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On the Effect of Delayed Feedback Information of Network Performance

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ON THE EFFECT OF DELAYED FEEDBACK INFORMATION ON NETWORK PERFORMANCE

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ABSTRACT

The performance of a network subject to either state dependent or state independent flow control is investigated. In the state dependent case, the flow control policy is a function of the total number of packets for which the controller has not yet received an acknowledgment. In this case it is shown that the optimal flow control is a sliding window mechanism. The effect of the delayed feedback on the network performance as well as the size of the window are studied. The state independent optimal rate is also derived. The performance of the state dependent and state independent flow control policies are compared. Conditions for employing one of the two types of flow control policies for superior end-to-end network performance are discussed. All the results obtained are demonstrated using simple examples.

Index Terms: computer networks, delayed feedback control, flow control, optimization, routing.

1. Introduction

One of the most challenging problems in the design of communication protocols is the derivation of the most appropriate flow control mechanism. In the X.25 User-Network interface recommendation, and in the OSI layered architecture, the flow control used is the sliding window (bang-bang control). Such a flow control mechanism can be applied either between a data terminal equipment (DTE) and a data circuit-terminating equipment (DCE) or between two DTE's [10].

One of the versions of the sliding window mechanism requires that, for each of the delivered packets, an acknowledgment packet (token) is sent back to its source. The sliding window flow control mechanism is a reliable mechanism for flow control because the receiver (a DCE or a DTE) is in control of the flow of packets in the network.

A different flow control mechanism proposed in the literature is rate flow control (see, *e.g.*, [1]). With this mechanism, the rate with which the source sends packets into the network is controlled. This mechanism has recently received support from researchers in the internetworking community [6], [15], [16]. If we compare these two different flow control mechanisms from their implementation point of view, we see that the window flow control mechanism can be easily implemented in the form of a sliding window. Reliable rate flow control mechanisms require very large time windows [1].

Probably the most fundamental difference between these two flow control strategies is the amount of information that they are based upon. Window flow control is based on feedback information about the state of the network whereas the rate flow control does not take into account the state of the network. In most of the studies of flow control it was assumed that acknowledgment packets are instantaneously received by the source [2], [7]. In reality, the acknowledgment packets travel from destination back to source and as such they are also subject to time delay.

In this paper, the performance of a network that operates under the above mentioned flow control strategies is investigated. First, the optimal state dependent flow control with *delayed feedback information* about the network state is derived. Second, the results are compared with the optimal state independent (rate) flow control. In order to make the analysis tractable, the communication resources are modeled as a Jackson network with a forward as well as an acknowledgment network. Furthermore, it is assumed that the maximum rate with which the source sends packets into the network is bounded. The optimal flow control mechanism which maximizes the average throughput under the condition that the expected packet time delay in the forward network does not exceed an upper bound is derived. Related work in the area of state dependent resource allocation is presented in [2] [3], [4], [5] and [9].

This paper is organized as follows. In Section 2, the formulation of the problem is introduced. In Section 3, it is shown that the optimal state dependent flow control with delayed feedback information under the previous optimization criterion is a window flow control mechanism. Furthermore, the effect of the delayed feedback information on the network performance is studied. An example that sheds light on the derived theoretical results is given. In Section 4, a number of conclusions are drawn and a number of issues that need further study are discussed.

2. The Statement of the Problem

The user, through a controller with capacity c, wishes to optimally utilize the resources of a Jackson network consisting of a forward and an acknowledgment network (see Figure 2.1). It is assumed that there is no interference between the forward and the acknowledgment network.

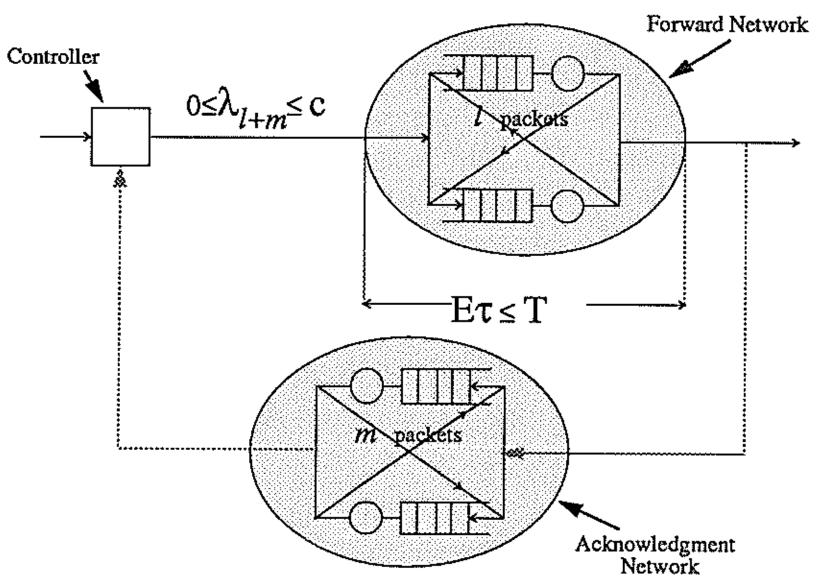

Fig. 2.1. A Jackson network with delayed acknowledgments.

Each of the I processors of the forward network has an infinite number of buffers and serves packets at each node with an exponential service rate. There are J processors in the acknowledgment network. Let μ^i be the service rate of the i^{th} processor, $1 \leq i \leq M$, with $M = I + J$. Let $\mathbf{R} = [r^{ij}]$ be the $(M+1) \times (M+1)$ routing matrix $(0 \leq i \leq M,\ 0 \leq j \leq M)$. Using this notation, packets join the network at node i with probability r^{0i}. Upon completion of service at node i, packets leave the network with probability r^{i0} or are routed from node i to node j with probability r^{ij}. It is assumed that the topology of the network does not change with time *and that, at the time a packet reaches its destination, an acknowledgment packet begins its way from destination to source.*

The evolution of the queueing network is described by the stochastic vector $\mathbf{Q}_t = (Q_t^1, \cdots, Q_t^{M-1}, Q_t^M)$, where Q_t^i refers to the number of packets at node i, $1 \leq i \leq M$.

