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Essays on Applied Economic Theory

Haibo Xu
Washington University in St. Louis

WASHINGTON UNIVERSITY IN ST. LOUIS

Department of Economics

Dissertation Examination Committee:
David Levine, Chair
Marcus Berliant
John Nachbar
Paulo Natenzon
B. Ravikumar
Maher Said

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Haibo Xu

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Introductory Chapter

My dissertation devotes to the study of decisions of individuals and their effects on economic and social outcomes, especially in situations where individuals have conflicting interests and interact under uncertainty. The dynamics of information transmission, reputation formation, and private/public learning in various situations is the main focus of this dissertation.

In Chapter 1, I study a dynamic game in which a financial expert seeks to optimize the utilization of her private information either by information disclosure to an investor or by self-using. The investor may be aligned or biased: an aligned investor always cooperates on the disclosed information, whereas a biased investor may strategically betray the expert. I characterize the joint dynamics of the expert's information disclosure and the investor's type revelation and show that, by the process of gradual information disclosure, the expert can significantly alleviate the hold-up effect exerted by the biased investor. In particular, I show that the equilibrium dynamics of the players' interactions is unique. I also examine how the expert can further improve her utilization of information by committing to a deadline or by committing to a particular pattern of information disclosure.

In Chapter 2, I develop a reputational cheap talk model in which an expert acquires and conveys information and a decision maker takes a payoff-relevant action. The expert may be aligned or biased: an aligned expert cares about the decision maker's payoff and would like to be known as aligned, whereas a biased expert always distorts information toward a particular direction. My main finding shows that the aligned expert's reputational concern may have a non-monotonic effect on his incentive to acquire information; that is, he acquires better information if and only if his reputational concern is moderate. Another finding shows that, although the biased type of expert only distorts information transmission, the existence of this type may actually increase the decision maker's payoff. I also examine how delegation may affect the players' decisions and payoffs in this essay and show that even with the rights to better use the information ex post, the aligned expert's information acquisition incentive may be weakened ex ante. Finally, I show that the decision maker prefers communication to delegation whenever informative communication with information acquisition is feasible.

In Chapter 3, I study a dynamic agency problem in which a principal and an agent interact on a project with initially unknown quality. A key feature in this problem is that the agent's hidden actions can give rise to hidden information about the project quality, which enables the agent to benefit from manipulating the principal's learning process. In particular, the agent's attempt on belief manipulation varies in his own assessment about the project quality. I examine how the principal can structure the provision of incentives by resorting to relationship termination. Relationship termination has two opposing effects: it destroys the surplus that the principal can obtain from the relationship continuation, but it also lowers the informational rents that the agent can capture from the belief manipulation. I show that in equilibrium the optimal rule of relationship termination follows a cut-off strategy: it is introduced in the contracts only when the expected relationship value is higher than a threshold value. In consequence, the dynamic agency cost presents a non-monotonic relationship with the project quality. I also examine how a limitation on the principal's payment ability shifts the agent's incentive on belief manipulation backwardly.

In Chapter 4, I consider a multilateral bargaining game in which a manager negotiates sequentially with several workers to share the units of surplus. The novel feature of my setup is that the manager can determine the ordering of her bargaining opponents endogenously. I show that double-sided hold-up effects arise in this game: the workers can hold up the manager by coordinating their moves, whereas the manager can hold up the workers by switching between the opponents. The interaction of these two effects gives rise to multiple equilibria, some of which present inefficient delays. Moreover, the delay may be bounded away from zero even if the time interval betIen two offers becomes arbitrarily small.

Chapter 1 A Model of Dynamic Information Disclosure

1.1 Introduction

As Hayek (1945) claimed more than half a century ago, the central issue in a variety of economic and social interactions is how to utilize information efficiently.[1] Theoretical and practical developments have contributed many effective and commonly used tools to help achieve this efficiency, including patent protection, contractual enforcement, and property rights allocation. However, these tools are often unavailable, resulting in a hold-up problem that discourages the utilization of information. For example, if an initially uninformed party has learned valuable information from another party, then his incentive to pay for the information weakens, as now himself is informed (Arrow, 1962). I provide an equilibrium analysis of information utilization in this paper and address how a process of gradual information disclosure helps to alleviate the hold-up problem.

To gain better understanding of this study, consider the scenario that a financial expert, who knows about some investment opportunities, seeks cooperation from a fund manager to optimize the utilization of her information. Being aware that the fund manager may be more motivated to seize all the information value rather than to establish a cooperative relationship, the financial expert may strategically slow down the release of her information to reduce the risk of being exploited. As the uncertainty about the fund manager's motives is gradually resolved, the financial expert eventually becomes confident enough to release all her information.

For another example, consider the scenario that an international auto company aims to enter the market of a developing country by cooperating with a local company and transferring its technology. However, because this country lacks an established law system, the

[1] See Hayek (1945), page 519-520: "The economic problem of society is thus not merely a problem of how to allocate 'given' resources..., it is a problem of the utilization of knowledge not given to anyone in its totality."

auto company's technology is under the risk of being leaked. In response, the auto company may choose to transfer some preliminary technology first, which provides an opportunity to learn about its partner. Contingent on the local company's reactions to these preliminary transfers, the auto company can decide to transfer more technology or exit the market.

These scenarios share some similarities. First, information is divisible and can be transmitted or disclosed in parts, which allow the values of those parts to be realized separately. For instance, preliminary technology can also generate revenues for the auto company. Second, contractual enforcement on information disclosure may be unreliable or even absent, which causes the potential hold-up problem. As a result, the parties' interactions must be self-enforcing, as in the case of the financial expert and the fund manager. Finally, timing cost can be an important factor that affects the utilization of information. Investment opportunities lose their values rapidly over time in a volatile stock market, whereas the auto company may lose market shares to its competitors if it delays the transfer of its technology.

Taking the first scenario as the prominent example in this paper, I develop a dynamic game that examines the gradual disclosure of information and its effects on the players' behaviors and the payoffs. A financial expert is endowed with an amount of private information that is valuable in the stock market, but she can only utilize it inefficiently on her own because of her limited access to the market. An investor has the potential to maximize the value of the expert's information, but he lacks the relevant information. As a result, efficient utilization of information requires information disclosure between both parties. Interactions go as follows. In each period, the expert may choose to self-use some information, in which case the investor is inactive. Alternatively, the expert may disclose some information to the investor, who can then either cooperate, which is mutually beneficial, or betray, which benefits only himself. The investor is either aligned or biased. An aligned investor always cooperates, whereas a biased investor may strategically betray. The financial expert is initially uncertain about the investor's type, so she must learn about it over time. Given the discounting cost, the expert's goal is to optimize the payoff from her information when external contracts are infeasible.

The financial expert faces two main trade-offs in determining her utilization of information. The first trade-off is whether information should be kept for self-use or disclosed. Although self-use of information yields substantial efficiency loss, the timing cost and the rents captured by the biased investor must be considered when choosing to disclose information. If the investor intends to betray, then self-use is preferred. If the investor's cooperation can be induced, then disclosure is preferred. If the expert chooses to disclose, then the second trade-off is the timing of information disclosure. A longer process of disclosure is more costly in time, but it safeguards the information from the biased investor. A shorter process of disclosure saves timing cost, but it provides better betrayal opportunity to the biased investor.

I construct an equilibrium in which the expert's trade-offs are resolved by a finite sequence of cut-off values, which represent the expert's beliefs about the investor's type. If the investor is highly aligned and therefore unlikely to betray, then the expert should disclose information faster. If the investor is moderately aligned, then the expert should slow down the process of information disclosure to weaken the biased investor's incentive for betrayal. Finally, if the investor is sufficiently biased, then the expert should not disclose any information but instead keep it for self-use, because any disclosure is too costly. This characterization gives an explicit insight about when and how the expert can alleviate the hold-up problem by employing a gradual disclosure of her information.

Moreover, I show that the equilibrium of this game is "essentially" unique.[2] The critical determinant of equilibrium uniqueness is that the completion of information disclosure is endogenously determined. Specifically, if there is an equilibrium (other than the equilibrium I construct) that requires the biased investor to betray with a higher probability in a particular period, after observing cooperation the expert believes that the investor is more aligned

[2]The equilibrium is "essentially" unique, because multiple equilibria can arise if the expert's initial belief about the investor's type is at some cut-off values, and can arise in some off equilibrium path of play. This will be much clear in the following analysis.

and thereby prefers to speed up her information disclosure in the continuation game, but then the biased investor should actually cooperate with certainty in the current period. Conversely, if there is an equilibrium (other than the equilibrium I construct) that requires the biased investor to betray with a lower probability in a particular period, after observing cooperation the expert is still very cautious about the investor and thereby prefers to slow down her information disclosure in the continuation game, which implies that the biased investor should betray with certainty in this period. In equilibrium, the biased investor's response is unique for (almost) any amount of information disclosure by the expert. As a result, the expert's problem is much like a decision problem in that she chooses the optimal plan of information disclosure from all feasible plans, which is also unique.

In many circumstances, the time period for information disclosure is limited. I examine how the existence of a deadline affects the expert's information utilization and, specifically, how the expert's payoff is improved if she commits to a deadline. If the deadline period is reached, the expert's choice is restricted in a way that no gradual disclosure of information, and therefore no gradual learning about the investor's type, is allowed in the future. Such a restriction lowers the expert's *ex post* payoff. However, expecting that the expert is more willing to disclose all her remaining information in the deadline period even her posterior belief is not sufficiently high, the biased investor can effectively decrease his betrayal probabilities in the periods before the deadline period. This decrease in betrayal probabilities, in turn, increases the expert's *ex ante* payoff. I show that, with moderate initial beliefs, the expert's equilibrium payoff is strictly improved if she commits to a proper deadline.

I also examine the effects on the expert's information utilization if she can fully commit to a particular process of information disclosure. In equilibrium, the optimal process with commitment has a property that the biased investor is induced to cooperate in all periods except the last one. In other words, the amount of information disclosed in the final period serves as a reward to the biased investor for exchanging his cooperation up to that period. The expert's problem in determining the optimal process is to trade off between the scale of the reward and the timing cost to deliver it. Consequently, by fully committing to a proper process, the expert's payoff can be improved.

In addition to the examples aforementioned, this game is also applicable to many other situations. For instance, if the valuable information refers to research ideas, then the game can address the building and termination of relationship between scientists. More broadly, if what the expert possesses is some sort of valuable assets, the game can be interpreted as a contribution game, in which one party contributes inputs and another party contributes productivity.

On a technical level, I deal with a reputation game in which the action space (the amount of remaining information to be utilized) varies over time and the timing structure (the completion of information utilization) is endogenously determined. As a result, part of my contributions lies in the detailed construction of the equilibrium and the verification of the equilibrium uniqueness, which offer novel insights to the study of games with similar technical properties.

1.2 Literature

My emphasis on the divisibility of the expert's information and its implications for relationship dynamics is related to Baliga and Ely (2010). Baliga and Ely (2010) consider a model in which a principal uses torture to extract information from an agent who may or may not be informed. In equilibrium, the informed agent initially resists but eventually concedes, and his divisible information is gradually extracted. In their paper, the equilibrium rate of information extraction is determined by the severity of the torture cost; therefore, the gradualism of the information extraction is essentially a constraint to the principal's problem. In contrast, in my paper, gradualism of information disclosure is the expert's optimal solution to alleviate the hold-up problem that she faces; that is, the expert can, but in equilibrium she optimally chooses not to, disclose all information in a single period.

Hörner and Skrzypacz (2011) develop a model in which an agent who knows a state of nature can gradually reveal this state to a firm in exchange for payments. They address the equilibrium that maximizes the agent's *ex ante* incentives to learn about the state of nature and show that, in such an equilibrium, revealing information gradually increases the agent's payoff and the process of information revelation always exhausts all the time periods. My study shows the complementary result that gradual information disclosure could be beneficial to the information possessor, but the underlying setup is quite different. Specifically, while discounting cost and outside option with self-use of information are crucial to my findings, they have no roles in their paper. These differences enable me to offer new insights to many real-world situations.

Gradualism also appears as the means to alleviate the hold-up effects in the literature on contribution games, including Admati and Perry (1991), Gale (2001), Lockwood and Thomas (2002), Marx and Matthews (2002), and Compte and Jehiel (2004). A key feature in my work is that gradual information disclosure arises due to asymmetric information about the investor's type, which is absent in these papers. Watson (1999, 2002) studies a contribution game with two-sided incomplete information and shows that the relationship between partners generally starts small and grows over time. In his papers, the amounts of contributions along the time horizon are pre-determined before the game starts. As a result, the players' actions at any given time are binary; the players either follow the pre-determined amount or betray. In my paper, the expert's action space on information disclosure is a continuum in each period, and the amounts of disclosure are determined during the process of play.

The gradual revelation of the investor's type is analytically related to the literature on reputation games, including Kreps and Wilson (1982), Milgrom and Roberts (1982), and Fudenberg and Levine (1989, 1992), as well as the literature on the war of attrition with incomplete information, including Abreu and Gul (2000), and Damiano, Li and Suen (2012). In these papers, the stage game is repeated and the only variables that change over time are the beliefs about the informed players' types. In my paper, both the belief and the stage game vary over time as the amount of remaining information decreases.

Anton and Yao (1994) show that an inventor can appropriate a sizable share of an idea's market value from a buyer if the inventor threatens to reveal the idea to a competitor in the event that the buyer defaults. Alternatively, Anton and Yao (2002) show that a seller can use partial disclosure to signal the full value of an idea and benefit from the buyers' competition for ownership of this idea. These papers allow enforceable contracts, but the timing structure of information disclosure is pre-determined. In contrast, my work focuses on the endogenous timing structure of information disclosure instead of any explicit contracts.

In most of these papers, except Baliga and Ely (2010), private information takes the forms of states or types, which are intrinsically indivisible. Therefore, dynamic information disclosure in these papers refers to a sequence of *probabilities* that a state or a type is gradually revealed. My paper focuses on the divisibility of information, and therefore dynamic information disclosure refers to a sequence of *amounts* that information is gradually revealed, as explained in the following analysis.

1.3 The model

I consider a dynamic game involving two players: a financial expert (she or E) and an investor (he or I). At the beginning of the game, the expert is endowed with an amount $Y_0 > 0$ of information, which refers to some investment opportunities that can be exploited in the stock market. A key feature regarding this amount of information is that, although the number Y_0 is common knowledge between the players, the detailed contents of the information is initially known only to the expert. Thus, the investor must learn the relevant contents from the expert first to take actions with the information. For simplicity, I assume that the expert's information is perfectly divisible. Time is discrete and goes to infinity. Both players are risk-neutral and share a common discount factor $\delta \in (0, 1)$. A potential explanation for the factor δ is that information in the stock market loses its value over time if it is not utilized immediately.

Actions and payoffs are as follows. In period t, if the amount of remaining information is $Y_t > 0$ and the relationship between the players is still ongoing, then the expert can either use an amount $x \leq Y_t$ by herself or disclose an amount $y \leq Y_t$ to the investor.[3] If an amount x is self-used, in this period the expert and the investor's payoffs are x and 0, respectively. After the realization of payoffs, the game extends to the next period with remaining information $Y_{t+1} = Y_t - x$. On the other hand, if an amount y is disclosed, then the investor can choose to cooperate or betray. Cooperation generates a "success" and gives payoffs $\alpha_E y$ and $\alpha_I y$ to the expert and the investor, whereas betrayal results in a "failure" and gives payoffs 0 and $\beta_I y$ to the expert and the investor. After the realization of payoffs, the game extends to period $t+1$ with information $Y_{t+1} = Y_t - y$. The parameters satisfy

$$\alpha_E > 1,\ \beta_I > \alpha_I > 0, \text{ and } \alpha_E + \alpha_I \geq \beta_I,$$

which indicate that, while the investor's cooperation is both socially efficient and preferred to self-use by the expert, the investor can benefit more from betrayal. This tension is the driving force underlying the players' interactions. In addition, I assume that the relationship is terminated whenever the investor betrays.[4] As a result, the expert's only choice after the relationship termination is to self-use her remaining information. The game ends when all the information has been utilized.

Some simplifications regarding the expert's information are adopted in the above setup. First, different units of information are equally valuable, which is reflected in the linear payoff functions. Second, information is not re-utilizable in the sense that any part of

[3] In the equilibrium I show later, whenever the expert self-uses her information, she self-uses all of it. Allowing the expert to disclose and self-use information simultaneously would not change this equilibrium property, therefore it has no effect on my qualitative findings.

[4] Alternatively, I can assume that, even after a betrayal, the expert can continue to disclose information to the investor. However, information disclosure after a betrayal exerts a lump-sum cost that is high enough to outweigh any benefit from potential cooperation by the investor. Thus, in equilibrium the expert optimally chooses to self-use her remaining information after a betrayal.

information can be exploited only once. Third, both self-use and disclosure of information are observable, so the amount of remaining information in each period is commonly known. Finally, information is not cumulative to the investor, therefore whenever an amount of information is disclosed, the investor must utilize it immediately.[5] While these simplifications make my analysis tractable, none of them is essential to my qualitative findings on the dynamics of information disclosure.

The investor may be *aligned* or *biased*. An aligned investor is non-strategic and always cooperates whenever an amount of information is disclosed to him. Conversely, a biased investor is strategic and may betray the expert. The expert is initially uncertain about the investor's type and holds a prior belief $\mu_0 \in (0,1)$ that the investor is aligned. Denote by μ_t as the expert's belief in period t. For notational simplicity, I use μ and Y as to refer to the expert's belief and information, respectively, when an explicit indication of time can be omitted.

A history at the beginning of period t summarizes all players' actions up to this period. A strategy of the expert specifies the amount of information she self-uses or discloses in period t as a function of each history. A strategy of the investor specifies the action he takes in period t as a function of each history and the amount of information disclosed by the expert in this period. The solution concept in this study is Perfect Bayesian Equilibrium (PBE). A strategy profile and a belief updating system consist of an equilibrium if each player's strategy maximizes his/her payoff and if the expert's belief updating follows Bayes' rule whenever possible. In particular, if in period t the expert's belief is μ_t and the biased investor betrays with probability p_t after a positive information disclosure, Bayes' rule requires that, after observing a success, the belief μ_{t+1} in period $t+1$ is as follows:

$$\mu_{t+1} = \frac{\mu_t}{\mu_t + (1-\mu_t)(1-p_t)},$$

[5] This assumption has no loss of generality in our setup. I provide a detailed explanation in the Conclusions.

whereas, after observing a failure, the belief μ_{t+1} drops to 0 in period $t+1$.

In the remainder of this section, I introduce some useful assumptions and notations. Let

$$q = \frac{\beta_I - \alpha_I}{\delta\beta_I} \text{ and } \mathbf{q}^k = \sum_{j=0}^{k} q^j$$

for $k \geq 0$. The superscript "k" in $\mathbf{q}^k$ is a number indicator, whereas the superscript "j" in q^j is the power of q. The role of q is that, if the expert discloses an amount y of information in period t and discloses an amount qy of information in period $t+1$ (based on a success in period t), the investor is indifferent to betraying in these two consecutive periods, which is an important condition in the analysis of equilibrium.[6] By definition, $\mathbf{q}^k$ is a function of k.

Assumption 1: $\alpha_E > 1 + q - \delta q$.

This assumption holds when α_E is relatively large, that is, the investor's cooperation is sufficiently appealing to the expert. Intuitively, it guarantees the existence of equilibrium in which information disclosure occurs, which is explained below.

Assumption 2: $(1-q)\alpha_E < 1 - \delta q$.

This assumption holds if δ is not too close to 1 when $q < 1$.[7] Intuitively, it implies that, because time discounting is costly, the expert prefers immediate self-use of information in period $t = 0$ to permanent cooperation with the investor, even when the latter is feasible.

Finally, let $\overline{k}$ be an integer to satisfy the inequalities

$$\frac{\alpha_E}{1 + (1-\delta)(\mathbf{q}^{\overline{k}} - 1)} > 1 \geq \frac{\alpha_E}{1 + (1-\delta)(\mathbf{q}^{\overline{k}+1} - 1)}.$$

By Assumption 1, I have $\overline{k} \geq 1$. By Assumption 2, $\overline{k}$ is finite. In the next section I show that the periods of information disclosure is bounded above by $\overline{k}+1$ in any equilibrium. Notice that $\overline{k}$ increases in α_E, which indicates that, from the expert's perspective, a longer process

[6] If the investor betrays and terminates the relationship in period t, his payoff is $\beta_I y$. If he cooperates in period t and betrays in period $t+1$, his payoff is $\alpha_I y + \delta\beta_I qy$. When $q = (\beta_I - \alpha_I)/\delta\beta_I$, $\beta_I y = \alpha_I y + \delta\beta_I qy$ holds.

[7] Notice that $\delta q = (\beta_I - \alpha_I)/\beta_I < 1$, so $1 - \delta q > 0$ always holds. If $q \geq 1$, Assumption 2 holds for any $\delta < 1$. But if $q < 1$, Assumption 2 holds only if δ is relatively small.

of information disclosure is acceptable if the investor's cooperation is more productive. Conversely, $\overline{k}$ decreases in q. The intuitive reasoning is that, when q is larger, the expert has to shift more information to the following periods to induce the biased investor's cooperation in the current period, which makes information disclosure more time costly and therefore less appealing to the expert. A decrease of δ has a similar effect as an increase of q on $\overline{k}$.

An immediate observation is that, if the players' interaction can only occur in period $t = 0$, then the expert discloses all her information to the investor if and only if $\mu_0 \geq 1/\alpha_E$, and her equilibrium payoff is $\mu_0 \alpha_E Y_0$ if $\mu_0 \geq 1/\alpha_E$ and Y_0 otherwise. Because of the potential hold-up effect exerted by the biased investor, the expert's willingness to disclose information is limited. In the next section, I explore how the expert can improve her payoff when a dynamic process of information disclosure is introduced.

1.4 Equilibrium analysis

I study the joint dynamics of the expert's information disclosure and the investor's type revelation in this section. Particularly, I show that a process of gradual information disclosure enables the expert to alleviate the hold-up effect and thereby increase her payoff.

1.4.1 Preliminary results

Before forwarding to the analysis of equilibrium properties, I present some preliminary results in this subsection. A definition is introduced first.

Definition 1 *A k-period scheme starting from period t is a scheme satisfying, with a sequence $y^k = (y_1, ..., y_{k-1}, y_k)$ and $1 \leq l \leq k$, (1) $Y_t = \sum_{l=1}^{k} y_l$; (2) $y_{l+1} = q y_l$ if $k > l \geq 1$; (3) amount y_l of information is disclosed in period $t - 1 + l$ if the relationship has not been terminated; and (4) amount $\sum_{j=l}^{k} y_j$ of information is self-used in period $t + l$ if the relationship is terminated in the previous period.*

By this definition, a k-period scheme essentially describes a strategy of the expert in the continuation game starting from period t. Specifically, this strategy makes the biased investor being indifferent to betraying in two consecutive periods when $k > 1$. I will show that, in equilibrium, if information disclosure occurs, then it follows a k-period scheme.

Denote $\mu_0^* = 1$. I have the following result.

Lemma 1 *There exists a unique sequence of values* $(\mu_0^*, \mu_1^*, ..., \mu_k^*, ..., \mu_{\overline{k}+1}^*)$ *satisfying*

(a) $V^k(\mu_k^*; Y) = V^{k+1}(\mu_k^*; Y) = \frac{\alpha_E Y}{1+(1-\delta)(\mathbf{q}^k - 1)} > Y$ *if* $1 \leq k \leq \overline{k}$, *and* $V^k(\mu_k^*; Y) = Y$ *if* $k = \overline{k} + 1$, *in which* $V^k(\mu; Y)$ *is recursively defined by*

$$V^1(\mu; Y) = \alpha_E \mu Y, \text{ and for } 2 \leq k \leq \overline{k} + 1,$$

$$\begin{aligned} V^k(\mu; Y) &= \frac{\min\{\mu, \mu_{k-1}^*\}}{\mu_{k-1}^*}\left(\frac{\alpha_E Y}{\mathbf{q}^{k-1}} + \delta V^{k-1}(\max\{\mu, \mu_{k-1}^*\}; Y - \frac{Y}{\mathbf{q}^{k-1}})\right) \\ &\quad + \left(1 - \frac{\min\{\mu, \mu_{k-1}^*\}}{\mu_{k-1}^*}\right)\delta\left(Y - \frac{Y}{\mathbf{q}^{k-1}}\right). \end{aligned}$$

(b) $\mu_0^* > \mu_1^* > ... > \mu_k^* > ... > \mu_{\overline{k}+1}^* > 0$.

The proof is shown in Appendix A. For $1 \leq k \leq \overline{k} + 1$, the cut-off values μ_k^* will define the evolution of the expert's beliefs after a series of successes in equilibrium, and the value functions $V^k(\mu; Y)$ will define the expert's equilibrium payoff with information disclosure.[8]

For a particular $V^k(\mu; Y)$, the superscript "k" indicates that the expert's information disclosure follows a k-period scheme. For an initial belief μ, the term $\min\{\mu, \mu_{k-1}^*\}/\mu_{k-1}^*$ is the probability that the investor cooperates in the first period of this scheme. In the term associated with this probability, $\alpha_E Y/\mathbf{q}^{k-1}$ is the expert's payoff from the investor's cooperation in the current period, and $\delta V^{k-1}(\max\{\mu, \mu_{k-1}^*\}; Y - Y/\mathbf{q}^{k-1})$ is her discounted payoff from the continuation game after observing the cooperation. On the other hand, the

[8] Notice that the subscript "k" in μ_k^* has no relation to the time period k. Instead, it only refers to one of the numbers $1, ..., \overline{k}, \overline{k} + 1$.

term $1-\min\{\mu,\mu_{k-1}^*\}/\mu_{k-1}^*$ is the probability that the investor betrays in the first period of this scheme, and the term $\delta(Y - Y/\mathbf{q}^{k-1})$ is the expert's discounted payoff from the continuation game after observing the betrayal.

Lemma 2 *Let $1 \leq k, l \leq \overline{k}+1$ and $k \neq l$. If $\mu \in (\mu_k^*, \mu_{k-1}^*)$, then $V^k(\mu;Y) > V^l(\mu;Y)$.*

The proof is shown in Appendix A. The property of the value functions will indicate that, for each belief μ (except the cut-off values μ_k^*), the process of information disclosure and the expert's equilibrium payoff are uniquely determined.

For $\overline{k} = 2$, I introduce Figure 1 to summarize the results presented in the previous lemmas. For instance, if $\mu \in (\mu_2^*, \mu_1^*)$, then $V^2(\mu;Y) > \max\{V^1(\mu;Y), V^3(\mu;Y)\}$. The blue bold envelope will capture the expert's equilibrium payoff from her information as a function of the belief μ. Notice that $V^2(\mu;Y)$ has a kink at μ_1^*, which reflects the change of its slopes when μ increases across μ_1^*. A similar explanation holds for $V^3(\mu;Y)$ with kinks at μ_2^* and μ_1^*.

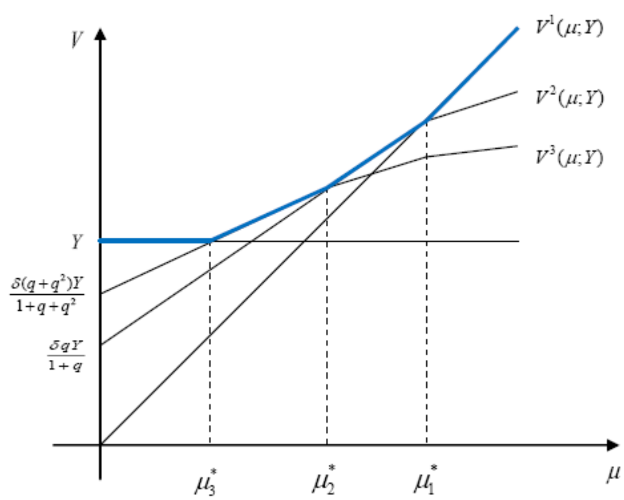

Figure 1: A description of the functions $V^k(\mu;Y)$ and the cut-off values μ_k^*, where $\overline{k}=2$. The blue bold envelope captures the value of dynamic information disclosure as a function of μ.

Consider two beliefs μ and μ', where $\mu \leq \mu'$. Let function $\varphi(\mu;\mu')$ satisfy

$$\mu' = \frac{\mu}{\mu + (1-\mu)(1-\varphi(\mu;\mu'))}.$$

Thus, $\varphi(\mu;\mu')$ is the biased investor's betrayal probability that makes the expert update her belief from μ to μ', after observing a success.

Definition 2 *Let $\mu \in [\mu_k^*, \mu_{k-1}^*]$ and $1 \leq k \leq \overline{k}+1$. A belief path $\boldsymbol{\mu}^k(\mu)$ is a sequence of updated beliefs satisfying $\boldsymbol{\mu}^k(\mu) = (\mu, \mu_{k-1}^*, ..., \mu_1^*, \mu_0^*)$ based on a series of successes.*

Whenever a failure is observed, the expert's belief drops to zero and stays there forever. In consequence, the only relevant belief updating path is the evolution of beliefs based on a series of successes. I will show that, if $\mu_0 \in [\mu_k^*, \mu_{k-1}^*]$ and $1 \leq k \leq \overline{k}+1$, then a pair of a k-period scheme and a belief path $\boldsymbol{\mu}^k(\mu_0)$ describes the players' behaviors and the expert's belief updating on the equilibrium path of play. Moreover, if the relationship is terminated in period t, the expert should self-use all remaining information in period $t+1$ to avoid the discounting cost in any equilibrium. From now on, I omit the description of strategies and beliefs after observing a failure.

1.4.2 Equilibrium results

In this subsection, I characterize the existence and uniqueness of equilibrium and explain their implications on the players' interactions and payoffs.

Before the detailed construction of equilibrium, I illustrate some key points briefly. An intuitive conjecture regarding the players' equilibrium behaviors is that, if the expert intends to disclose all remaining information Y_t in period t, then her belief μ_t should be large enough. As a result, if $\mu_0 < \mu_t$, the biased investor should use mixed strategy and betray with positive probability in some period $\tau < t$. Moreover, because of the timing cost, the expert prefers to disclose information as fast as she can, which implies that the biased investor should be indifferent between cooperating and betraying in any period $\tau < t$ in which information is disclosed. In consequence, if information disclosure occurs, it should follow a k-period scheme on the equilibrium path of play. I verify the correctness of these properties below.

The main difficulty in constructing an equilibrium is to describe the players' behaviors off the equilibrium path. Because of the observability of the expert's information disclosure, the concept of PBE requires that, after a deviation of the expert, the players' continuation play and the expert's belief updating should also consist of an equilibrium. Since the expert's information is divisible, before the game ends she has infinitely many deviations in each period. My work is to pin down the continuation play and belief updating after any of the expert's deviations and verify that no profitable deviation exists.

Proposition 1 *For any $\mu_0 \in (0,1)$ there exists an equilibrium in which the expert's payoff is $V^k(\mu_0; Y_0)$ if $\mu_0 \in [\mu_k^*, \mu_{k-1}^*]$ and is Y_0 if $\mu_0 \leq \mu_{\overline{k}+1}^*$, where $1 \leq k \leq \overline{k}+1$.*

I construct the equilibrium here, and show the proof in Appendix A. Suppose now the game is in period t with belief $\mu_t = \mu$ and information $Y_t = Y$, and the relationship has not been terminated.

Case 1: $\mu \in [\mu_k^*, \mu_{k-1}^*]$ with $1 \leq k \leq \overline{k}+1$.[9]

On the equilibrium path. Starting from period t, the expert's strategy follows a k-period scheme. The biased investor's strategy and the expert's belief updating are described by a belief path $\boldsymbol{\mu}^k(\mu)$. Specifically, in period t, the expert discloses $Y/\mathbf{q}^{k-1}$ and the biased investor betrays with probability $\varphi(\mu; \mu_{k-1}^*)$. The expert's payoff is $V^k(\mu; Y)$, which is measured in period t.

Off the equilibrium path. First, if the biased investor deviates to a probability p' in period t, where $p' \neq \varphi(\mu; \mu_{k-1}^*)$, then the expert continues to update her belief to μ_{k-1}^* after observing a success.[10]

Second, consider the expert's deviations in period t. There are three cases to consider: (1.1) an amount $y > Y/\mathbf{q}^{k-1}$ is disclosed; (1.2) an amount $y < Y/\mathbf{q}^{k-1}$ is disclosed; and (1.3) an amount $x \leq Y$ is self-used.[11]

[9] Notice that for $1 \leq k \leq \overline{k}$, μ_k^* can be drawn either from $[\mu_k^*, \mu_{k-1}^*]$ or from $[\mu_{k+1}^*, \mu_k^*]$, which implies that if $\mu = \mu_k^*$, I construct multiple equilibria for this belief. Similar construction applies to $\mu = \mu_{\overline{k}+1}^*$.

[10] The betrayal probability of the biased investor is unobservable to the expert. Therefore, the expert's belief updating does not need to follow Bayes' rule after the biased investor's deviation.

[11] Implicitly, I have $k > 1$ in case (1.1).

Consider case (1.1). If the expert discloses $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1}]$, where $1 \le l \le k-1$, then the biased investor betrays with probability $\varphi(\mu; \mu^*_{l-1})$. If a success is observed in the current period, (a) if $l = 1$, then in the next period the expert discloses all $Y - y$, and (b) if $l > 1$, then starting from the next period, with probability λ_l the play follows a pair of an $l-1$-period scheme and a belief path $\boldsymbol{\mu}^{l-1}(\mu^*_{l-1})$, and with probability $1-\lambda_l$ the play follows an l-period scheme and a belief path $\boldsymbol{\mu}^l(\mu^*_{l-1})$, where λ_l satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda_l \beta_I \frac{Y-y}{\mathbf{q}^{l-2}} + (1-\lambda_l)\beta_I \frac{Y-y}{\mathbf{q}^{l-1}}].^{12}$$

Consider case (1.2). If the expert discloses $y \le Y/\mathbf{q}^k$, the biased investor cooperates with certainty. Starting from the next period, the play follows a pair of a k-period scheme and a belief path $\boldsymbol{\mu}^k(\mu)$. If the expert discloses $y \in (Y/\mathbf{q}^k, Y/\mathbf{q}^{k-1})$, then the biased investor betrays with probability $\varphi(\mu; \mu^*_{k-1})$. If a success is observed in the current period, (a) if $k = 1$, then in the next period the expert discloses all $Y - y$, and (b) if $k > 1$, then starting from the next period, with probability λ_k the play follows a pair of a $k-1$-period scheme and a belief path $\boldsymbol{\mu}^{k-1}(\mu^*_{k-1})$, and with probability $1-\lambda_k$ the play follows a k-period scheme and a belief path $\boldsymbol{\mu}^k(\mu^*_{k-1})$, where λ_k satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda_k \beta_I \frac{Y-y}{\mathbf{q}^{k-2}} + (1-\lambda_k)\beta_I \frac{Y-y}{\mathbf{q}^{k-1}}].$$

Consider case (1.3). The investor is inactive in this case and the expert's belief satisfies $\mu_{t+1} = \mu$. Starting from the next period, the play follows a pair of a k-period scheme and a belief path $\boldsymbol{\mu}^k(\mu)$.

Case 2: $\mu \le \mu^*_{k+1}$.

On the equilibrium path. The expert self-uses all Y in period t, and her payoff is Y measured in period.

[12] It can be verified that $\lambda_l \in [0,1]$ and that it increases in y for $y \in (Y_t/\mathbf{q}^l, Y_t/\mathbf{q}^{l-1}]$. The mixing between the two schemes that the expert employs here is to keep the biased investor indifferent between betraying and cooperating in period t.

Off the equilibrium path. Only the expert's deviations in period t need to be considered. There are two cases: (2.1) an amount $y \leq Y$ is disclosed; and (2.2) an amount $x < Y$ is self-used.

Consider case (2.1). If the expert discloses $y \leq Y/\mathbf{q}^{\overline{k}+1}$, then the biased investor betrays with probability $\varphi(\mu; \mu^*_{\overline{k}+1})$. If a success is observed in the current period, starting from the next period with probability $\lambda_{\overline{k}+1}$ the play follows a pair of a $\overline{k}+1$-period scheme and a belief path $\boldsymbol{\mu}^{\overline{k}+1}(\mu^*_{\overline{k}+1})$, and with probability $1-\lambda_{\overline{k}+1}$ the expert self-uses all $Y-y$ in period $t+1$, where $\lambda_{\overline{k}+1}$ satisfies

$$\beta_I y = \alpha_I y + \delta \lambda_{\overline{k}+1} \beta_I \frac{Y-y}{\mathbf{q}^{\overline{k}}}.$$

If the expert discloses $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1}]$, where $1 \leq l \leq \overline{k}+1$, then the continuation play is the same as specified in case (1.1).

Consider case (2.2). The investor is inactive in this case and the expert's belief satisfies $\mu_{t+1} = \mu$. In the next period, the expert self-uses all $Y - x$.

The construction of the equilibrium is finished. To have some intuitive understandings, I illustrate the main features of the equilibrium with a simplified example.

Example: Consider a game with the parameters $\overline{k} \geq 2$, $q = 1$, $\mu_0 \in (\mu^*_3, \mu^*_2)$, and $Y_0 = Y$.

With these parameters, on the equilibrium path, the expert discloses information $Y/3$ to the investor in period $t = 0$, and she continues to disclose $Y/3$ in period $t = 1$ and $Y/3$ in period $t = 2$, based on a series of successes. In period $t = 0$, the biased investor betrays with probability $\varphi(\mu_0; \mu^*_2)$. In period $t = 1$, if information is disclosed, then he betrays with probability $\varphi(\mu^*_2; \mu^*_1)$. In period $t = 2$, if information is disclosed, then he betrays with certainty.

The key issue in constructing the equilibrium is how the biased investor should respond if the expert deviates. Keeping two inquiries in mind is helpful to the understanding of the equilibrium. The first one is, given that the biased investor is indifferent between cooperating and betraying in period $t = 0$ with disclosed information $Y/3$, should he strictly prefer to betray if the disclosed information is $Y/3+\epsilon$ in this period, where $\epsilon > 0$ but is arbitrarily small?[13] The answer is no. If he betrays with certainty in this case, the expert, after observing a success, knows that the investor is certain to be aligned and would therefore disclose all remaining information $2Y/3-\epsilon$ in the next period. But then the biased investor should cooperate with certainty in period $t=0$ so that he can betray in period $t=1$, which generates a contradiction.

In the equilibrium I construct, if the expert deviates to an amount $y \leq Y/4$ in period $t=0$, then the biased investor cooperates with certainty. If the deviation amount is $y \in (Y/4, Y/3)$, then the betrayal probability is $\varphi(\mu_0; \mu_2^*)$. If the deviation amount is $y \in (Y/3, Y/2]$, then the betrayal probability is $\varphi(\mu_0; \mu_1^*)$. Finally, if the deviation amount is $y > Y/2$, then the betrayal probability is 1. Figure 2 summarizes the belief updating system in period $t=0$ for this example.

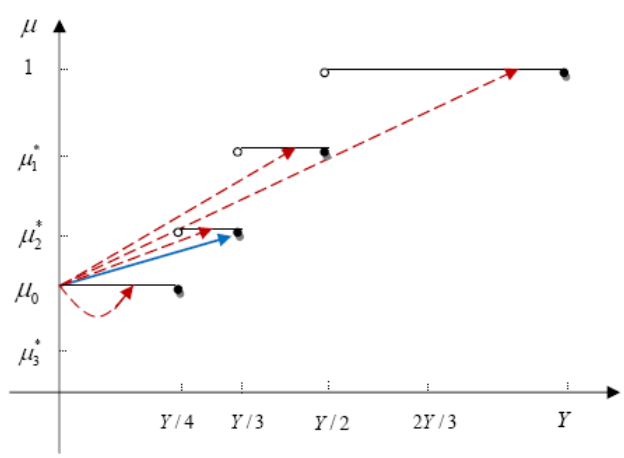

Figure 2: An example of the belief updating system, where $\mu_0 \in [\mu_3^*, \mu_2^*]$, $q=1$ and $\bar{k} \geq 2$. The blue line arrow represents the belief updating after cooperation on the equilibrium path. The red dashed arrows represent the belief updating after cooperation off the equilibrium path. The arrows for the belief updating after betrayal are omitted.

Consider two amounts y and y' of information disclosure by the expert in period $t=0$, where $y, y' \in (Y/4, Y/3]$ and $y > y'$. Notice that the biased investor's payoff from a betrayal is strictly higher with y. The second inquiry is, what makes the biased investor use the same

[13]From now on the term ϵ always refers to "$\epsilon > 0$ but is arbitrarily small."

betrayal probability $\varphi(\mu_0; \mu_2^*) \in (0,1)$ in response to the different amounts y and y'? The answer relies on the multiple equilibria at the cut-off value μ_2^*. In the continuation game that belief reaches μ_2^*, there are two equilibria, one in which the expert employs a 2-period scheme and one in which the expert employs a 3-period scheme. By mixing between these two equilibria, the biased investor can be induced to betray with a constant probability $\varphi(\mu_0; \mu_2^*)$ for any $y, y' \in (Y/4, Y/3]$. The detailed verification is seen in the proof.

Given the biased investor's responses, the expert's task is to choose the process of information disclosure that optimally balances the timing cost and the hold-up effect she faces. In equilibrium, a 3-period scheme maximizes her payoff from information disclosure, which is given by $V^3(\mu_0; Y)$.

The equilibrium I construct presents some of the main findings in this study. First, the expert can mitigate the hold-up problem exerted by the biased investor by disclosing her information gradually. Specifically, if information utilization is restricted in a one-shot game, the expert's payoff is $V^1(\mu_0; Y_0)$ if $\mu_0 \geq 1/\alpha_E$ and Y_0 otherwise. In contrast, in the dynamic game, if $\mu_0 \in [\mu_k^*, \mu_{k-1}^*] \cap (\mu_{\overline{k}+1}^*, \mu_1^*)$ with $2 \leq k \leq \overline{k}+1$, then the expert's equilibrium payoff is $V^k(\mu_0; Y_0)$, which is strictly larger than both $V^1(\mu_0; Y_0)$ and Y_0 by Lemma 1 and 2. Thus, for a non-empty set of initial beliefs, the expert can strictly benefit from the dynamics of information disclosure.

Second, if information disclosure occurs, then it is faster when the investor is more aligned. In other words, if $\mu_0 \in [\mu_k^*, \mu_{k-1}^*]$ and $1 \leq k \leq \overline{k}+1$, then information disclosure follows a k-period scheme in equilibrium. Intuitively, if the expert intends to induce the investor's cooperation by information disclosure, she trades off between the discounting cost and the rents captured by the biased investor. For $1 \leq k \leq \overline{k}$, this trade-off is resolved by the cut-off values μ_k^*, with the following indifference conditions:

$$V^{k+1}(\mu_k^*; Y) = \frac{\alpha_E Y}{\mathbf{q}^k} + \delta \frac{(\mathbf{q}^k - 1)}{\mathbf{q}^k} V^k(\mu_k^*; Y) = V^k(\mu_k^*; Y).$$

At the cut-off values μ_k^*, given the investor's equilibrium strategies, the expert is indifferent between a k-period scheme, which is less time costly but gives the biased investor greater rents (a payoff of $\beta_I Y/\mathbf{q}^{k-1}$), and a $k+1$-period scheme, which is more time costly but gives the biased investor less rents (a payoff of $\beta_I Y/\mathbf{q}^k$). Specifically, in the $k+1$-period scheme, the biased investor is induced to cooperate with certainty in the first period, and the continuation play after this period follows a k-period scheme with less remaining information. In equilibrium, it is optimal for the expert to use a longer process of information disclosure when the investor is less aligned.

