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Discounting of Hypothetical Monetary Outcomes that are Both Delayed and Probabilistic

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Abstract

The value of an outcome is affected both by the delay until its receipt (delay discounting) and by the likelihood of its receipt (probability discounting). Despite being well described by the same hyperboloid function, delay and probability discounting involve fundamentally different processes, as represented by the opposite effect amount has on the degree to which delayed and probabilistic rewards are discounted. Most of the previous research has studied the discounting of delayed and of probabilistic rewards separately, with little research examining more complex situations in which the rewards are both delayed and probabilistic. In the present experiment, participants made choices between a smaller reward that was both immediate and certain and a larger reward that was both delayed, and probabilistic. To examine the effect of amount, two larger, delayed and probabilistic rewards were used. A hyperboloid function provided excellent fits of probability discounting data at each delay and comparatively poorer fits for delay discounting data at each probability. In addition, an effect of amount on degree of discounting was consistently observed for probability discounting at each delay, but not for delay discounting at each probability. A hyperboloid model in which delay and probability were combined multiplicatively provided a good fit for the combined data. The results suggest that the hyperboloid is a good descriptor of more complicated decision making and that probability may be more heavily weighted than delay in determining people's choices.

Keywords: probability, delay, discounting, choice, hyperboloid, humans

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Discounting of Hypothetical Monetary Outcomes that are Both Delayed and Probabilistic

When alternatives differ on only one dimension, choice is relatively predictable: people prefer a reward that is larger rather than smaller, immediate rather than delayed, and certain rather than probabilistic. Choice becomes substantially more difficult when the alternatives differ on two or more dimensions and each alternative contains a preferred dimension. For example, consider the choice between $100 now and $200 in 3 years. Although a larger amount is preferred, so, too, a sooner reward is preferred. Many people might choose the immediate $100 despite the fact that $200 is objectively more. The delayed $200 may have less subjective value than the immediate $100. The devaluing of a reward because it is delayed in time is called delay discounting. Past research has consistently found that delay discounting is well described by a hyperbola function (e.g., Mazur, 1987):

$$V_d = A/(1 + kD), \tag{1}$$

where *V* is the subjective value of the delayed reward, *A* is its objective amount*, D* is the time until receipt of the reward, and *k* is a parameter reflecting the discounting rate. A two-parameter hyperboloid, in which the entire denominator is exponentiated by a scaling parameter, s_d, has been shown to provide superior fits than the simple hyperbola (Myerson & Green, 1995):

$$V_d = A/(1 + kD)^{s_d}. \tag{2}$$

Just as delay discounting involves the devaluing of a reward as a function of time to its receipt, probability discounting correspondingly involves the devaluing of a reward as a function of the likelihood, or odds-against, receiving it. For example, a 50% chance of winning $200 may be worth subjectively less than a guaranteed $100. As with delay discounting, the hyperboloid function provides an excellent description of probability discounting, but with *k* replaced with *h* and *D* replaced with *θ*, the odds-against receipt:

$$V_p = A/(1 + h\theta)^{s_p}. \tag{3}$$

Although the discounting of delayed and probabilistic rewards are well described by the same hyperboloid function, there are significant differences between delay and probability discounting. The most well-studied of these differences is the opposite effect that amount of reward has on the degree of discounting. In delay discounting, smaller rewards are discounted more steeply (i.e., the reward loses value more rapidly) than larger ones (termed the *magnitude effect),* whereas with probability, larger rewards are discounted more steeply (termed the *reverse magnitude effect*; (Estle, Green, Myerson, & Holt, 2006). Moreover, the parameters of Equations 2 and 3 are affected differently as amount of reward is varied (Estle et al., 2006; Myerson, Green, & Morris, 2011). In the case of delay discounting, the rate parameter, *k*, decreased as reward amount increased, whereas the exponent, *s*, of the hyperboloid discounting function showed no systematic change with increases in the amount of the delayed reward. In the case of probability discounting, however, the rate parameter, *h*, showed no systematic change whereas the exponent of the discounting function increased as reward amount increased.

Although much is known about how delay and probability affect decision making in isolation, little research has focused on choice behavior when the rewards are both delayed and probabilistic. For example, it is not known which, if any, effect amount has on discounting when a reward is both delayed and probabilistic. Nor is it known whether the hyperboloid discounting function would describe such choices and how the parameters of the function change as reward amount is varied. Given that amount has opposite effects on delay and probability, it may be that the effects of each cancel out. Alternatively, the effects of delay and probability on decision making may be additive. More generally, many everyday decisions involve outcomes that are both uncertain and delayed in time. If one chooses to smoke now, there is a possibility of getting cancer later on. In making an investment, there is the chance of a future payoff. The combined effects of delay and probability may reveal a pattern of behavior that is not predictable from their separate effects. It is therefore essential that we study outcomes that are both delayed and probabilistic to gain a more accurate understanding of everyday decision making.

Outside of discounting, a handful of studies have examined decisions involving both a delay and probability component. These experiments typically involved either adding a common delay to certain and probabilistic choices (Abdellaoui, Diecidue, & Onculer, 2011; Baucells & Heukamp, 2010; Keren & Roelofsma, 1995; Noussair & Wu, 2006; Sagristano, Trope, & Liberman, 2002; Weber & Chapman, 2005) or adding a common probability to immediate and delayed choices (Keren & Roelofsma, 1995; Weber & Chapman, 2005). The general consensus from these experiments is that people become more risk tolerant when rewards are further in the future. That is, probability matters more in the present than in the future. Although this is an intriguing finding, the choices in these experiments only varied along one dimension (delay or probability), in addition to differing in amount. It is difficult to understand the relative influence of delay and probability when the same delay or probability is added to both alternatives. When a delay is added to both a certain and a probabilistic alternative, for example, probability may carry more weight in influencing the decision because both rewards have the same delay (Prelec & Loewenstein, 1991). In contrast, if a probability is added to both an immediate and delayed alternative, delay may carry more weight. Examining alternatives that differ on both delay and probability may allow us to understand the relative influence of each in situations in which neither delay nor probability is made more salient.