$\mathbb{E}\gamma$ and $\mathbb{E}\tau$ denote the expected throughput and expected time delay, respectively, of the forward network. The controller attempts to maximize the average number of packets it sends into the network (expected throughput) such that the expected time delay of these packets in the forward network does not exceed a given upper bound T, that is,

$$\max_{\mathbb{E}\tau \leq T} \ \mathbb{E}\gamma \quad . \tag{2.1}$$

3. The Effect of Delayed Feedback Information on Network Performance

3.1. Optimal State Dependent Flow Control

In the sequel, the flow control problem of a Jackson network with nonzero acknowledgment delays is studied.

Let the $1 \times (M+1)$ matrix $\Theta \stackrel{def}{=} [\theta^0 \ \theta^1 \ \cdots \ \theta^M]$ be the solution of the traffic flow equations

$$\Theta = \Theta \mathbf{R} \quad ,$$

where $\theta^0 = 1$. Let

$$g_l \stackrel{def}{=} \sum_{k_1+k_2+\cdots+k_I=l} \prod_{j=1}^{I} \left(\frac{\theta^j}{\mu^j}\right)^{k_j} \quad , \tag{3.1}$$

for all l, $1 \leq l \leq N$, where $0 \leq k_i$, for $i = 1, \cdots, I$.

If l is the total number of packets in processors $1, 2, \cdots, I$, then the Norton equivalent, symbolized by ν_l, is given by

$$\nu_l \stackrel{def}{=} \frac{g_{l-1}}{g_l} \quad . \tag{3.2}$$

If, in addition, m is the total number of packets in processors $I+1, I+2, \cdots, M$, then the Norton equivalent of the processors serving the acknowledgment packets is symbolized by η_m and given by the equation

$$\eta_m = \frac{h_{m-1}}{h_m} \quad , \tag{3.3}$$

where

$$h_m = \sum_{k_{I+1}+\cdots+k_M=m} \prod_{j=I+1}^{M} \left(\frac{\theta^j}{\mu^j}\right)^{k_j} \quad . \tag{3.4}$$

In order to maximize the throughput of the forward network in such a way that the expected time delay does not exceed a given upper bound, a prime optimization method [8] for solving the problem is followed. If, at most N packets are permitted to enter the network, by using Norton's theorem [13], the original Jackson network (depicted in Figure 2.1) is first order equivalent with the Jackson network shown in Figure 3.1.

$\mathbb{E}\gamma_N$ and $\mathbb{E}\tau_N$ are the expected forward throughput and expected time delay, respectively, given that at any given moment no more that N packets can be in the network. In the sequel, we will determine the properties of the optimal flow control of a Jackson network with and without acknowledgment delays.

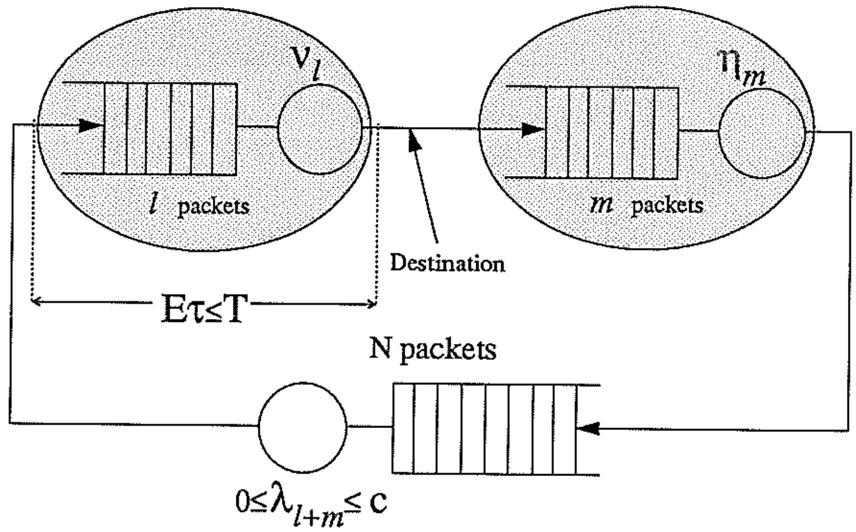

Fig 3.1. The first-order equivalent Jackson network with at most N packets.

3.1.1. Optimum Control without Acknowledgment Delay

In this section we derive the optimal state dependent flow control policy for the forward network without acknowledgment delay. Therefore, it is assumed that $\eta_m = \infty$, for all $m = 1, 2, ..., N$ (see Figure 3.1). A proof due to G. Weiss [14] is presented based upon policy iteration of the fact that with concave increasing service rates of the Norton's equivalent (as in the case of Jackson networks [11], [12], [13],) the optimal flow control is a window type mechanism. This result was first proven in [7] using a different method. The proof is presented here because the insight provided concerning the optimal flow control is used in the remainder of the paper.

Proposition 3.1. *If ν_k is a concave increasing function with respect to k, the optimal flow control is a window type mechanism with a random point, if it exists, corresponding to the last packet in the window, i.e.,*

$$\lambda_k = \begin{cases} c & \text{if } 0 \leq k \leq L-2 \\ 0 < \lambda_{L-1} \leq c & \text{if } k = L-1 \\ 0 & \text{if } L \leq k \quad . \end{cases} \tag{3.5}$$

Proof : Let us assume that under the current policy N packets can enter into the network. Let $(\lambda_0^*, \cdots, \lambda_{N-1}^*)$ correspond to the current control policy. We show how to choose a policy $(\lambda_0, \cdots, \lambda_{N-1})$, $c \geq \lambda_m > \lambda_m^*$, $\lambda_{m+1} < \lambda_{m+1}^*$, and $\lambda_k = \lambda_k^*$ for all k, $k \neq m$ and $k \neq m+1$, such that $\mathbb{E}\gamma_N \geq \mathbb{E}\gamma_N^*$ and $\mathbb{E}\tau_N \leq \mathbb{E}\tau_N^*$. Observe that

$$(p_0, \cdots, p_N) = ((1-x)p_0^*, \cdots, (1-x)p_{m-1}^*, (1+y)p_m^*, (1-z)p_{m+1}^*, \cdots, (1-z)p_N^*) \ .$$

