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Essays on Firm Strategies and Market Outcomes

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Essays on Firm Strategy and Market Outcomes

by

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Table of Contents

List of Figures

List of Tables

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ABSTRACT OF THE DISSERTATION

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Professor Marcus Berliant, Chair

In the first chapter of my dissertation, Aleksandr Yankelevich and I examine the effects of price matching guarantees on duopoly markets. We find that a commitment to price-match raises prices by altering consumer search behavior in three ways. First, price-matching diminishes firms' incentives to lower prices to attract consumers who have no search costs. Second, for consumers with positive search costs, price-matching lowers the marginal benefit of search, inducing them to accept higher prices. Finally, price-matching can lead to asymmetric equilibria where one firm runs fewer sales and both firms tend to offer smaller discounts than in a symmetric equilibrium. These price increases grow with the proportion of consumers who invoke price-matching guarantees and also in the level of equilibrium asymmetry.

The second chapter studies the effect of the complexity of consumers' preferences over a product on that product's market structure. I relate complexity of preferences to the number of dimensions of a Lancasterian characteristic space. Using a novel higher dimensional Hotelling model, I find that a fixed number of firms are likely to be better off competing over products with more complex preferences. Although firms face more intense competition in higher dimensional markets, the greater product differentiation afforded to them allows them to charge higher prices and earn higher profits. This result provides a clear

theoretical foundation for the observation that goods associated with more complex preferences typically display a greater variety of products sold. Additionally, I show that the behavior of more than two firms competing in more than one dimension differs wildly from that of firms typically studied in models of spatial competition.

The final chapter will examine firms' motives for implementing grandfather clauses that allow certain consumers to continue to access a service at a favorable, but no longer available price. Grandfather clauses permit firms to price discriminate between early adopters and new consumers in exchange for forfeiting the right to optimally set prices for early adopters. They may be used to thwart competition following a structural change, to respond to cost shocks, or to retain customers who consume another good from a multi-product firm. We analyze under what conditions firms might choose to offer grandfather clauses and what effects they have on welfare.

Chapter 1

Introduction

In my dissertation I seek to extend our knowledge of the ways that firms compete with one another in the marketplace. Particularly, I explore a variety of practices other than simply altering the price or quality of their product that firms use to gain an edge over their competition. In much of the economics literature it is supposed that firms can only gain long term advantage by selling either a superior product or a product they can produce at reduced cost. However, in equilibrium we observe that firms use a huge variety of special promotions, limited time offers, and other sorts of manipulation to gain advantage. Due to their persistence in equilibrium, it must be supposed that many of these marketing practices do confer advantages. My work seeks to understand the channels through which these advantages may be gained and whether or not the practices should be considered potentially harmful.

Each chapter of my dissertation focuses on a different strategy employed by firms. The second studies price matching guarantees, the third studies multi-axis product differentiation, and the last studies the use of grandfather clauses. The work is theoretical, and as such is mostly concerned with developing conceptual frameworks in which these phenomenon can be studied. Accordingly almost all of the techniques employed can be found in calculus or real analysis. More information on these works' relation to the literature, scope, and results can be found in the introductory section of each of the chapters.

The second chapter of my dissertation is coauthored with Aleksandr Yankelevich. While he contributed the majority of the work concerning the construction of the model and the proof of the main results, I contributed a thorough simulation of the environment over a restricted parameter space which is used extensively to explain the environment to the reader as well as forming the meat of section 2.6. Additionally, I helped motivate the model by referencing the concept of "deal-prone" consumers. Finally, I did significant work in the writing and editing of the paper.

Chapter 2

Price-Matching Guarantees

2.1 Introduction

Price-matching guarantees can be found in a variety of markets, including consumer electronics, office supplies, clothing, groceries and hotels. These guarantees typically come in the form of an offer by a firm to lower its price to that of a cheaper rival selling an identical good for consumers who can offer proof of the rival's price. Firms inform consumers about their price-matching guarantees in television advertisements, using print ads, and over the Internet. For instance, in one commercial, Walmart tells viewers that it will match rival prices eighteen times within the span of thirty seconds to remind consumers that its every day low prices are "Backed by [its] Ad Match Guarantee."[1] In a holiday ad, Toys "R" Us asks viewers why they would shop anywhere else for toys when "the highest concentration of the hottest toys is at Toys "R" Us, all with price-match guarantee."[2] Because of such marketing efforts, consumers who engage in repeated interactions with price-matching

[1]See "Walmart Match It Commercial." *youtube.com.* May 07, 2011. Retrieved February 28, 2015. ⟨http://www.youtube.com/watch?v=CIXOU7DQdS8⟩.

[2]See "Toys "R" Us–Holiday 2012 Commercial–'Toy Tracker Price Match'." *youtube.com.* November 04, 2012. Retrieved February 28, 2015.
⟨http://www.youtube.com/watch?feature=fvwp&v=3mQ0xmzeSfI&NR=1⟩.

brands, come to expect these guarantees. The expectation of a price-matching guarantee should influence the way a consumer shops, which in turn affects firm pricing. It is already well known that when consumers have additional information about firms, their shopping behavior can change.[3] Yet, in spite of the wealth of economics and marketing literature studying price-matching guarantees, the exploration of their effect on consumers' shopping incentives remains limited.

Economists initially viewed price-matching guarantees as being anti-competitive. This idea was first raised by Hay (1982) and then Salop (1986), who suggest that these guarantees allow firms to immediately retaliate against rival price cuts without actually listing lower prices or expending resources to learn about competitor prices. This can lead to tacit collusion in a non-cooperative equilibrium by removing firms' incentives to cut prices. Subsequently, this view was formalized in multiple settings: Bertrand oligopoly (Doyle 1988), differentiated products Stackelberg duopoly (Belton 1987), Hotelling duopoly (Zhang 1995), and differentiated products Bertrand duopoly where consumers incur hassle costs of applying price-matching guarantees (Hviid and Shaffer 1999).[4] Such models leave no room for consumers to make shopping decisions. Tacit collusion occurs because firms respond to each other, not to their customers.

An alternate line of reasoning posits that price-matching guarantees allow firms to price discriminate between consumers with limited price information and those who are informed about multiple price quotes. For example, Png and Hirshleifer (1987) show that price-matching guarantees allow firms to keep list prices high to extract welfare from uninformed consumers, while attracting informed consumers by offering to price-match the rival firm when it offers a lower price. Models with heterogeneous consumers have also been used to

[3] See for instance, Robert and Stahl (1993) and Janssen and Non (2008).

[4] More recently, Hviid and Shaffer (2010) have shown that price-matching guarantees can complement the price-increasing effect of a most-favored-customer clause when both are offered unilaterally by a single firm.

suggest that price-matching guarantees can lead to pro-competitive effects. For instance, Corts (1996) and Chen, Narasimhan, and Zhang (2001) show that when firms use price-matching guarantees to price discriminate, some or all consumers may end up paying lower prices and consumer welfare can increase. Using surveys of potential consumers, Jain and Srivastava (2000) and Srivastava and Lurie (2001) argue that consumers perceive stores that offer price-matching guarantees to have lower prices. Moorthy and Winter (2006) and Moorthy and Zhang (2006) build on this argument by constructing models of price-matching with respectively, horizontal and vertical firm differentiation, where consumers consider their location or service preferences when choosing where to purchase and consumers who are uninformed about prices use price-matching as a signal that influences their price expectation for a particular firm. They show that when the difference in production costs between the two firms is sufficiently large and the uninformed population is sufficiently small, price-matching guarantees can be used to signal a low price and consumer welfare improves for a range of parameters.

While each of the aforementioned models has shown that price-matching can alter firm pricing behavior, as Moorthy and Winter point out, and as we explore in detail in this article, another allocative effect of price-matching is its impact on consumers' incentive to invest in information about prices (i.e., to price shop). Price-matching models of tacit collusion preclude this effect by assuming that all consumers are perfectly informed about firm pricing decisions. Consequently, in these models, either price-matching leads to a symmetric monopolistic outcome, or in order to avoid the monopoly result, the authors assume that products are somehow differentiated. However, firms generally will not honor price-match guarantees on products that are not identical.[5] Price-matching models based

[5]An alternative interpretation casts product differentiation in terms of differences in firm location. But this interpretation is at odds with the idea of perfect price information unless all consumers can travel freely from store to store. Although this is conceivable in an on-line retail environment, price-matching guarantees often stipulate that on-line prices will not be matched.

on price discrimination assume that consumers are heterogeneous with respect to the amount of price information that they possess. However, differences in price information are exogenously imposed and consumers may be assumed to act in a way that is in contrast to what they would do were they allowed to engage in optimal shopping behavior.

We depart from the earlier literature by endogenizing the incentive to acquire price information and allowing consumers to engage in optimal price search using a duopoly version of Stahl's 1989 model of sequential consumer search in which firms first have an option to offer a price-matching guarantee before setting prices for a homogeneous good. There are two types of consumers in the market: those who face no opportunity cost of searching (referred to as shoppers) and those who do (non-shoppers). In Stahl-type models, it is well established that consumers follow a reservation price rule—continue searching only if the last price observed is greater than an endogenously determined reservation price—while firms randomize over lower prices to attract shoppers and over higher prices to realize greater profits from non-shoppers.

In this framework, we find that price-matching guarantees bring about a number of price-increasing changes in consumer search behavior. First, because shoppers freely observe every price, in Stahl's original model, the firm with the lowest listed price captures all of them. However, when consumers know that firms price-match, some shoppers can use a price-matching guarantee to obtain the lowest price at a firm listing a higher price. This option diminishes firms' incentive to lower prices because the lowest listed price no longer guarantees a firm will capture all shoppers. Recognizing this price-increasing effect, non-shoppers anticipate higher prices in firms they have not sampled. Hence, a second price-increasing effect arises from non-shoppers' willingness to pay a higher maximal price at the firm where they begin their search rather than pay the search cost to sample another firm's price.

As in Stahl's model, in equilibrium only shoppers sample the prices of both firms. Consequently, consumers who use price-matching guarantees (the shoppers) never expect them to yield a lower price. This stands in particular contrast to the Chen, Narasimhan, and Zhang (2001) model of consumer heterogeneity and competitive price-matching guarantees, where price-matching alters prices via information gains on the part of previously uninformed consumers, who may use the guarantees to secure lower prices. The fact that optimal searchers cannot realize such gains in our model seems to us rather sensible: price-matching can be a time consuming activity which only price conscious consumers with a lower opportunity cost of using their time should be expected to engage in. Moreover, because search is endogenous in our model, price-matching guarantees not only have the potential to diminish firm incentives to lower prices, but also to inhibit search activity in a way that raises prices even further.

In a recent article, Janssen and Parakhonyak (2013) also found that price-matching guarantees raise prices through their effects on consumer search. However, there are a number of important differences between this article and Janssen and Parakhonyak (2013), the foremost of which is that whereas we analyze a setting where price-matching policies are advertised by firms and invoked by consumers prior to purchase, Janssen and Parakhonyak explore the impact of posterior price-matching in which some consumers can get a discounted price if after having purchased from a firm that turns out to offer a price-match guarantee, these consumers acquire additional price information (e.g., from friends) that a different firm offers a lower price.

Unlike in our model (or that of Chen, Narasimhan, and Zhang 2001), Janssen and Parakhonyak cannot analyze how firms such as Walmart or Best Buy, with a reputation for price-matching, influence consumer behavior. Because in the model of Janssen and Parakhonyak, consumers do not learn whether or not a firm offers to price-match until they have sampled its price, price-matching guarantees do not affect search order. Moreover, Janssen and Parakhonyak suppose that price sensitive shoppers, whom we believe are natural candidates to invoke price-matching guarantees, never use these guarantees. Instead, as in Chen, Narasimhan, and Zhang (2001), price-changes in Janssen and Parakhonyak (2013) occur when *uninformed* consumers react to information gains that crop up because of price-matching guarantees—though unlike in Chen, Narasimhan, and Zhang (2001), consumers are "punished" with this additional information.

