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Essays on Inflation Targeting and Credit Markets

by

Jundong Zhang

A dissertation presented to the
Graduate School of Arts & Sciences
of Washington University in
partial fulfillment of the
requirements for the degree
of Doctor of Philosophy

May 2016

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Contents

List of Figures	v
List of Tables	vi
Acknowledgments	vii
Abstract	ix
1 Inflation Targeting with Incomplete Markets	1
1.1 Introduction	1
1.1.1 Inflation targeting	1
1.1.2 The use of money and consumer credit	2
1.1.3 Inflation and credit markets	4
1.1.4 Inflation and distributional effect	6
1.1.5 Main results	7
1.1.6 Recent related literature	10
1.2 A Simple Model	15
1.3 The General Model	20
1.3.1 Households	20
1.3.1.1 Cash agents	21
1.3.1.2 Credit agents	22
1.3.2 Monetary policy	24

1.3.3	Recursive equilibrium	25
1.3.4	Welfare loss of inflation	27
1.3.5	Characterizations	28
1.4	Calibration	32
1.4.1	Parameterization	32
1.5	Benchmark Monetary Policy: Helicopter Drops	33
1.5.1	Implications for credit markets	35
1.5.2	Welfare implications	38
1.5.2.1	The optimal inflation target	38
1.5.2.2	The decomposition of welfare loss	40
1.5.3	Distributional implications	45
1.5.4	The role of market incompleteness	47
1.5.5	A weaker default punishment with re-entry	49
1.6	Sensitivity Analysis	52
1.7	Alternative Monetary Policy: Open Market Operations	56
1.7.1	Implications for credit markets	58
1.7.2	Welfare implications	60
1.8	Conclusions	61
2	Inflation Targeting and Global Imbalances	65
2.1	Introduction	65
2.1.1	Motivation and recent related literature	65
2.1.2	Main results	67
2.2	Model Environments and Channels	68
2.2.1	Environments	68
2.2.2	Monetary policy	71
2.2.3	General equilibrium	72
2.2.4	Core channels	74

2.3	Quantitative Results	75
2.3.1	Benchmark	75
2.3.2	Asymmetry in the U.S. and emerging markets	78
2.4	Conclusion	79
	Bibliography	79
	A Appendix for Chapter 1	85
A.1	Data descriptions	85
A.2	Proof of Proposition 1	86
A.3	Proof of Proposition 2	86
A.4	Computation	87

List of Figures

1.1.1 Inflation targets	3
1.1.2 Percent of households with credit card balances	5
1.2.1 Gross real interest rate	20
1.5.1 The impacts of inflation targets on credit markets	36
1.5.2 The impact of inflation targets on aggregate money demand and inflation tax collection	37
1.5.3 The optimal inflation target and welfare loss	39
1.5.4 The distribution of welfare loss of -1.2% inflation compared to 3%	43
1.5.5 Impact of inflation targets on wealth inequality	46
1.5.6 Impact of inflation targets on the wealth level	47
1.5.7 Debt limit: benchmark vs state-contingent bonds model	50
1.7.1 The impacts of inflation on credit market: OMO 2	60
2.1.1 The U.S. external debt and portfolio equity plus FDI	66

List of Tables

1.5.1 Benchmark model vs data	34
1.5.2 The decomposition of welfare loss of -1.2% inflation compared to 3%	43
1.5.3 Benchmark vs re-entry	51
1.6.1 Quantitative effect of each parameter	55
1.6.2 Qualitative effects of each parameter	56
1.6.3 Sensitivity analysis	57
1.7.1 Open market operations vs helicopter drop	59
1.7.2 Welfare loss of monetary policy change (%)	62
2.3.1 Calibration	77
2.3.2 Benchmark results	78
2.3.3 Larger income volatility in emerging markets	79

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Dedicated to my parents and Bicheng

ABSTRACT OF THE DISSERTATION

Essays on Inflation Targeting and Credit Markets by

Jundong Zhang

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Costas Azariadis, Chair

The first chapter of the dissertation studies quantitatively and systematically the impacts of a wide range of inflation targets on credit markets, on social welfare and on wealth inequality. To this end, I develop a model featuring market segmentation, market incompleteness and limited commitment to financial contracts. Under incomplete markets, moderate inflation alleviates frictions in credit markets and thus improves social welfare. After calibrating the model to recent U.S. data, I report four major findings. First, as the inflation target increases, endogenous debt limits follow a humped shape with a flat tail. This coincides with the empirical relationship between inflation and credit market activities. Second, social welfare also takes on a humped shape followed by a flat tail, which leads to an optimal inflation rate of 3%. The sizable welfare loss (0.61%) at the Friedman rule inflation and the negligible welfare loss (0.006%) at 2% inflation explain in some sense why central banks in some leading developed countries maintain their inflation targets at 2-3%. Third, the optimal inflation with a complete set of financial assets is lower than that with an incomplete set of financial assets. This result is consistent with the fact that developed countries tend to keep lower inflation targets than developing countries. Fourth, the calibrated model generates a well matched Gini coefficient of wealth at 0.72 and implies that wealth inequality increases slowly with inflation rates.

The second chapter of the dissertation studies how inflation targeting affects the U.S. holdings of net foreign assets and explains two facts: the U.S. negative net position in bonds and positive net position in portfolio equity and FDI. I extend the model in the first chapter

to a two-country open economy model consisting of the U.S. and emerging markets (EM). Facing idiosyncratic income risks, credit agents in each country hold a portfolio of risk-free bonds and risky productive assets with idiosyncratic investment shocks. Financial integration allows credit agents to trade both kinds of assets globally. The only difference between the two countries is that the U.S. maintains a lower inflation target than EM. Calibrated to recent U.S. data, the model generates a higher debt limit for the U.S., where agents can borrow more than those in EM. With zero net bond supply in the world, the U.S. borrows from EM. This explains the U.S. negative net position of bonds. On the other hand, U.S. credit agents enjoy better risk sharing due to a larger debt limit. Therefore, the U.S. credit agents' consumption is less volatile than that of their foreign counterparts. This leads to a smaller covariance between the return from risky equity and tomorrow's consumption, and thus the U.S. credit agents require a lower risk premium on risky equity. As a result, they value those risky assets more highly and in effect buy them from EM.

Chapter 1

Inflation Targeting with Incomplete Markets

1.1 Introduction

1.1.1 Inflation targeting

Inflation targeting has become the cornerstone of monetary policy for central banks in many leading developed countries as well as in some developing countries. For example, the Bank of England has maintained an inflation target of 2 percent since 2003; the Reserve Bank of Australia has kept its own target of 2 to 3 percent since 1993; the Central Bank of Brazil has maintained its inflation target at 4.5 percent with 1.5 percent tolerance intervals since 2005.

Inflation targeting has some obvious advantages, as mentioned in Bernanke and Mishkin (1997): first, an explicit number for inflation target improves accountability of central banks and thus helps reduce the likelihood of time-inconsistency. Second, a stable relationship between money growth and inflation is not essential to the success of inflation targeting which uses all available information to achieve a desirable outcome. Third, inflation targeting makes central banks focus on what they can do over the long term, i.e., maintain inflation, instead

of focusing on short-term improvements in the cyclical behavior of the macroeconomy.

Central banks that use inflation targeting tend to set the numerical target at a small positive number. For example, the Board of Governors of the Federal Reserve Bank has been conducting monetary policy to help maintain an inflation rate of 2 percent over the medium to long term, which differs a great deal from the Friedman rule of zero nominal yields. Is 2 percent a good inflation target? If so, why? From Figure 1.1.1, we also find that developed countries tend to set lower inflation targets than developing countries. Is this phenomenon just a coincidence or is it theoretically justifiable?

1.1.2 The use of money and consumer credit

After the Danish government proposed in May 2015 that some stores could stop taking cash starting from 2016 in order to facilitate the shift to credit cards, debit cards and other payment systems, one might be ready to predict the death of cash. After all, with the increasing use of credit and debt cards for transactions and the rapid growth of electronic commerce, would it not be realistic to assume that U.S. consumers will stop using any physical payments such as cash or check in the future?

Nothing could be further from the truth. Evidence from the Diary of Consumer Payment Choice (DCPC), conducted in October 2012, suggests otherwise. Cash for the U.S. consumers is still a staple payment method in many aspects (number / share of transactions, spending category, consumer demographics and payments preference). 43 percent of consumers report that a debit card is their payment instrument of choice. The number is 22 percent for credit cards, 30 percent for cash and only 3 percent for checks.

As a matter of fact, market segmentation seems to prevail in the U.S. payments system. A large literature talks about the implications of market segmentation, in which only a fraction of people are active in or have access to financial markets. For example, Vissing-Jorgensen (2002) presents empirical evidence that limited asset market participation is important for estimating the elasticity of intertemporal substitution. Alvarez et al. (2002) study the

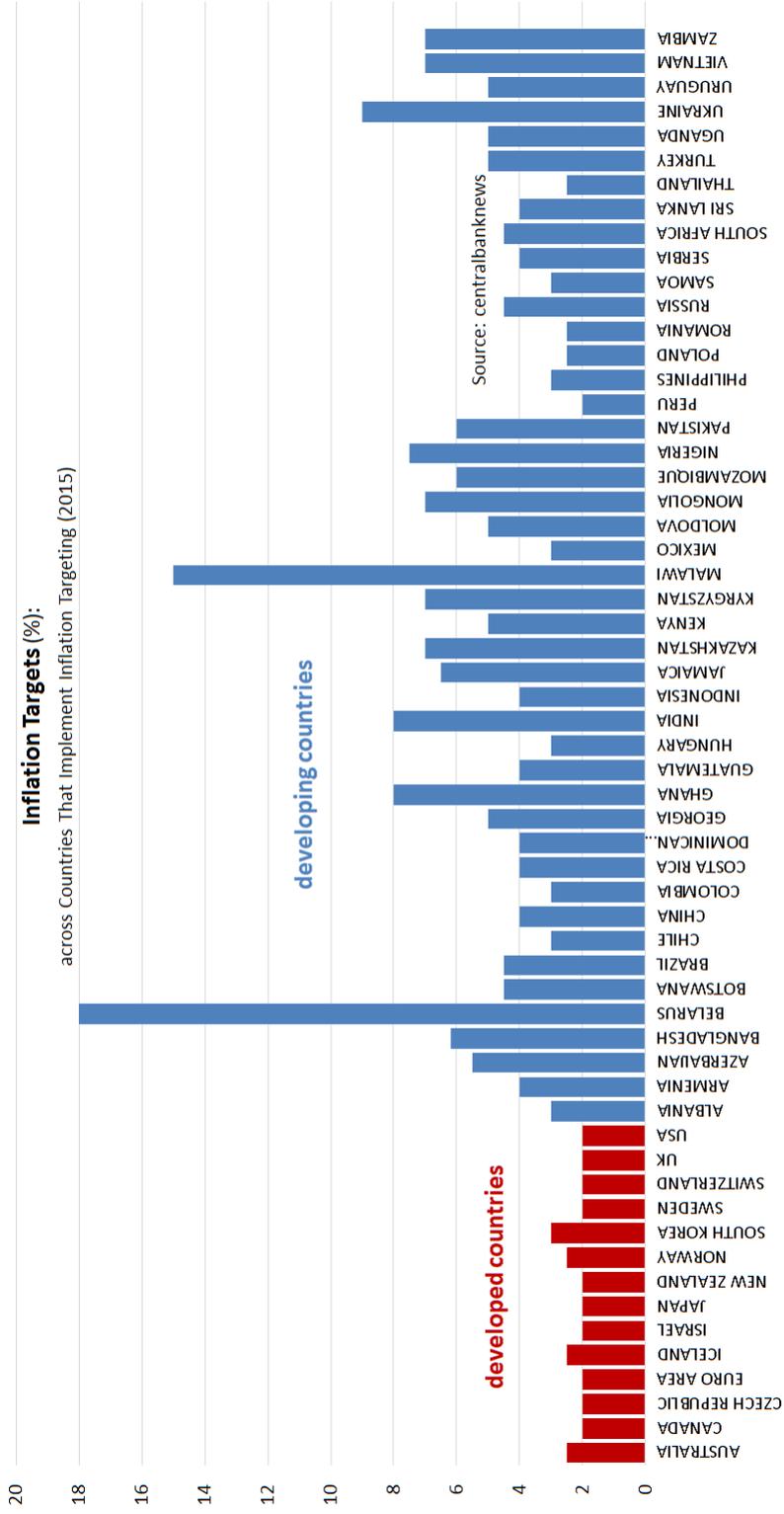


Figure 1.1.1: Inflation targets

effect of money injections on interest rates and exchange rates with endogenously segmented markets. Chien et al. (2011) study the impact of heterogeneous trading technologies on asset pricing and wealth distribution.

What do we know about the fraction of households who cannot or choose not to access to financial markets in the U.S.? Mulligan and Sala-i-Martin (2000) use the Survey of Consumer Finances (SCF) and show that 59 percent of U.S. households do not hold any interest-bearing assets. These households can be viewed as ones who only use cash or checks for transactions. I call them non-participants. In addition, by looking into the households with credit card balances in the SCF from 1989 to 2013, I find an average fraction of around 43 percent for credit users as shown in Figure 1.1.2.

Assume these are the households who have access to financial markets. The average percent of non-participants is 57 percent by this measurement, very close to the number in Mulligan and Sala-i-Martin (2000). Apart from the data from the SCF, the 2013 FDIC annual report shows that 20 percent of the U.S. households are unbanked (they don't have a bank account) and 7.7 percent are underbanked (they have one bank account but have used one of the alternative financial systems other than a bank in the past 12 months). If I define the unbanked or underbanked households as non-participants, the percentage of these households is smaller (27.7 percent) than that from the SCF.

1.1.3 Inflation and credit markets

To understand how inflation affects the behavior of households who participate in or abstain from credit markets, we need to go into links that connect inflation with credit market performance. The relationship between inflation and the performance of credit markets remains one of the most celebrated issues in modern macroeconomics. A large literature addresses this issue, both theoretically and empirically. For example, Huybens and Smith (1999) develops a model to show a negative relationship between inflation and financial market activities,

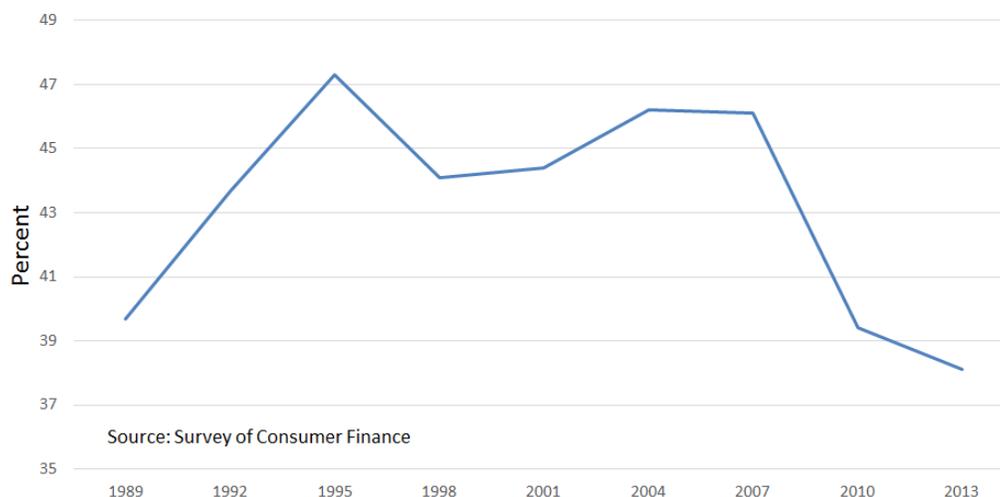


Figure 1.1.2: Percent of households with credit card balances

while Antinolfi et al. (2014) present a model to predict inflation can relax borrowing constraint and thus facilitate financial market activities. Boyd et al. (2001) empirically assess the predictions made by Huybens and Smith (1999) and find a significantly negative relationship between inflation rates and the performance of financial sectors. They conclude that inflation has a non-linear effect on financial market performance. When inflation is from moderate to high, the activities of the credit market contract as inflation goes up. When inflation reaches some threshold value (15 percent), the impact of inflation on the credit market diminishes rapidly. The paper doesn't estimate the effect of inflation on the credit market when inflation is very low.¹

In order to get a sense of the relationship between U.S. inflation and credit market activities, I run a piecewise linear regression with the breakpoint at 1.9 percent inflation,² by using the data of the U.S. consumer credit / income³ and the U.S. inflation rates, from 1947Q1 to 2014Q4. In addition, According to Sherman (2009), “*In 1978, the national landscape of usury regulation changed fundamentally with the Supreme Court's decision in Marquette*

¹They use cross-country data. The sample mean of inflation is 13.8%. For the first quartile, the mean of inflation is as high as 5.3%.

²I used the method of nonlinear least squares and obtained the value of the breakpoint that yields the best fitting statistical model.

³This paper focuses on unsecured debt and consumer credit is a dominant part of unsecured debt.

National Bank v. First of Omaha Service Corp. ... The competitive wave of deregulation was hugely beneficial to the credit card industry.” It implies that the act in 1978 during the financial deregulation process started to boost credit card industry. Based on this fact, I added a dummy variable of financial deregulation ($deregulation = 1_{(t \geq 1978)}$) to the piecewise linear regression as follows:

$$consumer\ credit/income = \underset{(0.000)}{0.099} + \underset{(0.000)}{0.762} \times \pi - \underset{(0.000)}{1.096} \times (\pi - 0.019) \times 1_{(\pi \geq 0.019)} + \underset{(0.000)}{0.043} \times deregulation.$$

(·) indicates the p-value at 0.05 significance level. From this regression, we can see that when inflation is lower than 1.9 percent, there is a significantly positive relationship between inflation and consumer credit / income; when inflation is higher than 1.9 percent, there is a significantly negative relationship. The aim here is not to find the specific value, but to show from data that there exists such a positive value which can serve as a breakpoint, and that there are reversed tendencies over the ranges of inflation below and above this point. By combining the empirical findings above, I can claim that the credit market activity in terms of consumer credit shows a humped shape followed by a flat tail w.r.t. inflation. To facilitate the following analysis, I define three zones of inflation in terms of the credit market performance: “credit expansion zone” at low inflation, “credit contraction zone” at moderate to high inflation and “inactive zone of inflation” at very high inflation.

1.1.4 Inflation and distributional effect

Monetary policy is not intended to benefit some group of people at the cost of other groups by redistributing wealth / income among them. It is, however, hard for central banks to ignore the distributional effect of monetary policy. Integrating household inequality issue into the policy making process may help us better understand the dynamics of the macroeconomy and implement monetary policy as well. Central bankers from the European Central Bank

and the Bank of England acknowledge that loose monetary policy may raise inequality due to higher asset prices. Coibion et al. (2012) outline five potential channels through which monetary policy might affect inequality. Although it is still very challenging to disentangle and quantify these channels empirically, distributional concerns are becoming an important factor for judging the soundness of monetary policy.