Additionally, most of the previous experiments involved only one or a few choice trials. For example, Keren and Roelofsma (1995) and Weber and Chapman (2005) examined the effect of adding a common delay to choices in the Allais Paradox. Additionally, most prior research did not find the point of indifference between alternatives in a choice, and instead examined the percentage of people who selected each alternative. Although a few studies did obtain indifference points (Ahlbrecht & Weber, 1997; Weber & Chapman, 2005), as is done in discounting experiments, none of these mapped out a discounting function.

To date, there has been only one experiment in discounting that explicitly studied outcomes that were both delayed and probabilistic (Yi, de la Piedad, & Bickel, 2006). In this experiment, participants made choices between a delayed lottery with a known probability of winning and a certain, smaller amount of money to be paid immediately. To analyze their results, Yi et al. converting probability to delay using Rachlin, Raineri, and Cross's (1991) constant of proportionality. By systematically comparing delayed and probabilistic rewards, Rachlin et al. determined that a given odds-against multiplied by 35.5 produced a subjectively equivalent delay. For example, a probability of 40% (an odds-against of 1.5) is equivalent to 53.25 days. Yi et al. added the equivalent and explicit delays to create what they termed a composite delay. A 40% chance of receiving a reward in 30 days, for example, is subjectively equivalent to receiving a reward in

83.25 days. Although they found that the hyperbolic function provided a good description of the data, this analysis nonetheless focused on delay. Because they converted probability to delay, it may not be surprising that they found a magnitude effect consistent with delay discounting in that participants showed lower rates of discounting with larger lotteries. Although Yi et al. mention that they also converted delay to probability, they did not elaborate on their findings and so it is unknown whether a reverse magnitude effect was observed. Moreover, it is becoming increasingly clear that delay and probability involve fundamentally different processes (Green & Myerson, 2004; Myerson et al., 2011), and that one process cannot be reduced to the other. Consequently, it is essential to examine combined delay and probability discounting from the perspective that they are distinct processes rather than to assume one can be converted to the other.

The present experiment examined, within the discounting framework, the effects on decision making when choices are between an immediate, certain reward and a larger reward that is both delayed and probabilistic. Unlike previous work, no assumptions were made about the dominate process, and choices varied on three dimensions (amount, delay, and probability of receipt). We were interested in whether the hyperboloid function, which has been shown to provide a good fit for separate delay and probability discounting data, also would provide a good description of choices involving both types of outcomes. Given that amount has opposite effects on delay and probability discounting, we also examined whether amount would have an effect when delay and probability were combined, and importantly, what that effect might look like.

Method

Fifty participants (31 females, 19 males; mean age = 21.27) were recruited from the Washington University Department of Psychology Human Subjects Pool. They received either partial course credit or payment for their participation. Participants were tested individually in a small room with a computer.

Participants were instructed that they would be asked to make a series of choices between two hypothetical amounts of money. One amount could be received immediately and was guaranteed whereas the other amount was larger, but was both delayed and probabilistic. For example, a participant might be asked to choose between $300 right now for certain, or an 80% chance of $800 in 6 months. Five delays (0 [immediate], 1 month, 6 months, 2 years, and 5 years) were crossed with five probabilities (10%, 25%, 40%, 80%, and 100%), at each of two amounts ($800 and $40,000). Each condition involved a combination of a single delay and single probability. When the delay was 0, the procedure reduced to a standard probability discounting task; when the probability was 100%, the procedure reduced to a standard delay discounting task. The combination of a 0 delay and 100% probability was not used.

Half of the participants experienced all of the conditions within the $800 amount followed by all of the conditions within the $40,000 amount. The order was reversed for the other participants. Within a single amount, the order of each delay-probability condition was randomized. This 5 (delay) x 5 (probability) x 2 (amount) design consisted of a total of 48 conditions (excluding the 0 delay and 100% probability condition from both amounts).

To determine the indifference points (i.e., the amount of immediate-certain reward that is approximately equivalent in value to the delayed-probabilistic amount), participants made five choices for each delay-probability-amount condition. The amount of the larger reward was held constant at either $800 or $40,000, depending on the amount condition. On the first choice of each condition, the smaller amount was half of the larger, delayed and probabilistic amount (i.e., $400 in the $800 conditions, and $20,000 in the $40,000 conditions). For each subsequent choice in a condition, the amount of the smaller reward was adjusted based on the participant's previous choice (Du, Green, & Myerson, 2002). The size of each subsequent adjustment was half that of the preceding adjustment. For example, when given the choice between $400 right now for sure and an 80% chance of $800 in 6 months, if the participant chose the $400, the amount of the smaller reward would be decreased to $200 on the next trial. If the participant instead had chosen the larger amount, the smaller reward would be increased to $600 on the next trial. The adjusting procedure continued for the four remaining trials within a condition. If the choice on the second trial was between $600 right now for certain and an 80% chance of $800 in 6 months, and the participant chose the $600, the smaller amount on the third trial would be reduced to $500 (half the previous adjustment). After the fifth choice, the indifference point was determined to be the amount of smaller reward that would have been presented had there been an additional trial. This titrating procedure converged on an immediate, certain amount that was approximately subjectively equivalent to the larger, delayed and probabilistic reward. Before the experimental session began, participants were given four practice trials involving one amount and two delays crossed with two probabilities. The values used were similar, but not identical to those used in the experimental trials.

