Modern Assessment of the High-Energy Background Environment at Small Atmospheric Depths Using the X-Calibur X-Ray Polarimeter and Its Implications

Rashied Baradaran Amini
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Modern Assessment of the High-Energy Background Environment at Small Atmospheric Depths Using the X-Calibur X-Ray Polarimeter and Its Implications

by

Rashied Baradaran Amini

A dissertation presented to
The Graduate School
of Washington University in
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Doctor of Philosophy

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ABSTRACT OF THE DISSERTATION

Modern Assessment of the High-Energy Background Environment at Small Atmospheric Depths Using the X-Calibur X-Ray Polarimeter and Its Implications

by

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Doctor of Philosophy in Physics

Washington University in St. Louis

Professor Henric Krawczynski, Chair

Since the discovery of cosmic rays, Earth’s upper atmosphere at depths of 1-10 g cm\(^{-2}\) has been used for balloon-borne observations probing the high-energy universe. However, the interaction of cosmic rays with the earth atmosphere generates numerous particles, each with their own flux and interaction physics that contribute to instrument background. In Fall 2014, the X-Calibur X-ray polarimeter designed and built at Washington University in St. Louis was launched from Ft. Sumner, New Mexico. While no astrophysical observation was performed as a result of a failure in telescope mechanisms, X-Calibur was able to record hours of instrument background with an energy resolution of \(\sigma_E \sim\) keV at a depth of 3.45 g cm\(^{-2}\).

Using the 2014 X-Calibur data, existing observations of atmospheric \(\gamma\)-rays, hadrons, and leptons, and the MEGAlib/GEANT particle transport simulation environments, I develop and perform preliminarily validation of detailed background models in the regime of 1 keV-100 GeV. The models are generally constructed by involving experimental data with atmospheric mass and composition models rather than integration of source/emission
functions or complete Monte Carlo simulation of cosmic rays entering at the top of the atmosphere. Notably, these models are derived as functions of energy, off-zenith angle to degree precision, geomagnetic latitude, and solar modulation factor enabling predictions of instrument background for future X-Calibur flights.

These background models are also used to optimize the shielding configuration through additional passive material or active components as constrained by the existing active shield.
Chapter 1

Introduction

*The material particle nature of primary cosmic radiation has been confirmed, although the processes turned out to be extraordinarily more complicated than we had assumed.*

Walther Bothe (Bothe, 1954)

Although astronomers have observed nearly all of the universe’s electromagnetic spectrum, the observation of its polarization has generally been limited to energies below the X-ray regime. Observations of X-ray polarization would provide additional information that can help constrain the physics of accreting black holes, pulsars, relativistic jets, and fundamental physics. Our group at Washington University in St. Louis developed and built the X-Calibur X-ray polarimeter in order to observe polarized X-ray emission in the 20-60 keV hard X-ray regime. X-Calibur was first flown on September 18, 2014 over Ft. Sumner, New Mexico. Although X-Calibur was successfully launched and operated nominally, a failure in the pointing mechanism meant X-Calibur was unable to observe astrophysical sources. The hours dedicated to observing astrophysical sources were instead spent on observing the high-energy background that effects all balloon-borne high-energy instruments.

The high-energy background that exists at altitudes of ∼40 km is a mixture of cosmic radiation that penetrates the atmospheric and secondary particles that are produced in
cosmic ray air showers. In the 1960s and 1970s, high-energy astrophysics was still nascent and many balloon-borne observations were conducted to understand the nature of atmospheric radiation and the astrophysical particles that produced them. Since the late 1970s, there have been limited efforts to observe and reconstruct spectra of these particles.

Yet these particles contribute to the X-Calibur background, reducing the signal-to-noise ratio of astrophysical observations. X-Calibur’s September 2014 flight has produced several hours of uninterrupted background data, providing a statistically significant and well-resolved data set for reconstructing the high-energy background at small atmospheric depths, \( \approx 1-10 \text{ g cm}^{-2} \).

However, this reconstruction is neither simple nor direct. First, the high-energy background is composed of a myriad of particles each with its own zenith angle-dependent spectrum. The X-Calibur detector only registers events with knowledge of their timing and deposited energy; it is essentially blind to the nature of what induced the event.

It is possible to work around this ambiguity by deriving models for each background constituent and validating them by comparing the simulated detector response to X-Calibur flight data. One method for deriving these models uses calculations on the reported observations of each background constituent. The results from the high-energy balloon-borne observations from the 1960s and 1970s provided a strong empirical basis for such calculations. However, the reported observations were made under different observational conditions and targeted specific particles and energy ranges. Moreover, not all observations are consistent and the complete interpretation of the data would require a detailed simulation of each instrument and its surrounding mass. Thus, these results cannot be directly applied to estimate X-Calibur’s background environment.

Yet X-Calibur is not spherically symmetric and only provides an unobstructed line-of-sight through its aperture. The X-Calibur’s background events are the result of rare cases
where incident particles penetrate through the shield or generate local secondaries that deposit energy in the detector. The effect of penetrating radiation can be simulated with particle transport code that accounts for interaction physics and material properties. The computational capability to perform 3D particle transport did not exist during the 1970s when most of the reports in the literature on high-energy atmospheric background were published. Computation affords us an opportunity to reevaluate past experimental data as the inputs to simulations of the high-energy background as observed by X-Calibur.

Thus I arrive at the questions motivating this dissertation: Can X-Calibur’s 2014 observational data be used to validate and constrain background model predictions? Can this model be used to reasonably predict the background for future X-Calibur flights? If so, how can the detector be configured to minimize the background event rate?

Answering these questions produced work that unites the observational and theoretical atmospheric particle physics in a wider model comprised of many, covalidated independent models. This work was inspired by three papers that laid the foundation for my approach. The work by Gehrels characterizing the background of the Low Energy Gamma-ray Spectrometer (LEGS) based on his calculations of different background components that also include local activation (Gehrels et al., 1984; Gehrels, 1985). The work of Kole in his dissertation on the effect of the thermal neutron background on X-ray polarization measurements with PoGOLite (Kole, 2014). Finally, the work of Shaw et al. (2003) who used existing models to estimate the background on the space-based Burst and Transient Source Experiment (BATSE). My method of numerical fitting of archival data and introducing terms to account for rigidity cutoff and solar modulation was inspired by Costa et al. (1984) who used fits of previously reported atmospheric γ-ray data.

This semi-empirical method for developing background models represented by Costa and this work contrasts with the Monte Carlo (MC) method of Kole (2014), Takada et al.
(2011), and Lei et al. (2004a). In their works, the passage of and generation of atmospheric secondary radiation by cosmic rays incident on the top-of-the-atmosphere is simulated. The MC simulation environment handles physical interactions and outputs the resulting simulated detector event information. The principle assumptions underlying this method are an empirically determined incident cosmic ray flux and atmospheric mass model that are used as input to the simulation. Kole’s model provides further definition of the atmospheric neutron flux; Takada’s model describes secondary $\gamma$-rays at high zenith angles; and, Lei’s Quotid Atmospheric Radiation Model (QARM) package predicts fluxes for neutrons, $\gamma$-rays, protons, neutrons, and muons.

These two approaches are complimentary methods of constraining the same spectra. The semi-empirical method is limited by the availability of observational data. MC simulations or analytical calculations must be used to fill gaps in observation. Although the MC simulation method uses empirical inputs, the simulation output needs to be verified empirically.

The spectra predicted by semi-empirical models have the advantage of being intrinsically validated if the observational parameters used in the prediction are within the scope of past data. For instance, as will be discussed in the following, it is reasonable to assume the observed spectrum of cosmic ray protons over Ft. Sumner will not change over time, provided observational conditions are similar. Additionally, nearly all past experiments report the observed variation in zenith-entrant integrated or differential spectra as balloons ascend and descend. This spectral variation as a function of atmospheric depth can be convolved with an atmospheric mass model, describing depth as a function of zenith angle, to determine spectral variation as a function of zenith angle. This contrasts with the available models of Kole and Lei that describe spectra semi-isotropically, above and below limb. The resulting difference in reported fluxes near limb can be large, as described in Chapter 5.
However, accurately applying archival data requires special consideration. First, careful review of the instruments and experimental conditions producing past measurements is required to ensure uniformity of model inputs. Second, a single measurement cannot be used to verify the parameters of every different spectral model. For a complete model validation, many observations over many atmospheric depths, solar modulations, and rigidity cutoffs are required to constrain model parameters. This contrasts with the MC simulation method as all generated spectra are calculated consistently from the same simulation. But a simulation is required for each observational condition. Developing a complete model describing all possible conditions requires many computationally-intensive simulations.

A validated background model can then be used to predict its influence for future balloon flights. X-Calibur is currently anticipating a flight from McMurdo Station, Antarctica in 2018. Wind conditions over Antarctica allow for extended balloon flights but the low rigidity cutoff near the magnetic south pole leads to the fluxes of charged cosmic rays and their atmospheric secondaries being several factors greater than over Ft. Sumner. In Chapter 4, the mechanisms leading to this effect are described in detail.

In order to improve observational efficiency, I suggest a method by which the shielding of X-Calibur can be modified to reduce the background rate. Determining the optimal shielding configuration is a high-dimension problem. Sampling five different shielding thicknesses for eight options to modify X-Calibur’s shielding results in nearly $10^{20}$ combinations of shields that require individual, computationally-burdensome simulations. I therefore developed a procedure to parametrically sample this space to estimate the resulting background event rates for conditions over Antarctica. Using the calculated relationships between additional shielding mass and background event rate, I am able to use linear optimization to determine the shielding configuration resulting in the lowest background event rate as a function of shield mass.
1.1 New Contributions

My doctoral work led to the development of new models describing the angle-dependent spectra of contributions to the high-energy background at small atmospheric depths. Whereas recent work focuses on estimating backgrounds through simulations, my methodology uses archival data and theory. The most similar past works are the characterization of BATSE backgrounds by Shaw et al. (2003) and the analytical atmospheric $\gamma$-ray model of Costa et al. (1984). However, Shaw et al. (2003) characterized the space radiation environment and Costa et al. (1984) developed an analytical model for $\gamma$-rays. My approach convolves a global atmospheric mass model with how a zenith-entrant spectrum varies as a function of depth for dominant background contributions to develop a complete background model for balloon flights from Ft. Sumner and McMurdo, Antarctica.

My methodology is simpler and more accessible than MC methods as it does not require computationally-intensive simulations. As the works of Kole and Lei determine spectra for particles incident above and below zenith angles of $90^\circ$, I develop spectral models that more precisely describe zenith angle-dependence. This is particularly relevant for instruments that are sensitive to aperture or limb-entrant flux. Furthermore, my method relies on archival data and simple analytical methods. Avoiding the complexity of MC simulations makes instrument design optimization more accessible with significantly reduced computational requirements. My work offers updates to the atmospheric $\gamma$-ray model of Costa et al. (1984) based on data reported by Schöenfelder et al. (1977), Akyüz et al. (1997), Kinzer et al. (1978), and Ryan et al. (1977) for spectral zenith angle-dependence and the fluxes above 20 MeV.

The results of my simulations show the primary contributions to the background over Ft. Sumner are atmospheric $\gamma$-rays and neutrons. These two background components contribute 61.6% of all events. Assuming an identical detector configuration, predictions of
the background over McMurdo vary by a factor of \(\sim 6\), with a greater observed background during solar minima. The primary contributions to the background in regions of low rigidity cutoff are from atmospheric \(\gamma\)-rays, positrons, and electrons.

The methodology I develop in Chapter 5 is validated through comparison with X-Calibur’s 2014 experimental data. Although this provides confidence in the model’s ability to predict background as would effect X-ray and \(\gamma\)-ray astrophysical observations over Ft. Sumner at small atmospheric depths, additional experimental verification is required to validate its predictions in other contexts. Such experiments are discussed in Chapter 8.

In light of the paucity of literature on shielding optimization for astrophysical instruments, I offer a new means of determining optimal shielding configuration. In my literature review, only Swartz et al. (2000) described optimizing shielding configuration for high-altitude balloon experiments. However, the work is limited to 1D simulations that were done to understand shielding effects, not to perform an optimization of the complete shield. An explicit shielding optimization study performed by the A Toroidal LHC ApparatuS (ATLAS) collaboration was conducted iteratively. As a result, the study took over three years of simulations (Baranov et al., 2005). However, similar calculations have been used to determine biological radiation effects. Wilson et al. (2000) calculated the efficacy of multiple shielding materials for radiation dose and cell transformation, but no constraints were defined and optimization was not performed. Other terrestrial radiation safety studies, such as Djordjevich et al. (1990), used linear optimization to minimize dose of radiation therapy. The most similar work I found in the literature used linear optimization to design the shielding of radiotherapy vaults (Newman and Asadi-Zeydabadi, 2008).

A selection of additional work performed during the course of my doctoral program is included in this dissertation. My experimental measurement of the point spread function
of the InFOCµS grazing incidence X-ray mirror and background estimates for the PolSTAR X-ray polarimeter are included in Chapter 3.

1.2 Overview of Dissertation

The program of the dissertation is as follows:

- **Chapter 2: The Promise of X-Ray Polarimetry.** Herein I discuss the history of X-ray polarimetry and the X-Calibur’s science program.

- **Chapter 3: The X-Calibur X-Ray Experiment.** Herein I discuss X-Calibur’s design, operation, implementation, and ways it can be modified for future measurement campaigns.

- **Chapter 4: Particle Physics at Small Atmospheric Depths.** Herein I discuss the physics involved in the transport of primary particles and emission of secondary particles at balloon altitudes.

- **Chapter 5: Background Models.** Herein I develop models of the high-energy background constituents following a review of the experiments and a description of the theory of radiation that underlies the models. Chapter 5 shows the derivation of these models, which are summarized in Appendix A.

- **Chapter 6: Results.** Herein I compare results of my models and QARM with X-Calibur’s measurements of background over Ft. Sumner. Moreover, I predict the best and worst case background conditions for a future flight over McMurdo.

- **Chapter 7: X-Calibur Shielding Optimization.** Herein I use routines of optimization to predict optimal shielding configurations for conditions over McMurdo.
• **Chapter 8: Summary.** Herein I provide a summary of the work, its implications, and opportunities for future research.
Chapter 2

The Promise of X-Ray Polarimetry

Every X-ray encodes information about the physics underlying its emission and its propagation from the source to the observer. In order to decode this information, a suitable telescope and detector are required. A focusing X-ray mirror and pixelated detector can decode angular information. Solid state detectors, such as those made from CZT, can decode the spectral information by measuring the ionization charge above each pixel. In the soft X-ray regime, diffraction gratings can also be used to decode the energy.

Decoding polarization imposes physical and technical challenges. Polarization can only be measured for an ensemble of photons and not for individual photons. Moreover, polarization cannot be directly observed. Instead, an intermediary process such as scattering or pair production is required to convert polarization to something that can be detected. This secondary process complicates the reconstruction of its initial energy, which must also be done indirectly. Second, each intermediary process is only effective within a certain energy band as scattering, absorption, and pair production cross-sections vary with energy. Third, relying on a secondary process reduces observational efficiency as compared to imagining or spectroscopic instruments. This is due to systematic errors introduced by more complex instrument design, the probabilistic nature of scattering or pair production processes, and
a lower observable event rate due to incomplete conversion of incident photons through scattering or pair production.

Despite these challenges, X-ray polarimetry is one of the next major goals in astrophysics as polarization can aid in understanding the processes involved in black hole systems, astrophysical jets, pulsars, supernova remnants along with the potential to answer remaining questions in fundamental physics. In this chapter I review the physics behind polarization and the new science achievable through X-ray polarimetry.

2.1 Physics of Polarization

Transverse waves have two degrees of freedom for oscillation. One is in the oscillation of its amplitude as the wave propagates and the other is the rotation of the wave vector in the plane perpendicular to its propagation. The frequency of the first oscillation corresponds to the energy of the wave while the latter to its polarization. The general equation of motion of electromagnetic waves defining propagation and polarization is Equation 2.1:

\[
\nabla \times [\mu (\nabla \times \vec{E})] = \frac{\omega^2}{c^2} \epsilon \vec{E}(r)e^{i\omega t}.
\]

Here, \(\epsilon\) and \(\mu\) are the permittivity and permeability tensors of the medium, \(\vec{E}\) is the oscillating electric field. In vacuum, these tensors have only on-diagonal components. However, birefringent media possess off-diagonal terms that effect propagation as a function of a photon polarization. The optical axis of a birefringent medium is defined as rotated basis in which the terms are on-diagonal. Photons with the plane of polarization perpendicular to the optical axis are ordinary-mode, denoted by the subscripts \(\perp\) or \(O\), photons while those with parallel polarization are extraordinary-mode, denoted by \(\parallel\) or \(X\), photons. We express the
Polarization occurs on the plane orthogonal to propagation. The polarization \( \vec{a} = \vec{a}_1 + \vec{a}_2 \), where \( \vec{a}_1/2 \) are its component vectors. The polarization vector \( \vec{a} \) rotates with its propagation in time \( i\omega t \) along the ellipse with its phase angle defined by \( \phi \). The direction of rotation, shown as \( +/− \), is defined by the Stokes parameter \( V \) calculated in Equation 2.4. The major axes of the ellipse are given by \( \vec{a}_i \) and \( \vec{a}_j \). The linear polarization angle angle \( \chi \) is defined as the difference in phase between \( a \) and \( \hat{a}_1 \). Separately, ellipticity of polarization is defined by \( \psi = \arctan(a_j/a_i) \).

polarization of \( \vec{E} \) using the Jones formalism:

\[
\vec{E} = E_0 \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} e^{i(\hat{k} \cdot \hat{r} − \omega t)} \equiv \vec{E}_0(r)e^{−i\omega t},
\]  

(2.2)

where \( a_1 \) and \( a_2 \) are the amplitudes along the basis vectors \( \hat{a}_1 = (1, 0) \) and \( \hat{a}_2 = (0, 1) \) defining the plane of the rotating, oscillating electric field perpendicular to the propagation direction of the electromagnetic wave. The amplitudes \( a_1 \) and \( a_2 \) can be complex and satisfy the equation \( |a_1|^2 + |a_2|^2 = 1 \). In this plane we can further define \( \hat{a} \) as shown in Figure 2.1 where \( a_1 = \cos \phi \) and \( a_2 = \sin \phi \).
Based on the Jones formalism, described in Lohse (2005), the Stokes parameters can be defined using the polarization vector \( \vec{a} \) as unit vector and its adjoint, \( \hat{a}^\dagger \), and the Pauli matrices, \( \sigma_\mu \), defined in Equation 2.3:

\[
\sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2.3}
\]

\[
I = \hat{a}^\dagger \hat{a} = |a_1|^2 + |a_2|^2;
\]

\[
Q = \hat{a}^\dagger \sigma_1 \hat{a} = |a_1|^2 - |a_2|^2;
\]

\[
U = \hat{a}^\dagger \sigma_2 \hat{a} = 2 \mathbb{I}(a_1^* a_2);
\]

\[
V = \hat{a}^\dagger \sigma_3 \hat{a} = 2 \Re(a_1^* a_2), \tag{2.4}
\]

The Stokes parameter \( I \) defines the intensity. \( Q \) and \( U \) define linear polarization along \( \hat{a}_1 \) and the axis rotated counterclockwise by 45° from \( \hat{a}_1 \), respectively. Elliptical polarization exists where \( |Q| \) and \( |U| \neq 1 \), or alternatively \( \psi > 0 \). \( V \) defines the direction of the rotation of \( \vec{a} \), with \( V > 0 \) being counterclockwise when looking into the beam. The Stokes parameters are defined by the product of complex conjugate amplitudes, implying they are time-averages over the period of the waves.

Note that these Stokes parameters are defined for a 100% polarized monochromatic wave. The definition can be generalized for a partially polarized quasi-monochromatic beam of light. Assuming an incoherent superposition of wavepackets, their Stokes parameters (indexed by \( i \)) can be added to obtain the Stokes parameters of the ensemble. The beam’s net Stokes parameters with the polarization fraction \( P \), and net polarization angle \( \Phi \), are
shown in Equation 2.5 (Chandrasekhar, 2013; Kislat et al., 2015):

\[ Q = \sum_{i=1}^{N} Q_i \]
\[ U = \sum_{i=1}^{N} U_i \]
\[ V = \sum_{i=1}^{N} V_i \]
\[ P = \frac{\sqrt{Q^2 + U^2 + V^2}}{I} \]
\[ \Phi = \frac{1}{2} \arctan \left( \frac{U}{Q} \right), \]

where \( I \) is the total intensity of the beam. These Stokes parameters share the relationship:

\[ I^2 \leq Q^2 + U^2 + V^2. \] (2.6)

In the case of a monochromatic, coherent beam \( \sqrt{Q^2 + U^2 + V^2} = 1 \) and \( P = 1 \), though when the beam is partially polarized \( P < 1 \).

Several physical mechanisms are responsible for the emission of polarized X-rays. Below I describe these interactions using the formalism of Greiner and Reinhardt (2013). The mechanisms by which polarimeters detect polarization are discussed in the following chapter.

### 2.1.1 Scattering

In general, scattering processes can produce polarization radiation. When \( E_{\gamma} \sim m_e c^2 \), the \( \gamma - e^\pm \)-collision can no longer be considered elastic and the original Thompson cross-section requires modification. Compton scattering becomes the dominant astrophysical mechanism for generating polarized hard X-rays wherein incident photons scatter in collisions with free, charged particles, like electrons. Figure 2.2 describes Compton scattering where \( \hat{\epsilon} \)
Figure 2.2: An incident photon with polarization $\epsilon$ scatters off an electron. The resulting photon has a polarization $\epsilon'$, where the angle between incident and resulting polarizations can be decomposed into a polar angle $\theta$ and azimuthal angle $\phi$.

and $\epsilon'$ are the photon polarization before and after scattering. The polarization-dependent Klein-Nishina formula is:

$$
\frac{d\hat{\sigma}}{d\Omega_{k'}}(\lambda, \lambda') = \alpha^2 \frac{1}{4m_0^2} \frac{\omega'^2}{\omega^2} \left[ \frac{\omega'}{\omega} + \frac{\omega}{\omega'} + 4 (\epsilon \cdot \epsilon'^2) - 2 \right].
$$

(2.7)

Here $m_0$ is the rest mass of the electron while $\omega$ and $\omega'$ are the incoming and outgoing photon frequency, respectively. The term $4 (\epsilon \cdot \epsilon'^2) - 2$ reduces to $-2 \sin^2(\theta) \cos^2(\phi)$ when averaging over both helicities, where $\theta$ is the scattering angle and $\phi$ is the azimuthal angle difference between $\epsilon$ and $\epsilon'$. The scattering angle and relative polarization angle are related such that as $\theta \to 90^\circ$ the outgoing beam becomes completely polarized:

$$
P = \frac{1 - \cos^2(\theta)}{1 + \cos^2(\theta)},
$$

(2.8)

where $\Pi$ is the polarization fraction of emission.

Compton scattering is considered to be responsible for the non-thermal spectrum of accreting black holes, where thermally emitted photons are upscattered, or Comptonized, by high-energy electrons in the corona acquiring net polarization. The polarized coronal X-rays then reflect off the accretion disk which imparts a polarization based on the incidence angle. This reflection and change in polarization is also calculable using the Klein-Nishina formula.
2.1.2 Cyclotron and Synchrotron Radiation

Moving charged particles within a magnetic field will undergo a constant acceleration and radiate. Although the particle does not accelerate along the magnetic field lines, it gyrates in a spiral owing to the Lorentz force while its velocity along the magnetic field line is unaffected. The motion in the plane perpendicular to the magnetic field is characterized with gyrofrequency $\frac{qB}{mc}$. Cyclotron radiation is emitted by non-relativistic particles while synchrotron is emitted by relativistic particles. Up till kinetic energies of $\beta \approx 0.2$, cyclotron assumptions apply. At higher energies, relativistic beaming, or preferential emission along the direction of propagation, occurs through synchrotron radiation. The radiation spectrum is given by the Fourier transform of the time evolution of the beamed radiation pattern from the gyrating particle. A graphical description of the process is shown in Figure 2.3.

Cyclotron radiation emission can be approximated as a dipole, with no radiation emitted along the axis of acceleration and a maximum in the plane perpendicular to the axis.
of acceleration. Per Figure 2.3, emission is linearly polarized in the $n - a$ plane, resulting in varying polarization angle depending on the location of the observer.

### 2.1.3 Bremsstrahlung

Bremsstrahlung, or free-free radiation, is the process by which charged particles radiate and lose energy in another charged particle’s electric field. For instance, as a high-energy electron deflecting due to a nucleus’s field will emit a photon. White dwarfs, neutron stars, and accreting black holes all emit using bremsstrahlung through the hot plasmas that surround them (Rybicki and Lightman, 2008). In Earth’s stratosphere, electron bremsstrahlung in the presence of atomic and nuclear fields accounts for the majority of the $\gamma$-ray flux.

The bremsstrahlung cross-section is proportional to the outgoing photon’s polarization (Olsen and Maximon, 1959):

$$d\sigma \propto \left( \frac{\vec{\epsilon} \cdot \vec{p}_f}{k \cdot \vec{p}_f} - \frac{\vec{\epsilon} \cdot \vec{p}_i}{k \cdot \vec{p}_i} \right)^2.$$  \hspace{1cm} (2.9)

As the electrons will have varying momenta as they travel in a non-homogeneous plasma, bremsstrahlung emission is typically observed as unpolarized. Nonetheless, it is a major source of X-rays which are polarized through Compton, synchrotron, or polarization-dependent absorption. Detailed discussion on polarization of bremsstrahlung can be found in Olsen and Maximon (1959) and Gluckstern et al. (1953).

### 2.2 Motivations Behind X-Ray Polarimetry

To date, exploration of the hard X-ray sky, 2-70 keV, has been limited to several missions that left as many new questions as they answered. By observing the polarization
signatures of these sources it may be possible to answer these new questions. Below, I summarize the value of X-ray polarimetry in understanding accreting black hole systems, relativistic jets, and magnetars, in order to understand their mechanisms of emission and underlying fundamental physics.

Additionally, investigations of supernova remnants, pulsar wind nebulae, and gamma ray bursts (GRBs) polarization may also provide opportunities to understand their emission physics. Notably, GRBs and accreting black hole systems may produce jets that can be described by the same mechanism (Lee et al., 2000).

2.2.1 Accreting Black Hole Systems and General Relativity

Accreting stellar mass and supermassive black hole systems are similar in their anatomy. Emission from the system is powered by the conversion of gravitational potential energy to kinetic and thermal energy in the accretion disk. As a result, thermal accretion disk emission is the principal emission from these accreting black holes. As inner disk temperature scales $T \propto M_{BH}^{-1/4}$, thermal emission peaks in X-ray from stellar mass black hole accretion disks and in optical/UV from supermassive black hole accretion disks (Novikov and Thorne, 1973). As mass inspirals into the black hole’s event horizon, the system must conserve angular momentum. It is theorized that collimated jets are formed along the axis of the black hole’s rotation carrying away angular momentum. The plasma jet is highly relativistic and emits synchrotron radiation indicating presence of a magnetic field. Additionally, photons entering the corona are boosted through inverse Compton interactions with the hot plasma.

A Newtonian thin disk model for accretion was first presented by Shakura and Sunyaev (1973). Although Newtonian generalizations cannot accurately describe dynamics in the vicinity of a black hole, major results included that accretion disks must have scale heights
significantly smaller than their radii (Shakura and Sunyaev, 1973). Their calculations were extended by Novikov and Thorne (1973) who formulated a disk model using general relativity. Novikov and Thorne’s description of thermal emission from the disk is steady state and does not account for transient effects (Thorne and Price, 1975). The conditions where the Novikov and Thorne description are valid are reviewed by McClintock et al. (2011).

Recently, progress in disk modeling has been made by developing numerical models of accretion through magnetohydrodynamic simulations that track the accretion disk plasma and magnetic field while accounting for radiative emission and general relativistic effects (McKinney, 2006; McKinney and Blandford, 2009; Narayan et al., 2016).

However, observed spectral features, like the bright emission from AGNs and hard X-ray emission from stellar- and supermassive black holes, indicate the presence of a corona in the black hole system that produces non-thermal radiation (For review of non-thermal theories see Romero et al., 2010). The simplest coronal model is the lamp post model, shown at the top of Figure 2.4 that posits a point-like corona suspended above the black hole (Dovčiak, 2004). Coronal emission in this model can reflect off the disk, imparting a linear polarization. The observed coronal light at infinity is a combination of directly observed light and reflected, polarized light. For fixed coronal luminosity and spectrum, the observed spectrum and linear polarization fraction varies as a function of coronal height and inclination.

Spherical coronae may have outer radii that extend beyond the innermost stable orbit (ISCO) and overlapping inner regions of the accretion disk (Schnittman and Krolik, 2010b). The spherical model is motivated by advection-dominated accretion flow generating gas within the overlapping region. As a result, the optically thin spherical coronae broadens the thermal emission spectrum of the inner regions of the disk through Comptonization.
Figure 2.4: Anatomy of an accreting black hole system: (a.) Lamp post corona, (b.) Spherical corona, and (c.) wedge corona. Coronal emission and reemission is reflected by the accretion disk and linearly polarized as a function of incident angle. Light passing nearer to the black hole is more effected by general relativistic effects.

Coronal thermal emission is also observed. The polarization signature of the spherical corona is limited by how far the outer radius extends and the inclination angle of the accretion disk.
As a third possibility, the wedge corona shown in Figure 2.4 may extend from the ISCO to the edge of the accretion disk. The resulting spectrum is a combination of broadened thermal and non-thermal emission of the disk and corona (Li et al., 2009; Schnittman and Krolik, 2010b; Dovčiak et al., 2012).

In summary, varying disk and coronal models will result in different observational signatures. The observed spectrum is some combination of thermal and non-thermal spectrum, some of which is reflected and/or downscattered resulting in broadening. The polarization angle and fraction will vary based on corona geometry and inclination angle (Schnittman and Krolik, 2010b). Due to the strong gravity environment and general relativistic effects, the path a photon follows can be determined through solving the geodesic equation assuming for a metric, initial location, and wavevector. On account of the strong gravity, photons emitted at different locations within the black hole system will arrive at different times. Using time of arrival differences of events from the same spectral feature to constrain accreting black hole parameters is the reverberation mapping technique, extensively reviewed by Uttley et al. (2014).

As trajectories passing closer to the black hole experience longer paths, corona geometry and any asymmetries of the disk system will effect the timing of events arriving at the detector. This time lag can be used to reconstruct the system’s geometry through the reverberation mapping technique (Peterson and Horne, 2004). The time lag is uncovered by Fourier transforming the observed time series for a given spectral feature (Uttley et al., 2014). The observed time lag increases as inclination angle nears $i = 0^\circ$ or corona-disk separation increases. This is illustrated in Figure 2.5, where Fe Kα emission from the disk illuminated by corona non-thermal X-rays will take a longer path if it travels closer to the black hole.
Figure 2.5: *Left* Fe Kα line frequency lag at three inclinations. As inclination and lag increases with inclination as trajectories become more effected by the gravity well. *Right* Energy lag as a function of inclination. Here, lag increases along with broadening and bluing of the spectrum as inclination increases. Both plots assume a lamp post corona model. Excerpted from Hoormann (2016).

A feature of accreting stellar mass black holes are High/Low Frequency Quasi-Periodic Oscillations (H/LFQPO) where total flux varies on the timescale of seconds to minutes but the spectrum remains unchanged. It is expected that the period of observed variation in emission corresponds to time scales of specific dynamics; two leading theories have been developed to explain HFQPO and LFQPO. One theory for origin of LQPOs is the presence of Lense-Thirring precession of the accretion disk, wherein space-time becomes twisted in high gravity regimes resulting in the low frequency oscillations from the inner accretion disk precessing like a solid body (Ingram et al., 2009; Ingram et al., 2015). In a second model for QPOs, it is theorized that HQPO vary in amplitude on the order of seconds originate within the rapidly spinning inner accretion disk, near the ISCO, where local “hot spots” due to MHD instabilities emit thermal radiation at a higher temperature than the rest of the disk (Beheshtipour et al., 2016).
2.2.1.1 Recent Investigations

Several black hole emission models have been developed to fit observational data. Of these, KERRBB and REFLION have been integrated with XSPEC to provide readily available and rapid estimates of observable spectra. Li et al. (2005) produced KERRBB that simulates Kerr black hole thermal spectra accounting for “frame-dragging, Doppler boost, gravitational redshift, and bending of light by the gravity of the black hole” (Arnaud, Accessed 08/2016a; Li et al., 2005). Ross and Fabian (2005) developed REFLION that simulates observable reflection spectra using a parametric grid defined by an independent ionization parameter, corona power law spectrum index, and Fe abundance assuming an optically thick disk and slab corona (Arnaud, Accessed 08/2016b). Since 2005, several others have studied the polarization signature of emission. Li extended his work from KERRBB to determine how inclination can be predicted using the thermal continuum polarization (Li et al., 2009). This was also the subject of Schnittman and Krolik (2010a). However, these models still are unable to describe observed phenomena. Gou et al. (2011) and later Tomsick et al. (2014) used updated REFLION and KERRBB to describe Cygnus X-1 without conclusive results for disk inclination.

Corona models also provide new insights into observable non-thermal signatures. In 2004, Dovčiak et al. (2004) provided first results of the polarization signature of the reflected non-thermal spectrum of AGNs. Matt et al. (1991) first introduced the lamp post model (“Model 1A”) in describing the observed X-ray bump in Seyfert 1 galaxies (see also Dovčiak, 2004). Dovčiak later described the polarization signature of lamp post coronae in 2012 (Dovčiak et al., 2012). Schnittman and Krolik (2010b) compared reflection spectra of spherical, wedge, and inhomogeneous (clumpy) corona geometries including the polarization signatures. Together, these provide many theoretical predictions that can be validated by polarimetric observations as shown in Figure 2.6.
Since time lag of the Fe Kα line was first observed in NGC 4151, its use in reverberation mapping for constraining many different black hole system parameters was soon to follow (Zoghbi et al., 2012). Risaliti et al. (2013) used reverberation mapping to constrain black hole spin rate on NGC 1365 independently of Brenneman et al. (2013). Walton et al. (2014) used reverberation mapping to better define parameters of the inner disk. Marinucci et al. (2014); Cackett et al. (2014); Wilkins et al. (2016) demonstrated the use of reverberation mapping for corona modeling.

For stellar mass black holes exhibiting QPO signatures, polarimetric data can be used to constrain their underlying physics and strong gravity effects in the inner accretion region. Separately, the hot spot model for HFQPO was defined by Schnittman (2005) and its polarization signature studied by Beheshtipour et al. (2016). The polarization signature of LFQPOs was described by Ingram et al. (2015).
2.2.2 Blazar Jets

It is theorized that the relativistic jets generated by massive accreting objects are powered by extracting the enormous angular moment of the rapidly spinning black hole as it extended horizon or differently rotating accretion disk results in large electric fields from the motional $\vec{v} \times \vec{B}$ electromagnetic force. However, the physical mechanism that converts momentum and produces such collimation is not definitively understood. Although polarized jet emission is observable through the electromagnetic spectrum, polarized X-rays may offer one of the best glimpses into the geometry of the magnetic fields of the inner accretion disks and the processes near the black hole where the jet is launched.

The source of blazar emission is undoubtedly an accreting, rapidly spinning black hole; however, it is uncertain whether it emerges through mass transfer from the accretion disk or from the black hole’s ergosphere (Simulations predicting both are discussed in Komissarov et al., 2007). Observable jet emission is highly polarized indicating the presence of a magnetic field radiating via the cyclotron/synchrotron process. It is theorized that the same relativistic electrons emitting synchrotron radiation upscatter their own synchrotron through the inverse-Compton interactions. This process is known as synchrotron-self Compton emission (SSC, Stern and Poutanen, 2004). In flaring jets it is further theorized that a burst of synchrotron and SSC results from particle acceleration by magnetic dissipation or from shocks. The swing in X-ray polarization angle and change in fraction is predicted by Zhang et al. (2015). Although it is suspected that the charged particles responsible for the SSC emission are leptons, it is possible they are secondaries resulting from hadronic cascades, rather than directly accelerated primaries. Determination of polarization degree in hard X-rays can distinguish between the two (Zhang et al., 2014). In addition to SSC emission, it is possible that energetic electrons can Comptonize ambient speed photons from the accretion
region, resulting in so-called external Compton hard X-ray or $\gamma$-ray emission. Krawczynski (2011) indicates X-ray polarimetry will further distinguish the origin of seed photons.

Measuring the polarization of the synchrotron X-ray emission from blazars can tell us about the geometry and field configuration at the base of the jet. A rotating, helical magnetic field sweeping through the regions of X-ray emission in the jet would produce smooth swings of polarization angle in synchrotron continuum emission (Pushkarev et al., 2005; Lyutikov et al., 2005; Marscher et al.; Marscher, 2014). It was also argued by Krawczynski (2011) that these swings in polarization angle would be more prominent in X-ray than radio or optical due to the smaller size of X-ray emitting regions resulting in more uniform and rapid flares (Krawczynski et al., 2013).

### 2.2.3 Magnetars and Fundamental Physics

Two classes of neutron stars exist whose emission indicates the presence of very strong magnetic fields $B \geq 10^{14}$ G. These magnetars are theorized to radiate using primarily power transferred from their magnetic fields as opposed to the emission-induced spin down of other pulsars. Anomalous X-ray Pulsars (AXP) were discovered to be similar to ordinary pulsars except their luminosity, thus their spin down derivative $d\omega/dt$, was estimated to be higher than other pulsars. Soft Gamma Repeaters (SGRs) were first discovered as transient $\gamma$-ray objects but were soon found to have the X-ray signature of a pulsar similar to that of AXP. (The history and physics of both are described in Woods and Thompson, 2006; Harding and Lai, 2006) We can solve for the surface dipole magnetic field based on observed spin down assuming a magnetic coupling mechanism is responsible (Harding and Lai, 2006):

$$B_s = \left(\frac{3I c^3 P \dot{P}}{2\pi^2 R^6}\right)^{1/2} \approx 2 \times 10^{12} G (P \dot{P}_{15})^{1/2}.$$  \hfill (2.10)
This results in surface fields $B_s \geq 10^{14}$ G where $\dot{P}_{15} = \dot{P}/10^{-15} \text{ s}^{-1}$ and $I \approx 10^{45} \text{ g cm}^2$. The magnetic field transfers energy to electrons as it sweeps past them, which then radiate through cyclotron/synchrotron and also through Comptonizing thermal photons.

For magnetars and pulsars, photon emission occurs not in a vacuum but an electron-rich plasma in a strong magnetic field. As a result, the media properties impact the propagation of photons based on their polarization as as dictated by the plasma and QED dielectric tensors. The dependence of propagation of light based on polarization is known as birefringence.

Closer to the surface of the magnetar, cyclotron absorption and Thompson scattering both contribute to polarization mode-dependent emission from magnetar atmospheres. When the dielectric tensor is rotated such that $\hat{z} = \hat{B}$, the resulting equations indicate that the plasma environment becomes opaque to X-mode photons while remaining transparent to O-mode photons (Lai and Ho, 2002; van Adelsberg and Lai, 2006; Harding and Lai, 2006).

The propagation of photons is further effected by the QED vacuum polarization owing to the strongly magnetized environment. The QED vacuum has its own dielectric tensor. Thus, the dielectric and permeability tensors are the summed effects of both the plasma and QED vacuum: $\epsilon = \epsilon_0 + \epsilon_{\text{Plasma}} + \epsilon_{\text{QED}}$. Because these terms are a function of $N$, $\vec{B}$ the propagation of photons will vary as a function of the magnetic field structure, magnitude, and altitude.

There exist locations where the magnetic field and plasma density result in the cancellation of contributions of the plasma and QED dielectric tensors. This cancellation of effects is defined as vacuum resonance. In vacuum resonance there exists a probability for O- and X-mode photons to convert to the other mode:

$$P_{O\rightarrow X} = e^{-\frac{\pi}{2} \frac{\vec{E} \cdot \vec{E}_{\text{ad}}}{\epsilon}} ,$$

$$P_{X\rightarrow O} = 1 - P_{X\rightarrow O},$$

(2.11)
Figure 2.7: Polarization evolution and mode conversion near magnetars for photons with $E > E_{ad}$. X-mode photons decouple first from the plasma, after which X-mode attenuation is negligible. At vacuum resonance, a fraction of X-mode photons per Equation 2.11 will convert to O-mode. These continue to attenuate until they later decouple from the plasma. O-mode photons are significantly attenuated due to their coupling to the plasma, but convert to X-mode at vacuum resonance which is no longer attenuated. Note conversion at vacuum resonance is not necessarily complete as implied in the figure. Adapted from Harding and Lai (2006).

where $E_{ad} = 2.5 \tan^{2/3} \theta_{k_B}(1/H)^{1/3}$ keV is the adiabatic energy threshold where photons propagate without spontaneous conversion. The effect of plasma on propagation diminishes with plasma density, resulting to a decoupling prior to the decoupling of the effects of the QED tensor. The effect of the plasma medium and QED effects on the propagation of photons is shown in Figure 2.7.

