Spring 5-15-2016

# Essays on Theories of Social Influence

Jiemai Wu
*Washington University in St. Louis*

WASHINGTON UNIVERSITY IN ST. LOUIS
Department of Economics

Dissertation Examination Committee:
John Nachbar, Chair
Mariagiovanna Baccara
Paulo Natenzon
Brian Rogers
Jonathan Weinstein

# Essays on Theories of Social Influence

by
Jiemai Wu

A dissertation presented to the
Graduate School of Arts and Sciences
of Washington University in
partial fulfillment of the
requirements for the degree
of Doctor of Philosophy

May 2016
St. Louis, Missouri

# Table of Contents

# List of Figures

# List of Tables

# Acknowledgments

Six years of PhD seem short when the home department is indeed a home. During this part of my life, I am incredibly fortunate to have met the most amazing mentors and friends. They truly made me a better person.

I deeply thank my advisor, John Nachbar, for years of encouragement and support. In this perhaps overly competitive profession, John always expresses his curiosity with humbleness and shares his wisdom with kindness. He shaped my definition of a scholar. I thank Paulo Natenzon, who helped me launch my first research project, for his continuous guidance and contagious enthusiasm that transformed struggles into adventures. I also thank Brian Rogers and Jonathan Weinstein for their insightful advice and constant encouragement, and I thank Adam Brandenburger for sharing his warm friendship as well as his love of language. I am extremely fortunate to have met these wonderful people at the beginning of my career.

I am also grateful for the company of my friends, Yunfei Cao, Mushegh Harutyunyan, Inkee Jang, Heechun Kim, Junmin Liao, Sunha Myong, Jaevin Park, Wenting Song, Bicheng Yang, Jundong Zhang, and many more. Flashbacks of time spent with them always put a smile on my face.

Lastly, I thank my parents and Xiaoyuan Huang for their continuous support and appreciation.

Jiemai Wu

Washington University in St. Louis

May 2016

Dedicated to John Nachbar.

ABSTRACT OF THE DISSERTATION

Essays on Theories of Social Influence

by

Jiemai Wu

Doctor of Philosophy in Economics

Washington University in St. Louis, 2016

Professor John Nachbar, Chair

This dissertation studies strategic social influence from a theoretical perspective.

The first chapter extends Bikhchandani, Hirshleifer and Welch's informational cascade model by introducing two types of players: experts with high signal accuracy and laymen with low signal accuracy. If a small enough fraction of laymen are present in the population, the probability of having a correct cascade is strictly higher than if no laymen are present. This is because the presence of laymen makes experts less eager to follow suit, which increases the amount of private information revealed.

The second chapter asks the following question: when a decision maker's (DM) choice depends on the information provided by persuaders, does the DM benefit from that information? I address this question in the context of a Bayesian persuasion game in which independent persuaders with no private information try to persuade a DM by gathering information using verifiable tests. All persuaders want the DM to switch her action from a default action to a new action, but whether it is optimal for the DM to switch depends on the state of the world. The persuaders strategically design tests that may be biased towards the new action and that best respond to the test designs of the other persuaders. I show that although the DM never gains from the information when there is only one persuader, there always exist equilibria in which the DM strictly gains when there are more than one persuader, even if these persuaders share identical preferences towards the new action. Moreover, these beneficial equilibria always feature noisy tests that never perfectly reveal the true

state. This paper shows that neither competition nor disagreement among the persuaders is necessary to facilitate a high level of information revelation in a persuasion game.

The third chapter discusses a situation in which one's consumption of a harmful tempting good (e.g., cigarettes, heroin) is affected by one's friend. Using Gul and Pesendorfer's temptation framework, I assume that having an addict as a friend makes the good more tempting. In this setting, I discuss the strategic interaction between players when they can endogenously choose friends. I show that there exist equilibria in which a player chooses a low consumption level in order to win the friendship of another. However, none of these equilibria are subgame perfect.

# Chapter 1

# Helpful Laymen in Informational Cascades

## 1.1 Introduction

It is conventional wisdom that smarter, more experienced people make better decisions. However, even a group of experts can cluster on the wrong choice. As shown by Bikhchandani, Hirshleifer, Welch (1992), and Banerjee (1992), when an agent observes both a private signal and the sequence of actions by previous agents, he can decide it is optimal to follow the choice of previous agents, even if his private signal indicates the opposite. This phenomenon, in which agents ignore their private information, is called an "informational cascade." In particular, if agents early in the sequence picked the wrong choice because of faulty private signals, the entire sequence of agents will follow the wrong choice, making it a "wrong informational cascade." Wrong informational cascades have been successfully replicated in experiments (Anderson and Holt 1997, Celen and Kariv 2004).

If even a group of experts aren't exempt from wrong cascades, how much worse will people do if the population also includes laymen, who have poorer signals than the experts? The answer is: they may end up doing better, with a lower frequency of wrong cascades.

Specifically, this paper analyzes games in a Bikhchandani-Hirshleifer-Welch setting with two types of players that differ in private signal accuracy. Suppose a group of consumers face the same choice, for example, whether to buy an iPhone or an Android phone. Assume one platform works better than the other, but it's hard to tell which is better with absolute certainty. Some consumers, whom I call experts, receive accurate private signals that indicate the correct choice most of the time. Others, whom I call laymen, are less familiar with the smartphone industry, and have poorer private signals that are incorrect more often. This paper shows that one can always find a small enough fraction of laymen, such that the probability of wrong cascades strictly decreases if those laymen are present among a group of experts. The reason comes from the discreteness of individuals: a player rationally follows suit and starts a cascade only after observing a fixed number of people making identical choices in a row. Let's call this number the "cascade triggering number". While adding a small fraction of laymen only decreases the overall information quality by a little, the cascade triggering number can discretely jump up by 1. That is, a player requires one more count of evidence before rationally ignoring his private signal in decision-making, and this increased hesitation delays the start of a cascade. As a result, the delay allows more information to be revealed to the public, which enables later players to make better choices.

This paper is closely related to Bikhchandani, Hirshleifer and Welch's paper on informational cascades (1992). Their paper includes a scenario in which each player's (potentially different) signal accuracy is public information, and discusses how sensitive cascades are with respect to the order of players, e.g. whether an expert decides first. My paper instead assumes anonymity of heterogeneous players, and shows how cascades are sensitive to a change in the entire player distribution, e.g. how many experts there are in the population. Other papers that address the topic of signal accuracy in cascades includes Sasaki's experimental study (Sasaki 2005), which shows that if the order of the players is linked with their ranking of signal accuracy, then there is a higher frequency of cascades when experts choose first. A paper by Pastine I. and Pastine T. (2006) provides examples to show that when homogeneous players' conditional signal probabilities are asymmetric for good and bad states, it is possible that the probability of correct cascades is not monotonic in players' common signal accuracy. My paper, on the other hand, studies heterogeneous players with symmetric conditional signal probabilities, and I show that the non-monotonicity of Pr(correct cascade) with respect to the fraction of experts not only exists, but also persists for any parameter specification.

Other studies have extended the cascade literature in different dimensions of heterogeneity. Smith and Sorensen (2000) discussed the possibility of confounded learning when players have opposite preferences with respect to the true state of the world. Goeree, Palfrey, and Rogers (2006) assume that a player's payoff is partly determined by a private preference shock that is independent of the true state. They conclude that as long as the support of such shock is rich enough, players will always learn and converge to the true state asymptotically. Other papers explored the possibility of players being exposed only to a (potentially different) subset of past action history (Banerjee and Fudenberg 2004, Acemoglu, Daleh, Lobel, and Ozdaglar 2011). All these papers assume a homogeneous signal distribution.

My paper is organized as follows: Section 2 lays out the the model. Section 3 summarizes the learning dynamics of the game. Section 4 derives the necessary and sufficient conditions of a cascade. Section 5 introduces the main theorem which states that it's always possible to have a higher probability of correct cascades by having some laymen among experts. A discussion of the robustness of the results is included at the end.

## 1.2 Model set up

There are two states of nature, $V \in \{v^H, v^L\}$, with equal prior probabilities $P(v^H) = P(v^L) = \frac{1}{2}$. An infinite sequence of i.i.d. players, $i = 1, 2, 3, ...$, enter with an exogenous order. Players differ in types $t \in \{expert, layman\}$, and $\pi \in [0, 1]$ is the probability that player $i$ is an expert $\forall i$.

Each player $i$ receives a private signal $S_i \in \{H, L\}$ conditional on the true state $V$, and player $i$'s type $t_i$. The conditional probabilities are summarized in Table 1. An *expert* receives a more accurate private signal than a *layman*. Type $t_i$ and signal $S_i$ are both private information.

"Experts" (fraction $\pi$) have high signal precision:

| Expert | $P(S_i = H\|V)$ | $P(S_i = L\|V)$ |
|---|---|---|
| V $= v^H$ | $p_E$ | $1 - p_E$ |
| V $= v^L$ | $1 - p_E$ | $p_E$ |

"Laymen" (fraction $1 - \pi$) have low signal precision:

| Layman | $P(S_i = H\|V)$ | $P(S_i = L\|V)$ |
|---|---|---|
| V $= v^H$ | $p_L$ | $1 - p_L$ |
| V $= v^L$ | $1 - p_L$ | $p_L$ |

$\frac{1}{2} \leqslant p_L < p_E < 1$

Table 1.1: Conditional private signal distribution

A player faces two choices: *adopt* or *reject*. The payoff of rejection is 0. The payoff of adoption is 1 when $V = v^H$, and $-1$ when $V = v^L$. In other words, a player wishes to adopt when $V = v^H$, and reject when $V = v^L$.

Once a choice is made, it becomes public information for later players. Therefore, each rational player $i$ observes the past action history $\{A_1, A_2, A_3 ... A_{i-1}\}$, his private type $t_i$, a private signal $S_i$, the fraction of experts $\pi$, and then chooses to adopt or reject.

Finally, player $i$'s strategy when he's indifferent requires extra specification. Here I focus on the cases in which the player randomly chooses adoption with a fixed probability when indifferent. Under this tie-breaking rule, there is a unique perfect Bayesian equilibrium in which player $i$ will

$$\textit{Adopt} \text{ if } P(v^H|A_1, ..., A_{i-1}, S_i, t_i) > P(v^L|A_1, ..., A_{i-1}, S_i, t_i);$$

$$\textit{Reject} \text{ if } P(v^H|A_1, ..., A_{i-1}, S_i, t_i) < P(v^L|A_1, ..., A_{i-1}, S_i, t_i);$$

$$Adopt \text{ with probability } z \in [0,1] \text{ if } P(v^H|A_1, ..., A_{i-1}, S_i, t_i) = P(v^L|A_1, ..., A_{i-1}, S_i, t_i).$$

For example, if $z = 0.5$, the player flips a coin when indifferent. The main result of this paper is robust to almost every arbitrary tie-breaking rule. Section 5 and 6 include the detailed discussion.

As in the previous literature, an *informational cascade for type t* is said to occur when it is optimal for a type-$t$ player to follow the choice of the preceding player regardless of his private signal. A *full informational cascade* occurs when it is optimal for a player to follow the choice of the preceding player regardless of his private signal *and* type.

It's intuitive that a cascade for experts occurs later than a cascade for laymen. Compared with laymen, experts are more confident in their private signals, so stronger evidence is needed to convince an expert to ignore his own signal and follow suit. Hence, a full cascade begins exactly when an expert enters a cascade.

## 1.3 Learning dynamics

To describe the dynamics of the game, let $\{l_n\}_{n=0}^{\infty}$ be a sequence of the *public likelihood ratio* where $l_0 \equiv 1$ and

$$l_n \equiv \frac{P(A_1,\ A_2,\ ...,\ A_n|v^H)}{P(A_1,\ A_2,\ ...,\ A_n|v^L)} \text{ for } n = 1, 2, 3, ...$$

Hence the decision rule for player $i$ can be rewritten as:

$$Adopt \text{ if } l_{i-1} \cdot \frac{P(S_i|v^H, t_i)}{P(S_i|v^L, t_i)} > 1;$$

$$Reject \text{ if } l_{i-1} \cdot \frac{P(S_i|v^H, t_i)}{P(S_i|v^L, t_i)} < 1;$$

$$Adopt \text{ with probability } z \in [0,1] \text{ if } l_{i-1} \cdot \frac{P(S_i|v^H, t_i)}{P(S_i|v^L, t_i)} = 1.$$

Conditional on the true state being 1, $\{l_n\}$ is a Markov chain:

Let

$$L_1 = \frac{1-p_E}{p_E},\ L_2 = \frac{1-p_L}{p_L},\ L_3 = \frac{p_L}{1-p_L},\ L_4 = \frac{p_E}{1-p_E}.$$

Since $\frac{1}{2} \leqslant p_L < p_E < 1$, $L_1 < L_2 < 1 < L_3 < L_4$. I use these four numbers as cutoff values to describe the evolution of $l_n$.

*When $l_n < L_1$:* No private signal can outweigh the strong public belief in favor of $v^L$. In this case a wrong full cascade of rejection occurs, and $l_{n+1} = l_n$ with probability 1.

*When $l_n = L_1$:* The current player is indifferent (and thus chooses adoption with probability $z$) only if he is an expert with signal $H$. Otherwise, he chooses rejection regardless of his signal. Therefore,

*a.* the current player *adopts* with probability $z\pi p_E$, and $l_{n+1} = l_n \cdot \dfrac{p_E}{1-p_E}$;

*b.* the current player *rejects* with probability $1 - z\pi p_E$, and $l_{n+1} = l_n \cdot \dfrac{1-z\pi p_E}{1-z\pi(1-p_E)}$.

*When $l_n \in (L_1, L_2)$:* The current player adopts only if he is an expert with signal $H$. Otherwise, he chooses rejection regardless of his signal. Therefore,

*a.* the current player *adopts* with probability $\pi p_E$, and $l_{n+1} = l_n \cdot \dfrac{p_E}{1-p_E}$;

*b.* the current player *rejects* with probability $1 - \pi p_E$, and $l_{n+1} = l_n \cdot \dfrac{1-\pi p_E}{1-\pi(1-p_E)}$.

The transition of $l_n$ when it falls in $\{L_2\}$, $(L_2, L_3)$, $\{L_3\}$, or $(L_3, L_4)$ can be deduced in a similar fashion. For each interval, pin down the type-signal combinations that lead to an adoption (respectively, rejection). Conditional on the action of the current player $A_i \in \{adopt, reject\}$, derive

$$l_{n+1} = l_n \cdot \frac{P(\text{type-signal combo that choose } A_i \mid v^H)}{P(\text{type-signal combo that choose } A_i \mid v^L)},$$

and the transitional probabilities accordingly. Finally, finish with the last possible scenario:

*When $l_n > L_4$:* No private signal can outweigh the strong public belief in favor of $v^H$. In this case a correct full cascade of adoption occurs, and $l_{n+1} = l_n$ with probability 1.

## 1.4 Conditions for a full cascade

The transition of the public likelihood ratio $l_n$ describes how the game evolves; however, it requires much calculation to tell if a cascade has started in an arbitrary game. There's a quicker way to spot the rise of a cascade. Let's start with the following Lemma.

**Lemma 1.** $\forall \pi \in (0,1)$, *if a sequence of consecutive identical actions are observed from the beginning of the game, a cascade for laymen starts after exactly* 1 *player, and a cascade for experts (hence, a full cascade) starts after exactly $N$ players, where $N$ is the smallest integer larger than*

$$\frac{\ln\left[\frac{p_E}{1-p_E} \cdot \frac{\pi(1-p_E)+(1-\pi)(1-p_L)}{\pi p_E+(1-\pi)p_L}\right]}{\ln \frac{1-\pi(1-p_E)}{1-\pi p_E}} + 1.$$

Note that $N$ is a decreasing function of $\pi$: the more experts there are in the population, the less evidence is needed to convince an expert to follow suit. For $\pi = 1$, $N = 2$. As $\pi \to 0$, $N \to \infty$ $\forall p_L, p_E$. Figure 1 in Section 5 also shows a plot of $N$ for $p_L = 0.55$ and $p_E = 0.95$.

*Proof.* I here prove the lemma for cascades of adoption. The proof for cascades of rejection is symmetric. Let $M$ be the smallest integer such that if player $M+1$ is a layman, and if all players $1, ..., M$ choose adoption, player $M+1$ also chooses adoption regardless of his private signal (i.e. he is in a cascade). Similarly, let $N$ be the smallest integer such that if player $N+1$ is an expert, and if all players $1, ..., N$ choose adoption, player $N+1$ also chooses adoption regardless of his private signal. Then, $M$ is simply the smallest integer s.t. $l_M > L_3$, and $N$ is the smallest integer s.t. $l_N > L_4$, where $L_3 = \frac{p_L}{1-p_L}$, $L_4 = \frac{p_E}{1-p_E}$, as defined in Section 3. Since

$$l_M = \left[\frac{\pi p_E + (1-\pi)p_L}{\pi(1-p_E) + (1-\pi)(1-p_L)}\right]^M,$$

$l_M > L_3$ implies

$$M > \frac{\ln \frac{p_L}{1-p_L}}{\ln \frac{\pi p_E+(1-\pi)p_L}{\pi(1-p_E)+(1-\pi)(1-p_L)}} \equiv M^*.$$

Note that $M^* < 1$ $\forall p_E, p_L$ s.t. $\frac{1}{2} \leqslant p_L < p_E < 1$. Therefore $M = 1$.

Similarly, since

$$l_N = \left[\frac{\pi p_E + (1-\pi)p_L}{\pi(1-p_E) + (1-\pi)(1-p_L)}\right]\left[\frac{1-\pi(1-p_E)}{1-\pi p_E}\right]^{N-1},$$

$l_N > L_4$ implies

$$N > \frac{\ln[\frac{p_E}{1-p_E} \cdot \frac{\pi(1-p_E)+(1-\pi)(1-p_L)}{\pi p_E+(1-\pi)p_L}]}{\ln \frac{1-\pi(1-p_E)}{1-\pi p_E}} + 1.$$

This completes the proof of the lemma. □

The above lemma identifies the start of a cascade when there is a sequence of identical actions from the very beginning of the game. The next proposition identifies the start of a cascade in a general case.

**Proposition 1.** *(Necessary and sufficient conditions for a full cascade) Following any history, if no cascade has yet started,*

*1. at least 1, and at most 2 consecutive identical actions are needed to trigger a cascade for laymen;*

*2. at least $N$, and at most $N+1$ consecutive identical actions are needed to trigger a cascade for experts (and therefore, a full cascade).*

*Hence, a full cascade occurs with probability 1 as the number of players goes to $\infty$.*

For example, let “A” denote “adopt” and let “R” denote “reject”. If $N = 4$, then the proposition implies that no full cascade has started after action history “RRAARAAA”. On the other hand, if the action history is “RRRAAAAA”, a full cascade of adoption must be in action. In other words, “$N$ consecutive identical actions” is the necessary condition for a full cascade; “$N + 1$ consecutive identical actions” is the sufficient condition.

*Proof.* I here prove the proposition for cascades of adoption. The proof for cascades of rejection is symmetric.

If a sequence of adoption starts from the beginning of the game, see Lemma. Otherwise, suppose a sequence of adoption starts after some history that ends with a rejection:

... (some history)..., R, A, A, A, A, A...

Denote the player associated with the rejection in the above example as player $k$.

Lemma implies that when $l = l_0 = 1$ (as at the beginning of the game), exactly $N$ adoptions are needed to trigger a cascade for experts (and therefore, a full cascade).

The fact that player $k$ chooses to reject implies that after receiving his private signal, player $k$'s private posterior probability for $v^L$ is at least 0.5. The public learning process captures this information by having $l_{k+1} \leqslant 1 \; \forall l_k$ whenever player $k$ rejects. Therefore, since exactly $N$ adoptions are needed to start a cascade when $l = 1$, in this less favorable scenario with $l_{k+1} \leqslant 1$, *at least* $N$ adoptions are needed to trigger a cascade for experts after player $k$ .

Next let's focus on player $k + 1$, the first player who chooses adoption in the sequence. Similarly, $l_{k+2} \geqslant 1 \; \forall l_{k+1}$ whenever player $k + 1$ adopts, so we need *at most* $N$ more adoptions (which means $N + 1$ adoptions in total) to trigger a cascade for experts.

The proof of cascades for laymen is obtained by simply replacing $N$ with 1 in the argument above.

Finally, since $N$ is finite, the probability of having a sequence of identical actions with the required length converges to 1, so a full cascade occurs with probability 1 in the limit. To formally show this, let $M$ denote the number of players. Note that if starting from somewhere in the sequence, there are $N+1$ consecutive experts all receiving signal $H$, they will all choose adoption and this starts a cascade. Therefore,

$$
\begin{aligned}
P(cascade) &> P(\text{there exists } N+1 \text{ consecutive experts with signal } H) \\
&\geqslant 1 - \left[1 - (\pi p_E)^{N+1}\right]^{\frac{M}{N+1}} \\
&\to 1 \text{ as } M \to \infty.
\end{aligned}
$$

□

## 1.5 Probability of correct cascades

### 1.5.1 Example

If a few consecutive players happen to receive wrong signals, a cascade starts where everyone later in the sequence chooses the wrong option even when they receive correct signals. Such unfortunate events always occur with a positive probability. From a welfare point of view, therefore, it is meaningful to study the probability of landing on a correct cascade, and in particular, how this probability changes with the demographics of the population.

