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Equilibrium Implications of Social Utility and Reference Dependence

Kangkang Wang
Washington University in St. Louis

WASHINGTON UNIVERSITY IN ST. LOUIS

Olin Business School

Dissertation Examination Committee:
Chakravarthi Narasimhan, Chair
Dmitri Kuksov, Co-Chair
Tat Y. Chan
Baojun Jiang
John Nachbar
Sherif Nasser

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ABSTRACT OF THE DISSERTATION

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Professor Chakravarthi Narasimhan, Chair

Professor Dmitri Kuksov, Co-Chair

It is well-known that consumers can deviate from the assumptions of standard economic theory in many ways. My dissertation examines the strategic implications of some behavioral anomalies of consumers in an equilibrium framework. In particular, I focus on how these behavioral anomalies could affect the optimal strategy and profitability of firms. The dissertation is comprised of two essays. In the first essay, I study the implications of consumer utility having a social component for competitive strategy of the firms and propose a model to explain the randomness of fashion hits observed in the fashion market. In the second essay, I investigate how firm pricing decision and profitability are affected by the presence of reference prices and consumer loss aversion.

The first essay looks at the fashion industry, where product demand is largely driven by consumers' desire to signal their social status through product usage, and theoretically explains why fashion hits appear to be randomly created. The fashion industry is characterized by its unpredictability and apparent randomness of fashion hits, i.e., no one is able to predict ahead of time which product will become the fashion statement about one's social status. In this essay, I consider fashion as a means consumers use to signal belonging to the high class and propose an analytical model of fashion hits in the presence of competition and consumers able to coordinate on which product to use. I show that consistently with the observed market phenomenon, in equilibrium, consumer coordination involves randomization between products chosen, i.e., in randomness of fashion hits. Analyzing optimal consumer choice, I derive that whenever low-type consumer demand for a product is positive, a price increase results in a higher probability of high-type con-

sumers choosing this product but lower low-type consumer demand. I also show that although high-type consumers may prefer (higher) prices that would lead to complete separation of the high- and the low-type consumers through product use, in equilibrium, firms always price as to attract positive demand from low-type consumers. The equilibrium price and profits turn out to be non-monotone in the low-type consumer valuation of being recognized as belonging to the high class. Equilibrium profits first increase and then decrease in this valuation.

The second essay focuses on another consumer deviation from standard economic theory – consumers evaluate prices relative to a reference point and exhibit loss aversion, i.e., consumer propensity to buy is more negatively affected by prices above the reference point than it is positively affected by prices below the reference point. Consumer loss aversion is a well-established phenomenon in marketing. The objective of this essay is to analytically examine how the competitive strategy and profitability are affected by the presence of consumer loss aversion. I develop a two-period model of competing firms and consumer search behavior affected by the reference price formed from past prices. While assuming that consumer loss aversion increases consumer propensity to search for lower prices, I find that it does not necessarily lead to lower prices and profits in a competitive environment. Specifically, consumer loss aversion could lead to higher prices and profits when consumer valuation is sufficiently high relative to their search costs and the proportion of consumers with positive search costs is in an intermediate range. I also show that even when forward-looking firms fully incorporate the negative effect of current price promotions on future profits, not only they still find promotional pricing optimal, but the optimal magnitude of promotions should not necessarily be reduced.

Chapter 1

A Model of the ”It” Products in Fashion

“Though this be madness, yet there is method in’t.”
— Hamlet Act 2, scene 2, 205-206.

1.1 Introduction

As opposed to utilitarian products, fashion products are bought mostly for their social value, i.e., for their propensity to impress on others their user’s desirability, such as taste, wealth or otherwise belonging to a popular social group. For example, in his article for Encyclopedia of Social Sciences, Sapir (1931) called fashion *“an outward emblem of personal distinction or of membership in some group to which distinction is ascribed,”* and Simmel (1957) maintained that *“fashion…satisfies people’s needs for group cohesion and the need for individual elevation from the society.”* In other words, fashion products are examples of status goods: products whose purpose is to signal that their user belongs to the desirable “high class” (Veblen 1899, Blumer 1969, Pesendorfer 1995).

At the same time, a characteristic distinguishing fashion from other status goods is an apparent randomness of fashion hits: numerous new designs come out of fashion houses every season, but only a few products turn out to be popular among consumers. *“Say what you will about haute* *couture, it's nothing if not unpredictable,"* asserts Susannah Frankel (1999), a fashion editor for *The Independent*. Fashion hits appear to emerge randomly: no one seems to be able to predict who will create the next hit product. Robinson (1961) quotes a trade association director as saying *"Thanks to fashion, this is an industry without statistics,"* and Barnett et al. (2010) summarize the same sentiment by describing luxury apparel as *"an unpredictable market in which the success or failure of any new product is a matter of chance largely immune to predictive analysis"*.

To illustrate the unpredictability of fashion hits, consider the phenomenon of the "it bag" in the fashion handbag market.[1] "It bag" refers to the bag that is a popular best seller and is deemed a must-have product for the social success often unavailable by the time it is widely recognized as such. There is a constant competition for the it bag among top fashion houses. In 1998, Fendi created the first bag referred to as the "it bag" by the popular press – the "original" Fendi Baguette. With such a successful debut, one could expect the same company would be able to build on its success and create the it bags of the next seasons. However, in the years that followed, the it bag of the season was made by Prada in 1999, Dior in 2000, Balenciaga in 2001, Luella in 2002, Louis Vuitton in 2003, and so on (Rumbold 2007). Although all fashion houses strive to create the it bag, not only they do not know whether their product will become the it bag or not, they don't even seem to be able to do much to affect the choice. For example, Betts (2006) quotes Stuart Verves, the designer behind the "suddenly hot British brand Mulberry," as saying *"creating an 'It' bag is just dumb luck. ... You have to wait for your time"* to support her point that *"the creative coup is often more the result of serendipity than science."*

This observation that randomness seems to be at the heart of fashion motivates our research. If one is to make predictions of and recommendations for the optimal firms' behavior in the fashion industry, one needs to understand the rules that govern the randomness of the fashion hits. This paper analyzes the fashion hit selection, and shows that although it is inherently random, the probability of a product becoming the fashion hit is in equilibrium uniquely defined. It then builds on this result to make predictions of and recommendations for the firms' optimal profit-maximizing

[1] The term "it bag" gained popularity in late 1990's – see, e.g., the Wikipedia entry at http://en.wikipedia.org/wiki/It_bag.

strategy in a competitive market.

1.1.1 The Fashion Industry

Products of the fashion industry are usually classified by season, especially so for high fashion. Fashion houses release their new designs at Fashion Weeks, normally about six months ahead of the season. For example, Fall/Winter collections are usually showcased in these events between January and April. Journalists and select other fashion influencers, such as editors of prestigious fashion magazines, attend these events and gather information about new designer collections. They screen these collections and share their opinions on what is going to be "hot" in the upcoming season through word-of-mouth communication, fashion media, such as fashion magazines, fashion blogs or TV programs. Consumers decide what to buy when the products actually arrive at the market, which is several months later than the Fashion Week events. At that time, many consumers may not know exactly which product will be the most popular, but main contenders are not that difficult to guess.[2] A particular product that catches on among consumers and becomes a symbol of "in" is often referred to as the "it" product.

It is important to note that while not released to consumers, prices of fashion products are usually decided much earlier than the product goes on sale and before the "it" product is determined. For example, according to Eileen Balaban-Eisenberg, the executive V.P. of the Connaught Group, New York, fashion designers make the product price information available for the key influencers in the fashion world even before the fashion shows at which the products are introduced, although this price information is not yet made available for the general public at the time.[3]

Note that some status goods, such as diamonds, luxury watches, cars, and certain handbags are well known to be status goods. Owning any sufficiently expensive product is enough to convince others of the high status of the owner. For example, having a Lamborghini may be all that is needed to impress one's wealth on others. For fashion products, however, what matters is whether one has

[2]See for example a blog discussion http://forum.purseblog.com/handbags-and-purses/whats-the-new-it-bag-for-2010-a-539341.html about the it bag of 2010 (the it bag turned out to be Mulberry Alexa).

[3]From a personal interview of Eileen Balaban-Eisenberg with the authors.

the right product. The recognition of a person as "in" crucially depends on the specific product she uses.

1.1.2 Information Asymmetry and Consumer Coordination

Not everyone has equal access to the information about what the right product is. The ability to acquire timely and accurate information is an important characteristic that differentiates one consumer from another in the fashion market. For example, while many blogs cover fashion, it is hard to discern which ones are run by the knowledgeable fashion mavens. Those with good access to fashion information know what is in trend and buy accordingly. Those with poor ability to acquire information can only guess at what to buy. Of course, they could also wait to find out from the popular press what turned out to be fashionable, but buying late is usually synonymous with buying nothing as far as the fashion statement is concerned, since popular designs often become sold-out for several months.[4] Once they are widely available and recognized, the popularity often spells doom for the prestige.[5]

The following story by Chow (2009) is an example of the importance of information access for consumers. In 2009, the Swedish fashion brand H&M in cooperation with designer Jimmy Choo developed a special line of products under the name "Jimmy Choo for H&M." With the fashion news columnist buzz that the line is going to be a hit, consumers "in the know" lined up in front of the store throughout the night before the scheduled release on November 14th, 2009, so that they could be the first ones to get in. By 1 pm that day, most of the special collection products were no longer available. This example illustrates that good access to fashion information is essential for consumers to get the "it" product before it is gone.

A product may often become fashionable due to a relatively small group of "high-class" consumers adopting or recommending it. For example, editors of fashion magazines or popular blogs, the apparel and accessory choices by movie stars or other celebrities, a collective chatter of con-

[4] Reasonably attractive fashion hits of one season often do enjoy popularity for several years, but no longer as a fashion statement.

[5] As in Yogi Berra's complaint about Ruggeri's restaurant, "Nobody goes there anymore. It's too crowded."

nected "fashion mavens," or trend-setters at elite clubs could spark a tipping point in the fashion trend. Hush Puppies popularized by a few enthusiasts (and the actor/fashion designer Isaac Mizrahi) in 1994 (Gladwell 2000) and an up-to-6-month shortage of UGG boots in late 2003 after a number of actresses were seen wearing them (Grant 2003) are some examples of the role of the small group of opinion leaders. Fashion editors are also important agents in the fashion industry since their choices and recommendations have considerable impact on the fashion trends. In fact, some industry insiders argue that fashion editors are the single most important influencer of fashion.[6] But the role of the consumer coordination has been largely ignored in the academic literature.

Note that since the purpose of fashion use by those belonging to the high class (henceforth referred to as the high-type consumers) is to distinguish themselves from those belonging to the low class (henceforth referred to as the low-type consumers), and the purpose of the fashion use by the low-type consumers is to mimic the high-type ones, the high-type consumers have the incentive to coordinate on their choice within themselves, but not to coordinate with the low-type consumers, while the low-type consumers have an incentive to guess (or adapt) to the high-type coordination, but do not have the incentive to coordinate with each other. It is therefore natural, that the coordination is designed as to be recognizable by the high class but not by the low class. In other words, information asymmetry in which the high class has superior access to fashion information is a result of the individual incentives as opposed to an arbitrary possibility.[7]

The interaction between the competitive firms and the high-type consumer coordination in the fashion industry is in the center of our research. Our model shows how the randomness of fashion hits comes as a result of high-type consumer coordination and their optimal response to the low-type consumer behavior. As we have stated before, the first objective of this model is to understand the driving forces and the rules of randomness of fashion hits. The second objective is to predict

[6] Or as Eileen Balaban-Eisenberg puts it, "fashion editors do not predict trends, they dictate them."

[7] Following similar logic, Bourdieu (1984) argued that the changing customs and lifestyle of the high class society can be explained through their desire to distinguish themselves from the low class in a way that is difficult to imitate. Specifically, Bourdieu considered the role of tastes and manners developed from childhood. We extend this logic to the context of contemporary fashion marketplace where consumers use products whose prices are set by firms. We thank an anonymous reviewer for suggesting to us this interesting parallel.

and make recommendations for the optimal competitive firm strategy in such a market.

1.1.3 Summary of Results

In the model formally introduced in Section 1.3, we consider two firms competing in a fashion market with two consumer types: the high and the low type. The high-type consumers are the "socially desired" type, so that all consumers would prefer being recognized as of the high type than as of the low type. Consumer types are their private knowledge and are not directly observable by other consumers. Other consumers however form expectations of a given consumer type based on the product she uses. The high-type consumers are allowed to coordinate their choices as to maximize their expected utility. The game starts with each firm introducing one product. Then the high-type consumers engage in the coordination about which product if any should be adopted. This coordination results in a recommendation of the choice to make. Only the high-type consumers know this recommendation. In the next stage, all consumers make the product purchase decision. Keeping in line with the idea that fashion is used mainly for social reasons, we assume that consumers derive utility of a product only through its effect on how this choice affects other consumers' beliefs that the consumer in question is of the high type.[8] Since one of the main contributions of our paper is to analyze the role of the consumer coordination, one of the important benchmarks we consider is the model without consumer ability to coordinate. This benchmark differs from the above model only in that the coordination stage is absent, and hence the high type consumers have neither the ability to coordinate nor an informational advantage over the low type consumers. The main results are as follows.

First, we find that in equilibrium, the coordination will always involve randomness of product recommendations. In other words, the consumer ability to coordinate always leads to randomness of fashion hits. Nevertheless, the probability of each product being selected as the "it" product is uniquely defined as a function of the product's price. Thus, unlike in the case without coordination, the expected sales are predictable and the expected profit functions can be uniquely derived.

[8] As we discuss in Section 1.3, such product value can be justified through the effect of product ownership on who the consumer will interact with in the matching game following product choice.

Another implication of the coordination in this model is that market for fashion always exists (without the coordination, market may not exist if the valuation of the low-type consumers for being recognized as "in" is higher than that of the high-type consumers).

Second, we find that the expected high-type consumer demand and the probability of a product becoming the "it" product (i.e., the desirable and the most popular one) increase in own price if and only if the equilibrium demand from the low-type segment is positive. On the other hand, the equilibrium demand from low type consumers decreases in price. We also find that the equilibrium price may be such that the product attracts strictly positive expected demand from the low-type consumers. Thus, in view of the previous result, in the neighborhood of the equilibrium price, the high-type consumers may appear to prefer the higher-priced products: even though they do consider the direct effect of a higher price as a negative factor in choosing a product, they may end up selecting the product with higher probability if its price is increased from the equilibrium one. Looking at the welfare of high-type consumers, we also find that the high-type consumers prefer the prices to be (weakly) higher than their optimal level for the firms.

Furthermore, we find that equilibrium prices and profits are not monotonic as functions of the low-type consumer valuation of being "in". In particular, as this valuation increases from zero to infinity, the equilibrium price first increases, then decreases, then increases again, and finally decreases. On the other hand, even though the total demand increases as the valuation increases, the profit first increases and then decreases. In other words, there is a level of low-type valuation of matching with the high-type consumers which is optimal for the industry.

Finally, as compared with the monopoly case, we find that competition may increase price. This happens when low-type consumer valuation for being recognized as "in" is not too high.

The rest of the paper is organized as follows. Section 1.2 discusses some academic literature related to fashion and status goods. Section 1.3 fully specifies the model. Section 1.4 presents the analysis of the consumer strategy. Section 1.5 discusses the full equilibrium of the game and provides insights into the market dynamics, and Section 1.6 discusses the robustness of the results to some assumptions. Section 1.7 concludes.

1.2 Related Literature

Our paper is most related to the economic and marketing analysis of fashion and fads prevalent in many socially-influenced markets. “How can success in cultural markets be at once strikingly distinct from average performance, and yet so hard to anticipate for profit-motivated experts armed with extensive market research?” ask Salganik, Dodd and Watts (2006). Although it is possible that some difficulty in predicting the outcomes is due to performance being a convex function of quality (Rosen 1981), Salganik, Dodd and Watts (2006) experimentally support the idea that social influence results in inherent randomness of choice. The extant literature offers two underlying mechanisms for seemingly random consumer coordination on a particular product within a category. One is the information cascade resulting from sequential consumer choice under incomplete information about the product. For example, Bikhchandani, Hirshleifer and Welch (1992) argue that the convergence behavior we observe in fashion and fads can be explained through an informational cascade model. In their model, individuals act in sequence and each individual makes her choice based on the action of all the people acting before her and the private signal she receives. In the equilibrium, it is optimal for later consumers to follow the behavior of the very early ones and ignore their own information. This leads to the convergence of consumer behavior. A fundamental building block of the information cascade models is the uncertainty about the intrinsic value of the product and the individual consumers having partial and private information about this value.[9]

Another stream of literature going back to the essay of Veblen (1899) emphasizes the role of fashion as a consumer-to-consumer signalling device. The idea is that people can signal their membership in a particular social class by their knowledge of the ways people of that class behave. Karni and Schmeidler (1990) studying the variation in the demand for fashion products from a purely consumer perspective without incorporating firms in the model, consider three product colors and two types of consumers. Consumers of one type prefer more consumers of the same type and fewer of the other type to use the same color as them, while consumers of the other type

[9]Information cascades are not confined to fashion. For example, Zhang (2010) empirically finds significant information cascades in the US kidney replacement market.

prefer more of both types of consumers to use the same color as them. In the equilibrium, there is a dynamic variation in demand for different colors, which mimics the dynamics of fashion cycles: consumers of the former type change colors as to avoid being mimicked by consumers of the other type. Pesendorfer (1995) proposes a dynamic game between a monopoly firm and consumers to explain the occurrence of fashion cycles. Consumers are of two types and everyone prefers to be matched with the desired type of consumers. The game is played for infinitely many periods and in each period, consumers' payoffs are determined by the type of people they are matched with, which then depends on the fashion design they use. Fashion cycle arises in the equilibrium. As Pesendorfer notes, there is an inherent issue in modeling competition in a market for signalling devices. In competition in most markets, if products are not differentiated, the result is pricing at the marginal cost of one of the firms and the sales going to the product with the lowest marginal cost. If marginal costs of both firms are equal, profits are zero and therefor the split of market share is inconsequential for either profits or consumer welfare. On the other hand, in a market for signalling devices, a price reduction may rob the product of its value. Therefore, competition may not result in prices declining to marginal costs and any combination of market shares may constitute an equilibrium.[10] Thus, the extant literature on status goods mostly concentrates on the monopoly case. As we will see, considering the consumer coordination allows us to model competition, which is an important aspect of fashion industry, in a way that results in the expected profits and market shares uniquely defined in equilibrium.

Our paper adopts this second approach of looking at fashion as a device used by consumers to convey information about their type to other consumers. In difference to the above literature, we model competition and the role and implications of high-type consumer coordination on which product to buy. Yoganarasimhan (2012) also considers informational structure of the consumer

[10]This indeterminism of profit expectations and a lower bound on prices is also implied by the models in Bagwell and Berheim (1996) and Becker (1991), although if consumer types are horizontally differentiated, so that consumers of each type prefer to identify with own type, products may have social value without being expensive or unpredictable (Kuksov 2007). However, in the latter case, the products are self-expressive but not status goods. Competition in status goods can also be modeled through assuming high-enough heterogeneity among high-type consumers with competitors targeting distinct market segments (Kuksov and Xie 2012), but the above-discussed problem arises when heterogeneity is not high enough. The model in this paper shows how one can model competition between undifferentiated status goods.

market similar to our model (i.e., high type consumers have better knowledge of what product is the "it") and provides excellent examples of randomness in fashion. Her paper assumes that fashion hits are exogenously random with a given probability distribution and focuses on the question of whether the monopoly firm should reveal the information about which product is a hit to all consumers or not. In difference, we endogenize the randomness of fashion hits (i.e., derive the probability of a product becoming the "it" product) and consider firms competing in price. Our model contributes to understanding of fashion markets in particular by proposing a theory both to explain the randomness of fashion hits and to predict the likelihood of a product becoming a fashion hit. In a related paper, Wernerfelt (1990) considers competition in advertising and assumes that the product-consumer type connection is a probabilistic function of advertising spending.

Since the signaling value of the status goods depends on the mass and the distribution of users, our paper is also related to the literature on consumers' desire for uniqueness and conformity (e.g., Leibenstein 1950, Becker 1991, Amaldoss and Jain 2005a, 2005b, and Balachander and Stock 2009), but relates these desires to the type association. Again, that literature does not consider the role of the consumer coordination and thus does not obtain the randomness of fashion as a result.

Consumer desire to communicate their type to each other provides a different but related perspective to the extensive literature on costly signalling done for the purpose of selling a product or skill (e.g., Spence 1973, Milgrom and Roberts 1986, Balachander and Srinivasan 1994, Desai and Srinivasan 1995, Moorthy and Srinivasan 1995, Simester 1995, Anderson and Simester 1998, Kalra et al. 1998, Desai 2000, Soberman 2003).

1.3 Model Setup

The market consists of two firms offering one product each and a unit mass of consumers of two types: the high-type consumer segment has mass α and the low-type one has mass $1 - \alpha$. We assume that consumers are interested in "projecting" a high-type image of themselves or, in other words, in convincing other consumers that they are of high type. As the consumer type is not directly observed, consumers try to signal their type through the product choice. We assume that the signalling value of the product is the only utility consumers derive from the products. This captures the notion that the fashion is a social device and its utility is derived from social interactions.

It is convenient to "rationalize" the utility of projecting an image through social interaction that occurs after product choice and in which the payoff depends on the type of consumers a given consumer interacts with. Specifically, following Pesendorfer (1995), we model the social value of a product as coming from a "matching game." In this game, consumers are randomly paired up with other consumers using the same product and the payoff of a match between consumers of types j and k depends on j and k only ($j, k = 1, 2$), i.e., consumers of the same type are homogenous. We normalize any consumer's payoff from matching with a low-type consumer to 0, and payoff of the high-type consumer matching with a high-type consumer to 1. We denote the remaining parameter – the payoff of the low type consumer from pairing up with a high type consumer by V. Consumer's objective in product choice is to maximize the expected payoff in the matching game net of the product's price. Therefore, the utilities of purchasing Product k for a high- and low-type consumer are given by

$$U_h(k) = q_k - p_k \quad \text{and} \quad U_l(k) = q_k V - p_k, \tag{1.1}$$

respectively, where q_k is the fraction of high-type consumers among users of product k. To keep notation parsimonious, we will refer to the option of "neither" product as product $k = 0$ with the associated price of this option being $p_0 = 0$.

The above assumption that consumers are paired up with each other according to their product use can be justified as an equilibrium strategy of consumers individually deciding on whether to accept a pairing. Namely, in equilibrium, one product will carry a better image than the other, i.e., it will have a higher proportion of high type consumers among its users. Since consumer type is private information, the only criterion that consumers can use to decide whether to accept a proposed pairing with another consumer is his/her product choice. So consumers who have the product with the higher image will only want to accept another consumer using the same product, since such strategy is feasible and results in a higher probability of pairing up with a high type consumer. Consumers with the lower image product would like to pair up with consumers using the higher image product, but given the above, such a strategy would not result in a pairing, since matching can only occur when both consumers are willing to do so. Consumers with the lower image product can only pair up among themselves, too (see Burdett and Cole 1997 for an explicit consideration of consumer acceptance strategy in a matching game which proves the above matching pattern).

It is worthwhile to note that the above formalization of consumer utility as coming from the matching game is mathematically equivalent to the assumption that consumers derive social utility defined by Equation (1.1) from convincing the other consumers that they are high-type with probability q_k. In other words, motivation of the social utility through the matching game, although theoretically appealing, is not essential. Alternatively, we could postulate that consumers derive utility from projecting the image of high type and this utility is proportional to the how precise that image is.

Turning back to the model, we assume that the high type consumers have the ability to coordinate in such a way that the outcome is unobserved by the low type consumers.[11] This assumption is crucial for our analysis. To formally model the high-type consumer coordination, we assume that the high-type consumers have access to the coordinating device which displays the purchase recommendation as to maximize the high-type consumer utility.[12] We will call this device "the coordinator."

To be consistent with the in-type homogeneity assumption, we further assume that all high-type consumers perfectly observe the coordinator's recommendation before the purchase decision, but

[11] According to Robinson (1961, p. 384), "The true fashion leaders comprise a much smaller and more esoteric group than is commonly supposed. They will have often lost interest in a designer by the time his name is known to the general public".

[12] Note that since the high-type consumer population is a priori homogenous, there is no issue of trading off utility across high-type consumers.

the low-type consumers acquire no information about it until after the purchase decision stage. We assume that right before the matching game, low-type consumers also learn the recommendation. Although they can no longer purchase the recommended product, they could possibly discard an item and not use it in the matching stage even if they have purchased it. This assumption is not essential for the main results.

The firms maximize their individual profits and have a constant and equal marginal product cost, which we normalize to zero. The game sequence and the decisions are as follows:

1. Firms 1 and 2 simultaneously introduce products and prices.
2. The coordinator recommends "product 1," "product 2," or "neither." The high-type consumers observe this recommendation, while the low-type consumers do not.
3. Consumers make purchase decisions.
4. Low-type consumers observe the recommendation and each consumer decides which product to use, if any, out of the products purchased. The "matching game" is then played out and consumers receive the utility according to Equation (1.1).

To fully define the payoffs in the matching game, we assume that choosing a product not chosen by anybody else results in zero payoff.[13] We use subgame-perfect Nash equilibrium as the solution concept, and solve the model by backward induction. Since our objective is to analyze the potential role of the coordination, we will focus on equilibria in which the high-type consumers follow the recommendation, i.e., we assume that if a product is recommended by the coordinator, all high-type consumers buy it, and if the device chooses "neither," no high-type consumers buy either product. This is a natural assumption since the coordinator is chosen to maximize the high-type consumer utility and therefore it is always optimal for the high-type consumers to follow its recommendation.[14]

[13]This assumption simplifies notation but is not necessary for any results: alternatively, and with no change in equilibrium results for all but a zero mass of consumers, we could define this payoff to be any value.

[14]An equilibrium in which high-type consumers completely ignore the recommendation also always exists, since if all consumers ignore any recommendation, there is no reason for any consumer to follow any recommendation. Considering such an equilibrium would amount to ignoring the role of the coordination. We will also discuss and rule out the possibility that high-type consumers follow the recommendation with probability $q \in (0, 1)$, or, equivalently, only some (positive mass) consumers follow it while the rest ignore it.