This intricate coupling of plasma and QED effects result in a strong polarization signature. Determining the polarization fraction and angle of magnetar X-ray emission will
determine the structure of their magnetic fields, confirm models of the plasma environment in their photospheres, and could, for the first time, validate QED’s predictions for vacuum polarization.

The strong magnetic fields of magnetars may allow for coupling of dark matter to photons, resulting in a detectable signature. Axions, a scalar CP-violating particle, was introduced by Peccei and Quinn (1977) (PQ) as a biproduct of the mechanism resolving the Strong CP problem of quantum chromodynamics. Since PQ, additional theories predicting axion-line particles (ALP) have emerged predicting similar scalar particles with a weak coupling to the electromagnetic force. ALPs couple to photons with the term $\mathcal{L} = g_{a\gamma} a E \cdot B$, where $g_{a\gamma}$ is the axion-photon coupling constant. For PQ axions, mass is related to the coupling constant by $m_a \approx \frac{6 \times 10^6 \text{GeV}}{g_{a\gamma}}$ eV. (Graham et al., 2016).

PQ axions or ALPs could form a candidate for cold dark matter owing to their theorized prevalence, low mass, and weak coupling. In solving Einstein’s equations with an additional pseudoscalar field and taking into account QCD effects, one can calculate the mass obtained by the axion through the QCD phase transition and determine the relic abundance given by $\Omega \approx 1$ needed to explain the predicted dark matter abundance of the universe.

Notably, the coupling factor is weak suppressing the $a E$ interaction when not in the presence of strong electromagnetic fields. Next, this coupling is directly proportional to $\sin \theta_{kB}$ and only couples to O-mode photons. In magnetar atmospheres it is possible sufficient interactions occur that effect the polarization signature providing experimental proof of the existence of ALP.

Perna et al. (2012) reported results of a study that estimated the observable polarization signature of axion-photon coupling at magnetars. A fundamental issue in detecting ALPs at magnetars is that where the axion-photon coupling is least suppressed, the portion of the atmosphere is most opaque to O-mode photons. As a result, the signature is weakened by
absorption. Nonetheless, Perna et al. (2012) reports that a signature may still be detectable \(\approx 4\) keV.
Chapter 3

The X-Calibur Experiment

The X-Calibur detector uses a low-Z scattering element and high-Z CZT pixel grid arrays to absorb scattered X-rays. The combination of focusing X-ray optics, low-Z scatterer, and high-Z absorber results in high scattering efficiency and excellent energy resolution. Though a certain configuration was developed, tested, and deployed for its Fall 2014 flight, modifications and enhancements to X-Calibur’s scatterer, CZT detectors, electronics, and shielding have been studied for future implementation.

3.1 X-Ray Polarimeters

X-ray polarimetry is still a largely unexplored field. NASA launched a single dedicated X-ray observatory in 1978. The OSO-8 Experiment detected linearly polarized 2.8 keV and 5.6 keV X-rays from the Crab pulsar and nebula and set upper limits on the linear polarization fraction of roughly a dozen bright X-ray sources (Weisskopf et al., 1978; Bunner, 1978). The OSO-8 Bragg reflection polarimeter had very narrow energy bandpass, 2.4-2.8 keV, and thus achieved only a modest sensitivity. Recently developed X-ray polarimeters observe a wider bandpass and achieve better sensitivities. At 2-10 keV energies, gas pixel detectors (GPD) and time projection chambers (TPC) are being used to infer the polarization.
Figure 3.1: Left shows the mass attenuation coefficients for Si while right shows it for CZT.

by measuring the angle of the photoelectrons ejected as X-rays interact with the target gas. The three orbiting X-ray polarimeters being considered for launch by NASA and ESA, IXPE, PRAXyS, and XIPE, are photoelectric effect polarimeters (Weisskopf et al., 2008; Jahoda et al., 2014; Soffitta et al., 2013). Between 10 keV and 1 MeV, or about twice the electron rest mass, Compton scattering interactions dominate all other interactions in low-Z materials within the X-Calibur science band as shown in Figure 3.19. Beyond 1 MeV, pair production begins to occur from which photon polarization can also be reconstructed. Figure 3.1 illustrates the mass attenuation coefficient for these interactions within Si and CZT, materials that are often used as the sensitive material in X-ray detectors.

3.1.1 Photoelectric Polarimeters

Photoelectric absorption is the dominant photon-matter interaction in the soft X-ray regime. In a photoelectric absorption interaction, a photon is absorbed while an electron is ejected from an atom. While the polar angle of the ejected electron’s trajectory is a function of photon energy, its azimuthal angle is a function of the photon’s polarization per:

$$\frac{d\sigma}{d\Omega} \propto \frac{Z^5 \sin^2 \theta \cos^2 \phi}{(1 - \beta \cos \theta)^4},$$

(3.1)
where $Z$ is atomic number, $\theta$ is the polar angle, $\phi$ is the azimuthal angle, and $\beta$ is electron velocity in units of $c$. The incident photon’s polarization can be reconstructed by tracking the ejection angle of the liberated photoelectron. This can be done by providing a medium for the photoelectron to interact and lose energy as its trajectory and energy deposition is measured.

The PRAXyS polarimeter uses a TPC to track these photoelectrons. An incident photon ejects a photoelectron. The photoelectron then ionizes gas atoms along its path, generating lower energy electrons which drift toward a cathode grid aligned in a plane parallel to the optical axis. These secondary electrons accelerate further as they pass through a high-voltage grid close to the cathode resulting in an ionization cascade. Ultimately, the electrons deposit their charge on conductive readout strips (Remillard et al., 2000). These strips record the coordinates of the ionization path in one dimension perpendicular to the optical axis. Drift time measurements are used to record the coordinates in the other direction. This entire process is depicted in Figure 3.2.

PRAXyS uses pure dimethyl ether (DME) at 0.25 bar as its gain gas and 128-channel Si strip detectors of 121 $\mu$m pitch. This configuration results in an energy resolution of $\sigma_E = 24(20)\%$ and polarization modulation parameter $\mu = 40(50)\%$ for $E = 4.5(6.4)$ keV, respectively (Kitaguchi et al., 2015).

An alternative method of tracking photoelectrons is using a GPD. The GPD and TPC both use gas-electron ionization cascades to generate a signal. The GPD’s method of detection differs as the bias voltage is applied along the optical axis as opposed to perpendicular. This way GPDs can track polarization and also image the source. The imaging angular resolution is based on the half energy width (HEW) of the footprint from secondary electrons. When paired with its Berera Observatory-developed mirror, XIPE’s Efficient X-Ray Photoelectric Polarimeter (EXP) instrument is reported to have an angular
Figure 3.2: Principle of detection for a TPC. A photoelectrically-ejected electron creates secondary electrons through ionizing detector gas as it continues through the chamber. The energies of these new secondaries are less than that of the primary electron. These induce events nearer to the initial photoelectric event depositing more charge on the strips than those further away. Through drift TOF and gain from the avalanche effect it is possible to determine the trajectory of the initial electron, thereby the polarization of the incident photon. Adapted from Kitaguchi et al. (2015).

Resolution of approximately 24” and energy resolution of 20% at 5.9 keV. The XIPE gain gas is different from PRAXyS, using a mixture of 80% DME/20% He. A noble gas is added to moderate the number of secondary electrons, reducing noise at the expense of a higher energy threshold. XIPE’s Medium Energy Solar Polarimeter (MESP) GPD uses a 60% DME/40% Ar mixture raising its energy range. While EXP’s energy range is 2-10 keV, MESP’s is 15-35 keV. (All values reported by Soffitta et al., 2013).

The major design challenge of photoelectric polarimeters is their sensitivity as absorption efficiency and electron drift length need to be balanced. The drift length is inversely proportional to the energy loss, given by the Bethe equation. The denser or higher-Z the medium, the more photoelectric events will occur increasing photon conversion; however, the shorter mean free path of the drift electrons means the cascade can be quenched too quickly, reducing signal amplification.
3.1.2 Scattering Polarimeters

As the photoelectric absorption gives way to coherent and incoherent scattering at higher photon energies, the mechanism of detection must change to effectively measure polarization. Scattering interactions at astrophysical sources discussed in Section 2.1 are no different from scattering interactions in a detector and the same principles used to create polarized light can be used to reconstruct its polarization. In its most general form, an incident X-ray enters the scattering medium within a detector where it scatters. The scattered X-ray photon will have a trajectory defined by $\theta$ and $\phi$ per Equation 2.7 as a function of incident polarization.

Scattering detectors face a challenge similar to photoelectric detectors. A higher-$Z$ medium scatters a larger fraction of incident photons but also absorbs a higher fraction owing to photoelectric interactions. As a result, the energy threshold is increased on account of absorption. A lower-$Z$ medium will reduce the scattering probability and require a larger detector. In general, multiple scattering of the initially scattered X-ray may obfuscate its original trajectory leading to greater uncertainty in polarization reconstruction.

Scattering polarimeters can use active scattering elements to derive a coincidence signal and to measure the location of the scattering event. In order to minimize uncertainty in location, the scattering medium can be segmented. The scintillation light of each segment is observed by a photomultiplier tube (PMT), silicon photomultiplier (SPM), or avalanche photomultiplier (APD).

For scattering polarimeters, it is possible to reconstruct the energy of Compton interactions through the proportionality of scintillation yield and deposited energy (Nikl, 2006):

$$N_\gamma \propto \frac{dE}{dx} \propto \rho_{ab} \frac{Z_{ab}}{A_{ab}} \frac{1}{\beta^2},$$

(3.2)
where $N\gamma$ is the number of scintillation photons, $-dE/dx$ is the electron energy loss per unit distance, $Z_{ab}$ and $A_{ab}$ are the absorber’s atomic number and mass, respectively, and $\beta$ is the electron velocity in units of $c$. Although scintillation yield is proportional to $Z$, in practice the selection of the scintillator must account for its effects of multiple scattering due to higher-$Z$ materials. As a result of Equation 3.2, low-$Z$ scintillators have poor energy resolutions.

One example of a scattering polarimeter is PoGOLite which was flown in July 2011, March 2013, and June 2016. In PoGOLite, triggers in two of the 61 hexagonal “fast scintillators” defines a hit (where “fast” refers to scintillation decay time). The azimuthal scattering angle is defined as the angle from the fast scintillator with the highest energy deposition to the lowest energy deposition.

To obtain better energy resolution of events, X-Calibur uses a high-$Z$ CZT array to detect X-rays scattered by a low-$Z$ scattering stick. An incident X-ray first scatters in a low-$Z$ medium while the scattered photon deposits its energy in CZT pixel array that surrounds the scatterer. Such a configuration allows for good energy resolution from the CZT, while the scatterer also serves as an active, coincidence detector where scintillation photons are detected by a PMT.

The azimuthal scattering angle is given by the line connecting the location of the triggered CZT pixel and the optical axis of the X-ray mirror. Linear polarization fraction and polarization angle can be inferred by analyzing the modulation of the azimuthal scattering angle distribution. The CZT detectors are sufficiently thick to absorb all of the X-rays in the 25-60 keV energy band. Since the mirror focuses the X-rays onto a single scintillator, effectively a single pixel, the configuration has the disadvantage that it cannot image astrophysical objects.
The INTErnational Gamma-Ray Astrophysics Laboratory (INTEGRAL) uses several techniques to reconstruct hard X-ray polarization (Teegarden et al., 1997). The SPectrometer for INTEGRAL (SPI) consists of Ge detectors. The SPI instrument can measure polarization by analyzing photons triggering two Ge detectors within a segmented array (Kalemci et al., 2004; Götz, 2012). The IBIS instrument is a coded mask aperture imager using two layers of detectors. The trajectory of a photon scattered in the first layer ISGRI (CdTe) can be tracked on the second layer, PICsIT (CsI) (Forot et al., 2008).

### 3.1.3 Pair Production Polarimetry

At $E_\gamma > 2m_e c^2$, $\gamma \rightarrow e^- e^+$ pair production can also be used to determine polarization. Specifically, the preferred plane of electron-positron trajectories generated by pair production is a function of the polarization of the photon.

Like all the other methods discussed there exists a challenge in detector design where increasing the pair production cross-section to improve sensitivity will also increase the scattering cross-section for the produced electron-positron pair that will lead to multiple scatterings and ambiguity reconstructing polarization.

Several methods exist to perform this measurement as surveyed by Bloser et al. (2004). Separately, Rolf Buehler (2010) investigated the possibility of reconstructing polarization in Fermi data. Reconstruction is confounded by multiple scattering at $E_{e^-} \sim 100$ MeV, small opening angles at GeV energies, and a relatively large detector pitch per radiation length in the detector.

As studied by myself and others, a future Fermi-like detector that decreases the radiation length of the pair producing medium and increases instantaneous pitch resolution
by reducing radiation length per meter and/or reducing pitch size may be able to reconstruct polarization at $\gamma$-ray energies (Jim Buckley and the JPL A-Team, 2015).

### 3.2 Hard X-Ray Astronomy with CZT Detectors

In the past several decades numerous hard X-ray and $\gamma$-ray observations have been performed with balloon-borne or spaceborne proportional counters or scintillation detectors. Unlike proportional counters, like the Proportional Counter Array (PCA) on the Rossi X-Ray Timing Explorer (RXTE), that lose detection efficiency at higher energies as interaction cross-sections in the gas medium diminish, detectors such as Sodium Iodide (NaI) or Bismuth Germanium Oxide (BGO) provide sufficiently large cross-sections at high energies (Glasser et al., 1994). Although scintillation and solid state germanium detectors have been traditionally used as a primary instrument for X-ray balloon observations, they have several limitations. Namely, they require relatively large active volumes due to their low-Z composition. Germanium detectors have required active cooling adding engineering complexity as well as a large, passive volume which may produce secondaries and more instrument background (Gehrels, 1985; Andersson et al., 2005). As scintillation detectors cannot distinguish how events enter their volumes, their angular resolution has typically been limited to $\alpha \gtrsim 1^\circ$ (Grindlay, 1998; Teegarden et al., 1985). These issues can be overcome at the expense of increased complexity through segmenting the detector and using focusing optics or a coded aperture mask (Soffitta et al., 2013; Kalemci et al., 2004).

Modern experiments achieve a much better sensitivity owing to focusing hard X-ray optics, and solid state detectors, such as Cadmium Zinc Telluride (CZT) or Cadmium Telluride (CdTe). The development of focusing mirrors enables collecting photons over large areas and focusing them on small detector elements achieving excellent signal to background
ratios. The first balloon-borne focusing hard X-ray telescopes were InFOCµS, HERO, and HEFT, flown for the first time in 2001, 2001, and 2005, respectively. The HERO experiment used gas proportional counters in the focal plane (Ramsey et al., 2002), InFOCµS, and HEFT used CZT detectors (Tueller et al., 2012; Baumgartner et al., 2003; Harrison et al., 2000; Chen et al., 2006).

The first test of CZT detectors in space-like environments were carried out with the PoRTIA experiment, flown in June 1995 (Parsons et al., 2004). PorTIA was flown over Palestine, Texas and Alice Springs, Australia in three different configurations with respect to passive and active shielding. It observed the spectra of multiple background environments using several shielding configurations. The HEXIS experiment performed similar measurements with a fully active CsI(Na) shield in 1997 (Slavis, 1999). The EXIST team worked on the development of a wide field of view coded mask X-ray telescope (Barthelmy et al., 2005; Goldwurm et al., 2001), but the results concerning the detector background are not directly relevant here as the background of a wide field of view instrument is strongly dominated by the aperture flux. SWIFT’s Burst Alert Telescope made use of a coded mask for the first spaceborne CZT instrument with a detector area of 5240 cm². Launched in 2012, the spaceborne NuSTAR telescope combines 10 m focal length focusing X-ray optics with CZT detectors (Harrison et al., 2013). Currently, SWIFT’s BAT and NuSTAR are the only CZT instruments in space.

### 3.3 X-Calibur’s Principle of Detection

X-Calibur’s general principle of detection is illustrated in Figure 3.3. A beam of X-rays from an astrophysical target enter the detector with a set of true Stokes parameters outlined in Equation 2.4. Independent of polarization, X-rays scatter or photoelectrically
Figure 3.3: X-Calibur’s principle of detection for the configuration flown during Fall 2014. Left: The detector is comprised of a scattering element, rings of pixelated CZT detectors arranged around the element, and a PMT coupled to the scatterer, serving as a coincidence detector. An X-ray with a linear polarization with respect to a reference coordinate system, shown as a black dotted line, enters the detector and Compton scatters within a scatterer/scintillator element. The scattered X-ray then impinges upon the CZT that records its position and energy. As some X-rays will backscatter, one ring of CZT detectors is placed in front of the scatterer to capture those events. Right: the beam will scatter and deposit energy in the CZT detectors surrounding the scattering element. The distribution of the azimuthal scattering angles depends on the beam’s linear polarization and polarization fraction.

absorb within the scatterer at a depth that depends on the interaction cross-sections, shown in Figure 3.19, and X-ray energy. The photons of an unpolarized beam will have azimuthally symmetric polarization and scatter without preferred direction onto the CZT surrounding the scatterer. Linearly polarized X-rays will scatter preferentially in a direction perpendicular to their electric field vector. Experimental data observing unpolarized and polarized beam detection are shown in Figure 3.4.

A figure of merit for the polarimeter is the modulation factor, $\mu$, defined by the minimum and maximum counts, $C_{\text{Min}}$ and $C_{\text{Max}}$, when observing a 100% polarized beam as in Figure 3.4:

$$\mu = \frac{C_{\text{Max}} - C_{\text{Min}}}{C_{\text{Max}} + C_{\text{Min}}}.$$  \hspace{1cm} (3.3)

For a given beam, we can define the minimum detectable polarization (MDP) with a 99% confidence level:

$$\text{MDP}(t) = \frac{4.29}{\mu R_s} \sqrt{\frac{R_S + R_B}{t}}.$$  \hspace{1cm} (3.4)
Figure 3.4: X-Calibur raw experimental data from the 40 keV beam at the Cornell High Energy Synchrotron Source (CHESS). *Left* shows an unpolarized beam while *right* shows a 100% polarized beam. Pixel maps are shown for all CZT rings. The color gives the number of hits per pixel on a linear scale. Figure excerpted from Beilicke et al. (2014).

where $R_S$ is the event rate from the beam, $R_B$ is the event rate from background, and $t$ is the observation time.

An alternative analysis method is discussed in Krawczynski (2011) based on a maximum likelihood analysis. Kislat et al. (2015) discusses the reconstruction of polarization fraction and angle energy spectra based on deconvolution techniques.
3.4 The X-Calibur Experiment

The X-Calibur experiment flown in Fall 2014 was comprised of the InFOCμS X-ray telescope and the X-Calibur polarimeter. The high-level specifications are given in Table 3.1.

3.4.1 InFOCμS Telescope and Support Structure

The InFOCμS telescope was developed jointly by NASA GSFC and Nagoya University to image hard X-rays in the 20-80 keV band (Ogasaka et al., 2005a). With a Wolter-I optical design, it is comprised of 255 shells of the paraboloid primary and hyperboloid secondary mirrors that focus a parallel X-ray beam as shown in Figure 3.5. The shell surfaces are made of Pt/C multilayers. X-rays reflect off the Pt/C interfaces and the layer spacing assures a coherent superposition of the partial waves from the different layers. At higher energies, X-rays begin to penetrate the shell layers reducing the effective area of the mirror. The mirror’s effective area as a function of energy and off-axis angle is shown in Figure 3.6.

Table 3.1: Summary of X-Calibur and InFOCμS specifications. The MDP for Crab is calculated for an observation time of 5 h at Ft. Sumner.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telescope Focal Length</td>
<td>8 m</td>
</tr>
<tr>
<td>(Telescope) Effective Area (30 keV)</td>
<td>50 cm²</td>
</tr>
<tr>
<td>PSF 75% Containment Radius</td>
<td>2.5’</td>
</tr>
<tr>
<td>Field of View</td>
<td>10’</td>
</tr>
<tr>
<td>CZT Area</td>
<td>128 cm²</td>
</tr>
<tr>
<td>Science Energy Band</td>
<td>20-60 keV</td>
</tr>
<tr>
<td>Detector Configuration</td>
<td>5 rings (5 mm) &amp; 3 rings (2mm)</td>
</tr>
<tr>
<td></td>
<td>4x(8x8) px/ring</td>
</tr>
<tr>
<td>Energy Resolution (FWHM)</td>
<td>0.05 E (5 mm) &amp; .11 E (2 mm)</td>
</tr>
<tr>
<td>MDP</td>
<td>11.9%</td>
</tr>
</tbody>
</table>
Figure 3.5: InFOCµS schematic. Far-field light is focused by reflecting off the primary and secondary nested mirror shells at grazing incidence. These X-rays are focused at the focal plane 8 m away. Figure adapted from Shibata et al. (2005).

The mirror is supported by a 8 m long truss. Whereas the 2014 flight used the InFOCµS truss and pointing system with an inertial pointing system, the 2016 flight uses a new carbon fiber-aluminum truss pointed by the Wallops Arc Second Pointer (WASP) (Rhodes et al., 2012a). For the 2014 flight, the telescope truss was held by a high-pressure ball joint, enabling pointing with an accuracy of 15-45" RMS in elevation and azimuth. Electrical and data interface is performed by a Mercotac 830-SS rotating ring contact (Beilicke et al., 2015; Mercotac, Accessed 3/2016).

3.4.1.1 Experimental Characterization of InFOCµS

During flight, the truss may deflect due to non-uniform thermal loading from exposure to sunlight causing the its focal point to drift from the optical axis. If not corrected, this offset between the true and expected focal point will result in the scattering plane no longer being normal to the detector plane. This will result in systematic errors in reconstructing the polarization fraction and angle. The offset can be corrected provided there is knowledge of truss deflection, the effect of off-axis pointing, and the telescope point spread function.
Figure 3.6: Left: Optimal InFOC\(\mu\)S effective area. Right: Encircled flux for 20 and 60 keV monochromatic beams, in black and teal, respectively. Both charts adapted from Shibata et al. (2005).

(PSF). Two techniques were used in X-Calibur’s recent 2016 flight to track deflection. One method tracks the deflection of LEDs mounted near the detector using a camera mounted on the InFOC\(\mu\)S mirror. The second method tracks the translation and rotation of the ellipsoid of a laser projected from the mirror assembly on a CCD chip near the detector.

To prepare for X-Calibur’s Fall 2016 flight, I visited NASA GSFC in May 2016 to measure InFOC\(\mu\)S’s PSF using their 100m X-ray beam line. These measurements had two objectives. The first objective was to verify the state of InFOC\(\mu\)S’s mirror after three balloon flights since its last PSF measurements. The second objective was to use a CZT detector in order to determine the PSF’s energy dependence.

A single X-Calibur 5mm CZT 8x8 pixel array was selected to image the beam at its focal point. However, the pixel array’s instantaneous field of view, 0.559”, is greater than the predicted pointing stability, 0.33”, let alone truss deflection. In order to sufficiently resolve the beam, I developed a technique for taking subpixel measurements by translating the pixel array through at points on a grid of subpixel spacing. The PSF is then proportional to the total number of counts per grid point divided by the exposure time of that grid point.
Figure 3.7: InFOCμS energy-dependent PSF as characterized in 2016. The spatial units are that of grid coordinates [0.5 mm] and flux is cm$^{-2}$.

Prior to the experiment, the subpixel measurement technique was validated with a toy simulation, and the detector was calibrated using Eu 152 and Am 241 sources. Results from the Am 241 calibration were used for flat-fielding pixel response. For the experiment, a bremsstrahlung 50 keV X-ray source was positioned 100 m from the InFOCμS mirror. The mirror was positioned $\sim 8.3$ m away from the detector to account for the divergence of the X-ray source. The beam-line was first centered using an X-ray sensitive CCD camera. The CCD image was compared to the image from the centered position of the CZT array to validate that the CZT was properly centered for my measurements.
Figure 3.8: InFOCμS encircled flux for four energy bins. The angle offset $\alpha$ from the optical center assumes the optical center for events recorded within a given energy bin.

For grid measurements, I used a relatively high beam current to generate good statistics for each coordinate. As a result, some events above 55 keV were registered owing to multiple events occurring within the pixel integration time. This implies that a similar effect affected lower energies as well. To image the PSF, 97 five-minute beam runs were taken with a grid spacing of 0.5 mm, corresponding to $\theta = 0.2''$. The energy-dependent PSF was reconstructed from these runs as seen in Figure 3.7 and the encircled flux as a function of distance to the optical center is shown in Figure 3.8.

There are two key findings from this experiment. First, there is good agreement between the past measurement of the InFOCμS PSF and the current measurement. Namely, encircled flux of both are nearly identical as can be seen on Figures 3.6 and 3.8. Second, the optical center in the detector plane is shown to slightly vary as energy is increased. At 44-48 keV, the optical center is shown to be $0.2 \pm 0.283''$ away from the optical center at 20-24 keV.

While at GSFC, a separate test was performed to determine the efficacy of an CCD camera mounted on the mirror assembly for checking alignment. The camera was installed and removed three times with images taken each time it was in place to determine the
consistency of the camera image center. We observed that the center was consistently shifted by about $\sim 1$ cm. As a result, the CCD camera was glued into its position.

### 3.4.2 X-Calibur Instrument

The polarimeter, shown in Figure 3.9, consists of a scattering element made of a 14 cm long and 1.3 cm diameter plastic scintillator read out by a photomultiplier surrounded by four panels each carrying $8 \times 2 \times 2 \text{ cm}^2$ CZT pixel array detectors. In the following, I refer to a detector ring as a set of four CZT detectors surrounding the scattering element on the four sides at a given depth along the optical axis. The ring R1 is the closest to the instrument aperture while R8 is the furthest. The plastic scatterer extends in height from the base of R8 to the top of R2. With this configuration, X-rays backscattered from the top of the stick impinge on R1. To minimize systematic errors associated with instrument asymmetries, the polarimeter is rotated continuously at $\sim 2$ RPM about the optical axis. The angle between the rotating polarimeter structure and the non-rotating mounting structure is read out by a code wheel with an accuracy of $1^\circ$. The design, properties, and calibration of X-Calibur are detailed in Beilicke et al. (2014) and summarized here.

#### 3.4.2.1 CZT and Readout Electronics

X-Calibur’s rings R1-R5 are 5 mm thick CZT detectors while rings R6-R8 are 2mm thick. Each thickness offers its own advantage over the other. The 5 mm CZT offers greater energy resolution at the expense of more susceptibility to background as compared to the 2 mm CZT.

Each detector is contacted with a 64-pixel anode grid of 2.5 mm pixel pitch and a monolithic cathode facing the scattering element. There are a total of 2048 data channels.
The cathodes of R1-R5 are biased at -500 V while cathodes of thinner R6-R8 are biased at -150 V. Each 64-pixel detector is bonded to a ceramic chip carrier that is plugged into the read-out electronic boards. These are comprised of a pair of Application Specific Integrated Circuits (ASICs) and Analog-Digital Converters (ADCs). Events are read out using two 32-channel self-triggering ASICs measuring the charge deposited in each pixel. Each ASIC reads in data from 32 pixels corresponding to one half of the 64-pixel detector. The ASICs are operated at medium amplification of 28.5 mV/fC and a signal peaking time of 0.5 µs. The ASIC readout noise is a function of temperature, as shown on Figure 3.11. For each side of the polarimeter, the ADC is read out by a single harvester board that relays the data to
a PC-104 computer with a rate of 6.25 Mbps. The anode side of one of the CZT detectors and a detector-ASIC board package are shown in Figure 3.10.

The detection of > 20 keV photons triggers the data acquisition. The ASICs acquire data over a 10 \( \mu s \) window after triggering; in this window, data from multiple pixels can be recorded. Following the triggering of a specific ASIC, a 130 \( \mu s \) dead time exists for that ASIC to guarantee the uninterrupted digitization and transfer of the detected amplitude information. The dead time only effects the particular ASIC being read out; all other ASICs continue to take data.

### 3.4.2.2 Anti-Coincidence Detection and Shielding

The polarimeter and the frontend readout electronics are positioned inside an active CsI(Na) shield that also serves as an anti-coincidence detector (ACD). The CsI is hermetically sealed in 5 mm Al casing. The active shield has 2.7 cm side wall thickness and 5 cm rear wall thickness “closed” at the front by a 1.4 cm thick tungsten plate and collimator. The collimator has a tapered interior with a diameter of 4.150 cm at the opening and 3.175 cm
at its base. A 5 mm thick tungsten plug at the base of the shield accommodates the wire harness while not permitting a direct LOS to the CZT detectors for incident radiation. The CsI shield is read out by four Hamamatsu R6233 PMTs biased at ≈ +800 V. A programmable digitizer flags CZT events that coincide in a 6.1 µs anti-coincidence window.

3.4.2.3 Scatterer/Scintillator

The polyvinyltoluene scattering element (H:C=5.17:4.69, hZi=3.4, ρ = 1 g cm⁻³, decay time 2.1 ns) performs two functions. First, it scatters incident X-rays into the CZT. Second, it acts as a coincidence detector (CD) as scintillation light produced from X-ray interactions is detected by a Hamamatsu R7600U-200 photomultiplier tube (PMT). True source events are defined as events with a CD trigger occurring within the 6.1 µs following a CZT event without the triggering of the ACD. Given its position deep within the detector and small geometry factor, the stick provides an excellent means of discriminating against background.
To increase scintillation light collection efficiency, the stick is wrapped in white Tyvek and the end facing the aperture is covered by a thin layer of Al.

### 3.4.3 Data Analysis

X-Calibur events are triggered by pixel events in the CZT. Up to nine simultaneous triggered channels are stored. Each recorded X-Calibur event contains an event number, a GPS time stamp, the code wheel angle, a list of CZT detector pixels with their digitized pulse heights, and flags from the CD and ACD. The \texttt{PlotChannels} package has been written in part to reconstruct unrotated, single- and multi-pixel events from raw ASIC and rotation wheel data. Through the channel calibration procedure outlined in Beilicke et al. (2014), the event energy is defined as $E = \sum_i E_i$ for $i$ pixels included in the event. Cuts can be made on single and multipixel events. X-Calibur flight data used in validating my background models was first processed through \texttt{PlotChannels}.

### 3.4.4 The Fall 2014 Flight of X-Calibur

On September 24, 2014, X-Calibur was launched from Ft. Sumner, New Mexico. The experiment ascended for two and a half hours and floated for about five hours at a flight altitude of 39,000 m and local atmospheric depth of 3.45 g cm$^{-2}$. During the flight a fault in the telescope’s pointing system prevented sustained astrophysical observations. Approaching inclement weather forced the mission to terminate before the pointing issue could be resolved. The X-Calibur instrument was recovered after the flight and was observed to be undamaged.

The balloon was launched at 51600 s Modified Julian Date (MJD) and the ascent to the observing depth of 3.45 g cm$^{-2}$ took about 9200 s. Instrument and telemetry data was taken on ascent and is shown on Figure 3.12. The Pfotzer maximum, defined as the altitude
where the greatest background flux is observed, is energy-dependent. X-Calibur data shows that maxima range between 60-100 mbar, corresponding to a GPS altitude of 20-23 km, for 50-4000 keV. The energy dependence of the Pfotzer maximum is shown in Figure 3.13. I discuss the Pfotzer maximum and its implications for assessment of atmospheric model errors in Section 4.2.

Telemetry data for the telescope truss ceased at 66565 s though instrument data was recorded until 77000 s. For most of the flight, the telescope and detector assembly rotated about the gondola axis at a rate of $\approx 0.2$ Hz while the telescope was pointed at an average elevation angle of $9.26^\circ$ above limb. This included a several minute test of the pointing
system where the optical axis was raised to an elevation angle of $51^\circ$. The cycling of truss elevation, corresponding to the elevation of X-Calibur’s aperture, is shown in Figure 3.14.

Otherwise, the polarimeter performed as expected resulting in five hours of uninterrupted measurements of the high-energy background at the observing altitude. The polarimeter rotation about the optical axis was unaffected by the pointing system failure though it was commanded to stop after 1.9 ks of rotation at the observing altitude.

I use and evaluate the data from 8.8 ksec of active observation time at a depth of approximately $3.45 \text{ g cm}^{-2}$ for validating the background models derived in Chapter 5. The average event rate was 297.47 Hz for all events and 127.44 Hz for single pixel events. The time-averaged energy spectra of single pixel events for various trigger conditions are shown in Figure 3.15.

The post-flight analysis showed that a setting of the flight data acquisition system resulted in additional dead time that was not attributable to the ASIC dead time. To account for this additional dead time, I adjusted observation time by removing all time between events where $\Delta t > 0.2$ s.
3.5 Potential Modifications to X-Calibur

Although the 2014 X-Calibur flight did not perform astrophysical observations it did validate the detector in its operational environment and provided the data used in this dissertation. X-Calibur’s second flight launched from Ft. Sumner on September 17, 2016 while there are additional flight opportunities launching from McMurdo Station in 2018 and as a space-based observatory. For the Antarctic flight and space observatory, potential modifications can be made to improve the instrument’s performance.

3.5.1 CZT and ASIC Modifications

Advancement in semiconductor technology has resulted in improved capabilities of ASICs which read out the channel’s change in voltage during a pixel event. While use of a pixelated anode allows for detection of events in two spatial dimensions, material non-uniformity and electron-hole trapping hampers efforts to better constrain the event with only channel voltage data. Zhang, et al. have developed 3D ASICs which achieve better...
Figure 3.15: Observed flux by X-Calibur at flight, 3.45 g cm$^{-2}$, in October 2014. Solid line, dashed line, and dotted line represent all single-pixel events, events passing the ACD veto trigger, and events coinciding with the CD trigger while passing the ACD veto trigger. While no minimum threshold was set for observations, data below 20 keV is not used in this study and is shown in green. X-Calibur science investigations utilize polarization measurements in the range $20 < E < 60$ keV, shown in grey.

Energy resolution and background suppression through their use of timing to reconstruct event depth (Zhang et al., 2004; Zhang et al., 2015; Zhang et al., 2012; Zhang et al., 2014). The principle of operation of 3D ASICs is described in Figure 3.16.

The continued refinement of 3D CZT/ASICs have led to the development of the Brookhaven National Lab’s H3Dv3 ASIC, with 128 negative charge amplification channels and 2 positive charge amplification channels (Zhang et al., 2012). The Krawczynski group requested a modified version of this ASIC with preamplifiers optimized for the X-Calibur energy range. Its adoption would provide several distinct advantages over the ASICs currently used in X-Calibur. Their low-noise characteristics allow for a 2 keV energy threshold rather than the current 20 keV threshold. The determination of the 3D event location allows for additional background discrimination as X-rays are expected to interact close to the detector surface. Currently, X-Calibur ASICs have 32 channels and two ASIC boards are required for each 8×8 CZT pixel array. As shown in Figure 3.17, modified H3DV3
Figure 3.16: *Left*: schematic of a 3D ASIC where a photon deposits energy twice within the CZT to initiate a two-pixel event. The incoming photon first interacts in the cathode but then twice in the CZT. In each CZT interaction, an electron-hole cloud pair forms that move to the anode and cathode, respectively, inducing charge on the readout contacts. The voltage ramping and time delay seen on the anode and cathode are a function of $z_i$. *Right*: ASIC timing diagram for what occurs left. The ASIC routine begins as soon as the cathode trigger is raised. Charge continues to build for a time $t_{\text{delay}} \approx 1 \mu s$ after the cathode trigger. After $t_{\text{delay}}$, the hold circuit allows voltages to be read out. Figure adapted from Zhang et al. (2004).

ASICs with 128 channels would allow for one ASIC per two CZT detectors, allowing for a more compact design. This additional space may allow for future shielding modifications.

### 3.5.2 Background Suppression

One of the major goals of this dissertation is to find new methods of improving X-Calibur’s observational performance by reducing the background event rate. Though the background rate over Ft. Sumner and other locations with relatively high geomagnetic rigidity cutoffs is unlikely to effect X-Calibur observation, flights over locations with near zero cutoff will see a dramatic rise in background as detailed in Chapter 5. To limit its contribution we consider a parametric space of potential shielding modifications. The methods and results of this study are reported in Chapter 7.
The current configuration utilizes an active, CsI shield to surround the base and sides of the eight ring polarimeter and a passive Tungsten top and collimator above the polarimeter. Although many modifications can be made to improve performance, these are constrained by programmatic limitations of schedule and budget as well as mechanical constraints of mass and gondola integration. Potential modifications must acknowledge these constraints. The parametric space of these modifications are reviewed in this section and shown in Figure 3.18. In Chapter 7, I perform simulations incorporating these modifications to determine optimal shield design for a future flight over Antarctica.

To reduce background from side-entrant flux, I evaluate the incorporation of additional passive shielding outside and inside the active shield. All other things being equal, it is well known that it is preferential to place passive shielding outside of the actively shielded volume. In this configuration, secondary events and events due to activation in an exterior passive shield can be vetoed by the active shield. However, mounting shielding outside of
the active shield requires more surface area for equivalent thickness and solid angle coverage, resulting in increasing mass for equivalent coverage. External mounting would also require re-engingeering the mechanical support structure whereas internal mounting may be trivially simple provided the required volume is available, as through the reconfiguration of the ASICs. The base of the active shield has a similar design trade with the additional constraint that all cabling is run through the tungsten plug located at the base.

To limit the entry of background through the front portion of the shield facing the X-ray mirror, the existing tungsten plate may be increased in thickness or replaced with a new active shield. Additionally, the collimator geometry may be varied to reduce the number of stray X-rays otherwise contributing to aperture flux. A final variation in configuration would be the removal of CZT detectors that demonstrate poor noise performance based on their thickness or positioning within the shield. While this may improve signal to noise ratio it would reduce the signal rate.
Figure 3.19: Mass attenuation for: left, Eljen polyvinyltoluene scatterer and right, LiH scatterer. Pair production is not shown here.

3.5.3 PolSTAR

Developing X-Calibur into a space observatory provides unique opportunities and challenges that require redesign to optimize the X-Calibur detector concept for spaceflight. From an engineering perspective much can technically be shared with a previous NASA Explorer-class mission, NuSTAR, thus the X-Calibur redesign is named PolSTAR. However, NuSTAR’s focal plane imaging CZT detector assembly has a collecting area of 16 cm$^2$, considerably less than PolSTAR’s 136 cm$^2$.

Without atmospheric attenuation that limits the observable spectrum to $> 20$ keV, PolSTAR can perform observations over a broad 3-80 keV energy band, making it possible to address many of the science questions discussed in Chapter 2. No longer limited by atmospheric attenuation the energy threshold is lowered by selecting LiH instead of polyvinyltoluene for the scattering element. Photoelectric and Compton cross-sections are equal at 9.6 keV in LiH as shown in Figure 3.19. To reduce the number of channels for data handling and to reduce the size of the polarimeter for mechanical compliance with the NuSTAR heritage, the polarimeter for PolSTAR has four detector rings as opposed to eight. An additional pixel array is located perpendicular to the optical axis below the scatterer that
Figure 3.20: Physical configuration of PolSTAR is based on the NuSTAR spacecraft with the major difference being the replacement of NuSTAR’s CZT focal plane array with the PolSTAR scattering polarimeter. The active shield around the polarimeter is omitted in this illustration.

may perform reference imaging. Unlike NuSTAR, it uses a single mirror for the telescope reducing the mass budget.
While our atmosphere provides the air and water necessary to maintain life on earth it also serves as a radiation shield, continuously absorbing the energy of cosmic rays that would otherwise endanger life on earth. Cosmic rays have energies ranging from $\approx 10^6 - 10^{20}$ eV. Their energy is absorbed in the atmosphere not by a single interaction but through an event cascade of particle creation, annihilation, decay, inelastic scattering, absorption, and radiation. Trajectories of air showers run the entire length of the atmosphere for some particles. Some secondary particles, like neutrinos, reach the surface without depositing energy while others leave the atmosphere, constituting earth’s radiation albedo.