We find that the more *informed* consumers (shoppers) invoke price-matching guarantees, the more powerful the price-increasing effects of price-matching. In particular, we study what happens when a proportion of shoppers chooses not to invoke guarantees, possibly as a result of hassle costs of doing so. As this proportion decreases in number, average prices increase, and if both firms offer price-matching guarantees, prices can reach the monopoly level in the limit where all shoppers invoke the guarantees. This result lends theoretical support to the empirical finding by Dugar and Sorensen (2006), that the market price varies inversely with the number of positive hassle cost buyers. Thus, as Dugar and Sorensen point out, firms that advertise price-match guarantees, but at the same time make them difficult to invoke, may be using them primarily to price discriminate among consumers rather than to achieve tacit collusion. Conversely, a regulator observing a trend in ease of use of price-matching guarantees might be concerned that they are being used to facilitate collusion.[6]

Finally, an additional finding in this article is that price-matching guarantees may lead to a multitude of asymmetric equilibria where otherwise homogeneous firms have different pricing strategies. In such equilibria, one firm sells to more shoppers, whereas the other plays a pricing strategy that leads it to sell to more non-shoppers. As the disparity in the proportion of each consumer segment that firms serve grows, firm profits increase at the

[6] Recently, certain brick and mortar businesses have begun matching on-line competitors. For instance, Toys "R" Us states that it will match Walmart.com, Amazon.com, and other selected online competitors. Moreover, the increasing use of smartphone technology makes it easier to offer proof of a rival's lower price.

expense of consumers. The higher the proportion of non-shoppers a firm serves, the more profit it will lose from these "captive" consumers by lowering its price to attract shoppers, and the less inclined it is to do so. The upward shift in this firm's price distribution implies that the firm that focuses on catering to shoppers does not need to lower prices as much to expect to capture the same proportion of them and its price distribution shifts upward as well. Hence, the more asymmetry that price-matching entails, the greater the welfare loss to consumers.

The remainder of this article is organized as follows. Section 2.2 sets up the model and equilibrium concept. Section 2.3 characterizes consumer search behavior. Section 2.4 solves for equilibrium when price-matching is imposed exogenously. Section 2.5 characterizes the complete market equilibrium when shoppers do not direct their search (that is, shoppers' search path is random). Section 2.6 numerically examines the consequences of asymmetric equilibria that prevail when shopper search is "directed" (and asymmetric). Section 2.7 concludes. All formal proofs are contained in a Supplemental Appendix.

2.2 Model and Equilibrium

With the exception of the framework for the acquisition of price information, our modeling assumptions are standard in the price-matching literature. Two firms, labeled 1 and 2, sell a homogeneous good. Firms face no capacity constraints and have an identical constant cost of 0 of producing one unit of the good. There is a unit mass of almost identical consumers with inelastic (unit) demand and valuation $v > 0$ for the good.

Consumers are a priori uninformed about prices, but they can learn about them through search. Following Stahl (1989), we assume that a proportion $\mu \in (0,\ 1)$ of the consumers have 0 search cost. These consumers are viewed as having no opportunity cost of time and are henceforth referred to as shoppers. The remaining $1 - \mu$ consumers, called non-shoppers, pay search cost $c \in (0, v)$ for each firm they visit with the exception of the first.[7] Search is sequential with costless revisits. After observing the price at the first firm for free, consumers decide whether or not to search the next one or to exit the market altogether. Consumers who have visited both firms may freely choose the cheapest price observed.[8]

In a model without price-matching, shoppers freely sample both prices and always buy from the firm with the lower listed price, but in a model where firms publicly announce offers to price-match, after sampling both prices, shoppers might choose to invoke a price-matching guarantee to purchase the product from a firm with the higher listed price. A shopper might, for instance, wish above all to procure the product at the lowest price, but given the opportunity, to do so at a particular firm (perhaps because of store brand preference or favorable store characteristics that are unrelated to the product being purchased). Such a shopper could then first sample the competing price and if necessary, invoke a price-match guarantee at the preferred firm rather than going back to the competitor. Alternatively, certain shoppers who have ended their shopping trip at the higher-priced firm may be deal-prone (e.g., Lichtenstein, Netemeyer, and Burton 1990; DelVecchio 2005), valuing not only the ability to secure the lowest price, but also the opportunity to purchase a good at a discount off the listed price. These shoppers strictly prefer the price-match "deal" to a return trip to the lower priced firm.[9]

[7]The assumption that the first visit is free is standard in the literature and we interpret it to mean that the non-shopper initially believes that he must have the good and treats the cost of the first visit as sunk. Janssen, Moraga-Gonzàlez, and Wildenbeest (2005) analyze a sequential search model with costly initial visits.

[8]One way to interpret the search cost is as a cost of finding out the price in a particular firm for the first time rather than as the cost of traveling there. Janssen and Parakhonyak (2014) show that when second visits are costly in a Stahl oligopoly search model, firms nevertheless use pricing strategies that are identical to the perfect recall case.

[9]A third explanation for invoking price-matching guarantees follows if instead of interpreting shoppers as having no opportunity cost of time, we treat them as individuals who read sales ads (Varian 1980) or as users of price-comparison sites (Janssen and Non 2008). In this case, if we think of some shoppers as gravitating toward their local firm unless its competitor offers the lower price, then such shoppers could use a price-matching guarantee to avoid traveling to the non-local firm. We thank a reader for this suggestion.

However, in reality, not all shoppers would choose to price-match given the opportunity. Regardless of how different consumers value their time, some simply do not pay attention to price-matching announcements,[10] whereas others may find the act of keeping and tracking price ads or the additional other activity needed to procure a price-match in some way distasteful—or as the literature often terms it, a hassle. Moreover, even if all shoppers observe price-matching announcements and find the application of a price-matching guarantee to be a hassle free activity, some shoppers may be unable to invoke a guarantee due to the discretion of a store worker who is unwilling to provide the match even though her employer has announced a price-matching policy.[11]

In order to account for the possibility that some shoppers would prefer to invoke a price-matching guarantee at the last store visited while others would prefer to purchase at a store that lists the lower price, we assume that when the two firms offer different prices, $\theta_S \in [0,\ 1]$ of shoppers face some impediment to using price-matching guarantees (e.g., they are unaware that the guarantees are available or find price-matching a hassle) and always purchase from the firm with the lower listed price instead.[12] The remaining $1 - \theta_S$ shoppers will invoke a price-matching guarantee at the last firm they stopped in when one is available and necessary to obtain the lower price there and purchase from the firm with the lower listed price otherwise.[13]

[10]It is not uncommon in the advertising literature to assume that some portion of consumers will remain unaware of certain attributes of a product or firm even when they are familiar with the product's price. For instance, Meurer and Stahl (1994) assume that buyers are aware of the prices of all existing products, but may not know which product offers the best match. Firms may advertise to inform consumers whether the product is a good match, but some consumers remain uninformed in equilibrium.

[11]For instance, Bloomberg has reported that workers at certain chains known for their price-matching policies, nevertheless may not execute the policies consistently. See Dudley, R., Rupp, L. "Price Matching Criticized From Wal-Mart to Toys 'R' Us: Retail." *Bloomberg.* April 30, 2013. Retrieved February 28, 2014. ⟨http://bloom.bg/ZhGPis⟩.

[12]We may, as done in Hviid and Shaffer (1999), treat this impediment as a hassle cost $z \geq 0$ of invoking a price-matching guarantee. However, because no shopper would ever pay z—it does not appear in any equation—it suffices to treat θ_S as a type of tie-breaking rule.

[13]In principle, when one firm charges a lower price and its competitor offers a price-matching guarantee, $1 - \theta_S$ represents the proportion of shoppers who are indifferent between these two firms (the remaining θ_S strictly preferring the firm charging a lower price due to impediments to price-matching). Our

When $\theta_S = 1$ (no shoppers invoke price-matching guarantees), it is easy to show that the equilibrium outcome reduces to that in Stahl (1989), such that we will generally limit our analysis to $\theta_S < 1$. Although we analyze equilibrium when $\theta_S = 0$ (see Proposition 4), there is reason to believe that empirically, θ_S is higher.[14] Therefore, throughout our analysis, we often focus on equilibria where $\theta_S \in (0,\ 1)$.

In order to capture the idea that a firm may develop a reputation for price-matching, we suppose that the game proceeds in three stages, as follows:

1. In stage one, firms simultaneously decide whether to adopt a price-matching guarantee. A firm that has adopted such a guarantee pre-commits itself to sell the good at the minimum listed price to consumers who have observed both prices and are hence able to invoke the guarantee. A firm that has not committed to price-match at this stage does not offer customers a price-matching guarantee in the price search stage.[15]

2. In the second stage, each firm's price-matching decision is known to all agents in the model. Firms then simultaneously choose prices, taking into consideration their beliefs about rival firm strategies as well as consumer search behavior. A pricing strat-

tie-breaking rule presumes that such shoppers purchase from the last firm they visit as long as they can obtain the lower price there. One may rationalize this assumption by supposing that θ_S subsumes not only those shoppers who face an impediment to invoking a price-matching guarantee, but also indifferent shoppers who always purchase from the firm listing the lower price. Alternatively, we could assume that $1 - \theta_S$ shoppers obtain some additional "deal value" from invoking a price-matching guarantee. Doing so would require certain modifications to our profit equations that would nevertheless preserve all of our findings.

[14] To our knowledge, Moorthy and Winter (2006) provide the only published data measuring the frequency with which price-matching guarantees are invoked in a retail setting. They find that redemption rates rarely surpass 10 percent in their retailer survey and average 5.8 percent, with a median of 5 percent. In contrast, recent empirical studies which seek to estimate search costs find that the percentage of consumers who search more than one firm is significantly higher than 10 percent. For instance, Moraga-Gonzàlez and Wildenbeest (2008) estimate that between 70 and 78 percent of consumers in the market for personal computer memory chips search more than once. De los Santos (2012) finds that 24 percent of searches leading to a transaction in the on-line book market visit more than one bookstore. This suggests that not all shoppers would invoke price-match guarantees when they could.

[15] In practice, some firms may offer price-matching guarantees without advertising an intent to price-match, in which case some consumers may unexpectedly happen upon the guarantees during the search process. Such firms are outside the scope of this article. Janssen and Parakhonyak (2013) study firms that make their price-matching decisions simultaneously with their pricing decisions, exclusively focusing on price-matching firms that don't announce their intent to price-match.

egy consists of a price distribution F_i, where $F_i(p)$ represents the probability that firm i offers a price no higher than p. The lower bound and upper bound of the support of the distribution for firm i are denoted as $\underline{p}_i$ and $\bar{p}_i$, respectively.

3. After prices have been realized, consumers choose optimal search strategies given their beliefs about each F_i.

Throughout, parameters v, c, μ, and θ_S, as well as the rationality of all agents in the model are commonly known.

The equilibrium concept used is Sequential Equilibrium. Intuitively, we can think of consumers who observe an off-equilibrium price at the first firm they sample as treating such deviations as mistakes when forming beliefs about the remaining firm's strategy. That is, consumers believe that unsampled firms play their equilibrium strategies at all information sets.

2.3 Consumer Search Behavior

Shoppers freely search both firms before making their purchase decision. For non-shoppers, it is well known that in models such as the one we have set up, the optimal search rule is to sample firms in ascending order of magnitude of the reservation price associated with searching each firm, with equal reservation prices implying indifference (this is known as Weitzman's 1979 Pandora's Rule). Moreover, the optimal stopping rule is for a non-shopper who has freely observed the price at firm j to continue search if and only if the observed price is higher than a reservation price, r_i, which makes him indifferent between searching firm i and stopping. This reservation price is then defined as the solution to[16]

$$\int_{\underline{p}_i}^{r_i} (r_i - p)\mathrm{d}F_i(p) = \int_{\underline{p}_i}^{r_i} F_i(p)\mathrm{d}p = c \tag{2.1}$$

[16] The first equality follows from integration by parts as long as there is no mass at $\underline{p}_i$, which according to Proposition 1 below, is always the case.

Note that reservation price r_i corresponds to non-shoppers who begin their search at firm j and vice versa because non-shoppers who begin at firm j must decide whether or not to search firm i based on the price they observed at firm j and their beliefs about firm i's pricing strategy.

Suppose that $r_1 = r_2$ in equilibrium—that is, non-shoppers are indifferent regarding the search order. If both firms choose the same action in the first stage—both match, or neither does—in the absence of additional a priori information about the firms, it is natural to suppose that the initial search would be random and that half of each type of consumer would visit each firm first. However, if one firm matches while the other does not and all consumers randomize their first search, as will be seen in Section 2.4, firms will set prices such that non-shoppers would prefer to search the non-matching firm first, a contradiction. Therefore, indifference requires some consumers to place greater probability on sampling the non-matching firm in equilibrium. Moreover, even if both firms choose the same first stage action, equilibria where heterogeneous consumers who are indifferent sample the two firms with different probabilities may exist (such as where one firm ends up selling to more shoppers and the other to more non-shoppers). To account for such asymmetries, we suppose that in equilibrium, shoppers and non-shoppers search firm 1 with respective probabilities β_S and β_N, where $\beta_S, \beta_N : \mathcal{B} \to [0, 1]$ and where $\mathcal{B} = [0, v] \times (0, 1) \times [0, 1]$ is the Cartesian product of the intervals that contain parameters c, μ, and θ_S. For concision, going forward, we omit the arguments on β_S and β_N. We note that in an equilibrium where $r_1 \neq r_2$, so that non-shoppers strictly prefer to begin search at a particular firm, β_N equals 0 or 1.