1.4 Consumer Choice and the Optimal Coordination Rule

We devote special attention to the consumer and the coordination strategies rather than just to the equilibrium predictions of the full game, because fully considering all possibilities of the subgame starting from the coordination stage would allow us not only to predict the equilibrium outcome of the full game, but also to analyze the profit-maximizing strategy of a firm conditional on its expectation of the other firm's choices.

Note that the coordination can ensure high-type consumer payoff of α by the "neither" recommendation, which would be optimal for all high-type consumers to follow. Therefore, the coordinator would not be providing the best recommendation to the high-type consumers if it were to ever recommend a product with price above $1-\alpha$. Given that the low-type consumers can expect this part of the coordination strategy, they will have no demand for such a product either. Thus, price above $1-\alpha$ guarantees zero sales and profits. It will be soon clear that each firm can guarantee positive sales by pricing sufficiently low, and therefore, pricing above $1-\alpha$ is a strictly dominated strategy. It will imply that pricing at 0 is also a strictly dominated strategy. Therefore, assuming without loss of generality that $p_2 \leq p_1$, we restrict our analysis in this section to the consumer strategies under the following condition:[15]

$$0 < p_2 \leq p_1 < 1 - \alpha. \tag{1.2}$$

Solving the game by backward induction, let us consider first consumer product use in Stage 4. Let us denote the low-type consumer demand for product k by x_k. If the coordinator recommended product $k \neq 0$, the users of product k will consist of all high-type consumers and those low-type consumers who purchased it. Note that the low-type consumers who bought product $j = 3 - k$ will chose not to use it. Therefore $q_k = \frac{\alpha}{\alpha + x_k}$ and $q_0 = 0$. The equilibrium value of q_j is either

[15]When $p_k = 1 - \alpha$, there is always an equilibrium where high-type consumers never coordinate on Product k and therefore Product k receives zero demand. However, there are other equilibria, of which the most favorable to Firm k will be defined by continuity from Proposition 1. As we will see in the following section, the latter outcome is the only one that can be an equilibrium outcome of the full game. Therefore, to avoid discussing these multiple equilibria throughout the text, we restrict our attention to $p_k < 1 - \alpha$ in this section.

not defined (if the low-type consumers who purchased product j all decide not use the product) or is zero (if some low-type consumers bought product j and at least one of them decides to use it), but the matching game payoff from using product j is zero in either case. If the coordinator recommended $k = 0$, i.e., not to buy either product, all consumers will end up not using either product in the matching game. This would lead to $q_1 = q_2 = 0$ (or undefined) and $q_0 = \alpha$.

In Stage 3, high-type consumers follow the recommendation, but a low-type consumer have to make her decision based on her expectation of the coordination and her expectation of the proportion of other low-type consumers buying each product. She can choose product 1, product 2, both or neither.

A low-type consumer purchase of product $k \neq 0$ gives her the expected matching utility of $q_k V$ when product k is coordinated upon by the high-type consumers and 0 when product $3 - k$ is coordinated upon. This incremental utility from the product choice does not depend on whether she also decides to purchase product $3 - j$ or not, which means that low-type consumer choice to buy or not each of the two products can be treated as two separate decisions. Specifically, the consumer will purchase one unit of product k, $k = 1, 2$, if and only if $U_l(k) \geq U_l(0)$. However, the equilibrium low-type consumer demands for the two products end up being inter-related because the high-type consumer coordination on product k and the low-type consumer expectation of this coordination are dependent on the high-type consumer trade-off between adopting product k and $j = 3-k$. Note that $U_l(k)$ decreases in the mass of low-type consumers who decide to buy product k. Given the low-type consumer expectation of the probability with which product k is coordinated upon, the low-type consumer demand for either product will therefore be determined by equating $U_l(k) = U_l(0)$ whenever this equation leads to an internal solution.

To determine the equilibrium coordination strategy (Stage 2), one needs to take into account high-type consumer expectation of the low-type consumer behavior in the following stage, given p_1 and p_2. Since the best choice to coordinate upon depends on the expected low-type consumer decisions and low-type consumer decisions depend on low-type consumer expectations of the coordination, the equilibrium coordination strategy follows from the simultaneous solution to high-

	Coordination strategy	Low-type consumer demand
Area 1	$\delta_k = \frac{p_k}{V(p_k+\alpha)}$	$x_k = \frac{\alpha(1-\alpha-p_k)}{\alpha+p_k}$
Area 2	$\delta_k = \frac{1}{2} + \frac{(2-V)(p_k-p_j)}{p_1+p_2+\sqrt{(p_1-p_2)^2(V-1)^2+4p_1p_2}}$	$x_k = \frac{\alpha V}{2p_k} - \alpha + \frac{\alpha V(2-V)(p_k-p_j)}{p_k(p_1+p_2+\sqrt{(p_1-p_2)^2(V-1)^2+4p_1p_2})}$
Area 3	$\delta_1 = \frac{V-p_2-V(p_1-p_2)}{V(1-p_1+p_2)}$; $\delta_2 = \frac{p_2}{V(1-p_1+p_2)}$	$x_1 = 0$; $x_2 = \frac{\alpha(p_1-p_2)}{1-p_1+p_2}$
Area 4	$\delta_1 = 0$; $\delta_2 = 1$	$x_1 = 0$; $x_2 = \max\{0, \frac{\alpha(V-p_2)}{p_2}\}$

Table 1.1: Coordination Strategy and Low-Type Demand for $p_1 > p_2$ ($k = 1, 2; j = 3 - k$).

and the low-type consumers' equilibrium conditions. A-priori, the coordination strategy must be one of the following: (1) follow a pure strategy and choose one of the options; (2) randomize between two options, e.g., picking product 1 and product 2 or between product 1 and neither, and (3) randomize between all three options. We will show that given prices satisfying Equation (1.2), choosing "neither" only or randomizing between one of the products and "neither" is never optimal. Therefore, the equilibrium coordination conditions come from the indifference between the two products or between all the three options, or from choosing one product only.

In equilibrium, for each product, there must be some low-type consumers who do not buy it. This is because if all low-type consumers buy a product in equilibrium, the net-of-price payoff for the high-type consumers of buying that product must be smaller than α and hence it is strictly optimal to coordinate on neither instead of this product.

Furthermore, we will show that whenever the low-type consumer demand for each of the two products is positive, it is optimal to randomize the coordinated choice across all the three options. We will also show that the equilibrium pricing strategy is such that in expectation, each product receives positive demand from the low-type consumers, although when the equilibrium pricing strategy is mixed, the optimal coordination strategy is to randomize between the two products only, and for the low-type consumers to only buy the lower-priced product.

Let δ_k denote the probability that the coordinator picks Product k, and x_k denote the size of low type consumers who purchase Product k, $k = 1, 2$. Then the following proposition characterizes the optimal coordination strategy and the consumer behavior conditional on the prices of the two products.

Proposition 1. *Given two prices satisfying Equation (1.2), we have*

1. *("Low price area") If $\alpha(1-V)(p_1+p_2)+(2-V)p_1p_2 < \alpha^2V$, the coordination randomizes between all the three options and low-type consumer demand is positive for both products.*

2. *("Medium price area") If $\alpha(1-V)(p_1+p_2)+(2-V)p_1p_2 \geq \alpha^2V$ but $p_2(1-V+p_1)+(p_1-V)(1-p_1) < 0$, the coordination randomizes between Products 1 and 2 and low-type consumer demand is positive for both products;*

3. *("High price area") If $p_2(1-V+p_1)+(p_1-V)(1-p_1) > 0$ but $p_1 < 1-p_2/V+p_2$, the coordination randomizes between Products 1 and 2 and the low-type consumer demand is positive only for Product 2, and*

4. *("Very high price area") If $p_1 \geq 1-p_2/V+p_2$, the coordination will choose Product 2 with probability 1 and Product 1 receives no demand;*

For $p_1 > p_2$, the coordination choice probabilities and the low-type consumer demands for the two products in the four areas above are stated in Table 1.1.[16]

Proof. See the Appendix. □

Note that some areas may be empty for some V or α. Area 4 is not empty only when $V < 1-\alpha$. The other three areas are not empty for $V < 2(1-\alpha)$. For $V \geq 2(1-\alpha)$, the medium- and high-price regions disappear as well. It is then immediately apparent that for V above this cut-off, the full equilibrium of the game must involve the coordination selecting "neither" with positive probability. Area 1 is the only one where the coordination will select "neither" with positive probability. This is because the high-type consumers can only get the payoff of α if the coordination picks neither. The coordinator must recommend a product to make the high-type consumers obtain a higher payoff. However, when prices are low enough, no randomizing between the two products would prevent the low-type consumers from buying, and the coordination ends

[16]As we derive in the appendix, for $p_1 = p_2$, the equilibrium of the consumer choice subgame can be different from the one stated in Table 1.1. Specifically, in Areas 3 and 4, any subgame in which the coordination picks the two products with probabilities δ_1 and $1-\delta_1$, where $\delta_1 \in [0,1]$ for Area 4 and $\delta_1 \in [p_1/V, 1-p_1/V]$ for Area 3, and low-type consumer demand is zero for either product, is a possible equilibrium outcome. However, as we show in Section 5, such equilibria of the consumer choice subgame cannot be reached with a positive probability in a subgame-perfect equilibrium of the full game.

up having to result in "neither" with a positive probability. Figure 1.1 illustrates the areas in Proposition 1 and how they change with V.

Insert Figure 1.1 here.

To understand how parameter values affect the equilibrium, note that in Area 1, the probability of coordinating on neither increases in α. This is because when the fraction of the high type consumers becomes larger, the utility from pairing with a random consumer (i.e., pairing with a high-type consumer with probability α and a low-type consumer with probability $1-\alpha$) increases, making the neither option more attractive. The probability of the coordination picking "neither" also increases in V. This is because as the low types' valuation for social interactions increases relative to that of the high type consumers, it becomes more and more difficult to create a fashion hit without letting the low type consumers buy. In fact, as V goes to infinity, the probability that the cootrdinator recommends any product tends to zero. Also note that as a product's price decreases to zero, the product's penetration of the low-type consumer segment approaches full even as its probability of being chosen by the coordinator tends to zero. Examining the coordination outcome probabilities in Table 1.1, one can also see the following effect of price.

Corollary 1. *An increase in a product's price leads to the high-type consumers coordinating on it with a higher probability but fewer low-type consumers buying it in all regions except the higher priced product in Areas 3 and 4 above. In Area 3, an increase in the higher-priced product leads to lower probability of it being coordinated upon, while in Area 4, higher priced product has zero demand.*[17]

Proof. Immediately follows from Table 1.1. □

The intuition for this result is as follows. If one of the prices is increased, the coordinator is trading off between the cost of the product and the changed expected utility of matching. The direct effect is that the higher price reduces the utility of the high type consumers and the coordinator's

[17]In the following section, we will prove that a pure-strategy equilibrium price never falls in Area 3 or 4.

preference for that product should decline. However, an increase in price also inhibits the low-type consumers from buying, which enhances the high-type consumer utility indirectly through the effect on the endogenous product value. This encourages the high-type consumers to coordinate on that product more frequently. The above corollary states that whenever there is such a tradeoff (i.e., whenever the low-type consumer demand for the product is positive), the effect of the higher expected matching utility always dominates the cost effect. This results in the high-type consumers coordinating on that product with a higher probability, but such that the demand from low-type consumers still decreases. This reasoning only fails to apply for the higher-priced product in the high-priced area because in that area, the higher-priced product receives zero demand from the low-type consumers.

As we have noted before, together with the equilibrium where high-type consumers follow the recommendation, there is also an equilibrium where high-type consumers ignore it (i.e., technically, coordination failure is always an equilibrium). Could there also be an equilibrium in between, so that some but not all high-type consumers coordinate, or equivalently, high-type consumers follow the recommendation of the coordinator with probability $q \in (0, 1)$? In such an equilibrium, high-type consumers would have to be indifferent between following the recommendation and another choice. One can argue that such outcome should not be expected since it is not stable in the following sense: if a slightly higher proportion of high-type consumers follow the recommendation, all high type consumers strictly prefer to follow the same choice with probability 1. The same argument could be applied to slightly lower proportion of consumers following the recommendation, in which case, the optimal could be to "do the opposite of the recommendation." However, such an outcome could in a sense still be thought as following the recommendation once the recommendation is redefined to mean the opposite of what it states. The possibility of an equilibrium where high-type consumers are indifferent between following the recommendation and not is then based on a knife-edge balance between the fraction of consumers who end-up following it and the fraction who respond to the recommendation in a precise way as to counter-balance the incentive to follow.[18] While the existence of such strategy is not easy to rule out, we show in the Appendix that there are no equilibria where some high-type consumers follow the recommendation and some do not follow it in the sense that they make their choice independently of the recommendation (i.e., although some do not follow, they do not modify their behavior as to in aggregate to counter-balance the incentive to follow).

1.5 Pricing Strategy and the Equilibrium of the Full Game

An interesting consequence of the total demand implied by Table 1.1 is that the tradeoff between price and demand for the profit maximization of a firm in this model is not necessarily the usual negative relationship between price and demand. In this model, lower price may lead to a lower total demand. This is because as Proposition 1 states, in all but one region, higher price increases the product's probability of being chosen although decreasing the demand from the low-type consumers. This is a result of fashion being a signaling device and its value endogenously coming from what it means in social interaction. When low type consumers are buying more, signaling becomes more difficult and hence the value of signaling decreases for the high type consumers. Hence there is a tension between the demand from high types and that from low types, and the overall effect on demand is a priori not clear. Theoretically, it is possible that higher price may create higher demand. For example, in the low-price region, when $V < \alpha$, the total demand would increase with the product's own price.

If there is a negative relationship between price and demand, then the firm has to tradeoff between the two. Since the demand in our model is continuous (except for when prices are equal in Area 3), lowering or raising price doesn't generate a discontinuous gain or loss of demand. In this case, the firm faces the nontrivial choice between charging a high price to create the premium image

[18] Note that in the equilibrium of Proposition 1, high-type consumers strictly prefer following the recommendation to any other choice, and thus will still prefer to do so if a small fraction of high-type consumers are made to deviate and not follow. On the other hand, in the equilibrium where nobody follows the recommendation although the recommendation is random according to Proposition 1, if a small proportion of high-type consumers could coordinate to follow it, they would strictly prefer to do so. In other words, the optimality of nobody following the recommendation is based on that exactly nobody follows it.

	Profits
Area 1	$\pi_k = \frac{\alpha p_k(V+p_k-Vp_k-\alpha V)}{V(\alpha+p_k)}$
Area 2	$\pi_k = \alpha(\frac{V}{2} - \frac{p_k}{2} + \frac{(2-V)(p_k-p_j)(p_k+V)}{p_1+p_2+\sqrt{(p_1-p_2)^2(V-1)^2+4p_1p_2}})$
Area 3	$\pi_1 = \frac{\alpha p_1(V+Vp_2-p_2-Vp_1)}{V(1-p_1+p_2)},\ \pi_2 = \frac{\alpha p_2(p_2+Vp_1-Vp_2)}{V(1-p_1+p_2)}$
Area 4	$\pi_1 = 0,\ \pi_2 = max\{\alpha p_2, \alpha V\}$

Table 1.2: Profit Functions Given Prices $p_1 > p_2$ ($k = 1, 2; j = 3 - k$).

(or, in the model, increase the probability that the product will end up having a premium image) and charging a lower price to generate more sales. Note that this tradeoff is not present in the absence of the high-type consumers' ability to coordinate as the absence of premium image would necessarily also imply absence of consumer demand from either type. The consumer ability to coordinate effectively makes the image a continuous variable through the probabilistic coordination strategy.

1.5.1 Equilibrium Price

The equilibrium coordination and consumer strategies reported in Table 1.1, result in the firms' profits reported in Table 1.2. As we will show below, as V decreases from infinity to 1, the equilibrium is in pure strategies with prices moving from low-price to medium-price area defined in Proposition 1. For $V < 1$, the equilibrium is in mixed strategies with prices in the high price region. A pure-strategy equilibrium cannot be in the high price region, because in that region, slightly undercutting the competitor's price is always better than pricing equally with the competitor.

To intuitively understand the driving forces leading to the equilibrium price strategies, observe that when V is high enough (Region 4 in Proposition 2 below), the medium and high price areas do not exist and therefore the equilibrium is derived from the first-order conditions on the profits in Area 1. Slightly below that (Region 3 in Proposition 2 below), the solution to the first-order conditions comes up to the binding constraint $1 - \alpha$, above which the coordinator would prefer to give up on trying to separate the high- and low- type consumers and thus the price becomes $1 - \alpha$.

When V is lower (Region 2 in Proposition 2 below), the medium price boundary crosses the boundary $p_1 = p_2 = 1 - \alpha$ and firm k's incentives to decrease price p_k from $p_1 = p_2 = 1 -$

α change. Specifically, the first-order conditions of the medium price area suggest decreasing price while the first-order conditions of the low-price area suggest increasing price. Therefore, the equilibrium prices lie on the boundary between these two areas.

When V is even lower (Region 1 in Proposition 2 below), the first-order conditions of the medium-price range suggest a pair of prices strictly within the medium price range. However, to make sure the first-order conditions of the medium-price region define the globally optimal responses, we also need to compare the profits given these prices to the profits from the optimal deviation to the high-price region, which is now not empty. That optimal deviation to higher price that makes the pair of prices rest in the high-price region results in lower profits when $V > 1$ and higher profits when $V < 1$.

Thus, when $V < 1$, responding to the solution of the first-order conditions of the medium-price area by a price in the high-price area is always better than responding by the optimal price in the medium price area. As we stated before, a pure-price-strategy equilibrium cannot exist in the high-price region, and thus for $V < 1$, only a mixed-price-strategy equilibrium exists.

To understand the nature of this mixed strategy equilibrium (for $V < 1$), consider the following process of price adjustment starting from the solution of the first-order equations in the medium-price area. First, the optimal response to such price is a higher price that put the two prices in the high-price area. The optimal response to that high price is to slightly undercut it. The optimal response to the new price is to again slightly undercut it (still resulting in a pair of prices within the high-price area). The undercutting argument continues until one firm finds it better to charge a much higher price than to undercut (this condition is reached before the prices move to the medium price area). But then the other firm will undercut again and the foregoing analysis repeats. This suggests that for $V < 1$, the equilibrium price is a mixed strategy within high price area. The explicit solution for the mixed strategy equilibrium is not analytically tractable.

The following proposition formalizes the above considerations.

Proposition 2. *Let* $V_1 = 2 + \frac{\alpha}{2} - \frac{\sqrt{\alpha^2+16\alpha}}{2}$ *and* $V_2 = 2 + \frac{\alpha}{2} - \frac{\sqrt{\alpha^2+8\alpha}}{2}$*. When* $V > 1$*, the symmetric equilibrium prices and profits are reported in Table 1.3, and the corresponding equilibrium*

Region of V	Prices	Profits
1. $[1, V_1]$	$p_k = \frac{V(2-V)}{2+V}$	$\pi_k = \frac{\alpha V^2}{2+V}$
2. $(\max\{1, V_1\}, \min\{V_2, 2-2\alpha\})$	$p_k = \frac{\alpha V}{2-V}$	$\pi_k = \frac{\alpha V(2-\alpha-V)}{2(2-V)}$
3. $[2-2\alpha, \max\{1+\alpha\}, V_2]$	$p_k = 1-\alpha$	$\pi_k = \frac{\alpha(1-\alpha)^2}{V}$
4. $[\max\{1+\alpha, V_2\}, \infty]$	$p_k = \sqrt{\frac{\alpha(V-\alpha)}{V-1}} - \alpha$	$\pi_k = \alpha + \alpha^2 - \frac{2\alpha(\alpha+\sqrt{\alpha(V-1)(V-\alpha)})}{V}$

Table 1.3: Equilibrium Prices and Profits ($k = 1, 2$, $j = 3 - k$).

Region of V	Editor's Strategy	Low-Type Consumer Demand
1. $[1, V_1]$	$\delta_k = 1/2$	$x_k = 1 - \alpha - V/2$
2. $(\max\{1, V_1\}, \min\{V_2, 2-2\alpha\})$	$\delta_k = 1/2$	$x_k = 1 - \alpha - V/2$
3. $[2-2\alpha, \max\{1+\alpha, V_2\}]$	$\delta_k = \frac{1-\alpha}{V}$	$x_k = 0$
4. $[\max\{1+\alpha, V_2\}, \infty]$	$\delta_k = \frac{1}{V}(1 - \sqrt{\frac{\alpha(V-1)}{V-\alpha}})$	$x_k = \sqrt{\frac{\alpha(V-1)}{V-\alpha}} - \alpha$

Table 1.4: Equilibrium Editor's Choice Probabilities δ_k and Low-type Consumer Demand x_k ($k = 1, 2$, $j = 3 - k$).

editor's strategy (δ_k for $k = 1, 2$) and the low-type consumer demand are reported in Table 1.4. Furthermore, the equilibrium is unique unless $V \in (\max\{1, V_1\}, \min\{V_2, 2-2\alpha\})$, in which case there are also asymmetric equilibria. The equilibrium price decreases in V while equilibrium low-type consumer demand increases in V for $V \in [1, V_1] \bigcup [V_2, \infty]$ and the equilibrium price increases in V while the low-type consumer demand decreases in V in the symmetric equilibrium when $V \in (\max\{1, V_1\}, \min\{V_2, 2-2\alpha\})$. Restricting attention to the symmetric equilibrium, the equilibrium profit increases with V until $2 - \sqrt{2\alpha}$ and then decreases, thus achieving the highest value for $V \in (\max\{1, V_1\}, \min\{V_2, 2-2\alpha\})$. When $V < 1$, there are no pure strategy equilibria.

Proof. See the Appendix. □

Figures 1.2 and 1.3 illustrate the equilibrium prices and profits, respectively, as a function of V when $\alpha = 1/4$. The graphs for other parameter values are similar.

Insert Figure 1.2 and 1.3 here.

Note that the equilibrium prices in Region 1 correspond to the medium-price area in Proposition 1, in which case "neither" is coordinated upon with positive probability by the high-type consumers. The equilibrium prices in Region 2 lie on the boundary between the low-price and medium-price area, and those in Region 3 and 4 corresponds to the low-price area. In the latter two areas, the high-type consumers always coordinate on one of the two products and the low type consumers buy both. Note that when $\alpha > 1/3$, we have $V_1 < 1$ and therefore Region 1 does not exist. When $\alpha < 1/3$, Region 3 disappears.

We now turn to the implications of Proposition 2. First note that it implies that there is a market for fashion for any V including $V > 2$. When $V > 2$, one could speculate that the low type consumers could just buy both products while the high-type consumers buy one of them even if the coordinator fully randomizes between the two products. This would imply that the high-type consumers are better off not buying anything and the coordinator would always select neither. However, for a high V, the coordinator is still able to make the products worthwhile to the high-type consumers by choosing them with sufficiently low probability. In equilibrium, although the probability of the coordinator selecting one of the products tends to zero as V tends to infinity, the market (total demand) for fashion products actually expands, as the lower expected demand from high-type consumers is more than compensated by the increased demand from the low-type consumers.

Proposition 2 also implies that even without the restriction of symmetric equilibrium, the equilibrium price is non-monotone in V since the unique equilibrium price is decreasing in V in Regions 1 and 4 and is higher at the upper end of Region 1 than at the lower end of Region 4. Since prices are equal to zero when $V = 0$, the equilibrium price first increases, then decreases, then increases again, and finally decreases asymptotically approaching $\sqrt{\alpha} - \alpha$ as $V \to \infty$ (see Figure 1.2 for an illustration for $\alpha = 1/4$). It is also interesting to observe that among all possible equilibria in Region 2, the symmetric one given in Proposition 2 is the optimal one for the industry in the sense that total industry profit is the highest among all potential equilibria. However, the symmetric equilibrium is not Pareto optimal. In fact, while the lower-price firm is worse off in any asymmetric equilibrium than in the symmetric equilibrium, the high-price firm is not only better off than the lower-price firm but also better off than it would be in the symmetric case.

It could be interesting to compare the equilibrium prices to the ones preferred by the high-type consumers and the socially-optimal ones. For this purpose, note that the preferred price by the high-type consumers is $p_1 = p_2 = V/2$ when $V < 2(1-\alpha)$ and the coordinator is indifferent between any prices otherwise. At $p_1 = p_2 = V/2 < 1-\alpha$, the coordinator will only pick the two products and randomize between them with equal probability. The low type consumers buy neither product, but they are just indifferent between buying and not buying. The above prices are the best for the high-type consumers as they prefer to have the cheapest "prestige" product that only the high type consumers will buy. A socially optimal price is $p_1 = p_2 = V/2$ for $V < 1$ and $p_1 = p_2 = 0$ for $V > 1$. This is because when low type consumers' matching utility is relatively low compared with that of the high type consumers, it is socially optimal for them to be separated from the high-type consumers to ensure that the high type consumers realize the highest matching utility (note that the fact that this separation comes at a price is not important for the consideration of social optimality since the price only results in wealth transfer and does not affect the sum of consumer surplus and the firms' profits). When the opposite is true, then the social optimal outcome should ensure that the low types are matched with high-type consumers as often as possible. Thus, for $V < 1$, the coordinator's preference is the same as the socially optimal outcome, but for $V > 1$, the high-type consumers prefer higher prices than socially optimal.

Thus, we find that equilibrium prices are lower and more low type consumers buy than what the high-type consumers would prefer. This reflects the tension between the high-type consumers (in a sense, the core demographics of the fashion firms) and the firms' interests. The high-type consumers would prefer to exclude the low type consumers from adopting the it product while firms want to attract the low-type consumers to increase sales and profit. The equilibrium prices are, however, higher than the socially optimal level. When $V > 1$, the socially optimal outcome requires that the low type consumers be involved in pairing up with high-type consumers to the maximum extent. Yet in equilibrium, high-type consumers have a higher chance of pairing with high-type consumers than the low-type consumers have.