As will be discussed in Section 4.1, the cosmic ray flux and the atmosphere itself is effected by the sun and earth’s magnetic field. The sun goes through periodic, 11-year cycles where its sunspot number fluctuates. The sunspot number is proportional to the magnetic field magnitude, thus at solar maximum the magnetic field is strongest. The cycle responds to the variation of solar modulation, an effect that varies the flux of incident galactic and anomalous cosmic rays entering the heliosphere. Near earth, cosmic rays and solar protons are deflected by earth’s magnetic field. The deflection effect results in a rigidity cutoff $R_{\text{cutoff}} \approx 0-60$ GeV that varies as a function of location and orientation of the observer.
Figure 4.1: Qualitative description of the effects of solar modulation and geomagnetic rigidity cutoff on flux of cosmic rays observed at earth. First, solar modulation deflects cosmic rays with energy per nucleon less than the solar modulation factor. Of what remains, particles with less energy per nucleon than the local rigidity cutoff are deflected by earth’s magnetic field.

To remove the blinds of atmospheric attenuation, astrophysics missions are lofted to small atmospheric depths, 1-10 g cm$^{-2}$, at an approximate altitude of 35-40 km above sea level. Although the atmospheric mass at small depths results in low interaction probabilities for particles and photons, primary cosmic rays enter the atmosphere with a flux independent of zenith angle. As zenith angle increases so does the distance from the top-of-the-atmosphere to the observer, resulting in a greater effective atmospheric depth, interaction probability, and consequently secondary production. At greater depths, the atmospheric attenuation of cosmic x-rays increase as does the local production of secondaries. The highest flux of charged particles in the atmosphere occurs at the Pfotzer maximum, typically around 20-25 km in altitude.

4.1 Effects of the Solar Cycle and Geomagnetism

Both solar modulation and the geomagnetic rigidity cutoff reduce the observed flux by reducing the flux of incoming primary cosmic rays which consequently reduce the production of atmospheric secondaries. This joint effect is summarized in Figure 4.1. At earth, the effect of solar modulation is strictly a function of the solar cycle while geomagnetic rigidity cutoff is
4.1.1 Solar Cycle Effects

The transport of cosmic rays from their source to an earthbound observer is affected locally by the solar cycle and earth’s magnetic field. The 11-year solar cycle itself is a result of the 22-year Babcock solar dynamo cycle (Babcock, 1961). Through the convection of plasma within a differentially rotating solar atmosphere, a magnetic field emerges. As a result of the sun’s physical scale and dynamic nature of convection, local variability produces a magnetic field that must be described to orders beyond dipole. At solar maxima, the Babcock model predicts a poloidal dipole field at its weakest magnitude and a toroidal quadripolar field at its strongest. This increased toroidal magnetic field causes magnetic pressure to equal and sometimes exceed the local gas pressure. This results in magnetic flux lines rising beyond the photosphere. These coronal loops are observable with plasma transiting along the field lines.
The solar magnetic field itself co-rotates with the sun, affecting the outflow of energetic charged particles leaving the corona. This interaction creates a heliospheric current sheet (HCS) in the shape of a spiral as shown in Figure 4.2 (Parker, 1958). As galactic cosmic rays enter the heliosphere they interact with the HCS, reducing their energy and modulating their passage (Parker, 1965). This modulation is defined as the solar modulation factor $\phi$ that limits the cosmic ray flux within the solar magnetosphere. The modulation factor is at a local maximum during each solar maximum. Similarly, the modulation factor is at a minimum at solar minima. This is empirically observed through the neutron flux detected on earth’s surface as primary cosmic rays create secondary neutrons in the atmosphere. Per Usoskin et al. (2011), the neutron flux at the surface is related to the modulation factor by:

$$N = \sum_i N_i = \sum_i \int_{K_{ci}}^{\infty} J_i(K, \phi) Y_i(K) \, dK,$$

(4.1)

where $N$ are total counts, $i$ reflects the different species of cosmic rays (e.g. Hydrogen, Helium, etc.), $J$ is the modulated cosmic ray flux $[\text{m}^2 \text{s sr GeV/nucleon}^{-1}]$, $K$ is the energy per nucleon [GeV/nucleon], $K_{ci}$ is the energy corresponding to the local geomagnetic rigidity cutoff discussed below, and $Y$ is the specific yield function associated with species, $i$. The cosmic flux is then related to the modulation factor by:

$$J_i(\phi, K) = J_{\text{LIS},i}(K + \Phi_i) \frac{K(K + 2K_r)}{(K + \Phi_i)(K + \Phi_i + 2K_r)},$$

(4.2)

where $\Phi_i$ is the true modulation defined as $\Phi_i = \frac{Z_i}{A_i} \phi$, $K_r = 0.938 \text{ GeV/nucleon}$, and $J_{\text{LIS}}$ is defined as the local interstellar flux, unmodulated by the sun (Usoskin et al., 2011). This is further defined as:

$$J_{\text{LIS},i}(\phi, K) = \frac{1.9 \times 10^4 P(K)^{-2.78}}{1 + 0.4866 P(K)^{-2.58}},$$

(4.3)

where $P(K) = \sqrt{K(K + 2K_r)}$ (Burger et al., 2000).
Figure 4.3: Monthly solar modulation factors from 1937 to 2012 as catalogued in Usoskin et al. (2011) exhibit a periodic yet chaotic cycle, akin to terrestrial weather.

Solar modulation shares the same period as the solar cycle, with minima of $\approx 250$ MV and maxima of $\approx 1250$ MV as seen in Figure 4.3. Variation in solar modulation has two effects on balloon-borne and low earth orbiting observatories. First, as solar modulation increases primary cosmic ray flux decreases. PAMELA data taken from July 2006 to December 2009, when the solar modulation varied from $\phi = 423$ MV to 255 MV. During this period, the integrated flux from 82 MeV to 48 GeV changed by 22.18%. Figure 4.4 shows the variation in spectrum from 1997 to 2002 as observed by BESS over Lynn Lake, Canada (Shikaze et al., 2007). An increase in the primary cosmic ray flux itself directly results in an increase of the secondary flux of all species as detailed below. A second effect of the solar cycle and varying solar modulation is atmospheric depletion due to an increase in cosmic ray flux. Atmospheric depletion is a result of sputtering and scattering interactions of impinging cosmic rays and high-energy secondaries. Depletion is moderated by geomagnetic effects as described in atmospheric models. For instance this work uses the COSPAR International Reference Atmosphere 2012 (CIRA 2012) that provides atmospheric profiles for solar minima, maxima, and mean activity (COSPAR, 2012).
Figure 4.4: Variation in primary, H+ cosmic ray flux, from 1997 to 2002, as observed by BESS over Lynn Lake, Canada ($R \approx 0.4$ GV Shikaze et al., 2007). Solar modulation factors shown in the legend are per Usoskin et al. (2011). The model values shown are those based on the results of my work described in Section 5.4.2.

For balloon missions the effect of atmospheric depletion is solely a function of atmospheric depth - the more depleted the atmosphere is, the higher the altitude above sea level a balloon may be lofted for a given balloon pressure. Provided measurements are taken at an identical atmospheric depth through a solar cycle, depletion should not noticeably affect observed fluxes.

4.1.2 Geomagnetic Effects

Although solar modulation and the geomagnetic rigidity cutoff both limit cosmic ray fluxes at earth, the mechanisms for doing so are different. An instrument’s proximity to the center of earth’s dipole field results in the variation of the rigidity cutoff as a function of geocentric distance, geomagnetic latitude, and orientation. The canonical work describing the effects of geomagnetism was written by Störmer and summarized by Smart and Shea (Störmer, 1956; Smart and Shea, 1985). For a charged particle in a 3D dipole field, the
equations of motion that define the particle’s acceleration as a function of its velocity vector and magnetic field vector read:

\[
\begin{align*}
\frac{dv_r}{dt} &= \frac{e(v_\theta B_\phi - v_\phi B_\theta)}{mc} + \frac{v_\theta^2}{r} + \frac{v_\phi^2}{r}, \\
\frac{dv_\theta}{dt} &= \frac{e(v_\phi B_r - v_r B_\phi)}{mc} + \frac{v_\phi v_r}{r} + \frac{v_\phi^2}{r \tan \theta}, \\
\frac{dv_\phi}{dt} &= \frac{e(v_r B_\theta - v_\theta B_r)}{mc} + \frac{v_\theta v_r}{r} + \frac{v_\theta v_\phi}{r \tan \theta},
\end{align*}
\]  

(4.4)

where \( \vec{v} \) and \( \vec{B} \) are decomposed into their spherical components in the same coordinate system. No closed form solutions exist, though Störmer defined a special case with respect to the energy per nucleon, defined as the rigidity cutoff (Smart and Shea, 1985):

\[
R_{\text{cutoff}}(\lambda, l, \theta, \zeta) = \frac{M \cos^4 \lambda}{l^2[1 + \sqrt{1 - \sin \theta \sin \zeta \cos^3 \lambda}]^2},
\]  

(4.5)

where \( M=59.6 \) GV is the maximum rigidity cutoff factor, \( \lambda \) is the geomagnetic latitude, \( l = 1 + \frac{a}{r_e} \) where \( \frac{a}{r_e} \) is the ratio of altitude above sea level to earth’s radius, \( \theta \) is the orientation’s off-zenith angle with respect to the local zenith angle, and \( \zeta \) is the orientation with respect to azimuthal angle as measured clockwise from local magnetic north. Note, that this formula applies to positively charged particles, such as cosmic ray protons. For negatively charged particles, the dependence on azimuthal angle is reversed. The result of Störmer’s equation is a unique geometry where rigidity cutoff varies proportional to \((\sin \theta \sin \zeta)^{-1/2}\) in allowed and forbidden cones as illustrated in Figure 4.5.

I evaluate the effect at all geomagnetic latitudes for zenith oriented and limb-oriented toward the east and west in Figure 4.6. Figure 4.6 shows the forbidden, mean, and lowest (allowable) rigidities as a function of geomagnetic latitude. While eastward facing at limb results in the highest effective rigidity cutoff, this is significantly more pronounced at the geomagnetic equator than at other latitudes. Similarly, the ratio between maximum and
Figure 4.5: **Left**: Qualitative description of Störmer cones and coordinate system with respect to an instrument. Forbidden region to the East has a higher rigidity cutoff while allowed region to the West has lower rigidity cutoff, meaning that more particles are incident from the West. **Right**: Rigidity cutoff at float altitude above Ft. Sumner, NM.

Figure 4.6: Maximum, minimum, and vertical rigidity cutoff as a function of geomagnetic latitude at 40 km altitude. A pink line denotes Ft. Sumner’s geomagnetic latitude.

minimum cutoff is pronounced at equatorial latitudes and approaches $\approx 1$ at the magnetic pole.

The physical effect of rigidity cutoff is simple. At lower cutoff rigidities, more flux enters the atmosphere resulting in more secondary flux. As X-Calibur rotated on the cardinal plane, its background flux was azimuthally averaged. Thus to estimate the cutoff at a location, I determined the average cutoff over the solid angle subtended by the sky per Equation 4.6:

$$\bar{R} = \frac{\int R(\theta, \zeta) \lambda, d\Omega}{\int d\Omega}.$$  

(4.6)
Figure 4.7: The trajectory of reentrant albedo as trapped charged particles. Splash albedo below the local cutoff leaving the atmosphere follows a helical path along the magnetic field line and reenters at the magnetic conjugate. For simplicity, the figure shows the magnetic pole to be equivalent to the geographical pole.

Otherwise, a higher fidelity model can account for the flux’s angle dependence as input for atmospheric attenuation and secondary generation.

A second geomagnetic effect is on albedo radiation, or atmospheric secondaries. The two types of albedo are splash and reentrant. Splash albedo is defined as atmospheric secondaries with an upward velocity. For charged splash albedo produced near the top of the atmosphere, it may be possible for the particle to escape the atmosphere with only negligible energy loss. When the particle’s kinetic energy is less than the local rigidity cutoff, the splash albedo may travel “for free” to the geomagnetic conjugate along the magnetic field line, e.g. a proton with $K < 3.7$ GeV may travel to $\lambda = 43^\circ$S from $\lambda = 43^\circ$N. When it arrives at $\lambda = 43^\circ$N it will enter the atmosphere as reentrant albedo. Due to the non-uniformity of earth and its magnetic field, reentrant albedo trajectories occupy a range of angles near zenith. This process is summarized in Figure 4.7.
4.2 Definition of Atmosphere

Modeling the behavior of high-energy particles in earth’s atmosphere first requires defining the atmosphere. Over the past half century, many models have been developed to describe the relationship between density, temperature, composition, and other atmosphere parameters as a function of altitude. Global atmospheric models describe the totality of the atmosphere as opposed to range models that describe a portion of the atmosphere. Reported global and range models have been summarized by the American Institute of Aeronautics and Astronautics and NASA (AIAA, Accessed 7/2016; GSFC, Accessed 7/2016).

Atmospheric models may be constructed either theoretically or empirically. At higher altitudes, where data has historically been more scarce, the models are analytical. Both the International Standard Atmosphere and US Standard Atmosphere describe the atmosphere theoretically (NASA and Bureau, 1962; NOAA and USAF, 1976; organization for standardization, 1975). The Jaccina model describes the atmosphere above 125 km (Jacchia, 1965). Empirical models are constructed through analysis of remote sensing and flight data, as exemplified by the NASA Global Reference Atmosphere Model (GRAM) (Justus and Leslie, 2008). Some global models, like CIRA, are a patchwork of validated theoretical and empirical range models that together form a global model (COSPAR, 2012).

CIRA 2012, the reference atmosphere used in this dissertation, uses NRLMSISE-00 for structure, neutral temperature, and composition (Picone et al., 2002) and Jacchia-Bowman 2008/GRAM 2007 for mass density above 120 km (Bowman et al.; Justus and Leslie, 2008) in addition to other models describing temperature and wind profiles. CIRA 2012 lists diurnally-averaged depth and partial pressure profiles up to 900 km altitude for average solar mean, maximum, and minimum activity at the equator.
For all calculations involving atmospheric depth and composition I rely on CIRA 2012’s density profile and use the geometric convention in Figure 4.8. In Figure 4.8, $R_e$ is earth’s radius, $a$ is the geocentric radius of the instrument, $l$ is the line-of-sight (LOS) path with $L$ being the distance from instrument to position along LOS, $\alpha$ is the angle from instrument to the position along LOS that varies for path integration, and $\theta$ is the zenith angle along which the detector aperture is oriented. $L$ is defined as a function of these variables in Equation 4.7:

$$L(a, l, \theta) = \sqrt{a^2 + 2al \cos \theta + l^2 \cos^2 \theta + l^2 \sin^2 \theta}. \quad (4.7)$$

I use the relation above to take the path integral using Equation 4.8 along the LOSs from zenith to earth limb. At $\theta < 60^\circ$ the depth is relatively unchanged while $60^\circ < \theta < 96^\circ$ shows a geometric growth as the path through the atmospheric becomes increasingly larger. Figure 4.9 compares the result of this calculation for several global atmospheric models with
Global models for NRLMSISE-00 and MSISE-90 are described in a report by GSFC (Accessed 7/2016). For each, integration is performed from 41.69 km to the model-defined top-of-the-atmosphere. This is 900, 999, and 250 km for CIRA 2012, NRLMSISE-00, and MSISE-90, respectively.

respect to a fixed geometric altitude.

\[ \tau(\theta) = \int_{A_{\text{balloon}}}^{A_{\text{max}}} \rho(L(A_{\text{balloon}}, l, \theta))dl, \]  

(4.8)

where \(dl\) is the incremental path along \(L\) which is defined in Figure 4.8, where \(A_{\text{balloon}}\) is the geocentric distance of the balloon and \(A_{\text{max}}\) is the top-of-the-atmosphere, and the density function \(\rho(L)\) is defined with respect to geocentric distance rather than altitude.

Throughout the following chapter, I use an interpolation of the result of this integration. Table A.2 shows the result of evaluating for \(\tau(\theta) \ [\text{g cm}^{-2}]\) in Equation 4.8 in radians and degrees [°].

My key assumption in using atmospheric models is that balloon flights are at constant atmospheric depth rather than geometric altitude. This is a realistic assumption as float altitude is determined by balloon gas density, balloon leakage, and temperature variation.
One practical consequence of this assumption is that it reduces the impact of systematic errors of the atmospheric mass model by relying on the relative atmospheric density profile rather than absolute. Moreover, while atmospheric models account for variation in diurnal cycle, season, solar cycle, and location on earth, I can neglect their impact by further assuming the profile is unchanged. Finally, I assume the composition at altitudes above the balloon can be described by the mean atmospheric composition. While this is not true, its effects are minor given the relative difference in interaction cross-section of atmospheric species at low densities.

### 4.3 Production of Secondary Particles

Primary cosmic rays are the fundamental energy source that drive the generation of atmospheric secondaries that contribute to X-Calibur’s background. Although there are many interactions that generate atmospheric secondaries, in this section we overview the specific mechanisms that produce the dominant contributions to the X-Calibur background rate.

#### 4.3.1 Secondary X-Rays and Gamma Rays Production

During the 1970s, numerous balloon-based observations were performed to determine the diffuse cosmic X-ray background (CXB) which required an understanding of the atmospheric X-ray and \( \gamma \)-ray flux for background subtraction. Work by Makino (Makino, 1970), Litchi (Lichti et al., 1975), Schönfelder (Schönfelder et al., 1977) (Schönfelder et al., 1980), Ling (Ling, 1975), Kinzer (Kinzer et al., 1978) (Kinzer et al., 1974), Graser (Graser and Schönfelder, 1977), Ryan (Ryan et al., 1977), Costa (Costa et al., 1984), Gehrels (Gehrels, 1985), and Dean (Dean et al., 1989) describe the atmospheric \( \gamma \)-ray flux above \( \sim 300 \) keV.
Since these papers, little work has been done to understand the flux \( E_\gamma \lesssim 300 \text{ keV} \) or more precise experimental verification of the angular and depth dependence of the atmospheric \( \gamma \)-ray flux despite the numerous instruments flown capable of these measurements as reviewed in Chapter 3.

The atmospheric \( \gamma \)-ray flux has been modeled semi-empirically by Ling (1975), Graser and Schönfelder (1977), Schönfelder et al. (1977), and Lichti et al. (1975). Angular dependence of the flux is unfolded from observation. Gamma-ray source terms that define one specific type of emission, \textit{e.g.} production by electron bremsstrahlung, are calculated as a function of photon energy and atmospheric depth. This source term is then integrated over a solid angle and path to estimate flux as a function of energy and angle of arrival. However, there are unavoidable systematic errors in unfolding source terms from observations as they were made with scintillating detectors that offer coarse angular resolution. For instance, Ling and Gruber (1977) used a collimated NaI/CsI scintillator with a 50° (FWHM) field of view. So although the flux’s angular dependence can be unfolded through rotation of the detector in flight, the detector itself will permit photons with varying directions of arrival. Through this method conflicting predictions have emerged, as in the case of Graser and Schönfelder (1977) and Schönfelder et al. (1977). Their predictions for flux as a function of zenith angle between 1-10 MeV can be seen in Figure 5.15.

There are four dominant interactions involved in the production and transport of X-rays and \( \gamma \)-rays in the atmosphere. At hard X-ray and greater energies, electron bremsstrahlung due to cosmic ray and secondary electrons is the dominant contribution to the secondary \( \gamma \)-ray spectrum. At around 511 keV, electron-positron annihilation produces \( \gamma \)-rays of \( > 511 \text{ keV} \). Beyond 20 MeV, downscattered contributions to the spectrum from \( \pi^0 \rightarrow \gamma \gamma \) become the dominant contribution to the spectrum below \( m_{\pi^0}/2 = 67 \text{ MeV} \) at \( \approx 40-50 \text{ MeV} \). At Compton energies, downscattering modifies the emission spectra. The contributions of electron bremsstrahlung and \( \pi^0 \)-decay are shown in Figure 4.10.
Figure 4.10: The angle-averaged secondary γ-ray spectrum above 10 MeV at $R_{\text{cutoff}} = 4.5$ GV and $\tau = 1$ g cm$^{-2}$ as adapted from Kinzer et al. (1974).

The spectral components in Figure 4.10 are angle-averaged but the underlying reality is more complicated. The direction of each component’s generating primary particles’ momentum, the degrees of freedom available in interactions, and conservation of momentum results in a different flux, energy-, and angle-dependence for each component of the net secondary γ-ray flux. For $e^-e^+$-annihilation and bremsstrahlung, the varying momenta of incident particles allows for wider range of outgoing γ-ray momenta than in $\pi^0$ decay that constrains their momenta to $p_\pi = 2E_\gamma \cos \theta$, where $\theta$ is the angle with respect to $\vec{v}_\pi$.

For $\pi^0$ decay, the sources are distinctly hadronic and occur very high in the atmosphere for H$^+$ cosmic rays, resulting in a predominantly downward flux. Some contribution to the $\pi^0$ decay spectrum may exist from other hadronic products of high-$Z$ cosmic ray interactions or particularly high-energy splash albedo. The sources of high-energy electrons are numerous: hadronic interactions and decays, pair production, Compton scattering, and ionization. Due to the atmospheric mass along the path near the horizon, bremsstrahlung contributions are dominant just above and below limb. This effect is more pronounced at energies $>150$ MeV as it becomes less likely for parent hadrons to survive along the path from large zenith angles or for high-energy $\pi^0$ to be formed from upward splash albedo.
As there are many possible interactions producing high-energy charged particles, there is a theoretical lack of constraint on bremsstrahlung sources and their spectra along with varying contribution of the $\pi^0$ decay as a function of angle. This has contributed to disagreements in the upward $\gamma$-ray flux in theoretical models of Ling, Ryan, Graser, and Schöenfelder. This conflict can be resolved explicitly through particle transport simulation or implicitly through further empirical constraints as is the approach in this dissertation. These approaches are described in more detail in Chapter 5.

The $e^- e^+ \rightarrow \gamma \gamma$ annihilation spectrum can be defined as a function of the atmospheric electron flux excess described in the following subsection. Symmetry dictates there should be an equivalent spectrum of secondary electrons and positrons but newly formed positrons quickly annihilate with electrons in the atmosphere. The angular dependence of the annihilation spectrum is a function of the differential cross-section in the center-of-mass frame:

$$\frac{d\sigma}{d\Omega} \propto 2 \csc^2 \theta - 1$$  \hspace{1cm} (4.9)

where the emission preference is along the incoming positron momentum vector axis and where positrons are produced and survive along their path to the instrument.

### 4.3.2 Secondary Lepton Production

Like cosmic ray protons and atomic nuclei, there exist cosmic ray electrons and positrons. Below $\sim 1$ GeV, positrons have shorter radiation lengths and typically annihilate prior to relevant atmospheric depths while electrons have a considerably longer radiation length. However the flux of primary cosmic ray electrons is relatively small as compared
to the secondary electron flux. These secondary electrons are produced from cosmic ray-induced air showers along with pair production of $\gamma$-rays. Similarly, positrons and muons $\mu^\pm$ are also produced.

In my review, the definitive work in describing secondary electron and positron fluxes is Daniel and Stephen’s 1974 paper (DS74) that first determined relevant source terms for production spectra and then evaluated a 1D numerical transportation of the produced particles through the atmosphere (Daniel and Stephens, 1974). These source terms were developed as a function of geomagnetic latitude and atmospheric depth for both solar maximum and minimum. While the work in DS74 also represents the most comprehensive theory for the secondary $e^\pm$ flux, the $\gamma$-ray models discussed in the previous subsection have been better corroborated with experimental data. The source terms used in DS74’s for electron production include decay of short-lived particles: $\pi^\pm \rightarrow \mu^\pm + \nu$, where $\mu^\pm \rightarrow e^\pm + \nu + \bar{\nu}$. DS74 accounts for $\kappa$ by accommodating it within calculations of the $\pi$ spectrum using muon flux at sea level. “Knock-on electrons” are those ejected through primary or secondary particle interactions with bound electrons in atmospheric atoms and molecules.

Beyond the theoretical work in DS74, numerous experiments were used to validate the DS74 model as well as in this work for the electron and positron flux. Verma (1967), Israel (1969), Webber (1968), Stephens (1970), and Beuermann (1971). Secondary electron flux is identified by subtracting the calculated attenuated cosmic ray electron flux from observed electron flux.

The secondary electron spectrum and its components appear in Figure 4.11. As there are many potential sources generating knock-on electrons, their distribution is dominated by the interaction primary particle’s sources and the atmospheric mass between interaction and instrument. What ultimately reaches the detector is governed by the Bethe equation
for energy loss through atomic excitation and ionization (Olive et al., 2014):

\[
-\left\langle \frac{dE}{dx} \right\rangle = \rho K \frac{Z}{A} \frac{1}{\beta^2} \left[ \frac{1}{2} \log \frac{2m_e c^2 \beta^2 \gamma^2 K_{\text{max}}}{I^2} - \beta^2 - \frac{\delta}{2} \right]
\]

where \( K = 4\pi N_A r_e^2 m_e c^2 = 0.307 \text{ MeV cm}^2 \text{ g}^{-1} \), \( Z/A \) references the medium in which the charged particle is being propagated with \( I \) being the mean excitation energy, \( \beta, \gamma, \) and \( K_{\text{max}} \) referencing the particle kinetic energy, and \( \delta \) being the density correction factor. \( K_{\text{max}} \) is further defined:

\[
K_{\text{max}} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma \frac{m_e}{M} + \left( \frac{m_e}{M} \right)^2}, \tag{4.11}
\]

where \( M \) is particle mass. Secondary electrons will be generated where the primary flux and mass along the path is greatest - leading to the highest flux near limb. The least flux will come from zenith where mass along the path is at a minimum. Flux from nadir is limited by the momenta of primary particles in secondary electron producing interactions and the increasing mass per unit length along paths below limb.

Electrons produced by \( \pi^- \) and \( \kappa^- \)-decay are then oriented along the momentum of the population of those primary particles, resulting in a flux essentially coming from zenith to limb.

Secondary positrons cannot be generated through ionization, and its spectrum is attributable to \( \pi^+ \) and \( \kappa^+ \)-decay. This leads to a comparative excess of secondary electrons despite charge symmetry over \( 4\pi \) sr; however, their splash albedo fluxes as measured in space by AMS are similar (Alcaraz et al., 2000).

Atmospheric muons are generated through the decay of short-lived \( \pi^\pm \) and \( \kappa^\pm \). Although muons are the dominant contribution to the total background at sea level, they are less likely to have been generated prior to arriving at small atmospheric depths. For this reason the flux is still relatively small and more likely to arrive at larger zenith angles.
Experimental work by the Balloon-borne Experiment with a Superconducting Spectrometer (BESS) collaboration has observed the primary cosmic ray flux and muon flux at small depths over Lynn Lake, Canada through the solar cycle and over Ft. Sumner (Motoki et al., 2003; Abe et al., 2003). Brunetti measured the pion and muon flux (Brunetti et al., 1996). BESS results show near symmetry of $\mu^+$ and $\mu^−$.

The rest mass of $\tau^\pm$ is prohibitively high for a tau flux to be produced in a quantity that would meaningfully contribute to background.

### 4.3.3 Secondary Hadron Production

The three major hadronic species produced in the upper atmosphere are protons, neutrons, and pions. Though others are produced as well, the stability and relatively low rest mass of these species result in them being the dominant contributions to the hadronic background.

Atmospheric protons are created from breakup of high-$Z$ cosmic rays and collisions of cosmic ray primaries with atomic nuclei in the atmosphere. The models for cosmic ray
Figure 4.12: BESS observation of protons over Ft. Sumner at various depths from its 2001 flight. The effect of rigidity cutoff is clear, with a soft cutoff occurring near 3.7 GeV. Above the cutoff, both atmospheric and cosmic ray primary protons contribute to the flux. But below the forbidden Störmer cone’s rigidity cutoff, 2.2 GV, only secondary protons contribute. This contribution increases at depth increases due to the atmosphere prompting the production of secondary protons.

Figure 4.12 shows the proton spectrum over Ft. Sumner during BESS’s 2001 flight. Through the local rigidity cutoff over Ft. Sumner and space-based observations of the cosmic ray continuum spectrum, it is possible to isolate the downward component of atmospheric protons whose components are secondary, downward protons and reentrant albedo. This was done by Papini, et al. using data from the MASS instrument flown in a manner similar to BESS (Papini et al., 1993).

Reentrant albedo component exists on account of splash albedo with energies below the local cutoff. Understanding splash albedo flux can constrain the contribution of reentrant albedo. Experiments reported by Verma (1967) and Wenzel et al. (1975) show...
the contribution of splash albedo. Verma provides extrapolated top-of-the-atmosphere contributions. As depth increases at small atmospheric depths, the contribution of reentrant albedo diminishes while splash albedo increases. Additional work by Papini, Grimani, and Stephens (Papini, 1995) develops an analytical model of atmospheric protons similar to the work of Stephens in his work on atmospheric electrons and $\gamma$-rays and the authors describing the atmospheric $\gamma$-ray flux.

Atmospheric neutrons are produced from the spallation of primary cosmic rays and atmospheric gas. Due to degrees of freedom available in many-body kinetics, the neutron spectrum is broad. The neutron free path in the atmosphere is large owing to its neutral charge. Neutrons lose energy through elastic scattering and nuclear recoil ionization and become thermalized such that the mean neutron kinetic energy asymptotically approaches mean atmospheric particle kinetic energy. The long paths, multiple scattering, and thermalization further broadens the spectrum as the distribution of thermal neutrons becomes increasingly isotropic with each collision.

The canonical work in atmospheric neutrons by Armstrong et al. (1973) uses MC transport modeling to determine the neutron flux at the top of the atmosphere. Kole’s atmospheric neutron model most directly guided this work (Kole et al., 2015). Kole used a flux of incident cosmic ray protons and alpha particles impinging an atmospheric model and tracked the production of neutrons to model the neutron flux as a function of geomagnetic latitude, atmospheric depth, and solar cycle.

With their low rest mass, pions, $\pi^\pm$ (139.6 MeV) and $\pi^0$ (135.0 MeV), are the most common mesons created by cosmic ray collisions in the atmosphere. Both $\pi^+$ and $\pi^-$ have a mean lifetime of 26 ns while $\pi^0$’s mean lifetime is 0.084 fs. Even at relativistic speeds, their ephemeral existence results in relatively low fluxes. The secondary $\pi^-$ flux was observed by Brunetti (Brunetti et al., 1996) and shown to be insignificant for the purpose of this study.
4.3.4 Local Activation

A final contributor to instrument background rates is local activation, \textit{i.e.} the emission from excited or otherwise unstable nuclei within or near the detector, such as those that have captured protons or neutrons. The activation spectrum is a function of the background particle spectra and local material with their absorption cross-sections. Excited nuclei enter metastable states and reemit on timescales corresponding to the nuclei; for some species reemission is essentially instant while for others it may take years. In this dissertation, I do not account for the contribution of activation to the X-Calibur background rate as it was found to have negligible contribution.
Chapter 5

Background Models

Understanding the effect of the background is crucial for designing and operating suborbital and orbital high-energy instruments. There is no shortage of observations of background constituents but they cannot be directly used in estimating their effect on an instrument. From these observations, I developed models for the dominant sources of background including: cosmic ray and secondary protons, cosmic ray and secondary electrons, secondary positrons and muons, and cosmic ray and secondary $\gamma$-rays. To do so, it was necessary to account for the conditions under which observations had been made such that they can be applied in estimating backgrounds for X-Calibur.

In this chapter, I develop background models for observational conditions above Ft. Sumner and describe their underlying assumptions. In Chapter 6, I show that these models accurately account for the background spectra observed by X-Calibur in 2014 over Ft. Sumner after simulating the detector response. I follow a similar approach in developing background models describing incident spectra over Antarctica.

Because these models are validated at the X-Calibur observing depth of $\tau=3.45$ g cm$^{-2}$ over Ft. Sumner, the following derivation reflects these observational conditions. All analytical equations are defined for $\tau=3.45$ g cm$^{-2}$ unless explicitly mentioned. As detailed below, I assume that the effects of solar modulation are less pronounced above Ft. Sumner.
owing to its rigidity cutoff. However, spectra over Antarctica are highly dependent on solar modulation. As it is difficult to accurately predict solar conditions years in advance, I develop models reflecting conditions during solar maxima and minima as best and worst case scenarios. However, the methods I follow below can readily be applied to shallow depths and different rigidities.

In this chapter I begin by describing the three different approaches for developing models of the high-energy environment at small atmospheric depths. I then outline simulation cases and high-level assumptions underlying my models. I then develop models for each background type. For each model, I describe model-specific assumptions, the approach for their derivation, and, where possible, cross-validate the results with existing literature. For each background type, I develop models describing the three conditions: Ft. Sumner, Antarctica during solar maxima, and Antarctica during solar minima. I conclude the chapter by comparing the spectra over Ft. Sumner determined by my models to those calculated by the QARM model.

Appendix A summarizes the all the equations derived in this chapter.

5.1 Approaches to Model Development

Through the literature three different approaches, each accompanied by its assumptions and advantages, have been used to assess the spectra of radiation in the atmosphere. A first method for developing background models is through analytical equations motivated by the physics of primary and secondary interactions constrained by empirical data. With the source term defined, it is possible to estimate the flux of background through how it is generated and attenuates in the atmosphere by determining source terms as a function of production type (e.g. $\pi^0 \rightarrow \gamma\gamma$), cross-sections, cosmic ray source, branching ratios, etc. A second
method to generate background models is by MC simulation. In the simulation, cascades generated by cosmic rays incident on a global atmospheric model are generated and tracked to determine the secondary radiation that is produced and what cosmic ray flux remains unattenuated. The third method which is the basis of this work is through semi-empirical fitting of experimental data.

5.1.1 Analytical Models

For primary radiation, namely $\gamma$-rays or cosmic rays that are enter the atmosphere, analytical models are simple. Their incident flux is degraded by transport through the atmosphere due to inelastic scattering and absorption for photons and energy loss and inelastic collisions for cosmic rays. However, atmospheric secondary radiation is more complicated because one must account for presence of the primary particle and attenuation of secondaries as they propagate to the instrument. Accommodating for these effects requires a series of integrals to determine where secondaries are produced and how they attenuates along their path to the instrument. The solution to the transport equation is given by an integral over the source term $s(E', z)$ giving the secondary flux production as a function of energy $E$ and depth $z$ (or sometimes density):

$$\frac{dF(E, h)}{d\Omega} = \int_l S(E', z) \rho(z) e^{-\int_0^l \mu(E') \rho(l) dl} dl,$$

where the general geometry is defined in Figures 4.8 and 5.1; $S(E', z)$ is the source term; $h$ is the atmospheric depth; $z$ is the depth as a length from top of the atmosphere; $\mu(E')$ is the attenuation as a function of energy; and $l$ is the path from source to instrument that can be calculated using Equation 4.7.
Figure 5.1: The geometry used in source term path integrals. Within the atmosphere, secondary flux is generated from a unit volume as a function of energy and atmospheric depth. The fraction of generated flux incident on the atmosphere is attenuated as a function of energy and local density along the path.

The definition of the source term $S(E', z)$ may vary based on the method of secondary production. For example, $\gamma$-ray production will require terms for both bremsstrahlung and $\pi^0$ decay as how and where the interaction occurs in the atmosphere differs. These source functions can be derived analytically using assumptions for cosmic ray fluxes and relevant interaction cross-sections, but contributions from secondaries create an additional degree of freedom that is not readily constrainable from first principles. For example, the bremsstrahlung emission of secondary $\gamma$-rays may be due to electrons from cosmic rays, reentrant albedo, locally produced electron secondaries, etc. Each contributing primary source would require its own set of assumptions thus resulting in compounding errors.

Another method of determining source terms is empirically through observing the variation of a spectrum as a function of depth (henceforth, depth curve). Provided contaminating background is subtracted from data, a source term can be unfolded from a depth curve, as attenuation properties and depth are well constrained by laboratory work and the balloon’s barometer. Although there remains ambiguity over the mechanism that
produced each observed particle, the resulting source term can be used to constrain analytical models. For strict background model development, an analytical exposition is not necessary but provides a concrete scientific understanding of the physics resulting in observed spectra. A strictly empirical source term can be developed as the case in Dean et al. (1989).

5.1.2 Simulation-Derived Models

Background models have also been developed directly through particle transport simulation by tracking the particles produced and attenuated as a result of primary cosmic rays. Particle transport simulation packages like GEANT (Agostinelli et al., 2003), MCNP (Goorley et al., 2012), and FLUKA (Ferrari et al., 2005) work by seeding primary particles with a specific energy and momentum using a MC method and then checking potential interactions at each step in a particle’s trajectory. Where and when an interaction occurs, its outcome is calculated using relevant equations from an associated physics package. Figure 5.2 shows different results from GEANT, MCNP, and FLUKA in their estimation of the secondary neutron spectrum as cosmic rays are transported through an atmospheric mass model using the same assumptions. The spectrum generated by MCNP, the package used by QARM, shows a normalization twice that of the spectrum generated by GEANT. Differences in computational and numerical methods employed by transport packages result in the introduction of systematic errors and, in this case of Figure 5.2, uncertainty in calculated spectrum.

For atmospheric backgrounds, several assumptions are required to calculate beyond the selection of the simulation package. First, a model for the cosmic ray spectrum needs to be defined. Second, a global atmospheric model needs to be defined with respect to relevant observation conditions. Because some of the observed spectra depend on interactions that occur below the instrument’s altitude, the local global model needs to be properly defined.
for observational conditions including solar modulation, latitude, and geometric altitude. For a complete atmospheric radiation model, simulations must be performed for all relevant solar modulation conditions and all latitudes and rigidities sufficient to at least generate an interpolated model. Because these simulations must account for a relevant observable solid angle and statistical significances needs to be achieved, a large volume and total, primary particle number must be simulated resulting in long computational times. Finally, after a simulation is completed, particle tracking must be performed with respect to a specific atmospheric depth. Notably, one major benefit for a global atmospheric radiation model like the Quotid/QinetiQ Atmospheric Radiation Model (QARM) is ensured self-consistency in estimates of the background as the same primary spectra and physics code is used for all constituents.\footnote{N.B. since beginning this work QARM was taken offline and recently made online as Models for Atmospheric Ionising Radiation Effects (MAIRE).}

In the literature, Takada et al. (2011) developed a simulation-based model for secondary X-rays, Kole et al. (2015) developed a model for secondary neutrons, and Lei et al. (2004c) developed QARM that generates combined primary and secondary spectra for $\gamma$-rays,
protons, neutrons, electrons, and muons. Details of the QARM model and how it was implemented for my comparison are discussed in Section 5.9. Kole’s neutron model was published while Lei et al. have created a web application to run QARM and generate results (Kole, 2014; Lei, Accessed 4/2016). Both their models generate semi-isotropic spectra, binned as downward ($0^\circ < \theta < 90^\circ$) and upward ($90^\circ < \theta < 180^\circ$) components.

5.1.3 Semi-Empirical Fitted Models

Semi-empirical fitted models provide a simple and elegant solution in developing a global radiation model. Over the past half-century, many observations of high-energy radiation have been made at small atmospheric depths. Background subtracted depth curves have been reported along with their observational conditions. For each background constituent, empirical depth curves can be mapped to reflect off-zenith entrant flux using the line-of-sight (LOS) mass from the instrument to the top-of-the-atmosphere. The result of these mappings are angle-dependent spectra that are continuous in the angle-domain. Where angle-dependent observations have been reported, interpolations of the data generate models defining angle- and depth-dependent spectra. This was the method of Costa et al. (1984) in their fitting of Schöenfelder’s results (Schöenfelder et al., 1977, 1980). Each model can be readily extended to describe a range of rigidities and solar modulations either contingent on the availability of observational data or through first principles. This approach is a similar approach to Graser and Schöenfelder (1977) and Lichti et al. (1975) in their calculations of source terms for atmospheric $\gamma$-ray production. However, because the goal of this effort is only to estimate background it is unnecessary to develop source terms that increase uncertainty.

The primary concern in developing such a radiation model describing multiple background constituents based on different experiments and theories is that the inputs to
the model must be made self-consistent. For instance, drift of the magnetic poles\textsuperscript{2}, varying estimates for calculating solar modulation over the same time period, and conflicting reports of flux must be accounted for. These concerns are addressed where they arise as I define each model.