Suppose $p_E = 0.95$, $p_L = 0.55$. That is, experts receive correct signals 95% of the time, and laymen only receive correct signals 55% of the time. Also assume $z = 0.5$, which means players flip a coin when indifferent. Plot A in Figure 1 describes how $N$, the length of the sequence of identical actions needed to trigger a cascade, changes with $\pi$. Plot B, which is the result of 100,000 Monte Carlo simulation trials, shows the frequency of correct cascades for each $\pi$.

Observe that in plot A, $N$ decreases with $\pi$. The more experts there are, the less it

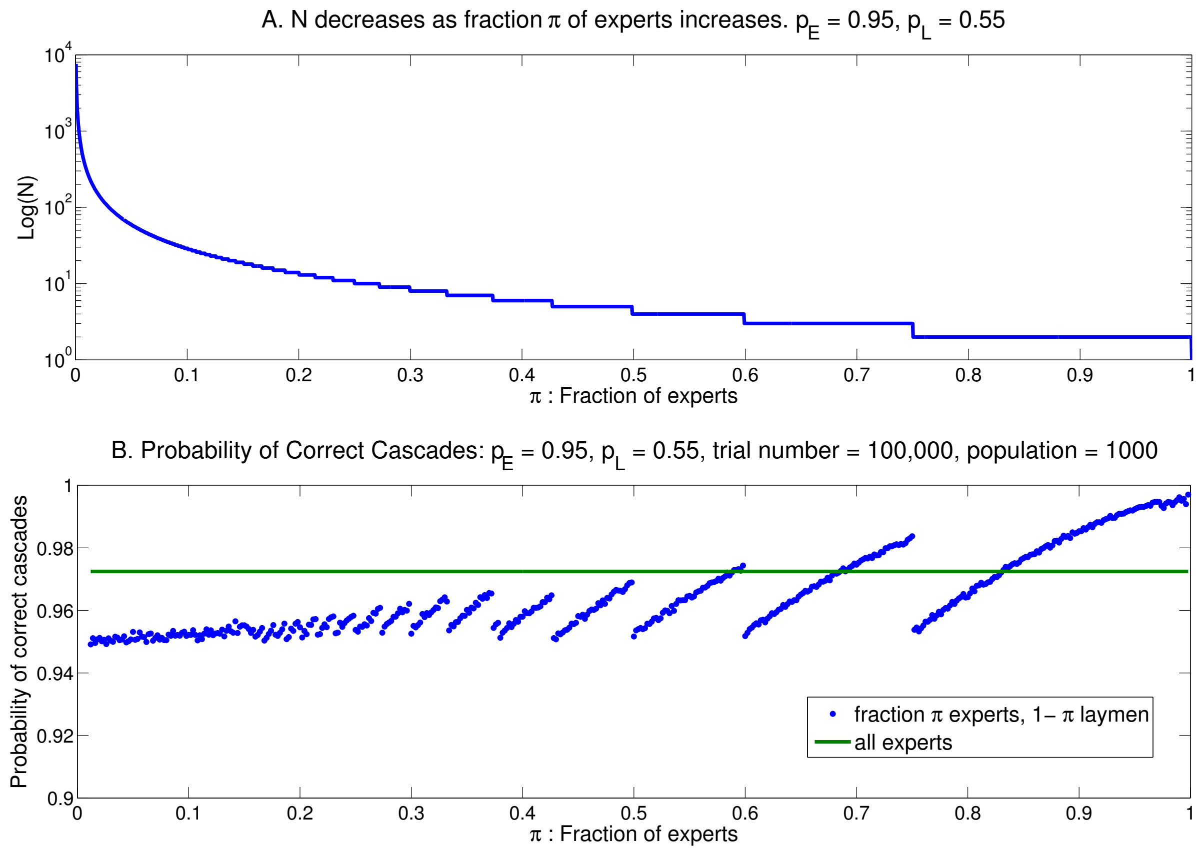



Figure 1.1: Probability of correct cascades exhibits discontinuous drop when integer N decreases

takes to trigger a full cascade. Let $f(\pi) \equiv N$ evaluated at $\pi$. Since $N$ only takes integer values, $f$ is a discontinuous function of $\pi$, and so I define a set of "turning points" $\{\pi_N\}_{N=2}^{\infty}$ s.t. $f(\pi_N) = N$ and $\lim_{\epsilon \to 0} f(\pi_N - \epsilon) = N + 1$. In the example, $\pi_2 = 1$, $\pi_3 \approx 0.75$, $\pi_4 \approx 0.6$, $\pi_5 \approx 0.5$.

In plot B, observe that the probability of correct cascades $p_{correct}(\pi)$ (blue dots in the picture) increases in $\pi$ until it drops when $\pi$ reaches $\pi_N$ for some $N$. A particularly interesting fact is that, for a range of $\pi \in (0.59, 0.6) \cup (0.69, 0.75) \cup (0.83, 1)$, $p_{correct}(\pi) > p_{correct}(1)$ (blue dots above the green line). When about 40% of the players are laymen, the probability of landing on the correct cascade is even higher than the case in which all players are experts.

What explains the drops in $p_{correct}$? And why is $p_{correct}$ higher when laymen are present?

To answer the first, note that the likelihood function of each sequence of type-signal realization (i.e. player 1 is an expert with H, player 2 is a layman with L, etc.) is continuous at each $\pi_N$, and therefore its left-sided limit at $\pi_N$ is equal to its value at $\pi_N$. However, $f(\pi_N) = \lim_{\epsilon \to 0} f(\pi_N - \epsilon) - 1$, which implies that although the overall population composition and signal quality at $\pi_N$ and $\pi_N - \epsilon$ are almost the same, players wait one less period to start a cascade at $\pi_N$. With an earlier start of the cascade, all later players' decisions are based on a smaller set of information, which, inevitably, lead to more mistakes and a lower $p_{correct}$.

The reason for a higher $p_{correct}$ when laymen are present follows the same logic. When there are laymen around, experts wait longer before following suit, and everyone benefits from this little hesitation. To be more specific, when all players are experts, suppose the first player adopts. Then even if the second player receives signal L, he's indifferent between adoption and rejection, and will therefore choose randomly. In contrast, when there is a small group of laymen, the expert second player with signal L no longer trusts the first player as much as himself, and instead strictly follows his own signal to reject. This added bit of conservativeness sends out a clearer message to later players; they know the second player adopts if and only if the signal is H. Because later players now have better information to work with, they end up making the correct choice more often. The theorem in the next section generalizes this idea.

### 1.5.2 Theorem on probability of correct cascades

In the example, when 40% of the players are laymen, the probability of correct cascades is higher than the case in which no players are laymen. The following theorem generalizes this result by showing that for any values of $p_E$ and $p_L$, there always exists a small enough fraction, such that if this fraction of laymen are present, the probability of correct cascades is higher than if no laymen are present.

**Theorem 1.** *Let $p_{correct}(\pi)$ be the probability of correct cascades when fraction $\pi$ of the* *population are experts. Then* $\forall p_E, p_L$ *with* $\frac{1}{2} \leqslant p_L < p_E < 1$, $\exists\ \underline{\pi} \in (0,1)$ *s.t.* $\forall \pi \in (\underline{\pi}, 1)$,

$$p_{correct}(\pi) > p_{correct}(1).$$

*Proof.* It suffices to prove that $\lim\limits_{\pi \to 1} p_{correct}(\pi) > p_{correct}(1)$.

Let $l_\pi(A_1, A_2)$ denote the public likelihood ratio after two actions $A_1, A_2 \in \{\text{A, R}\}$ when fraction $\pi$ are experts. Similarly define $l_{expert}(A_1, A_2)$ when $\pi = 1$: all players are experts.

$\lim\limits_{\pi \to 1} l_\pi(\text{A, A}) = \left(\dfrac{p_E}{1-p_E}\right)^2 > l_{expert}(\text{A, A}) = \dfrac{p_E\left[p_E + z\left(1-p_E\right)\right]}{\left(1-p_E\right)\left(1-p_E+zp_E\right)} > L_4 \Rightarrow$ cascade of adoption

$\lim\limits_{\pi \to 1} l_\pi(\text{R, R}) = \left(\dfrac{1-p_E}{p_E}\right)^2 < l_{expert}(\text{R, R}) = \dfrac{\left(1-p_E\right)\left[1-p_E+\left(1-z\right)p_E\right]}{p_E\left[p_E+(1-z)(1-p_E)\right]} < L_1 \Rightarrow$ cascade of rejection

$\lim\limits_{\pi \to 1} l_\pi(\text{A, R}) = \lim\limits_{\pi \to 1} l_\pi(\text{R, A}) = l_{expert}(\text{A, R}) = l_{expert}(\text{R, A}) = 1 \Rightarrow$ back to the origin

For both scenarios, the only possible action history that can trigger a correct cascade is a pair of correct actions following several pairs of opposite actions (A, R) or (R, A). Therefore,

$$\begin{aligned}
p_{correct}(1) &= \frac{1}{2}\sum_{k=0}^{\infty}[\text{P(opposite action pair} \mid v^H)]^k \cdot \text{P(A, A} \mid v^H) \\
&\quad + \frac{1}{2}\sum_{k=0}^{\infty}[\text{P(opposite action pair} \mid v^L)]^k \cdot \text{P(R, R} \mid v^L) \\
&= \frac{1}{2}\sum_{k=0}^{\infty}[p_E(1-p_E)]^k \cdot p_E \cdot [p_E + z \cdot (1-p_E)] \\
&\quad + \frac{1}{2}\sum_{k=0}^{\infty}[p_E(1-p_E)]^k \cdot p_E \cdot [p_E + (1-z) \cdot (1-p_E)] \\
&= \frac{1}{2}\sum_{k=0}^{\infty}[p_E(1-p_E)]^k \cdot p_E \cdot (p_E + 1) \\
&= \frac{p_E(p_E+1)}{2(1-p_E+p_E^2)}.
\end{aligned}$$

On the other hand,

$$\begin{aligned}
\lim_{\pi\to 1} p_{correct}(\pi) &= \frac{1}{2}\sum_{k=0}^{\infty}\lim_{\pi\to 1}[\text{P(opposite pair} \mid v^H)]^k \cdot \text{P(A, A} \mid v^H) \\
&\quad + \frac{1}{2}\sum_{k=0}^{\infty}\lim_{\pi\to 1}[\text{P(opposite pair} \mid v^L)]^k \cdot \text{P(R, R} \mid v^L) \\
&= \frac{1}{2}\sum_{k=0}^{\infty}\lim_{\pi\to 1}[\text{P("H, L" or "L, H"} \mid v^H)]^k \cdot \text{P(H, H} \mid v^H) \\
&\quad + \frac{1}{2}\sum_{k=0}^{\infty}\lim_{\pi\to 1}[\text{P("H, L" or "L, H"} \mid v^L)]^k \cdot \text{P(L, L} \mid v^L) \\
&= \sum_{k=0}^{\infty}[2p_E(1-p_E)]^k \cdot p_E^2 \\
&= \frac{p_E^2}{1-2p_E+2p_E^2}.
\end{aligned}$$

When $p_E \in (\frac{1}{2}, 1)$,

$$\frac{p_E(p_E+1)}{2(1-p_E+p_E^2)} < \frac{p_E^2}{1-2p_E+2p_E^2},$$

therefore $\lim_{\pi\to 1} p_{correct}(\pi) > p_{correct}(1)$.

□

From a slightly different point of view, as $\epsilon \to 0$, at each $\pi = \pi_N - \epsilon$ the game resembles one in which all players choose according to their private signals when they are indifferent, i.e. choosing adoption if and only if the private signal is H when the posterior probabilities are equal. Let's call such player a "*non-conformist*" for future reference. This tie-breaking rule helps to reveal more signals, and leads to a higher $p_{correct}$. At each $\pi = \pi_N + \epsilon$, the game resembles one where all players copy the previous player when indifferent, i.e. when the posterior probabilities are equal, a player chooses adoption if and only if the previous player chooses adoption. Fewer signals are revealed in this case, resulting in a lower $p_{correct}$. When $\pi = \pi_N$, players randomize when indifferent, resulting in a middle case with a medium $p_{correct}$.

In particular, note that the derivation of$\lim_{\pi \to 1} p_{correct}(\pi)$ is independent of the player's strategy when indifferent. Moreover, $\lim_{\pi \to 1} p_{correct}(\pi) = p_{correct}^{non-conformist}(1)$, the probability of correct cascades when all players are experts *and* non-conformists, and this is true for all tie-breaking strategies. Therefore, $\lim_{\pi \to 1} p_{correct}(\pi) > p_{correct}(1)$ because $p_{correct}^{non-conformist}(1) > p_{correct}(1)$, i.e. games with non-conforming experts are more likely to have correct cascades than games with experts who randomize when indifferent. In this sense, the randomization tie-breaking rule is "inferior" to the non-conformist tie-breaking rule. The following corollary shows that the result of the last theorem holds for all ties-breaking rules that are different from the non-conformist rule.

**Corollary 1.** *For any tie-breaking strategy $\tau \neq \tau^{non-conformist}$, let $p_{correct}^{\tau}(\pi)$ be the probability of correct cascades when fraction $\pi$ of the population are experts, then*

$\forall p_E, p_L$ *with* $\frac{1}{2} \leqslant p_L < p_E < 1$, $\exists\ \underline{\pi} \in (0,1)$ *s.t.* $\forall \pi \in (\underline{\pi}, 1)$,

$$p_{correct}^{\tau}(\pi) > p_{correct}^{\tau}(1).$$

*Proof.* $\forall \tau \neq \tau^{non-conformist}$, $\lim_{\pi \to 1} p_{correct}^{\tau}(\pi) = p_{correct}^{non-conformist}(1) > p_{correct}^{\tau}(1)$, and the result follows.

See Appendix for the proof of $p_{correct}^{non-conformist}(1) > p_{correct}^{\tau}(1)$.

□

## 1.6 Conclusion

This paper extends the Bikhchandani-Hirshleifer-Welch informational cascades model by incorporating heterogeneity in private signal accuracy. I conclude that in the unique perfect Bayesian equilibrium associated with a tie-breaking rule under which players randomize when indifferent, the probability of correct cascades is higher when a small enough group of laymen are present among a population of experts. The corollary in Section 5 shows that this result is robust to all tie-breaking rules except what I have called the "non-conformist" rule (i.e. choose according to own signal when indifferent). But even in cases where the non-conformist tie-breaking rule is adopted, as long as players are discrete, the intuition in section 5.1 carries on, and the probability of correct cascades is still non-monotonic with discontinuous jumps resembling those in Figure 1. Therefore, for all tie-breaking rules including the non-conformist rule, there exist cutoff expert fractions from which adding a small fraction of laymen makes correct cascades more frequent.

Although much of the discussion involves cases in which players are indifferent, quantitatively they make a big difference. As seen in the example in section 5.1, a population with 40% laymen and 60% experts land on the correct cascade more frequently than a population of 100% experts, even though the signal accuracy differs dramatically for the two player types (0.55 vs. 0.95). From this perspective, it confirms how easily an informational advantage can be outweighed by insufficient learning when people follow suit.

## 1.7 Appendix

### Nonconformist tie-breaking rule yields the highest probability of correct cascade

Consider homogeneous players with signal accuracy $p = P(H \mid v^H) = P(L \mid v^L)$ who adopt an arbitrary tie-breaking rule. I here prove that the probability of having a correct cascade is the highest if all players adopt a nonconformist tie-breaking rule.

**Definition.** A tie-breaking rule is defined by $\tau \equiv \{l_h\}_{h \in \mathcal{H}}$, where $\mathcal{H}$ is the set of all possible history such that the next player following such history can be indifferent. $l_h$ denotes the probability that if the player is indeed indifferent after history $h$, he picks an action in accordance to his private signal.

For example, if the first player chose $A$, the second player will be indifferent if he receives signal $L$. In this case $l_A$ denotes the probability that the second player chooses $R$. Similarly, $l_{ARR}$ denotes the probability that the 4th player with signal $H$ chooses $A$ when the action history $h = ARR$. For a counter example, $AA \notin \mathcal{H}$ because a cascade of adoption starts after $AA$, and the next player will not be indifferent regardless of his signal.

$\{l_h\}$ fully characterizes a tie-breaking rule because a player's strategy only depends on the previous action history and his private signal. Moreover, if $h \in \mathcal{H}$, only one of the two signals induces indifference, so it's sufficient to define the probabilities as functions of action history only.

**Definition 1.** Let $\tau^{nonconf}$ denote the nonconformist tie-breaking rule: always follow own signal when indifferent. I.e., $l_h = 1$ for all $h$.

**Proposition.** *Let $P^{correct}(\tau)$ denote the unconditional probability of correct cascades when the tie-breaking rule is $\tau$. Then for all $\tau \neq \tau^{nonconf}$,*

$$P^{correct}(\tau) < P^{correct}(\tau^{nonconf}).$$

*Proof.* It suffices to only consider tie-breaking rules with $l_h > 0\ \forall h$. If after history $h$ the next player is indifferent and he always copies the last player's action ($l_h = 0$), then his action conveys no information to the later players, and his presence has no effect on $P^{correct}$. Therefore, simply delete such player from the sequence, and only focus on the games played by players who follow own signal with positive probability when indifferent.

I prove the proposition in 3 steps. First, I show that $P^{correct}$ strictly increases in $l_A$ and $l_R$. I then show that $P^{correct}$ strictly increases in $l_h$ for all $h \in \mathcal{H}$. Finally, conclude that $P^{correct}$ is maximized only when $l_h = 1$ for all $h$, which corresponds to the nonconformist tie-breaking rule.

*Claim 1:* $\dfrac{\partial P^{correct}(\tau)}{\partial l_A} > 0$ and $\dfrac{\partial P^{correct}(\tau)}{\partial l_R} > 0$.

*Proof of Claim 1:*

$$
\begin{aligned}
P^{correct}(\tau) &= \frac{1}{2}P(AA \text{ or } ARAA \text{ or } RAAA \text{ or } ARARAA...|v^H) \\
&+\frac{1}{2}P(RR \text{ or } ARRR \text{ or } RARR \text{ or } ARARRR...|v^H) \\
&= \frac{1}{2}\left\{p\left[p+(1-p)l_A\right]+p(1-p)l_Ap\left[p+(1-p)l_{ARA}\right]+...\right\} \\
&+\frac{1}{2}\left\{p\left[p+(1-p)l_R\right]+(1-p)pl_Ap\left[p+(1-p)l_{ARR}\right]+...\right\}
\end{aligned}
$$

where $p = P(H|V = 1) = P(L|V = 0)$. Therefore,

$$
\begin{aligned}
\frac{\partial P^{correct}(\tau)}{\partial l_A} &= \frac{1}{2}\left\{p\left[p+(1-p)\right]+(1-p)p^2\left[p+(1-p)l_{ARA}\right]+...\right\} \\
&+\frac{1}{2}\left\{(1-p)p^2\left[p+(1-p)l_{ARR}\right]+...\right\} \\
&= \frac{1}{2}(2p-1)\{p(1-p)+\left[p(1-p)\right]^2(l_{ARA}+l_{ARR}) \\
&+\left[p(1-p)\right]^3\left[l_{ARA}(l_{ARARA}+l_{ARARR})+l_{ARR}(l_{ARRAA}+l_{ARRAR})\right] \\
&+...\} \\
&> 0
\end{aligned}
$$

since $p > \frac{1}{2}$.

A similar argument proves $\frac{\partial P^{correct}(\tau)}{\partial l_R} > 0$. //

*Claim 2:* $\frac{\partial P^{correct}(\tau)}{\partial l_h} > 0$ for all $h \in \mathcal{H}$.

*Proof of Claim 2:* Proof by induction. First,

$$\begin{aligned}
sgn\left(\frac{\partial P^{correct}}{\partial l_{ARA}}\right) &= sgn\left[P(AR)\cdot\frac{\partial P^{correct}}{\partial l_A}\right]=1\\
sgn\left(\frac{\partial P^{correct}}{\partial l_{RAA}}\right) &= sgn\left[P(RA)\cdot\frac{\partial P^{correct}}{\partial l_A}\right]=1\\
sgn\left(\frac{\partial P^{correct}}{\partial l_{ARR}}\right) &= sgn\left[P(AR)\cdot\frac{\partial P^{correct}}{\partial l_R}\right]=1\\
sgn\left(\frac{\partial P^{correct}}{\partial l_{RAR}}\right) &= sgn\left[P(RA)\cdot\frac{\partial P^{correct}}{\partial l_R}\right]=1
\end{aligned}$$

where $sgn(\cdot)=1$ means a positive value.

Note that any history $h\in\mathcal{H}$ can be written as $h=ARh'$ or $h=RAh'$ for some history $h'\in\mathcal{H}$. So if $sgn\left(\frac{\partial P^{correct}}{\partial l_{h'}}\right)=1$, then $sgn\left(\frac{\partial P^{correct}}{\partial l_h}\right)=sgn\left[P(AR)\cdot\frac{\partial P^{correct}}{\partial l_{h'}}\right]$ or $sgn\left(\frac{\partial P^{correct}}{\partial l_h}\right)=sgn\left[P(RA)\cdot\frac{\partial P^{correct}}{\partial l_{h'}}\right]$. In both cases the sign is positive.