We choose x, y, and z, such that

$$\sum_{k=0}^{N} p_k^* \ = \ 1$$

and

$$\mathbb{E}Q_N \ = \ \mathbb{E}Q_N^* \quad .$$

In other words,

$$x \sum_{k=0}^{m-1} p_k^* + z \sum_{k=m+1}^{N} p_k^* = y p_m^*$$

and

$$x \sum_{k=0}^{m-1} k p_k^* + z \sum_{k=m+1}^{N} k p_k^* = y m p_m^* \quad .$$

It is easy to see that if $y > 0$, then $x > 0$ and $z > 0$. Letting

$$\alpha_k = \begin{cases} \frac{x p_k^*}{y p_m^*} & \text{if } 0 \leq k < m \\ \frac{z p_k^*}{y p_m^*} & \text{if } m < k \leq N \end{cases} \quad ,$$

we have $\alpha_k > 0$, for $k \neq m$, with

$$\sum_{k \neq m} \alpha_k = 1$$

and

$$\sum_{k \neq m} k \alpha_k = m \quad .$$

By the concavity of ν_k with respect to k,

$$\sum_{k \neq m} \nu_k \alpha_k \leq \nu_m \quad .$$

Thus,

$$\mathbb{E}\gamma_N \geq \mathbb{E}\gamma_N^* \quad , \tag{3.6}$$

and

$$\mathbb{E}\tau_N \leq \mathbb{E}\tau_N^* \quad .$$

Furthermore, since $z > 0$,

$$p_N < p_N^* \quad .$$

∎

3.1.2. Optimum Control with Acknowledgment Delay

Let us assume that at any given moment, at most N packets can be unacknowledged (see Figure 3.2). The *equivalent controller* that describes the behavior of the original controller together with the acknowledgment network is a state dependent processor that corresponds to their Norton equivalent (see Figure 3.3). Let δ_l be the equivalent state dependent arrival rate into the forward network when there are l packets in the forward network. Then,

$$\delta_l \stackrel{\text{def}}{=} \frac{\sum_{n=1}^{N-l} \prod_{k=1}^{n} (\frac{\lambda_{l+k-1}}{\eta_k}) \eta_n}{1 + \sum_{n=1}^{N-l} \prod_{k=1}^{n} (\frac{\lambda_{l+k-1}}{\eta_k})} \quad , \tag{3.7}$$

for all $l \leq N$.

From Equation (3.7), we see that δ_l is a function of $\lambda_l, \lambda_{l+1}, \cdots, \lambda_{N-1}$, for all $l \leq N$. In particular δ_{N-1} is only a function of λ_{N-1}.

Notice that δ_l is the average throughput of a closed network with $N-l$ packets. This network consists of two queueing systems with rates η_m and λ_{l+m}, respectively. Because the service rate η_m is concave increasing, the maximum throughout of this closed network is achieved when the arrival process follows a window flow control policy. (See Proposition 3.1 and Equation 3.7.)

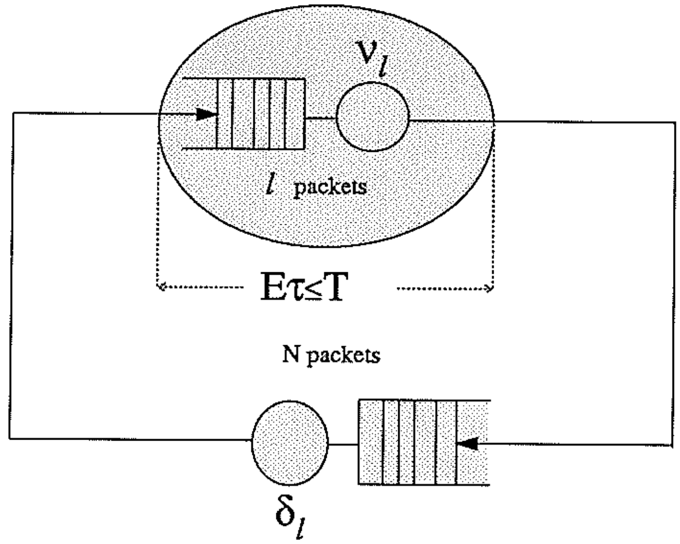

Fig. 3.2. The equivalent forward network with the equivalent controller.

Proposition 3.2 *The optimal flow control policy of a Jackson network with acknowledgment delay is of the form*

$$\lambda_l = \begin{cases} c & \text{if } 0 \leq l < L-1 \\ 0 < \lambda_{L-1} \leq c & \text{if } l = L-1 \\ 0 & \text{if } L \leq l \ . \end{cases} \tag{3.8}$$

L and λ_{L-1} take the highest feasible values that do not violate the time delay constraint $\mathbb{E}Q - T\mathbb{E}\gamma \leq 0$.

Proof : If l is the total number of packets inside the forward network, then its service rate ν_l is a concave increasing function with respect to l, for $l \geq 0$. From Proposition 3.1 we conclude that the optimal flow control is a window flow control with respect to the equivalent arrival rates δ_k, for $k \geq 0$. Therefore, under the time delay constraint $E\tau \leq T$, each variable δ_k, for $k \geq 0$, must take the highest possible value. From the discussion that follows Equation (3.7), we conclude that δ_k takes the highest possible value when the variables $\lambda_k,\ \lambda_{k+1}, \cdots$, take their highest possible values, for $k \geq 0$.

Therefore, we first assign to λ_0 its highest possible value; we next assign to λ_1 its highest possible value; and so on for $\lambda_2, \lambda_3, \cdots$. The only condition that must be maintained is $\mathbb{E}\tau \leq T$. Therefore the optimal control is a window mechanism with at most one random point at the end of the window.

∎

In what follows an iterative algorithm for the computation of the optimal flow control is given.

Algorithm 3.3.

Step 0: *$L := 1$. Set $\lambda_0 := c$ and $\lambda_k := 0$ for all k, $k \geq 1$. Check to see whether $\mathbb{E}\tau <= T$. If yes, continue to Step 1. Otherwise stop; no packets can enter into the network.*

Step 1: *$L := L+1$. Set $\lambda_k := c$ for all k, $0 \leq k \leq L-1$. Check whether $\mathbb{E}\tau \leq T$. If yes, repeat Step 1. Else, find the exact value of λ_{L-1} (which is between 0 and c), with which* *the last packet should be accepted and which results in $\mathbb{E}\tau = T$; the resulting flow control is the optimal flow control; stop.*

3.1.3. The Effect of Feedback Information on the Performance of the Optimal Control

In this section we study the effect of the delayed feedback information on the optimal window flow control.