Most of the DSGE literature studies the effect of monetary policy on the macroeconomy in a representative-agent setting, ignoring the distributional implications across households. By introducing heterogeneity among households, I can explore the potential distributional effect of monetary policy. With ex-ante heterogeneity in trading technology caused by market segmentation and ex-post heterogeneity in income realizations, monetary policy causes income redistribution among households. Romer and Romer (1999) find links between inflation and the well-being of the poor. Higher inflation improves the welfare of the poor in the short-run while lower inflation enhances it in the long-run. Doepke and Schneider (2006) estimate the effect of U.S. inflation on the value of nominal assets. The change in inflation causes wealth redistribution across different sectors and groups and thus generates winners and losers in wealth positions.

1.1.5 Main results

To explain the central bank's choice of an inflation target, I build a dynamic stochastic general equilibrium model in an environment of market segmentation. The model features two frictions (asset market incompleteness and limited commitment on debts) which monetary policy can alleviate or worsen. Through this model, I can analyze quantitatively the effect of inflation targeting on the credit market, on social welfare, and on wealth inequality.

The paper studies an endowment economy with a constant aggregate income, populated by a continuum of agents with measure one who receive idiosyncratic income shocks each period over an infinite horizon. Those agents are exogenously divided into two groups: cash agents and credit agents. For cash agents, money is the only asset they can hold and plays

two roles. First, they need it to purchase consumption goods, that is, they are subject to a cash-in-advance constraint. Second, money serves as a store of value with which cash agents smooth consumption against idiosyncratic income shocks, as in Bewley (1980). The group of cash agents is made up of households who do not or cannot participate in bond market, because they defaulted in the past and thus have been excluded from the credit market⁴ or they choose to opt out due to a prohibitive entry cost.⁵

Credit agents, on the other hand, can hold state-noncontingent bonds in addition to money to hedge against income shocks. Since money is a dominated asset,⁶ credit agents will only hold bonds in equilibrium to take advantage of the higher rate of return on bonds. By issuing bonds, credit agents can borrow but cannot commit to repay. Debts are unsecured and borrowers may repudiate their debts. Defaulting credit agents are excluded from the credit market forever and reduced to cash agents with the minimum money holding. No credit agents default in equilibrium due to a participation constraint.⁷ Therefore, in order for credit agents to remain solvent, the continuation value of solvency needs to be no less than the continuation value of default. Limited commitment on debt gives rise to endogenous debt limits within which debts are always repaid. This endogenously generated debt limit plays a central role in changing the welfare of credit agents and thus social welfare because credit agents rely a lot on credit markets for risk sharing.

By studying stationary equilibria across a wide range of inflation targets,⁸ I obtain results on how inflation influences the credit market, social welfare and wealth inequality over a long term. The major innovation in this paper is the refinement and calibration of an empirically supported transmission channel that leads from inflation to the credit market (inflation-

⁴For example, households who have poor credit records or low credit scores have difficulties applying for credit cards.

⁵Annual fees for credit cards are one kind of entry cost.

⁶I can ensure this statement as long as inflation is no less than the Friedman rule inflation.

⁷This is just a simplifying assumption. In general, default may be desirable when financial markets are incomplete. See Dubey et al (2005). On the other hand, a complete market generates much higher debt limits than the data. Even very weak punishment cannot completely fix this problem.

⁸This paper focuses on long-run effect of expected inflation targets. I assume once the central bank announces its inflation target, the economy will stay at that inflation rate forever. In this regard, there is a range of economies with different values of inflation for us to do cross-economy comparisons.

credit channel), as suggested in a theoretical contribution by Antinolfi et al. (2014). The debt limits implied by this channel are consistent with the three zones uncovered by empirical work connecting inflation with credit market performance.

This paper develops a modified cash-in-advance (CIA) constraint with which cash agents can obtain additional money by selling their current-period endowments. As I mentioned earlier, the default value is determined by the cash agents' value with the minimum money holding, or the bottom cash agents' value, which depends on two competing effects: inflation tax and the distributive channel leading wealth from the top to bottom cash agents through the lump-sum transfer. At low inflation, the former effect dominates due to large money holding. Therefore, the default value decreases with inflation when inflation is low. The story will be different when inflation is moderate to high. Due to the modified Cash-in-advance constraint, money demand can drop to zero as opposed to a large positive lower bound with a standard Cash-in-advance constraint. With money demand dropping to zero when inflation rises from moderate to high values, the negative effect of inflation tax diminishes accordingly and thus the positive distributive effect starts to dominate. Therefore, the default value starts to increase with inflation when inflation is moderate to high. As a result, the U-shaped default value generates hump-shaped debt limits.

When inflation rises from a low rate, the value of default drops, which discourages borrowers from renegeing on their debts. This raises debt limits, deepens the credit market and supports the “credit expansion zone” found in the empirical evidence. When inflation is moderate, higher inflation makes default more attractive and thus borrowers have more incentives to renege, which lowers the debt limit, as described by the “credit contraction zone”. When inflation is very large, cash agents won't hold money any more and inflation targeting loses all impact on the economy. This is what I call the “inactive inflation zone”. To sum up, debt limits implied by the calibrated model show a humped shape followed by a flat tail w.r.t. inflation.

Once we understand the relation between debt limits and inflation, we can study the

effect of inflation on the welfare of credit agents and thus on aggregate social welfare. The welfare of credit agents depends largely on the size of debt limits. On the one hand, larger debt limits raise real interest rates and improve the welfare of creditors who earn higher interest payments from bonds. On the other hand, larger debt limits allow debtors to smooth consumption better against adverse income risks. Therefore, larger debt limits make all credit agents better off. According to the analysis above, these limits show a humped shape w.r.t inflation, so does the welfare of credit agents. I weigh each group by their relative population, which makes social welfare simply be the population weighted average of the welfare of the two groups. In the “credit expansion zone”, cash agents lose due to increasing inflation tax while credit agents gain due to looser credit markets as inflation rises. The combination of the two competing welfare effects generates an inflation rate in this zone that maximizes social welfare.

To sum up, the monetary policy of inflation targeting can alleviate frictions in credit markets to improve credit market conditions and thus social welfare only by inflating the economy moderately. How much the monetary policy inflates the economy depends on the degree of frictions in credit markets, which arise from an incomplete set of financial assets and limited commitment to financial contracts in this paper. Azariadis et al. (2015) also study what monetary policy can do to alleviate or overcome frictions in credit markets. They conclude that the monetary policy of nominal GDP targeting at the zero lower bound can overcome the frictions caused by non-state contingent nominal contracts by credibly promising to increase the price level.

1.1.6 Recent related literature

The Friedman rule tends to be infeasible in economies with frictions caused by private information, lack of commitment, search, market incompleteness or borrowing constraints. Moderate inflation in this environment of frictions helps relax incentive, participation or borrowing constraints, thus improving social welfare (see Levine, 1990 and Smith, 2002). Molico

(2006) uses a random matching model and shows that small amounts inflation improve social welfare by decreasing the dispersion of wealth and prices when shocks are persistent. My paper is mainly in line with the literature in which frictions result from limited commitment or market incompleteness and households receive idiosyncratic income, liquidity or preference shocks each period.

In seminal work, Bewley (1980) introduces precautionary money demand which helps people self-insure against idiosyncratic shocks. The Bewley model has been studied recently in the literature about monetary policy. To my best knowledge, Imrohorglu (1992) is the first paper to assess quantitatively the welfare cost of inflation at 5% and 10% rates as opposed to zero inflation in an economy where agents hold money to smooth consumption against idiosyncratic income shocks. Wen (2015) develops an analytically tractable Bewley model with production to study again the welfare cost of inflation. He finds that agents in his model are willing to forgo consumption by 3 - 4% to avoid 10% annual inflation, based on U.S. data. In his model, inflation erodes the self-insurance value of money and raises the volatility of consumption for low-income households. Erosa and Ventura (2002), Akyol (2004), Algan and Ragot (2010), Sunel (2013), Camera and Chien (2014) and Herman and Pugsley (2014) extend the Imrohorglu (1992) model to a portfolio choice framework in which people hold money and risk-free bonds simultaneously for self-insurance. Erosa and Ventura (2002) finds that inflation has important distributional effects across households since inflation acts as a regressive consumption tax; in effect, they focus on how the distributional effects of inflation respond to different transaction cost structures. Akyol (2004), however, finds that a 10% inflation rate can maximize social welfare in an endowment economy. In his paper, the positive inflation is optimal because it induces a bond demand that improves risk-sharing and redistributes resources from high-income to low-income agents. Algan and Ragot (2010) show that the long-run neutrality of inflation on capital accumulation obtained in complete market models no longer holds when households face binding credit constraints. They provide a quantitative rationale for the observed hump-shaped relationship between inflation

and capital accumulation. Sunel (2013) studies the distributional and welfare consequences of disinflation in emerging market countries. Camera and Chien (2014) study systematically the distributional impacts of long-run inflation in heterogeneous agent economies with endogenous labor supply and cash-in-advance constraints. In their paper, small departures from zero inflation have the biggest impacts on the distributions on income, wealth and consumption. Herman and Pugsley (2014) explore the welfare effect of the Volcker disinflation. One of the important contribution of my paper is that I refine theoretically and calibrate quantitatively a mechanism of how inflation impacts the credit market. The debt limits implied by this mechanism coincide with the three zones in empirical evidence on the relationship between inflation and credit market activities. Based on recent U.S. data, I find the optimal inflation to be 3 percent. This finding has two illustrative implications: first, a 3 percent target is close to the average inflation rate in the U.S. between 1980 and 2006, 3.9 percent; second, the sizable welfare loss at the Friedman rule inflation and the negligible welfare loss at 2 percent inflation compared to the optimal one, 3 percent, explain in some sense the Federal Reserve practice of maintaining a 2 percent inflation rate over the medium to long term. In this sense, 2 percent inflation target is a relatively reasonable one for the U.S. economy in terms of social welfare.

Another stream of literature related to this paper is endogenous market-incompleteness due to limited commitment. Kehoe and Levine (1993) propose the model of debt-constrained but complete asset markets. Alvarez and Jermann (2000) provide a complementary approach that pins down the endogenous solvency constraints recursively, and delivers the same results as Kehoe and Levine (1993). Zhang (1997) develops ways to endogenize the borrowing constraint with incomplete markets. The core concepts among these papers are the same: given the default punishment that borrowers will be excluded from credit market forever once they renege on their debts, agents will borrow up to a limit within which debts are guaranteed to be repaid. Based on the Lagos-Wright framework, Carapella and Williamson (2015) create an endogenous role for government debt by imposing limited commitment on

credit. They demonstrate government debt plays a role as collateral and helps discourage default. This has similar implications as my model does when the central bank implements monetary policy through open market operations rather than by helicopter drops of money.

In addition, a large amount of theoretical and empirical literature studies the effect of inflation on the economy (for example, financial market, output, etc.). Bullard and Keating (1996) find empirically there is no association between long-run inflation and real output except for some low inflation countries in which permanent inflation shocks increase real output permanently. Azariadis and Smith (1996) develop a monetary growth model with information asymmetry and predict the empirical pattern of the non-monotonic relationship between long-run output levels and inflation. Moreover, credit rationing emerges when inflation is high enough. Kandel et al. (1996) test the Fisher hypothesis and find a negative relationship between ex-ante real interest rates and expected inflation. Barr and Campbell (1997) estimate expected future real interest rates and inflation rates using the U.K. data and find a strongly negative correlation between real interest rates and expected inflation over a short horizon. Huybens and Smith (1999) present a model to predict that inflation and financial market activity are strongly negatively correlated. Boyd et al. (2001) find empirically a nonlinear effect of inflation on the performance of financial sectors and the negative correlation between inflation and the financial market activity when inflation is moderate to high. Antinolfi et al. (2007) build a model to show dollarization can affect the relationship between inflation and output. They model an economy in which inflation and output are positively correlated at low levels of inflation while the pattern is reversed at high levels of inflation due to the substitution of dollars for deposits issued by domestic banks, and thus a reduction in financial intermediation and investment. This paper is close to Antinolfi et al. (2014), who formulate a central bank's choice of an optimal inflation target to maximize discounted stationary utility for a heterogeneous population of infinitely lived households in an economy with limited commitment, segmented markets, constant aggregate income and complete markets. They show that the optimal inflation is positive because inflation relaxes

credit constraints and improves risk-sharing.

Based on the previous related literature, this paper proposes two major innovations. For the stream of literature on the relationship between inflation and financial market performance, theoretical papers predict either positive or negative relation between inflation and credit market activities. This paper generates three empirically supported credit zones in a unified model through a transmission channel leading from inflation to credit markets. I call this channel the inflation-credit channel, which gives rise to a comprehensive conclusion of the effect of inflation on credit market activities. This is the first innovation. Next, based on how inflation alleviates or worsens the frictions in credit markets, which is governed by that channel, this paper answers the two questions about inflation targeting. The quantification of welfare loss of inflation through an endogenous credit market is another innovation, which distinguishes my paper from the literature that studies welfare loss of inflation in the framework of the Bewley model. By endogenizing debt limits, this paper is able to study the effect of inflation targets on credit markets and wealth inequality, in addition to social welfare, and thus generates richer policy implications of inflation targeting. In addition to these two innovations, this paper also compares two monetary policy regimes, helicopter drops and open market operations, to see how debt limits and the optimal inflation target react to policy instruments. Besides, in order to deliver a comprehensive policy implications, this paper also studies quantitatively the role of market incompleteness and the severity of default punishment.

The paper is organized as follows. In section 1.2, I start with a simple deterministic model which introduces concepts and builds intuition. In section 1.3, I present the full-blown model, define the concepts of recursive general equilibrium and formalize the welfare loss of inflation. Section 1.4 develops the calibration process and picks parameter values. Section 1.5 reports benchmark results under a monetary policy of helicopter drops. These results include the impact of inflation target on credit market activities, social welfare and wealth inequality. Next, I examine the role of market incompleteness by studying the same

problem, now with a complete set of financial assets. At the end of this section, I impose a weaker punishment on default to better match the debt limit / income ratio. Section 1.6 reports the results of sensitivity analysis. In section 1.7, I compare open market operations with helicopter drops.

1.2 A Simple Model

In this section, I provide a theoretical model to show that there exists a positive inflation target that maximizes social welfare in an economy with segmented markets and limited commitment on debts. To obtain analytical solutions, I develop a deterministic and simplified version of the fully-fledged quantitative model used in the next section.

I set up an endowment economy populated by four agents indexed by $i = 1, 2, 3, 4$. Agents $i = 1$ and 2 only use non-negative amounts of money (cash agents) while agents $i = 3$ and 4 only trade risk-free bonds (credit agents).⁹ Time is discrete and denoted by $t = 1, 2, 3, \dots$. All agents share the same utility form:

$$\sum_{t=0}^{\infty} \beta^t \log(c_t^i),$$

with $0 < \beta < 1$. Individual endowments are periodic:

$$(e_t^1, e_t^2) = (e_t^3, e_t^4) = \begin{cases} (1 + \alpha, 1 - \alpha) & t = 0, 2, 4, \dots \\ (1 - \alpha, 1 + \alpha) & t = 1, 3, 5, \dots \end{cases}$$

Thus, individual income shares fluctuate deterministically and aggregate endowment is constant at one unit. The parameter α denotes income risk or dispersion.

First, we look at cash agents' stationary choice problem at $t = 0$:

⁹Credit agents actually can hold both money and bonds but choose to trade bonds in equilibrium because the rate of return on bonds dominates the rate of return on money. For simplicity, I assume they hold bonds only. The results won't change.

$$v^{ca} = \frac{1}{1 - \beta^2} \max\{c_H^{ca}, c_L^{ca}\} (\log(c_H^{ca}) + \beta \log(c_L^{ca})), \quad (1.2.1)$$

where

$$c_H^{ca} = 1 + \alpha - m, \quad (1.2.2)$$

$$c_L^{ca} = 1 - \alpha + \frac{m}{1 + \pi}, \quad (1.2.3)$$

where $m \geq 0$ is the real balances and π is the inflation rate controlled by the central bank.

The F.O.C. for high-income cash agent is:

$$\frac{1}{c_H^{ca}} = \frac{\beta}{1 + \pi} \frac{1}{c_L^{ca}}. \quad (1.2.4)$$

The F.O.C. for low-income cash agent is:

$$\frac{1}{c_L^{ca}} \geq \frac{\beta}{1 + \pi} \frac{1}{c_H^{ca}}, \quad = \text{if } m > 0. \quad (1.2.5)$$

In any equilibrium where $1 + \pi > \beta$ (this is satisfied in most cases), we have $x_H > x_L$. By combining equations (1.2.2), (1.2.3) and (1.2.4), we obtain:

$$m = \frac{(1 + \pi + \beta)\alpha + \beta - 1 - \pi}{1 + \beta}. \quad (1.2.6)$$

Since this paper focuses on monetary equilibrium, the money holding m is required to be positive. Thus, we impose an upper bound on the inflation rate:

$$\pi < \frac{\alpha + \alpha\beta + \beta - 1}{1 - \alpha}. \quad (1.2.7)$$

We will apply this parameter restriction in the numerical analysis later in this section. We plug equation (1.2.6) into equations (1.2.2) and (1.2.3) and obtain:

$$v^{ca} = \frac{1}{1 - \beta^2} \left(\log \left(\frac{2 + \pi - \pi\alpha}{1 + \beta} \right) + \beta \log \left(\frac{(2 + \pi - \pi\alpha)\beta}{(1 + \pi)(1 + \beta)} \right) \right). \quad (1.2.8)$$

Next, we look at credit agents' problem:

$$v^{cr} = \frac{1}{1 - \beta^2} \max_{\{c_H^{cr}, c_L^{cr}\}} (\log(c_H^{cr}) + \beta \log(c_L^{cr})), \quad (1.2.9)$$

where

$$c_H^{cr} + a_H = 1 + \alpha + Ra_L, \quad (1.2.10)$$

$$c_L^{cr} + a_L = 1 - \alpha + Ra_H, \quad (1.2.11)$$

where R is gross rate of return on risk-free bonds and a denotes bond holdings. In equilibrium, $a_H = -a_L > 0$. Borrowers have limited commitment on their debts and thus may repudiate them. Defaulting credit agents are excluded from credit market forever and reduced to cash agents. In order for them not to default, the continuation value of solvency needs to be no less than the default value. Therefore, credit agents are subject to a participation constraint:

$$v^{cr} \geq v^{ca}.$$

According to Alvarez and Jermann (2000), this participation constraint can be written equivalently as an endogenous borrowing constraint:

$$a_H + \bar{a}_H \geq 0, \quad (1.2.12)$$

$$a_L + \bar{a}_L \geq 0, \quad (1.2.13)$$

where $(\bar{a}_H, \bar{a}_L) > 0$ are the largest debt limits within which debts are repaid for sure.

A deterministic equilibrium is defined by a set of policy functions (c^{ca}, m, c^{cr}, a) , a set of value functions v^{ca} and v^{cr} , a set of prices (π, R) and the debt limit \bar{a} such that:

1. Given prices (π, R) and debt limit \bar{a} , the policy functions solve the household problem of the two groups; v^{ca} and v^{cr} are the corresponding value functions.
2. The debt limit \bar{a} satisfies:

$$\bar{a} = \min \{ \bar{a}_L, \bar{a}_H \},$$

where

$$\bar{a}_L = \{ -a_L : v_L^{cr} = v_L^{ca} \},$$

$$\bar{a}_H = \{ -a_H : v_H^{cr} = v_H^{ca} \}.$$

That is, \bar{a} is the tighter debt limit that guarantees solvency in any future idiosyncratic state.