Because shoppers will obtain the lowest price regardless of where they begin their search, to add structure to our model, in a number of propositions below we focus on equilibria where shoppers' search path is random ($\beta_S = 1/2$). Moreover, from Propositions 1 to 6, we focus on equilibria where $r_1 = r_2$ (non-shoppers are indifferent between which firm to sample first, though the search path is not necessarily random), whereas in Proposition 7 we characterize equilibria where $r_1 \neq r_2$. In Section 2.6 we examine equilibria where $\beta_S \neq 1/2$.

2.4 Firm Pricing Strategies

Working backwards, in this section, we derive equilibria in the four possible subgames that follow firms' price-matching decisions: the subgame where neither firm price-matches, the subgame where both firms price-match, and the two subgames where only one firm matches. In Section 2.5, we compare the outcomes that prevail in each of the pricing subgames to determine firms' optimal price-matching policies.

Proposition 1 below states that in general, firms do not play pure pricing strategies in equilibrium.[17] It also places limitations on the way that firms may price in equilibrium, and consequently on the way that consumers search. It tells us that regardless of firms' price-matching decisions, firms will generally not offer a price higher than the largest possible price of their competitors, nor a price high enough to induce non-shoppers to search more than one firm. The proof, which borrows heavily from standard mass shifting arguments found in Narasimhan (1988), Stahl (1989) and Janssen and Non (2008), is highly involved. Therefore, in the Supplemental Appendix, we provide a sketch of the intuition that references these articles in addition to a complete proof.[18]

Proposition 1. *Suppose that all consumers are indifferent regarding which firm to sample first and that in the event that both firms offer price-matching guarantees, $\theta_S \neq 0$. Then in equilibrium, firms play mixed pricing strategies over the same supports. The supports do not contain any breaks, they are bounded from above by $\bar{p} = \min\{v,\ r\}$ where $r = r_1 = r_2$,*

[17] The lone exception occurs when both firms offer price-matching guarantees on the equilibrium path and $\theta_S = 0$. This is explored in Proposition 4.

[18] As is discussed in footnote 24, the qualification that consumers are indifferent regarding which firm to sample is not technically necessary, but is useful for the purpose of exposition.

and at most one firm may have one mass point at $\bar{p}$. If a mass point exists in equilibrium, then non-shoppers who sample firm i first, must stop searching after observing a price of r_j unless $v < r_j$.

As will be seen below, the equilibrium outcome is symmetric if and only if all consumers choose their first price sample at random ($\beta_S = \beta_N = 1/2$). In this case, because according to Proposition 1, at most one firm may have a mass point in equilibrium, when the equilibrium outcome is symmetric, there are no mass points and both firms always run sales—that is, they price below $\bar{p}$ with probability 1. Alternatively, the equilibrium may be asymmetric, in which case one firm has a mass point at $\bar{p}$. Note that because according to Proposition 1 a price equal to r and strictly higher than v is only observed off the equilibrium path, in equilibrium, non-shoppers whose first observation equals r (and who are hence indifferent between stopping and searching the next firm) stop. Therefore, because firms never price above r, as in Stahl (1989), non-shoppers only sample one price in equilibrium. This means that in equilibrium, price-matching can only impact non-shoppers indirectly because a consumer cannot use a price-matching guarantee without observing a second price.[19]

In the following two subsections, we analyze equilibria in subgames where both firms make the same price-matching decision. We conclude that in a subgame where both firms offer price-matching guarantees, consumers expect higher prices and firms expect higher profits than in a subgame without price-matching.

[19]In this respect, we differ from Janssen and Parakhonyak (2013), who assume that an exogenous proportion of non-shoppers discovers the rival firm's price post-purchase without further search. These non-shoppers are then assumed to invoke a price-matching guarantee at the first store if the rival's price is lower.

2.4.1 Neither Firm Price-Matches

Following Stahl (1989), Astorne-Figari and Yankelevich (2014) show that in the subgame without matching, there is a unique Sequential Equilibrium where both firms distribute prices over support $[(1-\mu)\,\bar{p}/(1+\mu),\ \bar{p}]$ with distribution $F\,(p) = (1+\mu)\left(1-\underline{p}/p\right)/\,(2\mu)$, where $\bar{p} = \min\{v,\ r^*\}$ and r^*, the equilibrium reservation price, is defined as

$$r^* = \begin{cases} r\,(\mu,\ c) = c\left(1 - \frac{1-\mu}{2\mu}\ln\frac{1+\mu}{1-\mu}\right)^{-1} & \text{if} \quad r\,(\mu,\ c) \le v \\ \infty & \text{otherwise} \end{cases} \tag{2.2}$$

In equilibrium, regardless of shoppers' search order, shoppers will always purchase from the firm with the lower listed price. On the other hand, non-shoppers randomly choose to sample one firm and because $\bar{p} = \min\{v,\ r^*\}$, they purchase from the first firm sampled without observing the price of the other firm.[20] Thus, as in Stahl (1989) firms randomize over lower prices to attract shoppers, and over higher prices to extract greater profits from captive non-shoppers.

2.4.2 Both Firms Price-Match

When both firms offer price-matching guarantees, the expected profit that each firm obtains from shoppers differs markedly from that when price-matching is not available. Consider the expected profit equation for firm 1 when both firms offer to price-match:

$$\begin{aligned} &\mathrm{E}[\pi_1\,(p_1,\ F_2\,(p_1))] = (1-\mu)\,\beta_N p_1 \\ &+\mu\left\{p_1\,(\beta_S\theta_S + 1 - \beta_S)\,[1 - F_2\,(p_1)] + (1-\theta_S)\,(1-\beta_S)\,\mathrm{E}\,[p_2|p_2 < p_1]\,F_2\,(p_1)\right\} \end{aligned} \tag{2.3}$$

[20] Hence, the equilibrium outcome is the same for all $\beta_S \in [0,\ 1]$ (β_S is absent in Equation (2.2) and $F(p)$ above), whereas $\beta_N = 1/2$ in equilibrium. Suppose this were not the case. From Astorne-Figari and Yankelevich (2014), we know that when $\beta_N \neq 1/2$ (λ in their article), the price distribution of the firm that more non-shoppers choose to sample first, first order stochastically dominates that of its rival. This would imply that the reservation price associated with the rival firm is lower, such that all non-shoppers would prefer to sample the firm with the dominated distribution first, a contradiction.

Suppose that firm 1 sets price p_1. From Proposition 1, we know that the $(1-\mu)\beta_N$ non-shoppers who sample firm 1 first stay there and pay p_1. When neither firm offers to price-match, with probability $1-F_2(p_1)$, $p_1 < p_2$ and firm 1 sells to every shopper in the market. Otherwise it does not sell to shoppers. However, when both firms offer price-matching guarantees, as can be observed by decomposing the second line of Equation (2.3), the order in which firms are sampled matters and firm 1 cannot expect to capture every shopper by listing a lower price.

When $p_1 < p_2$, firm 1 captures $\mu(1-\beta_S)$ shoppers who previously sampled firm 2 and discovered a lower price in firm 1 as well as $\mu\beta_S\theta_S$ shoppers who first sample the price of firm 1 and upon learning the price of firm 2, purchase from firm 1 rather than invoke a price-matching guarantee at firm 2. Each of these shoppers pay p_1 to firm 1. The remaining shoppers invoke a price-matching guarantee at firm 2. However, although firm 1 may lose some shoppers that it would have captured with a lower price sans price-matching guarantees, by offering to price-match, it hedges its losses when it ends up with a higher price than firm 2 because some shoppers will invoke a price-match guarantee at firm 1. In particular, with probability $F_2(p_1)$, we know that $p_1 \geq p_2$. In this case, shoppers who sample p_1 first all purchase from firm 2, but the $\mu(1-\theta_S)(1-\beta_S)$ shoppers who visit firm 2 first will invoke a price-matching guarantee upon sampling firm 1 and paying firm 2's expected price to firm 1 (that is, $\mathrm{E}\left[p_2|p_2 < p_1\right]$).

In Proposition 2 we characterize the Sequential Equilibria for the subgame where both firms offer price-matching guarantees and $\theta_S \in (0, 1]$. The proposition indicates that price-matching can influence prices in three ways: (i) directly by altering firms' price distribution functions, (ii) indirectly via the reservation price, and (iii) through its effect on the sampling order of individual consumers, as represented by β_S and β_N. Proposition 2 characterizes equilibria where $\beta_S \geq 1/2$. Equilibria where $\beta_S < 1/2$ follow analogously.

Proposition 2. *Suppose that firms are exogenously required to offer price-matching guarantees, all consumers are indifferent regarding which firm to sample first, and* $\theta_S \in (0,\ 1]$. *Then in equilibrium, both firms distribute prices over support*

$$[\underline{p}, \bar{p}] = \left[\min\{v,\ r^*\} \left[\frac{(1-\mu)\beta_N}{\mu(\beta_S\theta_S + 1 - \beta_S) + (1-\mu)\beta_N} \right]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}}, \min\{v, r^*\} \right]$$

and the equilibrium reservation price, $r^* = r_1^* = r_2^*$ *equals*

$$r^* = \begin{cases} r(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N) & if \quad r(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N) \le v \\ \infty & otherwise \end{cases},$$

$$r(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N) = c\left\{ 1 - \frac{[(1-\mu)\beta_N]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}}}{\mu(1-\beta_S)(1-\theta_S)} \right.$$
$$\left. \times \left\{ [\mu(\beta_S\theta_S + 1 - \beta_S) + (1-\mu)\beta_N]^{\frac{(1-\beta_S)(1-\theta_S)}{1-\beta_S(1-\theta_S)}} - [(1-\mu)\beta_N]^{\frac{(1-\beta_S)(1-\theta_S)}{1-\beta_S(1-\theta_S)}} \right\} \right\}^{-1}$$

Suppose that firm 1 has a mass point at $\bar{p}$. *Then it distributes prices according to*

$$F_1(p) = \left\{ 1 + \frac{(1-\mu)(1-\beta_N)}{\mu[(1-\beta_S)\theta_S + \beta_S]} \right\} \left[1 - \left(\frac{\underline{p}}{p} \right)^{\frac{(1-\beta_S)\theta_S+\beta_S}{\theta_S}} \right]$$

over $[\underline{p}, \bar{p})$, *while firm 2 distributes prices according to*

$$F_2(p) = \left[1 + \frac{(1-\mu)\beta_N}{\mu(\beta_S\theta_S + 1 - \beta_S)} \right] \left[1 - \left(\frac{\underline{p}}{p} \right)^{\frac{\beta_S\theta_S+1-\beta_S}{\theta_S}} \right]$$

In equilibrium, the expected prices for the two firms equal each other.

Following Stahl (1989), the proof of this proposition proceeds by using firms' indifference between all actions in the supports of their distribution functions to solve for distributions F_1 and F_2 and then by applying the distributions to non-shoppers' optimal stopping rule (Equation (2.1)) to solve for the reservation price. The difference in this article is that when firms offer to price-match, the changes in consumer shopping behavior following Equation (2.3) substantively alter firms' indifference conditions via their expected profit equations. This changes firms' price distributions as well as non-shoppers' reservation price.

Non-shopper indifference between which firm to sample first leads to the following set of equations:

$$\begin{aligned} &\int_{\underline{p}}^{r_2} F_2(p)\mathrm{d}p = \int_{\underline{p}}^{r_2} F_1(p)\mathrm{d}p \\ \Leftrightarrow &\int_{\underline{p}}^{r_2} p\mathrm{d}F_2(p) = \lim_{x\to r_2-}\int_{\underline{p}}^{x} p\mathrm{d}F_1(p) + \bar{p}\left[1 - \lim_{x\to r_2-} F_1(x)\right] \end{aligned} \tag{2.4}$$

The first equation follows from Weitzman's (1979) Pandora's Rule and non-shoppers' indifference (so $r_2 = r_1 = r$); the second, which sets the expected price of the two firms equal to each other, follows from integration by parts together with the fact that $\bar{p} = \min\{v,\ r\}$ in equilibrium. Using the expected price equality, $\mathrm{E}_1[p] = \mathrm{E}_2[p]$, we can now implicitly solve for β_N as a function of the remaining parameters.