5.2 Model Cases and Assumptions

In order to describe the impact of background on X-Calibur flights above Ft. Sumner and Antarctica at different points in the solar cycle, I develop background models appropriate for the simulation cases shown in Table 5.1. To validate my background models, I simulate the conditions above Ft. Sumner in September 2014 for two different elevation angles. For the future flights I simulate a telescope pointing at a zenith angle of $\theta=55^\circ$, a representative value given the allowable observable elevation is constrained to $\theta < 65^\circ$ due to balloon and structure intersecting the star tracker LOS. As the rigidity cutoff is greater than the solar modulation potential over Ft. Sumner, I assume no variation in background.

\textsuperscript{2}Although slow, this becomes relevant when comparing observations performed over the same location nearly half a century apart.

Table 5.1: Simulated test cases of the Fall 2014 and future X-Calibur flights. I assume upper and lower limits on the solar modulation potential for Fall 2014 while using an estimate for Fall 2016. I consider approximate solar maximum and minimum cases for the planned Fall 2018 flight at Antarctica.

<table>
<thead>
<tr>
<th>Location</th>
<th>Geomagnetic Latitude</th>
<th>Date</th>
<th>Elevation Angle</th>
<th>$\phi$ [MV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ft. Sumner, NM</td>
<td>42°</td>
<td>September 2014</td>
<td>0° (\approx 780)</td>
<td>9.26° (\approx 780)</td>
</tr>
<tr>
<td>Ft. Sumner, NM</td>
<td>42°</td>
<td>September 2016</td>
<td>35° (\approx 500)</td>
<td>55° (\approx 500)</td>
</tr>
<tr>
<td>Antarctica</td>
<td>-80° &lt; $\lambda$ &lt; -90°</td>
<td>Fall 2018</td>
<td>35°</td>
<td>250</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>35°</td>
<td>1200</td>
</tr>
</tbody>
</table>
I define two assumptions which underlie the development of background models. First, I neglect the effects of the solar modulation potential where it is less than the lowest observed cutoff. Observations of the earth’s γ-ray albedo over a solar cycle by Harris et al. (2003) show minimal variation. The 511 keV flux increased by a factor of 1.05 and 1.09 for \( R_{\text{cutoff}} > 7 \) and \( R_{\text{cutoff}} < 7 \) GV, respectively.

As pertains to this work, I estimate \( 630 < \phi < 930 \) MV for Fall 2014 from the observed modulation in C, O, Mg, Si, and Fe fluxes (Wiedenbeck and Binns). Because \( R_{\text{cutoff}} = 3.7^{+0.8}_{-0.5} \) GV over Ft. Sumner, the minimum bound is well above any solar modulation potential.

In locations such as McMurdo where the solar modulation factor is larger than the local rigidity cutoff, solar activity has a direct effect on cosmic particle spectra and consequently the production of all secondaries. Although it may be possible to predict the future solar modulation potential affecting X-Calibur’s 2018 flight from McMurdo through time series analysis, I choose to evaluate best and worst cases over Antarctica where \( \phi = 1200 \) and 250 MV, respectively (Sturrock, 2003; Haubold et al., 2014). A final effect of solar activity is the relative depletion of earth’s atmosphere. At lower solar modulation the flux of cosmic rays incident upon the atmospheric increases. As the flux increases, so does the total kinetic energy imparted on the atmosphere, resulting in a higher rate of atmospheric mass loss.

Table 5.2: Comparison of unscattered, “on-beam” CXB flux assuming fixed atmospheric depth verses geometric altitude for the CIRA 2012 atmosphere models under solar maximum, mean, and minimum conditions. The unscattered CXB flux is calculated at a fixed atmospheric depth and geometric altitude at different points of the solar cycle at the top and bottom of the table, respectively.

<table>
<thead>
<tr>
<th>Model</th>
<th>Solar Period</th>
<th>Depth [g cm(^{-2})]</th>
<th>Altitude [km]</th>
<th>Flux [N (s cm(^{-2})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Depth</td>
<td>Solar Maximum</td>
<td>3.45</td>
<td>42.74</td>
<td>2.292(4)</td>
</tr>
<tr>
<td></td>
<td>Solar Mean</td>
<td>3.45</td>
<td>41.69</td>
<td>2.292(0)</td>
</tr>
<tr>
<td></td>
<td>Solar Minimum</td>
<td>3.19</td>
<td>40.93</td>
<td>2.292(1)</td>
</tr>
<tr>
<td>Fixed Altitude</td>
<td>Solar Maximum</td>
<td>3.86</td>
<td></td>
<td>1.987</td>
</tr>
<tr>
<td></td>
<td>Solar Mean</td>
<td>3.45</td>
<td>41.69</td>
<td>2.292(0)</td>
</tr>
<tr>
<td></td>
<td>Solar Minimum</td>
<td>3.19</td>
<td></td>
<td>2.52(1)</td>
</tr>
</tbody>
</table>
that changes the atmospheric mass profile. However, I am able to neglect this variation by normalizing the altitude of flights to constant atmospheric depth rather than geometric altitude. I test this assumption by calculating the total flux of unscattered cosmic X-ray background (CXB) for equatorial atmospheres at solar minimum and maximum using fixed depth and geometric altitude. As shown in Table 5.2, this assumption results in nearly identical results at different points of the solar cycle.

My second assumption is that a spectrum’s angular dependence can be approximated by mapping observed depth curves through integrating the atmospheric mass from instrument to top-of-the-atmosphere as a function of zenith angle. Specifically, if a function $F(\tau, \theta = 0^\circ)$ defines background flux as a function of atmospheric depth entering from zenith, I can substitute $\tau(\theta)$, described by Equation 4.8, in place of $F$’s dependence on an independent $\tau$ that also allows me to generalize the function for other zenith angles. Such functions have been established in literature by zenith-pointed balloon observations where depth curve data are obtained across of a range of atmospheric depths as the balloon ascends. This method provides only a first order estimate as the the same atmospheric depth along paths of different zenith angles correspond to LOSs through different layers of the atmosphere, with unequal geometric length. In some cases, such as muon decay, geometric distance rather than atmospheric depth is a relevant parameter. Thus the attenuation and the generation of secondaries on these paths will be different.

The spectra that result from the following background models are summarized in Figure 5.21. These models provide angle-dependent spectra from zenith to nadir. Finally, an additional source of background is local activation from captured particles. In particular the high-$Z$ Cd and Te within the detector have large neutron capture cross-sections over the relevant spectrum of thermal neutrons (Gicking and Krane, 2011; Tomandl et al., 2003). Activation in the CZT detector can be an important issue as a local decay may trigger an event without triggering a veto in the active shield. However, these rates typically matter
for an orbiting space-based instrument. Given the relatively short duration of the X-Calibur balloon flight, I do not consider the contribution to background event rate from activation.

5.3 Approach

The ultimate goal is to model the zenith-angle spectrum of the dominant contributions to the X-Calibur background rate. I describe the angle-dependent energy spectra with equations of the form:

$$F(E, \theta) = s(E, \theta) f_0(E),$$

(5.2)

where the angle-dependent spectrum $F(E, \theta)$ is the product of a unitless shaping function $s(E, \theta)$ and the reference intensity spectrum $f_0(E)$. Throughout the literature, balloon observations of zenith-entrant flux measured during balloon ascent have been reported. From these, I develop a function $f(E, \tau)$, for flux as a function of depth, which I map with my atmospheric mass model $\tau(\theta)$ described in Equation 4.8. The function $\tau(\theta)$ will be used often in what follows. Those wishing to develop similar models can perform the integration for $\tau(\theta)$ or use an interpolation of the values in Table A.2.

I will assume the following empirical form for the shaping functions:

$$s(E, \theta) = \tau(\theta)^{\Gamma(E)},$$

(5.3)

where $\Gamma(E)$ is a power law index that varies as a function of energy. This empirical method differs from the analytical method in Equation 5.1 where one solves the transport equation. This allows the shaping to reflect the variation in spectrum as zenith angle is varied. This approach follows Beuermann in his description of the variation of $\gamma$-ray, electron, and positron flux (see Fig. 7 in Beuermann (1971)).
Wherever possible, the approach for each background takes advantage of archival data or validated models. For instance, BESS observations over Ft. Sumner and Lynn Lake, Canada define spectra and variation of flux as a function of depth for protons, $\mu^\pm$, $\alpha$, and $\pi$. These models are derived in an identical manner. However, because BESS data are only reported for particles arriving from near zenith, I have to develop my own models for splash albedo, which is incident from below limb.

There is no single canonical reference or experiment that defines the angle-dependent spectra of atmospheric electrons and positrons. While I primarily rely on the semi-empirical model of (Stephens, 1970, DS74), I use other reports and models to extend DS74’s description of electrons and positron. Similarly, I follow the work of Costa et al. (1984) in estimating the spectrum of atmospheric $\gamma$-rays. However, it lacks a description of $\gamma$-rays with $E > 10$ MeV which have significant contribution to the background event rate. Separately, I develop first-order estimates of the CXB and cosmic electron spectra, wherein the former is cross-validated with the work of Gehrels (1985) and Takada et al. (2011). Kole (2014) developed a simulation-based model describing atmospheric neutrons which is used rather than developing a model based on the work of Armstrong et al. (1973).

Occasionally, some papers will report data in plots rather than tables. For instance, the results of the DS74 semi-empirical model are describe in various figures. These figures are digitized using Plot Digitizer (Huwaldt, 2013). When digitized data is used, no analysis is done to assess fit quality. Where archival data is used, the $\sigma_{RMS}$ of my fit is report.

Finally, I contextualize these models by showing the event rates resulting from background simulations of the background models derived in this work, reflecting conditions over Ft. Sumner in 2014 with an elevation angle of $\approx 9.24^\circ$, in Table 5.3. Atmospheric $\gamma$-rays and neutrons are predicted to have the largest contribution to the background within the X-Calibur science band and over all energies, accounting for 70.77% and 61.64% of events,
Table 5.3: Event rate from simulations of background models derived in this work for conditions reflecting those experienced by X-Calibur during its 2014 flight. Event rates are defined as multi-pixel events within the X-Calibur science band and all events. The total of these events are shown at bottom and are compared to the flight result, which is shown adjacent in parenthesis.

<table>
<thead>
<tr>
<th>Model</th>
<th>$20 \leq E &lt; 60$ keV [Hz]</th>
<th>All Events [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmospheric $\gamma$-rays</td>
<td>8.17</td>
<td>111.96</td>
</tr>
<tr>
<td>Neutrons (Kole)</td>
<td>8.31</td>
<td>73.89</td>
</tr>
<tr>
<td>Cosmic Protons</td>
<td>3.53</td>
<td>53.33</td>
</tr>
<tr>
<td>Positrons</td>
<td>1.70</td>
<td>24.99</td>
</tr>
<tr>
<td>$\mu^+$</td>
<td>0.95</td>
<td>18.99</td>
</tr>
<tr>
<td>Attenuated CXB</td>
<td>0.82</td>
<td>10.30</td>
</tr>
<tr>
<td>$\mu^-$</td>
<td>0.52</td>
<td>11.27</td>
</tr>
<tr>
<td>Albedo Protons</td>
<td>0.49</td>
<td>7.41</td>
</tr>
<tr>
<td>Secondary Electrons</td>
<td>0.38</td>
<td>5.63</td>
</tr>
<tr>
<td>Unattenuated CXB</td>
<td>0.03</td>
<td>0.55</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>16.73 (27.20)</strong></td>
<td><strong>206.36 (297.47)</strong></td>
</tr>
</tbody>
</table>

respectively. Notably, the neutron model is derived in Kole (2014) and is the only model that is not derived in this work. Further analysis and detail discussion of simulation results can be found in the following chapter.

### 5.4 Cosmic and Secondary Protons

The significant proton contributions to the detector background are cosmic protons, reentrant albedo, and secondary protons. The general form of the their spectra is shown in Equation 5.4, where $s(\theta)$ is a unitless shaping function that defines the contribution from zenith to limb and $f(E)$ is the spectrum.

$$F_p(E, \theta) = s_p(\theta)f_p(E) \tag{5.4}$$
5.4.1 Above Ft. Sumner, NM

To estimate the flux above Ft. Sumner, I use results reported by BESS from its 2001 flight over Ft. Sumner (Abe et al., 2003). I divide the observed proton flux into three regimes. Regime I, $E \lesssim 2.5$ GeV, describes incident proton flux with energies below the allowed Störmer cone. Namely, this is assumed to be reentrant albedo or secondary protons. Region II, $2.5 \lesssim E \lesssim 3.7$ GeV, encompasses flux of energies above the allowed Störmer cone and below the average, rigidity cutoff. This regime has a mixed composition of cosmic primaries and other secondaries. Regime III, $E \gtrsim 3.7$ GeV, is assumed to be comprised only of primary cosmic ray protons.

First, I develop analytical models for the proton intensity spectrum $f_{p,FtS}(E)$ in the three regimes by fitting to BESS observations of the proton flux over Ft. Sumner at $\tau \approx 4.58$ g cm$^{-2}$ and matching boundary conditions. The BESS data and my fit are shown in Figure 5.3: The BESS-observed proton spectrum over Ft. Sumner, in open circles, and its fit, as a solid line. The data reflects BESS observation of zenith-entrant protons at 4.58 g cm$^{-2}$. Note that the energy breaks in the spectrum correspond approximately to the definition of the three regimes mentioned in the text.
Figure 5.4: The proton spectrum over Ft. Sumner is divided into three regimes based on the sources of emission. Each panel shows the reported BESS data with the fits.

5.3, while Equation A.3 defining $f_{p,FtS}(E)$ is given in Appendix A. This fit of Equation A.3 to the data shows an error of $\sigma_{RMS}=3.170 \text{ (s m}^2\text{ GeV sr)}^{-1}$ over all regimes.

I next determine the zenith angle dependence of the proton spectrum. Because the origin of each regime’s protons is different, the angle dependence of each regime needs to be derived independently. To do so I use the data taken by BESS as it ascended to its maximum float altitude. During ascent data was taken on the flux incident from zenith. For the maximum and minimum energy of each regime, I fit power laws functions to the variation in flux as a function of atmospheric depth and average them. The power law indices are:

$$\Gamma_{p,FtS}(E) = \begin{cases} 0.743 & E \leq 2.16 \text{ GeV} \\ 0 & 2.16 < E \leq 3.35 \text{ GeV} \\ -0.075 & E > 3.35 \text{ GeV} \end{cases} \quad (5.5)$$

Example BESS data describing the variation in flux as a function of depth and their fits are shown in Figure 5.4. The error of the $\Gamma$ fits in Figure 5.4 are $\sigma_{RMS}=4.60$, 2.18, 6.26, 3.43, 0.72, and 0.353 for energy bins 0.50, 1.08, 3.69, 4.31, 5.86, and 9.29 GeV, respectively. For regime II, an arbitrary fit of $\Gamma=0$ was found to sufficiently describe this regime with an average factor of 1.32 greater error than the individual fits for 3.69 and 4.31 GeV.
Although these power law indices $\Gamma_{p,F_{15}}(E)$ were determined based on the observation of zenith-entrant flux as BESS’s altitude rose, I can use a change of variable to determine the variation in flux as a function of zenith angle. Namely, I begin with:

$$f(E, \tau) = f_0 \tau^{\Gamma(E)}, \quad (5.6)$$

where the varying zenith-entrant flux $f(E, \tau)$ is determined by an initial condition $f_0$, i.e. the minimum depth where the observation was made, the current depth $\tau$, and the power law variation $\Gamma$ defined in Equation 5.5.

I use BESS’s minimum depth as a reference depth $\tau_0 = 4.58$ g cm$^{-2}$ and the spectrum defined at this depth in Equation A.3 as the reference spectrum $f_0$. Dividing Equation 5.6 by the reference spectrum results in:

$$s_{p,0}(E, \theta) = \left[ \frac{\tau(\theta)}{\tau_0} \right]^{\Gamma(E)}, \quad (5.7)$$

where $\tau(\theta)$ is the atmospheric depth as a function of zenith angle determined from the atmospheric mass model. $\Gamma(E)$ are those given in Equation 5.5. Thus using this mapping I am able to determine the zenith-angle dependence of the incident proton flux for each of the three regimes. Thus $s_{p}(E, \theta)$ is unitless and is used as a normalization factor to modulate the proton spectrum as zenith angle increases.

Prior to determining $s_{p,0}(E, \theta)$, I determine normalization factors to adjust $\tau_0$ to 3.45 from 4.55 g cm$^{-2}$. This is done for each regime by using their respective $\Gamma_{p}(E)$. The normalization factors are:

$$m_I = 1.022$$
$$m_{II} = 1$$
$$m_{III} = 0.81, \quad (5.8)$$
Figure 5.5: The function $s_p(E, \theta)$ is defined separately for regimes I/II and III. In order to better estimate the flux between zenith in limb, I multiply their forms of Equation 5.7 as I assume low-energy protons are produced through the energy loss of high-energy proton along the same path.

and are used in Equation 5.10. They are sensible given that the lower atmospheric depth will result in less production of low-energy secondaries and while allowing more high-energy cosmic protons to penetrate.

However, BESS only observed protons arriving from near zenith and not from limb. The use of each regime’s power law must be justified at limb. For the high-energy protons of regime III, I assume that sufficient energy loss occurs along their paths from top-of-the-atmosphere to the detector so that neither the mass profile of the path nor the zenith angle has an effect on their passage through the atmosphere.

For low-energy protons of regime I and II, $E \lesssim R$, I assume that the production of secondary protons arriving from zenith angles between zenith and limb is dependent on the flux of cosmic rays passing through equivalent depths. Alternatively expressed, through conservation of momentum I assume that secondary protons incident from limb are ultimately due to the energy loss of cosmic protons entering from near limb. I thus neglect the fraction of scattered secondary and degraded cosmic ray protons which contribute to the limb-entrant
flux through multiple or large-angle scattering. Thus I estimate the zenith angle-dependence flux of E<3.7 GeV protons by scaling their flux with the zenith-angle dependence of cosmic ray protons in the following manner:

\[ s_p(E < 3.7 \text{ GeV}, \theta) = s_{p,0}(E > 3.7 \text{ GeV}, \theta, ) \cdot s_{p,0}(E < 3.7 \text{ GeV}, \theta). \]  

(5.9)

Here, \( s_{p,0}(E > 3.7 \text{ GeV}, \theta) \) is the form of Equation 5.7 that describes regime III while \( s_{p,0}(E < 3.7 \text{ GeV}, \theta) \) is the form that describes regime I and II. Their product is \( s_p(E < 3.7, \theta) \) as can be seen in Figure 5.5.

Accounting for the above, I further specify Equation 5.7 and \( s_p(E, \theta) \) takes the following form for the three regimes:

\[
s_{p, \text{FtS}}(E, \theta) = \begin{cases} 
  m_I \tau(\theta)^{\Gamma(E_I) + \Gamma(E_{III})} & E \leq 2.16 \text{ GeV} \\
  m_{II} \tau(\theta)^{\Gamma(E_{II}) + \Gamma(E_{III})} & 2.16 < E \leq 3.35 \text{ GeV} \\
  m_{III} \tau(\theta)^{\Gamma(E_{III})} & E > 3.35 \text{ GeV}
\end{cases}
\]  

(5.10)

where the units for \( \tau(\theta) \) can be either radians or degrees [°] per interpolating Table A.2. The zenith angle-dependent spectrum of downward protons over Ft. Sumner can then be expressed as the product of Equations A.3 and 5.10. Finally, although BESS observed at depths as great as 50 g cm\(^{-2}\), this approach should be restricted for use \( \tau \lesssim 10 \text{ g cm}^{-2} \) owing to the validation of my model at 3.45 g cm\(^{-2}\) and the small variation in mass profile between \( \tau = 3.45 \) and 10 g cm\(^{-2}\).

### 5.4.2 Above Antarctica

A similar approach was used to describe the downward proton spectrum over Antarctica by adopting the results of BESS taken over Lynn Lake, Canada between 1997-2002 (Shikaze...
et al., 2007). However, as the rigidity cutoff at Lynn Lake is $R \approx 0.4$ GV, solar modulation must now be accounted for. In order to maintain consistency with other background models, I use the catalogue of solar modulation factors in Usoskin et al. (2011) for each BESS flight rather than those reported by BESS. Table 5.4 lists the dates, BESS-reported solar modulations, and Usoskin-reported solar modulations for those dates.

As can be observed in Figure 4.4, the shape of the proton spectrum and its normalization change over the course of a solar cycle such that a single shaping function will not accurately describe its effect. Rather than the regime-based approach in assessing downward protons over Ft. Sumner, I determine angle-dependent spectra by evaluating the variation of the flux in each energy bin through the solar cycle. Doing so results in a different form of the final equation:

$$F_{p, \text{McM}}(E, \theta, \phi, \tau) = f(E, \phi)^{\Gamma(E, \theta, \phi)} \Gamma(E, \theta, \phi) s_p(E, \phi, \theta).$$  \hspace{1cm} (5.11)

Here, the angle-dependent spectra $F_{p, \text{McM}}(E, \theta, \phi, \tau)$ and its contributing factors are all functions of solar modulation $\phi$. The unitless power law index $\Gamma$ is determined through fitting the change in flux at each depth, energy bin, and BESS flight solar modulation so that the spectrum angle-dependence is preserved in varying solar conditions.

First, I define $f(E, \phi)$ as the spectrum observed by BESS as a function of solar modulation. This was determined by measuring the change in observed flux at the smallest energy bin.

Table 5.4: The solar modulation factors as reported by Shikaze et al. (2007) and Usoskin et al. (2011) during BESS flights from 1997-2002 over Lynn Lake, Canada.

<table>
<thead>
<tr>
<th>Date</th>
<th>$\phi_{\text{BESS}}$ [MV]</th>
<th>$\phi_{\text{Usoskin}}$ [MV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 1997</td>
<td>491</td>
<td>409</td>
</tr>
<tr>
<td>July 1998</td>
<td>591</td>
<td>514</td>
</tr>
<tr>
<td>August 1999</td>
<td>658</td>
<td>609</td>
</tr>
<tr>
<td>August 2000</td>
<td>1300</td>
<td>1057</td>
</tr>
<tr>
<td>August 2001</td>
<td>1109</td>
<td>904</td>
</tr>
</tbody>
</table>

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atmospheric depth for each energy bin of each BESS flight. A polynomial least-squares fit was calculated for each bin per \( g_E(\phi) = a + b \phi + c \phi^{-0.25} \), which was found to be more effective than the linear fit expected from Coulombic repulsion \( F \propto V \), where \( V \) is electrostatic potential. The quality of both fits are notably affected by the measurements in August 2000 and 2002 that observed different total fluxes over periods of nearly identical solar modulation according to Usoskin. As detailed in Figure 4.4, the solar modulation calculated by Usoskin was 1057 and 1058 MV by Usoskin while BESS had estimated 1300 and 1109 MV for the same flights, respectively. Although the BESS-reported solar modulation factors better qualitatively describe the effect of modulation on flux, I use Usoskin in order to maintain consistency with all other models derived in this chapter.

As a result, the fits for solar maxima are poor unless one of these data sets is removed from fitting. Removing the 2000 BESS data set yields a better fit. From this fit, I extrapolate the spectra for solar maxima and minima and fit the result to determine the intensity spectrum of downward protons for these conditions: \( f_{p,\text{Max}} \) and \( f_{p,\text{Min}} \), respectively. These intensity spectra are defined in Equations A.7 and A.8 given in Appendix A. Their units are \((m^2 \text{ GeV sr})^{-1}\).

The analytical description of the zenith angle-dependent spectra’s dependence on energy at low rigidities is similar to the description over Ft. Sumner, wherein a power law \( \Gamma \) is used to modulate shaping. However, to reflect the changing zenith angle-dependence of the spectra over the solar cycle, the power law \( \Gamma \) is now a function \( \Gamma(E, \phi) \) rather than a constant. To determine \( \Gamma(E, \phi) \), I begin by adapting BESS proton flux data catalogued by Shikaze et al. (2007) for the flights during 1999 and 2000. For each flight, I follow the exact procedure as downward protons over Ft. Sumner to determine \( \Gamma \) describing flux as a function of depth.
Figure 5.6: Calculated and fitted $\Gamma(E, \phi)$ power law indicies. Power law indicies above 0 indicate that flux increases along with zenith angle, while $\Gamma < 0$ indicates it diminishes. The blue and red points reflect $\Gamma$ calculated from BESS data, which were then used in the linear fit described by Equation 5.12. The black and teal dots indicate $\Gamma$ values estimated for solar maxima and minima, respectively, using the linear fit. These points are then fitted with the polynomial described by Equation 5.13.

However, rather than dividing the spectrum into three regimes to determine singular values of $\Gamma$, I determine it for each energy bin. Doing this for 1999 and 2000 BESS data, provides data points describing the change in $\Gamma$ for each energy bin as solar modulation changed between both flights. This is the definition of $\Gamma(E, \phi)$ which can estimate changes in the spectrum as solar change by using Equation 5.7. It is analytically defined by using a linear fit between the $\Gamma(E)$ determined for 1999 and 2000 per:

$$\Gamma(E, \phi) = \frac{\Gamma(E, 1057) - \Gamma(E, 609)}{1057 - 609} \phi + \frac{1057 \Gamma(E, 609) - 609 \Gamma(E, 1057)}{1057 - 609}$$  (5.12)
I then fit this table of power laws indices describing spectral variation as a function of depth. These functions are fitted as a power series:

\[ \Gamma(E) = \sum_{i=-k}^{k'} c_i E^i, \]  

(5.13)

where \( \Gamma \) is the power law index and \( k \) and \( k' \) range from some negative value to positive value to account for both growth and attenuation. Attenuation at \( E > 18.5 \text{ GeV} \), the highest reported energy from BESS, is assumed to be constant. Using this relationship between BESS’s 1999 and 2000 results, I perform a linear fit at each energy bin. I extrapolate from the fit to determine power law indices for each energy bin at solar maximum and minimum, which are also fitted. Equations A.10 and A.9 define \( \Gamma(E, \phi_{\text{Max}}) \) and \( \Gamma(E, \phi_{\text{Min}}) \) and are located in Appendix A. They are shown in Figure A.11 along with \( \Gamma \) values calculated from BESS data.

I can now develop a formal shaping function:

\[ s_p(E, \phi, \theta) = \left( \frac{\tau(\theta)}{\tau_0} \right)^{\Gamma(E, \phi)}, \]  

(5.14)

where \( \tau_0 \) is the reference depth from which the observation was made. For BESS, I determine \( \tau_0 = 4.55 \text{ g cm}^{-2} \) by averaging the minimum atmospheric depths of all Lynn Lake flights. Like for protons over Ft. Sumner, either radians or degree \([\degree]\) may be used for \( \tau(\theta) \).

Figure 5.7 shows the unitless shaping factor \( s(E, \theta) \) for both solar minima and maxima, shown on the left and right, respectively. Notably, at solar minimum nearly all flux is entrant above \( \theta \approx 30^\circ \). At solar minima the flux becomes considerably more isotropic. It also appears to begin converging to the result of Figure 5.5. Namely, near \( \theta \approx 90^\circ \) and at low energies, the scaling factor begins to increase dramatically. In Figure 5.7 this is seen as factor values above 2.00 are clipped near limb and at energies below 2 GeV. In Figure 5.5 factors increase...
Figure 5.7: Shaping function for the proton spectrum over Antarctica at solar minimum and maximum on left and right, respectively. Values are unitless and normalized to observation at that energy at $\tau_0$.

to over $\sim 10$ near limb for low-energy protons. Given that these two derivations were based on different data sets with different methods employed, this confirmation validates the self-consistency of the general approach.

### 5.4.3 Upward Secondary Protons

Few observations have been made of secondary protons arriving from below limb. Notable measurements of splash albedo protons have been made by Verma (1967) and Wenzel et al. (1975) through the 1960s. Both use well-collimated instruments to observe protons incident from approximately nadir and their results were not generalized to secondary protons entrant between limb and nadir. In order to determine the spectrum and angular distribution of these protons, I merge Wenzel’s 1967 observations of protons above Palestine, TX with the predictions of my proton model at limb. As the effect of proton splash albedo on X-Calibur
background rate was found to be very small owing to its spectrum, I nominally assume that Palestine, TX and Ft. Sumner, NM have comparable rigidities.

Using the methodologies discussed above, I develop a model to reconstruct the observable proton flux during Wenzel’s measurement. In Figure 5.8 I show Wenzel’s data along side my fit:

$$f_{\text{Wenzel, FtS}}(E) = 234 \pm 7.9 \ e^{-6.019 \ E \ E^{-0.096}}$$
$$f_{\text{Wenzel, FtC}}(E) = 1250 \pm 35 \ e^{-11.836 \ E \ E^{-0.102}}$$

(5.15)

where $E$ is in units of GeV and $f$ is in units of $(s \ m^2 \ GeV \ sr)^{-1}$. The measurement was taken at $\bar{\tau}=4.3 \ g \ cm^{-2}$, I increase the fit’s normalization by a factor of 1.178 to adjust to $\tau=3.45 \ g \ cm^{-2}$. The data above Ft. Chuchill reflects conditions at 1.9 g cm$^2$ and $\phi=671$ MV, per Usoskin et al. (2011). To reflect the relevant depth I multiply by a factor of 0.642 based on observations of BESS.
Figure 5.9: The angle-dependent spectra $F_{p, FTS}(E, \theta)$ plotted with several different angle cuts exposing the change in spectra near nadir. At limb $\theta \approx 1.57$, the boundary conditions of this model are defined by the boundary condition of limb-entrant downward protons of Equations A.3 and 5.10. At nadir, the model is defined by the observations of Wenzel et al. (1975).

I then take the ratio of Wenzel’s nadir-entrant flux, $f_{\text{Wenzel}}$, with the predicted limb-entrant flux, $F_{p, FTS}$, for each energy bin. A fit of this ratio is made between where the data sets overlap, 185 $< E < 300$ MeV:

$$j_p(E) = \frac{f_{\text{Wenzel}}(E)}{F_{p, FTS}(E, \theta_{\text{limb}})}.$$  \hspace{1cm} (5.16)

As Ft. Churchill ($\lambda=67^\circ$) reflects conditions at $\approx 0$ GV rigidity cutoff, I assume Wenzel’s measurement reflects conditions above Antarctica. Taking the ratios of my model results for Ft. Sumner and $\phi = 655$ MV where the rigidity cutoff is negligible, I evaluate Equation 5.16. Fitting the results yields:

$$j_{p, FTS}(E) = 0.139 \ e^{-6.019E} \ E^{0.956}$$

$$j_{p, McM}(E) = 0.020(2) \ e^{-6.029E - 0.147E^{-1}} \ E^{-0.809}.$$  \hspace{1cm} (5.17)
However, the spectrum at limb will not converge to the nadir-entrant spectrum immediately past limb. I introduce a new term in Equation 5.18 that modulates the zenith angle and energy dependence of the convergence. Thus, Equation 5.16 is used as an argument to an arbitrary boundary condition such that the limb-entrant spectrum begins to converge to the Wenzel reported spectrum at $\frac{\pi}{8}$ beyond the definition of limb. This convergence is modeled as by the function:

$$1 + \frac{j(E) - 1}{1 + e^{4\pi[\theta - (\theta_{\text{limb}} + \pi/8)]}},$$

where zenith angle $\theta$ is defined in radians. The factor of $4\pi$ and the phase shift of $\pi/8$ are also arbitrarily chosen as to limit the upward contribution as the production of the upward flux will be limited beyond limb. The resulting equation for the angle-dependent spectrum beyond $\theta_{\text{limb}}$ is:

$$F_p(E, \theta > \theta_{\text{limb}}) = F_p(E, \theta_{\text{limb}}) \times \left[1 + \frac{j(E) - 1}{1 + e^{4\pi[\theta - (\theta_{\text{limb}} + \pi/8)]}}\right],$$

where $f_0(E)$ is the reconstructed spectrum at limb for a given energy and downward proton model and $\theta$ is in radians. The upward proton spectrum over Ft. Sumner is plotted for various angle cuts in Figure 5.9. The units for all $F_p(E)$ are $(s \text{ m}^2 \text{ GeV sr})^{-1}$ and where $E$ is in GeV.

### 5.5 Secondary Neutrons

I use the atmospheric neutron model of Kole (2014) which incorporates altitude, the solar modulation potential, and geomagnetic latitude based on numerous observations. I use the input parameters defined in Table 5.1 as inputs to this model for each simulation case to determine an isotropic flux of thermal neutrons. Specifically, the depth for neutron models over Ft. Sumner and Antarctica is $h=3.45 \text{ g cm}^{-2}$, where $h$ is Kole’s symbol for atmospheric
depth $\tau$. The magnetic latitudes are $\lambda=47.4^\circ$ and $\lambda=80^\circ$, respectively. For solar modulation over Ft. Sumner, I assume $\phi=750$, corresponding to $s=0.58$, which is Kole’s symbol for solar modulation that is defined as $s=0$ for $\phi=250$ MV and $s=1$ for $\phi=1200$ MV.

5.6 Secondary and Cosmic Leptons

Cosmic protons and heavier nuclei interactions with the upper atmosphere induce air shower cascades generating secondary electrons and muons along with their antiparticles in the regime from keV to GeV. In addition to secondaries, cosmic electrons also contribute to the background. Equation 5.20 is the general form of the lepton background models:

$$F_{lep}(E, \theta) = f_{lep}(E) \ s_{lep}(\theta),$$

(5.20)

where $f_{lep}(E)$ has units of (s cm$^2$ GeV sr)$^{-1}$ and $s_{lep}(\theta)$ is a unitless shaping function that defines the spectrum’s angular dependence. This is modified for the upward fluxes of muons in a manner similar to that of upward protons.

For secondary electrons, the main modes of production are “knock-on”$^3$ collisions, $\pi^- \rightarrow \mu^-$-decay, and pair production. As discussed in Section 5.7, the dominant source of electron ejection and Compton scattering events are upward atmospheric $\gamma$-rays, which are abundant at small atmospheric depths. I therefore assume the “knock-on” component has a modest dependence on zenith angle. I assume there exists a generally downward distribution of $\pi$ production due to the conservation of momentum of the incident primaries. Notably, the $\pi^\pm \rightarrow \mu^\pm \rightarrow e^\pm$ interaction chain typically occurs at depths 20-200 g cm$^{-2}$, implying directions of arrival from within $60^\circ \lesssim \theta \lesssim \theta_{\text{limb}}$ (Boezio et al., 2000). To determine electron

$^3$“Knock-on” is used by Beuermann as a catch-all for ejected electrons with sufficient energy to cause additional ionizations.

Secondary positrons, similar to electrons, are produced though weak interactions, muon decay, and pair production. However there are no “knock-on” positrons. Symmetry arguments, calculations in DS74 and Stephens (1981), and High Energy Antiproton Telescope (HEAT) measurement reported in DuVernois et al. (2001) indicate that for the same sources and means of production there is 1:1 ratio of secondary positrons and electrons. To determine the positron spectrum and its shaping, I rely on DS74, Stephens, and Duvernois (Daniel and Stephens, 1974; Stephens, 1981; DuVernois et al., 2001). The spectra of downward muons over Ft. Sumner and Lynn Lake was determined by BESS (Motoki et al., 2003; Abe et al., 2003). As a result, their spectra and shaping are calculated identically to the downward proton flux in Section 5.4. However, as there is little data on upward fluxes of $e^+$ and $\mu^\pm$, I develop new semi-empirical models for their flux and shaping functions between limb and nadir.

I do not evaluate the contribution from cosmic ray positrons, as I assume they will annihilate prior to reaching relevant depths. However, as the cosmic positron spectrum has been established similar methods can be used to determine their contribution (Adriani et al., 2013).

5.6.1 Above Ft. Sumner, NM

5.6.1.1 Secondary Electrons

I use the results of the DS74 spectral model to determine the angle-dependent spectra over Ft. Sumner. However, the results are reported as figures which were traced using Plot
Digitizer. Moreover, the results were reported for $R_{\text{cutoff}} = 0, 1,$ and 4.5 GV, not the 3.7 GV over Ft. Sumner.

To use his results, I first begin by adjusting the spectrum to reflect a depth of $\tau = 3.45$ g cm$^{-2}$ and $R = 3.7$ GV. To adjust for depth, I determine the total flux as a function of depth for electrons with $E > 10$ MeV shown in DS74’s Figure 15. The difference in normalization is 3.30. All electron and positron intensity spectra $f(E)$ over Ft. Sumner have already incorporated this term.

The spectra of $R_{\text{cutoff}} = 0, 1,$ and 4.5 GV are most different at $E < 38$ MeV. I take the ratio of normalization of the $R_{\text{cutoff}} = 0$ and 1 GV flux to $R_{\text{cutoff}} = 4.5$ GV flux at each energy bin. From these, I determine a linear fit parameterizing the variation in flux as a function of rigidity cutoff:

$$f(E, R_{\text{cutoff}} = 3.7 \text{ GeV}) = f(E, R_{\text{cutoff}} = 4.5 \text{ GeV}) \left[ 1 - 0.8 \frac{\Delta N}{\Delta R} E \right], \quad (5.21)$$

where the slopes are $\Delta N/\Delta R$ and $f(E, R=4.5 \text{ GeV})$ is the spectrum reported by DS74. The slopes $\Delta N/\Delta R$ show an inverse proportion with energy as the as seen in Figure 5.10. The relationship confirms that as rigidity cutoff increases, the secondary electron flux that results from cosmic ray showers diminishes. This effect is most pronounced at lower electron energies. From its fit, shown in Equation A.16, I determine slopes for each energy bin to determine the spectrum for $R=3.7$ GV. The same procedure is followed for positrons which show a different relationship. A polynomial fit is made and is given in Equation A.17. Beyond $E > 0.7$ GeV, I assume $\Delta N_{e^+}/\Delta R = -0.05$ to represent a decreasing variation over rigidity cutoff as energy increases. The exact value has a negligible effect on the final result. Both equations defining $\Delta N/\Delta R$ are shown in Appendix A.
The modulation of the secondary electron, left, and positron, right, flux at \( \sim \text{MeV} \) energies by change in rigidity cutoff, \( \frac{dN}{dR} \), is inversely proportional to energy. Open circles show the variation as slopes of linear fits of DS74 model results relating flux between \( R_{\text{cutoff}} = 0, 1, \) and 4.5 GV spectra. My fit is shown as a line.

The resulting spectrum of downward electrons over Ft. Sumner at 3.45 g cm\(^{-2}\) can be characterized by Equation 5.22:

\[
f_{e^-}(E, \theta \leq \theta_{\text{limb}}) = \begin{cases} 
1.520 \times 10^{-5} E^{-2.282} & 0.002 \leq E < 0.05 \text{ GeV} \\
9.036 \times 10^{-4} \left[ \left( \frac{E}{0.583} \right)^{3.30} + \left( \frac{E}{0.583} \right)^{1.18} \right]^{-1} & E \geq 0.05 \text{ GeV},
\end{cases}
\]

where units are in (s cm\(^2\) GeV sr\(^{-1}\)).

The contribution of the \( \pi^- \)-decay chain to the electron flux diminishes past limb, resulting in the primary contributions coming from "knock-on" and pair production. This results in a harder spectrum without the \( E > 50 \text{ MeV} \) contributions from \( \pi^- \) decay. The different spectra of downward and upward atmospheric electrons have been observed. Here, I divide the upward spectrum into regimes above and below a phenomenological spectral break at 200 MeV. Verma (1967) reports observation of splash albedo electrons over Palestine, Texas, with a potential spectral break between 100-300 MeV. Per Israel (1969) in his observation above Ft. Churchill, Canada, a break in the splash albedo electron spectrum may exist between 20-100 MeV. QARM model output reflecting conditions above Ft. Sumner in October 2014 suggests a break appears near 200 MeV. Above 200 MeV, data from Verma
shows a spectral index $\Gamma = 1.20(0)^{+2.54}_{-0.15}$, Israel $\Gamma = 2.718^{+2.17}_{-0.414}$, and QARM $\Gamma = 3.06$. I select $\Gamma = 3$ as the spectra index of the upward electron flux.

The equation defining the spectrum of secondary electrons over Ft. Sumner at $3.45 \text{ g cm}^{-2}$ is:

$$f_{e^-}(E, \theta > \theta_{\text{limb}}) = 2.330 \times 10^{-5} E^{-3}$$

where units are again in $(s \text{ cm}^2 \text{ GeV sr})^{-1}$.