3. The bond market clears:

$$a_H + a_L = 0$$

4. By Walras' Law, consumption goods market also clears automatically. That is, the sum of expected value of consumption goods of both groups of agents, equals to the expected aggregate endowment of one unit:

Since in equilibrium, only low-income credit agent borrow and may be potentially financially constrained, the second constraint (1.2.13) comes into play and suppose it is binding:

$$a_H = -a_L = \bar{a}_L = \bar{a}. \quad (1.2.14)$$

The F.O.C. for high-income credit agent is:

$$\frac{1}{c_H^{cr}} = \beta R \frac{1}{c_L^{cr}}. \quad (1.2.15)$$

The F.O.C. for low-income credit agent is:

$$\frac{1}{c_L^{cr}} \geq \beta R \frac{1}{c_H^{cr}}, \quad = \text{if } a_L > -\bar{a}.$$

By combining equations (1.2.10), (1.2.11), (1.2.14) and (1.2.15), we obtain:

$$\bar{a} = \frac{\beta R + \alpha \beta R + \alpha - 1}{(1 + R)(1 + \beta R)}. \quad (1.2.16)$$

From equations (1.2.10), (1.2.11) (1.2.15) and (1.2.16), we obtain:

$$c_L^{cr} = \frac{2\beta R}{1 + \beta R}, \quad c_H^{cr} = \frac{2}{1 + \beta R}.$$

These expressions lead to the value function:

$$v^{cr} = \frac{1}{1 - \beta^2} \left(\log \left(\frac{2}{1 + \beta R} \right) + \beta \log \left(\frac{2\beta R}{1 + \beta R} \right) \right). \quad (1.2.17)$$

By letting $v^{cr} = v^{ca}$, we can pin down R and recover the debt limit a from equation (1.2.16). In order to demonstrate the fundamental mechanism behind the model, I conduct a simple numerical analysis to simplify the problem.¹⁰ Since one major question is to find an optimal inflation target other than the Friedman rule inflation, it is important to study how the economy will respond to different inflation rates controlled by the central bank. Let $\beta = 0.96, \alpha = 0.20$. In order for inequality (1.2.7) to hold, π needs to be less than 0.45. Here

¹⁰We can do comparative static analysis directly without parameterization. Parameter values will help us check if $m \geq 0$.

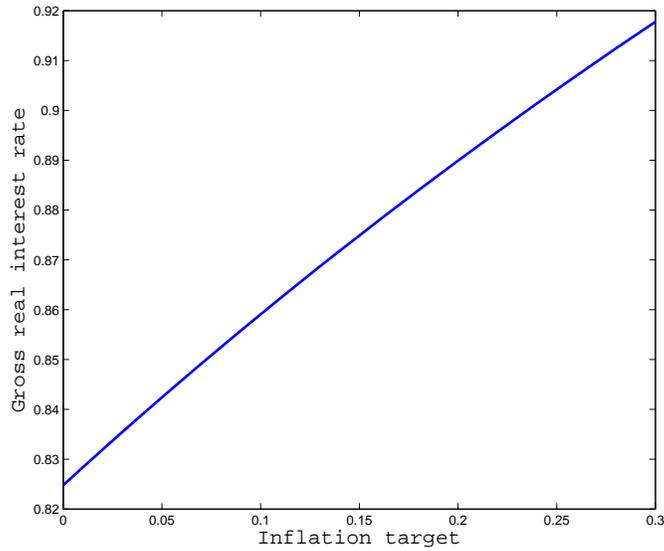


Figure 1.2.1: Gross real interest rate

I let inflation targets π range from 0 to 30%, with 1% increment.

First, we obtain R as a function of π by solving the nonlinear equation $v^{cr}(R(\pi)) - v^{ca}(\pi) = 0$. Figure 1.2.1 shows the relationship between inflation and the gross real interest rate, which is smaller than one over the given range of the inflation.

Next, by applying comparative static analysis to the problem above, we easily find that v^{ca} is decreasing in π while v^{cr} , the debt limit a and the yield R are increasing. When all agents are treated equally, social welfare is simply a linear combination of v^{ca} and v^{cr} , which is maximized at a positive inflation rate lying between 0 and 30%.

1.3 The General Model

1.3.1 Households

This paper studies an endowment economy with constant aggregate consumption goods. The economy is populated by a continuum of agents with measure one who receive idiosyncratic

income shocks to their endowments each period (discrete time) over an infinite horizon. Each period, agents receive an endowment e_t , in terms of the perishable consumption goods. The income process $\{e_t\}$ is independently and identically distributed across agents and follows a two-state first order Markov process over time with $e \in \{e^l, e^h\}$. The stationary transition probability is denoted by $pr(e'|e) = \Pr[e_{t+1} = e' | e_t = e] > 0$. This paper focuses on stationary equilibrium and there are no aggregate shocks to the endowment.

Agents are exogenously divided into two groups: cash agents with mass ω and credit agents with mass $1 - \omega$. For cash agents, money is the only asset they can hold and plays two roles. First, they need money to purchase consumption goods. In this regard, cash agents are subject to a cash-in-advance constraint. Second, money serves as a store of value with which cash agents smooth consumption against idiosyncratic income shocks. Credit agents, on the other hand, can hold state-noncontingent bonds in addition to money to hedge against income shocks. Since there is neither a cash-in-advance constraint nor liquidity service needs for credit agents, they only hold bonds in equilibrium due to the higher rate of return on bonds. By trading bonds, credit agents can borrow but with limited commitment. That's to say, debts are unsecured and borrowers may renege on their debts. Defaulting credit agents are excluded from the credit market forever and reduced to cash agents. There is no default in equilibrium. Therefore, in order for them to remain solvent, the continuation value of solvency needs to be no less than the default value. Limited commitment on debts gives rise to an endogenous debt limit within which debts are guaranteed to be repaid.

1.3.1.1 Cash agents

Let c^{ca} and $m/(1 + \pi)$ denote start-of-period consumption and real balance for cash agents, where π is inflation rate. Assume agents carry over real balances from the previous period to the current period; the real value of money holdings is discounted by $1/(1 + \pi)$. Denote with a prime in the next period's variables.

I make an assumption on the timing arrangement for cash agents: they make decisions

before receiving the lump-sum transfer from government. Cash agents purchase consumption goods subject to a modified cash-in-advance constraint. In addition to the money carried over from the previous period, they can also use extra money by liquidating their current endowment e . With the timing arrangement above, the endowment can change the CIA constraint but the transfer cannot.

Given the current state (e, m) and inflation rate π , cash agents choose $(c^{ca}, m') \geq 0$ to maximize expected life-time utility. The problem has a recursive representation:

$$v^{ca}(e, m) = \max_{\{c^{ca}, m'\}} u(c^{ca}) + \beta E v^{ca}(e', m'), \quad (1.3.1)$$

$$\text{s.t. } c^{ca} + m' = e + \frac{m}{1 + \pi} + tr, \quad (1.3.2)$$

$$\text{and } c^{ca} \leq \frac{m}{1 + \pi} + e, \quad (1.3.3)$$

$$m' \geq 0 \quad (1.3.4)$$

where $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, tr is lump-sum transfer from government to each agent. Assume all agents receive the same lump-sum transfer.

1.3.1.2 Credit agents

I analyze this group in the same way as cash agents and let c^{cr} and a denote start-of-period consumption and bond holdings for credit agents. Given the current state (e, a) and a risk-free interest rate r , they choose (c^{cr}, a') to maximize expected life-time utility subject to a participation constraint due to limited commitment on debts. The problem has a recursive representation:

$$v^{cr}(e, a) = \max_{\{c^{cr}, a'\}} u(c^{cr}) + \beta E v^{cr}(e', a'), \quad (1.3.5)$$

$$\text{s.t. } c^{cr} + \frac{a'}{1+r} = e + a + tr, \quad (1.3.6)$$

$$\text{and } v^{cr}(e, a) \geq v^{def}(e) \quad \forall e \in \{e^l, e^h\}, \quad (1.3.7)$$

where $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and $v^{def}(e)$ is the default value. The inequality (1.3.7) implies credit agents will repay their debts for sure in any future state. Recall that defaulting credit agents are excluded from credit markets forever and reduced to cash agents, with the minimum money holdings initially (they can still accumulate money in the future). Therefore, the defaulting value can be pinned down as follows:

$$v^{def}(e) = v^{ca}(e, m_{min}), \quad (1.3.8)$$

where m_{min} is the lower bound of money holdings across all the cash agents in equilibrium.

In order to write the problem as a recursive form, I transform the participation constraint into a 'not-too-tight' endogenous borrowing constraint as in Alvarez and Jermann (2000). The major difference is that credit agents trade state-noncontingent bonds compared to state-contingent bonds in theirs. Due to the market incompleteness for credit agents, the continuation value of repaying debts must be preferable to the default value in each possible state next period. Hence, the endogenous borrowing limit is accordingly state-noncontingent in that it takes the tightest one among all the possible endowment values. Mathematically,

$$A = \min_{\{e\}} \{-a(e) : v^{cr}(e, a) = v^{def}(e)\}. \quad (1.3.9)$$

That is, A is the tightest debt limit that guarantees solvency in any future idiosyncratic income state.

Thus, the participation constraint (1.3.7) can be reduced equivalently to an endogenous borrowing constraint:

$$a' + A \geq 0. \tag{1.3.10}$$

As long as the amount of borrowing doesn't surpass the debt limit A , credit agents will repay their debts for sure.

1.3.2 Monetary policy

The central bank implements monetary policy by “helicopter drop” to maintain its long-term inflation target.¹¹ It controls the growth rate γ of aggregate supply of money in real terms,¹² denoted by M . For the time being, assume there is no government debt, taxes or spending. The law of motion for M is:

$$M' = \frac{M}{1 + \pi} + \gamma \frac{M}{1 + \pi}. \tag{1.3.11}$$

The quantity of newly issued money in real terms is $\gamma \frac{M}{1 + \pi}$, which is the lump-sum transfer, tr , equally transferred to each agent in both groups. We can see in stationary equilibrium, $\pi = \gamma$.

¹¹In section 1.7, the central bank conducts open market operations as an alternative monetary policy to maintain its inflation target.

¹²Controlling the growth rate of aggregate money in real terms is equivalent to controlling that in nominal terms. In other words, the growth rate of aggregate money in nominal terms is also γ . To see this, we start with aggregate money supply in nominal terms, \tilde{M} . Suppose the central bank controls the growth rate γ of aggregate money in nominal terms. Then the law of motion for \tilde{M} is: $\tilde{M}' = (1 + \gamma) \tilde{M}$. Let $M' = \tilde{M}'/P$. By dividing P on both sides, we have

$$M' = (1 + \gamma) \frac{M}{1 + \pi}$$

, exactly same as equation 1.3.11.

1.3.3 Recursive equilibrium

Now we are in a position to define a recursive equilibrium. The state of a cash agent $(e, m) \in E \times M$, with $M = [0, \infty)$ and $E = \{e^l, e^h\}$. Let $\mathcal{P}(E)$ denote the power set of E , $\mathcal{B}(M)$ denote the Borel σ -algebra of M . Define the subset of possible states $(\mathbb{M}, \mathbb{E}) \subseteq \mathcal{B}(M) \times \mathcal{P}(E)$. Denote the set $S^{ca} = \mathbb{M} \times \mathbb{E}$. The state of a credit agent $(e, a) \in E \times B$, with $B = [-A, \infty)$ and $E = \{e^l, e^h\}$. Let $\mathcal{B}(B)$ denote the Borel σ -algebra of B . Define the subset of possible states $(\mathbb{B}, \mathbb{E}) \subseteq \mathcal{B}(B) \times \mathcal{P}(E)$. Finally denote the set $S^{cr} = \mathbb{B} \times \mathbb{E}$.

A stationary recursive equilibrium is defined by a set of policy functions $c^{ca}(e, m)$, $m'(e, m)$, $c^{cr}(e, a)$ and $a'(e, a)$; a set of value functions $v^{ca}(e, m)$ and $v^{cr}(e, a)$; a set of prices $\{\pi, r\}$; the debt limit A ; and two stationary distributions $\phi^{ca}(e, m)$ and $\phi^{cr}(e, a)$, such that:

1. Given prices $\{\pi, r\}$ and debt limit A , the policy functions solve the household problem of the two groups; $v^{ca}(e, m)$ and $v^{cr}(e, a)$ are the corresponding value functions.
2. The debt limit A satisfies:

$$A = \min_{\{e\}} \{-a(e) : v^{cr}(e, a) = v^{def}(e)\}. \quad (1.3.12)$$

That is, A is the tightest debt limit that guarantees solvency in any future idiosyncratic state.

3. The bond market clears:

$$\int \int a'(e, a) d\phi^{cr}(e, a) = 0. \quad (1.3.13)$$

4. The money market clears:

$$\int \int m'(e, m) d\phi^{ca}(e, m) = M. \quad (1.3.14)$$

5. By Walras' Law, consumption goods market also clears automatically. That is, the sum of expected value of consumption goods of both groups of agents, equals to the expected aggregate endowment of one unit:

$$\omega \int \int c^{ca}(e, m) d\phi^{ca}(e, m) + (1 - \omega) \int \int c^{cr}(e, a) d\phi^{cr}(e, a) = 1. \quad (1.3.15)$$

6. The policy functions and the transition matrix of the income process generate a probability distribution P over the state space for cash and credit agents, respectively:

(a) for cash agents:

$$P^{ca}((e, m), (e', m'(e, m))) = \sum_{m':(e', m'(e, m)) \in S^{ca}} p(e'|e), \quad (1.3.16)$$

the probability of transiting from state (e, m) to a state in the set S^{ca} .

(b) for credit agents:

$$P^{cr}((e, a), (e', a'(e, a))) = \sum_{a':(e', a'(e, a)) \in S^{cr}} p(e'|e), \quad (1.3.17)$$

the probability of transiting from state (e, a) to a state in the set S^{cr} .

7. The distributions ϕ^{ca} and ϕ^{cr} are stationary:

$$\phi^{ca}(S^{ca}) = \int \int P^{ca}((e, m), S^{ca}) d\phi^{ca}, \forall S^{ca}, \quad (1.3.18)$$

$$\phi^{cr}(S^{cr}) = \int \int P^{cr}((e, a), S^{cr}) d\phi^{cr}, \forall S^{cr}. \quad (1.3.19)$$

1.3.4 Welfare loss of inflation

The welfare criterion this paper applies here is simply the sum of the expected discounted life-time utilities under the equilibrium stochastic consumption streams of both groups of agents as in Aiyagari and McGrattan (1998). The aggregate welfare function, denoted by W , weighs each agent equally and thus weighs each group by their mass. W is represented by:

$$W := \omega \int \int v^{ca}(e, m) d\phi^{ca}(e, m) + (1 - \omega) \int \int v^{cr}(e, a) d\phi^{cr}(e, a). \quad (1.3.20)$$

Using this criterion, I compute the aggregate welfare with a range of inflation targets, $W(\pi)$. Denote the highest welfare by W^* and call the corresponding inflation rate the optimal inflation target, π^* . For an agent at time zero, the welfare loss of inflation π relative to the optimal inflation π^* can be calculated in terms of certainty equivalence of consumption:

$$\Delta(\pi) := 1 - \left[\frac{W(\pi)}{W^*} \right]^{1/(1-\sigma)}, \quad (1.3.21)$$

which is in consumption goods unit. In this regard, it is the compensating variation from an ex-ante perspective. Since $\Delta(\pi)$ is non-negative by definition, it implies that each agent is ex-ante indifferent between inflation π^* and inflation π with consumption increased by $\Delta(\pi)$. One can imagine of this as the percentage of consumption one would receive additionally for sure as compensation, at the start of the economy, to stay at π rather than move to π^* ; or the percentage of consumption one would commit to give up, at the start of the economy, in order to stay at π^* rather than move to π .

1.3.5 Characterizations

Conjecture that the value functions v^{ca} and v^{cr} exist and are differentiable. Let $u_c := \frac{\partial u(c^{ca})}{\partial c^{ca}}$ or $\frac{\partial u(c^{cr})}{\partial c^{cr}}$, $v_m^{ca} := \frac{\partial v^{ca}}{\partial m}$ and $v_a^{cr} := \frac{\partial v^{cr}}{\partial a}$. Let also $\lambda^{ca}, \mu^{ca}, \mu^{cr}$ be Lagrangian multipliers on the constraints (1.3.2), (1.3.3) and (1.3.10). For cash agents:

$$(c^{ca}) : u_c = \lambda^{ca} + \mu^{ca},$$

$$(m') : \beta E v_m^{ca} = \lambda^{ca}.$$

The envelope theorem implies:

$$v_m^{ca} = (\lambda^{ca} + \mu^{ca}) / (1 + \pi).$$

By combining these equations, we have:

$$\beta E [u_c / (1 + \pi)] = u_c - \mu^{ca}.$$

Additionally:

$$\mu^{ca} (m / (1 + \pi) + e - c^{ca}) = 0.$$

This is an economy with occasionally binding CIA constraints. When $\mu^{ca} = 0$, the cash agents' problem is reduced to the standard Bewley problem. When $\mu^{ca} > 0$, the modified CIA constraint influences the intertemporal decision making. To be more specific, cash agents will exhaust all available money at hand, which is the real balance carried over from last period, plus current income. The question is then: when does an agent face a binding CIA constraint? As mentioned before, money serves two roles for cash agents: medium of exchange and self-insurance buffer that smoothes consumption. In this incomplete-market environment, cash agents need to accumulate positive amounts of money when receiving high

income shocks (lucky times), and spend it to smooth their consumption when receiving low income shocks (unlucky times). When an agent receives a long enough series of low income shocks in a row, he / she is highly likely to spend as much money accumulated during lucky times as possible to the point where the CIA constraint is just binding. Otherwise (when lucky times persist or unlucky times are short), agents don't face binding CIA constraint.

Proposition 1 *There exists a positive cut-off value of inflation π_{nm} below which all equilibria are monetary equilibria (the economy is in the “active zone of inflation”). When inflation is above π_{nm} , all equilibria are non-monetary equilibria (the economy is in the “inactive inflation zone”).*

Proof. See the Appendix.

The crucial linkage between the two groups is the continuation value of default which equals the value of cash agent with the minimum stock of money. In order to study the effect of long-term inflation on credit agents' welfare and thus on social welfare, it is important to figure out first the impact of inflation targets on the welfare of cash agents. The inflation tax rate is denoted by $\tau_\pi = \frac{\pi}{1 + \pi}$, which is increasing in inflation π . The deadweight loss from this inflation tax is accordingly $m \cdot \tau_\pi$.

In order to figure out the endogenous debt limits, we need first study the property of the default value for credit agents, which is determined by the bottom cash agents' value according to the punishment. The bottom cash agents' value depends on two competing effects: inflation tax and the distributive channel leading wealth from the top to the bottom cash agents through the lump-sum transfer. To start with, we analyze the bottom cash agents' problem with a standard CIA constraint for comparison.