If $\beta_S = 1/2$, it is readily seen that the unique value of $\beta_N \in [0,\ 1]$ that solves $\mathrm{E}_1[p] = \mathrm{E}_2[p]$ also equals 1/2. In this case, $\mathrm{E}_1[p] = \mathrm{E}_2[p] = \mathrm{E}[p]$ where

$$\mathrm{E}[p] = \frac{\bar{p}(1-\mu)^{\frac{2\theta_S}{1+\theta_S}}}{\mu(1-\theta_S)}\left[(1+\mu\theta_S)^{\frac{1-\theta_S}{1+\theta_S}} - (1-\mu)^{\frac{1-\theta_S}{1+\theta_S}}\right] \tag{2.5}$$

It also follows that, $F_1(p) = F_2(p)$, and consequently, that neither firm has a mass point at $\bar{p}$. This establishes existence. Although the symmetric equilibrium is perhaps the most natural one when both firms price-match—consumers choose the first price sample at random in the absence of any information differentiating the two firms—the equilibrium is not unique, and in Section 2.6, we numerically explore the existence of equilibria where $\beta_S > 1/2$ (and as a consequence, $\beta_N > 1/2$).

Our next two results—Propositions 3 and 4—make more precise the effect that price-matching has on firms' price distribution functions and non-shoppers' reservation price.

Proposition 3. *Suppose that firms are exogenously required to offer price-matching guarantees and* $\beta_S = \beta_N = 1/2$*.*

1. *Then in equilibrium, a firm price distribution with a lower proportion of shoppers* μ *or lower proportion of shoppers who ignore price-matching guarantees* θ_S *dominates one with a higher proportion in the sense of first-order stochastic dominance.*

2. *If* $r^* < v$*, then* r^* *is decreasing in* μ *and in* θ_S*.*

Because $\bar{p} = \min\{v, r^*\}$, as was the case when firms did not offer price-matching guarantees, in any equilibrium described in Proposition 2, non-shoppers choose a firm to search first and make a purchase there, whereas all shoppers search both firms and obtain the lower price. However, contrary to the subgame without matching, the lower price is not always obtained at the firm with the lower listed price. For instance, as noted above, if firm 1 has the lower listed price, the $\mu(1 - \beta_S)$ shoppers who search firm 2 first all purchase from firm 1, but of the $\mu\beta_S$ shoppers who search firm 1 first, $\mu\beta_S\theta_S$ purchase from firm 1 while $\mu\beta_S(1 - \theta_S)$ obtain a price-match at firm 2. This inability to capture all shoppers at the lower listed price diminishes firms' incentive to set lower prices—and more so the lower θ_S.

The first-order stochastic dominance relationship in Part (1) of Proposition 3 together with Equation (2.1) imply that the lower θ_S, the higher the equilibrium reservation price. When $\beta_S = \beta_N = 1/2$, a simple application of l'Hôpital's rule will show that Equation (2.2) is the limit of the equilibrium reservation price in Proposition 2 as θ_S approaches one (as the proportion of shoppers who invoke a price-matching guarantee goes to zero). According to Proposition 3, Part (2), for any $\theta_S < 1$, the latter reservation price is never lower than the former. This means that a price that could induce a non-shopper to search in an equilibrium without price-matching might no longer do so in an equilibrium with matching—that is, non-shoppers may be willing to accept higher prices in lieu of search in a symmetric equilibrium with price-matching. Thus, when consumers engage in optimal search, the price increasing effect of price-matching guarantees is exacerbated—search is inhibited over a wider range of prices for a subset of the population.

Together, Propositions 2 and 3 tell us that price-matching guarantees do not offer a price benefit to consumers because a portion of consumers never uses them, while the remainder could procure the lower listed price with or without them.[21] This stands in contrast to existing studies such as those of Chen, Narasimhan, and Zhang (2001) and Janssen and Parakhonyak (2013), in which price-matching guarantees potentially allow consumers who would otherwise remain relatively uninformed to pay a lower listed price. Our findings appear to us more cogent because individuals who find search costly are likely to find satisfying the non-pecuniary requirements that many firms impose on price-matching customers costly as well, so we would not expect such individuals to rely on price-matching guarantees (one might for instance suppose that in addition to paying search cost c, all non-shoppers find price-matching a "hassle," though this is unnecessary because non-shoppers observe a single price in equilibrium). Conversely, an individual who does not find the use of a price-matching guarantee a hassle is also likely to be an individual who is willing to shop around for price.

More generally, Proposition 3 tells us that the greater the proportion of shoppers who invoke price-matching guarantees, the higher the price that any consumer is likely to face. This is consistent with the experimental results of Dugar and Sorensen (2006), who find that as the number of positive hassle cost buyers in the market is reduced, average market price approaches the monopoly price in a monotonic fashion. The next proposition says that when $\theta_S = 0$—that is, all shoppers invoke price-matching guarantees when they are available—price-matching leads to a unique monopolistic equilibrium.

[21]The only potential benefit accrues to deal-prone shoppers who invoke price-matching guarantees in terms of any intrinsic value that they derive from the deal.

Proposition 4. *Suppose that firms are exogenously required to offer price-matching guarantees and $\theta_S = 0$. Then there exists a unique Sequential Equilibrium where both firms set price v and $r_1^* = r_2^* = v + c$.*[22]

Intuitively, when $\theta_S = 0$, lower prices do not attract additional customers. In particular, shoppers who encounter a higher price at the second firm they search will use a price-matching guarantee at the second firm rather than go back to the first firm to obtain a lower price. As a result, firms extract all consumer welfare by pricing at v.

At the opposite extreme, when $\theta_S = 1$, no one invokes price-matching guarantees. In this case, if we set $\beta_S = \beta_N = 1/2$ and substitute $\theta_S = 1$ into Equations (2.3) and firms' price distribution functions in Proposition 2, we obtain respectively, firms' profits and price distributions in the subgame without matching. Proposition 3 shows that the distribution when $\theta_S = 1$ is strictly dominated by any symmetric distribution where some shoppers invoke price-matching guarantees. This immediately leads to the following result:

Corollary 1. *In a symmetric equilibrium where both firms offer price-matching guarantees and some shoppers invoke them, expected prices and profits are higher than in an equilibrium without price-matching.*

2.4.3 Only Firm 1 Price-Matches

In this subsection we examine equilibria that arise when firm 1 is exogenously required to offer price-matching guarantees while firm 2 is required not to. The analysis when only firm 2 is exogenously required to offer price-matching guarantees is analogous.

[22]We note that the assumption of costless first visits is not innocuous here. Janssen, Moraga-Gonzàlez, and Wildenbeest (2005) show that when non-shoppers have to pay for every price quote, full participation, which is presumed here, requires v to be no lower than firms' reservation prices.

Proposition 5. *Suppose that firm 1 is exogenously required to offer price-matching guarantees while firm 2 is required not to and that all consumers are indifferent regarding which firm to sample first. Then in equilibrium, both firms distribute prices over support*

$$[\underline{p}, \bar{p}] = \left[\min\{v, r^*\}\left\{\frac{(1-\mu)(1-\beta_N)}{\mu[(1-\beta_S)\theta_S + \beta_S] + (1-\mu)(1-\beta_N)}\right\}, \min\{v, r^*\}\right]$$

and the equilibrium reservation price, $r^* = r_1^* = r_2^*$ equals

$$r^* = \begin{cases} r(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N) & if \quad r(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N) \leq v \\ \infty & otherwise \end{cases},$$

$$r(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N) = c\{1 - \frac{(1-\mu)(1-\beta_N)}{\mu[(1-\beta_S)\theta_S + \beta_S]} \times \ln\{1 + \frac{\mu[(1-\beta_S)\theta_S + \beta_S]}{(1-\mu)(1-\beta_N)}\}\}^{-1}$$

Firm 1 distributes prices according to

$$F_1(p) = \left\{1 + \frac{(1-\mu)(1-\beta_N)}{\mu[(1-\beta_S)\theta_S + \beta_S]}\right\}\left(1 - \frac{\underline{p}}{p}\right)$$

while firm 2 distributes prices according to

$$F_2(p) = \left[1 + \frac{(1-\mu)\beta_N}{\mu}\right]\left[1 - \left(\frac{\underline{p}}{p}\right)^{\frac{1}{\beta_S + \theta_S - \beta_S\theta_S}}\right]$$

over $[\underline{p}, \bar{p})$ with a mass point at $\bar{p}$. In equilibrium, the expected prices for the two firms equal each other.

As in the equilibria described in Proposition 2, non-shoppers randomly choose a firm to search first and make a purchase there. We continue our focus on equilibria where shoppers randomly choose their initial price sample ($\beta_S = 1/2$). When $\beta_S = 1/2$, it must be that $\beta_N < 1/2$—implying that more non-shoppers purchase from the non-matching firm. Suppose this were not the case. Then, firm 1, which offers price-matching guarantees, expects to sell to more shoppers than firm 2, and to at least as many non-shoppers. But this would lead firm 1 to place greater probability on higher prices than firm 2, breaking the equality between reservation prices (or alternatively, expected prices) needed to make consumers indifferent between which firm to sample first.

Even assuming $\beta_S = 1/2$, the equilibrium value of β_N cannot be solved for explicitly as a function of μ and θ_S by using expected price equality $\mathrm{E}_1[p] = \mathrm{E}_2[p]$ (the expressions for $\mathrm{E}_1[p]$ and $\mathrm{E}_2[p]$ are written in full in the proof of Proposition 5 in the Supplemental Appendix). However, because $\mathrm{E}_1[p]$ and $\mathrm{E}_2[p]$ are both smooth functions in μ and θ_S, we are able to verify existence of an equilibrium value of $\beta_N \in [0,\ 1/2)$ numerically on a grid. To do so, we compute β_N for 10,000 $(\mu,\ \theta_S)$ pairs spaced evenly over the parameter space $\mu \times \theta_S \subset (0,\ 1) \times (0,\ 1)$. We employ this approach in Figures 2.1 through 2.4.[23] The shaded region in Figure 2.1 represents the set of μ and θ_S such that $\beta_S = 1/2$ and the equilibrium described in Proposition 5 exists, whereas Figure 2.4(a) displays the value of β_N that leads to equilibrium for each of the $(\mu,\ \theta_S)$ pairs in Figure 2.1. Figure 2.4(a) makes apparent that $\beta_N \in [0,\ 1/2)$, and moreover that β_N increases in θ_S, but decreases in μ. When θ_S grows or μ shrinks, firm 1's ability to maintain higher prices via shoppers who invoke price-matching guarantees diminishes. In equilibrium, β_N must then grow to make consumers indifferent between which firm to sample first.

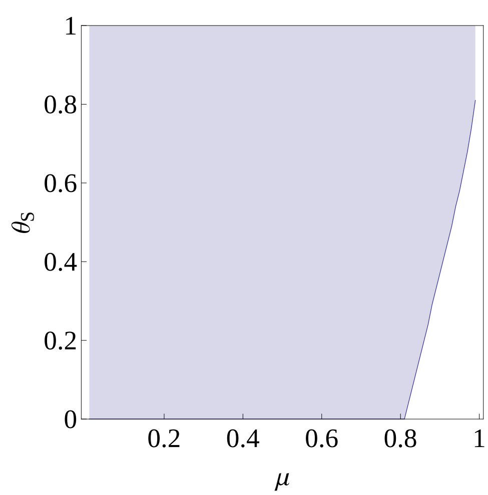
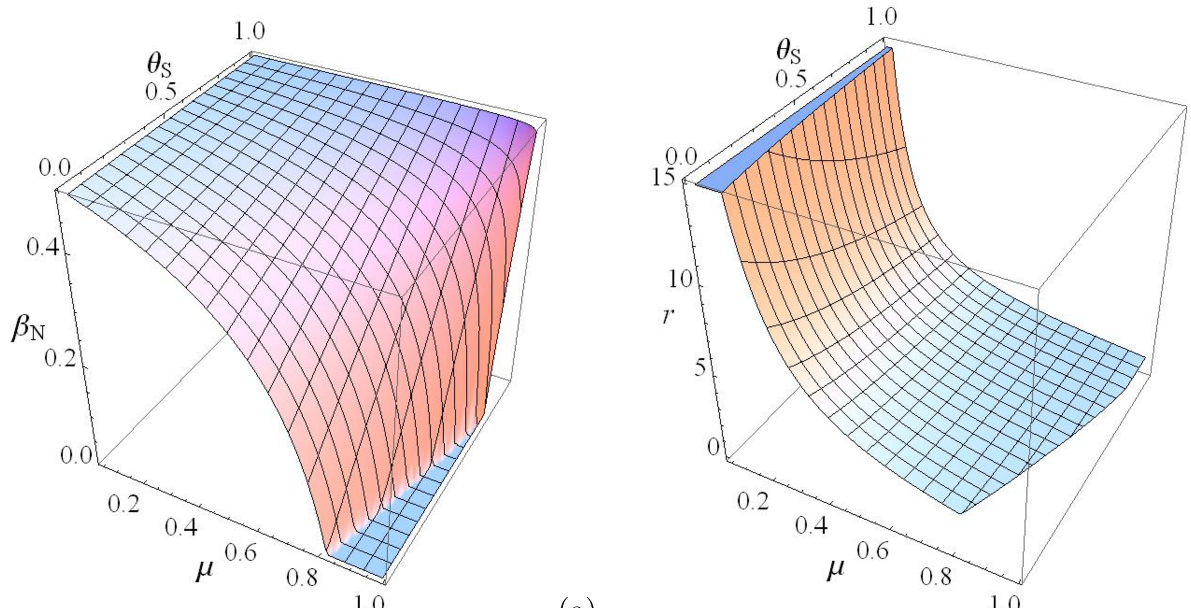


Figure 2.1: Set of μ and θ_S Such That Equilibrium With $\beta_S = 0.5$ and $\beta_N \in (0,\ 0.5)$ Exists When Only Firm 1 Matches.