Although the preference structure specified in this model is different in nature from the one used in the classic pricing games, an upward shift in V could still be interpreted as an overall increase in consumer valuation. This increase, however, doesn't lead to a higher equilibrium price. This is because when the low type consumers' valuation for signaling (V) increases, it becomes more attractive to the firm to sell to the low type consumers, and the consumer value of the product declines. In fact, Proposition 2 implies that when V is sufficiently high, the equilibrium demand from the high type consumers decreases but that from the low-type consumers increases.

As Proposition 2 indicates, the profit implications of an increase in low-type consumer valuation V of pairing with high-type consumers are also not straightforward. Equilibrium profit of each of the two firms, increases in V when $1 \leq V \leq V_1$. This is because in that area, although higher V leads to lower price, the larger demand resulting from more low-type consumers buying more than offsets the profit effect of the lower price and overall profit increases. On the other hand, when $V > V_2$, an increase in low type's valuation for signaling is detrimental to the firms. As low-type consumer valuation for matching with the high-type consumers increases, it becomes more and more difficult for the coordinator to keep low-type consumers from buying and as a result, low type demand for each product increases in V. A consequence of this is a decrease in the potential value of the product to either consumer type at any given pair of prices, and results in the optimal price declining so that the price decline effect dominates the market expansion effect and thus the profit of each firm decreases. In Region 2, the equilibrium price increases in V but total demand decreases in V. These two opposing forces first lead to an increase and then to a decrease in profits as V increases.

1.5.2 The Role of Consumer Coordination

To understand the role played by consumer coordination in the fashion industry, let us compare the above results to the results we would obtain in the model if consumers were not allowed to set up a coordination mechanism. The model of the latter case is the same as the main model except that

Stage 2 does not exist. The time line of the game then becomes the following:

1. Firms 1 and 2 simultaneously set prices.
2. All consumers make purchase decisions.
3. The "matching game", in which consumers with the same product are randomly matched into pairs and utility of matching is realized.

If we keep the assumption that using a product not used by anybody else results in zero payoff, then nobody buying any product is always an equilibrium for any V and any prices. However, as we show below, allowing a small number of high-type consumers to coordinate on an option would result in positive equilibrium demand for some V.[19] This assumption is equivalent to the assumption that each possible consumer-choice option $k = 0, 1,$ and 2 is "seeded" with a demand from high-type consumers, i.e., there is a zero-mass of high type consumers who (non-strategically) choose this option in the matching game, which we will adopt. As we mentioned above, in the absence of this assumption, the set of equilibria expands even further and thus our point that without the consumer coordination mechanism, it is not possible to predict equilibrium firm payoffs, remains valid. This assumption ensures that a consumers will always have a match regardless of the product he/she uses in the matching game and seems to be the strongest assumption one can make to reasonably restrict equilibria in the above game. With this assumption, we have the following lemma:

Lemma 1. *In the model with no consumer coordination mechanism, and assuming that each consumer-choice option is "seeded" with zero-mass of high-type consumer demand, we have:*

1. *When $V > 1$, in equilibrium, there is no market; i.e., in equilibrium, each product receives zero demand regardless of price.*

[19] Conceptually speaking, allowing low-type consumers to coordinate is not useful because both high- and low-type consumers try to stay away from low-type consumers. If one uses the interpretation of the consumer value as coming from trying to convince other consumers that she is a high type rather than deriving this value from the matching game, an argument in the spirit of Intuitive Criterion can be used instead to rule out the belief that off-equilibrium-path product use can be by a low type when $V < 1$, i.e., the assumption we made can be partly justified by signalling arguments.

2. *When $1-\alpha < V < 1$, any price $p_k \in [1-\alpha, 2-\alpha-\frac{1-\alpha}{V}]$ may have positive demand in equilibrium. If prices are equal and within this range, any split of market share is an equilibrium one. No market is also an equilibrium outcome.*

3. *When $V < 1-\alpha$, $p_1 = p_2 = V$ with any split of market share is an equilibrium outcome.*

Proof. See Appendix. □

In other words, without a coordination mechanism, the market either does not exist, or the market shares and profits are not predicted by the equilibrium. As a corollary to the above lemma and Proposition 2, we have the following result.

Proposition 3. *While without a coordination mechanism, in equilibrium, the market for fashion may not exist for some parameter values (e.g., when $0 < V < 1$), with the high-type coordinator, in equilibrium, the market for fashion exists for any $V > 0$.*

Proof. See Appendix. □

In the model without a coordination mechanism, market may not exist when the low type consumer valuation for signaling is high due to the difficulty of keeping the low type consumers from buying the same product as the high-type consumers. High-type consumers setting up a coordinating mechanism solves this problem. Thus, for $V > 1$, consumer coordination may be viewed as ensuring the existence of the market for fashion given the assumption that the high-type consumers (relative to the low-type consumers) have a superior access to their coordinator. To take advantage of this informational difference, the high-type coordinator must not choose its recommendations deterministically. Indeed, while too wide difference in prices could technically counter-balance the above incentive to randomize, we have shown that in equilibrium, the prices are neither high enough nor sufficiently different from each other to make a deterministic coordination strategy optimal.

Proposition 4. *In the model with a coordination mechanism, in a pure-price-strategy equilibrium, there will always be randomness in the selection of fashion hits.*

Proof. This claim immediately follows from Proposition 2. □

Since the high type consumers follow their coordinator's choice, the fashion hit will also appear random. This is consistent with the popular observation that fashion hits materialize randomly. Our model suggest however, that the hits are not arbitrary in the sense that they are random because the high-type consumers intentionally adopt a coordination strategy which randomizes according to a uniquely-defined rule. If the coordination strategy were deterministic, the low type consumers would be able to expect the outcome and purchase the product that is slated to become the "it". The outcome would be the same as in the case without the coordinating mechanism and would not be optimal for the high-type consumers. Thus, the optimal coordination strategy must not be deterministic. Since the utility of the high type consumers is directly affected by the number of low type consumers who use the same product, she attempts to reduce the low-type consumer purchases of the hit product through randomizing. This exact formula for this randomization is unique and depends on the product prices. In other words, there is a method behind the seeming madness of fashion whimsy.

1.5.3 Effect of Competition

Let us now examine the effect of competition on the fashion industry. To do this, we consider a monopoly firm offering two products in the same market and compare the results to the duopoly market above.

Since this game following the price-setting stage is the same as the one in the main model, the analysis of the consumer coordination and purchase strategies applies to this game. The only difference is that when solving for the optimal prices in Stage 1, the objective function of the firm is the sum of the two products' profits rather than the profit from just one product. This leads to the following result.

Proposition 5. *If both products are managed by a monopolist, when* $1 \leq V < 2 + \frac{\alpha}{2} - \frac{\sqrt{\alpha^2+8\alpha}}{2}$*, the monopolist sets prices* $p_1 = p_2 = \frac{\alpha V}{2-V}$*. These prices are lower than in the competitive market, and* *the coordination strategy involves randomizing between the two products with equal probability. The profit of the monopoly is* $\frac{\alpha V(1-\alpha-V)}{2-V}$. *When* $V \geq 2+\frac{\alpha}{2}-\frac{\sqrt{\alpha^2+8\alpha}}{2}$, *the optimal monopoly price is the same as the competitive one.*

Proof. See Appendix. □

Note that in this model, the monopoly charges such prices that the demand from low type consumers for both products is positive. We know from Proposition 1 that an increase in the own price of a product will lead it to be chosen with higher probability. Since the coordinator has to trade-off between the two products, this will inevitably lead to the other product being coordinated upon with lower probability and earning lower profit. This will hurt the total profit of the monopoly firm, and the monopoly finds it optimal to keep prices at a lower level. This is the intuition for the result that when V is small and therefore prices are in Region 1, competition drives up the equilibrium prices. One may find this result surprising since normally one would expect competing firms' incentive to increase own market share to lead to a lower price. In this model, on the contrary, it is in the best interest of the industry to keep prices lower than the competitive incentives would suggest, since an increase of one product's price hurts the profitability of the other product. Note that compared with the competitive case, the monopoly case is farther away from what is desired by the high-type consumers ($p_1 = p_2 = V/2$), but if $V > 1$, it is closer to the socially optimal outcome ($p_1 = p_2 = V/2$).

Another somewhat surprising observation from the comparison of the results of the monopoly and competitive cases, is that they lead to exactly the same outcomes in Regions 2, 3 and 4. In other words, when V is relatively large, firms essentially do not compete in prices. The intuition for this is that when V is large, the only relevant case is the low-price region. In this case, the revenue from a product does not depend on the price of the other product. Therefore, firms are effectively competing with the "neither" option instead of competing with each other, and therefore the outcomes of the competitive and industry-maximizing strategies are the same.

1.6 Robustness

Several assumptions in the model warrant further consideration. Given that the notion of the high-type consumer coordination plays an essential role in this model, let us consider the assumptions we made about the coordinator construct. The following questions may arise: 1) what are the possible recommendation choices? 2) what is the objective of the coordinator? and 3) is the coordination essentially accomplished by one monolithic entity or is there a role for multiple coordinators of the same kind? In this section, we consider relaxing the assumptions we have made on each of the above points.

1.6.1 Restricting the Coordinator Recommendations to Products

Especially when the role of coordinator is played by a single agent, such as a fashion magazine editor, one may argue that abstaining from recommendation is either practically difficult or technically not a valid action.

There are several reasons the null recommendation (i.e., the recommendation not to buy) could be a viable outcome of the coordination. One is that if the coordination is performed by a group, it may fail to converge to a single opinion. Failure to convey consistent report of agreement, i.e., different recommendations coming from different fashion mavens, could be interpreted as a null recommendation. In the case of a single coordinating agent, such as a fashion magazine editor, one may argue that null recommendation could be implied by certain choices of the editor: such as a recommendation of style instead of a product or a recommendation of multiple products as equally desirable. Also, multiple editors working together could implement null recommendation by recommending different products (see Section 1.6.3 for further discussion of the potential roles of multiple coordinators).

At the same time, it is also interesting to consider how the results would change if null recommendation is not a feasible recommendation. For that purpose, note that in some parameter value range (small enough V), in equilibrium, neither is never recommended. Therefore, restricting the recommendation space in that cases results in no change in the outcomes. When the equilibrium involved recommending neither with positive probability, the coordinator was indifferent between null recommendation and a product recommendation. Since restricting the editor not to make null recommendation would "tip-off" low-type consumers to the higher probability of a product recommendation, it would increase low-type demand (assuming high-type consumers always follow the recommendation) and thus decrease the expected payoff of the high-type consumers. Therefore, the high-type consumers would then strictly prefer if the recommendation is not followed. This could lead to high-type consumers following the recommendation no longer an equilibrium, and the market for fashion disappearing. Since such an outcome is not optimal for the firms, firms would then have the incentive to avoid setting prices that make null recommendation optimal. As Proposition 1 implies, this would require prices to be higher. However, when V is sufficiently high, the price required becomes so high that it is optimal for high-type consumers not to buy, and therefore, market does not exist for high enough V.

The above consideration leads to the following outcomes when neither recommendation is not possible (formal proof is in the Appendix). 1) When $1 < V \leq \min\{V_2, 2-2\alpha\}$, the equilibrium is the same as in the main model; 2) When $V_2 < V \leq 2-2\alpha$, in equilibrium, prices are $p_1 = p_2 = \alpha V/(2-V)$, i.e., higher than the ones predicted by the original model, and 3) When $V > 2-2\alpha$, there is no demand for either product (regardless of price).

1.6.2 Coordinating vs. Predicting the Best-Seller

In the model, we introduced the coordinator as an an abstraction of the coordinating process of the high-type consumers. Within the model assumptions, it is also possible to interpret the coordinator as an arbitrarily chosen representative of the high type consumers (since they are homogenous). On the other hand, if the coordinator is actually a physical person, she might have other incentives. For example, fashion magazine editors and fashion mavens could strive to correctly predict the future outcomes. Let us consider what happens if we add the latter motivation to the objective function of the coordinator.

For this purpose, note that since in equilibrium where the recommendation matters at all, the high-type consumers follow the recommendation, the only way for the recommendation of a product not to also be a correct prediction of the product with the higher demand is for the demand from low-type consumers for one of the products to exceed that of the other product by more than the market size of the high-type consumers. Since the low-type consumer demand decreases in price, this could happen when the price difference is high enough and the coordinator recommends the more expensive product. If the coordinator's desire to predict demand in such a case is high enough (relative to her desire to maximize the high-type consumer utility), she would then for sure recommend the cheaper product. But this would mean that the recommendation is no longer unpredictable for the low-type consumers and therefore, the outcome is equivalent to the outcome of the model without a consumer coordination mechanism. In other words, when $V > 1$, the market does not exist. Since such an outcome is not optimal for the firms, in equilibrium, each firm would prefer to set its price close enough to the competitor's and thus the equilibrium price of the original model is still the only equilibrium as far as the coordinator always recommends one of the products.

If the original model predicted null recommendation with positive probability, the coordinator was indifferent between null recommendation and a recommendation of a product. In this case, adding to the coordinator's objective function any incentive to predict a product would mean that the coordinator would always recommend a product rather than "neither."

Thus, the result of adjusting the coordinator's objective function to include an incentive to predict a product's dominant market share correctly, results in the same outcome as in the above considered model where the null recommendation is not allowed: conceptually, the only difference is that the market no longer exists for high enough V.

1.6.3 Multiple Coordinators

One may argue that the reality of the fashion marketplace is that there are multiple coordinators who rarely recommend the same product. How can this be reconciled with the assumption of a single coordinating entity, or can the model accommodate multiple coordinators? Since our assumption of a single high-type consumer segment was a simplification of real marketplace, a simple answer could be that different coordinators serve different high-type segments. A more challenging objective is to allow multiple coordinators serve the same high-type segment.

One possibility is the following. Suppose, for example, the recommendations are observable by all consumers at the same time. In the case of one coordinator, the information asymmetry between consumer types then disappears. Multiple coordinators, however, allow restoring information asymmetry and the results of our model. Specifically, it could be that high-type consumers, covertly from low-type consumers, could coordinate on one of the coordinators. Note that if the coordinators act independently, sometimes their recommendations would coincide and then low-type consumers would know what "the real" recommendation to the high-type consumers was. This would then depress the high-type consumer willingness to pay for the recommended product. On the other hand, if coordinators could coordinate with each other, they would then be interested in coordinating on opposite recommendations, whether they know which one of them is picked by the high-type consumers or not. This is because due to randomization, they are indifferent between different recommendations, but should strictly prefer to keep the recommendation unknown to the low-type consumers. Note that since the assumption of information asymmetry between high- and low-type consumers is essential to our model, the observation that it is difficult for an outsider to see the "common denominator" of the recommendations could be viewed in evidence of support of our model.

1.7 Further Discussion and Conclusion

It is a common adage that the fashion industry is fast-changing and highly unpredictable. While in many markets the consumer value of a product or service is slowly changing through time, the value of a potentially fashionable item may drastically change from month to month. Understanding this industry is further complicated by the significant influence of fashion mavens, celebrities, and fashion magazine editors. In this paper, we presented an equilibrium analysis of competitive and intrinsically random fashion market explicitly modeling consumer coordination mechanism. As a result, we are able to understand the role and regularities of the random fashion changes and suggest how firms' strategy in a fashion market where consumer coordination is important is different from one in more steady status good markets.

The first consequence of the presence of consumer coordination is that in equilibrium, fashion selection is always probabilistic as opposed to deterministic. This is because when deterministic strategy could work, one firm undercutting the other's price by a small amount would lead it to gain market share significantly larger than a half. This would not happen without a coordinating mechanism: as Pesendorfer (1995) and Bagwell and Bernheim (1996) show in such case, there is a point reducing price below which leads to loss of all sales since the product can no longer signal status. However, when the coordination is possible, some status value of such product is preserved by the optimal recommendation strategy that picks this product with a lower probability.

Another but related implication of the role of consumer coordination is that competition may lead to prices that result in a strictly positive demand from the low-type segment. At first glance, this may seem counter-intuitive, since given the assumed two-segment structure of the market, the preferred solution for the high-type consumers is the cheapest status good that would fully separate them from the low-type consumers (Pesendorder 1995, Bagwell and Bernheim 1996). Within our model, this outcome is feasible when $V < 2$ ($p_k = V/2$). Since one could expect firms to try to raise prices, one can understand how prices could become higher. Why would the presence of the coordinator acting on behalf of high-type consumers leads instead to lower prices then the best ones for the high-type consumers? The intuition for this result is that if one of the two prices is reduced, the coordinator cannot commit to choose the higher priced product with higher probability. If low-type consumers believed that the editor would do so and abstained from buying, the editor would then choose the cheaper product with probability one. Therefore in order for the price reduction not to be optimal, the equilibrium low-consumer demand for the cheaper product must be positive, and the result is that both products receive positive expected demand from high and low type consumers.

We also find that with consumer coordination, the fashion market may exist regardless of the relative value the high- and low-type consumers place on projecting the image of the high type. This is in contrast to the result that market may not exist in the model without consumer coordination when low type consumer's valuation for signaling is too large. The reason for this result is again that by not choosing either product most of the time, the coordination allows the products to still play a role of status symbols in the remaining instances.

While supporting the notion that fashion hits are random, our model implies that fashion hits are not completely arbitrary, but set according to a unique probability distribution and affected by firms' pricing strategies. In particular, this allows us to show that whenever the demand from low type consumers for a product is positive, an increase in the price of that product always leads to higher probability of becoming the it product, but lower demand from the low type consumers. This suggests that for the coordinator's decision of which product to recommend, reducing demand of the low type consumers is a more important consideration than the price of the product.

Combining the result about the uniqueness of the equilibrium coordination strategy with the existence of the market, one obtains that while appearing arbitrary or frivolous, the coordination in fact plays a stabilizing role in the fashion industry. Understanding the process of the fashion hit selection allows us to further consider and predict firms' profits and their profit-maximizing strategies. For example, we observe that in some range of the low-type consumers' valuation for signaling, competition between firms drives up market price. Furthermore, the equilibrium price zigzags in the low-type consumers' valuation for signaling when the low-type consumer valuation for signalling increases from zero to infinity, as firms trade-off between attracting the increased demand of the low-type consumers and increasing the expected status of their products. In particular, the equilibrium price is also non-monotone in the fraction α of the high-type consumers. The equilibrium profit first increases and then decreases in V, implying that there is an optimal-for-the-industry value of low-type consumer valuation of signalling. When V is large enough, the monopoly case and competitive case lead to the same equilibrium outcomes, since firms are competing with the outside option rather than with each other. As V tends to infinity, the equilibrium price asymptotes to $\sqrt{\alpha} - \alpha$ and profits asymptote to $(\sqrt{\alpha} - \alpha)^2$, so that for large V, the optimal-for-the-industry amount of high-type consumers in the market approaches $1/\sqrt{2}$.

The are a few considerations begging further research. For the sake of simplicity, we abstracted away from all actions of the firms that could influence the emergence of fashion hits except for the price. While one point of view that the whimsical opinion of fashion editors, such as the one famously portrayed in the Hollywood movie "The Devil Wears Prada," may be far more important for setting the trend than the firm's promotional efforts, another is that firms can actively affect consumer coordination through marketing efforts. It would be natural to consider what role advertising may play in the emergence of fashion hits.[20] One could see advertising as potentially having at least one of the following two roles. First, it could increase awareness of a product among high-type consumers therefore reducing the probability of miss-communication in disseminating the outcome of the high-type consumer coordination. In this role, clearly, firms would have incentive to advertise and advertising could increase the probability of the coordinator selecting the product. This would be especially true when the coordinator is not indifferent between a recommendation of a product and a recommendation not to buy, i.e., when low-type consumer valuation of projecting a high-type image is not too high. Second, one could alternatively assume that high-type consumers can learn the coordination outcome perfectly, but advertising increases product awareness among low-type consumers. Even in this case, one can show that in equilibrium, firms would advertise. The intuition is that as we have shown, firms do have an incentive to achieve positive sales to low-type consumer segment.

Some other simplifying assumptions we made were consumer within-segment homogeneity, complete un-observability of consumer type, and the perfect correlation between consumer desirability (type) and consumer access to the recommendation. As we have shown, in equilibrium, for each product, some but not all low-type consumers buy it. If their valuation V for projecting the

[20]For example, Wernerfelt (1990) assumes that advertising affects the probability of a product being adopted as an image product, and Dukes and Liu (2010) consider how advertising could play a role in coordinating a channel with competing manufacturers.

high-type image would be heterogeneous, then consumers with higher valuation would be the ones buying the products. We would then observe that some lower valuation low-type consumers do not buy either product and some high valuation ones buy both. The difference with the main model would be that now the demand from the low-type consumers would be determined by the V of the indifferent consumer. As price decreases, the relevant V will also decrease, but the conceptual implications would remain the same. Imperfect access of high-type consumers to the recommendation would increase the benefit of not buying (since some high-type consumers would then not buy) and thus decrease the demand for products. Furthermore, if some consumer desirability attributes are observable without product use, then the model should be thought of as applicable within each "slice" of consumers according to the desirability attribute. In other words, the effects of product use could be considered conditional on the observable characteristics. Again, the more consumer desirability is observed, the less incentive consumers would have to buy products for the purpose of projecting an image.

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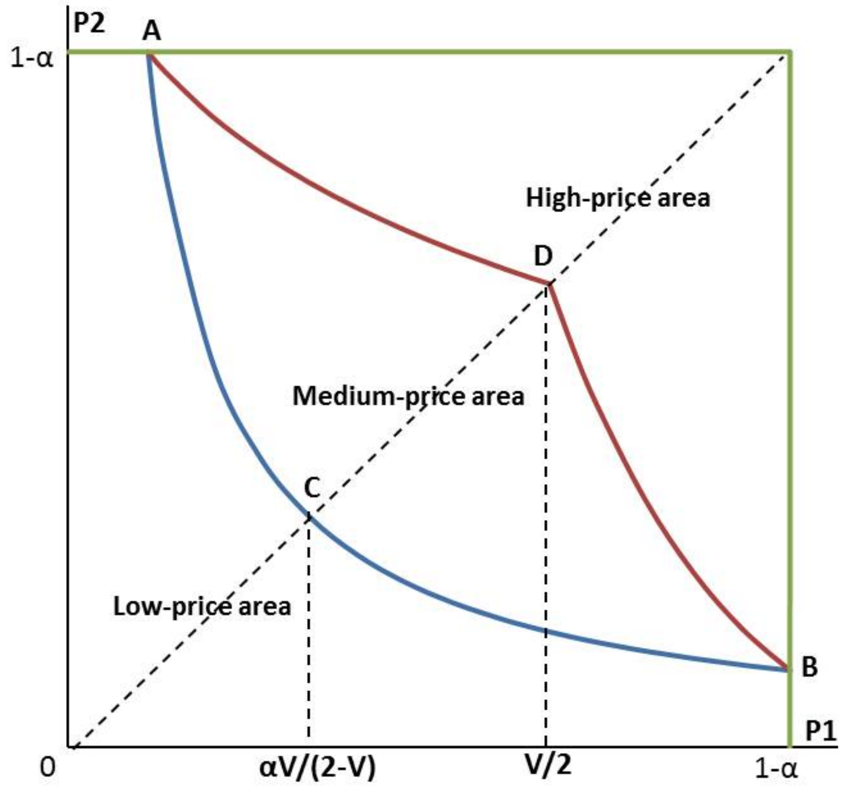

Figure 1.1: Plot of areas defined in Proposition 1 (to scale for $\alpha =$, $V = 1$).

Notes about Figure 1.1:

1) The line ACB will always be below ADB as long as the medium-price and high-price area exist.

2) When prices are symmetric, $P1 = P2 = \alpha V/(2-V)$ marks the boundary between the low-price and medium-price area, and $P1 = P2 = V/2$ separates the medium- and high-price area.

3) As V increases, line ACB and ADB will shift upward and rightward, but will always be symmetric around the 45 degree line. Also, the low-, medium- and high-price area will all exist as long as $V < 2 - 2\alpha$. When $V > 2 - 2\alpha$, both the medium- and high-price area disappear. Also, for $V < 1 - \alpha$, there is an additional "very high prices" area above the high-price area.

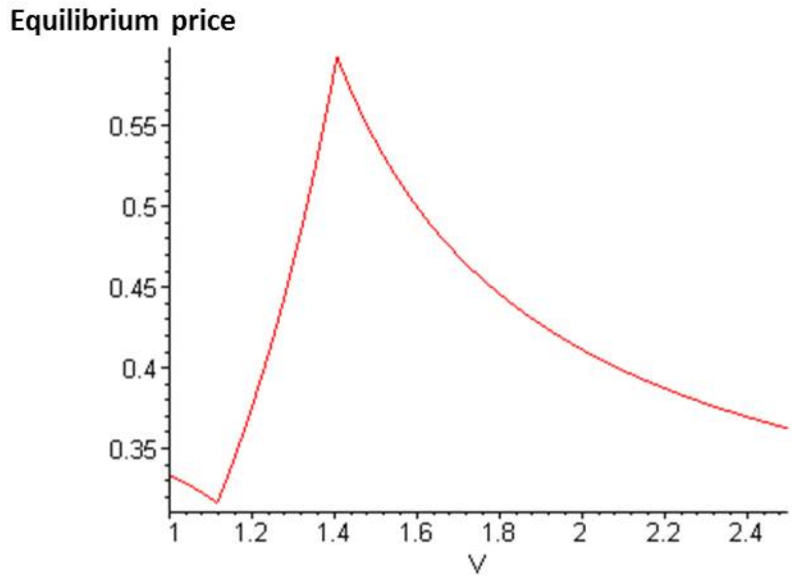

Figure 1.2: Change of equilibrium price with V when $\alpha =$. Before the first kink, the equilibrium price is in the medium-price area; it reaches the low-price area at the first kink, stays on the boundary between medium- and low-price areas till the second kink, and then is inside the low-price area (see Proposition 1 for the description of the areas).