Lemma 3.4. *For a given optimal flow control policy, the state dependent rate of the aggregated controller δ_l for all l, $0 \le l \le L-1$, is*
(i) an increasing function with respect to the controller rates λ_k for all k, $l \le k \le L-1$,
(ii) an increasing function with respect to the service rates of the processors of the acknowledgment network,
(iii) a concave decreasing function with respect to l, $0 \le l \le L-1$.
Proof : When l packets are in the forward network δ_l is the Norton equivalent of a network consisting of the original controller and the Norton equivalent of the acknowledgment network (see Figure 3.1, Figure 3.2 and Equation (3.7)). The result follows from Proposition 7.1 (i) (see the Appendix). To prove the second statement note that the Norton equivalent of the acknowledgment network is an increasing function with respect to the service rate of any of its constituent service processors [11]. The result then follows from Proposition 7.2 (i). The fact that δ_l is decreasing is a direct conclusion of Proposition 7.1 (i). Concavity can be shown based on [11].

∎

Lemma 3.5 below demonstrates the way the window size is affected due to variations in the service rates of the processors of the acknowledgment network.

Lemma 3.5. *The window size of the optimal flow control is*
(i) decreasing with respect to the maximum controller rate c,
(ii) increasing with respect to the service rate of any of the processors of the forward network,
(iii) decreasing with respect to the service rate of any of the processors of the acknowledgment network.
Proof : (i) From Proposition 3.2 the optimal flow control is of the form $\lambda_l = c$ for $0 \le l \le L-1$, and $0 < \lambda_{L-1} \le c$. Let $c^* > c$ and let us change the values of the optimal arrival rates to $\lambda_l^* = c^*$ for $0 \le l \le L-1$, $\lambda_{L-1}^* = \lambda_{L-1}$, and $\lambda_l^* = 0$ for $l \ge L$. From Lemma 3.4 (i) we conclude that the service rates of the aggregated controller δ_l, $0 \le l \le L-1$, increase, and from Proposition 7.1 (v) the expected time delay of the packets in the forward network increases. But under the previous optimal policy $\mathbb{E}\tau = T$. Therefore under the new control policy the time delay constraint is violated. As a result for the optimal flow control $\lambda_{L-1}^* < \lambda_{L-1}$, and the result follows. The proof of (iii) is based on Lemma 3.4 (ii) and arguments similar to the ones used above.
(ii) The network operates under the optimal flow control. Without affecting the flow control policy we increase the service rate of any of the processors in the forward network. The service rate of equivalent controller δ_l, $0 \le l \le L-1$, does not change. From Lemma 3.4 (iii) and Proposition 7.2 (iv) we conclude that the expected time delay of the packets in the forward network decreases. The result then follows from Proposition 3.2.

∎

Lemma 3.6. *For a given upper bound on the average time delay T of the forward network, the optimum average throughput $\mathbb{E}\gamma$ is*
(i) an increasing function with respect to the maximum capacity c of the original controller,

(ii) an increasing function with respect to the service rates of the processors in the forward network,
(iii) an increasing function with respect to the service rates of the processors of the acknowledgment network.

Proof :

(i) Let $c^* > c$. Then the set of the control policies that can be implemented when $0 \leq \lambda_l \leq c^*$, for $l \geq 0$, is a superset of the control policies that can be implemented when $0 \leq \lambda_l \leq c$ for $l \geq 0$. The result then follows.

(ii) The network operates under the optimal flow control. Without affecting the flow control policy we increase the service rate of any of the processors in the forward network. The service rate of the equivalent controller δ_l, $0 \leq l \leq L-1$, does not change. From Lemma 3.4 (iii), Proposition 7.2 (i), and Proposition 7.2 (iv) we conclude that the network throughput increases and that the expected time delay of the packets in the forward network decreases. The result then follows from Proposition 3.2.

(iii) From Lemma 3.4 (ii) we know that the state dependent service rate of the aggregate controller is an increasing function with respect to the service rate of any of the processors in the acknowledgment network. From Proposition 3.2, we know that higher maximum arrival rates δ_l^*, $0 \leq l \leq L-1$, result in greater network throughput for a given T. The result then follows.

■

The above results show that as the congestion in the acknowledgment network builds up, the effective arrival rate of packets in the forward network decreases. Therefore, the network throughput decreases whereas the size of the optimal window increases. Notice that if the service rate of the processors of the acknowledgment network drops below a critical value, the acknowledgment network becomes the bottleneck of the flow control protocol. In the limiting situation in which the service rates of the processors of the acknowledgment network tend to zero, the network throughput tends to zero whereas the size of the optimal window tends to infinity.

3.2. Optimal State Independent Flow Control of a Jackson Network

Let λ be the state independent rate with which packets enter the forward network, and $\mathbf{R_f} = [r^{ij}]$ be the $I \times I$ routing matrix in the forward network ($1 \leq i \leq I$, $1 \leq j \leq I$). The $1 \times I$ matrix $\Theta = [\theta^1 \cdots \theta^I]$ is the solution of the traffic flow equations of the forward network, *i.e.*,

$$\Theta = \Lambda + \Theta \mathbf{R_f} \quad , \tag{3.9}$$

or

$$\Theta = \Lambda(\mathbf{I} - \mathbf{R_f})^{-1} \quad .$$

Here Λ denotes the load vector of the input traffic flows

$$\Lambda = \lambda[r^{01} \cdots r^{0I}] \quad .$$

With

$$\alpha = [\alpha^1 \cdots \alpha^I] \stackrel{\text{def}}{=} [r^{01} \cdots r^{0I}](\mathbf{I} - \mathbf{R_f})^{-1} \quad ,$$

we obtain

$$\theta^j = \alpha^j \lambda \quad ,$$

for all j, $j = 1, 2, \cdots, I$.

The time delay amounts to

$$\mathbb{E}\tau = \frac{\mathbb{E}Q}{\mathbb{E}\gamma} = \frac{\mathbb{E}Q^1 + \cdots + \mathbb{E}Q^I}{\lambda} \quad .$$

Equivalently,

$$\mathbb{E}\tau = \sum_{j=1}^{I} \frac{\alpha^j}{\mu^j - \alpha^j \lambda} \quad . \tag{3.10}$$

The expected time delay is an increasing function of the external arrival rate. Therefore, the maximum feasible arrival rate λ that achieves the upper bound constraint T

$$\sum_{j=1}^{I} \frac{\alpha^j}{\mu^j - \alpha^j \lambda} \leq T \tag{3.11}$$

is optimal. We have, thus, shown the following:

Lemma 3.7. *The maximum feasible rate λ is given by:*

$$\sum_{j=1}^{I} \frac{\alpha^j}{\mu^j - \alpha^j \lambda} = T \quad . \tag{3.12}$$