Finally, information disclosure occurs only if the investor is somewhat aligned; that is, when $\mu_0 \geq \mu_{\overline{k}+1}^*$. If $\mu_0 < \mu_{\overline{k}+1}^*$, compared with the outside option of self-use, even the most effective process of information disclosure is too costly to the expert. In equilibrium, the cut-off value $\mu_{\overline{k}+1}^*$, which is determined by the indifference condition

$$V^{\overline{k}+1}(\mu_{\overline{k}+1}^*; Y) = Y,$$

solves the expert's problem of when information should be disclosed to the investor.

My second main result is about the equilibrium uniqueness of this game.

Proposition 2 *The expert's equilibrium payoff is unique in this game. That is, the payoff is $V^k(\mu_0; Y_0)$ if $\mu_0 \in [\mu_k^*, \mu_{k-1}^*]$ and is Y_0 if $\mu_0 \leq \mu_{\overline{k}+1}^*$, where $1 \leq k \leq \overline{k}+1$.*

As shown in the proof in Appendix A, the equilibrium of this game is "essentially" unique. If the initial belief μ_0 satisfies $\mu_0 \in (\mu_k^*, \mu_{k-1}^*)$ and $1 \leq k \leq \overline{k}+1$, then the players' interactions follow a unique pair of a k-period scheme and a belief path $\boldsymbol{\mu}^k(\mu_0)$. If $\mu_0 < \mu_{\overline{k}+1}^*$, then the expert's information utilization is uniquely determined by self-use. However, the uniqueness of equilibrium is only in the sense of "essential" because multiplicity of equilibrium arises if the initial belief is at the cut-off values and arises in some off equilibrium path of play. Nevertheless, the expert's equilibrium payoff is unique for any initial belief.

The endogenous completion of the expert's information disclosure is the main determinant of the equilibrium uniqueness. In a particular period, if there is a putative equilibrium that requires the biased investor to betray with a probability larger than the probability he plays in the equilibrium I constructed, the expert becomes more optimistic about the investor's type and would speed up her information disclosure in the event of a success. However, expecting that the process of disclosure is faster in the continuation game after a success, the biased investor should strictly prefer to cooperate in the current period. A similar argument also shows that there is no other equilibrium in which the biased investor betrays with a probability less than the probability he plays in the equilibrium I constructed. As a result, given the biased investor's unique response, the expert's problem regarding her information utilization degenerates to a restricted decision problem, which results in a unique process of information disclosure.

1.5 Extensions

I consider some extensions in this section. My main focus is how the expert can increase her payoff if she has some or full commitment power in determining her information utilization.

1.5.1 Committing to a deadline

In many circumstances, the time period for information disclosure is limited. For instance, investment opportunities in the stock market may be valuable only before the implementation of some new regulation policies. In this subsection, I explore the effects of a deadline on the expert's information utilization, especially how the expert can benefit from committing to a deadline.

Definition 3 *Let period T be the deadline period; therefore, information disclosure is feasible only in period $t \leq T$.*

In other words, at most $T+1$ periods are available for information disclosure.[14] If the deadline period T is reached, the continuation game becomes a one-shot game, and the expert discloses all remaining information only if her belief satisfies $\mu \geq 1/\alpha_E$. On the other hand, in the previous section I have seen that, if no deadline exists, the expert discloses all remaining information in a single period only if her belief satisfies $\mu \geq \mu_1^*$, where $\mu_1^* > 1/\alpha_E$. This difference is central to the understanding of a deadline's effects on the expert's information utilization.

Intuitively, if period T is reached, then the expert cannot further benefit from gradual information disclosure and thereby her *ex post* payoff is reduced by this restriction. To see this, notice that for a belief in the range $(\mu_{\overline{k}+1}^*, \mu_1^*)$, the expert's payoff from a multi-period information disclosure is strictly larger than her payoff from a single-period disclosure or self-use. However, expecting that in the deadline period the expert is willing to disclose all remaining information even her belief is not sufficiently large (only $\mu_T \geq 1/\alpha_E$ is required), the biased investor can lower his betrayal probabilities in the periods before the deadline. In turn, the expert's *ex ante* payoff can be increased. For example, if $T=1$ and $\mu_0 \in [\max\{\mu_2^*, 1/\alpha_E\}, \mu_1^*)$, by disclosing an amount $Y_0/(1+q) - \epsilon$ and thereby inducing the investor's full cooperation in period $t=0$, the expert can guarantee a total payoff sufficiently close to $\alpha_E Y_0[1+\delta q \mu_0]/(1+q)$, which is strictly larger than the equilibrium payoff $V^2(\mu_0; Y_0)$ without a deadline. My next result generalizes these intuitions.

Proposition 3 *For any $\mu_0 \in [\mu_k^*, \mu_{k-1}^*)$ and $2 \leq k \leq \overline{k}+1$, if the expert commits to a deadline $T = k-1$, there exists an equilibrium in which her payoff is strictly larger than $V^k(\mu_0; Y_0)$.*

How does the presence of a deadline have effects on the expert's information utilization depends on the expert's initial belief about the investor's type. In the equilibrium I construct in the proof, for any deadline $T = k-1$ and $2 \leq k \leq \overline{k}+1$, the belief set $(0,1)$ is partitioned

[14] I assume that the expert's self-use of information is not limited by the deadline T. Thus, even in period $t > T$, self-use of information is feasible.

into three intervals by two cut-off values, $\underline{\mu}_k$ and $\overline{\mu}_k$, which satisfy $0 < \underline{\mu}_k < \overline{\mu}_k < 1$. If $\mu_0 \geq \overline{\mu}_k$, the expert's information disclosure follows an l-period scheme, where $1 \leq l < k$, which never reaches the deadline. The reason is that, although such a scheme requires that the expert's belief is at least μ_1^* for her to disclose all remaining information in the lth period of this scheme, slowing down the process of disclosure until the deadline is reached is too time costly. For example, if the initial belief satisfies $\mu_0 > \mu_1^*$, then the expert discloses all of her information in period $t = 0$, no matter what the deadline is. If $\mu_0 \in [\underline{\mu}_k, \overline{\mu}_k]$, then the expert's information disclosure follows a k-period scheme. The reason is that, for this range of initial beliefs, a scheme reaching the deadline (contingent on a series of successes) can effectively lower the biased investor's betrayal probabilities during the process of disclosure. Finally, if $\mu_0 \leq \underline{\mu}_k$, then the expert self-uses her information. Intuitively, if the initial belief is relatively low, for the expert to update her belief to at least $1/\alpha_E$ in the limited time (at most k periods), the betrayal probabilities during the process of disclosure need to be sufficiently large, which makes information disclosure unattractive. For example, if $T = 1$ but $\overline{k}$ is relatively large, then $\mu_{\overline{k}+1}^* < \underline{\mu}_2$ holds and the expert with a belief $\mu_0 \in [\mu_{\overline{k}+1}^*, \underline{\mu}_2)$ is crowded out of information disclosure by the presence of a deadline.

Most importantly, if the expert has the ability to commit to a deadline, her payoff can be strictly improved if the initial belief satisfies $\mu_0 \in [\mu_k^*, \mu_{k-1}^*)$ and $2 \leq k \leq \overline{k} + 1$. The reasoning is straightforward. Compared with the k-period scheme in the equilibrium without a deadline, by choosing a deadline $T = k - 1$, the same k-period scheme of information disclosure can induce the biased investor to be more cooperative and therefore increase the expert's payoff.

I describe the expert's belief updating system with a deadline $T = 2$ in the following figure. If $\mu_0 \geq \overline{\mu}_3$, then the expert employs a 1-period scheme or a 2-period scheme of information disclosure. If $\mu_0 \in [\underline{\mu}_3, \overline{\mu}_3]$, then the expert employs a 3-period scheme that reaches the deadline based on a series of successes. Finally, if $\mu_0 \leq \underline{\mu}_3$, then the expert self-uses her information. Notice that in this example $[\mu_3^*, \mu_2^*] \in [\underline{\mu}_3, \overline{\mu}_3]$.

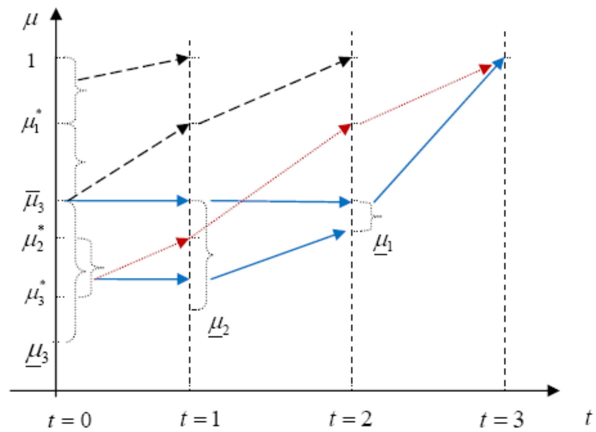

Figure 3: An example of the belief updating system with $T = 2$. The blue line arrows represent the belief updating after cooperation such that the deadline is reached. The black dashed arrows represent the belief updating after cooperation such that the deadline is not reached. The red dotted arrows represent the belief updating after cooperation if no deadline exists. The arrows for the belief updating after betrayal are omitted.

The property of payoff improvement is not limited to the set $[\mu^*_{\overline{k}+1}, \mu^*_1)$ of initial beliefs. For example, in the proof, I show that $\underline{\mu}_{\overline{k}+1} < \mu^*_{\overline{k}+1}$. As a result, the expert with an initial belief $\mu_0 \in (\underline{\mu}_{\overline{k}+1}, \mu^*_{\overline{k}+1})$ can also benefit from gradual information disclosure by committing to a deadline $T = \overline{k}$.

1.5.2 Optimal Commitment

Instead of merely committing to a deadline, in some circumstances the expert can commit to a sequence $y^\tau = (y_0, y_1, ...y_\tau)$ of information disclosure, in which y_t is the amount of information to be disclosed in period $t \leq \tau$ contingent on that the relationship is ongoing, and τ may or may not be finite.[15] For instance, an auto company's technology transfer to its partner may follow a pre-committed schedule. I explore the properties of the expert's optimal commitment in this section.

[15] In this dynamic game, committing to a sequence y^τ is equivalent to the principal having full commitment.

Define

$$\overline{V}^1(\mu; Y) = \alpha_E \mu Y,$$

and for $k \geq 2$,

$$\overline{V}^k(\mu; Y) = \frac{\alpha_E Y}{\mathbf{q}^{k-1}} + \delta q \frac{\mathbf{q}^{k-2}}{\mathbf{q}^{k-1}} \overline{V}^{k-1}(\mu; Y).$$

The superscript "k" also refers to the notion of "a k-period scheme."

I simplify the expert's optimal commitment problem with the following arguments. First, because of the linearity of her payoff function, if the expert can benefit from committing to a sequence y^τ, then in the optimal commitment all information should be committed; that is, $\sum_{j=0}^{\tau} y_j = Y_0$. Second, given the optimally committed sequence y^τ, the biased investor should cooperate with certainty in any period $t < \tau$. The reason is that, if the biased investor betrays with probability $p_t = 1$ and terminates the relationship in period $t < \tau$, then the designing of the sub-sequence $(y_{t+1}, y_{t+2}, ...y_\tau)$ has no chance to induce his cooperation. Therefore, the expert is better off if she re-allocates the amount $\sum_{j=t+1}^{\tau} y_j$ proportionally to the amounts in the sub-sequence $(y_0, y_1, ...y_t)$. On the other hand, if the biased investor betrays with probability $p_t \in (0, 1)$ in period $t < \tau$, then it is better for the expert to disclose only $y_t - \epsilon$ in period t for the sake of the investor's full cooperation, and then re-allocate ϵ proportionally to the future periods. It can be verified that, in the limit, the expert's optimal commitment is reduced to the following problem:

$$\max_{k \in \mathbb{N}} \overline{V}^k(\mu_0; Y_0)$$

$$s.t. \overline{V}^k(\mu_0; Y_0) \geq Y_0.$$

That is, the expert optimally commits to a k-period scheme of information disclosure, contingent on the result that her payoff from this scheme is larger than the payoff from self-use. Let $k^*(\mu_0)$ denote the solution to this problem. I have the next result.

Proposition 4 *For any $\mu_0 \in [\mu_k^*, \mu_{k-1}^*)$ and $2 \leq k \leq \overline{k} + 1$, with the optimal commitment,* *the expert's payoff is strictly larger than* $V^k(\mu_0; Y_0)$.

The key property of the optimal commitment is that, when the final period of the $k^*(\mu_0)$-period scheme is reached, the expert agrees to disclose all remaining information no matter what her belief about the investor's type is in this period. This commitment enables the expert to induce the biased investor's full cooperation during the first $k^*(\mu_0) - 1$ periods of information disclosure, which is qualitatively similar to, but effectively stronger than, the scenario of committing to a deadline.

Given this property, in determining the optimal commitment, the expert essentially trades off between the length of the disclosure process and the amount of information to be captured by the biased investor in the final period. In the proof, I show that, if the expert's initial belief is moderate, say $\mu_0 \in [\mu^*_{\overline{k}+1}, \mu^*_1)$, she can strictly benefit from her optimal commitment. Moreover, $k^*(\mu_0)$ (weakly) decreases in μ_0, which indicates that the higher the betrayal probability is in the final period, the longer the process of information disclosure should be. Besides, for any $\mu_0 \in [\mu^*_k, \mu^*_{k-1})$ and $2 \leq k \leq \overline{k} + 1$, $k^*(\mu_0) \leq k$ and the inequality is strict for some initial beliefs μ_0. In consequence, the process is (weakly) faster in the optimal commitment scenario than in the scenario without commitment. Finally, I show that the expert may benefit from information disclosure even when her initial belief μ_0 is less than $\mu^*_{\overline{k}+1}$.

1.6 Conclusions

In this paper, I study a dynamic game in which an expert can utilize her private information either by self-use or by information disclosure to an investor. The investor has the potential to realize the value of the expert's information in a more efficient way, but he may have incentive to hold up the expert for his own benefit. My main finding is that, in the unique equilibrium, the expert can mitigate the investor's hold-up effect by employing a process of gradual information disclosure. I also address how the expert can increase her equilibrium payoff by committing to a deadline or by committing to a particular pattern of information disclosure.

I briefly discuss some assumptions adopted in this study. First, if the expert has no outside option or Assumption 2 does not hold, then in equilibrium the process of information disclosure goes to infinity when the expert's initial belief about the investor's type converges to zero.[16] However, this process is less realistic, because amounts of information disclosed in the very beginning (or in the very end) must also converge to zero if $q \geq 1$ (or $q < 1$). Second, if the expert's information is not equally valuable or perfectly divisible, then the ordering of different pieces of information to be disclosed may also matter in equilibrium. Finally, given the aligned investor's non-strategic behavior, assuming that information is not cumulative to the investor is without loss of generality. The reason is that any action on information accumulation reveals the investor's type as biased and therefore causes the expert to stop information disclosure.[17] Because of time discounting and linear payoff function, the biased investor can not benefit from information accumulation.

One potential extension of this game is to consider the case in which the investor's action is not observable and his cooperation can generate a success only with a probability of less than one. In other words, the expert's information utilization is not only subject to adverse selection but also to moral hazard. As a result, after obtaining a payoff zero in a period, the expert's belief does not drop to zero. My conjecture is that there exists a positive value of belief such that, if the expert's belief is less than this value, then she resorts to self-use of information. However, a complete characterization of this setup is more complicated.

Another extension is to consider the case in which the amount Y_0 of information is initially unknown to the investor. For instance, the expert may be of a low type $Y_0 = Y_L > 0$ with probability $\theta \in (0, 1)$ and may be of a high type $Y_0 = Y_H > Y_L$ with probability $1 - \theta$. The dynamics of information disclosure becomes more subtle because of the type pooling

[16] Technically, the constructions of cut-off values μ_k^* and value functions $V^k(\mu; Y)$ in Lemma 1 are not restricted by the condition $V^k(\mu; Y) \geq Y$, and I have an infinite sequence such that $k \to +\infty$ and $\mu_k^* \to 0$.

[17] Given Assumption 2, it can be verified that if $\mu = 0$, then the expert should self-use her information instead of seeking cooperation from the biased investor.

and separating of the experts. Intuitively, a low type of expert may attempt to pretend to be a high type in order to delay the investor's betrayal, whereas a high type of expert may mimic a low type's behavior to fasten the investor's type revelation. Alternatively, I may also assume that the amount of information self-used by the expert is not observable to the investor. In this case, the expert may use her outside option more strategically because of the endogenous generation of private types. For instance, it is not necessarily true that whenever the expert prefers to self-use *some* information, she self-uses *all* information immediately. I leave these questions open for future research.

Appendix A. Proofs.

The proof of Lemma 1.

Proof. The proof is by induction. Consider $V^1(\mu; Y)$ and $V^2(\mu; Y)$ for any possible value $\mu_1^* \in (0, 1]$. I have $V^1(0; Y) < V^2(0; Y)$ and $V^1(1; Y) > V^2(1; Y)$. Notice that both of the value functions are continuous and strictly increasing in μ, but the slope of $V^1(\mu; Y)$ is strictly larger than the slope of $V^2(\mu; Y)$ for any μ.[18] Therefore, given $\overline{k} \geq 1$, I have a unique μ_1^* satisfying $V^1(\mu_1^*; Y) = V^2(\mu_1^*; Y) = \frac{\alpha_E Y}{1+q-\delta q} > Y$. In particular, $\mu_1^* = \frac{1}{1+q-\delta q}$. It can be verified that $V^1(\mu; Y) > V^2(\mu; Y)$ for $\mu > \mu_1^*$, and $V^1(\mu; Y) < V^2(\mu; Y)$ for $\mu < \mu_1^*$.

Now suppose that for any $k - 1 = 1, \cdots, \overline{k} - 1$ there is a unique μ_{k-1}^* satisfying (a) and (b). By induction I show that there is a unique μ_k^* satisfying (a) and (b). First I have

$$V^k(0; Y) = \delta \frac{(\mathbf{q}^{k-1} - 1)Y}{\mathbf{q}^{k-1}} < \delta \frac{(\mathbf{q}^k - 1)Y}{\mathbf{q}^k} = V^{k+1}(0; Y).$$

Second, consider $\mu = \mu_{k-1}^*$. For any possible value $\mu_k^* \leq \mu_{k-1}^*$, I have

$$\begin{aligned} V^{k+1}(\mu_{k-1}^*; Y) - V^k(\mu_{k-1}^*; Y) &= \frac{\alpha_E Y}{\mathbf{q}^k} + \delta V^k(\mu_{k-1}^*; \frac{(\mathbf{q}^k - 1)Y}{\mathbf{q}^k}) - V^k(\mu_{k-1}^*; Y) \\ &= \frac{\alpha_E Y}{\mathbf{q}^k} + (\delta \frac{(\mathbf{q}^k - 1)}{\mathbf{q}^k} - 1)V^k(\mu_{k-1}^*; Y) \\ &= \frac{\alpha_E Y}{\mathbf{q}^k}(1 - \frac{1 + (1-\delta)(\mathbf{q}^k - 1)}{1 + (1-\delta)(\mathbf{q}^{k-1} - 1)}) \\ &< 0, \end{aligned}$$

in which the second equality holds because $V^k(\mu_{k-1}^*; Y)$ is linear in Y. Since for any possible value $\mu_k^* \in (0, \mu_{k-1}^*]$ both $V^k(\mu; Y)$ and $V^{k+1}(\mu; Y)$ are continuous and strictly increasing in μ, and $V^k(\mu; Y)$ has a larger slope, there is a unique $\mu_k^* \in (0, \mu_{k-1}^*)$ satisfying (1) $V^{k+1}(\mu_k^*; Y) = V^k(\mu_k^*; Y)$, (2) $V^k(\mu; Y) > V^{k+1}(\mu; Y)$ for $\mu \in (\mu_k^*, \mu_{k-1}^*]$, and (3) $V^k(\mu; Y) < V^{k+1}(\mu; Y)$ for

[18] Technically, the slope of $V^2(\mu; Y)$ at the kink μ_1^* is not defined. However, because $V^2(\mu; Y)$ is continuous in μ, this kink does not affect the comparison between these two value functions. I omit this special case. Similar treatment is applied to the other value functions.

$\mu < \mu_k^*$. Moreover, the statement of (2) can be augmented to have $V^k(\mu;Y) > V^{k+1}(\mu;Y)$ for $\mu > \mu_k^*$. To see this, consider $\mu \in [\mu_{k-1}^*, \mu_{k-2}^*]$. I have

$$V^k(\mu;Y) = \frac{\alpha_E Y}{\mathbf{q}^{k-1}} + \delta \frac{(\mathbf{q}^{k-1} - 1)}{\mathbf{q}^{k-1}} V^{k-1}(\mu;Y)$$

and

$$V^{k+1}(\mu;Y) = \frac{\alpha_E Y}{\mathbf{q}^k} + \frac{\delta \alpha_E q Y}{\mathbf{q}^k} + \delta^2 \frac{(\mathbf{q}^k - 1 - q)}{\mathbf{q}^k} V^{k-1}(\mu;Y).$$

By the conditions that $\alpha_E > 1 + (1-\delta)(\mathbf{q}^k - 1)$ and $V^{k-1}(\mu;Y) \geq \frac{\alpha_E Y}{1+(1-\delta)(\mathbf{q}^{k-1}-1)}$, it is direct to see that $V^k(\mu;Y) > V^{k+1}(\mu;Y)$ for $\mu \in [\mu_{k-1}^*, \mu_{k-2}^*]$. Apply this argument recursively, I have $V^k(\mu;Y) > V^{k+1}(\mu;Y)$ for $\mu > \mu_k^*$.

Moreover, because

$$V^{k+1}(\mu_k^*;Y) = \frac{\alpha_E Y}{\mathbf{q}^k} + \delta \frac{(\mathbf{q}^k - 1)}{\mathbf{q}^k} V^k(\mu_k^*;Y) = V^k(\mu_k^*;Y),$$

I have

$$V^k(\mu_k^*;Y) = V^{k+1}(\mu_k^*;Y) = \frac{\alpha_E Y}{1 + (1-\delta)(\mathbf{q}^k - 1)} > Y,$$

in which the inequality holds because $\alpha_E > 1 + (1-\delta)(\mathbf{q}^k - 1)$ for $k \leq \bar{k}$.

Now consider $V^{\bar{k}+1}(\mu;Y)$. Because

$$V^{\bar{k}+1}(0;Y) = \delta \frac{(\mathbf{q}^{\bar{k}} - 1)Y}{\mathbf{q}^{\bar{k}}} < Y,$$

$$V^{\bar{k}+1}(\mu_{\bar{k}}^*;Y) = \frac{\alpha_E Y}{1 + (1-\delta)(\mathbf{q}^{\bar{k}} - 1)} > Y,$$

and $V^{\bar{k}+1}(\mu;Y)$ is continuous and strictly increasing in μ for any $\mu \in [0, \mu_{\bar{k}}^*]$, there is a unique $\mu_{\bar{k}+1}^* \in (0, \mu_{\bar{k}}^*)$ satisfying $V^{\bar{k}+1}(\mu_{\bar{k}+1}^*;Y) = Y$. It can be further verified that $V^{\bar{k}+1}(\mu;Y) > Y$ if and only if $\mu > \mu_{\bar{k}+1}^*$.

The proof of Lemma 1 is ended. The remainder of this proof solves the cut-off values μ_k^*

explicitly. For $1 \leq k \leq \overline{k}$, let

$$\Phi^k = \mathbf{q}^k \alpha_E - \delta(\mathbf{q}^k - 1)(1 + (1-\delta)(\mathbf{q}^{k+1} - 1))$$

and

$$\Psi^k = \mathbf{q}^k \alpha_E - \delta(\mathbf{q}^k - 1)(1 + (1-\delta)(\mathbf{q}^k - 1)).$$

Given $\alpha_E > 1 + (1-\delta)(\mathbf{q}^k - 1)$ for $k \leq \overline{k}$, I have $0 < \Phi^k < \Psi^k$. Notice that μ_1^* can be represented as $\mu_1^* = \frac{1}{1+q-\delta q}\frac{\Phi^0}{\Psi^0}$. For $2 \leq k \leq \overline{k}$, By the condition

$$\begin{aligned} V^k(\mu_k^*; Y) &= \frac{\mu_k^*}{\mu_{k-1}^*}(\alpha_E \frac{Y}{\mathbf{q}^{k-1}} + \delta V^{k-1}(\mu_{k-1}^*; Y - \frac{Y}{\mathbf{q}^{k-1}})) + (1 - \frac{\mu_k^*}{\mu_{k-1}^*})\delta(Y - \frac{Y}{\mathbf{q}^{k-1}}) \\ &= \frac{\alpha_E Y}{1 + (1-\delta)(\mathbf{q}^k - 1)}, \end{aligned}$$

I can solve μ_k^* recursively as

$$\mu_k^* = \frac{1}{1 + (1-\delta)(\mathbf{q}^k - 1)} \prod_{j=0}^{k-1} \frac{\Phi^j}{\Psi^j}.$$

Finally, for $k = \overline{k} + 1$, by the condition

$$V^{\overline{k}+1}(\mu_{\overline{k}+1}^*; Y) = \frac{\mu_{\overline{k}+1}^*}{\mu_{\overline{k}}^*}(\alpha_E \frac{Y}{\mathbf{q}^{\overline{k}}} + \delta V^{\overline{k}}(\mu_{\overline{k}}^*; Y - \frac{Y}{\mathbf{q}^{\overline{k}}})) + (1 - \frac{\mu_{\overline{k}+1}^*}{\mu_{\overline{k}}^*})\delta(Y - \frac{Y}{\mathbf{q}^{\overline{k}}}) = Y,$$

I have

$$\mu_{\overline{k}+1}^* = \frac{1 + (1-\delta)(\mathbf{q}^{\overline{k}} - 1)}{\Psi^{\overline{k}}} \prod_{j=0}^{\overline{k}-1} \frac{\Phi^j}{\Psi^j}.$$

■

The proof of Lemma 2.

Proof. First consider $1 \leq k \leq \overline{k}$. In the proof of Lemma 1 I have seen that, if $l = k+1$, $V^k(\mu_k^*; Y) = V^{k+1}(\mu_k^*; Y)$ and $V^k(\mu; Y) > V^{k+1}(\mu; Y)$ for any $\mu > \mu_k^*$. Now consider $l = k+2$

(in case $k+2 \leq \overline{k}$). I have seen that $V^{k+1}(\mu^*_{k+1}; Y) = V^{k+2}(\mu^*_{k+1}; Y)$ and $V^{k+1}(\mu; Y) > V^{k+2}(\mu; Y)$ for $\mu > \mu^*_{k+1}$. Because $\mu \in (\mu^*_k, \mu^*_{k-1})$ and $\mu^*_k > \mu^*_{k+1}$, by transitivity I have $V^k(\mu; Y) > V^{k+2}(\mu; Y)$. Recursively, I can show that for any $l > k$, the inequality $V^k(\mu; Y) > V^l(\mu; Y)$ holds for any $\mu \in (\mu^*_k, \mu^*_{k-1})$.

By applying a similar argument, I can show that for any $k \leq \overline{k}+1$ and any $l < k$, $V^k(\mu; Y) > V^l(\mu; Y)$ holds for any $\mu \in (\mu^*_k, \mu^*_{k-1})$. ■

The proof of Proposition 1.

First I introduce a technical result, which will be used in the proof of the equilibrium.

Lemma A1 *For $\mu \in [\mu^*_k, \mu^*_{k-1}]$ and information Y, with $2 \leq k \leq \overline{k}+1$ and $2 \leq j \leq k$, I have the following inequality*

$$V^j(\mu; Y) > \frac{\mu}{\mu^*_{j-2}}(\alpha_E \frac{Y}{\mathbf{q}^{j-1}} + \delta V^{j-1}(\mu^*_{j-2}; Y - \frac{Y}{\mathbf{q}^{j-1}})) + (1 - \frac{\mu}{\mu^*_{j-2}})\delta(Y - \frac{Y}{\mathbf{q}^{j-1}}).$$

Proof. Let $Z^j(\mu; Y)$ denote the term on the right side of the inequality. By extending the value function $V^{j-1}(\cdot; \cdot)$, I have

$$\begin{aligned}
V^j(\mu; Y) &= \frac{\mu}{\mu^*_{j-1}}\delta[\frac{\mu^*_{j-1}}{\mu^*_{j-2}}(\alpha_E \frac{qY}{\mathbf{q}^{j-1}} + \delta V^{j-2}(\mu^*_{j-2}; Y - \frac{(1+q)Y}{\mathbf{q}^{j-1}})) \\
&\quad + (1 - \frac{\mu^*_{j-1}}{\mu^*_{j-2}})\delta(Y - \frac{(1+q)Y}{\mathbf{q}^{j-1}})] \\
&\quad + \frac{\mu}{\mu^*_{j-1}}\alpha_E \frac{Y}{\mathbf{q}^{j-1}} + (1 - \frac{\mu}{\mu^*_{j-1}})\delta(Y - \frac{Y}{\mathbf{q}^{j-1}})
\end{aligned}$$

and

$$\begin{aligned}
Z^j(\mu; Y) &= \frac{\mu}{\mu^*_{j-2}}\delta[\frac{\mu^*_{j-2}}{\mu^*_{j-2}}(\alpha_E \frac{qY}{\mathbf{q}^{j-1}} + \delta V^{j-2}(\mu^*_{j-2}; Y - \frac{(1+q)Y}{\mathbf{q}^{j-1}})) \\
&\quad + (1 - \frac{\mu^*_{j-2}}{\mu^*_{j-2}})\delta(Y - \frac{(1+q)Y}{\mathbf{q}^{j-1}})] \\
&\quad + \frac{\mu}{\mu^*_{j-2}}\alpha_E \frac{Y}{\mathbf{q}^{j-1}} + (1 - \frac{\mu}{\mu^*_{j-2}})\delta(Y - \frac{Y}{\mathbf{q}^{j-1}}).
\end{aligned}$$

Therefore,

$$V^j(\mu;Y) - Z^j(\mu;Y) = (\frac{\mu}{\mu^*_{j-1}} - \frac{\mu}{\mu^*_{j-2}})[\alpha_E \frac{Y}{\mathbf{q}^{j-1}} + \delta^2(Y - \frac{(1+q)Y}{\mathbf{q}^{j-1}}) - \delta(Y - \frac{Y}{\mathbf{q}^{j-1}})] > 0$$

because $\mu^*_{j-1} < \mu^*_{j-2}$ and $\alpha_E > 1 + (1-\delta)(\mathbf{q}^{\overline{k}} - 1)$. ■

The proof of the equilibrium.

Proof. As having been stated before, the observability of the expert's information disclosure requires that, after a deviation by the expert, the players' continuation play and the expert's belief updating should also consist of an equilibrium. In particular, notice that the biased investor's choice to cooperate or betray not only depends on the amount of information disclosed in the current period, but also depends on the expected payoff he can obtain in the continuation game.

Case 1: $\mu \in [\mu^*_k, \mu^*_{k-1}]$ with $1 \leq k \leq \overline{k} + 1$.

On the equilibrium path, the expert's information disclosure follows a k-period scheme. If $k = 1$ so the expert discloses all Y in period t, it is optimal for the biased investor to betray with probability $\varphi(\mu; \mu^*_0) = 1$. If $k > 1$, the biased investor is indifferent between betraying in period t with a payoff $\beta_I Y/\mathbf{q}^{k-1}$ and betraying in period $t+1$ with a total payoff $\alpha_I Y/\mathbf{q}^{k-1} + \delta\beta_I qY/\mathbf{q}^{k-1}$, so his betrayal probability $\varphi(\mu; \mu^*_{k-1})$ is optimal. Moreover, the expert's belief updating follows Bayes' rule and her payoff is $V^k(\mu; Y)$.

Consider the players' deviations in period t. Notice that off the equilibrium path the expert's belief updating is not required to follow the Bayes' rule, so she can continue to update her belief as she does on the equilibrium path. Given such a response by the expert, there is no profitable deviation from the probability $\varphi(\mu; \mu^*_{k-1})$ for the biased investor.

Consider case (1.1) that the expert deviates to an amount $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1}]$, where $1 \leq l \leq k-1$. If $l = 1$, it is direct to verify that the biased investor should betray with probability $\varphi(\mu; \mu^*_0) = 1$ and after observing a success the expert should disclose $Y - y$ in period $t+1$. Given these responses, the expert's deviation payoff, which is denoted by D, is

$$D = \mu[\alpha_I y + \delta\alpha_I(Y-y)] + (1-\mu)\delta(Y-y).$$

Because D is linear in y, the optimal deviation is $y = Y/(1+q)+\epsilon$ or $y = Y$.[19] If $y = Y/(1+q)+\epsilon$, by Lemma A1 and Lemma 2, I have

$$D \leq \lim_{\epsilon \downarrow 0} D = Z^2(\mu; Y) \leq V^2(\mu; Y) \leq V^k(\mu; Y)$$

with $2 < k \leq \overline{k}+1$ and $\mu \in [\mu_k^*, \mu_{k-1}^*]$. Therefore, it is not a profitable deviation for the expert. If $y = Y$, by Lemma 2 I have

$$D = V^1(\mu; Y) \leq V^k(\mu; Y)$$

with $2 < k \leq \overline{k}+1$ and $\mu \in [\mu_k^*, \mu_{k-1}^*]$. So it is not a profitable deviation for the expert.

If $l > 1$, after observing a success the belief is updated to μ_{l-1}^*. Because at this belief there is a continuation equilibrium with an $l-1$-period scheme and there is another continuation equilibrium with an l-period scheme (notice that these two equilibria give the expert the same payoff $V^{l-1}(\mu_{l-1}^*; Y-y)$), by mixing them with probabilities λ_l and $1-\lambda_l$ respectively, the biased investor is indifferent between betraying in period t and betraying in period $t+1$, so it is optimal for him to betray in period t with probability $\varphi(\mu; \mu_{l-1}^*)$. Given these responses, the expert's deviation payoff is

$$D = \frac{\mu}{\mu_{l-1}^*}(\alpha_E y + \delta V^{l-1}(\mu_{l-1}^*; Y-y)) + (1-\frac{\mu}{\mu_{l-1}^*})\delta(Y-y).$$

Because D is linear in y, the optimal deviation is $y = Y/\mathbf{q}^l+\epsilon$ or $y = Y/\mathbf{q}^{l-1}$. If $y = Y/\mathbf{q}^l+\epsilon$,

[19] Without further conditions, it is indeterminate whether D increases in y or not.

by Lemma A1 and Lemma 2 I have

$$D \leq \lim_{\epsilon \downarrow 0} D = Z^{l+1}(\mu; Y) \leq V^{l+1}(\mu; Y) \leq V^k(\mu; Y)$$

with $1 < l \leq k-1$ and $\mu \in [\mu_k^*, \mu_{k-1}^*]$. So it is not a profitable deviation for the expert. If $y = Y/\mathbf{q}^{l-1}$, by Lemma 2 I have

$$D = V^l(\mu; Y) \leq V^k(\mu; Y)$$

with $1 < l \leq k-1$ and $\mu \in [\mu_k^*, \mu_{k-1}^*]$. So it is not a profitable deviation for the expert.

Consider case (1.2) that the expert deviates to an amount $y < Y/\mathbf{q}^{k-1}$. If $y \leq Y/\mathbf{q}^k$ and the biased investor cooperates for sure in period t, $\mu_{t+1} = \mu_t$ and the play of a pair of a k-period scheme and a belief path $\boldsymbol{\mu}^k(\mu)$ consists of an equilibrium starting from period $t+1$. Given this continuation play, the biased investor actually should cooperates for sure because

$$\beta_I y \leq \alpha_I y + \delta \beta_I (Y - y)/\mathbf{q}^{k-1}$$

for any $y \leq Y_t/\mathbf{q}^k$. Given these responses, if the expert discloses $y \leq Y/\mathbf{q}^k$, her payoff is

$$D = \alpha_E y + \delta V^k(\mu; Y - y).$$

Because D is linear in y, the optimal deviation is $y = 0$ or $y = Y/\mathbf{q}^k$. If $y = 0$, I have

$$D = \delta V^k(\mu; Y) < V^k(\mu; Y),$$

so it is not a profitable deviation for the expert. If $y = Y/\mathbf{q}^k$, by Lemma 2 I have

$$D = V^{k+1}(\mu; Y) \leq V^k(\mu; Y)$$

with $\mu \in [\mu_k^*, \mu_{k-1}^*]$. So it is not a profitable deviation for the expert.

If $y \in [Y/\mathbf{q}^k, Y/\mathbf{q}^{k-1})$, after observing a success the belief is updated to μ_{k-1}^*. Because at this belief there is a continuation equilibrium with a $k-1$-period scheme and there is another continuation equilibrium with a k-period scheme, by mixing them with probabilities λ_k and $1-\lambda_k$ respectively, the biased investor is indifferent between betraying in period t and betraying in period $t+1$, so it is optimal for him to betray in period t with probability $\varphi(\mu; \mu_{k-1}^*)$. Given these responses, the expert's deviation payoff is

$$D = \frac{\mu}{\mu_{k-1}^*}(\alpha_E y + \delta V^{k-1}(\mu_{k-1}^*; Y-y)) + (1 - \frac{\mu}{\mu_{k-1}^*})\delta(Y-y).$$

Because D is linear in y, the optimal deviation is $y = Y/\mathbf{q}^k$ or $y = Y/\mathbf{q}^{k-1} - \epsilon$. If $y = Y/\mathbf{q}^k$, by Lemma 2 I have

$$D \leq V^{k+1}(\mu; Y) \leq V^k(\mu; Y)$$

with $\mu \in [\mu_k^*, \mu_{k-1}^*]$. So it is not a profitable deviation for the expert. If $y = Y/\mathbf{q}^{k-1} - \epsilon$, I have

$$D \leq \lim_{\epsilon \downarrow 0} D = V^k(\mu; Y).$$

So it is not a profitable deviation for the expert.

Consider case (1.3) that the expert deviates to self-use an amount $x \leq Y$. Because $\mu_{t+1} = \mu_t$ and the continuation play starting from period $t+1$ is a pair of a k-period scheme and a belief path $\boldsymbol{\mu}^k(\mu)$, in period t the expert's deviation payoff D satisfies

$$D = x + \delta V^k(\mu; Y-x) \leq V^k(\mu; Y),$$

which is because $V^k(\mu; Y) \geq Y$ for $\mu \in [\mu_k^*, \mu_{k-1}^*]$ and $k \leq \overline{k}+1$. So it is not a profitable deviation for the expert.

Case 2: $\mu \leq \mu_{\overline{k}+1}^*$.

On the equilibrium path the investor is inactive and the expert's payoff is Y.

Consider case (2.1) that the expert deviates to disclose an amount $y \leq Y$. If $y \leq Y/\mathbf{q}^{\overline{k}+1}$, after observing a success the belief is updated to $\mu^*_{\overline{k}+1}$. Because at this belief there is a continuation equilibrium with a $\overline{k}+1$-period scheme and there is another continuation equilibrium in which all $Y-y$ is self-used, by mixing them with probabilities $\lambda_{\overline{k}+1}$ and $1-\lambda_{\overline{k}+1}$ respectively, the biased investor is indifferent between betraying and cooperating in period t, so it is optimal for him to betray in period t with probability $\varphi(\mu;\mu^*_{\overline{k}+1})$. Given these responses, the expert's deviation payoff is

$$D = \frac{\mu}{\mu^*_{\overline{k}+1}}\alpha_E y + \delta(Y-y).$$

Because D is linear in y, the optimal deviation is $y=0$ or $y = Y/\mathbf{q}^{\overline{k}+1}$. If $y=0$, I have

$$D = \delta Y < Y,$$

so it is not a profitable deviation for the expert. If $y = Y/\mathbf{q}^{\overline{k}+1}$, I have

$$D - Y = \frac{\mu}{\mu^*_{\overline{k}+1}}\alpha_E \frac{Y}{\mathbf{q}^{\overline{k}+1}} - \frac{1+(1-\delta)(\mathbf{q}^{\overline{k}+1}-1)Y}{\mathbf{q}^{\overline{k}+1}} \leq 0$$

because $\mu \leq \mu^*_{\overline{k}+1}$ and $\alpha_E \leq 1+(1-\delta)(\mathbf{q}^{\overline{k}+1}-1)$. So it is not a profitable deviation for the expert.

If $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1}]$, where $1 \leq l \leq \overline{k}+1$, the proof is same to the one described in case (1.1), and I can verify that

$$D \leq V^{\overline{k}+1}(\mu;Y) \leq Y$$

for any deviation payoff D. So it is not profitable for the expert.

Consider case (2.2) that the expert deviates to self-use an amount $x < Y$. His deviation payoff satisfies

$$D = x + \delta(Y-x) < Y.$$

Therefore, it is not a profitable deviation for the expert. ■

The proof of Proposition 2.

Proof. The proof is done by induction. Suppose now the game is in period t with belief $\mu_t = \mu$ and information $Y_t = Y$. Let $\overline{W}(\mu; Y)$ be the expert's largest equilibrium payoff measured in period t.

Case 1: $\mu \geq \mu^*_{\bar{k}+1}$.

First consider $\mu \geq \mu^*_1$. In any equilibrium, the expert's payoff is no less than $V^1(\mu; Y) = \alpha_E \mu Y$, which is obtained by disclosing all information Y in period t.

Claim 0.1 There is no equilibrium in which the expert's payoff is strictly larger than $V^1(\mu; Y)$.

Suppose not. Then there exists a $\mu' \geq \mu^*_1$ such that there is an equilibrium in which the expert's largest equilibrium payoff satisfies $\overline{W}(\mu'; Y) > V^1(\mu'; Y)$. First, in this equilibrium the expert does not self-use an amount $x \leq Y$ in period t. Because if she does so, her largest equilibrium payoff should satisfy $\overline{W}(\mu'; Y) \leq x + \delta \overline{W}(\mu'; Y - x)$, which requires $\overline{W}(\mu'; Y) \leq Y < V^1(\mu'; Y)$ because of the necessary linearity of $\overline{W}(\mu'; Y)$ in Y. Second, in this equilibrium the expert does not disclose an amount $y > Y/(1+q)$ in period t. Because if she does so, the biased investor betrays for sure in this period and the expert's largest equilibrium payoff satisfies

$$\overline{W}(\mu'; Y) = \mu'[\alpha_E y + \delta \alpha_E (Y - y)] + (1 - \mu')\delta(Y - y) < V^1(\mu'; Y),$$

in which the inequality is shown in the proof of the previous proposition.