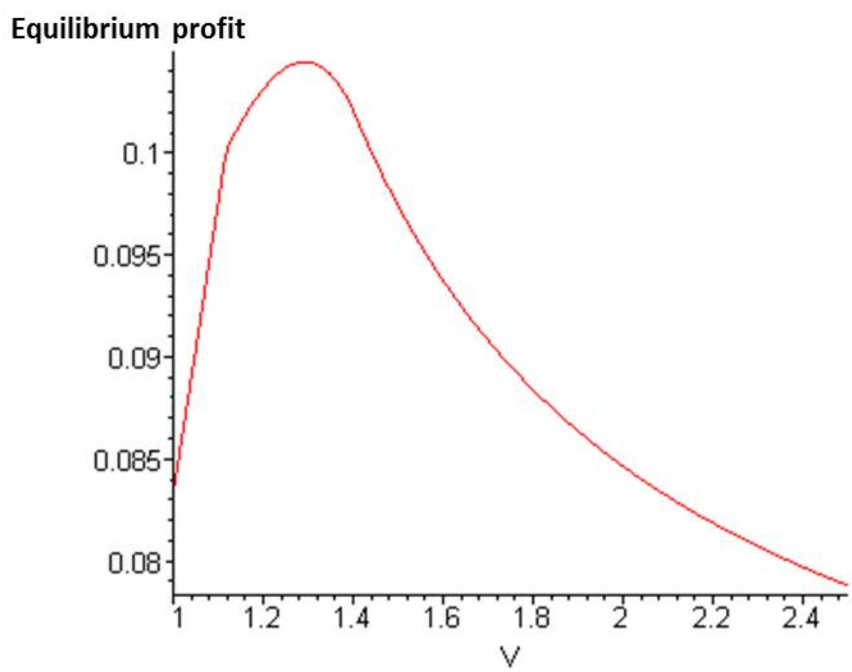

Figure 1.3: Change of equilibrium profit with V when $\alpha = 1/4$.

Appendix

Proof of Proposition 1

Let us first consider the case all high-type consumers follow the coordinators recommendation. We will then prove that this must be the case when prices are not equal and can not happen with a positive probability of the full game.

Since the coordinator works for the high type consumers, hereafter we use the term "high type consumers' payoff" and "coordinator's payoff" interchangeably. Note that in equilibrium, it can not be that all low-type consumers buy one of the products since otherwise the coordinator would strictly prefer not to choose it, making it strictly optimal for low-type consumers not to buy it either.

When the coordinator randomizes between the two products only, demand from the low type consumers can not be 0 for both products unless prices are equal, since otherwise the coordinator will deviate to the cheaper product. When the low type consumers only buy one product, then it must be the cheaper product. Thus, when the coordinator randomizes between products 1 and 2, only two cases are possible: a fraction of low type consumers buy each product, or a fraction of low types buy Product 2 only.

Proof of item 1. Consider the case when the coordinator is indifferent between recommending Product 1, Product 2 and neither. Her payoff is $\alpha/(\alpha + x_k) - p_k$ when picking Product k ($k = 1, 2$) and α when picking neither. This implies $\alpha/(\alpha+x_k)-p_k = \alpha$. In this case, the low-type consumer demand for each product must be positive. This is because the coordinator's payoff of choosing a product no low type consumers buy is greater than α and thus the coordinator will not choose neither if the low-type demand for one of the products is zero. Therefore in this case, the low type consumer demand must be positive (but less than maximal) for each of the products. Therefore the following indifference conditions on the low-type consumer choice must be satisfied:

$$V\delta_k \frac{\alpha}{\alpha + x_k} - p_k + (1 - \delta_1 - \delta_2)\alpha = (1 - \delta_1 - \delta_2)\alpha, \quad \text{for } k = 1, 2. \tag{1.3}$$

Solving these equations leads to the results given in the first line of Table 1.1. The solution for x_1, x_2, δ_1 and δ_2 need to satisfy the boundary conditions $0 \leq x_k \leq 1 - \alpha$, $\delta_k > 0$ and $1 - \delta_1 - \delta_2 > 0$. As we have shown above, if the last condition is satisfied, the others are satisfied as well. The last condition ($1 - \delta_1 - \delta_2 > 0$) is equivalent to the condition specified in Item 1, and the proof of Item 1 together with the corresponding results in Table 1.1 is complete.

Proof of item 2. Consider now the case when the coordinator randomizes between Products 1 and 2 only and the low-type consumers generate positive demand for both products. The coordinator's indifference between the two products and the low-type consumer indifference between all three options imply

$$\frac{\alpha}{\alpha + x_1} - p_1 = \frac{\alpha}{\alpha + x_2} - p_2, \quad \text{and} \quad V\delta_k \frac{\alpha}{\alpha + x_k} - p_k = 0, \tag{1.4}$$

respectively. Finally, the coordinator will only pick the two products with positive probability: $\delta_1 + \delta_2 = 1$. Solving these 4 equations yields the results given in Table 1.1. Note that the boundary conditions $x_1 \geq 0$ and $\delta_1 + \delta_2 = 1$ (i.e., that the coordinator prefers not to choose "neither") result in the two inequalities defining area of Item 2 ($x_2 \geq 0$ is never binding when $x_1 \leq 0$ is not).

Proof of item 3. Now consider the possibility that the coordinator is indifferent between the two products, but low-type consumer demand is only positive for the lower priced one. Then the indifference condition for the coordinator and the low-type consumers imply

$$\frac{\alpha}{\alpha + x_1} - p_1 = \frac{\alpha}{\alpha + x_2} - p_2, \quad \text{and} \quad V\delta_2 \frac{\alpha}{\alpha + x_2} - p_2 = 0, \tag{1.5}$$

respectively. Low type demand for Product 1 is 0: $x_1 = 0$. In addition, we have that the coordinator never choose neither with positive probability: $\delta_1 + \delta_2 = 1$. Solving these 4 equations results in equations of the Area 3 row of Table 1.1. The boundary conditions $\delta_1 > 0$ and $x_1 = 0$ (i.e., low-type consumers prefer not to buy Product 1) is equivalent to the condition in Item 3 and the other boundary conditions are not binding in this case.

Proof of item 4. When the coordinator chooses both products with positive probability, only the foregoing three cases are possible. Therefore, when they are not satisfied, the equilibrium must be that the coordinator only picks one product with positive probability. There can be no equilibrium where the coordinator randomizes only between one of the products and neither, since in that case the deviation payoff of choosing the other product is strictly larger than α. Therefore if the coordinator does not choose one of the products with positive probability, her strategy must be to choose Product 2 with probability 1.

When $p_2 \geq V$, the low-type consumers will not buy either product regardless of coordinator's choice and it is optimal for the coordinator to select Product 2 with probability 1. Note that when $p_2 > V$, the condition in Item 4 is satisfied (as far as $p_1 \geq p_2$, which is true by the choice of notation).

When $p_2 < V$ and $\delta_2 = 1$, a positive fraction of low types will buy Product 2. The indifference condition of the low types implies:

$$\frac{\alpha}{\alpha + x_2} V - p_2 = 0, \tag{1.6}$$

which yields $x_2 = \frac{\alpha(V - p_2)}{p_2}$ and the high type (and the coordinator)'s payoff is $\frac{(1-V)p_2}{V}$. The payoff to the coordinator by deviating to Product 1 is $1 - p_1$. Therefore the coordinator will not deviate if and only if $\frac{(1-V)p_2}{V} > 1 - p_1$, which is equivalent to the condition in Item 4. This completes the proof of Item 4.

To rule out the remaining potential outcome – that of the coordinator choosing "neither" with probability one, – consider the following. If in equilibrium the coordinator chooses neither with probability 1 and low-type consumers have rational expectations, none of the low type consumers will buy any product. But then the coordinator's payoff of choosing Product k is $1 - p_k$, which is greater than her payoff α of choosing "neither." Thus the coordinator would strictly prefer to deviate.

To show that the equilibrium is unique (i.e., it can not be that two of the above items lead to an equilibrium for a particular pair of prices), it suffices to note the following: when $p_1 < 1 - p_2/V + p_2$ (and given Equation (2)), the coordinator must choose both products with positive probability. Proving this by contradiction, assume an equilibrium where the coordinator only picks Product k ($k = 1, 2$) with positive probability. Then it must be (the cheaper) Product 2 because in such equilibrium, no low-type consumers will buy Product $3 - k$ and thus the coordinator will strictly prefer to choose Product $3 - k$ if it is cheaper. Suppose in equilibrium the coordinator only chooses Product 2 with positive probability and some low-type consumers buy it. Then the low type consumers must be indifferent between buying and not buying it, which means $V\alpha/(\alpha+x_2)-p_2 = 0$. The coordinator's payoff is $\alpha/(\alpha + x_2) - p_2 = \frac{(1-V)p_2}{V}$. If she deviates to Product 1, her payoff would be $1 - p_1$. When $p_1 < 1 - p_2/V + p_2$ holds, $\frac{(1-V)p_2}{V} < 1 - p_1$ and the coordinator will deviate. Thus, when $p_1 < 1 - p_2/V + p_2$ and the coordinator only chooses Product 2 with positive probability, no low-type consumers should be buying it. Since the low types don't buy, it means $p_1 \geq p_2 \geq V$. However, this again contradicts the condition $p_1 < 1 - p_2/V + p_2$. Thus, when $p_1 < 1 - p_1/V + p_1$, the coordinator must choose both products with positive probability.

The consideration of equal prices that is needed to complete the proof of Proposition 1 is straightforward.

"Partial" Following of the Coordinator's Recommendation

Here, we consider and rule out the possibility of an equilibrium in which high-type consumers follow the recommendation with some probability $q \in (0, 1)$ or, alternatively, in which only some fraction $q \in (0, 1)$ of high-type consumers follow the coordinator in equilibrium. Specifically, relaxing the assumption that all high-type consumers follow the coordinator, consider the following modification of the consumer game once prices are set. First, high-type consumers decide whether to follow the coordinator and (in the same stage) the coordinator makes the recommendation. Those who decided to follow it, do so, while the rest of high-type consumers make a decision on what (if anything) to buy independent of the recommendation, i.e., follow a strategy not affected by the recommendation. This assumption of independence of the consumer decision to follow the recommendation from the outcome of the recommendation rules out the potential equilibria where the coordinator does not select one of the options because none of the consumers would follow it (and so, although some consumers always follow the coordinator on the equilibrium path, the equilibrium path is affected by the coordinator's fear of becoming irrelevant due to being ignored off equilibrium path). To be specific about the coordinator's objective function, assume that the coordinator maximizes the payoff of those who end up following her. We assume that the purchase decision of those who chose not to follow is independent of the coordinator's recommendation to rule out the possibility that those who do not follow act to counter-balance the coordinator's recommendation to make it irrelevant. If the purchase choice of those who do not follow the coordinator would be affected by the recommendation, they essentially still would coordinate through the coordinator's recommendation by defining an alternative "language" of what to do depending on the coordinator's recommendation.[21]

For succinctness, let us call those consumers who choose to follow the editor's recommendation "the followers." Note that a consumer is not a priori classified as a follower or a non-follower, since the consumer's strategy could involve randomizing over being a follower or not and choosing to be a follower with a probability $q \in (0, 1)$. In other words, whether a consumer is a follower is defined based on her realized choice of whether to follow.

Consider a potential equilibrium of the above game where the mass of followers is strictly positive but also strictly smaller than α, denote the total high-type consumer demand for Product i conditional on the coordinator's recommendation j by z_{ij} and denote the mass of low-type consumers using Product i in the matching game by x_{ij}, $i, j = 0, 1$, or 2. Note that $z_{1j} + z_{2j} + z_{0j} = \alpha$ and $x_{1j} + x_{2j} + x_{0j} = 1 - \alpha$. Note again, that we do not assume here that consumers adopt a

[21] Although completely ruling out such possibilities is complicated, conceptually they are difficult to justify since as far as consumers still structure their decision based on the coordinator's recommendation, it would be optimal (in fact, one can show, in equilibrium, strictly preferable) for all of them to instead directly follow the coordinator's recommendation.

priori different pure strategies with some adopting the strategy to follow and some adopting the strategy to ignore. Equally consistent with the above notation is the case of high-type consumers using a mixed strategy in following the editor, so that the realization of the mixed strategy leads to the above levels of the demand from the high-type segment. Denote the payoff of the followers when the coordinator recommends Product j by $U_f(j)$. In equilibrium, these must be equal for all choices j the coordinator selects with positive probability. Since it is our goal to rule out equilibria with partial following as defined above, we will be looking to rule out different types of such equilibria until we have none left.

First, it cannot be that some low-type consumers do not use a product they bought in the matching game when some high-type consumers bought it. Proving this by contradiction, suppose some low-type consumers bought Product i but do not use it when there are some high-type consumers who bought the same product. Without loss of generality, let $i = 1$. Then the low-type consumers who bought but do not use Product 1 either use neither or use Product 2. In the former case, low-type consumers must be indifferent between using and not using Product 1 in the matching game, which means $z_{1j}/(z_{1j} + x_{1j}) = z_{0j}/(z_{0j} + x_{0j})$ (note that the high-type consumers always use the product they bought since they do not learn anything between the product choice stage and the matching game stage and thus, buying a product and not using it could not be an individually rational strategy for a high-type consumer). This equation also implies $z_{0j} > 0$. Therefore, the high-type consumers must also be indifferent between buying Product 1 and not buying, which requires $z_{1j}/(z_{1j} + x_{1j}) - p_1 = z_{0j}/(z_{0j} + x_{0j})$. However, this contradicts the previous equation. Thus, the only possibility not yet ruled out is that some low-type consumers bought both products and decide to use Product 2. This means the low-type consumers must be indifferent between using Product 1 and 2. By the argument similar to the above, the high-type and low-type consumers cannot be both indifferent between the two products unless $p_1 = p_2$. Furthermore, if $p_1 = p_2 = p$, the low-type consumer behavior under consideration cannot be optimal when the coordinator recommends Product 1. To see this, note that since followers always do what the coordinator recommends and non-followers behave independently, high type demand for Product 1 is the highest when the coordinator recommends it and lowest if she does not. So if Product 1 is given up in the matching game in favor of Product 2, then the coordinator could not have possibly recommended Product 1. The coordinator could not have recommended Product 2 either. To see this, note that high type demand for Product 2 is highest when the coordinator recommends it and hence the payoff of using it should also be highest. So if $z_{12}/(z_{12}+x_{12}) = z_{22}/(z_{22}+x_{22})$, then it must be true that $z_{1j}/(z_{1j}+x_{1j}) \geq z_{2j}/(z_{2j}+x_{2j})$ for any Product j chosen with positive probability in the equilibrium. This means using Product 1 weakly dominates using Product 2 no matter what product the coordinator recommends. Since prices are equal, the low types should just buy Product 1 instead of buying two products, which leads to contradiction. So if Product 1 is given up in the matching game in favor of Product 2, the recommendation could not have been Product 2 either. Then the only possibility left is that the coordinator has recommended neither. Suppose this is the case and the coordinator also chooses Product 2 with positive probability in the equilibrium. Then according to the previous analysis, it must be true that $x_{12} > x_{10}$ and $x_{22} < x_{20}$. Also, $z_{12} = z_{10}$ and $z_{22} > z_{20}$. This means $z_{12}/(z_{12}+x_{12}) < z_{22}/(z_{22}+x_{22})$ and the low types should give up Product 1 in the matching game in favor of Product 2 when the coordinator recommends Product 2. However, from the previous analysis we know that this is not possible. So if the coordinator has recommended neither, it cannot also choose Product 2 with positive probability in the equilibrium. Suppose instead that the coordinator also chooses Product 1 in the equilibrium. Then in the equilibrium, the payoff of using Product 1 in the matching game is at least as high as that of using Product 2 under all recommendations. This means low types should only buy Product 1 instead of both products, leading to contradiction. So if the coordinator has recommended neither, the only possibility is that it recommends neither with probability 1. However, if this is the case, then purchasing two products is redundant for the low types, since they lead to exactly the same payoff. Therefore, we have ruled out all possible scenarios in which the low types could possibly give up a product in the matching game when the high types bought it. This means if a low type consumer bought a product, then she must use it in the matching game if at least some high-type consumers use it.

Second, it cannot be that when the coordinator recommends Product 1, some high types select Product 2 or vice versa. The proof is as follows. Suppose some high types select Product 2. Then $U_f(1) = z_{11}/(z_{11} + x_{11}) - p_1 = z_{21}/(z_{21} + x_{21}) - p_2$. Given the above proven point, if the coordinator deviates to Product 2, low type demand for Product 2 will not change, but high type demand for Product 2 will increase since the followers now switch from buying Product 1 to buying Product 2. So $U_f(2) = (z_{21} + \epsilon)/(z_{21} + \epsilon + x_{21}) - p_2 > U_f(1)$, where ϵ is the size of followers. This means the coordinator will deviate to Product 2 and hence it could not be an equilibrium.

Given the previous argument, if the high types do not completely follow the coordinator, then when the coordinator recommends Product 1 (or Product 2), those who do not follow must all select neither. However, this cannot happen in the equilibrium. This is because if it happens, then $U_f(1) = z_{11}/(z_{11} + x_{11}) - p_1 = z_{01}/(z_{01} + x_{01})$. Since $z_{11} + z_{01} = \alpha$ and $x_{11} + x_{01} = 1 - \alpha$, it must be true that $z_{11}/(z_{11} + x_{11}) > \alpha > z_{01}/(z_{01} + x_{01})$. So $U_f(1) < \alpha$. If the coordinator deviates to recommending neither, followers will select neither and non-followers' behavior remains unchanged. Now if $z_{10} = 0$, i.e., all high types select neither, then low types will also select neither in the matching game, in which case $U_f(0) = \alpha > U_f(1)$. If $z_{10} > 0$, then $x_{10} > 0$, but since $z_{10} < z_{11}$, it must be true that $x_{10} < x_{11}$. So some low types will choose not to use Product 1, which means they must be indifferent between using and not using the product, i.e., $z_{10}/(z_{10} + x_{10}) = z_{00}/(z_{00} + x_{00}) = \alpha$. So $U_f(0) = z_{00}/(z_{00} + x_{00}) = \alpha > U_f(1)$. In both cases, the coordinator will strictly prefer to deviate to recommending neither. So the equilibrium could not hold.

Finally, we show that it cannot be that some high types buy Product 1 or 2 when the coordinator recommends neither. Similar to the proof of the second point, this is because if the coordinator deviates to recommending Product 1 (or 2), high type demand will increase while the low type demand remains the same. So the followers will be strictly better off. We have thus excluded all possible cases where the high-type consumers do not completely follow the coordinator.

Proof of Proposition 2

Since the higher priced product receives zero demand in Area 3 of Proposition 1, equilibrium prices must fall in Areas 1, 2, or 3 defined in Proposition 1, in all of which the coordinator chooses both products with positive probability. Let us consider separately possibility that the equilibrium prices fall within each of these areas. Table 1.1 results in the profit functions reported in Table 1.2.

Low-price Area (Area 1). When prices are within this area, we have

$$\frac{\partial \pi_i}{\partial p_k} = p_k^2 + 2\alpha p_k + \alpha V - V p_k^2 - 2\alpha V p_k - \alpha^2 V \tag{1.7}$$

When $V \leq 1$, we have $\frac{\partial \pi_i}{\partial p_k} > 0$ and therefore firms always benefit from increasing price. So in this case equilibrium prices cannot be in Area 1. When $V > 1$, $\frac{\partial \pi_i}{\partial p_k}$ is positive for small p_k but negative for large p_k. The optimal price, denoted by p_{low}, is given by

$$p_{low} = \frac{\alpha - \alpha V + \sqrt{\alpha^2 + \alpha V^2 - \alpha V - \alpha^2 V}}{V - 1}, \tag{1.8}$$

and $p_1 = p_2 = p_{low}$ is a candidate for the equilibrium price.

Medium-price Area (Area 2). Simultaneous solution of the first-order conditions implied by the profit functions in Area 2 leads to

$$p_1 = p_2 = p_{med} = \frac{V(V-2)}{V+2}, \tag{1.9}$$

which is therefore also a candidate for the equilibrium.

High-price Area (Area 3). Since the profit functions are not symmetric, we need to examine them separately. For Product 2, $\frac{\partial \pi_2}{\partial p_2}$ is always positive, so the firm having the lower price strictly prefers to raise it. For Product 1, $\frac{\partial \pi_1}{\partial p_1}$ is positive for small p_1 and negative for large p_1. the optimal price is found by solving for the first-order condition $\frac{\partial \pi_1}{\partial p_1} = 0$. Denote this optimal price by p_{high}:

$$p_{high} = \frac{V + Vp_2 - \sqrt{Vp_2 + Vp_2^2}}{V} \tag{1.10}$$

Note that in this area, there could not be a pair of prices $p_2 < p_1$ since Firm 2 would strictly prefer to raise price, and $p_1 = p_2$ can not be equilibrium because one of the firms deviating to slightly lower price would result in that firm strictly increasing both the demand from low type consumers and the probability of being chosen by the coordinator (which becomes strictly above 1/2). Therefore, an equilibrium cannot exist in pure price strategies falling within this area. However, p_{high} is useful when checking for deviations that would bring prices within Area 3.

In summary, a pure price strategy equilibrium of this game could be $p_1 = p_2 = p_{low}$, $p_1 = p_2 = p_{med}$ or the prices at the boundary between Area 1 and 2 (if the boundary conditions are not satisfied for one of the first two possibilities).

When $p_1 = p_2 = p_{med}$, the profit function for each firm is

$$\pi_1 = \pi_2 = \pi_{med} = \frac{\alpha V^2}{V + 2}$$

When one firm charges p_{med} and the other firm responds by the "high-optimal price", then the deviating firm will obtain

$$\pi_{meddev} = \frac{\alpha(2 - V - \sqrt{2 - V}\sqrt{2 - V^2 + 3V})(V^2 - 3V - 2 + \sqrt{2 - V}\sqrt{2 - V^2 + 3V})}{\sqrt{2 - V}\sqrt{2 - V^2 + 3V}(2 + V)} \tag{1.11}$$

$\pi_{med} < \pi_{meddev}$ when $V < 1$ and $\pi_{med} > \pi_{meddev}$ when $V > 1$. $\pi_{med} = \pi_{meddev}$ when $V = 1$. So $p_1 = p_2 = p_{med}$ cannot be an equilibrium price when $V < 1$.

When $p_1 = p_2 = p_{med}$, low type consumer's demand for each product is $x_1 = x_2 = \frac{\alpha(3V-2)}{2(2-V)}$ and high type consumers' equilibrium payoff is $\frac{(V-2)^2}{V+2}$. Three boundary conditions need to be satisfied.

1. $x_k > 0$. This is satisfied when $V \geq 1$.

2. $x_k \leq 1-\alpha$. This is satisfied for $V < \frac{2(2-\alpha)}{2+\alpha}$.

3. Incentive constraint of the coordinator. Need to check that the coordinator's equilibrium payoff is greater than α, which requires $V < V_1$, where V_1 is defined in the Proposition 2. This is a stronger condition than the previous one.

This concludes the proof that $p_1 = p_2 = p_{med}$ is the equilibrium of the game when $1 \leq V \leq V_1$, which is Region 1 in Table 1.3 and 1.4 .

$p_1 = p_2 = p_{low}$ could also be an equilibrium of the game. In order for this equilibrium to hold, the boundary condition in Item 1 of Proposition 1 must be satisfied, i.e., we must have

$$\alpha(1-V)(p_1+p_2) + (2-V)p_1p_2 < \alpha^2 V$$

When $V > V_2$, this condition is satisfied for $p_1 = p_2 = p_{low}$. When $V > 1+\alpha$, $p_{low} < 1-\alpha$. So when $V > \max\{1+\alpha, V_2\}$, we have that $p_1 = p_2 = p_{low}$ is the local equilibrium of the game. When $2-2\alpha < V < \max\{V_2, 1+\alpha\}$, $p_1 = p_2 = 1-\alpha$ is the equilibrium instead of $p_1 = p_2 = p_{low}$.

When $V > \max\{1+\alpha, V_2\}$, $p_1 = p_2 = p_{low}$ satisfies the boundary condition, but we still need to check for deviations. Suppose one firm (say firm 1) charges p_{low} and the other (firm 2) decides whether to deviate. We could be in the low, medium or high price case depending on the price of firm 2. By definition of p_{low}, deviation within the low price case is never profitable. The cutoff price between the low price case and medium price case is given below

$$c_1 = \frac{\alpha(\alpha - \alpha V + (1-V)\sqrt{\alpha(\alpha+V^2-V-\alpha V)})}{\alpha V - \alpha - (2-V)\sqrt{\alpha(\alpha+V^2-V-\alpha V)}} \tag{1.12}$$

If $p_2 \leq c_1$, we are in the low price case. If $p_2 > c_1$, we are in the medium price case. $\frac{\partial \pi_2}{\partial p_2}$ when $p_2 \geq c_1$, so profit decreases in the medium price case and deviation within this area cannot be profitable. In the high price area, the most profitable deviation is to the "high optimal" price. However, the deviation profit is always smaller than the equilibrium profit, so the firm won't deviate. This suggests whenever $V \geq V_2$, $p_1 = p_2 = p_{low}$ is the global equilibrium of the game.

Following similar steps, it could be checked that $p_1 = p_2 = 1 - \alpha$ is the global equilibrium of the game when $2 - 2\alpha < V < \max\{V_2, 1 + \alpha\}$. This proves Region 3 and 4 in Table 1.3 and 1.4.

Since $V_1 < V_2$ always holds, $[1, V_1]$ and $[\max\{1 + \alpha, V_2\}, \infty)$ never intersects each other, so $p_1 = p_2 = p_{low}$ and $p_1 = p_2 = p_{med}$ cannot be the equilibrium at the same time. This suggests they are the unique equilibrium of the game in their respective region.