3.3. An Example

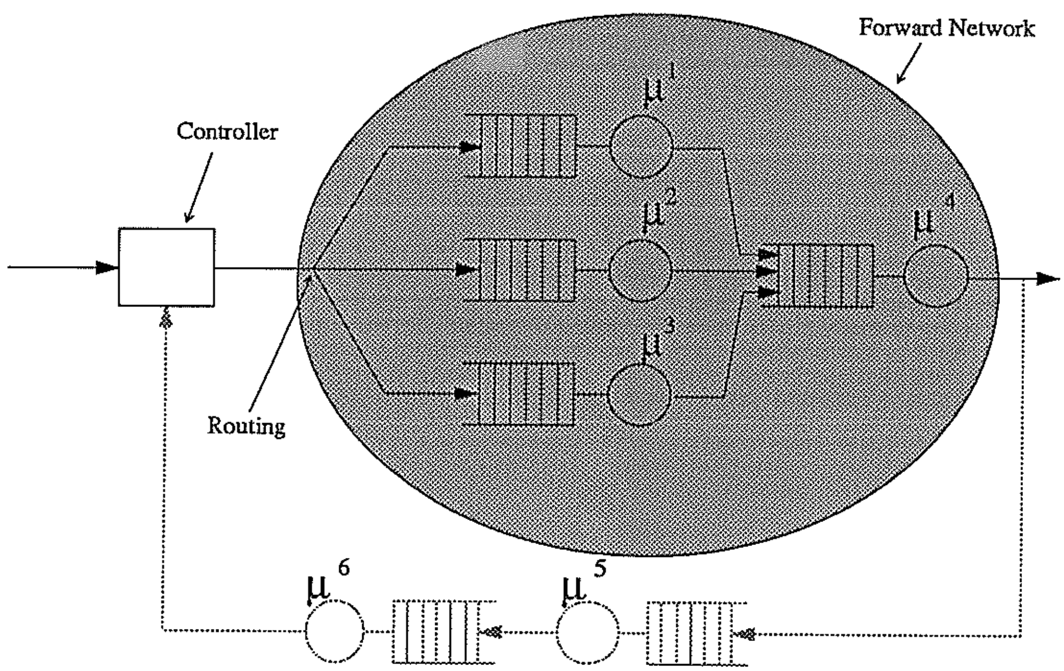

Fig. 3.3. A network of processors with delayed feedback subject to state dependent flow control.

In the sequel the effect of delayed feedback information on network performance is examined. Consider the network of processors depicted in Figure 3.3. The service rates of the processors are $\mu^1 = 2.0$ *packets/sec*, $\mu^2 = 1.0$ *packet/sec*, $\mu^3 = 0.5$ *packet/sec*, and $\mu^4 = 3.0$ *packets/sec*. For the case in which the network operates under state dependent flow control, acknowledgment packets are sent back to the source. In our example the acknowledgment network is represented by two processors in tandem with service rates μ^5 *packet/sec* and μ^6 *packet/sec*. Packets arrive into the network with state dependent arrival rate λ_k, where 0.0 *packets/sec* $\leq \lambda_k \leq c$ *packets/sec*. The packets are routed to server μ^1 with probability $\frac{4.0}{7.0}$, to server μ^2 with probability $\frac{2.0}{7.0}$ and to server μ^3 with probability $\frac{1.0}{7.0}$. Network performance results are labeled with the letter W or R, indicating whether the results were obtained using window control or rate control, respectively. The results show the dependence of the average throughput of the forward network and the window size on two parameters, namely the maximum controller capacity c, and the service rate of the processors in the acknowledgment network. Although the average throughput of the forward network and window size also depend on the service rate of the processors in the forward network, we have not included the corresponding curves because of space constraints.

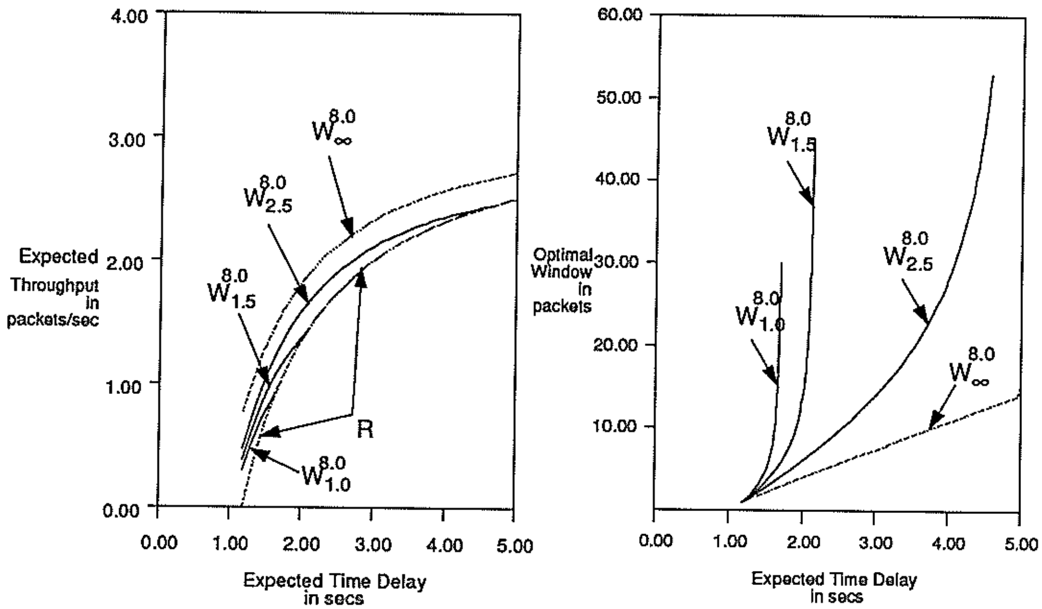

Fig. 3.4. The effect of the acknowledgment network on the average throughput and on the optimal window size.