Results

Two ANOVAs comparing delay and probability discount rates using area under the curve (to be described later) found no effect of order of presentation of the $800 and $40,000 amount conditions, both *F*s < .063 (both *p*s > .803). The data, therefore, were collapsed for the remaining analyses.

The hyperboloid function (Equations 2 and 3) was fit both to delay discounting data at each probability individually, and to probability discounting data at each delay individually for a total of 10 discounting curves at each of the two larger, delayed and probabilistic amounts. For delay discounting at each probability, the data were separated into five groups according to probability, each of which consisted of five indifference points. Discounting as a function of delay was observed for each probability group individually. A similar procedure was used to assess probability discounting at each delay. In order to fit the hyperboloid function to data involving both delayed and probabilistic rewards, the raw values need to be transformed. The hyperboloid predicts that at a 100% probability, the subjective value, v_p, will be equal to the objective amount, A. However, when the outcome is certain, but also delayed, v_p should not be equal to A. The same logic applies when there is no delay to receiving a reward, but it is probabilistic. Amount, A, must be transformed in order to incorporate both delay and probability into the hyperboloid function.

To fit the hyperboloid to probability discounting data at each delay, each indifference point was divided by the indifference point when the outcome had a 100% probability but was delayed. For example, the indifference point for an outcome that was delayed by 6 months and had a probability of 40% was divided by the indifference point at a 6 month delay and 100% probability. With this approach, indifference points were normalized to their certain equivalent at the same delay. The same procedure was used to fit delay discounting data at each probability. For example, the indifference point at a 6 month delay and 40% probability would be divided by the indifference point when the delay was 0 and the probability was 40%.

Figure 1 shows mean indifference points and best-fitting curves for the $800 (top panels) and $40,000 (bottom panels) conditions. The left panels show probability discounting at each delay, and the right panels show delay discounting at each probability. As can be seen in the two left panels, probability discounting curves at each delay significantly overlap. This suggests that although people discount according to the likelihood of the outcome, they do not distinguish among the different delays. In contrast, the delay discounting curves (right panels) are less steep and much flatter. This further supports the claim that people do not discount much by delay when making choices involving both delay and probability.

The hyperboloid provided excellent fits to the mean probability discounting data at each delay (all R^2s > .99; see Table 1). In addition, *s* was less than 1.0 in all 10 conditions, and significantly so in nine of those conditions (the exception being the $40,000-2 year condition, t (48) = 2.2823, p = .107), all other nine *t*s > 3.76 (all *p*s < .032). Although the hyperboloid fit the mean delay discounting data well at many probabilities, overall it provided poorer fits than the probability discounting data, and several R^2s were extremely low.

The area under the curve (AuC) was computed for each condition to provide a supplementary, atheoretical measure of the degree of discounting (Myerson, Green, & Warusawitharana, 2001). With a simple hyperbola, the discounting rate parameters, *k* and *h*, can be compared across conditions. However, because the hyperboloid also contains a non-independent parameter, *s*, comparison of *k* and *h* across conditions is difficult. Area under the curve provides an alternative way to compare discounting by summing the area under connecting indifference points. The AuCs are normalized so as to range from 1.0 (indicating no discounting) to 0.0 (indicating complete discounting). To examine the effect of amount, indifference points were converted into proportions of the objective, larger amounts ($800 or $40,000). Unlike fitting the hyperboloid function to the data, AuC is theoretically neutral and thus, no assumptions are made that people discount in a particular way. Indeed, indifference points at times did not decline and occasionally even showed an increase when examining delay discounting (see Figure 1). Taking the proportion of the immediate (certain) equivalent at each probability (delay), as was done with the analyses described earlier, occasionally yielded indifference points greater than 1.0, which artificially inflates AuC.

As can be seen in the left panel of Figure 2, a magnitude effect for delay discounting was observed at the 100% probability condition (greater discounting for smaller, delayed rewards), as has been reliably reported in the discounting literature. However, the magnitude effect was not consistently found at the other probabilities. An ANOVA revealed overall no significant effect of amount, $F(1, 49) = 1.863$, $p = .179$. In contrast, a marginally significant effect of amount, $F(1, 49) = 3.790$, $p = .057$ was obtained for the probability discounting data, with a reverse magnitude effect (greater discounting for larger, probabilistic rewards) observed for all but the longest delay (see the right panel of Figure 2). The AuC analysis also revealed the differences in the extent to which people discount by delay and probability. AuC values were very low for probability discounting across each delay, implying the value of the outcome was discounted by probability to a great extent regardless of the delay. In contrast, AuC for delay discounting showed a much larger range of values across the five probabilities.

Finally, the data were fitted to a hyperboloid function that combines delay and probability multiplicatively:

$$V = A/[(1 + kD)^{s_d} * (1 + h\theta)^{s_p}], \quad (4)$$

where, as in the previous equations, V is the subjective value, A is the objective (non-discounted) value, k is the discounting rate parameter for delay, h is the discounting rate parameter for probability, D is the delay, θ is the odds-against, and the exponents, s_d and s_p, are the scaling/weighting parameters for delay and probability, respectively.