Therefore, if such an equilibrium exists, it is necessary that the expert discloses $y \leq Y/(1+q)$ in period t. Let $S_1 = \{\mu \geq \mu^*_1 | \overline{W}(\mu; Y) > V^1(\mu; Y)\}$ and consider $\mu' \in S_1$. Let $\mu'_{t+1} \geq \mu'$ be the belief in period $t+1$ after observing a success in period t, and let $W(\mu'_{t+1}; Y - y)$ be the expert's continuation payoff at belief μ'_{t+1} in this equilibrium, which is measured in period $t+1$. Then the payoff $\overline{W}(\mu'; Y)$ satisfies

$$\begin{aligned}\overline{W}(\mu'; Y) &= \frac{\mu'}{\mu'_{t+1}}(\alpha_E y + \delta W(\mu'_{t+1}; Y-y)) + (1 - \frac{\mu'}{\mu'_{t+1}})\delta(Y-y) \\ &\leq \frac{\mu'}{\mu'_{t+1}}(\alpha_E y + \delta \overline{W}(\mu'_{t+1}; Y-y)) + (1 - \frac{\mu'}{\mu'_{t+1}})\delta(Y-y)\end{aligned}$$

for any $y \leq Y/(1+q)$ and $\mu'_{t+1} \geq \mu' \geq \mu_1^*$. Because $\overline{W}(\mu'; Y)$ is necessarily linear in y, it reaches the maximum either at $y = 0$ or $y = Y/(1+q)$. If $y = 0$, I have $\overline{W}(\mu'; Y) \leq Y$, which contradicts $\overline{W}(\mu'; Y) > V^1(\mu'; Y)$. So it is only possible that $y = Y/(1+q)$. Because

$$V^1(\mu'; Y) \geq \frac{\mu'}{\mu'_{t+1}}(\alpha_E \frac{Y}{1+q} + \delta V^1(\mu'_{t+1}; \frac{qY}{1+q})) + (1 - \frac{\mu'}{\mu'_{t+1}})\delta \frac{qY}{1+q}$$

for any $\mu'_{t+1} \geq \mu' \geq \mu_1^*$, to have $\overline{W}(\mu'; Y) > V^1(\mu'; Y)$ it is necessary that

$$\overline{W}(\mu'_{t+1}; \frac{qY}{1+q}) \geq W(\mu'_{t+1}; \frac{qY}{1+q}) > V^1(\mu'_{t+1}; \frac{qY}{1+q}),$$

so $\mu'_{t+1} \in S_1$. Recursively, in any future period l with beliefs $\mu'_l \geq \mu'_{l-1}$, where $l > t$, the continuation payoff $W(\mu'_l; Y_l)$ measured in period l with remaining information $Y_l > 0$ should satisfy $W(\mu'_l; Y_l) > V^1(\mu'_l; Y_l)$ and $\mu'_l \in S_1$. However, because in each period the disclosed information is no more than $1/(1+q)$ of the total remaining information, it can be verified that

$$\overline{W}(\mu'; Y) \leq \frac{\alpha_E Y}{1+q} \sum_{h=0}^{+\infty} \frac{(\delta q)^h}{(1+q)^h} = V^1(\mu_1^*; Y) \leq V^1(\mu'; Y).^{20}$$

Therefore S_1 is empty and there is no equilibrium in which the expert's payoff is strictly larger than $V^1(\mu; Y)$ when $\mu \geq \mu_1^*$.

Claim 0.2 If and only if $\mu = \mu_1^*$ that there exists an equilibrium in which the expert does not disclose all Y in period t but her payoff is also $V^1(\mu; Y)$.

First, I have shown above that if the expert discloses $y \in (Y/(1+q), Y)$ or self-uses

[20] Term $\frac{\alpha_E Y}{1+q} \sum_{h=0}^{+\infty} \frac{(\delta q)^h}{(1+q)^h}$ is the payoff that the expert gets if (1) in each period the disclosed information is $1/(1+q)$ of the total remaining information, and (2) the biased investor cooperates for sure in each period.

$x \leq Y$ in period t, her payoff is strictly less than $V^1(\mu; Y)$. Second, if there exists such an equilibrium and the expert discloses $y \leq Y/(1+q)$ in period t, her payoff should have the form

$$\overline{W}(\mu; Y) \leq \frac{\mu}{\mu_{t+1}}(\alpha_E y + \delta V^1(\mu_{t+1}; Y - y)) + (1 - \frac{\mu}{\mu_{t+1}})\delta(Y - y) \leq V^1(\mu; Y)$$

for any $\mu \geq \mu_1^*$ and an updated belief μ_{t+1} in period $t+1$ after observing a success in period t. It can be verified that the equalities hold simultaneously if and only if $\mu_{t+1} = \mu = \mu_1^*$ and $y = Y/(1+q)$. Notice that for $\mu = \mu_1^*$, I have shown in Proposition 1 that there is an equilibrium in which the expert discloses $y = Y/(1+q)$ in the first period.

Claim 0.3 If $\mu < \mu_1^*$, there is no equilibrium in which the expert discloses all Y in period t, therefore the biased investor's equilibrium payoff is strictly less than $\beta_I Y$.

To see this, notice that if $\mu < \mu_1^*$ and the expert discloses $y = Y/(1+q) - \epsilon$ in period t, the biased investor's betrayal probability p should satisfy $p \leq \varphi(\mu; \mu_1^*)$. This is because that, if $p > \varphi(\mu; \mu_1^*)$, then after observing a success the belief μ_{t+1} in period $t+1$ satisfies $\mu_{t+1} > \mu_1^*$ and the expert will disclose all $Y - y$ at this belief μ_{t+1}. But then the biased investor strictly prefers to cooperate in period t because $\alpha_I y + \delta\beta_I(Y - y) > \beta_I y$, which contradicts $p > \varphi(\mu; \mu_1^*)$. Given $p \leq \varphi(\mu; \mu_1^*)$, by disclosing $y = Y/(1+q) - \epsilon$ in period t and disclosing $Y - y$ in period $t+1$ if a success is observed in period t, the expert can guarantee a payoff D satisfying

$$D \geq \frac{\mu}{\mu_1^*}(\alpha_E y + \delta V^1(\mu_1^*; Y - y)) + (1 - \frac{\mu}{\mu_1^*})\delta(Y - y) > V^1(\mu; Y)$$

when $\epsilon \to 0$. So if $\mu < \mu_1^*$, there is no equilibrium in which the expert discloses all Y in period t.

For $\mu \geq \mu_{\overline{k}+1}^*$ and $1 \leq k \leq \overline{k} + 1$, I introduce some properties.

Property 1 ($P1$) *If* $\mu \in (\mu_k^*, \mu_{k-1}^*)$, *there is a unique equilibrium, in which the expert's payoff is* $V^k(\mu; Y)$ *and the biased investor's payoff is* $\beta_I Y/\mathbf{q}^{k-1}$.

Property 2 ($P2$) *If* $\mu = \mu_k^*$*, there are multiple equilibria, in which the expert's payoff is* $V^k(\mu; Y)$ *and the biased investor's payoff ranges from* $\beta_I Y/\mathbf{q}^k$ *to* $\beta_I Y/\mathbf{q}^{k-1}$ *if* $1 \le k \le \overline{k}$ *and ranges from* 0 *to* $\beta_I Y/\mathbf{q}^{\overline{k}}$ *if* $k = \overline{k} + 1$*.*

Property 3 ($P3$) *If* $\mu < \mu_k^*$*, the biased investor's equilibrium payoff is strictly less than* $\beta_I Y/\mathbf{q}^{k-1}$*.*

I have seen that these properties are true for $\mu \ge \mu_1^*$. In particular, $P2$ holds for $\mu = \mu_1^*$ because of the mixing between the two equilibria, in one of which a 2-period scheme is employed and in the other a 1-period scheme is employed.

Suppose these properties hold for $\mu \in [\mu_k^*, \mu_{k-1}^*)$ and $1 \le k \le \overline{k}$. Now consider $\mu \in [\mu_{k+1}^*, \mu_k^*)$. Let y be the amount of information disclosed in period t and p be the biased investor's betrayal probability in this period.

Claim 1.1 In any equilibrium the expert's payoff is at least $V^{k+1}(\mu; Y)$.

Step 1.1.1 If $y < Y/\mathbf{q}^k$, then $p \le \varphi(\mu; \mu_k^*)$. Suppose not, then after observing a success the belief μ_{t+1} in period $t+1$ satisfies $\mu_{t+1} > \mu_k^*$. By $P1$, if the biased investor cooperates in period t his total payoff is at least $\alpha_I y + \delta\beta_I(Y - y)/\mathbf{q}^{k-1}$, which is strictly larger than the payoff $\beta_I y$ by betraying. So he should cooperate for sure in period t. A contradiction.

Step 1.1.2 There exists an $\eta_{k+1} > 0$ such that if $y \in (Y/\mathbf{q}^k - \eta_{k+1}, Y/\mathbf{q}^k)$ then $p = \varphi(\mu; \mu_k^*)$. Suppose not, so $p < \varphi(\mu; \mu_k^*)$ for any $y < Y/\mathbf{q}^k$, then after observing a success the belief μ_{t+1} in period $t+1$ satisfies $\mu_{t+1} < \mu_k^*$. If $y = Y/\mathbf{q}^k - \epsilon$ and $\epsilon \to 0$, the biased investor's payoff by betraying in period t converges $\beta_I Y/\mathbf{q}^k$, while by $P3$ his payoff by cooperating in period t is strictly less than $\alpha_I Y/\mathbf{q}^k + \delta\beta_I q Y/\mathbf{q}^k = \beta_I Y/\mathbf{q}^k$, so he should betray for sure in period t. A contradiction.

Step 1.1.3 The expert's equilibrium payoff is at least $V^{k+1}(\mu; Y)$. To see this, notice that if $y = Y/\mathbf{q}^k - \epsilon$, where $0 < \epsilon < \eta_{k+1}$, the expert's payoff is

$$\frac{\mu}{\mu_k^*}(\alpha_I y + \delta V^k(\mu_k^*; Y - y)) + (1 - \frac{\mu}{\mu_k^*})\delta(Y - y).$$

This payoff strictly increases in y and converges to $V^{k+1}(\mu;Y)$ when $\epsilon \to 0$. So in equilibrium the expert's payoff is at least $V^{k+1}(\mu;Y)$.

I introduce a new property here, which is stronger than $P3$.

Property $3'$ ($P3'$) *If $\mu < \mu_k^*$, the biased investor's equilibrium payoff is no larger than $\beta_I Y/\mathbf{q}^k$.*

With Claim 1.1 shown above, this property holds for $\mu < \mu_1^*$. This is because that, if $\mu < \mu_1^*$, then the disclosed information y in period t should satisfy $y \leq Y/(1+q)$ in any equilibrium. If the biased investor weakly prefers to betray with y then his equilibrium payoff is $\beta_I y \leq \beta_I Y/(1+q)$. If he strictly prefers to cooperate in period t then the continuation game starting from period $t+1$ is same to the one starting from period t, except the remaining information is $Y-y$ (or $Y-x$ if the expert self-uses x in period t). In any case it can be verified that $P3'$ holds.

Claim 1.2 There is no equilibrium in which the expert's payoff is strictly larger than $V^{k+1}(\mu;Y)$.

Step 1.2.1 Similar to the argument shown for the case $\mu \geq \mu_1^*$, if there is an equilibrium in which the expert self-uses an amount $x \leq Y$ or discloses an amount $y > Y/(1+q)$ in period t, the expert's payoff in such an equilibrium can not be strictly larger than $V^{k+1}(\mu;Y)$. Moreover, if and only if $\mu = \mu_{k+1}^*$ and $x = Y$ there is an equilibrium in which the expert's payoff equals to $V^{k+1}(\mu;Y)$.

Step 1.2.2 For $2 \leq l \leq k$, if $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1})$ then $p = \varphi(\mu;\mu_{l-1}^*)$. To see this, first notice that if $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1})$ then $p \geq \varphi(\mu;\mu_{l-1}^*)$. Suppose not, so $p < \varphi(\mu;\mu_{l-1}^*)$. Then by $P3'$ the biased investor's payoff is no more than $\alpha_I y + \delta\beta_I(Y-y)/\mathbf{q}^{l-1}$ by cooperating in period t, which is strictly less than $\beta_I y$ by betraying, therefore he should betray for sure in period t. A contradiction. Second, if $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1})$ then $p \leq \varphi(\mu;\mu_{l-1}^*)$. Suppose not, so $p > \varphi(\mu;\mu_{l-1}^*)$. Then by $P1$, the biased investor's payoff is no less than $\alpha_I y + \delta\beta_I(Y-y)/\mathbf{q}^{l-2}$ by cooperating in period t, which is strictly larger than $\beta_I y$ by betraying, therefore he should cooperate for sure in period t. A contradiction. Thus, I conclude that if $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1})$

then $p = \varphi(\mu; \mu^*_{l-1})$. Modify this argument slightly, I can show that if $y = Y/\mathbf{q}^{l-1}$ then $p \in [\varphi(\mu; \mu^*_{l-1}), \varphi(\mu; \mu^*_{l-2})]$.

Step 1.2.3 For $2 \leq l \leq k$, if $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1}]$ then there is no equilibrium in which the expert's payoff is strictly larger than $V^{k+1}(\mu; Y)$. Suppose not, then the expert's largest payoff in such an equilibrium satisfies

$$\overline{W}(\mu; Y) \leq \frac{\mu}{\mu^*_{l-1}}(\alpha_I y + \delta V^{l-1}(\mu^*_{l-1}; Y - y)) + (1 - \frac{\mu}{\mu^*_{l-1}})\delta(Y - y) \leq V^l(\mu; Y) < V^{k+1}(\mu; Y),$$

which contradicts the requirement that $\overline{W}(\mu; Y) > V^{k+1}(\mu; Y)$. A contradiction.

Step 1.2.4 If $y < Y/\mathbf{q}^k$, then there is no equilibrium in which the expert's payoff is strictly larger than $V^{k+1}(\mu; Y)$. In Claim 1.1 I have seen that it is true for $y < Y/\mathbf{q}^k$ and $p = \varphi(\mu; \mu^*_k)$. Now consider $y < Y/\mathbf{q}^k$ and $p < \varphi(\mu; \mu^*_k)$. Define $S_{k+1} = \{\mu \in [\mu^*_{k+1}, \mu^*_k) | \overline{W}(\mu; Y) > V^{k+1}(\mu; Y)\}$. Similar to the argument shown for the case $\mu \geq \mu^*_1$, I can show that the set S_{k+1} is empty. Moreover, I can show that $\overline{W}(\mu; Y) = V^{k+1}(\mu; Y)$ only if $\mu = \mu^*_{k+1}$ and $y = Y/\mathbf{q}^{k+1}$ for $2 \leq k \leq \overline{k}$.

Step 1.2.5 If $y = Y/\mathbf{q}^k$ then $p = \varphi(\mu; \mu^*_k)$. By $P3'$, I can show that if $y = Y/\mathbf{q}^k$ then $p \geq \varphi(\mu; \mu^*_k)$. However, if $p > \varphi(\mu; \mu^*_k)$, the expert's payoff is strictly less than $V^{k+1}(\mu; Y)$. So in equilibrium it is necessary that $p = \varphi(\mu; \mu^*_k)$ when $y = Y/\mathbf{q}^k$.

I also pin down what η_{k+1} is in Claim 1.1 by $P1$ and $P3'$. It can be verified that, if $y < Y/\mathbf{q}^{k+1}$ then $p = 0$, and if $y \in (Y/\mathbf{q}^{k+1}, Y/\mathbf{q}^k]$ then $p = \varphi(\mu; \mu^*_k)$. So $\eta_{k+1} = q^{k+1}Y/(\mathbf{q}^k \cdot \mathbf{q}^{k+1})$.

Thus, if $\mu \in (\mu^*_{k+1}, \mu^*_k)$, the unique equilibrium is consisted by a pair of a $k+1$-period scheme and a belief path $\boldsymbol{\mu}^{k+1}(\mu)$. In this equilibrium the expert's payoff is $V^{k+1}(\mu; Y)$ and the biased investor's payoff is $\beta_I Y/\mathbf{q}^k$. If $\mu = \mu^*_{k+1}$, by Step 1.2.1 and Step 1.2.4, there are multiple equilibria, in which the expert's payoff is $V^{k+1}(\mu; Y)$, while the biased investor's equilibrium payoff ranges from $\beta_I Y/\mathbf{q}^{k+1}$ to $\beta_I Y/\mathbf{q}^k$ if $0 \leq k \leq \overline{k} - 1$ and ranges from 0 to $\beta_I Y/\mathbf{q}^{\overline{k}}$ if $k = \overline{k}$. So properties $P1$ and $P2$ are true for $\mu \geq \mu^*_{k+1}$. Moreover, property $P3'$ has also been verified for $\mu \geq \mu^*_{\overline{k}+1}$. The only remaining part is to show that for $\mu < \mu^*_{\overline{k}+1}$

the biased investor's equilibrium payoff is no larger than $\beta_I Y/\mathbf{q}^{\overline{k}+1}$, which will be completed by the proof for the next case.

Case 2: $\mu < \mu^*_{\overline{k}+1}$.

By self-using all Y in period t, the expert can guarantee a payoff Y. I show that if the expert does not self-use all Y in period t, there is no equilibrium in which the expert's payoff is no less than Y. To see this, first I can show that, with an argument similar to the one shown for the case $\mu \geq \mu^*_1$, if the expert only self-uses an amount $x < Y$ or discloses an amount $y > Y/(1+q)$ in period t, her payoff is strictly less than Y. The remainder of the proof considers $y \leq Y/(1+q)$.

Claim 2.1 If the expert discloses $y > Y/\mathbf{q}^{\overline{k}+1}$, her payoff is strictly less than Y.

Step 2.1.1 For $2 \leq l \leq \overline{k}+1$, if $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1})$ then $p = \varphi(\mu; \mu^*_{l-1})$, and if $y = Y/\mathbf{q}^{l-1}$ then $p \in [\varphi(\mu; \mu^*_{l-1}), \varphi(\mu; \mu^*_{l-2})]$. The proof replicates the one shown in Step 1.2.2.

Step 2.1.2 For $2 \leq l \leq \overline{k}+1$, if $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1}]$ then there is no equilibrium in which the expert's payoff is no less than Y. Suppose not, then the expert's largest payoff in such an equilibrium satisfies

$$\begin{aligned}\overline{W}(\mu; Y) &\leq \frac{\mu}{\mu^*_{l-1}}(\alpha_I y + \delta V^{l-1}(\mu^*_{l-1}; Y-y)) + (1 - \frac{\mu}{\mu^*_{l-1}})\delta(Y-y) \\ &\leq V^l(\mu; Y) < V^{\overline{k}+1}(\mu; Y) < V^{\overline{k}+1}(\mu^*_{\overline{k}+1}; Y) = Y\end{aligned}$$

which contradicts the requirement that $\overline{W}(\mu; Y) \geq Y$. A contradiction.

Claim 2.2 If the expert discloses $y \leq Y/\mathbf{q}^{\overline{k}+1}$, her payoff is no larger than Y.

Suppose not. Define $S_{\overline{k}+1} = \{\mu < \mu^*_{\overline{k}+1} | \overline{W}(\mu; Y) > Y\}$. With an argument similar to the one shown for the case $\mu \geq \mu^*_1$, I can show that the set $S_{\overline{k}+1}$ is empty.

Claim 2.3 If the expert discloses $y \leq Y/\mathbf{q}^{\overline{k}+1}$, her payoff is strictly less than Y.

Suppose not. Then for some $\mu' < \mu^*_{\overline{k}+1}$ there is an equilibrium in which the expert's payoff satisfies $\overline{W}(\mu'; Y) = Y$ by disclosing an amount $y \leq Y/\mathbf{q}^{\overline{k}+1}$ in period t. Notice that

$\overline{W}(\mu';Y)$ takes the form

$$\overline{W}(\mu';Y) \leq \frac{\mu'}{\mu_{t+1}}(\alpha_E y + \delta\overline{W}(\mu_{t+1};Y-y)) + (1-\frac{\mu'}{\mu_{t+1}})\delta(Y-y),$$

in which μ_{t+1} is the belief in period $t+1$ after observing a success in period t. For the equality to hold, it is necessary that (a) $\mu' = \mu_{t+1}$, (b) $y = Y/\mathbf{q}^{\overline{k}+1}$ and (c) $\overline{W}(\mu_{t+1};Y-y) = Y-y$. If (c) is realized by self-using an amount $x \leq Y-y$ in period $t+1$, I have seen that it should be the case $x = Y-y$. However, in this case the biased investor should betray for sure in period t, so $\mu_{t+1} = 1 > \mu'$, which makes the equality infeasible. If (c) is realized by continuing to disclose, in a recursive way it should be the case that (b') the principal discloses $1/\mathbf{q}^{\overline{k}+1}$ of the remaining information in each period and (a') the biased investor cooperates for sure in each period. However, (a') contradicts with (b') because in period t the biased investor's payoff by betraying is $\beta_I Y/\mathbf{q}^{\overline{k}+1}$, which is strictly larger than the permanent cooperation payoff $\frac{\alpha_I Y_t}{\mathbf{q}^{\overline{k}+1}}\sum_{h=0}^{+\infty}\frac{\delta^h(\mathbf{q}^{\overline{k}+1}-1)^h}{(\mathbf{q}^{\overline{k}+1})^h}$.

Therefore, the equilibrium is unique when $\mu < \mu^*_{\overline{k}+1}$, in which the expert self-uses all Y in period t. Because in this case the investor's payoff is zero, property $P3'$ also holds for $\mu < \mu^*_{\overline{k}+1}$.

Finally I pin down the biased investor's response if the expert discloses $y \leq Y/\mathbf{q}^{\overline{k}+1}$. First, his betrayal probability p satisfies $p \geq \varphi(\mu;\mu^*_{\overline{k}+1})$. If not, the next period's belief after a success is strictly less than $\mu^*_{\overline{k}+1}$ and the expert would self-use all $Y-y$ for sure. But then the biased investor should betray for sure in period t. A contradiction. Second, if $y < Y/\mathbf{q}^{\overline{k}+1}$, his betrayal probability p satisfies $p \leq \varphi(\mu;\mu^*_{\overline{k}+1})$. If not, by $P1$ his payoff is at least $\alpha_I y + \delta\beta_I(Y-y)/\mathbf{q}^{\overline{k}}$ by cooperating in period t, which is strictly larger than $\beta_I y$ by betraying. So he should cooperate for sure. A contradiction. Third, if $y = Y/\mathbf{q}^{\overline{k}+1}$, by $P1$ and $P3'$, it can be verified that $p \in [\varphi(\mu;\mu^*_{\overline{k}+1}), \varphi(\mu;\mu^*_{\overline{k}})]$. ■

The proof of Proposition 4.

Proof. Consider the expert's problem:

$$\max_{k\in\mathbb{N}} \overline{V}^k(\mu_0; Y_0) \qquad s.t. \qquad \overline{V}^k(\mu_0; Y_0) \geq Y_0.$$

Let $k^*(\mu_0)$ be the solution to this problem.

Notice that, for $k \geq 2$, the value function $\overline{V}^k(\mu; Y)$ also has the expression

$$\overline{V}^k(\mu; Y) = \frac{\alpha_E Y}{\mathbf{q}^{k-1}} \textstyle\sum_{l=0}^{k-2} (\delta q)^l + \dfrac{(\delta q)^{k-1}}{\mathbf{q}^{k-1}} \alpha_E \mu Y.$$

For a particular k, if there is a $\widetilde{\mu}$ satisfying $\overline{V}^{k+1}(\widetilde{\mu}; Y) = \overline{V}^k(\widetilde{\mu}; Y)$, I can verify that $\overline{V}^k(\mu; Y) > \overline{V}^{k+1}(\mu; Y)$ for $\mu > \widetilde{\mu}$ and $\overline{V}^k(\mu; Y) < \overline{V}^{k+1}(\mu; Y)$ for $\mu < \widetilde{\mu}$. Moreover, I can explicitly solve that $\overline{V}^{k+1}(\widetilde{\mu}; Y) = \overline{V}^k(\widetilde{\mu}; Y) = \frac{\alpha_E Y}{1+(1-\delta)(\mathbf{q}^k - 1)}$.

Let $\widetilde{k}$ satisfy

$$\overline{V}^{\widetilde{k}}(0; Y) < \overline{V}^{\widetilde{k}+1}(0; Y) \quad \text{and} \quad \overline{V}^{\widetilde{k}+1}(0; Y) \geq \overline{V}^{\widetilde{k}+2}(0; Y).$$

It can be verified that $\widetilde{k} \geq 1$ and for any $1 \leq j \leq \widetilde{k}$, I have $\overline{V}^j(0; Y) < \overline{V}^{j+1}(0; Y)$. The implication of $\widetilde{k}$ is that, for any $k \geq \widetilde{k} + 1$, if there is a $\widetilde{\mu}$ satisfying $\overline{V}^{k+1}(\widetilde{\mu}; Y) = \overline{V}^k(\widetilde{\mu}; Y)$, then $\widetilde{\mu} \leq 0$. Thus, for any $\mu_0 \in (0, 1)$, $k^*(\mu_0) \leq \widetilde{k} + 1$ is necessary for the expert's optimal commitment.

Similar to the proof shown for Lemma 1, it can be verified that, if $\widetilde{k} < \overline{k}$, there is a unique sequence $(\widetilde{\mu}_0, \widetilde{\mu}_1, ..., \widetilde{\mu}_k, ..., \widetilde{\mu}_{\widetilde{k}+1})$ satisfying (1) $\widetilde{\mu}_{k-1} > \widetilde{\mu}_k$ for $1 \leq k \leq \widetilde{k} + 1$, (2) $\widetilde{\mu}_0 = 1$, $\widetilde{\mu}_1 = \mu_1^*$, $\widetilde{\mu}_{\widetilde{k}+1} = 0$, (3) $\overline{V}^{k+1}(\widetilde{\mu}_k; Y) = \overline{V}^k(\widetilde{\mu}_k; Y) = \frac{\alpha_E Y}{1+(1-\delta)(\mathbf{q}^k - 1)} > Y$ for $1 \leq k \leq \widetilde{k}$ and $\overline{V}^{\widetilde{k}+1}(0; Y) \geq Y$. Thus, for any $\mu_0 \in [\widetilde{\mu}_k, \widetilde{\mu}_{k-1}]$ and $1 \leq k \leq \widetilde{k} + 1$, $k^*(\mu_0) = k$ and the expert's optimal commitment payoff is $\overline{V}^k(\mu_0; Y_0)$. Moreover, for $2 \leq k \leq \widetilde{k} + 1$ I can show that $\widetilde{\mu}_k < \mu_k^*$ by showing $\overline{V}^k(\mu_k^*; Y) > V^k(\mu_k^*; Y)$ recursively. Therefore, the process of information disclosure with an optimal commitment is (weakly) faster than the process of information disclosure without a commitment.

On the other hand, if $\widetilde{k} \geq \overline{k}$, there is a unique sequence $(\widetilde{\mu}_0, \widetilde{\mu}_1, ..., \widetilde{\mu}_k, ..., \widetilde{\mu}_{\overline{k}+1})$ satisfying (1) $\widetilde{\mu}_{k-1} > \widetilde{\mu}_k$ for $1 \leq k \leq \overline{k}+1$, (2) $\widetilde{\mu}_0 = 1$, $\widetilde{\mu}_1 = \mu_1^*$, $\widetilde{\mu}_{\overline{k}+1} \geq 0$, (3) $\overline{V}^{k+1}(\widetilde{\mu}_k; Y) = \overline{V}^k(\widetilde{\mu}_k; Y) = \frac{\alpha_E Y}{1+(1-\delta)(\mathbf{q}^k-1)} > Y$ for $1 \leq k \leq \overline{k}$ and $\overline{V}^{\overline{k}+1}(\widetilde{\mu}_{\overline{k}+1}; Y) = Y$. Thus, for any $\mu_0 \in [\widetilde{\mu}_k, \widetilde{\mu}_{k-1}]$ and $1 \leq k \leq \overline{k}+1$, $k^*(\mu_0) = k$ and the expert's optimal commitment payoff is $\overline{V}^k(\mu_0; Y_0)$. Similarly, I have $\widetilde{\mu}_k < \mu_k^*$ for $2 \leq k \leq \overline{k}+1$.

No matter $\widetilde{k} < \overline{k}$ or $\widetilde{k} \geq \overline{k}$, for any $\mu_0 \in [\mu_k^*, \mu_{k-1}^*)$ and $2 \leq k \leq \overline{k}+1$, it can be shown recursively that $\overline{V}^k(\mu_0; Y_0) > V^k(\mu_0; Y_0)$. Combined with $\overline{V}^{k^*(\mu_0)}(\mu_0; Y_0) \geq \overline{V}^k(\mu_0; Y_0)$, I have $\overline{V}^{k^*(\mu_0)}(\mu_0; Y_0) > V^k(\mu_0; Y_0)$, which proves the statement in the proposition.

The main difference between the case $\widetilde{k} < \overline{k}$ and the case $\widetilde{k} \geq \overline{k}$ is that, in the first case the expert can strictly benefit from an optimal commitment for any initial belief $\mu_0 \in (0, \mu_1^*)$, whereas in the second case the expert can strictly benefit from an optimal commitment if and only if $\mu_0 \in (\widetilde{\mu}_{\overline{k}+1}, \mu_1^*)$, where $\widetilde{\mu}_{\overline{k}+1}$ may be strictly larger than 0. ■

Appendix B. **The proof of Proposition 3.**

The proof is done by the following lemmas.

Lemma B1 *There exists a sequence of values* $(\underline{\mu}_1, ..., \underline{\mu}_k, ..., \underline{\mu}_{\overline{k}+1})$ *satisfying*

(a) $H^k(\underline{\mu}_k; Y) = Y$ *if* $1 \leq k \leq \overline{k}+1$, *in which* $H^k(\mu; Y)$ *is recursively defined by*

$$H^1(\mu; Y) = \alpha_E \mu Y, \quad \text{and for } 2 \leq k \leq \overline{k}+1,$$

$$\begin{aligned} H^k(\mu; Y) &= \frac{\min\{\mu, \underline{\mu}_{k-1}\}}{\underline{\mu}_{k-1}}\left(\frac{\alpha_E Y}{\mathbf{q}^{k-1}} + \delta H^{k-1}(\max\{\mu, \underline{\mu}_{k-1}\}; Y - \frac{Y}{\mathbf{q}^{k-1}})\right) \\ &\quad + \left(1 - \frac{\min\{\mu, \underline{\mu}_{k-1}\}}{\underline{\mu}_{k-1}}\right)\delta\left(Y - \frac{Y}{\mathbf{q}^{k-1}}\right). \end{aligned}$$

(b) $1 > \underline{\mu}_1 > ... > \underline{\mu}_k > ... > \underline{\mu}_{\overline{k}+1} > 0$.

Proof. Consider property (a). By the definition of $H^k(\mu; Y)$, I have $\underline{\mu}_1 = 1/\alpha_E$. Recursively, for $2 \leq k \leq \overline{k}+1$ and $H^k(\underline{\mu}_k; Y) = Y$, I can show that

$$\underline{\mu}_k = \frac{1}{(\alpha_E)^k} \prod_{j=0}^{k-1}(1 + (1-\delta)(\mathbf{q}^j - 1)).$$

Because $\alpha_E > 1 + (1-\delta)(\mathbf{q}^{\overline{k}} - 1)$, property (b) holds. ■

Remark B1 *In the equilibrium I will construct, the cut-off values* $\underline{\mu}_k$ *and the value functions* $H^k(\mu; Y)$ *work as follows. If the number of remaining periods feasible for information disclosure is* k, *where* $1 \leq k \leq \overline{k}+1$, *the cut-off value* $\underline{\mu}_k$ *determined by* $H^k(\underline{\mu}_k; Y) = Y$ *is the threshold between information disclosure and self-use of information in the current period; that is, information disclosure occurs if and only if the current belief* μ *satisfies* $\mu \geq \underline{\mu}_k$.

Lemma B2 *For* $1 \leq k \leq \overline{k}+1$, *the following properties hold:*

(a) $H^k(\mu;Y)$ *strictly increases in* μ,

(b) $\underline{\mu}_k < \mu^*_k$,

(c) for $\mu \in [\mu^*_k, \mu^*_{k-1})$, $H^k(\mu;Y) = V^k(\mu;Y)$ *if* $k = 1$ *and* $H^k(\mu;Y) > V^k(\mu;Y)$ *if* $k \geq 2$.

Proof. Consider property (a). First notice that $H^1(\mu;Y)$ strictly increases in μ. Now suppose that, for $1 \leq k-1 \leq \overline{k}$, $H^{k-1}(\mu;Y)$ also strictly increases in μ. Consider $H^k(\mu;Y)$. I have

$$H^k(\mu;Y) = \frac{\mu}{\underline{\mu}_{k-1}} \frac{\alpha_E Y}{\mathbf{q}^{k-1}} + \delta(Y - \frac{Y}{\mathbf{q}^{k-1}})$$

if $\mu \leq \underline{\mu}_{k-1}$, and

$$H^k(\mu;Y) = \frac{\alpha_E Y}{\mathbf{q}^{k-1}} + \delta H^{k-1}(\mu; Y - \frac{Y}{\mathbf{q}^{k-1}})$$

if $\mu > \underline{\mu}_{k-1}$. In either case, $H^k(\mu;Y)$ strictly increases in μ.

Consider properties (b) and (c). First notice that $\underline{\mu}_1 < \mu^*_1$ and $H^1(\mu;Y) = V^1(\mu;Y)$ hold. Suppose that both (b) and (c) hold for $\mu \in [\mu^*_k, \mu^*_{k-1})$ and $1 \leq k \leq \overline{k}$. Now consider $\mu \in [\mu^*_{k+1}, \mu^*_k)$. Here I introduce a value function as

$$\Delta^{k+1}(\mu;Y) = \frac{\mu}{\mu^*_k}(\alpha_E \frac{Y}{\mathbf{q}^k} + \delta H^k(\mu^*_k; Y - \frac{Y}{\mathbf{q}^k})) + (1 - \frac{\mu}{\mu^*_k})\delta(Y - \frac{Y}{\mathbf{q}^k}).$$

By the assumption of induction, I have $\Delta^{k+1}(\mu;Y) \geq V^{k+1}(\mu;Y)$. If $\mu \leq \underline{\mu}_k$ and $\mu^*_k \leq \underline{\mu}_{k-1}$ over here, by extending $H^{k+1}(\mu;Y)$ and $\Delta^{k+1}(\mu;Y)$ as

$$\begin{aligned} H^{k+1}(\mu;Y) \quad = \quad & \frac{\mu}{\underline{\mu}_k}\delta[\frac{\underline{\mu}_k}{\underline{\mu}_{k-1}}(\alpha_E \frac{qY}{\mathbf{q}^k} + \delta H^{k-1}(\underline{\mu}_{k-1}; Y - \frac{(1+q)Y}{\mathbf{q}^k})) \\ & + (1 - \frac{\underline{\mu}_k}{\underline{\mu}_{k-1}})\delta(Y - \frac{(1+q)Y}{\mathbf{q}^k})] \\ & + \frac{\mu}{\underline{\mu}_k}\alpha_E \frac{Y}{\mathbf{q}^k} + (1 - \frac{\mu}{\underline{\mu}_k})\delta(Y - \frac{Y}{\mathbf{q}^k}), \end{aligned}$$

$$\begin{aligned}\Delta^{k+1}(\mu;Y) &= \frac{\mu}{\mu_k^*}\delta[\frac{\mu_k^*}{\underline{\mu}_{k-1}}(\alpha_E\frac{qY}{\mathbf{q}^k}+\delta H^{k-1}(\underline{\mu}_{k-1};Y-\frac{(1+q)Y}{\mathbf{q}^k})) \\ &\quad +(1-\frac{\mu_k^*}{\underline{\mu}_{k-1}})\delta(Y-\frac{(1+q)Y}{\mathbf{q}^k})] \\ &\quad +\frac{\mu}{\mu_k^*}\alpha_E\frac{Y}{\mathbf{q}^k}+(1-\frac{\mu}{\mu_k^*})\delta(Y-\frac{Y}{\mathbf{q}^k}),\end{aligned}$$

I have

$$H^{k+1}(\mu;Y)-\Delta^{k+1}(\mu;Y)=(\frac{\mu}{\underline{\mu}_k}-\frac{\mu}{\mu_k^*})(\alpha_E\frac{Y}{\mathbf{q}^k}-\delta(Y-\frac{Y}{\mathbf{q}^k})+\delta^2(Y-\frac{(1+q)Y}{\mathbf{q}^k}))>0$$

because $\underline{\mu}_k < \mu_k^*$ and $\alpha_E > 1+(1-\delta)(\mathbf{q}^{\overline{k}}-1)$. If $\mu > \underline{\mu}_k$ or $\mu_k^* > \underline{\mu}_{k-1}$, by modifying the above argument slightly, I can also show that $H^{k+1}(\mu;Y)-\Delta^{k+1}(\mu;Y) > 0$. Therefore, I have

$$H^{k+1}(\mu;Y) > \Delta^{k+1}(\mu;Y) \geq V^{k+1}(\mu;Y) \geq Y$$

for $\mu \in [\mu_{k+1}^*, \mu_k^*)$ and $1 \leq k \leq \overline{k}$. Because $H^{k+1}(\mu;Y)$ strictly increases in μ, I have $\underline{\mu}_{k+1} < \mu_{k+1}^*$. This completes the proof of (b) and (c). ■

Remark B2 *In the equilibrium I will construct, given a deadline $T=k-1$, the expert's equilibrium payoff is no less than $H^k(\mu_0;Y_0)$ if $\mu_0 \geq \underline{\mu}_k$. By properties (b) and (c) shown in this lemma, such an equilibrium payoff is strictly larger than $V^k(\mu_0;Y_0)$ if $\mu_0 \in [\mu_k^*, \mu_{k-1}^*)$ and $2 \leq k \leq \overline{k}+1$.*

Lemma B3 *For $2 \leq k \leq \overline{k}+1$, there is a number $\bar{l}(k)$, which is a function of k, and a sequence $(\mu_{k,1}^*, ..., \mu_{k,\bar{l}(k)-1}^*, \mu_{k,\bar{l}(k)}^*)$ satisfying*

(a) for $3 \leq k \leq \overline{k}+1$ and $2 \leq l \leq \bar{l}(k)$, $\mu_{k,1}^* = \mu_1^*$, $\underline{\mu}_k < \mu_{k,\bar{l}(k)}^*$, $\mu_{k,l}^* < \mu_{k,l-1}^*$ and $\mu_{k,l}^* < \mu_{k-1,l-1}^*$.

(b) $U^{k,l}(\mu;Y) \gtreqless U^{k,l+1}(\mu;Y)$ if $\mu \gtreqless \mu_{k,l}^*$ and $1 \leq l < \bar{l}(k)$, and $U^{k,l}(\mu;Y) \gtreqless H^k(\mu;Y)$ if

$\mu \gtreqless \mu^*_{k,l}$ *and* $l = \bar{l}(k)$*, in which* $U^{k,l}(\mu;Y)$ *is recursively defined by*

$$U^{k,1}(\mu;Y) = \mu\alpha_E Y \quad \textit{and for } 2 \le l \le \bar{l}(k),$$

$$\begin{aligned} U^{k,l}(\mu;Y) \;&=\; \frac{\min\{\mu,\mu^*_{k-1,l-1}\}}{\mu^*_{k-1,l-1}}(\alpha_E\frac{Y}{\mathbf{q}^{l-1}} + \delta U^{k-1,l-1}(\max\{\mu,\mu^*_{k-1,l-1}\};Y-\frac{Y}{\mathbf{q}^{l-1}})) \\ &+(1-\frac{\min\{\mu,\mu^*_{k-1,l-1}\}}{\mu^*_{k-1,l-1}})\delta(Y-\frac{Y}{\mathbf{q}^{l-1}}). \end{aligned}$$

The interpretation of value function $U^{k,l}(\mu;Y)$ is as follows. The superscript "k" indicates the number of periods feasible for information disclosure, whereas the superscript "l" indicates that an l-period scheme of information disclosure is employed.

Proof. For $2 \le k \le \bar{k}+1$, notice that $U^{k,1}(\mu;Y) = U^{k,2}(\mu;Y)$ holds if $\mu = \mu^*_1$. Therefore, denote $\mu^*_{k,1} = \mu^*_1$. Also, denote $\mu^*_{k,0} = 1$.

If $k = 2$, $U^{2,1}(\mu^*_{2,1};Y) = H^2(\mu^*_{2,1};Y)$ holds. I define the sequence by a single element sequence $(\mu^*_{2,1})$. It can be verified that all the other properties in (a) and (b) hold.

Consider $k = 3$. Notice that $U^{3,2}(\mu;Y) = V^2(\mu;Y)$, so $U^{3,2}(\mu;Y)$ strictly increases in μ. I also have $U^{3,2}(\mu;Y) > H^3(\mu;Y)$ when $\mu \ge \mu^*_{2,1}$, and $U^{3,2}(0;Y) < H^3(0;Y)$. Because $U^{3,2}(\mu;Y)$ always has a larger slope, there is a unique $\mu^*_{3,2}$ satisfying $U^{3,2}(\mu;Y) = H^3(\mu;Y)$ and $0 < \mu^*_{3,2} < \mu^*_{2,1}$. Define the sequence as $(\mu^*_{3,1},\mu^*_{3,2})$. It can be verified that all the other properties in (a) and (b) hold. Moreover, because $H^3(\mu^*_2;Y) > V^2(\mu^*_2;Y)$, I have $\mu^*_{3,2} > \mu^*_2$.

Suppose that for $2 \le k \le \bar{k}$ a sequence $(\mu^*_{k,1},...,\mu^*_{k,\bar{l}(k)-1},\mu^*_{k,\bar{l}(k)})$ satisfying (a) and (b) has been defined. Now I define the sequence for $k+1$. First, for any $2 \le j \le \bar{l}(k)$, there is a unique $\mu_{k+1,j}$ satisfying $U^{k+1,j}(\mu;Y) = U^{k+1,j+1}(\mu;Y)$ and $0 < \mu_{k+1,j} < \mu^*_{k,j-1}$. To see this, notice that if $\mu \ge \mu^*_{k,j-1}$ I have

$$U^{k+1,j}(\mu;Y) = \alpha_E\frac{Y}{\mathbf{q}^{j-1}} + \delta U^{k,j-1}(\mu;Y-\frac{Y}{\mathbf{q}^{j-1}})$$

and

$$U^{k+1,j+1}(\mu;Y) = \alpha_E \frac{Y}{\mathbf{q}^j} + \delta U^{k,j}(\mu; Y - \frac{Y}{\mathbf{q}^j}).$$

By the assumption of induction, $U^{k,j-1}(\mu;Y) \geq U^{k,j}(\mu;Y)$ if $\mu \geq \mu^*_{k,j-1}$, so $U^{k+1,j}(\mu;Y) > U^{k+1,j+1}(\mu;Y)$ if $\mu \geq \mu^*_{k,j-1}$. On the other hand, I have $U^{k+1,j}(0;Y) < U^{k+1,j+1}(0;Y)$. Because both $U^{k+1,j}(\mu;Y)$ and $U^{k+1,j+1}(\mu;Y)$ strictly increase in μ and the former has a larger slope in μ, there is a unique $\mu_{k+1,j}$ satisfying the conditions.

Second, there is a unique μ' satisfying $U^{k+1,\bar{l}(k)}(\mu;Y) = H^{k+1}(\mu;Y)$ and $0 < \mu' < \mu^*_{k,\bar{l}(k)-1}$. To see this, notice that if $\mu \geq \mu^*_{k,\bar{l}(k)-1} > \underline{\mu}_k$, I have

$$H^{k+1}(\mu;Y) = \alpha_E \frac{Y}{\mathbf{q}^k} + \delta H^k(\mu; Y - \frac{Y}{\mathbf{q}^k}).$$

Because $U^{k,\bar{l}(k)-1}(\mu;Y) \geq U^{k,\bar{l}(k)}(\mu;Y) > H^k(\mu;Y)$ if $\mu \geq \mu^*_{k,\bar{l}(k)-1}$, I have $U^{k+1,\bar{l}(k)}(\mu;Y) > H^{k+1}(\mu;Y)$ if $\mu \geq \mu^*_{k,\bar{l}(k)-1}$. On the other hand, I have $U^{k+1,\bar{l}(k)}(0;Y) < H^{k+1}(0;Y)$. Because both $U^{k+1,\bar{l}(k)}(\mu;Y)$ and $H^{k+1}(\mu;Y)$ strictly increase in μ and the former has a larger slope in μ, there is a unique μ' satisfying the conditions.