When $V_1 < V < \min\{2 - 2\alpha, V_2\}$, neither $p_1 = p_2 = p_{med}$ nor $p_1 = p_2 = p_{low}$ is the equilibrium since the boundary conditions are violated. Under this condition, the prices at the boundary between the low prices and medium price case are the equilibria of the game. In particular, there is a symmetric pure-strategy equilibrium in which

$$p_1 = p_2 = p_{sym} = \frac{\alpha V}{2 - V}, \ \pi_1 = \pi_2 = \pi_{sym} = \frac{\alpha V(\alpha + V - 2)}{2(V - 2)} \tag{1.13}$$

Without loss of generality, assume that firm 1 charges p_{sym} and firm 2 decides whether to deviate. For firm 2, $\frac{\partial \pi_2}{\partial p_2} > 0$ when $p_2 < p_{sym}$ (low price case) and $\frac{\partial \pi_2}{\partial p_2} < 0$ when $p_2 > p_{sym}$ (medium price case). So firm 2 won't deviate to these two areas. It remains to check for deviations to the high price case. The high optimal price for firm 2 is given by

$$p_{symdev} = \frac{2 + \alpha V - V - \sqrt{\alpha(2 + \alpha V - V)}}{2 - V}, \tag{1.14}$$

and the deviation profit for firm 2 is

$$\pi_{symdev} = \frac{(\alpha - \sqrt{\alpha(2 + \alpha V - V)})(V - \alpha V - 2 + \sqrt{\alpha(2 + \alpha V - V)})}{(2 - V)\sqrt{\alpha(2 + \alpha V - V)}}. \tag{1.15}$$

However, the high optimal price may not be feasible. If $p_{symdev} > 1 - \alpha$, the deviation price becomes $1 - \alpha$. Let $c_2 = \frac{2(1-2\alpha)}{1-\alpha}$. Then if $V < c_2$, $p_{symdev} < 1 - \alpha$ and vice versa.

Denote the profit of firm 2 if it deviates to $1 - \alpha$ by $\pi_{symdev2}$. Then $\pi_{symdev2} = \frac{\alpha(1-\alpha)}{2}$. $\pi_{sym} - \pi_{symdev2}$ is concave in V and $\pi_{sym} - \pi_{symdev2} > 0$ at both $V = c_2$ and $V = V_2$. This implies deviation to $1 - \alpha$ is never profitable. For the same reason, deviation to the high optimal price is not profitable either whenever it is feasible. This proves $p_1 = p_2 = p_{sym}$ is the equilibrium.

Consider the case where p_1 and p_2 change by a small amount from p_{sym}, but still at the boundary between the low price and medium price case. Since the profit functions and their derivatives are all continuous in price, when the changes are small enough, the equilibrium profit is still larger than the deviation profits. This shows that there are multiple equilibria in this region, which proves Region 2 in Table 1.3 and 1.4 in the proposition.

Proof of Lemma 1

Throughout this proof, by high-type consumer strategy and payoffs, we will mean the strategy and payoffs of those high-type consumers who are free to make a choice (remind that there is also a zero mass of high-type consumers that "seed" each possible consumer product choice $k = 0, 1$, and 2 which are therefore assumed to act non-strategically). Given this, one also has to interpret "zero demand" for a product as at most zero-mass of consumers choosing this product.

Proof of Item 1 (the case of $V > 1$). When $V > 1$, the low-type consumers will buy whenever the high-type ones buy. For any prices, consider nobody buying product $k = 1, 2$ except a zero mass of low-type consumers to make the proportion of high-type consumers $q_k = \alpha + p_k/V$. In this case, the high-type consumer payoff is α and low-type ones' is αV. Deviation payoff of buying Product k is $\alpha + p_k/V - p_k$ for the high type and αV for the low type consumers. Since $V > 1$, we have $\alpha + p_k/V - p_k < \alpha$ and neither type will deviate. This proves that zero demand for each product is always an equilibrium of the consumer choice subgame.

To show that this is the only equilibrium for any consumer choice subgame, let x_k and z_k denote low type and high type consumers' equilibrium demand for Product k respectively, $k = 0, 1, 2$. If $x_0 = 1 - \alpha$, then $z_k > 0$, $k = 1, 2$ could not be an equilibrium since the low types will buy whenever the high types buy. Therefore, in such case, the equilibrium both products receive zero demand. If $0 < x_0 < 1 - \alpha$, low type consumers must be indifferent between buying and not buying. Without loss of generality, assume $x_1 > 0$. Then this means $q_1 V - p_1 = q_0 V$. High type consumer payoff of buying Product 1 and neither is $q_1 - p_1$ and q_0 respectively and given

$V > 1$, we have $q_1 - p_1 < q_0$. So the high types will deviate and therefore no equilibrium with $0 < x_0 < 1 - \alpha$ exists. If $x_0 = 0$, then $z_0 = 0$, or the high types will deviate to neither. Without loss of generality, assume $x_1 > 0$. Then the payoff of buying Product 1 is $q_1 V - p_1$ to high types and $q_1 - p_1$ to low types. In order for low types not to deviate to neither, there must be a zero mass of low type consumers buying neither product to make $q_0 V = q_1 V - p_1$. However, this means the high types will deviate to neither and hence equilibrium couldn't exist for $x_0 = 0$. This proves nobody buying any product is the unique equilibrium of consumer choice subgame for any pair of prices (p_1, p_2). This completes the proof of Item 1.

Proof of Item 2 (the case of $1 - \alpha \leq V \leq 1$). When analyzing the consumer choice subgame, first we consider the subgame equilibrium with positive sales. There may also exist an subgame equilibrium with no market, which will be analyzed afterwards. Let us assume without loss of generality that $p_1 \geq p_2$.

When $p_1 > p_2 \geq V$, all high types buying the cheaper product and low types making no purchase constitutes an subgame equilibrium. To see this, note that prices exceed low types' possible valuation (if $p_2 = V$ only a zero mass of low-type consumers can possibly buy Product 2, otherwise low-type consumer valuation of Product 2 becomes strictly below V), so it's optimal for them not to buy anything. Given that low types don't buy, use of any product in the matching game ensures equal payoff to the high types, so they strictly prefers to buy the cheaper one.

In the special case of $p_1 = p_2 \geq V$, low types still won't make any purchase and high types obtain equal payoff of $1 - V > 0$ from using any one of the two products. So it's optimal for all high types to buy a product, but the split of market share could be arbitrary.

When $V \geq p_1 > p_2$, consider the following subgame equilibrium: all high type consumers buy Product 1, while low type's demand for Product 1 is $x_1 = \alpha V / p_1 - \alpha$ if $p_1 \geq \alpha V$ and $1 - \alpha$ if $p_1 \leq \alpha V$; further, $x_2 = 0$, but a zero mass of low type consumers buy Product 2 such that the proportion of high type consumers among Product 2 users is equal to $\frac{\alpha}{\alpha + x_1} - \frac{1}{V}(p_1 - p_2)$. Under this strategy, $U_l(1) = U_l(2) = \frac{\alpha}{\alpha + x_1} - p_1$ and $U_h(1) - U_h(2) = (p_1 - p_2)(1/V - 1) > 0$, so neither low- or high-type consumers will deviate. Therefore the above strategy constitutes an subgame equilibrium.

When $p_1 \geq V > p_2$ and $p_1 < 1 - (1/V - 1)p_2$, following similar analysis, the following strategy can be shown as an subgame equilibrium: all high-type consumers buy Product 1 and $x_1 = x_2 = 0$, but there is a zero mass of low type consumers buying Product 2 such that the proportion of high types consumers among all Product 2 users is equal to p_2/V.

When $p_1 \geq V > p_2$ and $p_1 > 1 - (1/V - 1)p_2$, it could be shown that the following strategy is an equilibrium of the subgame: all high type consumers buy Product 2; low type's demand for Product 1 is zero and that for Product 2 is equal to $\alpha V/p_2 - \alpha$ if $p_2 > \alpha V$ and $1 - \alpha$ otherwise.

We now turn to our discussion of the possible no-market consumer choice subgame equilibrium. Nobody buying any product is an equilibrium if $p_1, p_2 \in [1 - \alpha, 1]$. To check deviation, notice that high types consumers' equilibrium payoff is equal to α, and the maximal payoff they could obtain by deviating to a purchase is $1 - p_i < \alpha$. The inequality comes from the condition $p_i > 1 - \alpha$. So the high type consumers won't deviate. The same argument applies to low type consumers as well.

We now prove the statement in the Lemma. For any price $p_2 < 1-\alpha$, there is a price $p_1 > 1-\alpha$ (any price $(1 - \alpha < p_1 < 1 - (1 - 1/V)p_2$ will do) which would result in all high-type consumers buying Product 1 and Product 2 receiving zero demand. It is then straightforward to check that such response is strictly optimal unless Product 2 already receives zero demand. Therefore, a price below $1 - \alpha$ receiving positive demand can not be an equilibrium outcome. Furthermore, an optimal response to a price $p_1 > 2 - \alpha - \frac{1-\alpha}{V}$ is a price just below $(1 - p_1)V/(1 - V)$, which given the assumed condition on p_1 would, according to the optimal consumer strategy derived in Item 3, result in Product 1 having zero demand and Product 2 being bought by all high-type consumers. Therefore, a price above $2 - \alpha - \frac{1-\alpha}{V}$ cannot receive positive demand in equilibrium. Finally, any price $p_1 = p_2 \in [1 - \alpha, 2 - \alpha - \frac{1-\alpha}{V}]$ may be an equilibrium outcome with positive demand since unilateral deviation from it to any price in $[1 - \alpha, 1]$ could result in no market (which is one of the equilibria of such consumer subgame as discussed above) and responding by a price below $1 - \alpha$ leads to the unique consumer choice equilibrium with the deviating firm receiving zero demand and the other firm receiving demand from all high-type consumers. Similarly, prices $p_1 = p_2 = 1 - \alpha$ with zero demand for either product is also an equilibrium because a deviating firm is guaranteed to receive zero demand. This completes the proof of Item 2.

Proof of Item 3 (the case of $V < 1 - \alpha$). First, note that in the consumer choice subgame of the current case, the equilibrium consumer strategies with positive sales specified in the proof of Item 2 still constitute a consumer-choice subgame equilibrium since the only constraint the proof required was $V < 1$. This means when $p_1 = p_2 = V$, low types not buying and high types buying any one of the two products constitutes a subgame equilibrium. When $p_1 = V$ and $p_2 < V$, consumers only buy Product 1 and Product 2 receives no demand. When $p_1 > V$ and $p_2 = V$, there will be no demand for Product 1. It then follows straightforward that prices $p_1 = p_2 = V$ with all high-type consumers and only them buying one of the products (with an arbitrary split of market share) is an equilibrium since, as shown above, deviation to either lower or higher price results in no sales. This completes the proof of Item 3.

Proof of Proposition 3

From Proposition 1, case (4) cannot happen with positive probability in the equilibrium since at least one of the firms is getting zero sales and profit. For all the other cases, the sales and profit of both firms is positive. So in the equilibrium, both firms must enjoy positive sales and profit.

Proof of Proposition 5

We'll first explore the change of total monopoly profit with respect to prices.

Low price area. In the low price case, the profit of the individual product doesn't depend on the price of the other product. So maximizing total profit is the same as maximizing the profit for individual products. From the proof of Proposition 1, $p_1 = p_2 = p_{low}$ is the optimal prices provided they satisfy the boundary condition. If it doesn't satisfy the boundary condition, the optimal prices will be on the boundary. Let

$$p_{cut}(p_2) = \frac{\alpha(V\alpha + p_2V - p_2)}{2p_2 + \alpha - p_2V - V\alpha} \tag{1.16}$$

Then any pair of prices ($p_{cut}(p_2)$,p_2) in on the boundary of the low-price case and the medium-price case. The total profit for charging ($p_{cut}(p_2)$,p_2) is

$$\pi_{Boun}(p_2) = \frac{\alpha(2V\alpha p_2 - p_2V^2\alpha + \alpha V^3 - V^2\alpha - 2p_2V^2 - p_2^2V^2 + p_2V^3 - 2p_2^2 + 3p_2^2V)}{V(V\alpha + Vp_2 - \alpha - 2p_2)} \tag{1.17}$$

with

$$\frac{\partial \pi_{Boun}}{\partial(p_2)} = \frac{\alpha(p_2+\alpha)(V-1)(2-V)(p_2V + \alpha V - 2p_2)}{V(V\alpha + p_2V - \alpha - 2p_2)^2} \tag{1.18}$$

We have $\frac{\partial \pi_{Boun}}{\partial(p_2)} > 0$ when $p_2 < \frac{\alpha V}{2-V}$ and $\frac{\partial \pi_{Boun}}{\partial(p_2)} < 0$ otherwise, where $\frac{\alpha V}{2-V}$ is the symmetric price on the boundary. This suggests profit of the monopoly firm is maximized when price are symmetric on the boundary, or at

$$p_1 = p_2 = p_{sym} = \frac{\alpha V}{2-V}. \tag{1.19}$$

The condition for $p_1 = p_2 = p_{low}$ to hold was already identified in Proposition 1. Combining the above derivations, we can conclude that within the low-price case, the profit is maximized at $p_1 = p_2 = p_{sym}$ when $V < v2$ and $p_2 = p_2 = p_{low}$ when $V \geq v_2$. The profit at $p_2 = p_2 = p_{sym}$ is given by

$$\pi_{optboun} = \frac{\alpha V(2-\alpha-V)}{2-V}. \tag{1.20}$$

Medium price area. The total profit of the monopoly is given by

$$\pi_{Med} = \frac{\alpha[(p_1+p_2)(1-V) + 2V^2 - \sqrt{(p_1^2+p_2^2)(V-1)^2 + 2p_1p_2(1+2V-V^2)}]}{2V}. \tag{1.21}$$

It could be shown that $\frac{\partial \pi_{Med}}{\partial p_1} < 0$ and $\frac{\partial \pi_{Med}}{\partial p_2} < 0$ for $V < 2$. So the monopoly firm will lose profit by raising any of the prices. Then the optimal prices in the medium-price case is also on the boundary of the low-price and medium-price case. Following the same discussion as in the previous section, the optimal prices will be the same as those in the low-price case.

High price area. In the high price case, total profit of the two products is given by

$$\pi_{High} = \frac{\alpha(p_1^2 V + p_2^2 V + p_1 p_2 - p_2^2 - p_1 V - 2p_1 p_2 V)}{V(p_1 - p_2 - 1)} \tag{1.22}$$

and

$$\frac{\partial \pi_{High}}{\partial p_1} \propto (p_2^2 V + 3p_2 V - 2p_1 p_2 V + V - 2p_1 V + p_1^2 V - p_2). \tag{1.23}$$

The right hand side decreases in p_1 and is positive at $p_1 = 1 - \alpha$ when $V > 1$. So raising the price of the more expensive product is always desirable. In order to maximize total profit, the monopoly will set $p_1 = 1 - \alpha$. We have:

$$\frac{\partial \pi_{High}}{\partial p_2} \propto (p_1 - 2p_2 + 2p_1 p_2 - p_1^2 - p_2^2). \tag{1.24}$$

The right-hand side decreases with p_2. So for a given p_1, profit first increase and then decreases with p_2. The optimal p_2 is given by

$$p_{lowopt}(p_1) = p_1 - 2 + \sqrt{1 - p_1}. \tag{1.25}$$

When $p_1 = 1 - \alpha$, $p_{lowopt}(1 - \alpha) = \sqrt{\alpha} - \alpha$. We need to check whether this price satisfies the boundary conditions for the high-price case.

When $p_1 = 1 - \alpha$, in order for the high-price case to hold, $p_2 > \frac{\alpha(\alpha+V-1)}{2-\alpha-V}$ must be satisfied. Comparing these two values, we found that $\sqrt{\alpha} - \alpha$ satisfies the boundary condition when $V < 2 - \alpha - \sqrt{\alpha}$, but is out of bound for $V > 2 - \alpha - \sqrt{\alpha}$.

To conclude the high-price case, when $V < 2 - \alpha - \sqrt{\alpha}$, the optimal prices are $p_1 = 1 - \alpha, p_2 =$

$\sqrt{\alpha} - \alpha$ and the optimal profit is

$$\pi_{opthigh} = \frac{\alpha(2V + 2\sqrt{\alpha} - 1 - \alpha - 2V\sqrt{\alpha})}{V}. \tag{1.26}$$

When $V < 2 - \alpha - \sqrt{\alpha}$, the optimal prices are $p_1 = 1 - \alpha, p_2 = \frac{\alpha(\alpha+V-1)}{2-\alpha-V}$ and the optimal profit is

$$\pi_{opthigh2} = \frac{\alpha(V^3 + V\alpha^2 + 2V^2\alpha - 2\alpha^2 + 3\alpha - 3V^2 - 4V\alpha + 3V - 2)}{V(\alpha + V - 2)}. \tag{1.27}$$

Direct comparison of $\pi_{optboun}$ with $\pi_{opthigh}$ and $\pi_{opthigh2}$ shows that $\pi_{optboun} > \pi_{opthigh}$ and $\pi_{optboun} > \pi_{opthigh2}$ in their respective region. So the global equilibrium of the monopoly case is $p_1 = p_2 = p_{Sym}$ for $V < v_2$ and $p_1 = p_2 = p_{Low}$ for $V > v_2$.

Equilibrium Outcome When Coordinator cannot Recommend "Neither"

The equilibrium of the consumer choice subgame in the medium, high and very high price area remain the same, because the coordinator doesn't pick the null option in the equilibrium in these areas. In the low price area, the coordinator picks the null option with positive probability, which is not allowed in the current model. So it's no longer the equilibrium. Also, that the coordinator selects the null option with positive probability means that she cannot achieve higher payoff than α by selecting the two products only. So when the coordinator's choice is limited to the two products only, high types's equilibrium payoff will be lower than α. The high types can be better off by not following the coordinator and not buying any product, in which case nobody buys and high types's payoff is exactly equal to α in the equilibrium. This means in the low price area, nobody buying is the equilibrium of the consumer choice subgame.

The foregoing analysis implies that the equilibrium specified in Proposition 2 will only be affected in the region where the equilibrium lies in the low price area in the main model. To see this, note that the only difference between the current model and the main model is in the low price area. So the only potentially profitable deviation from the equilibrium in Proposition 2 is to deviate to the low price area. However, such strategy results in no market and zero profit. So whenever the equilibrium lies in the areas other than the low price area, firms will not deviate from the strategy in Proposition 2. When the equilibrium lies in the low price area, the equilibrium will be different since the equilibrium of the consumer choice subgame is now different. This means for $1 < V \leq \min\{V_2, 2-2\alpha\}$, the equilibrium will be the same as the main model, since it's not in the low price area in this region. For $V > 2-2\alpha$, the low price area spans the whole parameter space and nobody buying is the equilibrium in any consumer choice subgame. So there is no market in the equilibrium. For $V_2 < V \leq 2-2\alpha$, the strategy in Proposition 2 leads to zero profit. So firms will deviate to higher price and the equilibrium is on the boundary between the low and medium price area, which is the lowest prices that result in positive profit. So the equilibrium prices are $p_1 = p_2 = \frac{\alpha V}{2-V}$. This proves the statement made in the main text.

Chapter 2

The Bright Side of Loss Aversion[1]

"If life seems jolly rotten
There's something you've forgotten [. . .]
Always look on the bright side of life."
— Monty Python's Lyrics

2.1 Introduction

Marketing practitioners and researchers have long recognized that consumer purchase decisions are affected by reference prices (Markowitz 1952) and are disproportionally affected by losses compared to gains (Galanter and Pliner 1974), i.e., consumers respond more strongly to prices higher than their reference point than to prices lower than their reference point.

A case in point is a recent J.C. Penny's ad campaign associated with its decision to reduce the frequency and the amount of price promotions. A characteristic ad of this campaign starts with a woman on a shopping trip hysterically screaming "NO!" when she sees a "70% off" promotion that was valid only yesterday. Then another woman screams "NO!" when she is passing by the window of a store and notices a "62% off" sign on the same jacket she is wearing. The ad then continues to show consumers frantically clipping coupons and a consumer walking away from a line waiting for a store to open after seeing that discounts are (only) 43% off. It ends with the announcement

[1]This is joint work with Dmitri Kuksov

of the company's new pricing campaign "Enough.is.enough."[2]

This ad illustrates that J.C. Penney recognizes that consumers are not only concerned about the value obtained (price paid given product) but also distressed by the foregone opportunities of promotions not taken advantage of. Put in another way, consumer purchase decisions are reference-dependent. Exposure to sales and promotional activities lowers the reference prices in consumers' minds and these reference prices matter above and beyond of how they should if consumers were fully rational, especially when the prices are above their reference price.

The prominence of this negative side of reference prices – that consumers are more sensitive to the perceived loss of paying above the reference price than to the perceived gain of paying below the reference price is a well-established phenomenon in the marketing literature and is known as loss aversion (Galanter and Pliner 1974; Kahneman and Tversky 1979).[3]

If loss aversion is present in consumer purchase decisions, firms must take it into account in their pricing and promotion decisions. The objective of this research is to investigate how firms should optimally respond to the presence of consumer loss aversion and how consumer loss aversion affects industry profitability. In particular, we focus on the effect of consumer loss aversion on the optimal pricing and promotional strategy of firms. Promotions are an important component of a firm's marketing mix decision and are widely adopted by firms as a competitive tool. J.C. Penney, for example, featured nearly 600 sales in 2011.[4] Granted, this was before the afore-mentioned change in its strategy. But even the new J.C. Penney pricing policy involves some promotions, only fewer of them, not be as deep, and "better timed". Since promotions have a strong effect on the formation of reference prices, the effects of reference prices and loss-aversion must be carefully

[2]"J.C. Penney gets rid of hundreds of sales", available at http://finance.yahoo.com/news/j-c-penney-gets-rid-151954852.html

[3]In aggregate, loss aversion could be observationally similar to the asymmetric effect of negative vs. positive reviews on the future sales. For example, a price increase by Netflix in 2011 lead to fierce emotional reaction from its customers and received considerable media coverage (Wingfield and Stelter 2011). One could imagine, a price cut would not have received as much attention and consumer response. Another example is the well-known price discrimination case by Amazon in 2000. Consumers were angered when they found out that Amazon was selling DVDs at different prices to different consumers and they were particularly distressed to find that they paid a higher price than others (Streitfeld 2000).

[4]"J.C. Penney gets rid of hundreds of sales", available at http://finance.yahoo.com/news/j-c-penney-gets-rid-151954852.html

considered when designing a promotional strategy.

It is easy to see how loss aversion could negatively affect a firm's profit. Loss aversion implies depressed consumer valuation for a product or higher intention of consumers to search for a better deal if lower prices have been observed in the past. This should lead the firm to lower prices to compensate consumers for the perceived loss and therefore, lower profits. Taking a step back, expecting this negative effect in the future and attempting to ameliorate it implies that a firm could be effectively restricted from profitable (in the short term) price promotions. Intuitively, constraints on the strategy space are also a negative factor for profitability.

However, constraints on the strategy space could be beneficial in a competitive environment. There are two reasons for this possibility. First, a firm may benefit from competitor being constrained and second, constraints on its own strategy (when known to the competitor) could be beneficial due to their strategic effect on competitor's decisions.

The objective of this paper is to better understand how consumer loss aversion affects firms' pricing and promotional strategies as well as profitability over time, and in particular, whether and under what conditions loss aversion may possibly result in higher overall profits. In other words, is there a bright side of loss aversion for the firms? And practically, how should firms adapt their strategy in view of consumer loss aversion?

In markets where firms adopt price promotions and price dispersion exists, consumers often need to conduct search to find the best prices. But search is costly. There are various reasons for search being costly to consumers (see, e.g., Smith et al. 1999): external search costs include the monetary investment required to obtain information and the opportunity cost of time, while internal search costs include the mental efforts required to integrate information and make the optimal decision. For instance, in the case of shopping in physical stores, consumers need to travel to the store to find out the price. In the case of online shopping, consumers need to spend time and efforts browsing websites and checking prices. They also need to learn how to make the best use of search engines, price-comparison sites and other information sources to find out the best offer. Empirical researchers in marketing have identified significant and positive search costs among consumers using data from various product categories including auto insurance, books and computer chips (e.g., Hong and Shum 2006; Moraga-Gonzalez and Wildenbeest 2008; Honka 2010). Search cost is also likely to vary among consumers for a number of reasons, such as difference in opportunity cost of time and difference in enjoyment of shopping.

To analyze the implications of consumer loss aversion for the optimal strategy of firms under realistic assumptions on the marketplace, we develop a two-period model of duopoly competition where consumers need to search to know prices. Consumer decisions are affected by a reference price, which is determined by the prices of the previous period. We assume that the reference price affects consumers' search intention and they are more sensitive to price increases than to price decreases, which captures the notion of loss aversion.[5] We compare this model to a model without consumer loss aversion and examine the implications of consumer loss aversion on the optimal pricing and promotional decisions of firms. Clearly, reference prices could have a positive effect on profits if consumers are sensitive to both gains and losses, since high reference prices could positively affect consumer valuation or negatively affect their intension to search. Likewise, low reference prices could clearly have a negative effect on profits. The standard argument for the possibility of positive effect of reference prices is through the benefits of high reference prices. However, the idea of loss aversion is that the effect of low reference prices should be stronger. Therefore, one of the questions we would like to address is whether the aforementioned seemingly negative aspect of reference prices could be desirable from firms' point of view. For this objective and to simplify the model, we consider the model without the positive aspect of (high) reference prices.

We show that although loss aversion has a negative effect on second-period prices and profits, the forward-looking behavior of competing firms leads to higher expected first-period prices and profits than what they would be in the absence of consumer loss aversion. The effect of loss aversion on the first-period pricing has a direct and a strategic component. The direct effect is that foreseeing the negative effect of current price promotions on future profit, each firm optimally

[5]We also show that the results are robust when the reference prices have a direct effect only on the willingness to pay.

promotes less. The strategic effect – i.e., the optimal response of each firm to the direct effect on the other firm's pricing – is that firms price higher because they are less concerned about price competition with the other firm. The direct and strategic effects reinforce each other in the first period to raise prices and profits.[6] At the same time, we show that the above effects never invalidate the optimality of promotional pricing: some promotions are always optimal in both the first and the second period. This result underscores the dilemma faced by competing firms: even though they recognize the negative long-term implications of promotional activities, they cannot (and should not) stay away from them altogether.

We further show that the combined direct and strategic effects of the forward-looking behavior of competing firms could be strong enough to result in higher total profit of each firm. Examining the conditions under which consumer loss aversion could be beneficial, we find that firms could benefit when the following two conditions are both satisfied. First, consumer valuation for the products must be high enough relative to the consumer search costs, so that the equilibrium prices in the absence of loss aversion are constrained by consumer search intention rather than by consumer valuation. In particular, we show that if the highest price charged in the absence of loss aversion is constrained by consumer valuation, then consumer loss aversion always leads to lower total profit. Second, the proportion of consumers with positive search cost must be in an intermediate range.