Therefore, by induction, conclude that $\frac{\partial P^{correct}}{\partial l_h}>0$ for all $h\in\mathcal{H}$. //

Finally, Claim 2 implies that $P^{correct}(\tau)$ is maximized when $l_h=1$ or all $h\in\mathcal{H}$, which is only true for $\tau=\tau^{nonconf}$. □

# Chapter 2

# Beneficially Imperfect Persuaders

## 2.1 Introduction

When a decision maker receives selective information from biased persuaders, does she benefit from that information? I consider this question within the framework of a Bayesian persuasion game - persuaders with no private information influence a decision maker by collecting information endogenously. Suppose these persuaders want the decision maker to switch from her default action to a new action regardless of the true state of the world. Then, the decision maker gains nothing if she receives information from only one such persuader (Kamenica and Gentzkow, 2011). If one persuader is useless, can the decision maker gain from consulting more persuaders? If so, what requirements need to be satisfied? This paper shows that, in fact, very little is needed for the decision maker to gain from a persuasion game with more than one persuader. As long as the persuaders collect information independently, then, even if all persuaders are identically biased towards the same action, there always exist strict equilibria with high payoffs for the decision maker. Moreover, all of these equilibria feature persuaders that collect only noisy information.

Consider the following example.

Ann is the leader of an investment group. Her group members, Bob, Charles, and Dan, want her to hire their friend, Evan. Ann is willing to hire Evan only if he has sufficient quantitative skills, but her group members are happy to work with Evan even if he does not possess those skills. Although Bob, Charles, and Dan know Evan's personality very well, they do not know how quantitatively skilled he is. Because Ann is too busy to meet Evan herself, she asks some or all members to independently give Evan a test and then to truthfully report back. Based on the original test questions and Evan's test results, Ann hires if she thinks there is a high enough chance that Evan is skilled.

In a perfect Bayesian equilibrium, if Ann never strictly prefers to hire Evan, then she does not gain from the tests. This is always the case when the members design tests that always pass a skilled person. Call these "quasi-revealing" tests. In equilibria with quasi-revealing tests, Ann hires only when Evan passes all tests, as a single failure perfectly identifies Evan as unskilled. The members pick easy tests with high enough passing rates for an unskilled person, so that straight passes leave Ann exactly indifferent (in which case, she hires). In expectation, Ann never gains from these tests.

When Ann asks only Bob to test Evan, Bob's optimal test design is quasi-revealing. Consequently, Ann does not gain from Bob's report.

Suppose Ann asks all three members to test Evan independently, and the members can conduct only twenty-minute oral tests. Given the constrained time and format, Evan can get stuck on an easy question even if he is skilled. Therefore, Evan's performance, at most, *suggests* rather than reveals his true skills. In other words, quasi-revealing tests are not feasible. In this case, because a failure does not perfectly identifies Evan as unskilled, there exist perfect Bayesian equilibria in which Ann hires after only two passes. In these equilibria, the tests are designed so that each pass is very informative. Note that when Evan, in fact, passes all three tests, Ann strictly prefers to hire him. Therefore, in expectation, Ann strictly gains.

The existence of equilibria with a high payoff for Ann does not rely on exogenous constraints on the information environment such as the one described above. In fact, there always exist strict equilibria in which the members endogenously design tests that sometimes fail a skilled person. Ann's low standards (e.g. two passes instead of three) give members the incentive to choose those tests. In these equilibria with endogenously noisy tests, Ann always strictly gains.

The results of this paper also apply to other scenarios. For example, consider salesmen that promote a product by offering free trials of its selected features or doctors that use selected medical tests to suggest a surgery as opposed to a conservative treatment. This paper also offers new insights on the effects of competition versus collusion among endogenous information providers. It emphasizes that even when information providers are identical in every dimension and externalities are absent, there always exist robust equilibrium outcomes with more information revealed than the collusive outcome.

The remainder of the paper is organized as follows. Section 2 discusses related papers. Section 3 provides a numerical example. Section 4 contains a formal development, and Section 5 concludes with a discussion.

## 2.2 Related papers

This paper is closely related to two papers on Bayesian persuasion games: Kamenica and Gentzkow (2011) and Gentzkow and Kamenica (2016). I extend Kamenica and Gentzkow (2011) by introducing multiple independent persuaders. This extension gives rise to a wide range of equilibrium outcomes. In particular, if there is only one persuader, as in Kamenica and Gentzkow (2011), the decision maker never benefits from the collected information. This is because in any single-persuader equilibrium, the persuader can and, indeed, will design his test in such a way that a pass leaves the decision maker precisely indifferent. Since the decision maker never strictly prefers to switch her action, the test never strictly increases her expected utility. In contrast, in games with multiple independent persuaders, there always exist equilibria in which the decision maker strictly prefers to switch her action. The numerical example in Section 3 further elaborates on this comparison. Moreover, in the single-persuader equilibria studied in Kamenica and Gentzkow (2011), if the decision maker chooses the default action, she is always certain of her choice. This is no longer the case in equilibria with multiple independent persuaders.

Gentzkow and Kamenica (2016) study a model of multiple persuaders under an information environment that is Blackwell-connected, i.e., an environment in which each persuader can unilaterally deviate to induce any feasible distribution of belief that is more informative. Note that, the information environment studied in this paper is not Blackwell-connected because of the independence of the persuaders[1]. For comparison, an environment is Blackwell-connected if the tests chosen by the persuaders can be arbitrarily correlated. In this case, misaligned incentives among persuaders are necessary for persuaders to reveal sufficient information that benefits the decision maker. If persuaders share identical preferences, then, regardless of the total number of persuaders, all strict equilibria[2] of the game are outcome-equivalent to the single-persuader game, and the decision maker never gains. This is why heterogeneous preferences among persuaders are necessary to induce more information revelation in their paper: a persuader will reveal additional information as a means

[1]For example, let the true state be $H$ or $L$ with equal probabilities. Suppose the first persuader chooses a test with $\Pr(pass|H) = 0.8$ and $\Pr(pass|L) = 0.2$; the second persuader chooses an uninformative test with $\Pr(pass|H) = \Pr(pass|L) = 1$. The induced posterior belief for the state $H$ is 0.8 with probability 0.5 and 0.2 with probability 0.5. Blackwell-connectedness requires that, given the strategy of the first persuader, the second persuader can unilaterally deviate to a different test, so that the two tests induce a posterior belief of 0.9 with probability 0.5 and 0.1 with probability 0.5. However, since the two persuaders choose tests independently, such belief distribution is unattainable by a unilateral deviation. If the second persuader deviates to a more informative test so that he can sometimes induce a posterior belief of 0.9 or 0.1, then it is always possible that his test fails when the test from the first persuader passes, or vice versa. Either case induces a posterior belief between 0.8 and 0.2 with positive probability.

[2]While there do exist equilibria that generate a higher payoff for the decision maker, they all require that persuaders choose excessively informative tests when indifferent. For example, full revelation is always an equilibrium, assuming that persuaders reveal the true state when indifferent. However, it is a rather unnatural prediction, since all persuaders prefer a less informative outcome. See Section 6 of Gentzkow and Kamenica (2016) for more details.

to induce his preferred action only when persuaders differ in their preferences. In contrast, when persuaders are independent, I show that even if persuaders have identical preferences, there always exist strict equilibria in which the persuaders choose relatively informative tests that strictly benefit the decision maker. The persuaders act in this way not because they are competing for different actions, but because when they choose those tests, the Bayesian decision maker switches her action upon seeing relatively few counts of passes.

There are papers on cheap talk persuasion games with multiple persuaders (e.g., Battaglini, 2002; Ambrus and Takahashi, 2008; Ambrus and Lu, 2010). However, note that for any game in which the persuaders' preferences are state-independent, if the decision maker observes only the outcome of the test and, not the design, the only equilibrium is a trivial one in which the persuaders always conduct completely uninformative tests that never fail, and the decision maker is never persuaded (Sobel, 2011). Therefore, in this paper, it is crucial that the decision maker observes both the design and the outcome of the test.

Alternatively, in persuasion games with state-independent persuaders, if the persuaders can conduct only truthful and unbiased tests, but a test can be hidden from the decision maker if the outcome is unfavorable, then there also exist non-trivial equilibria in which the decision maker is sometimes persuaded (e.g. Bhattacharya and Mukherjee, 2013; Felgenhauer and Schulte, 2014; Hart, Kremer, and Perry, 2015). A key distinction is that the persuaders in those papers have private information (persuaders report after they see the test outcomes), whereas the persuaders in this paper do not (persuaders unconditionally commit to their test choices, and they always report the test outcomes). Due to this difference, noise does not benefit the decision maker in their settings. If persuaders report only after they collect evidence about the true state, they have an incentive to collect as much evidence as possible and report only good evidence. Therefore, limiting the accuracy of the collected information only weakens the reported good evidence and discourages the persuaders from collecting evidence overall. Both of these effects harm the decision maker. On the contrary, in this paper, the decision maker observes both good and bad evidence. Noise in the tests encourages the persuaders to collect more evidence in general because when they do so, the decision maker puts a smaller weight on the reported negative evidence in equilibrium. This results in a higher level of information revelation and an increased expected utility for the decision maker.

Many assumptions in this paper are similar to those in the standard voting literature, such as Feddersen and Pesendorfer (1998). But there is one crucial difference that leads to very different results. The decision maker in this paper does not commit to any decision rule that is based only on test outcomes. In the voting literature, the decision maker takes a certain action if the number of votes passes an exogenous threshold, regardless of the voting strategy (e.g., the unanimity rule or the majority rule). In contrast, the decision maker in this paper chooses the action that best responds to both the test outcomes and the test design. In particular, if the decision maker were to commit to a fixed outcome-based standard (e.g., two passes out of three tests), the persuaders would simply choose uninformative tests that never fail. If that were the case, the decision maker would rather ignore the persuaders and always choose the default action. This outcome is undesirable for both the decision maker and the persuaders.

## 2.3 A numerical example

For a better understanding of this paper's main results, I revisit the motivating example from the Introduction in a numerical context. The job candidate, Evan, is either quantitatively skilled or unskilled with $\Pr(\textit{skilled}) = \Pr(\textit{unskilled}) = 0.5$. Each group member gets payoff 1 if Evan is hired, and 0 otherwise. Ann's payoff is summarized by the table below.

| | skilled | unskilled |
|---|---|---|
| hire | 1 | 0.2 |
| not hire | 0.8 | 1 |

In other words, Ann is willing to hire Evan if and only if the probability that he is quantitatively skilled is at least 0.8.

### 2.3.1 A single persuader

Suppose that Ann asks only Bob to test Evan. The test reports either pass or failure. For any test design, only two numbers matter for the decision-making: $\Pr(pass \,|\, skilled)$ and $\Pr(pass \,|\, unskilled)$. To maximize the unconditional probability of hiring, the uniquely optimal test design for Bob has

$$\Pr(pass \,|\, skilled) = 1 \text{ and } \Pr(pass \,|\, unskilled) = 0.25.$$

Ann hires if and only if Evan passes the test. However, since $\Pr(skilled \,|\, pass) = 0.8$, Ann is merely indifferent between hiring and not hiring when Evan passes the test. Therefore, in expectation, Ann does not strictly benefit from this test. Indeed, Ann's expected utility is 0.9, which is the same level that she gets if she completely ignores the test and never hires Evan (this is her default choice under her prior belief). When Ann's decision depends on Bob's test, although she never misses a skilled person, she hires an unskilled person too often.

### 2.3.2 Multiple persuaders with quasi-revealing tests

Now, suppose that Ann asks Bob, Charles, and Dan to independently test Evan, and the three members can choose any test design. Just as in the previous case with Bob only, there are equilibria in which all members choose tests that always pass a skilled person. Here is a symmetric example: each member chooses a test with the same pair of conditional probabilities

$$\Pr(pass \,|\, skilled) = 1 \text{ and } \Pr(pass \,|\, unskilled) = 0.63.$$

Ann hires if and only if Evan passes all tests because a single failure perfectly reveals that Evan is unskilled. To maximize the unconditional probability of hiring, members choose tests with a high pass rate for an unskilled person, so that Ann is indifferent when Evan passes all the tests. Again, since Ann never strictly prefers to hire Evan, she never strictly gains from the tests. Indeed, her expected utility in this equilibrium is still 0.9 - the lowest possible amount.

In fact, Ann's expected utility never exceeds 0.9 as long as the equilibrium tests never fail a skilled person. In these cases, Ann hires only when Evan passes all the tests, and the persuaders always choose tests with high enough $\Pr(pass \mid unskilled)$, so that Ann is indifferent when all tests are passed. Therefore, for Ann to have a higher expected utility, the equilibrium tests must sometimes fail a skilled person.

### 2.3.3 Multiple persuaders with noisy tests

One way to make the members sometimes fail a skilled person is by introducing exogenous noise. As described in the Introduction, suppose there is a restriction on the duration and the format of the tests, so that the members can, at most, learn Evan's noisy *conditions* during the tests, as opposed to his true skills. For example, even if Evan is skilled, he might happen to be very sleepy during Bob's test, thus creating a bad condition for his performance on that particular test. Let $gc$ and $bc$ denote good conditions and bad conditions, respectively. Assume that Evan's conditions during the three tests are i.i.d. with $\Pr(gc \mid skilled) = \Pr(bc \mid unskilled) = 0.85$. The members' strategy is to choose $\Pr(pass \mid gc)$ and $\Pr(pass \mid bc)$ instead. With the exogenous noise, the members can never design informative tests that have $\Pr(pass \mid skilled) = 1$.[3]

There are equilibria in which Bob, Charles, and Dan each designs a test such that Evan always passes when *conditions* are good. Here, I describe the two symmetric equilibria of this type.

[3]The only test design that has $\Pr(pass \mid skilled) = 1$ is $\Pr(pass \mid gc) = \Pr(pass \mid bc) = 1$, but in this case the test is completely uninformative because it is passed unconditionally.

*Equilibrium 1:* each member chooses a test with the same pair of conditional probabilities

$$\Pr(pass \mid gc) = 1 \text{ and } \Pr(pass \mid bc) = 0.51.$$

Ann hires if and only if Evan passes all three tests, in which case she is indifferent. Ann's expected utility is 0.9, and the unconditional probability of hiring is 0.497.

*Equilibrium 2:* each member chooses a test with the same pair of conditional probabilities

$$\Pr(pass \mid gc) = 1 \text{ and } \Pr(pass \mid bc) = 0.03.$$

Ann hires if and only if Evan passes at least two tests, and she is indifferent when he passes exactly two tests. Ann's expected utility is 0.96, and the unconditional probability of hiring is 0.514.

While Ann's expected utility remains at 0.9 in the first equilibrium, her expected utility is strictly higher in the second. The crucial difference is that in the second equilibrium, Ann no longer requires three passes to hire. She adopts a lower standard of two passes because in this equilibrium, a failure reveals only that the conditions were bad during one test, which does not necessary imply that Evan is unskilled. Meanwhile, the test designs in this equilibrium feature a very low $\Pr(pass \mid bc)$; each pass is relatively positive, and two passes outweigh a failure. The members in this equilibrium do not deviate to a higher $\Pr(pass \mid bc)$ because that would raise Ann's standard from two passes to three, making the deviation a costly move.

Interestingly, the members also prefer the second equilibrium since the probability of hiring is higher. The benefit of Ann's low standard outweighs the cost of a low $\Pr(pass \mid bc)$. This, in fact, is generally true when Ann requires a high posterior belief to hire or when the conditions are very noisy. I illustrate this with the generalized comparative statics below.

*Generalized comparative statics*

Replace Ann's payoff table with

| | skilled | unskilled |
|---|---|---|
| hire | 1 | $1 - p_d$ |
| not hire | $p_d$ | 1 |

where $p_d \in \left(\frac{1}{2}, 1\right)$. Let $\Pr(gc \,|\, skilled) = \Pr(bc \,|\, unskilled) = p$, where $p \in (p_d, 1)$.

To better visualize the outcomes of the two symmetric equilibria with $\Pr(pass \,|\, gc) = 1$, consider the following figures. Figure 1 assumes that $p_d = 0.8$ and $p \in (0.8, 1)$. Figure 2 assumes that $p_d \in (0.5, 0.8)$ and $p = 0.8$. Label the equilibria by Ann's standard for hiring. The equilibrium labeled "standard = 2" is the one in which Ann hires after two passes. The equilibrium labeled "standard = 3" is the one in which Ann hires after three passes.

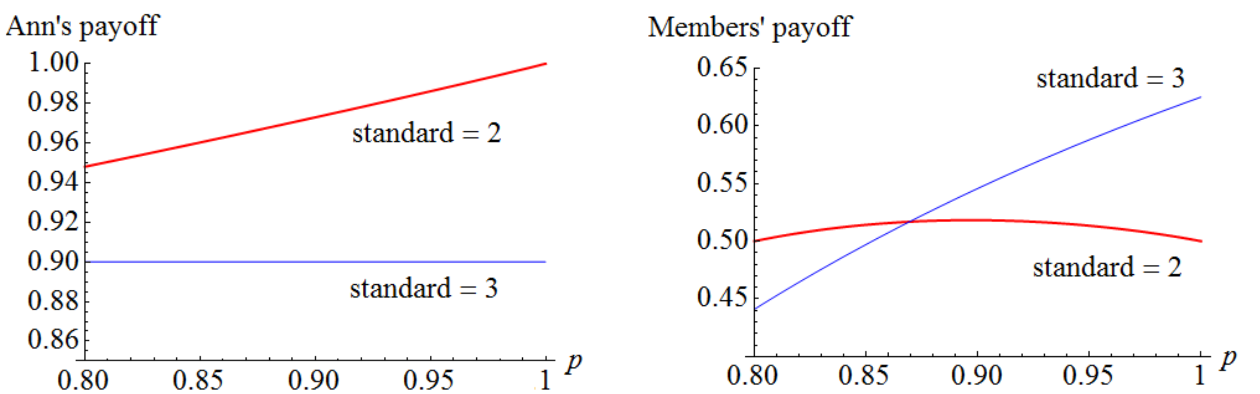



Figure 2.1: $p_d = 0.8$ and $p \in (0.8, 1)$

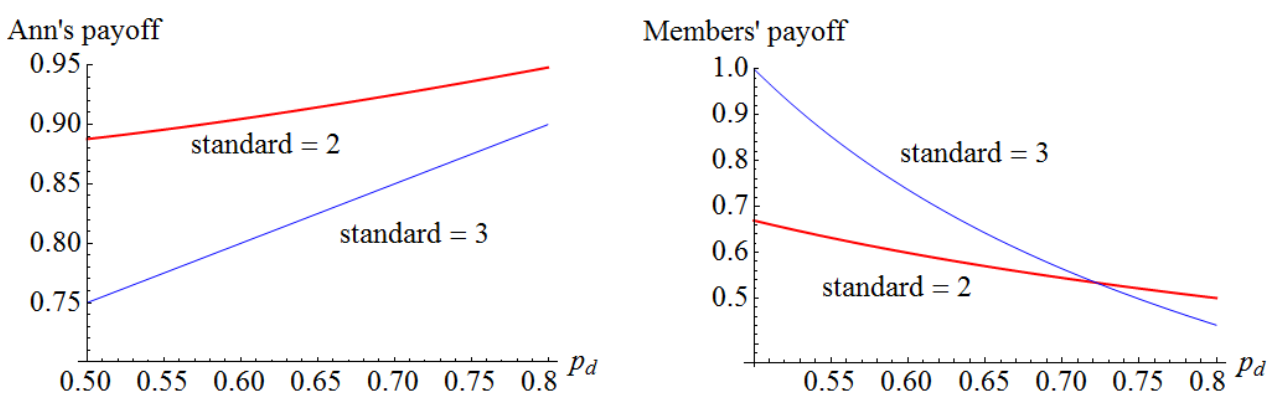



Figure 2.2: $p_d \in (0.5, 0.8)$ and $p = 0.8$

The figures illustrate the comparison between the equilibrium outcomes. First, as discussed earlier, Ann always prefers the equilibrium with a low standard (in the plots on the left, the red line is always above the blue). More surprisingly, the interviewers also prefer this equilibrium when $p$ is low or when $p_d$ is high (in the plots on the right, the red line is above the blue when $p_d = 0.8$ and $p < 0.87$ or when $p = 0.8$ and $p_d > 0.72$). On the one hand, when $\Pr(pass \mid bc)$ is low, Evan is less likely to pass. On the other hand, Ann requires one fewer pass to hire. In general, the latter benefit outweighs the former cost when $p$ is low or when $p_d$ is high because, in these cases, Evan is more likely to fail in equilibrium, so the benefit of a low standard dominates.

### 2.3.4 Discussion

This numerical example illustrates the main ideas of the paper well, but it has its limitation. For example, the three-persuader case fails to characterize the wide range of equilibria associated with a larger number of persuaders.[4] The example does not discuss the asymmetric equilibria, either.

More importantly, the example covers only equilibria with exogenous noise. Theorems 7 and 8 below endogenize these results by showing that, in fact, there always exist equilibria in which persuaders endogenously choose noisy tests that sometimes fail in the good state. Moreover, those equilibria are always associated with a relatively low standard from the decision maker, and, consequently, the decision maker always gains from the tests in those cases.

The next section addresses all of these issues.

[4] When there are 50 persuaders, for example, there exist 24 symmetric equilibria with $\Pr(pass \mid gc) = 1$, and the decision maker's payoff in these 24 cases is monotonically decreasing with the standard that she adopts in each of them.

## 2.4 A general approach

In this section, I study a general case with $n$ persuaders. To develop the main results, Section 4.2 first shows that the decision maker never benefits if equilibrium tests can perfectly reveal the true state. Therefore, when the decision maker does benefit, the equilibrium tests must be noisy. Section 4.3 shows that having exogenous noise embedded in the testing technology indeed gives rise to a wide range of equilibria with high payoffs for the decision maker. Finally, Section 4.4 endogenizes the results in Section 4.3 by showing that persuaders can endogenously design noisy tests in a noise-free environment, and the decision maker always benefits whenever this is the case.