I determine a shaping parameter $\eta$ for electrons by using DS74’s results presented as a figure illustrating the change in flux as a function of depth. A fit to his results for the variation in flux for $> 10$ and $> 100$ MeV. I fit both functions which I define as $\eta(E, \tau)$, that is piecewise on a $100$ MeV boundary condition. At $3.45 \text{ g cm}^{-2}$ above Ft. Sumner, I define $\eta_{e^-}(E, \theta)$:

$$\eta_{e^-}(E, \tau) = \begin{cases} 
\frac{9.151 \times 10^4}{E^{2.77} + \tau^{2.77}} & E \leq 100 \text{ MeV} \\
47844.2 & E > 100 \text{ MeV},
\end{cases}$$

where $\tau < 115 \text{ g cm}^{-2}$, the depth at $\theta = 90^\circ$. Between $\theta = 90^\circ$ and limb, I set $\tau = 115 \text{ g cm}^{-2}$ irrespective of actual depth to reflect the loss of production from the $\pi$-decay chain. In order to develop $s_{e^-}$ from $\eta_{e^-}$ the function must be mapped with atmospheric depth as a function of zenith angle and then normalized to the flight depth of $3.45 \text{ g cm}^{-2}$ as shown in Equation 5.26.

Unfortunately, the zenith angle dependence of secondary electrons past limb has not been measured. Verma (1967) reports that over Palestine, TX the splash albedo electron flux is about a factor of two greater than the reentrant albedo electron flux between $37$-$344$ MeV. He calculates that the reentrant flux is greater than the local secondary flux by about a factor of five. However, given the instrument’s field of view and relatively small atmospheric
Figure 5.11: The zenith angle dependence of secondary electrons relative to zenith-entrant flux for electrons of $E \leq 100$ MeV and $E > 100$ MeV. Secondary positrons of all energies are represented by electrons of $E > 100$ MeV as they are produced through similar processes.

To provide an estimate that may describe the zenith angle-dependence of upward electrons, I generalize the upward electron flux by shifting $\eta$ around limb:

$$\eta(\theta > \theta_{\text{limb}}) = \eta(\pi - \theta).$$  \hfill (5.25)

What results is a near sinusoidal dependence on zenith-angle wherein flux is less beyond the maximum value at limb than right above it. I now define $s_{e^-}(E, \tau(\theta))$ in Equation 5.26 for $\tau = 3.45 \ \text{g cm}^{-2}$:

$$s_{e^-}(E, \theta) = \begin{cases} 
\frac{\eta(\tau(\theta))}{\eta(E, 3.45 \ \text{g cm}^{-2})} & \theta \leq \theta_{\text{limb}} \\
\frac{\eta(\tau(\pi - \theta))}{\eta(E, 3.45 \ \text{g cm}^{-2})} & \theta > \theta_{\text{limb}}
\end{cases}$$  \hfill (5.26)

where $\theta$ is in units consistent with the definition of $\tau(\theta)$ and $E$ is in MeV, referencing Equation 5.24. The variation of $s_{e^-}(E, \theta)$ above limb is shown in Figure 5.11.
5.6.1.2 Secondary Positrons

A similar approach can be used to determine functions describing the net positron flux. For positrons, the results reported by Codino et al. (1997) have shown good agreement between observations and the estimates of the DS74 and Stephens (1981) models. I use the reported downward positron flux in DS74 adjusting for the rigidity cutoff and depth of our 2014 flight. According to DS74 and Beuermann (1971), electrons with $E \gtrsim 100$ MeV and positrons of all energies share approximately the same spectrum and normalization. Assuming this to be from symmetry in production, I develop the shaping function of the positron spectrum above limb using the DS74 model for the electron spectrum as a function of depth above 100 MeV to be representative for the positron spectrum. However, there have been limited investigations of the upward positrons that can constrain their spatial distribution and normalization. Per DuVernois et al. (2001), there is an approximate 1:1 ratio of upward electrons and positrons at $\sim 1$ GeV, well above the energy of “knock-on” electrons. I use this data to extend the shaping function of electrons to that of positrons.

$$f_{e^+}(E) = \begin{cases} 2.120 \times 10^{-2} \left[ \frac{E}{0.0583} \right]^{1.493} + \left( \frac{E}{0.058} \right)^{-0.374} \right]^{-1} & 0.002 \leq E < 0.5 \text{ GeV} \\ 8.242 \times 10^{-5} E^{-2.924} & E \geq 0.5 \text{ GeV} \end{cases} \quad (5.27)$$

As the means of producing positrons and electrons above $\sim 100$ MeV are the same, the shaping function for positrons of all energies is defined by $\eta_{e^-}$:

$$\eta_{e^+}(E) = \eta_{e^-}(E > 100 \text{ MeV}) \quad (5.28)$$
Figure 5.12: The variation of $\mu^-$, left, and $\mu^+$, right, spectra as a function of depth are nearly identical. My fitted power laws descriptions are shown as dotted lines along side BESS data represented by circles.

### 5.6.1.3 Secondary Muons

The derivation of the spectrum and shaping functions of muons is different than of electrons and positrons as they do not share the same means of production. Moreover, as there have been no observations of their upward fluxes, I must determine a sensible method of their estimation.

In order to determine the spectrum of downward $\mu^\pm$, I again use the direct results of BESS over Ft. Sumner (Abe et al., 2003). An approach similar to Section 5.4.1 is followed. First, power laws indicies $\Gamma(\tau)$ to describe the growth in flux as a function of depth are developed. As atmospheric muons are essentially secondaries, I see flux increase with depth for $\mu^-$ and $\mu^+$ as shown on Figure 5.12 along with their fits. I define $\Gamma_{\mu^-} = 0.814 \pm 0.0589$ and $\Gamma_{\mu^+} = 0.873 \pm 0.0317$ as I average across energy bins for $\mu^-$ and $\mu^+$, respectively. Table 5.5 describes the quality of specific fits. With this, I adjust the observed zenith-entrant flux observed at BESS’s minimum observing between $\tau=4.58$ and $3.45 \text{ g cm}^{-2}$.

BESS observation reported in Abe et al. (2003) only extends to 2.5 GeV. To fit the continuum spectrum I determine the spectral index at $E > 2.5$ GeV from BESS results over
Lynn Lake (Motoki et al., 2003). Combining results, I determine the zenith-entrant flux at our depth over Ft. Sumner:

\[
f_{\mu^-}(E) = \begin{cases} 
3.393 \pm 1.842 \ E^{-1.319} & 0.1 \leq E < 1.7 \ \text{GeV} \\
6.604 \pm 2.570 \ E^{-2.574} & E \geq 1.7 \ \text{GeV}
\end{cases} \tag{5.29}
\]

\[
f_{\mu^+}(E) = \begin{cases} 
3.983 \pm 1.996 \ E^{-1.225} & 0.1 \leq E < 1.9 \ \text{GeV} \\
8.063 \pm 2.840 \ E^{-2.324} & E \geq 1.9 \ \text{GeV}
\end{cases} \tag{5.30}
\]

where units are in (cm\(^2\) s GeV sr\(^{-1}\)).

The results for the zenith angle-dependence of the downward muon flux can be computed in a similar manner to protons:

\[
s_{\mu^\pm}(\theta < \theta_{\text{limb}}) = \left[ \frac{\tau(\theta)}{3.45} \right]^{\Gamma_{\mu^\pm}}, \tag{5.31}
\]

but there is no direct constraint on their upward flux. The dominant source of muons is through \(\pi^\pm\)-decay. Consequently, I derive results for muons based on their \(\pi^\pm\) parent particles.

I determine an estimated \(\mu^\pm\) spectrum peak based on the theoretical spectral peak of \(\pi^\pm\) calculated in Stephens (1981), verified by experimental results of Brunetti et al. (1996) and Codino et al. (1997), and kinematics. Assuming negligible momentum transfer to \(\nu\) and \(\bar{\nu}\) in these interactions, \(E_{\text{peak}} = 284\ \text{MeV}\) for \(\mu^\pm\). While there is no experimental data

Table 5.5: Power law fits describing variation of muon flux over Ft. Sumner shown along with their RMS error.

<table>
<thead>
<tr>
<th>GeV</th>
<th>(\Gamma)</th>
<th>(\sigma_{RMS})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu^+)</td>
<td>0.50</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>2.2</td>
<td>0.90(0)</td>
</tr>
<tr>
<td>(\mu^-)</td>
<td>0.50</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>2.2</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>8.41</td>
<td>0.75</td>
</tr>
</tbody>
</table>
available to validate this estimate for $\mu^-$, I can compare to QARM model output where $E_{\text{Peak}} = 285$ MeV.

Using this peak, I estimate the upward flux. I determine phenomenological mapping functions $j_{\mu, \text{FtS}}$ and $x_{\mu, \text{FtS}}$ to relate the results of upward proton flux to the upward muon flux. Although the production of muons is a tertiary process for protons, I assume that the upward flux of protons directly relates to the upward flux of muons. Thus, I shift $j_p(E)$ based on the energy difference between the peaks of the downward muon and proton spectra at limb. Equation 5.32 defines $j_{\mu}(E)$ for muons past limb at Ft. Sumner based on Equation 5.16:

$$ j_{\mu, \text{FtS}}(E) = j_{\mu, \text{FtS}}(E - 0.159 \text{ GeV}). $$

(5.32)

I use the same scaling function described in Equation 5.18 to determine the angle-dependent spectrum past limb:

$$ F_{\mu}(E, \theta > \theta_{\text{limb}}) = F_{\mu}(E, \theta_{\text{limb}}) \times \left[ 1 + \frac{j_{\mu}(E) - 1}{1 + e^{4\pi[\theta - (\theta_{\text{limb}} + \pi/8)]}} \right]. $$

(5.33)

Here, units of $F_{\mu}$ are $(s \, m^2 \, \text{GeV sr})^{-1}$ and units of $\theta$ are in radians.

### 5.6.2 Above Antarctica

Backgrounds over Antarctica are calculated in a similar manner. For electrons and positrons, the spectra are defined by DS74’s results for “high solar modulation,” or solar minima.\(^4\) However, to develop an accurate estimate of the secondary electron and positron fluxes, the solar modulation values assumed by DS74 need to be determined.

\(^4\)**N.B.** I note that the differential spectrum of negatrons at 0.0 GV rigidity cutoff for 1.0 g cm\(^{-2}\) shown in Figure 12 of DS74 is not consistent with the integrated spectra at solar maximum or minimum at the same depth in Figure 15. I believe the normalization of the differential spectrum was reduced by a factor of 10 for illustration. A similar reduction was used for positron differential fluxes in the same figure.
For $\mu^\pm$, BESS data exists and is scaled through the prescription for cosmic and secondary protons as in Section 5.4. Shaping functions at Antarctica are defined with reported growth curves with similar adjustments made as at Ft. Sumner to account for limb and upward flux.

### 5.6.2.1 Secondary Electrons

The spectrum of secondary electrons at $R_{\text{cutoff}} = 0$ GV, corresponding to the magnetic poles, and during high solar modulation is described by DS74. A fit to this spectrum is defined in Equation 5.35. To effectively estimate the incident spectra for $\phi_{\text{Max}}$ and $\phi_{\text{Min}}$, I need to determine the true solar modulations values used in DS74’s calculation of the high solar modulation spectrum.

Assuming the flux of secondary leptons in the atmosphere is directly proportional to the flux of incident cosmic ray protons, I reconstruct the DS74 solar modulation values by using my model for protons over Antarctica. For secondary electrons, DS74’s model scales for solar modulation based on the observations reported by Beedle and Webber (1968) and Schmidt (1972) for low and high modulation, respectively. Using the table of historic solar modulation values in Usoskin et al. (2011), I determine values of $\phi \approx 400$ and 900 MV for as having coincided with the experiments of Beedle and Webber (1968) and Schmidt (1972). Next, I use these solar modulation values to develop an expression for the incident proton spectrum at $R_{\text{cutoff}} = 0$ GV using the model of protons over Antarctica defined in the previous section. Taking the ratio of integrated proton spectra from 0.1-100 GeV results in a difference in normalization of 2.19. This is reasonably close to the ratio of $\approx 1.9$ found in DS74.

Continuing this approach, I use the proton models for $\phi_{\text{Max}}$ and $\phi_{\text{Min}}$ to determine their ratios with respect to the assumed high solar modulation $\phi \approx 900$ MV assumed by DS74.
The resulting normalization factors $m$ to be used in Equations 5.35 and 5.36 are:

\[ m_{\text{Max}} = 0.64 \]
\[ m_{\text{Min}} = 3.59, \]

for $\phi_{\text{Max}}$ and $\phi_{\text{Min}}$, respectively. The electron intensity spectrum is thus:

\[
f_{e^-, 0}(E) = \begin{cases} 
5.538 \times 10^{-5} m E^{2.58} & E \leq 0.032 \ \text{GeV} \\
0.0256 m \frac{E}{2.74(0)E^{-1} + 18.768E^{-2}} & E > 0.032 \ \text{GeV},
\end{cases}
\]

with $f_{e^-, 0}(E)$ in units of $(\text{s m}^2 \text{ GeV sr})^{-1}$.

The shaping of the secondary electron spectrum $s_{e^-}(E, \theta)$ is dictated by Equation 5.26 as little variation was demonstrated between the depth dependence of flux at low rigidities during low solar modulation and $R=4.5$ GV. Instead, more variation is reported between high and low solar activity at $R=0$. DS74 reports that the incident flux remains constant till $\tau \gtrsim 10 \ \text{g cm}^{-2}$ during low modulation while it increases by about a factor of two from 0 to 10 g cm$^{-2}$ during high solar modulation. Figure 5.7 showing my predictions of the shaping function $s_p(E, \theta)$ show strong, uniform intensity from near zenith during low solar modulation.

### 5.6.2.2 Secondary Positrons

The spectrum of secondary positrons shares the same normalization factors $n$ as secondary electrons defined above. The positron spectrum is defined in Equation 5.36. Their shaping function is is defined by Equation 5.26 while accounting for the positron restriction
on $\eta$ in Equation 5.28.

$$f_{e^+,0}(E) = \begin{cases} 
0.32 E^{-0.38+175.24/E^{2.04}} & E \leq 0.466 \text{ GeV} \\
1.166 \times 10^{-3} E^{-3.13} & E \leq 0.032 \text{ GeV},
\end{cases}$$

(5.36)

where units again are $(\text{s m}^2 \text{ GeV sr})^{-1}$.

## 5.6.2.3 Secondary Muons

I follow a similar approach as in 5.4.2 to determine the spectra and angular distribution of downward muons at solar minimum and maximum. However, the BESS reported muon spectra are the average of their measurements of BESS 1997, 1998, and 1999 Lynn Lake flights. The reported spectra for $\mu^+$ and $\mu^-$ are first fitted with $\sigma_{\text{RMS}}=0.10$ and 0.01 $(\text{s m}^2 \text{ GeV sr})^{-1}$, respectively. Next, I use the variation in flux between the BESS flights in 1997 and 1999 per reported BESS energy bin to determine a linear fit between the two points to determine the flux for each energy bin as a function of solar modulation. I use this linear fit to extrapolate spectra for $\phi_{\text{Min}}$ and $\phi_{\text{Min}}$ from the 1999 data (Motoki et al., 2003). Muon zenith-entrant spectra are defined by Equations A.36, A.37, A.38, and A.39. These are all located in Appendix A.1.5.

The muon shaping function defines the angular dependence of downward flux. The $\Gamma$ shaping power indicies are the same as in Equation 5.31. Past the limb, the spectrum is determined in the manner of muons over Ft. Sumner.

## 5.6.3 Cosmic Electrons

Separate from the contribution of secondaries, I account for the background contribution from cosmic electrons over Ft. Sumner and Antarctica. The form of the zenith
angle-dependent spectrum of cosmic electron follows Equation 5.2. For the spectrum of
cosmic electrons incident on the top-of-the-atmosphere, I fit the spectrum observed by AMS
(Aguilar et al., 2014). A good broken power law fit was found by fitting the spectrum in
four energy domains as per Equation A.30. The fit to this spectrum is $\sigma_{\text{RMS}} = 0.202 \text{ (s m}^2 \text{ GeV sr)}^{-1}$, are shown in Figure 5.13.

In order to determine the shaping function for cosmic electrons, it’s necessary to
determine how they are attenuated once they enter the atmosphere. To do so, I calculate the
attenuation coefficient $\bar{\mu}$ assuming that energy loss occurs primarily due to bremsstrahlung
and pair-production and mean atmospheric composition using Equation A.18 (See Eq. 32.26-
32.28 in Olive et al., 2014). The resulting attenuation coefficient $\bar{\mu}$ and atmospheric mass
model are used to determine the shaping function for cosmic electrons:

$$s_{\text{cosmic e}^-}(\theta) = e^{-\mu_{e^-} \tau(\theta) L(\theta)}, \quad (5.37)$$

where $\mu_{e^-}$ is the attenuation factor for electrons and $L(\theta)$ is path length from instrument
to top-of-the-atmosphere as a function of zenith angle (not a characteristic length as in

Figure 5.13: The cosmic electron spectrum as observed by AMS, open circles, as compared
to my fit (Aguilar et al., 2014).
Equation A.18). I only account for unattenuated cosmic electrons arriving at the detector and therefore restrict the lower bound of the spectrum of electrons to $E_{\text{min}} = \max(R, \phi)$. The final equation for cosmic electron flux at the balloon is defined in Equation 5.38 where $E > E_{\text{Min}}$.

$$F_{\text{cosmic}}(E, \theta) = f_{0, \text{cosmic}}(E) e^{-\mu_e} \tau(\theta)$$  \hspace{1cm} (5.38)

### 5.7 Secondary X-Rays and Gamma Rays

Our model of the atmospheric $\gamma$-ray background is based on the angle-dependent spectrum defined in Costa et al. (1984) which combines results of Ling, Schönfelder, Ryan, Graser (Ling and Gruber, 1977; Schönfelder et al., 1977; Ryan et al., 1977; Graser and Schönfelder, 1977). It describes the zenith angle-dependence of the spectrum from 1-10 MeV. However, results based on those described in Chapter 7 indicate $\gamma$-rays of $E > 10$ MeV make a substantial contribution to the background event rate. Therefore it is necessary to extend the Costa model. I extend the domain of the model by incorporating observations of atmospheric $\gamma$-rays $10 < E < 200$ MeV as reported in Kinzer et al. (1978). Moreover, I incorporate results of Akyüz et al. (1997), published after Costa, and Ryan et al. (1977) to refit the spectrum’s angle-dependence. A comparison of my modifications to the Costa model along with pertinent and experimental results from 1.5-10 MeV is shown in Figure 5.15.

Equation 5.39 shows the general form of the equation defining the atmospheric $\gamma$-ray spectrum. It is similar to others in this paper with a unitless shaping factor term and the angle-averaged intensity spectrum. It also includes an additional term to account for the...
rigidity cutoff dependence of the spectrum:

\[
F_{\text{atm}} \gamma(E, \theta, R, \phi) = f_{\text{atm}} \gamma,0(E) \ s_\gamma(\theta) \ \left( \frac{4.5 \ GeV}{\text{Max}(R, \phi)} \right)^{\alpha(E)},
\]

where the units of \( F_{\text{atm}} \gamma \) and \( f_{\text{atm}} \gamma,0 \) are \((s \ cm^2 \ MeV \ sr)^{-1}\), \( s_\gamma \) is unitless, and \( R \) and \( \phi \) are the local rigidity cutoff and solar modulation factor respectively.

To extend the Costa model, I begin by modifying its intensity spectrum. First, I set its normalization to that of Costa’s at \( E < 1 \) MeV, which had been defined by Ling and Gruber (1977). I divide the spectrum with two spectral breaks at 40.1 and 152 MeV, which describe the influence of the \( \pi^0 \)-decay contribution. For \( 0.5 < E < 40.1 \) MeV, I fit the spectrum index to the results of Schöenfelder et al. (1980); Kinzer et al. (1978), and Ryan et al. (1977). For \( 40.1 < E < 152 \) and \( E > 152 \) MeV, I fit the spectrum index to results of Kinzer et al. (1978) and Ryan et al. (1977). The resulting atmospheric \( \gamma \)-ray intensity spectrum is defined in Appendix A in Equation A.41 where units are \((s \ cm^2 \ keV \ sr)^{-1}\). Figure 5.14 compares these observations, my model, and the model of Costa. When fitting to the selected data \( E < 10 \) MeV, my model and the Costa model show \( \sigma_{RMS} = 3.920 \times 10^{-4} \), when fitting to selected data \( E > 10 \) MeV, my model shows \( \sigma_{RMS} = 1.475 \times 10^{-6} \). Units of \( \sigma_{RMS} \) are \((s \ cm^2 \ keV \ sr)^{-1}\) for both.

I next modify the shaping term of the Costa model by refitting his function with experimental data. Specifically, I refit by incorporating recent results of Akyüz et al. (1997) and past observations for the regime \( 1.5 < E < 10 \) MeV (Schöenfelder et al. (1977); Graser and Schöenfelder (1977), and Ryan et al. (1977)). I preserve the form of Costa’s shaping term and only introduce new normalization factors and phase shifts. Although this modification results in an overconstrained fit it has the benefit of relying on Costa’s validation.
Figure 5.14: Comparison of observed and modeled atmospheric $\gamma$-ray spectra occurring over Palestine, TX at $\tau$=1 g cm$^{-2}$. I show results of Graser and Schönfelder (1977); Ryan et al. (1977); Schönfelder et al. (1980), and Kinzer et al. (1974) as compared to the fitted models of Costa et al. (1984) and author’s.

Costa defines his shaping function piecewise over three zenith-angle regimes, $0 \leq \theta < 0.698$, $0.698 \leq \theta < 1.92$, and $1.9199 \leq \theta < \pi$. At $\theta < 0.698$ [rad], Costa’s defines the shaping function $R(E, \theta, \tau) = \tau$. However, based on the data of Schönfelder et al. (1977) and Ryan et al. (1977) it should be $\sim (0.7 \pm 0.20_{-0.34}) R(E, \theta < 0.698, \tau)$. Which I approximate as $R(E, \theta, \tau) = 0.5 \tau$. Next, I shift the zenith-angle boundary at $\theta = 1.9199$ to occur closer to limb, which better fits the data of Schönfelder et al. (1977), existing models of Graser and Schönfelder (1977) and Ling and Gruber (1977), and the physics at limb where cosmic ray hadron-initiated production of $\gamma$-rays ought to fall off rather than increase. I now define the second regime to be $0.698 \leq \theta < \frac{5\pi}{9}$. At $\theta \geq \frac{5\pi}{9}$, I set qualitative boundaries such that a new fitted shaping function can be centered within the measurements of Ryan et al. (1977); Schönfelder et al. (1977), and Akyüz et al. (1997), the latter of which has less uncertainty. Here, for simplicity I use the measurements of Akyüz et al. (1997) at Ft. Sumner, only, as opposed to including those over Alice Springs, Australia, which are at a higher rigidity cutoff.
Figure 5.15: Comparison of experimental results for and models of integrated atmospheric γ-ray flux as a function of angle at $\tau = 2.5 \text{ g cm}^{-2}$. Experimental results are from Ryan et al. (1977); Akyüz et al. (1997) and Schöenfelder et al. (1977). Models are from Schöenfelder et al. (1977); Graser and Schöenfelder (1977); Ling and Gruber (1977); Costa et al. (1984) and this work.

$(\lambda = -29^\circ)$. I provide the following zenith angle constraints:

\begin{align*}
R(E, \theta = \frac{5\pi}{8}) &= 0.88 \ R_0(E, \theta = 1.92) \\
R(E, \theta = \frac{13\pi}{8}) &= 0.6 \ R_0(E, \theta = \frac{13\pi}{8}) \\
R(E, \theta = \pi) &= 0.5 \ R_0(E, \theta = \pi)
\end{align*}

(5.40)

where $R_0$ is Costa’s original shaping equation. The first constraint shifts the maximum of the distribution closer to limb while reducing the total flux by a factor of 0.88 to better reflect the results of the experiment of Schöenfelder et al. (1977). The latter two values limit the flux contribution at large zenith angles so it better reflects results of Akýuz and Ryan.

To provide freedom for the fit, I offer factors of the form $A_1 \ \theta + A_2$ and $B_1 \ \theta^2 + B_2 \ \theta + B_3$ to be multiplied with $R_0(E, \ 0.698 < \theta \leq 1.92)$ and $R_0(E, \ 1.92 < \theta \leq \pi)$, respectively. Additionally, I constrain the fit to ensure continuity of flux at $\theta$ boundaries. Finally, because data on the zenith-dependence of $> 10$ MeV γ-ray is limited, I assume that
Figure 5.16: Comparison of the results for the angle-dependent spectrum of the Costa model, dotted lines and my modifications to it, solid lines. Results for 1 g cm\(^{-2}\) appear left while 10 g cm\(^{-2}\) appear right. All assume \(R_{\text{cutoff}} = 4.5\) GV.

the shaping function of energies above 250 MeV per \(R(E > 250\) MeV, \(\theta > 5\pi/9\), \(\tau\)) = \(R(E > 250\) MeV, \(\theta > 5\pi/9\), \(\tau\)). The resulting shaping function equations are described in Equations A.42-A.43 located in Appendix A.

Costa cites the results of Thompson et al. (1981) and Frontera et al. (1981) for determining the parameter \(\alpha\), which describes the increase in spectrum normalization as rigidity cutoff changes. As I extend the spectrum used in the model, I extend the work Thompson and Frontera to adjust for varying rigidity cutoff (Frontera et al., 1981). Costa suggests \(\alpha=1.05\) to describe “intermediate energies.” I modify \(\alpha\) in Equation A.44 as reported \(\alpha=0.96\) at \(E\approx300\) keV to \(\alpha=1.13\) at \(E>30\) MeV (Costa et al., 1984).

While this scaling is of little impact accounting for the difference between Palestine and Ft. Sumner, a significant difference exists between Palestine and Antarctica where rigidity cutoff effects are effectively nonexistent. In my simulations at Antarctica I substitute the value for solar modulation for geomagnetic rigidity cutoff as the lower bound for primary cosmic ray spectra. Otherwise, I make no assumptions on the solar modulation dependence of the atmospheric \(\gamma\)-ray spectrum despite indication of its effect in Harris et al. (2003). In Harris, the mean flux at the 511 keV annihilation line between periods of high and low solar
activity varies with a factor of 1.104, for $R < 7$ GV. However, rigidities $< 7$ GV encompass both Ft. Sumner and Antarctica and leave the problem underconstrained.

5.8 Cosmic X-rays and Gamma-Rays

From hard X-ray to $\gamma$-rays, the primary source of the cosmic backgrounds are galactic diffuse emission and active galactic nuclei, giving an approximately isotropic distribution across the sky above the atmosphere (Gruber et al., 1999; Ajello et al., 2008; Ackermann et al., 2015). At balloon altitudes the CXB is zenith angle-dependent owing to the absorption and scattering of the CXB photons in the atmosphere. While measurements of the CXB at balloon altitudes exist (summarized in Gehrels, 1985), these were generally performed to measure the CXB and not its background. Notably, Schönenfelder et al. (1977) develop an approach to estimate the effects of the atmosphere on CXB which I use this section. Separately, spaceborne observation of the CXB has yielded precise measurements of its spectra, extending from keV to GeV, which I use in my analysis.

I consider two components to the CXB contribution to the background at small atmospheric depths. One component, $F_{\text{CXB}, 0}$, represents the CXB that enters the atmosphere and arrives at the detector without being attenuated, whether through absorption or inelastic scattering. The other component, $F_{\text{CXB}, s}$, represents CXB photons that arrive at the detector after inelastic scattering(s) in the atmosphere. The relative proportion of these two contributions depend on the photon path through the atmosphere and its energy. If they are summed and energy loss from completely attenuated photons (pair production or photoabsorption), is accounted for, the will result in the total CXB flux. This can be summarized by the following equation

$$F_{\text{CXB}}(E) = F_{\text{CXB}, 0}(E, \theta) + F_{\text{CXB}, s}(E, \theta) + \Delta, \quad (5.41)$$
where $\Delta$ is the loss from completely attenuated photons. In the following I neglect $\Delta$ and assume $F_{\text{CXB}}(E) = F_{\text{CXB}, 0}(E, \theta) + F_{\text{CXB}, s}(E, \theta)$.

To determine the CXB contribution to the background, I follow an approach similar to my derivation of the angle-dependent spectrum of cosmic electrons. However, this time I account for $\gamma$-rays that have been inelastically scattered. I define two types of photons with respect to the detector: “on-beam” incident photons that are unscattered or elastically scattered and “off-beam” incident photons that have been Compton scattered at least once. Through this division I am able to determine the angle-dependent spectrum of unattenuated photons and use the results of Gehrels (1985) and my atmospheric $\gamma$-ray model to determine the latter.

I use the CXB measurements summarized by Ackermann et al. (2015) to determine the proportion of total photons incident on the atmosphere that are on-beam. The fit to Ackermann’s CXB spectrum is define in Equation A.46 in Appendix A in units of $(\text{s cm}^2 \text{ keV sr})^{-1}$. Figure 5.17 shows the data and its fit, with $\sigma_{\text{RMS}} = 3.710 \times 10^{-4}$ $(\text{s cm}^2 \text{ keV sr})^{-1}$. With this intensity spectrum, I determine the on-beam angle-dependent CXB spectrum by
Figure 5.18: On-beam CXB spectrum as a function of zenith angle and energy with units in (s cm$^2$ keV)$^{-1}$.

mapping the attenuation equation with my atmospheric model per:

$$F_{CXB}(E, \theta) = f_{CXB,0}(E)e^{-\mu(E) \tau(\theta)},$$

(5.42)

where $\tau(\theta)$ is atmospheric depth as a function of zenith angle and $L(\theta)$ is the length of the path from instrument to top-of-the-atmosphere as a function of zenith angle. The attenuation factor $\mu(\bar{E})$ in units of cm$^2$ g$^{-1}$ is the attenuation based on mean atmospheric composition and is obtained from XCOM (Xcom, 2010). The resulting spectrum is shown in Figure 5.18 as a contour plot.
To determine the off-beam spectrum I assume conservation of flux wherein:

\[ F_\gamma(E, \theta) = F_{\text{atm}} \gamma(E, \theta) + F_{\text{CXB}}^0(E, \theta) + F_{\text{CXB}}^s(E, \theta). \]  (5.43)

Here \( F_\gamma \) is the total observed \( \gamma \)-ray flux, which is comprised of atmospheric \( \gamma \)-rays \( F_{\text{atm}} \gamma \), unattenuated CXB \( F_{\text{CXB}}^0 \), and inelastically scattered CXB \( F_{\text{CXB}}^s \).

For all energies and angles above limb where the total observed \( \gamma \)-ray spectrum is the sum of atmospheric \( \gamma \)-rays, on-beam CXB, and off-beam CXB. For the total observed spectrum, I take the observed total downward spectrum of Gehrels over Palestine, TX at 3.5 g cm\(^{-2}\), Equation 5.44, applied semi-isotropically.

\[
F_\gamma(E) = \begin{cases} 
2.19 \times 10^2 \ E^{0.7} & 0.024 \geq E < 0.035 \ \text{MeV} \\
5.16 \times 10^{-2} \ E^{-1.81} & E \leq 0.035 \ \text{MeV}
\end{cases}
\]  (5.44)

I use the angle-dependent spectrum from my atmospheric \( \gamma \)-ray model adjusted to \( R_{\text{cutoff}} = 4.5 \) GV to represent the rigidity cutoff over Palestine. The angle-dependent on-beam CXB spectrum is determined by Equation 5.42. To determine the off-beam component I subtract the modeled spectra from the total observed Gehrels spectrum for each angle and energy bin. At higher energies and near limb where the atmospheric \( \gamma \)-ray model over-predicts flux relative to Gehrels, I truncate the off-beam CXB flux model. A comparison of all spectra is shown in Figure 5.19 for 49, 364, and 2000 KeV, respectively.

I validate this approach by comparing my results to Takada et al. (2011) who reported results of their Monte Carlo transport simulation to estimate CXB incident on a balloon-borne detector. First, I directly compare to total observed flux from on- and off-beam CXB and atmospheric \( \gamma \)-ray flux from \( 0^\circ < \theta < 10^\circ \) in Table 5.6. Note that Takada’s fluxes include secondary fluxes, e.g. reemission, and therefore exceed our fluxes. I also evaluate the scattering fraction of the CXB, defined as \( \lambda \). I define the scattering fraction \( \lambda \) as the
Figure 5.19: Comparison of the observed total downward $\gamma$-ray spectrum reported in Gehrels (1985), estimated atmospheric $\gamma$-ray and on-beam CXB spectra, and the resulting off-beam CXB determined by subtraction. Left, center, and right show results for 49, 364, and 2000 keV, respectively.

off-beam component over all CXB photons incident upon the detector:

$$\lambda(\tau, \theta, E) = \frac{F_s(\tau, \theta, E)}{F_o(\tau, \theta, E) + F_s(\tau, \theta, E)},$$

(5.45)

where $F_o$ and $F_s$ are functions of atmospheric depth, $\tau$, zenith angle, $\theta$, and energy, $E$. In Figure 5.20 I compare our results to those of Takada as a function of zenith angle for various energy bins.

Table 5.6: The ratio of the total CXB flux to secondary $\gamma$-ray flux within $0^\circ < \theta < 10^\circ$ based on our model and simulations by Takada et. al.. For errors on the Takada model see Table 3 Takada et al. (2011)

<table>
<thead>
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<th></th>
<th>125-250 keV</th>
<th>250-550 keV</th>
<th>550-1250 keV</th>
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<tr>
<td>Our model</td>
<td>0.765</td>
<td>0.780</td>
<td>0.775</td>
</tr>
<tr>
<td>Takada</td>
<td>0.753</td>
<td>0.755</td>
<td>0.743</td>
</tr>
</tbody>
</table>
Figure 5.20: Comparison of values of $\lambda$ as derived by our analytical model for the on-beam CXB. This chart reflects values for an atmospheric depth of 8 g cm$^{-2}$ which can be directly compared to Figure 19 in Takada et al. (2011). I choose the same energy bins as Takada. The scattering fraction is shown by the chart over the line. The dotted, black line proportional to $\cos^{-1}[\theta]$ at the bottom is shown to guide the eye.

5.9 QARM Model

The QARM model determines the spectra of each background constituent by simulating cosmic ray protons and $\alpha$ and solar protons incident on a model atmosphere. Incident particles are tracked to determine their energy loss and the generation of secondaries, which are also tracked.

5.9.1 QARM Simulation Details

QARM has two options for determination of the incident cosmic ray spectra: the ISO-15390 and “JSC/NASA” (Badhwar-ONeill) models (Nymmik, 2006; O’Neill, 2010). These models are reviewed in Adams Jr et al. (2009). There is no natively implemented model for incident solar protons and a user must import SPENVIS output (ESA, Accessed 07/2016).
QARM’s atmospheric model is based on MSIS-90 model Hedin (1991). It is divided into 310 shells from sea level to 100 km altitude with composition and temperature calculated for each shell based on MSIS-90.

The local geomagnetic rigidity cutoff is calculated by MAGNETOCOSMICS (Desorgher, 2006). Rigidity cutoff maps have been calculated for the 1950, 1960, 1970, 1980, 1990, 2000, 2005 epochs for $K_p$-indices 0 to 6+, which accounts for different geomagnetic storm conditions that effect the local geomagnetic field. Only the vertical rigidity cutoff is calculated to reduced computation, thus the incidence of particles within the Störmer allowed cone and rejection of those in the forbidden cone are not accounted for.

Simulations are performed using MCNPX that only properly simulate $\alpha$ with $E<1$ GeV (Egdorf et al., 2003; Lei et al., 2004c). Response matrices are developed per simulation results for proton, neutron, charged pion, charged muon, electron, and $\gamma$-ray for different initial conditions. Reporting of electrons, muons, and pions do not distinguish between either particle or antiparticle.

5.9.2 QARM Models for X-Calibur Flight Conditions

I accessed QARM on January 22, 2016 to generate models for atmospheric radiation corresponding to the conditions above Ft. Sumner during the X-Calibur flight on September 22, 2014. I attempted to access QARM again in early February in order to calculate QARM models representative of $\phi_{\text{Min,Max}}$ over Antarctica, only to notice it had been taken offline. My inputs are shown in Table 5.7. $K_p$-index was found in the archives of the Adolf-Schmidt-Observatory (GFZ, Accessed 1/2016).
Figure 5.21 compares generated QARM spectra with angle-average spectra from my own models and Kole’s neutron model. These figures describe the atmospheric radiation environment over Ft. Sumner per conditions in Table 5.7.

Although the theory and the mechanism behind QARM is well described, how it unfolds simulations to determine spectra is not clear to the user. Consequently, interpreting its output may not describe the how spectra were determined. However, because my models were developed using A notable difference in the models is the treatment of limb-entrant particles. Due to greater mass along the LOS to limb, secondary particle flux generally rises several factors above what is seen at zenith. At X-Calibur’s altitude, limb occurs beyond $\theta = 90^\circ$. Under my assumptions, the flux continues to increase between $90^\circ$ and $\theta_{\text{limb}}$ with a spectrum that is often different. Thus, for lepton models there is a significant discrepancy from these limb contributions.

A second visible difference is in the comparative excess of $\gamma$-rays and electrons below $\sim 10$ MeV. It is possible this exists because energy loss, e.g. through scattering and bremsstrahlung. Doing so would likely result in softer power laws. However, it is unclear what the source of difference is in our models describing electrons entering from above limb.

A concern with the QARM estimates is the lack of prediction for the positron spectra. The detector response to positrons share more in common with $\gamma$-rays than electrons, as

<table>
<thead>
<tr>
<th>QARM Option</th>
<th>Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date</td>
<td>September 22, 2014</td>
</tr>
<tr>
<td>Latitude</td>
<td>34.4731</td>
</tr>
<tr>
<td>Longitude</td>
<td>104.2422</td>
</tr>
<tr>
<td>Depth</td>
<td>3.45 g cm$^{-2}$</td>
</tr>
<tr>
<td>Incident Radiation</td>
<td>GCR</td>
</tr>
<tr>
<td>$K_p$</td>
<td>4</td>
</tr>
<tr>
<td>GCR Model</td>
<td>JSC/NASA</td>
</tr>
</tbody>
</table>

Table 5.7: QARM inputs describing the 2014 X-Calibur flight.
positrons will very likely annihilate when encountering dense matter. Per the electron spectra shown in Figure 5.21, it appears that the QARM prediction for the electron spectrum assumes the inclusion of positrons. For detectors that are sensitive to $\gamma$-rays, it is unclear how an incident spectra without positrons would reflect observational conditions.
Figure 5.21: Comparisons of background spectra for QARM and models used in this work for protons, neutrons, $e^\pm$, $\mu^\pm$, and $\gamma$ shown from top to bottom, respectively. In all frames, the open circles denote QARM output. Asterisks in the legends results from the models derived in this chapter. Left and right columns show the background entrant from above and below $\theta = 90^\circ$, respectively.
Chapter 6

Results

Predicting the background event rate can help develop an effective observation program for future X-Calibur flights. If the observed background spectrum can be reasonably determined, the signal-to-noise ratio (SNR) of observations can be predicted with higher accuracy. However, the high-energy background is not monolithic and is comprised of a variety of particle types each with their own angle-dependent spectra. The net background and relative fraction of contributions may vary based on flight altitude, rigidity cutoff, and solar activity. Models describing the various background components for flights over Ft. Sumner and Antarctica have been derived in Chapter 5.

In this chapter, I determine the background rate at the detector through particle transport modeling. The angle-dependent spectra determined in Chapter 5 are used as input to a simulation that accounts for their transport through the passive material of the X-Calibur detector and the response of the CZT, ACD, and CD detectors.

I validate these models by showing the extent to which they describe the 2014 X-Calibur flight data using particle transport simulations. I then determine predictions for best and worst cases of background rate that would affect flights over Antarctica.
Figure 6.1: The mass model of the X-Calibur instrument and surrounding structure used for simulations in the Geomega environment. Black represents Al; green, Tungsten HD17; blue, CsI; red, CdZnTe detector; and yellow, plastic scintillator. PMTs and other nearby structures are not modeled.

6.1 X-Calibur Model Implementation

A high-level mass model of the 2014 X-Calibur configuration was modeled in Geomega, a part of the MEGAlib package built on the GEANT4 and CERN Root packages (Zoglauer et al., 2006). The mass model is depicted in Figure 6.1. MEGALib was chosen as it provides a simple platform for simulating source flux as a function of geometry and energy.

For simplicity, the readout electronics and cabling are not incorporated into the mass model and simplified geometries for the mechanical support and pressure vessel are used. The mass model includes neither the telescope truss nor the X-ray optics.