Besides explaining and predicting the effect of loss aversion on firm's profits, our model also helps in identifying implications of loss aversion on the optimal pricing and promotional strategy of firms in a competitive environment. First, it shows that due to detrimental impact of current promotions on future profits, competing firms should somewhat restrain from promotions, but not forego them altogether. It also indicates that in the presence of consumer loss aversion, when the regular prices are constrained by consumers' intention to search for competitor's price rather than by consumer valuation, a firm should not only change its promotional strategy but also increase its regular price. As a result, the optimal depth of promotions from the (new) regular price could be

[6]Furthermore, less frequent or lower depth promotions also imply lower incentives for consumers to search, and the reduced propensity of consumers to search may also imply higher optimal prices and thus even higher profits.

even higher than the optimal depth of promotions from the regular price in the absence of consumer loss aversion. In other words, if the regular prices are kept exogenously fixed, consumer loss aversion implies optimality of lower/less promotions, but keeping the regular price endogenous could mean optimality of higher/more promotions.

Extending the model to multiple periods, we illustrate how consumer loss aversion may result in ever-increasing price pressure over time with profits declining to zero, but also illustrate through an extension of the model how the equilibrium price distribution could exhibit a cyclical pattern of decreasing price over time for some number of periods followed by prices "resetting" to a higher level, after which the decreasing price trend resumes. The latter equilibrium pattern materializes, for example, when the market has an (arbitrarily small) segment of consumers who face search costs but do not exhibit loss aversion behavior or who are loyal to each firm. It is also interesting to note that the timing at which prices reset to the higher level in this model is probabilistic as both competing firms follow mixed strategies in whether to set a high price in hopes that the industry dynamics would "reset" and only when both firms set the high price, the dynamics is reset. In other words, when one of the firms (ex ante optimally) decides to charge a higher price to try to reset the industry dynamic, it may (ex post) regret doing so.

The remainder of the paper is organized as follows. The following section discusses the extant literature related to references price and loss aversion effects, as well as literature on modeling competitive markets with consumer search. We then formally define the model in Section 2.3 and briefly discuss the main assumptions. The model is analyzed in Section 2.4. This analysis is then followed by a discussion of the main assumptions of the model and analysis of some model extensions in Section 2.5 for the purpose of understanding the robustness of the model predictions and to speculate how the results could change or be extended to other realistic scenarios. Section 2.6 concludes with a recap of the paper's major findings and contributions.

2.2 Related Literature

This paper is most closely related to the literature on the effects of reference price and loss aversion, as well as to the literature on price dispersion under competition. An assumption frequently adopted in standard economic theory is that people are fully rational and hence should only care about the final payoff they expect to obtain. Alternatively, Markowitz (1952) was one of the early researchers to define utility with respect to reference points rather than to the final outcome. Galanter and Pliner (1974) showed that the pain people experience from losing some amount of money appears to be greater than the joy they experience from winning the same amount. Kahneman and Tversky (1979) integrated these ideas into their definition of the consumer value function. According to the Prospect Theory they suggested, the value function of consumers is defined relative to a reference point and represents an option as a loss (gain) if the price is higher (lower) than the reference point. Moreover, it postulates that consumers should be more sensitive to losses than to gains, i.e., that consumers are loss averse. These effects have been confirmed in a number of experimental studies.

Empirical research with observed consumer purchase data (e.g., Winer 1986; Kalyanaram and Winer 1995) has found consistent evidence of reference price effect in consumer purchase decisions. There is also empirical evidence of consumer loss aversion (e.g., Putler 1992; Hardie et al. 1993; Kalyanaram and Winer 1995, and Mazumdar, Raj and Sinha 2005).[7]

Empirical studies showed that reference price could be determined by past prices (e.g., Winer 1986; Lattin and Bucklin 1989; Bolton, Warlop and Alba 2003), competitors' prices (Hardie et al. 1993; Rajendran and Tellis 1994) and future price expectations (Kalwani and Yim 1992) and be affected by various factors such as purpose of the trip (Thaler 1985), frequency of sales (Kalwani

[7] Although others found that such effect is attenuated when accounting for consumer heterogeneity in price responses (Krishnamurthi, Mazumdar and Raj 1992; Bell and Lattin 2000), the loss aversion effect still seems to remain in many categories (Bell and Lattin 2000). Furthermore, even when loss aversion disappears at population level when consumer heterogeneity is accounted for, it may still prevail within some consumer segments (Krishnamurthi, Mazumdar and Raj 1992; Bell and Lattin 2000). Therefore, even if loss aversion is not a universal phenomenon, it is important to understand how pricing and promotional strategy should be adapted for markets/categories in which loss aversion is present as opposed to the ones where it is not as economically important.

and Yim 1992), store characteristics (Biswas and Blair 1992), the amount of information provided by retailers (Lynch and Ariely 2000) and prices paid by other consumers (Feinberg, Krishna and Zhang 2002). In some studies, past prices observed have been demonstrated to be the strongest determinant of consumers' internal reference price (Briesch et al. 1997; Mazumdar, Raj and Sinha 2005). In this paper, as we concentrate on the pricing and promotion decisions in a two-period model, we focus on the implications of past-price-determined reference price. Reference price effect has been found to influence many aspects of consumers' purchase decision, such as brand choice (e.g., Kalyanaram and Winer 1995), purchase quantity (Krishnamurthi, Mazumdar and Raj 1992), purchase timing (Bell and Bucklin 1999) and search behavior (Grewal, Monroe and Krishnan 1998). Again, as we abstract from non-price variables and for simplicity consider single-unit individual consumer demand in this paper, we model the behavioral implications of reference price effect through consumer search (and consequently, purchase) behavior. While the extant behavioral and empirical research focused more on how the reference price impacts consumer behavior and less on how it affects or should affect the firm's strategy, in the current research, we aim to understand how consumer loss aversion should affect the optimal strategy of a firm.

There is some analytical literature examining the implications of reference price effect on firm behavior. Greenleaf (1995) studies the optimal dynamic pricing policy of a monopoly when demand is affected by the reference price effect. Kopalle, Rao and Assunção (1996) extend the analysis to multiple products and the competitive case. Both papers show that reference price effect may lead to high-low pricing in the equilibrium but only when the gain effect of the reference point is greater than the loss effect; in other words, when loss aversion is not present in the aggregate demand function. Our paper differs in two aspects. First, we model the effect of reference price on individual consumer utility. Second, we focus on loss aversion, which is a condition under which the above papers do not find promotions to be optimal. The reason is that in those models, promotions are not optimal in the absence of reference point effects. Alternatively, we start with the marketing environment where promotions are optimal in a one-shot game and ask whether they would persist and how the pricing strategy and profits would be impacted by the presence of loss aversion when firms set prices over time. We then find that not only promotions would then persist, but their magnitude could even increase relative to the case of no loss aversion.

Given the marketing environment we study, another closely related stream of research is the analytical research on price dispersion in competitive markets. Varian (1980) models oligopolistic price competition among identical firms when consumers are heterogeneous in their access to information. Narasimhan (1988) looks at the competitive promotion strategy of firms when each firm has a loyalty segment of consumers. In these models, the price distribution is constrained upward by consumer product valuation. Stahl (1989) analyzes a similar model but where uninformed consumers are not exogenously prevented from being informed but rather face a search cost to find the second price. Our model builds on Stahl's by introducing consumer loss aversion effect and the dynamic (two-period) game structure.[8]

Finally, our paper is related to the increasing body of marketing research that looks at the strategic implications of behavioral anomalies (see Narasimhan et al. 2004). For example, Cui et al. (2004) investigate channel coordination problem when channel members care about fairness and show that if channel members care about the equitable distribution of outcomes in addition to their own payoffs, optimal channel coordination may be achieved. Orhun (2009) examines the product line design problem when consumers exhibit choice set-dependent preferences, i.e., their valuation for one product depends on its attribute levels relative to other products in the choice set. She finds that when consumers' preference is choice set-dependent, the firm may choose more different products than justified from the price discrimination motive alone. Narasimhan and Turut (2012) examine the implications of reference-dependant preferences on the incentives to differentiate and imitate in innovation.

[8]Other research examining competitive price promotions includes Lal (1990), who analyzes an oligopolistic pricing model in an infinite horizon repeated game and discovers that competition can be interpreted as a long-run collusion between national brands to fight a third firm, Rao (1991) who models duopoly competition between a national brand and a private label and finds that the national brand optimally promotes to defend market share from the private label, and Lal et al. (1996) who investigate the manufacturers' incentives to adopt trade dealing and how it is affected by retailer's ability to forward buy. Note that price promotions may also be optimal in the absence of competitive rationale. For example, Jeuland and Narasimhan (1985) provide an explanation for temporary price cuts by sellers as a mechanism to discriminate between buyers with more and less intense demand. However, as we will argue the surprising effect of increased profits due to loss aversion may only be obtained under competition.

2.3 Model

There are two undifferentiated firms selling one product each in each of the two time periods. Firms have equal variable production cost normalized to 0 and there is no fixed cost associated with selling the product. Consumers of unit total mass have single-unit demand in each of the two periods and value each product equally at V. In each period, consumers may have to incur a search cost to observe the price of a product. There are two consumer segments that differ in their search cost: consumers in one segment have zero search cost and thus always observe both prices, while consumers in the other segment have a positive search cost s per product (firm) and thus have a non-trivial search decision to make. The size of the first segment of consumers is $\beta \in (0, 1)$ and the size of the second one is $2\alpha = 1 - \beta$. In addition, as a simplification and consistent with previous literature on consumer search costs, we assume that consumers in the second segment are able to obtain the first search for free.[9]

In each of the two periods, first, the two firms simultaneously decide what price p_{jt} to charge (j indices firms and t indices time periods), and then, consumers decide whether to search for price and which product to buy. Firms are forward-looking and do not discount future profits (i.e., have a discount factor equal to 1).

To model consumer loss aversion, we assume that consumers are affected by a reference price formed during the previous period. In the first period, we consider V as already incorporating the effect of the reference price.[10] In the second period, we assume that the reference price p^r equals the lower price charged by the two firms in the first period. Although realistically, the reference price could be between the higher and the lower price, assuming it to be equal to the lower price makes the loss aversion effect stronger and simplifies the analytical tractability. Note that an implicit assumption here is that in the second period, consumers know both prices charged

[9] This assumption is not essential in our case. Without this assumption, a fraction of consumers in the segment with positive search costs could abstain from entering the market, but our main results are not affected by this possibility. Furthermore, for high enough V, the first search cost is always optimal to incur and thus it has no effect on the decisions at all.

[10] Assuming different values for V in different periods does not change the main results as far as the valuation does not depend on the firm's actions under consideration.

by the two firms in the first period. While in Section 5, we argue for the robustness of the model results to the alternative assumption that in the second period, consumers only know the prices they have searched for in the first period, we consider the assumption of knowledge of both prices as realistically capturing the notion that new information is learned over time and that it is easier to learn information over time rather than when pressed with the need to purchase. For example, consumers can learn between purchase occasions through promotional mailings or TV ads that come at the time periods in which consumers do not desire to purchase. Consumers may also happen to acquire past price information in their casual conversations with other consumers, come across other consumers' purchase experience when looking for product reviews online (since the product is not storable across periods, it is not essential whether the demand in the two periods comes from the same set of consumers or not), etc[11]. In Section 2.5, we discuss an extension of the model in which consumers are only informed of the price they observed in the first period and hence have heterogeneous reference prices in the second period. We show that the main results of the model are robust to this alternative assumption.

We model the asymmetric effect of reference price (loss aversion) by assuming that consumers respond differently to prices above the reference price (loss) and prices below the reference price (gain). Consumers search for the other price for sure if the observed price is higher than the reference price and make economically-justified search decision if the reverse is true. Since observing a higher price incentivizes consumers to continue searching, this assumption implies that consumers are more sensitive to price changes in the losses domain than in the gain domain. In the gain domain, consumers essentially behave the same as when there is no reference price (loss aversion) effect, which means we have ruled out the direct positive effect of reference prices. We do this for several reasons. First, the extant empirical and behavioral research argued that losses have a stronger effect on consumer behavior. Second, the loss and gain affects are separate effects and

[11]Consumers may also have incentives to learn prices when they are not in need of the product but have more free time. The motivation for conducting such post-purchase or between-purchase search, however, could be different from that of a buying decision. The purpose, for example, could be to gather information for future purchase, to resolve post-purchase dissonance, or just to satisfy one's curiosity. We do not explicitly model such behavior because it does not provide extra vitality into the main findings of the model.

considering them one at a time in an analytical model is useful to more clearly see the implications of each. Third, investigating the strategic implications of consumer loss aversion requires comparing the current model with a model in which consumers are not loss averse (i.e., equally sensitive to losses and gains). Since the difference between the two models should only lie in the negative domain, the current assumption best serves this purpose. Finally, one of our goals is to see whether and under what conditions the negative aspect of reference price (i.e., the one in the domain of losses) could be beneficial to firms. It is clear that the positive aspect (when the prices are in the domain of gains) could be beneficial. Focusing on just the negative domain of the effect allows us to disentangle the two forces and more effectively address our research objectives.

We further assume that the reference price only affects consumers' search behavior but not their valuation for the products. This assumption is not essential but simplifies the analytical derivation. In Section 2.5, we consider a variation of the model where consumer search behavior is not affected but, instead, their utility is decreased if they have to pay a price above their reference price, and show that the main implications do not change.

2.4 Analysis

We use subgame perfect Nash equilibrium as the solution concept and thus solve the game through backward induction. Let $F_{jt}(p)$ denote the CDF of the equilibrium price distribution of firm j in period t and denote its upper and lower limits by $\overline{p}_{jt}$ and $\underline{p}_{jt}$, respectively. Note that a pure price strategy (i.e., one without promotions) would correspond to $\overline{p}_{jt} = \underline{p}_{jt}$. Since the two firms are symmetric and the equilibrium profits and price distributions also turn out to be the same (i.e., asymmetric equilibria do not exist), we will sometimes omit the subscript indexing firms from the profit and price notation, and will use the subscript to refer to the time period. The following Section 4.1 derives and discusses the optimal strategies in the second period; Section 4.2 analyzes the optimal strategies in the first period; Section 4.3 derives the outcomes of a benchmark model where consumer loss aversion does not exist, and Section 4.4 derives the implications of consumer loss aversion by comparing the outcomes of the main model to the outcomes of the benchmark model. Section 5 then discusses the importance of the main assumptions of the model for the main results and considers a number of model modifications in order to establish robustness and extend the implications to other market settings.

2.4.1 Second-Period Pricing

The second-period game is similar to the one in the models of Varian (1980), Narasimhan (1988) and Stahl (1989). While the analysis is similar to that of Stahl (1989), it may be useful to briefly recap the argument to understand the intuition for the forces at play.

Let us first consider the optimal consumer search strategy and purchase decision. Consumers with zero search cost always look at both prices and buy from the firm with the lower price as far as this price is at or below V.[12] Consumers with search cost s start with making one search. Half of them observe the price of Firm 1 and the other half observe the price of Firm 2.[13]

Without loss of generality, suppose that a consumer searched Product 1 first and observed price p_{12} (where index 2 refers to the second period and index 1 refers to the firm). By assumption, if $p_{12} > p^r$, then she will search for the other price p_{22}. If $p_{12} < p^r$, then she needs to decide whether to search by comparing the expected search cost s and the expected benefit of search determined by her expectation of Firm 2's pricing strategy $F_{22}(p)$. Clearly, if the consumer prefers to search after observing p_{12} at a certain level, she will also search if she observed p_{12} at a higher level. Also note that the equilibrium price distributions $F_{j2}(p)$ should be such that consumers with search cost s do not search after observing the highest price in the distribution ($\overline{p}_{j2}$). This is because if they do, they will for sure find a lower price and thus $\overline{p}_{j2}$ guarantees zero profit, while a price of $s/2$

[12]Just as in the above mentioned models, due to the absence of mass points in the price distribution, it is inconsequential for the firms' pricing strategy and profits what happens in the event of a price tie. But to be specific, one can postulate that in the event of a tie, the demand is split equally.

[13]There are several ways to justify this conclusion. The most direct one is to assume a random search technology. However, such assumption is not necessary since the only equilibrium where consumers strategically select where to search first is one in which the first searches are equally split between the two firms. This is because the firm with higher proportion of first searches (if such exist) will set an on-average higher price, which will make it strictly optimal for consumers to search first at the other firm.

guarantees positive profit.[14]

Given that consumers with search cost s do not search after observing the highest (and therefore, any) price in the price distribution, the model then reduces to one in Varian (1980) and implies

$$F_{j2}(p) = F_2(p) = \frac{(1-\alpha)p - \overline{p}_2}{\beta p}, \tag{2.1}$$

where $\overline{p}_2$ is the highest price in the second-period price distributions. It remains to derive the upper limit of the price distribution $\overline{p}_2$. First, this price must be at or below consumer valuation for the products V. In addition, as argued above, the highest price $\overline{p}_2$ must satisfy the condition of no-search by consumers with search cost s. This condition implies $\overline{p}_2 < p^r$ and such that the benefit of search after observing it is at or below s. The latter condition implies

$$\int_{\underline{p}_2}^{\overline{p}_2} (\overline{p}_2 - p)\, dF_2(p) \leq s, \tag{2.2}$$

where the lower limit of the price distribution $\underline{p}_2$ can be derived from Equation (2.1) as

$$\underline{p}_2 = \frac{\alpha \overline{p}_2}{1-\alpha}. \tag{2.3}$$

It therefore follows that $\overline{p}_2$ has to be at or below

$$V_{\mathrm{s}} = \frac{\beta}{\beta - \alpha \ln(1/\alpha - 1)} s. \tag{2.4}$$

The following lemma summarizes the above discussion:

Lemma 2. *Suppose consumers have a reference price p^r in the second period of the game, then in equilibrium, firms set prices according to the price distribution defined by Equation (2.1) where $\overline{p}_2 = \min\{V, p^r, V_s\}$ and have expected profits of $\pi_2 = \alpha \overline{p}_2$ each.*

[14]The standard argument applies to show that the highest price can not be a mass point, since undercutting by small enough amount to capture the full demand of consumers with zero search costs would then be strictly preferable. The above argument also implies that there is no pure-strategy equilibria.

Proof. For complete proof, see Appendix. □

Lemma 2 sheds light on the short-term effects of consumer loss aversion and previously-formed reference prices. V_s corresponds to the cutoff point of V in the benchmark model without loss-aversion (see Section 4.3 for further discussion of this benchmark) between the case where the equilibrium price distribution is bounded by consumer incentive to search and the one where the equilibrium price distribution is bounded by consumer valuation for the products. V_s is also equal to the upper bound of the equilibrium price distribution when it is bounded by consumers' search intention. An implication of Lemma 2 is that when the reference price is high, the equilibrium in the second period is the same as that in the model without loss aversion. This comes from the assumption that relative to the model without consumer loss aversion, consumers are only affected by losses. When the reference price is relatively low, the equilibrium price distribution will be "cropped" by the reference price p^r. The intuition is that lower reference price induces consumers to search for lower prices, which intensifies price competition between firms. One can also see from the lemma that the equilibrium profits of firms decrease as the reference price decreases, which reflects the long-term effect of consumer loss aversion.

Given the downward pressure on prices, one could speculate that if the reference price decreases, the price range would be getting compressed to the lower part of the price distribution. However, while the effect of reference prices on consumer behavior is only for the potential prices above the reference price, as it is clear from Equations (2.1) and (2.3), the effect of reference prices on pricing extends to the lower prices so that the whole distribution of prices shifts downward. We summarize this result as the following proposition.

Proposition 6. *Lower reference prices both restrict the ability of firms to obtain demand from higher prices and lead to optimality of stronger promotions, i.e., to a downward shift of the lower limit of the optimal price range.*

Proof. See Appendix □

The intuition for this result is that as firms' profitability is constrained by the reference price for prices at the high end of the optimal price range, the lower prices become more attractive. The competition for demand at lower prices then drives prices down even further.

2.4.2 First-Period Pricing

After examining the effect of consumer loss aversion on the second-period pricing, we can come back and solve the first-period pricing game. Since consumers are making a repeat purchase decision (consumers cannot or are unwilling to stockpile the product) and their buying decision in the first period does not affect the price they will observe in the second period, they do not have to account for future decision-making in their first-period decision. Consequently, in the first-period pricing game, consumers' strategy will be the same as in the model without consumer loss aversion and is similar to the one discussed in the previous subsection. Specifically, consumers with zero search cost will always observe both prices and buy from the firm with the lower price. Consumers with search cost s will observe a price for free first and decide whether to search for the other price based on the price they observed and the expectation of the price charged by the other firm. They will search if and only if the expected savings exceed the search cost s.

Firms' decision, however, is different from the one in the second-period pricing game. Since second-period prices and profits are affected by the reference price determined in the first period, firms need to take into account the effect of their first-period pricing decision on the demand and the competitor's price in the following period. Without loss of generality consider the first-period pricing problem of Firm 1. Suppose Firm 1 expects Firm 2 to price in the first period according to the distribution $F_{21}(p)$ (where 1 indexes the first period and 2 indexes the firm). Then, the objective of Firm 1 is to choose p_{11} to maximize the following profit function

$$\pi_1^s(p_{11}|F_{21}(p)) + \int_{p_{11}}^{\overline{p}_{21}} \alpha \min\{p_{11}, V_{\mathrm{s}}\}\, dF_{21}(p) + \int_{\underline{p}_{21}}^{p_{11}} \alpha \min\{p, V_{\mathrm{s}}\}\, dF_{21}(p), \tag{2.5}$$

where the first term $\pi_1^s(p_{11}|F_{21}(p))$ is the profit of Firm 1 if the second period were absent and given that Firm 2 prices according to distribution $F_{21}(p)$, and the other two terms capture the future profit of Firm 1 and reflect the dynamics of the model. The second term captures Firm 1's profit in the second period if p_{11} becomes the reference price and the last term captures its profit if the opposite is true, i.e., if the first-period's price of Firm 2 is lower and p_{21} becomes the reference price. It is easy to see that the sum of the latter two terms decreases as p_{11} decreases. This means if a firm lowers price now, its expected profit in the second period will also decrease. This concern leads firms to price less aggressively in the first period, and therefore results in higher prices. Firms have to tradeoff between immediate profit and future profit by taking into account the effect of loss aversion on consumer search behavior.

While firms now have an addition dis-incentive of reducing price in the first period, the first period pricing game still has no pure strategy equilibrium. The intuition is that when prices are equal, each firm obtains demand from half of the market. Given this, if one firm lowers price by an arbitrarily small amount, it will capture the whole market in the current period but lose some profit in the future period. Since the incremental profit in the current profit is discontinuous while the change of profit in the second period is continuous, firms will still find it optimal to undercut rather than to charge equal prices. Therefore no single-price strategy can be optimal in equilibrium.

To solve for the equilibrium in the first period, we need to consider $p < V_{\text{s}}$ and $p > V_{\text{s}}$ (where V_{s} is defined by Equation (4)) separately, since firms' pricing incentive is different in these two regions. When $p < V_{\text{s}}$, the reference price p^r in the second period is below V_{s} and thus, expected second-period profit is equal to αp^r. Equation (2.5) then simplifies to $p + [1 - F_1(p)](1-\alpha)p + \int_{\underline{p}_1}^{p} \alpha t \, dF_1(p)$ (subscript for firm is suppressed). This means the equilibrium price distribution in this region has the following form

$$F_{j1}(p) = F_1(p) = \frac{1 - (\underline{p}_1/p)^{\frac{1-\alpha}{\beta}}}{1-\alpha}, \text{ for } p < V_{\text{s}} \tag{2.6}$$

where $\underline{p}_1$ is the lower bound of the equilibrium price distribution in the first period. When $p > V_{\text{s}}$, the reference price p^r could be higher than V_{s} if the price charged by the other firm is also higher than V_{s}. In this case, second-period profit is equal to αV_{s}. Under this condition, Equation (2.5) simplifies to $p + [1 - F_1(p)]\beta p + \int_{\underline{p}_1}^{V_\text{s}} \alpha t \, dF_1(p) + [1 - F_1(V_\text{s})]\alpha V_\text{s}$, which leads to the following equation

$$F_{j1}(p) = F_1(p) = \frac{(1-\alpha)p - \alpha \overline{p}_1}{\beta p}, \text{ for } p > V_\text{s} \tag{2.7}$$

where $\overline{p}_1$ is the upper bound of the equilibrium price distribution in the first period. It then remains to derive the upper and lower limits of the price distribution. Similar to the argument in the analysis of the second-period pricing game, consumers with search cost s must find it optimal not to search for the other price at the top price $\overline{p}_1$, which implies the following condition

$$\int_{\underline{p}_1}^{\overline{p}_1} (\overline{p}_1 - p) \, dF_2(p) \leq s \tag{2.8}$$

Also, the equilibrium price distribution $F_1(p)$ must be continuous at V_s, which implies

$$\frac{1 - \left(\underline{p}_1 / V_\text{s}\right)^{\frac{1-\alpha}{\beta}}}{1-\alpha} = \frac{(1-\alpha)V_\text{s} - \alpha \overline{p}_1}{\beta V_\text{s}} \tag{2.9}$$

Equations (2.8) and (2.9) then imply $\overline{p}_1 \leq Z$, where Z is the solution to the following equation

$$\left(\frac{\beta V_\text{s}}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} [(1-\alpha)Z - \alpha V_\text{s}]^{\frac{\beta}{1-\alpha}} = Z(\alpha \ln V_\text{s} + 2 - 3\alpha - \alpha \ln Z) - s\beta - \alpha V_\text{s} \tag{2.10}$$

These findings are summarized in the following lemma.