Third, there is a unique μ'' satisfying $U^{k+1,\bar{l}(k)+1}(\mu;Y) = H^{k+1}(\mu;Y)$ and $0 < \mu' < \mu^*_{k,\bar{l}(k)}$. To see this, notice that if $\mu \geq \mu^*_{k,\bar{l}(k)} > \underline{\mu}_k$, I have $U^{k+1,\bar{l}(k)+1}(\mu;Y) > H^{k+1}(\mu;Y)$ because $U^{k,\bar{l}(k)}(\mu;Y) \geq H^k(\mu;Y)$. On the other hand, I have $U^{k+1,\bar{l}(k)+1}(0;Y) < H^{k+1}(0;Y)$. Because both $U^{k+1,\bar{l}(k)+1}(\mu;Y)$ and $H^{k+1}(\mu;Y)$ strictly increase in μ and the former has a larger slope in μ, there is a unique μ'' satisfying the conditions.

Finally, if $\mu'' < \mu_{k+1,\bar{l}(k)}$, define $\bar{l}(k+1) = \bar{l}(k) + 1$ and $\mu^*_{k+1,\bar{l}(k+1)} = \mu''$, and define $\mu^*_{k+1,j} = \mu_{k+1,j}$ for $1 \leq j \leq \bar{l}(k)$. If $\mu'' \geq \mu_{k+1,\bar{l}(k)}$, define $\bar{l}(k+1) = \bar{l}(k)$ and $\mu^*_{k+1,\bar{l}(k+1)} = \mu'$, and define $\mu^*_{k+1,j} = \mu_{k+1,j}$ for $1 \leq j < \bar{l}(k)$. With this construction of the sequence, It can be verified that all the other properties in (a) and (b) hold. ■

Remark B3 *If the deadline T satisfies $T = k - 1 > 0$, an important feature is that, if the expert's initial belief is relatively large, she prefers to disclose all information before the deadline is reached. For instance, if $\mu_0 \geq \mu^*_1$, the expert discloses all Y_0 in period*

$t = 0$ no matter what the deadline is. Therefore, I need to characterize the cut-off values and value functions that describe the expert's utilization of information for any initial belief. The systems derived in Lemma B1 and Lemma B3 provide a full description of the expert's behaviors, which will be shown in the equilibrium. Also notice that the value $\overline{\mu}_k$ I introduced in the main analysis equals $\mu^*_{k,\bar{l}(k)}$ here.

Lemma B4 *For $2 \leq k \leq \overline{k}+1$, there exists an equilibrium in which the expert's payoff is $U^{k,l}(\mu;Y)$ if $\mu \in [\mu^*_{k,l}, \mu^*_{k,l-1}]$ and $1 \leq l \leq \bar{l}(k)$, is $H^k(\mu;Y)$ if $\mu \in [\underline{\mu}_k, \mu^*_{k,\bar{l}(k)}]$, and is Y if $\mu \leq \underline{\mu}_k$.*

Proof. Suppose now the game is in period t with belief $\mu_t = \mu$ and information $Y_t = Y$, and the number of remaining periods for information disclosure is k, where $2 \leq k \leq \overline{k}+1$. Let y be the amount of information disclosed in period t, and let x be the amount of information self-used in period t. Consider the strategy profile and belief updating system described as follows.

Case 1: $\mu \in [\mu^*_{k,l}, \mu^*_{k,l-1}]$ and $1 \leq l \leq \bar{l}(k)$.

On the equilibrium path. Starting from period t, the expert's information disclosure follows an l-period scheme. In the first period of the l-period scheme, the biased investor betrays with probability $\varphi(\mu; \max\{\mu, \mu^*_{k-1,l-1}\})$. After observing a success in period t, the expert's belief updates to $\max\{\mu, \mu^*_{k-1,l-1}\}$ in period $t+1$. In this case, the expert's payoff is $U^{k,l}(\mu;Y)$.

Off the equilibrium path. For any deviation by the biased investor, after observing a success the expert continues to update her belief to $\max\{\mu, \mu^*_{k-1,l-1}\}$ in period $t+1$. The remainder considers the expert's deviations.

First, consider $y > Y/\mathbf{q}^{l-1}$. If $y \in (Y/\mathbf{q}^{j-1}, Y/\mathbf{q}^{j-2}]$ and $2 \leq j \leq l$, the biased investor betrays with probability $\varphi(\mu; \mu^*_{k-1,j-2})$.[21] If a success is observed in period t, (a) if $j = 2$, the expert discloses $Y - y$ in the next period, and (b) if $j > 2$, starting from the next period

[21] Here I have $\mu^*_{k-1,j-2} > \mu^*_{k,l-1}$ because $\mu^*_{k-1,l-2} > \mu^*_{k,l-1}$.

the Case 1-equilibrium with a $j-2$-period scheme is played with probability λ_j and the Case 1-equilibrium with a $j-1$-period scheme is played with probability $1-\lambda_j$, in which λ_j satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda_j \beta_I \frac{Y-y}{\mathbf{q}^{j-3}} + (1-\lambda_j)\beta_I \frac{Y_t - y}{\mathbf{q}^{j-2}}].$$

That is, λ_j makes the biased investor being indifferent between cooperating and betraying in period t.[22]

Second, consider $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1})$. (a) If $\mu \geq \mu^*_{k-1,l-1}$, the biased investor cooperates for sure. Starting from period $t+1$ the Case 1-equilibrium with an $l-1$-period scheme is played. (b) If $\mu < \mu^*_{k-1,l-1}$, the biased investor betrays with probability $\varphi(\mu; \mu^*_{k-1,l-1})$. After observing a success in period t, if $\bar{l}(k-1) > l-1$, starting from the next period the Case 1-equilibrium with an $l-1$-period scheme is played with probability λ_l and the Case 1-equilibrium with an l-period scheme is played with probability $1-\lambda_l$, in which λ_l satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda_l \beta_I \frac{Y-y}{\mathbf{q}^{l-2}} + (1-\lambda_l)\beta_I \frac{Y-y}{\mathbf{q}^{l-1}}];$$

if $\bar{l}(k-1) = l-1$, starting from the next period the Case 1-equilibrium with an $l-1$-period scheme is played with probability λ'_l and the Case 2-equilibrium with a $k-1$-period scheme is played with probability $1-\lambda'_l$, in which λ'_l satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda'_l \beta_I \frac{Y-y}{\mathbf{q}^{l-2}} + (1-\lambda'_l)\beta_I \frac{Y-y}{\mathbf{q}^{k-2}}].$$

Third, consider $y \leq Y/\mathbf{q}^l$. (a) If $\mu \geq \mu^*_{k-1,l-1}$, the biased investor cooperates for sure. Starting from period $t+1$ the Case 1-equilibrium with an $l-1$-period scheme is played. (b) If $\bar{l}(k-1) = l$ and $\mu \in [\mu^*_{k-1,l}, \mu^*_{k-1,l-1}]$, the biased investor cooperates for sure. Starting from period $t+1$ the Case 1-equilibrium with an l-period scheme is played. (c) Consider

[22] By "Case 1-equilibrium", I mean the equilibrium described in Case 1. Similarly, "Case 2-equilibrium" refers to the equilibrium described in Case 2. The difference between these two classes of equilibria is that, in the equilibrium described in Case 1 the deadline is never reached, whereas in the equilibrium described in Case 2 the deadline is reached based on a series of successes.

$\mu \in [\mu^*_{k-1,j+1}, \mu^*_{k-1,j}]$ such that $l \leq j \leq \bar{l}(k-1) - 1$. If $y \leq Y/\mathbf{q}^{j+1}$, the biased investor cooperates for sure and starting from period $t+1$ the Case 1-equilibrium with a $j+1$-period scheme is played. If $y \in (Y/\mathbf{q}^{j+1}, Y/\mathbf{q}^{j}]$, the biased investor betrays with probability $\varphi(\mu; \mu^*_{k-1,j})$. After observing a success in period t, starting from the next period the Case 1-equilibrium with a j-period scheme is played with probability λ_j and the Case 1-equilibrium with a $j+1$-period scheme is played with probability $1-\lambda_j$, in which λ_j satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda_j \beta_I \frac{Y-y}{\mathbf{q}^{j-1}} + (1-\lambda_j)\beta_I \frac{Y-y}{\mathbf{q}^{j}}].$$

(*d*) Consider $\mu \in [\underline{\mu}_{k-1}, \mu^*_{k-1,\bar{l}(k-1)}]$. If $y \leq Y/\mathbf{q}^k$, the biased investor cooperates for sure. Starting from period $t+1$ the Case 2-equilibrium with a $k-1$-period scheme is played. If $y \in (Y/\mathbf{q}^k, Y/\mathbf{q}^{\bar{l}(k-1)}]$, the biased investor betrays with probability $\varphi(\mu; \mu^*_{k-1,\bar{l}(k-1)})$. After observing a success in period t, starting from the next period the Case 1-equilibrium with an $\bar{l}(k-1)$-period scheme is played with probability λ_k and the Case 2-equilibrium with a $k-1$-period scheme is played with probability $1-\lambda_k$, in which λ_k satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda_k \beta_I \frac{Y-y}{\mathbf{q}^{\bar{l}(k-1)-1}} + (1-\lambda_k)\beta_I \frac{Y-y}{\mathbf{q}^{k-2}}].$$

Finally, if the expert self-uses x instead of disclosing y, starting from period $t+1$ the Case 1-equilibrium is played if $\mu \geq \mu^*_{k-1,\bar{l}(k-1)}$, the Case 2-equilibrium is played if $\mu \in [\underline{\mu}_{k-1}, \mu^*_{k-1,\bar{l}(k-1)})$, and $Y-y$ is self-used in period $t+1$ if $\mu < \underline{\mu}_{k-1}$.

Case 2: $\mu \in [\underline{\mu}_k, \mu^*_{k,\bar{l}(k)}]$.

On the equilibrium path. Starting from period t, the expert's information disclosure follows a k-period scheme. In the first period of the k-period scheme, the biased investor betrays with probability $\varphi(\mu; \max\{\mu, \underline{\mu}_{k-1}\})$. After observing a success in period t, the expert's belief updates to $\max\{\mu, \underline{\mu}_{k-1}\}$ in period $t+1$. In this case, the expert's payoff is $H^k(\mu; Y)$.

Off the equilibrium path. For any deviation by the biased investor, after observing a success the expert continues to update her belief to $\max\{\mu, \underline{\mu}_{k-1}\}$ in period $t+1$. The remainder considers the expert's deviations.

First, consider $y > Y/\mathbf{q}^{k-1}$. (a) If $y \in (Y/\mathbf{q}^{k-1}, Y/\mathbf{q}^{\bar{l}(k-1)}]$, the biased investor betrays with probability $\varphi(\mu; \mu^*_{k-1,\bar{l}(k-1)})$. After observing a success in period t, starting from the next period the Case 1-equilibrium with an $\bar{l}(k-1)$-period scheme is played with probability λ'_k and the Case 2-equilibrium with a $k-1$-period scheme is played with probability $1-\lambda'_k$, in which λ'_k satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda'_k \beta_I \frac{Y-y}{\mathbf{q}^{\bar{l}(k-1)-1}} + (1-\lambda'_k)\beta_I \frac{Y-y}{\mathbf{q}^{k-2}}].$$

(b) If $y \in (Y/\mathbf{q}^{j}, Y/\mathbf{q}^{j-1}]$ such that $2 \leq j \leq \bar{l}(k-1)$, the biased investor betrays with probability $\varphi(\mu; \mu^*_{k-1,j-1})$. After observing a success in period t, starting from the next period the Case 1-equilibrium with a $j-1$-period scheme is played with probability λ'_j and the Case 1-equilibrium with a j-period scheme is played with probability $1-\lambda'_j$, in which λ'_j satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda'_j \beta_I \frac{Y-y}{\mathbf{q}^{j-2}} + (1-\lambda'_j)\beta_I \frac{Y-y}{\mathbf{q}^{j-1}}].$$

(c) If $y > Y/(1+q)$, the biased investor betrays for sure. After observing a success, the expert discloses $Y-y$ in period $t+1$.

Second, consider $y < Y/\mathbf{q}^{k-1}$. (a) If $\mu \geq \underline{\mu}_{k-1}$, the biased investor cooperates for sure. Starting from period $t+1$ the Case 2-equilibrium with a $k-1$-period scheme is played. (b) If $\mu < \underline{\mu}_{k-1}$, the biased investor betrays with probability $\varphi(\mu; \underline{\mu}_{k-1})$. After observing a success in period t, starting from the next period the Case 2-equilibrium with a $k-1$-period scheme is played with probability λ''_k and the expert self-uses all $Y-y$ in period $t+1$ with probability $1-\lambda''_k$, in which λ''_k satisfies

$$\beta_I y = \alpha_I y + \delta \lambda''_k \beta_I \frac{Y-y}{\mathbf{q}^{k-2}}.$$

Finally, if the expert self-uses x instead of disclosing y, starting from period $t+1$ the Case 2-equilibrium is played if $\mu \in [\underline{\mu}_{k-1}, \mu^*_{k-1,\bar{l}(k)})$, and $Y-y$ is self-used in period $t+1$ if $\mu < \underline{\mu}_{k-1}$.

Case 3: $\mu \leq \underline{\mu}_k$.

On the equilibrium path. The expert self-uses $x = Y$ and her payoff is Y.

Off the equilibrium path. Consider the expert's deviations. (a) If $y \leq Y/\mathbf{q}^{k-1}$, the biased investor betrays with probability $\varphi(\mu; \underline{\mu}_{k-1})$. After observing a success in period t, starting from the next period the Case 2-equilibrium with a $k-1$-period scheme is played with probability λ''_k and the expert self-uses all $Y-y$ in period $t+1$ with probability $1-\lambda''_k$, in which λ''_k is denoted in the previous case. (b) If $y \in (Y/\mathbf{q}^{k-1}, Y/\mathbf{q}^{\bar{l}(k-1)}]$, the biased investor betrays with probability $\varphi(\mu; \mu^*_{k-1,\bar{l}(k-1)})$. After observing a success in period t, starting from the next period the Case 1-equilibrium with an $\bar{l}(k-1)$-period scheme is played with probability λ'_k and the Case 2-equilibrium with a $k-1$-period scheme is played with probability $1-\lambda'_k$, in which λ'_k is denoted in the previous case. (c) If $y \in (Y/\mathbf{q}^j, Y/\mathbf{q}^{j-1}]$ such that $2 \leq j \leq \bar{l}(k-1)$, the biased investor betrays with probability $\varphi(\mu; \mu^*_{k-1,j-1})$. After observing a success in period t, starting from the next period the Case 1-equilibrium with a $j-1$-period scheme is played with probability λ'_j and the Case 1-equilibrium with a j-period scheme is played with probability $1-\lambda'_j$, in which λ'_j is denoted in the previous case. (d) If $y > Y/(1+q)$, then the biased investor betrays for sure. After observing a success, the expert discloses $Y-y$ in period $t+1$. (e) If the expert self-uses $x < Y$, in the next period she self-uses $Y-x$. ■

Remark B4 *The verification that the strategy profile and belief updating system consist of an equilibrium is similar to the one shown for Proposition 1. So I omit the details. The key property is that, given the biased investor's strategy, if $\mu \in [\mu^*_{k,l}, \mu^*_{k,l-1}]$ and $1 \leq l \leq \bar{l}(k)$, the expert prefers an l-period scheme of information disclosure to any other scheme or self-use; if $\mu \in [\underline{\mu}_k, \mu^*_{k,\bar{l}(k)}]$, the expert prefers a k-period scheme of information disclosure to any other scheme or self-use; and if $\mu \leq \underline{\mu}_k$, the expert prefers self-use to any scheme of* *information disclosure.*

Lemma B5 *If $T = k-1$, $\mu \in [\mu_k^*, \mu_{k-1}^*)$ and $2 \leq k \leq \overline{k}+1$, there exists an equilibrium in which the expert's payoff is strictly larger than $V^k(\mu; Y)$.*

Proof. First, by Lemma B2, I have $H^k(\mu; Y) > V^k(\mu; Y)$ if $\mu \in [\mu_k^*, \mu_{k-1}^*)$ and $2 \leq k \leq \overline{k}+1$. Second, because $\mu_k^* > \underline{\mu}_k$, the expert's payoff is at least $H^k(\mu; Y)$ in the equilibrium I have constructed in Lemma B4. Therefore, the statement is true. ■

References

[1] Abreu, Dilip., and Gul, Faruk. 2000. "Bargaining and Reputation." Econometrica, 68(1): 85-117.

[2] Admati, Anat R., and Perry, Motty. 1991. "Joint Projects without Commitment." Review of Economic Studies, 58(2): 259-276.

[3] Anton, James J., and Yao, Dennis A. 1994. "Expropriation and Inventions: Appropriable Rents in the Absence of Property Rights." American Economic Review, 84(1): 190-209.

[4] Anton, James J., and Yao, Dennis A. 2002. "The Sale of Ideas: Strategic Disclosure, Property Rights, and Contracting." Review of Economic Studies, 69(3): 513-531.

[5] Arrow, Kenneth. 1962. "Economic Welfare and the Allocation of Resources for Inventions." In R. Nelson (ed.) The Rate and Direction of Inventive Activity: Economic and Social Factors (Princeton, NJ: Princeton University Press).

[6] Baliga, Sandeep., and Ely, Jeffrey C. 2010. "Torture." Working paper.

[7] Compte, Olivier., and Jehiel, Philippe. 2004. "Gradualism in Bargaining and Contribution Games." Review of Economic Studies, 71(4): 975-1000.

[8] Damiano, Ettore., Li, Hao., and Suen, Wing. 2012. "Optimal Deadlines for Agreements." Theoretical Economics, 7(2): 357-393.

[9] Hayek, Friedrich. 1945. "The Use of Knowledge in Society." American Economic Review, 35(4): 519-530.

[10] Fudenberg, Drew., and Levine, David K. 1989. "Reputation and Equilibrium Selection in Games with a Patient Player." Econometrica, 57(4): 759-778.

[11] Fudenberg, Drew., and Levine, David K. 1992. "Maintaining a Reputation when Strategies are Imperfectly Observed." Review of Economic Studies, 59(3): 561-579.

[12] Gale, Douglas. 2001. "Monotone Games with Positive Spillovers." Games and Economic Behavior, 37(2): 295-320.

[13] Hörner, Johannes., and Skrzypacz, Andrzej. 2011. "Selling Information." Working paper.

[14] Kreps, David M., and Wilson, Robert. 1982. "Reputation and Imperfect Information." Journal of Economic Theory, 27(2): 253-279.

[15] Lockwood, Ben., and Thomas, Jonathan P. 2002. "Gradualism and Irreversibility." Review of Economic Studies, 69(2): 339-356.

[16] Marx, Leslie M., and Matthews, Steven A. 2000. "Dynamic Voluntary Contribution to a Public Project." Review of Economic Studies, 67(2): 327-358.

[17] Milgrom, Paul., and Roberts, John. 1982. "Predation, Reputation, and Entry Deterrence." Journal of Economic Theory, 27(2): 280-312.

[18] Watson, Joel. 1999. "Starting Small and Renegotiation." Journal of Economic Theory, 85(1): 52-90.

[19] Watson, Joel. 2002. "Starting Small and Commitment." Games and Economic Behavior, 38(1): 176-199.

Chapter 2 Reputational Concern with Endogenous Information Acquisition

2.1 Introduction

Information acquisition and information transmission are essential activities in a variety of organizations, markets and societies. Uninformed principals who have the authority to make the final decisions must rely on these forms of activities performed by experts for the sake of decision optimality. For instance, when making policies, government officials often seek advice from social scientists. For the advice to be valuable, the scientists may need to acquire the relevant information first. Investors may ask for suggestions from their financial consultants if they are uncertain about the qualities of the projects and may leave it to the consultants to determine about how informative their reports should be. A pervasive feature underlying these expert-decision maker interactions is that the experts' advice and suggestions are often non-verifiable and thus explicit contracts contingent on them are infeasible. Instead, the experts may be self-incentivized by their reputational concerns for being aligned with the decision makers' objectives (i.e., whether they are perceived as caring about the decision makers' payoffs). This is quite common in practice, and helps to explain how these scientists, consultants, analysts and politicians are motivated and rewarded.

Initiated by the seminal works of Crawford and Sobel (1982) and Joel Sobel (1985), the literature on reputational cheap talk games has generated many insightful findings on how the experts' motivations to convey information truthfully may be affected by their reputational concerns. However, most of the studies in this field assume that the experts' information is exogenously given, and little attention has been paid to the potential effects of reputational concerns on the experts' information acquisition decisions. To obtain a better understanding of the roles that experts play in our society, I endogenize the experts' information acquisition in the current study and examine the interactions and welfare consequences of the experts' information acquisition, information transmission and reputational concerns. Specifically, I mainly address the following questions. Would the experts acquire better information if they are more concerned about their reputations? Would the decision makers benefit from the existence of a type of expert who is certain to contribute nothing in terms of information acquisition and transmission? If delegation is feasible, would the decision makers prefer communication to delegation or the inverse?

I develop a reputational cheap talk model in this paper. An expert first decides whether costly but more accurate information should be acquired. Afterwards, the expert receives a signal and sends a message to the decision maker. Based on this message, the decision maker then takes an action that is payoff-relevant to both parties. The expert may be aligned or biased. An aligned expert cares about the decision maker's welfare and would like to be known for being aligned with the decision maker's objectives. In contrast, for the sake of simplicity, a biased expert is assumed to be non-strategic in that he never acquires better information and is in favor of sending a particular message. The incorporation of reputational concern in the aligned expert's payoff captures the idea that experts may be motivated by how they are perceived by others instead of by explicit compensation schemes.

The aligned expert trades off between three factors: *reputation effect*, *informativeness effect* and information acquisition cost. Sending a message unfavored by the biased type always separates the aligned expert from the other type perfectly, regardless of what the nature state is. Thus, this message causes the aligned expert to obtain a larger payoff from reputation building. This effect is known as the reputation effect. In contrast, if the aligned expert aims to reveal his information truthfully, he benefits from the information acquisition because better information enables the decision maker to take better actions and form more precise posterior beliefs. This effect is known as the informativeness effect. One of my main findings shows that reputational concern may have a non-monotonic effect on the aligned expert's information acquisition incentive; that is, the aligned expert acquires better information if and only if his reputational concern is moderate. The reasoning behind this result is as follows. If the reputational concern is relatively low, the informativeness effect is outweighed by the acquisition cost, and thus, the aligned expert's attempt to perform information acquisition is restricted. If the reputational concern is relatively high, the aligned expert is too incentivized to exploit the reputation effect by sending the message unfavored by the biased type, so better information is worthless to him. Only in the moderate range of the reputational concern does the informativeness effect dominate both the acquisition cost and the reputation effect such that the aligned expert has the proper incentive to acquire better information. Additionally, I show that the aligned expert's decisions regarding information acquisition and transmission are complementary: the more truthfully the information is transmitted *ex post*, the stronger the aligned expert's incentive is to acquire better information *ex ante*. This relationship effectively captures the experts' behaviors in many real-world situations.

Abstracting from the detailed models, most of the existing studies on reputation games show that an individual's reputational concern has a monotonic effect on his/her behavior; that is, with an increase in the reputational concern, the individual's incentive to take a decision unfavored by the other individuals either decreases (e.g., Joel Sobel (1985), Benabou and Laroque (1992) and Wei Li (2010)) or increases (e.g., Morris (2001), Ely and Valimaki (2003) and Ely, Fudenberg and Levine (2008)). The novel finding of a non-monotonic effect in my study bridges the existing results and further contributes to my understanding of how an individual's behavior may be affected by his/her concerns with building a reputation.

Another main finding in this study is that, although the biased type of expert contributes nothing in terms of information acquisition and transmission, the existence of this type of expert may actually improve the decision maker's payoff. A crucial property in my model is that if the information acquisition cost is moderate, the aligned expert will attempt to acquire better information if and only if he is compensated in the form of both better actions and more accurate beliefs. From the perspective of the decision maker's welfare, if it is certain that the expert is aligned, the decision maker can benefit from the expert's truthful information transmission, but this benefit may not be maximized if better information is not acquired. Conversely, if biased experts are present, the decision maker suffers from the information loss caused by these experts. However, she may benefit from the better information conveyed by the aligned expert if this expert performs information acquisition. If the probability of an expert being biased is relatively low, the information loss introduced by the biased type is exceeded by the increased gain from the better information transmitted by the aligned type. Thus, the decision maker's payoff can be improved. This finding enables us to starkly compare my results with the results in the aforementioned papers, in which the decision maker's payoff is inevitably reduced by the existence of the biased experts because of the lack of endogenous information acquisition.

Instead of eliciting information through communication, the decision maker may delegate her decision rights to the expert. For instance, in real-world practice, a government may grant the pricing rights to a regulated firm, and a financial consultant may have much discretion on investment decisions. In this study, I also examine the effects of delegation on the expert's information acquisition incentive and on the decision maker's choice regarding when delegation should be employed. Delegation may be *unrestricted* such that the expert can take any action from the original action space. Alternatively, delegation may be *restricted* such that the decision maker can optimally design the delegation set. If the biased expert always takes the largest permissible action, the key feature under delegation is that the aligned expert can always separate from the biased type perfectly by taking the payoff-relevant action. Thus, the reputation effect vanishes, and the informativeness effect is limited. I show that, compared with unrestricted delegation, optimally restricted delegation always reduces the aligned expert's incentive to acquire better information, which contradicts the result in Dezso Szalay (2005). Additionally, compared with communication, delegation (whether unrestricted or restricted) may reduce the aligned expert's information acquisition incentive, which is a counter-example to the result in Aghion and Tirole (1997). Finally, compared with the result in Wouter Dessein (2002), which finds that the decision maker always prefers delegation to communication whenever informative communication is feasible, my result is more biased towards communication in the sense that whenever there exists an equilibrium with information acquisition, the decision maker strictly prefers communication to delegation. This result could be further enhanced if I impose more restrictive conditions.

2.2 Literature

This paper belongs to the growing literature on cheap talk initiated by Crawford and Sobel (1982). Joel Sobel (1985) was the first to consider a reputational cheap talk model in which the aligned expert conveys information truthfully and the biased expert attempts to appear aligned. Benabou and Laroque (1992) extend Sobel's model by incorporating noisy signals. The reputational concerns in these papers are "good" in the sense that the biased expert's incentive to manipulate information is restricted. I primarily build on and borrow from Stephen Morris (2001). Morris endogenizes the role of the aligned expert and shows that no information can be conveyed in equilibrium if the aligned expert is too concerned about his reputation. He refers to this phenomenon as "political correctness". Ely and Valimaki (2003) and Ely, Fudenberg and Levine (2008) show that the reputational concern of the long-run player who is interacting with a sequence of short-run players could lead to the market being shut down and the loss of all surplus. The authors refer to this effect as "bad reputation". In these papers, the expert's information is exogenously given, and the focus is on how the information is revealed. In contrast, I endogenize the quality of information in this study and mainly consider the effects of reputational concern on the expert's information acquisition incentive.

Some recent papers have introduced a third party to the reputational cheap talk games. Wei Li (2010) analyzes a model in which an intermediary between the expert and the decision maker exists. She finds that the biased expert and the biased intermediary's reporting truthfulness are strategic complements. Li and Mylovanov (2008) and Durbin and Iyer (2009) develop models in which the expert may acquire information by himself or follow an interest group's recommendation in exchange for an access fee, and thus endogenize the source of the expert's bias. Another strand of the literature on reputational cheap talk, which is not so close to ours, focuses on the situations in which the expert has an incentive to build a reputation regarding his ability. (e.g., Scharfstein and Stein (1990), Brandenburger and Polak (1996), Gilat Levy (2004), Andrea Prat (2005), Ottaviani and Sorensen (2006a, b), Gentzkow and Shapiro (2006), Wei Li (2007) and Giuseppe Moscarini (2007)).

My paper also relates to the literature on information acquisition. Dezso Szalay (2005) studies a model in which the expert acquires costly information and then chooses an optimal action. He shows that it may be desirable for the principal to restrict the expert's discretion to improve the information acquisition incentive, even if the expert is perfectly aligned. Hao Li (2001) derives a similar insight within a group decision framework in which optimally designed conservatism increases experts' attempts to collect evidence and improves the quality of the group decision. Dur and Swank (2005), Gerardi and Yariv (2008) and Che and Kartik (2009) show that it could be optimal for the decision maker to hire experts with different preferences and opinions because these experts have stronger incentives to collect relevant information. Because none of these papers are concerned about the expert's reputation, they differ substantially from my study.

Finally, my paper refers to the literature on delegation. Aghion and Tirole (1997) note that delegation always increases the agent's incentive to acquire information because he can better employ the information in the action taking stage. Wouter Dessein (2002) finds that the decision maker prefers delegation to communication whenever informative communication is feasible. Alonso and Matouschek (2008) generalize the delegation literature and characterize the properties of optimal delegation. My findings in this paper complement these results regarding the understanding of when the decision maker should delegate her rights to the expert. For more papers regarding delegation, see, among others, Holmstrom

(1977, 1984), Melumad and Shibano (1991), Tymofiy Mylovanov (2008), Krishna and Morgan (2008) and Kovac and Mylovanov (2009).

2.3 The model

I consider a communication game involving two players: an expert (referred to as E or he) and a decision maker (referred to as DM or she). The nature state is binary: $\theta \in \{0, 1\}$. Each state occurs with probability 1/2. Before he receives a signal $s \in \{0, 1\}$, the expert has private access to an information acquisition technology that can increase the accuracy p of the signal. With effort $e = 1$, which costs $c > 0$, the signal s will be equal to the state θ with probability $p = p_1$; with effort $e = 0$, which costs 0, the signal s will be equal to the state θ with probability $p = p_0$. I assume $1/2 < p_0 < p_1 < 1$, so the signal is informative but imperfect in both cases.[23] Afterwards, the expert receives a signal s and sends a message $m \in \{0, 1\}$ to the decision maker. Based on the message, the decision maker takes an action $a \in [0, 1]$ to maximize her payoff. For instance, this action could be a decision about how much capital should be invested or what the optimal policy should be. Finally, the decision maker learns the nature state θ and forms her posterior belief about the expert's type based on the information in her possession.

The decision maker's payoff depends on the nature state and her action, which is represented by the quadratic loss function

$$\Pi = -(a - \theta)^2.$$

Notice that with this payoff function, the decision maker's optimal action is equal to the probability that she attaches to the state $\theta = 1$. There are two types of experts: an aligned

[23] I assume $p_0 > 1/2$. A possible justification for this assumption is that without information acquisition, the expert's signal is informative because of his experience or expertise. For instance, if a financial expert is consulted about a particular investment decision, his judgment is informative because of his past research on the general economic environment. However, if he investigates the context of this particular decision and obtains additional information, his judgment will be more informative.

expert (referred to as A) with prior probability $\lambda \in (0,1)$ and a biased expert (referred to as B) with prior probability $1-\lambda$. The aligned expert and the decision maker share the same preference regarding the current action, and his payoff is given by

$$U_A = -(a-\theta)^2 + \mu\phi(\lambda) - ec,$$

in which $\phi(\lambda)$ is the decision maker's posterior belief that the expert is aligned, $\mu \geq 0$ is the aligned expert's reputational concern attached to the posterior belief, and ec is the information acquisition cost. Intuitively, the degree of reputational concern μ may represent the aligned expert's future opportunities to provide consultations or find employment. I use a reduced form of reputation building in this study to capture the idea that the higher the reputational concern is, the stronger the expert's attempt to appear aligned will be because of the complementarity between μ and $\phi(\lambda)$. In the following analysis, I also refer to the first term in this payoff function as the aligned expert's *current gain* and refer to the second term as his *reputational gain.* The biased expert is assumed to be non-strategic in that he does not acquire better information and always sends the message $m = 1$.

I look for Perfect Bayesian Equilibria (PBE) in this paper. For the aligned expert, his strategy σ_A consists of five probabilities: $\sigma_A = \{\alpha_A, (x_{A,}w_A), (y_A, z_A)\}$. α_A is the probability that he acquires better information. Based on the acquisition decision $e = 0$, x_A (w_A) is his truthful reporting probability that he sends message $m = 1$ ($m = 0$) if his signal is $s = 1$ ($s = 0$). Similarly, based on the acquisition decision $e = 1$, y_A (z_A) is his truthful reporting probability that he sends message $m = 1$ ($m = 0$) if his signal is $s = 1$ ($s = 0$). The decision maker's strategy σ_{DM} is the action a_m she takes if she receives the message m. Besides, after observing the realized nature state θ the decision maker forms posterior belief $\phi(\lambda)$ about the expert's type. A strategy profile σ^* and belief updating system ϕ^* consist of an equilibrium if each player's strategy maximizes his/her payoff and if the posterior belief follows Bayes' rule whenever possible. For the sake of simplicity, I restrict my attention to the equilibria in which the aligned expert uses pure strategy on his acquisition decision.

My paper belongs to the literature on cheap talk games, and has a babbling equilibrium in which the decision maker learns nothing about the nature state from the expert's message. Specifically, the biased expert's non-strategic role implies that his message is uninformative. If the aligned expert does not acquire better information and always sends message $m = 0$ regardless of his signal, the decision maker's optimal action is to take $a_0^* = a_1^* = 1/2$. Expecting to have no effect on the decision maker's action, the aligned expert has no incentive to deviate from this strategy. Thus, the existence of equilibrium is guaranteed in this game. The interesting question that arises here is when equilibria other than the babbling one exist. I introduce a definition as follows.

Definition 4 *An equilibrium $\{\sigma^*, \phi^*\}$ is an informative equilibrium (IE) if $a_m^* \neq a_{m'}^*$ for $m \neq m'$ on the equilibrium path.*

This definition simply says that the decision maker's optimal action is responsive to the message that she received.[24] In the next section, I mainly describe how the existence of informative equilibria, especially the ones with information acquisition, is affected by the aligned expert's reputational concern.

2.4 Communication

The communication game can be represented by three stages: the stage of information acquisition, the stage of information transmission, and the stage of action taking and belief updating. The equilibrium behaviors of the players can be identified by the backward induction approach.

[24] My definition of informative equilibrium is consistent with those in some studies (e.g., Gilat Levy (2004) and Wei Li (2010)) but differs slightly from those in some other studies (e.g., Stephen Morris (2001)). Precisely, my definition is *stronger* than the one in Morris (2001). In my study, an equilibrium is informative only if the decision maker can learn something about the nature state, whereas in Morris (2001), an equilibrium is informative if the decision maker can learn something about the nature state, or the expert's type, or both. There is no significant effect on my main findings if I adopt the definition in Morris (2001).

2.4.1 Preliminary analysis

Given any strategy of the expert, the decision maker's optimal response in the last stage is uniquely determined. Specifically, if $\Pr_A(m = l|\theta = k)$ represents the expected probability from the decision maker's perspective that the aligned expert will send a message $m = l$ if the state is $\theta = k$, where k, $l \in \{0, 1\}$,[25] given the non-strategic role of the biased expert, I have the optimal action a_m^* as

$$a_0^* = \frac{\Pr_A(m=0|\theta=1)}{\Pr_A(m=0|\theta=1)+\Pr_A(m=0|\theta=0)} \quad \text{and} \quad a_1^* = \frac{\lambda \Pr_A(m=1|\theta=1)+(1-\lambda)}{\lambda \Pr_A(m=1|\theta=1)+\lambda \Pr_A(m=1|\theta=0)+2(1-\lambda)},$$

and I have the equilibrium belief $\phi^*(\lambda|m, \theta)$ as

$$\phi^*(\lambda|0,0) = 1 \quad \text{and} \quad \phi^*(\lambda|1,0) = \frac{\lambda \Pr_A(m=1|\theta=0)}{\lambda \Pr_A(m=1|\theta=0)+(1-\lambda)},$$
$$\phi^*(\lambda|0,1) = 1 \quad \text{and} \quad \phi^*(\lambda|1,1) = \frac{\lambda \Pr_A(m=1|\theta=1)}{\lambda \Pr_A(m=1|\theta=1)+(1-\lambda)}.$$

In appendix A, I provide a general description of the optimal actions to take and the posterior beliefs to form. This description helps to generate some preliminary results.

Lemma 3 *For any informative equilibrium in which $a_1^* > a_0^*$, the aligned expert sends $m = 0$ with probability one on the equilibrium path if his signal is $s = 0$; for any informative equilibrium in which $a_1^* < a_0^*$, the aligned expert sends $m = 1$ with probability zero on the equilibrium path if his signal is $s = 1$.*

A crucial property in this communication game is that, regardless of what the nature state is, the aligned expert can always separate from the biased type perfectly by sending message $m = 0$ and thus obtain a larger reputational gain if $\mu > 0$. This property could be

[25] To save notations, the detailed expression of this probability is relegated in appendix A.

observed from the inequality

$$\mu\phi^*(\lambda|m=0,\theta=k)=\mu>\mu\lambda\geq\mu\phi^*(\lambda|m=1,\theta=k)\quad\text{for}\quad k\in\{0,1\}.$$

I denote this property as the *reputation effect.* Meanwhile, given a particular signal, if sending message $m=0$ also increases the aligned expert's current gain, he should certainly do so. Lemma 3 captures these arguments.

Lemma 4 *For any informative equilibrium in which $a_1^*<a_0^*$, there exists another informative equilibrium in which $a_1'>a_0'$.*

There is no restriction on how the meaning of a message is conveyed by the expert and deduced by the decision maker. Potentially, an equilibrium with $a_1^*<a_0^*$ may exist in this game; that is, the decision maker rationally infers that the probability of the state being $\theta=1$ is higher if the message she received is $m=0$ rather than $m=1$. The finding in the above lemma demonstrates that whenever such a "reversely inferred" equilibrium exists, another "obversely inferred" equilibrium exists. Thus, without loss of generality, I can narrow my attention to the set of equilibria in which $a_1^*\geq a_0^*$.

Lemma 5 *If there is information acquisition in an equilibrium, then this equilibrium is informative.*

Information acquisition is socially valuable if the increased accuracy of the signal enables the decision maker to take better actions that are preferred by both players. However, if the aligned expert expects his information transmission to have no effect on the decision maker's action, he is actually better off by saving the acquisition cost and simply obtaining the largest reputational gain. The result in Lemma 5 highlights the necessity of equilibrium informativeness to the expert's attempt to acquire better information, which is one of the driving forces in my study.

I are primarily interested in the existence of equilibria with information acquisition and how this existence varies with the expert's reputational concern. The aforementioned results substantially simplify my investigation of these equilibria. Particularly, if an equilibrium with information acquisition exists, by Lemma 5 I know that it is necessarily informative, and by Lemma 4 it is without loss of generality to focus on the case that $a_1^* > a_0^*$. Finally, by Lemma 3 I only need to focus on the informative equilibria in which the aligned expert conveys his information truthfully if his signal is $s = 0$. The main results will be derived in the next subsection.

2.4.2 Equilibrium analysis

One question remains in the stage of information transmission: when does the aligned expert have an incentive to tell the truth (at least with a positive probability) after observing a signal $s = 1$? With the simplifications derived above, an equilibrium is informative if $x_A^* > 0$ (in case $\alpha_A^* = 0$ in equilibrium) or $y_A^* > 0$ (in case $\alpha_A^* = 1$ in equilibrium). Intuitively, if the aligned expert always sends message $m = 0$ regardless of his information while the biased expert always sends message $m = 1$ by assumption, the decision maker can learn nothing about the nature state, and thus, the equilibrium is necessarily babbling. If the received signal is $s = 1$, for the aligned expert to have $x_A^* > 0$ or $y_A^* > 0$ he must be compensated by the current gain by sending message $m = 1$ because of the reputation effect by sending message $m = 0$. This trade-off is resolved by the following proposition.

Let

$$\underline{\mu} = \frac{p_0^3 + (1-p_0)^3 - p_0(\frac{1-\lambda p_0}{2-\lambda})^2 - (1-p_0)(\frac{1-\lambda+\lambda p_0}{2-\lambda})^2}{1 - \frac{p_0^2\lambda}{p_0\lambda+(1-\lambda)} - \frac{(1-p_0)^2\lambda}{(1-p_0)\lambda+(1-\lambda)}},$$

$$\overline{\mu} = \frac{p_1^3 + (1-p_1)^3 - p_1(\frac{1-\lambda p_1}{2-\lambda})^2 - (1-p_1)(\frac{1-\lambda+\lambda p_1}{2-\lambda})^2}{1 - \frac{p_1^2\lambda}{p_1\lambda+(1-\lambda)} - \frac{(1-p_1)^2\lambda}{(1-p_1)\lambda+(1-\lambda)}}.$$

I have $0 < \underline{\mu} < \overline{\mu}$.

Proposition 5 *For an informative equilibrium to exist, the aligned expert's reputational concern μ must be relatively low: if $\alpha_A^* = 0$ on the equilibrium path, $\mu \leq \underline{\mu}$; if $\alpha_A^* = 1$ on the equilibrium path, $\mu \leq \overline{\mu}$.*

Fix the accuracy of the signals and assume that the decision maker responds optimally. If the aligned expert receives a signal $s = 1$, his payoff by sending message $m = 1$, which is denoted as v_1 in the proof, increases with his truthful reporting probability. To see this, notice that all the terms a_1^*, $\phi^*(\lambda|1, 0)$ and $\phi^*(\lambda|1, 1)$ increase in x_A (or y_A) and v_1 increases in these terms (as long as $a_1^* \leq p_i$ if $\alpha_A^* = i$ in equilibrium). Intuitively, the higher the truthful reporting probability on the signal $s = 1$ is, the less noisy the message $m = 1$ is. Thus, the decision maker can more precisely deduce information about the nature state and the expert's type, which are both beneficial to the aligned expert. I denote this payoff-increasing property as the *informativeness effect*.

If the aligned expert's reputational concern is relatively low, the aforementioned informativeness effect outweighs the reputation effect, and thus, an informative equilibrium may exist. However, if the reputational concern is too large, the aligned expert becomes too incentivized by the desire to build his reputation, and the reputation effect eventually outweighs the informativeness effect, which causes the communication to become inevitably uninformative. This finding captures the spirit described by Morris (2001) as "political correctness" and the one denoted by Ely and Valimaki (2003) and Ely, Fudenberg and Levine (2008) as "bad reputation".

Additionally, $\underline{\mu} < \overline{\mu}$ implies that, keeping other things unchanged, the higher the signal accuracy p is, the more supportable it is for an equilibrium to be informative. This could be seen from the fact that v_1 is an increasing function of p, which uncovers another source of the informativeness effect.

I introduce an assumption for the next result.

Assumption: $2p_1 + 1 - 4p_0 \leq 0$.

This condition reduces the number of potential deviations I need to consider for an equilibrium with information acquisition to exist, and its detailed role is explained in the proof of the next proposition. Because it has no qualitative effect on the remaining findings, I assume that it holds in the remainder of this study.