Table 5.1 lists the simulated test cases. For flights over Ft. Sumner, I simulate the background for four zenith angles. Telemetry from a portion of the 2014 X-Calibur flight indicates an average elevation angle of 9.26° (θ=80.74°), though the elevation angle was raised up to 51° during that portion of the flight as can be seen in Figure 3.14. To describe
the 2014 flight, I simulate cases at elevation angles of 0° and 9.26°. To simulate the 2016 X-Calibur flight, I simulate elevation angles of 35° and 55°. For cases over Antarctica, I only simulate an elevation angle of 35°. For all flights, the azimuthal orientation about the balloon gondola axis is not modeled.

The CZT energy resolution is modeled in MEGALib. I first fit the laboratory-measured energy resolution as a function of energy in Figure 3.11. The fit for resolution is determined to be

\[
\sigma(E) = \left[2.8891 + 5.236 \times 10^{-4} \frac{E}{\text{keV}}\right] \text{keV},
\]

a linear approximation of the energy resolution response from 20 – 1000 keV at -5°C. This result is incorporated into the MEGALib CZT .det file as to describe the resolution as a Gaussian function. Separately, I assume there are no failure rates for all detectors.

MEGALib simulations were performed for each background constituent separately. Within the MEGALib environment, event selection is performed using defined trigger conditions for all X-Calibur detectors: CZT, ACD event flagging with the CsI shield, and CD event flagging with the scattering stick. Each is set with appropriate event number threshold and event threshold energy.

In reality, the energy threshold for the CsI ACD is \(\gtrsim 200\) keV based on laboratory experiments where radioactive sources were placed outside of the CsI shield. Beilicke et al. (2015) reports that isotopes \(^{241}\)Am, \(^{152}\)Eu, and \(^{137}\)Cs produce \(\approx 5\%\), \(\approx 28\%\), and \(\approx 46\%\) event veto rates at their \(\gamma\)-ray peaks of 59.9, 121.8, and 661.6 keV\(^5\). Moreover, because scintillation light must travel from its location of emission to the triggering PMT, the energy threshold increases as distance to the PMT increases. MEGALib does not perform ray tracing and therefore cannot accurately model the physics behind the triggering of scintillator detectors.

\(^5\)The \(^{152}\)Eu \(\gamma\)-ray spectrum extends well beyond the peak at 121.8 keV that may contribute to event rate. It has additional prominent spectral lines at 244.7, 344.3, 778.9, 964.0, 1085.8, 1112.0, and 1408.0 keV. (Peterson)
Figure 6.2: Varying the ACD energy threshold in MEGALib changes the observed spectrum of albedo $\gamma$-rays. With an increased threshold, the number of events passing the veto condition increases. The spectrum of passing events can be compared to the dotted line representing all events.

I determine a simulated energy threshold in the ACD comparable to 2014 flight data by running tests of several different threshold energies. For example, Figure 6.2 shows the spectrum of albedo $\gamma$-rays over Antarctica at solar minimum for various ACD trigger thresholds. Comparing simulation results over Ft. Sumner with X-Calibur’s observation, I conclude that 1500 keV is an appropriate simulated energy threshold for the MEGALib environment assuming an identical scintillation response in all portions of the CsI shield. Varying the energy threshold of the scintillating stick used for the CD between 1-5 keV was found to have little effect on the background flux. Within COSIMA, the threshold condition for the CZT detectors was set at 1 keV to accept nearly all CZT events. The correct CZT energy threshold was implemented in post-processing as discussed below.

Particle transport modeling is performed by COSIMA, which interprets user inputs for GEANT4, which it calls. Based on input trigger conditions, COSIMA tracks particles depositing energy exceeding the energy threshold for sensitive volumes and outputs relevant trajectory and event data to .sim files. All COSIMA simulations assumed a detector
configuration that CZT, ACD, and CD detectors were independent so that all event data could be recorded. COSIMA/GEANT4 was configured to simulate electromagnetic interactions with the Livermore physics package. Hadronic interactions were simulated using the QGSP-BIC-HP package.

These COSIMA .sim files are then used as inputs to REVAN, which folds detector response, including energy resolution, detector failure rate, etc., with the raw events in the .sim file. The options I used for the REVAN analysis are shown in Table 6.1. REVAN outputs .tra files which contain event information, including event position, observed energy, and the physics that initial interaction that deposited energy within the detector. A script I developed parses .tra files to extract data into a .csv format which is loaded into Mathematica. Events from all MEGALib output for a flight scenario are joined, tagged with event type (i.e. photoelectric, Compton, pair-production, muon event), event time, energy deposited, deposition location, and initial background type. These event lists capture multi-pixel events as are no native cuts on single-pixel data within MEGALib.

Additional post-processing is performed to further improve simulation fidelity. To simulate the low energy detector performance, the MEGALib output is folded with the X-Calibur energy threshold response per detector ring averaged over the entire detector (Beilicke et al., 2014). The energy threshold cumulative distribution function (CDF) was determined to be:

$$E_{\text{threshold}} = \frac{1}{1 - e^{-0.356 \cdot (E - 20.983 \text{ keV})}} \text{ keV}. \quad (6.2)$$

Table 6.1: REVAN options chosen for analysis.

<table>
<thead>
<tr>
<th>REVAN Option</th>
<th>Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coincidence Search</td>
<td>Coincidence merged in simulation</td>
</tr>
<tr>
<td>Clustering</td>
<td>24 adjacent voxels</td>
</tr>
<tr>
<td>Electron Tracking</td>
<td>Pearson correlation</td>
</tr>
<tr>
<td>Compton Tracking</td>
<td>Reconstruction with energy recovery</td>
</tr>
</tbody>
</table>
The above CDF expression is then used to randomly veto simulated events based on their energy, *e.g.* at 20.983 keV, 50% of simulated events are rejected. However, applying the CDF reported in Beilicke et al. (2014) does not properly adjust simulated data to match flight observation near threshold. It is unclear why the discrepancy exists though it has negligible effect on validation. The energy threshold was increased by 3.9 keV to simulated data to match flight data.

Neither REVAN settings nor post-processing methods for defining multi-pixel event rejection based on coincidence windows were found to accurately reproduce the single-pixel background spectrum observed by X-Calibur in 2014. This deviation implies some systematic difference in counting of single-pixel events. Figure 6.3 shows the result of event time coincidence window ($\Delta t < 10\mu s$) post-processing of simulated multi-pixel events as compared to the single-pixel event output from PlotChannels. Moreover, PlotChannels multi-pixel output does not return event pixel data except for the total number of pixels in an event. To compare simulation to flight data event processing that cuts on single-pixel events, I determine a scaling factor for each energy bin in the flight single-pixel and multi-pixel spectrum. This was done for each combination of CZT-only, CZT-ACD, and CZT-ACD-CD flagged events. The result of this scaling is shown in Figure 6.3. However, this procedure is only valid where the incident background spectrum is similar to Ft. Sumner. For this reason, predictions of background over Antarctica are based on multi-pixel events.

Finally, the ASIC dead time is modeled through post-processing to reject events that cannot be recorded as the ASICs are reading out CZT charge data.
Figure 6.3: Comparison of 2014 X-Calibur flight and simulated single- and multi-pixel TC$_1$ spectra in dashed and solid lines. Spectra reflect CZT triggering, only. The simulated spectra are based on my background models and assumes a 9.26$^\circ$ elevation angle. Simulated single-pixel spectra reflect two methods of post-processing: scaling to fix flight response and coincidence window. Note that the latter does not accurately match the observed single-pixel spectrum.

6.2 Model Validation Using the 2014 X-Calibur Flight

In order to estimate the background rate for future X-Calibur flights and determine signal-to-noise for candidate observation targets, I first validate the approach I have taken in Chapter 5 by comparing 2014 X-Calibur data to simulated backgrounds based on my models and QARM.

The models derived in the previous chapter are validated at a high-level owing to the numerous background contributions and the limited set of experimental data to validate against. Specifically, I show that these models provide a good estimate of the background over Ft. Sumner for hard X-ray and $\gamma$-ray instruments. Where generalizations were made in the models to account for lack of experimental data, e.g. fluxes past limb, the resulting uncertainties are unlikely to be significant as the dominant $\gamma$-ray and neutron backgrounds have been more definitely studied. This level of validation provides some degree of confidence
in their ability to describe background fluxes over Antarctica or at other atmospheric depths; however, further experiment is required to refine and validate the assumptions in extending the models to conditions different from Ft. Sumner.

To validate the models, I compare observed single-pixel and multi-pixel spectra. I next compare the relative distribution of events on the detector. Finally, I compare the relative contribution of background components in my models with the results of QARM simulations.

For all tests, I analyze results for CZT-only, CZT-ACD, and CZT-ACD-CD trigger conditions. Table 6.2 defines trigger conditions TC_I, TC_H, and TC_III used in this chapter and in Chapter 7.

Separately, the effects of aperture flux will be discussed in Chapter 7.

### 6.2.1 Spectral Analysis

I first validate my background models and the resulting simulated spectra by comparing them to the X-Calibur observed spectra and the simulated spectra based on the QARM background model. Figure 6.4 shows the multi-pixel spectra of simulations for all three trigger conditions, assuming an elevation angle of 9.26°, and of X-Calibur’s observation.

Table 6.2: Trigger conditions (TC_x) used in analysis. In all cases, an energy deposition within a single pixel of the CZT triggers the detector. Energy thresholds for scintillators in MEGALib are not based on total energy deposition rather than detection of scintillation light. The same trigger condition definitions are used in Chapter 7 though the TC_H ACD energy threshold is lowered to 150 keV.

<table>
<thead>
<tr>
<th>ACD Trigger [keV]</th>
<th>CD Trigger [keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC_I None</td>
<td>None</td>
</tr>
<tr>
<td>TC_H 1500</td>
<td>None</td>
</tr>
<tr>
<td>TC_III 1500</td>
<td>4</td>
</tr>
</tbody>
</table>

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Figure 6.4: Comparison of 2014 X-Calibur flight and simulated spectra, based on output of models defined in this work and QARM background models for trigger conditions TC_1 (top), TC_II (center), and TC_III (bottom). All simulations assume an elevation angle of 9.26°. Shaded region of simulated spectra indicate statistical, ±1σ uncertainty from the simulation - not any systematic or other errors. The bottom panel showing the spectra for TC_III shows raw simulated spectra, lightly shaded, and as normalized to flight, darkly shaded.
For TC₁, there is excellent agreement between the observed spectrum and both simulated spectra. For a simulated elevation angle of 9.26°, the fitted spectral indices of the simulated spectra and observation are nearly identical. The most visible discrepancy is the energy of the spectral break, where the flight spectrum’s occurs at about 150 keV while the peaks simulated energy spectra are both found at about 200 keV. The spectral power law index of the observation between 20 keV and the break is 0.125 and from the break to 4300 keV it is 1.347. Accounting for the different spectral breaks, the power law indices for my model are 0.079 and 1.415. For QARM they are 0.026 and 1.461(4). The proximity of these indices indicates the agreement between all results. Results for a 0° elevation angle, not shown, are less consistent with the observed spectrum. Here, both simulated models show a higher event rate. While normalizing over the entire observed spectrum, my model and QARM show 12.2(3)% and 40.7% greater normalizations, respectively.

For TC₂, I show that the simulated ACD energy threshold results in spectra that are similar to the 2014 observations. A notable change with respect to TC₂ is that the peak near 20 keV has become further pronounced. This peak is reflected in the simulated spectrum of my model but not in QARM’s spectrum. Otherwise, both simulated spectra share similar relatives features as TC₁. For TC₃, simulated spectra have a lower normalization than the observed spectrum by about a factor of two. A χ²-fit across each simulated spectrum shows that these simulations predict only 54% of the observed residual background. This indicates that the modeled energy threshold of the scintillating stick is too high or that there is some other systematic modeling issue. In the bottom pane of Figure 6.4, I show the raw simulated spectra as more lightly shaded than the spectra with fitted normalizations. The spectra’s shapes are different as well. Both simulated spectra can appear to be a single power law, with indices 0.786 and 0.736 for results of my background model and QARM, respectively.

To further understand the discrepancies, I evaluate the contribution of each background type to the spectrum for all three trigger conditions. Figure 6.5 shows the simulated
Figure 6.5: Observed hadronic (left column), leptonic (center column), and γ-ray (right column) spectra for trigger conditions TC_I (top), TC_H (center), and TC_III (bottom). I include the sum of all component spectra based on my model for the given trigger condition on all plots. All plots are based on multi-pixel events and assume an elevation angle of 9.26°.

spectrum of each component compared to the total simulated spectrum for results of my model and QARM. Figure 6.6 shows the normalized contribution to the event rate within the X-Calibur science energy band. From these figures, it is clear that neutrons and γ-rays, neutral background components, are the dominant background components in all trigger conditions. Their fraction increases when ACD and CD triggers are accounted for.
Figure 6.6: Simulated relative contribution of background constituents to the event rate in the X-Calibur band. Left shows the result from my model while right shows the result of QARM for a 9.26° elevation angle. The systematic differences of the models are described in Section 5.9.

Comparing Figures 6.4, 6.5, and 6.6, I show what accounts for the differences between observation and simulation in order to understand potential systematic errors. Near the X-Calibur energy threshold, neutrons are the dominant flux. The simulated excess, visible in results of my model and QARM, is primarily generated by neutrons and γ-rays. A dip between the energy threshold and the spectral break at \( \sim 200 \) keV, in addition to an excess around the break, is seen in nearly all simulated spectra. Although it is not seen due to binning in Figure 6.5, a prominent 511 keV-line appears in the proton, positron, \( \mu^+ \), and γ-ray simulated spectra. These are responsible for the sharp line in the summed spectrum that was not observed by X-Calibur. Finally, the tail of the observed spectrum is similar but not identical to the simulated spectra. The observed double peaks at 2.5 and 4 MeV roughly correspond to peaks at 1.5 and 3 MeV in the simulated spectra. The contributions resulting in these peaks are from the proton and \( \mu^+ \) spectra.

These differences imply that potential systematic errors exist in developing background models and/or the description of X-Calibur in the simulation environment. First, it is possible that the correct rigidity cutoff over Ft. Sumner has not been used in developing
Figure 6.7: Variation of the cosmic proton contribution to background as rigidity cutoff varies. Spectra were determined using response matrices discussed in Chapter 7.

background models. Because the excess near the spectral break is not directly attributable to any specific source, I investigate the spectral features in the tail. The spectra of protons and muons, whose production are directly related to the proton flux, are determined by the local rigidity cutoff. The cutoff over Ft. Sumner has been reported to be 4.2-4.5 GV in the late 1990s (Abe et al., 2003; Barwick et al., 1997). For my models, I had used $R=3.7$ GV calculated by QARM/MAGCOS in order to cross-validate with QARM output. I qualitatively assess the effect of a higher cutoff by changing the lower limit of the cosmic proton spectrum in Equation A.3 from 3.7 to 4.5 GeV. Figure 6.7 compares the background spectrum induced by cosmic protons as the cutoff increases. The result is a moderation of the double peaked feature of the tail of the spectrum - however the location of peaks does not change with rigidity cutoff.

The 511 keV-line and its identical presence in component spectra of both models imply a systematic error relating to the simulation environment as a whole. Local $\gamma$-ray production from annihilation is inevitable and is seen in simulated spectra. However, it is possible that something is not described in the simulation environment that reduces its intensity. First, it
Figure 6.8: The excess single-pixel spectrum near 511 keV with respect to a fitted power law spectrum is shown for 2mm and 5mm observation and simulation using this work. The areas corresponding to the X-Calibur energy resolution (FWHM) for both detector thicknesses are shown as shaded regions while a reference, pink vertical line is shown at 511 keV.

is possible that motion, in altitude and rotation, and other passive mass near the detector has smoothed the line or degraded its spectrum. In Figure 6.8, I show the excess near 500 keV by power law spectra fitted around the excess contributions for the observed single-pixel spectrum in the 2 mm and 5 mm thick CZT and simulated single-pixel spectrum over all rings. For the observed spectrum, the region corresponding to the energy resolution appears as a shaded region around respective peaks. The excess in the 2014 X-Calibur spectrum is observed between 350-500 keV rather than a sharp line as seen in simulated spectra.

This lack of a line is most likely attributable to unmodeled mass or other systematic simulation errors. This mass would result in additional $dE/dx$, lowering and broadening the spectral peak. The shift of the peak, $\approx 20$ keV, is substantially less for simulations than for the observation, $\approx 60-120$ keV. It is unlikely X-Calibur’s energy resolution near the peak, shown as the shaded region in Figure 6.8, is responsible for the broadening.

Comparing Figure 6.5 and Figure 6.6 shows general agreement between the models. Differences include the relatively larger contribution of $\gamma$-rays and the smaller contribution
of muons in QARM with respect to my model. In both models, the most events are due to \( \gamma \)-rays. My model shows the relatively minor contribution of CXB as compared to atmospheric \( \gamma \)-rays. Per Figure 5.21, both the model of Kole and QARM predict very similar neutron spectra; however, with respect to the other assumptions made, they are the largest contributor to events below 30 keV. Notably, my model shows the contribution of positron-initiated events dominates leptonic backgrounds despite the extended spectrum cosmic and secondary electrons.

Finally, it is not clear what is contributing to the difference between my model and QARM predictions of the incident muon spectrum. I applied the same approach in estimating the fluxes of protons and neutrons. This approach results in estimates of the proton flux that are slightly lower than QARM’s, yet the muon flux is higher by about a factor of five with a far steeper spectrum below 1.9 GeV. As the BESS data was taken by observing near zenith and its observation showed substantial increase in flux as depth increased, considerable flux from zenith can be expected. It is possible that QARM’s modeling of atmospheric interactions induces energy losses which degrade the spectrum.

### 6.2.2 Spatial Event Distribution

I further validate my background model by comparing how events are distributed in the detector. In this case, I determine the agreement by gauging the proportionality of the spatial distribution of events rather than the detection rate predictions.

First, I determine the background flux in the 5 mm and 2mm rings of the detector. Table 6.3 gives the 25-60 keV background rates for the entire CZT and the 5 and 2 mm thick CZT detectors separately for flight, our model, and results of the QARM model. Values for both models are shown after normalizing to the flight spectrum above 20 keV for each
trigger condition. After normalization there is general agreement, though both my model and QARM predict higher fluxes on the 5 mm CZT rings closest to the aperture.

Description of event deposition in the detector as a function of zenith angle, *i.e.* in the plane seen on the right side of Figure 3.3, provides another useful metric for comparison. Showing a similar spatial distribution of events indicates that the zenith angle-dependence of incident flux predicted by the models provides a reasonable description of the events that ultimately trigger the CZT. I use the codewheel position to determine the pixel angle with respect to zenith in the observed data. I show the binned flux for the observation and simulations in Figure 6.9. Although the simulated spectra are more intense above 150 keV, they are in general agreement with the observed data.

Table 6.3: Observed background flux in 25-60 keV, with and without ACD and CD trigger conditions, as compared to simulated flux from my model and QARM. Flux of single-pixel events is listed for the whole detector, all 5 mm rings, and all 2 mm rings. Simulated fluxes are normalized based on flux upon the entire instrument. Units are in cts s$^{-1}$ cm$^{-2}$.

<table>
<thead>
<tr>
<th>2014 Observation</th>
<th>2014 Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All CZT</td>
</tr>
<tr>
<td>TC$_I$</td>
<td>0.127</td>
</tr>
<tr>
<td>TC$_{II}$</td>
<td>4.1$\times 10^{-2}$</td>
</tr>
<tr>
<td>TC$_{III}$</td>
<td>2.5$\times 10^{-3}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>My Model</th>
<th>2 mm</th>
<th>QARM</th>
<th>2 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 mm</td>
<td>My Model</td>
<td>2 mm</td>
<td>QARM</td>
<td>2 mm</td>
</tr>
<tr>
<td>TC$_I$</td>
<td>0.12</td>
<td>0.09(0)</td>
<td>0.11</td>
<td>0.09</td>
</tr>
<tr>
<td>TC$_{II}$</td>
<td>4.0$\times 10^{-2}$</td>
<td>2.3$\times 10^{-2}$</td>
<td>2.7$\times 10^{-2}$</td>
<td>1.7$\times 10^{-2}$</td>
</tr>
<tr>
<td>TC$_{III}$</td>
<td>3.1$\times 10^{-3}$</td>
<td>1.4$\times 10^{-3}$</td>
<td>2.6$\times 10^{-3}$</td>
<td>2.1$\times 10^{-3}$</td>
</tr>
</tbody>
</table>
Figure 6.9: Event angle distribution of observed and simulated single-pixel events of my model and QARM, from left to right, respectively. The event angle $\theta$ is the angle with respect to zenith. The unit for all panels is Hz.

### 6.2.3 Validation Through Detector Orientation

The background model developed in Chapter 5 can be further validated by comparing the simulated data to X-Calibur flight data as a function of elevation angle and rotation along azimuth.

The elevation angle of the detector offers an opportunity to investigate the zenith angle-dependence of the background models as X-Calibur was raised up to an elevation angle of $\approx 55^\circ$ during its 2014 flight. For simulations, I assumed an averaged elevation angle of $9.26^\circ$ based on telemetry from only a portion of the X-Calibur flight. Simulations of background incident on a mass model raised to $9.26^\circ$ are closer to the experimental data than those at $0^\circ$ elevation. The $\sigma_{\text{RMS}}$ of simulations of $0^\circ$ and $9.26^\circ$ elevation angles with respect to the observed spectrum are shown on Table 6.4.

Although X-Calibur was raised to $\approx 55^\circ$, this was done for such a short duration that an uncertainty in event rate $\sim \pm 40$ Hz exists beyond elevation angles of $15^\circ$. Results of simulations at elevation angles shown in Table 5.1 for QARM and the background model cannot be meaningfully validated. Results for simulations are plotted against experimental data in Figure 6.10.
Figure 6.10: Background rate as a function of elevation angle as determined by 2014 data and background simulations using my model, filled circles, and QARM, open circles. The shaded region encloses the flight results with the ±1σ statistical uncertainties. Analysis of flight data does not incorporate truss roll angle.

6.3 Predictions of Background Effects Over Antarctica

Balloon flights over Antarctica benefit from the slow, circling wind patterns and stable climate allowing for long-duration missions. However, the low cutoffs near the magnetic pole provide little protection from high-energy cosmic rays. For X-ray missions, these primary cosmic rays and the secondaries particles they produce result in higher background rates than at higher rigidities. In order to predict what these higher rates will be, I simulated backgrounds over Antarctica corresponding to solar maxima and minima based on the models

Table 6.4: Comparison of $\sigma_{\text{RMS}}$ between simulation and observation of 0° and 9.26° elevation angles. The spectrum break is assumed to be 220 keV for uniformity in calculations and units of $\sigma_{\text{RMS}}$ are in (s cm$^2$ keV)$^{-1}$.

<table>
<thead>
<tr>
<th>Elevation Angle</th>
<th>20-220 keV</th>
<th>0.22-10 MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>This Work</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0°</td>
<td>$9.11 \times 10^{-7}$</td>
<td>$2.12 \times 10^{-6}$</td>
</tr>
<tr>
<td>9.26°</td>
<td>$5.47 \times 10^{-7}$</td>
<td>$1.18 \times 10^{-7}$</td>
</tr>
<tr>
<td><strong>QARM</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0°</td>
<td>$7.99 \times 10^{-6}$</td>
<td>$8.25 \times 10^{-7}$</td>
</tr>
<tr>
<td>9.26°</td>
<td>$1.82 \times 10^{-6}$</td>
<td>$7.32 \times 10^{-7}$</td>
</tr>
</tbody>
</table>
derived in Chapter 5. From these simulations, I have determined background rates, resulting
ASIC dead time, their spectra, and the zenith angle distribution.

During X-Calibur’s flight over Ft. Sumner in 2014, over two million events were
recorded with a multi-pixel event rate of 297 Hz, of which only 1.8% were single-pixel
events meeting TC\textsubscript{III} and depositing 20-60 keV. The background rate over Antarctica may
result in excessive detector dead time.

Results of simulations over Antarctica at solar maximum and minimum predict multi-
pixel event rates of 1484 and 6565 Hz, corresponding to mean times between events of 673 and
152 $\mu$s, respectively. These values do not reflect ASIC dead time. In order to simulate ASIC
dead time an event deposition code was developed for Mathematica where CZT volumes
corresponding to each ASIC are modeled. When an event occurs within an ASIC, a flag is
set. Events occurring within 134$\mu$s, corresponding to the dead time after ASIC integration
and before ASIC readout is complete, are rejected. Events not rejected and the total number
of rejected events are exported for analysis.

For an 8-ring CZT configuration identical to what was flown in 2014, this analysis
indicates that flights over Antarctica will record 99.768% and 99.005% of events at solar

| Table 6.5: Background multi-pixel event rates for the 2014 X-Calibur flight and simulations
within the 20-60 keV X-Calibur science band and 20-1000 keV compared. All simulation-
derived values assume an elevation angle of 35°. Values for trigger condition TC\textsubscript{III} have been
adjusted as described in Section 6.2.1. All units are in Hz. |
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ft. Sumner</td>
<td>This Work</td>
<td>QARM</td>
<td>Antarctica</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Observation</td>
<td></td>
<td></td>
<td>Solar Maxima</td>
<td>Solar Minima</td>
</tr>
<tr>
<td>20-60 keV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TC\textsubscript{I}</td>
<td>27.2</td>
<td>25.0</td>
<td>30.7</td>
<td>86.3</td>
<td>382.</td>
</tr>
<tr>
<td>TC\textsubscript{II}</td>
<td>9.38</td>
<td>9.1</td>
<td>8.0</td>
<td>21.2</td>
<td>92.6</td>
</tr>
<tr>
<td>TC\textsubscript{III}</td>
<td>0.58</td>
<td>0.67</td>
<td>0.52</td>
<td>1.78</td>
<td>6.80</td>
</tr>
<tr>
<td>20-1000 keV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TC\textsubscript{I}</td>
<td>212.15</td>
<td>225.5</td>
<td>300.6</td>
<td>841.</td>
<td>3950</td>
</tr>
<tr>
<td>TC\textsubscript{II}</td>
<td>63.80</td>
<td>68.0</td>
<td>74.2</td>
<td>197.</td>
<td>926.</td>
</tr>
<tr>
<td>TC\textsubscript{III}</td>
<td>5.31</td>
<td>5.2</td>
<td>5.0</td>
<td>13.2</td>
<td>63.4</td>
</tr>
</tbody>
</table>
maxima and minima, respectively, due to the saturation effect. The background multi-pixel event rate for non-rejected events are shown in Table 6.5 and are compared to X-Calibur’s 2014 observation along with the results of the simulations discussed in Section 6.2.

As the spectrum of cosmic rays incident on the atmosphere varies with the solar cycle and local rigidity cutoff, so will the spectra of the secondaries that result from their air showers and the relative contribution of background constituents to the background rate. Figure 6.11 shows the spectra that result from simulations for solar maxima and minima as compared to the spectra observed by X-Calibur over Ft. Sumner in 2014, for reference.

For both solar conditions, the resulting spectra are similar to that observed over Ft. Sumner though their normalization change. With their higher solar modulation, solar maxima result in backgrounds that are nearly a factor of three greater than Ft. Sumner for each trigger condition. Solar minima results in significantly higher backgrounds. Notably, as flux increases so does the efficiency of the CD and ACD in rejecting background events. The ratio of event rates shown in Table 6.5 show the rate of background events passing TC_{III} and between 20-1000 keV are 2.5%, 2.0%, and 1.8% for conditions over Ft. Sumner and solar maxima and minima over Antarctica, respectively.
Figure 6.12: The simulated observed spectra of hadrons, leptons, and γ-rays over Antarctica from left to right, respectively, and for trigger conditions TC\textsubscript{I} (top), TC\textsubscript{II} (center), and TC\textsubscript{III} (bottom). The spectra at solar maxima are shown as dotted lines while the spectra at solar minima are shown as solid lines. All spectra are for multi-pixel events.

Table 6.6: Observed background flux in 20-60 keV over Antarctica, for all trigger conditions. Only multi-pixel events are shown. Units are in s\(^{-1}\) cm\(^{-2}\).

<table>
<thead>
<tr>
<th></th>
<th>Solar Maxima</th>
<th>Solar Minima</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>5 mm</td>
</tr>
<tr>
<td>TC\textsubscript{I}</td>
<td>0.674</td>
<td>0.730</td>
</tr>
<tr>
<td>TC\textsubscript{II}</td>
<td>0.166</td>
<td>0.190</td>
</tr>
<tr>
<td>TC\textsubscript{III}</td>
<td>7.41 \times 10^{-3}</td>
<td>8.66 \times 10^{-3}</td>
</tr>
</tbody>
</table>
Figure 6.13: Relative contributions of backgrounds above Antarctica for solar maxima and minima, left and right, respectively, and for trigger conditions TC$_I$ (top), TC$_{II}$ (center), and TC$_{III}$ (bottom).

The spectra of each background constituent and its relative contribution to the total background for all trigger conditions are shown on Figures 6.12 and 6.13. Over all locations, the contribution of atmospheric $\gamma$-rays is the largest and their relative proportion has increased over Ft. Sumner. In contrast to the contributions over Ft. Sumner, the contributions of positrons and electrons are far larger while the contribution of neutrons and protons have diminished at both solar maxima and minima. This may be attributable to the generation of atmospheric secondaries by much larger flux of lower energy cosmic ray protons which are less likely to penetrate the shield and deposit energy. The flux of GeV
protons $\gtrsim 4$ GeV would remain generally unchanged. Gamma-rays and positrons, which pair produce as they impinge on the detector, have the highest fraction of events that pass trigger conditions.

Finally, the simulations indicate that the distribution of events along the detector over Antarctica is different from that over Ft. Sumner. In comparing Tables 6.3 and 6.6, the ratio of flux upon the 5 mm to 2 mm CZT detectors has increased. The distribution of the events with respect to zenith angle has changed as well. Figure 6.14 shows that the influence of limb is reduced with the majority of events coming from $\theta < 60^\circ$ and near nadir.

### 6.4 X-Calibur Observation Strategy

Using the background rates determined in the previous sections, I can continue to determine their effect on X-Calibur’s observation of astrophysical sources. For all
astrophysical observations, pointing the telescope such that the source lies at the center of the PSF is important to increase the signal-to-noise ratio (SNR). However, uncertainty in pointing accuracy and the truss’s deflection implies that without modest adjustment of the field of view it will be unclear if the source is truly centered.

In order to determine the effect of pointing uncertainty, I have developed a model that folds source emission with the InFocμs PSF reported by Okajima (2001). The model projects the PSF on 6×6 points across a 25×25 arcmin² area of the sky with the source centered. This grid spacing corresponds to a slight overlap of the 12 arcminute (FWHM) field of view. For each point, the flux from the mirror intersecting the scattering element is summed. Event rate for different sources are scaled based on the simulations of Kislat which predict that Crab generates a 1.1 Hz signal rate (TCIII). This scaling assumes a uniform detector response above 20 keV. Table 6.7 lists the signal rates used in this section’s analysis. I use background rates corresponding to multi-pixel events within 20-60 keV meeting TCIII. These rates are reported in Table 6.5.

As observation time of a source increases so does the statistical significance σ of the observation. They are related by:

\[ \sigma = \frac{r_{\text{on}} - r_{\text{off}}}{\sqrt{r_{\text{on}} + r_{\text{off}}}} \sqrt{t}, \quad (6.3) \]

Table 6.7: X-Calibur candidate source event rates were determined by integrating the product of spectra and InFocμs’s effective area and scaling the result with Crab simulations. Spectra for these sources are based on the reports of Wilms et al. (2006); Strickman et al. (1979); dal Fiume et al. (1998). This calculation assumes trigger condition TCIII.

<table>
<thead>
<tr>
<th>Source</th>
<th>Event Rate [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyg X-1: Hard State</td>
<td>3.6</td>
</tr>
<tr>
<td>Cyg X-1: Soft State</td>
<td>2.5</td>
</tr>
<tr>
<td>Crab</td>
<td>1.1</td>
</tr>
<tr>
<td>Her X-1</td>
<td>0.21</td>
</tr>
</tbody>
</table>
where $r_{on}$ is the sum of the source and background rate observed when the source is directly centered and $r_{off}$ is the sum of the source and background rate when it is not. Assuming a required significance of $\sigma = 2$ to confirm the source is aligned with the telescope’s optical axis, I solve for $t$ at each sky coordinate. The results for the extreme cases of Cyg X-1 in its hard state and Her X-1 for observation over Ft. Sumner and Antarctica are shown in Figures 6.15 and 6.16, respectively.

However, the analysis to verify that the telescope is pointed directly on-source is different than establishing significant statistics for the scientific analysis. Here, the
observational requirements are set by the goal of the specific observation. The required observation time is a function of these goals, which are unique for each target.

The accreting stellar mass black hole Cygnus X-1 has been observed to have two states, soft and hard, based on the description of its different spectra Thorne and Price (1975). In the hard state, a low accretion rate results in an inner disk that is hotter and optically thinner than the outer disk. Two components are theorized to describe its spectra. The first is due to Comptonization of cold disk thermal photons, which obfuscates the initial polarization of photons. The second component arises from coronal X-rays near the inner disk reflecting off the disk. This reflection would impart a polarization fraction and angle relative to the
disk’s inclination. The soft state is characterized by its thermal emission below 10 keV which is not observed in the hard state. Moreover, no jets are observed (Hanke, 2007). X-Calibur observation in 20-60 keV would constrain Cygnus X-1’s coronal parameters and provide follow up for several measurements. Notably, it constrain on polarization in a regime below INTEGRAL’s initial 250-400 keV polarimetric observation (Laurent et al., 2011) in its hard state. Observations in Cygnus X-1’s soft state by NuSTAR and Suzaku have been used to model its reflection component in addition to constraining spin and inclination (Tomsick et al., 2014). As fits were inconclusive, X-Calibur can provide clarity through polarized observation as indicated by the decomposed soft state spectrum in Figure 2.6.

The Crab pulsar wind nebula’s X-ray spectrum is dominated by synchrotron emission generated by its magnetic field as it rotates. It is unclear whether this emission is primarily from the region between the last open magnetic field lines (slot-gap model) or the region between where the field lines would be perpendicular to the rotational axis and the light cylinder where plasma corotational velocity would exceed c (outer-gap, see Hirotani, 2008, for further discussion and references.). Polarimetry by X-Calibur can distinguish between the two through the different in polarization fraction. Additionally, mapping the time variation of polarization fraction as the magnetic field rotates can be used to reconstruct its magnetic field and what may responsible for its jet-torus structure (Volpi et al., 2009; Komissarov and Lyubarsky, 2003). Crab’s polarization has been observed by OSO-8 below 10 keV and later by INTEGRAL above 200 keV (Forot et al., 2008; Weisskopf et al., 1978). However, they were unable to constrain emission beyond optical observations (Volpi et al., 2009). Although X-Calibur is not an imaging polarimeter to resolve these features, observation within 20-60 keV would provide useful information to further study the Crab nebula.

Her X-1 is an eclipsing binary X-ray pulsar that is strongly magnetic, with a dipole field measured to be $\approx 4.6 \times 10^{12}$ G (Brecher and Ulmer, 1978). As a strongly magnetic, accretion-powered pulsar it provides opportunities to investigate the magnetic field structure and
plasma and vacuum resonances as discussed in Section 2.2.3. X-Calibur would provide the first X-ray polarimetric observations of Her X-1 since OSO-8 (Silver et al., 1979). Although Her X-1’s magnetic field is less than $B_{QED}$, observational signatures that indicate the physics of plasma and vacuum resonance and their relation to transmission may be present. Kii indicates that signature may be observable below 30 keV. Both van Adelsberg and Lai (2006) and Fernández and Davis (2011) indicate the signatures are strongest below 10 keV. Regardless, observations near 20 keV can constrain future polarimetric observations.

Finally, Figure 6.17 shows the significance achieved for 3600 and 7200 s of observation above Ft. Sumner and Antarctica.
Chapter 7

X-Calibur Shielding Optimization

Flying high altitude balloons in Antarctica offers the distinct advantage of longer times aloft and consequently longer observation times. However, in light of the predictions made in Section 6.3, this benefit is offset by the very low rigidity cutoff near the magnetic south pole. The design of X-Calibur’s ACD and CD triggering mitigates the influence of background but the increase in flux results in higher background rates after ACD and CD triggers. Moreover, it can overwhelm the detector with so many background events that ASIC dead time reduces the overall observational efficiency regardless of the trigger conditions. Therefore, it is important to determine if there are effective means of reducing the influence of background further.

The purpose of this chapter is to determine which of these means will be most effective in reducing the X-Calibur background rate over Antarctica. It not immediately clear which potential shielding options result in an optimal configuration. First, additional shielding may increase the secondary background in X-Calibur’s 20-60 keV science band. Second, the optimal shielding strategy varies based on the background component. For instance, optimal shielding against \(\gamma\)-rays would require high-Z material while low-energy neutrons would require a combination of high- and low-Z material (Rokni et al., 2007). This combination of high- and low-Z material itself would result in the production of secondary neutrons from
incident, high-energy protons. Another concern is that the addition of more high-Z material will result in the production of bremsstrahlung γ-rays from incident electrons.

Moreover, the determined means must also be realistic to implement. For instance, active shields, like X-Calibur’s CsI shield that protects the CZT from the sides and below, are very effective at reducing background by vetoing CZT events that coincide with the detection of scintillation light in the CsI. However, compared to passive shielding like tungsten they are less dense, require additional electronics, and are more expensive. Any shield optimization must reflect the engineering and programmatic constraints in addition to the physical constraints.

I have outlined several options for modifying X-Calibur’s shielding in Section 3.5.2. Adjusting the thickness of the shield creates a very large space of potential shielding configurations. Sampling only five thicknesses for eight sections of the shield results in $\sim 10^{20}$ different combinations. The simulation of these combinations would be impractical.

There is little work reported on the subject of shield optimization for astrophysical experiments. Swartz et al. (2000) is the most pertinent in its study of the variation of lead slab thickness for attenuating the high-energy background of hard X-ray balloon-borne instruments. However, their approach assumed a linear detector response and the shield configuration itself was not optimized. Baranov et al. (2005) discusses an iterative optimization approach used to reduce background radiation from the ATLAS experiment. Such an iterative approach cannot efficiently explore a large space of options. However, research in radiation medicine and nuclear reactor design have employed efficient methods of optimization across a large parameter space (Djordjevich et al., 1990; Newman and Asadi-Zeydabadi, 2008; Mahlers, 1991).

In the following, I optimize the shield configuration for a constant total shield mass. For each design option, I simulate the dominant incident background components to construct
a detector response matrix. Folding incident spectra with the responses matrices, I am able to determine the variation in background rate as the shield parameters are varied. A minimization routine calculates the optimal shielding configuration given constraints on total mass and allowable combinations of shielding design options.

Separately, another method to mitigate the effect of background is modifying the implementation of X-Calibur’s ASICs or reduction of detector rings. Doing so would reduce total detector dead time; however, this is not investigated here.

7.1 Limitations of Background Reduction

Although it is possible to optimize X-Calibur’s shield, there are fundamental limits to how much the background rate can be reduced. One limitation is the detector’s aperture, which permits both astrophysical X-rays and background flux to enter the detector. An additional limitation is the probabilistic nature of radiation-material interactions, where even very thick $4\pi$ sr shielding will not completely shield background. In general, neutral radiation, i.e. $\gamma$-rays and neutrons, is most capable of penetrating shielding. A final limitation is practical considerations of shielding mass and geometry. In this chapter, I simply assume a total mass that is allocated for shielding.

The shield optimization strategy will vary depending on whether more background is attributable to aperture flux or penetrating radiation. To determine the limiting physics, I simulate the X-Calibur detector to estimate the contribution of aperture flux.

First, I replace the mass model of the collimator with a solid tungsten cylinder of the same dimensions that effectively plugs the aperture and simulate the background over Antarctica during solar minimum. Figure 7.1 shows the result of these simulations as
Figure 7.1: Background simulations of the X-Calibur detector over Antarctica during solar minima were also performed using a solid tungsten cylinder in place of the existing collimator, effectively plugging the aperture. *Left* shows the spectra for all trigger conditions with and without the tungsten plug. The shaded regions in this panel show ±1σ statistical uncertainty of the “plugged” results. *Right* shows the aperture flux ratio for each trigger condition. Shaded regions in this panel show ±1σ statistical uncertainty from both standard and “plugged” mass models. All results assume a 35° elevation angle.

compared with simulations of the same background and the standard mass model. The left panel compares the spectra of both simulations while the right panel shows the ratio of events attributable to aperture flux. The estimated aperture flux is shown in Table 7.1, which can be compared to the rightmost column in Table 6.5.