Lemma 3. *The equilibrium prices in the first period of the main model are in symmetric mixed strategies $F_1(p)$ with upper limit $\overline{p}_1$ and lower limit $\underline{p}_1$.*

1. *If $V \geq V_s$, then*

$$F_1(p) = \begin{cases} \frac{1}{1-\alpha} + \frac{\alpha^2 V_s - \alpha(1-\alpha)\min\{V,Z\}}{(1-\alpha)\beta} V_s^{\frac{\alpha}{\beta}} p^{-\frac{1-\alpha}{\beta}}, & \text{if } \underline{p}_1 \leq p < V_s \\ \frac{(1-\alpha)p - \alpha \min\{V,Z\}}{\beta p}, & \text{if } V_s \leq p < \overline{p}_1 \end{cases} \tag{2.11}$$

where $\overline{p}_1 = \min\{V, Z\}$, $\underline{p}_1 = V_s \left(\frac{\alpha(1-\alpha)\min\{V,Z\} - \alpha^2 V_s}{\beta V_s}\right)^{\frac{\beta}{1-\alpha}}$ and Z is defined in Equa-

tion (2.10).

2. *If* $V < V_s$*, then* $F_1(p) = \frac{1}{1-\alpha}[1 - \alpha\,(\frac{V}{p})^{\frac{1-\alpha}{\beta}}]$, $\bar{p}_1 = V$ *and* $\underline{p}_1 = \alpha^{\frac{\beta}{1-\alpha}} V$.

Total profits of each firm across the two periods are equal to $\underline{p}_1$.

Proof. For complete proof, see Appendix. □

The Lemma above states the equilibrium of the first-period pricing game. Similar to the second-period pricing game, the equilibrium prices and consumer strategy are different depending on the value of V. The distribution of mixed strategy is either bounded by consumers' incentive to search when V is large or by consumer valuation for the products when V is small. To understand the former case, when V is large, firms are not able to charge full consumer valuation due to competition, and so, V becomes irrelevant in determining the equilibrium prices. Consumers with search cost s are exactly indifferent between searching for the other price and not when they face the highest price of the distribution. At the top price $\overline{p}_1$, consumers with zero search cost will always buy from the other firm and demand only comes from consumers with search cost s. If consumers with search cost s strictly prefer to continue searching at this price, then the firm will also lose these consumers and get zero profit, which is clearly not optimal. On the other hand, if consumers with search cost s strictly prefer not to search at the top price, then if the firm raises price by a small amount, they will still not search. By raising price, the firm is able to secure the same demand but increase profit. So in the equilibrium, the price distribution must be bounded by the search incentive of consumers with search cost s.

In the latter case, when the equilibrium price distribution is bounded by consumer valuation for the products, consumers strictly prefer not to search for the other price for any price in the distribution, including the top price V. This is because when V is small, firms' ability to raise price is constrained by V rather than the pressure of competition. When price is equal to V, consumers with search cost s strictly prefer not to search for the other price and just buy the current product at price V. Although firms still have incentive to raise price to fully take advantage of consumers' search intention, the constraint of consumers valuation prevents them from doing so.

One way to interpret mixed strategy is to view it as promotions. While the exact price distributions in the first period are complex, one of the conceptual implications is that taking into account consumer loss aversion never completely eliminates the optimality of price promotions, only that their magnitude and distribution need to be adjusted to account for their negative impact on future profits. We summarize this conceptual implication in the following proposition.

Proposition 7. *Forward-looking behavior of firms in the presence of loss aversion never eliminates the optimality of promotions (even in the first period).*

Proof. Immediately follows from the finding in Lemma 3 that the equilibrium of the first-period pricing game is in non-degenerate mixed strategies. □

The intuition for the above proposition is that no matter how strong the future negative effect of current promotions may be, it is continuous in the depth of the promotions and decreases to zero as the price reduction is decreased to zero. On the other hand, if promotions were absent, the (short-term) benefit of a promotion coming from attracting the segment of consumers who observe both prices is positive and bounded away from zero no matter how small the promotion is. In what follows, we will see an even stronger result: in some cases, loss aversion may lead to optimality of stronger promotions in the first period. To see this, we need to not only consider how loss aversion affects the optimal promotion depth but also how it affects the upper bound on prices (i.e., the regular price) in the first period.

2.4.3 Benchmark: A Model without Consumer Loss Aversion

As a benchmark case, consider a two-period model in which consumers do not exhibit loss aversion in the second period, while everything else remains the same as in the main model. In this benchmark model, consumers make economically-justified search decisions for all prices and hence do not have asymmetry in their response to prices above and below their reference price. Since consumers have a linear utility function, this assumption essentially implies no effect of reference price on consumer purchase decision. Therefore, the model reduces to a twice repeated static game in which firms compete on price and consumers have heterogeneous search costs. Since there is no dynamics between periods, we only need to solve the single-period game with consumer search. Following the same logic as the derivations in the second-period pricing game, the unique equilibrium of this game can be derived to be in symmetric mixed strategy pricing following the same distribution $F_b(p)$ on $p \in [\underline{p}_b, \bar{p}_b]$ (subscript b stands for the benchmark case) in the two periods, where

$$F_b(p) = \frac{(1-\alpha)p - \alpha \min\{V, V_{\mathrm{s}}\}}{\beta p} \text{ with } \bar{p}_b = \min\{V, V_{\mathrm{s}}\} \text{ and } \underline{p}_b = \frac{\alpha \min\{V, V_{\mathrm{s}}\}}{1-\alpha}. \tag{2.12}$$

The equilibrium profits of each firm are $\pi_b = \alpha \min\{V, V_{\mathrm{s}}\}$ in each period.

Similar to the main model, in the benchmark model, the equilibrium price distribution is either bounded by consumers' incentive to search or by their valuation for the products. When $V > V_{\mathrm{s}}$, the equilibrium price distribution is determined by consumers' incentive to search for a better price and is not dependent on V. Consumers with search cost s are exactly indifferent between searching and not searching for the other price at the top of the distribution. When $V \leq V_{\mathrm{s}}$, the equilibrium price distribution is bounded by consumer valuation for the products V and consumers with search cost s strictly prefer not to search for the other price and will just buy from the firm they searched first. As we will see in the following section, the different dynamics underlying the two types of equilibrium strategies will lead to drastically different implications of consumer loss aversion.

2.4.4 Effect of Consumer Loss Aversion

Comparing the results in Lemma 2 and 3 to the results of the benchmark model, we can see the effect of consumers loss aversion on firm pricing.

Proposition 8. *With consumer loss aversion, average first-period equilibrium prices and profits of firms are higher than the ones in the model without consumer loss aversion. Average equilibrium prices and profits in the second period are always lower than the ones in the model without consumer loss aversion.*

Proof. Follows from comparison of the results in Lemma 2 and Lemma 3 with the solution of the benchmark model. □

The intuition of this result has been more or less discussed in the previous analysis. Although second-period profit is lowered due to the effect of consumer loss aversion, the forward-looking behavior of firms softens competition in the first period and results in higher prices. This effect has two components – a direct component and a strategic component. The direct component is that each firm has less incentive to cut its own price when consumers are loss averse. In the second period, lower reference price leads to lower profits. Foreseeing this, firms should optimally promote less in the first period. Note that this component has a positive effect on that firm's price but negative on its profit since it constraints prices to be higher than short-term optimal. However, this direct effect of loss aversion on one firm's price leads to higher profits of the other firm. Since firms in competition benefit from overall rise in prices, this effect equally applied to both firms benefits each of the firms. The strategic component is that each firm is allowed to price higher and engage less in price competition since its competitor is now less concerned about cutting prices. The two components together drive up the equilibrium prices and profits of firms in the first period when consumers are loss averse.

In the second period, since the equilibrium price distribution is always "cropped" by the reference price, the profits are capped by the profit in the benchmark model as well. So average prices and profits are always lower than the ones in the benchmark model. Whether overall profits are higher or lower is not immediately clear. Comparing the equilibrium profits of firms in the two models, we have the following proposition.

Proposition 9. *When $V \geq V_s$ such that the equilibrium price is bounded by search incentive in the model without consumer loss aversion, consumer loss aversion could lead to higher firm profits. On the other hand, when $V < V_s$ such that the equilibrium price distribution is bounded by consumer valuation in the model without consumer loss aversion, consumer loss aversion always leads to lower firm profits.*

Proof. Follows from comparison of Lemma 2 and Lemma 3 with the solution of the benchmark model. □

As has been mentioned in the previous section, the effect of consumer loss aversion differs for the two cases in the benchmark model. When V is small such that equilibrium price is constrained by consumer valuation for the products in the benchmark model, loss aversion always hurts firms. In this case, due to the concern for future profit, the first-period profit of firms is not everywhere equal in the equilibrium price distribution. However, the difference in expected second-period profit offsets this effect and overall, firms are indifferent between charging different prices in the distribution, including the top price. When the equilibrium price distribution is bounded by consumer valuation, this top price is the same in the model with and without consumer loss aversion. At this price, first-period profit remains the same as in the benchmark model yet second-period profit is strictly lower when consumers are loss averse. So overall, firms cannot possibly enjoy higher profit. When V is large such that equilibrium price is constrained by consumer search intention rather than consumer valuation, the upper bound of the equilibrium price distribution also shifts upward, which opens the possibility of higher firm profits. Therefore, only in this area we can (and do) observe higher firm profits when consumers are loss averse.

Insert Figure 2.1 here.

Figure 2.1 illustrates how the effect of loss aversion depends on the parameter values. Our model has three parameters: α, V and s. However, the results in Lemmas 2 and 3 and those of the benchmark model show that s could be normalized to 1, or equivalently, we only need to consider two parameters – α and V/s. Therefore we plot everything in terms of α and V/s. The top plot of Figure 2.1 demonstrates how the effect of loss aversion depends on value of V/s. The grey line in the figure shows the percentage change of profit in the main model relative to the benchmark model. If this percentage is positive, then it means loss aversion increases firm profit. If it is negative, then it reduces firm profit. When V/s is small enough, the equilibrium price distribution is bounded by consumer valuation, so firms could not benefit from consumer loss aversion. Only when V/s is large enough could consumer loss aversion lead to higher prices and profits in the first period. Also, when V/s is large enough, the equilibrium price distribution does not depend on consumer valuation, so the line is flat for large enough V/s.

The bottom plot of Figure 2.1 shows how the effect of consumer loss aversion depends on value of α, which is less straightforward than the previous case. Again, the grey line is the percentage change of profit in the main model relative to the benchmark model. On one hand, since consumers with zero search cost are always informed of the prices, their search behavior does not have much impact on the intensity of competition between firms. Firms' incentive to raise price and hence the additional profits in the first period mainly come from consumers with search cost s. So as the size of consumers with search cost s increases, consumer loss aversion is more likely to benefit firms. As an extreme case, when $\alpha = 0$, there is no consumer with search cost s and all consumers have zero search cost. In this case, both the models with and without consumer loss aversion will result in zero price and profit. On the other hand, however, higher value of α could also reduce the benefit of consumer loss aversion. This is because for a given V, a lower value of α could lead to a search-bounded equilibrium while a higher value of α could result in a valuation-bounded equilibrium in the model without consumer loss aversion. Since the profit-increasing effect of consumer loss aversion could only be observed for search-bounded equilibrium, it is possible that larger α could actually eliminate this effect. We summarize the above findings as the following proposition.

Proposition 10. *The profit-increasing effect of consumer loss aversion is monotonic in V, i.e., either the profit is always lower than in the case of no loss aversion, or it is higher when V is large enough. The profit-increasing effect of consumer loss aversion, however, is not monotonic in α.*

Proof. See Appendix. □

Insert Figure 2.2 here.

Figure 2.2 illustrates the effect of consumer loss aversion over the two-dimensional space of α and V/s. The grey area in the plot is where loss aversion leads to higher profit and the white (light) area is where loss aversion leads to lower profit. As discussed earlier, the profit-increasing effect of loss aversion is monotonic in V/s. This point is further demonstrated in Figure 2.3. For small values of α, loss aversion leads to universally lower profit for firms. For large values of α, loss aversion increases firm profit as long as V/s is also large enough. This cutoff point of α is approximately equal to 0.316. As long as α is larger than this cutoff value, we are able to observe firms benefiting from loss aversion for some values of V/s.

In the benchmark model without consumer loss aversion, it is, in equilibrium, optimal for firms to offer promotions to consumers. In the model with consumer loss aversion, even though firms have the tendency to raise price and engage less in price competition, they do not give up promotions completely. Hence it is interesting to investigate how consumer loss aversion affects firms' optimal promotion strategy and how this is related to the profit implications of loss aversion. In order to do this, we first define two terms qualitatively reflecting the firms' promotion strategies. We define absolute depth of promotion as the difference between the highest and lowest price of the equilibrium price distribution and relative depth of promotion as the the highest price divided by the lowest price. The following proposition summarizes our findings in this respect.

Proposition 11. *The relative depth of promotion in the first period is always lower when consumers are loss averse. The absolute depth of promotion in the first period is lower when loss aversion leads to higher profit, but may be higher when loss aversion leads to lower profit.*

Proof. From comparison of Lemma 3 to the solution of the benchmark model. □

Proposition 11 offers insights into how firms' promotional strategies are affected by consumer loss aversion. When loss aversion increases firm profitability, firms always promote less in the equilibrium compared with the benchmark model, no matter how promotion depth is measured. In the parameter range where loss aversion decreases firm profitability, however, in could be optimal for the firms to promote more if measured by the absolute depth of promotions offered in the equilibrium. The intuition for consumer loss aversion potentially resulting in deeper promotions is that the regular prices of the products are endogenously determined. The concern for the negative effect of price promotions on future profits discourages firms from cutting prices in the first period. So if the regular prices of the products were kept exogenously fixed, consumer loss aversion would definitely imply lower/less promotions in the equilibrium. However, since the regular prices of the products are also allowed to be adapted to optimally respond to competitor's pricing strategy, firms are able to charge higher regular prices because their competitor is now pricing less aggressively. So the overall effect could be that higher/more promotions are optimal.

2.5 Extensions, Robustness and Further Discussion

In order to better understand the practical implications of the model analyzed in the previous sections, it is important to understand the role of the main conceptual assumptions we made, and understand whether the results would remain the same or how they would change if we modify the way the conceptual assumptions are formulated in the model. In this section, we discuss the main assumptions of the model and examine a number of model extensions.

2.5.1 Discussion of the main assumptions

The model formulated in Section 2.3 is characterized by four notable features: (*i*) multiple periods (dynamic game with firms having to respond to past and future price setting behavior); (*ii*) competitive market; (*iii*) short-term optimality of promotions (i.e., use of promotions is optimal in a static model without consumer loss aversion), and (*iv*) at least some consumers have positive search cost. We will start with a discussion why the above features are important for our main results, i.e., why the results may not hold without any one of these assumptions, and argue that they represent realistic features of most markets, and then continue with consideration of how the implications could be affected if these assumptions are formulated differently.

In Section 2.3, we model the effect of loss aversion through a two-period dynamic model with forward-looking firms. A dynamic framework is needed because consumer loss aversion has both short-term and long-term effect on firm pricing and profitability. The short-term effect is that the prices and profits of firms will be lowered because they need to compensate for consumer loss. The long-term effect, as discussed in Section 2.4, is that prices will be higher. So the total effect of consumer loss aversion extends beyond a single purchase occasion that a static model can capture. If we were to model consumer loss aversion in a static model, then firms would always be hurt by the presence of consumer loss aversion. Also, the long-term and strategic effect of consumer loss aversion would be missing from the model. Therefore, we need a two-period model to fully examine the implications of consumer loss aversion.

We model loss aversion in a competitive setting for several reasons. First, it captures the reality in the marketplace. Competition is present in most product categories and consumers are essentially deciding between different products rather than between buying and not buying. Examples of such situations include groceries, electronic products, sporting goods, and etc. Second, a competitive model is more likely to allow us to observe interesting implications of consumer loss aversion. In a multi-period monopoly model, the monopoly firm will find it optimal to charge a single price in each period and under standard assumptions, the price is non-increasing over time. Therefore, as far as the reference price is determined by past prices, loss aversion will not affect the equilibrium of the game because consumers do not experience loss in the equilibrium.[15] Thus, competition is essential in our setting in order to possibly see a beneficial impact of loss aversion on profits.

The short-run optimality of price promotions (non-degenerate mixed-strategy price equilibrium in the model without consumer loss aversion) is also essential for our results. Without promotions, the presence of consumer loss aversion will only hurt firm profit. For example, consider a two-period competitive model with differentiated products in which consumers are distributed on a Hotelling line and have demand for one unit product in each period. In the second period, consumers are affected by a reference price, which is equal to the lower price in the first period. They suffer disutility if price is higher than the reference price and enjoy additional utility if price is

[15]If one assumes that consumers are forward-looking and form reference price based on expectation of future prices, loss aversion could hurt the firm's profits. This is because consumers expect lower prices in the future and their valuation for the product is lowered. Firms need to lower price to compensate for consumer loss and hence profit is reduced.

lower than the reference price. The equilibrium prices of the standard Hotelling model without loss aversion are in pure strategy and are equal to the transportation cost t for both firms. In other words, promotions are not optimal without consumer loss aversion. In the foregoing model with consumer loss aversion, the second-period equilibrium prices will always be lower than t no matter what the reference price is. If the reference price is low, firms have to lower price to compensate for consumer loss, while if the reference price is high, the additional utility intensifies price competition and drives down prices. In the first period, the equilibrium will be the same as in the static model. Firms do not deviate because raising price does not affect the equilibrium of the second period and cutting price leads to lower reference price and lower profit in the second period. Thus, if promotions are not optimal in the static game with no loss aversion, firms cannot be better off than the situation without loss aversion.

Prior researches have proposed various models to explain price dispersion in the market. Among them, heterogeneity in search costs among consumers is found to lead to mixed strategy in firm pricing. We follow this stream of literature and assume consumers need to search to observe prices in order to validate the optimal use of price promotions in the equilibrium, whose importance has been discussed above. Some of the consumer search models in the literature result in price distribution in the equilibrium that is bounded by consumer valuation for the products (e.g., Varian 1980; Narasimhan 1988), while others imply an equilibrium that is bounded by consumers' intention to search. As discussed in Section 2.4, consumer loss aversion could lead to higher profits only if the equilibrium price distribution is bounded by consumers' search intention. So consumer search plays an important role both for modeling purpose and for the main conceptual results of the paper.

2.5.2 Loss aversion affecting consumer valuation

In Section 2.3, we model loss aversion in terms of its effect on consumer search behavior. An alternative way to capture loss aversion is to assume that it affects consumer valuation for the products. As we will argue, the framework we adopted in Section 2.3 is closely related to this alternative framework. To see this, consider an alternative formulation of consumer loss aversion in which consumers' utility of buying from firm i is equal to

$$V - p_i - I\gamma(p_i - p^r), \tag{2.13}$$

where $\gamma > 0$ is a parameter reflecting the magnitude of the loss aversion impact on consumer utility, and I is the indicator of the domain of losses (i.e., $I = 1$ if $p_i \geq p^r$, and $I = 0$ otherwise). Such a formulation means consumers experience a disutility that is proportional to the difference in price if and only if the observed price is higher than the reference price (the domain of loss). In both frameworks, consumers respond more strongly to losses than to gains. The only difference is whether the effect is through consumer search behavior or their utility.

What will the equilibrium look like in the second period in the new framework? If the reference price is above the equilibrium price range of the benchmark model, then the equilibrium remains the same as in the benchmark model because consumers are not affected by gains. If the reference price overlaps with the price range in the benchmark model, then prices will be forced down, but will be higher than the equilibrium prices in the main model. In this case, higher observed price increases the pain experienced by consumers. Hence they have stronger incentive to search for a lower price compared with when they are not loss averse. The same conceptual force as in the main model intensifies competition between firms and lowers prices. This is why the current framework and the search framework are closely related to each other – in both frameworks, prices are driven down due to stronger consumer intention to search. However, since consumers' intention to search in the current framework is not as strong as in the main model, the equilibrium second-period prices turn out to be higher than those in the main model as well.

Furthermore, when γ tends to infinity, the second-period prices will converge to the equilibrium prices of the main model. The argument is as follows. Since the disutility from loss decreases as price is reduced towards the reference price, firms have incentive to minimize the loss experienced by consumers due to the pressure of competition. As the magnitude of the disutility (parameter γ) becomes larger, such incentive will become stronger and the upper bound of the equilibrium price distribution will become closer and closer to the reference price p^r. As an extreme case, if γ is infinitely large, consumers will not purchase as long as price is above the reference point and the price distribution will be bounded by the reference price in the equilibrium. This is equivalent to our main model in which consumers conduct exhaustive search if price is higher than the reference point. Similar argument then applies to the first period prices and profits. So as γ increases, the outcome of the utility framework model will converge to that of the main model. Therefore the main results of our main model hold in this alternative framework as long as γ is large enough. This finding is formally summarized in the following proposition.

Proposition 12. *The main model where loss aversion affects consumer search behavior is equivalent to the limiting case of the model in which loss aversion affects consumer utility of purchase.*

Firms' overall profit under the current framework can be either higher or lower than that in the effect-on-search framework. To see that it could be lower than the main model, note that when γ is set equal to zero, reference price effect disappears and firm profit will be equal to that in the benchmark model. Obviously it will be lower than the main model in those cases where loss aversion is found to increase firm profit under the search framework. In other situations, firm profit could also be higher under the current framework. As has been discussed previously, equilibrium price and profit in the second period will be higher under the current framework. When first-period equilibrium price is bounded by consumer valuation, firm profit in the first period is the same under the two frameworks. So overall profit is higher than the search framework if first-period equilibrium price is bounded by consumer valuation.

What will happen if consumers respond to both gains and losses under the current framework? In this case, when consumers search, if they find a lower price above the reference point, then they suffer less pain. If they find a lower price below the reference point, then there is even additional gain. So compared to the case where consumers are only sensitive to losses, their incentive to search for lower price is even stronger. So second-period prices and profits will be lower than the ones when consumers are only sensitive to losses, but closer to the ones in the main model. Yet the conceptual forces behind the intuition of the main mode results still remain and hence the main results should be robust to this change, too.

2.5.3 Competition over more than two periods

In Section 2.3, we model the effect of consumer loss aversion in a two-period game. Although we believe such a formulation is sufficient to reveal the dynamic effect of loss aversion, it may be interesting to discuss what additional insights could be obtained if we model the effect of loss aversion over more time periods. Consider a variation of the model in Section 2.3 with more than two time periods and in each period other than the first one, consumers are affected by a reference price equal to a weighted average of past prices. Other assumptions are the same as in the main model.

The last period of this game is the same as in the main model. So last-period profit is always lower than without consumer loss aversion. In each period in between, firms maximize their expected profit from that period on. In addition to the incentives in the static model, firms are also affected by two dynamic effects due to consumer loss aversion. On one hand, the reference price formed from past prices influences consumer search behavior in the current period and escalates competition between firms. This effect will drive down the average price in the market compared with the model without consumer loss aversion. On the other hand, firms are concerned about the impact of their current pricing decision on future profit. Since lowering price results in lower reference price and lower profit in the future, firms are less incentivized to engage in price competition. This effect will drive up the average price in the market. Whether the combined force of the two effects will result in higher or lower price in the market should depend on the value of the reference price. If the reference price is high, then the former effect should be very weak relative to the latter and we should expect the average price to be higher than that in the benchmark model. If the reference price is low, however, the first effect will be very strong and average price is likely to be lower than that in the benchmark model. In the first period, firms are only affected by the concern of the effect of current pricing on future profits. Price will be higher than the benchmark model for sure. Except the last period, profit in each period should be higher than the benchmark model with some probability. So in the multi-period model, firms could still be better off with consumer loss aversion if the potential increase in profit in earlier periods can compensate for the decrease in profit in the last period.

One of the implications of the main model is that prices will keep declining over time on the equilibrium path. In a multi-period model, we will still arrive at this result if it is assumed that the reference price is equal to the lower price charged by the two firms in the previous period. Although firms have the incentive to raise price due to the concern for future prices, equilibrium price will still be bounded by the reference price. Deviation by either firm to a higher price results in zero profit in the current period and same profit in the future. So neither firm will deviate. Since the reference price is determined by the lower price in the previous period, the observed prices will decline on the equilibrium path. Note that since the incentive to promote comes from capturing the segment of consumers with positive mass while the long-term effect is continuous in the price change, no matter how important the long-term effect is, the range of prices can dwindle to zero only if the highest price charged declines to zero. Therefore, over time, prices will approach marginal costs. In other words, this version of multi-period game results in similar implication as the durable goods monopoly game even though we have repeated demand.

One of the assumptions we have made for simplicity is that products are undifferentiated. Let us consider a slightly modified version of the model. In the modified model, we assume that there is a small fraction ϵ (where ϵ is arbitrarily small) of consumers loyal to each of the firms, or alternatively, a small fraction of consumers have high search costs (even after the loss aversion effect is taken into consideration). Among the rest consumers, $(1-\epsilon)\beta$ have zero search cost and $(1-\epsilon)(1-\beta) = (1-\epsilon)2\alpha$ have search cost s. The loyal consumers only buy from their preferred firm and are not affected by loss aversion. The rest of the model is the same as the foregoing model. In this model, while prices are at high enough levels, firms would optimally price as to not induce the larger segment of consumers with positive search costs to search. Since the size of the loyal consumers is very small by assumption, they do not affect the optimal pricing strategy of firms. So when prices are at reasonably high level, the equilibrium prices will be the same as the model without the loyal consumers. However, when prices decline to a sufficiently low level, it becomes optimal to "give up" on the consumers who exhibit loss aversion effect, and set prices high as to only sell to this small fraction of loyal consumers. At some point, both firms happen to set price at this high level and therefore the reference price resets to this high level.[16] After this, since the reference price is now at a high level, the optimal pricing will resume on the downward trend. This result is formally summarized in the following proposition.

Proposition 13. *In the multi-period model with an arbitrarily small segment of loyal consumers, equilibrium prices will exhibit a cyclical pattern, i.e., they repeatedly decline over time for some periods before being reset to a higher level.*

2.5.4 Alternative reference-price formation mechanisms

Several assumptions are made in Section 2.3 with respect to the formation and consumer effects of reference prices. We believe they capture the essence of loss aversion effect while being specified in a way to simplify analytical tractability. However, one may argue for alternative ways to model consumer response to reference prices. We now turn to the discussion of alternative specifications of the reference price effect on consumer choice made in the main model and how they will influence the main results of the paper.