Proposition 6 *There is a non-empty set* $(\underline{c}, \overline{c})$ *such that for any* $c \in (\underline{c}, \overline{c})$ *an equilibrium with information acquisition exists if and only if* $\mu \in [\underline{\mu}_c, \overline{\mu}_c]$*, where* $[\underline{\mu}_c, \overline{\mu}_c] \subset (0, \overline{\mu})$*.*

I provide a complete description of the existence of an equilibrium with information acquisition in the proof, in which the cut-off values $\underline{c}$, $\overline{c}$, $\underline{\mu}_c$ and $\overline{\mu}_c$ are identified. Nevertheless, the finding is more interesting if the acquisition cost c is in the range $(\underline{c}, \overline{c})$. In this scenario, reputational concern actually has a non-monotonic effect on the aligned expert's information acquisition incentive: he acquires better information if and only if his reputational concern is moderate. If the aligned expert intends to reveal his information truthfully (at least with positive probabilities), the informativeness effect is larger if better information is acquired. Intuitively, analogous to the reason for $\underline{\mu} < \overline{\mu}$, truthful revelation of information based on more accurate signals can induce the decision maker to take better actions and to form more precise beliefs. However, if the reputational concern is too low, this larger informativeness effect caused by information acquisition is not enough to cover the acquisition cost, which implies that the aligned expert's incentive to acquire better information is restricted. Conversely, if the reputational concern is too high, the aligned expert knows that he will send message $m = 0$ to exploit the large reputation effect, regardless of what the signal is. Thus, better information is worthless to him. Only if the reputational concern is in the moderate range may the informativeness effect from the information acquisition outweigh both the acquisition cost and the reputation effect. This establishes the result in the proposition.

One property worth mentioning is the complementarity between the information acquisition and the information revelation. Lemma 5 demonstrates that information may be acquired only if the information is not totally neglected, the proof of Proposition 6 further shows that the aligned expert's incentive to acquire information is strongest if he aims to reveal the better information honestly without any distortion. Thus, in this communication game the better the *ex post* usage of information, the more information is acquired *ex ante*.

Abstracting from the detailed setups, most of the existing studies on reputation games show that an individual's reputational concern has a monotonic effect on his/her behavior. Specifically, with an increase in the reputational concern, this effect is either "good" in the sense that the individual's attempt to push an agenda unfavored by the other players is mitigated (e.g., Joel Sobel (1985), Benabou and Laroque (1992) and Wei Li (2010)), or "bad" in the sense that the individual's attempt to push such an agenda is intensified (e.g., Morris (2001), Ely and Valimaki (2003) and Ely, Fudenberg and Levine (2008)). The novel finding of the non-monotonic effect in this paper combines these two branches and enables us to understand the relationship between an individual's reputational concern and his/her behavior in a more subtle way.

2.4.3 Welfare analysis

The existence of the biased type of expert introduces information/action distortions in the studies on both good reputation games and bad reputation games. From the viewpoint of welfare, the decision maker's payoff is unambiguously lowered in these papers. An interesting question that arises here is whether the decision maker may actually benefit from the possible existence of a biased expert if information is endogenously acquired. I address the welfare analysis in this subsection.

One of the well-known properties in the literature on cheap talk games is the multiplicity of equilibria, which may complicate the welfare comparison among different scenarios.[26] To avoid such a problem, I restrict my attention to the most informative equilibrium, which is formally defined as follows.

[26] For instance, if a babbling equilibrium in scenario X is compared with an informative equilibrium in scenario Y, it is highly possible that the decision maker prefers the latter scenario. Conversely, if an informative equilibrium in scenario X is compared with a babbling equilibrium in scenario Y, the decision maker may prefer the former scenario.

Definition 5 *An informative equilibrium* $\{\sigma^*, \phi^*\}$ *is a most informative equilibrium (MIE), if there is no other equilibrium* $\{\sigma', \phi'\}$ *such that* $\alpha'_A \geq \alpha^*_A$, $x'_A \geq x^*_A$, $w'_A \geq w^*_A$, $y'_A \geq y^*_A$, $z'_A \geq z^*_A$ *on the equilibrium paths and if at least one of the inequalities is strict.*

Intuitively, an equilibrium is most informative if there is no other equilibrium in which the aligned expert acquires and truthfully conveys more information. Apparently, if $\alpha^*_A = y^*_A = z^*_A = 1$ in an equilibrium, the amount of information being acquired and transmitted is maximized. In this case, this equilibrium is a most informative equilibrium. Using this definition, I derive the following result.

Proposition 7 *Suppose* $c \in (\underline{c}, \overline{c}]$. *Compared with the non-existence of the biased type, the decision maker's most informative equilibrium payoff is improved by the existence of the biased type if* $\mu \in [\underline{\mu}_c, \overline{\mu}_c]$ *and* $\lambda \in (\lambda^*, 1)$.

I define λ^* in the proof and can verify that $0 < \lambda^* < 1$. The aligned expert's reputational concern plays two roles in this communication game. It may cause the aligned expert to distort his information transmission for the sake of the reputation effect, but it may also induce him to acquire better information for the sake of the informativeness effect. If it is commonly known that the expert is aligned, the reputation effect vanishes, while the informativeness effect may be limited if the acquisition cost is relatively large and no information is thus acquired. If the expert can be biased, the role played by the aligned expert's reputational concern becomes crucial to the decision maker's payoff. On the one hand, the biased type of expert always distorts the information conveyed in this game. More importantly, according to Proposition 5, the aligned expert's reputational concern may further cause him to produce an endogenous distortion. The combination of these two distortions could be seen as a cost of the existence of the biased type. On the other hand, Proposition 6 states that even if acquisition cost c is in the range $(\underline{c}, \overline{c}]$, better information may be acquired in this game if the decision maker is uncertain about the expert's type. This could be seen as a benefit of the existence of the biased type. The result in Proposition 7 shows that from the decision maker's perspective, the net effect of these cost and benefit may be positive if the aligned expert's reputational concern is in the information acquisition zone and if the possibility of an expert being biased is relatively negligible.

The potential welfare-increasing effect induced by the existence of the biased type differs substantially from the results in the aforementioned papers and may contribute some general insights that extend beyond the detailed setup considered in the current study. For instance, in many environments, even though the decision makers (or the relevant individuals) have the abilities to learn the types of the experts (or the other individuals) perfectly before the interactions unfold, they may strategically keep their uncertainty about the types to better motivate the experts.

2.5 Delegation

Instead of eliciting relevant information through communication and taking actions by themselves, the decision makers may delegate their decision rights to the experts in many organizations. For example, government officials make regular decisions on behalf of the public, and shareholders grant significant discretion to company managers. Moreover, to further limit the agency costs, the decision makers can optimally restrict the experts' decision sets, as frequently seen in real life cases in which regulated firms are permitted to set prices only below some price caps. In this section, I address how the players' decisions and payoffs may be affected by delegation.

The timing of the delegation game proceeds as follows. At the beginning of the game, the decision maker announces a delegation set $A \subseteq [0, 1]$. Afterwards, the nature state θ is realized, and the expert decides whether better information should be acquired. Information acquisition increases the accuracy of the signal in the same manner described previously. Then the expert receives a signal s and takes an action a from the delegation set. Finally, the decision maker learns the nature state and forms posterior beliefs about the expert's type.

A natural modification of the biased expert's role is to assume that he always takes the largest permissible action from the delegation set. Apparently, under this assumption, the biased expert never acquires information in any equilibrium. For the aligned expert and the decision maker, the strategies and equilibria are defined similarly to those in section 2.3 except that the decision maker's belief updating system $\phi(\lambda|a,\theta)$ now depends on the action a taken by the expert instead of the message m sent by him. I consider both *unrestricted* delegation and *restricted* delegation in this study. Delegation is unrestricted if the delegation set is $A = [0,1]$, and delegation is restricted if the delegation set is $A = [\underline{a}, \overline{a}]$ in which the decision maker determines whether $\underline{a} > 0$ or $\overline{a} < 1$. The latter case captures the widespread usage of price caps or wage floors in many regulated organizations.[27]

A direct result in this delegation game is as follows.

Corollary 1 *Regardless of whether the delegation set is unrestricted or restricted, the aligned expert's reputational gain is constant in equilibrium.*

The intuition for this result is that, if the expert is granted the decision rights, the aligned expert now can separate from the biased expert by taking the payoff-relevant actions instead of sending messages. More importantly, because the delegation set is always a continuum, this separation is perfect. Thus, the aligned expert always earns the largest reputational gain under delegation. This property implies that under delegation, the reputation effect vanishes, and the aligned expert only trades off between the acquisition cost and the informativeness effect with respect to the current gain if he decides to acquire information. I characterize the technical results of this delegation game in the next lemma.

[27] Generally, the decision maker could employ other forms of restrictions on delegation, such as veto-based delegation (e.g., Tymofiy Mylovanov (2008)). Alternatively, because the signal space is binary in my setup, the decision maker may directly design a two-element action set for the expert to choose from. However, this sort of restriction may be too strong for real-world observations, especially for situations in which the action a to be taken is approximately equal to, for example, the amount of optimal investment, tax rate or price. Hence, in this study, my focus on a weaker restriction such that the decision maker can only decide the ceiling and floor of the delegation set.

Lemma 6 *(1) If the delegation set is unrestricted, the aligned expert acquires better information if and only if $c \leq c_u$.*

(2) If the delegation set is optimally restricted, the aligned expert acquires better information if and only if $c \leq c_r$. The optimally restricted delegation set is $A = [0, \overline{a}_0]$ if $c > c_r$ and is $A = [0, \overline{a}_1]$ if $c \leq c_r$.

The cut-off values are also denoted in the proof. Compared with unrestricted delegation, a top-down restriction on the delegation set benefits the decision maker in that the action distortion introduced by the biased type is mitigated. However, the cost of this restriction is twofold: (i) the action (i.e., p_0 or p_1) maximizing the aligned expert's current payoff (and thus the decision maker's payoff in case she knows that the expert is aligned) after the signal $s = 1$ may become unavailable; and (ii) because of the effect in (i), the aligned expert's incentive to acquire information may be reduced. The decision maker optimally trades off these benefit and cost, which results in the equilibrium outcomes characterized in (2).

Importantly, because $c_r < c_u$ for any $\lambda < 1$, the property in (ii) actually occurs if $c \in (c_r, c_u]$, which implies that, compared with the unrestricted delegation, the optimally restricted delegation reduces the aligned expert's incentive to acquire better information. Dezso Szalay (2005) shows that the agent (in his paper) may have a stronger motivation to collect socially valuable information if his action choices are optimally restricted, and Hao Li (2001) derives a similar result within a group decision framework. My result complements these findings regarding the understanding of how an individual's information acquisition incentive may be affected by the design of the optimal action choices.

I also obtain the following result.

Corollary 2 *Compared with communication, delegation (whether unrestricted or restricted) may reduce the aligned expert's incentive to acquire better information.*

A widely accepted principle proposed by Aghion and Tirole (1997) states that if the decision rights are delegated to the individual who has access to the relevant information about the nature state, he is always more motivated to acquire better information than he would be under communication. Intuitively, if the individual can determine the usage of the information *ex post*, his demand for better information increases *ex ante*. In this study, I provide a counter-example to this principle, which is depicted in Corollary 2. Under communication, if the aligned expert intends to convey his information truthfully, the high accuracy of the signals also benefits his reputation-building efforts, which enable him to acquire more costly information in some equilibria. In contrast, under delegation, the aligned expert only acquires less costly information in equilibrium because the reputational gain is constant. However, because the aligned expert's reputational concern may also destroy his acquisition incentive, as I emphasized in the communication game, a general comparison between the communication and the delegation is more ambiguous.

Proposition 8 *The decision maker strictly prefers communication with the most informative equilibrium to the optimally restricted delegation if and only if* $c \in (c_r, \overline{c}]$ *and* $\mu \in [\underline{\mu}_c, \overline{\mu}_c]$.

From the decision maker's perspective, whether her decision rights should be delegated depends on the role played by the aligned expert's reputational concern under communication. Particularly, if the aligned expert acquires better information and the acquisition cost is covered by the increase in both current gain and reputational gain under communication, delegation actually reduces the aligned expert's information acquisition incentive, which causes the decision maker to obtain a lower payoff. Although Wouter Dessein (2002) claims that the decision maker always prefers delegation to communication whenever an informative equilibrium exists under communication (with the leading example in Crawford and Sobel (1982)), my result in Proposition 8 is relatively biased towards communication.[28] If I further assume that $\underline{\mu} \leq \mu'$,[29] the result in Proposition 5 can be enhanced such that the

[28] If I only compare communication with unrestricted delegation, which is the main focus in Dessein (2002), then the decision maker will prefer communication to a larger extent. Moreover, If $\lambda < 1/(2 - 4p_1(1 - p_1))$, the decision maker's payoff under unrestricted delegation is lower than $-1/4$ because of the action distortion introduced by the biased expert. In this scenario, the decision maker always prefers communication to unrestricted delegation because any of her equilibrium payoffs under communication is no lower than $-1/4$.

[29] μ' is defined in the proof of Proposition 2.

decision maker prefers communication with the most informative equilibrium to delegation whenever informative equilibrium exists under communication.[30] Nevertheless, because the detailed setup in my paper substantially differs from that of Dessein (2002), my finding here should be viewed as a complement to his result.

2.6 Conclusions

A common property of many organizations is that experts are incentivized by their reputational concerns instead of explicit monetary payments. In this paper, I study how these concerns may influence the experts' attempts to acquire information, which few studies on reputation games have examined. My main insight shows that for proper information acquisition cost, the aligned expert acquires better information if and only if his reputational concern is moderate. Another insight is that, although the only role played by the biased experts is to distort information transmission, the existence of this type of expert may actually improve the decision maker's payoff. Additionally, I also examine the potential effects of delegation on the players' decisions and show that the decision maker prefers communication to delegation whenever informative communication with information acquisition exists. These novel findings enable us to obtain a better understanding of the possible consequences of an individual's desire to build a reputation.

One potential extension of this paper is to generalize the effort and signal accuracy levels. For instance, consider an effort cost function $c(p)$ that satisfies $c(1/2) = 0$, $c(1) = +\infty$, $c' > 0$ and $c'' > 0$. I expect that the qualitative results in my paper will be unchanged with this cost function. However, the aligned expert's optimal acquisition effort may increase and decrease more smoothly if his reputational concern increases. In this case, the optimal acquisition

[30] $\underline{\mu} \leq \mu'$ holds if $\frac{p_0}{(p_1\lambda+(1-\lambda))(p_0\lambda+(1-\lambda))} - \frac{(1-p_0)}{((1-p_1)\lambda+(1-\lambda))((1-p_0)\lambda+(1-\lambda))} \geq 0$, which is true if p_0 is not too close to 1/2. If $\underline{\mu} \leq \mu'$ (although it is not necessary) and if an informative equilibrium exists, then I can verify that the aligned expert always tells the truth without any distortions in the most information equilibrium. With this property, the decision maker always weakly prefers communication to delegation, and this preference is strict in some scenarios (e.g., if $c \in (c_r, \overline{c}]$ and $\mu \in [\underline{\mu}_c, \overline{\mu}_c]$).

effort better justifies the non-monotonic effect derived in this paper.

Another extension is to allow the biased expert to be strategic and also have reputational concerns. For instance, let him have a payoff function $\eta a + \mu_B \phi(\lambda) - ec$, in which $\eta > 0$ is a weight attached to the action taken by the decision maker and $\mu_B \geq 0$ is his reputational concern with appearing aligned. If the ratio η/μ_B is sufficiently large, which implies that the biased expert is mainly interested in inducing a higher action instead of building a reputation, I can expect my main findings to remain true. However, if this ratio is relatively small, new issues may arise. Intuitively, with an increase in the biased expert's reputational concern, the aligned expert will find it more difficult to benefit from the reputational gain. This increased difficulty may cause him to reduce information acquisition but may also cause him to reduce information distortion, and the net effect would be highly ambiguous. A potentially meaningful result is that the decision maker's payoff may be reduced if the biased expert becomes more concerned about his reputation.

Finally, I may let the expert be a long-run player who interacts with a sequence of short-run players and thus endogenize the expert's reputational concern. I leave these extensions for future studies.

Appendix A: A general description of the decision maker's optimal actions and posterior beliefs.

In this appendix, I first relax my restrictions on the expert's strategies and provide a complete description of the decision maker's equilibrium behaviors in the last stage of the communication game. Particularly, I assume here that the biased expert is also strategic. Thus, a strategy σ_B of the biased expert also consists of five probabilities, and each probability has the same meaning as the one in the aligned expert's strategy σ_A. Moreover, the expert may also use mixed strategy on his information acquisition decision.

For $i \in \{A, B\}$, let

$$\begin{aligned}
\Pr_i(m=1|\theta=1) &= \alpha_i[p_1 y_i + (1-p_1)(1-z_i)] + (1-\alpha_i)[p_0 x_i + (1-p_0)(1-w_i)], \\
\Pr_i(m=1|\theta=0) &= \alpha_i[(1-p_1) y_i + p_1(1-z_i)] + (1-\alpha_i)[(1-p_0) x_i + p_0(1-w_i)], \\
\Pr_i(m=0|\theta=0) &= \alpha_i[p_1 z_i + (1-p_1)(1-y_i)] + (1-\alpha_i)[p_0 w_i + (1-p_0)(1-x_i)], \\
\Pr_i(m=0|\theta=1) &= \alpha_i[(1-p_1) z_i + p_1(1-y_i)] + (1-\alpha_i)[(1-p_0) w_i + p_0(1-x_i)].
\end{aligned}$$

Given the strategies σ_A and σ_B, from the perspective of the decision maker, if the state is $\theta = k$, then the expected probability that the type i of expert will send a message $m = l$ is $\Pr_i(m = l|\theta = k)$, where k, $l \in \{0, 1\}$. Thus, I have the optimal action a^*_m as follows:

$$\begin{aligned}
a^*_1 &= \tfrac{\lambda \Pr_A(m=1|\theta=1) + (1-\lambda)\Pr_B(m=1|\theta=1)}{\lambda \Pr_A(m=1|\theta=1) + (1-\lambda)\Pr_B(m=1|\theta=1) + \lambda \Pr_A(m=1|\theta=0) + (1-\lambda)\Pr_B(m=1|\theta=0)}, \\
a^*_0 &= \tfrac{\lambda \Pr_A(m=0|\theta=1) + (1-\lambda)\Pr_B(m=0|\theta=1)}{\lambda \Pr_A(m=0|\theta=1) + (1-\lambda)\Pr_B(m=0|\theta=1) + \lambda \Pr_A(m=0|\theta=0) + (1-\lambda)\Pr_B(m=0|\theta=0)}.
\end{aligned}$$

Also, I have the equilibrium belief $\phi^*(\lambda|m, \theta)$ as follows:

$$\begin{aligned}
\phi^*(\lambda|1,1) &= \tfrac{\lambda \Pr_A(m=1|\theta=1)}{\lambda \Pr_A(m=1|\theta=1) + (1-\lambda)\Pr_B(m=1|\theta=1)}, \\
\phi^*(\lambda|1,0) &= \tfrac{\lambda \Pr_A(m=1|\theta=0)}{\lambda \Pr_A(m=1|\theta=0) + (1-\lambda)\Pr_B(m=1|\theta=0)}, \\
\phi^*(\lambda|0,0) &= \tfrac{\lambda \Pr_A(m=0|\theta=0)}{\lambda \Pr_A(m=0|\theta=0) + (1-\lambda)\Pr_B(m=0|\theta=0)}, \\
\phi^*(\lambda|0,1) &= \tfrac{\lambda \Pr_A(m=0|\theta=1)}{\lambda \Pr_A(m=0|\theta=1) + (1-\lambda)\Pr_B(m=0|\theta=1)}.
\end{aligned}$$

In the main equilibrium analysis, the biased expert is assumed to be non-strategic in a way such that he does not acquire better information and always sends the message $m = 1$. In this case, the optimal actions and posterior beliefs can be modified with the terms $\alpha_B = 0$, $w_B = 0$ and $x_B = 1$.

Appendix B: Proofs.

The proof of Lemma 3.

Proof. First suppose that there exists an informative equilibrium in which $\alpha_A^* = 0$. So the accuracy of the signal is p_0.

With the assumption that the biased expert does not acquire better information and always sends the message $m = 1$, if in this equilibrium $a_1^* > a_0^*$, I can verify that $\phi^*(\lambda|0,0) = \phi^*(\lambda|0,1) = 1$, $\phi^*(\lambda|1,0) \leq \lambda$, $\phi^*(\lambda|1,1) \leq \lambda$, and $1 - p_0 \leq a_0^* < a_1^* \leq p_0$ for any equilibrium strategies x_A^* and w_A^*. Consider the case that the aligned expert has a signal $s = 0$. By sending message $m = 1$, his payoff, which is denoted by v_1, is

$$v_1 = -p_0(a_1^* - 0)^2 - (1 - p_0)(a_1^* - 1)^2 + \mu[p_0\phi^*(\lambda|1,0) + (1 - p_0)\phi^*(\lambda|1,1)].$$

While by sending message $m = 0$, his payoff, which is denoted by v_0, is

$$v_0 = -p_0(a_0^* - 0)^2 - (1 - p_0)(a_0^* - 1)^2 + \mu.$$

This is because that from the expectation of the aligned expert, the nature state is $\theta = 0$ with probability p_0 and is $\theta = 1$ with probability $1 - p_0$. Given the conditions derived above, I have $v_0 > v_1$, which implies that in this equilibrium the aligned expert should send message $m = 0$ with probability one if his signal is $s = 0$.

Conversely, if in this equilibrium $a_1^* < a_0^*$, I have $\phi^*(\lambda|0,0) = \phi^*(\lambda|0,1) = 1$, $\phi^*(\lambda|1,0) \leq \lambda$, $\phi^*(\lambda|1,1) \leq \lambda$, and $1 - p_0 \leq a_1^* < a_0^* \leq p_0$ for any equilibrium strategies x_A^* and w_A^*. Consider the case that the aligned expert has a signal $s = 1$. By sending message $m = 1$, his payoff, which is denoted by u_1, is

$$u_1 = -p_0(a_1^* - 1)^2 - (1 - p_0)(a_1^* - 0)^2 + \mu[p_0\phi^*(\lambda|1,1) + (1 - p_0)\phi^*(\lambda|1,0)].$$

While by sending message $m = 0$, his payoff, which is denoted by u_0, is

$$u_0 = -p_0(a_0^* - 1)^2 - (1 - p_0)(a_0^* - 0)^2 + \mu.$$

Similarly, I have $u_0 > u_1$, which implies that in this equilibrium the aligned expert should send message $m = 1$ with probability zero when his signal is $s = 1$.

Modifying this argument slightly, I can show that the statement also holds if $\alpha_A^* = 1$ in this informative equilibrium. ■

The proof of Lemma 4.

Proof. Suppose there exists an informative equilibrium in which $a_1^* < a_0^*$. Let this equilibrium be denoted by the strategy profile $\sigma^* = (\sigma_A^*, \sigma_{DM}^*)$ and belief updating system $\phi^*(\lambda|m, \theta)$. Consider another strategy profile $\sigma' = (\sigma'_A, \sigma'_{DM})$ such that $\alpha'_A = \alpha_A^*$, $x'_A = 1 - w_A^*$, $w'_A = 1 - x_A^*$, $y'_A = 1 - z_A^*$, $z'_A = 1 - y_A^*$, $a'_1 = 1 - a_1^*$, $a'_0 = 1 - a_0^*$, and another belief updating system $\phi'(\lambda|m, \theta)$ such that $\phi'(\lambda|0, 0) = \phi'(\lambda|0, 1) = 1$, $\phi'(\lambda|1, 0) = \phi^*(\lambda|1, 1)$, $\phi'(\lambda|1, 1) = \phi^*(\lambda|1, 0)$. I need to show that this strategy profile σ' and belief updating system ϕ' also consist of an equilibrium.

First, notice that given the aligned expert's strategy σ'_A, the new belief updating system $\phi'(\lambda|m, \theta)$ satisfies the Bayes' rule and the decision maker's strategy σ'_{DM} is her best response. To see this, I take $\phi'(\lambda|1, 0)$ as an example. Suppose in the original equilibrium $\{\sigma^*, \phi^*\}$ I have $\alpha_A^* = 0$. With the result in Lemma 3, I have $x_A^* = 0$ and

$$\phi^*(\lambda|1, 1) = \frac{(1 - p_0)(1 - w_A^*)\lambda}{(1 - p_0)(1 - w_A^*)\lambda + (1 - \lambda)}.$$

Given the strategy σ'_A, for the belief $\phi'(\lambda|1, 0)$ to follow the Bayes' rule, it is sufficient and necessary to have the equation

$$\phi'(\lambda|1, 0) = \frac{\lambda[(1 - p_0)x'_A + p_0(1 - w'_A)]}{\lambda[(1 - p_0)x'_A + p_0(1 - w'_A)] + (1 - \lambda)}. \tag{1}$$

Since I construct the strategy profile σ' in a way such that $\phi'(\lambda|1,0) = \phi^*(\lambda|1,1)$, $x'_A = 1 - w^*_A$ and $w'_A = 1 - x^*_A$, the equation of (1) actually holds. Thus, the belief $\phi'(\lambda|1,0)$ follows the Bayes' rule. Similarly, I can verify that the construction of the other beliefs also follows the Bayes' rule and the decision maker's actions to take are optimal.

Second, notice that given the decision maker's strategy σ'_{DM} and belief updating system $\phi'(\lambda|m,\theta)$, the aligned expert's strategy σ'_A is his best response. To see this, I consider a potential deviation as an example. Suppose $\alpha^*_A = 0$ in the original equilibrium. By the informativeness of this equilibrium and the result in Lemma 3, it is necessary that $w^*_A < 1$ and $x^*_A = 0$. If the aligned expert has a signal $s = 0$, the inequality

$$\begin{aligned} & -p_0(a^*_1 - 0)^2 - (1-p_0)(a^*_1 - 1)^2 + \mu[p_0\phi^*(\lambda|1,0) + (1-p_0)\phi^*(\lambda|1,1)] \\ \geq\ & -p_0(a^*_0 - 0)^2 - (1-p_0)(a^*_0 - 1)^2 + \mu \end{aligned} \tag{2}$$

holds because the requirement of equilibrium behaviors; that is, the aligned expert has a (weakly) larger payoff by sending message $m = 1$ than by sending message $m = 0$. Now, given the new strategy profile σ' and belief updating system ϕ', I need to show that if the aligned expert has a signal $s = 1$, he (weakly) prefers to sending message $m = 1$ than sending message $m = 0$ because of $x'_A = 1 - w^*_A > 0$. That is, I need the inequality

$$\begin{aligned} & -p_0(a'_1 - 1)^2 - (1-p_0)(a'_1 - 0)^2 + \mu[p_0\phi'(\lambda|1,1) + (1-p_0)\phi'(\lambda|1,0)] \\ \geq\ & -p_0(a'_0 - 1)^2 - (1-p_0)(a'_0 - 0)^2 + \mu \end{aligned} \tag{3}$$

to hold. By the construction of σ' and ϕ', the inequality of (3) actually holds whenever the inequality of (2) holds. Similarly, by considering all the possible deviations, I can show that if the aligned expert has no incentive to deviate from his strategy σ^*_A in the pair of σ^* and ϕ^*, he has no incentive to deviate from his strategy σ'_A in the pair of σ' and ϕ', which establishes the optimality of his strategy σ'_A.

So the strategy profile $\sigma' = (\sigma'_A, \sigma'_{DM})$ and belief updating system $\phi'(\lambda|m,\theta)$ also consist of an informative equilibrium. Especially, in this equilibrium $a_1' > a_0'$. ■

The proof of Lemma 5.

Proof. Suppose not. Then there exists a non-informative equilibrium with $a_1^* = a_0^* = 1/2$ in which the aligned expert acquires better information. Notice that the aligned expert's payoff in this equilibrium is bounded above by $-1/4+\mu-c$, in which μ represents the largest reputational gain by separating from the biased type perfectly. However, given the decision maker's strategy, if the aligned expert deviates to effort $e = 0$ and always sends message $m = 0$, his deviation payoff is $-1/4+\mu$, which is larger than $-1/4+\mu-c$. A contradiction. Thus, if in an equilibrium the aligned type acquires better information, this equilibrium is necessarily informative. ■

The proof of Proposition 5.

Proof. First suppose that on the equilibrium path $\alpha_A^* = 1$. By the lemmas above, if the aligned expert's strategy is σ_A, the decision maker's optimal actions are

$$a_1^* = \tfrac{(1-\lambda)+\lambda p_1 y_A}{2(1-\lambda)+\lambda y_A} \quad \text{and} \quad a_0^* = \tfrac{1-p_1 y_A}{2-y_A}.$$

Also, her posterior beliefs are

$$\begin{aligned} \phi^*(\lambda|0,0) = 1 \quad \text{and} \quad \phi^*(\lambda|1,0) &= \tfrac{(1-p_1)y_A\lambda}{(1-p_1)y_A\lambda+(1-\lambda)}, \\ \phi^*(\lambda|0,1) = 1 \quad \text{and} \quad \phi^*(\lambda|1,1) &= \tfrac{p_1 y_A\lambda}{p_1 y_A\lambda+(1-\lambda)}. \end{aligned}$$

I summarize some useful properties here, which could be easily verified.

Remark 1 *For any $p_1 \in (\frac{1}{2}, 1)$ and $y_A \in [0,1]$, I have $1-p_1 \le a_0^* \le \frac{1}{2} \le a_1^* \le p_1$, $\frac{\partial a_1^*}{\partial y_A} > 0$, $\frac{\partial a_1^*}{\partial p_1} > 0$. $\frac{\partial a_0^*}{\partial y_A} < 0$, $\frac{\partial a_0^*}{\partial p_1} < 0$. $\frac{\partial \phi^*(\lambda|1,0)}{\partial y_A} > 0$, $\frac{\partial \phi^*(\lambda|1,1)}{\partial y_A} > 0$.*

For the equilibrium to be informative, it is necessary that $y_A > 0$, so the aligned expert tells truth with positive probability when his signal is $s = 1$. My work here is to check when this condition holds in equilibrium.

Consider the case that $s = 1$. From the view of the aligned expert, with probability p_1 the state is $\theta = 1$ and with probability $1 - p_1$ the state is $\theta = 0$. Let v_0 be his payoff by sending $m = 0$, I have

$$v_0 = -p_1(a_0^* - 1)^2 - (1 - p_1)(a_0^* - 0)^2 + \mu - c,$$

and let v_1 be his payoff by sending $m = 1$, I have

$$v_1 = -p_1(a_1^* - 1)^2 - (1 - p_1)(a_1^* - 0)^2 + \mu[p_1\phi^*(\lambda|1,1) + (1 - p_1)\phi^*(\lambda|1,0)] - c.$$

Notice that for any $a_0^* \in [1 - p_1, 1/2]$, $\partial v_0/\partial a_0^* > 0$. Together with $\partial a_0^*/\partial y_A < 0$ derived in Remark 1, v_0 is decreasing in y_A and is minimized at $y_A = 1$. Define this value as $v_0(1)$, I have

$$v_0(1) = -p_1^3 - (1 - p_1)^3 + \mu - c.$$

Similarly, I can show that for any $a_1^* \in [1/2, p_1]$, v_1 is increasing in y_A and is maximized at $y_A = 1$. Define this value as $v_1(1)$, I have

$$v_1(1) = -p_1(\frac{1 - \lambda p_1}{2 - \lambda})^2 - (1 - p_1)(\frac{1 - \lambda + \lambda p_1}{2 - \lambda})^2 + \mu[\frac{p_1^2\lambda}{p_1\lambda + (1 - \lambda)} + \frac{(1 - p_1)^2\lambda}{(1 - p_1)\lambda + (1 - \lambda)}] - c.$$

Given the cut-off value $\overline{\mu}$, it is easy to show that if $\mu \leq \overline{\mu}$, I have $v_1(1) \geq v_0(1)$, so sending message $m = 1$ with probability one could be the aligned expert's equilibrium behavior when his signal is $s = 1$, or say, $y_A = 1$. On the other hand, if $\mu > \overline{\mu}$, sending $m = 0$ always results the aligned expert with a payoff larger than the payoff by sending $m = 1$, so in this case the equilibrium can not be informative.

However, notice that if $\mu < \overline{\mu}$, $y_A = 1$ is not the only choice for an equilibrium to be informative. Specifically, there exists a $y_A'(\mu) \in (0, 1)$ satisfying $v_1(y_A'(\mu)) - v_0(y_A'(\mu)) = 0$,[31]

[31] Abuse the notations slightly, I use $y_A'(\cdot)$ as a function of μ, and use $v_1(\cdot)$ and $v_0(\cdot)$ as functions of $y_A'(\mu)$.

which means that there is another candidate for an equilibrium to be informative such that the aligned expert randomizes between the two messages after observing a signal $s = 1$. Also notice that $\partial y'_A(\mu)/\mu > 0$. This is because if the reputational concern increases, the aligned expert has stronger incentive to send message $m = 0$ after observing the signal $s = 1$ to earn the reputational gain by perfectly separating from the biased type. To counteract this incentive, it is necessary to increase his current payoff if he sends $m = 1$ and reduce this payoff if he sends $m = 0$, which could be done by increasing y'_A.

Now suppose that on the equilibrium path $\alpha^*_A = 0$. Change the notations correspondingly and repeat the proof derived above, I can show that for the existence of informative equilibrium, it is necessary that $\mu \leq \underline{\mu}$. ■

The proof of Proposition 6.

Proof. By Lemma 5 I know that if in an equilibrium the aligned expert acquires better information, this equilibrium must be informative. Also, by Proposition 5 I know that if in an informative equilibrium there is information acquisition, it is necessary that $\mu \leq \overline{\mu}$. So in this proof I focus my attention on $\mu \in [0, \overline{\mu}]$.

I first assume that there exists an informative equilibrium with information acquisition, and derive the decision maker's optimal responses. After that I characterize the necessary and sufficient conditions for this equilibrium to hold.

If such an equilibrium exists, given the aligned expert's strategy σ^*_A, the decision maker's equilibrium actions are

$$a^*_1 = \tfrac{(1-\lambda)+\lambda p_1 y^*_A}{2(1-\lambda)+\lambda y^*_A} \quad \text{and} \quad a^*_0 = \tfrac{1-p_1 y^*_A}{2-y^*_A},$$

and her equilibrium posterior beliefs follow:

$$\phi^*(\lambda|0,0) = 1 \quad \text{and} \quad \phi^*(\lambda|1,0) = \tfrac{(1-p_1)y^*_A\lambda}{(1-p_1)y^*_A\lambda+(1-\lambda)},$$
$$\phi^*(\lambda|0,1) = 1 \quad \text{and} \quad \phi^*(\lambda|1,1) = \tfrac{p_1 y^*_A\lambda}{p_1 y^*_A\lambda+(1-\lambda)}.$$

Because of the informativeness of this equilibrium, I have $y_A^* > 0$. With these actions and beliefs, I can express the aligned expert's equilibrium payoff as:

$$\begin{aligned} E_\theta U_A &= \frac{1}{2}\{-p_1(a_0^*-0)^2-(1-p_1)(a_0^*-1)^2+\mu\}+\frac{1}{2}\{-p_1(a_1^*-1)^2-(1-p_1)(a_1^*-0)^2 \\ &\quad +\mu[p_1\phi^*(\lambda|1,1)+(1-p_1)\phi^*(\lambda|1,0)]\}-c. \end{aligned}$$

The term in the first $\{\cdot\}$ is the aligned expert's expected payoff if he observes a signal $s=0$, and the term in the second $\{\cdot\}$ is his expected payoff if he observes a signal $s=1$. Now I consider this expert's possible deviations.

Case 1: Deviating to $e=0$ and revealing his information truthfully. Let D_1 be his deviation payoff in this case, and D_2, D_3, D_4 for the following cases. Then,

$$\begin{aligned} D_1 &= \frac{1}{2}\{-p_0(a_0^*-0)^2-(1-p_0)(a_0^*-1)^2+\mu\} \\ &\quad +\frac{1}{2}\{-p_0(a_1^*-1)^2-(1-p_0)(a_1^*-0)^2+\mu[p_0\phi^*(\lambda|1,1)+(1-p_0)\phi^*(\lambda|1,0)]\}. \end{aligned}$$

Case 2: Deviating to $e=0$ and always sending message $m=0$. Then,

$$D_2 = \frac{1}{2}\{-(a_0^*-0)^2-(a_0^*-1)^2\}+\mu.$$

Case 3: Deviating to $e=0$ and always sending message $m=1$. Then,

$$D_3 = \frac{1}{2}\{-(a_1^*-0)^2-(a_1^*-1)^2\}+\frac{1}{2}\mu\{\phi^*(\lambda|1,1)+\phi^*(\lambda|1,0)\}.$$

Case 4: Deviating to $e=0$ and always lying. Then,

$$\begin{aligned} D_4 &= \frac{1}{2}\{-p_0(a_1^*-0)^2-(1-p_0)(a_1^*-1)^2+\mu[p_0\phi^*(\lambda|1,0)+(1-p_0)\phi^*(\lambda|1,1)]\} \\ &\quad +\frac{1}{2}\{-p_0(a_0^*-1)^2-(1-p_0)(a_0^*-0)^2+\mu\}. \end{aligned}$$

I derive some useful properties here, which could be easily verified.

Remark 2 *For any* $y_A^* > 0$*, I have* $\frac{\partial(a_1^*-a_0^*)}{\partial y_A^*} > 0$*,* $\frac{\partial(a_1^*+a_0^*)}{\partial y_A^*} < 0$*,* $\frac{\partial(\phi^*(\lambda|1,1)-\phi^*(\lambda|1,0))}{\partial y_A^*} > 0$*,* $\frac{\partial(\phi^*(\lambda|1,1)+\phi^*(\lambda|1,0))}{\partial y_A^*} > 0$*.*

Notice that for any $y_A^* > 0$, $D_4 < D_1$ holds, so I can ignore case 4. Moreover, with the assumption that $2p_1 + 1 - 4p_0 \leq 0$, $D_3 \leq D_1$ holds, so I can also ignore case 3.[32] If the aligned expert deviates to $e = 0$ and uses mixed strategy in the information transmission stage, his deviation payoff is bounded above by $\max\{D_1, D_2\}$, so without loss of generality, I only need to consider the first two deviations.

By Proposition 5, I know that there are two scenarios to consider: (1) $y_A^* = 1$. (2) $y_A^* \in (0,1)$ such that the aligned expert is indifferent between sending $m = 0$ and sending $m = 1$ after observing the signal $s = 1$. I derive the existence conditions separately for these two scenarios.

Scenario 1: $y_A^* = 1$.

For notational simplicity, I continue to use a_0^*, a_1^*, $\phi^*(\lambda|1,0)$ and $\phi^*(\lambda|1,1)$ in the analysis of this scenario. But remember that all of them should be modified to incorporate the value $y_A^* = 1$. Denote

$$\begin{aligned}
&\mu' = \tfrac{(2p_0-a_0^*-a_1^*)(a_1^*-a_0^*)}{1-p_0\phi^*(\lambda|1,1)-(1-p_0)\phi^*(\lambda|1,0)},\\
&\underline{c} = (p_1 - p_0)(a_1^* - a_0^*),\\
&\overline{c} = (p_1 - p_0)(a_1^* - a_0^*) + \tfrac{\mu'}{2}(p_1 - p_0)(\phi^*(\lambda|1,1) - \phi^*(\lambda|1,0)).
\end{aligned}$$

With $\overline{\mu}$, which could be represented as $\overline{\mu} = \frac{(2p_1-a_0^*-a_1^*)(a_1^*-a_0^*)}{1-p_1\phi^*(\lambda|1,1)-(1-p_1)\phi^*(\lambda|1,0)}$ in this scenario, I have the properties such that $0 < \underline{c} < \overline{c}$ and $0 < \mu' < \overline{\mu}$. Besides, the slopes of these payoffs satisfy the condition $\frac{\partial D_1}{\partial \mu} < \frac{\partial E_\theta U_A}{\partial \mu} < \frac{\partial D_2}{\partial \mu}$. When $\mu = \mu'$, $D_1 = D_2$ holds. So $D_1 > D_2$ for $\mu < \mu'$ and $D_1 < D_2$ for $\mu > \mu'$.

[32] Here is the only reason I want to introduce the assumption. If $2p_1 + 1 - 4p_0 > 0$, there is a λ' such that for any $\lambda < \lambda'$, there exists a μ_λ such that $D_3 > D_1$ for $\mu < \mu_\lambda$. In this scenario, I need to modify the proof and compare $E_\theta U_A$ with D_3. This complicates the notations, but has no effect on the qualitative results.

Compare $E_\theta U_A$ with $\max\{D_1, D_2\}$. If $c > \overline{c}$, I have $E_\theta U_A < \max\{D_1, D_2\}$ for any μ, which implies that the aligned expert always has incentive to deviate, so this informative equilibrium with information acquisition does not exist. If $c \leq \overline{c}$, there exists a $\overline{\mu}_c \in [\mu', \overline{\mu}]$ such that for any $\mu \leq \overline{\mu}_c$, I have $E_\theta U_A \geq D_2$, and there exists a $\underline{\mu}_c \in [0, \mu']$ such that for any $\mu \geq \underline{\mu}_c$, I have $E_\theta U_A \geq D_1$. This implies that for any $\mu \in [\underline{\mu}_c, \overline{\mu}_c]$ this informative equilibrium with information acquisition exists. Specifically, I have

$$\overline{\mu}_c = \frac{(2p_1 - a_0^* - a_1^*)(a_1^* - a_0^*) - 2c}{1 - p_1\phi^*(\lambda|1,1) - (1-p_1)\phi^*(\lambda|1,0)} \quad \text{and} \quad \underline{\mu}_c = \max\{0, \frac{2c - 2(p_1 - p_0)(a_1^* - a_0^*)}{(p_1 - p_0)(\phi^*(\lambda|1,1) - \phi^*(\lambda|1,0))}\}.$$

Notice that $\underline{\mu}_c > 0$ if and only if $c > \underline{c}$.

Scenario 2: $y_A^* \in (0,1)$ such that the aligned expert is indifferent between sending $m = 0$ and sending $m = 1$ after observing the signal $s = 1$.

Similarly, I define

$$\begin{aligned}
\widetilde{\mu} &= \frac{(2p_0 - a_0^* - a_1^*)(a_1^* - a_0^*)}{1 - p_0\phi^*(\lambda|1,1) - (1-p_0)\phi^*(\lambda|1,0)}, \\
\underline{\underline{c}} &= (p_1 - p_0)(a_1^* - a_0^*), \\
\overline{\overline{c}} &= (p_1 - p_0)(a_1^* - a_0^*) + \tfrac{\widetilde{\mu}}{2}(p_1 - p_0)(\phi^*(\lambda|1,1) - \phi^*(\lambda|1,0)).
\end{aligned}$$

in which a_0^*, a_1^*, $\phi^*(\lambda|1,0)$ and $\phi^*(\lambda|1,1)$ now should be modified to incorporate the equilibrium value $y_A^* \in (0,1)$.