Figure 7.1 shows that aperture flux constitutes a significant fraction of the TC_I event rate at energies <5 MeV. However, the TC_{II} and TC_{III} event rate does not appear to be influenced by aperture flux. Further analyzing the component spectra in Figure 7.2, it is clear that the TC_{III} spectra are dominated by γ-rays and neutrons with a similar observed

Table 7.1: Background rates due to aperture flux over Antarctica during solar minima. Rates account for multi-pixel events and assume an elevation angle of 35°. The uncertainty described is the ±2σ statistical uncertainty.

<table>
<thead>
<tr>
<th></th>
<th>20-60 keV [Hz]</th>
<th>20-1000 keV [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC_I</td>
<td>255±16</td>
<td>2691±52</td>
</tr>
<tr>
<td>TC_{II}</td>
<td>&lt; 2.3</td>
<td>&lt; 31.08</td>
</tr>
<tr>
<td>TC_{III}</td>
<td>&lt; 0.88</td>
<td>&lt; 3.32</td>
</tr>
</tbody>
</table>
Figure 7.2: The simulated observed spectra of background over Antarctica during solar minima assuming a “plugged” shield. Panels show spectra of hadrons, leptons, and $\gamma$-rays, from left to right, respectively, for trigger conditions TC$_I$ (top), TC$_II$ (center), and TC$_{III}$ (bottom). The spectra can be directly compared to the spectra using standard mass model in Figure 6.12.

spectra than those shown using simulations of the standard X-Calibur mass model shown in Figure 6.12.

The consequence of these findings shows that the penetration of the shield is the limiting factor for TC$_II$ event rate and not the aperture flux. Thus in this chapter I evaluate modifications to the shielding rather than varying the collimator or the size of the aperture.
7.2 Approach

To reduce the complexity of the optimization, I reduce the size of the parameter space. First, I determine dominant background components to simulate based on the results of Section 6.3. Second, for each shielding configuration I determine the parameter space available for modification. Third, for each shielding option, I develop functions that relate the variation in mass to background rate (mass-rate functions). Finally, these relationships are used to determine the optimal selection of shielding options.

I compare the results of the parametric approach with those from full simulations in Section 7.3 and validate the parametric approach in Section 7.4 by simulating response of an optimal shield design with the backgrounds used in Section 6.3.

In this analysis, I limit the optimization routine to trigger conditions TC\textsubscript{I} and TC\textsubscript{II} as analysis of TC\textsubscript{III} requires excessive simulation resources.

7.2.1 Parametric Mass Model

To perform the optimization analysis I select several independent options for shielding. While many options exist, I only evaluate options that permit for practical and limited modification of the current X-Calibur design. The tenable options are shown in Figure 3.18. From these, I assume X-Calibur will use a 5-ring detector rather than its current 8 rings.

The remaining options are for modifying or adding to the top, sides, and base of the current active shield. For each of these three options there exist three potential variations: material, solid angle subtended, and thickness. The terminology used in this chapter describing the varying shield parameters is as follows: a shield design case $X.y.z$ is defined by specifying all options, $X$, and suboptions, $y$, along with a thickness, $z$. 

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Figure 7.3: The shield cases shown in this figure exemplify the suboptions used in the minimization. Options A and B describe variation of the top shield; options C and D describe the variation of the side shield; and, option E describes variation of the base shield. In all figures, red represents CZT, blue, CsI, light green, tungsten, black, aluminum, and yellow, polyvinyltoluene. These models can be compared to the existing X-Calibur model in Figure 6.1. All illustrations are Geomega output and do not show X-Calibur’s external aluminum housing which is incorporated in the mass model.
I simplify the parameter space by assuming all active and passive shields will be composed of CsI and tungsten, respectively. This is assumed given their existing use in X-Calibur shield configuration and their high density and availability. I do not evaluate relevant neutron shielding materials as the reduction of neutron flux requires long paths through low-Z media, which is impractical for X-Calibur.

The current shielding configuration, shown in Figure 6.1 in the previous chapter can be compared to the mass models I simulated for the shielding optimization in Figure 7.3. Option A models the use of a “partial” top shield that does not cover the existing CsI side shield. This contrasts with the current X-Calibur design and option B, whose top shield extends past the outer radius of the CsI side shield. Within option A, there are suboptions for a passive, tungsten top shield, A.a, and an active, CsI top shield, A.b. Option C evaluates the impact of a passive side shield to augment the background reduction by the existing CsI side shield. Suboption C.a evaluates the effect of an internally mounted shield while C.b evaluates the effect of an externally mounted shield. Suboption C.a extends from the top to the bottom of the side shield, while C.b extends from the top of the side shield to the bottom of the base, CsI shield. Option D is similar to C.b, but it only extends to the bottom of the side shield. Suboption E.a and E.b evaluate the effect of an internal and external passive, tungsten base shield, respectively. Both allow for dimensions of the existing tungsten plug located at the base of X-Calibur. For all externally mounted shielding, I assume there are no voids which may be required for mechanical interfacing.

For each suboption, a set of thicknesses is selected so the effect on background rate can be determined as the thickness is varied. Initial thickness values are selected based on scaling of the dimensions of the existing shield. From this point, the thickness is varied proportion to the attenuation, $\propto e^{-\mu x}$. As the attenuation of $\gamma$-rays is less in CsI, a larger thickness is assumed. To reduce simulation time, I only simulate one thickness for B.a and D.a, which
Table 7.2: The shielding cases selected for analysis are indexed in this table along with their associated shield thicknesses. Cases are identified by X.y.z, where X indicates the shielding option, y, the suboption, and z, the case. Each option is otherwise identical except for the identified parameters. All passive shield options assume elemental tungsten - not Tungsten HD17 - while the active shield option assumes CsI. The listed shield thicknesses are in cm.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>a</td>
<td>1.39</td>
<td>1.65</td>
<td>1.96</td>
<td>2.34</td>
<td>2.78</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b</td>
<td>1.05</td>
<td>1.48</td>
<td>2.1</td>
<td>2.97</td>
<td>4.2</td>
<td>5.9</td>
</tr>
<tr>
<td>B</td>
<td>a</td>
<td>Passive</td>
<td>1.39</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>a</td>
<td>Internal</td>
<td>0.098</td>
<td>0.19</td>
<td>0.38</td>
<td>0.75</td>
<td>1.475</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>External</td>
<td>0.098</td>
<td>0.19</td>
<td>0.38</td>
<td>0.75</td>
<td>1.475</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>a</td>
<td>External</td>
<td>0.38</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>E</td>
<td>a</td>
<td>Internal</td>
<td>0.098</td>
<td>0.19</td>
<td>0.38</td>
<td>0.75</td>
<td>1.475</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>External</td>
<td>0.098</td>
<td>0.19</td>
<td>0.38</td>
<td>0.75</td>
<td>1.475</td>
<td>2.9</td>
</tr>
</tbody>
</table>

can be directly compared to design cases A.a.1 and C.b.2, respectively, which share the same thickness.

Figure 7.3 shows examples of each suboption while Table 7.2 lists the thicknesses associated with each case. For all cases, a slightly simpler model is assumed than that of X-Calibur. Elemental tungsten is used in place of tungsten HD17, an energy threshold of 150 keV is assumed for the ACD, and aluminum that surrounds the active shield and instrument is not modeled.
7.2.2 Incident Spectra and Their Simulation

I develop response matrices that can be used for varying input spectra rather than evaluate the detector response to the spectra defined by the equations in Chapter 5. Response matrices allow for quick assessment of detector response to a variety of observational conditions rather than simulating each spectrum individually. In order to determine the shielding configuration for a worst case scenario, I evaluate the detector response to incident spectra for solar minima conditions over Antarctica.

The results in Section 6.3 indicate that the largest contributions to flux at Antarctica in descending order are $\gamma$-rays, positrons, protons, neutrons, and electrons. As discussed above, the neutron contribution was not selected for analysis.

A response matrix is generated for each selected background component, detector case per Table 7.2, and trigger conditions $TC_1$ and $TC_{11}$. To generate each response matrix, I simulate the detector response to a series of different monochromatic, isotropic beams with a flux of $1$ particle s$^{-1}$ cm$^{-2}$. An incident spectrum can then be folded with the response matrix to calculate the observed spectrum. Because the response matrices are simulated assuming an isotropic background, I simplify the zenith-angle dependence of incident spectra for solar maximum and minimum over Antarctica. For $\gamma$-rays, I average the flux from zenith to nadir to calculate the isotropic flux. For electrons and positrons, I assume isotropic fluxes that are averages of their downward fluxes. For protons, the isotropic flux is half of the angle-averaged downward flux. This simplification is made because upward protons demonstrate little contribution to the total background.

To further minimize computation, I develop response matrices for only $\gamma$-rays, protons, and electrons. The effect of shielding on positrons can be simplified by assuming all positron interactions are annihilations near the surface of the shield. To validate the assumption, I
Figure 7.4: The observed spectrum of background events when fluxes of $\gamma$-rays and positrons are folded with their respective response matrices for design case Aa.1. The teal line represents the result of folding the spectrum with the positron response matrix and the black dotted line represents the result of the $\gamma$-ray response matrix.

To calculate the mean free path of positrons in tungsten and CsI. The annihilation cross section is (Heitler, 1954):

$$\sigma(Z, E) = \frac{\pi r_e Z}{\gamma + 1} \left\{ \ln(\gamma + \sqrt{\gamma^2 - 1}) \frac{\gamma^2 + 4\gamma + 1}{\gamma^2 - 1} - \frac{\gamma + 3}{\sqrt{\gamma^2 - 1}} \right\}$$  \hspace{0.5cm} (7.1)

where $\gamma \approx E/m_e c^2$ and $r_e$ is the classical electron radius. Evaluating the mean free path of a 1 GeV positron in CsI yields $\sim 10^{-15}$ cm. Even if Bhaba scattering occurs, the positron is very likely to annihilate without penetration. To verify this, I applied identical incident positron spectra to two response matrices simulated to determine the detector response to $\gamma$-ray and positron fluxes. Figure 7.4 shows the observed spectra, which verifies identical detector response below 500 keV. Thus, I estimate the positron contribution by folding its spectrum with the $\gamma$-ray response matrix per $F_{\text{incident } e^+} M_{\gamma} = F_{\text{observed } e^-}$.

To further verify the accuracy of these response matrices, I compare results from the previous chapter to response matrix output. For $\gamma$-rays, protons, electrons, and positrons
Figure 7.5: Black lines show the simulated spectra for the X-Calibur detector over Antarctica during solar minimum while teal dots indicate the result of folding identical incident spectra with the response matrices simulated for design case A.a.1. Simulated spectra reflect a detector area of 128 cm$^2$ while response matrix spectra reflect a detector area of 80 cm$^2$. Because the incident spectra folded with response matrices are not modeled with their true zenith-angle dependence and mass model of option A.a.1 varies with what was simulated in Chapter 6, normalization of design case A.a.1 spectra have been adjusted for easier comparison.

during solar minimum over Antarctica, I compare the observed spectra simulated with the X-Calibur detector with the response matrix result for design case A.a.1. Figure 7.5 shows this comparison. Because the incident spectra folded with response matrices are not modeled with their true zenith-angle dependence and mass model of option A.a.1 varies with what was simulated in Chapter 6, the qualitative similarity of spectra show reasonableness of this approach.
Figure 7.6: Background spectra calculated by folding the incident $\gamma$-ray flux at solar minima above Antarctica with response matrices for trigger condition TC_{II}. All design case spectra shown represent the minimum and maximum thicknesses for each suboption.

### 7.2.3 Optimization

For each suboption in A, C, and E, I determine the relationship of background rate and shielding thickness and mass. For each suboption, the observed rate within for each design case is calculated by integrating the detector response between 20-60 keV. A function $r(m)$ describing the relationship between this rate and the associated shield mass and weight can then be fitted to the resulting data.

What follows is a series of decisions to reduce the parametric space prior to minimization. Suboptions within A, C, and E are compared to one another to determine which suboption provides better overall performance. Additionally, I compare design cases A.a.1 to B.a.1 and C.b.3 to D.a.1 to determine the effect of shield coverage for a constant thickness. If design case B.a.1 or D.a.1 is found to be more effective, the relationship of suboption A.a. and C.b are scaled to the respective values.

For $\gamma$-rays, protons, and positrons and each trigger condition, I can minimize the event rate subject to the constraint of additional shielding mass. The goal of the minimization can...
be formalized as:

\[
\begin{align*}
\text{Minimize:} & \\
R &= \frac{1}{J} \sum_i \sum_j r_i(m_j) \\
\text{Subject to:} & \\
M &\leq m_{\text{top,0}} + 65 \text{ kg} \\
M &= \sum_j m_j. \\
\end{align*}
\] 

The parametrized background rate \( R \) is the sum over the background rate \( r_i \) from each component. The component background rate is a function of shielding mass for the top, side, and base represented by \( j \). However, each \( r_i(m_j) \) represents the total background rate for background entering all portions of the shield. Therefore, the contributions in \( J \) need to be normalized by \( 1/J \). For instance, for the electron background I define a parametrized rate \( r_i \) for contribution from folding the incident electron spectrum with response matricies based on simulations of shield options varying the top shield, side shield, and bottom shield \( (J=3) \).

The functions \( r_i(m_j) \) are determined by observing the variation in background rate as shield thickness is increased for a given suboption. I develop \( r_i(m_j) \) by fitting the results of the integrations. For instance, Figure 7.6 illustrates the spectra of the minimum and maximum shield thicknesses for options A, C, and E for both trigger conditions. Rates \( r_{\text{top}}, r_{\text{side}}, \) and \( r_{\text{base}} \) represent the event rate’s dependence on shielding mass for each suboption.

Because I assume option A replaces the existing X-Calibur tungsten HD17 shield top, the existing top shield mass \( m_{\text{top,0}} \) is subtracted from the mass growth constraint. The masses of each shield configuration is based on the geometry of the shield. Additional constraints are discussed in the following section. The minimization is performed using Mathematica’s \text{NMinimize} function using the simulated annealing algorithm (Carr, Accessed 8/2016). To
verify that global minima are found, several runs of \texttt{NMinimize} are run with different initial search positions.

### 7.3 Results

In order to properly optimize the shielding configuration, I develop mass-rate functions for each suboption that describe the change in background rate as shield thickness is varied. I evaluate these mass-rate functions to report the shielding response to incident $\gamma$-rays, positrons, protons, and electrons spectra. These mass-rate functions are then used to minimize event rate subject to constraint of total mass described in Equation 7.2. Event rate minimizations are performed for four optimization scenarios, OS$_\alpha$, OS$_\beta$, OS$_\gamma$, and OS$_\delta$. These different optimization scenarios reflect different constraints on the minimization arising from different engineering and programmatic constraints.

#### 7.3.1 Parametric Analysis

Figures 7.7-7.9 show the mass-rate functions of each design option for incident $\gamma$-rays, protons, and electrons spectra. I omit the results of positrons due to their similarity to the $\gamma$-ray mass-rate functions. In all figures, results for TC$_1$ appear on top while results for TC$_{11}$ with an energy threshold of 150 keV appear on the bottom. The jagged features in these figures can be attributed to carrying only the uncertainties from the response matrices.

For all particle types, the greatest variation in event rate occurs in Option C owing to the large solid angle subtended by the side shield. Although the response matrices assume isotropic flux, this reasonably approximates the background incident upon the side shield over Antarctica. Per Figure 6.14, the dominant contributions to incident flux are incident
from near zenith and nadir. At low elevation angles, rotation results in the angle-averaging of this flux. At high elevation angles, the flux incident from limb will be averaged. Additionally, suboption C.a significantly increases the total rate even after the veto condition as a result of secondaries generated within the shield. Options A and E indicate a weaker dependence on shielding mass than C.

Options B and D represent shielding modifications with increased solid angle coverage of the CZT detector compared with similar, respective suboptions A.a and C.b. For most background types, B.a offers a modest improvement as compared to A.a. For similar thickness, design case B.a.1 offers a reduction in γ-ray events by a factor of 1.09 compared to A.a.1. However, this comes with a considerable mass penalty as design case B.a.1 is a factor of 3 more massive.

Separately, decreasing the height of the external shield as in suboption D.a does not result in a statistically significant increase in flux over C.b. For the same thickness $\tau = 0.38$ cm, the γ-ray-induced background rate increases by a factor of 1.022, within statistical uncertainty, while the shield mass is reduced by a factor of 1.09, or 1.33 kg.

The mass-rate relation is similar for γ-rays and positrons. In comparing suboptions A.a and A.b, the effect of the active shield is clear. For TC$_I$, tungsten offers slightly better attenuation though this is not the case for TC$_II$. At 22 kg mass, suboption A.a has factors of 0.925 and 0.963 less background rate from γ-rays and positrons. However, under TC$_II$ when the veto from the active shield is used, A.b offers factors of 0.825 and 0.602 less rate than A.a at 22 kg for γ-rays and positrons, respectively. For option C, an external shield is shown to be far more effective under TC$_II$. Evaluated at 30 kg, the external shield of suboption C.b has half the event rate of C.a. Results for option E show a lack of sensitivity between the two suboptions.
The mass-rate relation for protons is also similar to γ-rays and positrons except that the variation between suboptions is notably less. For most suboptions, an initial increase in rate coincides with an increase in shielding mass due to secondary production. However, rate begins to diminish at a certain mass threshold, \( \sim 5-10 \) kg, owing to attenuation.

Option A’s electron mass-rate relationship shows that flux first begins to decrease to a minimum and increase as shielding mass is added. This effect is attributable to the critical energy \( E_C \) of tungsten and CsI, defined as the energy where the bremsstrahlung cross-section becomes greater than the ionization cross-section. At Antarctica, 95.2\% of the atmospheric electron flux has energies below \( E_{C, W} = 7.62 \) MeV and 96.7\% below \( E_{C, CsI} = 11.17 \) MeV (PDG, Accessed 8/2016). As tungsten has a higher bremsstrahlung cross-section and lower critical energy, its production of γ-rays is greater resulting in a slightly higher event rate. For options C and E, the shielding response to electrons is the inverse of that of γ-rays and positrons as internal shields are shown to be consistently more effective under both trigger conditions.
Figure 7.8: The variation of the event rate contributions of $\gamma$-rays, protons, and electrons as shielding mass is varied for suboption C.a and C.b. Top shows the results for trigger condition TC\textsubscript{I} and bottom shows the results for trigger condition TC\textsubscript{II}. The shaded regions show the $\pm 1\sigma$ statistical uncertainties from folding incident spectra with the response matrix uncertainties. For each suboption, an analytical function is fitted to the mass-rate results and is shown as a dotted line. The result for design case D.a.1’s full side shield is shown as bounding carets.

I further analyze the thickness-rate functions to extrapolate functions to describe tungsten and CsI shields that fully cover the top of the side shield rather than only the interior diameter as suboptions B.a and B.b. Figure 7.10 shows how design cases A.a. and A.b are transformed to form suboptions B.a and A.b based on design case B.a.1. While the figure shows the transformation for the $\gamma$–ray response under TC\textsubscript{I}, the shift is applied independently for each background type and condition. In the following, I assume that option D offers superior mass-rate performance and I adjust all fits for option C to reflect partial coverage.
Figure 7.9: The variation in event rate contributions of γ-rays, protons, and electrons as shielding mass is varied for suboptions E.a and E.b for trigger condition TCII. The shaded regions show the ±1σ statistical uncertainties from folding incident spectra with the response matrix uncertainties. For each suboption, an analytical function is fitted to the mass-rate results and is shown as a dotted line.

7.3.2 Shield Optimization

These mass-rate relationships are fitted to polynomial, exponential ($A \ e^{B \ m}$), and exponential-power law ($A \ e^{B \ m \ C}$) functions so that they can be used for minimization. These are shown as dotted lines along side the shaded regions in Figures 7.7-7.9 that indicate the region of ±1σ uncertainty from simulation statistics. While some calculated mass-flux variation may indicate other physics than those described in the previous section, I assume these fits to be generalizations and that future simulations can further explore the parametric region near the calculated minimum.

Although the general form of the minimization shown in Equation 7.2, I consider four optimization scenarios, OS$_{\alpha-\delta}$, each with unique constraints reflecting different practical concerns in reconfiguring the shield. Optimization scenario OS$_{\alpha}$ constrains the minimization by forcing mutual exclusivity of suboptions within options for top, side, and base shields. For example, I define than an external side shield and internal side shield cannot both be used for a point design. Optimization scenario OS$_{\beta}$ evaluates a composite top shield that consists of a
Figure 7.10: Mass-rate functions for option B, right, are estimated by shifting their thickness-rate relationship using case B.a.1 as a reference coordinate, left. Both plots show the shift for $\gamma$-ray rates under trigger condition TC$_I$.

Table 7.3: Summary of optimization scenarios. Options not referenced for optimization scenarios OS$_{\alpha}$, OS$_{\beta}$, and OS$_{\gamma}$ retain the constraint of mutually exclusive suboptions.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>OS$_{\alpha}$</td>
<td>All suboptions mutually exclusive.</td>
</tr>
<tr>
<td>OS$_{\beta}$</td>
<td>A tungsten/CsI (A.a/A.b) composite top shield is allowed for option A.</td>
</tr>
<tr>
<td>OS$_{\gamma}$</td>
<td>Only a tungsten top shield (either A.a or B.a) is allowed for options A and B.</td>
</tr>
<tr>
<td>OS$_{\delta}$</td>
<td>Only suboptions A.b, C.b, and E.b are evaluated but $m_{A,b}$ and $m_{E,b}$ are not independently constrained.</td>
</tr>
</tbody>
</table>

tungsten shield, suboption A.a, on top of a CsI shield, suboption A.b. Because the event rate is now a function of both suboptions, it is a non-linear optimization. Scenario OS$_{\gamma}$ assumes that the top shield will only be comprised of tungsten (suboptions A.a and B.a). Scenario OS$_{\delta}$ selects only the most effective suboptions, A.b, C.b, and E.b, and removes constraints of the maximum allowable mass of the top and base shields. This is done to determine if a lower minimum exist at masses greater than what had been simulated. For easy reference, these optimization scenarios are summarized in Table 7.3. Additional assumptions and constraints used in shield optimization are detailed in Appendix B.

To further understand how the optimal shield configuration is effected by veto efficiency, I define the parameter $f_{TC}$:

$$f_{TC} = \frac{R_I}{R_I + R_{II}},$$

(7.3)
Figure 7.11: The results of shield optimization assuming completely exclusive suboptions: top left, all options mutually exclusive; top right, a composite top shield; bottom left, a shield only comprised of tungsten; and bottom right, where top and bottom shield masses are unconstrained. The line at $f_{TC}=0.132$ shows the estimated veto efficiency of X-Calibur as compared to the condition used in these simulations.

where $R_{I/II}$ are the total event rates for TC$_{I/II}$. Minimizations are performed for $0 \leq f_{TC} \leq 1$. The physical interpretation of $f_{TC}$ is veto efficiency because as $f_{TC}$ increases, it implies more events are passing through the active shield without raising the active shield veto in TC$_{II}$. I consider two resulting systematic interpretations of $f_{TC}$. First, assuming a fixed shield configuration, increasing $f_{TC}$ implies diminishing efficiency. Second, assuming variable shield configuration and fixed background spectra, a varying $f_{TC}$ will show how the optimal shield configuration will vary. If true shield efficiency can be determined, then the optimal point design will be at its $f_{TC}$. 

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Figure 7.12: Left shows the event rate \( r(m_{A,b}) + r(m_{E,b}) \) as masses \( m_{A,b} \) (active, CsI top shield) and \( m_{E,b} \) (external base shield) are independently varied. Right shows \( r(m_{A,b}) + r(m_{C,b}) \) as masses \( m_{A,b} \) (active, CsI top shield) and \( m_{C,b} \) (external side shield) are independently varied. A strong gradient forces a minimization toward A.b while a strong gradient toward a potentially global minimum prioritizes C.b mass growth. All rates are assuming \( f_{TC} = 0 \) and are divide by two to average the contributions. The scales of both plots describe the background event rate [Hz]. Note that they are different scales.

The minimized suboption masses and their resulting event rates are shown in Figures 7.11 and 7.14 as a functions of \( f_{TC} \). The optimal masses of shielding components at \( f_{TC} \sim 0 \) are shown in Table 7.4.

The top left of Figure 7.11 shows the result of minimization performed assuming mutual exclusivity of all suboptions, the top right shows results for a composite top, the bottom left shows results for a tungsten top, and the bottom right shows results when top and bottom shield masses are independently unconstrained.

For all four minimizations and nearly all \( f_{TC} \), the external side shield of suboption C.b dominates mass growth indicating a gradient, seen in Figure 7.12, which continues steadily past the boundary condition \( m_{C,b,\text{max}} \). The only condition in which the external side shield is not preferential to suboption C.a’s internal side shield is when \( f_{TC}=1 \). Similarly, the active, CsI top shield A.b is almost universally selected.
Figure 7.13: The function \(r_A(m_{A,a}, m_{A,b})\), defined in Equation B.3, as \(m_{A,a}\) (tungsten) and \(m_{A,b}\) (CsI) masses are independently varied. Background rate \(r_A(m_{A,a}, m_{A,b})\) is defined for the composite top shield of optimization scenario OS\(_\beta\). The scale describes the background event rate [Hz].

The active, CsI top shield, external, tungsten side shield, and external tungsten base shield, suboptions A.b, C.b, and E.b, respectively, compose the most effective shield. I show their mass-mass-rate topographies in Figure 7.12. In \(r(m_{A,b}) + r(m_{E,b})\) shown on the left, the reason for the strong preference toward \(m_{C,b}\) as in the optimizations in Figure 7.11 becomes clear. Suboption C.b is a very strong constrain to the mass growth of other suboptions as a gradient toward a global minima, nearly invariant in both \(m_{A,b}\) and \(m_{E,b}\), pulls the total mass until it reaches the boundary condition. However, the steepest gradient of \(m_{A,b} \times m_{C,b}\) and \(m_{A,b} \times m_{E,b}\) occurs when \(m_{A,b}\) begins to increase. Near \(\sim 25\) kg a trajectory would result in the sole increase of \(m_{E,b}\). Before this can occur, the constraint of total mass and the effectiveness of suboption C.b restrict the continued growth of \(m_{E,b}\). Though as \(f_{TC}\) varies, the local minimum shifts slightly resulting in a slow variation of optimal \(m_{A,b}\) and \(m_{E,b}\).

For scenario OS\(_\beta\), I plot Equation B.3 as a function of \(m_{A,a}\) and \(m_{A,b}\) to determine why suboption A.a’s passive top shield is never selected under nominal veto trigger thresholds. The topography of \(m_{A,b} \times m_{A,a}\) in Figure 7.13 is similar to that of \(m_{A,b} \times m_{E,b}\). What is
most striking is the relative stability of the local minima near $m_{A,b} \approx 37$ and $m_{A,a} \approx 20$ kg. Given the gradient in $m_{A,b} \times m_{C,b}$, there will always be preference toward placing mass in the external side shield of C.a than the passive top shield of A.a. This occurs even in OS$_\beta$, where the two top shield suboptions are not mutually exclusive.

From the above, I draw two major conclusions. First is the immediate value of an active shield. Scenarios OS$_\alpha$, OS$_\beta$, and OS$_\delta$ all select an active shield (OS$_\gamma$ does not permit an active shield). Observing the topography it is clear that for minimal mass growth it offers the most accessible decrease in the TC$_{II}$ event rate. The second conclusion is the secondary priority of the external side shield. Although each minimization in Figure 7.11 implies the importance of the side shield, it is only on account of the relatively lax constraint on total shielding mass growth. Otherwise, the first 25 kg of additional shielding would result in arriving at a local event rate minima if it is used on an active shield top and any additional mass would be used on shielding via the $m_{C,b} \perp m_{A,b}$ saddle point.

Additionally, I note that suboption E.b’s external base shield is universally selected with no instances of an internal base shield being more effective. Similarly, only under extreme cases are an internal side shield or a passive top shield of suboptions C.a or A.a, respectively, selected. Similarly option B, which represents the full top shield of the existing X-Calibur shield configuration, is found to be consistently less mass effective than a partial top shield. Option D, describing a shorter external side shield, was not used in the minimization. Because side shield mass grows more slowly in height than radius, $\Delta m_D \propto \Delta h$ and $\Delta m_B \propto (\Delta r)^2$, the mass-rate relations in Figure 7.8 show its preference. The rather obvious result being that adding shielding to locations where shielding is thickest will be less mass effective than adding it anywhere else.

On Figure 7.14, the background rates that result from all minimizations are relatively close to one another and significantly lower the background event rate. This is the case for
Figure 7.14: The background rates that result from the optimized shield configurations as a function of $f_{TC}$. The background rates of optimization scenario OS$_\alpha$ are shown on left. Right shows the rates of all other optimization scenarios as normalized to the results of OS$_\alpha$. On both, I compare results of this chapter to those reported based on simulation of the existing X-Calibur configuration over Antarctica during solar minima in Section 6.3. Those results are shown as open circles at $f_{TC}=1$ and 0 and connected by a dotted line. The pink line at $f_{TC}=0.132$ on both plots shows the estimated veto efficiency of X-Calibur based on observation over Ft. Sumner.

Both trigger conditions, which are described by $f_{TC}=1$ and 0 for TC$_I$ and TC$_II$, respectively.

This is because following the selection of an active, CsI top shield, every mass-rate gradient toward a further reduced rate is comparatively shallow.

All optimization scenarios demonstrate a factor of $\approx 1.7$ less background rate than simulated observation above Antarctica during solar minima. Interestingly, the major

Table 7.4: Masses of shield suboptions determined for all optimization scenarios. Masses determined for $f_{TC}=0$ reflect idealized active veto performance while masses for $f_{TC}=0.133$ reflect existing X-Calibur shield performance. The total mass is defined by the constraint in Equation 7.2. Mass $m_{A,a}$ represents a tungsten top shield; $m_{A,b}$, an active, CsI top shield; $m_{C,b}$, an externally-mounted, tungsten side shield; and $m_{E,b}$, an externally-mounted, tungsten bottom shield.

<table>
<thead>
<tr>
<th></th>
<th>$f_{TC}=0$</th>
<th></th>
<th></th>
<th>Rate [Hz]</th>
<th>$f_{TC}=0.133$</th>
<th></th>
<th></th>
<th></th>
<th>Rate [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>OS$_\alpha$</td>
<td>N/A</td>
<td>28.0</td>
<td>63.2</td>
<td>2.1</td>
<td>695</td>
<td>N/A</td>
<td>16.4</td>
<td>63.2</td>
<td>13.7</td>
</tr>
<tr>
<td>OS$_\beta$</td>
<td>N/A</td>
<td>10.5</td>
<td>63.2</td>
<td>19.5</td>
<td>704</td>
<td>N/A</td>
<td>13.6</td>
<td>63.2</td>
<td>16.5</td>
</tr>
<tr>
<td>OS$_\gamma$</td>
<td>18.8</td>
<td>N/A</td>
<td>63.2</td>
<td>11.22</td>
<td>746</td>
<td>19.1</td>
<td>NA</td>
<td>63.2</td>
<td>11.0</td>
</tr>
<tr>
<td>OS$_\delta$</td>
<td>N/A</td>
<td>28.0</td>
<td>63.2</td>
<td>2.1</td>
<td>695</td>
<td>N/A</td>
<td>16.4</td>
<td>63.2</td>
<td>13.7</td>
</tr>
</tbody>
</table>
increase in event rate for OS$_\beta$ near $f_{TC} \approx 1$ shows a region of unoptimized space. This event rate jump is attributable to the condition forcing inclusion of the active shield. As the active top shield mass $m_{A,b}$ continues to grow as $f_{TC}$ increases, no mass is allocated to the tungsten shield above it until $f_{TC} \approx 0.9$. However, at this point the difference in CsI and tungsten interaction cross-sections becomes too large and the constraint on including a CsI top shield becomes so burdensome that a new minimum reducing the mass of C.b is formed which increases event rate.

Separately, to better assess the optimal shielding configuration for X-Calibur without modification of veto sensitivity, I estimate the existing $f_{TC}$. In this chapter, I have assumed that the MEGALib energy threshold of 150 keV used in the response matrices results maps to $f_{TC}=0$. Thus I compare the relative increase in background event rate from simulations of the existing X-Calibur configuration with active shield thresholds of 150 and 1500 keV. I calculate $f_{TC}=0.133$. Lowering the X-Calibur threshold energy will reduce $f_{TC}$ and improve background rejection per Figure 7.14 roughly along the dotted lines shown in the figure. Increasing shielding as shown in Figure 7.14 will lower those dotted lines. Table 7.4 shows optimal shield component masses for $f_{TC}=0$ and 0.133 to reflect results if the existing shield threshold is not adjusted.

### 7.4 Point Design Evaluation

To better understand the performance of an optimized detector, I select the shielding configuration found to minimize the event rate in optimization scenarios OS$_\alpha$ and OS$_\delta$ at $f_{TC} = 0$. A mass model depicting it is shown in Figure 7.15. Directly from the minimization I estimate event rates for observation above Antarctica during solar minima.
Figure 7.15: Geomega model of the optimized reconfiguration of X-Calibur. The tungsten HD17 top shield has been replaced with an active, CsI top shield of nearly equivalent mass. A new cylindrical, tungsten side shield is mounted around the existing CsI side shield. A new tungsten annular plate is mounted underneath the CsI shield and around the existing tungsten plug.

For $20 \leq E \lesssim 10^4$ keV, the current shielding gives $R_I=4119$ and $R_{II}=666$ Hz. Furthermore, I estimate its $R_{III}$ event rate within 20-60 keV. I scale based on the ratio of the $R_{II}$ calculated through simulation and minimization. This mass-rate scaling model predicts an background event rate within the X-Calibur science band of 3.3 Hz.

To determine the proximity of these estimates to a complete simulation, I apply the results of the minimization to the existing X-Calibur mass model. The existing 28.3 kg, tungsten top shield is removed and replaced with the optimization of suboption A.b, a CsI active shield of 28.0 kg. The external, 63.2 kg tungsten side shield of C.b and external tungsten base shield of 2.1 kg are also added. No other modification of X-Calibur is performed. I simulate the conditions during solar minima over Antarctica with the detector
at a 35° elevation angle. I assume for this simulation that the threshold condition for raising the veto flag is the deposition of 1500 keV within any segment of the active shield.

However, it bears mentioning that there are several major gaps in fidelity in evaluating the predictions of the minimized model and the simulation. Namely, the distribution of incident flux is not controlled for in the minimization and the “baseline” X-Calibur model that formed the basis of response matrix simulations did not actually reflect the configuration of the real X-Calibur.

I analyze the results of the simulation identically to what was reported in Section 6.3. Figure 7.16 shows the simulated spectrum for TC_I, TC_{II}, and TC_{III} and compares them to the results of Section 6.3 based on the existing X-Calibur design. Background event rates for different energy bands are shown in Table 7.5. For ease of comparison, I have included the rates predicted in 6.3 here. The contributions to the total background of specific particle types is shown in Figure 7.17.

Comparing the rates and observed spectra between the two designs show a significant improvement in reducing the total background rate, which was to be expected by the addition
of 65 kg of high-Z shielding. The total event rate with TC\textsubscript{I} drops from 5190 Hz to 1719 Hz, near that of the estimated background rate during solar maxima, 1163 Hz. Within the X-Calibur science band, backgrounds are lowered but the reduction in event rate is less dramatic, only \(~\sim\)1-3 Hz less contribution to the background passing TC\textsubscript{III}.

The addition of shielding mass has resulted in the change of spectrum as well. A soft tail now emerges in the TC\textsubscript{II} spectra from the contributions of electron bremsstrahlung in tungsten and annihilating positrons. However, this contribution occurs beyond the X-Calibur energy range but is also effectively vetoed such that it does not appear in the TC\textsubscript{II} spectrum. Otherwise, \(\gamma\)-rays are the dominant contribution as they were with the existing X-Calibur design.

Finally, I note that the estimates made using the the approach of event rate minimization provided good insight into the results of simulating this one point design. The shielding optimization was performed minimizing the TC\textsubscript{II} event rate, which was found to be 666 Hz. The simulated TC\textsubscript{II} event rate was 714±11 Hz, similar to what had been predicted. The estimated 3.3 Hz event rate for TC\textsubscript{III} events within the X-Calibur science band is just outside the statistical uncertainty of the simulated rate of 4.23 Hz. However,

Table 7.5: Background multi-pixel event rates for the optimized shield and the existing X-Calibur configuration for solar minima above Antarctica. All values assume an elevation angle of 35°. Values for trigger condition TC\textsubscript{III} have been adjusted as described in Section 6.2.1.

<table>
<thead>
<tr>
<th></th>
<th>Optimized Design [Hz]</th>
<th>Current Design [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-60 keV</td>
<td>TC\textsubscript{I}</td>
<td>98.1(0)±4.2</td>
</tr>
<tr>
<td></td>
<td>TC\textsubscript{II}</td>
<td>59.3±3.2</td>
</tr>
<tr>
<td></td>
<td>TC\textsubscript{III}</td>
<td>4.23±0.86</td>
</tr>
<tr>
<td>20-1000 keV</td>
<td>TC\textsubscript{I}</td>
<td>854±12</td>
</tr>
<tr>
<td></td>
<td>TC\textsubscript{II}</td>
<td>536±10</td>
</tr>
<tr>
<td></td>
<td>TC\textsubscript{III}</td>
<td>29.1±2.3</td>
</tr>
<tr>
<td>All Events</td>
<td>TC\textsubscript{I}</td>
<td>1719±17</td>
</tr>
<tr>
<td></td>
<td>TC\textsubscript{II}</td>
<td>714±11</td>
</tr>
<tr>
<td></td>
<td>TC\textsubscript{III}</td>
<td>50.1±3.9</td>
</tr>
</tbody>
</table>
lack of modeling fidelity can be attributed to the high estimate for event rate. As shown in Figure 7.18, the simulated results demonstrate a very strong dependence on zenith-angle that was not optimized for. Instead the distribution seen for the current design in Figure 6.9 has been dramatically reduced, with events coming from \( \theta < 30^\circ \) effectively shielded against.

### 7.5 Discussion

The results of this study have several implications. First, the goal of the study was to posit that parametric shielding studies can provide results comparable to iterative samplings of the parameter space and considerably more prescriptive than one-dimensional trade space studies. With this approach, there are significant gaps in fidelity that are unavoidable. These
Figure 7.18: The reconstructed distribution of event rate as a function of angle to zenith and energy. This distribution is similar to that in Figure 6.9, though modifications to shield have effectively reduced the number of events coming beyond $\theta = 30^\circ$. Units are in Hz.

included the isotropic description of incident spectra, the simplification of the existing mass model, and the linear and non-linear (for OS$_{\beta}$) minimization of independently simulated shield elements. Despite this, I have shown this approach of parameter selection, limited simulation, and optimization can accurately estimate the simulated detector response to variations in shielding configuration. While the global minimum background event rate that determined through full sampling of the shielding parameter space is unknown, the results of this method have predicted the event rates of the simulated optimized design to factors $< 1.5$.

Analyzing four different constraint scenarios resulted in two identical solutions. The result of these minimizations show that the inclusion of an active, CsI shield top provides the most mass-effective means of shielding to $\sim$28 kg. Once this minimum is reached, an approximate saddle in the $m_{C,b} \times m_{A,b}$ plane results in the preferential growth of an external side shield fitted around the existing active, CsI side shield.
Simulating X-Calibur with a shielding configuration determined through minimization shows that the optimized shielding configuration results in a 70% lower TC\textsubscript{I} event rate than the existing X-Calibur design. However, the TC\textsubscript{II} event rate predicted through the optimization approach is only 60% lower. This implies that there is a diminishing return to the addition of shielding in the explored parameter space. The result is validated by of the optimization and the mass-rate topographies that show the design at a minimum.

Yet these minimizations were performed on the TC\textsubscript{II} event rate, which rejects events that coincide with a veto flag from the ACD. This event rate condition does not fully describe the actual event recording process, which also uses a CD to accept events passing through X-Calibur’s scattering element. This simplification was made on account of the effectiveness of the coincidence flag and the large statistics required to optimize shielding with respect to TC\textsubscript{III}. As a result, the simulation of the optimized point design shows a decrease in TC\textsubscript{III} event rate that is less than the minimized TC\textsubscript{II} rate. Because explored shielding options are readily minimized without significant improvement to the TC\textsubscript{III} rate, other avenues which improve the CD response should be investigated.
Chapter 8

Summary

*With four parameters I can fit an elephant, and with five I can make him wiggle his trunk.*

John Von Neumann (Dyson, 2004)

This dissertation has several key results that can support the development and operation of future high-energy balloon observations. A new set of background models are developed that allow for rapid and accurate estimation of angle-dependent spectra above Ft. Sumner and Antarctica. The same methodologies I have used in this dissertation can be applied to determine background spectra over other geomagnetic rigidity cutoffs.

