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The Trajectory Method for Evaluating Periodic Bursty Traffic

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THE TRAJECTORY METHOD FOR EVALUATING PERIODIC BURSTY TRAFFIC

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ABSTRACT

Asynchronous Transfer Mode (ATM) has been expected to be the basis for the next generation communication networks. The point of queueing analysis for ATM networks is to evaluate the behavior of bursty traffic. Periodicity of the traffic is another significant characteristics of some applications, and is difficult to treat by means of conventional queueing theories. This report describes the trajectory method for evaluating the queueing behavior of periodic bursty traffic, which has not been studied yet. The method is based on calculation of the difference between arriving and departing traffic. The results obtained by this method include packet loss rate and queue length distribution. And it is shown that periodic bursty traffic has, in general, moderate queueing characteristics which is between bulk arrival model and M/M/1 model which are too optimistic and too pessimistic, respectively.

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1. INTRODUCTION

1.1. Motivation

Asynchronous Transfer Mode (ATM) has been proposed as the basis for the next generation of public communication networks [3,8,9,11]. As an ATM switch handles many kinds of traffic in a single entity, i.e., multi-service capability, queueing analyses of ATM networks for various kinds of traffic must be done.

Especially for ATM networks the point of queueing analysis is to evaluate the behavior of bursty traffic. In fact, many services that should be handled on ATM networks are bursty, for example coded video and high resolution images, such as those obtained from digital X-ray machines, computerized tomography scanners, etc. Typical peak rates for these services range from 10kp/s to 350kp/s, average rates range from 100p/s to 20kp/s, and the typical time period for a burst ranges from a few milliseconds to a few seconds.

ATM networks require traffic valves at the access points [1,10]. Figure 1 shows a typical example of a traffic valve, that is called the pseudo-buffer mechanism. The usre's packets drive a simulation of a buffer, which in turn determines which of the user's traffic is permitted to enter the network. A subscriber assigns parameters to the pseudo-buffer according to his requirements. In particular he specifies the maximum rate λ_p at which packets enter the pseudo-buffer, the rate λ_a at which they leave and the pseudo-buffer capacity B. The first valve restricts the peak rate of the input traffic to λ_p. Packets that pass through the first valve are accumulated at the pseudo-buffer. If the pseudo-buffer is full when a packet attempts to enter, it is discarded.

The pseudo-buffer mechanism allows a wide variety of user behaviors. The "most bursty" behavior possible for a user is to alternate between active and idle periods; specifically the user can transmit at rate λ_p until the pseudo-buffer becomes full then go idle until the pseudo-buffer is empty, then go active again, repeating in a periodic fashion. In this paper we analyze the queueing behavior that results from multiplexing many such sources. In our view, this is an important limiting case for the class of behaviors permitted by an ATM network and forms an appropriate basis for accepting or rejecting new connections.

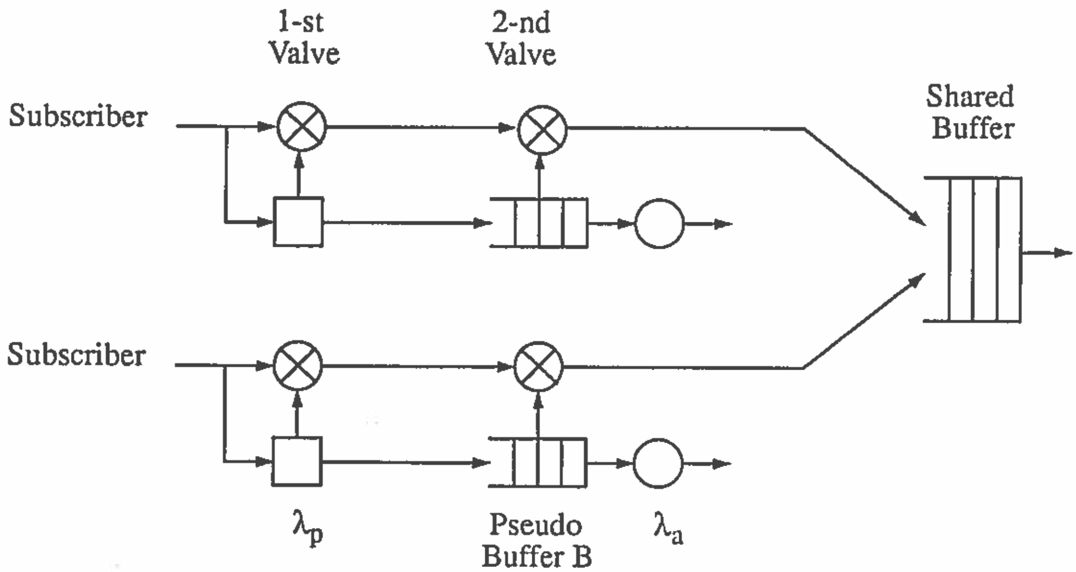

Figure 1: Traffic valve; pseudo-buffer mechanism

1.2. Other analyses

There has been earlier research studying the queueing behaviors of packet networks carrying bursty traffic. Some research evaluates ATM network performance by simulation [2]. In general simulations are apt to take excessive CPU time to obtain accurate results with respect to the lower packet loss rate region, typically less than 10^{-6}.

On the other hand, conventional analytical evaluations for ATM networks are based on Markov processes, even if the arrival traffic is bursty, and they require the solution of large transient matrices [1]. In this case, if the transient matrices are too large, or, as it happens occasionally, if they are multi-dimensional, it is difficult to solve them. Moreover, these analyses fail to take into account the effect of the traffic valve, which is clearly a crucial element in the behavior of these networks.

Some research approximates queueing behavior by using diffusion process models, but they don't consider periodicity nor finite buffer size [5,6].

Here is a brief comparison of several results of conventional analyses. The M/M/1 model is the most frequently referred to, and is based on Poisson arrivals [4]. The result is too optimistic because it fails to account for bursty traffic. Bulk arrival analysis is another alternative, but it is too pessimistic as it assumes that all packets in a burst arrive at the same instant [7]. The third analysis is the convolution method which deals with periodic and bursty traffic [12]. It gives call acceptance criterion, but not performance parameters.

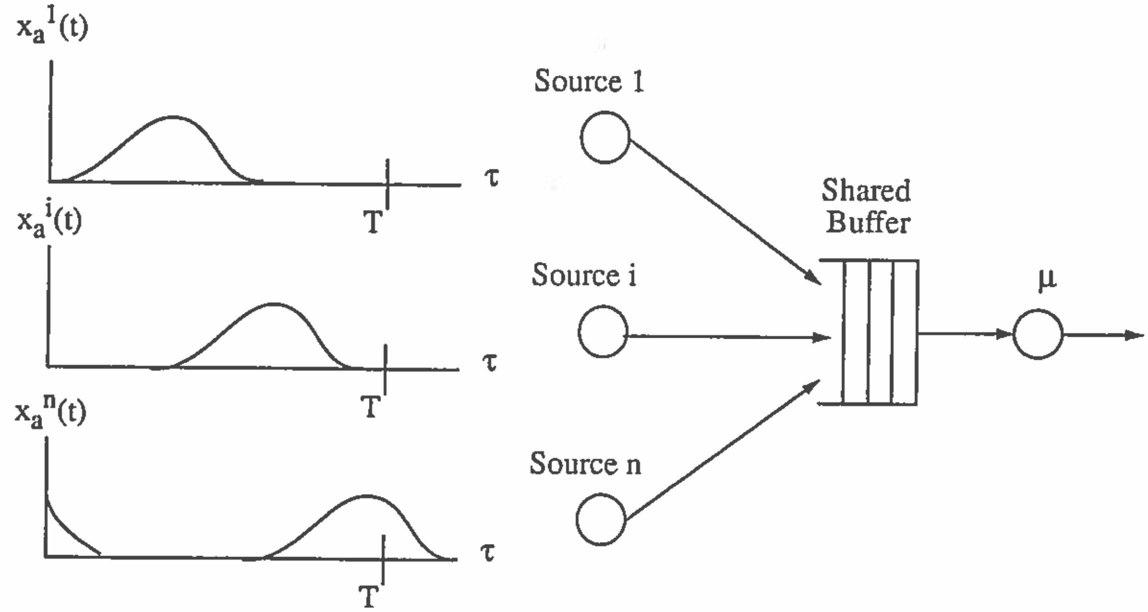

Figure 2: Queueing model

1.3. Goal

The goal of this study is as follows;

(a) To obtain analytical methods to evaluate periodic bursty traffic, which might lead to theoretical analysis for arbitrary traffic in an ATM network environment.