By a similar argument, it could be shown that for $c > \overline{\overline{c}}$, this informative equilibrium with information acquisition does not hold; and for $c \leq \overline{\overline{c}}$, there exists a $\underline{\underline{\mu}}_c \in [0, \widetilde{\mu}]$ and a $\overline{\overline{\mu}}_c \in [\widetilde{\mu}, \overline{\mu}]$ such that this informative equilibrium with information acquisition holds if and only if $\mu \in [\underline{\underline{\mu}}_c, \overline{\overline{\mu}}_c]$. Specifically, for any $c \leq \overline{\overline{c}}$ I have

$$\overline{\overline{\mu}}_c = \frac{(2p_1 - a_0^* - a_1^*)(a_1^* - a_0^*) - 2c}{1 - p_1\phi^*(\lambda|1,1) - (1-p_1)\phi^*(\lambda|1,0)} \quad \text{and} \quad \underline{\underline{\mu}}_c = \max\{0, \frac{2c - 2(p_1 - p_0)(a_1^* - a_0^*)}{(p_1 - p_0)(\phi^*(\lambda|1,1) - \phi^*(\lambda|1,0))}\}.$$

Now compare these two scenarios. Given the properties I derived in Remark 1 and 2, I have $\overline{\overline{c}} < \overline{c}$ and for any $c \leq \overline{c}$, $[\underline{\underline{\mu}}_c, \overline{\overline{\mu}}_c] \subset [\underline{\mu}_c, \overline{\mu}_c]$ (if $c \in (\overline{\overline{c}}, \overline{c}]$, let $[\underline{\underline{\mu}}_c, \overline{\overline{\mu}}_c]$ be defined by the empty set). This implies that for the existence of equilibrium with information acquisition, the condition derived in scenario 1 is sufficient and necessary. ■

The proof of Proposition 7.

Proof. Let

$$\lambda^* = \frac{1 + 4p_0(p_0 - 1)}{1 + 2p_0(p_0 - 1) + 2p_1(p_1 - 1)}.$$

If it is commonly certain that the expert is aligned, that is, $\lambda = 1$, the aligned expert's reputational gain is always μ, no matter what his decisions on information acquisition and transmission are. Modify the proof of Proposition 6 slightly, I can show that if $c \in (\underline{c}, \overline{c}]$, the most informative equilibrium is the one in which the aligned expert does not acquire better information, but he reveals his information truthfully, so $\alpha_A^* = 0$, $x_A^* = w_A^* = 1$. In this equilibrium, the decision maker's payoff is $E_\theta \Pi = p_0(p_0 - 1)$.

Consider $\lambda < 1$. By the proof of Proposition 6, I know that if $\mu \in [\underline{\mu}_c, \overline{\mu}_c]$ for any $c \in (\underline{c}, \overline{c}]$, the most informative equilibrium is the one in which the aligned expert acquires better information and reveals his information truthfully, so $\alpha_A^* = 1$, $y_A^* = z_A^* = 1$. In this equilibrium, the decision maker's payoff is $E_\theta \Pi = -\frac{1-\lambda}{2(2-\lambda)} + \frac{\lambda}{2-\lambda} p_1(p_1 - 1)$.

For $\lambda \in (\lambda^*, 1)$, I have $-\frac{1-\lambda}{2(2-\lambda)} + \frac{\lambda}{2-\lambda} p_1(p_1 - 1) > p_0(p_0 - 1)$. Thus, if $c \in (\underline{c}, \overline{c}]$, the decision maker's most informative equilibrium payoff is improved by the existence of the biased type if $\mu \in [\underline{\mu}_c, \overline{\mu}_c]$ and $\lambda \in (\lambda^*, 1)$. ■

The proof of Corollary 1.

Proof. If $\mu = 0$, apparently $\mu\phi^*(\lambda|a, \theta) = 0$ for any equilibrium action a and belief formation $\phi^*(\lambda|a, \theta)$. So in this case the aligned expert's reputational gain is constant.

If $\mu > 0$, fix the accuracy p of the signal and let $a^*(s) \in A$ defines the action that maximizes the expert's current gain when his signal is $s \in \{0, 1\}$, that is,

$$a^*(s) \in \arg\max_{a \in A} \{-p(a - s)^2 - (1 - p)(a - 1 + s)^2\}.$$

Denote $\widehat{a}$ as the largest element in A, so the biased expert always takes $\widehat{a}$. If $a^*(s) \neq \widehat{a}$ for the signal $s \in \{0,1\}$, taking $a^*(s)$ when the signal is s maximizes the aligned expert's current gain, and separates him from the biased type perfectly thus maximizes his reputational gain. In these scenarios, $a^*(s)$ is taken in equilibrium and the aligned expert's reputational gain is constant, that is, μ. If $a^*(s) = \widehat{a}$ for the signal $s \in \{0,1\}$, taking $\widehat{a}$ causes a reputational lose for the aligned expert because of $\phi^*(\lambda|\widehat{a},\theta) \leq \lambda$ for any θ. Instead, there exists an $\underline{\epsilon} > 0$ such that for any $\epsilon \in (0, \underline{\epsilon}]$, if the aligned expert takes $\widehat{a} - \epsilon$, then again he can separate from the biased type perfectly and have

$$\begin{aligned} & -p((\widehat{a}-\epsilon)-s)^2 - (1-p)((\widehat{a}-\epsilon)-1+s)^2 + \mu \\ > \quad & -p(\widehat{a}-s)^2 - (1-p)(\widehat{a}-1+s)^2 + \lambda\mu, \end{aligned}$$

which implies that the loss on the current gain by taking action $\widehat{a} - \epsilon$ is outweighed by the increased benefit on the reputational gain. So in these scenarios the aligned expert's also separates from the biased type perfectly, and his reputational gain is constant. ■

The proof of Lemma 6.

Proof. I denote the cutoff values first. Let

$$\overline{a}_0 = \tfrac{1-\lambda+\lambda p_0}{2-\lambda} \quad \text{and} \quad \overline{a}_1 = \tfrac{1-\lambda+\lambda p_1}{2-\lambda}.$$

I have $\overline{a}_1 > \overline{a}_0$. Also let

$$c_u = (p_1 - p_0)(p_1 + p_0 - 1),$$

and

$$c_r = \begin{cases} p_0(1-p_0) - \frac{2p_1(p_1-1)(\lambda^2-2)+(1-\lambda)^2}{2(\lambda-2)^2} & \text{if} \quad \lambda \geq \frac{2p_0-1}{p_1+p_0-1} \\ \frac{1}{2}(p_1-p_0)(p_1+p_0+\frac{2\lambda p_1-2}{2-\lambda}) & \text{if} \quad \lambda < \frac{2p_0-1}{p_1+p_0-1} \end{cases}.$$

If the delegation set is unrestricted, then the set is $A = [0,1]$. By Corollary 1, the aligned expert's reputational gain is μ. I consider the limiting payoffs that $\epsilon \to 0$ if $a^*(s) = \widehat{a}$ in the remainder of this paper. Now consider the aligned expert's information acquisition decision. If better information is acquired, he takes $a^* = p_1$ if the signal is $s = 1$ and takes $a^* = 1 - p_1$ if the signal is $s = 0$. Thus his expected payoff is $p_1(p_1 - 1) + \mu - c$ in this case. If better information is not acquired, he takes $a^* = p_0$ if the signal is $s = 1$ and takes $a^* = 1 - p_0$ if the signal is $s = 0$. Thus his expected payoff is $p_0(p_0 - 1) + \mu$ in this case. Comparing these two cases, the aligned expert acquires better information if and only if $c \leq c_u$.

If the delegation set is optimally restricted, I have the decision maker's problem as

$$\max_{\{\underline{a},\overline{a}\}} \lambda\{\frac{1}{2}[-p(\underline{a}^*)^2-(1-p)(\underline{a}^*-1)^2]+\frac{1}{2}[-p(\overline{a}^*-1)^2-(1-p)(\overline{a}^*)^2]\}+(1-\lambda)\{-\frac{1}{2}(\overline{a}-1)^2-\frac{1}{2}\overline{a}^2\}$$

in which p is the accuracy of the signal, $\underline{a}^* \geq \underline{a}$ is the optimal action taken by the aligned expert if his signal is $s = 0$, and $\overline{a}^* \leq \overline{a}$ is the optimal action taken by the aligned expert is his signal is $s = 1$. It can be easily verified that let $\underline{a} = 0$ is (weakly) optimal for the decision maker because it can induce the aligned expert to take action $\underline{a}^* = 1-p$ if his signal is $s = 0$. Thus, in the optimally restricted delegation set, the floor could be $\underline{a} = 0$.

On the other hand, $\overline{a}^* = \overline{a}$ should hold in equilibrium. If it is not, which means that given $\overline{a}$ and p the aligned expert's optimal action $\overline{a}^* \geq 1/2$ is lower than $\overline{a}$ after observing a signal $s = 1$, the decision maker can further lower $\overline{a}$ to reduce the action distortion introduced by the biased type. Thus, I only need to consider $\overline{a}^* = \overline{a}$ in equilibrium. First Order Condition of the decision maker's problem shows that $\overline{a} = (1 - \lambda + \lambda p)/(2 - \lambda)$. So, if the decision maker can expect that the aligned expert acquires better information, then $\overline{a}$ should be $\overline{a}_1$, otherwise it is $\overline{a}_0$.

Now suppose that there exists an equilibrium in which $\overline{a} = \overline{a}_1$ and the aligned expert acquires information. I need to derive the conditions for this equilibrium to hold. In this equilibrium, the aligned expert's expected payoff is

$$E_\theta U_A = \frac{1}{2}p_1(p_1 - 1) + \frac{1}{2}[-p_1(\overline{a}_1 - 1)^2 - (1 - p_1)(\overline{a}_1 - 0)^2] + \mu - c.$$

If he deviates to $e = 0$, then after signal $s = 0$ he takes action $1 - p_0$, while after signal $s = 1$ he takes action p_0 if $p_0 \leq \overline{a}_1$ and takes action $\overline{a}$ if $p_0 > \overline{a}_1$. Thus, two cases should be considered.

Case 1: $\lambda \geq \frac{2p_0-1}{p_1+p_0-1}$. In the case I have $p_0 \leq \overline{a}_1$ and so after the deviation the aligned expert takes action p_0 if his signal is $s = 1$. His deviation payoff is

$$D_1 = p_0(p_0 - 1) + \mu.$$

If $c \leq c_r$, the payoff $E_\theta U_A$ is larger than D_1, so the equilibrium actually exists.

Case 2: $\lambda < \frac{2p_0-1}{p_1+p_0-1}$. In this case I have $p_0 > \overline{a}_1$ and so after the deviation the aligned expert takes action $\overline{a}_1$ if his signal is $s = 1$. His deviation payoff is

$$D_2 = \frac{1}{2}p_0(p_0 - 1) + \frac{1}{2}[-p_0(\overline{a}_1 - 1)^2 - (1 - p_0)(\overline{a}_1 - 0)^2] + \mu.$$

Similarly, if $c \leq c_r$, the payoff $E_\theta U_A$ is larger than D_2, so the equilibrium also exists in this case. ■

The proof of Corollary 2.

Proof. Suppose $c \leq \overline{c}$ and $\mu \in [\underline{\mu}_c, \overline{\mu}_c]$, so under communication, there is information acquisition in the most informative equilibrium.

Notice that $\underline{c} = \frac{1}{2-\lambda}(2p_1 - 1)(p_1 - p_0)$ and both $\underline{c}$ and $\overline{c}$ converge to $\frac{1}{2}(2p_1 - 1)(p_1 - p_0)$ if $\lambda \to 0$, which would be less than $c_u = (p_1 - p_0)(p_1 + p_0 - 1)$. But if $\lambda \to 1$, $\underline{c}$ converges to $(2p_1 - 1)(p_1 - p_0)$, which is larger than c_u. So there is a $\widehat{\lambda}$ such that if $\lambda > \widehat{\lambda}$, compared with communication, unrestricted delegation reduces the aligned expert's information acquisition incentive.

On the other hand, I have $\underline{c} > c_r$ for any λ. So compared with communication, optimally restricted delegation always reduces the aligned expert's information acquisition incentive. ■

The proof of Proposition 8.

Proof. Notice that $c_r < \underline{c}$. By Proposition 6, if $c \in (c_r, \overline{c}]$ and $\mu \in [\underline{\mu}_c, \overline{\mu}_c]$, there exists an informative equilibrium with information acquisition. Thus, the most informative equilibrium is the one in which the aligned expert acquires better information and reveals his information truthfully. In this equilibrium, the decision maker's payoff is

$$E_\theta \Pi = -\frac{1-\lambda}{2(2-\lambda)} + \frac{\lambda}{2-\lambda}(p_1^2 - p_1). \tag{4}$$

On the other hand, given $c > c_r$, the optimally restricted delegation set is $[0, \overline{a}_0]$ and there is no information acquisition under this delegation. So in this equilibrium the decision maker's payoff is

$$E_\theta \Pi = -\frac{1-\lambda}{2(2-\lambda)} + \frac{\lambda}{2-\lambda}(p_0^2 - p_0). \tag{5}$$

Apparently, the payoff of (4) is strictly larger than the payoff of (5) if $p_1 > p_0 > 1/2$. So if $c \in (c_r, \overline{c}]$ and $\mu \in [\underline{\mu}_c, \overline{\mu}_c]$, the decision maker strictly prefers communication with the most informative equilibrium to the optimally restricted delegation.

Conversely, suppose $c \notin (c_r, \overline{c}]$ or $\mu \notin [\underline{\mu}_c, \overline{\mu}_c]$ (if $[\underline{\mu}_c, \overline{\mu}_c]$ does not exist, let it be defined by the empty set). If $c = c_r$ and $\mu \in [\underline{\mu}_c, \overline{\mu}_c]$, then the decision maker's most informative equilibrium payoff under communication is still the payoff of (4). However, under the optimally restricted delegation, the aligned expert acquires better information and the decision maker's payoff is also the payoff of (4). So in this scenario, the decision maker is indifferent between communication and the optimally restricted delegation.

If $c \notin [c_r, \overline{c}]$ or $\mu \notin [\underline{\mu}_c, \overline{\mu}_c]$, there is no informative equilibrium with information acquisition under communication. Thus, in this scenario the decision maker's largest payoff is the payoff of (5), which is obtained in the equilibrium in which the aligned expert does not acquire better information, but he reveals his information truthfully. However, under the optimally restricted delegation, the decision maker's payoff is either the payoff of (4) or the payoff of (5). So with these parameter ranges, the decision maker weakly prefers the optimally restricted delegation to the communication. Together with all the analysis, $c \in (c_r, \overline{c}]$ and $\mu \in [\underline{\mu}_c, \overline{\mu}_c]$ are both sufficient and necessary for the decision maker to strictly prefer communication with the most informative equilibrium to the optimally restricted delegation.

■

References

[1] Aghion, Philippe., and Tirole, Jean. 1997. "Formal and Real Authority in Organizations." Journal of Political Economy, 105(1): 1-29.

[2] Alonso, Ricardo., Matouschek, Niko. 2008. "Optimal Delegation." Review of Economic Studies. 75(1): 259-293.

[3] Benabou, Roland., and Laroque, Guy. 1992. "Using Privileged Information to Manipulate Markets: Insiders, Gurus, and Credibility." Quarterly Journal of Economics, 107(3): 921–958.

[4] Brandenburger, Adam., and Polak, Ben. 1996. "When Managers Cover Their Posteriors: Making the Decisions the Market Wants to See." RAND Journal of Economics, 27(3): 523-541.

[5] Che, Yeon-Koo., and Kartik, Navin. 2009. "Opinions as Incentives." Journal of Political Economy, 117(5): 815-860.

[6] Crawford, Vincent P., and Sobel, Joel. 1982. "Strategic Information Transmission." Econometrica, 50(6): 1431–1451.

[7] Dessein, Wouter. 2002. "Authority and Communication in Organizations." Review of Economic Studies, 69 (4), 811–838.

[8] Dur, Robert., and Swank, Otto. 2005. "Producing and Manipulating Information." Economic Journal. 115(01): 185-199.

[9] Durbin, Erik, and Iyer, Ganesh. 2009. "Corruptible Advice." American Economic Journal: Microeconomics, 1:2, 220-242.

[10] Ely, Jeffrey C., and Valimaki, Juuso. 2003. "Bad Reputation." Quarterly Journal of Economics, 118(3): 785–814.

[11] Ely, Jeffrey., Fudenberg, Drew., and Levine, David K. 2008. "When Is Reputation Bad?" Games and Economic Behavior, 63(2): 498–526.

[12] Farrell, Joseph., and Gibbons, Robert. 1989. "Cheap Talk with Two Audiences." American Economic Review, 79(5): 1214–1223.

[13] Gentzkow, Matthew., and Shapiro, Jesse M., 2006. "Media Bias and Reputation." Journal of Political Economy, 114(2): 280-316.

[14] Gerardi, Dino., and Yariv, Leeat. 2008. "Costly Expertise." American Economic Review, 98(2): 187-193.

[15] Holmstrom, Bengt. 1977. "On Incentives and Control in Organizations." (Ph.D. Thesis, Stanford University).

[16] Holmstrom, Bengt. 1984. "On the Theory of Delegation." in M. Boyer, and R. Kihlstrom (eds.) Bayesian Models in Economic Theory (New York: North-Holland) 115–141.

[17] Krishna, Vijay., and Morgan, John. 2001. "A Model of Expertise." Quarterly Journal of Economics, 116(2): 747–775.

[18] Krishna, Vijay., and Morgan, John. 2008. "Contracting for Information under Imperfect Commitment." RAND Journal of Economics. 39(4): 905-925.

[19] Kovac, Eugen., and Mylovanov, Tymofiy. 2009. "Stochastic Mechanisms in Settings without Monetary Transfers: The Regular Case." Journal of Economic Theory, 144(4): 1373-1395.

[20] Levy, Gilat. 2004. "Anti-Herding and Strategic Consultation." European Economic Review, 48(3): 503-525.

[21] Li, Hao. 2001. "A Theory of Conservatism." Journal of Political Economy, 109(3): 617-636.

[22] Li, Ming., and Mylovanov, Tymofiy. 2008. "Credibility for Sale: The Effect of Disclosure on Information Acquisition and Transmission." Unpublished.

[23] Li, Wei. 2007. "Changing One's Mind When the Facts Change: Incentives of Experts and the Design of Reporting Protocols." Review of Economic Studies, 74(4): 1175–1194.

[24] Li, Wei. 2010. "Peddling Influence through Intermediaries." American Economic Review, 100(3): 1136-1162.

[25] Melumad, Nahum D., and Shibano, Toshiyuki. 1991. "Communication in Settings with No Transfers." RAND Journal of Economics, 22 (2), 173–198.

[26] Morgan, John., and Stocken, Phillip C.. 2003. "An Analysis of Stock Recommendations." RAND Journal of Economics, 34(1): 183–203.

[27] Morris, Stephen. 2001. "Political Correctness." Journal of Political Economy, 109(2): 231-265.

[28] Moscarini, Giuseppe. 2007. "Competence Implies Credibility." American Economic Review, 97(1): 37-63.

[29] Mylovanov, Tymofiy. 2008. "Veto-Based Delegation." Journal of Economic Theory, 138(1): 297-307.

[30] Ottaviani, Marco., and Sorensen, Peter Norman. 2006a. "Professional Advice." Journal of Economic Theory, 126(1): 120–142.

[31] Ottaviani, Marco., and Sorensen, Peter Norman. 2006b. "Reputational Cheap Talk." RAND Journal of Economics, 37(1): 155–175.

[32] Prat, Andrea. 2005. "The Wrong Kind of Transparency." American Economic Review, 95(3): 862–877.

[33] Prendergast, Canice., and Stole, Lars. 1996. "Impetuous Youngsters and Jaded Old-Timers: Acquiring a Reputation for Learning." Journal of Political Economy, 104(6): 1105–1134.

[34] Scharfstein, David S., and Stein, Jeremy C. 1990. "Herd Behavior and Investment." American Economic Review, 80(3): 465–479.

[35] Sobel, Joel. 1985. "A Theory of Credibility." Review of Economic Studies, 52(4): 557–573.

[36] Szalay, Dezso. 2005. "The Economics of Clear Advice and Extreme Options." Review of Economic Studies, 72(4): 1173-1198.

Chapter 3 Learning, Belief Manipulation and Optimal Relationship Termination

3.1 Introduction

I study a dynamic environment in which a principal owns a project with initially unknown quality and an agent is employed repeatedly to implement this project. The project can be either good or bad. In each period, if effort is exerted by the agent, a good project succeeds, and thereby produces a positive return, with higher probability than a bad project does. However, if the agent shirks, then the project is certain to fail independent of its type, which results in a zero return. Because of the unobservability of the agent's effort, any reward or punishment provided by the principal needs to be performance based. The purpose of this paper aims to examine the equilibrium resolution of the quality uncertainty as well as the optimal provision of incentives in the absence of long-term contracts.

The key feature in this principal-agent relationship is that the agent's hidden action can generate hidden information. For instance, if the agent deviates to shirk when he is believe to put in effort, the players' assessments about the project quality diverge in the event of a failure: the principal is more pessimistic than the agent is. Because of the nature of the project that a success generated by a bad project is more costly and thereby should be rewarded more, such a belief-misalignment enables the agent to extract informational rents from the continuation interactions. In consequence, the principal needs to provide high powered incentives to induce the agent's proper actions, either by monetary transfers or other instruments. I first show that, if only monetary payments are feasible, the agent's attempts on belief manipulation, correspondingly his informational rents, increase in his own assessment about the project quality.

This paper investigates how the principal can counteract the agent's belief manipulation incentive by introducing relationship termination; that is, an instrument other than monetary payments. From the viewpoint of the principal, terminating the relationship in the event of a failure has two opposing effects. On one side, it destroys the surplus that the principal can obtain from a relationship continuation. On the other side, by reducing the agent's continuation payoff to zero, it effectively weakens the agent's incentive to extract informational rents by inducing a belief misalignment. In consequence, whether relationship termination should be introduced in a spot contract depends on the principal's trade-off between these two effects. My main finding is that, in equilibrium, the principal's optimal provision of incentives contains relationship termination if and only if her belief about the project quality is above a threshold. In other words, relationship termination is more likely to happen when the expected relationship value is relatively high. Notice that in this environment the instrument of relationship termination is a *substitute* to the instrument of monetary payments: the principal optimally chooses relationship termination even when monetary payments are able to provide all the incentives.

The principal's optimal rule of relationship termination has several implications on the agency cost as well as on the relationship dynamics. First, in equilibrium the agent's informational rent is non-monotonic in the common belief about the project quality: it increases when the belief is below the threshold but drops to zero when the belief is above the threshold. Second, a failure after a series of successes is more intolerable, in the sense that more likely to trigger the relationship termination, than a failure after a series of failures is. Finally, if the prior probability of the project being good is lower than the threshold, then there is a cut-off time such that the relationship is stable before this time but is subject to potential termination after this time.

In many circumstances the principal's ability to make monetary payments is limited. For instance, a government's expenditure may not exceed its fiscal budget in a particular time period. In this paper I also examine how such a limitation has effects on the principal's contractual arrangements. A key factor is that the principal must resort to relationship termination more frequently in the events of failures if she can not reward the agent sufficiently in the events of successes. I show that one period's limitation on payments may give rise to a backward-transmitting effect on the principal's provision of incentives: if the agent expects that he is more likely to be motivated by punishment instead of reward in the future, he has stronger attempt to deviate to capture the informational rents today. In consequence, the principal has to provide stronger incentive today, which may cause a new limitation on monetary payments. Notice that relationship termination serves as a *complement* to monetary payments in this environment.

3.2 Literature

My paper is related to the growing literature on career concerns initiated by Holmstrom (1999). Holmstrom (1999) studies a model in which an agent's talent is revealed over time through observations of performance, and examines how this agent's concern for a career may have beneficial or detrimental effects on his decisions. Bonatti and Horner (2012) develop a model similar to Holmstrom (1999), except that the agent's effort is complementary to his talent. They show that the equilibrium effort levels at different times are strategic substitutes and the dynamics of effort is single-peaked over time. Klein and Mylovanov (2012) consider a reputational cheap talk model and investigate the conditions for an agent's career concerns to overwhelm any myopic incentives to distort his actions and reports. In all these papers, the agent's compensation is determined by the competitive market instead of any explicit output-contingent contracts. Alternatively, Gibbons and Murphy (1992) and Meyer and Vickers (1997) incorporate explicit contracts in the context of career concern models and explore the interactions between implicit incentives and explicit incentives. Prat and Jovanovic (2012) consider a model in which both the agent and the principal can fully commit to a long-term contract. They show that incentives become easier to provide as the uncertainty about the agent's quality is gradually resolved.

Among this literature, my study is most closely related to Bhaskar (2012). In a model with a finite time horizon, Bhaskar (2012) shows that the agent's attempt to manipulate the principal's belief becomes stronger when the time horizon lengthens. I complement his result by showing that, with an infinite time horizon, the agent's incentive on belief manipulation increases when his own belief is more optimistic. Most importantly, I show that the principal can effectively reduce the dynamic agency cost by introducing relationship termination as a substitute to monetary transfers, which is absent in Bhaskar (2012) as well as in the aforementioned papers.

My study is also related to the literature on strategical experimentation. Bergemann and Hege (2005) and Horner and Samuelson (2012) develop venture capital financing models that mainly differ in the allocation of the bargaining power between the principal and the agent. They show that equilibrium financing stops too early compared to the socially efficient stopping time. In a sequential testing context without financing, Gerardi and Maestri (2012) investigate the optimal long-term contracts for the principal to induce the agent's information acquisition and truth reporting. I share the similarity with these papers that the quality of the project is learned over time and the agent's hidden action may generate hidden information. However, because in these papers at most one success can be achieved or reported, the structures of incentive provision are quite different.

Finally, my study is related to the papers on dynamic moral hazard and dynamic adverse selection, especially to the rachet effect discussed in Laffont and Tirole (1988). Laffont and Tirole (1988) show that in a repeated agency problem the rachet effect leads too much pooling of the agent's choices in the beginning of the relationship. My main departure is that, in my model, the agent has to learn his own type gradually and thereby his incentive to manipulate the principal's belief varies over time.

3.3 The model

Consider a dynamic game in which time is discrete and indexed by $t = 0, 1, ..., \infty$. There are two players: a principal (P or she) and an agent (A or he). The principal owns a project with initially unknown quality. The project is good with prior probability $\mu_0 \in (0,1)$ and is bad with prior probability $1-\mu_0$. The agent is hired to implement the project repeatedly and in each period he can either work or shirk. If the agent works, a good project succeeds with certainty whereas a bad project only succeeds with probability $p \in (0,1)$. If the agent shirks, then the project fails for sure no matter what the type it is. A success generates a return $R > 0$ to the principal, but a failure only generates a return 0. Both of the players are risk neutral and share a common discount factor $\delta \in (0,1)$.

I consider a particular circumstance in which the players can not commit to long-term contracts except that their agreements on relationship termination are credible. In period t, if the relationship is still ongoing, the time structure is as follows. The principal maximizes her expected payoff by providing a spot contract $\varphi_t = \{b_t^S, b_t^F; d_t^S, d_t^F\}$ to the agent, in which b_t^i is the monetary payment to the agent and d_t^i is the probability of relationship continuation if the realized outcome is $i \in \{S, F\}$, where S indicates "success" and F indicates "failure". Given the contract φ_t, the agent decides whether to accept or reject it, which is denoted by $a_t = 1$ and $a_t = 0$ correspondingly. If he accepts the contract, then he further decides whether to work, which is indexed by $e_t = 1$ and costs $c > 0$, or shirk, which is indexed by $e_t = 0$ and costs 0. After that, an outcome $i \in \{S, F\}$ is realized and payment b_t^i is delivered. With probability d_t^i the relationship extends to the next period and with the complementary probability $1 - d_t^i$ the game is ended. Both of the players' outside option values are normalized to be zero. In particular, the agent is protected by limited liability. Thus, $b_t^i \geq 0$ for any $i \in \{S, F\}$. I assume that $pR > c$, which indicates that even a bad project is socially valuable. The agent's choice on working or shirking is unobservable to the principal. In consequence, the principal's contract proposal can only be contingent on the realized outcomes.

The uncertainty about the project quality is resolved over time. An important feature in this game is that the agent's hidden action gives rise to hidden information and the players' assessments about the quality may diverge. Let μ_t be the principal's belief, or the *public* belief, and $\widehat{\mu}_t$ be the agent's *private* belief about the project quality at the beginning of period t. Define

$$\mu_t^S = \frac{\mu_t}{\mu_t + (1-\mu_t)p} \text{ and } \mu_t^F = 0.$$

Thus, if the principal expects that the agent works in period t, after observing a success or a failure, her posterior belief is represented by μ_t^S or μ_t^F respectively. In a similar manner, I can define the agent's posterior beliefs by $\widehat{\mu}_t^S$ and $\widehat{\mu}_t^F$ corresponding to the realized outcomes. I call a belief as a *common* belief when $\mu_t = \widehat{\mu}_t$.

Contingent on the game has not been terminated, a public history h_t summarizes all the observable decisions and realized outcomes up to period t; that is, for $i \in \{S, F\}$, $h_t = \{\varphi_0, a_0, i_0, ..., \varphi_{t-1}, a_{t-1}, i_{t-1}\}$. In addition, the agent has a private history $\widehat{h}_t$ due to his unobservable choices on working and shirking; that is, $\widehat{h}_t = \{\varphi_0, a_0, e_0, i_0, ..., \varphi_{t-1}, a_{t-1}, e_{t-1}, i_{t-1}\}$. Let H_t and $\widehat{H}_t$ be the sets of all public and private histories. Thus, the evolution of the players' posterior beliefs can be denoted by $\mu_t := \mu_t(h_t)$ and $\widehat{\mu}_t := \widehat{\mu}_t(\widehat{h}_t)$. I examine Markovian Equilibrium in this study. A Markovian strategy of the principal specifies a contract φ_t in period t that depends on the public belief μ_t; that is, $\varphi_t := \varphi(\mu_t)$. A Markovian strategy of the agent specifies actions a_t and e_t in period t that depend on the public belief μ_t and his private belief $\widehat{\mu}_t$; that is, $a_t := a(\widehat{\mu}_t; \mu_t)$ and $e_t := e(\widehat{\mu}_t; \mu_t)$. A strategy profile and a belief updating system consist of a Markovian Equilibrium if, given the other player's strategy, each player's strategy maximizes his/her payoff, and belief updating follows Bayes' rule whenever possible. I restrict my attention to the equilibria in which the agent only takes pure actions on the equilibrium path. Notice that by this requirement, the players' beliefs necessarily coincide on the equilibrium path. In consequence, belief misalignment can occur only when the agent deviates.

Because $pR > c$, the first-best policy requires that the relationship continues forever and the agent works in each period. Let $S(\mu_t)$ be the expected surplus from the first-best policy if the players has a common belief μ_t in period t. Thus, I have

$$S(\mu_t) = \frac{1}{1-\delta}((\mu_t + (1-\mu_t)p)R - c).$$

Alternatively, I demonstrate $S(\mu_t)$ as the players' expected relationship value, or relationship value for short. Notice that $S(\mu_t)$ increases in μ_t, which implies that the higher the players' common assessment about the project quality is, the larger is the relationship value.

3.4 Sequential contracts without relationship termination

In this section, I consider the baseline model in which any agreement on relationship termination is not credible; that is, it is commonly known that in each period the principal can propose a new contract and the agent can choose to accept or reject. Thus, the principal's contract $\varphi(\mu_t)$ only specifies wage payments $b^S(\mu_t)$ and $b^F(\mu_t)$. I mainly address how the agent's belief about the project quality affects his incentive to manipulate the principal's belief.

I start the analysis by arguing that, in any equilibrium, the agent participates and works in each period. If there is an equilibrium in which the agent rejects the principal's contract proposal $\varphi(\mu_t)$ in period t at a common belief μ_t, then no information is generated in this period and the common beliefs follow $\mu_{t+1} = \mu_t$. In consequence, the principal should offer the same contract in period $t+1$, $\varphi(\mu_{t+1}) = \varphi(\mu_t)$, and the agent should reject it. Recursively, there is no working, thus no positive surplus to share, starting from period t. However, if the principal deviates to a contract proposal $\widetilde{\varphi}(\mu_t)$ in period t with $\widetilde{b}^S(\mu_t) = c/p + \epsilon$ and $\widetilde{b}^F(\mu_t) = 0$, where $\epsilon > 0$ but is arbitrarily small, the agent should accept it and works, which gives both players positive payoffs. This establishes my argument.

Let $V(\widehat{\mu}_t; \mu_t)$ be the agent's equilibrium payoff depending on his private belief $\widehat{\mu}_t$ and the principal's belief μ_t. In period t, the equilibrium contract $\varphi(\mu_t)$ has to satisfy the agent's incentive comparability constraint, which is given by

$$\begin{aligned} V(\mu_t;\mu_t) &= (\mu_t + (1-\mu_t)p)(b^S(\mu_t) + \delta V(\mu_t^S;\mu_t^S)) \\ &\quad +(1-\mu_t-(1-\mu_t)p)(b^F(\mu_t) + \delta V(0;0)) - c \\ &\geq b^F(\mu_t) + \delta V(\mu_t;0). \end{aligned} \tag{IC-1}$$

Notice that on the equilibrium path the players' beliefs coincide. If the agent works in period t, with probability $\mu_t+(1-\mu_t)p$ the project succeeds, the agent obtains current wage payment $b^S(\mu_t)$ and his discounted continuation value is $\delta V(\mu_t^S;\mu_t^S)$; with probability $1-\mu_t-(1-\mu_t)p$ the project fails, the agent obtains current payment $b^F(\mu_t)$ and his discounted continuation value is $\delta V(0;0)$. If the agent shirks, the project fails for sure, and his current wage payment is $b^F(\mu_t)$. Most importantly, because the agent's shirking is an unobservable deviation in equilibrium, the principal believes that the project is bad with certainty in the event of a failure, whereas the agent's private belief is unchanged, $\widehat{\mu}_{t+1} = \widehat{\mu}_t = \mu_t$. Thus, the agent's discounted continuation payoff is represented by $\delta V(\mu_t;0)$ when he shirks.

Moreover, the equilibrium contract $\varphi(\mu_t)$ also needs to satisfy the agent's individual rationality constraint

$$V(\mu_t;\mu_t) \geq 0, \tag{IR-1}$$

and the limited liability constraint

$$b^S(\mu_t),\, b^F(\mu_t) \geq 0. \tag{LL-1}$$

Let $\Pi(\mu_t)$ be the principal's equilibrium payoff depending on her belief μ_t. In period t, the principal's problem is

$$\Pi(\mu_t) = \max_{\varphi(\mu_t)} (\mu_t + (1-\mu_t)p)(R - b^S(\mu_t) + \delta\Pi(\mu_t^S)) + (1-\mu_t-(1-\mu_t)p)(0 - b^F(\mu_t) + \delta\Pi(0))$$

$$s.t.\ \ \text{(IC-1), (IR-1), (LL-1).} \tag{P-1}$$

Because of limited liability, the agent can guarantee a non-negative payoff in any equilibrium by shirking in each period; that is, $V(\widehat{\mu}_t; \mu_t) \geq 0$ for any $\widehat{\mu}_t$ and μ_t. As a result, I can omit the constraint (IR-1). If in period t the common belief is $\mu_t = 0$, then the problem degenerates to a repeated moral hazard problem thereafter. By standard argument, I can show that the principal provides a stationary contract $\varphi(0) = \{c/p, 0\}$ in each period $\tau \geq t$ and captures all surplus. Thus, I have

$$\Pi(0) = \frac{pR - c}{1 - \delta}, \quad \text{and} \quad V(0; 0) = 0.$$

These findings help to characterize the agent's payoff $V(\mu_t; \mu_t)$.

Lemma 7 *Without relationship termination, in equilibrium the constraint (IC-1) satisfies*

$$V(\mu_t; \mu_t) = \delta V(\mu_t; 0) = \frac{\delta(1-p)c}{(1-\delta)p}\mu_t. \tag{1}$$

The proofs of the results are relegated in the appendix. This result shows that the agent can earn informational rents in equilibrium because of his ability to manipulate the principal's beliefs. Intuitively, if the principal infers that the project is bad with certainty after observing a failure, she would propose a stationary contract $\varphi(0) = \{c/p, 0\}$ in any period thereafter, which provides the most promising reward on successes. By manipulating the principal's inference strategically, the agent obtains positive payoffs in the event that the project is actually good. In addition, the more optimistic the agent himself is, the stronger is his attempt on belief manipulation, which could be seen from the fact that $V(\mu_t; 0)$ is an increasing function of μ_t.

I proceed to solve for the wage payment $b^S(\mu_t)$ in the equilibrium condition (1), which can be expanded as

$$V(\mu_t;\mu_t) = (\mu_t + (1-\mu_t)p)(b^S(\mu_t) + \delta V(\mu_t^S;\mu_t^S)) - c = \frac{\delta(1-p)c}{(1-\delta)p}\mu_t.$$

Replace $V(\mu_t^S;\mu_t^S)$ recursively I obtain

$$b^S(\mu_t) = \frac{c}{\mu_t + (1-\mu_t)p} + \frac{\delta(1-p)c\mu_t}{p(\mu_t + (1-\mu_t)p)}. \tag{2}$$

If the project is implemented only once with common belief μ_t, the principal's equilibrium contract is given by $\underline{\varphi}(\mu_t) = \{\underline{b}(\mu_t), 0\}$ with $\underline{b}(\mu_t) = c/(\mu_t + (1-\mu_t)p)$, which induces the agent's working but leaves him no rent. In contrast, in my dynamic setup with repeated project implementation, to counteract the agent's attempt on belief manipulation, the principal needs to provide a wage premium in the event of a success; that is, $b^S(\mu_t)$ in (2) satisfies $b^S(\mu_t) > \underline{b}(\mu_t)$. Moreover, due to the fact that the agent's manipulation attempt increases in μ_t, the principal's expected wage payment $(\mu_t + (1-\mu_t)p)b^S(\mu_t)$ also increases in μ_t.

With the characterization of the equilibrium contract, the principal's equilibrium payoff is given by

$$\Pi(\mu_t) = (\mu_t + (1-\mu_t)p)(R - b^S(\mu_t) + \delta\Pi(\mu_t^S)) + (1 - \mu_t - (1-\mu_t)p)\delta\Pi(0).$$

Plug in $b^S(\mu_t)$ and expand $\Pi(\mu_t^S)$ recursively, I have

$$\Pi(\mu_t) = \frac{(\mu_t + (1-\mu_t)p)R - c}{1-\delta} - \frac{\delta(1-p)c}{(1-\delta)p}\mu_t. \tag{3}$$

Notice that this payoff is actually the difference between the social surplus $S(\mu_t)$ and the agent's rents $V(\mu_t;\mu_t)$. I summarize the results in the following proposition.

Proposition 9 *Without relationship termination, the principal's equilibrium contract* $\varphi(\mu_t)$

is given by

$$b^F(\mu_t) = 0 \quad and \quad b^S(\mu_t) = \frac{c}{\mu_t + (1-\mu_t)p} + \frac{\delta(1-p)c\mu_t}{p(\mu_t + (1-\mu_t)p)},$$

and the players' equilibrium payoffs are given by

$$V(\mu_t;\mu_t) = \frac{\delta(1-p)c}{(1-\delta)p}\mu_t \quad and \quad \Pi(\mu_t) = \frac{(\mu_t + (1-\mu_t)p)R - c}{1-\delta} - \frac{\delta(1-p)c}{(1-\delta)p}\mu_t.$$

In this section, the principal can only resort to monetary transfers to induce working from the agent, and I show that the agent obtains substantial rents by his ability to create a belief misalignment. I address the effects on the players' actions and payoffs when the principal can introduce socially inefficient relationship termination in the next section.

3.5 Sequential contracts with relationship termination

In many real-world companies the wage contracts mostly are subject to renegotiation, however, it is less likely to see that a fired employee may be re-hired by the same employer. In this section I consider the circumstance that the players' agreements on relationship termination are credible. As a result, in addition to the wage payments $b^S(\mu_t)$ and $b^F(\mu_t)$, the principal's contract proposal $\varphi(\mu_t)$ also specifies the probabilities of relationship continuation, $d^S(\mu_t)$ and $d^F(\mu_t)$, contingent on the outcomes. I examine the optimality for the principal to introduce relationship termination.

Slightly modify the argument presented in the previous section, I can show that in equilibrium the agent accepts the principal's contract and works in each period as long as the relationship is ongoing. In consequence, the equilibrium contract $\varphi(\mu_t)$ proposed in period t

satisfies the agent's incentive comparability constraint

$$\begin{aligned} V(\mu_t;\mu_t) &= (\mu_t + (1-\mu_t)p)(b^S(\mu_t) + \delta d^S(\mu_t) V(\mu_t^S;\mu_t^S)) \\ &\quad +(1-\mu_t)(1-p)(b^F(\mu_t) + \delta d^F(\mu_t) V(0;0)) - c \\ &\geq b^F(\mu_t) + \delta d^F(\mu_t) V(\mu_t;0). \end{aligned} \tag{IC-2}$$

The implications of this constraint are similar to the ones of constraint (IC-1), except that here the continuation probabilities are incorporated. Moreover, $\varphi(\mu_t)$ also satisfies the agent's individual rationality constraint

$$V(\mu_t;\mu_t) \geq 0, \tag{IR-2}$$

the limited liability constraint

$$b^S(\mu_t),\, b^F(\mu_t) \geq 0, \tag{LL-2}$$

and the feasibility constraint

$$d^S(\mu_t),\, d^F(\mu_t) \in [0,1]. \tag{Fe-2}$$

With a belief μ_t in period t, the principal's problem can be stated as

$$\begin{aligned} \Pi(\mu_t) &= \max_{\varphi(\mu_t)} (\mu_t + (1-\mu_t)p)(R - b^S(\mu_t) + \delta d^S(\mu_t)\Pi(\mu_t^S)) \\ &\quad +(1-\mu_t)(1-p)(0 - b^F(\mu_t) + \delta d^F(\mu_t)\Pi(0)) \\ &\quad s.t. \;\; \text{(IC-2), (IR-2), (LL-2), (Fe-2).} \end{aligned} \tag{P-2}$$

Again, the constraint (IR-2) could be ignored because of limited liability. If in period t the principal's belief is $\mu_t = 0$, I can verify that in equilibrium the principal offers a stationary contract $\{c/p, 0; 1, 1\}$ starting from this period and the relationship is never terminated. Thus, their payoffs are given by $\Pi(0) = (pR - c)/(1-\delta)$ and $V(0;0) = 0$. With this observation, I establish the following result.

Lemma 8 *With relationship termination, in equilibrium the constraint (IC-2) satisfies*

$$V(\mu_t;\mu_t) = (\mu_t + (1-\mu_t)p)(b^S(\mu_t) + \delta V(\mu_t^S;\mu_t^S)) - c = \delta d^F(\mu_t)V(\mu_t;0) = \frac{\delta(1-p)c}{(1-\delta)p}\mu_t d^F(\mu_t). \tag{4}$$

Condition (4) states that $d^S(\mu_t) = 1$, thereby in equilibrium it is never optimal to terminate the relationship after a realized success. Intuitively, punishing the agent in the event of a success by relationship termination can only induce him to shirk with higher probability, which worsens the principal's provision of incentives. Compared to condition (1) in Lemma 7, the crucial ingredient of condition (4) is that, in equilibrium, whether the agent can obtain positive rents depends on whether the relationship termination would be triggered by a failure; that is, what $d^F(\mu_t)$ is.