In Figure 3.4, the curve W_k^x depicts the network performance under state dependent flow control and state independent routing when $\mu^5 = \mu^6 = k$ *packets/sec* and $c = x$ *packets/sec*. Curve R shows the network performance when the network is subject to state independent flow control. In Figure 3.4, the optimal window size is depicted as a function of the upper bound of the time delay constraint T. The sets of curves shown in Figure 3.4 illustrate three important points: (1) Network performance deteriorates as the acknowledgment delays increase. (See Lemma 3.5 (iii) and Lemma 3.6 (iii).) (2) The window size increases as acknowledgment delays increase. (3) *If* a given end-to-end time delay is achievable using both window flow control and state independent flow control, then network throughput is greater using window flow control. However, using a window flow control, a given end-to-end time delay may not be achievable. For example, as we see in Figure 3.4, when $\mu^5 = \mu^6 = 1.5$ *packets/sec*, the maximum achievable end-to-end time delay is roughly 2.0 secs. We see from Figure 3.4 that if an acceptable end-to-end time delay is 3.0 secs, then the achievable network throughput using rate flow control exceeds that which can be achieved using window flow control. This is an example of a situation in which *the delayed feedback information represents the effective bottleneck of the flow control mechanism. As a result, the state independent flow control becomes a more effective alternative to the state dependent flow control.* In Figure 3.5, curve $W_{1.0}^k$ depicts the network performance under state dependent flow control and state independent routing when $\mu^5 = \mu^6 = 1.0$ *packets/sec*, and $c = k$ *packets/sec*. In Figure 3.5, the optimal window size is depicted as a function of the upper bound of the time delay constraint T. The sets of curves shown in Figure 3.5 show that as the controller capacity increases, the network throughput increases and the optimal window size decreases. (See Lemma 3.6 (i) and Lemma 3.5 (i).) In Figure 3.6, we demonstrate the way in which network performance is affected when the service rate of a processor in the acknowledgment network is varied and when the end-to-end expected time delay is held fixed at 1.7 secs.

Summarizing the results presented thus far, we have compared the performance of networks operating under either window or rate flow control and have studied the dependence of the optimal window flow control on different network parameters.

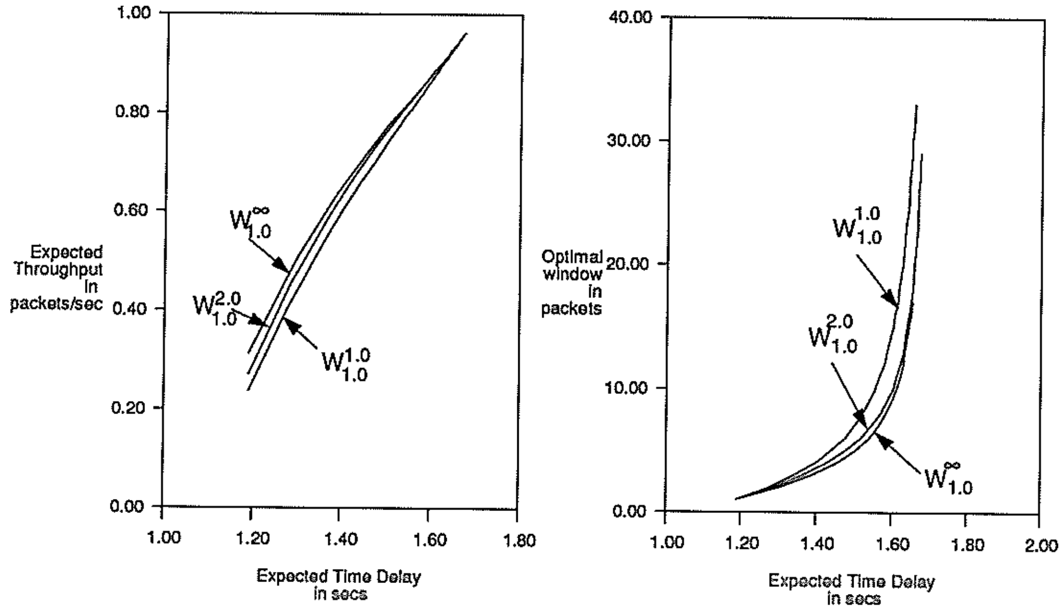

Fig. 3.5. The effect of the capacity constraint on the average throughput and on the optimal window size.

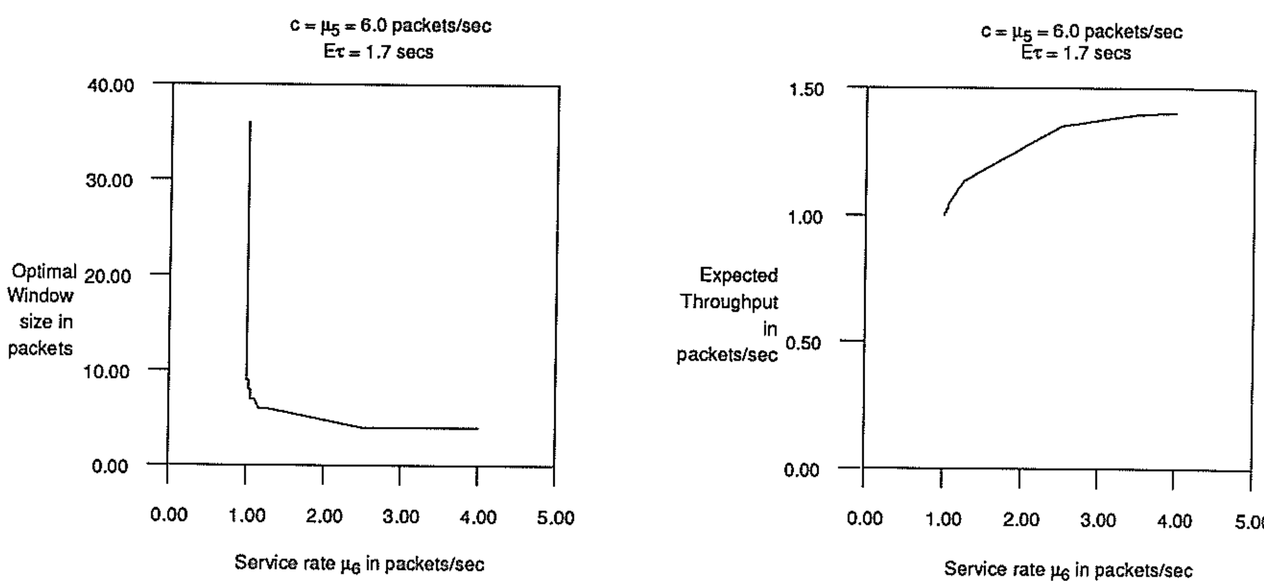

Fig. 3.6. The effect of the acknowledgment network on the network performance

In practical situations we would like to have some idea of the size of the optimal flow control window as quickly as possible. In the sequel we introduce two ways of approximating the optimal window size. Both approximations are computationally less complex that Algorithm 3.3. We study the accuracy of these approximations through examples. Our motivation for trying to assess rapidly the optimal window size arises from our recognition that the size of the window flow control cannot exceed the size of the available buffer utilized for its implementation. Thus we would like to decide as quickly as possible whether to use a window or rate flow control given a maximum available buffer size.