### 2.4.1 Model setup

There are two states of the world: $L$ and $H$. [5] There are $n$ persuaders and a decision maker. The decision maker can choose one of two actions, $a_L$ or $a_H$. (Think of $a_L$ as "not hire" and $a_H$ as "hire" in the motivating example.) Her preference is described by a utility function $u$ that depends on her action and the true state: $u(a_L, L) = u(a_H, H) = 1$, $u(a_H, L) = 1 - p_d$, and $u(a_L, H) = p_d$, for some $p_d \in (\frac{1}{2}, 1)$. With these preferences, the decision maker prefers $a_H$ iff. the posterior probability for state $H$ is above $p_d$. Thus $p_d$ can be viewed as the decision maker's "threshold of doubt." I assume here that the decision maker chooses $a_H$ when she is indifferent.

The persuaders, on the other hand, all prefer that the decision maker chooses $a_H$, regardless of the true state. Their preference can be represented by a common utility function $v$ with $v(a_H) = 1$, and $v(a_L) = 0$.

The persuaders and the decision maker share a common prior: $\Pr(H) = \Pr(L) = \frac{1}{2}$. Each persuader $i$ can perform an endogenous test on an i.i.d. *condition* $c_i \in \{c_H, c_L\}$ that is correlated with the true state. $\Pr(c_H|H) = \Pr(c_L|L) = p$. Assume that $p \in [p_d, 1]$; the

[5] The main result of the paper is robust when the state space is a continuum; see discussion in section 5.E.

decision maker prefers $a_H$ if she directly observes $c_H$. A test is a garbling of the condition that generates a message $m_i \in \{pass,\ fail\}$ with some probabilities conditional on the condition $c_i$. The strategy of each persuader is to choose a test - i.e., a pair of conditional probabilities $(x_i, y_i)$, where $x_i \equiv \Pr(pass|c_L)$ and $y_i \equiv \Pr(pass|c_H)$. Each persuader chooses his test as a best response to the tests of the other persuaders. The decision maker observes both the tests $((x_1, y_1), ..., (x_n, y_n))$ and their outcome messages $(m_1, ..., m_n)$.

The timeline of the game is summarized below.

1. $N$ persuaders simultaneously choose tests $(x_1, y_1), ..., (x_n, y_n)$.

2. Nature chooses the state of the world.

3. Conditional on the state of the world, nature also chooses a condition $c_i$ for each persuader.

4. Each test generates an outcome $m_i$ conditional on $c_i$.

5. After observing all persuaders' choices of test, as well as the test outcomes, the decision maker Bayesian updates her belief about the true state and chooses an action $a$.

Let $\mathbf{t} \equiv ((x_1, y_1), (x_2, y_2), ..., (x_n, y_n))$ denote the persuaders' tests. Let $U(\mathbf{t})$ denote the expected utility of the decision maker and $V(\mathbf{t})$ the expected utility of each persuader before they see the test outcomes. Let $\underline{U} \equiv \frac{1}{2}(1 + p_d)$ be *the decision maker's expected utility when she receives no information from any persuader.* (In this case, she always chooses $a_L$.) Then, $U(\mathbf{t}) \geq \underline{U}$ for all $\mathbf{t}$ because she can always ignore the persuaders' information to guarantee $\underline{U}$. On the other hand, let $\overline{U} \equiv 1$ be *the decision maker's expected payoff when she learns the true state.* Then, $U(\mathbf{t}) \leq \overline{U}$ for all $\mathbf{t}$.

### 2.4.2 Equilibria with quasi-revealing tests

Suppose $p = 1$, i.e. exogenous noise does not exist and the tested condition *is* the true state. In equilibrium, the persuaders never design tests that can perfectly reveal state $H$ because it is always profitable to sometimes pool state $L$ as state $H$. There indeed exist equilibria with tests that can perfectly reveal state $L$ if $y_i = 1$ and $x_i < 1$. I call them *quasi-revealing tests*. The goal of this section is to show that when an equilibrium features quasi-revealing tests, the decision maker expects to gain nothing. It does not take long too see that *this is uniquely the case when there is only one persuader* (Kamenica and Gentzkow, 2011). However, the same statement does not automatically extend to the case of multiple persuaders because, despite it being an unlikely outcome, having all persuaders reveal the true state is always an equilibrium. To see why, note that each persuader $i$ is indifferent regarding his test choice when at least one other persuader reveals the true state. A fully-revealing equilibrium exists if, in this case of indifference, persuader $i$ always reveals the true state, too. However, this equilibrium is always Pareto dominated by some less revealing equilibrium. Moreover, if there is a tiny chance that persuaders $1, 2, ..., n-1$ do not reveal the true state, persuader $n$ strictly prefers not to. The fully-revealing equilibrium also vanishes easily when, for example, an infinitesimal cost occurs if an persuader conducts a fully-revealing test. Therefore, I impose an assumption to avoid this unnatural outcome.

**Assumption 1.** When $(x_j, y_j) = (0, 1)$ for some $j \neq i$, persuader $i$ chooses some $(x_i, y_i) \neq (0, 1)$.

Under assumption 1, when persuaders choose quasi-revealing tests, the decision maker never gains. To see why, first note that in these equilibria, a single failure perfectly reveals state $L$ and the decision maker chooses $a_H$ only when all tests are passed. Therefore, to maximize the probability of $a_H$, the persuaders design tests with sufficiently high probability of passes, so that the decision maker is exactly indifferent when all tests are passed. As a result, the decision maker never strictly prefers to choose $a_H$, and her expected utility is simply $\underline{U}$ - what she would get if she were to always choose $a_L$. Only the persuaders benefit from the tests.

**Proposition 2.** *Assume that Assumption 1 holds. When $p = 1$, in equilibria with $y_i = 1$*

*for all $i$,*

$$\frac{1}{x_1 \cdot \ldots \cdot x_n} = \frac{p_d}{1 - p_d},$$

*and the decision maker chooses $a_H$ iff. all tests are passed.*

*The expected utility of persuaders is*

$$V(\mathbf{t}) = \frac{1}{2}\left(1 + \frac{1 - p_d}{p_d}\right).$$

*The expected utility of the decision maker is*

$$U(\mathbf{t}) = \underline{U}.$$

All proofs not included in this section are in the Appendix.

Proposition 1 suggests that the decision maker's expected utility is higher than $\underline{U}$ only in equilibria with tests that never reveal the true state. Section 4.3 and 4.4 show that those equilibria indeed exist when $n > 2$.

### 2.4.3 Equilibria with exogenous noise

In this section, tests that perfectly reveal the true state are not feasible due to exogenous noise ($p < 1$). When there is only one persuader, the presence of exogenous noise does not improve the equilibrium outcome for the decision maker.[6] However, when there are more persuaders, exogenous noise gives rise to a range of equilibria with high payoffs for the decision maker. For simplicity, throughout this section I focus only on equilibria with $y_i = 1$ for all $i$ - i.e., equilibria with tests that never fail given condition $c_H$. Theorem 8 in Section 4.4 shows that relaxing this restriction only strengthens the main result.

[6] Simply treat the tested noisy condition $c$ as the true state and apply Proposition 1. The optimal test for the unique persuader leaves the decision maker just indifferent when the test is passed, inducing an expected utility of $\underline{U}$ for her.

#### 2.4.3.1 Symmetric equilibria

Suppose $p < 1$. As illustrated in the numerical example in Section 3, there are multiple symmetric equilibria with $y_i = 1$ for all $i$. The number of those equilibria increases linearly with the number of persuaders. For each integer $k$ strictly larger than $\frac{n}{2}$ and weakly smaller than $n$, there exists a symmetric equilibrium in which persuaders choose tests in such a way that the decision maker chooses $a_H$ if and only if at least $k$ tests are passed. Theorem 1 formalizes this and explains why a persuader strictly prefers to choose the same test as everyone else.

**Theorem 2.** *Let $p < 1$. Given any $n$, and any integer $k \in (\frac{n}{2}, n]$, there exists a strict symmetric equilibrium in which $(x_i, y_i) = (x, 1)$ for all $i$, where*

$$x = \frac{p - (1-p)\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{k}}\left(\frac{p}{1-p}\right)^{\frac{n-k}{k}}}{p\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{k}}\left(\frac{p}{1-p}\right)^{\frac{n-k}{k}} - (1-p)},$$

*and the decision-maker chooses $a_H$ if and only if the number of passes is at least $k$. Moreover, $x$ is increasing in $k$.*

*Proof.* Given $n$, $k$, suppose that all persuaders choose the test $x$, as specified above; then

$$\frac{\Pr(H|\text{exactly } k \text{ passes})}{\Pr(L|\text{exactly } k \text{ passes})} = \left[\frac{p + (1-p)x}{(1-p) + px}\right]^k \left(\frac{1-p}{p}\right)^{n-k} = \frac{p_d}{1-p_d}. \tag{2.1}$$

The decision maker is indifferent (and, hence, chooses $a_H$) when exactly $k$ out of $n$ test outcomes are passes. Since more passes suggest a higher probability of state $H$, the decision maker chooses $a_H$ if and only if the number of passes is at least $k$. Moreover, $x$ is an increasing function of $k$. When persuaders choose less informative tests (higher $x$), more passes are needed to persuade the decision maker (higher $k$).

The same logic in the proof of Proposition 1 explains why, in this equilibrium, each persuader $i$ strictly prefers to choose $y = 1$: a deviation to $(x', y')$ for some $y' < 1$ is profitable only when it induces the decision maker to choose $a_H$, even if $n-k+1$ tests, including the test from persuader $i$, have failed. However, this is never the case for any $x' \leq y' < 1$. Therefore, a downward deviation in $y'$ always strictly decreases the unconditional probability of $a_H$ because it lowers the passing rate and makes each pass less positive.

Next, I show that a persuader strictly prefers to choose $x$ when everyone else chooses $x$.

Suppose that persuader $i$ deviates to a test with $x' > x$. The game outcome is affected only when the test outcome of $i$ is a pass. Given the higher passing rate when the condition is low, a pass from $i$'s test is less informative. This implies that the decision maker now strictly prefers $a_L$ when exactly $k$ persuaders (including $i$) report passes. In other words, when $i$ reports a pass, the decision maker needs to see at least $k$ more passes from the other persuaders in order to choose $a_H$. To find out the decision maker's exact response to the deviation, consider the extreme case in which $x' = 1$. That is, persuader $i$ deviates to the least informative test that is never failed. Under this extreme case, persuader $i$ is completely uninformative, and the decision maker chooses an action based only on information delivered by the other persuaders. Moreover, among the rest of the $n-1$ persuaders, $k$ passes and $n-k-1$ failures are sufficient to induce action $a_H$. Therefore, when $i$ reports a pass, the decision maker requires exactly $k$ more passes from the other persuaders to choose $a_H$, and this is true for all $x' \in (x, 1]$. Knowing how the decision maker responds to an upward deviation in $x$, it is most profitable for the persuader to choose $x' = 1$. Then, the new expected utility for the persuaders is equal to the probability of having at least $k$ passes among the rest of the $n-1$ persuaders. Since those $n-1$ persuaders are still choosing test $x$, $\Pr(\text{at least } k \text{ passes}|n-1 \text{ tests with } x) < \Pr(\text{at least } k \text{ passes}|n \text{ tests with } x)$. Therefore, any deviation to some $x' > x$ strictly decreases $i$'s expected utility.

Suppose that persuader $i$ deviates to a test with $x' < x$. Again, the game outcome is affected only when the test outcome of $i$ is a pass. A deviation to a more informative test with lower $\Pr(pass|s_L)$ can be profitable only if it induces the decision maker to choose $a_H$ upon seeing fewer passes - i.e., she chooses $a_H$ when there are only $k-1$ passes. I show that this never happens. Suppose that $i$ deviates to the most informative test, $x' = 0$, and his test outcome is a pass. Suppose, further, that among the rest of the $n-1$ persuaders, $k-2$ report passes, and $n-k+1$ report failures. Then, the posterior likelihood in this case is

$$\begin{aligned}\frac{\Pr(H)}{\Pr(L)} &= \left(\frac{p}{p-1}\right)\left[\frac{p+(1-p)x}{(1-p)+px}\right]^{k-2}\left(\frac{1-p}{p}\right)^{n-k+1} \\ &= \left[\frac{p+(1-p)x}{(1-p)+px}\right]^{k-2}\left(\frac{1-p}{p}\right)^{n-k} \\ &< \frac{p_d}{1-p_d}\end{aligned}$$

by equation (1). Even if $i$ deviates to the most informative test, the decision maker still chooses $a_H$ only when at least $k$ tests are passed, which implies that a downward deviation in $x$ always strictly decreases the persuader's utility. Therefore, given that all the other persuaders choose the $(x, 1)$, it is uniquely optimal for persuader $i$ to choose $(x, 1)$, as well. □

Given $n$, Theorem 1 implies that a symmetric equilibrium can always be identified by the minimum fraction of passes, $\alpha \equiv \frac{k}{n}$, that induces the action $a_H$. Henceforth, I will call $\alpha$ the "standard" of a symmetric equilibrium.

**Definition 2.** A symmetric equilibrium has *standard* $\alpha$ if the decision maker chooses $a_H$ if and only if the fraction of passes is at least $\alpha$.

How does the decision maker rank her payoff among the many symmetric equilibria? The answer: her equilibrium payoff decreases with the equilibrium standard, and her payoff is strictly higher than $\underline{U}$ in all symmetric equilibria except the one with standard $\alpha = 1$. Moreover, fixing standard $\alpha < 1$, as the number of persuaders converges to infinity, the decision maker eventually learns the true state. However, if $\alpha = 1$, she never learns the true state, and her expected utility is always $\underline{U}$.

**Theorem 3.** *Let $p < 1$, and fix $n$. Let $\mathbf{t}_\alpha$ represent the tests chosen in the symmetric equilibrium with $y_i = 1$ $\forall i$ and standard $\alpha$. Then, $\alpha' > \alpha$ implies $U(\mathbf{t}_{\alpha'}) < U(\mathbf{t}_\alpha)$. Moreover,*

$U(\mathbf{t}_1) = \underline{U}$.

*Proof.* Let $\alpha'$ and $\alpha$ be the standards of two symmetric equilibria. All else equal, $\alpha' > \alpha$ must imply that the persuaders choose a less informative test in the former equilibrium - i.e., $x' > x$. This is why the decision maker requires a higher fraction of passes to be persuaded. Theorem 1 also shows that$\alpha$ is an increasing function of $x$. As a result, less information is revealed to the decision maker in the equilibrium with standard $\alpha'$, and this harms her. Therefore, $U(\mathbf{t}_{\alpha'}) < U(\mathbf{t}_\alpha)$.

When $\alpha = 1$, the decision maker chooses $a_H$ only when all tests are passed. Moreover, it is optimal for the persuaders to choose tests in such a way that the decision maker is exactly indifferent when all tests are passed. In other words, the decision maker is indifferent between $a_H$ and $a_L$ whenever $a_H$ is chosen. This implies that her expected utility is equivalent to the amount when she chooses $a_L$ unconditionally - that is, $\underline{U}$. Therefore, I conclude that $U(\mathbf{t}_1) = \underline{U}$. □

What happens when the number of persuaders converges to infinity? On the one hand, as $n$ increases, the decision maker aggregates information from more persuaders, but, on the other hand each persuader chooses a less informative test (a higher $x$). Theorem 3 shows that if the equilibrium standard $\alpha$ is less than 1, the former effect dominates, and the decision maker eventually learns the true state. In contrast, recall that if the equilibrium standard is 1, the decision maker's payoff remains at $\underline{U}$ for all $n$. In this case, the equilibrium test converges to the uninformative one ($x = 1$) as the number of persuaders expands.

**Theorem 4.** *Let $p < 1$. As $n \to \infty$, in all symmetric equilibria with $y_i = 1$ $\forall i$ and some standard $\alpha < 1$, the decision maker "learns the true state" - i.e., her expected utility $U(\mathbf{t}_\alpha)$ converges to $\overline{U}$.*

*Proof.* As $n \to \infty$, the actual fraction of passes converges to the expected fraction, which is equal to $\Pr(pass|H)$ when the state is $H$ and $\Pr(pass|L)$ when the state is $L$. Moreover, $x < 1$ in any equilibrium with standard $\alpha < 1$; hence, $\Pr(pass|H) > \Pr(pass|L)$. Therefore, in the limit, the decision maker can always distinguish the two states and choose the action that exactly matches the true state. As a result, $U(\mathbf{t}_\alpha) \to \overline{U}$ when $\alpha < 1$. □

How do the persuaders rank their payoff among the many symmetric equilibria? As illustrated by the numerical example in Section 3, among all symmetric equilibria, the persuaders can also strictly prefer the one with the most informative test. Theorem 4 shows that asymptotically, for a range of parameters, the persuaders indeed have the same ranking over equilibria as the decision maker: they prefer the state-revealing symmetric equilibria over the non-revealing one.

**Theorem 5.** *Given $p_d \in \left(\frac{1}{2}, 1\right)$ and $p \in (p_d, 1)$, in all symmetric equilibria with $y_i = 1\ \forall i$ and standard $\alpha < 1$, persuaders' expected utility $V(\mathbf{t}_\alpha)$ converges to $\frac{1}{2}$ as $n \to \infty$. In the symmetric equilibrium with $y_i = 1\ \forall i$ and standard $\alpha = 1$, $V(\mathbf{t}_1)$ converges to*

$$f(p_d, p) \equiv \frac{1}{2}\left[\left(\frac{p_d}{1-p_d}\right)^{\frac{p-1}{2p-1}} + \left(\frac{p_d}{1-p_d}\right)^{\frac{-p}{2p-1}}\right].$$

*Moreover, there exist $B \in [0,1]^2$ s.t. when $(p_d, p) \in B$, $f(p_d, p) < \frac{1}{2}$, i.e., the persuaders strictly prefer the state-revealing symmetric equilibria.*

The proof of Theorem 4 (in Appendix) shows that $f$ is decreasing in $p_d$ and increasing in $p$, and it monotonically converges to a value less than $\frac{1}{2}$ when $p_d \to p$ or when $p \to p_d$.

Figure 3 illustrates that $(p_d, p) \in B$ when they are sufficiently close, i.e. the persuaders strictly prefer the state-revealing equilibria when the decision maker is relatively picky (relatively high $p_d$) or when the testing environment is relatively noisy (relatively low $p$). In these cases, for the persuaders, the benefit of a lower standard $\alpha$ dominates the cost of a lower $x$. This is because when $p_d$ and $p$ are close, $x$ is generally low across all equilibria in order for passes to be persuasive. As a result, failures are frequent across equilibria with all levels of standards, and this makes the benefit of a lower standard dominating.

While this paper does not have an analytical analogy for the cases with finite $n$, numerically, the same result holds for small $n$. Cases of $n \leq 20$ exhibit a universal pattern

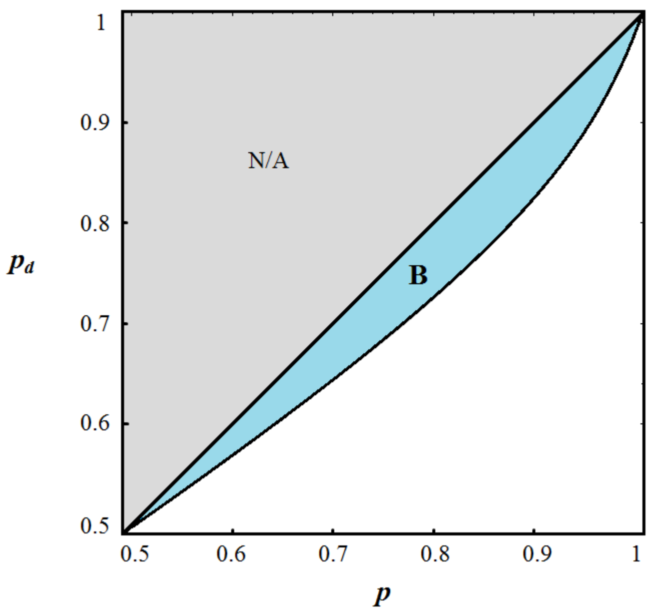



Figure 2.3: Persuaders prefer the state-revealing symmetric equilibria over the non-revealing symmetric equilibrium asymptotically iff. $(p_d, p) \in B$.

numerically: for a fixed $p$, there exists $\bar{p}_d$ such that when $p_d > \bar{p}_d$, the persuaders have the highest expected utility in the symmetric equilibrium with the lowest standard $\alpha$; for a fixed $p_d$, there exists $\underline{p}$ such that when $p < \underline{p}$, the persuaders also strictly prefer the equilibrium with the lowest $\alpha$.

#### 2.4.3.2 Asymmetric equilibria

To show that the results in Section 4.3.1 are robust, I relax the restriction of symmetric equilibria and discuss general results when persuaders can choose different tests. Similar to the results for symmetric equilibria, there exist asymmetric equilibria with different persuader-specific standards. Just as before, the decision maker strictly benefits from the tests if and only if her standard for $a_H$ permits some failures.

I start by defining the analogy of an equilibrium standard in the asymmetric setting. When persuaders can choose different tests, the standard is no longer a number like $\alpha$, but a persuader-specific set. Given the test choices, the decision maker chooses $a_H$ if and only if the set of persuaders with passed tests belongs to the set.

**Definition 3.** Given test choices $\mathbf{t}$, let $a \subseteq \{1, 2, ..., n\}$ denote the set of persuaders whose tests are passed. Then, $s \in \mathcal{P}(\{1, 2, ..., n\})$ is called the *standard set* for a given equilibrium when the decision maker chooses $a_H$ if and only if $a \in s$.