The combined hyperboloid was derived from the individual hyperboloid functions. Recall Equation 2, which describes the subjective value of a delayed outcome: $V_d = A/(1 + kD)^{s_d}$. To normalize Equation 2, V_d becomes a proportion of A: $V_d{}' = V_d/A = 1/(1 + kD)^{s_d}$. To incorporate probability into delay discounting, the subjective value of the delayed outcome, V_d', is transformed into a proportion of the subjective value of the probabilistic outcome, V_p: $V = V_d{}'/V_p = 1/(1 + kD)^{s_d}$. This same transformation was done in the earlier analyses when examining delay discounting at each probability. Because $A/(1 + h\theta)^{s_p}$ is equal to V_p, it can replace V_p in the equation: $V_d{}'/[A/(1 + h\theta)^{s_p}] = 1/(1 + kD)^{s_d}$. When rearranged by multiplying the left side by A and dividing by $(1 + h\theta)^{s_p}$, delay and probability combine multiplicatively to form Equation 4.

When fit to the raw data, the combined hyperboloid provided excellent fits at both amounts (R^2s > .99). Table 2 shows the parameter values and fits for both amounts. Consistent with previous discounting findings, both s_d and s_p were significantly less than 1.0 at each amount, indicating that the hyperboloid provided a better fit than a simple hyperbola (Myerson & Green, 1995). When comparing the estimates of all four of the parameters for the two amounts simultaneously, none of the parameter changes from the $800 to $40,000 condition was significantly different from 0, indicating no difference between the two amounts (all *t*s < 1.64, all *p*s > .11). The lack of a significant change may be due to low power from testing too many parameters simultaneously. This is especially possible given that the effect of amount on discounting is opposite in delay and probability discounting (Green, Myerson, & Ostaszewski, 1999).

Figure 3 shows two three-dimensional graphs of the discounting curves at each delay and probability as predicted by the combined hyperboloid (Eq. 4). To assess if a simpler model would provide an equally good fit, the data also were fit to a two-parameter hyperbola with no *s*:

$$V = A/[(1 + kd) * (1 + h\theta)], \tag{5}$$

and a three-parameter hyperboloid in which the entire denominator is exponentiated to a single *s*:

$$V = A/[(1 + kd) * (1 + h\theta)]^s. \tag{6}$$

The four-parameter combined hyperboloid (Eq. 4) provided a significantly better fit than Equation 5 for both the $800 ($F(2, 50) = 95.29$, $p < .01$) and $40,000 ($F(2, 50) = 65.89$, $p < .01$) amount conditions, as well as when the function was fit to the amounts simultaneously ($F(2, 50) = 30$, $p < .01$). Similarly, Equation 4 provided a significantly better fit than Equation 6 at both the \$800 ($F(1, 50) = 39.71$, $p < .01$) and \$40,000 ($F(1, 50) = 9.64$, $p < .01$) conditions, and when the function was fit to the amounts simultaneously ($F(1, 50) = 9.64$, $p < .01$).

Discussion

The current experiment systematically varied both delay and probability in a combined discounting procedure. This is the first study to examine both the effect of probability on delay discounting and the effect of delay on probability discounting within the discounting framework. The results clearly demonstrated that the hyperboloid function provided an excellent description of more complicated decision-making situations in which the outcomes are both delayed and probabilistic. Equation 2 provided excellent fits of the probability discounting data at each delay. The *s* parameter was significantly less than 1.0 in all cases, consistent with previous findings showing that the hyperboloid provides a better description of delay and probability discounting than does a simple hyperbola (McKerchar et al., 2009; Ostaszewski, Green, & Myerson, 1998).

In addition, a novel model was proposed that describes discounting of rewards that are both delayed and probabilistic. A hyperboloid model in which delay and probability are combined multiplicatively (Eq. 4) provided an excellent fit to the combined data. Its statistically superior fit over a hyperbola that combined delay and probability (Eq. 5) is consistent with findings that a hyperboloid provides a better fit for simple delay and probability discounting (Green & Myerson, 2004). In addition, Equation 4 provided statistically better fits than another hyperboloid in which the entire denominator was exponentiated to a single, overall *s* (Eq. 6). This, too, is consistent with findings from individual delay and probability discounting experiments that show that the *s* parameter behaves differently for delay and probability discounting across different amounts (Estle et al., 2006; Myerson et al., 2011); thus, a separate *s* exponent would be needed for delay and for probability to accurately describe discounting involving both types of outcomes.

The finding that delay and probability combine multiplicatively stands in contrast to Killeen's (2009) proposal of an additive model of discounting. Killen argued that the subjective value of a good is derived by adding the utility of the good to the disutility of the delay. It should be noted that Killen did not specifically address discounting involving both delayed and probabilistic rewards, but his argument would imply that the disutility of both the delay and the odds-against receipt are to be subtracted from the utility of the good. Not only did a multiplicative model provide good fits of discounting of both delayed and probabilistic outcomes in this experiment, but also its derivation is mathematically tractable from the hyperboloid used to describe simpler discounting.

A second finding was that the fit of the hyperboloid function (Eq. 2) to the probability discounting data provided a superior fit than when fit to the delay discounting data. The hyperboloid provided R^2s of .99 for probability discounting data at each delay and at each amount. Although the hyperboloid fit the delay discounting data well at many of the probabilities, there was a wide variation in fits, and overall they were much poorer. In addition, Figure 1 shows that people discounted by probability in a manner typically observed in other discounting experiments. That is, they discounted to a greater extent at lower odds-against, and to an increasingly lesser extent at larger odds-against (Green & Myerson, 2004; Myerson, Green, Hanson, Holt, & Estle, 2003; Rachlin et al., 1991). In contrast, the delay discounting curves were flatter, with little discounting observed. Interestingly, Figure 1 also shows that the probability discounting curves at each delay overlapped almost completely. This finding suggests that although people were discounting by probability in the manner typically observed, they were not differentiating among the delays.