(b) To apply the method to periodic bursty traffic.

(c) To observe characteristics of periodic bursty traffic.

2. PERIODIC TRAFFIC MODELING

For the purpose of queueing analyses with respect to periodic bursty traffic, the queueing model depicted in Figure 2 is adopted.

There are n sources connected to a shared buffer whose buffer size is B. The i-th source transmits packets to the shared buffer at the rate of $x_a^i(t)$, which has period T with phase ϕ_i. Each source is identical except for the phases, which are randomly and independently distributed over the period T. Let $X_a^i(t)$ be the number of packets from the i-th source up to time t, i.e.;

$$X_a^i(t) = \int_0^t x_a^i(\tau)d\tau.$$

Total number of arrivals from n sources can be defined as $X_a(t)$, and

$$X_a(t) = \sum_{i=1}^{n} X_a^i(t).$$

If the phase ϕ_i of all sources are given, $X_a(t)$ is completely determined. Then queueing behavior such as waiting queue length and packet loss rate can be evaluated, if the buffer size B and service rate $\mu(t)$ are given.

The objective is to determine the probability of a phase relationship which leads to packet loss and waiting queue, or fraction of packet loss and waiting queue.

3. PACKET LOSS AND QUEUE LENGTH

We assume that the service rate $\mu(t)$ is deterministic, i.e.;

$$\mu(t) = \text{constant rate} = \mu.$$

This is reasonable, because ATM networks are expected to operate without flow control. We define variables as follows;

$X_a(t)$ = number of packets that have arrived up to time t,
$X_e(t)$ = number of packets that have entered buffer up to time t,
$X_d(t)$ = number of packets that have left buffer up to time t,
$L(t)$ = number of packets lost up to time t,
$Q(t)$ = number of packets in buffer at time t.

These quantities are shown in Figure 3 as an example.

$X_a(t)$ has slope μ at time t_1. It is increasing more rapidly than the service rate μ from t_1 to t_2, and at t_2 waiting queue $Q(t_2)$ reaches B. As tangent of $X_a(t)$ is still greater than μ between t_2 and t_3, the buffer overflows during the period and packet loss occurs, which is expressed as $L(t)$. At time t_3 the tangent of $X_a(t)$ has the quantity of μ again, $X_e(t)$ begins to follow $X_a(t)$ up to time t_4. Though $X_d(t)$ has been constantly increasing at the rate of μ up to time t_4, it catches up with $X_e(t)$ at time t_4.

The relationship of the quantities stated above is as follows;

$$\begin{aligned}
L(t) &= X_a(t) - X_e(t) \\
Q(t) &= X_e(t) - X_d(t) \\
\frac{dX_d(t)}{dt} &= \begin{cases} \mu & \begin{array}{l}\text{if } B > 0 \text{ and } Q(t) > 0 \\ \text{or } B = 0 \text{ and } \frac{dX_a(t)}{dt} \geq \mu\end{array} \\ \frac{dX_a(t)}{dt} & \text{otherwise} \end{cases} \\
\frac{dX_e(t)}{dt} &= \begin{cases} \mu & \begin{array}{l}\text{if } B > 0 \text{ and } Q(t) = B \\ \text{or } B = 0 \text{ and } \frac{dX_a(t)}{dt} \geq \mu\end{array} \\ \frac{dX_a(t)}{dt} & \text{otherwise} \end{cases}
\end{aligned}$$

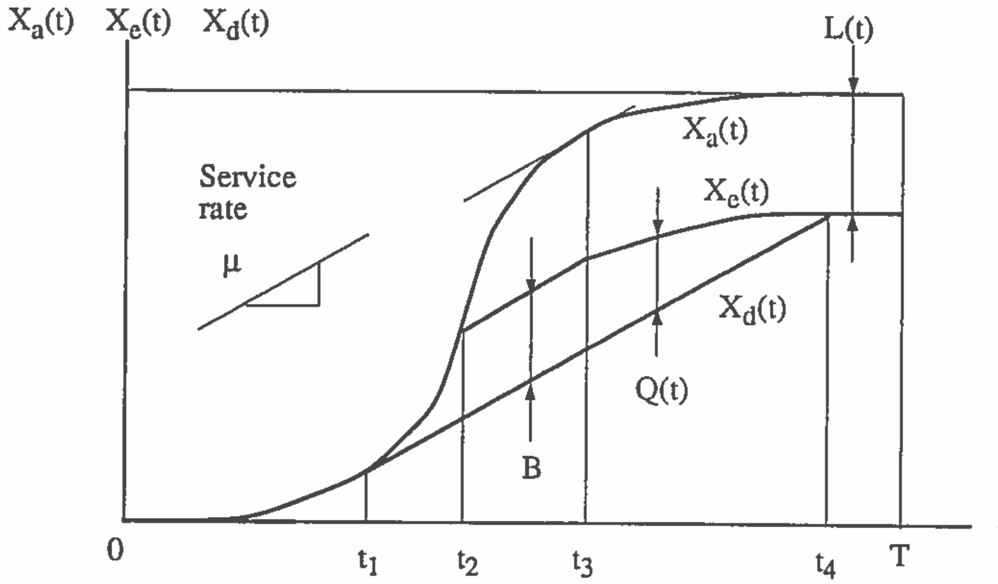

Figure 3: Trajectory of arrivals

The third and the fourth equations can be modified as follows by combining the conditions;

$$\frac{dX_d(t)}{dt} = \begin{cases} \mu & \text{if } Q(t) > 0 \text{ or } \frac{dX_a(t)}{dt} \geq \mu \\ \frac{dX_a(t)}{dt} & \text{otherwise} \end{cases}$$

$$\frac{dX_e(t)}{dt} = \begin{cases} \mu & \text{if } Q(t) = B \text{ and } \frac{dX_a(t)}{dt} \geq \mu \\ \frac{dX_a(t)}{dt} & \text{otherwise} \end{cases}$$

These equations describe the queueing behavior of the given model. The packet loss rate and time-average queue length for the given $X_a(t)$ are expressed as follows, respectively;

$$\begin{aligned} &\text{Packet loss rate for given } X_a(t) \\ &\quad \equiv Pr(loss \mid given\ X_a(t)) \\ &\quad = \frac{L(T)}{X_a(T)} \\ &\text{Time average queue length for given } X_a(t) \\ &\quad \equiv Q(given\ X_a(t)) \\ &\quad = \frac{1}{T}\int_0^T Q(t)dt \end{aligned}$$

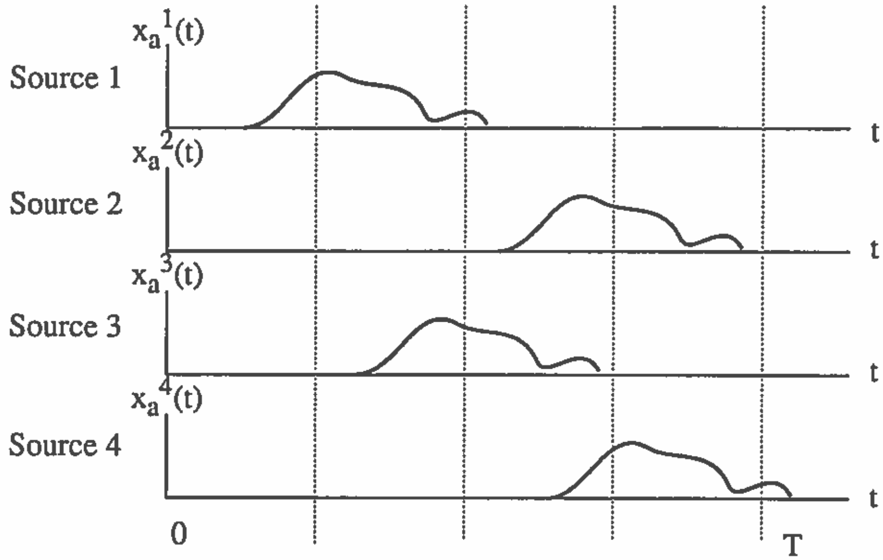

Figure 4: Intervals in period

4. PROBABILITY OF TRAJECTORY

So far the packet loss rate and time-average queue length are analyzed, when the number of packets $x_a(t)$ that have arrived up to time t is given. We refer to each particular choice of $x_a(t)$ as a trajectory. To determine the probability of a trajectory assume that the period $(0, T)$ is divided into m intervals, and let k_j be the number of sources whose bursts start in the j-th interval. Figure 4 shows an example. In the figure, the period $(0, T)$ is composed of four intervals, and source 1, 2, 3 and 4 start at the first, third, second and third interval, respectively.