*Remark* 1. Since the decision maker is Bayesian, given the model setup, a standard set must satisfy this: if $a_1 \in s$ and $a_1 \subset a_2$, then $a_2 \in s$. That is, more passes cannot be less persuasive. The analogy of a higher standard for symmetric equilibria is a smaller standard set in the asymmetric setting.

Here are two equilibria in which three persuaders choose different tests.

**Example** Let $p = 0.8$, $p_d = \frac{2}{3}$, $n = 3$.

Equilibrium 1: persuaders choose $\mathbf{t}_1 = \left(\left(\frac{2}{7}, 1\right), (1, 1), (1, 1)\right)$. The decision maker's standard set is $s = \{\{1, 2, 3\}\}$. $V(\mathbf{t}_1) = 0.643$, $U(\mathbf{t}_1) = 0.833 = \underline{U}$.

Equilibrium 2: persuaders choose $\mathbf{t}_2 = \left((0, 1), (0, 1), \left(\frac{2}{7}, 1\right)\right)$. The decision maker's standard set is $s = \{\{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$. $V(\mathbf{t}_2) = 0.569$, $U(\mathbf{t}_2) = 0.927 > \underline{U}$.

In the first equilibrium, only the first persuader chooses an informative test, and the decision maker chooses $a_H$ if and only if the first test is passed. I verify that this is, indeed, an equilibrium. For the first persuader, given that the other persuaders are completely uninformative, the game resembles one in which he is the unique persuader, and, in this case, his optimal test is $x_1 = \frac{2}{7}$. For the second (or the third) persuader, deviating to a lower $x_2$ is profitable only if doing so expands the standard set from $\{\{1, 2, 3\}\}$ to $\{\{2, 3\}, \{1, 2, 3\}\}$. However $\{2, 3\}$ will never be in a standard set because a pass from the second persuader will never offset a failure from the first persuader, given that the third persuader is uninformative. Therefore, there is no profitable deviation for any persuader.

In the second equilibrium, all persuaders choose informative tests. both the first and the second persuaders choose truthful tests that perfectly reveal their conditions, while the third persuader does not. I verify that this is also an equilibrium. For the first (or the second) persuader, the utility-maximizing test that induces $s' = \{\{2,3\},\{1,2,3\}\}$ is $\left(\frac{2}{7},1\right)$, which yields

$$V\left(\left(\frac{2}{7},1\right),(0,1),\left(\frac{2}{7},1\right)\right) = 0.518 < V(\mathbf{t}_2).$$

The utility-maximizing test that induces $s" = \{\{1,2,3\}\}$ is $\left((1,1),(0,1),\left(\frac{2}{7},1\right)\right)$, which yields

$$V\left((1,1),(0,1),\left(\frac{2}{7},1\right)\right) = 0.386 < V(\mathbf{t}_2).$$

Therefore, there is no profitable deviation for the first (or the second) persuader. For the third persuader, the most profitable deviation is $(1,1)$, and the standard set becomes $\{\{1,2,3\}\}$. This yields

$$V((0,1),(0,1),(1,1)) = 0.34 < V(\mathbf{t}_2).$$

Hence, there is no profitable deviation for any persuader.

Note that in the example above, the expected utility of the decision maker is equal to $\underline{U}$ in equilibrium 1 and higher than $\underline{U}$ in equilibrium 2. This is, in fact, a general feature of asymmetric equilibria: the decision maker's expected utility is strictly higher than $\underline{U}$ if and only if the standard set is strictly larger than $\{\{1,2,...,n\}\}$. The same intuition from Theorem 2 applies here: in the case of $s = \{\{1,2,...,n\}\}$, the decision maker never strictly prefers to choose $a_H$, and, therefore, she is as well off as always choosing $a_L$ unconditionally.

**Theorem 6.** *Given an equilibrium with tests* $\mathbf{t}$, *if* $s_{\mathbf{t}} = \{\{1,2,...,n\}\}$, *then* $U(\mathbf{t}) = \underline{U}$. *If* $s_{\mathbf{t}} \supsetneq \{\{1,2,...,n\}\}$, *then* $U(\mathbf{t}) > \underline{U}$.

The numerical example also shows that the persuaders have a higher expected utility in equilibrium 1 (only one informative test) than in equilibrium 2 (more than one informative test). Theorem 6 shows that, in general, among all equilibria that feature the smallest standard set $\{\{1,2,...,n\}\}$, persuaders have the highest expected utility in the equilibrium with only one informative test: $x_i < 1$ and $y_i = 1$ for some $i$, and $x_j = y_j = 1$ for all

$j \neq i$. When two or more persuaders independently choose informative tests ($x < 1$), the probability that the decision maker chooses $a_H$ in an equilibrium is lower. This is due to the independence of test choices. Note that in an equilibrium, the probability of $a_H$ is negatively associated with the average posterior belief when the test outcomes do not fall under the standard set. The lower the latter average posterior belief is, the higher the former probability for persuasion can be. This is because the decision maker is more willing to choose $a_H$ upon seeing weakly informative passes when those passes can assure her that the very worst cases did not happen. However, when persuaders independently choose informative tests, the average posterior belief when the test outcomes do not meet the standard is bounded from below. Among the many outcome combinations not included in the standard set, the ones that are a balanced mixture of passes and failures are more frequent than the ones with mostly failures. Therefore, the high average posterior in these cases prevents persuaders from choosing relatively high $x$, which leads to a low probability for persuasion.

**Definition 4.** When $n = 1$ (benchmark), let $(x_B, 1)$ denote the persuader's equilibrium test choice[7], and let $V_B$ denote his expected utility.

**Theorem 7.** *Assume that $p < 1$. Fix $n \geq 2$. In equilibria with $y_i = 1\ \forall i$, if the standard set induced by equilibrium tests $\mathbf{t}$ is $\{\{1, 2, ..., n\}\}$, then $V(\mathbf{t}) \leq V_B$. Moreover, the equality holds if and only if there exists $i$ s.t. $x_i = x_B$ and $x_j = 1$ for all $j \neq i$.*

Due to the complexity of asymmetric equilibria, whether Theorem 6 extends to equilibria with larger standard sets remains an open question. Numerical analysis shows that for small $n$, symmetric equilibria always yield a lower expected utility than $V_B$ for the persuaders, regardless of the equilibrium standard. An analysis of asymmetric equilibria with $n = 3$ also shows that the persuaders have the highest expected payoff when the equilibrium features a

[7]Specifically, $x_B = \dfrac{p - \left(\frac{p_d}{1-p_d}\right) + \left(\frac{p_d}{1-p_d}\right)p}{\left(\frac{p_d}{1-p_d}\right)p - 1 + p}$.

unique informative test. These evidence suggests that Theorem 6 can indeed extend beyond equilibria with the smallest standard set.

### 2.4.4 Equilibria with endogenous noise

Section 4.3 shows that when exogenous noise is present, there are equilibria in which the decision maker strictly gains. I now endogenize this result. This section mostly focuses on the case of $p = 1$ (no exogenous noise). I show that there always exist strict equilibria with high payoffs for the decision maker. In these equilibria, the persuaders endogenously choose noisy tests with $\Pr(pass|H) < 1$ even when tests with $\Pr(pass|H) = 1$ are feasible. Moreover, whenever an equilibrium features some tests with $\Pr(pass|H) < 1$, the decision maker's expected utility must be strictly higher than $\underline{U}$. The following examples illustrate the main intuition.

**Example 1.** Let $p = 1$, $n = 2$, and $p_d = 0.64$. There exists an equilibrium in which both persuaders choose $\mathbf{t}_1 = \mathbf{t}_2 = (0.1, 0.8)$ and the decision maker chooses $a_H$ if any only if at least one test is passed.

**Example 2.** Let $p = 1$, $n = 3$, and $p_d = \frac{81}{113}$. There exists an equilibrium in which all persuaders choose $\mathbf{t}_1 = \mathbf{t}_2 = \mathbf{t}_3 = (0.2, 0.9)$ and the decision maker chooses $a_H$ if and only if at least two tests are passed.

I verify that both examples are strict equilibria in the Appendix. To see why it is strictly profitable for a persuader to choose a test that sometimes fail state $H$, note that a deviation to $\Pr(pass|H) = 1$ has three effects. First, the probability of a pass is higher. Second, a pass induces a higher posterior belief. Third, a failure induces a lower posterior belief. In both equilibria, a deviation to $\Pr(pass|H) = 1$ is not profitable because the penalty from the third effect outweighs the benefits from the first two effects: following this deviation, a failure perfectly reveals state $L$ and the decision maker never chooses $a_H$ when she sees a failure from the persuader that deviated. This decreases the overall probability of $a_H$.

Now focus on the fact that, in both examples, the decision maker can choose $a_H$ even when some test is failed. (Indeed, the decision maker's relatively low standard is precisely the reason why the persuaders are willing to choose noisy tests.) As discussed in earlier sections, this implies that the decision maker strictly prefers $a_H$ when all tests are passed and, therefore, her expected utility must be strictly higher than $\underline{U}$.

The following theorems generalize the examples. Theorem 7 shows that equilibria with endogenous noise exist universally. Theorem 8 shows that the decision maker always gains in equilibria with endogenous noise.

**Theorem 8.** *Let $p = 1$ and $n > 1$. For all $p_d \in \left(\frac{1}{2}, 1\right)$, there exists a strict equilibrium in which the persuaders choose tests that never perfectly reveal the true state.*

The proof of Theorem 7 is done in two steps. First, I show that for $n = 2$ and for all $p_d$, there exist strict equilibria in which both persuaders choose endogenously noisy tests. Then, I extend the result and argue that some of these equilibria can be extended into an $n$-persuader equilibrium in which the first two persuaders play the same strategy as in the two-persuader equilibrium, while the rest persuaders choose uninformative tests that are always passed independent of the true state.[8] See the formal proof in the Appendix.

Next, I show tests that sometimes fail given state $H$ (or condition $c_H$, if $p < 1$) appear only if the decision maker can choose $a_H$ even in the presence of some failures, and, therefore, the decision maker always gains in those equilibria. If, in equilibrium, the decision maker chooses $a_H$ only when all tests are passed, the persuaders always find it optimal to choose $y_i = 1$ for all $i$, inducing an expected utility of $\underline{U}$ for the decision maker.

**Theorem 9.** *For all $p_d \in \left(\frac{1}{2}, 1\right)$, and $p \in [p_d, 1]$, if in an equilibrium $y_i < 1$ for some $i$, then the standard set $s \supsetneq \{\{1, ..., n\}\}$, and the decision maker's expected utility is strictly higher than $\underline{U}$.*

[8] Whether symmetric equilibria with endogenously noisy tests exist for arbitrary $n$ remains an open question.

*Proof.* I first show that *if the standard set in a given equilibrium is* $s = \{\{1, ..., n\}\}$*, then the equilibrium tests must satisfy* $y_1 = 1$ *for all* $i$*.*

If in an equilibrium, the decision maker chooses $a_H$ iff. all tests are passed, then the persuaders must choose tests such that the decision maker is indifferent when all tests are passed. That is, tests $(x_i, y_i)$ satisfy

$$\prod_i^n \left[\frac{y_i p + x_i (1-p)}{y_i (1-p) + x_i p}\right] = \frac{p_d}{1-p_d}.$$

Suppose that $y_i < 1$ for some $i$. persuader $i$ is strictly better off when he increases $y_i$ because doing so increases the probability of a pass from $i$ and makes a pass from $i$ more positive. The standard set does not shrink after the deviation since $s$ is already the smallest standard set. Therefore, such a deviation strictly increases the probability of $a_H$, and persuader $i$ must choose $y_i = 1$ in such an equilibrium.

Since $s = \{\{1, ..., n\}\}$ implies that $y_i = 1$ for all $i$, when $y_i < 1$ for some $i$, it must be the case that $s \supsetneq \{\{1, ..., n\}\}$. That is, the decision maker is indifferent when some tests are failed. Therefore, when all tests are passed, the decision maker must strictly prefer $a_H$. This implies that her expected utility must be strictly higher than $\underline{U}$. □

Finally, it is worth mentioning that although equilibria with endogenous noise do exist and are always beneficial for the decision maker, they are never the best equilibria for the persuaders. Kamenica and Gentzkow (2011) imply that in an environment without exogenous noise, the best equilibrium for the persuaders is outcome-equivalent to the one in which the first persuader chooses a test as if he is the only persuader, and the rest choose uninformative tests that are always passed unconditionally.

## 2.5 Discussion

***A. Single persuader with multiple tests***

The games that this paper studies feature multiple identical persuaders, each of whom independently conducts one test. One might ask what happens if there is only one persuader in charge of all these independent tests. In this case of one persuader, the equilibrium is unique: the persuader chooses tests $\mathbf{t}$ that maximize $V(\mathbf{t})$. The outcome is simply equivalent to the the best equilibrium outcome for the multiple persuaders in this paper. When exogenous noise is absent, the unique persuader optimally design tests so that only one of them is informative, and the decision maker does not gain from the tests (Kamenica and Gentzkow, 2011). However, if exogenous noise exists and the tests on each noisy condition must be identical, the decision maker can expect a better outcome. As Theorem 4 implies, if the decision maker is sufficiently picky or if the environment is sufficiently noisy, the unique persuader optimally designs relatively informative tests that induce a high payoff for the decision maker.

Overall, when there is a single persuader conducting multiple tests (or, alternatively, when all persuaders collaborate), outcomes with a high payoff for the decision maker are more rare. Given that all the persuaders' preferences are perfectly aligned and externalities are absent, this comparison emphasizes that merely preventing perfect coordination among the persuaders is sufficient to induce high levels of information revelation.

***B. Correlated tests***

In this paper, I assume that the tests are independent, which is a natural assumption for many real-world scenarios, including the interview example in the Introduction. Consider, instead, that the persuaders may choose correlated tests. In this case, the decision maker never gains regardless of whether exogenous noise is present. Specifically, if there is no exogenous noise, regardless of the number of persuaders, all strict equilibria[9] are outcome-equivalent to the equilibrium with a single persuader and a zero-gain for the decision maker. When there is exogenous noise, the persuaders are better off when there are more of them, and the decision maker never gains from their collected information.[10]

Therefore, the absence of perfect correlation is crucial for the existence of strict equilibria with a high payoff for the decision maker. While this paper studies the case of perfect independence, the same intuition carries over in the case of partial correlation: as long as the persuaders cannot perfectly coordinate on the number of test failures, they have the incentive to design tests with fairly informative passes so that the decision maker adopts a relatively low standard. This generates a high level of information revelation and a high payoff for the decision maker.

In general, as Li and Norman (2015) point out, the assumption of independence gives rise to an even wider range of predictions when persuaders have different preferences. Their paper provides an example in which adding an independent persuader may strictly harm the decision maker.

### *C. Sequential persuaders*

The persuaders in this paper choose tests simultaneously. Suppose, instead, that they choose the tests in a sequence. Then, the unique subgame perfect equilibrium is outcome-equivalent to the best equilibrium for the persuaders in the simultaneous game. If there is no exogenous noise, results in Kamenica and Gentzkow (2011) predict that the first persuader optimally

[9] While other equilibria - e.g. the fully revealing equilibrium in which all persuaders reveal the true state - exist, they reply on tie-breaking rules that feature excessive information revelation. These equilibria are Pareto dominated by the described strict equilibria.

[10] To see why, note that a game with $n$ identical persuaders choosing correlated tests is analogous to a game in which one persuader conducts a test on $n$ i.i.d. conditions jointly. Moreover, a test on $n$ i.i.d. conditions is analogous to a test on a single condition with better accuracy (a higher $p$). In the latter case, the equilibrium features a test with a higher passing rate under a bad condition (a higher $x$) and, hence, a higher expected utility for the persuader. The increased $x$ is supported by the increased informativeness of a failure. Since the decision maker either chooses her default action $a_L$ or is indifferent between $a_L$ and $a_H$ when she chooses $a_H$ after seeing a pass, the decision maker is as well off as always choosing $a_L$ unconditionally. Hence, the information collected from the test does not increase the decision maker's expected utility.

chooses a test design as if he is the only persuader, and all later persuaders choose uninformative tests that are always passed. The decision maker does not gain in this equilibrium. Results in Section 4.3.2 show that this may be the case when exogenous noise is present, as well.

### *D. Asymmetrically constrained testing technology*

The assumption of exogenous noise is symmetric in this paper. That is, $\Pr(c_H|H) = \Pr(c_L|L) = p$. However, the symmetry is sufficient, but not necessary for the existence of equilibria with high expected utility for the decision maker. To see why, note that the symmetric constraint puts an upper bound on both posterior probabilities $\Pr(H|pass)$ and $\Pr(L|fail)$. In equilibrium, only the latter constraint on $\Pr(L|fail)$ binds since the persuaders endogenously choose tests such that $\Pr(H|pass) \leq p_d < p$ anyway. Therefore, relaxing the constraint on $\Pr(c_L|L)$ does not change the equilibrium outcome. The main results of this paper hold as long as $\Pr(c_H|H) < 1$.

### *E. Continuous state space*

The main result of the paper does not hinge on the assumption of the binary state space. Suppose the true state is a continuous variable $z \in \mathbb{R}$. If the action space of the decision maker is still $\{a_H, a_L\}$ and the persuaders still strictly prefer $a_H$ regardless of the true state, then the decision maker strictly gains only if there are multiple persuaders conducting noisy tests.

To see why this is true, first note that, in equilibrium, the persuaders endogenously choose coarse test designs even when the state space is a continuum. As Kolotilin (2015) shows, when there is one persuader, the optimal test is one such that the persuader reports only a "pass" when $z \geq z^*$ and a "failure" when $z < z^*$, where $z^*$ is a threshold chosen such that the decision maker is indifferent when the persuader reports a pass. When there are multiple persuaders, it is also an equilibrium that each persuader chooses this same test with threshold $z^*$, and all tests yield the same outcome. In these cases, the decision maker does not gain from the test(s) because she either plays her default action if the test(s) failed, or is indifferent if the test(s) passed.

On the contrary, when test outcomes never perfectly reveal the true state, there exist multi-persuader equilibria in which the persuaders choose relatively informative tests with a relatively higher threshold, and the decision maker chooses $a_H$ even if some tests failed. This implies that, in expectation, the decision maker strictly gains since she strictly prefers $a_H$ when all tests passed. The existence of these equilibria results from the same intuition as in the binary-state case: when the persuaders cannot perfectly coordinate on the test outcomes, they have an incentive to design relatively informative tests so that a few failures can be outweighed by strong passes.

For example, when there are two persuaders, there exists a symmetric equilibrium in which both persuaders choose to report "pass" when $z \geq \overline{z}$ and "fail" when $z < \overline{z}$, where $\overline{z}$ is a relatively high[11] threshold chosen in such a way that the decision maker is indifferent when one persuader reports "pass" and the other reports "failure". The decision maker chooses $a_H$ if and only if there is at least one pass. To check this is indeed an equilibrium, note that when the second persuader is using this strategy, it suffices to check that it is not profitable for the first persuader to always report "pass" regardless of the state: [12] following such a deviation, the decision maker chooses $a_H$ if and only if the second persuader reports "pass", but this leads to a lower probability of $a_H$, since previously the decision maker also chooses $a_H$ when the second persuader reports "failure" but the first reports "pass." Therefore, this deviation cannot be profitable.

[11]compared to the threshold for the single persuader game, $z^*$

[12]To see why, first note that since a "pass" is sufficient to guarantee $a_H$, the first persuader has no incentive to design a test whose outcomes can be more positive than a "pass" by implementing higher threshold(s). Alternatively, if he deviates in such a way that the most positive test outcome is less positive than a "pass", then the decision maker responds by choosing $a_H$ only if the second persuader reports "pass". Therefore in this category it is best to deviate to an uninformative test described above.

### *F. Implications for the decision maker*

If, in reality, a decision maker relies her decision on persuaders' endogenously collected information, this paper suggests that independently consulting more than one persuader can make the decision maker better off even in a worst case scenario, that is, even if those persuaders all have the same extreme bias towards a certain action and side transfers to induce information revelation are not feasible. Focusing only on the best equilibrium for the decision maker, her expected utility increases substantially even if she has merely two persuaders instead of one, as suggested by the proof of Theorem 7. Even if there is reason to believe that the actually equilibrium selected is the biased persuaders' favorite equilibrium, there are still methods for the decision maker to apply in order to induce a high payoff. As Theorem 4 suggests, by restricting the persuaders' ability to collect perfect information and by requiring all persuaders to use the same test design, the decision maker can guarantee a high payoff, as the most informative equilibrium is Pareto dominant.

Overall, this paper emphasizes that the lack of perfect coordination among multiple persuaders is sufficient to increase the payoff for the decision maker. If, in addition, the decision maker can find persuaders whose preferences are more aligned with hers, or if the decision maker can induce information revelation through payment transfers, then this paper predicts that she should expect an even higher payoff.

## 2.6 Appendix

### 2.6.1 Proof of Proposition 1

When $y_i = 1$ for all $i$, and $\frac{1}{x_1 \cdot \ldots \cdot x_n} = \frac{p_d}{1-p_d}$, the decision maker is indifferent when all tests are passed:

$$\begin{aligned}\frac{\Pr(H|pass_1,...,pass_n)}{\Pr(L|pass_1,...,pass_n)} &= \prod_{i=1}^{n} \frac{\Pr(pass_i|H)}{\Pr(pass_i|L)} \\ &= \frac{1}{x_1 \cdot \ldots \cdot x_n} \\ &= \frac{p_d}{1-p_d}.\end{aligned}$$

I first show that no persuader has an incentive to deviate to some $y < 1$.