The AuC analyses supported these findings. The degree of discounting for probability discounting did not differ systematically or by much across the different delays, whereas the degree of discounting for delay discounting varied widely depending on the probability (see Fig. 2). AuC also revealed a consistent reverse magnitude effect for probability discounting, and the absence of a magnitude effect for delay discounting. Although a reverse magnitude effect was observed in which the larger, probabilistic amount was discounted more than the smaller, certain amount, the difference between the two amount conditions was extremely small. Rather than a lack of distinguishing between the two amounts, this finding is likely due to a floor effect. At the $800 amount, participants already were discounting the larger amount to such an extent that there was very little room to discount much further. The important findings are that the reverse magnitude effect was present at all but the longest delay and that there was a lack of a consistent magnitude effect for delay discounting. In sum, the present results suggest that probability is more heavily weighted than delay in determining people's choices when the outcomes are both probabilistic and delayed.

The finding that probability was such a dominating process is surprising given previous research involving both delay and probability. Other researchers have suggested that people become more risk tolerant with increased delay (e.g., Baucells & Heukamp, 2010; Sagristano et al., 2002), implying that the certain equivalent of a probabilistic outcome changes depending on the delay. It then would be expected that the probability discounting rate would change across the different delay conditions. In the current experiment, however, there was no difference among the delays, suggesting that delay had little, if any, impact on the degree of probability discounting.

There are several critical procedural differences between experiments that may account for this discrepancy. Previous experiments have varied only one dimension (delay or probability), in addition to amount. In contrast, the current experiment employed a procedure in which the larger reward was both delayed and probabilistic, while the smaller reward was both immediate and certain. It may be that when a delay is added to both the certain and the probabilistic rewards, as was the case in previous experiments (e.g., Weber & Chapman, 2005), preference switches from the certain to the probabilistic reward, whereas when a delay is added only to the probabilistic reward, as in the current study, preference does not change.

Previous experiments also used fewer choice trials and examined the percentage of people who chose each alternative, rather than obtaining indifference points. The current experiment not only used a larger range of probabilities and delays, but obtained certain-immediate equivalents to the delayed and probabilistic reward for each participant at each delay-probability-amount combination. Furthermore, Weber and Chapman (2005), who also examined delayed and probabilistic choice, found discrepancies between single and joint evaluations of choice alternatives as well as differences between the Allais Paradox and a common ratio procedure, despite the similarities between the two procedures. They found no effect on the Allais Paradox when a delay was added, but did find an effect of delay using a common ratio procedure.

A recent experiment by Abdellaoui et al. (2011) also calculated the number of participants who were risk-seeking and risk-averse at each delay condition, and reported that the number of risk-averse individuals decreased with delay. The absolute decrease in percentage of people who were risk-averse, however, was quite low. When a probabilistic reward could be received immediately, the percentage of risk-averse participants was 77, and decreased to only 75% and 67% with delays of 6 and 12 months, respectively. Although statistically significant, most participants' preferences remained unchanged throughout all time delays. Similarly, Noussair and Wu (2006) argued that a large percentage of individuals are more risk tolerant in the future, but out of 103 cases, only 31 showed greater risk tolerance for a lottery in the future than for a lottery in the present. In contrast, in 61 of the cases, there was no difference. Although there are certainly individual differences, this finding suggests risk preferences remain rather consistent across time, a result that is not dissimilar from the present findings.

In addition to examining the percentages of risk-averse individuals, Abdellaoui et al. (2011) analyzed choices using a canonical model and found that risk preference, but not the utility of the outcome, is affected by time. Their canonical model is similar to prospect theory (Kahneman & Tversky, 1979) in that it includes both a probability weighting function and a utility function, but is adjusted to incorporate delay. Using this model, they reported that utility remained stationary across time and that it is only the probability weight that changed with time. That is, sensitivity to probability, but not the subjective evaluation of the outcome, changed across time. This finding, at the very least, suggests that the interaction between delay and probability is very complicated and more research is needed to more fully understand and differentiate the respective contributions of delay and likelihood.

The current findings also differed from the results obtained by Yi et al. (2006). Yi et al. observed a magnitude effect consistent with delay discounting in which the smaller amount was discounted to a greater extent than the larger amount, whereas this effect was not consistently observed in the current study. The likely reason for this discrepancy is the difference in analyses. Yi et al. assumed that probability and delay were equivalent processes and converted probability to delay in order to analyze the summed composite delay. Given the recent evidence suggesting delay and probability are distinct processes (e.g., Estle et al., 2006), they were not assumed to be interchangeable in the current experiment, and instead probability was incorporated into delay discounting by normalizing indifference points to their certain equivalent. Because the procedures used were similar across the two experiments, the difference in analyses likely produced the different findings.

Beyond this finding, the differences in assumptions and analyses make these two experiments difficult to compare. By converting probability to delay, Yi et al. (2006) forced their analysis and focus of study to be on delay discounting. In contrast, the current experiment examined both the effects of probability on delay discounting as well as the effects of delay on probability discounting. By fitting the raw data to a hyperboloid that incorporated delay and probability, we could observe the behavior of both the delay and probability parameters across amounts. Importantly, our analyses allowed us to examine the relative influence of delay and probability on decision-making within the discounting framework.

One potential concern with the current experiment is the lack of delay discounting at the 100% probability condition, in which the task is reduced to a typical delay discounting task. Although the hyperboloid function provided an excellent fit for these data, Figure 1 shows relatively flat discounting curves for the 100% probability conditions. This finding stands in stark contrast to that obtained under typical delay discounting procedures. One possible reason for the present finding is that the order in which the conditions were studied was completely randomized. The 100% probability conditions were intermixed with all other probability conditions, and both probability and delay had the possibility of changing in every condition. It may be that too many aspects of the choices were changing across questions and that participants simply chose to focus primarily on one aspect. If this were the case, it nonetheless suggests that probability is the more important component when decisions involve both delayed and probabilistic rewards.