Under a standard CIA constraint ($c \leq m / (1 + \pi)$), money holdings cannot drop too far below the average endowment if cash agents wish to consume a decent amount of goods. To achieve this, cash agents must maintain a large real balance and pay large inflation tax. Although real money holding m decreases in inflation, the amount of the decrease is limited and tends to be smaller than the increase in tax rate τ_π . Therefore, the loss from inflation tax

$m \cdot \tau_\pi$ is monotonically increasing with inflation. As a result, the negative effect of inflation tax always dominates the positive distributive effect. Consequently, the bottom cash agent's budget set accordingly shrinks with inflation, so does their value.¹³ In a word, the bottom cash agents always suffer when inflation increases.

With the modified CIA constraint, however, the story will be different. Money holding can drop largely to zero thanks to the additional money that cash agents obtain by liquidating their current endowments. This loosens the CIA constraint and thus the bottom cash agents are able to better protect themselves against inflation tax. In this environment, the two competing effects dominates in turn as inflation rises. At low inflation, the negative effect of inflation tax dominates due to large money holding, which leads to the decreasing default value. With money demand dropping to zero when inflation rises from moderate to high values, the negative effect of inflation tax diminishes accordingly and thus the positive distributive effect starts to dominate, which leads to the increasing default value. Therefore, the U-shaped default value just determines the hump-shaped debt limits.

Proposition 2 *For some parameter values, there exists a positive cut-off value of inflation π_{it} . When inflation is below π_{it} , the inflation tax $\frac{\pi}{1+\pi}m$ is increasing with the rate of inflation. When inflation lies between π_{it} and π_{nm} , the inflation tax is decreasing. When inflation is above π_{nm} , the inflation tax drops to zero.*

Proof. See the Appendix.

Next we analyze credit agents' problem:

$$u_c = \beta E u_{c'} + \mu^{cr},$$

$$\mu^{cr} (a' + A) = 0.$$

and

¹³Cash agents make decisions by taking the transfer as given.

$$A = \min_{\{e\}} \{ -a(e) : v^{cr}(e, a) = v^{def}(e) \}.$$

This is a standard incomplete-market model except the endogenously generated debt limit A . Given A , credit agents become financially constrained when receiving a long enough series of low income shocks. The value of A is vital in solving credit agents' problem and assessing social welfare. Equation (1.3.9) shows how to determine A . For each inflation target, there exists an associated default value and so does a debt limit A . Since v^{cr} is strictly increasing in bond holdings a and the default value shows a U-shape w.r.t inflation, the lower bound for bond position \underline{a} also takes on a U-shape w.r.t. inflation. Because $A = -\underline{a}$, the debt limit A shows a humped shape. In other words, small inflation improves the activity of credit market and large inflation hurts it. When inflation is low, an expansionary monetary policy improves the credit market and induces more risk sharing among credit agents. When inflation is already high, conducting expansionary monetary policy tightens up the credit market and makes credit agents worse off. In words, the debt limit depends on the incentive to opt out, which is determined by the continuation value of default. Given the U-shaped welfare of cash agents and the hump-shaped welfare of credit agents, there exists an inflation target that maximizes social welfare, i.e., the convex combination of the values of both groups.¹⁴ Intuitively, credit agents rely a lot on credit markets for risk sharing. Moderate inflation benefits credit agents by relaxing credit markets but meanwhile hurts cash agents by imposing inflation tax. Therefore, the central bank needs to inflate the economy in order to relax credit markets to the point at which the (weighted) marginal benefit from improvements in credit markets just equals to the (weighted) marginal cost of inflation.

¹⁴I treat the two groups equally.

1.4 Calibration

1.4.1 Parameterization

We calibrate the model based on the data from the U.S. economy for the sample period 1980-2006.¹⁵ Since this paper focuses on the effects of long-term inflation targets, the model period is one year. Following Deaton (1991), Aiyagari (1994) and Heaton and Lucas (1996), I estimate the income process as a first-order autoregressive process in the natural logarithm of income by using PSID data.

$$\log(e_{t+1}) = \text{const} + \rho \log(e_t) + \epsilon_t, \quad (1.4.1)$$

where ϵ_t is i.i.d. normal with mean zero and standard deviation ε . The persistence ρ and volatility ε are free from any structural modeling assumptions, so they can be estimated separately from the statistical model above. Based on PSID data, I obtain $\rho = 0.564$ and $\varepsilon = 0.244$. I then follow Tauchen (1986) to approximate this AR(1) process to a two-state (e_l, e_h) Markov transition chains with transition process Pr . The remaining parameters are subjective discount factor β , risk aversion σ and the relative population of cash agents ω . In the model, cash agents are those who can only hold non-interest-bearing assets such as currency, demand deposit and checking account, while credit agents can have access to consumer credit and hold interest-bearing financial assets. With regard to data, there is a wide range of the percentage of cash agents or credit agents based on different database or measurements. Based on the Survey of Consumer Finance (SCF), on one hand, if we look at data on financial assets that are non-interest-bearing, Mulligan and Sala-i-Martin (2000) show 59% of U.S. households don't hold any nonmonetary financial assets, who can be viewed as cash agents in our model. On the other hand, if we look at data on consumer credit (credit card is the major form and tools for consumer credit), the average percentage

¹⁵Since this paper focuses on the long-term effects of inflation target in a stationary equilibrium, I omit the data in the period as of 2007 when the economy has been deviating from the stationary trend.

of U.S. households having credit card balance from 1989 to 2013 is 43%. This means 57% of U.S. households are classified as cash agents in our model. Either way shows nearly 60% U.S. households are cash agents by the given definition. Based on the Survey of Consumer Payment Choice, cash and check payments weigh 37% in total consumer payments, which implies the relative population of cash agents is 37%. I jointly calibrate these three parameters by matching three targets (sample average): real interest rate 2.3%, the ratio of M1 to GNP 16.0% and the ratio of credit constrained (rcc) households 15.8%. I summarize all parameters as follows:

$$(e_l, e_h) = (0.756, 1.244), \quad Pr = \begin{bmatrix} 0.782 & 0.218 \\ 0.218 & 0.782 \end{bmatrix}, \quad \beta = 0.967, \quad \sigma = 1.225, \quad \omega = 0.40$$

1.5 Benchmark Monetary Policy: Helicopter Drops

In order to assess the long-term effects of inflation targets on credit markets and social welfare, below I conduct two types of experiments based on a computational analysis of steady states derived from the calibrated model above. This section presents the benchmark results from the experiment in which the central bank conducts helicopter drop (HD) to maintain its inflation target. Later on I will conduct another experiment in which the central bank implements monetary policy through open market operations (OMO).

The range of inflation targets is from -1.2% ¹⁶to 50%, with 1% increments. I study systematically the effects of this range of inflation on credit markets, social welfare and inequality. First of all, I compare the equilibrium values of aggregate and distributional variables implied by the calibrated model above with their empirical counterparts in the U.S. data. The first three variables in Table 1.5.1 are the calibration targets and thus are exactly matched to the corresponding targets. The fourth one, the Gini coefficient of wealth,

¹⁶This inflation is obtained by the definition of Friedman rule inflation. Under -1.2% inflation, the corresponding nominal interest rate (real interest rate plus inflation) is zero.

Table 1.5.1: Benchmark model vs data

	Model	Data
M1/GNP	16.0%	16.0%
ratio of credit constrained people	15.8%	15.8%
real interest rate	2.3%	2.3%
Gini coefficient of wealth	0.72	[0.65, 0.79]
inflation rate	3% (optimal)	3.9% (average)
percentage of cash agents	0.40	[0.37, 0.59]
debt limits / income ratio	0.99	0.33

indicates the highly matched distributional fact in terms of wealth across all the households. The model generates the optimal inflation rate of 3%, a little lower than the average inflation rate in the U.S. between 1980 and 2006, 3.9%. According to FRBNY Consumer Credit Panel / Equifax, the average credit / income ratio between 1999 and 2014 is 0.326, while this variable implied by the model is three times as large as the data. Regardless of the relatively big quantitative disparity between the model prediction and data on the ratio of credit limit and income, the model predicts pretty well the qualitative effect of inflation on the credit market activity, in a manner consistent with the empirical evidence. The high debt limit / income ratio implied by the model is caused by the severe default punishment of permanent exclusion from the credit market, which discourages borrowers from repudiating their debts to a large degree. Later on in the end of this section I will introduce the probability of re-entering into the credit market upon defaulting. With the chance of re-entry, debt limit can be lowered to the level of data. For the moment, the model without re-entry serves as the benchmark. In the remainder of this section, I describe in details the impacts of inflation on both aggregate and distributional variables.

1.5.1 Implications for credit markets

We start by looking at the implications of inflation for credit markets. In the model, the impact of inflation targets on credit markets is nonmonotonic. In order to facilitate the following discussion based on different ranges of inflation, I define three consecutive zones of inflation targeting based on how inflation affects debt limits which are consistent with the empirical finding. Figure 1.5.1 shows that deflation depresses credit limits and even crowds out private credit to some degree, while expansionary monetary policy facilitates the credit market activity by relaxing borrowing constraints when inflation is small. I call this range (low) of inflation the “credit expansion zone” of inflation targeting. The debt limit peaks at 1.00 in the units of the consumption good (this number can also be interpreted as the ratio of debt limit and aggregate endowment) at 4% inflation. Beyond this point, monetary expansions start to reduce borrowing capacity. I call this range (moderate) of inflation the “credit contraction zone”. The debt limit at the Friedman rule inflation (-1.2%) is 0.58, 42.0% lower than the maximum debt limit; the debt limit at the end of “credit contraction zone” (13% inflation) is 0.39, 61.0% lower than the maximum debt limit. Starting from 14% inflation (high), inflation targeting has a vanishing effect on debt limits compared to the previous two zones. In this regard, I call it the “inactive inflation zone”.¹⁷ Accordingly, I call the first two zones the “active zone” (from -1.2% to 13% inflation) as a whole. In the following discussion, the focus is on the “active zone” of inflation targeting unless stated otherwise. This model prediction is consistent with the empirical relationship between inflation and credit market activity in section 1.

In the “credit expansion zone”, the cash agents’ value drops due to increasing inflation tax as inflation goes up. Recall that once credit agents renege on their debts, they are excluded from credit markets and reduced to cash agents forever. In this regard, their outside option

¹⁷The U.S. has never had inflation rate larger than 15% during the post-war period.

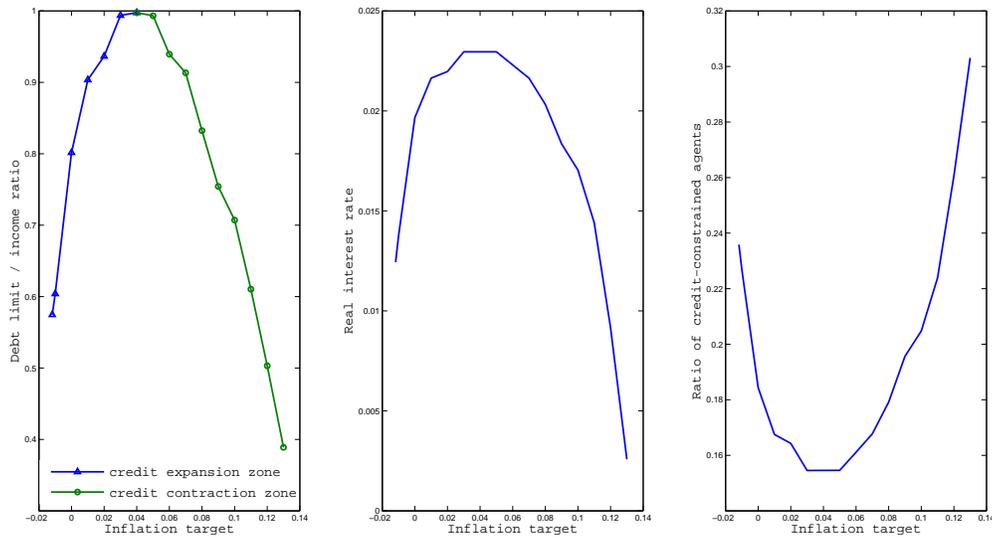


Figure 1.5.1: The impacts of inflation targets on credit markets

value is connected with the cash agents' value. Because the value of default decreases with inflation in this zone, credit agents have less incentive to default as inflation goes up which raises the debt limit and eases the credit market. According to the analysis in section 1.3.5, cash agents become better off in the “credit contraction zone” due to less inflation tax imposed on them. Thanks to the modified CIA constraint, these agents are able to obtain additional money by selling their current-period endowment. Therefore, they just carry over much less money from last period than they do under the standard CIA constraint, and thus suffer less from inflation tax, as shown in Figure 1.5.2. When inflation is from moderate to high, the budget set of cash agents grows with inflation due to lower inflation tax and thus smaller loss from inflation tax, which improves the welfare of the bottom cash agents. When the default value goes up, credit agents have more incentives to opt out. This leads to tightened credit markets and lower debt limits.

In the “active zone” defined above, real interest rates show a humped shape w.r.t. inflation. They first increase with inflation and reach the maximum of 2.3% at 4% inflation,

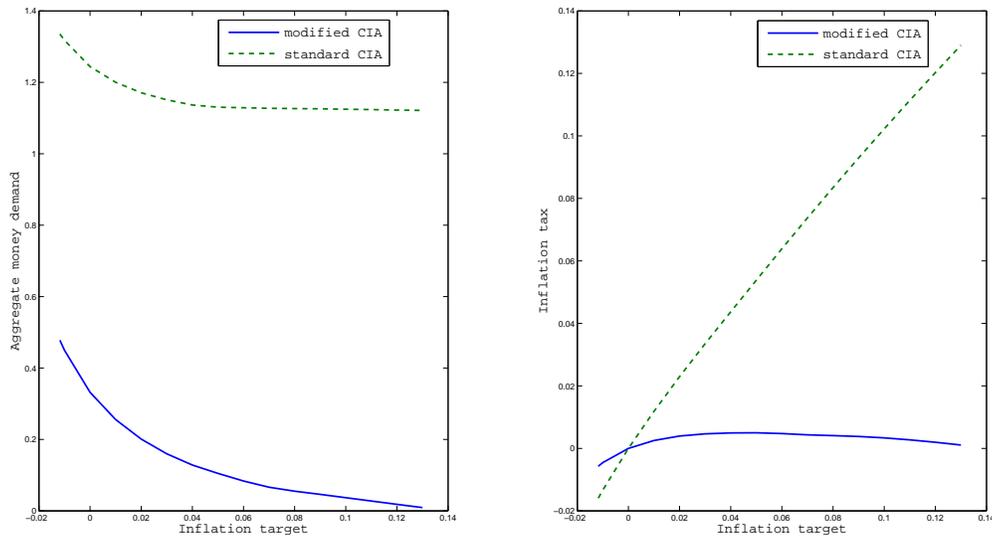


Figure 1.5.2: The impact of inflation targets on aggregate money demand and inflation tax collection

beyond which they enter into the “credit contraction zone” and start to decrease. The real interest rate is 1.2% at the Friedman rule inflation and 0.3% at the end of the “credit contraction zone”. The results about real interest rates in the “credit contraction zone” are consistent with much empirical literature. Kandel et al (1996) empirically find a negative relationship between ex-ante real interest rate and expected inflation. In addition, Boyd and Champ (2003) document that there is a strong negative relationship between inflation and real Treasury bill rate. Those empirical papers, however, don’t assess the relationship when inflation is very low or even negative (the “credit expansion zone”). In line with the findings in literature on limited commitment on debts such as Kehoe and Levine (1993), the real interest rate moves together with endogenous debt limits. That is, it increases with inflation in the “credit expansion zone”. Credit-constrained agents in our model are those credit agents who want to borrow more to smooth consumption but cannot. They are financially constrained by the participation constraint due to limited commitment. In this way, no one defaults in equilibrium. In data, 15.8% of people are turned down for credit. Intuitively, the number of credit-constrained agents is negatively related to the endogenous debt limit, as

shown in Figure 1.5.1. In the “credit expansion zone”, the ratio of credit-constrained agents decreases from 23.6% to 15.5% as the debt limit relaxes, while the ratio rises to 30.3% in the “credit contraction zone”.

1.5.2 Welfare implications

This subsection contains the key result of this paper, the welfare implications of inflation targets. Different inflation targets will affect the amount of risk sharing, the smoothness of life-time consumption streams, the distribution of wealth for both groups of agents, as well as the whole economy. This section seeks to assess welfare effects for each group and the whole economy.

1.5.2.1 The optimal inflation target

To start with, we look at the big picture and measure the aggregate social welfare at each possible inflation target.¹⁸ From section 1.2 and section 1.3.5, we already know that there exists an optimal inflation target that maximizes social welfare from a qualitative view. In this section I find the specific optimal inflation quantitatively using the calibrated model based on U.S. data. Before showing some quantitative results, I will clarify briefly the mechanism that gives rise to an optimal inflation target. The welfare of cash agents has a U-shape w.r.t. inflation, which generates a hump-shaped debt limits. Since the welfare of credit agents depends largely on the amount of risk sharing permitted by prevailing debt limits, credit agents are better off first and then deteriorate as inflation increases. When I treat both groups equally and take into consideration the trade-offs between the improvements in credit market and the dead-weight loss from the inflation tax, the optimal inflation emerges that maximizes social welfare.

Using the measurement of welfare loss described in section 1.3.4, Figure 1.5.3 shows the

¹⁸The inflation target ranges from the Friedman rule inflation (-1.2%) to 50%, with 1% increment. Similarly, I only focus on the “active zone” (from -1.2% to 13% inflation) of inflation targeting unless stated otherwise.

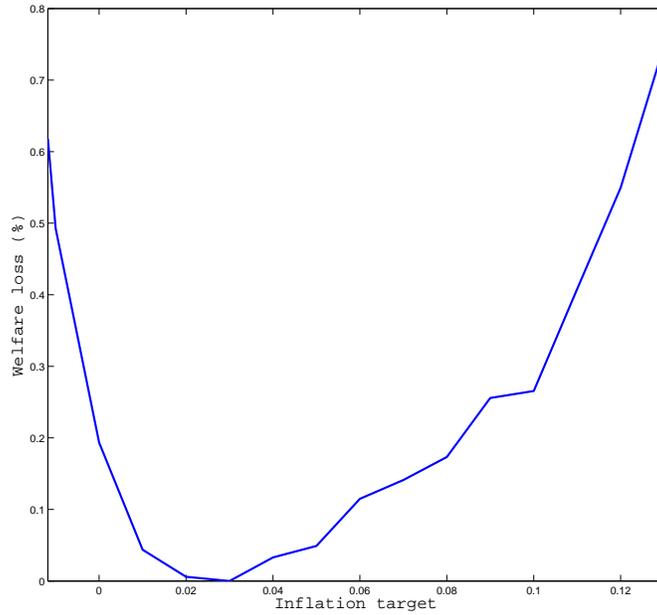


Figure 1.5.3: The optimal inflation target and welfare loss

optimal inflation target in this model economy is 3%. The welfare loss of the Friedman rule inflation as opposed to 3% inflation is 0.61%. This means that the average U.S. household would commit to give up 0.61% of its consumption to stay in 3% inflation rather than move to the Friedman rule inflation. In other words, the average ax-ante welfare loss of shifting to the Friedman rule stationary equilibrium from the 3% inflation optimum rate is 0.61%. Usually, the optimal inflation tends to differ from the inflation (4%) that generates the maximum debt limit, although the two rates are close to each other. The inflation generating the maximum debt limit can only give credit agents the highest welfare. Since cash agents also matter for aggregate welfare, and they prefer low inflation or deflation, the optimal inflation is no larger than the rate maximizing the debt limit.