[23]An annotated program that performs these calculations is available upon request from the authors.



(a) (b)

β_N $r\,(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N)$

Figure 2.2: (a) β_N and (b) $r\,(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N)$

Because firm 2 has a mass point at $\bar{p}$, it may be thought of as running fewer sales than firm 1—for instance, $\bar{p}$ may be interpreted as a manufacturer's suggested retail price and a discount from that price could be called a sale. However, even though firm 2 runs sales less frequently than firm 1, because expected prices are equal in equilibrium, when firm 2 does run sales, it will tend to offer greater discounts than firm 1. In the next section, we will consider how firm profits (and hence expected prices) in the equilibria of this subsection compare to profits when neither or both firms price-match.

Proposition 6 represent the comparative static counterpart to Proposition 3 when only firm 1 offers a price-matching guarantee.

Proposition 6. *Suppose that firm 1 is exogenously required to offer price-matching guarantees while firm 2 is required not to, $\beta_S = 1/2$, and $\beta_N \in (0,\ 1/2)$.*

1. *Then in equilibrium, for firm 1 a price distribution with a lower proportion of shoppers, μ, dominates one with a higher proportion in the sense of first-order stochas-*

tic dominance if and only if $\frac{\partial \beta_N}{\partial \mu} > -\frac{1-\beta_N}{\mu(1-\mu)}$. *A distribution with a lower proportion of shoppers who ignore price-matching guarantees,* θ_S, *dominates one with a higher proportion if and only if* $\frac{\partial \beta_N}{\partial \theta_S} > -\frac{1-\beta_N}{1+\theta_S}$. *Moreover, expected prices for both firms are decreasing in* μ *if and only if* $\frac{\partial \beta_N}{\partial \mu} > -\frac{1-\beta_N}{\mu(1-\mu)}$ *and decreasing in* θ_S *if and only if* $\frac{\partial \beta_N}{\partial \theta_S} > -\frac{1-\beta_N}{1+\theta_S}$.

2. *If* $r^* < v$, *then* r^* *is decreasing in* μ *if and only if* $\frac{\partial \beta_N}{\partial \mu} > -\frac{1-\beta_N}{\mu(1-\mu)}$ *and decreasing in* θ_S *if and only if* $\frac{\partial \beta_N}{\partial \theta_S} > -\frac{1-\beta_N}{1+\theta_S}$.

By applying the implicit function theorem to $\mathrm{E}_1[p] - \mathrm{E}_2[p] = 0$, we can determine $\frac{\partial \beta_N}{\partial \mu}$ and $\frac{\partial \beta_N}{\partial \theta_S}$. Unfortunately, this yields a pair of highly unwieldy equations, and as an alternative, we numerically compute the reservation price over the set of μ and θ_S for which an equilibrium of the type described in Proposition 5 exists (the shaded region in Figure 2.1) to determine whether or not the inequalities in Proposition 6 hold. These computations are displayed in Figure 2.2(b), which shows the equilibrium reservation price decreasing over all μ and θ_S, as in Proposition 3, Part (2). Because $r(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N)$ is a smooth function in μ and θ_S, Figure 2.2(b) tells us that the inequalities in Proposition 6 always hold in the shaded region in Figure 2.1, so that the equilibrium reservation price is increasing with the proportion of shoppers who invoke price-matching guarantees and moreover, according to Proposition 6, Part (1), so is the expected price.

Although our focus thus far has been on equilibria where consumers who have no price information are indifferent between which firm to sample first, as can be observed in Figure 2.1, assuming $\beta_S = 1/2$, there is a subset of μ and θ_S where such an equilibrium does not exist. In Proposition 7, we characterize the equilibrium that prevails throughout the remainder of the $\mu \times \theta_S$ parameter space when $\beta_S = 1/2$. We then show that the equilibria characterized in Propositions 5 and 7 partition this space.

Proposition 7. *Suppose that firm 1 is exogenously required to offer price-matching guarantees while firm 2 is required not to and* $\beta_S = 1/2$.

1. *An equilibrium in which non-shoppers all prefer to sample firm 1 first does not exist.*

2. *For sufficiently low* θ_S *and sufficiently high* μ*, there exists an equilibrium where all non-shoppers prefer to sample firm 2 first, in which both firms distribute prices over support*

$$\left[\underline{p}, \bar{p}\right] = \left[\min\left\{v, r_2^*\right\}\left[\frac{2\left(1-\mu\right)}{2-\mu\left(1-\theta_S\right)}\right], \min\left\{v, r_2^*\right\}\right]$$

and the equilibrium reservation, r_2^**, equals*

$$r_2^* = \begin{cases} r_2\left(\mu,\ \theta_S,\ c\right) & if \quad r_2\left(\mu,\ \theta_S,\ c\right) \leq v \\ \infty & otherwise \end{cases},$$

$$r_2\left(\mu,\ \theta_S,\ c\right) = c\frac{1-\theta_S}{1+\theta_S}\left\{\left[\frac{2\left(1-\mu\right)}{2-\mu\left(1-\theta_S\right)}\right]^{\frac{2}{1+\theta_S}} + \frac{2\mu}{2-\mu\left(1-\theta_S\right)} - 1\right\}^{-1}$$

Firm 1 distributes prices according to

$$F_1\left(p\right) = \frac{2-\mu\left(1-\theta_S\right)}{\mu\left(1+\theta_S\right)}\left(1-\frac{\underline{p}}{p}\right)$$

while firm 2 distributes prices according to

$$F_2\left(p\right) = 1 - \left(\frac{\underline{p}}{p}\right)^{\frac{2}{1+\theta_S}}$$

over $\left[\underline{p}, \bar{p}\right)$ *with a mass point at* $\bar{p}$*. In equilibrium, firm 1's reservation price and expected price is no lower than those of firm 2.*

Regarding Part (1), we already knew that when $\beta_S = 1/2$, it must be that $\beta_N < 1/2$, so we should not expect an equilibrium where non-shoppers all prefer to sample firm 1 first to exist (which would imply $\beta_N = 1$). With regard to Part (2), it is worth noting that the price distributions in Proposition 7 are the limits of the price distributions in Proposition 5 as $\beta_N \to 0$, holding $\beta_S = 1/2$. Additionally, assuming that v is not binding, in equilibrium, $r_1(\mu,\ \theta_S,\ c)$ equals

$$r_1(\mu,\ \theta_S,\ c) = c + r_2(\mu,\ \theta_S,\ c)\frac{2(1-\mu)}{\mu(1+\theta_S)}\ln\left[\frac{2-\mu(1-\theta_S)}{2(1-\mu)}\right] \tag{2.6}$$

As a consequence, $r_1(\mu,\ \theta_S,\ c) > r_2(\mu,\ \theta_S,\ c)$ occurs if and only if

$$c > r_2(\mu,\ \theta_S,\ c)\left\{1 - \frac{2(1-\mu)}{\mu(1+\theta_S)}\ln\left[\frac{2-\mu(1-\theta_S)}{2(1-\mu)}\right]\right\} \tag{2.7}$$

where it can be observed that c divided by the bracketed expression in the right hand side of Inequality (2.7) is the limit of the equilibrium reservation price in Proposition 5 as $\beta_N \to 0$, holding $\beta_S = 1/2$. This means that when $\beta_S = 1/2$, the equilibria in Propositions 5 and 7 partition the space $\mu \times \theta_S \subset (0,\ 1) \times (0,\ 1)$. That is, when shopper sampling order is random in an equilibrium where only firm 1 offers to price-match, non-shoppers are either indifferent regarding which firm to sample first or they strictly prefer to sample firm 2 first. In the latter case, Equation (2.4) indicates that non-shoppers expect to pay a lower price at the non-matching firm, so that for a subset of parameters, the non-matching firm behaves like a "low price" competitor to its price-matching rival.[24]

2.5 Market Equilibrium

In the first stage of the game, firms must decide whether or not to make a price-matching announcement. For each firm, this decision depends on a comparison of the profits that it expects to obtain in each of the pricing subgames discussed in the previous section. In this section, we continue to focus on equilibria where $\beta_S = 1/2$; that is, shoppers randomly choose which firm's price to sample first before moving on to the second firm. This

[24]The claims proving Proposition 1 in the Supplemental Appendix suggest that when $\beta_N = 0$, additional equilibria may exist on the set $\mu \times \theta_S$ when the assumption regarding consumer indifference is omitted from the proposition. These equilibria can be ruled out numerically. For instance, if firm 1 is the matching firm and $\beta_N = 0$, Proposition 1 sans the indifference assumption posits an equilibrium in which $r_2 < r_1$ in which both firms have a mass point at the upper bound of their supports. However, numerically, we have found that for any value of $\Pr(p_2 = \bar{p}_2) \in (0,\ 1)$, $r_1 < r_2$ in such an equilibrium, a contradiction (additional detail is available upon request from the authors). Thus, when $\beta_S = 1/2$, we need only to concentrate on the equilibria described in Propositions 5 and 7.

follows naturally if shoppers observe either that both firms offer price-matching guarantees, or that neither one does, in which case there is no information about the prices of the individual firms to be gleaned from the first stage. However, even if one firm announces its guarantee to match the other firm's price, while the other does not, at the end of their search, shoppers will nevertheless procure the lowest price available, so that randomization is a reasonable search strategy. Going forward, we assume that v is large enough not to be binding on firm supports (and thus, as a result of Proposition 4, we must assume that $\theta_S \neq 0$).

Fortunately, when $\beta_S = 1/2$, because the stage-two pricing equilibria when both firms make the same stage-one matching decision are symmetric, as displayed in the payoff matrix labeled Table 2.1, our analysis boils down to a pair of profit comparisons over the set $\mu \times \theta_S$: (i) a comparison of the matching firm's expected profit when only one firm offers a price-matching guarantee $\mathrm{E}\,\pi_M^{MN} = \mathrm{E}\,\pi_M^{NM}$ against the symmetric expected profit when neither firm price-matches $\mathrm{E}\,\pi^{NN}$, and (ii) a comparison of the non-matching firm's expected profit when only one firm offers a price-matching guarantee $\mathrm{E}\,\pi_N^{MN} = \mathrm{E}\,\pi_N^{NM}$ against the symmetric expected profit when both firms price-match $\mathrm{E}\,\pi^{MM}$. Because all expected profit functions are smooth, we can follow the approach of Chen, Narasimhan, and Zhang (2001) by numerically comparing the difference between $\mathrm{E}\,\pi^{NN}$ and $\mathrm{E}\,\pi_M^{MN}$ as well as the difference between $\mathrm{E}\,\pi_N^{MN}$ and $\mathrm{E}\,\pi^{MM}$ over the range of relevant parameters—in this case, parameter space $\mu \times \theta_S$—to derive all possible equilibria of the complete game.

Table 2.1: Stage-One Profit Comparison

	Match	Don't Match
Match	$\mathrm{E}\,\pi^{MM}$, $\mathrm{E}\,\pi^{MM}$	$\mathrm{E}\,\pi_M^{MN}$, $\mathrm{E}\,\pi_N^{MN}$
Don't Match	$\mathrm{E}\,\pi_N^{NM}$, $\mathrm{E}\,\pi_M^{NM}$	$\mathrm{E}\,\pi^{NN}$, $\mathrm{E}\,\pi^{NN}$

As should be expected from Figure 2.1, the expressions for the expected profits of firms in equilibria where only one firm offers a price-matching guarantee vary depending on whether non-shoppers are indifferent regarding which firm to sample first (as in Proposition 5) or all prefer to sample the non-matching firm first (as in Proposition 7). Suppose that, only firm 1 announces a price-matching guarantee. The expected profits of firm 1 in these two cases are respectively,

$$\mathrm{E}\,\pi_M^{MN} = \frac{2\,(1-\mu)\,(1-\beta_N)\,[\mu + (1-\mu)\,\beta_N]\,\bar{p}^{MN}}{2\,(1-\mu)\,(1-\beta_N) + \mu\,(1+\theta_S)}, \tag{2.8}$$

$$\mathrm{E}\,\pi_M^{M\widetilde{N}} = \frac{2\mu\,(1-\mu)\,\bar{p}^{M\widetilde{N}}}{2-\mu\,(1-\theta_S)} \tag{2.9}$$

where the superscript MN ($M\widetilde{N}$) refers to equilibria where non-shoppers are indifferent regarding which firm to sample first (prefer to sample the non-matching firm first). In contrast, the symmetric expected profit when neither firm price-matches is $\mathrm{E}\,\pi^{NN} = (1-\mu)\,\bar{p}^{NN}/2$.