Note that $k_1 + k_2 + \cdots + k_m = n$, and assume all phases are equally likely. Then

$$Pr(k_1, \ldots, k_m) = \binom{n}{k_1, \ldots, k_m} \left(\frac{1}{m}\right)^n$$

This multinominal coefficient represents the probability of the combination that the numbers of phases in the j-th interval is k_j . In other words the $Pr(k_1, \cdots, k_m)$ is the probability of the trajectory $X_a(t)$ which is composed of $x_a^i(t)$, and the phase of $x_a^i(t)$ satisfies the description above.

To compute packet loss rate and queue length, assume all sources that start in the given interval, start at the beginning of the interval. Then the packet loss rate is given as the summation of the product of packet loss rates for the given $X_a(t)$ and the probability of the $X_a(t)$ trajectory. The average queue length is also given as the summation of the products of the time-average queue length for the given $X_a(t)$ and the probability of the

$X_a(t)$ trajectory as follows;

$$\begin{aligned}
&\text{Packet loss rate}\\
&\quad \equiv Pr(loss)\\
&\quad = \sum_{k_1,\ldots,k_m} Pr(loss \mid given\ X_a(t))Pr(k_1,\ldots,k_m)\\
&\quad = \sum_{k_1,\ldots,k_m} \frac{L(T)}{X_a(T)} \binom{n}{k_1,\ldots,k_m} \left(\frac{1}{m}\right)^n
\end{aligned}$$

$$\begin{aligned}
&\text{Average queue length}\\
&\quad \equiv \bar{Q}\\
&\quad = \sum_{k_1,\ldots,k_m} \bar{Q}(given\ X_a(t))Pr(k_1,\ldots,k_m)\\
&\quad = \sum_{k_1,\ldots,k_m} \frac{1}{T}\int_0^T Q(t)dt \binom{n}{k_1,\ldots,k_m} \left(\frac{1}{m}\right)^n
\end{aligned}$$

5. EXAMPLES

Following is an example of the computation using the trajectory method. Let the number of sources be two and let each source have uniform peak rate λ_p during the active period. During the idle cycle period the arrival rate is zero and assume that the duty-cycle of each source is 50%, i.e., each average arrival rate λ_a is $\lambda_p/2$. Other assumptions for this example are;

service rate $\mu(t)$ is deterministic and it is equal to λ_p,
period T is composed of 4 intervals,
buffer size is B.

There are three possible phase combinations with respect to two sources, i.e., the phase difference is 0, $T/4$, or $T/2$. Note that a phase difference $T/4$ is equivalent to $3T/4$. Figure 5 shows these combinations and the corresponding trajectories for $X_a(t)$, $X_e(t)$, $X_d(t)$.

The probability of each trajectory is also shown in Figure 5 . In the last case where the phase difference is $T/2$, there is no packet loss nor waiting queue. Packet loss rate $Pr(loss)$ and time-average queue length $\bar{Q}$ are given as follows;

$0 \leq B < \frac{\lambda_a T}{2}$

$$Pr(loss) = \frac{\lambda_a T - B}{2\lambda_a T} \times \frac{1}{4} + \frac{(\lambda_a T/2) - B}{2\lambda_a T} \times \frac{1}{2} = \frac{1}{4} - \frac{3B}{8\lambda_a T}$$

$$\bar{Q} = \frac{B}{2} \times \frac{1}{4} + \frac{4\lambda_a BT - B^2}{8\lambda_a T} \times \frac{1}{2} = \frac{6\lambda_a BT - B^2}{16\lambda_a T}$$

$\frac{\lambda_a T}{2} \leq B < \lambda_a T$

$$Pr(loss) = \frac{\lambda_a T - B}{2\lambda_a T} \times \frac{1}{4} + 0 \times \frac{1}{2} = \frac{\lambda_a T - B}{8\lambda_a T}$$

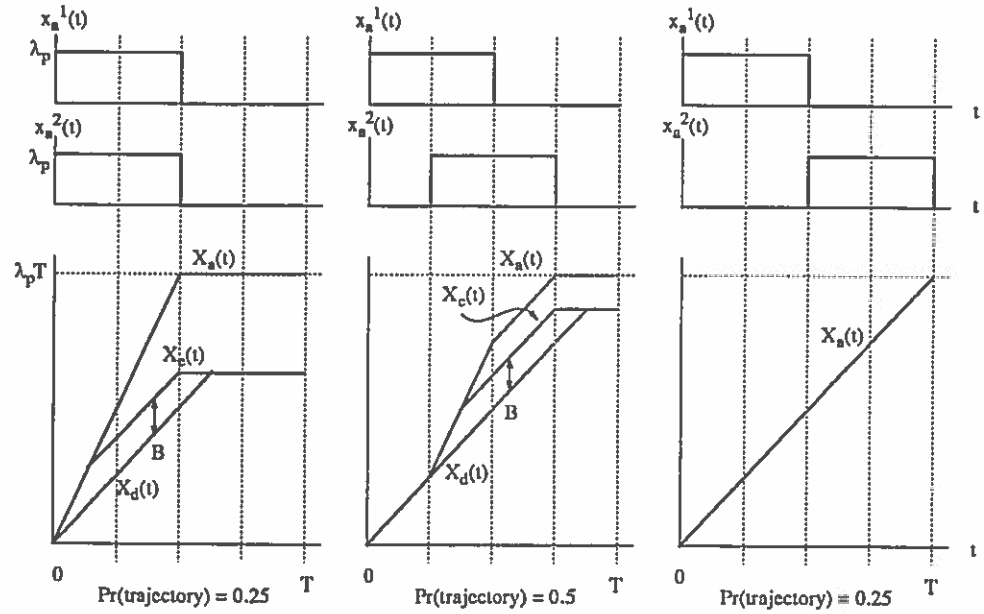

Figure 5: Calculation example

$$\bar{Q} = \frac{B}{2} \times \frac{1}{4} + \frac{\lambda_a T}{4} \times \frac{1}{2} = \frac{\lambda_a T + B}{8}$$

$\lambda_a T \leq B$

$$Pr(loss) = 0$$

$$\bar{Q} = \frac{\lambda_a T}{2} \times \frac{1}{4} + \frac{\lambda_a T}{4} \times \frac{1}{2} = \frac{\lambda_a T}{4}$$

6. CALCULATION ALGORITHM

The basic algorithm used to calculate packet loss and queue length is presented in Table 1. The algorithm calculates packet loss rate, time-average queue length and probability of a given trajectory of arrival $X_a(t)$. The possible trajectories are provided by combining all possible values of k_i, where $i \in [1, m]$. Appendix 1 shows more details with respect to the for-loop which calculates $X_e(t)$, $X_d(t)$ and the fraction of queue length within the given interval i.

The origin of the trajectory is not necessarily given at $t = 0$, because $X_a(t)$ is cyclic and has period T. If the origin is selected at which the waiting queue length is zero, it is sufficient to calculate packet loss and queue length from the origin during period T. To find the origin at which the waiting queue is empty, we use the following theorem;

THEOREM 6.1.
Given;

periodic arrival $x_a(t) > 0$,

$X_a(t) \equiv \int_0^t x_a(\tau)d\tau$ *the trajectory of arrivals,*

which is the number of packets which have arrived up to time t,

$\mu(t) = \mu$ *uniform service rate,*

time-average arrival rate $\bar{X}_a \equiv \frac{X_a(t+T)-X_a(t)}{T} < \mu$,

then;

there is at least a moment at which the waiting queue is empty.

The proof of the theorem is presented in Appendix 2.

The operation "Get origin" in Table 1 finds the point at which the waiting queue is empty. The algorithm implies calculation of the trajectory of $X_e(t)$ and $X_d(t)$ from $X_a(t)$, and that of queue length fraction from these quantities.