Another factor that might have contributed to the present finding is that in the conditions in which the larger reward was to be received for sure, the choice presented to the participant stated explicitly that it had a 100% probability of receipt. In typical delay discounting tasks, the participant chooses between a smaller, immediate and a larger, delayed reward; there is no mention of the odds of receiving the rewards, though it is implied that the receipt is guaranteed. Explicitly stating the probability of receiving the reward may have shifted the participant's attention away from the delay. This finding is similar to a phenomenon found in the delay discounting literature termed the *hidden zero effect* (Magen, Dweck, & Gross, 2008). When choosing whether to receive a smaller, immediate or a larger, delayed reward, one is also choosing whether to receive nothing now or nothing later. For example, in a typical delay discounting task, one might choose between \$100 now or \$200 in 6 months. An equivalent choice that also makes explicit what one would not receive would be between \$100 now and \$0 in 6 months or \$0 now and \$200 in 6 months. Magen et al. found that participants chose the larger alternative more often (i.e., they discounted less) when the explicit zeros were added. Given that very little delay discounting was observed in any of the probability conditions, it is plausible that people tended to focus more on the explicit probability, and this led to decreased discounting.

The randomization of the conditions in which several different aspects of the choice situation (delay, probability, and amount) were constantly changing might have been too overwhelming for participants. Their focus on probability over delay, although in accord with our argument that probability is more dominant, may not accurately reflect how people make decisions. It remains to be seen whether our findings would remain if fewer changes occurred between conditions. Randomization of all conditions was used in the current experiment so as not to implicitly make either delay or probability more salient. However, differences might be obtained if conditions were studied in a more structured sequence. For example, participants could experience all of the 100% probability conditions at each delay before moving on to a new probability. This would create a series of delay discounting tasks at various probabilities. Because probability remains the same across several conditions while delay changes, delay may become more salient. The reverse procedure also can be done by creating a series of consecutive probability discounting tasks at each delay. In this case, because probability is the only thing changing between conditions, participants may focus more on the likelihood of receipt rather than waiting time. We currently are conducting such experiments in our laboratory. If probability continues to dominate choice in discounting tasks despite efforts to make delay more salient, such a result would provide additional support for the argument that probability is more important when making decisions involving both delay and probability. Of most importance, the present findings demonstrate that a multiplicative hyperboloid model provides an excellent description of choices involving rewards that are both delayed and probabilistic.

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Figure Captions

Figure 1. Mean indifference points and best fitting curves from Equation 2 (Delay Discounting) and Equation 3 (Probability Discounting). The left panels show probability discounting at each delay and the right panels show delay discounting at each probability. The top two figures correspond to the $800 conditions and the bottom two correspond to the $40,000 conditions.

Figure 2. Mean area under the curve (AUC) for delay discounting data at each probability (Panel 1) and probability discounting data at each delay (Panel 2).

Figure 3. Three-dimensional graphs of the discounting curves as predicted by Equation 4, for the $800 and $40,000 conditions.

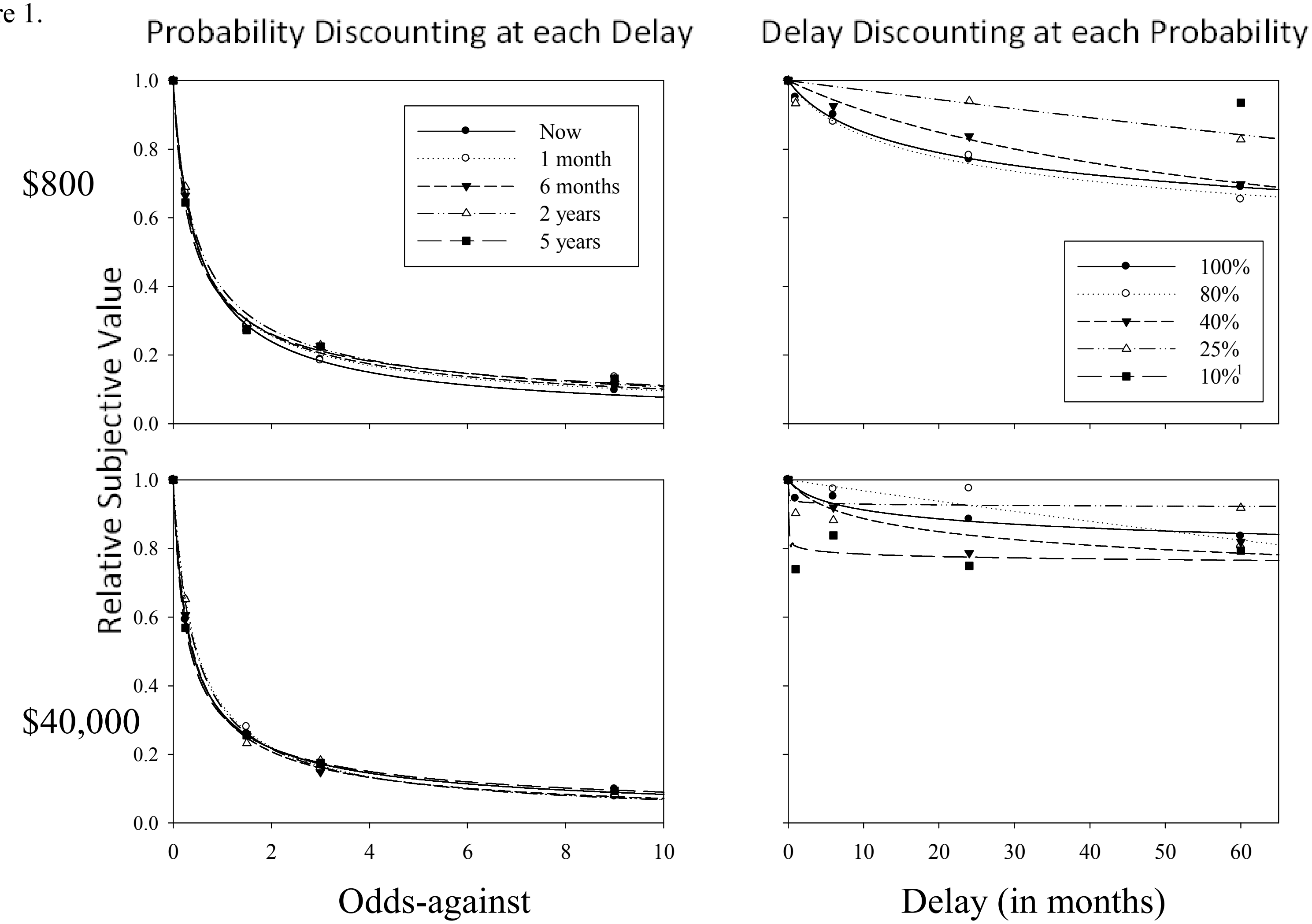
Table 1. Mean discounting rate parameter (*h* and *k*), *s* exponent, and proportion of variance accounted for (R^2) by Equation 2 for delay discounting data and by Equation 3 for probability discounting data at each amount.