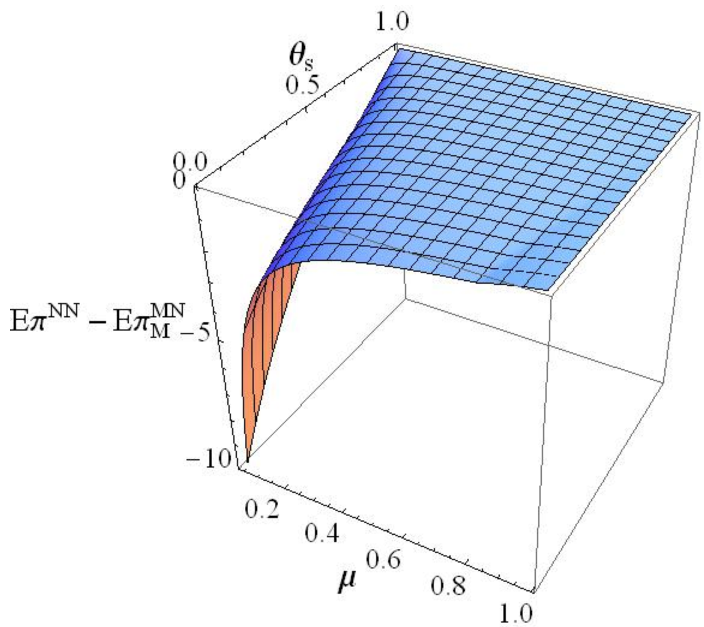


Figure 2.3: $\mathrm{E}\,\pi^{NN} - \mathrm{E}\,\pi_M^{MN}$ over $\mu \times \theta_S \subset (0,\ 1) \times (0,\ 1)$ (assuming $c = 1$).

Figure 2.3 maps the difference between $\mathrm{E}\,\pi^{NN}$ and $\mathrm{E}\,\pi_M^{MN}$ (or $\mathrm{E}\,\pi_M^{M\tilde{N}}$ as appropriate) over parameter space $\mu \times \theta_S$. From the figure, it can be observed that $\mathrm{E}\,\pi_M^{MN}$ (or $\mathrm{E}\,\pi_M^{M\tilde{N}}$ as appropriate) is always greater than $\mathrm{E}\,\pi^{NN}$ in the interior of $\mu \times \theta_S$: that is, the price-matching firm never wants to deviate to not matching when its rival does not match (or conversely, the situation where neither firm matches is not an equilibrium of the complete game). Moreover, as the figure makes evident, the difference in profit grows in the proportion of shoppers who invoke price-matching guarantees. The one exception to $\mathrm{E}\,\pi_M^{MN},\ \mathrm{E}\,\pi_M^{M\tilde{N}} > \mathrm{E}\,\pi^{NN}$ occurs when $\theta_S = 1$. In this case, even though a firm may offer a price-matching guarantee, consumers do not invoke it, and as a result, consumer search and firm pricing are precisely the same as if price-matching were not an option.

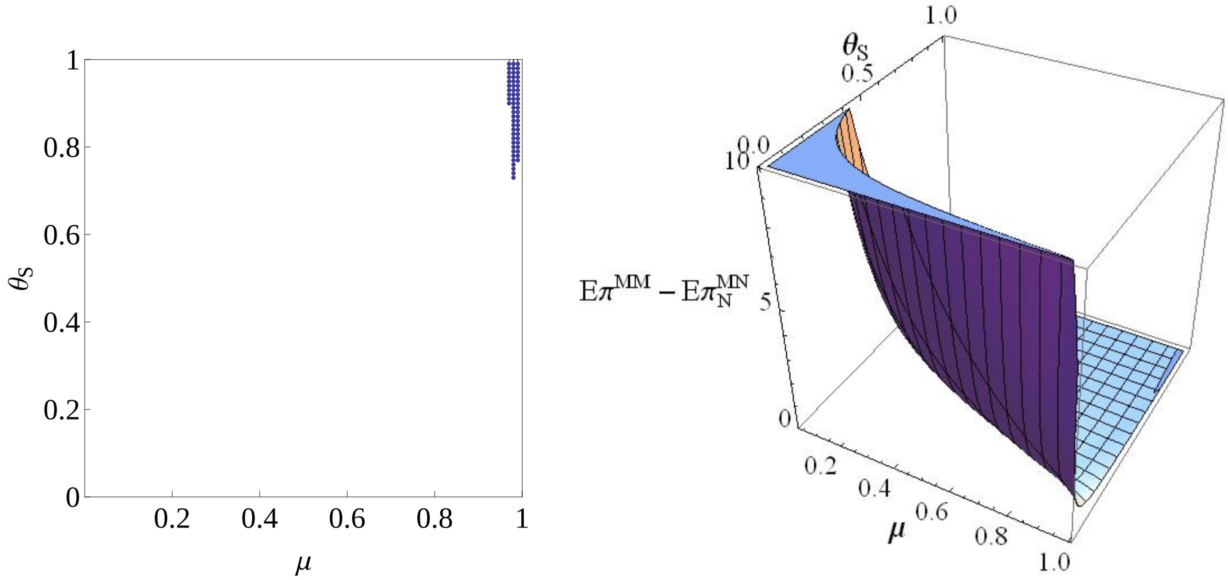


Figure 2.4: $\mu \times \theta_S \subset (0,\ 1) \times (0,\ 1)$: (a) Shaded Region Implies $\mathrm{E}\,\pi_N^{MN} - \mathrm{E}\,\pi^{MM} \geq 0$; (b) $\mathrm{E}\,\pi^{MM} - \mathrm{E}\,\pi_N^{MN}$ or $\mathrm{E}\,\pi^{MM} - \mathrm{E}\,\pi_N^{M\widetilde{N}}$ (assuming $c = 1$).

Like Figure 2.3, Figure 2.4(a), which compares the expected profit of the non-matching firm when only one firm offers a price-matching guarantee with the symmetric expected profit when both firms price-match, shows that price-matching is usually—though not always—a best response to a competitor's price-matching announcement. The non-matching firm analogues of Equations (2.8) and (2.9) are respectively, $\mathrm{E}\,\pi_N^{MN} = 2\,(1-\mu)\,(1-\beta_N)\,\bar{p}^{MN}$ and $\mathrm{E}\,\pi_N^{M\widetilde{N}} = (1-\mu)\,\bar{p}^{M\widetilde{N}}$. Comparing these to the symmetric expected profit when both firms price-match,

$$\mathrm{E}\,\pi^{MM} = \left(\frac{1-\mu}{1+\mu\theta_S}\right)^{\frac{2\theta_S}{1+\theta_S}} \frac{(1+\mu\theta_S)\bar{p}^{MM}}{2}, \tag{2.10}$$

we observe that with the exception of a small region in which both μ and θ_S are close to 1, $\mathrm{E}\,\pi^{MM}$ is greater than $\mathrm{E}\,\pi_N^{MN}$ (or $\mathrm{E}\,\pi_N^{M\widetilde{N}}$ as appropriate) over $\mu \times \theta_S$.[25] In particular, with regard to equilibria where non-shoppers prefer to sample the non-matching—and in

[25]Smoothness of the profit functions thereby implies a subset of parameter values over which one firm strictly prefers to price-match while the other firm is indifferent between offering and not offering price-matching guarantees. The subset is a curve that partitions $\mu \times \theta_S$.

that case, lower expected price—firm, Figure 2.4(a) tells us that because for most μ and θ_S, $\mathrm{E}\,\pi^{MM} > \mathrm{E}\,\pi_N^{M\tilde{N}}$, announcing a price-matching guarantee is generally a profitable alternative to being a non-matching, "low price" competitor to a price-matching firm. Figure 2.4(b), which maps the actual profit difference compared in Figure 2.4(a), shows that the difference is decreasing in θ_S. When θ_S grows, fewer shoppers invoke price-matching guarantees, narrowing the distinction between offering and not offering these guarantees.

The following result summarizes the discussion above.[26]

Market Equilibrium. *Suppose that shoppers sample prices at random and $\theta_S \in (0,\ 1)$. In the equilibrium of the complete game, either both firms will offer price-matching guarantees or only one firm will offer a price-matching guarantee on the equilibrium path. In any equilibrium outcome where firms are given the option of offering price-matching guarantees, firm profits and expected prices are higher than when price-matching is not an option.*

2.6 Asymmetric Equilibria (Numerical Analysis)

Although equilibria where $\beta_S = 1/2$ are intuitively appealing and mathematically tractable, because β_S and β_N are endogenously determined, equilibria where $\beta_S \neq 1/2$ also exist, even if both firms announce an intent to price-match on the equilibrium path. Although a complete investigation of such equilibria would be quite lengthy, they warrant some discussion, particularly when only a single firm announces an intent to price-match. To simplify the exposition, we restrict the discussion to equilibria where non-shoppers are indifferent regarding which firm to sample first.

[26]The final sentence in the result below relies on the comparison of $\mathrm{E}\,\pi^{NN}$ and $\mathrm{E}\,\pi_M^{MN}$ along with Corollary 1.

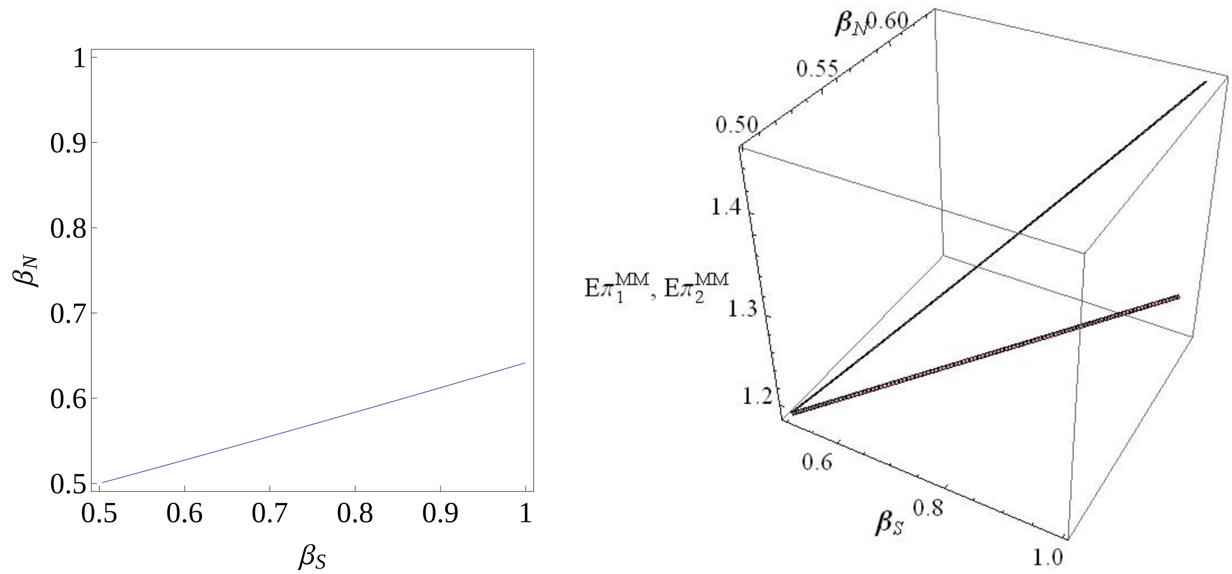


Figure 2.5: Both Firms Match and $\mu = \theta_S = 0.5$: (a) Equilibrium $\beta_S > 0.5$ and $\beta_N > 0.5$; (b) Equilibrium Profits (assuming $c = 1$).