Suppose that persuader $j$ deviates to some $\left(x_j^{'}, y_j^{'}\right)$ s.t. $y_j^{'} \in [x_j^{'}, 1)$. (If $y_j^{'} < x_j^{'}$, then relabel "pass" as "fail" and vice versa so that $y_j^{'} > x_j^{'}$.) There are three effects of a downward deviation of $y_j$. First, the ex ante probability of a pass decreases. Second, a lower $y_j$ makes a pass from $j$ less persuasive since

$$\frac{\Pr(pass_j|H)}{\Pr(pass_j|L)} = \frac{p \cdot y_j + (1-p) \cdot x_j}{(1-p) \cdot y_j + p \cdot x_j}$$

is increasing in $y_j$ . Third, a lower $y_j$ makes a failure from $j$ less negative since

$$\frac{\Pr(fail_j|H)}{\Pr(fail_j|L)} = \frac{1-[p \cdot y_j + (1-p) \cdot x_j]}{1-[(1-p) \cdot y_j + p \cdot x_j]}$$

is decreasing in $y_j$.

Therefore, a decrease in $y_j$ is profitable only if it induces the decision maker to choose $a_H$ even if test $j$ is failed. However, this is never the case since it requires that

$$\frac{1-y_j}{1-x_j} \cdot \prod_{i \neq j}^{n} \frac{1}{x_i} \geq \frac{p_d}{1-p_d},$$

but $\frac{1}{x_1 \cdot ... \cdot x_n} = \frac{p_d}{1-p_d}$ implies that the required inequality never holds for $y_j^{'} \geq x_j^{'}$. Hence, persuader $j$ is strictly worse off after a decrease to $y_j' < 1$.

I next show that a deviation in $x_i$ also strictly decreases persuader $i$'s expected utility for all $i$.

Suppose that persuader $i$ chooses $x_i^{'} > x_i$. In this case a pass from $i$ is less positive, and the decision maker chooses $a_L$ even if all tests are passed, making the persuaders strictly worse off.

Suppose that persuader $i$ chooses $x_i^{'} < x_i$. Because $y_j = 1$ for all $j$, it is still the case that the decision maker chooses $a_H$ only when all tests are passed, since each failure directly reveals state $L$. Therefore, such a downward deviation only decreases the probability of $a_H$ because a pass from $i$ is less likely. Again, the persuaders are strictly worse off.

Therefore, the proposed strategies indeed constitute an equilibrium. In this equilibrium, the expected utility of the persuaders is simply the expected probability that all tests are passed, $V = \frac{1}{2}\left(1 + \frac{1-p_d}{p_d}\right)$. The decision maker either chooses $a_L$, or is indifferent between $a_H$ and $a_L$ when she chooses $a_H$. Hence, her expected utility is equal to what she would get from always choosing $a_L$: $\underline{U}$.

### 2.6.2 Proof of Theorem 4

When $\alpha = 1$, by Theorem 3, the decision maker learns the true state, and she chooses $a_H$ if and only if the true state is $H$. Therefore, $V(\mathbf{t}_\alpha)$ converges to $\frac{1}{2}$, the ex ante probability for state $H$.

When $\alpha = 1$, $\mathbf{t}_1 = ((x, 1), ..., (x, 1))$, where

$$x = \frac{p - (1-p)\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}}}{p\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}} - (1-p)},$$

$$\begin{aligned}
\Pr(a_H|H) = [\Pr(pass|H)]^n &= [p + (1-p)x]^n \\
&= \left[ p + (1-p) \cdot \frac{p - (1-p)\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}}}{p\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}} - (1-p)} \right]^n \\
&\to \left(\frac{p_d}{1-p_d}\right)^{\frac{p-1}{2p-1}} \text{ as } n \to \infty.
\end{aligned}$$

$$\begin{aligned}
\Pr(a_H|L) = [\Pr(pass|L)]^n &= [(1-p) + px]^n \\
&= \left[ 1 - p + p \cdot \frac{p - (1-p)\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}}}{p\left(\frac{p_d}{1-p_d}\right)^{\frac{1}{n}} - (1-p)} \right]^n \\
&\to \left(\frac{p_d}{1-p_d}\right)^{\frac{-p}{2p-1}} \text{ as } n \to \infty.
\end{aligned}$$

Hence,

$$V(\mathbf{t}_1) = \frac{1}{2} \cdot \Pr(a_H|H) + \frac{1}{2} \cdot \Pr(a_H|L) \to \frac{1}{2} \left[ \left(\frac{p_d}{1-p_d}\right)^{\frac{p-1}{2p-1}} + \left(\frac{p_d}{1-p_d}\right)^{\frac{-p}{2p-1}} \right] \equiv f(p_d, p),$$

with

$$\frac{\partial f}{\partial p_d} = \frac{\left(\frac{p_d}{1-p_d}\right)^{\frac{p-1}{2p-1}} \left(1 + \frac{p}{p_d} - 2p\right)}{2(1-2p)\, p_d (1-p_d)} < 0,$$

$$\lim_{p_d \to \frac{1}{2}} f = 1,$$

$$\lim_{p_d \to p} f = \frac{1}{2} \cdot p^{\frac{-p}{2p-1}} (1-p)^{\frac{1-p}{2p-1}} < \frac{1}{2}.$$

$$\frac{\partial f}{\partial p} = \frac{\left(\frac{p_d}{1-p_d}\right)^{\frac{-p}{2p-1}}\left[1+\left(\frac{p_d}{1-p_d}\right)\right]\ln\left(\frac{p_d}{1-p_d}\right)}{2\left(2p-1\right)^2} > 0,$$

$$\lim_{p\to p_d} f = \frac{1}{2}\cdot p_d^{\frac{-p_d}{2p_d-1}}\left(1-p_d\right)^{\frac{1-p_d}{2p_d-1}} < \frac{1}{2},$$

$$\lim_{p\to 1} f = \frac{1}{2}\left(1+\frac{1-p_d}{p_d}\right) > \frac{1}{2}.$$

Therefore, $\forall p_d \in \left(\frac{1}{2}, 1\right)$, there exist $\overline{p}$ s.t. $p \in (p_d, \overline{p})$ implies $f\left(p_d, p\right) < \frac{1}{2}$. Define region $B$ accordingly.

### 2.6.3 Proof of Theorem 5

When $s_{\mathbf{t}} = \{\{1, 2, ..., n\}\}$, the decision maker chooses $a_H$ only when all tests are passed. Thus, she either chooses $a_L$ or is indifferent between $a_H$ and $a_L$ when she chooses $a_H$. Therefore, her payoff is always $\underline{U}$ - the payoff she gets when she chooses $a_L$ unconditionally.

### 2.6.4 Proof of Theorem 6

Theorem 6 is proved in two steps. I first show that the result is true for the case of two persuaders ($n = 2$) and then extend the result to an arbitrary $n$.

**Lemma 2.** *Assume that $p < 1$. When $n = 2$, let $\mathbf{t} = \left(\left(x_1, 1\right), \left(x_2, 1\right)\right)$ denote the tests in an equilibrium. Then, $V(\mathbf{t}) \leq V_B$. Moreover, the equality holds if and only if $\mathbf{t} = \left(\left(1, 1\right), \left(x_B, 1\right)\right)$ or $\left(\left(x_B, 1\right), \left(1, 1\right)\right)$.*

*Proof.* Since $p_d > \frac{1}{2}$, the standard set for $n = 2$ must be $\{\{1, 2\}\}$. Therefore, the persuaders' optimal strategy is to choose $\mathbf{t}$ such that the decision maker is exactly indifferent when she sees two passes:

$$\frac{\Pr(H|pass, pass)}{\Pr(L|pass, pass)} = \frac{\Pr(pass, pass|H)}{\Pr(pass, pass|L)} = \frac{\left[p+(1-p)x_1\right]\left[p+(1-p)x_2\right]}{\left(1-p+px_1\right)\left(1-p+px_2\right)} = \frac{p_d}{1-p_d} \tag{2.2}$$

$$x_2^*(x_1) = \frac{p^2 - (\frac{p_d}{1-p_d} - 1)p(1-p)x_1 - \frac{p_d}{1-p_d}(1-p)^2}{\left(\frac{p_d}{1-p_d} - 1\right)p(1-p) + \left[\frac{p_d}{1-p_d}p^2 - (1-p)^2\right]x_1} \tag{2.3}$$

There are infinitely many equilibria when $n = 2$, as long as $x_1$ and $x_2$ satisfy equation (3). Since $V(\mathbf{t}) = \frac{1}{2}\left[\Pr(a_H|H) + \Pr(a_H|L)\right]$, to prove the lemma, it is sufficient to show that the conditional probabilities

$$\Pr(a_H|H) = \Pr(pass, pass|H)$$

$$\Pr(a_H|L) = \Pr(pass, pass|L)$$

are both maximized exactly when $x_1 = 1,\ x_2 = x_B$, or $x_1 = x_B,\ x_2 = 1$.

Notice that the expected posterior likelihood when the decision maker chooses $a_L$,

$$l_{a_L} \equiv \frac{1 - \Pr(pass, pass|H)}{1 - \Pr(pass, pass|L)} = \frac{1 - \left(\frac{p_d}{1-p_d}\right) \cdot \Pr(pass, pass|L)}{1 - \Pr(pass, pass|L)},$$

strictly decreases with $\Pr(pass, pass|L)$, as well as $\Pr(pass, pass|H) = \frac{p_d}{1-p_d} \cdot \Pr(pass, pass|L)$. In other words, $\Pr(a_H|H)$ and $\Pr(a_H|L)$ are maximized exactly when $l_{a_L}$ is minimized. For arbitrary $(x_1, x_2)$,

$$l_{a_L} = \frac{(1 - x_1x_2)(1-p)^2 + (2 - x_1 - x_2)p(1-p)}{(1 - x_1x_2)p^2 + (2 - x_1 - x_2)p(1-p)},$$

which decreases in both $x_1$ and $x_2$ because $(1-p)^2 < p^2$. Moreover,

$$\lim_{x_1 \to 1} l_{a_L} = \lim_{x_2 \to 1} l_{a_L} = \frac{1-p}{p}. \tag{2.4}$$

Since $\frac{1-p}{p}$ is precisely the posterior likelihood when the decision maker chooses $a_L$ (i.e. the test is failed) in the benchmark case, (4) implies that both $\Pr(a_H|H)$ and $\Pr(a_H|L)$ are maximized exactly when $x_1 = 1$ or $x_2 = 1$; that is, the equilibria associated with the highest payoff for the persuaders must have one of the persuaders choosing an uninformative test and the other choosing the benchmark test. In either case, the maximized conditional payoff is equal to the persuader's conditional payoff in the benchmark case, and, hence, $V(\mathbf{t}) = V_B$. □

Remark: The unique symmetric equilibrium with $x_1 = x_2$ generates the lowest payoff for the persuaders.[13]

**Theorem 6.** Assume that $p < 1$. Fix $n \geq 2$. In equilibria with $y_i = 1\ \forall i$, if the standard set induced by tests $\mathbf{t}$ is $\{\{1, 2, ..., n\}\}$, then $V(\mathbf{t}) \leq V_B$. Moreover, the equality holds if and only if there exists $i$ s.t. $x_i = x_B$ and $x_j = 1$ for all $j \neq i$.

*Proof.* Start with $n = 3$. Fix the test choice of the third player, $x_3$. Let $l_i$ denote the posterior likelihood ratio $\dfrac{\Pr(H|pass)}{\Pr(L|pass)} = \dfrac{p + (1-p)x_i}{1 - p + px_i}$ after a single pass from test $i$. Let $A \equiv \frac{p_d}{1-p_d}$. Given $x_3$, if a pair of $(x_1, x_2)$ solves

$$\max_{x_1, x_2} \Pr(a_H|H)\ \Pr(a_H|L)$$

$$s.t.\ l_1 \cdot l_2 \cdot l_3 = A$$

then $(x_1, x_2)$ maximizes the payoff of the persuaders. Rearranging the constraint into $l_1 \cdot l_2 = \dfrac{A}{l_3}$, the optimization problem is identical to the problem in Lemma 1 with $\hat{A} = \dfrac{A}{l_3}$. Therefore, applying Lemma 1, in the equilibrium with the highest expected utility for the persuaders, one of the persuaders must choose an uninformative test. Without loss of generality, assume that $x_1 = 1$ and $x_2 > 0$. Given $x_1 = 1$ and $x_2 > 0$ as best responses to $x_3$, now find the optimal $x_3$ that maximizes the persuaders' expected utility.

[13] Let $V|H \equiv \Pr(a_H|H)$, $V|L \equiv \Pr(a_H|L)$, $A \equiv \frac{p_d}{1-p_d}$, then $V|H''(x_1) = -\dfrac{2A^2(p-1)p(2p-1)^3}{[(A-1)p^2(x_1-1) - x_1 + p(A-1+2x_1)]^3} > 0$; $V|L''(x_1) = -\dfrac{2A(p-1)p(2p-1)^3}{[(A-1)p^2(x_1-1) - x_1 + p(A-1+2x_1)]^3} > 0$. Moreover, $V|H'(x_1) = V|L'(x_1) = 0$ if and only if $x_2 = x_1 = \dfrac{(p-1)\sqrt{A} + p}{p - 1 + p\sqrt{A}}$.

Since persuader 1 is completely uninformative, deleting him from the game does not change the equilibrium outcome. An iterated application of Lemma 1 implies that $(x_2, x_3) = (1, x_B)$ or $(x_B, 1)$. Although the decision maker consults three persuaders, only one of them chooses an informative test. The same logic applies to any $n \geq 3$. Applying Lemma 1 and deleting uninformative persuaders iteratively until there is only one persuader left who follows the benchmark strategy, $(x_B, 1)$. Hence, $V(\mathbf{t}) = V_B$ for all $n > 2$. □

### 2.6.5 Proof of Example 1

Let $p = 1$, $n = 2$, and $p_d = 0.64$. I verify that $\mathbf{t} = ((0, 1, 0, 8), (0.1, 0.8))$ and $s(\mathbf{t}) = \{\{1\}, \{2\}, \{1, 2\}\}$ is an equilibrium.

When players choose the above strategies, $V(\mathbf{t}) = 0.575$. Given $(x_2, y_2) = (0.1, 0.8)$, I show that persuader 1 is strictly worse off if he deviates to some $\left(x_1^{'}, y_1^{'}\right) \neq (0.1, 0.8)$. I use $\mathbf{t}'$ to denote the tests after persuader 1's deviation.

a. If a deviation by persuader 1 induces $s(\mathbf{t}') = \{\{1\}, \{2\}, \{1, 2\}\}$, then $\left(x_1^{'}, y_1^{'}\right)$ must satisfy

$$\frac{8\left(1 - y_1^{'}\right)}{1 - x_1^{'}} \geq \frac{16}{9} \text{ and } \frac{2y_1^{'}}{9x_1^{'}} \geq \frac{16}{9}.$$

Among all deviations that satisfy the above inequalities, persuaders' payoff is uniquely maximized when $\left(x_1^{'}, y_1^{'}\right) = (x_1, y_1) = (0.1, 0.8)$.

b. If a deviation by persuader 1 induces $s(\mathbf{t}') = \{\{2\}, \{1, 2\}\}$, then $V(\mathbf{t}') = \Pr(\text{pass from persuader 2}) = 0.45 < V(\mathbf{t})$.

c. If a deviation by persuader 1 induces $s(\mathbf{t}') = \{\{1\}, \{1, 2\}\}$, then $\left(x_1^{'}, y_1^{'}\right)$ must satisfy

$$\frac{2y_1^{'}}{9x_1^{'}} \geq \frac{16}{9} \Rightarrow x_1^{'} \leq \frac{1}{8} \text{ and } y_1^{'} \leq 1.$$

In this case, $V(\mathbf{t}')$ is maximized at $\left(x_1^{'}, y_1^{'}\right) = \left(\frac{1}{8}, 1\right)$ and $V\left(\frac{1}{8}, 1\right) = 0.5625 < V(\mathbf{t})$.

d. If a deviation by persuader 1 induces $s(\mathbf{t}') = \{\{1, 2\}\}$, then

$V(\mathbf{t}') \leq \Pr(\text{pass from persuader 2}) = 0.45 < V(\mathbf{t})$.

Therefore, conclude that persuader 1 is strictly worse off after a deviation to some $\left(x_1^{'}, y_1^{'}\right) \neq (0.1, 0.8)$.

### 2.6.6 Proof of Example 2

Let $n = 3$, $p_d = \frac{81}{113}$, and $p = 1$. I verify that $\mathbf{t} = ((0.2, 0.9), (0.2, 0.9), (0.2, 0.9))$ and $s(\mathbf{t}) = \{\{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$ is an equilibrium.

When players choose the above strategies, $V(\mathbf{t}) = 0.538$. Given $(x_2, y_2) = (x_3, y_3) = (0.2, 0.9)$, I show that persuader 1 is strictly worse off if he deviates to some $\left(x_1^{'}, y_1^{'}\right) \neq (0.2, 0.9)$. I use $\mathbf{t}'$ to denote the tests after persuader 1's deviation.

Among all deviations that induce $s(\mathbf{t}') = \{\{1,2,3\}\}$, the most profitable is $\left(x_1^{'}, y_1^{'}\right) = (1, 1)$, which yields $V(\mathbf{t}') = 0.425 < V(\mathbf{t})$.

If a deviation by persuader 1 induces $s(\mathbf{t}') = \{\{1,2,3\}, \{2,3\}\}$, then $\left(x_1^{'}, y_1^{'}\right)$ must satisfy

$$\frac{0.9^2}{0.2^2} \cdot \frac{1 - y_1^{'}}{1 - x_1^{'}} \geq \frac{p_d}{1 - p_d}.$$

Among all deviations that satisfy the above inequality, persuaders' payoff is maximized when $\left(x_1^{'}, y_1^{'}\right) \to (1, 1)$, and $V(\mathbf{t}') \to 0.425 < V(\mathbf{t})$.

If a deviation by persuader 1 induces $s(\mathbf{t}') = \{\{1,2,3\}, \{1,2\}, \{1,3\}\}$, then $\left(x_1^{'}, y_1^{'}\right)$ must satisfy

$$\frac{0.9}{0.2} \cdot \frac{0.1}{0.8} \cdot \frac{y_1^{'}}{x_1^{'}} \geq \frac{p_d}{1 - p_d}.$$

Among all deviations that satisfy the above inequality, persuaders' payoff is maximized when $\left(x_1^{'}, y_1^{'}\right) = (\frac{2}{9}, 1)$, and $V(\mathbf{t}') = 0.535 < V(\mathbf{t})$.

If a deviation by persuader 1 induces $s(\mathbf{t}') = \{\{1,2,3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$, then $\left(x_1^{'}, y_1^{'}\right)$ must satisfy

$$\frac{y_1^{'}}{x_1^{'}} \geq \frac{0.9}{0.2}, \ \frac{1 - y_1^{'}}{1 - x_1^{'}} \geq \frac{0.1}{0.8}.$$

Among all deviations that satisfy the above inequalities, persuaders' payoff is uniquely maximized when $\left(x_1^{'}, y_1^{'}\right) = (x_1, y_1) = (0.2, 0.9)$.

If $\{1\} \in s(\mathbf{t}')$, then $\left(x_1^{'}, y_1^{'}\right)$ must satisfy

$$\frac{y_1^{'}}{x_1^{'}} \cdot \frac{0.1^2}{0.8^2} \geq \frac{p_d}{1-p_d} \Rightarrow \frac{y_1^{'}}{x_1^{'}} \geq 162.$$

Among all deviations that induce $s(\mathbf{t}') = \{\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$, the most profitable is $\left(x_1^{'}, y_1^{'}\right) = (\frac{1}{162}, 1)$, which yields $V(\mathbf{t}') = 0.503 < V(\mathbf{t})$.

Among all deviations that induce $s(\mathbf{t}') = \{\{1\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$, the most profitable is $\left(x_1^{'}, y_1^{'}\right) = (\frac{7}{1295}, \frac{162}{185})$, which yields $V(\mathbf{t}') = 0.511 < V(\mathbf{t})$.

$\{2\} \notin s(\mathbf{t}')$ and $\{3\} \notin s(\mathbf{t}')$ for all $\mathbf{t}'$. Therefore, conclude that that persuader 1 is strictly worse off after a deviation to some $\left(x_1^{'}, y_1^{'}\right) \neq (0.2, 0.9)$.

### 2.6.7 Proof of Theorem 7

To prove that equilibria with endogenously noisy tests exist universally, I first start with the case of two persuaders and then extend it to the case of $n$persuaders.