Table 1: Calculation algorithm

```
Predicate Packet loss and queue length
   for k_1 ∈ [0, n] ⇒
      for k_2 ∈ [0, n − k_1] ⇒
         ⋮
         for k_{m−1} ∈ [0, n − Σ_{j=1}^{m−2} k_j] ⇒
            k_m := n − Σ_{j=1}^{m−1} k_j;
            Get origin ;
            { Decide point on given trajectory, at which queue length is 0 }
            for i ∈ [1, m] ⇒
               Get X_e(iT/m) ;
               Get X_d(iT/m) ;
               Get (m/T) ∫_{(i−1)T/i}^{iT/m} Q(t)dt ;
               Get Q̄(given X_a(t)) := Q̄ + (m/T) ∫_{(i−1)T/m}^{iT/m} Q(t)dt ;
            rof ;
            Get Pr (loss | given X_a(t)) ;
            Get Pr (k_1, ..., k_m) ;
            Pr(loss) := Pr(loss) + Pr(loss | given X_a(t)) × Pr(k_1, ..., k_m) ;
            Q̄ := Q̄ + Q̄(given X_a(t)) × Pr(k_1, ..., k_m) ;
         rof ;
         ⋮
      rof ;
   rof ;
end ;
```

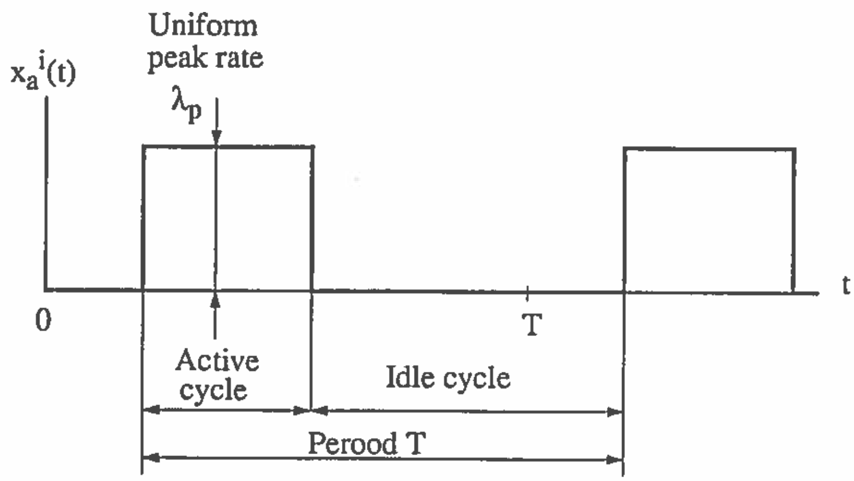

Figure 6: Source model

7. NUMERICAL RESULTS

This section shows some numerical computation results which depict characteristics of periodic bursty traffic. The graphs shown hereafter are obtained on the condition that the period $[0, T]$ is divided into 8 intervals and each source begins to send uniform rate bursts at the start of the interval. The model of source is depicted in Figure 6. Here is a summary of parameters;

n = number of sources,
λ_p = peak rate of each source,
λ_a = average rate of each source,
μ = deterministic service rate,
T = cyclic period of each source, normalized by $1/\mu$,
d = duty cycle of each source = λ_a/λ_p,
ρ = offered load normalized by $\mu = d \times \lambda_p \times n$,
B = buffer size.

Figure 7 shows the relationship between packet loss rate and offered load where duty-cycle of each source is fixed at 1/8. The offered load varies according to variable peak rate of each source. The period of each source is 1024, where the unit time of the period is defined as the reciprocal of the uniform serving rate μ. The buffer size B is zero.

The curves in Figure 7 have uneven points. These discontinuities are caused by the nature of uniform rate burstiness and their periodicity. Here is an intuitive explanation. Figure 8 shows a certain $x_a(t)$ which may happen with a certain probability. As shown in Figure 8, assume that service rate μ is around the step which is multiple of a single source peak rate. The hatched area corresponds to packet loss. As the peak rate of each source increases such that the multiple of the peak rate crosses the service rate, the hatched

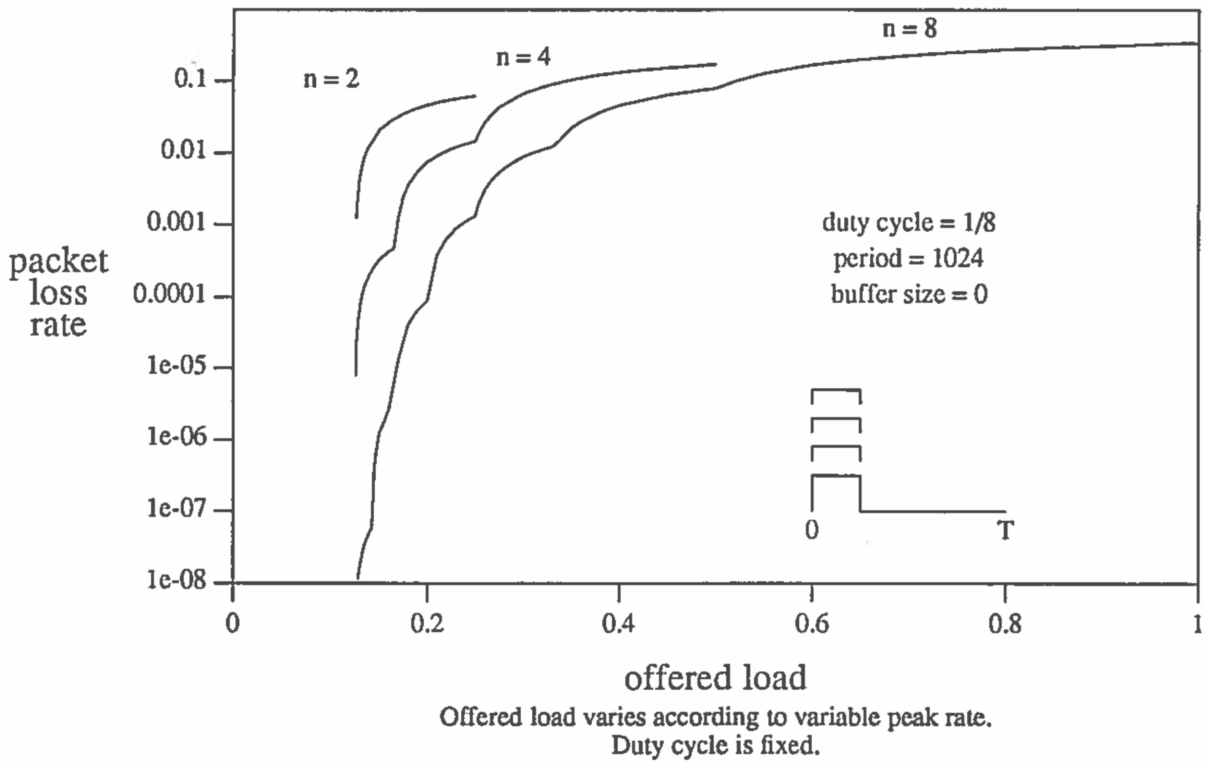

Figure 7: Packet loss rate vs offered load (B=0)