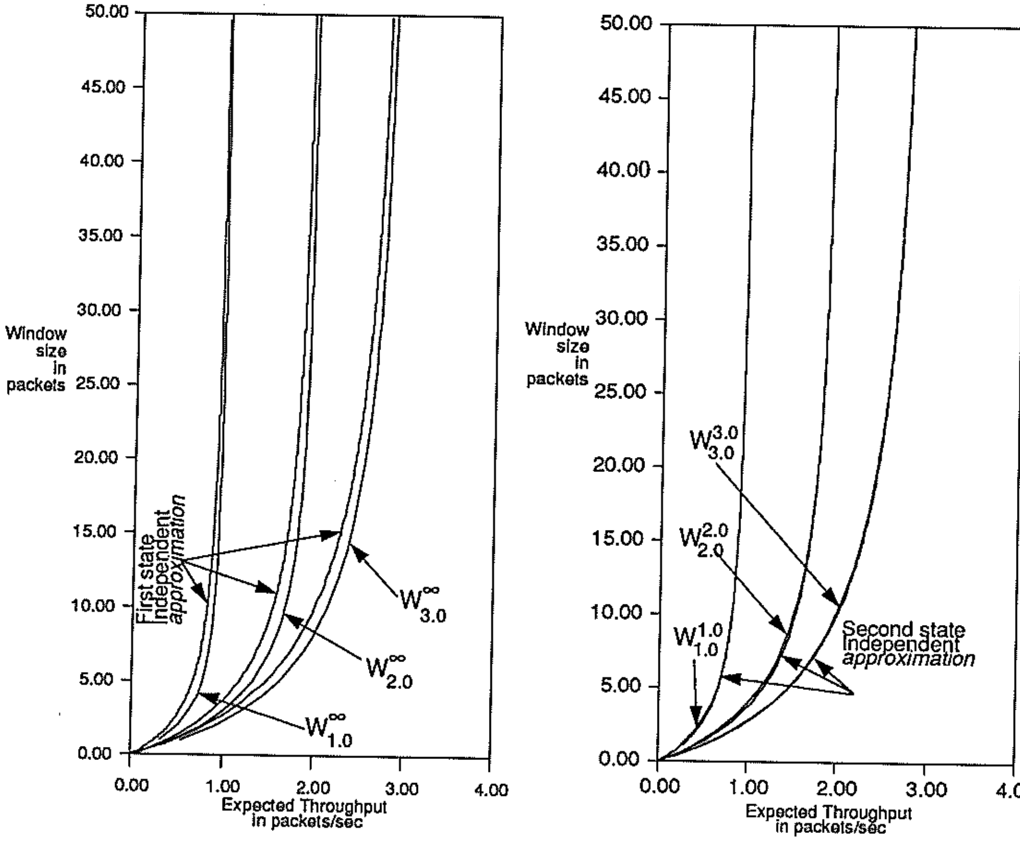

Fig. 3.7. Approximate computation of the optimal window size.

We first approximate the optimal window size by the optimal window size corresponding to a network operating with $c = \infty$. The $c = \infty$ approximation gives a lower bound on the size of the optimal window. (See Lemma 3.5 (i) and Figure 3.5.) We next approximate the optimal window size by the expected number of packets in both the forward and acknowledgment networks operating under state independent flow control and under the constraint that the expected time delay of the packets in the forward network do not exceed a given upper bound T. We see from Figure 3.7 that the second approximation is quite accurate and thus can be used in practical applications. From numerous examples, we observe that the approximation appears to provide an upper bound for the required window size. Concluding, we believe that the examples studied in this section reveal realistic and computationally effective approximations that can be used in practice for the determination of the effectiveness of rate-based flow control as well as window-based flow control. We notice that the congestion of the acknowledgment network and its effect on the optimal window size can be effectively approximated by either a network operating with $c = \infty$ or an open network (consisting of both the forward and the acknowledgment networks) operating under state independent flow control.

When the original network was approximated by an open network, we noticed that the optimal window was approximately equal to the expected number of packets in the forward and acknowledgment networks. Notice that at server j of the acknowledgment network, the expected number of packets is

$$EQ^j = \frac{\alpha^j \lambda}{\mu^j - \alpha^j \lambda} \quad .$$

Therefore if congestion builds up in server j of the acknowledgment network, in order to keep the throughput at the same level, the user should increase its window size. Simple calculations reveal that if the service rate of the server j is changed from μ^j_{old} to μ^j_{new}, the user can keep its throughput at the same level if the network user changes its window size according to the equation

$$EQ^j_{new} - EQ^j_{old} = \frac{\mu^j_{old} - \mu^j_{new}}{(\mu^j_{old} - \alpha^j \lambda) \times (\mu^j_{new} - \alpha^j \lambda)} \quad .$$

The previous equation also reveals that if the service rate decreases, the user should increase the window size, whereas if the service rate increases, the user should decrease the window size. Obviously this approximation does not take into account the effect of changes in the end-to-end expected time delay. It can be used, however, as a first approximation of the effect of congestion on the user's window size.

If, on the other hand, the user wishes to keep its window size fixed and at the same time maintain its throughput, the user should reduce its effective rate through that particular resource. This can be done by reducing the acknowledgment rate through the utilization of selective acknowledgment or by the introduction of additional resources into the network.

4. Conclusions

In the present paper the effect of acknowledgment delays was studied in detail. We first showed that when congestion arises in the forward network, the size of the optimal window of state dependent flow control that maximizes the performance of the network *decreases*. On the other hand, when congestion appears in the acknowledgment network, the size of the optimal window *increases*. We further proved that under heavy congestion in the acknowledgment network, the state independent flow control avoids the acknowledgment network congestion and thus gives better performance. Whereas the congestion that arises in the forward network affects both window and rate flow control, the congestion that appears in the acknowledgment network affects only the state dependent (window) flow control.

The problem of flow control remains a subtle issue in computer communications. A number of questions require further study. Is it really advantageous to have an end-to-end control all the time? The previous analysis suggests that it would be advantageous to have an end-to-end window flow control as long as there is no congestion and as long as the optimal window size is acceptable. Whenever local congestion affects the acknowledgment packets, local control procedures are needed to attempt to eliminate or by-pass the congestion.