	Probability Discounting					Delay Discounting				
	Now	1 Month	6 Months	2 Years	5 Years	100%	80%	40%	25%	10%
\$800										
h/k	2.731	3.553	3.859	3.451	5.069	0.218	0.213	0.032	4.77E-06	-0.017
s	0.767	0.653	0.624	0.626	0.557	0.141	0.154	0.334	600.549	0.070
R^2	0.999	0.997	0.998	0.998	0.997	0.991	0.977	0.987	0.569	0.000
\$40,000										
h/k	5.293	2.815	4.165	3.065	6.736	0.603	4.82E-06	0.387	1.82E+07	2.19E+07
s	0.622	0.798	0.703	0.779	0.570	0.047	667.906	0.075	0.004	0.013
R^2	0.999	0.999	0.999	0.998	0.999	0.904	0.858	0.875	0.231	0.815

Table 2. Parameter estimates for Equation 4 at each amount.

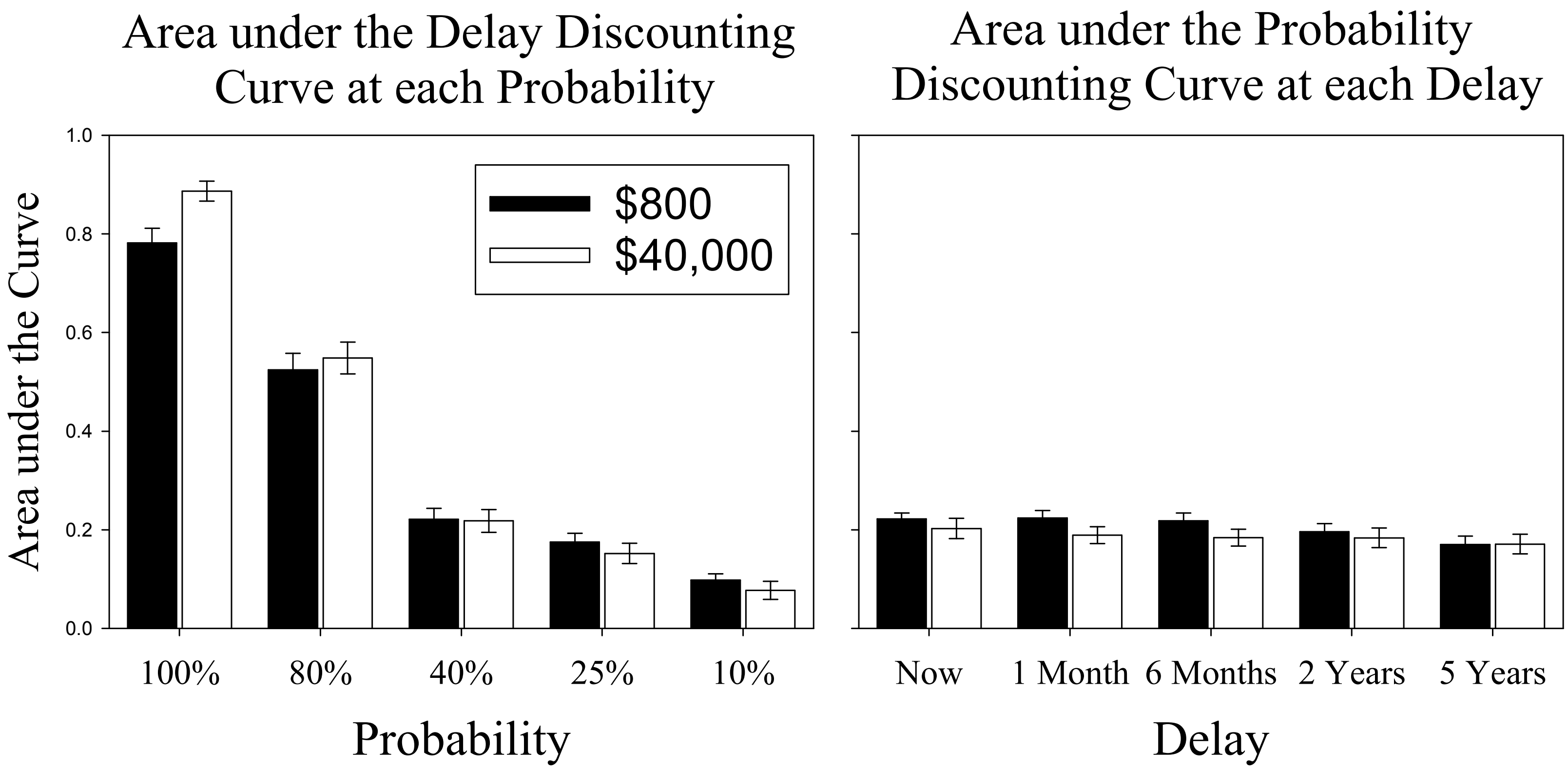
	\$800	\$40,000	difference
k	0.167	0.140	-0.027
h	3.637	4.278	0.641
s_d	0.155	0.083	-0.072
s_p	0.650	0.683	0.033
R^2	0.997	0.997	