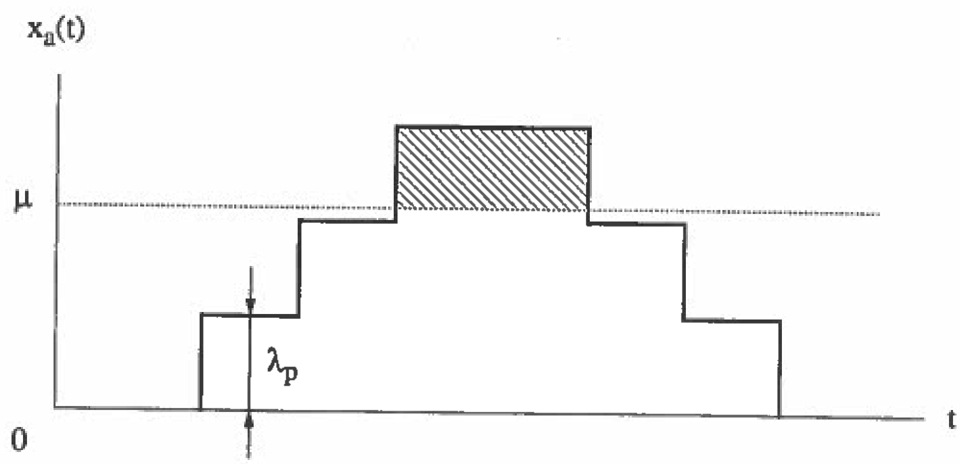

Figure 8: Step configuration of $x_a(t)$

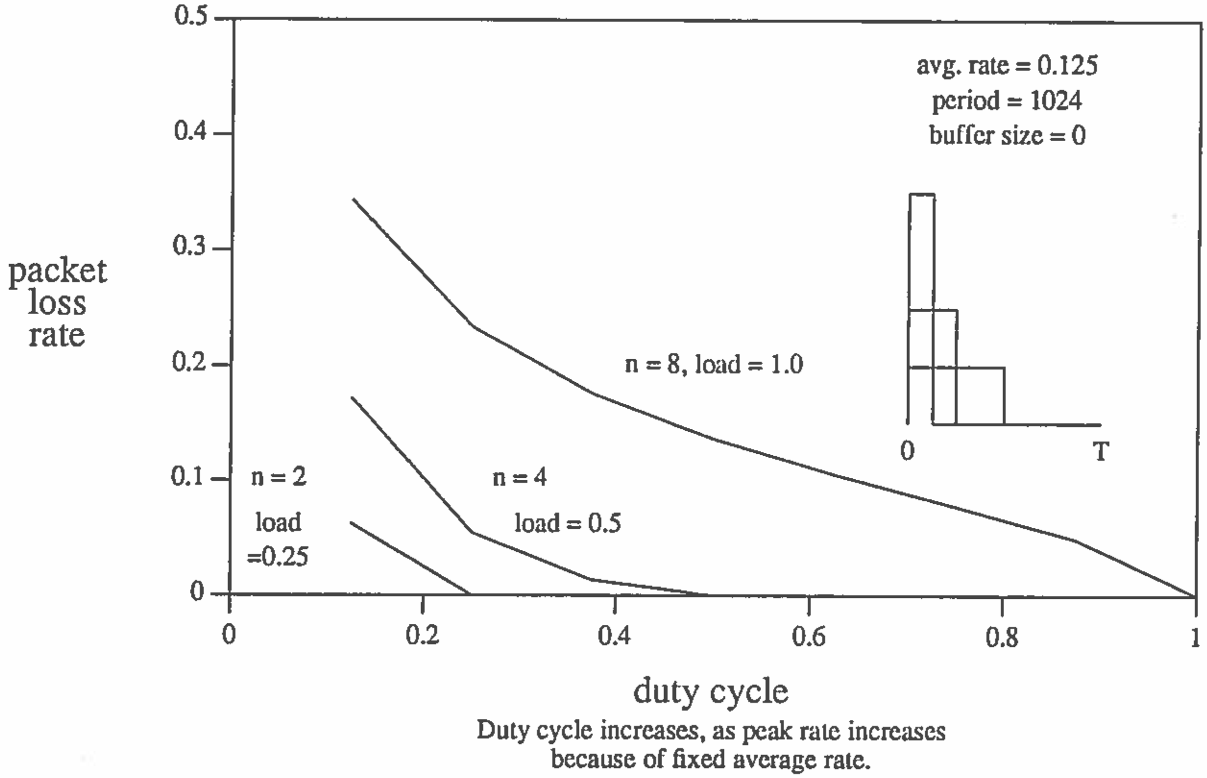

Figure 9: Packet loss rate vs duty cycle

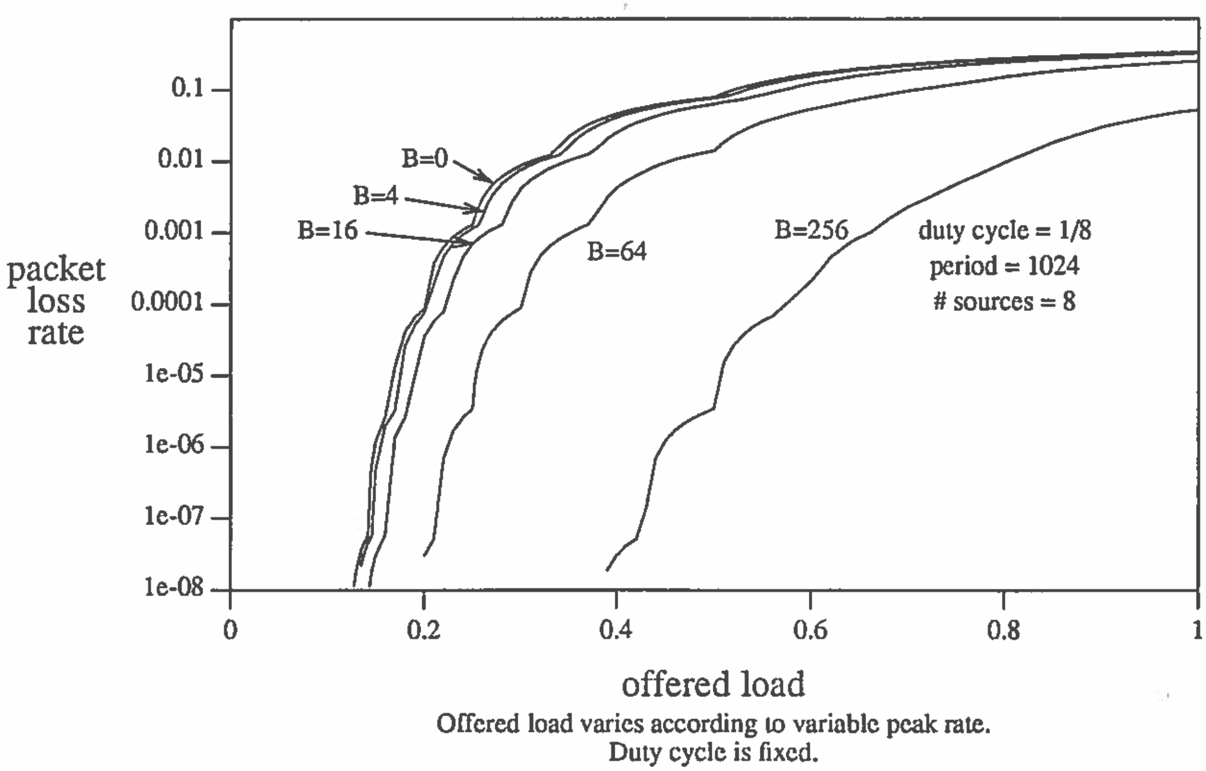

Figure 10: Packet loss rate vs offered load

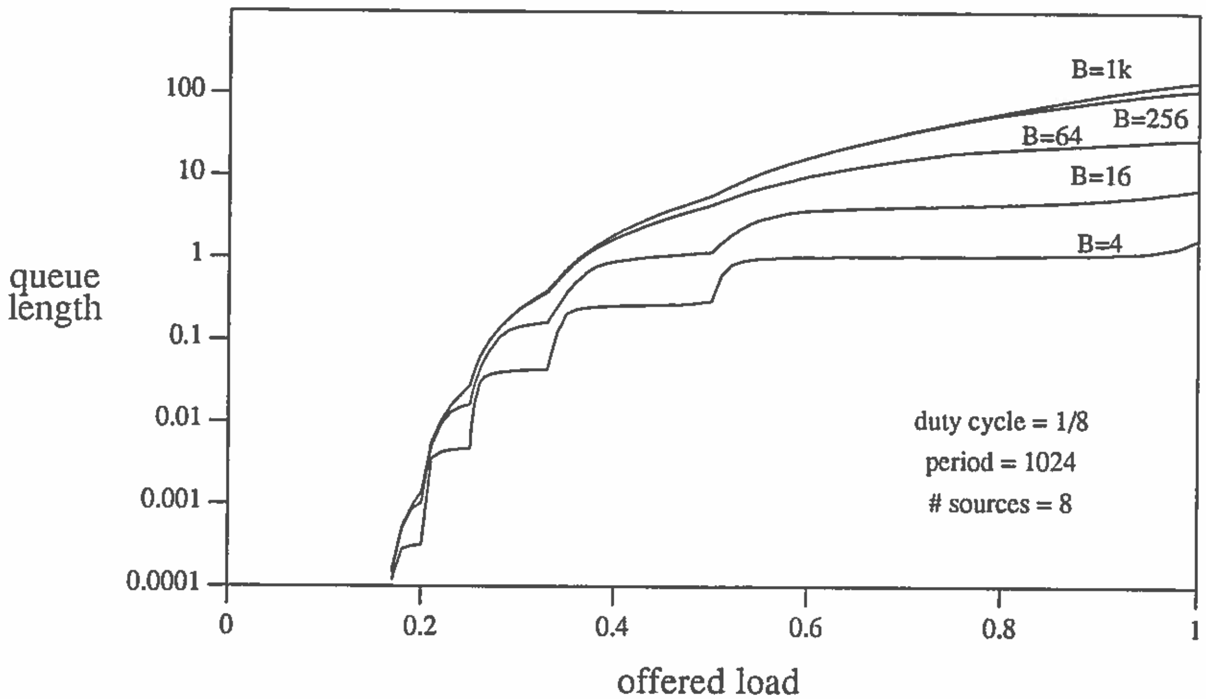

Figure 11: Time average queue length vs offered load

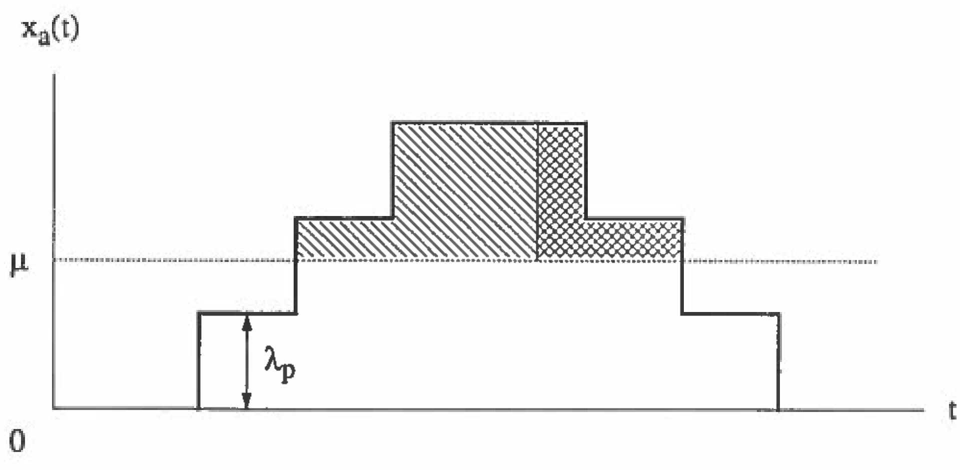

Figure 12: Step configuration of $x_a(t)$

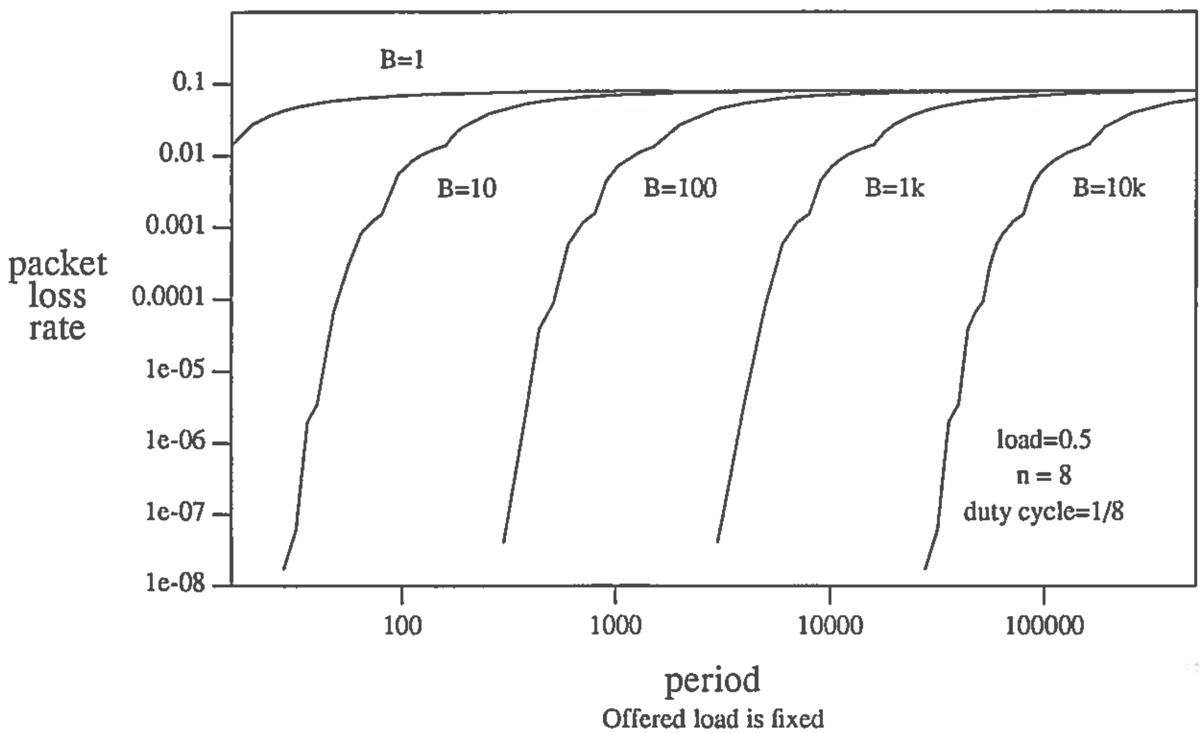

Figure 13: Packet loss rate vs period T

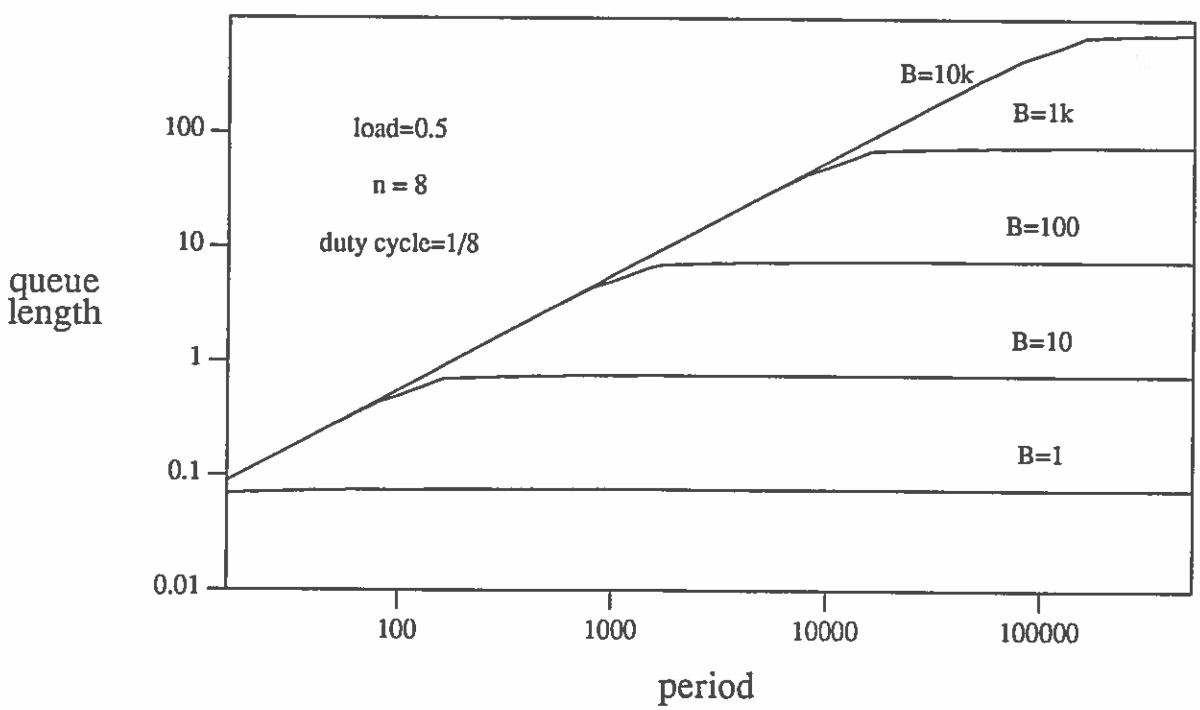

Figure 14: Time average queue length vs period T

area increases unevenly because of the sudden increase of the width of the hatched area. Therefore packet loss rate increases unevenly when peak rate increases across the multiples of the service rate. In the case that the number of sources is 8, the number of peak rates which cause discontinuities is 7, because there may be seven possible steps in trajectories $X_a(t)$ at most. In general, if discontinuity occurs when the offered load is ρ, then $\rho = \mu/i\lambda_p$ for some integer $i \in [2, n]$.

In Figure 7, as $\lambda_p = \mu$, and $n = 8$, the points of discontinuity are $1/2, \ldots, 1/8$. These points exactly match those in the graph.

Figure 9 shows the relation of packet loss rate and duty-cycle of each source. Though the average rate of each source is fixed, the duty-cycle of each source has been changed by varying the peak rate. It is obvious that the smaller the duty-cycle is, the more bursty the source is. The result shows that burstiness leads to packet loss.