5. Acknowledgments

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7. Appendix

Monotonicity Properties for a Controlled Finite Birth-Death Process

In this appendix we analyze the behavior of the closed network depicted in Figure 7.1. If k is the total number of packets in the upper processor, the upper and the lower processors serve packets with state dependent rates μ_k and λ_k, respectively. Let $\mathbb{E}\gamma_N$, $\mathbb{E}Q_N^c$ and $\mathbb{E}\tau_N^c$ be the expected throughput, the expected number and the expected time delay of the packets in the lower processor of the network in Figure 7.1, respectively. Similarly, let $\mathbb{E}Q_N$ and $\mathbb{E}\tau_N$ be the expected number and the expected time delay of the packets in the upper processor of the network in Figure 7.1. Observe that

$$\mathbb{E}\gamma_N = \frac{\mathbb{E}Q_N}{\mathbb{E}\tau_N} = \frac{\mathbb{E}Q_N^c}{\mathbb{E}\tau_N^c} = \frac{N}{\mathbb{E}\tau_N + \mathbb{E}\tau_N^c} \quad .$$

The results presented here follow a methodology suggested by G. Weiss in [14].

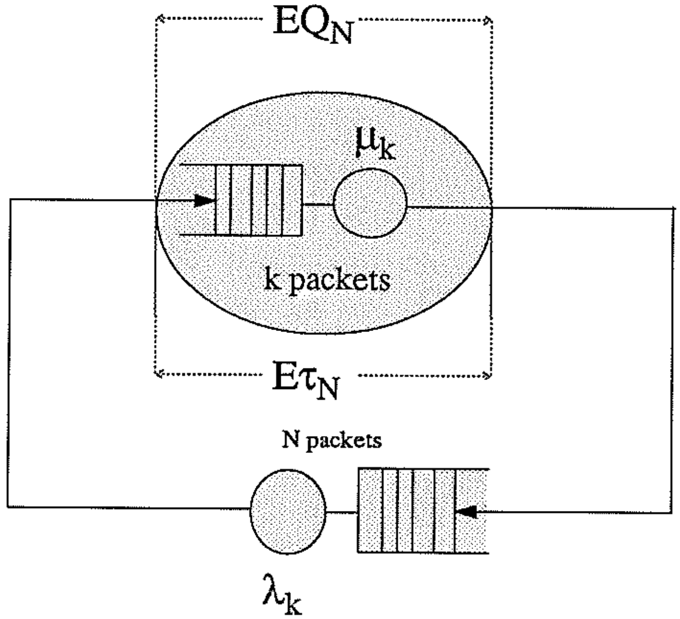

Fig. 7.1. A single class network with at most N packets subject to state dependent flow control.

Proposition 7.1

(i) If μ_k is an increasing function of k, the expected throughput $\mathbb{E}\gamma_N$ is increasing in λ_k for $k = 0, 1, \cdots, N-1$.

(ii) The expected number of packets $\mathbb{E}Q_N$ is increasing in λ_k for $k = 0, 1, \cdots, N-1$.

(iii) The expected number of packets $\mathbb{E}Q_N^c$ is decreasing in λ_k for $k = 0, 1, \cdots, N-1$.

(iv) If μ_k is an increasing function of k, the expected time delay of the packets in the lower processor $\mathbb{E}\tau_N^c$ is decreasing in λ_k for $k = 0, 1, \cdots, N-1$.

(v) If $\frac{k}{\mu_k}$ is an increasing function of k, the expected time delay $\mathbb{E}\tau_N$ is increasing in λ_k for $k = 0, 1, \cdots, N-1$.

Proof: Let $\mathbf{p}^* = (p_0^*, \cdots, p_N^*)$ correspond to the control $\lambda^* \stackrel{\text{def}}{=} (\lambda_0^*, \cdots, \lambda_N^*)$ and let $\mathbf{p} = (p_0, \cdots, p_N)$ correspond to the control $\lambda \stackrel{\text{def}}{=} (\lambda_0, \cdots, \lambda_N)$. Let us assume that $\lambda_i \geq \lambda_i^*$ for all $i \leq N-1$. Then

$$\frac{p_k^*}{p_{k-1}^*} = \frac{\lambda_{k-1}^*}{\mu_k} \leq \frac{\lambda_{k-1}}{\mu_k} = \frac{p_k}{p_{k-1}} \quad ,$$

for $k = 1, \cdots, N-1$, from which it follows that

$$\sum_{i=k}^{N} p_i^* \leq \sum_{i=k}^{N} p_i \quad .$$

Since the μ_k's are increasing,

$$\sum_{k=1}^{N} \mu_k p_k^* \leq \sum_{k=1}^{N} \mu_k p_k \quad ,$$

which completes the proof of (i).

(ii) $\mathbb{E}Q_N = \sum_{k=1}^{N} k p_k^*$. The arguments of (i) hold here if μ_k is substituted for k.

(iii) $\mathbb{E}Q_N^c = N - \mathbb{E}Q_N$. The statement is then true because of (ii).

(iv) This statement holds because $\mathbb{E}\tau_N^c = \frac{\mathbb{E}Q_N^c}{\mathbb{E}\gamma_N}$.

(v) $\mathbb{E}\tau_N$ is a weighted average of $\frac{k}{\mu_k}$, for $k = 1, \cdots, N$, with weights $q_k^* = \frac{\mu_k p_k^*}{\sum_{i=1}^{N} \mu_i p_i^*}$. The arguments of (i) hold if q_k^* is substituted for p_k^*, and the statement follows.

■

In a similar way we can prove the following relations.

Proposition 7.2

(i) If λ_k is a decreasing function of k, the expected throughput $\mathbb{E}\gamma_N$ is increasing in μ_k for $k = 0, 1, \cdots, N-1$.

(ii) The expected number of packets $\mathbb{E}Q_N^c$ is increasing in μ_k for $k = 0, 1, \cdots, N-1$.

(iii) The expected number of packets $\mathbb{E}Q_N$ is decreasing in μ_k for $k = 0, 1, \cdots, N-1$.

(iv) If λ_k is a decreasing function of k, the expected time delay of the packets in the network $\mathbb{E}\tau_N$ is decreasing in μ_k for $k = 0, 1, \cdots, N-1$.

(v) If $\frac{N-k}{\lambda_k}$ is a decreasing function of k, the expected time delay $\mathbb{E}\tau_N^c$ is increasing in μ_k for $k = 0, 1, \cdots, N-1$.