I proceed to the determination of $d^F(\mu_t)$. Define

$$\mu^* = \frac{p^2R - pc}{p^2R - pc + c}.$$

It can be verified that $\mu^* \in (0,1)$. Replacing $b^S(\mu_t)$ in the principal's problem (P-2) by condition (4), I have the new problem as

$$\begin{aligned}\Pi(\mu_t) \quad = \quad & \max_{\varphi(\mu_t)} (\mu_t + (1-\mu_t)p)(R + \delta\Pi(\mu_t^S) + \delta V(\mu_t^S;\mu_t^S)) - c \\ & +(1-\mu_t)(1-p)\delta d^F(\mu_t)\Pi(0) - \delta d^F(\mu_t)V(\mu_t;0)\end{aligned}$$

subject to (LL-2) and (Fe-2). A direct observation is that the relationship continuation probability $d^F(\mu_t)$ has no effect on $\Pi(\mu_t^S)$ and $V(\mu_t^S;\mu_t^S)$, so I only need to examine the last two terms. On the equilibrium path of play, a failure reveals the project quality (being bad) perfectly and the principal's continuation payoff is uniquely determined by $\delta\Pi(0)$ if the relationship is maintained. In consequence, at the beginning of period t, if the principal intends to continue the relationship in the event of a failure, her expected payoff from this event is $(1-\mu_t)(1-p)\delta\Pi(0)$, which decreases in μ_t. However, such a relationship continuation enables the agent to obtain informational rents $\delta V(\mu_t;0)$ by belief manipulation, which increases in μ_t, and the principal has to pay a wage premium to counteract the agent's belief manipulation incentive. The principal trades off between these two opposing effects in determining the optimal rule of relationship termination and in equilibrium she follows a cut-off strategy as

$$\begin{aligned} d^F(\mu_t) = 1 \quad \text{if} \quad \mu_t \leq \mu^* \quad &\Leftrightarrow \quad \delta V(\mu_t;0) \leq (1-\mu_t)(1-p)\delta\Pi(0), \\ d^F(\mu_t) = 0 \quad \text{if} \quad \mu_t > \mu^* \quad &\Leftrightarrow \quad \delta V(\mu_t;0) > (1-\mu_t)(1-p)\delta\Pi(0). \end{aligned} \tag{5}$$

This optimal rule of relationship termination gives rise to the following insight.

Corollary 3 *In equilibrium, socially inefficient relationship termination is introduced in the contracts if and only if when the relationship value is relatively high.*

Recall that the players' relationship value $S(\mu_t)$ strictly increases in μ_t, thereby $\mu_t > \mu^*$ is equivalent to $S(\mu_t) > S(\mu^*)$. Intuitively, when the principal is sufficiently optimistic about the project quality, terminating the relationship in the event of a failure only leads to a negligible loss of future surplus on expectation, but it can reduce the informational rents captured by the agent substantially. Conversely, when the principal is extremely pessimistic about the project quality, relationship termination after a failure causes a substantial loss of future surplus on expectation, but can only save a few rents captured by the agent. As a result, relationship termination is introduced by the principal if and only if she is optimistic enough about the project quality.

Denote by

$$\mu_* = \frac{p\mu^*}{1 + p\mu^* - \mu^*}.$$

In other words, if $\mu_t = \mu_*$, then $\mu_t^S = \mu^*$. With the condition (4) shown in Lemma 8 and the optimal rule of relationship termination in (5), the principal's equilibrium wage payments in the events of successes and the agent's informational rents are given by

$$\begin{array}{llll} b^S(\mu_t) = \frac{c}{\mu_t+(1-\mu_t)p} & \text{and} \quad V(\mu_t;\mu_t) = 0 & \text{if} & \mu_t > \mu^*, \\ b^S(\mu_t) = \frac{c}{\mu_t+(1-\mu_t)p} + \frac{\delta(1-p)c}{(1-\delta)p}\frac{\mu_t}{\mu_t+(1-\mu_t)p} & \text{and} \quad V(\mu_t;\mu_t) = \frac{\delta(1-p)c}{(1-\delta)p}\mu_t & \text{if} & \mu_* < \mu_t \le \mu^*, \\ b^S(\mu_t) = \frac{c}{\mu_t+(1-\mu_t)p} + \frac{\delta(1-p)c}{p}\frac{\mu_t}{\mu_t+(1-\mu_t)p} & \text{and} \quad V(\mu_t;\mu_t) = \frac{\delta(1-p)c}{(1-\delta)p}\mu_t & \text{if} & \mu_t \le \mu_*. \end{array} \tag{6}$$

If $\mu_t \le \mu_*$, the wage payment $b^S(\mu_t)$ in (6) is the same to the one shown in (2), and the determinant of it can be explained in a similar way. If $\mu_t > \mu^*$, the wage payment $b^S(\mu_t)$ in (6) is the same to $b^*(\mu_t)$ shown in the previous section, which is because that now the agent can not benefit from belief manipulation.

The interesting case is the wage payment $b^S(\mu_t)$ in (6) when $\mu_* < \mu_t \le \mu^*$. For any common belief μ_t from the set $(\mu_*, \mu^*]$, I have $\mu_t^S > \mu^*$ and $\mu_t^F = 0$. In either case, the agent's equilibrium rent in the continuation game starting from period $t+1$ is zero. In consequence, in period t the agent has the last chance to obtain positive rent $\delta V(\mu_t; 0)$ and all rent should be paid in the wage payment $b^S(\mu_t)$ contingent on a success. In particular, if δ is sufficiently large, such a wage payment could be large enough to exceed the current project return R. Also, if $\delta > 1/2$, then in this case $b^S(\mu_t)$ strictly increases in μ_t.

Another insight can be deduced from the agent's equilibrium rents.

Corollary 4 *With optimal relationship termination, the agent's incentive on belief manipulation, as well as his equilibrium rent, is non-monotonic in μ_t.*

To see this, notice that with optimal relationship termination $V(\mu_t; \mu_t)$ first increases then decreases (drops to zero) when μ_t increases. This insight indicates that, from the agent's perspective, to start a project with a relatively pessimistic common assessment may be more promising to him. In addition, since the agency cost of this principal-agent problem is captured by $\delta V(\mu_t; 0)$ when $\mu_t \le \mu^*$ and by $(1-\mu_t)(1-p)\delta\Pi(0)$ when $\mu_t > \mu^*$, I also have a non-monotonic relationship between the agency cost and the common belief μ_t in equilibrium.

For any $\mu_t \in (0, \mu^*]$ in period t, let $\lambda(t) \in \mathbb{N}$ be a number satisfy

$$\frac{\mu_t}{\mu_t + (1-\mu_t)p^{\lambda(t)-1}} \leq \mu^* < \frac{\mu_t}{\mu_t + (1-\mu_t)p^{\lambda(t)}}.$$

The implication of $\lambda(t)$ is that, if in period t the common belief is $\mu_t \in (0, \mu^*]$, it takes at least $\lambda(t)$ successes for the updated belief to be strictly larger than μ^*. For example, if $\mu_t \in (\mu_*, \mu^*]$, then $\lambda(t) = 1$. For notational simplicity, I also let $\lambda(t) = \infty$ if $\mu_t = 0$. With this definition, the principal's equilibrium payoff $\Pi(\mu_t)$ is given by

$$\Pi(\mu_t) = \frac{(1-p)\mu_t + (1-\delta)p}{(1-\delta)(1-\delta p)}(R - \delta c) - c$$

if $\mu_t > \mu^*$, and

$$\Pi(\mu_t) = \frac{(1-p)\mu_t + (1-\delta)p}{(1-\delta)(1-\delta p)}(R - \delta c) - c + \frac{\delta(1-\mu_t)(1-p)(1-(\delta p)^{\lambda(t)})}{(1-\delta)(1-\delta p)}(pR - c) - \frac{\delta(1-p)c}{(1-\delta)p}\mu_t$$

if $\mu_t \leq \mu^*$.[33]

I introduce the following result.

Proposition 10 *If $\mu_t > 0$, the principal's equilibrium payoff with optimal relationship termination is strictly larger than her equilibrium payoff without relationship termination; if $\mu_t = 0$, her equilibrium payoffs are the same in these two cases.*

This result can be verified by directly comparing the principal's equilibrium payoffs with and without relationship termination. I omit the detailed calculation here. Because relationship termination is an instrument other than wage payments for the principal to incentivize the agent and it is employed with positive probability when $\mu_t > 0$, it is quite intuitive that

[33] In the above analysis I assume the tie-breaking rule that if the principal is indifferent between relationship termination and relationship continuation, she continues the relationship with probability one; that is, if $\mu_t = \mu^*$, then $d^F(\mu_t) = 1$. Alternatively, if the principal terminates the relationship with positive probability when she is indifferent, then the equilibrium values $b^S(\mu^*)$, $b^S(\mu_*)$, $V(\mu^*, \mu^*)$ and $\Pi(\mu_*)$ should be slightly modified. However, all the other values are unaffected. Because $\mu_t = \mu^*$ is of measure zero, using this tie-breaking rule is without loss of generality. I keep this assumption in the next section.

the principal's payoff can be improved if relationship termination is credible. Particularly, in this circumstance relationship termination serves as a substitute to monetary payments: the principal optimally employs relationship termination even monetary payments are able to provide all the incentives.

Corollary 5 *If $\mu_0 \leq \mu^*$, the players' relationship is stable in the first $\lambda(0) - 1$ periods, but it may be subject to termination after these periods.*

This insight describes the relationship dynamics. Intuitively, the first $\lambda(0) - 1$ periods of the relationship can be interpreted as a "honey-moon" phase in which failures are tolerable in the sense that no termination is required. In contrast, if the project quality is still uncertain after this phase, failures become intolerable and termination is triggered in these events.

3.6 Extensions

I consider some extensions in this section. Specifically, I investigate how the principal's optimal provision of incentives is affected if the principal's ability to make a monetary payment is limited, and if there is no limited liability on the agent.

3.6.1 Limited wage payments by the principal

In many circumstances a principal's ability on wage payments is limited. For instance, a government's feasible monetary transfer in a particular time period is constrained by its fiscal budget, and a company's monetary payment is restricted by its borrowing ability. In this subsection, I assume that in any period the principal's maximal wage payment is R and examine how this limitation has effects on the players' relationship interaction in equilibrium.[34]

[34] Assuming an upper bound on wage payments other than R would give rise similar qualitative analysis.

Denote by

$$\chi(\mu) = \frac{c}{\mu + (1-\mu)p} + \frac{\delta(1-p)c}{(1-\delta)p} \frac{\mu}{\mu + (1-\mu)p}.$$

In the previous section I have show that, if no limitation on wage payments is exerted, in equilibrium $b^S(\mu_t) < R$ for any $\mu_t > \mu^*$ or $\mu_t \leq \mu_*$, which could be seen from (6). If $b^S(\mu_t) \leq R$ also holds for any $\mu^* < \mu_t \leq \mu^*$, then exerting a payment limitation has no effect on the equilibrium contracts. For the analysis here to be non-trivial, I introduce the following assumption.

Assumption 3: $\chi(\mu^*) > R$.

The inequality holds when δ is sufficiently large. With limited wage payments, the constraint (LL-2) is modified as

$$0 \leq b^S(\mu_t),\, b^F(\mu_t) \leq R. \tag{LL-2'}$$

In consequence, now the principal's problem is

$$\begin{aligned} \Pi(\mu_t) \quad = \quad & \max_{\varphi(\mu_t)} (\mu_t + (1-\mu_t)p)(R - b^S(\mu_t) + \delta d^S(\mu_t)\Pi(\mu_t^S)) \\ & +(1-\mu_t)(1-p)(0 - b^F(\mu_t) + \delta d^F(\mu_t)\Pi(0)) \\ \\ & s.t. \;\; \text{(IC-2), (IR-2), (LL-2'), (Fe-2).} \end{aligned} \tag{P-2'}$$

I have the following result.

Lemma 9 *With limited wage payments and Assumption 3, in equilibrium there exists $\mu_t \leq \mu^*$ such that $b^S(\mu_t) = R$ and $d^F(\mu_t) < 1$.*

Compared to the results in the previous section, Lemma 9 indicates that, if the principal's ability on wage payments is limited, relationship termination may arise in the contractual arrangements even when the players' assessment about the project quality is relatively pessimistic. The intuitive reasoning is that, to incentivize the agent to work, the principal has to punish the agent more heavily in the events of failures if she can not reward him sufficiently in the events of successes.

Since on the equilibrium path of play the players' common belief either goes up gradually or drops to 0 and stays at it forever, without the limitation on payments there is at most one period's wage that can exceed R, as I have seen in the previous section. Nevertheless, an interesting implication is that exerting the limitation on payments can actually have longer effects on the principal's provision of incentives.

To see this, consider a common belief μ_t in period t such that $\mu_t^S \in (\mu_*, \mu^*]$ and $\chi(\mu_t^S) > R$. The proof of Lemma 9 shows that, if the belief is μ_t^S in period $t+1$, in equilibrium the principal's contract satisfies $b^S(\mu_t^S) = R$ and $d^F(\mu_t^S) < 1$, and the agent's payoff is $V(\mu_t^S; \mu_t^S) = (\mu_t^S + (1-\mu_t^S)p)R - c$. In other words, the limitation on the principal's payment is binding and the relationship is terminated with positive probability in the event of a failure. Now consider the belief μ_t in period t. In equilibrium, the constraint (IC-2) can be expressed by

$$V(\mu_t; \mu_t) = (\mu_t + (1-\mu_t)p)(b^S(\mu_t) + \delta((\mu_t^S + (1-\mu_t^S)p)R - c)) - c = \frac{\delta(1-p)c}{(1-\delta)p}\mu_t d^F(\mu_t).$$

If I have the further assumption that $\chi(\mu_t) - \delta((\mu_t^S + (1-\mu_t^S)p)R - c) > R$ (which holds if δ is sufficiently large), then the principal's contract in period t should satisfy $b^S(\mu_t) = R$ and $d^F(\mu_t) < 1$. In consequence, in both periods t and $t+1$ the principal's wage payments can be limited.

The reasoning goes as follows. Expecting that his continuation payoff $V(\mu_t^S; \mu_t^S)$ in period $t+1$ is lowered because of the principal's limited ability to make a payment, the agent has stronger attempt to shirk in period t when his belief is μ_t. To counteract this attempt, the principal needs to raise the wage payment $b^S(\mu_t)$ sufficiently, which may cause another payment restriction in period t. With backward induction, the larger $\chi(\mu)$ is, the longer the effect that the payment limitation can have.

From the principal's perspective, there are two types of relationship termination if a payment limitation is exerted. If $\mu > \mu^*$, relationship termination continues to be a *substitute* to wage payments. In contrast, if $\mu \leq \mu^*$, relationship termination becomes a *complement* to wage payments because the latter is not enough to incentivize the agent fully.

3.6.2 No limited liability on the agent

In this subsection I examine how important the assumption on limited liability is to my findings in the previous sections. Although assuming that the agent can not be punished by negative wage payments is quite reasonable in a variety of situations, in many others the agent can actually bear such punishments. Without limited liability, if the common belief in period t is μ_t, the principal's problem is described by

$$\begin{aligned} \Pi(\mu_t) &= \max_{\varphi(\mu_t)} (\mu_t + (1-\mu_t)p)(R - b^S(\mu_t) + \delta d^S(\mu_t)\Pi(\mu_t^S)) \\ &\quad +(1-\mu_t)(1-p)(0 - b^F(\mu_t) + \delta d^F(\mu_t)\Pi(0)) \end{aligned}$$

$$s.t. \quad \text{(IC-2), (IR-2), (Fe-2).} \tag{P-2”}$$

I have the following result.

Lemma 10 *Without limited liability on the agent, $\Pi(\mu_t) = S(\mu_t)$ is the principal's equilibrium payoff, and $V(\mu_t;\mu_t) = 0$ is the agent's equilibrium payoff. The relationship is never terminated.*

To see that this result holds, consider the following contractual arrangement proposed by the principal in period t when the common belief is μ_t:

$$d^S(\mu_t) = d^F(\mu_t) = 1,\ b^S(\mu_t) = \tfrac{c}{p} \text{ and } b^F(\mu_t) = 0 \qquad \text{if } \mu_t = 0;$$

$$d^S(\mu_t) = d^F(\mu_t) = 1,\ b^S(\mu_t) = \tfrac{c}{\mu_t+(1-\mu_t)p} + \tfrac{(1-\mu_t)(1-p)}{\mu_t+(1-\mu_t)p}\tfrac{\delta(1-p)c}{(1-\delta)p},\ b^F(\mu_t) = -\tfrac{\delta(1-p)c}{(1-\delta)p} \qquad \text{if } \mu_t > 0.$$

With this contractual arrangement, I also have $V(\mu_t;0) = (1-p)c\mu_t/((1-\delta)p)$. In period t, if the agent shirks his deviation payoff satisfies $b^F(\mu_t) + \delta V(\mu_t;0) \le 0$, while if he works his payoff satisfies $V(\mu_t;\mu_t) = 0$. As a result, the agent has no incentive to shirk in this period. Because both constraints (IR-2) and (FE-2) are satisfied, the above contractual arrangement is an equilibrium arrangement, and the players' payoffs are given by $\Pi(\mu_t) = S(\mu_t)$ and $V(\mu_t;\mu_t) = 0$. Moreover, because this arrangement is always accepted by the agent and it gives the principal all social surplus, in any equilibrium the players' payoffs should satisfy $\Pi(\mu_t) = S(\mu_t)$ and $V(\mu_t;\mu_t) = 0$, which establishes the result in Lemma 10.

In any period t with common belief μ_t, the agent's hidden action enables him to manipulate the principal's belief and thereby earn informational rents in the continuation play of the game. However, these informational rents are necessarily bounded above. Without limited liability, by designing a contract that punishes the agent severely enough in the event of a failure; that is, letting $b^F(\mu_t)$ be sufficiently negative, the principal can effectively counteract the agent's attempt on belief manipulation. In consequence, the agent's equilibrium payoff can be lowered to make the constraint (IR-2) be binding. Because no relationship termination is required, the principal captures all the relationship value in equilibrium. This result illustrates how essential the assumption on limited liability is to my results in the previous sections.

Without limited liability, an alternative way for the principal to capture social surplus is to sell the project to the agent. Apparently, by making a take-it-or-leave it offer $S(\mu_0)$ in period 0, the principal's payoff is maximized. However, if the ownership of the project can not be changed freely, as is often the case, then my characterization of the contractual arrangement above gives some insights for the principal's provision of incentives.

3.7 Conclusions

In this paper I study a dynamic agency problem in which both the principal and agent learn about the project quality over time. Because of the unobservability of his choices on working and shirking, the agent has the potential to manipulate the principal's learning process and thereby benefit from the misalignment on beliefs. In particular, the agent's attempt on belief manipulation varies in his own learning process. I examine how the principal can structure the optimal provision of incentives by a combination of monetary payments and relationship termination and show that, in equilibrium, relationship termination is introduced in the contracts only when the expected relationship value is relatively high. The optimal rule of relationship termination also gives rise to some implications on the dynamics of the agent's informational rents, of the principal's agency cost, as well as of the relationship evolvement. In addition, I examine how the contractual arrangements may be affected if the principal's ability to make wage payments is limited.

One potential extension of this study is to allow the principal (or both players) to have long-term commitment power. In this circumstance, because the principal can not renew the contract to incorporate the generated information, the agent's attempt on belief manipulation may be weakened. However, I expect that such an attempt would not vanish. The reason is, since on average a success generated by a bad project should be compensated more, the agent still can benefit from a privately more optimistic belief about the project quality. Another extension is to generalize the information structure in a way such that a good project can only succeed with probability strictly between p and 1 when the agent works. As a result, the players' posterior beliefs go down gradually in the events of failures. The challenge is that in this circumstance the agent's informational rents can not be explicitly formulated, which makes the problem of the optimal trade-off between monetary payments and relationship termination to be more involved. However, I expect that my main insights continue to hold even in this generalized setup.

In this paper I only consider the equilibria in which the agent uses pure strategy on the equilibrium path of play. Effort can be put in to examine the existence of equilibria in which the agent uses mixed strategy. In addition, there may also exist non-Markovian equilibria.

I leave these questions for the future research.

Appendix: proofs.

The proof of Lemma 7.

Proof. Notice that the equations hold when $\mu_t = 0$. In the remainder of this proof I consider the case $\mu_t > 0$.

Step 1. $b^F(\mu_t) = 0$ should hold in equilibrium, otherwise the principal can reduce the payment in the event of a failure and strictly increase her payoff without violating any constraint.

Step 2. $V(\mu_t; 0) = \frac{(1-p)c}{(1-\delta)p}\mu_t$. To see this, notice that if in period t the principal believes that the project is bad with probability one, her belief remains at it forever no matter what the agent acts. With this belief, her optimal strategy is to propose a stationary contract $\varphi(0) = \{c/p, 0\}$ in each period $\tau \geq t$, which maximizes her expected payoff.

With any private belief $\widehat{\mu}_\tau \geq 0$ in period $\tau \geq t$, the agent should work with the contract $\varphi(0)$, which is because the current payoff $(\widehat{\mu}_\tau + (1-\widehat{\mu}_\tau)p)c/p - c$ by working is larger than the current payoff 0 by shirking in period τ, and this period's action has no effect on the contract provision in the future. Thus, with private belief $\widehat{\mu}_t = \mu_t$, the agent's payoff $V(\mu_t; 0)$ is given by

$$\begin{aligned} V(\mu_t; 0) &= (\mu_t + (1-\mu_t)p)(\frac{c}{p} + \delta V(\mu_t^S; 0)) \\ &\quad + (1 - \mu_t - (1-\mu_t)p)(0 + \delta V(0; 0)) - c. \end{aligned}$$

Abusing the notation slightly, define a sequence $(\mu_t, ..., \mu_{t+l}, ...)$ satisfying $\mu_{t+l+1} = \mu_{t+l}^S$ for this step, the agent's payoff $V(\mu_t; 0)$ can be reformulated as

$$V(\mu_t; 0) = (\frac{c}{p} - \delta c)(\frac{\mu_t}{\mu_{t+1}} + \delta\frac{\mu_t}{\mu_{t+2}} + \delta^2\frac{\mu_t}{\mu_{t+3}} + ...) - c.$$

It can be verified that $\mu_t/\mu_{t+l} = \mu_t + (1-\mu_t)p^l$, thus, I have

$$V(\mu_t;0) = \frac{(1-p)c}{(1-\delta)p}\mu_t.$$

Step 3. If $b^S(\mu_t) > 0$ in equilibrium, then $V(\mu_t;\mu_t) = \delta V(\mu_t;0)$, otherwise the principal can reduce the payment $b^S(\mu_t)$ in the event of a success and strictly increase her payoff without violating any constraint.

Step 4. In equilibrium, for any $\mu_\tau \geq 0$ in period $\tau \geq t$, $b^S(\mu_\tau) > 0$. Suppose not. First consider the case that $b^S(\mu_\tau) = 0$ for any $\mu_\tau \geq 0$ in period $\tau \geq t$. But this simply implies that $V(\mu_t;\mu_t) \leq 0 < V(\mu_t;0)$, which contradicts the constraint (IC-1). Second consider the case that $b^S(\mu_\tau) > 0$ for some, but not all, beliefs μ_τ in equilibrium. Consider two consecutive periods $t+l$ and $t+l+1$ satisfying $b^S(\mu_{t+l}) = 0$ and $b^S(\mu^S_{t+l}) > 0$. By step 3, $b^S(\mu^S_{t+l}) > 0$ in period $t+l+1$ requires

$$V(\mu^S_{t+l};\mu^S_{t+l}) = \delta V(\mu^S_{t+l};0). \tag{A1}$$

On the other hand, the constraint (IC-1) in period $t+l$ should satisfy

$$V(\mu_{t+l};\mu_{t+l}) = (\mu_{t+l} + (1-\mu_{t+l})p)\delta V(\mu^S_{t+l};\mu^S_{t+l}) - c \geq \delta V(\mu_{t+l};0),$$

which requires

$$V(\mu^S_{t+l};\mu^S_{t+l}) \geq V(\mu^S_{t+l};0) + \frac{c}{\delta(\mu_{t+l} + (1-\mu_{t+l})p)} > \delta V(\mu^S_{t+l};0). \tag{A2}$$

Clearly, (A1) contradicts to (A2). Thus, the second case can not be true. ■

The proof of Lemma 8.

Proof. The reasoning for (1) $b^F(\mu_t) = 0$, (2) $b^S(\mu_t) > 0$, (3) constraint (IC-2) is binding, and (4) $V(\mu_t;0) = (1-p)c/((1-\delta)p)$ is the same to the one shown in the proof of Lemma 7. The remainder of this proof is to show that $d^S(\mu_t) = 1$; that is, the relationship should not be terminated in the event of a success.

Suppose that there exists an equilibrium in which, for some public belief μ_t, the equilibrium probability $d^S(\mu_t) < 1$. Consider a deviation by the principal such that $\widetilde{d}^S(\mu_t) = 1$ and other things unchanged. Clearly, all the constraints continue to hold. Because that $\Pi(\mu_t^S) + V(\mu_t^S;\mu_t^S) > 0$ is necessarily true in this equilibrium, if $\Pi(\mu_t^S) > 0$, then by such a deviation, the principal's payoff is strictly increased by an amount

$$(\mu_t + (1-\mu_t)p)\delta(\widetilde{d}^S(\mu_t) - d^S(\mu_t))\Pi(\mu_t^S).$$

If $\Pi(\mu_t^S) = 0$, then $V(\mu_t^S;\mu_t^S) > 0$ and such a deviation strictly increases the agent's payoff by an amount

$$(\mu_t + (1-\mu_t)p)\delta(\widetilde{d}^S(\mu_t) - d^S(\mu_t))V(\mu_t^S;\mu_t^S).$$

By reducing the wage payment $b^S(\mu_t)$ slightly, the principal can strictly increase her payoff without violating any constraint. Thus, in equilibrium $d^S(\mu_t) = 1$. ■

The proof of Lemma 9.

Proof. Notice that a necessary condition for $\chi(\mu^*) > R$ is $\delta > 1/2$. In addition, if $\delta > 1/2$, then $\chi(\mu)$ strictly increases in μ.

It can be verified that the result in Lemma 8 continues to hold even the principal's payment ability is limited. Thus, the incentive comparability constraint is

$$V(\mu_t;\mu_t) = (\mu_t + (1-\mu_t)p)(b^S(\mu_t) + \delta V(\mu_t^S;\mu_t^S)) - c = \frac{\delta(1-p)c}{(1-\delta)p}\mu_t d^F(\mu_t), \tag{A3}$$

and the principal's problem can be rearranged as

$$\begin{aligned}\Pi(\mu_t) \quad &= \quad \max_{\varphi(\mu_t)} (\mu_t + (1-\mu_t)p)(R + \delta\Pi(\mu_t^S) + \delta V(\mu_t^S;\mu_t^S)) - c \\ &\quad\quad +(1-\mu_t)(1-p)\delta d^F(\mu_t)\Pi(0) - \delta d^F(\mu_t)V(\mu_t;0)\end{aligned}$$

subject to (LL-2') and (Fe-2).

Step 1. Consider $\mu_t > \mu^*$. It is straightforward to see that the principal's equilibrium contract satisfies $d^F(\mu_t) = 0$ and $b^S(\mu_t) = c/(\mu_t + (1-\mu_t)p)$. Thus, the limitation on wage payments has no effect on the players' behaviors and payoffs when $\mu_t > \mu^*$.

Step 2. Consider $\mu_t \in (\mu_*, \mu^*]$. In this case, in equilibrium the principal chooses a probability $d^F(\mu_t)$ as large as possible, which is because $(1-\mu_t)(1-p)\Pi(0) - V(\mu_t; 0) \geq 0$ (the inequality is strict if $\mu_t < \mu^*$). Since $\mu_t^S > \mu^*$ and thereby $V(\mu_t^S; \mu_t^S) = 0$, if the agent's works in period t his payoff $V(\mu_t; \mu_t)$ in (A3) is bounded above by $(\mu_t + (1-\mu_t)p)R - c$. In consequence, his deviation payoff by shirking should be no more than this value.

Suppose $\chi(\mu_*) < R$. It can be verified that there is a $\underline{\mu} \in (\mu_*, \mu^*)$ such that

$$b^S(\mu_t) = R \text{ and } d^F(\mu_t) = \frac{(1-\delta)p}{\delta(1-p)c\mu_t}((\mu_t + (1-\mu_t)p)R - c) < 1$$

for $\mu_t \in (\underline{\mu}, \mu^*]$, and

$$b^S(\mu_t) \leq R \text{ and } d^F(\mu_t) = 1$$

for $\mu_t \in (\mu_*, \underline{\mu}]$. As a result, because of the limitation on wage payments, the principal has to terminate the relationship in the event of a failure with positive probability when $\mu_t \in (\underline{\mu}, \mu^*]$.

Alternatively, suppose $\chi(\mu_*) \geq R$. It can be verified that for any $\mu_t \in (\mu_*, \mu^*]$,

$$b^S(\mu_t) = R \text{ and } d^F(\mu_t) < 1$$

in equilibrium. Similarly, because of limited wage payments, the principal has to introduce relationship termination in the contract proposal when $\mu_t \in (\mu_*, \mu^*]$. ■

References

[1] Bergemann, Dirk., and Hege, Ulrich. 2005. "The Financing of Innovation: Learning and Stopping." RAND Journal of Economics, 36(4): 719-752.

[2] Bhaskar, V. 2012. "Dynamic Moral Hazard, Learning and Belief Manipulation." Working paper.

[3] Bonatti, Alessandro., and Horner, Johannes. 2012. "Career Concerns with Coarse Information." Working paper.

[4] Gerardi, Dino., and Maestri, Lucas. 2012. "A Principal-Agent Model of Sequential Testing." Theoretical Economics, forthcoming.

[5] Gibbons, Robert., and Murphy, Kevin J. 1992. "Optimal Incentive Contracts in the Presence of Career Concerns: Theory and Evidence." Journal of Political Economy, 100(3): 468-505.

[6] Holmstrom, Bengt. 1999. "Managerial Incentive Problems: A Dynamic Perspective." Review of Economic Studies, 66(1): 169-182.

[7] Horner, Johannes., and Samuelson, Larry. 2012. "Incentives for Experimenting Agents." Working paper.

[8] Klein, Nicolas., and Mylovanov, Tymofiy. 2012. "Will Truth Out?-An Advisor's Quest To Appear Competent." Working paper.

[9] Meyer, Margaret A., and Vickers, John. 1997. "Performance Comparisons and Dynamic Incentives." Journal of Political Economy, 105(3): 547-581.

[10] Laffont, Jean-Jacques., and Tirole, Jean. 1988. "The Dynamics of Incentive Contracts." Econometrica 56(5): 1153-1175.

[11] Prat, Julien., and Jovanovic, Boyan. 2012. "Dynamic Contracts when Agent's Quality is Unknown." Working paper.

Chapter 4 Multilateral Bargaining with an Endogenously Determined Procedure

4.1 Introduction

Conflicts and negotiations are central issues in various real-world interactions, and their significance has been long recognized from both theoretical and practical standpoints. Since Rubinstein (1982), the literature on alternative bargaining has deeply shaped our understanding of how conflicts resolution may be affected by the bargaining procedure, e.g., who makes an offer earlier and who can make offers at a higher frequency. However, most papers in this literature assume that the procedures are exogenously given and have paid little attention to how these procedures are determined. With a particular bargaining game, this paper aims to contribute to the literature by characterizing the endogeneity of bargaining procedures and their effects on equilibrium properties.

Specifically, I consider a multilateral bargaining game in which a manager negotiates with several workers sequentially, one at a time. When an agreement is reached by the manager and a worker, one unit of surplus is realized and shared by this pair of players. This negotiation process may also represent some other situations, for example, a company renews employment contracts with its senior managers, a manufacturer vertically integrates its component suppliers, or a country seeks to coordinate its international policies with those of other countries.

A key feature of my setup is that the manager can choose any (remaining) worker to bargain with, which implies that the ordering of her opponents to be at the bargaining table is endogenously determined. This point is important, because if a worker acts too aggressively, the manager can credibly threaten to move him backwards to the end of the bargaining sequence and make him suffer from the time-discounting cost. By doing so, she can significantly weaken this opponent's bargaining posture. This hold-up effect enables the manager to capture more rent from the total surplus, which is one of the main effects examined in this study.

Conversely, the workers can also hold up the manager and thus counteract her advantage in endogenously determining the bargaining sequence. Although the units of surplus are independent of one another and the manager can effectively stop the bargaining game at any time by making non-serious offers and rejecting any offer, each player knows that all of the agreements will be reached eventually in any equilibrium. This implies that during the negotiation process, if several agreements remain, one period of delay will cost the manager much more than a single worker, because the manager cares about the total units in the continuation game instead of the single one over which she is currently bargaining. In other words, the manager is relatively impatient, and this impatience enables the workers to coordinate their moves and extract more surplus from the agreements.

The interaction of these two hold-up effects results in the main properties of equilibrium in my multilateral bargaining setup. First, I derive the multiplicity of equilibrium and show that there is a set, which is represented by an interval, such that any value in this set could be the manager's equilibrium payoff. Intuitively, if the workers follow the same strategy, the manager is indifferent between any two of them to bargain with. By varying the probability of retaining the current opponent and adjusting the offers properly, I can construct an equilibrium payoff set for the manager. Second, I address the inefficiency of equilibrium. If a player expects to be rewarded in the continuation game by acting aggressively in the current period, her/his attempts to delay the agreements will increase, which will cause inefficient equilibria to arise. Moreover, I strengthen this finding by showing that the real-time delay of equilibrium may not vanish even if the time interval between two offers becomes arbitrarily small.

To verify the robustness of the findings described above, I extend the analysis to incorporate the asymmetry of workers. Specifically, the surplus of each agreement differs from others. Although the workers may no longer be able to follow the same strategy, and the manager must treat her opponents differently, the hold-up effects and equilibrium properties naturally carry over to this setup as long as the degree of asymmetry is not excessive and the players are relatively patient. However, if the asymmetry among the workers is substantial or the discount factor is sufficiently small, the manager may strictly prefer to negotiate with the workers by following a particular ordering, which results in a unique and efficient equilibrium.

4.2 Literature

In recent years, a growing number of studies have examined bargaining games in which a long-run player negotiates with several short-run players in a sequential manner. In Hongbin Cai (2000, 2003), because the ordering of the bargaining opponents is exogenously given, the long-run player must move on to another short-run player if no agreement is reached with the current one in a round. In a setup with one buyer and multiple sellers, several papers partially endogenize the bargaining ordering by considering that the buyer pre-determines a particular ordering of the sellers at the beginning of the game and sticks to it during the negotiation process. These papers are primarily concerned with determining how the optimal ordering may be affected by the nature of the negotiations, which may be public or private (see Noe and Wang (2004), and Krasteva and Yildirim (2010)), or by the degree of asymmetry between the sellers (see Marx and Shaffer (2007), Jun Xiao (2010), and Krasteva and Yildirim (2011)). My model complements these papers by further endogenizing the long-run player's role in determining the ordering of her opponents. Specifically, even during the game process, in my model the long-run player can bargain with any remaining short-run player without the restriction of a fixed ordering.

My paper is most analogous to an independent work by Duozhe Li (2011), in which the bargaining procedure is also endogenously determined. The models differ from each other in the details. In Li's random proposer setup, the only unit of surplus is realized after all of the agreements have been reached, which implies that the agreements are perfectly complementary. In contrast, in my deterministic but alternative proposer setup, the agreements are independent because each worker individually contributes one unit of surplus. Most importantly, Li mainly addresses how the ordering of reached agreements may be affected by various transfer schemes, contingent contracts or cash-offer contracts. In my setup, I focus on the interactions of the double-sided hold-up effects and characterize the equilibrium payoff set of the manager.

The current paper is also related to an earlier work by Shaked and Sutton (1984). In their paper a firm can choose to negotiate with any worker. However, given that only one job opportunity is available and that the workers are perfectly substitutive, in equilibrium the outside workers play no strategic roles in Shaked and Sutton's model. Instead, the firm only uses the outside workers to credibly threaten the inside worker's bargaining power. In contrast, in my model the strategic coordination of the workers' actions is essential to all of the main findings.

Several other papers on bargaining with complete information also characterize the multiplicity and inefficiency of equilibrium, e.g., Haller and Holden (1990), Fernandez and Glazer (1991), Busch and Wen (1995). A common feature of these papers is that the players' payoffs are determined not only by the shares accorded to each player in the event that an agreement is reached but also by the normal form game played if the negotiation breaks down in the current period. The additional normal form game contributes to the rise of multiplicity and inefficiency in these setups, which differs from my setup in a crucial way.

4.3 The model

In my model, a manager (M or she) bargains sequentially with n workers to share n units of surplus: one unit for each pair of the players M & i, in which $i \in N_0 = \{1, 2, ..., n\}$ represents a worker (he) and N_0 defines the set of workers at the beginning of the game. I assume $n \geq 2$. Let N_t be the set of the workers remaining in the continuation game starting in period t. All of the players are risk-neutral and share a common discount factor $\delta \in (0, 1)$.

A *round* of bargaining is denoted by an offer and a (potential) counter-offer. At the beginning of each round, the manager chooses a worker $i \in N_t$ to be her bargaining opponent. Worker i makes the first offer. If the offer is accepted, the players receive their proposed shares, worker i exits the game, and the manager moves on to the next round with a new opponent $j \in N_{t+1}$. If the offer is rejected, then the manager makes a counter-offer in the next period to worker i. If this counter-offer is accepted, then the game moves forward in the same manner as described above. Otherwise, at the beginning of the next round, the manager chooses to bargain with a worker $j \in N_{t+2}$. Importantly, j may or may not be i. The ability to determine endogenously the ordering of bargaining opponents captures the manager's bargaining advantage, which is the main focus of the current model. The game ends if the last agreement is reached. However, if the manager engages in perpetual disagreements with some workers, the game lasts indefinitely.[35]

The game considered in this study assumes complete and perfect information. A *history* of this game summarizes all of the actions that have been taken in the past.[36] A player's *strategy* is a function that specifies how she/he acts contingent upon the histories that have been reached. By convention, the *equilibrium* in my setup refers to the notion of Subgame Perfect Equilibrium, and an equilibrium *outcome* $v_0 = (v_0^M; (v_0^i)_{i \in N_0})$ describes each player's total payoff measured in period $t = 0$. I use the realized ordering of agreements to index the workers with numbers. For instance, v_0^1 represents the payoff of the worker who reaches the first agreement with the manager, v_0^2 represents the payoff of the worker who reaches the second agreement, and so on until v_0^n. Additionally, throughout the paper, an offer $x \in [0, 1]$

[35] The structure of this bargaining round can be generalized to incorporate more bargaining frictions. For instance, if the manager begins bargaining with a worker in period t, then depending on no agreement has been reached between them, the firm may change her opponent only after $T \geq 2$ periods have elapsed. To capture the main intuitions, in this model I focus on the case with $T = 2$.

[36] There are three categories of histories: the histories at the beginning of period t, the histories before the proposer makes an offer and the histories before the responder accepts/rejects the offer. Notice that the first two categories coincide with each other in Rubinstein's (1982) model, whereas in my model, they differ in the periods during which the manager has an opportunity to choose her opponent.

denoted in the players' strategies in any period t refers to the share that the manager receives in this period, regardless of whether the offer is made by the manager. Thus, if the ith agreement with an offer x_i is reached in period t_i in an equilibrium, the manager and the ith worker's equilibrium payoffs are

$$v_0^M = \sum_{i=1}^n \delta^{t_i} x_i \quad \text{and} \quad v_0^i = \delta^{t_i}(1 - x_i)$$

respectively. Besides, let $v_t = (v_t^M; (v_t^i)_{i \in N_t})$ represent the players' discounted payoffs in the continuation games starting in period t.

An equilibrium is inefficient if delay exists in this equilibrium, which means that at least one of the proposals is rejected on the equilibrium path and the final agreement is reached in period $t \geq n$.

In the remainder of this section, I present two preliminary results.

Lemma 11 *In any equilibrium of this multilateral bargaining game, if the penultimate agreement is reached in period $t-1$, then the last agreement is reached in period t with the offer $x = \delta/(1+\delta)$.*

After the penultimate agreement has been reached, only one worker remains, and the continuation game degenerates into the standard bilateral bargaining game introduced by Rubinstein (1982). Thus, the result is immediate. To save notations, I omit the descriptions of strategy profiles and outcomes in the continuation games in which a maximum of only one agreement can be reached.

Lemma 12 *In any equilibrium of this multilateral bargaining game, no perpetual disagreement exists between the manager and any worker.*

Proof. Suppose not. Then there is an equilibrium in which for a subset $N \subseteq N_0$, there is perpetual disagreement between the manager and any worker $i \in N$. Let t be the period during which the manager reaches her last agreement with the workers in the set $N_0 \backslash N$.

Then in period $t+1$, an offer with a share $x \in (0,1)$ between the manager and any worker $i \in N$ is profitable for both parties. Hence, this offer should be accepted. A contradiction. ■

This result indicates that, although the manager has the advantage that she can endogenously determine the bargaining procedure, this advantage is limited. More precisely, the manager has the ability to temporarily leave aside a worker and bargain with the others first. However, she can not credibly threaten to leave aside this worker forever.[37]

4.4 Equilibrium analysis

I derive the main equilibrium properties in this section. I introduce some equilibria in subsection 4.4.1 and highlight how the interaction of the two hold-up effects results in the multiplicity of equilibrium. In subsection 4.4.2, I explore the property of inefficiency and show that the delays in some equilibria may not vanish if the time interval between two offers converges to zero. Finally, in subsection 4.4.3, I show that the hold-up effects and the main properties carry over to the setup with asymmetric workers.

4.4.1 Multiplicity of equilibrium

The lemmas presented in the last section indicate that if $n = 2$, the agreements must be reached in two consecutive periods in any equilibrium. Because of the time-discounting between periods, the manager has strong attempt to finish all of the negotiations as quickly as possible. However, this attempt actually enhances the workers' bargaining powers. This finding will be clearly shown by Example 1.

Example 1 *An equilibrium with no worker-switching when* $n = 2$*.*

[37] This lack of commitment power is a central feature of my setup that differentiates our results from those of Shaked and Sutton's (1984) study.

(1) *If no agreement has been reached, worker i at the bargaining table makes an offer $x = \delta/(1+\delta)^2$ and rejects any offer larger than $y = (1+\delta-\delta^3)/(1+\delta)^2$; the manager makes an offer y, and rejects any offer less than x.*

(2) *If worker i rejects the manager's offer in period $t-1$, he is re-chosen to be at the bargaining table in period t.*

The proofs of this example and of the remaining results are shown in the appendix. In this equilibrium, the workers' strategies only depend on the maximum number of agreements to be reached, and the manager is indifferent with regard to her opponents. Thus, retaining the current opponent is a (weakly) best response. However, this *no worker-switching* property enables the workers to coordinate their bargaining postures and act aggressively.

To demonstrate this point more clearly, I show that the equilibrium payoff vector is $v_0 = ((\delta+\delta^2+\delta^3)/(1+\delta)^2; (1+\delta+\delta^2)/(1+\delta)^2, \delta/(1+\delta))$. If $\delta \to 1$, then $v_0 \to (3/4; 3/4, 1/2)$. Worker i, who signed the first agreement in period $t = 0$, receives a payoff close to 3/4. In Rubinstein's (1982) model, each player offers $1/(1+\delta)$ to himself/herself. However, in this example before any agreement is reached, worker i offers $[1+\delta^2/(1+\delta)]/(1+\delta)$ to himself at the bargaining table, as the surplus is $1+\delta^2/(1+\delta)$ instead of 1. The key is that the additional term $\delta^2/(1+\delta)$ is the discounted share that the manager can obtain from the second agreement (in equilibrium). Expecting to be retained until the agreement is reached enables worker i to internalize the manager's payoff from the later agreement and to capture more rent. In this manner, the workers can collectively hold up the manager.