In the case that the buffer size B is not zero, Figure 10 and Figure 11 show packet loss rate and queue length, respectively. The curves of queue length in Figure 11 have uneven points and they correspond to $\rho = \mu/i\lambda_p$, where $i = 2, \ldots, n$.

The uneven points in Figure 10 have different characteristics from those in Figure 7. Figure 12 depicts the situation. The hatched area corresponds to queue length, and if it is equal to B, the crosshatched area may appear and corresponds to packet loss. Queue length (hatched area) can vary while there is no packet loss (crosshatched area). This implies that uneven points in the queue length graphs correspond to $\rho = \mu/i\lambda_p$. Once queue length grows up to B, packet loss may occur by the increasing peak rate of each source. Consequently, the discontinuities in the loss rate curves occur at larger values of ρ when B increases.

Figure 13 and Figure 14 show packet loss rate and queue length, respectively, with respect to the period T. The larger the period is, the more bursty the traffic is. Consequently a large period T leads to more packet loss and larger queues.

8. CLOSING REMARKS

So far, basic ideas about the trajectory method have been described, and some examples are shown to analyze periodic bursty traffic.

It is interesting to compare the characteristics of periodic bursty traffic with those of other traffic models. The candidates are the bulk arrival model and the conventional M/M/1 model. In the bulk arrival model, the number of bursts arriving per unit service time is Poisson, while the number of packets in the burst is geometrically distributed. Figure 15 shows packet loss rates obtained using these three models. As is mentioned in the introduction section, the bulk arrival model is too pessimistic, and M/M/1 is too optimistic. It is interesting, though, that packet loss rate of M/M/1 is more than that of periodic bursty arrival, if both buffer size and offered load are small. This is because exceptionally high arrival rates can occur in the M/M/1 model under such conditions. But this phenomenon loses its importance in real applications. Because very high speed networks, such as ATM networks, require and can have large buffers to be tolerant of packet congestion.

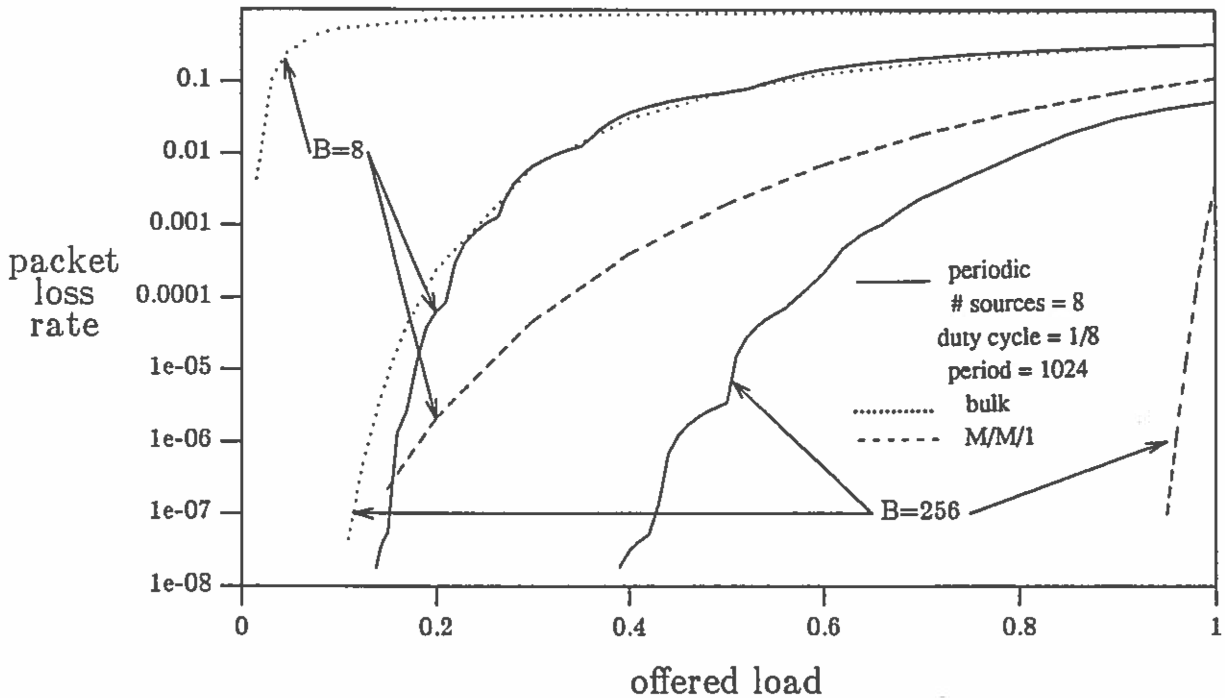

Figure 15: Comparison of packet loss rate

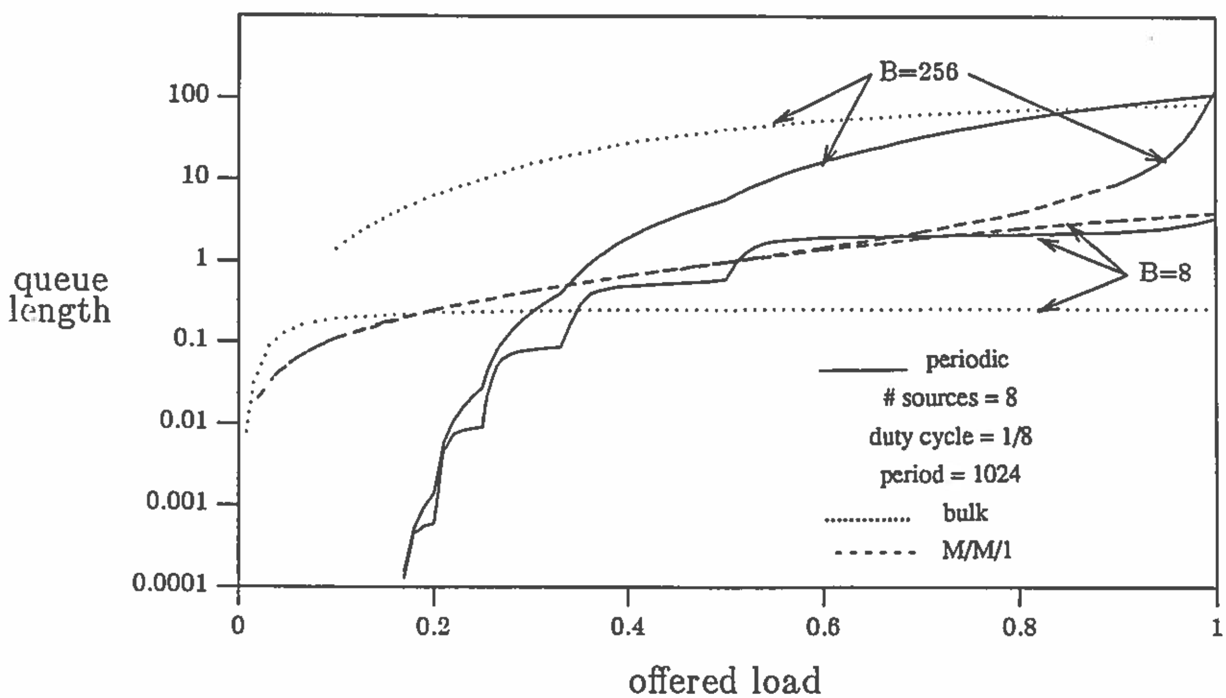

Figure 16: Comparison of queue length

Figure 16 depicts queue length characteristics. The bulk arrival model produces shorter waiting queues than other models do in high offered load region because of extremely high packet loss rate. Comparing periodic bursty traffic model and M/M/1 model in case of large buffer size, periodic bursty traffic produces longer waiting queues than M/M/1 does in high offered load region, and shorter waiting queues in low offered load region. The reason is similar to the previous one for Figure 15, i.e., exceptionally high arrival rates of the M/M/1 model cause the longer queues than the periodic bursty traffic does in the low offered load region, and in the high offered load region the burstiness of the periodic bursty traffic leads to the longer queues. In case of small size buffer, the periodic bursty traffic provides shorter waiting queue because of its higher packet loss rate.

Future works may include;

Comparison of analytical results with simulation.
Enhancement of method for more efficiency and accuracy.
Evaluation of packet loss rate and queue length distributions.
Investigation of closed form.
Other modeling, i.e., shared buffer, cascaded connected buffer, etc.
Evaluation of bursty traffic in networks.
Develop other method to analyze bursty traffic.