1.5.2.2 The decomposition of welfare loss

Inflation impacts different groups of agents in different ways. In order to figure out how each group contributes to aggregate welfare, we look into the detailed mechanism by decomposing the welfare loss of switching from 3% inflation to the Friedman rule inflation (-1.2%)¹⁹ by group and by wealth quartiles.²⁰ To this end, I use the same measurement described in section 1.3.4.

Before conducting the decomposition exercise, I separate and quantify the contributions of the two channels (endogenous debt limits and lump-sum transfers) that affect credit agents' welfare. Given any two inflation targets in the "active zone", say 3% and 12% inflation, credit agents gain 1.1% (certainty equivalence of consumption) from 12% to 3% inflation. In order to figure out how much each channel contributes to the welfare improvement, I restrain the general equilibrium effect by fixing the real interest rate at 12%. First, compute the welfare under the debt limit with 3% inflation and without transfers and denote it by W_1 ; second, compute the welfare under the debt limit with 3% inflation and with transfers and denote it by W_2 ; third, compute the welfare under the debt limit with 12% inflation and without transfers and denote it by W_3 ; lastly, compute the welfare under the debt limit with 12% inflation and with transfers and denote it by W_0 (the actual welfare at 12% inflation). As a consequence, the ratio of $(W_2 - W_1)$ to $(W_0 - W_1)$ is the contribution of the lump-sum transfer to the welfare improvement, while the ratio of $(W_3 - W_1)$ to $(W_0 - W_1)$ is that of the endogenous debt limit. In this example, the percentage contribution of the transfer is

¹⁹We can analyze the welfare loss from 3% inflation to both directions across the range of inflation targets. That's, in addition to the welfare loss from 3% to -1.2%, this paper can also study the welfare loss from 3% to 13%, the end of the "active zone". The reason why I focus on the former one is that for the U.S. economy, the sustained inflation undershooting (below the target of 2%) or even the high likelihood of deflation is much more urgent and potentially harmful than a high inflation rate. On the other hand, here I focus on the long-run effect of inflation targets, which means we ask how much compensating consumption agents (on average) would ask for to stay at -1.2% inflation instead of 3% if the economy could reach immediately the -1.2% inflation equilibrium without any transitions. To this end, we examine the welfare implications associated with 3% inflation economy and the -1.2% inflation economy long after the -1.2% inflation equilibrium has reached its permanent state.

²⁰We define wealth as the non-negative sum of money holdings and bond holdings. In this model economy with segmented markets, cash agents' wealth refers to money only and credit agents' wealth refers to bonds only.

31.1% and 69.1% for the debt limit (the two don't necessarily sum up to one). Therefore, the channel of the endogenous debt limit serves as a major driving force to change credit agents' welfare.

Table 1.5.2 shows the decomposition results of the welfare loss (negative value means welfare gains) by groups and wealth quartiles given a policy change. When the inflation target deviates from the optimal one 3% to the Friedman rule inflation -1.2%, the whole economy suffers 0.61% welfare loss. The group of credit agents has less risk sharing and suffers 1.71% welfare loss due to the tightened debt limit, while the group of cash agents gains 1.06% welfare due to a positive rate of return on money holdings. Figure 1.5.4 shows the distributions of the welfare loss for the whole economy and each group. For the group of cash agents, there are two competing effects: positive returns on money and government's lump-sum tax $\frac{\pi M}{1+\pi}$.²¹ Most cash agents enjoy welfare gains concentrating between 2.2% and 0%. For money-strapped agents, the positive contribution of returns on money to welfare is zero and they only suffer from the lump-sum tax. On the other side, each credit agent suffers a loss. How big the loss is depends on their wealth positions, which I will talk about soon. From Figure 1.5.4, we see that the welfare losses concentrate on the range between 0.2% and 3%. The opposite welfare effects of the two groups embody the redistributive channel between groups. With 3% inflation, the government collects inflation tax from cash agents and provides the uniform lump-sum transfer (in real terms) to each agent in both groups. Credit agents benefit from this inflation policy in two ways: a direct gift of resource transfers and also ample borrowing capacity. Both of these positive effects improve their welfare.²² By comparison, cash agents only suffer from a higher inflation tax without any benefits. With -1.2% inflation, the government collects lump-sum taxes from both groups of agents and pays real returns on money. In this way, credit agents suffer a lot from two aspects: tighter debt limits and taxes without any returns. Cash agents now become winners.

²¹This can be equivalently viewed as a negative transfer to both groups of agents.

²²As shown above, the effect of endogenous debt limit plays a dominant role in affecting credit agents' welfare.

Although they pay the same lump-sum tax as credit agents do, they gain more from receiving positive returns on money.

Does inflation hurt the poor, the middle class, or the rich? To answer this question, I fix the wealth distribution in the stationary equilibrium of 3% inflation and decompose the welfare loss of -1.2% inflation compared to 3% in terms of the fixed wealth quartiles. The second column in Table 1.5.2 shows the nonmonotonic welfare loss w.r.t. wealth. To be more specific, the welfare loss first decreases and then increases with wealth. Assume Q1, Q2, Q3 and Q4 are called by the poor, the lower middle, the upper middle and the rich, respectively. Then the poor, the lower middle and the rich go through welfare losses and only the upper middle enjoy welfare gains. The poor are mainly made up of debtors, 70% of whom turn to be credit constrained due to the tighter debt limit as well as the lump-sum tax and thus worse off, in spite of lower interest payments on debts. The lower middle are either debtors or cash agents who hold zero money and stay autarky. For the debtors (who face nonbinding borrowing constraint), they gain due to lower interest payments whose effect is larger than that of the lump-sum tax. For the money-binding agents,²³ they suffer from the lump-sum tax. Because the money-binding agents are the majority in the lower middle class, this group suffers a welfare loss in aggregate. The remaining cash agents (most of whom hold positive amounts of money) and a small amount of creditors constitute the upper middle. With more money at hand, cash agents in this group enjoy relatively considerable returns on money, which makes them better off despite the lump-sum tax. The creditors lose due to less returns on bonds as well as the lump-sum tax. Cash agents being the majority in this group makes the upper middle class better off on average. Lastly, creditors in the rich holding a large amount of bonds live worse off due to less returns and the lump-sum tax. The

²³Since these agents hold zero amount of money, the return on money is zero despite a positive rate of return.

Table 1.5.2: The decomposition of welfare loss of -1.2% inflation compared to 3%

	Benchmark			Wasteful Government		
	whole	cash agents	credit agents	whole	cash agents	credit agents
average	0.61%	-1.06%	1.71%	-0.47%	-2.39%	0.79%
Q1	0.76%	-0.18%	0.49%	-0.37%	-1.48%	-0.68%
Q2	0.43%	-0.67%	1.30%	-0.79%	-2.03%	0.25%
Q3	-0.22%	-1.11%	2.05%	-1.45%	-2.40%	1.16%
Q4	1.58%	-2.23%	3.20%	0.75%	-3.60%	2.51%

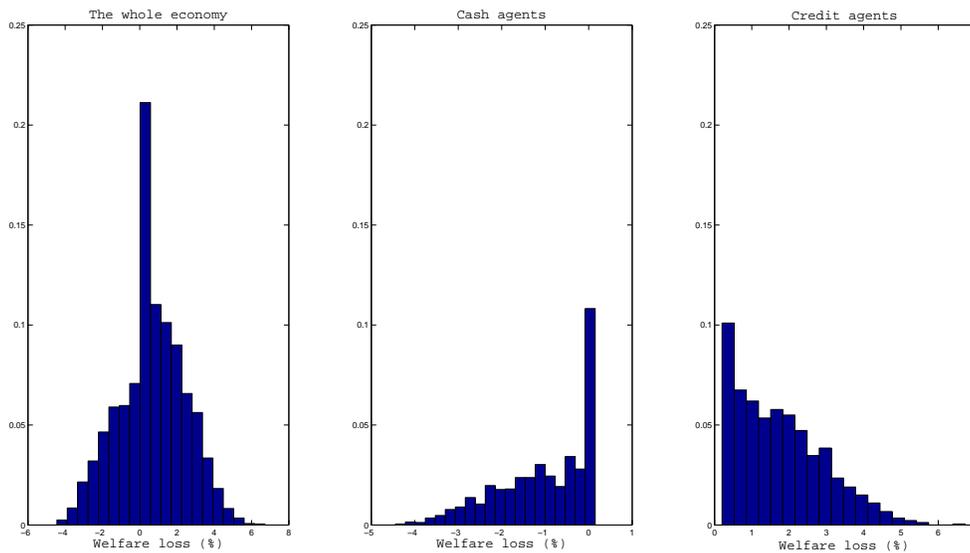


Figure 1.5.4: The distribution of welfare loss of -1.2% inflation compared to 3%

decomposition of welfare loss in terms of wealth quartiles implies a redistributive channel among each wealth group. More specifically, at -1.2% inflation target relative to 3%, the government actually redistributes resources from the poor, the lower middle and the rich to the upper middle class.

What if the government just discards all the inflation tax collection or uses it only to pay for the government spending? In this way, neither group gets the lump-sum transfer. With the tax collection discarded, the whole economy suffers a welfare loss of 2.02%. The right-half of Table 1.5.2 shows the detailed decomposition of welfare loss of -1.2% inflation compared to 3% when the government is wasteful. On average, the whole economy has welfare gains because the welfare improvement for cash agents from positive returns on money outweighs the loss for credit agents due to deteriorating credit conditions. As far as the decomposition by wealth quartiles is concerned, the qualitative differences between a benevolent government and a wasteful one lie in the bottom half of the whole economy and the first quartile in the group of credit agents. A wasteful government acts as a double-edged sword when it comes to inflation tax. On one hand, the government discards the inflation collection levied from cash agents when inflation is positive. On the other hand, it doesn't levy inflation tax any longer on either cash or credit agents when inflation is negative.²⁴ Therefore, after the lump-sum tax is removed at the Friedman rule inflation, only the positive effects analyzed above come into play and the three groups just mentioned (the bottom half of the whole economy and the first quartile in the group of credit agents) gain.

The welfare analysis above has important policy implications for inflation targeting policies that may have some influence on aggregate or group welfare. In recent years, the Board of Governors of the Federal Reserve has been conducting monetary policy to maintain an inflation rate of 2 percent over the medium to long term. Naturally, the question is: why 2 percent? Among others, social welfare is one way to go. Figure 1.5.3 shows a very tiny welfare loss (0.006%) of 2% inflation. The negligible welfare loss can justify in some sense

²⁴Neither group receives the equal lump-sum transfer under a wasteful government. When inflation is negative, the transfer actually becomes tax.

the practice that the Fed has been aiming at 2% inflation target in the past years. In addition, the debt limit with 2% inflation reaches 94.2% of that with 3% inflation, and 93.9% of the maximum debt limit with 4% inflation. A loose enough credit market also makes 2% a relatively desirable target.

1.5.3 Distributional implications

In this subsection I present the distributional implications of inflation targeting, with the focus on wealth inequality. Figure 1.5.5 shows the Gini coefficient of wealth for the whole economy, the group of credit agents and the group of cash agents, respectively. In the benchmark economy, the economy-wide Gini coefficient for wealth is 0.72, compared to 0.65 - 0.79 in the U.S. data. The blue upward sloping curve implies that inflation does worsen wealth inequality. In the “active zone”, the Gini coefficient rises from 0.65 to 0.80. For the group of cash agents, inflation targeting has a nonmonotonic impact on the within-group inequality, which widens the inequality very slightly and then narrows it down starting from 2% inflation.²⁵ By comparison, inflation has almost no effect on the within-group inequality of credit agents. When inflation is high, higher inflation makes money holdings uniformly converge to 0, thus making the money distribution more concentrated and reducing cash inequality largely. As money holdings converge and bond holdings disperse, with a relatively stable distribution, wealth gaps widen significantly with inflation.

Next, I report the equilibrium per capita wealth level by the quartiles of wealth. With regard to cash agents, the wealth (in the form of money) declines with inflation because the opportunity cost of precautionary savings rises with inflation. Despite the lower value of money, the availability of lump-sum transfers and the sale of current-period endowment provide cash agents with some opportunity to hedge against idiosyncratic income risks. These

²⁵With 13% inflation, each cash agent’s money holding approaches to zero, so the inequality is very close to zero.

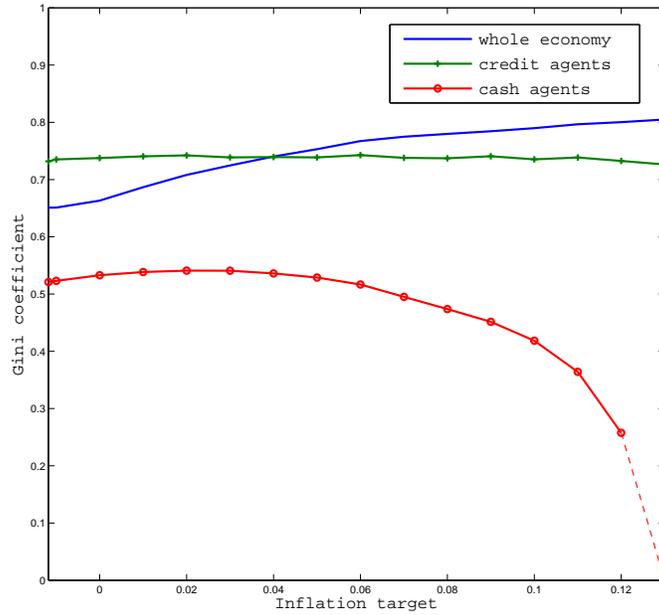


Figure 1.5.5: Impact of inflation targets on wealth inequality

factors, taken together, reduce the incentives to hold money. In particular, the steepest decline occurs in the top quartile of agents with a small departure from the Friedman rule inflation. Because most of the precautionary savings are concentrated among the money-rich people (Q1 in the middle figure in Figure 1.5.6), they suffer the largest wealth decline when inflation rises. For credit agents, the wealth (in the form of bonds) takes on a humped shape w.r.t. inflation in the top two quartiles, while the pattern is reversed in the bottom two quartiles. People in the top two quartiles change their bonds positions according to the profitability of those bonds, which is determined by real interest rates with the humped shape in Figure 1.5.1. Debtors in the bottom two quartiles would like to borrow as much as possible to smooth consumption. Therefore, the intertemporal decisions of those debtors depend largely on the amount of risk sharing or the maximum borrowing capacity. As a consequence, endogenous debt limits play a key role in wealth positions. Economy-wide wealth levels by wealth quartile are shown in the left figure in Figure 1.5.6. The wealth in the top two quartiles declines with inflation and the wealth in the second quartile is hardly

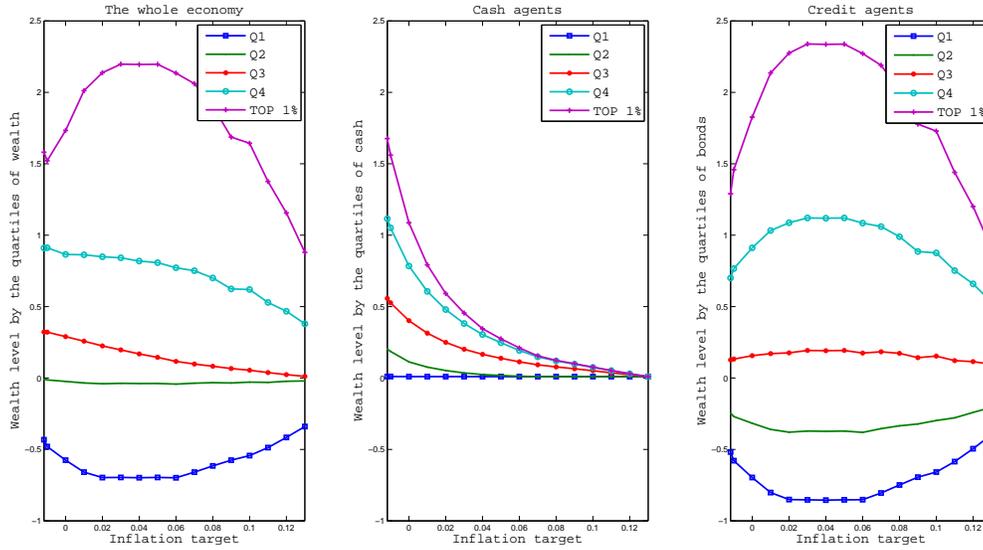


Figure 1.5.6: Impact of inflation targets on the wealth level

affected by inflation. Since all the people in the bottom quartile are debtors, the wealth position is purely determined by the first quartile of credit agents, as shown in the right figure in figure 1.5.6.

1.5.4 The role of market incompleteness

We now examine the role of market incompleteness through which inflation targets affect the credit market performance and social welfare. Suppose credit agents can trade a complete set of financial assets, i.e., state-contingent bonds, but they still have limited commitment to financial contracts. As in the benchmark, in order to ensure no default in equilibrium, credit agents are subject to the same punishment that they will be excluded from the credit market forever and reduced to cash agents if repudiating their debts. By Alvarez and Jermann (2000), the trading positions of the Arrow securities are bounded below by state-contingent limits. By trading the complete set of assets, credit agents now solve their problems as follows:

$$v^{cr}(e, a(e)) = \max_{\{c^{cr}, a'\}} u(c^{cr}) + \beta E v^{cr}(e(e'), a'(e; e')), \quad (1.5.1)$$

$$\text{s.t. } c^{cr}(e, a(e)) + \sum_{e'=\{e^l, e^h\}} q(e; e') a'(e; e') = e + a(e) + \gamma \frac{M}{1 + \pi}, \quad (1.5.2)$$

$$\text{and } v^{cr}(e, a) \geq v^{def}(e) \quad \forall e \in \{e^l, e^h\}. \quad (1.5.3)$$

Inequalities (1.5.3) can be expressed by two endogenous borrowing constraints:

$$a'(e; e^l) \geq \underline{a}(e^l) = \min \{a'(e^l) : v^{cr}(e^l, a') \geq v^{def}(e^l)\},$$

$$a'(e; e^h) \geq \underline{a}(e^h) = \min \{a'(e^h) : v^{cr}(e^h, a') \geq v^{def}(e^h)\}.$$

In equilibrium, the Arrow securities net out to zero:

$$\int \int \sum_{e'} q(e; e') a'(e, a; e') d\mu(e, a) = 0$$

and no-arbitrage condition implies:

$$q(e; e') = \frac{pr(e'|e)}{1 + r}$$

I solve this model using the same parameter values as in the benchmark. Figure 1.5.7 shows the debt limit / income ratio implied by the model with a complete set of assets. Accordingly, the debt limits become state-contingent and higher limits correspond to high shocks. As long as the state-contingent bond positions are above those corresponding limits, credit agents prefer to repay their state-contingent claims next period. The debt limits given tomorrow's shock being high are uniformly larger than those given low shocks because agents with high shocks tomorrow have less incentive to default. Compared to the benchmark, state-

contingent debt limits are averagely four times as large as the noncontingent debt limits. Intuitively, having access to the credit market with a complete set of financial assets is more attractive to agents than with an incomplete set of assets, given persistent and volatile income process. Therefore, credit agents with a complete set of assets have less incentive to default and thus enjoy larger debt limits. With a complete set of assets, the optimal inflation drops from 3% to 0. That is, social welfare peaks at zero inflation. One major role of inflation targeting with incomplete markets is to alleviate frictions in credit markets and thus improve social welfare by inflating moderately the economy. In the benchmark, there are two sources of frictions in credit markets: an incomplete set of financial assets and limited commitment to financial contracts. By introducing a complete set of financial assets, the first source of frictions is shut down. With fewer frictions, monetary policy doesn't have to inflate the economy as much as in the benchmark to maximize social welfare. If we measure financial sophistication by the degree of market completeness,²⁶ this result implies an economy with a higher level of financial sophistication should target a lower inflation rate, which coincides with the fact that developed countries tend to set lower inflation targets than developing countries.