One assumption we have made is that the reference price is determined by the lower price of the previous period. Several questions may be asked regarding this assumption. One is that all consumers may not have the same reference price. Consumers may not know both prices charged by firms in the previous period. Even if they know both of them, the product they actually purchased and the price paid then may be the more relevant determinant of the reference price. Another question is whether the lowest price in the market is the most important determinant of reference price. The empirical literature has identified several factors that consumers appear to be sensitive to in the formation of reference prices. These include price paid on last purchase

[16]This high level is not exactly V since for the possibility that both firms set the high price, it is optimal to price slightly below V and thus the price distribution at the top is also mixed.

occasion, brands' own past prices and current prices of competitors. In the context considered in this paper, these findings may translate to the following possibilities of reference price formation: 1) price paid by the consumer last period; 2) a weighted average of previous prices charged by the two firms; 3) a weighted sum of past prices and expectation of current prices.

The intuition behind the main results of the paper still holds under each of these alternative assumptions and hence the main results should not be affected. Two conceptual forces play a critical role. One is that second-period profit should be positively related to first-period prices so that firms are less incentivized to engage in price competition in the first period. The other is that firms need to be forward-looking so that the dynamics of the model can come into play. These two forces together compel firms to increase price in the first period. Obviously, the second force is not affected by how the reference price is formed. So we only need to consider how the alternative reference-price formation mechanisms affect the first force.

Under the first alternative assumption, consumers' reference prices are only determined by the product they purchased in the previous period. The reference prices are now heterogeneous among consumers. In the second period, some consumers will have a higher reference price than others. For ease of discussion, denote the higher reference price by p_h^r and the lower one by p_l^r. Similar to the main model, if both p_h^r and p_l^r are higher than the equilibrium price range of the benchmark model, then the equilibrium of the benchmark model should still constitute an equilibrium in the current setting. If either p_h^r or p_l^r is below the upper bound of the equilibrium price of the benchmark model, then price will be lowered because at least part of the consumers now will search and buy from the other firm when price is above their reference point. So price competition will be intensified between the two firms. This intuition is the same as the main model. In the main model, due to consumers' incentive to search, price in the second period is bounded by the reference price p^r. In the current case, conceptually the equilibrium price distribution could potentially be bounded by p_h^r or p_l^r. If the gap between p_h^r and p_l^r is small, then price should be bounded by p_l^r and the equilibrium is the same as the main model. If the gap between p_h^r and p_l^r is large, then firms will have incentive to abandon the customers will lower reference price by raising price up to p^r_h. On the equilibrium path, consumers with search cost s are expected to divide equally between the two firms. So in the second period, if p^r_h is less than $2p^r_l$, the equilibrium should remain the same as the main model and the solution to the whole model will also remain unchanged. In fact, this could happen in the equilibrium. For example, when $\alpha = 0.4$ and V/s is sufficiently large, the upper and lower bound of the equilibrium price distribution in the first period of the main model is equal to 6 and 4.36 respectively. This means $p^r_h < 2p^r_l$ will always be satisfied in the equilibrium. Also notice that under these parameters, firms will benefit from loss aversion in the main model and consequently, they will also benefit under this alternative assumption. So the main result of the paper is robust to this change.

We now turn to the discussion of the second alternative assumption. Under this assumption, the reference price in the second period depends on both prices in the previous period but is still homogeneous across consumers. The subgame equilibrium results derived for the second period of the main model still apply here because they only depend on the value of p^r, not on how it is formed. So in the second period, equilibrium price and profit still declines with the reference price p^r. Since p^r is equal to a weighted average of first period prices, it decreases if either one of the prices decreases. This means first-period price still positively correlates to second-period profit and this gives firms incentive to raise price in the first period. Since second-period reference price is not as low as in the main model, we expect the profit-lowering effect in the second period and the profit-enhancing effect in the first period to be weaker. However, qualitatively all effects remain the same as in the main model and the main results of the paper should be robust to this change. As an example, consider the special case where consumers' reference price in the second period is determined by the higher price in the first period. When price is bounded by consumers' incentive to search in the benchmark model, the upper bound of equilibrium price is equal to V_s. When loss aversion is present, this upper bound will be higher than V_s. When firms charge the upper bound price, the resulting reference price will always be larger than V_s, leading to the same equilibrium price and profit in the second period compared to the benchmark model. However, in the first period, expected profit will be higher than the benchmark model. So overall, firms will always benefit from loss aversion as long as price is bounded by consumer search in the benchmark model. Compared to the main model where the profit-increasing effect of loss aversion is only observed for some parameter values, this alternative assumption increases the area where it can happen. This shows the robustness of the results to this change of assumption.

The third assumption requires the reference price to be determined by both past prices and expected current prices. In the second period, the equilibrium price would be even lower than the main model. To see this, note that in the main model, second-period equilibrium price is bounded above by the reference price p^r. The expected price in the equilibrium is always lower than the reference price from previous period. So when past prices and current prices are combined to form the reference price, the resulting reference point is lower than the one determined by past prices alone. This means second-period profit will be driven even lower than in the main model, but firms have higher incentive to raise price in the first period. Overall, the intuition is quite similar to the main model and to the previous two cases as well.

2.6 Conclusion

One of the observed regularities of consumer behavior is that consumer preferences are reference-dependent. In particular, in a pricing context, consumers may be affected by reference prices in their purchase decisions, and may be more sensitive to losses than to gains. In this paper, we investigated the implications of this consumer loss aversion for the optimal pricing strategy and profitability of firms in a competitive environment.

While our model captures important aspects of the marketplace – competition, promotions, consumer lack of information and search costs, long- and short-term considerations of the firms and the tension between the immediate and future profits, - we abstracted away from non-pricing decisions. As expected, we found that second-period profit is reduced due to the effect of loss aversion, forwarding-looking behavior of firms leads to lower attractiveness of price promotions in the first period. Furthermore, less use of promotions by one firm allows the other firm to engage less in price competition and charge higher price. These two forces reinforce each other in driving up the equilibrium price in the first period and lead to the somewhat counter-intuitive result that overall profits may increase in the presence of consumer loss aversion. Furthermore, we identified conditions for this result: consumer valuation for the products being high enough relative to consumer search costs, so that the equilibrium price distribution is bounded by the consumer willingness to search as opposed to product valuation, and the fraction of consumers with higher search cost being in an intermediate range.

Although the presence of consumer loss aversion discourages firms from using price promotion in the first period, it never completely eliminates the optimality of promotions. Firms should still find it optimal to use price promotion as a competitive tool, but they may want to promote less. This finding underscores an important dilemma faced by competing firms – although price promotion leads to lower profit, they cannot and should not get away from it. The model also captures the notion that price promotions are "addictive" in the sense that once firms start to give discounts, they have to discount even more deeply in the next period to attract consumers. While the above results are not unexpected by practitioners, a somewhat more nuanced result is that when the regular price (i.e., the upper limit of the price distribution) is set as optimal given consumer behavior, the optimal maximal depth of price promotions (even in the first period) could be higher in the presence of consumer loss aversion.

We also extended the analysis of the model to considering other reasonable assumptions that may realistically capture the market characteristics and consumer behavior. We discussed the intuition and results of the model when loss aversion affects consumer valuation, with more time periods and under alternative formation processes of reference prices and showed that the main results of the paper are fairly robust to these extensions. One implication of the main model adapted to multiple periods is that prices will keep declining over time due to the effect of loss aversion. We show that if we slightly modify the model by adding an arbitrary small fraction of loyal consumers, then prices of the products will exhibit a cyclical pattern, which perhaps better corresponds to the observed regularities of the contemporary marketplace.

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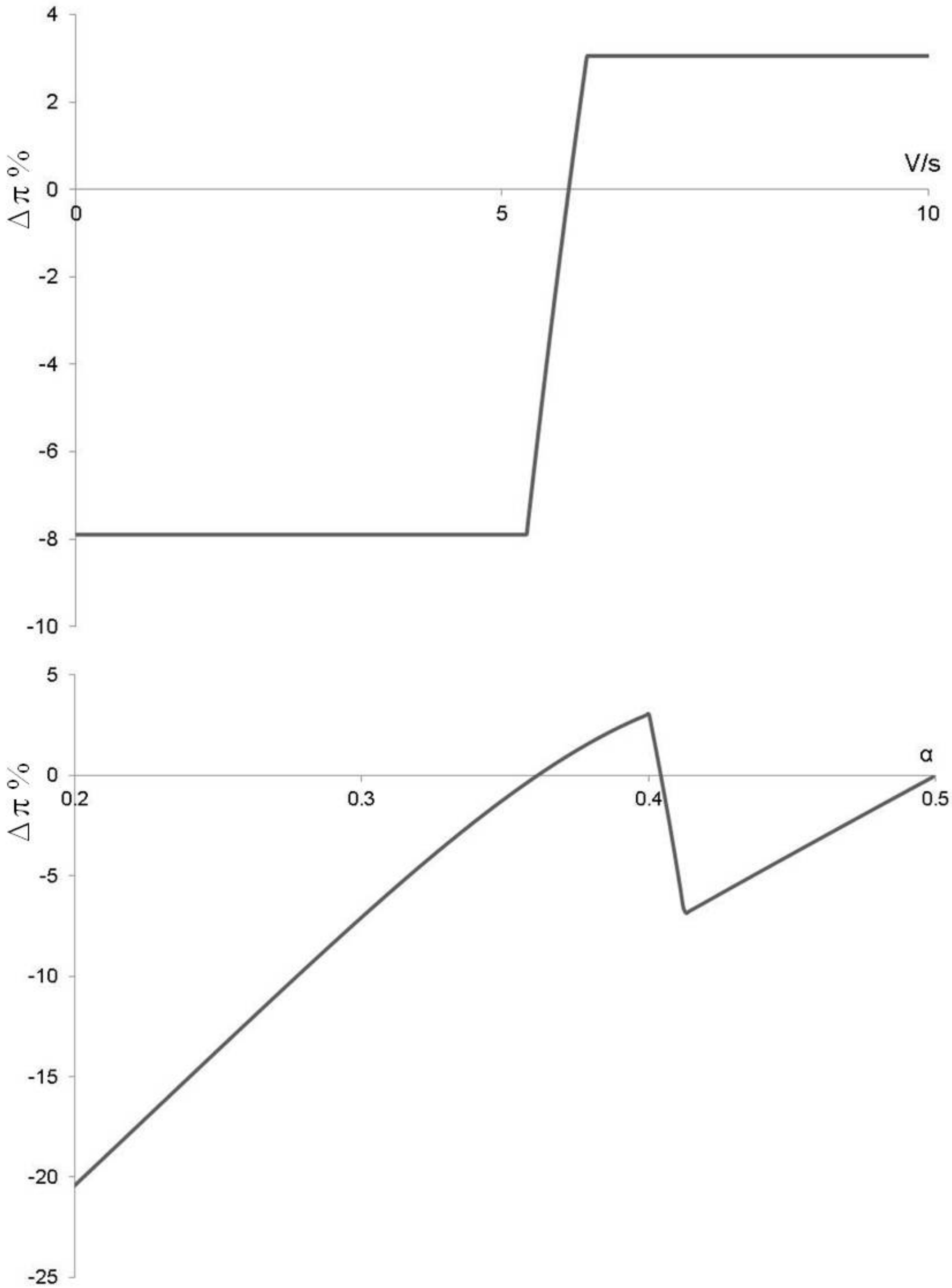

Figure 2.1: Percentage change in equilibrium profits due to consumer loss aversion as a function of α for $V/s = 6$ (top) and as a function of V/s for $\alpha = 0.4$ (bottom).

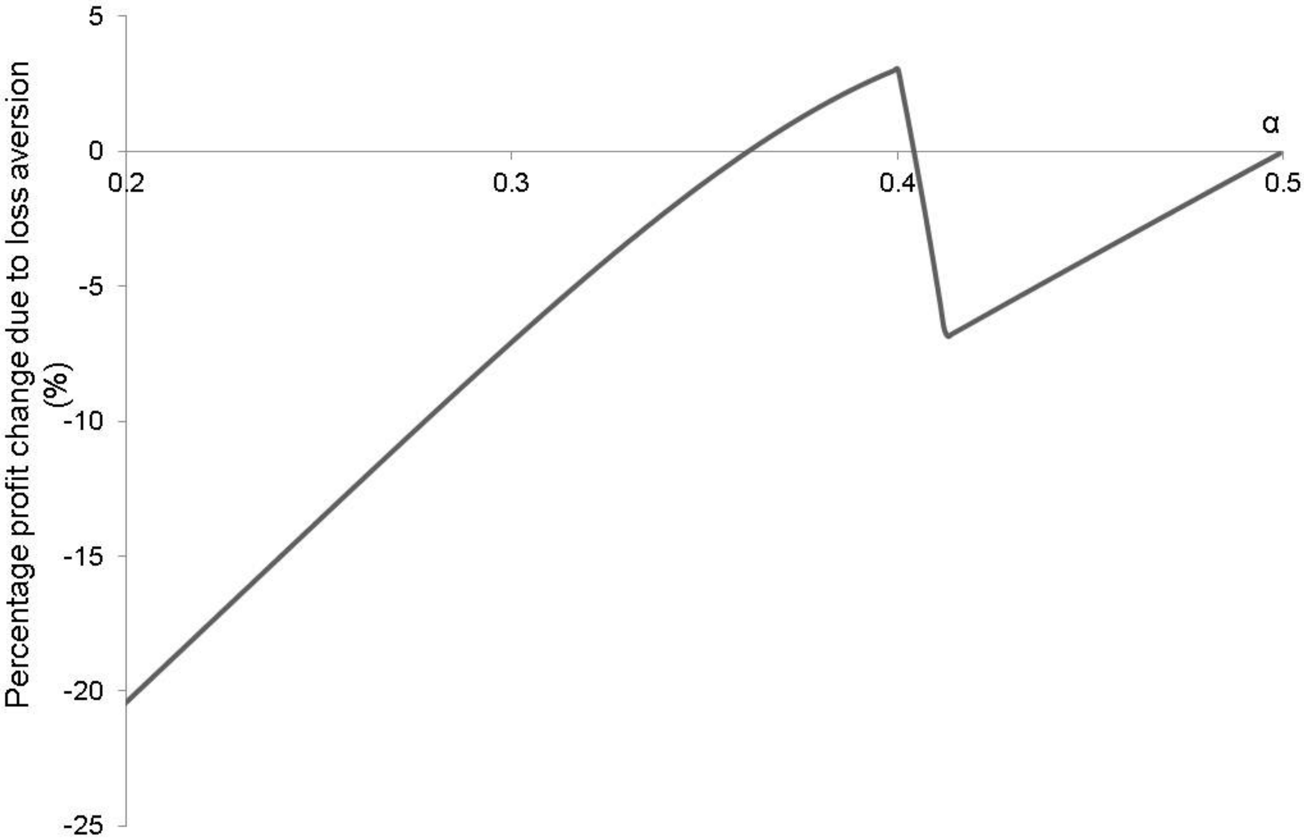

Figure 2.2: Region where loss aversion increases profits (axes: α and V/s)

Appendix

Proof of Lemma 2

First note that in any period, firms cannot charge more than V. Since p^r is drawn from the equilibrium price distribution of the first period, it must be true that $p^r \leq V$. First consider the case where $p^r \geq V_{\text{s}}$, which implies $V \geq p^r \geq V_{\text{s}}$. Need to show that $\bar{p_2} = \min\{p_r, V, V_{\text{s}}\} = V_{\text{s}}$. Under this condition, the equilibrium of the single-period model without consumer loss aversion is described by Equation (12). In this equilibrium, $\bar{p}_b = V_{\text{s}} < p^r$ and consumers are not affected by p^r. This means the equilibrium of the single-period model without consumer loss aversion still constitutes an equilibrium in the main model. In particular, $\bar{p_2} = V_{\text{s}}$ and $\pi_2 = \alpha V_{\text{s}}$.

Next consider the case where $p^r < V_{\text{s}}$. Need to show that $\bar{p_2} = \min\{p^r, V, V_{\text{s}}\} = p^r$. As discussed in the main text, it must be true that $\bar{p_2} \leq p^r$. We now show that $\bar{p_2} = p^r$. Suppose the contrary is true. Then the equilibrium constraints become the same as the ones used in solving the benchmark model. Solving these constraints will lead to the same equilibrium price distribution as the one described in Equation (12). In particular, we will get $\bar{p_2} = \min\{V, V_{\text{s}}\} > p^r$, which results in contradiction. This means in equilibrium, it must be true that $\bar{p_2} = p^r$.

In order to solve for $F_2(p)$, three conditions must be satisfied. 1) Expected profit must be the same for all prices charged with positive probability in the equilibrium, which leads to equation (2.14) below. 2) $\bar{p_2} = p^r$, which leads to equation (2.15) below. 3) Constraint on the lower bound $\underline{p}_2$ of the distribution, which leads to equation (2.16) below.

$$\alpha p + [1 - F_2(p)]\beta p = c_1 \tag{2.14}$$

$$F_2(\bar{p_2}) = 1 \tag{2.15}$$

$$F_2(\underline{p}_2) = 0 \tag{2.16}$$

Solving the three equations together leads to $F_2(p) = \frac{(1-\alpha)p - \alpha p^r}{\beta p}$, $\bar{p}_2 = p^r$ and $\underline{p}_2 = \frac{\alpha p^r}{1-\alpha}$. Both firms earn profit equal to $\alpha\bar{p}_2 = (1-\alpha)\underline{p}_2$. In order to verify that it is indeed an equilibrium, we need check 1)firms will not deviate to a lower price; 2)firms will not deviate to a higher price and 3)consumers with search cost s will not search in the equilibrium.

In order to check the first condition, notice that if a firm deviates to charging a price $p < \underline{p}_2$, it will get all consumers with search cost 0 and half of the consumers with search cost s with profit equal to $(1-\alpha)p$. Since $(1-\alpha)p < (1-\alpha)\underline{p}_2$, neither firm will deviate to charging a lower price. To check the second condition, if any firm deviates to a price $p > \bar{p}_2 = p^r$, consumers will search and will buy from the other firm for sure. So whoever deviates will get zero profit. This means condition (2) is satisfied. To check the third condition, we only need to check that consumers with search cost s do not search at the highest price of the distribution p^r. To do this, we compute the expected price implied by $F_2(p)$, $E(p) = \frac{\alpha p^r}{\beta} \ln \frac{1-\alpha}{\alpha}$. When consumers see the price p^r, their expected saving from searching for the other price is equal to $p^r - E(p) = p^r(1 - \frac{\alpha}{\beta} \ln \frac{1-\alpha}{\alpha}) = \frac{p^r}{V_{\mathrm{s}}} s$ and their cost of search is equal to s. When $p^r < V_{\mathrm{s}}$ (the case under consideration), $p^r - E(p) < s$ and hence consumers with search cost s will not search for the other price. This proves that the foregoing strategy profile constitutes an equilibrium of the game. So $\bar{p}_2 = p^r$ and $\pi_2 = \alpha p^r$. Thus we have considered all possible cases and concluded the proof.

Proof of Proposition 6

Need to show that the lower bound of the equilibrium price distribution of the second-period game is not higher than that of the benchmark model. From the proof of Lemma 2, if $p^r \geq V_{\mathrm{s}}$, then the equilibrium in the second period is the same as the benchmark model and $\underline{p}_2 = \underline{p}_b$. If $p^r < V_{\mathrm{s}}$, then in the equilibrium, $\underline{p}_2 = \frac{\alpha p^r}{1-\alpha}$. Since $\underline{p}_b = \frac{\alpha}{1-\alpha} \min\{V, V_{\mathrm{s}}\}$ and $p^r < \min\{V, V_{\mathrm{s}}\}$, $\underline{p}_2 < \underline{p}_b$. This concludes the proof.

Proof of Lemma 3

Proof of Item 1 As discussed in the main text, in the first period, prices have an upward pressure due to loss aversion. So the upper bound of the equilibrium price distribution will be higher than that of the benchmark model V_s. In the equilibrium, firms are indifferent between all prices charged with positive probability. When $p < V_s$, the expected profit of a firm charging price p is equal to $p + [1 - F_1(p)]\beta p + [1 - F_1(p)]\alpha p + \int_{\underline{p}_1}^{p} \alpha t \, dF_1(p)$. This translates into the following differential equation

$$p + [1 - F_1(p)](1 - \alpha)p + \int_{\underline{p}_1}^{p} \alpha t \, dF_1(p) = constant$$

Solving this differential equation leads to the following form of $F_1(p)$ for $p < V_s$

$$F_1^l(p) = \frac{1}{1-\alpha} + c_1 p^{-\frac{1-\alpha}{\beta}}, \ for\ p < V_s$$

When $p \geq V_s$, the expected profit of a firm charging price p is equal to $p + [1 - F_1(p)]\beta p + \int_{\underline{p}_1}^{V_s} \alpha t \, dF_1(p) + [1 - F_1(V_s)]\alpha V_s$, which leads to the following equation

$$p + [1 - F_1(p)]\beta p = c_2$$

Solving this equation, we get the following form of $F_1(p)$ for $p \geq V_s$.

$$F_1^h(p) = \frac{(1-\alpha)p - c_2}{\beta p}, \ for\ p \geq V_s$$

Similar to the benchmark model, when V is very large, the equilibrium price distribution is bounded by consumers' incentive to search. In this equilibrium, the following conditions need to be satisfied:

1. Condition on the lower bound of the distribution: $F_1^l(\underline{p}_1) = 0$.
2. Condition on the upper bound of the distribution: $F_1^h(\bar{p}_1) = 1$.
3. $F_1(p)$ must be continuous, which means $F_1^l(p)$ and $F_1^h(p)$ must equal each other at V_s: $F_1^l(V_s) = \delta$ and $F_1^h(V_s) = \delta$.
4. Consumers must be indifferent between search and not search at the top price $\bar{p}_1$: $\bar{p}_1 -$

$\int_{\underline{p}_1}^{\bar{p}_1} t\, dF_1(t) = s$

Solving these five equations together for $(c_1, c_2, \bar{p}_1, \underline{p}_1, \delta)$ leads to the result reported in the lemma with $\min\{V, Z\} = Z$. We checked that the constraints $\overline{p}_1 > V_{\mathrm{s}}$, $\underline{p}_1 < V_{\mathrm{s}}$ are indeed satisfied at this solution. Firms will not deviate to charging a lower price because doing so strictly decreases profit. They will not deviate to a higher price either, because in that case consumers will start searching and buy from the other firm. Whoever deviates to a higher price will get zero profit. So firms will not deviate. Also, in order for the equilibrium to hold, $\bar{p}_1$ cannot exceed consumer valuation for the product. So this is the equilibrium when $V \geq Z$.

When $V < Z$, the above equilibrium does not hold. Instead, the equilibrium price distribution will be bounded by V. The following conditions must be satisfied in the equilibrium.

1. Condition on the lower bound of the distribution: $F_1^l(\underline{p}_1) = 0$.
2. Condition on the upper bound of the distribution: $F_1^h(V) = 1$.
3. $F_1(p)$ must be continuous, which means $F_1^l(p)$ and $F_1^h(p)$ must equal each other at V_{s}: $F_1^l(V_{\mathrm{s}}) = \delta$ and $F_1^h(V_{\mathrm{s}}) = \delta$.

Solving these four equations for $(c_1, c_2, \underline{p}_1, \delta)$ leads to the result reported in the lemma. We checked that $\underline{p}_1 < V_{\mathrm{s}}$ is indeed satisfied at this solution. Firms will not deviate to other prices for the same reasons as in the previous case. Also, following similar steps as in the proof of Lemma 2, it could be verified that consumers with search cost s do not search for the other price in the equilibrium. This shows that the foregoing solution is indeed the equilibrium of the game and concludes the proof.

Proof of Item 2 When $V < V_{\mathrm{s}}$ is satisfied, the equilibrium price distribution in the benchmark model is bounded by V. Since firms have the tendency to raise price due to the effect of loss aversion, the equilibrium price will also bounded by V in the current model. The expected total profit of a firm charging price p is equal to $p + [1 - F_1(p)]\beta p + [1 - F_1(p)]\alpha p + \int_{\underline{p}_1}^{p} \alpha t\, dF_1(p)$. In the equilibrium, the expected profit must be the same for all prices charged with positive probability in the equilibrium, which leads to the following equation

$$p + [1 - F_1(p)](1 - \alpha)p + \int_{\underline{p}_1}^{p} \alpha t \, dF_1(p) = constant$$

Solving this differential equation results in the following form of $F_1(p)$

$$F_1(p) = \frac{1}{1-\alpha} + cp^{-\frac{1-\alpha}{\beta}}$$

Two other constraints must also be satisfied in the equilibrium: 1) $\bar{p}_1 = V$, which leads to the equation $F_1(V) = 1$; 3) constraint on the lower bound $\underline{p}_1$ of the distribution, which requires $F_1(\underline{p}_1) = 0$. Solving them together result in the equilibrium described in the lemma.

To verify that it is an equilibrium, we need to check that: 1)firms will not deviate to a lower price; 2)firms will not deviate to a higher price and 3)consumers with search cost s will not search in the equilibrium. If one firm deviates to a lower price $p < \underline{p}_1$, the expected profit is equal to p, which is smaller than the equilibrium profit, so they will not do so. If one firm deviates to a higher price, consumers will not buy since the price exceeds their valuation and the firm will get zero profit. So they will not do so either. To check that consumers with search cost s do not search in the equilibrium, we only need to show that they do not search at the top price V. We calculate the expected price in the equilibrium $E(p) = V^{\frac{\alpha}{1-\alpha}}(\alpha^{-\frac{\alpha}{1-\alpha}} - 1)$. The expected saving from searching for the other price is then equal to $V - E(p)$, which is smaller than s under the current condition. So consumers with search cost s will not search in the equilibrium. This concludes the proof.

Proof of Proposition 10

When $V < V_{\mathrm{s}}$, loss aversion never leads to higher profit according to Proposition 9. When $V_{\mathrm{s}} \leq V$, firms' equilibrium profit in the benchmark model is independent of V. The equilibrium profit in the main model reported in Lemma 3 increases in V until $V = Z$ and is independent of V when $V > Z$. So the difference between profit of the main model and that of the benchmark model is weakly increasing in V and hence the profit-increasing effect of loss aversion is monotonic in V. Figure 2.2 shows that this effect is not monotonic in α.