**Lemma 3.** *Let $p = 1$ and $n = 2$. $\forall p_d \in \left(\frac{1}{2}, 1\right)$, there exists an equilibrium in which $x_i \in (0,1)$ and $y_i \in (0,1)$ for $i = 1, 2$.*

*Proof.* It suffices to show that there exist a symmetric equilibrium with $x_1 = x_2 = x \in (0,1)$, $y_1 = y_2 = y \in (0,1)$, and $s = \{\{1\}, \{2\}, \{1,2\}\}$, such that $x$ and $y$ satisfy

$$\frac{y}{x} \cdot \frac{1-y}{1-x} = \frac{p_d}{1-p_d},$$

and each persuader's expected utility is

$$\begin{aligned} V^* &= \frac{1}{2}\left[2y(1-y) + y^2 + 2x(1-x) + x^2\right] \\ & \frac{1}{2}\left(2y - y^2 + 2x - x^2\right) \end{aligned}$$

I now go through each case of unilateral deviation to identify the conditions for $x$ and $y$ such that the proposed strategies indeed form an equilibrium.

a. Suppose persuader 1 deviates to some $(x_a, y_a)$ s.t. $s_a = \{\{1\}, \{2\}, \{1,2\}\}$. $s_a$ implies that $x_a$ and $y_a$ must satisfy

$$
\begin{aligned}
\frac{y_a}{x_a} \cdot \frac{1-y}{1-x} &\geq \frac{y}{x} \cdot \frac{1-y}{1-x}, \\
\frac{y}{x} \cdot \frac{1-y_a}{1-x_a} &\geq \frac{y}{x} \cdot \frac{1-y}{1-x}.
\end{aligned}
$$

The inequalities imply that

$$
\frac{y}{x} \cdot x_a \leq y_a \leq \frac{y - x + (1-y)x_a}{1-x_a},
$$

which in turn implies that

$$
x_a \leq x \text{ and } y_a \leq y.
$$

Following this deviation, persuader 1's expected utility is

$$
\begin{aligned}
V_a &= \frac{1}{2}\left[y_a(1-y) + x_a(1-x) + (1-y_a)y + (1-x_a)x + y_a y + x_a x\right] \\
&\quad \frac{1}{2}\left[(1-y)y_a + (1-x)x_a + x + y\right]
\end{aligned}
$$

Since $V_a$ is increasing in both $x_a$ and $y_a$, $V_a$ is maximized when $x_a = x$ and $y_a = y$, i.e. there is no profitable deviation to some $(x_a, y_a) \neq (x, y)$ s.t. $s_a = \{\{1\}, \{2\}, \{1,2\}\}$.

b. Suppose persuader 1 deviates to some $(x_b, y_b)$ s.t. $s_b = \{\{2\}, \{1,2\}\}$. $s_b$ implies that $x_b$ and $y_b$ must satisfy

$$
\frac{y}{x} \cdot \frac{1-y_b}{1-x_b} \geq \frac{y}{x} \cdot \frac{1-y}{1-x},
$$

and persuader 1's expected utility is

$$\begin{aligned} V_b &= \frac{1}{2}(y+x) \\ &< \frac{1}{2}\left[y+x+\left(y-y^2\right)+\left(x-x^2\right)\right] = V^*. \end{aligned}$$

Therefore, there is no profitable deviation to some $(x_b, y_b)$ s.t. $s_b = \{\{2\}, \{1,2\}\}$.

c. Suppose persuader 1 deviates to some $(x_c, y_c)$ s.t. $s_c = \{\{1,2\}\}$. Then, persuader 1's expected utility following this deviation is

$$\begin{aligned} V_c &= \frac{1}{2}(y \cdot y_c + x \cdot x_c) \\ &\leq \frac{1}{2}(y+x) \\ &\leq V_b \\ &< V^*. \end{aligned}$$

Therefore, there is no profitable deviation to some $(x_c, y_c)$ s.t. $s_c = \{\{1,2\}\}$.

d. Finally, suppose persuader 1 deviates to some $(x_d, y_d)$ s.t. $s_d = \{\{1\}, \{1,2\}\}$. $s_d$ implies that $x_d$ and $y_d$ must satisfy

$$\begin{aligned} &\frac{y_d}{x_d} \cdot \frac{1-y}{1-x} \geq \frac{y}{x} \cdot \frac{1-y}{1-x} \\ \Rightarrow &x_d \leq \frac{x}{y} \text{ and } y \leq 1. \end{aligned}$$

Following this deviation, persuader 1's expected utility is

$$\begin{aligned} V_d &= \frac{1}{2}(y_d + x_d) \\ &\leq \frac{1}{2}\left(1+\frac{x}{y}\right). \end{aligned}$$

A sufficient condition for $V_d \leq V^*$ is $\frac{1}{2}\left(1+\frac{x}{y}\right) \leq V^*$.

Summarizing the four cases of possible deviation, it suffices to prove the lemma if, for all

$p_d \in \left(\frac{1}{2}, 1\right)$, there exist $x \in (0,1)$ and $y \in (0,1)$ s.t.

$$\frac{y}{x} \cdot \frac{1-y}{1-x} = \frac{p_d}{1-p_d} \tag{2.5}$$

$$\left(2y - y^2 + 2x - x^2\right) - \left(1 + \frac{x}{y}\right) \geq 0 \tag{2.6}$$

Let $A \equiv \frac{p_d}{1-p_d}$, then (5) implies

$$y(x) = \frac{1}{2} + \left[\frac{1}{4} - A\left(x - x^2\right)\right]^{\frac{1}{2}},$$

and the left hand side of (6) can be rewritten as

$$g(x) \equiv (A+1)\left(x - x^2\right) + x + \frac{1}{2} + \left[\frac{1}{4} - A\left(x - x^2\right)\right]^{\frac{1}{2}}.$$

Note that $g(0) = 0$ and $g'(0) = 1 > 0$, which implies that there exist some positive $x$ close to 0 s.t. $y(x) \in (\frac{1}{2}, 1)$ and $g(x) \geq 0$. This completes the proof. □

Next, I prove Theorem 7 by showing that for any $n > 1$, there exist an equilibrium in which the first two persuaders play strategies specified in Lemma 2, and the rest persuaders choose tests that are always passed unconditionally.

**Theorem 7.** Let $p = 1$ and $n > 1$. For all $p_d \in \left(\frac{1}{2}, 1\right)$, there exists an equilibrium in which the persuaders choose tests that never perfectly reveal the true state.

*Proof.* It suffices to show that there exists an equilibrium in which $x_1 = x_2 = x \in (0,1)$, $y_1 = y_2 = y \in (0,1)$, and $x_i = y_i = 1$ for all $i > 2$.

Lemma 2 shows that there is no profitable deviation for the first or the second persuader. I here prove that without loss of generality, among all pairs of $(x, y)$ that support Lemma 2, there always exists some $(x, y)$ such that no deviation will bring the third persuader a higher expected utility than $V^*$ as well.

First, note that a unilateral deviation by the third persuader may be profitable only if a pass from his test dominates two failed tests from both the first and the second persuaders, i.e.,

$$\frac{(1-y)^2}{(1-x)^2} \cdot \frac{y_3}{x_3} \geq \frac{p_d}{1-p_d}.$$

Since $\frac{p_d}{1-p_d} = \frac{y}{x} \cdot \frac{1-y}{1-x}$, the above condition is equivalent to

$$\frac{y_3}{x_3} \geq \frac{y}{x} \cdot \frac{1-x}{1-y} \tag{2.7}$$

Since the decision maker is indifferent after one pass and one failure from the first two persuaders, it is impossible for the decision maker to choose $a_H$ when, in addition to one pass and one failure, the test from the third persuader fails as well. Therefore, it is sufficient to check only the following two cases of deviation:

a. The third persuader deviates to some $(x_a, y_a)$ such that $s_a = \{\{3\}, \{3,1\}, \{3,2\}, \{3,1,2\}\}$. In this case, it is most profitable for third persuader to choose $x_a \in (0,1)$ and $y_a = 1$, which yields an expected utility of $V_a = \frac{1}{2}(1 + x_a) = \frac{1}{2}\left(1 + \frac{x}{y} \cdot \frac{1-y}{1-x}\right)$.

By construction, $2y - y^2 + 2x - x^2 \geq 1 + \frac{x}{y}$ from (6) and $\frac{1-y}{1-x} \in (0,1)$. Hence, it must be the case that $V^* = \frac{1}{2}(2y - y^2 + 2x - x^2) > \frac{1}{2}\left(1 + \frac{x}{y} \cdot \frac{1-y}{1-x}\right) = V_a$. Conclude that it is not profitable for the third persuader to deviate in this way.

b. Persuader 3 deviates to some $x_b \in (0,1)$ and $y_b \in (0,1)$ s.t.

$s_b = \{\{1,2\}, \{3\}, \{3,1\}, \{3,2\}, \{3,1,2\}\}$. $s_b$ implies that, in addition to (7), $x_b$ and $y_b$ must also satisfy

$$\frac{1-y_b}{1-x_b} \geq \frac{x}{y} \cdot \frac{1-y}{1-x}. \tag{2.8}$$

Since $V_b = \frac{1}{2}\left[y_b + x_b + (1-y_b)\, y^2 + (1-x_b)\, x^2\right]$ increases in $x_b$ and $y_b$, $V_b$ is maximized when both (7) and (8) hold with equality, i.e.,

$$\frac{y_b}{x_b} = \frac{y}{x} \cdot \frac{1-x}{1-y} \text{ and } \frac{1-y_b}{1-x_b} = \frac{x}{y} \cdot \frac{1-y}{1-x},$$

which yields

$$x_b = \frac{x - xy}{x + y - 2xy} \text{ and } y_b = \frac{y - xy}{x + y - 2xy}.$$

$$V_b\left(x, y\right) = \frac{y + x^2 y - x^3 y + x\left(1 - 2y + y^2 - y^3\right)}{2\left(x + y - 2xy\right)}$$

$$V^* - V_b = \frac{x^3\left(1 - 3y\right) + \left(1 - y\right)^2 y + x^2\left(6y - 2\right) + x\left(1 - 6y + 6y^2 - 3y^3\right)}{-2y + x\left(4y - 2\right)}$$

Lemma 2 also specifies that $y(x) = \frac{1}{2} + \left[\frac{1}{4} - A\left(x - x^2\right)\right]^{\frac{1}{2}}$, therefore $V^* - V_b$ can be expressed as a single function of $x$, $h(x) \equiv V^* - V_b$, with

$$h(x) = -\frac{(x-1)x\left\{A(3x-1)\left[\sqrt{1+4A(-1+x)x} - 1\right] + (x-1)\left[1 + 3\sqrt{1+4A(-1+x)x}\right]\right\}}{4x\sqrt{1+4A(x-1)x} - 2\left[1 + \sqrt{1+4A(x-1)x}\right]}.$$

Note that $h(0) = 0$ and $h'(0) = 1$. In other words, there exists $x \in (0, 1)$ and $y \in (0, 1)$ s.t. $V^* - V_b > 0$ and no profitable deviation exists for third persuader. This completes the proof. $\square$

# Chapter 3

# The False Promise of Becoming a Better Person

## 3.1 Introduction

It is a recurring theme in fiction that a person can quit a bad habit in the pursuit of a friend or a love interest. To list a few examples, this includes movies "Knocked Up" (Ben quits marijuana for Alison), and "Yes Man" (Carl quits his habit of always saying "no" to make friends), as well as the TV series "How I Met Your Mother" (Barney quits sleeping around for Robin). This paper examines whether such hopeful perception is true. I focus on cases in which people consume some harmful, tempting good, and their resistance against temptation is negatively correlated with their friends' consumption. Moreover, friends are chosen endogenously. The good news: there indeed exist equilibria in which one exercises extra self control and chooses a low consumption level in order to win the friendship of another. The bad news: no such equilibrium is subgame perfect, i.e. those cases are not stable.

To be more precise, I adopt the temptation model of Gul and Pesendorfer (henceforth, "GP") (2001, 2004, 2007) and focus on people's consumption of some harmful tempting good,

$d$ (drugs, drinking, devil's food cake...). The special trait of the good $d$ is that, when an agent is kept away from it, he wishes to consume as little as possible. However, if the good is presented in front of the agent, it becomes so tempting that it's difficult to resist consumption. After going through a struggle of self-control, the agent will likely deviate from his previous plan and consume a positive amount.

I extend GP's model by claiming that in any society, no one is completely isolated when facing temptation. The strength of one's resistance against tempting goods is influenced by one's friend. Specifically, I assume that the more $d$ one and one's friend consumed in the past, the harder it is to resist consumption in the current period. Therefore, making friends with an "addict" will increase one's own consumption level, and making friends with someone with "a clean past" helps cut consumption down. The presence of such peer effects leads to a preference for friends with low past consumption.

Assume there is a strong player, say Alison, who has low past consumption, and a weak player, say Ben, who has high past consumption, Alison will reject Ben's friend request if his past consumption is too high. However, Ben can change the situation by choosing a low consumption today, so that he can win Alison's friendship back tomorrow. I show in this paper that such an equilibrium does exist. However, any equilibrium where Alison conditionally accepts Ben tomorrow depending on his low consumption today fails to be subgame perfect. This is because all equilibria of conditional friendship are enforced by Alison's threat to reject Ben tomorrow if his consumption is not low enough today. However, I show that this threat is not credible, as Alison is in fact willing to befriend Ben even if he consumed more than required. Knowing the non-credibility of the threat, Ben deviates to a higher consumption. Anticipating Ben's deviation, Alison will decline to participate in such conditional friendship agreement at the first place, even though Ben and Alison are both better off if they stick to the agreement.

This paper contributes to two literatures. Firstly, there is work in decision theory that has established models of self-control problems and the formation of addiction, but the major focus has been on individual decision-making (Becker and Murphy 1988, Gul and Pesendorfer 2001, 2004, 2007, Kopylov 2012), with little discussion on the impact of social interaction. The paper by Battaglini, Benabou, and Tirole (2005) is one of the very few that addresses self-control problems in peer groups. In their paper, peer effects exist only through informational spillovers. The probabilities for players to experience exogenously high or low self-control cost are correlated, therefore a player can update his belief on his own cost by observing the behavior of others. In this setting, they predict that a player prefers a friend who is worse at self-control, as this makes his own successes more encouraging and his failures less discouraging. In contrast to their paper, here I assume that a player's friends can directly change his own self-control cost, and I predict that a player prefers friends who are better at self-control, because hanging out with them makes it easier to resist temptation himself. Secondly, the focus of the peer effect literature has mostly been on empirical detection and measurement (Evans et al. 1992, Dishion et al. 1999, Sacerdote 2001, Lundborg 2006). Models of peer effects either assume a fix network (Ballester, Calvo-Armengol, and Zenou 2006; Calvo-Armengol, Patacchini, and Zenou 2009), or an endogenous network with no self-control problem (Badev, 2013). In particular, those papers do not assume that smoking, committing crime, or using drugs is necessarily harmful to the agents. The agents simply choose the lifestyle that maximizes their welfare; taking the cigarettes away from them only makes their life more miserable. On the contrary, in this paper I focus on the situation in which agents realize the harm of consuming a certain good, but simply can't resist the temptation. This coincides with observations of smokers who wish they could quit smoking, or drug addicts who wish they weren't addicted. The presence of such self-control problems justifies the possibility of a welfare improvement, and this paper focuses on how making good friends leads to such an improvement by making self-control easier.

The rest of the paper is organized as follows. Section 2 describes the set up of a two-stage, simultaneous-move game, followed by the characterization of equilibria in section 3. Section 4 illustrates why equilibria with conditional friendship cannot be subgame perfect.

Section 5 states that the weaker player always prefers to consume first when the timing of consumption is endogenous. A conclusion is included in Section 6.

## 3.2 Two-period game set up

Suppose there are 2 players, Alison and Ben, whose past consumption of a harmful tempting good is $d_0^A$, $d_0^B \in [0,1]$, respectively. Assume Alison consumed less than Ben in the past:$d_0^A \leq d_0^B$. There are two time periods. In each period, players observe each other's past consumption, decide whether to be friends, and then simultaneously choose the current period consumption with possible peer effect from the friend. The following time line summarizes the decision problem in details.

**Day 1**

1. Alison observes the past consumption level $d_0^B$ of Ben to decide if she wants to be friends with him, and vice versa. They become friends if and only if they *both* want to be friends. In this case, we say $A\heartsuit_{t=1}B$.

2. Alison chooses $d_1^A \in [0,1]$ to maximize

$$W^A(d_1^A) = W_1^A(d_1) + \beta W_2^A(d_1^A), \text{ where}$$

$$W_t^A = U(d_t^A) + \sigma_t^A \left[V(d_t^A) - V(1)\right] + \mathbf{1}_{A\heartsuit_t B} \cdot u_F \text{ for } t = 1, 2.$$

We assume that $U$ is decreasing and $V$ is increasing. $u_F > 0$ is the utility the player receives when he/she has a friend. For $t = 1, 2$,

$$\sigma_t^A = \begin{cases} \sigma(d_{t-1}^A, d_{t-1}^B) & \text{when A}\heartsuit_t\text{B} \\ \sigma(d_{t-1}^A, d_{t-1}^A) & \text{when players are not friends in } t \end{cases}$$

for some positive, increasing function $\sigma$.

Similarly, define Ben's maximization problem by switching the indices.

Intuitively, the commitment utility $U$ represents agent's ranking over consumption level when he's free of temptation (i.e., when he commits to consume exactly $d_1$ ex ante). A decreasing $U$ implies that the good is harmful. On the other hand, the temptation utility $\sigma_1 V$ measures the impulse to consume as much as possible when the good is present. An increasing $V$ implies that the good is tempting. Since $\sigma$ is also increasing, the good becomes more irresistible in the current period if the player consumed a large amount in the past, or if the player befriended someone who consumed a large amount in the past. In other words, $\sigma$ summarizes both the addictiveness of the good, and the peer effect from a friend. Figuratively, $U$ and $V$ are, respectively, an angel and a demon in the player's mind, and $\sigma$ represents the strength of the demon.

**Day 2**

1. Players decide whether to be friends after observing $d_1^A$ and $d_1^B$. We say $A \heartsuit_{t=2} B$ if the two players are friends in day 2.

2. Amy chooses $d_2^A \in [0,1]$ to maximize

$$W_2^A(d_2^A) = U(d_2^A) + \sigma_2^A \left[V(d_2^A) - V(1)\right] + \mathbf{1}_{A\heartsuit_{t=2}B} \cdot u_F.$$

   Ben simultaneously chooses $d_2^B$ likewise.

## 3.3 Equilibrium

Before characterizing the equilibria of the game, we start with a few comparative statics that help illustrate players' preferences for friend selection.

**Lemma 4.** *Player i's optimal consumption $d_t^{i*}$ increases in $\sigma_t^i$ for $i =$ Alison, Ben, and $t = 1, 2$.*

*Proof.* Let $\sigma_1^H \geq \sigma_1^L$. When $\sigma_1^i = \sigma_1^H$, let $d_1^H$ denote the optimal consumption in day 1, and let $W_2(d_1^H)$ denote the the maximized payoff in day 2 following $d_1^H$. When $\sigma_1^i = \sigma_1^L$, let $d_1^L$ denote the optimal consumption in day 1, and let $W_2(d_1^L)$ denote the the maximized payoff in day 2 following $d_1^L$.

$$U(d_1^H) + \sigma_1^H V(d_1^H) + \beta W_2(d_1^H) \geqslant U(d_1^L) + \sigma_1^H V(d_1^L) + \beta W_2(d_1^L) \tag{3.1}$$

$$U(d_1^L) + \sigma_1^L V(d_1^L) + \beta W_2(d_1^L) \geqslant U(d_1^H) + \sigma_1^L V(d_1^H) + \beta W_2(d_1^H) \tag{3.2}$$

Since LHS (1) - RHS (2) $\geq$ RHS (1) - LHS (2), we have

$$\left(\sigma_1^H - \sigma_1^L\right)\left[V(d_1^H) - V(d_1^L)\right] \;\; \geqslant \;\; 0.$$

As $V$ is increasing, $V(d_1^H) \geqslant V(d_1^L)$ if and only if $d_1^H \geqslant d_1^L$. Hence $\sigma_1^H \geqslant \sigma_1^L$ implies $d_1^H \geqslant d_1^L$, i.e. $d_1^*$ increases in $\sigma_1$.

Similarly define $d_2^H$ and $d_2^L$ for $\sigma_2^H \geq \sigma_2^L$.

$$U(d_2^H) + \sigma_2^H V(d_2^H) \geqslant U(d_2^L) + \sigma_2^H V(d_2^L) \tag{3.3}$$

$$U(d_2^L) + \sigma_2^L V(d_2^L) \geqslant U(d_2^H) + \sigma_2^L V(d_2^H) \tag{3.4}$$

Equations (3) and (4) imply that $d_2^*$ also increases in $\sigma_2$.[1] □

**Lemma 5.** *(1) If $i\heartsuit_{t=1}j$, then $d_1^i$ increases with $d_0^j$ and $W^i$ decreases with $d_0^j$.*

*(2) If $i\heartsuit_{t=2}j$, then an increase in $d_1^j$ increases both $d_1^i$ and $d_2^i$. Moreover, it increases $W_1^i$, decreases $W_2^i$, and decreases $W^i$.*

*Proof.* When $i\heartsuit_{t=1}j$, $\dfrac{\partial \sigma_1^i}{\partial d_0^j} \geq 0$ implies $d_1^{i'}(d_0^j) \geq 0$ by lemma 1. Moreover,

$$\frac{\partial W^i}{\partial d_0^j} = \frac{\partial \sigma_1^i}{\partial d_0^j} \cdot \left[V(d_1^{i*}) - V(1)\right] \leq 0 \tag{3.5}$$

[1]This statement is an application of Proposition 2 in “Harmful addiction”, Gul and Pesendorfer (2007)

since $\sigma$ and $V$ are both increasing.