1.5.5 A weaker default punishment with re-entry

The severity of default punishment plays a crucial role in determining the debt limit. In the benchmark, the punishment of permanent exclusion from the credit market makes borrowers have very weak incentives to renege on their debts. This accordingly generates a relatively high debt limit compared to data. Following Azariadis et al. (2015) and Bai and Zhang

²⁶Admittedly, this measurement is polarized because an economy has either low financial sophistication (with an incomplete set of financial markets) or high financial sophistication (with a complete set of financial assets) according to this measurement. There are no levels of financial sophistication in between. For simplicity, I assume developed countries have a complete set of financial assets while developing countries don't. By keeping everything else the same, I am interested in whether the level of financial sophistication can provide a potential explanation for the significant difference in inflation targets between developed countries and developing countries.

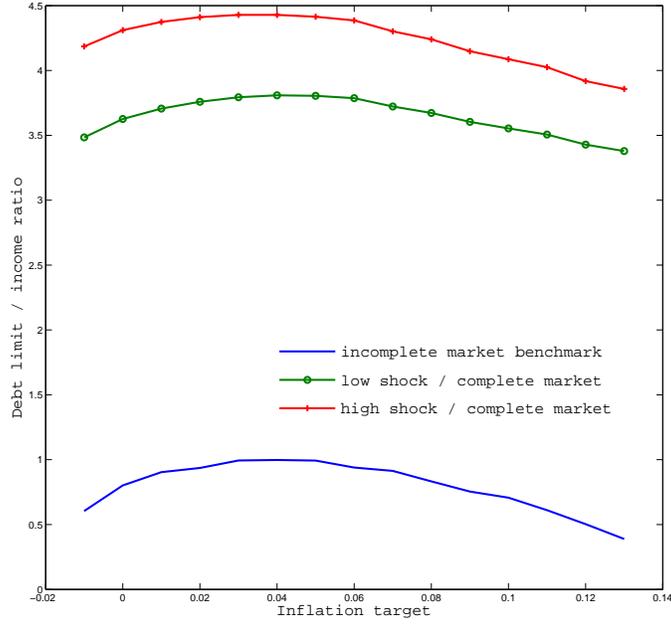


Figure 1.5.7: Debt limit: benchmark vs state-contingent bonds model

(2010), I introduce the probability of re-entering the credit market after default. Assume the re-entry probability is φ and defaulters have total debt relief and just receive zero amount of money. In other words, defaulter's initial bond position is zero if they can re-enter and their initial money holding is also zero. Therefore, the default value becomes:

$$V^{def}(e) = V^d(e, m_{min})$$

where

$$V^d(e, m) = \max_{\{c, m'\}} u(c) + (1 - \varphi) \beta EV^d(e', m') + \varphi \beta EV^{cr}(e', 0)$$

$$s.t. \quad c + m' = e + \frac{m}{1 + \pi} + tr$$

Table 1.5.3: Benchmark vs re-entry

	Benchmark	Re-entry
M1/GNP	16.0%	17.4%
ratio of credit constrained	15.8%	14.6%
real interest rate	2.3%	2.3%
Gini coefficient for wealth	0.72	0.64
optimal inflation rate	3%	2%
percentage of cash agents	0.40	0.40
debt limit / income ratio	0.99	0.33
government bonds / GDP	0	0.62
welfare loss	0.61%	0.32%

NOTE: “welfare loss” refers to the welfare loss of -1.2% inflation as opposed to 3%.

$$\text{and } c \leq \frac{m}{1 + \pi} + e$$

By targeting the debt limit / income ratio to 0.33 in data, the re-entry probability is set to 0.71. This means those defaulters, on average, are excluded from the credit market for 1.4 years, which is a very lenient punishment when compared to the permanent exclusion and thus generates a lower debt limit. According to the analysis in section 1.5.1, lower debt limits lead to lower real interest rates. In order to ensure $r = 2.3\%$ and all the parameters are the same as in the benchmark, the only way is to raise the level of aggregate bond supply, which is zero in the benchmark. In this endowment economy without considering fiscal policy, we can treat the government bonds / GDP ratio B as another parameter.²⁷ When $B = 0.62$,²⁸

²⁷In order to focus on the effect of pure monetary policy, I don't consider any tax levied by fiscal policy to balance the government budget. For simplicity, I assume the government prints additional money to pay for interest payments on government bonds and thus to balance its budget. Those additional money don't go to cash agents' hands but to credit agents who hold government bonds in the form of consumptions goods in equivalent values.

²⁸This number is close to the average ratio of the U.S. federal plus U.S. state debt divided to GDP over the postwar period, 0.67, in Aiyagari and McGrattan (1998).

$r = 2.3\%$. Table 1.5.3 shows the comparisons between the benchmark and re-entry model. First, introducing the probability of re-entry and positive aggregate bond supply doesn't affect cash agents' behavior. Second, a looser punishment does increase default incentive and thus lower the debt limit. Third, a tighter debt limit doesn't make more people credit constrained. On the contrary, it reduces the number of people who get credit constrained, because with positive aggregate bond supply, the bond distribution shifts to the right and more credit agents choose to hold a positive amount of bonds to better hedge against income risks. Lastly, the welfare loss of -1.2% inflation compared to 3% inflation in the re-entry model is only half of that in the benchmark. In the re-entry model, the debt limit under -1.2% inflation is 0.26, 21% lower than that under 3% inflation, while in the benchmark, the debt limit under -1.2% inflation is 42% lower than that under 3% inflation. Recall from section 1.5.2 that the debt limit serves as a major driving force behind the welfare of credit agents. Therefore, the smaller change in debt limits with the re-entry model causes smaller welfare losses.

1.6 Sensitivity Analysis

In this section I conduct sensitivity analysis to see how sensitive our major results are to the changes: 1. in the following parameters such as discount factor β , risk aversion σ , the persistence of income process ρ and the volatility of income process ε ; 2. in the severity of default punishment. In the model, by exposing agents to larger income risks or making them more sensitive to risks, these parameters influence the amount of money / bond that agents want to hold in order to hedge against income risks and smooth consumption. As a consequence, the optimal inflation rate, the corresponding welfare loss and distributional implications are also affected by these parameters.

We start with ρ . Consider a cash agent who is lucky enough to receive a consecutive series of high income shocks. He / she tends to accumulate more money holdings than some

unlucky cash agent who receive low income shocks in a row. When ρ rises, then the role of buffer stock of money is less important to the lucky guy and he / she starts to cut down on money holdings. In aggregate, the equilibrium real balance is reduced and average welfare increases, which raises the continuation value of default for a credit agent. Higher income persistence and larger outside option values make the credit market less important for credit agents as a hedge against income risks. This raises incentives to default and thus lowers debt limits. Smaller debt limits, in turn, reduce the welfare of credit agents.

With regard to ε , a greater risk exposure makes cash agents desire to hold more money as a buffer stock and thus increases aggregate real balances. This leads to lower default values. Credit agents now have weaker incentives to default, which raises debt limits. Higher debt limits and greater risk exposures are two competing effects on credit agents. When inflation is small, the negative effect of risks is dominant and credit agents' welfare decreases in risk exposures. When inflation is moderately large, the positive effect of relaxed debt limit dominates and the welfare increases when risk exposure does.

With larger risk aversion σ , cash agents want a smoother consumption profile, which stimulates the precautionary saving in the form of money. A smoother consumption stream makes cash agents better off. For credit agents, on one hand, the higher outside option value tempts them to default. On the other hand, they are more risk sensitive and thus rely more on credit market to smooth consumptions, which restrains their incentives to default. Since the risk aversion coefficient governs credit agents' behavior more directly and prominently, the latter effect outweighs and the debt limit goes up. When β is larger, people value future consumption more and accumulate more assets in either money or bond. Credit agents accordingly rely more on the credit market and become reluctant to default. This generates a larger debt limit.

The remainder of this section reports the quantitative effects of all these parameters. First, I conduct a sensitivity analysis by changing only one parameter at a time, as shown in Table 1.6.1. The result of this exercise confirms the narrative analysis above and separates

clearly the role of each parameter in influencing aggregate and distributional variables. Next, in order to make the comparison more meaningful, I study the effects of changing parameters ε , σ and ρ , by simultaneously adjusting β to ensure that equilibrium real interest rate is 2.3% with the optimal inflation rate to be 3% as in the benchmark case (whenever possible).

From Table 1.6.1, we can see in details the effect of each parameter. First of all, real interest rate is very sensitive to β and ρ . Other parameters have only very marginal impact on it. For other endogenous variables (debt limit, real balance, Gini coefficient of wealth and the range of “active zone”), the effects of these parameters show very clear patterns. As far as the optimal inflation rate is concerned, multiple competing effects of these parameters including ω make it complicated to tell whether there is a clear pattern. From Table 1.6.1, however, we see the optimal inflation target increases in risk aversion and income risk volatility and decreases in discount factor and the persistence of income process. For the sake of simplicity, I summarize the qualitative effects in Table 1.6.2.

Now I conduct the sensitivity analysis by adjusting β and ω to keep r at 2.3% and the optimal inflation at 3% whenever possible.²⁹ When controlling the general equilibrium effect caused by real interest rate, I arrive at the conclusions safely on the influence of these parameters and summarize them in Table 1.6.3, which are consistent with the results in Table 1.6.2. The sensitivity analysis above is illustrative in some reasons. First, β and ω don’t change much to ensure real interest rate and the optimal inflation the same as benchmark. In particular, ω , if adjusted, always lie in the range between 0.3 and 0.46, which is consistent with data: [0.3, 0.55]. Second, the debt limit / income ratio under 3% optimal inflation is around 1, with some quantitative disparity under different parameters, and more importantly, the qualitative effect of inflation on debt limit is unchanged regardless of parameter choices. Third, the Gini coefficient of wealth is insensitive to any parameter.

²⁹When $\rho = 0.9$, the maximum optimal inflation is 2%. To make comparisons more meaningful, we choose another parameter value $\rho = 0.6$ with which we can ensure $r = 2.3\%$ and $\pi^* = 3\%$ by adjusting β and ω .

Table 1.6.1: Quantitative effect of each parameter

	π	r	A	A_{max}	M	rcc	$Gini_{wealth}$	welfare loss	active zone
benchmark	3%	2.3%	0.99	1.00	0.16	15.8%	0.72	0.61%	[-1.2%, 13%]
$\sigma = 1.5$	3%	2.2%	1.13	1.23	0.22	13.3%	0.71	0.95%	[-1%, 18%]
$\sigma = 2$	3%	2.1%	1.36	1.69	0.33	10.8%	0.69	1.43%	[-0.7%, 25%]
	★5%	2.3%	1.50	1.69	0.25	9.6%	0.72	1.46%	[-0.7%, 25%]
$\beta = 0.95$	3%	3.0%	0.57	0.59	0.11	23.5%	0.72	0.31%	[-2.2%, 12%]
$\beta = 0.98$	3%	1.6%	1.75	1.89	0.22	8.0%	0.73	1.10%	[-0.6%, 15%]
	★2%	1.5%	1.63	1.89	0.29	8.8%	0.71	1.11%	[-0.6%, 15%]
$\varepsilon = 0.2$	3%	2.3%	0.69	0.69	0.09	19.3%	0.74	0.34%	[-1.5%, 10%]
	★1%	2.3%	0.68	0.694	0.16	19.3%	0.70	0.34%	[-1.5%, 10%]
$\varepsilon = 0.3$	3%	2.3%	1.38	1.48	0.27	12.6%	0.71	1.15%	[-0.9%, 18%]
$\rho = 0$	3%	3.2%	1.54	1.62	0.18	1.8%	0.73	0.33%	[-2.3%, 29%]
	★18%	3.1%	1.31	1.62	0.06	2.4%	0.79	0.81%	[-2.3%, 29%]
$\rho = 0.5$	3%	2.5%	1.04	1.10	0.17	11.4%	0.72	0.58%	[-1.5%, 15%]
	★4%	2.6%	1.09	1.10	0.14	10.8%	0.74	0.60%	[-1.5%, 15%]
$\rho = 0.9$	3%	0.5%	0.13	0.30	0.01	48.2%	0.79	0.02%	[-1.2%, 2%]
	★0%	1.0%	0.23	0.30	0.04	44.4%	0.71	0.08%	[-1.2%, 2%]

NOTE: the row starting with “★” indicates the optimal inflation rate has been changed as opposed to 3% in the benchmark case. All the variables in that row are the corresponding equilibrium values with the new optimal inflation. The first row of each parameter value (without “★”) shows the equilibrium values of each variable still with 3% inflation, given just one parameter change. To make comparisons consistent, “welfare loss” refers to the welfare loss of -1.2% inflation compared to the indicated inflation (either 3% or the new optimal inflation).

Lastly, the welfare loss don't change much with different parameters.

Table 1.6.2: Qualitative effects of each parameter

	r	A	M	rcc	$Gini_{wealth}$	welfare loss	active zone
σ	-	↗	↗	↘	↘	↗	widen and shift to the right
β	↘	↗	↗	↘	↗	↗	widen and shift to the right
ε	-	↗	↗	↘	↘	↗	widen and shift to the right
$\rho > 0$	↘	↘	↘	↗	↗	↘	narrow and shift to the left

NOTE: I consider the effects given an increase in these parameters.

1.7 Alternative Monetary Policy: Open Market Operations

Recall that in the benchmark experiment , there are no government bonds and all private debts sum up to zero. This section presents the results from the experiment in which the central bank conducts open market operations by selling or buying government bonds to achieve some monetary target. With government bonds considered, the government budget should be balanced in terms of units of consumption goods:

$$G_t + (1 + r) B_t + \frac{M_t}{1 + \pi} = T_t + B_{t+1} + M_{t+1}, \quad (1.7.1)$$

where The law of motion for the aggregate real balance supply is still:

$$M_{t+1} = \frac{M_t}{1 + \pi} + \gamma \frac{M_t}{1 + \pi}.$$

Table 1.6.3: Sensitivity analysis

	β	ω	A	M	rcc	$Gini_{wealth}$	welfare loss	active zone
benchmark	0.967	0.40	0.99	0.16	15.8%	0.72	0.61%	[-1.2%, 13%]
$\sigma = 1.5$	0.966	0.40	1.13	0.22	13.4%	0.71	0.76%	[-1%, 18%]
$\sigma = 2$	0.963	0.46	1.22	0.31	12.2%	0.69	1.10%	[-0.9%, 25%]
$\varepsilon = 0.2$	0.966	0.30	0.69	0.09	19.3%	0.74	0.50%	[-1.5%, 10%]
$\varepsilon = 0.3$	0.967	0.40	1.380	0.27	12.6%	0.71	1.15%	[-0.9%, 18%]
$\rho = 0.5$	0.970	0.40	1.19	0.18	9.7%	0.72	0.66%	[-1%, 16%]
$\rho = 0.9$	0.950	0.40	0.13	0.01	47.2%	0.79	0.01%	[-2.8%, -1%]

NOTE: β and ω in the second and third columns are values (adjusted if necessary) that ensure $r = 2.3\%$ and $\pi^* = 3\%$ in the new equilibrium if possible (in the case with $\rho = 0.9$ and $r = 2.3\%$, social welfare increases in inflation from -2.8% to any positive number and thus there does not exist an optimal inflation rate). To make comparisons consistent, “welfare loss” refers to the welfare loss of -1.2% inflation compared to 3% inflation.

In a stationary equilibrium, (1.7.1) can be rewritten as:

$$G + rB = T + \frac{\pi}{1 + \pi}M.$$

I assume $G = T$, which implies that the interest payments on government bonds are completely financed by the inflation tax collection. With this assumption, the results can get rid of the effects of fiscal policy on welfare and credit market and focus on the effects of monetary policy implemented by OMO. Then we have:

$$rB = \frac{\pi}{1 + \pi}M. \quad (1.7.2)$$

Now the government just uses seigniorage revenues collected from cash agents to pay interest on government bonds held by credit agents. Compared to the equal lump-sum

transfer in the benchmark experiment, what the government actually does through OMO is transfer resources from cash agents to credit agents. The budget constraints for both agents change accordingly:

$$\text{cash agents : } c^{ca} + m' = e + \frac{m}{1 + \pi}$$

$$\text{credit agents : } c^{cr} + \frac{a'}{1 + r} = e + a$$

The following part of this section reports the counterpart quantitative results and compare them with benchmark results caused by monetary policy of helicopter drop. First, I will use the same parameter values as in the benchmark; second, to control for the general equilibrium effect caused by real interest rate, I adjust β and ω (if necessary) to ensure $r = 2.3\%$ and $\pi^* = 3\%$ as in the benchmark. Table 1.7.1 reports major aggregate and distributional variables implied by two monetary policies. The two experiments play different roles in assessing the two regimes of monetary policies. When comparing the effects of inflation on welfare, I choose the first one (OMO 1); when comparing the effects of inflation on credit market, I choose the second one (OMO 2).³⁰

1.7.1 Implications for credit markets

To begin with, I present the differences in credit markets resulted from OMO 2 and the benchmark. By controlling real interest rate and the optimal inflation, I can safely compare the performance of credit markets based on different policies. The equilibrium debt limit reaches 1.111 by implementing open market operations as opposed to 0.994 in the benchmark. The aggregate bond holdings, which equal to the supply of government bonds, amount to

³⁰Since there is no significant change in wealth inequality, I skip the comparisons on the distributional implications.

Table 1.7.1: Open market operations vs helicopter drop

	OMO1	OMO 2	Benchmark
r	2.5%	2.3%	2.3%
π^*	3%	3%	3%
A	1.01	1.11	0.99
M	0.15	0.16	0.16
rcc	12.8%	11.5%	15.8%
$Gini_{wealth}$	0.72	0.72	0.72
B	0.18	0.21	0
welfare loss	0.13%	–	–

NOTE: OMO 1 refers to the first experiment on open market operations in which all the parameters keep the same as in the benchmark; OMO 2 refers to the second experiment in which $\beta = 0.97, \omega = 0.28$ to ensure $r = 2.3\%, \pi^* = 3\%$.