Unlike other models that may describe only one contribution to the total high-energy background or simulation-derived models that are computationally-intensive, mine were calculated semi-empirically. These models were developed by reviewing archival data. Where gaps existed in any one observation, data from other observations were used after taking special consideration to match observational parameters or assumptions. Other gaps were filled by making theoretical predictions which could be validated by other reports.

I validated these models by using their predictions for incident spectra over Ft. Sumner as input to a particle transport simulation that integrated the detector response. The result
Figure 8.1: The background rate for trigger conditions TC_I (left) and TC_{II} (right) as mass shielding is increased. Each point on the curve reflects a new optimal design point for a shield comprised of an active, CsI top shield, an externally-mounted tungsten side shield, and an externally-mounted tungsten bottom shield. Figure assumes \( f_{TC}=0.113 \).

Some of these simulations show excellent agreement with X-Calibur observation. They also show good agreement with the simulated detector response to incident spectra predicted by the QARM model. Using these models, I also predicted the observed background spectra and its implication for observation over Antarctica for solar minima and maxima, which describe the worst and best case observation conditions, respectively. The prediction for background event rate at solar minima over Antarctica indicate that long observational times are required to obtain significant science results. Results for predicted background rates over Ft. Sumner and Antarctica are shown in Table 8.1.

Table 8.1: Observed background rates in 20-60 keV over Antarctica, for all trigger conditions. Rates over Ft. Sumner reflect X-Calibur’s elevation angle of 9.26° while rates over Antarctica reflect an elevation angle of 35°. Rates reflect multi-pixel events. Units are in Hz.

<table>
<thead>
<tr>
<th></th>
<th>Ft. Sumner</th>
<th>Antarctica</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Solar Maximum</td>
<td>Solar Minimum</td>
<td></td>
</tr>
<tr>
<td>TC_I</td>
<td>25</td>
<td>86.3</td>
<td>382</td>
<td></td>
</tr>
<tr>
<td>TC_{II}</td>
<td>9.1</td>
<td>21.2</td>
<td>92.6</td>
<td></td>
</tr>
<tr>
<td>TC_{III}</td>
<td>0.67</td>
<td>1.78</td>
<td>6.8</td>
<td></td>
</tr>
</tbody>
</table>
To mitigate the background contribution to event rate, I evaluated the effect of modifications to X-Calibur’s shielding to determine an optimal design point. To do so I developed a parametric model of X-Calibur shield concepts which were simulated such that I could establish the change in event rate as a function of additional shielding mass. These mass-rate functions were used in an optimization routine that minimized background event rate as total shielding mass was constrained. To summarize findings, I show a linear reduction in event rate as mass is added for a design optimized for each mass point in Figure 8.1. The results indicate that additional mass would be best allocated first toward an active top shield, and next toward a passive side shield mounted external to the existing active side shield. However, the effectiveness of these modifications is limited by the influence of penetrating neutral radiation. The minimized data rates for all trigger conditions are shown in Table 7.5.

8.1 Opportunities for Future Work

Throughout my research toward this dissertation, I identified several opportunities for new measurements and studies that can improve the accuracy and applicability of the models I describe in Chapter 5.

Notably, the models that I have developed are only validated with respect to conditions over Ft. Sumner. In order to better describe the influence of protons near the rigidity cutoff over other geomagnetic latitudes further analysis is required. In Figure 5.3, protons in regime II, those with energies between 2.16-3.35 GeV, may be attributable to the allowed Störmer cone described in Section 4.1.2. As can be seen on Figure 4.6, while the rigidity cutoff for zenith entrant particles increases closer to the geomagnetic equator, $\Delta R = R_{\text{forbidden}} - R_{\text{allowed}}$ increases. The effect of this would be broadening of regime II. However, I found no
observational data that could constrain the behavior of the proton spectrum. Using data from BESS and Verma (1967), it is readily possible to predict the effect of the broadening of this regime as a function of latitude by matching boundary conditions set by the secondary and primary proton spectra. Nevertheless, a BESS-like measurement would provide constraint of the prediction.

Moreover, I have assumed azimuthal symmetry with respect to zenith. For the scope of this work, such a dependence was not required; however, other applications for an atmospheric radiation model may require the azimuthal dependence of spectra. This may be accomplished by mapping zenith-entrant spectra with rigidity cutoff defined in Equation 4.5. This would result in a second unitless shaping factor that would complement the shaping factors I have developed for spectra’s dependence on atmospheric depth as a function of zenith angle. Data reported by Thompson et al. (1981) on the observation of atmospheric \( \gamma \)-rays from space provides some constraint but further observations sensitive to variation in flux as a function of azimuth would be required for an effective validation.

Furthermore, additional constraints are needed on the upward component of atmospheric positrons of which no observations were found. Because positrons are one of the largest contributions to the background event rate predicted by my model, such an observation would significantly improve the accuracy of my model’s predictions. I had considered developing a constraint on this flux by estimating the contributions from \( \gamma \rightarrow e^+e^- \) and splash protons using those two models. However, the uncertainty from such a theoretical calculation may not offer more effective constraint than the general procedure what I used in Chapter 5. Thus, an observation of the upward positron flux would provide good constraint as to its influence on background event rate. Lastly, new observations of splash electrons and protons would also constrain the ambiguities left by the reports of Israel (1969); Verma (1967) and Wenzel et al. (1975).
An additional ambiguity that exists in my predictions regards the description of flux near limb. Per Figure 6.10, the X-Calibur event rate as a function of elevation angle shows a very ambiguous relation. This may be attributable to the rotation of the detector in azimuth, which could not be controlled for, in conjunction with the limited time spent at various elevations. Observational data taken over a range of elevation angles with an unambiguous background spectrum should provide sufficient data for constraining the influence of limb-entrant background.

My models and their predictions for Antarctica have not yet been validated. An X-Calibur flight over Antarctica will provide an opportunity to validate and refine these models to improve their accuracy. However, data from X-Calibur’s 2016 flight can be used to check my assumption for neglecting the effect of solar modulation when its potential is less than the rigidity cutoff. These results may also be compared to the results reported by Harris et al. (2003).

Finally, in Chapter 7 I have shown the effectiveness of linear optimization for shielding design in comparing its results with the particle transport simulation of the optimal point design. However, the non-linear mass-rate equation developed for optimization scenario $OS_\beta$ would benefit from simulation of a point design.
Appendix A

Formulae and Data Tables

This appendix summarizes the results of the model derivations in Chapter 5 as a reference for readers interested in estimating backgrounds over Ft. Sumner and Antarctica during solar minimum and maximum at 3.45 g cm$^{-2}$.

Background models developed in this work can be extended to other locations and atmospheric depths and formulae in this Appendix reproduced by following the procedures in Chapter 5 with appropriate modifications for rigidity, solar modulation, and atmospheric depth.

A.1 Formulae

For equations referencing atmospheric depth as a function of zenith angle, $\tau(\theta)$, the function can be evaluated by interpolating the table found in Section A.2. Solar minima and maxima are $\phi=250$ and 1200 MV, respectively. Downward refers to flux incident upon the detector from above $\theta_{\text{limb}}$ while upward is flux incident from below limb.
Additionally, all energies $E$ in formulae are of units specified in its intervals (e.g. in Equation A.3, $E$ is in the units of GeV). All fluxes are in (s cm$^2$ [energy] sr)$^{-1}$ unless otherwise noted.

### A.1.1 Downward Proton Models

#### A.1.1.1 Over Ft. Sumner

The data used to derive from model is reported in Abe et al. (2003). The form of the equation for the angle-dependent spectrum is:

$$ F(E, \theta) = m_{p,FtS}(E)f_{p,FtS}(E)s_{p,FtS}(E, \theta) \tag{A.1} $$

The constant $m(E)$ normalizes existing data to a depth of 3.45 g cm$^{-2}$:

$$ m_{p,FtS}(E) = \begin{cases} 
1.022 & E \leq 2.16 \text{ GeV} \\
1 & 2.16 < E \leq 3.35 \text{ GeV} \\
0.81 & E > 3.35 \text{ GeV}
\end{cases} \tag{A.2} $$

The intensity spectrum $f_{p,FtS}(E)$ is:

$$ f_{p,FtS}(E) = \begin{cases} 
18.477 \ E^{-1.05} & E \leq 1.09 \text{ GeV} \\
17.708 \ E^{-0.559} & 1.09 < E \leq 2.16 \text{ GeV} \\
0.243 \ E^{5.009} & 2.16 < E \leq 3.35 \text{ GeV} \\
201.752 \ E^{-0.566} & 3.35 < E \leq 4.14 \text{ GeV} \\
1595.182 \ E^{-2.013} & 4.14 < E \leq 10 \text{ GeV} \\
7760.167 \ E^{-2.7} & E > 10 \text{ GeV},
\end{cases} \tag{A.3} $$
where units are \((\text{s cm}^2 \text{ GeV sr})^{-1}\).

The shaping function \(s(E, \theta)\) is defined as:

\[
s_{p, FtS}(E, \theta) = \begin{cases} 
\frac{\tau(\theta)}{3.35} \Gamma(E) + \Gamma(E > 3.35) & E \leq 3.35 \text{ GeV} \\
\frac{\tau(\theta)}{3.35} \Gamma(E) & E > 3.35 \text{ GeV}
\end{cases}
\] (A.4)

And the shaping power index \(\Gamma(E)\) is:

\[
\Gamma_{p, FtS}(E) = \begin{cases} 
0.743 & E \leq 2.16 \text{ GeV} \\
0 & 2.16 < E \leq 3.35 \text{ GeV} \\
-0.075 & E > 3.35 \text{ GeV}
\end{cases}
\] (A.5)

A.1.1.2 Over Antarctica

The data used to derive this model is reported in Motoki et al. (2003). The form of the angle-dependent spectrum of downward protons over Antarctica takes the following form:

\[
F_{p, McM}(E, \theta, \phi, \tau) = f(E, \phi)^{\Gamma(E, \theta, \phi)} s_p(E, \phi, \theta).
\] (A.6)

Equations A.7 and A.8 define the intensity spectrum of zenith-entrant protons over Antarctica during solar maximima and minima, respectively.

\[
f_{p, \text{Min}}(E) = \begin{cases} 
492.219 E^{0.668} e^{-0.707 E} & 0.1 < E \leq 2.65 \text{ GeV} \\
444.65 E^{0.653} e^{-0.183 E} & 2.65 < E \leq 12 \text{ GeV} \\
8023.83 E^{-2.7} & E > 12 \text{ GeV}
\end{cases}
\] (A.7)
The units for the equations are $(s \text{ cm}^2 \text{ GeV sr})^{-1}$.

Equations A.9 and A.10 describe $\Gamma(E, \phi)$, the power laws for use in Equation A.6 that define the variation in flux as depth or zenith angle is increased. Although Chapter 5 defines the procedure for determining $\Gamma(E, \phi)$, here I present results for $\phi = 1200$ and 250 MV, respectively:

\[
\Gamma_{\text{Max}}(E) = \begin{cases} 
0.505 & 0.15 < E < 0.3 \text{ GeV} \\
-0.069 - 0.0277 E^{-2} + 0.265 E^{-1} & 0.3 \leq E < 11 \text{ GeV} \\
-1.450 \times 10^{-3} E & E \geq 11 \text{ GeV}
\end{cases}
\] (A.9)

\[
\Gamma_{\text{Min}}(E) = \begin{cases} 
0.171(0) - 0.485 E - 0.118 E^2 & 0.15 < E < 1.0 \text{ GeV} \\
-2.48 + 1.664 E^{-1} + 0.402(0) E & 1.0 \leq E < 18.83 \text{ GeV} \\
-0.0195 E^2 + 1.337 \times 10^{-5} E^4 & E \geq 18.83 \text{ GeV}
\end{cases}
\] (A.10)

The shaping function is:

\[
s_p(E, \phi, \theta) = \left[ \frac{\tau(\theta)}{\tau_0} \right]^{\Gamma(E, \phi)},
\] (A.11)

where $\phi$ in $\Gamma(E, \phi)$ refers to either to its value solar minima or maxima defined in Equation A.10 or A.9, respectively.
A.1.2 Upward Proton Models

This model is based on fitting the boundary condition at limb with the upward proton flux reported by Wenzel et al. (1975). The angle-dependent spectra of the upward protons flux takes the following form (same as Equation 5.19):

\[
F_p(E, \theta > \theta_{\text{limb}}) = F_p(E, \theta_{\text{limb}}) \times \left[ 1 + \frac{j_p(E) - 1}{1 + e^{4\pi\theta - (\theta_{\text{limb}} + \pi/8)}} \right]. \tag{A.12}
\]

The only additional parameter requiring definition is \( j_p(E) \), which defines the convergence of the downward spectrum at limb to the observations of Wenzel et al. (1975). The factor \( j_p(E) \) is unitless.

A.1.2.1 Over Ft. Sumner

\[
j_{p,FtS}(E/\text{GeV}) = 0.139 e^{-6.019E} E^{0.956} \tag{A.13}
\]

A.1.2.2 Over Antarctica

\[
j_{p,McM}(E/\text{GeV}) = 0.020(2) e^{-6.029E} - 0.147E^{-1} E^{-0.809} \tag{A.14}
\]

A.1.3 Secondary Electrons and Positrons

The spectra of downward secondary electrons and positrons is based on the model reported in Daniel and Stephens (1974). Their upward spectra is fitted to the data reported by Israel (1969) and Verma (1967). The angle-dependent spectra for downward secondary
electrons and positrons takes the following form:

\[ F_{lep}(E, \theta) = f_{lep}(E) \ s_{lep}(\theta), \tag{A.15} \]

To predict the electron and positron backgrounds over Antarctica, I introduce a scaling factor \( m_{\text{Max/Min}} \) to represent the variation in flux during solar maxima and minima. These are \( m_{\text{Max}} = 0.64 \) and \( m_{\text{Min}} = 3.59 \). These factors, derived through my proton model, are unitless and are identical for both electrons and positrons.

### A.1.3.1 Referenced Equations

In Section 5.6.1, I reference these formulae defining how spectra defined by Daniel and Stephens (1974) were adjusted to reflect conditions experienced by X-Calibur above Ft. Sumner.

\[
\frac{\Delta N_{e^-}}{\Delta R} = -0.043 - 5.712 \times 10^{-4} \ E^{-1} \tag{A.16}
\]

\[
\frac{\Delta N_{e^+}}{\Delta R} = \begin{cases} 
-0.802 - 3.178 \times 10^{-6} \ E^{-2} + 3.691 \times 10^{-3} \ E^{-1} & E < 0.7 \ \text{GeV} \\
+2.093 \ E - 1.510 \ E^2 & E > 0.7 \ \text{GeV}, \tag{A.17}
\end{cases}
\]

and \( E \) is in GeV.

To determine the cosmic electron shaping function, I determine the attenuation coefficient of high-energy electrons passing through the atmosphere using (Olive et al., 2014):

\[
\mu_{e^-} = 4\alpha r_e^2 \frac{N_A}{A} \left\{ Z^2[L - f(Z)] + Z \ L' \right\} \tag{A.18}
\]
in units \( \text{cm}^2 \ \text{g}^{-1} \), where \( \alpha \) is the fine structure constant, \( r_e \) is the classical electron radius [cm], \( N_A \) is Avogadro’s number, \( A \) is the atomic mass, \( Z \) is the atomic number, \( L \) and \( L' \) are
characteristic radiation lengths, and \( f(Z) \) is an empirically determined function. \( L, L' \), and \( f(Z) \) are defined in Olive et al. (2014). I determine \( \mu \) by taking an average of the radiation lengths of \( N_2, O_2, \) Ar, and \( CO_2 \) by weighting them with respect to their proportional mass contribution assuming sea level composition.

### A.1.3.2 Electrons Over Ft. Sumner

The intensity spectrum \( f_{e-}^{\text{FtS}}(E) \) of secondary downward electrons over Ft. Sumner is (same as Equation 5.22):

\[
f_{e-}^{\text{FtS}}(E, \theta \leq \theta_{\text{limb}}) = \begin{cases} 
1.520 \times 10^{-5} E^{-2.282} & 0.002 \leq E < 0.05 \text{ GeV} \\
9.036 \times 10^{-4} \left[ \left( \frac{E}{0.583} \right)^{3.30} + \left( \frac{E}{0.583} \right)^{1.18} \right]^{-1} & E \geq 0.05 \text{ GeV}, 
\end{cases}
\]

(A.19)

Below limb, the spectrum becomes:

\[
f_{e-}^{\text{FtS}}(E, \theta > \theta_{\text{limb}}) = 2.330 \times 10^{-5} E^{-3}
\]

(A.20)

where units are \((s \ cm^2 \ GeV \ sr)^{-1}\) for both.

I now define the unitless shaping factor \( s_{e-}(E, \tau(\theta)) \) in Equation A.21 for \( \tau=3.45 \text{ g cm}^{-2} \) (same as Equation 5.26):

\[
s_{e-}(E, \theta) = \begin{cases} 
\frac{\eta(\tau(\theta))}{\eta(E, 3.45 \text{ g cm}^{-2})} & \theta \leq \theta_{\text{limb}} \\
\frac{\eta(\pi-\theta)}{\eta(E, 3.45 \text{ g cm}^{-2})} & \theta > \theta_{\text{limb}}
\end{cases}
\]

(A.21)

where \( \eta(E, \theta) \) is (same as Equation 5.24):

\[
\eta_{e-}(E, \tau) = \begin{cases} 
9.151 \times 10^4 & E \leq 100 \text{ MeV} \\
47844.2 & E > 100 \text{ MeV},
\end{cases}
\]

(A.22)
However, past limb it becomes:

\[ \eta(\theta > \theta_{\text{limb}}) = \eta(\pi - \theta). \] \hspace{1cm} (A.23)

### A.1.3.3 Positrons Over Ft. Sumner

For positrons the intensity spectrum \( f_{e^+, \text{FtS}}(E) \) is (same as Equation 5.27):

\[
    f_{e^+, \text{FtS}}(E) = \begin{cases} 
        2.120 \times 10^{-2} \left[ \left( \frac{E}{0.0083} \right)^{1.493} + \left( \frac{E}{0.055} \right)^{-0.374} \right]^{-1} & 0.002 \leq E < 0.5 \text{ GeV} \\ 
        8.242 \times 10^{-5} E^{-2.924} & E \geq 0.5 \text{ GeV}
    \end{cases}
\] \hspace{1cm} (A.24)

The shaping factor \( s_{e^+, \text{FtS}}(E, \theta) \) is identical to Equation A.21. However, \( \eta_{e^+}(E) \) takes a different form owing to the similar sources of production of positrons and high-energy electrons (same as Equation 5.28):

\[ \eta_{e^+}(E) = \eta_{e^-}(E > 100 \text{ MeV}) \] \hspace{1cm} (A.25)

### A.1.3.4 Electrons Over Antarctica

First I define scaling values that change intensity spectrum normalization to reflect solar minima and maxima conditions (same as Eq. A.26). These values are for both the electron and positron backgrounds:

\[
    m_{\text{Max}} = 0.64 \\
    m_{\text{Min}} = 3.59
\] \hspace{1cm} (A.26)
The intensity spectrum $f_{e^-, \text{McM}}(E)$ of electrons over Antarctica is (same as Equation ??):

$$\begin{align*}
  f_{e^-, \text{McM}}(E) &= \begin{cases} 
    5.538 \times 10^{-5} \text{ m } E^{2.583} & E \leq 0.032 \text{ GeV} \\
    \frac{0.0256}{2.74(0)E^{1.1}+18.768E^{3.2}} & E > 0.032 \text{ GeV},
  \end{cases}
\end{align*}$$

(A.27)

with units (s cm$^2$ GeV sr)$^{-1}$. This spectrum is used for electrons incident from all directions.

The shaping function $s_{e^-}(E, \theta)$ and parameter $\eta(E)$ are identical their definitions in Equations A.21 and A.22, respectively.

A.1.3.5 Positrons Over Antarctica

The intensity spectrum $f_{e^+, \text{McM}}(E)$ of positrons over Antarctica is (same as Equation ??):

$$f_{e^+, \text{McM}}(E) = \begin{cases} 
  0.2 \times 10^{-3} E^{-3.13} & E \leq 0.466 \text{ GeV} \\
  1.166 \times 10^{-3} E^{-3.13} & E \leq 0.032 \text{ GeV}
\end{cases}$$

(A.28)

with units (s cm$^2$ GeV sr)$^{-1}$. This spectrum is used for electrons incident from all directions.

Its shaping function and parameter $\eta(E)$ are defined by Equations A.21 and A.25, respectively.

A.1.4 Cosmic Electrons

This model is based on fits to the AMS-reported cosmic electron spectrum in Aguilar et al. (2014). The formula defining the angle-dependent spectrum for cosmic electrons is (same as Equation 5.38):

$$F_{\text{cosmic } e^-}(E, \theta) = f_{0, \text{cosmic } e^-}(E) e^{-\mu_{e^-} \theta}$$

(A.29)
where \( f_{0, \text{cosmic } e^-} \) is the intensity spectrum, \( \mu_e^- = 9.182 \times 10^{-11} \text{ cm}^2 \text{ g}^{-1} \) is the attenuation factor for averaged atmospheric composition and \( \tau(\theta) \) is the atmospheric depth [g cm\(^{-2}\)] that is defined by interpolating the values in Table A.2.

The intensity spectrum \( f_{0, \text{cosmic } e^-} \) as reported in Aguilar et al. (2014) is described by:

\[
f_{\text{cosmic } e^-}(E) = \begin{cases} 
18.24 \times 10^{-0.683} & 0.65 \leq E < 1.46 \text{ GeV} \\
25.758 \times 10^{-1.596} & 1.46 \leq E < 3.62 \text{ GeV} \\
84.233 \times 10^{-2.517} & 3.62 \leq E < 7.1 \text{ GeV} \\
308.736 \times 10^{-3.18} & E \geq 7.1 \text{ GeV}, 
\end{cases} \tag{A.30}
\]

in units of \( (\text{m}^2 \text{ s GeV sr})^{-1} \).

### A.1.5 Muons

The downward muon spectrum is based on fits to the data reported by Shikaze et al. (2007) and Abe et al. (2003). The angle-dependent spectra of muons are derived identically to those of protons as both models were based on BESS observation. The form of their spectra is Equation A.15. The units of all spectra in this section are \( (\text{cm}^2 \text{ s GeV sr})^{-1} \).

The general form of the spectrum of all upward muons is (same as Equation 5.33):

\[
F_\mu(E, \theta > \theta_{\text{limb}}) = F_\mu(E, \theta_{\text{limb}}) \times \left[ 1 + \frac{j_\mu(E) - 1}{1 + e^{4\pi[\theta - (\theta_{\text{limb}} + \pi/8)]}} \right], \tag{A.31}
\]

where I define \( j_\mu(E) \) below.
A.1.5.1 Muons Above Ft. Sumner

The intensity spectra for negative and positive muons over Ft. Sumner was reported by Abe et al. (2003). It can be fitted with the following functions (same as Equations 5.29 and 5.30, respectively):

\[
f_{\mu^-}(E) = \begin{cases} 
3.393 \ E^{-1.319} & 0.1 \leq E < 1.7 \ \text{GeV} \\
6.60385 \ E^{-2.574} & E \geq 1.7 \ \text{GeV}
\end{cases} \tag{A.32}
\]

\[
f_{\mu^+}(E) = \begin{cases} 
3.983 \ E^{-1.225} & 0.1 \leq E < 1.9 \ \text{GeV} \\
8.063 \ E^{-2.324} & E \geq 1.9 \ \text{GeV}
\end{cases} \tag{A.33}
\]

The shaping function for both muons is identical:

\[
s_{\mu^\pm}(\theta < \theta_{\text{limb}}) = \left[ \frac{\tau_{\text{atm}}(\theta)}{3.45} \right]^{\Gamma_{\mu^\pm}} \tag{A.34}
\]

but the power law index \(\gamma\) varies as described in Table A.1.

The convergence parameter \(j_{\mu^\pm, \text{FtS}}(E)\) for use in Equation A.31 is:

\[
j_{\mu^\pm}(E) = j_{p, \text{FtS}}(E - 0.159 \ \text{GeV}), \tag{A.35}
\]

where \(j_{p, \text{FtS}}(E)\) is defined in Equation A.13.

Table A.1: Power law fits describing variation of muon flux as depth increases.

<table>
<thead>
<tr>
<th>(E) (GeV)</th>
<th>(\Gamma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu^+) 0.50</td>
<td>0.85</td>
</tr>
<tr>
<td>(\mu^+) 2.2</td>
<td>0.90(0)</td>
</tr>
<tr>
<td>(\mu^+) 8.41</td>
<td>0.75</td>
</tr>
<tr>
<td>(\mu^-) 0.50</td>
<td>0.84</td>
</tr>
<tr>
<td>(\mu^-) 2.2</td>
<td>0.86</td>
</tr>
</tbody>
</table>

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A.1.5.2 Muons Above Antarctica

The intensity spectra of muons over Antarctica is defined at solar minima and maxima:

\[
\begin{align*}
    f_{\mu^-, \phi_{\text{Max}}}(E) &= \begin{cases} 
    18.63 \, E^{-0.003} \, e^{-0.566 \, E} & E \leq 2.7 \, \text{GeV} \\
    16.849 \, E^{-1.121} \, e^{-0.118 \, E} & E > 2.7 \, \text{GeV},
    \end{cases} \tag{A.36} \\
    f_{\mu^-, \phi_{\text{Min}}}(E) &= \begin{cases} 
    17.84 \, E^{0.11} \, e^{-0.603 \, E} & E \leq 3 \, \text{GeV} \\
    23.59 \, E^{-1.49} \, e^{-0.072 \, E} & E > 3 \, \text{GeV},
    \end{cases} \tag{A.37} \\
    f_{\mu^+, \phi_{\text{Max}}}(E) &= \begin{cases} 
    22.904 \, E^{-0.019} \, e^{-0.57 \, E} & E \leq 2.28 \, \text{GeV} \\
    20.58 \, E^{-1.08} \, e^{-0.12 \, E} & E > 2.28 \, \text{GeV},
    \end{cases} \tag{A.38} \\
    f_{\mu^+, \phi_{\text{Min}}}(E) &= \begin{cases} 
    19.012 \, E^{-0.181} \, e^{-0.43 \, E} & E \leq 2.9 \, \text{GeV} \\
    22.85 \, E^{-1.35} \, e^{-0.075 \, E} & E > 2.9 \, \text{GeV}.
    \end{cases} \tag{A.39}
\end{align*}
\]

Their shaping function, power law indices \(\Gamma\), and convergence parameter \(j_{\mu^\pm}(E)\) are the same as at Ft. Sumner, defined by Equation A.34, Table A.1, and Equation A.35. Note that in using Equation A.35, the equation use for \(j_{p}(E)\) must correspond to the desired flight environment (e.g. solar minima over Antarctica).

A.1.6 Atmospheric \(\gamma\)-Rays

This model is based on the work of Costa et al. (1984). The intensity spectrum has been refitted to reflect the atmospheric \(\gamma\)-ray spectra reported by Kinzer et al. (1978) and Schöenfelder et al. (1980). The shaping term has been refitted to reflect data reported by Akyüz et al. (1997).
The angle-dependent spectrum of atmospheric $\gamma$-rays is defined by three factors defining the intensity spectrum $f_{\text{atm}}\gamma_0(E)$, shaping $s(E, \theta)$, and rigidity response with power law index $\alpha(E)$:

$$F_{\text{atm}}\gamma(E, \theta, R, \phi) = f_{\text{atm}}\gamma(E) s_\gamma(\theta) \left(\frac{4.5 \text{ GeV}}{\text{Max}(R, \phi)}\right)^{\alpha(E)},$$

where the units of $F_{\text{atm}}\gamma$ and $f_{\text{atm}}\gamma$ are $(\text{s cm}^2 \text{ MeV sr})^{-1}$. All other terms are unitless.

The intensity spectrum is defined by:

$$f_{\text{atm}}\gamma(E) = \begin{cases} 
5.454 \times 10^{-3} & 0.01 \leq E < 0.5 \text{ MeV} \\
6.445 \times 10^{-3} & 0.5 \leq E < 41.3 \text{ MeV} \\
9.044 \times 10^{-5} & 41.3 \leq E < 151.8 \text{ MeV} \\
0.422 E^{-2.13} & E \geq 151.8 \text{ MeV} 
\end{cases}$$

The shaping factor $s_\gamma(E, \theta)$ describes the spectrum’s angle-dependence. I define factors $A_1(\theta)$, $A_2(E, \theta, \tau)$, $A_3(\theta)$, and $B(E, \tau)$ for use in the shaping equation:

$$
\begin{align*}
A_1(E, \tau) &= 0.840232 - 0.95597 E^{3.696 \times 10^{-5}} \tau^{-8.002 \times 10^{-5}} \\
A_2(E, \tau) &= 1.66848 E^{3.696 \times 10^{-5}} \tau^{-8.002 \times 10^{-5}} - 0.586 \\
A_3(\theta) &= 2.8 \cos^6(0.95\theta - \frac{23\pi}{36}) + 0.999 \\
B(E, \tau) &= e^{-0.0143 E \tau^{0.1429} E^{-0.05}}
\end{align*}
$$

where $\theta$ is zenith angle, $\tau$ is the atmospheric depth of the instrument, and the units for $\theta$ are radians; $E$, MeV; and $\tau$, g cm$^{-2}$. 

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These factors can then be inserted in the definition of the shaping function:

\[
s(E, \theta, t) = \begin{cases} \\
\frac{t}{2} & 0 \leq \theta < 0.698 \\
(A_1(E, \tau) \theta + A_2(E, \tau)) A_3(\theta) & 0.698 \leq \theta < 1.74 \\
\times \left[ -0.253 e^{2.38 \theta} E^{-0.2 + 0.286 \theta} \tau^{1.57 - 0.818 \theta} \right] & 1.74 \leq \theta < \pi \\
\times (1 - 1.164 B(E, \tau) + 0.903) \theta^2 + (6.300 \\
\times B(E, \tau) - 4.770) \theta + (8.298 B(E, \tau) - 5.575) & \\
\times \left[ 18.046 E^{0.35} E^{(0.1577 - 0.082139 \theta) + 0.0082139} \right] & \\
\end{cases}
\]

where at E > 250 MeV and \( \theta > 1.74 \), E = 250 MeV should be used in place of the energy. Note that \( s(E, \theta, t) \) is not a smooth function, but is fitted to empirical data.

The power index \( \alpha(E) \) in the rigidity response term is defined as:

\[
\alpha(E) = \begin{cases} \\
0.018 E + 0.96 & 0.1 \leq E < 5 \text{ MeV} \\
0.0032 E + 1.05 & 5 \leq E < 30 \text{ MeV} \\
1.13 & E \geq 30 \text{ MeV} \\
\end{cases}
\]

(A.44)

A.1.7 Cosmic X-Ray Background

The unscattered CXB spectrum is based on fits to the spectrum reported by Ackermann et al. (2015). The scattered CXB spectrum is derived by subtracting the scattered component, atmospheric \( \gamma \)-rays, and unscattered component from the total \( \gamma \)-ray spectrum reported in Gehrels (1985).
A.1.7.1 Unscattered CXB

The general form of the CXB angle-dependent spectrum is (same as Equation 5.42):

\[
F_{\text{CXB}, \theta}(E, \theta) = f_{\text{CXB}, \theta}(E) e^{-\bar{\mu}(E) \tau(\theta)},
\]

(A.45)

where units of \( f_{\text{CXB}, \theta} \) are in \( \text{s cm}^{-2} \text{ keV sr}^{-1} \). The attenuation factor \( \bar{\mu}(E) \) is the interpolated mean attenuation factor of average atmospheric composition in units of \( \text{cm}^2 \text{ g}^{-1} \) (Xcom, 2010). The atmospheric depth \( \tau(\theta) [\text{g cm}^{-2}] \) is defined in Table A.2.

The intensity spectrum is fitted to the results of Ackermann et al. (2015):

\[
f_{\text{CXB}, \theta}(E) = \begin{cases} 
0.027 - 939.3 \ E^{-1.75} + 1390.4 \ E^{-1.5} & E \leq 125 \ \text{keV} \\
-708.5 \ E^{-1.25} + 136.0 \ E^{-1} - 2.37 \ E^{-0.5} & \\
-708.5 \ E^{-1.25} + 136.0 \ E^{-1} - 2.37 \ E^{-0.5} \\
-5.980 \times 10^{-6} + 85.3 \ E^{-1.75} - 48.2 \ E^{-1.5} \\
+9.96 \ E^{-1.25} - 0.80(0) \ E^{-1} \\
+0.0025 \ E^{0.5} + 4.42 \times 10^{-9} \ E^{-0.5} & 125 < E \leq 3000 \ \text{keV} \\
-1.33 \times 10^{-10} - 10.1 \ E^{-2} + 5.31 \ E^{-1.75} \\
-1.00(2) \ E^{-1.5} - 0.102 \ E^{-1.25} - 0.006 \ E^{-1} \\
+2.020 \times 10^{-4} \ E^{-0.75} - 3.79 \times 10^{6} \ E^{-0.5} \\
+3.60(3) \times 10^{-8} \ E^{-0.25} & 3.0 < E \leq 25.1 \ \text{MeV} \\
29.99 \ E^{-2.27} & E > 25.1 \ \text{MeV} 
\end{cases}
\]

(A.46)
A.1.7.2 Unscattered CXB

The total observable $\gamma$-ray background is defined as:

$$F_\gamma(E, \theta) = F_{\text{atm}} \gamma(E, \theta) + F_{\text{CXB}, 0}(E, \theta) + F_{\text{CXB}, s}(E, \theta),$$  \hspace{1cm} (A.47)

where $F_{\text{CXB}, 0}$ is the unattenuated CXB spectrum defined above and $F_{\text{CXB}, s}$ is the inelastically scattered CXB spectrum.

For $F_\gamma(E)$, I use the observed total downward spectrum of Gehrels over Palestine, TX at 3.5 g cm$^{-2}$:

$$F_\gamma(E) = \begin{cases} 2.1910^2 E^{0.7} & 0.024 \geq E < 0.035 \text{ MeV} \\ 5.1610^{-2} E^{-1.81} & E \leq 0.035 \text{ MeV} \end{cases}$$  \hspace{1cm} (A.48)

which is assumed to be semi-isotropic. The atmospheric $\gamma$-ray contribution is defined above.

Solving Equation A.47 for $F_{\text{CXB}, s}(E, \theta)$ produces the angle-dependent flux of attenuated CXB, which is can be used for all geomagnetic latitudes and solar conditions.
A.2 Tables of Values

Table A.2: Path length $L(\theta)$, as defined in Figure 4.8, and atmospheric depth $\tau(\theta)$ calculated from the CIRA 2012 atmospheric model (COSPAR, 2012). These are defined assuming an atmospheric depth of $\tau(0)=3.45$ g cm$^{-2}$ and 900 km representing top-of-the-atmosphere for moderate solar activity. Both $L(\theta)$ and $\tau(\theta)$ are functions of zenith angle $\theta$, in either radians or degrees [$^\circ$].

<table>
<thead>
<tr>
<th>$\theta$ [rad]</th>
<th>$\theta$ [$^\circ$]</th>
<th>$L(\theta)$ [km]</th>
<th>$\tau(\theta)$ [g cm$^{-1}$]</th>
</tr>
</thead>
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<td>0</td>
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<td>3.43467</td>
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<tr>
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<tr>
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Appendix B

Further Definition of Shield Optimization

Equation 7.2 provides a summary of the minimized background rate equation and constraints, but I have altered it to provide results that reflect different potential optimization scenarios and provide a more accurate minimized background event rate.

B.1 Changes to the Minimized Effective Background Rate \( R \)

In Equation 7.2, I defined the background rate being minimized as:

\[
R = \frac{1}{J} \sum_i \sum_j r_i(m_j).
\]

However, I change this equation to perform minimization as a function of \( f_{TC} = r_1/(r_1 + r_{II}) \) and to normalize rate so that it can be compared directly with the results in Section 6.3.

The form of the rate equation is now:

\[
R = \frac{1}{J} \left[ m_1 f_{TC} \sum_i \sum_j r_{i,1}(m_j) + m_{II} \left(1 - f_{TC}\right) \sum_i \sum_j r_{i,II}(m_j) \right], \quad \text{(B.1)}
\]
where subscripts I and II reference the use of results for TC_I and TC_{II} and m_{I/II} are normalization factors. Thus the first term is the relative contribution of TC_I events while the second term is that of TC_{II}.

In order to make a direct comparison to simulated results from Section 6.3, I use normalization factors $m_I=1.10$ and $m_{II}=1.24$. These are the ratio of all events over all energies to the contribution from $\gamma$-rays, positron, electron, and protons from the simulation of background over Antarctica during solar minima. However, as the simulations for Antarctica assumed a 1500 keV energy threshold for TC_{II}’s veto condition, this normalization does not account for the shield threshold. The implications of shield threshold are discussed in Chapter 7.

### B.2 Optimization Scenario-Specific Constraints

Optimization scenario OS_\alpha constrains the minimization by forcing mutual exclusivity of suboptions within options for top, side, and base shields. For example, an external side shield and internal side shield cannot both be used for a point design. For instance, the additional tungsten side shield of option C can only be placed inside or outside of the detector. This restriction allows the minimization to accurately reflect the conditions of the response matrix simulations. When the masses are restricted to the minimum and maximum simulated shield masses, each option is constrained by:

\[
m_{X,y, \text{min}} \leq m_{X,y} \leq m_{X,y, \text{max}} \quad \land \quad m_{X,y'} = 0
\]

\[\land
\]

\[
m_{X,y', \text{min}} \leq m_{X,y'} \leq m_{X,y', \text{max}} \quad \land \quad m_{X,y} = 0.
\]
Here \( X.y \) and \( X.y' \) represent different suboptions. Taking the top line of in Equation B.2 as an example, if \( m_{X,y} \) results in a local minimum given all other constraints, then \( m_{X,y'} = 0 \) kg. Moreover, \( m_{X,y,\text{min}} \leq m_{X,y} \leq m_{X,y,\text{max}} \), where the minimum and maximum masses are defined by based on the domain of shield component masses simulated, unless otherwise defined.

Optimization scenario \( \text{OS}_\beta \) evaluates a composite top shield that consists of a tungsten shield, suboption A.a, on top of a CsI shield, suboption A.b. However, no simulations were performed for a composite shield. For the purpose of estimation, I assume that the tungsten shield attenuates flux passing through it without modifying its spectrum. This allows event rate to be calculated by assuming that events that would result from passage of radiation through the tungsten top shield can then be further attenuated by the lower CsI shield. This results in a non-linear mass-rate function dependent on both \( m_{A,a} \) and \( m_{A,b} \):

\[
\begin{align*}
r_A(m_{A,a}, m_{A,b}) &= \frac{r_A(m_{A,a})}{r_A(m_{A,a,\text{min}})} r(m_{A,b}).
\end{align*}
\]

(B.3)

Here, \( m_{A,a,\text{min}} \) is the event rate when the minimum simulated mass for a tungsten top shield used. The factor \( r_A(m_{A,a})/r_A(m_{A,a,\text{min}}) \) modulates \( r(m_{A,b}) \) based on \( m_{A,a} \). All other things equal, the boundary where \( m_{A,a} = m_{A,a,\text{min}} \) would mean that using it is identical to \( m_{A,a} = 0 \) kg.

Scenario \( \text{OS}_\gamma \) assumes that the top shield will only be comprised of tungsten. Here, suboptions A.b and B.b are removed and all suboptions remain mutually exclusive. Scenario \( \text{OS}_\delta \) removes option B and suboptions A.a, C.a, and E.a and also allows variation of \( m_{A,b} \) and \( m_{E,b} \) above the limits placed by the maximum shield thicknesses simulated. This increases the available space so the potential for finding a new local maximum beyond the simulated cases. Although this region has not been simulated, the mass-rate functions are well-behaved for options A and E as seen in Figures 7.7 and 7.9.
B.3 Additional Assumptions

Finally, I further restrict the minimum thickness of the active, CsI top shield for all scenarios. I constraint the minimum allowed $m_{A,b}$ and $m_{B,b}$ corresponding to the minimum thickness of CsI, 1 cm, in the X-Calibur shield. This is done as ray trace simulations were not performed as to determine the effectiveness of thinner active shields.
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