Mathematically, multiple β_S and β_N pairs may prevail for any combination of the exogenous parameters because in each equilibrium involving subgames played after at least one firm reveals an intent to price-match, the solution for β_S and β_N is obtained using the single equation that sets the expected prices in the two firms equal to each other, making all consumers indifferent between which firm to search first. For instance, in the subgame where both firms offer price-matching guarantees, Figure 2.5(a) shows the set of all equilibrium $\beta_S > 1/2$ and $\beta_N > 1/2$ in the case that $\mu = \theta_S = 1/2$ while Figure 2.5(b) represents the expected profits of firms 1 and 2 for combinations of β_S and β_N in Figure 2.5(a). From 1.5(b), we can observe that as the equilibrium becomes more asymmetric from the standpoint that the absolute value difference between β_N and $1 - \beta_N$ increases, so do the profits of the two firms. Table 2.2 presents additional evidence of profits rising with the amount of asymmetry in the subgame where both firms offer price-matching guarantees for various combinations of μ and θ_S. In particular, for each combination of μ and θ_S, both firms' expected profits are higher when $\beta_N = 0.55$ (and β_S is endogenously determined accordingly) than when $\beta_N = 0.5$, assuming an equilibrium where non-shoppers are indifferent regarding which firm to sample first exists when $\beta_N = 0.55$. Likewise, expected profits are higher when $\beta_N = 0.6$ than when $\beta_N = 0.55$.[27]

Table 2.2: Firm Profits When Both Firms Match[a, b]

		$\beta_N = 0.50$	$\beta_N = 0.55$		$\beta_N = 0.60$		$\beta_S = 0.999$	
μ	θ_S	$\pi_1 = \pi_2$	π_1	π_2	π_1	π_2	π_1	π_2
.20	.20	10.75	10.79	11.75	N/A	N/A	10.80	12.03
.50	.20	3.12	3.15	3.37	3.18	3.62	3.24	4.32
.80	.20	1.06	1.09	1.13	1.11	1.20	1.23[c]	1.71[c]
.20	.50	4.27	N/A	N/A	N/A	N/A	4.29	4.59
.50	.50	1.19	1.20	1.29	1.22	1.38	1.23	1.46
.80	.50	0.35	0.36	0.38	0.37	0.40	0.42	0.57
.20	.80	2.65	N/A	N/A	N/A	N/A	2.66	2.73
.50	.80	0.71	0.72	0.77	N/A	N/A	0.72	0.77
.80	.80	0.19	0.19	0.20	0.20	0.22	0.21	0.23

[a] μ consumers have no cost of search (shoppers), while $1 - \mu$ have search cost $c = 1$ (non-shoppers). θ_S shoppers always ignore price-matching guarantees. β_S and β_N respectively represent the fraction of shoppers and non-shoppers who begin search at firm 1. Non-shoppers' valuation for the good is assumed to be strictly higher than their equilibrium reservation price.

[b] Equilibrium β_S varies with μ and θ_S for a given value of β_N and vice versa. N/A implies that an equilibrium where non-shoppers are indifferent regarding which firm to sample first does not exist for the given value of β_N. Rightmost column gives results for $\beta_S = 0.999$ to approximate profits with highest level of asymmetry.

[c] No equilibrium with $\beta_S = 0.999$. Results given are for $\beta_N = 0.999$ and $\beta_S = 0.80$.

[27] Moreover, as might be expected from Proposition 3, Table 2.2 suggests that expected profits decrease in μ and θ_S for each realization of β_N.

When β_N, $\beta_S > 1/2$, as can be inferred from Equation (2.3), firm 1 will capture a higher proportion of non-shoppers while firm 2 expects to capture more shoppers. Firm 1 sets higher prices and has fewer sales than in the symmetric case because when it serves a higher proportion of non-shoppers, it loses more profit from these captive consumers whenever it lowers its price. Even though firm 2 has fewer non-shoppers than in the symmetric case, it will tend to have higher prices as well because it no longer needs to lower prices as much to have the same probability of capturing the bulk of the shoppers as it did in the symmetric case. Thus, both firms have higher prices and expected profits than in the symmetric case.

Intuitively, an asymmetric equilibrium may result in the presence of price-matching because more shoppers may prefer to purchase at a particular firm (and so first sample the price of its rival), but are lexicographic, valuing a purchase at a lower price over a purchase at a preferred firm. Price-matching will allow some shoppers to purchase at a preferred firm at the lower price even when that firm does not list the lower price. For instance, if firm 2 ends up setting the higher price ex-post, $\mu\beta_S(1-\theta_S)$ shoppers nevertheless purchase there. The higher β_S, the greater the number of shoppers that end up making a purchase at firm 2. Ex-ante, given μ and θ_S, firms set prices to make non-shoppers indifferent between which firm to sample. A higher β_S entails a higher β_N in order that Equation (2.4) may hold.

Table 2.3, which corresponds to Table 2.2 for a subgame with a single matching firm, also suggests that expected profits are increasing in the absolute value difference between β_N and $1-\beta_N$ for both the matching and non-matching firm. In Section 2.5 we learned that when $\beta_S = 1/2$, price-matching (whether by one firm or both) always led to higher firm profits and prices relative to a regime where price-matching is forbidden. Tables 2.2 and 2.3 suggest that asymmetry in the search behavior of consumers will exacerbate these effects. Moreover, because we know that shoppers always obtain the lowest price in equilibrium whereas non-shoppers never invoke price-matching guarantees, firm profit increases must come at the expense of consumer welfare.

Table 2.3: Firm Profits When Only Firm 2 (Firm 1) Matches[a, b]

		$\beta_N = 0.55(0.45)$		$\beta_N = 0.60(0.40)$		$\beta_S = 0.999(0.001)$	
μ	θ_S	$\pi_1(\pi_2)$	$\pi_2(\pi_1)$	$\pi_1(\pi_2)$	$\pi_2(\pi_1)$	$\pi_1(\pi_2)$	$\pi_2(\pi_1)$
.20	.20	5.78	6.29	N/A	N/A	10.75	11.99
.50	.20	0.71	0.76	0.94	1.07	3.23	4.30
.80	.20	0.16	0.16	0.18	0.19	0.45[c]	0.63[c]
.20	.50	N/A	N/A	N/A	N/A	4.29	4.58
.50	.50	0.71	0.76	0.94	1.07	1.23	1.46
.80	.50	0.16	0.16	0.18	0.19	0.42	0.57
.20	.80	N/A	N/A	N/A	N/A	2.66	2.73
.50	.80	0.71	0.76	N/A	N/A	0.72	0.77
.80	.80	0.16	0.16	0.18	0.19	0.21	0.23

[a] μ consumers have no cost of search (shoppers), while $1-\mu$ have search cost $c = 1$ (non-shoppers). θ_S shoppers always ignore price-matching guarantees. β_S and β_N respectively represent the fraction of shoppers and non-shoppers who begin search at firm 1. Non-shoppers' valuation for the good is assumed to be strictly higher than their equilibrium reservation price.

[b] Equilibrium β_S varies with μ and θ_S for a given value of β_N and vice versa. N/A implies that an equilibrium where non-shoppers are indifferent regarding which firm to sample first does not exist for the given value of β_N. Rightmost column gives results for $\beta_S = 0.999$ ($\beta_S = 0.001$) to approximate profits with

highest level of asymmetry.

[c]No equilibrium with $\beta_S = 0.999$ ($\beta_S = 0.001$). Results given are for $\beta_N = 0.999$ ($\beta_N = 0.001$) and $\beta_S = 0.657$ ($\beta_S = 0.343$).

From Section 2.4, we know that when $\beta_S = 1/2$, beliefs regarding β_N must vary on and off the equilibrium path because whereas the unique value of β_N is $1/2$ when either both, or neither firm announces an intent to price-match, as seen in Figure 2.2(a), it turns out that $\beta_N \neq 1/2$ when one firm offers a price-matching guarantee while the other does not. In a subgame where both firms price-match, consumers cannot make any inference about firm pricing behavior ahead of the search process, so that randomization regarding the first sample seems to be the most reasonable approach. This is not necessarily the case after consumers have observed firms making opposite matching decisions. Thus, it seems sensible to think that beliefs regarding β_N might be more symmetric in an equilibrium where both firms match relative to one where only one does so. An insightful comparison then arises if we juxtapose the expected profits in the leftmost profit column in Table 2.2 with the rightmost column in Table 2.3, the former representative of (symmetric) randomization when both firms match and the latter approximating the "highest level of asymmetry" that might arise when only one firm matches. Among the (μ, θ_S) pairs in the two tables, we observe that only in the case of $\mu = 0.80$ and $\theta_S = 0.20$, is expected profit higher when both firms offer price-matching guarantees. As such, within the context of this model, there are two ways to interpret the real world observation that only a fraction of the firms producing the same good tend to offer price-matching guarantees: (i) most consumers are shoppers who do not invoke price-matching guarantees[28] or (ii) firms that choose not to price-match believe that a more symmetric (and potentially less profitable) equilibrium might prevail were they to offer price-matching guarantees.

[28]This interpretation proceeds from Figure 2.4(a), which shows that when $\beta_S = 1/2$ in all subgames, both firms offer price-matching guarantees in equilibrium unless μ and θ_S are both very high.

2.7 Conclusion

This article explores the effects that price-matching guarantees have on firms and consumers when consumers optimally search for price after learning firms' price-matching policies. Price-matching guarantees alter the shopping behavior of both types of consumers in our model in a way that encourages firms to raise prices. When consumers who have no cost of price search invoke price-matching guarantees at firms that list higher prices, firms are discouraged from lowering prices in order to attract such consumers. Understanding this price-increasing effect, consumers who face an opportunity cost of searching for price accept higher prices at already sampled firms because they anticipate that further search is less likely to yield a lower price. In addition, because consumers with no search costs may be able to obtain the lower price at either firm, there is a multiplicity of asymmetric equilibria where more asymmetry leads to higher expected prices and firm profits.

While the underlying mechanism driving the effects of price-matching in our model is new, this article is not orthogonal to the previous literature. The effect that price-matching has on consumer search leads to both welfare diminishing tacit collusion and price discrimination. Tacit collusion occurs because firms understand that a rival's "threat" to match a lower price entails a smaller benefit from any incremental price cut. This threat increases the greater the percentage of consumers who have observed both prices that invokes price-matching guarantees. Price discrimination occurs because consumers who have no cost of price search may use a price-match to secure a lower price from the firm listing the higher price while the firm's remaining customers pay the higher listed price. However, contrary to the result in signaling models of price-match, where ex-ante asymmetries persist ex-post, we find that price-matching alone is enough to generate an asymmetric equilibrium. In the model presented, asymmetries increase firm profits at the expense of consumer welfare, but it would be interesting to see how differences in firm production costs or branding influence search behavior when price-matching guarantees are in place. This is not immediately obvious from the above analysis. Ex-ante asymmetries have the potential to reduce asymmetries ex-post, but it is unclear if this is a good thing because it may entail more purchases from the higher cost firm.[29]

This study has implications for future empirical work. Recent empirical literature has focused primarily on comparisons of price observations between firms with and without price-matching guarantees and arrived at opposite results (Moorthy and Winter 2006; Arbatskaya, Hviid, Shaffer 2006). The results in this article suggest that such cross-sectional findings point purely to underlying cost or other differences between firms without telling us the overall welfare effect of adopting a matching policy. Under most combinations of parameters, the expected prices among otherwise homogeneous firms that differ in the adoption of a price-matching guarantee remain the same in our model. This suggests that what is really necessary is a welfare comparison of firms over time—before any have adopted price-matching policies, and after some or all have (see Hess and Gerstner 1991); although even this may not be foolproof, as the adoption of a matching policy may follow a change in production costs.

A survey test of our model could ask individuals who use price-matching guarantees to secure the lowest price if they would obtain that price regardless by purchasing somewhere else. An affirmative answer would validate the model by telling us that price-matching guarantees can keep consumers out of firms with lower listed prices.

[29] In an experimental study, Biswas et al. (2002) find that when consumers have preconceived notions of a store's price image, low-price guarantees may lead to heightened (lowered) intentions to sample the prices of other stores when the price image is high (low). In another, Mago and Pate (2009) show that increases in market prices brought about by price-matching guarantees are curtailed by firms' cost asymmetries.

Chapter 3

Preference Complexity and Multidimensional Competition

3.1 Introduction

Economists have studied the effects of product complexity on market structure for some time. There is a wide variety of papers that explore the consequences of complexity on such topics as international trade, innovation, and firm organization. However, all of these models treat complexity as an artifact of the good's creation; more complex goods are those which are more difficult or complicated to produce. In other words, economists have considered complexity solely from the point of view of the producer. In this paper I investigate another form of complexity which stems from the variation in consumers' preferences over the god. From this perspective, some goods, like forks, are simple. What consumers want from them is straightforward and there are not many different ways to achieve that goal. Preferences over other goods, like automobiles, are complex because consumers want many things from them which can be delivered in a variety of distinct ways.