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A. APPENDIX 1

The algorithm shown in Table 2 is used to calculate $X_e(t)$, $X_d(t)$ and the fraction of queue length within the interval i.

for $i \in (1, m) \Rightarrow$
if $X_e((i-1)T/m) == X_d((i-1)T/m) \Rightarrow$
if $X_a(iT/m) - X_a((i-1)T/m) < \mu T/m \Rightarrow$
$X_e(iT/m) := X_e((i-1)T/m) + X_a(iT/m) - X_a((i-1)T/m)$;
$X_d(iT/m) := X_e(iT/m)$;
$Q(iT/m) := 0$;
$qlt(iT/m) := 0$;
{ $qlt(t) = \int_{t-1}^{t} Q(t)dt$ }
fi ;
else if $\mu T/m < X_a(iT/m) - X_a((i-1)T/m) < \mu T/m + B \Rightarrow$
$X_e(iT/m) := X_e((i-1)T/m) + X_a(iT/m) - X_a((i-1)T/m)$;
$X_d(iT/m) := X_d((i-1)T/m) + \mu T/m$;
$Q(iT/m) := X_e(iT/m) - X_d(iT/m)$;
$qlt(iT/m) := \frac{T}{2m} Q(iT/m)$;
fi ;
else if $\mu T/m + B < X_a(iT/m) - X_a((i-1)T/m) \Rightarrow$
$X_e(iT/m) := X_e((i-1)T/m) + \mu T/m + B$;
$X_d(iT/m) := X_d((i-1)T/m) + \mu T/m$;
$Q(iT/m) := B$;
$qlt(iT/m) := \frac{T}{2m} B$;
fi ;
fi ;

if $X_e((i-1)T/m) > X_d((i-1)T/m) \Rightarrow$
if $X_a(iT/m) - X_a((i-1)T/m) < \mu T/m - Q((i-1)T/m) \Rightarrow$
$X_e(iT/m) := X_e((i-1)T/m) + X_a(iT/m) - X_a((i-1)T/m)$;
$X_d(iT/m) := X_e(iT/m)$;
$Q(iT/m) := 0$;
$qlt(iT/m) := \frac{1}{2} Q((i-1)T/m) \times \frac{Q(iT/m)T/m}{\mu T/m - X_a(iT/m) X_a((i-1)T/m)}$;
fi ;
else if $\mu T/m - Q((i-1)T/m) < X_a(iT/m) - X_a((i-1)T/m)$
$< \mu T/m - Q((i-1)T/m) + B \Rightarrow$
$X_e(iT/m) := X_e((i-1)T/m) + X_a(iT/m) - X_a((i-1)T/m)$;
$X_d(iT/m) := X_d((i-1)T/m) + \mu T/m$;
$Q(iT/m) := X_e(iT/m) - X_d(iT/m)$;
$qlt(iT/m) := \frac{T}{2m}(Q((i-1)T/m) + Q(iT/m))$;
fi ;
else if $\mu T/m - Q((i-1)T/m) + B < X_a(iT/m) - X_a((i-t)T/m) \Rightarrow$
$X_e(iT/m) := X_d((i-1)T/m) + \mu T/m + B$;
$X_d(iT/m) := X_d((i-1)T/m) + \mu T/m$;
$Q(iT/m) := B$;
$qlt(iT/m) := B\mu T/m - \frac{1}{2} \times \frac{(B - Q((i-1)T/m))^2}{X_a(iT/m) - X_a((i-1)T/m) - \mu T/m}$;
fi ;
fi ;
$\bar{Q} := \bar{Q} + \frac{1}{T} qlt(iT/m)$;
{ $\bar{Q}$ is time average queue length }
rof ;

Table 2: Precise algorithm

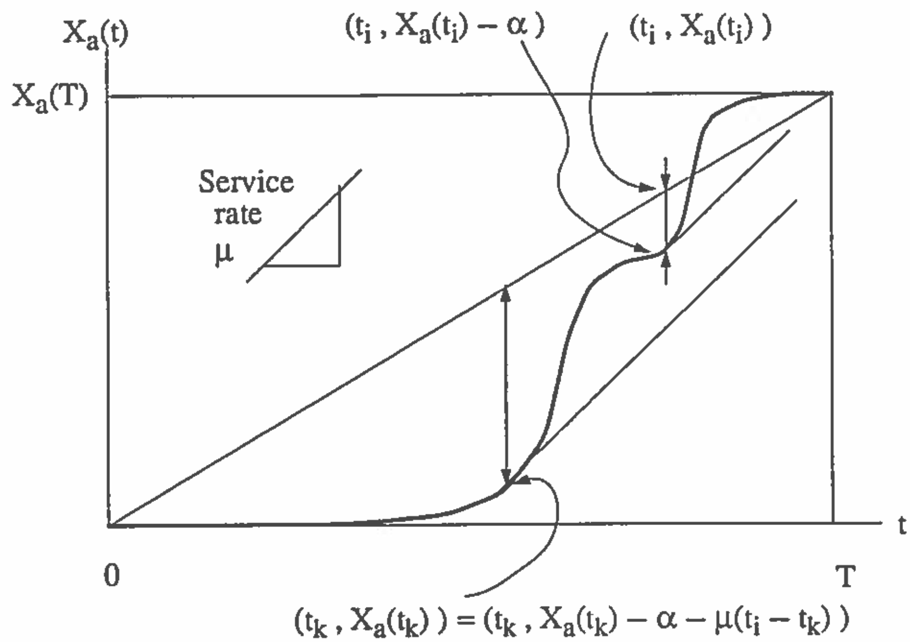

Figure 17: Tangent lines

B. APPENDIX 2

Theorem 1

Given;

periodic arrival $x_a(t) > 0$,

$X_a(t) \equiv \int_0^t x_a(\tau)d\tau$ the trajectory of arrivals,

which is the number of arrivals which have arrived up to time t,

$\mu(t) = \mu$ uniform service rate,

time-average arrival rate $\bar{X}_a \equiv \frac{X_a(t+T)-X_a(t)}{T} < \mu$,

then;

there is at least a moment at which waiting queue is zero length in the buffer with size B.

Proof ;

Case1: If $B = 0$, no waiting queue at all time.

Case2: If $B > 0$ and $x_a(t) < \mu$ at all time, an arrival cannot wait the next arrival in the buffer, because the server catches it before the next arrival enters the buffer.

Case3: If $b > 0$ and $\frac{d}{dt}X_a(t) = x_a(t) = \mu$ for some t, $X_a(t)$ contacts tangent lines whose slope is μ. Let the point $(t_i, X_a(t_i))$ be the contact point on the trajectory $X_a(t)$ with a μ-slope tangent line, where $i \in [1, m]$ and m is the number of contact points on $X_a(t)$ during the period T.

If a certain contact point $(t_i, X_a(t_i))$ maximizes $\bar{X}_a \times t_i - X_a(t_i)$, where $\bar{X}_a$ is average arrival rate, compared with $\bar{X}_a \times t_j - X_a(t_j)$ for any contact point $(t_j, \ X_a(t_j))$ other than

$(t_i, \quad X_a(t_i))$, i.e.,

$$\bar{X}_a \times t_i - X_a(t_i) \geq \bar{X}_a \times t_j - X_a(t_j), \tag{1}$$

then the point $(t_i, \quad X_a(t_i))$ on the trajectory gives zero queue length, i.e., $Q(t_i) = 0$.
Because:
Figure 17 shows the situation. If $Q(t_i) \neq 0$, i.e., the server has not caught up with the arrival at time t_i, there must be at least another tangent line which passes through the point $(t_i, \quad X_a(t_i) - \alpha)$, where $\alpha > 0$. And the line contacts the trajectory $X_a(t)$ at the point $(t_k, \quad X_a(t_k))$, where $t_k < t_i$.
The line must satisfy that;

$$X_a(t_k) = X_a(t_i) - \alpha - \mu(t_i - t_k) \tag{2}$$

Substituting Equation 2 to Inequality 1, we obtain;

$$t_i(\mu - \bar{X}_a) \leq t_k(\mu - \bar{X}_a) - \alpha \tag{3}$$

The Inequality 3 does not hold, because;

$$t_i > t_k > 0, \quad \mu > \bar{X}_a, \quad \text{and} \quad \alpha > 0.$$

Therefore, $Q(t_i) = 0$.
Q.E.D.

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