When $i\heartsuit_{t=2}j$, to see how $W_1^i$ and $W_2^i$ change with $d_1^j$ we need to determine how $d_1^{i*}$ changes with $d_1^j$ first. Note that the FOCs for a player's optimal consumption is

$$[d_2]\ U'(d_2) + \sigma_2(d_1)V'(d_2) = 0 \tag{3.6}$$

$$[d_1]\ U'(d_1) + \sigma_1 V'(d_1) + \beta \cdot \frac{\partial \sigma_2^i}{\partial d_1^i} \cdot [V(d_2) - V(1)] = 0 \tag{3.7}$$

When $d_1^j$ increases, from (5) we know that $d_2^*(d_1)$, hence $V(d_2^*(d_1))$ and the LHS of (6), increases for each value of $d_1$. The SOC at the maximum ensures that the LHS of (6), i.e. the marginal payoff of $d_1$, is decreasing in $d_1$. Therefore as LHS (6) increases for every value of $d_1$, the optimal value $d_1^{i*}$ also increases, which gives us

$$d_1^{i*'}(d_1^j) \geq 0 \tag{3.8}$$

By (6) and (7),

$$W_1^{i'}(d_1^j) = -\beta \cdot \frac{\partial \sigma_2^i}{\partial d_1^{i*}} \cdot d_1^{i*'}(d_1^j) \left[V(d_2^{i*}) - V(1)\right] > 0 \tag{3.9}$$

After an increase in $d_1^j$, $\frac{\partial \sigma_2^i}{\partial d_1^j}$ represents the total increase in $\sigma_2^i$, and $\frac{\partial \sigma_2^i}{\partial d_1^{i*}} \cdot d_1^{i*'}(d_1^j)$ represents the indirect increase in $\sigma_2^i$ due to an increased $d_1^{i*}$. Therefore, $\frac{\partial \sigma_2^i}{\partial d_1^j} - \frac{\partial \sigma_2^i}{\partial d_1^{i*}} \cdot d_1^{i*'}(d_1^j)$ represents the direct increase in $\sigma_2^i$, which is positive. We conclude that

$$W_2^{i'}(d_1^j) = \frac{\partial \sigma_2^i}{\partial d_1^j} \cdot \left[V(d_2^{i*}) - V(1)\right] \leq 0 \tag{3.10}$$

$$W^{i'}(d_1^j) = \left[\frac{\partial \sigma_2^i}{\partial d_1^j} - \frac{\partial \sigma_2^i}{\partial d_1^{i*}} \cdot d_1^{i*'}(d_1^j)\right] \cdot \beta \left[V(d_2^{i*}) - V(1)\right] < 0 \tag{3.11}$$

□

Therefore, by lemma 1 and lemma 2 we know that on both days, players want to befriend those with a low enough past consumption level. In particular, since we have assumed that $d_0^A \leq d_0^B$, if Alison makes friends with Ben on day 1, she earns the friendship benefit $u_F$, at the cost of an increased $\sigma_1$ (higher self-control cost). On the other hand, after making friends with Alison, Ben is better-off both due to the friendship benefit $u_F$, and a lowered $\sigma_1$ as well. The same argument works for day 2. Therefore, in an equilibrium the two players are friends in day 1 if and only if $d_0^B$ is low enough for Alison. Similarly, the player with a lower $d_1$ determines the condition for friendship on day 2. Here let's impose an assumption that when two players are friends, the player with the lower past consumption still has stronger resistance against temptation even after the mutual peer effect.

**Assumption 1.** For $t = 1, 2$, if $d_{t-1}^i \geq d_{t-1}^j$, then $\sigma_t^i \geq \sigma_t^j$ when $i \heartsuit_t j$.

We will assume A1 for the rest of this paper.

With A1, since Alison has a lower $d_0^A$ to start with, she chooses a lower $d_1$ than Ben even if they're friends in the first period. Therefore, Alison is still the player who sets the condition under which she and Ben are friends on day 2. The players' equilibrium friend-selection strategy is summarized below.

**Proposition 3.** *[Equilibrium friend-selection strategy] Ben always accepts Alison as a friend.*

*There exists $K_0, K_1 \in [0, 1]$ s.t. Alison accepts Ben on day 1 iff. $d_0^B \leq K_0$, and on day 2 iff. $d_1^B \leq K_1$.*

Fix the friendship status in $t = 1, 2$ (e.g. friends on both days, friends only on day 2), as well as player $j$'s consumption $d_1^j$, let $d_t^{i*}$ denote player $i$'s optimal consumption on day $t$. We have the following proposition stating the different classes of equilibria.

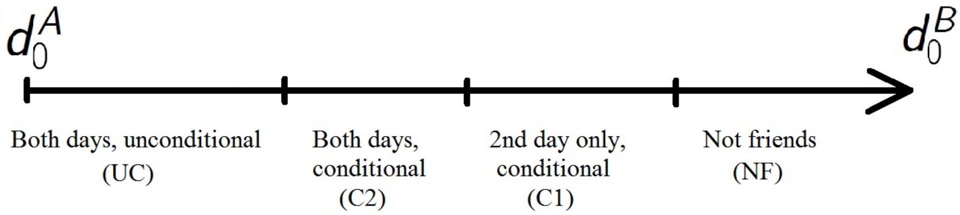
**Proposition 4.** *Fix $d_0^A$. $\exists d^{UC} \leq d^{C2} \leq d^{C1} \in [0, 1]$ such that*

*If $d_0^B \in [d_0^A, d^{UC}]$, $A \heartsuit_t B$ and $d_t^i = d_t^{i*}$ for $t = 1, 2$, $i = A, B$. [Unconditional friendship, "UC"]*

*If $d_0^B \in [d_0^{UC}, d^{C2}]$, $A \heartsuit_t B$ and $d_t^A = d_t^{A*}$ for $t = 1, 2$, and $d_1^B < d_1^{B*}$. [Conditional 2-day friendship, "C2"]*

*If $d_0^B \in [d^{C2}, d^{C1}]$, A and B are friends in day 2 only. $d_t^A = d_t^{A*}$ for $t = 1, 2$, and $d_1^B < d_1^{B*}$. [Conditional 1-day friendship, "C1"]*

*If $d_0^B \in [d^{C1}, 1]$, A and B are alone for both days. [No friendship, "NF"]*




*Proof.* Proposition 2 is a corollary of proposition 1. Note that Alison's payoff decreases with both $d_0^B$ and $d_1^B$ if she's friends with Ben on those days. When $d_0^B$ is sufficiently low, the increased self-control cost with Ben is lower than the friendship benefit, so Alison befriends Ben, and they optimally chooses their consumption levels (UC). When $d_0^B$ is not low enough for the unconditional friendship, although Ben has no control over $d_0^B$, he does have control over $d_1^B$, so there exists a class of equilibria in which although the net payoff of befriending Ben who optimally chooses $d_1^{B*}$ is negative, Alison can improve this payoff by asking Ben to choose a smaller $d_1^B = K_1 < d_1^{B*}$, so that the increased self-control cost with Ben doesn't surpass the friendship benefit. Since choosing a low $d_1^B$ is costly to Ben, he will only agree to such plan if $K_1$ is sufficiently high; otherwise the two players remain alone (UF).

The only thing left to specify is that suppose Ben has agreed to choose $d_1^B = K_1$, and that Alison agreed to be friends on day 2, does Alison also agree to be friends in day 1? Note that in this case Alison compares the payoff $W^A_{A\heartsuit_1 B}$ with the outside option of being alone, which gives a constant payoff independent of $d_0^B$. Since $W^A_{A\heartsuit_1 B}$ decreases in $d_0^B$ by lemma 2, Alison is friends with Ben if $d_0^B$ is sufficiently low (C2), and stays alone otherwise (C1). □

The following example confirms the existence of the C1 and C2 equilibria.

**Example 3.** Let $U(d) = -d^2,\ V(d) = \frac{3}{2} \cdot d^{1/2},\ \beta = 0.9,\ u_F = 0.1,\ d_0^A = 0.\ \sigma_t^i = 1 + d_{t-1}^i$ if player $i$ is alone; $\sigma_t^i = 1 + 0.6 d_{t-1}^i + 0.4 d_{t-1}^j$ if player $i$ is friends with $j$. Let $d_0^A = 0,\ d_0^B = 0.5$ or 0.6. Let Alison's requirement for day-2 friendship be $d_1^B = K_1 = 0.6$, which according to the construction of the problem is lower than what Ben would have chosen if he is guaranteed to be friends with Alison on day 2. In Table 1 we summarize the maximized payoff Alison and Ben can get for each possible friendship outcome given $d_1^B = K_1 = 0.6$. We conclude that we have a C2 equilibrium when $d_0^B = 0.5$, and a C1 equilibrium when $d_0^B = 0.6$.

| $K_1 = 0.6$ | | $d_0^B = 0.5$ | $d_0^B = 0.6$ |
|---|---|---|---|
| Alison | Alone in both periods (UF) | -1.45533 | -1.45533 |
| | $\heartsuit B$ in $t = 1$ only | -1.44848 | -1.46504 |
| | $\heartsuit B$ in $t = 2$ only (C1) | -1.37893 | **-1.37893** |
| | $\heartsuit B$ on both days (C2) | **-1.36472** | -1.38019 |
| Ben | Alone in both periods (UF) | -1.66051 | -1.69145 |
| | $\heartsuit A$ in $t = 1$ only | -1.49318 | N/A |
| | $\heartsuit A$ in $t = 2$ only (C1) | -1.55995 | **-1.59376** |
| | $\heartsuit A$ in both periods (C2) | **-1.3984** | N/A |

Table 3.1: Maximized payoff for each friendship outcome, given $d_1^B = K_1 = 0.6$.

## 3.4 C1 and C2 equilibria are not subgame perfect

We have just shown the existence of equilibria in which Ben exercises extra self-control and chooses a low day-1 consumption in order to win the friendship of Alison on day 2. However, in this section we are going to show that no such equilibrium is subgame perfect.

Intuitively, any conditional friendship agreement is enforced by Alison's threat to reject Ben if Ben chooses a higher day-1 consumption than the required $K_1$. Ex ante, if Alison anticipates that Ben chooses a level higher than $K_1$, she would rather stay alone on day 2, and choose her $(d_1^A, d_2^A)$ accordingly. However, if Alison is prepared to engage in a conditional friendship agreement, she will choose her day-1 consumption as a function of $K_1$, which will be a higher level than what she would have chosen if she were to be alone on day 2. Such a higher $d_1^A$ lowers Alison's "bargaining power" on day 2, in the sense that following a high day-1 consumption, Alison is now in fact willing to accept Ben with a $d_1^B$ above $K_1$. In other words, Alison's threat to reject Ben when $d_1^B > K$ is non-credible once the players have entered day 2. Seeing this, Ben has incentive to deviate to a consumption level higher than $K_1$, which lowers Alison's payoff. Anticipating Ben's deviation, Alison will not enter a conditional friendship agreement at the first place. The formal proof is provided below.

**Theorem 10.** *The C1 and C2 equilibria are not subgame perfect.*

*Proof.* We prove by contradiction. Suppose the C1 equilibrium is subgame perfect. This requires Alison to be indifferent between befriending and rejection Ben in $t = 2$, i.e.

$$W_2^A(\sigma_2^{A\heartsuit_{t=2}B}) = W_2^A(\sigma_2^{Alone}). \tag{3.12}$$

Denote Alison's consumption in the C1 equilibrium as $(d_1^{C1}, d_2^{C1})$, then

$$W_{C1}^A = W_1^A(\sigma_1^{Alone}, d_1^{C1}) + W_2^A(\sigma_2^{Alone}, d_2^{C1}).$$

But Alison's optimal consumption when she anticipates to be alone for both periods with $(\sigma_1^{Alone}, \sigma_2^{Alone})$ is $(d_1^{Alone}, d_2^{Alone}) \neq (d_1^K, d_1^K)$ because $\sigma_2^{A\heartsuit_{t=2}B} \neq \sigma_2^{Alone}$. Hence,

$$W_{K1plan}^A < W_{Alone}^A \;\Rightarrow\; \text{profitable deviation.}$$

Therefore, C1 cannot be an equilibrium, and we have a contradiction.

Similarly, to show the C2 equilibrium is not subgame perfect, either, we use "$A\heartsuit B$ in $t = 1$ only" as a profitable deviation for contradiction. □

**Example 4.** For a detailed illustration, let's show that the C1 equilibrium in example 1 is not subgame perfect. Use the same utility functions as before, that is, $U(d) = -d^2,\ V(d) = \frac{3}{2} \cdot d^{1/2},\ \beta = 0.9,\ u_F = 0.1.$ $\sigma_t^i = 1 + d_{t-1}^i$ if player $i$ is alone; $\sigma_t^i = 1 + 0.6d_{t-1}^i + 0.4d_{t-1}^j$ if player $i$ is friends with $j$. Let $d_0^A = 0$ and $d_0^B = 0.6$. Recall that in this case we have a C1 equilibrium in which the players are not friends on day 1, and Alison accepts Ben as a friend on day 2 conditional on $d_1^B \leq K_1 = 0.6$. However, after Alison has chosen $d_1^A(K_1)$ anticipating that she befriends Ben who consumed $K_1$, on the second day she in fact does not want to reject Ben no matter how high Ben's actual day-1 consumption is. Given $d_1^A(K_1)$, from (12) we can calculate that as long as $d_1^B \leq 1.76$, which is naturally satisfies since the upper bound for consumption is 1 in our set-up, Alison prefers having Ben as a friend on day 2 over loneliness. Therefore, $K_1 = 0.6$ is a non-credible threat for Ben, and he will deviate to a higher $d_1^B$ on day 1, which harms Alison. Anticipating Ben's deviation, Alison would rather quit the conditional friendship deal, and stays alone from day 1.

## 3.5 Endogenous timing

One crucial reason why C1 or C2 equilibria are not subgame perfect is because Alison cannot best-respond to Ben's deviation in the game. Since the players decide their consumption level simultaneously, by the time Alison observes Ben's deviation, she has already chosen her consumption, which is now suboptimal. However, we also know that if an equilibrium with conditional friendship exists, both Alison and Ben are better off in that equilibrium than being alone. Therefore, if players have the freedom to consume at different times, and if a C1 or C2 equilibrium does exist, Ben will strictly prefer to consume (at the required low level) before Alison, so she will indeed be friends with him on day 2, since the risk of Ben's deviation no longer exists. For the same reason, Alison strictly prefers to consumer later than Ben. The detailed timeline is described below.

On each day $t = 1, 2$, players still have only one chance to choose $d_1$, but now suppose instead that they can choose to consume either in the morning or in the afternoon. Hence we impose the new time line: At each $t = 1, 2$,

1. Observe $d^A_{t-1}$ and $d^B_{t-1}$. Players decide whether to be friends.
2. Given the friendship status, players choose whether they want to consume in the morning. If so, choose $d_t$ that maximizes $W$.
3. If a player didn't consume in the morning, he/she chooses $d_t$ in the afternoon to maximize $W$.

With this new time line, the simultaneous game is transformed into a sequential game when the players choose to consume at different times. In particular, when $d^B_0 \in [d^{UC}, d^{C1}]$ as described in proposition 2, Ben is better off with a conditional friendship agreement, therefore he has incentive to choose $d^B_1 = K_1$, as required by Alison, before Alison chooses her own consumption. This way, once Alison observes that Ben did not deviate from $K_1$, she will also best-respond by befriending Ben on day 2. On the other hand, Alison has the incentive to move after Ben, so that she can observe $d^B_1$ and best-respond to Ben's deviation by choosing a lower $d^A_1$ and not befriending Ben on day 2. Therefore we conclude:

*Claim* 1. If $d^B_0 \in [d^{UC}, d^{C1}]$ as specified in proposition 2, Ben chooses $d^B_1 = K_1 < d^{B*}_1$ in the morning, and Alison chooses $d^{A*}_1$ in the afternoon of day 1. The conditional friendship equilibrium (C1 or C2) is subgame perfect.

## 3.6 Conclusion

This paper extends Gul and Pesendorfer's temptation framework by introducing peer effects and social interaction into a player's decision when he faces a harmful tempting good. The strategic interaction of players, as well as their preferences for friends with low past consumption of the harmful tempting good, gives rise to a class of equilibria where a player chooses low consumption in order to win the friendship of another. However no such equilibrium is subgame perfect, because the potential friend's threat that enforces the low consumption is non-credible. As a result, although such conditional friendship is beneficial for both players, they choose to stay alone. This intuition also implies that when the timing of consumption is endogenous, the player with the worse self-control problem prefers to chooses his consumption first, so the other player can initiate the conditional friendship at no risk.

One possible extension of the paper is to introduce more players to the game, such that when B exercises extra self-control for A, B may require C to do so as well, or else B rejects C as a friend. This way we can expect a chained conditional friendship agreement, which lowers the consumption of many players at once, even though they're not mutually friends. Another extension is to allow for the concealment of past consumption. In this case whether players can benefit from hiding their identity, and whether they still exercise extra self-control for a friend while remaining anonymous is worth discussing.

# Bibliography


[1] Acemoglu, D., Dahleh, M. A., Lobel, I., and Ozdaglar, A. Bayesian learning in social networks. *Review of Economic Studies 78* (2011), 1201–1236.

[2] Ambrus, A., and Lu, S. E. Almost fully revealing cheap talk with imperfectly informed senders. *Games and Economic Behavior 88* (November 2014), 174–189.

[3] Ambrus, A., and Takahashi, S. Multi-sender cheap talk with restricted state spaces. *Theoretical Economics 3*, 1 (March 2008), 1–27.

[4] Anderson, L. R., and Holt, C. A. Information cascades in the laboratory. *The American Economic Review 87*, 5 (December 1997), 847–862.

[5] Badev, A. Discrete games in endogenous networks: Theory and policy. *Working paper* (2013).

[6] Ballester, C., Calvó-Armengol, A., and Zenou, Y. Who's who in networks. wanted: The key player. *Econometrica 74*, 5 (September 2006), 1403–1417.

[7] Banerjee, A., and Fudenberg, D. Word-of-mouth learning. *Games and Economic Behavior 46* (2004), 1–22.

[8] Banerjee, A. V. A simple model of herd behavior. *The Quarterly Journal of Economics 107*, 3 (August 1992), 797–817.

[9] Battaglini, M. Multiple referrals and multidimensional cheap talk. *Econometrica 70*, 4 (July 2002), 1379–1401.

[10] Battaglini, M., Bénabou, R., and Tirole, J. Self-control in peer groups. *Journal of Economic Theory 123*, 2 (August 2005), 105–134.

[11] Becker, G. S., and Murphy, K. M. A theory of rational addiction. *Journal of Political Economy 96*, 4 (August 1988), 675–700.

[12] Bhattacharya, S., and Mukherjee, A. Strategic information revelation when experts compete to influence. *The RAND Journal of Economics 44*, 3 (2013), 522–544.

[13] Bikhchandani, S., Hirshleifer, D., and Welch, I. A theory of fads, fashion, custom, and cultural change as informational cascades. *Journal of Political Economy 100*, 5 (October 1992), 992–1026.

[14] Calvó-Armengol, A., Patacchini, E., and Zenou, Y. Peer effects and social networks in education. *Review of Economic Studies 76*, 4 (2009), 1239–1267.

[15] Çelen, B., and Kariv, S. Distinguishing informational cascades from herd behavior in the laboratory. *American Economic Review 94*, 3 (June 2004), 484–498.

[16] Dishion, T. J., McCord, J., and Poulin, F. When inerventions harm: Peer groups and problem behavior. *American Psychologist 54*, 9 (September 1999), 755–764.

[17] Evans, W. N., Oates, W. E., and Schwab, R. M. Measuring peer group effects: A study of teenage behavior. *Journal of Political Economy 100*, 5 (October 1992), 966–991.

[18] Feddersen, T., and Pesendorfer, W. Convicting the innocent: The inferiority of unanimous jury verdicts under strategic voting. *The American Political Science Review 92*, 1 (March 1998), 23–25.

[19] Felgenhauer, M., and Schulte, E. Strategic private experimentation. *American Economic Journal: Microeconomics 6*, 4 (November 2014), 74–105.

[20] Gentzkow, M., and Kamenica, E. Competition in persuasion. *Working paper* (January 2016).

[21] Goeree, J. K., Palfrey, T. R., and Rogers, B. W. Social learning with private and common values. *Economic Theory 28* (2006), 245–264.

[22] Gul, F., and Pesendorfer, W. Temptation and self-control. *Econometrica 69*, 6 (November 2001), 1403–1435.

[23] Gul, F., and Pesendorfer, W. Self-control and the theory of consumption. *Econometrica 72*, 1 (January 2004), 119–158.

[24] Gul, F., and Pesendorfer, W. Harmful addiction. *Review of Economic Studies 74*, 1 (2007), 147–172.

[25] Hart, S., Kremer, I., and Perry, M. Evidence games: Truth and commitment. *Working paper* (March 2016).

[26] Kamenica, E., and Gentzkow, M. Bayesian persuasion. *American Economic Review 101* (October 2011), 2590–2615.

[27] Kolotilin, A. Experimental design to persuade. *Games and Economic Behavior 90* (March 2015), 215–226.

[28] Kopylov, I. Perfectionism and choice. *Econometrica 80*, 5 (September 2012), 1819–1843.

[29] Li, F., and Norman, P. On bayesian persuasion with multiple senders. *Working paper* (September 2015).

[30] Lundborg, P. Having the wrong friends? peer effects in adolescent substance use. *Journal of Health Economics 25*, 2 (March 2006), 214–233.

[31] Pastine, T., and Pastine, I. E. Signal accuracy and informational cascades. *Working paper* (2006).

[32] Sacerdote, B. Peer effects with random assignment: Results for dartmouth roommates. *The Quarterly Journal of Economics 116*, 2 (2001), 681–704.

[33] Sasaki, S. Signal qualities, order of decisions, and informational cascades: Experimental evidences. *ISER Discussion Paper*, 621 (October 2004).

[34] Smith, L., and Sørensen, P. Pathological outcomes of observational learning. *Econometrica 68*, 2 (March 2000), 371–398.

[35] Sobel, J. Giving and receiving advice. *Advances in Economics and Econometrics, Tenth World Congress 1* (2013).