The next example introduces the second hold-up effect and shows how the manager may benefit from her ability to determine the bargaining ordering endogenously.

Example 2 *An equilibrium with worker-switching when $n = 2$.*

(1) *If no agreement has been reached, worker i at the bargaining table makes an offer $w = \delta/(1+\delta)$ and rejects any offer larger than $z = 1-\delta^2/(1+\delta)$; the manager makes an offer z and rejects any offer less than w.*

(2) *If worker i rejects the manager's offer in period $t-1$, he is re-chosen to be at the bargaining table after the manager has bargained with worker j for one round.*

Because the proof is quite similar to the proof of Example 1, it is omitted. Given the workers' strategies, in this example the manager is also indifferent with regard to which worker she negotiates with. Thus, a (weakly) best response is to switch to the other opponent. By properly adjusting each player's proposal/acceptance rules, I can construct a new equilibrium. The outcome of this equilibrium is efficient, and the payoff vector is $v_0 = (\delta; 1/(1+\delta)1, \delta/(1+\delta))$. This finding more closely resembles Rubinstein's result; for instance, each worker offers $1/(1+\delta)$ to himself. Whereas the manager in Example 1 is held up by the workers, the manager in this case can threaten to switch to the other worker and thereby weaken the bargaining power of the current one. These two effects counteract each other in this equilibrium, and for $\delta \to 1$, the payoff vector $v_0 \to (1; 1/2, 1/2)$.

The manager's equilibrium payoffs differ substantially from each other in these two examples. Another intuitive explanation is that in Example 1, before the first agreement is reached, one period of delay implies a loss $1-\delta$ of the social surplus for worker i but a loss $1-\delta^2$ of the total social surplus for the manager because the manager's second agreement is also postponed.[38] These losses could be explained as the bargaining costs or the relative impatience of each party. Intuitively, the party with the higher bargaining cost or impatience has lower bargaining power in the negotiation process and earns lower shares from the agreements. Conversely, in Example 2, before the first agreement is reached, one period of delay also implies a loss $1-\delta^2$ of the social surplus for worker i if he rejects the manager's offer,[39] so the manager's bargaining disadvantage in Example 1 vanishes. This finding explains why the manager earns higher equilibrium payoff in Example 2 than the payoff in Example 1.

[38] In Example 1, since the worker i at the bargaining table is never replaced before the first agreement, from the worker i's perspective, the value 1 of the agreement becomes δ if it is delayed by one period, so the loss of the value is $1-\delta$. However, from the manager's perspective, the value $1+\delta$ of the two agreements becomes $\delta+\delta^2$ if the first agreement is delayed by one period, so the loss of the value is $1-\delta^2$.

[39] In Example 2, if the worker i at the bargaining table rejects the manager's offer in period t, he expects that he will be re-chosen at the bargaining table in period $t+2$ (in equilibrium). From his perspective, the value 1 of the agreement in period t will become δ^2 if he rejects the offer in the current period. So the loss is $1-\delta^2$ for such a rejection.

These two examples show that multiple equilibria may arise if the manager has to negotiate with several workers sequentially and if the ordering of her opponents is endogenously determined. I extend this analysis to the general case with $n \geq 2$ and show that there is a set, which is represented by an interval, such that any value in this set could be the manager's equilibrium payoff.

Let

$$\overline{W}_n = \tfrac{\delta(1-\delta^n)}{1-\delta^2} \quad \text{and} \quad \underline{W}_n = \textstyle\sum_{k=0}^{n-1} \delta^k (1 - V_{n-k})$$

in which $V_K = \sum_{h=1}^{K} \frac{\delta^{2h-2}}{(1+\delta)^h}$. Notice that $\overline{W}_n > \underline{W}_n$ for $n \geq 2$, and $\overline{W}_n \to \frac{n}{2}$, $\underline{W}_n \to \frac{2^n-1}{2^n}$ for $\delta \to 1$. Accordingly, I produce the following proposition.

Proposition 11 *For any value $w \in [\underline{W}_n, \overline{W}_n]$, there is an equilibrium in which the manager's payoff is w.*

In the proof of this proposition, for $w = \underline{W}_n$, the equilibrium I construct is a natural generalization of the equilibrium introduced in Example 1, and for $w = \overline{W}_n$, the equilibrium I construct is a generalization of the equilibrium introduced in Example 2. To save notations in the following analysis, I refer to these equilibria as the *no worker-switching equilibrium with n workers* and the *worker-switching equilibrium with n workers*, respectively.

I claimed above that if the workers follow the same strategy, the manager is essentially indifferent with regard to which worker she chooses to negotiate with. Not surprisingly, new equilibria could be constructed if the retaining probability is between zero and one and if the offers are properly adjusted. I derive these equilibria explicitly in the proof and show that any value between $\underline{W}_n$ and $\overline{W}_n$ could serve as the manager's equilibrium payoff.

Some other points are also worth mentioning. The set described here does not capture all of the equilibrium payoffs of the manager when $n \geq 2$. In the next subsection, after introducing the property of inefficiency, I will demonstrate that the manager's equilibrium payoff may actually exist outside of this set. Additionally, although the equilibrium concept used here is SPE, the equilibria that I considered in the examples and in the proposition satisfy the definition of Markovian Perfect Equilibrium because all of the strategies only depend on the payoff-relevant information (i.e. the maximum number of agreements to be reached). Thus, I can relax the assumption of information completeness and perfectness in a way that the workers can only observe the number of peers who have left, which is a better approximation of real-world situations.

4.4.2 Inefficiency of equilibrium

In addition to the multiplicity of equilibrium, another widespread concern in the literature is the efficiency of the bargaining outcomes. This issue is of particular interest given that delays are observed quite frequently in real-life bargaining processes. In this subsection, I explore how the interaction of the manager's ability to endogenize the bargaining ordering and the workers' abilities to coordinate their moves may give rise to inefficient outcomes.

Consider the following example.

Example 3 *An equilibrium with delay when $n = 2$.*

Phase 1: there are $2T > 0$ periods in this phase. In the lth period, where $l \in \{1, 2, \cdots, 2T\}$, the manager makes an offer $y = 1$ and rejects any offer less than $w_l = \delta^{2T+1-l}(x^* + \frac{\delta^2}{1+\delta}) - \frac{\delta^2}{1+\delta}$, whereas worker i makes an offer $x = 0$ and rejects any offer larger than $z_l = \frac{\delta^{2T+2-l}}{1+\delta}$. If the manager switches to worker $j \neq i$ in the l'th period, where $l' < 2T$, then the strategy profile of the no worker-switching equilibrium in Example 1 is played from this period onwards.

Phase 2: after Phase 1, the manager switches to worker j. The manager makes an offer $y^* = \delta^{2T+1}(1 - x^*)$ and rejects any offer less than x^*, whereas worker j makes an offer $x^* = \frac{\delta+\delta^3-\delta^{2T+2}(1+\delta)}{(1+\delta)(1-\delta^{2T+2})}$ and rejects any offer larger than y^*. If worker j rejects the manager's offer, then in the next period, the manager switches to worker i, and the game returns to Phase 1.

If δ and T satisfy the condition $w_1 + z_1 > 1$ and the game starts in Phase 1, the above strategy profile consists of an equilibrium. Notice that given the non-serious offers and acceptance rules in Phase 1, the first agreement is reached in period $t = 2T$. As a result, there is delay in equilibrium. In the proof of this example, I show that if the players are relatively patient, the condition holds for some T. Thus, the existence of this type of equilibrium is guaranteed.

Unlike the outcomes in Example 1 and 2, the worker who reaches the first agreement in this equilibrium has a lower payoff than the other worker, whereas the manager's payoff $w_1 + \delta^2/(1+\delta)$ is larger than the ones that she could obtain from those outcomes. Intuitively, if the manager can endogenize the bargaining ordering to credibly delay her opponent's opportunity to make a counter-offer, she can weaken her opponent's bargaining stance in the current round and enlarge the share she can obtain. This process is reflected by the interaction in Phase 2. This strategy provides another way for the manager to hold up the workers with her bargaining advantage. Delay arises in equilibrium if the first worker at the bargaining table plays relatively tough to avoid being the one who is most severely held up. This process is reflected by the interaction in Phase 1.

This equilibrium has a form that the first agreement is reached in an even period, which is the first period for the worker who is involved in this agreement to be chosen at the bargaining table. If $n = 2$, this is the only possible form for an equilibrium with delay. The reason is that, if the first agreement is reached between the manager and worker i in period $t > 0$ in any other form, this pair of players must have met and bargained in some periods before t. By deviating from and frontloading this offer (and implicitly the second agreement) in those periods, all of the players strictly benefit. This restriction shows that in a setup with only two workers, the workers are rather competitive to each other, and their abilities to hold up the manager collectively through inefficient delay is quite limited. Nevertheless, if there are more workers, other forms of delay may arise, and the workers' hold-up effect could become stronger, as shown by the next example.

Let δ^* solve $1 + \delta - \delta^3 - \delta^4 - \delta^5 = 0$ and suppose that $\delta > \delta^*$. I have $\delta^* < 1$.

Example 4 *An equilibrium with delay when $n = 3$.*

(1) *If an agreement is reached in period $t = 0$, then the strategy profile in the no worker-switching equilibrium with two workers is played in the continuation game starting in period $t = 1$.*

(2) *If no agreement is reached in period $t = 0$, then the strategy profile in the worker-switching equilibrium with three workers is played in the continuation game starting in period $t = 1$.*

(3) *In period $t = 0$, worker i at the bargaining table makes an offer $x \leq 1 - \delta^4/(1+\delta)$; the manager rejects any offer less than $y = (\delta + \delta^2 + \delta^3)/(1+\delta)^2$.*

The first agreement is reached in period $t = 1$ in this equilibrium. This example demonstrates how the workers may exert another hold-up effect on the manager through inefficient delays. The key of this hold-up effect is that, to trigger an effective punishment or reward based on the manager's behavior, multiple equilibria must exist in the continuation game. This scenario is feasible only if at least two workers remain in the continuation game. If $n = 2$, the continuation play after the first agreement is uniquely determined. Thus, in this case, whenever the manager has a chance to frontload the first agreement, she will do so. Conversely, if $n > 2$ and frontloading this agreement may cause the manager to be punished, the manager's attempt to do so will be restricted. The difference between these two situations shows that the workers can generally coordinate themselves in a more flexible manner if their group is relatively larger.

Regarding the property of equilibrium inefficiency, a widely considered problem is that the inefficiency may vanish if the real-time between two offers converges to zero. For instance, if the term "period" denotes a minute instead of a week or a month, the social cost of delay in the above mentioned examples may be viewed as negligible. One of my main results shows that equilibrium delay could be bounded away from zero even if the offers are made within arbitrarily close time frames. This finding is characterized by the following proposition.

Proposition 12 *Let $\delta = \exp(-\rho\Delta)$, in which Δ denotes the real-time interval between two* *offers, and ρ denotes the positive interest rate. In the multilateral bargaining game with $n \geq 2$, there may be an equilibrium with T periods of delay such that* $\lim_{\Delta \to 0}(\Delta T) > 0$.

I explicitly prove this proposition in the game with $n = 3$. First, I apply the same logic used in Example 4 to show that when δ increases, the possible periods of delay T expands. The reason is, when the discounting cost is relatively small to the manager, the equilibrium path on which the workers would coordinate in the continuation game becomes more crucial to the manager's current decisions. To avoid incurring potential punishment from the workers, the manager may have to accept long periods of delay. Second, I solve for the limiting case with $\delta \to 1$ ($\Delta \to 0$) and show that the real-time delay may not disappear. This proof can be easily modified to incorporate more workers. Intuitively, when the number n increases, the manager's payoff difference between the *worker-switching* and the *no worker-switching* equilibria becomes larger, and it is possible to construct an equilibrium with even longer periods of delay.

In the last subsection, I claimed that there is an interval in which any of the values could be the manager's equilibrium payoff. However, this interval does not capture all of the manager's equilibrium payoffs. For instance, I show that the manager's equilibrium payoff in Example 3 is actually larger than δ. If inefficient equilibria exist, the players may interact with each other in more complicated ways. I establish the following result.

Lemma 13 *In the multilateral bargaining game with $n \geq 2$, there may be an equilibrium in which the manager's payoff is larger than $\overline{W}_n$ and an equilibrium in which the manager's payoff is less than $\underline{W}_n$.*

Consider a game with $n = 3$. Notice that in Example 4, the manager's payoff is $\delta(1 - \delta^3)/(1 - \delta^2)$, which is exactly the same as her payoff in the *worker-switching equilibrium with three workers*. However, because of the social cost of delay, the workers' payoffs in Example 4 are lower than those in the corresponding *worker-switching equilibrium*. This finding shows that the manager enjoys a relative bargaining advantage in such an equilibrium with delay. By threatening to switch to this inefficient equilibrium in the continuation game if no agreement is reached in the first several periods of the game, the manager can further weaken the workers' bargaining postures and increase her payoff. Using this logic, I clearly construct an equilibrium in the proof and show that the manager's payoff may be larger than $\overline{W}_n$.

Conversely, I also construct an equilibrium in the proof to show that the manager's equilibrium payoff may be less than $\underline{W}_n$. If the manager is rewarded by the *worker-switching equilibrium* in the continuation game for accepting an offer in period $t = 0$ but is punished by the *no worker-switching equilibrium* in the continuation game for rejecting the offer in this period, the share being acceptable to the manager in period $t = 0$ could be substantially reduced. This finding implies that there are equilibria in which some workers' equilibrium payoffs could be higher than the payoffs that they would receive in the *no worker-switching equilibrium*. This property is important because if such an equilibrium is used as a new scheme to punish the manager for failing to reach an agreement in the first several periods, the manager's payoff can be further lowered. The proof contains a more detailed argument on this point.

Unfortunately, I can not easily provide a complete characterization of the manager's equilibrium payoff set by relying on this multilateral bargaining setup. The essential feature of the game considered here is that the exits of the workers and the ending of the game are endogenously determined, and the lack of a well-defined recursive structure substantially complicates the analysis. Another difficulty regarding the model is that the set of equilibria does not expand monotonically as δ increases. The intuition is as follows: for a putative equilibrium to exist, some of the "threats" may no longer be credible if the players become more patient. For instance, the equilibrium that I constructed to derive a payoff lower than $\underline{W}_n$ is valid only for a moderate value of δ (i.e. $\delta \in (\delta^*, \delta') \subset (0, 1)$).

Nevertheless, the qualitative properties of the double-sided hold-up problem are well captured by the inefficiency and multiplicity of equilibrium derived in this section.

4.4.3 Bargaining with asymmetric workers

Thus far, my analysis depended on the assumption that the workers are symmetric. Thus, if they follow the same strategy, the manager is indifferent with regard to which two workers she bargains with. However, this assumption is not necessarily true for my main results. In this subsection, I show that the double-sided hold-up effects may also arise if the workers are asymmetric.

Suppose that there are two workers, A and B, and that their contributions to the project are u_A and u_B, respectively, as valued in the periods during which the agreements are reached. Without loss of generality, I assume that $u_A \geq u_B > 0$. Consider the strategy profile in the following example.

Example 5

(1) If no agreement is reached: A makes an offer $x = \frac{\delta}{1+\delta}u_A - \frac{\delta^2}{(1+\delta)^2}u_B$*, and rejects any counter-offer larger than* $x' = \frac{1}{1+\delta}u_A - \frac{\delta^3}{(1+\delta)^2}u_B$*. When bargaining with A, the manager makes an offer* x' *and rejects any counter-offer smaller than* x*. B makes an offer* $y = \frac{\delta-\delta^2}{1+\delta}u_A + \frac{\delta^3}{(1+\delta)^2}u_B$ *and rejects any counter-offer larger than* $y' = (1-\delta)u_A + \frac{\delta^2}{(1+\delta)^2}u_B$*. When bargaining with B, the manager makes an offer* y' *and rejects any counter-offer smaller than* y*.*

(2) If no agreement is reached: if A rejects the manager's offer in period t*, he is re-chosen in period* $t+1$*. If B rejects the manager's offer in period* t*, he is re-chosen with probability* $p = \frac{(1+2\delta-\delta^2-\delta^3)u_B-(1+\delta-\delta^2-\delta^3)u_A}{(\delta+\delta^2-\delta^4)u_B-(\delta^2-\delta^4)u_A}$ *in period* $t+1$*.*

The strategy profile above consists of an equilibrium if $y' \leq u_B$ and $p \in [0,1]$. The manager's payoff is $v' = \frac{\delta}{1+\delta}u_A + \frac{\delta^3}{(1+\delta)^2}u_B$. This equilibrium corresponds to the *no worker-switching equilibrium* in Example 1, although a slight modification is needed to satisfy the manager's condition of indifference regarding her choice of opponent. This modification is needed because if worker B "believes" that after rejecting the manager's offer he will be re-chosen with a probability of one in the next period and acts aggressively based on this "belief" (e.g., makes an offer $\frac{\delta}{1+\delta}u_B - \frac{\delta^2}{(1+\delta)^2}u_A$), the manager will actually switch to worker A if $u_A > u_B$. Thus, worker B has to weaken his toughness and ask for a smaller share (notice that $\frac{\delta-\delta^2}{1+\delta}u_A + \frac{\delta^3}{(1+\delta)^2}u_B > \frac{\delta}{1+\delta}u_B - \frac{\delta^2}{(1+\delta)^2}u_A$ if $u_A > u_B$).

Now consider another strategy profile.

Example 6

(1) If no agreement is reached: A makes an offer $w = \frac{\delta^3}{1+\delta}u_A + \delta(1-\delta)u_B$, and rejects any counter-offer larger than $w' = \frac{\delta^2}{1+\delta}u_A + \frac{1+\delta-2\delta^2}{1+\delta}u_B$. When bargaining with A, the manager makes an offer w' and rejects any counter-offer smaller than w. B makers an offer $z = \frac{\delta+\delta^2-\delta^3}{1+\delta}u_B - \frac{\delta^2-\delta^3}{1+\delta}u_A$ and rejects any counter-offer larger than $z' = \frac{1+\delta-\delta^2}{1+\delta}u_B$. When bargaining with B, the manager makes an offer z' and rejects any counter-offer smaller than z.

(2) If no agreement is reached: if A rejects the manager's offer in period t, he is re-chosen in period $t+1$ with probability $q = \frac{(1+\delta-2\delta^2)(u_A-u_B)}{(\delta-\delta^4)u_A-(\delta^2-\delta^4)u_B}$. If B rejects the manager's offer in period t, he is re-chosen with probability 0 in period $t+1$.

This strategy profile consists of an equilibrium if $z \geq 0$ and $q \in [0,1]$. The manager's equilibrium payoff is $v'' = \frac{\delta+\delta^2-\delta^3}{1+\delta}u_B + \frac{\delta^3}{1+\delta}u_A$. This equilibrium corresponds to the *worker-switching equilibrium* in Example 2. Similarly, I need to adjust worker A's toughness to satisfy the manager's indifference condition if $u_A > u_B$.

Assume that $u_A \leq 2u_B$ and let $\widehat{\delta}$ solve $1-\delta^2-\delta^3=0$. In this case, if $\delta > \widehat{\delta}$, all of the aforementioned conditions hold, and both of the strategy profiles consist of equilibria. Furthermore, under these assumptions, I find that $v'' > v'$, which implies that the manager has multiple equilibrium payoffs. By jointly adjusting the workers' bargaining postures, I can show that all of the values between v' and v'' could be the manager's equilibrium payoffs. The argument is analogous to the proof of Proposition 11.

Now suppose that there is a third worker C with potential contribution u_C. Let $u_C \leq u_B \leq u_A$. Assume that u_A is sufficiently close to u_C and that δ is sufficiently close to 1. By modifying the proof of Example 4, I can show that a delay arises in equilibrium even if the workers are asymmetric.

However, new issues may arise if the players are impatient or if the workers' asymmetry is large. Consider a situation in which $u_A = 10u_B > 0$. I can easily verify that in any potential equilibrium, if the manager starts bargaining with worker A first, her payoff is no less than $\delta(1-\delta)u_A$. This result occurs because by rejecting the offer in period $t = 0$, the manager can make a counter-offer $(1-\delta)u_A$ in period $t = 1$. In turn, this counter-offer will undoubtedly be accepted by A. Conversely, if the manager bargains with worker B first, then her payoff is no larger than $\max\{\delta u_B + \delta^2 u_A/(1+\delta), \delta^2(u_A + \delta u_B)\}$. These terms indicate that after rejecting the offer in period $t = 0$, the manager can have a maximum total payoff of $u_B + \delta u_A/(1+\delta)$ in period $t = 1$ (such that her offer $x = u_B$ is accepted by B) or $u_A + \delta u_B$ in period $t = 2$ (in this case, her offer is rejected by B in period $t = 1$, but she is lucky enough to have all of the surplus starting in period $t = 2$). If δ is relatively small (e.g., $\delta < 2/5$), the manager strongly prefers to bargain with worker A first, and the equilibrium outcome will be unique.

However, deriving a complete analysis of a game with asymmetric workers and an endogenous bargaining procedure may be quite complicated. Generally, the equilibrium set varies with the degree of the workers' asymmetry and the discount factor δ. Nevertheless, I show that the multiplicity and inefficiency of equilibrium may exist if the players are sufficiently patient and their degree of asymmetry is relatively low. These findings show that my main results are robust even if I slightly perturb the structure of the original game.

4.5 Conclusions

In a multilateral bargaining model in which a manager sequentially negotiates with several workers, I addressed how the bargaining procedure is determined endogenously and its effects on the properties of equilibrium. Precisely, two hold-up effects arise in this model. On the one hand, because time-discounting matters, the workers can take advantage of the manager's attempt to end all of the negotiations as soon as possible and collectively hold up the manager by coordinating their moves. On the other hand, by endogenizing the ordering of her opponents, the manager can also hold up the workers and weaken their bargaining powers. The interaction of these two effects results in the multiplicity of equilibrium, and some of the equilibria exhibit delayed agreements, which imply that the outcomes are inefficient. Moreover, delay may not vanish if the time interval between two periods converges to zero.

Appendix

The proof of Example 1.

Proof. Before the first agreement is reached, because the two workers follow the same strategy, the manager is indifferent with regard to which worker to bargain with. Thus, if her offer in period $t-1$ is rejected by worker i, re-choosing this worker to be at the bargaining table in period t is a best response. This justifies her strategy in (2).

Consider the strategies in (1). Because the offer x (y) is acceptable to the manager (worker i), making an offer $x' > x$ is not a profitable deviation for worker i, while making an offer $y' < y$ is not a profitable deviation for the manager.

Consider the deviation that worker i makes an offer $x'' < x$ in period t. Because this offer is rejected by the manager, in the continuation game starting in period $t+1$ worker i has payoff $\frac{\delta+\delta^2+\delta^3}{(1+\delta)^2}$. However, if worker i makes the offer x in period t, his payoff is $\frac{1+\delta+\delta^2}{(1+\delta)^2}$ in this peirod. Because $\frac{\delta(\delta+\delta^2+\delta^3)}{(1+\delta)^2} < \frac{1+\delta+\delta^2}{(1+\delta)^2}$, deviating to an offer x'' is not profitable for worker i. Similarly, consider the deviation that the manager makes an offer $y'' > y$ in period t. Because this offer is rejected by worker i, in the continuation game starting in period $t+1$ the manager has payoff $\frac{\delta}{(1+\delta)^2} + \delta\frac{\delta}{1+\delta}$. However, if the manager makes the offer y, her payoff is $\frac{1+\delta-\delta^3}{(1+\delta)^2} + \delta\frac{\delta}{1+\delta}$ in period t. Because $\delta(\frac{\delta}{(1+\delta)^2} + \delta\frac{\delta}{1+\delta}) < \frac{1+\delta-\delta^3}{(1+\delta)^2} + \delta\frac{\delta}{1+\delta}$, deviating to y'' is not profitable for the manager.

Consider the acceptance/rejection strategies specified in (1). If worker i rejects the offer y in period t, then in period $t+1$ his payoff is $\frac{1+\delta+\delta^2}{(1+\delta)^2}$. But if he accepts the offer y in period t, his payoff is $\frac{\delta+\delta^2+\delta^3}{(1+\delta)^2}$ in this period. Because $\frac{\delta(1+\delta+\delta^2)}{(1+\delta)^2} = \frac{\delta+\delta^2+\delta^3}{(1+\delta)^2}$, deviating to reject the offer y is not profitable for worker i. If the manager rejects the offer x in period t, then in period $t+1$ her total payoff is $\frac{1+\delta-\delta^3}{(1+\delta)^2} + \delta\frac{\delta}{1+\delta}$. But if she accepts the offer x in period t, her payoff is $\frac{\delta}{(1+\delta)^2} + \delta\frac{\delta}{1+\delta}$ in this period. Because $\delta(\frac{1+\delta-\delta^3}{(1+\delta)^2} + \delta\frac{\delta}{1+\delta}) = \frac{\delta}{(1+\delta)^2} + \delta\frac{\delta}{1+\delta}$, deviating to reject the offer x is not profitable for the manager. ■

The proof of Proposition 11.

Proof. Let $w = \sum_{l=1}^{n} \delta^{l-1} w_l$ such that $w_l \in [1 - \sum_{h=1}^{n+1-l} \frac{\delta^{2h-2}}{(1+\delta)^h}, \frac{\delta}{1+\delta}]$ for $l < n$ and $w_n = \frac{\delta}{1+\delta}$ for $l = n$. Notice that if $w_l = 1 - \sum_{h=1}^{n+1-l} \frac{\delta^{2h-2}}{(1+\delta)^h}$ for any l, then $w = \underline{W}_n$. If $w_l = \frac{\delta}{1+\delta}$ for any l, then $w = \overline{W}_n$. So the decomposition of w is always feasible. The approach is to construct an equilibrium in which the manager gets share w_l from the lth agreement. For $l \leq n$, let

$$y_l = \frac{w_l}{\delta} + (1-\delta) \sum_{m=1}^{n-l} \delta^{m-1} w_{l+m},$$

and let p_l satisfy the condition

$$1 - y_l = \delta p_l (1 - w_l) + \delta^{n+1-l} (1 - p_l)(1 - w_n).$$

Consider the following strategy profile.

(1) In period t, if there are $n+1-l \geq 2$ agreements remaining, worker i at the bargaining table makes an offer w_l, and rejects any offer larger than y_l; the manager makes an offer y_l, and rejects any offer less than w_l.

(2) In period t, if there are $n+1-l \geq 2$ agreements remaining, and if worker i rejects the manager's offer in this period, with probability p_l he will be re-chosen at the bargaining table in period $t+1$ and with probability $1-p_l$ he will be re-chosen after the manager has bargained with every $j \in N_t \backslash \{i\}$ for one round.

Following the same logic in the proof of Example 1, I can show that given the values of y_l and p_l this strategy profile consists of an equilibrium, in which the manager's payoff is w. Specifically, y_l solves the condition that the manager is indifferent between accepting and rejecting the offer w_l in period t, and p_l solves the condition that worker i is indifferent between accepting and rejecting the offer y_l in period t. ■

The proof of Example 3.

Proof. If the players follow this strategy profile and the game starts in Phase 1, the first agreement is reached in period $t = 2T$ with an offer x^*.

Consider Phase 1. w_l and z_l are the manager and worker i's expected payoffs discounted in this lth period, so the acceptance rules are optimal. If the condition $w_1 + z_1 > 1$ holds, I have $w_l + z_l > 1$ for any $l \in \{1, 2, \cdots 2T\}$, which implies that the manager and worker i's total needs in the lth period is larger than 1, so no deviation from the proposal rules is profitable for any player. Finally, if the manager switches to worker j in the l'th period, where $l' < 2T$, she is punished by the *no worker switching equilibrium* starting from this period. Because $w_1 + z_1 > 1$ also implies $\delta^{2T+1-l}(x^* + \frac{\delta^2}{1+\delta}) \geq \frac{\delta+\delta^2+\delta^3}{(1+\delta)^2}$, in which the right-hand term represents the manager's deviation payoff, such a deviation is not profitable for her.

Now consider Phase 2. The proof is quite similar to the one of Example 1, so I can verify that there exists no profitable deviation from the proposal rules or acceptance rules.

Finally, I show that for some T and δ, $w_1 + z_1 > 1$. Consider $T = 2$. Then for $w_1 + z_1 > 1$, it is sufficient to have $\delta^6(1 + \delta)(1 + \delta^2) > 1 + \delta^2 + \delta^4$. Apparently, there is a $\underline{\delta} \in (0, 1)$ such that for any $\delta > \underline{\delta}$, the condition holds. So the existence of this equilibrium is guaranteed by a sufficiently large δ. ■

The proof of Example 4.

Proof. The strategy profile specified in (1) and (2) consists of an equilibrium in the continuation game starting in period $t = 1$, so I only need to check that no one has incentive to deviate from the strategies in (3).

If the manager rejects an offer y' in period $t = 0$, she has total payoff $1+\frac{\delta^2}{1+\delta}$ in period $t = 1$ in the continuation game. The discounted value of this payoff in period $t = 0$ is $\delta(1 + \frac{\delta^2}{1+\delta})$. If the manager accepts this offer y' in period $t = 0$, she has payoff $y' + \frac{\delta^2}{(1+\delta)^2} + \frac{\delta^3}{1+\delta}$. So if $y' < y = \delta - \frac{\delta^2}{(1+\delta)^2}$, the manager rejects this offer; and if $y' \geq y = \delta - \frac{\delta^2}{(1+\delta)^2}$, the manager accepts this offer.

If worker i makes an offer x in period $t = 0$, and it is rejected, worker i has payoff $\frac{\delta^3}{1+\delta}$ in period $t = 1$ in the continuation game. The discounted value of this payoff in period $t = 0$ is

$\frac{\delta^4}{1+\delta}$. If this offer x is accepted, then worker i has payoff $1-x$ in period 0. So worker i only makes an offer $x \leq 1-\frac{\delta^4}{1+\delta}$ in period $t=0$.

If $\delta > \delta^*$, I have $\frac{\delta^4}{1+\delta}+\delta-\frac{\delta^2}{(1+\delta)^2} > 1$, the manager and worker i's total needs in period $t=0$ is larger than one, thus delay arises in equilibrium. ■

The proof of Proposition 12.

Proof. The proof is by construction. In step 1, I construct an equilibrium with delay, and explore the relationship between the discount factor δ and the possible periods of delay $T=2T'$ in this equilibrium. In step 2, I derive the limit of real-time delay when $\delta \to 1$, or equivalently, $\Delta \to 0$. Because in a general case if T is odd, at least I have an even number $T-1$ of periods of delay. So without loss of generality, I can assume that T is even. Also, I have $V_K = \sum_{h=1}^{K} \frac{\delta^{2h-2}}{(1+\delta)^h}$ as defined before.

Step 1: Consider the game with $n=3$. The following strategy profile consists of an equilibrium with $2T'$ periods of delay if the condition $\delta^{2T'} \geq \frac{1}{\left(\delta^2+1\right)(\delta+1)^2}(\delta^4+\delta^3+2\delta^2+2\delta+1)$ holds:

(1) Let i^* denote the worker at the bargaining table in period $t=0$. If the manager switches to another worker $j \neq i^*$ in period t before the first agreement is reached, the strategy profile in Example 4 is played in the continuation game starting in period t.

(2) Consider that i^* has not been replaced. If the first agreement is reached in period $t < 2T'$, then the strategy profile in Example 1 is played in the continuation game starting in period $t+1$. If the first agreement is reached in period $t \geq 2T'$, then the strategy profile in Example 2 is played in the continuation game starting in period $t+1$.

(3) Before the first agreement is reached, worker i^* at the bargaining table makes an offer $x = 1-V_3$, and rejects any offer larger than $y \leq 1-\delta V_3$; the manager makes a non-serious offer such as $x' = 1$ in period $t < 2T'$ and an offer $x'' = 1-\delta V_3$ in period $t \geq 2T'$, rejects any offer less than $y' = \frac{\delta^{2T'-t+1}}{(\delta+1)^3}\left(\delta^2+\delta+1\right)^2$ in period $t < 2T'$ and rejects any offer less than

$y'' = 1 - V_3$ in period $t \geq 2T'$.

The proof of this step is as follows. First, given the strategies in (1) and (3), the manager has no incentive to switch to another worker j in period $t > 0$ before the first agreement is reached. Because such a deviation is strictly dominated by accepting the offer in period $t = 0$. Second, If the manager and worker i^* have incentive to reach an agreement before period $2T'$, it is optimal for them to reach it in period 0 instead of in period $t > 0$. Because if they reach the agreement with an offer x^* in period $t > 0$, a deviation in period $t = 0$ with an offer $\delta^t x^* + \frac{1-\delta^t}{2}$ will result all of the players with higher payoffs. Thus, I only need to check that the first agreement is not reached in period $t = 0$. Notice that in period $t = 0$, the largest share that worker i^* could offer to the manager is $1-\delta^{2T'}V_3$. If $\delta^{2T'} \geq \frac{\left(\delta^4+\delta^3+2\delta^2+2\delta+1\right)}{\left(\delta^2+1\right)(\delta+1)^2}$ holds, I have inequality

$$\delta^{2T'}(1 - V_3 + \frac{\delta^2}{1+\delta} + \frac{\delta^3}{1+\delta}) \geq 1 - \delta^{2T'}V_3 + \delta(1 - V_2) + \delta^2(1 - V_3).$$

This inequality implies that it's weakly better for the manager to reach the first agreement in period $2T'$ instead of accepting the most favorable offer in period $t = 0$. So the strategy profile demonstrated above consists of an equilibrium, in which the first agreement is reached in period $2T'$.

Step 2: Let $T = 2T'$ be the number of delayed periods derived in the step 1. In this step, I show that $\lim_{\Delta\to 0} \Delta T > 0$. Let the real number r satisfy $r - 1 < T = 2T' \leq r$ and $\delta^r = \frac{\left(\delta^4+\delta^3+2\delta^2+2\delta+1\right)}{\left(\delta^2+1\right)(\delta+1)^2}$. Replace δ with $\exp(-\rho\Delta)$ in this equality, I have the following condition

$$e^{-\rho\Delta r} = \frac{1}{\left(e^{-2\rho\Delta}+1\right)\left(e^{-\rho\Delta}+1\right)^2}\left(e^{-4\rho\Delta} + e^{-3\rho\Delta} + 2e^{-2\rho\Delta} + 2e^{-\rho\Delta} + 1\right).$$

Let RHS denote the term on the right hand side and LHS denote the term on the left hand side. From the condition above, I have

$$\lim_{\Delta\to 0} RHS = \frac{7}{8} = \lim_{\Delta\to 0} LHS,$$

which implies

$$\lim_{\Delta\to 0} \Delta(r-1) = \lim_{\Delta\to 0} \Delta T = \lim_{\Delta\to 0} \Delta r = \frac{1}{\rho}(\ln 8 - \ln 7).$$

Apparently, $\lim_{\Delta\to 0} \Delta T > 0$. Thus, even the time interval Δ between two offers becomes arbitrarily small, real-time delay can happen in the game with $n = 3$. ■

The proof of Lemma 13.

Proof.

Part 1: The manager's equilibrium payoff may be larger than $\overline{W}_n$. Consider the following strategy profile in the game with $n = 3$, in which i^* denotes the worker who is at the bargaining table in period $t = 0$:

(1) If no agreement is reached before period $t = 2$, the strategy profile in Example 4 is played, and worker i^* is retained at the bargaining table in period $t = 2$. If an agreement is reached in period $t < 2$, the strategy profile of the *worker-switching equilibrium with two workers* is played in the continuation game starting in period $t + 1$.

(2) If no agreement is reached in period $t = 0$, in period $t = 1$ worker i^* only accepts an offer $x \le 1 - \frac{\delta^5}{1+\delta}$, and the manager makes an offer $y = 1 - \frac{\delta^5}{1+\delta}$.

(3) In period $t = 0$, the manager only accepts an offer $x' \ge \frac{\delta+\delta^4-\delta^6}{1+\delta}$, and worker i^* makes an offer $y' = \frac{\delta+\delta^4-\delta^6}{1+\delta}$.

Notice that if no agreement is reached before period $t = 2$, the strategy profile denoted in (1) consists of an equilibrium, so I only need to check that the players' strategies in periods $t = 1$ and $t = 0$ satisfy the requirement of no profitable deviations. The key point is, if worker i^* rejects the manager's offer in period $t = 1$, his payoff is $\frac{\delta^4}{1+\delta}$ in the continuation game starting in period $t = 2$. So he accepts the manager's offer $y = 1 - \frac{\delta^5}{1+\delta}$ in period $t = 1$.

This acceptance implies that the manager's total payoff in period $t=1$ is $1-\frac{\delta^5}{1+\delta}+\frac{\delta^2}{1+\delta}+\frac{\delta^3}{1+\delta}$. The discounted value of this payoff is $\frac{\delta+\delta^2+\delta^3+\delta^4-\delta^6}{1+\delta}$ in period $t=0$, which is larger than $\frac{\delta(1-\delta^3)}{1-\delta^2}$.

Part 2: The manager's equilibrium payoff may be less than $\underline{W}_n$. Consider the game with $n=3$. This part of proof consists of two steps. Let δ^* solve $1+\delta-\delta^2-2\delta^3=0$ and δ' solve $1+\delta-\delta^3-\delta^4-\delta^5=0$. I have $0<\delta^*<\delta'<1$.

Step 1: The following strategy profile consists of an equilibrium if $\delta \geq \delta^*$:

(1) If an agreement is reached in period $t=0$, the strategy profile of the *worker-switching equilibrium with two workers* is played in the continuation game starting in period $t=1$. If no agreement is reached in period $t=0$, the strategy profile of the *no worker-switching equilibrium with three workers* is played in the continuation game starting in period $t=1$.

(2) In period $t=0$, worker i makers an offer $x=0$, and the manager accepts any offer $y \geq 0$.

The key of the proof is, if the manager accepts the offer $x=0$ in period $t=0$, she is "rewarded" by the *worker-switching equilibrium* in the continuation game, and her total payoff is $0+\delta\frac{\delta}{1+\delta}+\delta^2\frac{\delta}{1+\delta}=\delta^2$. If she rejects, she is "punished" by the *no worker-switching equilibrium* in the continuation game, and her total payoff in period $t=1$ is $\frac{1+2\delta+2\delta^2+\delta^3+\delta^4}{(1+\delta)^3}$, which is $\frac{\delta+2\delta^2+2\delta^3+\delta^4+\delta^5}{(1+\delta)^3}$ in period $t=0$. For $\delta \geq \delta^*$, $\delta^2 \geq \frac{\delta+2\delta^2+2\delta^3+\delta^4+\delta^5}{(1+\delta)^3}$, so the manager accepts the offer $x=0$ in period $t=0$. Notice that this equilibrium enlarges the first worker's payoff to an entire unit, which is important for the next equilibrium.

Step 2: The following strategy profile consists of an equilibrium if $\delta \in (\delta^*, \delta')$, in which i^* denotes the worker who is at the bargaining table in period $t=0$:

(1) If no agreement is reached before period $t=2$, the strategy profile in the step 1 is played in the continuation game starting in period $t=2$, and worker i^* is retained at the bargaining table in period $t=2$. If an agreement is reached in period $t<2$, the strategy profile of the *no worker-switching equilibrium with two workers* is played in the continuation

game starting in period $t+1$.

(2) If no agreement is reached in period $t=0$, in period $t=1$ worker i^* only accepts an offer $x \leq 1-\delta$, and the manager makes an offer $y = 1-\delta$.

(3) In period $t=0$, the manager only accepts an offer $x' \geq \frac{\delta-\delta^3-\delta^4+\delta^5}{(1+\delta)^2}$, and worker i^* makes an offer $y' = \frac{\delta-\delta^3-\delta^4+\delta^5}{(1+\delta)^2}$.

The key of the proof is, if no agreement is reached before period $t=2$, worker i^* can get 1 in the continuation game starting in period $t=2$. This enables him to bargain more aggressively in periods $t=1$ and $t=0$. Precisely, he rejects any offer larger than $1-\delta$ in period $t=1$. So in period $t=1$, the manager at most can have total payoff $1-\delta+\delta\frac{\delta}{(1+\delta)^2}+\delta^2\frac{\delta}{1+\delta} = \frac{1+\delta+\delta^4}{(1+\delta)^2}$ if she makes an acceptable offer to worker i^*. If the manager's offer in period $t=1$ is rejected, she has payoff δ^2 in period $t=2$ and the discounted value of this payoff in period $t=1$ is δ^3.Notice that if $\delta \in (\delta^*, \delta')$, $\frac{1+\delta+\delta^4}{(1+\delta)^2} > \delta^3$, so the manager actually makes the offer $y = 1-\delta$ in period $t=1$ if no agreement is reached in period $t=0$. This further implies that an offer $y' = \frac{\delta-\delta^3-\delta^4+\delta^5}{(1+\delta)^2}$ is enough for the manager to accept in period $t=0$. Notice that this equilibrium results the manager with payoff $\frac{\delta+\delta^2+\delta^5}{(1+\delta)^2}$, which is smaller than $\sum_{k=0}^{k=2}\delta^k(1-V_{3-k})$. ■

References

[1] Busch, Lutz-Alexander., and Wen, Quan. 1995. "Perfect Equilibria in a Negotiation Model." Econometrica. 63(3): 545-565.

[2] Cai, Hongbin. 2000. "Delay in Multilateral Bargaining under Complete Information." Journal of Economic Theory. 93: 260-276.

[3] Cai, Hongbin. 2003. "Inefficient Markov Perfect Equilibria in Multilateral Bargaining." Economic Theory. 22: 583-606.

[4] Fernandez, Raquel., and Glazer, Jacob. 1991. "Striking for a Bargain Between Two Completely Informed Agents." American Economic Review. 81(1): 240-252.

[5] Haller, Hans., and Holden, Steinar. 1990. "A Letter to the Editor on Wage Bargaining." Journal of Economic Theory. 52: 232-236.

[6] Krasteva, Silvana., and Yildirim, Huseyin. 2010. "On the Role of Confidentiality and Deadlines in Bilateral Negotiations." Working Paper.

[7] Krasteva, Silvana., and Yildirim, Huseyin. 2011. "Payoff Uncertainty, Bargaining Power, and the Strategic Sequencing of Bilateral Negotiations." Working Paper.

[8] Li, Duozhe. 2011. "One-to-Many Bargaining with Endogenous Protocol." Working paper.

[9] Marx, Leslie M., and Shaffer, Greg. 2007. "Rent Shifting and the Order of Negotiations." International Journal of Industrial Organization, 25(5): 1109-1125.

[10] Noe, Thomas., and Wang, Jun. 2004. "Fooling All of the People Some of the Time: a Theory of Endogenous Sequencing in Confidential Negotiations." Review of Economic Studies. 71: 855-881.

[11] Rubinstein, Ariel. 1982. "Perfect Equilibrium in a Bargaining Model." Econometrica. 50: 97-109.

[12] Shaked, Avner., and Sutton, John. 1984. "Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model." Econometrica: 52: 1351-1364.

[13] Stahl, Ingolf. 1972. Bargaining Theory. Stockholm: Stockholm School of Economics.

[14] Xiao, Jun. 2010. "Bargaining Order in a Multi-Person Bargaining Game." Working Paper.