Figure 1.



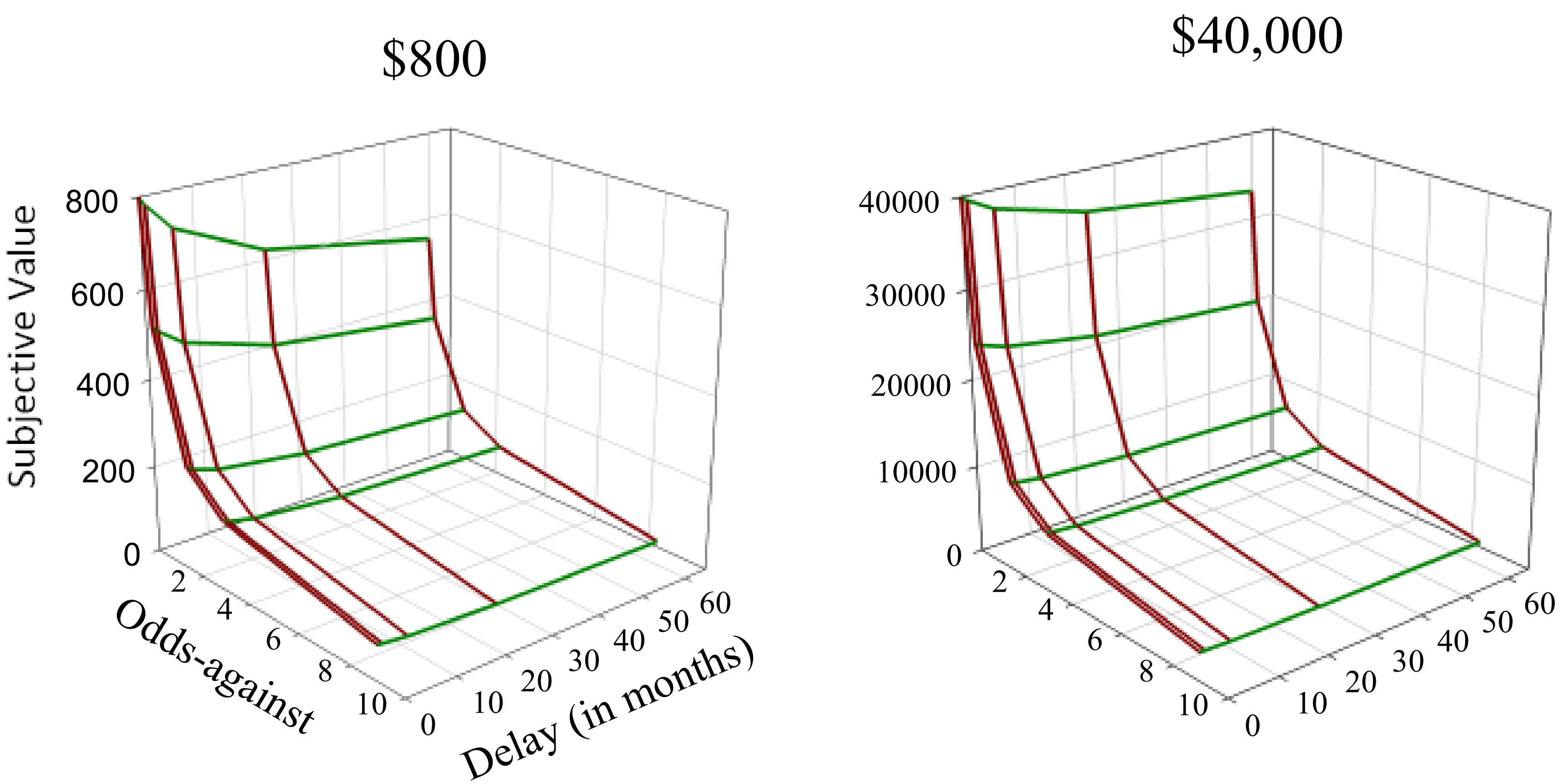
[1] The 10% line is not shown. Unlike in typical discounting where the subjective value decreases with time, the subjective value here increased with time. Thus, many of the values after the immediate delay were above 1.0 as the line curved up.

Figure 2.



Area under the Delay Discounting Curve at each Probability
Area under the Probability Discounting Curve at each Delay
Area under the Curve
1.0
0.8
0.6
0.4
0.2
0.0
$800
$40,000
100%
80%
40%
25%
10%
Probability
Now
1 Month
6 Months
2 Years
5 Years
Delay

Figure 3.



$800
$40,000
Subjective Value
800
600
400
200
0
40000
30000
20000
10000
0
Odds-against
2
4
6
8
10
Delay (in months)
0
10
20
30
40
50
60