). Introduction to RiskMetrics. 4th ed. New York: Morgan Guaranty Trust Company.

ENGLE, R.F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica* **50**(4), 987–1008.

BOLLERSLEV, T. (1986). Generalize Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics* **31**, 307–327.

NELSON, D.B. (1991). Conditional Heteroskedasticity in Asset Returns:A New Approach. *Econometrica* **59**(2), 347–370.

- JORION, P. (2006). Value at Risk: The New Benchmark for Managing Financial Risk. 3rd ed. New York: McGraw-Hill Education.
- BOX, G.E.P. AND JENKINS, G.M. (1970). Time Series Analysis: Forecasting and Control. San Francisco: Holder-Day.
- SHUMWAY, R.H. AND STOFFER, D.S. (2011). Time Series Analysis and Its Applications: With R Examples. 3rd ed. Springer Science & Business Media.
- WURTZ, D. AND CHALABI, Y. AND LUKSAN, L. (2006). Parameter Estimation of ARMA Models with GARCH/APARCH Errors: An R and SPlus Software Implementation. *Journal of Statistical Software* **55**, 28–33.