0.21 as opposed to 0 in the benchmark. With the introduction of government bonds, credit agents now can borrow more than in the case with lump-sum transfers. The reason lies in the changes in the incentive to default. Government bonds essentially serve as a way of transferring resources from cash agents to credit agents and thus it tends to make the value of solvency more desirable by relaxing the participation constraint. With less incentives to default, credit agents are able to enjoy a looser borrowing constraint. As in the benchmark, the debt limit implied by OMO 2 also takes on a humped shape w.r.t. inflation. In other words, small inflation relaxes credit limits and large inflation tightens them. The left panel in Figure 1.7.1 shows the impact of inflation on debt limits. When the government conducts open market operations to maintain inflation in this model, the supply of government bonds is endogenously determined by credit agents' aggregate demand for them. From the right panel of Figure 1.7.1, I conclude that implementing expansionary monetary policy over small inflation rates stimulates the demand for government bonds, which as a consequence raises their value. In contrast, expansionary monetary policy does the opposite for large inflation rates.

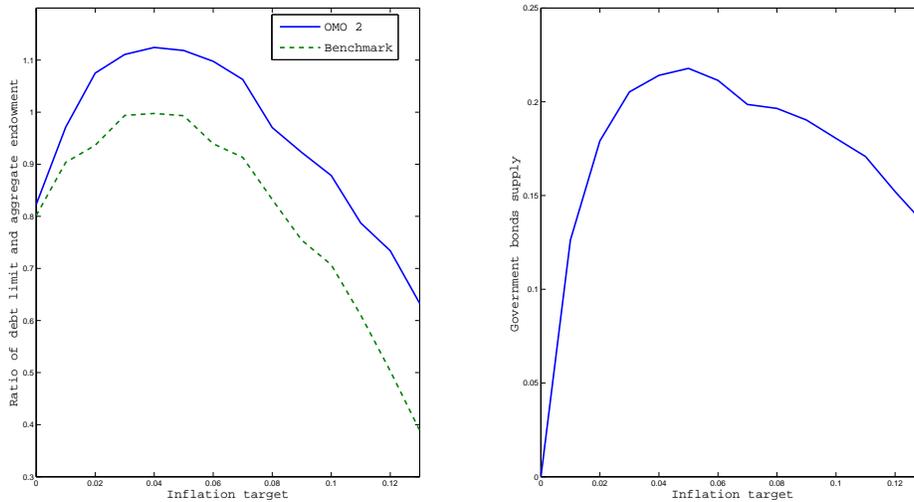


Figure 1.7.1: The impacts of inflation on credit market: OMO 2

1.7.2 Welfare implications

We know that the monetary policy of helicopter drops is less distortive than open market operations. Does this mean the economy will suffer a loss when the monetary policy is changed from the helicopter drop to OMO? The short answer is not necessarily. To answer this question more precisely, I still apply the same measurement of welfare loss as in the benchmark to each group and then the whole economy. Table 1.7.2 reports the welfare loss of the policy change at each inflation rate (negative values still mean welfare gains). As mentioned above, OMO essentially transfers resources from cash agents to credit agents by repaying interest payments to government bond holders. Intuitively, this benefits credit agents at the cost of cash agents, compared to the equal lump-sum transfer in the benchmark. As Table 1.7.2 shows, cash agents suffer losses while credit agents make gains at each inflation. Since cash agents populate 40% of the whole economy, the economy-wide welfare loss is a little complicated. The aggregate economy loses at some inflation rates and benefits at other inflation rates. Simply speaking, introducing government bonds improves welfare when

inflation is very small or relatively large, while it depresses welfare when inflation is moderate. In particular, the welfare loss of switching from 3% inflation in the benchmark to 3% under the case of OMO 1 is 0.13%; the welfare loss of switching from 3% inflation in the benchmark to 2% (the new optimal target) under the case of OMO 1 is 0.09%. Either way implies a welfare loss if the government conducts OMO rather than simply equal lump-sum transfer.

1.8 Conclusions

In this paper, I develop a dynamic stochastic general equilibrium model in an environment featuring market incompleteness, limited commitment and two assets (money and bond). With two segmented groups, cash and credit agents, this paper studies the impact of a range of inflation rates on the credit market, social welfare, and wealth inequality. Based on the quantitative results implied from the calibrated model, I answer the questions such as: Is 2% inflation target a good one for the U.S. economy? Why do developed countries keep lower inflation targets than developing countries? Does inflation hurt the rich or the poor? What potential harm will deflation do to the economy?

Previous studies either predict the negative or positive effect of inflation on the credit market activity. This paper manages to arrive at the comprehensive conclusion about the effects of inflation on the credit market. As a major indicator for the credit market activity, the debt limit shows a humped shape followed by a flat tail w.r.t. inflation. This finding is consistent with the empirical observation on the relationship between credit market activity and inflation. Different from most of previous related literature in which inflation is welfare-decreasing, this paper predicts an inflation rate of 3% that maximizes social welfare by using the U.S. data. Inflation rates either smaller or larger than 3% cause welfare loss. Particularly, the Friedman rule inflation not only leads to a welfare loss as big as 0.61% in consumption goods units, but also brings about a severe credit condition in which debt limit

Table 1.7.2: Welfare loss of monetary policy change (%)

	0	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%
whole	-0.02	-0.06	-0.09	0.13	0.12	0.13	0.09	0.10	0.07	0.00	0.00	-0.07	-0.15	-0.23
cash	0.00	0.31	0.48	0.55	0.57	0.57	0.53	0.47	0.44	0.42	0.39	0.35	0.29	0.22
credit	-0.03	-0.11	-0.18	-0.16	-0.18	-0.17	-0.20	-0.14	-0.18	-0.29	-0.25	-0.34	-0.44	-0.53

NOTE: the welfare loss are computed by changing from helicopter drop to OMO 1 while keeping the inflation unchanged. So it is inflation target-wise welfare loss.

is only 59% as large as that under 3% inflation. The results on the welfare analysis as well as the credit market performance are also consistent with the practice that the Fed has been conducting monetary policy to help maintain a long-term inflation rate of 2%. In addition, this paper makes some preliminary investigations on wealth inequality. On the average, higher inflation results in higher wealth inequality. In terms of the Gini coefficient, it concentrates around 0.7. When the inflation deviates from either 3% (model optimum) or 2% (real target) inflation to any small deflation, both the rich and the poor will lose and the upper group in the middle class will gain.

When credit markets are imperfect, inflation targeting becomes important. Moderate inflation alleviates frictions in credit markets and thus improves social welfare through the inflation-credit channel studied in the paper. Because both real and financial activities largely depend on credit market conditions, inflation targeting can have a significant impact on production, household saving / consumption decisions, portfolio choice and even international capital flows, based on this inflation-credit channel. For example, the general framework of this paper, if extended to a two-country open economy model (the U.S. and emerging markets, EM), can explain the U.S. negative net position in bond instruments and positive net position in portfolio equity and FDI (risky assets). Credit agents in each country hold a portfolio of risk-free bonds and risky productive assets to hedge against idiosyncratic income risks. With financial integration, credit agents can trade both kinds of assets globally. The only difference between the two countries is that the U.S. maintains a lower inflation target than EM. Calibrated to recent U.S. data, the model generates a higher debt limit for the U.S., where agents can borrow more than those in EM. With zero net bond supply in the world, the U.S. borrows from EM. This explains the U.S. negative net position in bonds. On the other hand, U.S. credit agents have better risk sharing due to a larger debt limit. Therefore, the U.S. credit agents' consumption is less volatile than that of their counterparts, which leads to a smaller covariance between the rate of return on risky assets and tomorrow's consumption and thus a lower risk premium on risky equity. With

globally integrated financial markets, the U.S. credit agents are more willing to hold those risky assets and in effect buy them from EM. This explains the U.S. positive net position in portfolio equity and FDI.

Chapter 2

Inflation Targeting and Global Imbalances

2.1 Introduction

2.1.1 Motivation and recent related literature

From the year of 1980 to 2012, the U.S. external debt increasingly accumulated from 4.3% to 33.3% of its GDP. On the other hand, the U.S. increased its position in net portfolio equity and foreign direct investment (FDI) from 0.7% to 2.9% of its GDP during the same period (See Figure 2.1.1).

A large number of literature pays attention to the sustainable and unprecedented global imbalances and explains this phenomenon. Differences in business cycle volatility or productivity, a “global saving glut” or “valuation effect” are potential causes for global imbalances. Among others, valuation effect refers to that the valuation of US net foreign assets has a stabilizing effect on the current account and thus positive net investment income flows to the US, as suggested in Hausmann and Sturzenegger (2006), Gourinchas and Rey (2007a,b) and Pavlova and Rigobon (2010). This paper is in line with the literature that focuses on the asymmetry in financial sectors: the asymmetry of the supply of assets (Caballero et al.

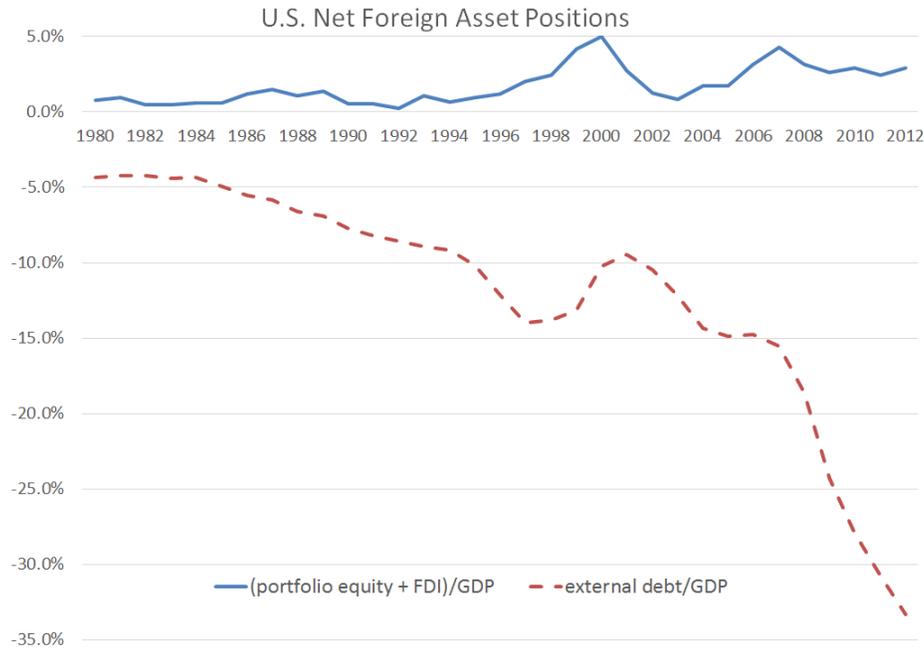


Figure 2.1.1: The U.S. external debt and portfolio equity plus FDI

(2008) and Pavlova and Rigobon (2010)), the asymmetry of idiosyncratic shocks (Angeletos and Panousi (2011)), or the asymmetry in financial development (Mendoza et al. (2009), Maggiori (2011), Chien and Naknoi (2015)).

This paper is closely related to Mendoza et al. (2009), Chien and Naknoi (2015) and Zhang (2015). Mendoza et al. (2009) develop a model to explain that differences in financial development affect the position and composition of a country’s net foreign assets. Chien and Naknoi (2015) construct a stochastic growth multi-country model in which agents face heterogeneous trading technologies and thus heterogeneous household portfolio choices within a country and across countries offer an explanation for global imbalances. Zhang (2015) sets up a model to study quantitatively how inflation targeting interacts with imperfect credit markets. In his paper, moderate inflation maximizes credit market activities and social welfare. Based on the previous closely related literature, this paper studies how monetary policy of inflation targeting affects global imbalances and explains the U.S. negative position in bonds and positive position in portfolio equity and FDI. The differences in financial development

is endogenously determined by each country's monetary policy.

2.1.2 Main results

I extend the model in Zhang (2015) to a two-country open economy model consisting of the U.S. and emerging markets (EM). Credit agents in each country hold a portfolio of risk-free bonds and risky productive assets to hedge against idiosyncratic income risks. Financial integration allows credit agents to trade both kinds of assets globally. In the benchmark, the only difference between the two countries is that the U.S. maintains a lower inflation target than EM. Calibrated to recent U.S. data, the model generates a higher debt limit for the U.S., where agents can borrow more than those in EM. With zero net bond supply in the world, the U.S. borrows from EM. This explains the U.S. negative net position of bonds. On the other hand, U.S. credit agents enjoy better risk sharing due to a larger debt limit. Therefore, the U.S. credit agents' consumption is less volatile than that of their foreign counterparts. This leads to a smaller covariance between the return from risky equity and tomorrow's consumption, and thus the U.S. credit agents require a lower risk premium on risky equity. As a result, they value those risky assets more highly and in effect buy them from EM.

The paper is organized as follows. In section 2.2, I present the model, define the recursive general equilibrium and explore the core channel that explains the U.S. net foreign asset holdings. In section 2.3, I first show the benchmark results and then extended results with larger income volatility for emerging markets. The last section concludes the paper.

2.2 Model Environments and Channels

2.2.1 Environments

This paper studies a two-country open economy with two units of consumption goods and two units of productive assets in the world.¹ The world economy consists of two economies: the U.S. and emerging markets. The only difference between the two economies is that the U.S. implements monetary policy to keep a lower inflation rate.² For the sake of convenience, I only describe the model environment in the U.S. hence after. The model environment in emerging markets is identical except its inflation rate. In the U.S. economy, a continuum of agents receive idiosyncratic income shocks to their endowments each period over an infinite horizon. At each period, an agent receives an endowment e_t , which is perishable consumption goods. The income process $\{e_t\}$ is independently and identically distributed across agents and follows a two-state first order Markov process over time with $e \in \{e^l, e^h\}$. The stationary transition probability is denoted by $pr(e'|e) = \Pr[e_{t+1} = e' | e_t = e] > 0$.

As in Zhang (2015), markets are segmented with cash agents and credit agents. Cash agents can only hold cash for two purposes: purchase of consumption goods and self-insurance. By comparison, credit agents can hold risk-free bonds and non-negative risky productive assets. By trading risk-free bonds, credit agents can borrow but cannot commit to repay and hence might renege on their debts. Defaulters are excluded from credit markets forever and reduced to cash agents with the minimum money holding.³ No credit agents default in equilibrium due to the participation constraint with which the solvency value is no less than the default value. In this way, inflation affects credit agents indirectly by affecting their outside option value, which determines endogenous credit limits. Limited commitment generates endogenous debt limits within which debts are always repaid. This endogenous debt limit plays a crucial role in affecting risk-sharing capacity for credit agents.

¹I assume the initial position of productive assets for each country is the same. So per capita productive assets is one unit in each country.

²I will discuss later about adding other dimensions of economy-wide difference.

³In this paper, I focus on the default risk of agents rather than countries.

Risky productive assets are subject to idiosyncratic investment shocks. By holding those risky assets, credit agents require risky premium on them.

First, let us look at cash agents' problem:

Let c^{ca} and $m/(1 + \pi)$ denote start-of-period consumption and real balance for cash agents, where π is inflation rate. Assume agents carry over real balances from the previous period to the current period; the real value of money holdings is discounted by $1/(1 + \pi)$. Denote with a prime in the next period's variables.

I make an assumption on the timing arrangement for cash agents: they make decisions before receiving the lump-sum transfer from government. Cash agents purchase consumption goods subject to a modified cash-in-advance constraint. In addition to the money carried over from the previous period, they can also use extra money by liquidating their current endowment e . With the timing arrangement above, the endowment can change the CIA constraint but the transfer cannot.

Given the current state (e, m) and inflation rate π , cash agents choose $(c^{ca}, m') \geq 0$ to maximize expected life-time utility. The problem has a recursive representation:

$$v^{ca}(e, m) = \max_{\{c^{ca}, m'\}} u(c^{ca}) + \beta E v^{ca}(e', m'), \quad (2.2.1)$$

$$\text{s.t. } c^{ca} + m' = e + \frac{m}{1 + \pi} + tr, \quad (2.2.2)$$

$$\text{and } c^{ca} \leq \frac{m}{1 + \pi} + e, \quad (2.2.3)$$

$$m' \geq 0 \quad (2.2.4)$$

where $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, tr is lump-sum transfer from government to each agent. Assume all agents receive the same lump-sum transfer.

Next, let us look at credit agents' problem:

let c^{cr} , a and k denote start-of-period consumption, bond holdings and risky assets for credit agents. Given the current state (e, a, k) and a risk-free interest rate r and the price for risky assets p , they choose (c^{cr}, a', k') to maximize expected life-time utility subject to a participation constraint due to limited commitment on debts.⁴ The problem has a recursive representation:

$$v_i^{cr}(e, a, k) = \max_{\{c^{cr}, a', k' \geq 0\}} u(c^{cr}) + \beta E v_i^{cr}(e', a', k'),$$

$$\text{s.t. } c^{cr} + \frac{a'}{1+r} + k'p_i = e + a + kp_i + zk^\theta + tr,$$

$$\text{and } v^{cr}(e, a, k) \geq v^{def}(e) \quad \forall e \in \{e^l, e^h\} \quad (2.2.5)$$

where

$$v^{def}(e) = v^{ca}(e, \underline{m}).$$

and z is the idiosyncratic investment shock and thus zk^θ is the output produced from idiosyncratic projects operated by credit agents. In order to write the problem as a recursive form, I transform the participation constraint into a 'not-too-tight' endogenous borrowing constraint as in Alvarez and Jermann (2000). The major difference is that credit agents trade state-noncontingent bonds compared to state-contingent bonds in theirs. Due to the market incompleteness for credit agents, the continuation value of repaying debts must be no less than the default value in any future state. Hence, the endogenous borrowing limit is accordingly state-noncontingent in that it takes the tightest one among all possible states. Mathematically,

⁴I assume the risk-free bond a is unsecured and thus cannot be collateralized by k .

$$A = \min_{\{e\}} \{-a(e) : v^{cr}(e, a, k) = v^{def}(e)\}. \quad (2.2.6)$$

That is, A is the tightest debt limit that guarantees solvency in any future idiosyncratic income state.

Thus, the participation constraint (2.2.5) can be reduced equivalently to an endogenous borrowing constraint:

$$a' + A \geq 0. \quad (2.2.7)$$

As long as the amount of borrowing doesn't surpass the debt limit A , credit agents will guarantee to repay their debts.

2.2.2 Monetary policy

The central bank implements monetary policy by “helicopter drop” to maintain its long-term inflation target. It controls the growth rate γ of aggregate supply of money in real terms,⁵ denoted by M . For the time being, assume there is no government debt, taxes or spending. The law of motion for M is:

$$M' = \frac{M}{1 + \pi} + \gamma \frac{M}{1 + \pi}. \quad (2.2.8)$$

The quantity of newly issued money in real terms is $\gamma \frac{M}{1 + \pi}$, which is the lump-sum transfer, tr , equally transferred to each agent in both groups. We can see in stationary equilibrium, $\pi = \gamma$. In this paper, the central bank controls inflation so inflation is given

⁵Controlling the growth rate of aggregate money in real terms is equivalent to controlling that in nominal terms. In other words, the growth rate of aggregate money in nominal terms is also γ . To see this, we start with aggregate money supply in nominal terms, \tilde{M} . Suppose the central bank controls the growth rate γ of aggregate money in nominal terms. Then the law of motion for \tilde{M} is: $\tilde{M}' = (1 + \gamma)\tilde{M}$. Let $M' = \tilde{M}'/P$. By dividing P on both sides, we have

$$M' = (1 + \gamma) \frac{M}{1 + \pi}$$

, exactly same as equation 2.2.8.

exogenously.

2.2.3 General equilibrium

This subsection gives the definition of recursive general equilibrium. The state of a cash agent $(e, m) \in E \times M$, with $M = [0, \infty)$ and $E = \{e^l, e^h\}$. Let $\mathcal{P}(E)$ denote the power set of E , $\mathcal{B}(M)$ denote the Borel σ -algebra of M . Define the subset of possible states $(\mathbb{M}, \mathbb{E}) \subseteq \mathcal{B}(M) \times \mathcal{P}(E)$. Denote the set $S^{ca} = \mathbb{M} \times \mathbb{E}$. The state of a credit agent $(e, a, k) \in E \times B \times K$, with $B = [-A, \infty)$ $K = [0, \infty)$ and $E = \{e^l, e^h\}$. Let $\mathcal{B}(B)$ and $\mathcal{B}(K)$ denote the Borel σ -algebra of B and K . Define the subset of possible states $(\mathbb{B}, \mathbb{K}, \mathbb{E}) \subseteq \mathcal{B}(B) \times \mathcal{B}(K) \times \mathcal{P}(E)$. Finally denote the set $S^{cr} = \mathbb{B} \times \mathbb{K} \times \mathbb{E}$.

A stationary recursive equilibrium is defined by a set of policy functions $c^{ca}(e, m)$, $m'(e, m)$, $c^{cr}(e, a, k)$, $a'(e, a, k)$ and $k'(e, a, k)$; a set of value functions $v^{ca}(e, m)$ and $v^{cr}(e, a, k)$; a set of prices $\{\pi, r, p\}$; the debt limit A ; and two stationary distributions $\phi^{ca}(e, m)$ and $\phi^{cr}(e, a, k)$, such that:

1. Given prices $\{\pi_i, r_1 = r_2, p_i\}$ and debt limit A_i , the policy functions solve the household problem of the two groups; $v_i^{ca}(e, m)$ and $v_i^{cr}(e, a, k)$ are the corresponding value functions.
2. The debt limit A satisfies:

$$A_i = \min_{\{e\}} \left\{ -a_i(e) : v_i^{cr}(e, a, k) = v_i^{def}(e) \right\}. \quad (2.2.9)$$

That is, A is the tightest debt limit that guarantees solvency in any future state.

3. The bond market clears:

$$\sum_{i=1,2} \int \int \int a'_i(e, a, k) d\phi_i^{cr}(e, a, k) = 0. \quad (2.2.10)$$

where $i = 1$ is the U.S. and $i = 2$ is emerging markets.

4. The risky assets market clears:

$$\sum_{i=1,2} \int \int \int k'_i(e, a, k) d\phi_i^{cr}(e, a, k) = 2.$$

the initial risky asset position is 0.5 unit for each country.

5. The money market clears:

$$\int \int m'_i(e, m) d\phi_i^{ca}(e, m) = M_i.$$

6. By Walras' Law, the world-wide consumption goods market also clears automatically.

That is, the sum of expected value of consumption goods of both groups of agents in each economy, equals to the expected aggregate endowment of one unit:

$$\omega \sum_{i=1,2} \int \int c_i^{ca}(e, m) d\phi_i^{ca}(e, m) + (1 - \omega) \sum_{i=1,2} \int \int \int c_i^{cr}(e, a, k) d\phi_i^{cr}(e, a, k) = 2. \quad (2.2.11)$$

7. The policy functions and the transition matrix of the income process generate a probability distribution P over the state space for cash and credit agents, respectively:

(a) for cash agents:

$$P^{ca}((e, m), (e', m'(e, m))) = \begin{cases} \sum_{e' \in \mathbb{E}} p(e'|e) & \text{if } (e', m'(e, m)) \in S^{ca} \\ 0 & \text{otherwise} \end{cases} \quad (2.2.12)$$

the probability of transiting from state (e, m) to a state in the set S^{ca} .

(b) for credit agents:

$$P^{cr}(s, (e', a'(s), k'(s))) = \begin{cases} \sum_{e' \in \mathbb{E}} p(e'|e) & \text{if } (e', a'(s), k'(s)) \in S^{cr} \\ 0 & \text{otherwise} \end{cases} \quad (2.2.13)$$

the probability of transiting from state $s \triangleq (e, a, k)$ to a state in the set S^{cr} .

8. The distributions ϕ_i^{ca} and ϕ_i^{cr} are stationary:

$$\phi_i^{ca}(S^{ca}) = \int \int P^{ca}((e, m), S^{ca}) d\phi_i^{ca}, \forall S^{ca}, \quad (2.2.14)$$

$$\phi_i^{cr}(S^{cr}) = \int \int \int P^{cr}((e, a, k), S^{cr}) d\phi_i^{cr}, \forall S^{cr}. \quad (2.2.15)$$

2.2.4 Core channels

After re-arranging F.O.C. equations with Envelope theorem for credit agents in either economy, I obtain the following equations:

$$u'(c) = \beta(1+r) E[u'(c(z'))] + E[\mu(z')] \quad (2.2.16)$$

$$u'(c) = \beta E[R'_i(z', k') \cdot u'(c(z'))] + E[\mu(z') \cdot R'_i(z', k')] \quad (2.2.17)$$

where $\mu(z')$ is the Lagrangian multiplier for the endogenous borrowing constraint and $R'_i(z', k') = \frac{p'_i + z'\theta k'^{\theta-1}}{p_i}$ is rate of return on risky productive assets. Assume the borrowing constraint is non-binding, then we have:

$$E[R'_i(z', k')] - (1+r) = -\frac{Cov(R'_i(z', k'), u'(c(z')))}{E[u'(c(z'))]} \quad (2.2.18)$$

The LHS can be positive under some parameter values. More specifically, with i.i.d. idiosyncratic investment shocks, there is a risk premium on holding productive assets. This statement is true when the rate of return of productive assets is negatively correlated to marginal utility of tomorrow's consumption, or positively related to tomorrow's consumption. The equation (2.2.18) is the key to understand the positive net position of the U.S. risky productive assets (portfolio equity). The economy with smaller covariance of return on risky assets and tomorrow's consumption, or lower risk premium, values productive assets more highly and thus buys those assets from the other economy.

2.3 Quantitative Results

2.3.1 Benchmark

I calibrate the model based on the data from the U.S. economy for the sample period 1980-2012. The paper studies the long-term effect of inflation on the U.S. holdings of net foreign assets, so the model period is one year. First, the U.S. average inflation during the calibration period is 3.9%; the emerging markets' GDP-weighted average inflation during the same period is 10.0%. So $\pi_{US} = 3.9\%$, $\pi_{EM} = 10.0\%$. Then following Deaton (1991) and Heaton and Lucas (1996), I estimate the endowment process using a AR(1) process in the natural logarithm of income based PSID data.

$$\log(e_{t+1}) = const + \rho \log(e_t) + \epsilon_t, \quad (2.3.1)$$

where ϵ_t is i.i.d. normal with mean zero and standard deviation ε . The persistence ρ and volatility ε are free from any structural modeling assumptions, so they can be estimated separately from the statistical model above. Based on PSID data, I obtain $\rho = 0.564$ and $\varepsilon = 0.244$. I then follow Tauchen (1986) to approximate this AR(1) process to a two-state (e_l, e_h) Markov transition chains with transition probability matrix Pr . I interpret

the endowment as labor income that accounts for 0.9 unit of consumption goods and the individual production using productive assets as capital income that accounts for 0.1 unit. By tauchen (1986), $(e_l, e_h) = (0.65, 1.15)$ and the transition probability is 0.71 (symmetric matrix). Next, I assume the i.i.d. investment shocks take two values with equal probability. $z = \bar{z}(1 \pm \Delta z)$. Because the per capita capital income, $\bar{z}k^\theta$, is 0.1 unit and per capita productive assets is one unit, \bar{z} is calibrated to 0.1. In addition, the variation of investment shocks Δz is set to 2.5 according to Mendoza et al. (2009). Therefore, $(z_l, z_h) = (-0.15, 0.35)$. Last, I jointly calibrate the remaining four parameters β , σ , ω , and θ to match four targets: real interest rate 2.3%, the ratio of M1 to GNP 0.16, the fraction of credit constrained households 15.8% and the U.S. equity premium 3.5%. I summarized the calibration in Table 2.3.1.

Before we move to quantitative results, let me first define the holdings of net foreign assets in risk-free bonds and risky productive assets (equity) respectively as follows:

$$NFA_i^{bond} = \int \int \int a'_i(e, a, k) d\phi_i^{cr}(e, a, k) \quad (2.3.2)$$

$$NFA_i^{equity} = \int \int \int [k'_i(e, a, k) - 1] d\phi_i^{cr}(e, a, k) \quad (2.3.3)$$

Based on the parameters values obtained above and the definition of net foreign assets, the model generates some implications on net foreign assets in bond and equity and also equity premium. With a larger debt limit, the U.S. credit agents can borrow more and thus the U.S. holds a negative position in bonds, -4.7% of its endowment, which explains 34.5% of the data. Due to the larger debt limit, the U.S. credit agents have better risk sharing and their consumption is less volatile than that of their counterparts, which gives rise to a lower covariance between rate of return on risky assets and tomorrow's consumption as equation (2.2.18) indicates. Therefore, the U.S. credit agents require lower risk premium on risky assets and actually buys them from emerging markets. The model implies the U.S. positive

Table 2.3.1: Calibration

parameters	value	description	note
<i>exogenously given parameters</i>			
π_{US}	3.9%	inflation rate	from data
π_{EM}	10.0%	inflation rate	from data
<i>model-free parameters</i>			
(e_l, e_h)	(0.65, 1.15)	income shocks	from PSID
Pr	$\begin{bmatrix} 0.71 & 0.29 \\ 0.29 & 0.71 \end{bmatrix}$	transition probability	from PSID
(z_l, z_h)	(-0.15, 0.35)	investment shocks	Mendoza et al. (2009)
<i>jointly-calibrated parameters</i>			
β	0.96	discount factor	$r = 2.3\%$
σ	1.53	risk aversion	M1/GNP=0.16
ω	0.40	mass of cash agents	credit-rationed ratio = 15.3%
θ	0.73	capital share	U.S. equity premium = 3.5%

position in equity, at 0.4% of its endowment, which explains 23.5% of data.

Table 2.3.2: Benchmark results

U.S. debt limit	EM debt limit
0.94	0.84
U.S. NFA_bonds	EM NFA_bonds
-4.1%	4.1%
U.S. NFA_equity	EM NFA_equity
0.4%	-0.4%
U.S. equity premium	EM equity premium
3.5%	3.9%

2.3.2 Asymmetry in the U.S. and emerging markets

In the benchmark, the two economies are identical except their inflation. Now I assume that the income volatility for emerging markets is 0.25 rather than 0.244. Then I repeat the quantitative procedures in the benchmark and obtain the following results in Table 2.3.3. Facing larger income volatility, cash agents in emerging markets choose to hold more money for self-insurance, which lower cash agents value and thus outside option value for credit agents. With lower incentive to default, credit agents have a larger debt limit than in the benchmark. The U.S. credit agents still have a larger debt limit than their counterparts, so the U.S. still holds a negative position in bonds. The direction of the change in the bond position, however, is ambiguous, and depends on two competing effects that act on credit agents in emerging markets. On one hand, the larger debt limit makes emerging markets' credit agents lend less to their counterparts and thus shrinks the level of the negative position. On the other hand, when the income process becomes more volatile, emerging markets' credit agents will save more to better self-insure themselves against the income risks. In equilibrium,

the former effect dominates the latter and thus the level of the negative position in bonds is lowered. With less advantage in risk sharing for the U.S. credit agents, they will buy less risky assets from emerging markets.

Table 2.3.3: Larger income volatility in emerging markets

U.S. debt limit	EM debt limit
0.94	0.93
U.S. NFA_bonds	EM NFA_bonds
2.0%	-2.0%
U.S. NFA_equity	EM NFA_equity
0.2%	-0.2%
U.S. equity premium	EM equity premium
3.7%	3.8%

2.4 Conclusion

This paper develops a stochastic two-country open economy model to study how different inflation rates affect the U.S. net foreign asset holdings in bonds and portfolio equity (plus FDI). Each country features market segmentation, market incompleteness and limited commitment to debt contracts. In the calibrated model, the humped debt limits w.r.t. inflation make the U.S. have larger borrowing capacity and thus borrow from emerging markets. The better risk sharing due to the larger debt limit yields less volatile consumption for the U.S. credit agents, who accordingly require lower risk premium on equity and thus buy them from emerging markets. The differences in the capacity of credit markets, endogenously determined by different inflation targets, explain well the U.S. negative position in bonds and positive position in portfolio equity and FDI.

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Appendix A

Appendix for Chapter 1

A.1 Data descriptions

- credit card limits: nationwide aggregate lines of credit cards, from FRBNY Consumer Credit Panel / Equifax, 1999Q1-2014Q4. The data are only available from 1999Q1.
- income: disposable personal income, from FRED, 1999Q1-2014Q4.
- real interest rate: federal fund rate minus inflation, from FRED, 1980-2006, annual data.
- M1: includes currency in circulation, demand deposits and checking account, from FRED, 1980-2006, annual data.
- GNP: gross national production, from FRED, 1980-2006, annual data.
- ratio of credit constrained households: fraction of households who are turned down for credit application, from the SCF, 1989-2013, annual data.

A.2 Proof of Proposition 1

Proof. High-income (lucky) agents would like to save in money to smooth consumption and also to purchase consumption goods if saving is desirable. We have known that the modified CIA constraints are always non-binding for them. Therefore, to figure out whether there is a non-monetary equilibrium and pin down the corresponding cut-off value of inflation if any, we look at the lucky agents' problem. Only when

$$-u'(1 + \alpha) + \frac{\beta}{1 + \pi} [pu'(1 + \alpha) + (1 - p)u'(1 - \alpha)] > 0, \quad (\text{A.2.1})$$

do lucky agents have motives to save in money to equate the inequality above. Therefore, equilibrium money demand is always positive when

$$\pi < \pi_{nm} = \beta \left[p + (1 - p) \frac{u'(1 - \alpha)}{u'(1 + \alpha)} \right] - 1.$$

When $\pi > \pi_{nm}$, even lucky agents would hold zero money and stay autarky (the modified CIA constraint can satisfy zero money holding), so does unlucky agents. ■

A.3 Proof of Proposition 2

Proof. Assume inflation takes positive value from 1%, 2%, 3%, The F.O.C. of lucky agents is

$$-u'(1 + \alpha - m) + \frac{\beta p}{1 + \pi} u' \left(1 + \alpha + \frac{m}{1 + \pi} \right) + \frac{\beta(1 - p)}{1 + \pi} u' \left(1 - \alpha + \frac{m}{1 + \pi} \right) = 0.$$

By taking implicit differentiation, we have $m'(\pi) < 0$ and $m''(\pi) > 0$. Therefore, Individual money demand m is a decreasing function of inflation π and thus inflation tax $\frac{\pi}{1 + \pi} m$ can be written as a function of π , $g(\pi) = \frac{\pi}{1 + \pi} m(\pi)$. Let $g'(\pi) = \frac{\pi(1 + \pi)m'(\pi) + m(\pi)}{(1 + \pi)^2} = 0$,

we have

$$\frac{m(\pi)}{\pi(1+\pi)} = -m'(\pi). \quad (\text{A.3.1})$$

The LHS is a decreasing function ranging from zero (due to the modified CIA constraint) to a positive upper bound: $\frac{m(1\%)}{1\% \times (1+1\%)}$. The RHS is also a decreasing function ranging from $-m'(\pi_{nm}) > 0$ to $-m'(1\%)$. Under some parameter values, the inequality

$$\frac{m(1\%)}{1\% \times (1+1\%)} > -m'(1\%)$$

can be satisfied. Therefore, the nonlinear equation (A.3.1) has a unique solution, π_{it} . When $\pi < \pi_{it}$, $g'(\pi) > 0$ so inflation tax is increasing in inflation; when $\pi_{it} < \pi < \pi_{nm}$, $g'(\pi) < 0$ so inflation tax is decreasing; when $\pi > \pi_{nm}$, all the cash agents choose to stay autarky and thus the inflation tax is zero due to the zero tax base. ■

A.4 Computation

The challenge of the computation is to find the fixed point of the endogenous debt limit in the framework of a heterogeneous agent model with market segmentation. I summarize the algorithm briefly as follows:

1. solve cash agents' problem:
 - (a) guess aggregate real balance M ;
 - (b) given π and M , solve the Bewley problem, subject to occasionally binding CIA, constraint by policy function iteration;
 - (c) simulate cash agents' consumption and real balance with $N^{ca} = 100000 \times 0.40 = 40000$ and $T = 5000$, by using policy functions obtained above and the income transition matrix;

- (d) discard the first 500 periods to obtain a stationary simulation panel;
 - (e) update M from the simulation panel above until M converges;¹
 - (f) obtain value function from the policy functions $v^{ca}(e, m)$.
2. obtain the default value:
- (a) pick the minimum real balance m_{min} by search of the simulation panel in step 1;
 - (b) let the default value $v^{def}(e)$ equal the cash agents' value $v^{ca}(e, m)$ with m_{min} .
3. solve for the debt limit A :
- (a) guess A first. Given A_n (n^{th} iteration), we solve a heterogeneous agent model with borrowing constraints;
 - i. given the lump-sum transfer found in step 1, and a current guess on r , solve the credit agents' problem and obtain policy functions by policy function iteration;
 - ii. generate credit agents' simulation panel on consumption and bonds with $N^{cr} = 100000 \times 0.60 = 60000$ and $T = 5000$, by using the policy functions obtained above and the income transition matrix;
 - iii. discard the first 500 periods to obtain a stationary simulation panel;
 - iv. update r using the bisection method to clear bond markets until aggregate bonds converge to zero;
 - (b) obtain the continuation value of solvency $v^{cr}(e, a)$ from the policy functions;
 - (c) update the debt limit A until it converges:
 - i. given $v^{cr}(e, a)$, calculate the new debt limit as follows:

$$A_{n+1} = \min_{\{e\}} \{ -\underline{a}(e) : v^{ca}(e, a) = v^{def}(e) \}$$

¹The convergence criterion for M is 10^{-6} ; the convergence criterion for value function is 10^{-8} ; the convergence criterion for aggregate bonds is 10^{-6} ; the convergence criterion for debt limits is 10^{-3} .

- ii. update A given some relaxation parameter.
4. Given the fixed point of A found in step 3, we re-compute credit agents' problem as above.