To distinguish technical complexity from Preference complexity, consider food products. Typically, food products are technically simple, but preference-complex. There is a great variety of characteristics that consumers care about in their food including the taste, health value, presentation, and ethical consequences of the production of their food, and each of these properties can be broken down into numerous sub categories. Health value, for instance, can be divided into fat content, cholesterol content, sodium content, as well as the presence of a variety of allergens. All of these characteristics apply to a good as technically simple as bread, which has been produced for thousands of years. For the rest of the paper, assume that complexity refers to preference complexity unless otherwise noted.
Consider the markets for milk and cheese. Whereas these goods are overall similar, there are many, many more individual varieties of cheese for sale than varieties of milk. A similar observation can be made about the markets for flour and bread. I argue that the most salient inherent difference in the markets for milk and cheese is the fact that cheese is much more complex. In this paper I make the case that more complex goods generate a greater variety of products in equilibrium. While it seems immediate to claim that more potential products should lead to more actual products, there's no guarantee that the demand will exist for all of the individual varieties. In section 4 I provide evidence that increasing the complexity of a good raises the price of all of the products in the market, allowing more to coexist in equilibrium.
To get this result, I embed a characteristic space (Lancaster 1966) inside a model of spatial competition. This creates a hypercube over which a number of firms compete, similar to the framework used by Irmen and Thisse (Irmen and Thisse 1998). Then the complexity of the product space can be represented by the number of dimensions of the hypercube. While Irmen and Thisse find that the number of dimensions in their model is largely irrelevant, this is a direct result of only studying a duopoly. I allow many firms to operate simultaneously and compare the nature of their competition as I vary the dimension of the

space. Because this model is the first to study the decisions of more than two firms competing over more than one dimension, I initially restrict attention to an exogenous location pricing game.

I find that the behavior of the model is markedly different from standard Hotelling models. Neither the principle of minimum differentiation (Hotelling 1929) or principle of maximum differentiation from (d'Aspremonte et. al. 1979) hold. In addition, I find that firms' prices decline directly in the number of their neighbors. I obtain these unusual results because my model allows for interactions between a firm's competition with each of its neighbors. In other multi-firm Hotelling models, such as the circular road (Salop 1979) or the hyper-pyramid (Von Ungern-Sternberg 1991), consumers live only on the edges between firms, so the results of competition have no direct effect on each other. Finally, when I relax the assumption of exogenous location on a square, I find that the standard location choice game for four firms does not have a symmetric pure strategy equilibrium.

The rest of the paper is organized as follows. Section 2 describes the model and provides a brief example. The basic case of the model is solved in section 3. An alternate specification of the model, useful for interdimensional comparisons, is solved in Section 4. In section 5, I depart from my previous work to study an endogeonus location model in 2 dimensions. Section 6 concludes.

3.2 The Model

The model I study is a variant of Hotelling's famous model. Hotelling conceived of the single axis of competition in his original model as location along a main street in a town. Most recent uses of Hotellings model have reimagined the axis of competition as a measure of product differentiation along a single characteristic. The model I study poses the question of what happens when firms compete over many characteristics of a good simultaneously and whether or not the number of individual characteristic is important.

There exists a good with n characteristics. As an example, consider pizza with its characteristics "Thin crust vs. deep dish" and "greasy vs. non-greasy". Each of these can be normalized to a spectrum [0,1] and varied independently of each other. Then, every possible type of pizza can be represented by a square. More generally, I look at an n-cube defined by $Z = [0,1]^n$. In order to exploit the high degree of inherent symmetry in a cube, I assume that 2^n symmetric firms compete on Z. Order the firms arbitrarily and label them firm 1 through firm 2^n. To avoid confusion, a good refers to the total market, where a product is a specific instance of the good given by a particular location vector.

There is a mass of consumers, normalized without loss to 1, distributed uniformly across the interior of the hypercube, with their location giving the exact configuration of the good that they most prefer. Consumers suffer quadratic disutility from consuming a product that deviates from their ideal with a standard, Euclidean notion of distance. Each agent purchases exactly one good from one of the firms. An agent located at x receives indirect utility according to $u_x(y,p)$ from purchasing good y at price p.

$$u_x(y,p) = -p - t\sum_{i=1}^{n}(x_i - y_i)^2$$

Above, t denotes equivalently the size of the hypercube or the intensity of preferences, while the summation is merely the square of the Euclidean distance from x to y. An agent at x will prefer firm A to firm B if and only if $u_x(A,p_A) \geq u_x(B,p_B)$.

Let firm i be located at point z^i and charge price p_i. Define $\mathbf{p} = (p_1, p_2, p_3 ...)$ as the vector of prices. The set of consumers that purchase from firm i is given by the set of consumers that individually prefer firm i to each other firm j. Define

$$S_i(\mathbf{p}) := \{y \in Z |\ p_i + t\sum_{k=1}^{n}(z_k^i - y_k)^2 \leq p_j + t\sum_{k=1}^{n}(z_k^j - y_k)^2\ \forall_j\}$$

Then the demand of firm i, $D_i(\mathbf{p})$ is the volume of this set, or $D_i(\mathbf{p}) = V(S_i(\mathbf{p}))$. It does not matter whether or not S_i is defined through weak or strict preference, as the set of indifferent consumers has no volume as long as firms are distinct. I assume that any product may be produced at the same constant marginal cost, which is normalized to zero without loss of generality, so the firms' profit function is given by $\pi_i(\mathbf{p}) = p_i D_i(\mathbf{p})$.

Since all firms are ex-ante symmetric and competing in a symmetric environment, I wish to restrict attention to symmetric equilibria. Towards that end, assume that firms position themselves in a reflectively symmetric arrangement.

Definition 1. *An set of points $A = \{z^i\}_{2^n}$ is called* reflectively symmetric *if* $\forall_{i,j}\exists_{z^k \in A}$ *such that* $z^k = (z^i_1, z^i_2 ... z^i_{j-1}, 1 - z^i_j, z^i_{j+1}, ... z^i_n)$.

If a set of points is reflectively symmetric, the position of all other points is uniquely determined by the position of a single point. Call that point z^1. Any reflectively symmetric set of points containing z^1 can be alternately defined by $\prod_{i=1}^n \{z^1_i, 1 - z^1_i\}$, the set of every possible vector z^k such that $z^k_i = z^1_i$ or $z^k_i = 1 - z^1_i$. For example, when n=2, every reflectively symmetric positioning can be characterized by $((a, b), (1-a, b), (1-a, 1-b), (a, 1-b))$ for some a and b. Geometrically speaking, points in a reflectively symmetric arrangement form a hyperrectangle, invariant to reflections over the coordinates.

Notice that $z^1_i < \frac{1}{2}$ if and only if $1 - z^i_i > \frac{1}{2}$. Then if I bisect each coordinate of the cube to create 2^n subcubes, exactly one point from each reflectively symmetric set of points will fall into each subcube. Relabel the point closest to the origin to be z^1 such that $z^1 \leq \frac{1}{2}$. Like every point in a reflectively symmetric set, z^1 will have n neighbors that differ from it in only one coordinate. The distance between any two neighbors, or the side length of the hyperrectangle that the set defines, will be $\delta_j = 1 - 2z^1_j$ for the relevant coordinate j.

For most of the paper, with the exception of section 5, I will assume an exogenous location. Firms are forced to locate in some reflectively symmetric positioning which can be summarized by the representative member, z^1. In the next section, I will solve a simple example in this model to give the reader a clear notion of how the model works.

3.2.1 An Example

Take n=2, t=1. Four firms locate in a reflectively symmetric manner on a square. Let their positioning be defined by $z^1 = (0,0)$, so that firms position on the vertices of the square. Due to symmetry, it suffices to solve the problem of a single firm. Without loss of generality, consider firm 1. It is safe to assume that all other firms are choosing a uniform price, p. The set of consumers indifferent between firm 1 and firm 2, located at point $(1,0)$, is given by the equation (3.1), which simplifies to (3.2).

$$p_1 + (x_1 - 0)^2 + (x_2 - 0)^2 = p + (1 - x_1)^2 + (x_2 - 0)^2 \tag{3.1}$$
$$x_1 = \frac{1 - p_1 + p}{2} \tag{3.2}$$

Both x_1 and x_2 must be weakly positive to lie in Z. To lie in $S_1(\mathbf{p})$, a point (x_1, x_2) must satisfy 3 constraints, one for each of firm 1's competitors. In the set of constraints that define $S_1(\mathbf{p})$ below, the first three are the constraints from firms located at (1,0), (0,1), and (1,1) respectively.

$$x_1 \leq \frac{1 - p_1 + p}{2}$$
$$x_2 \leq \frac{1 - p_1 + p}{2}$$
$$x_1 \leq \frac{2 - p_1 + p}{2} - x_2$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

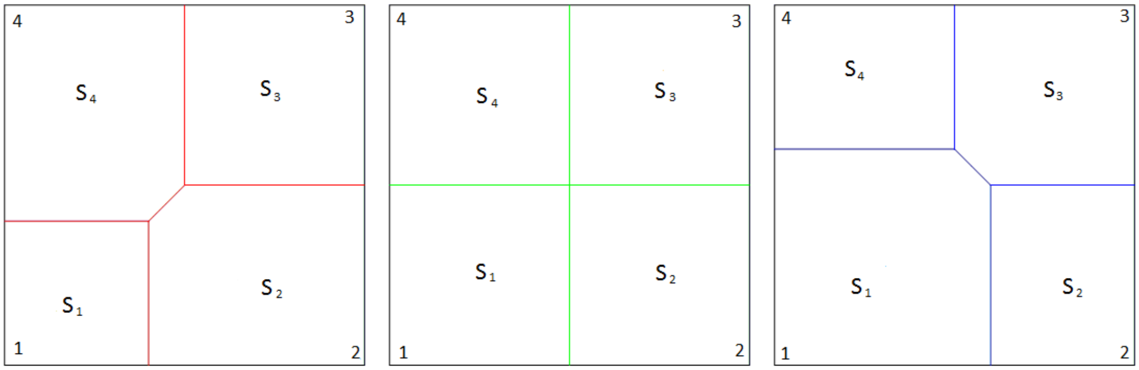


Figure 3.1: Possible shapes of $S_1 - S_4$

There are three possible configurations for $S_1(\mathbf{p})$ presented by Figure 3.1. Represented from left to right are the cases in which firm 1 charges a higher, equal, or lower price relative to the other three.

As you can see, the formula determining $V(S_1(\mathbf{p}))$ depends on whether or not $p_1 < p$.

$$\text{If } p_1 < p: \quad D_1(\mathbf{p}) = \frac{(1-p_1+p)^2}{4} - \frac{(p-p_1)^2}{8}$$
$$\text{If } p_1 \geq p: \quad D_1(\mathbf{p}) = \frac{(1-p_1+p)^2}{4}$$

Recall that firm 1's profit is given by $\pi_1(\mathbf{p}) = p_i D_i(\mathbf{p})$. Since the expressions are the same when $p = p_1$, the profit function is certainly continuous, but it is possibly non-differentiable at $p = p_1$, which must hold in equilibrium. For now, assume that the function is differentiable. In order to solve for p_1, I take the derivative of the profit function, set it equal to zero, and then impose that $p = p_1$. Although there may be different expressions for the right and left hand derivatives at $p = p_1$, differentiability assures us that they are equivalent. I use the right hand derivative for our calculations since it is simpler. Following this process, in a symmetric equilibrium $p = \frac{1}{2}$.

3.3 Price Equilibrium

Consider a product space with n characteristics. There are 2^n firms. In a symmetric equilibrium, all firms will charge the same price and capture the $\frac{1}{2}$x$\frac{1}{2}$x$\frac{1}{2}$... hypercube that contains them.

Lemma 1. *The profit function, $\pi_j(\mathbf{p})$ is everywhere differentiable in p_1.*

Proof. See Appendix B. □

I'd like to take a moment to discuss the general strategy of finding equilibria in this environment. Because I am interested in symmetric equilibria, I can assume that all firms are choosing the same price and then consider the problem of a potential deviant. Without loss, I can assume that the potential deviant is firm 1, located closest to the origin. Then, I must find firm 1's profit function in terms of the candidate price, p, and his deviation, p_1. Due to the differentiability of the profit function, I always assume that the price of the firm whose problem is being considered is greater than or equal to the price of other firms, as this greatly simplifies the expressions involved. Knowing the profit function, I can take the first order condition with respect to p_1, then set $p = p_1$ and solve to find the equilibrium price.

The only difficult part of this process is finding an expression for $D_1(\mathbf{p})$. Recall that $S_1(\mathbf{p})$ is a region defined by the constraints $0 \leq x_i$ for each i and also, for each j, (3.3).

$$p_1 + t\sum_{k=1}^{n}(x_k - z_k^1)^2 \leq p_j + t\sum_{k=1}^{n}(x_k - z_k^j)^2 \tag{3.3}$$

z_k^j can take one of two forms: either z_k^1, or $(1 - z_k^1)$. Letting $I_j = \{k \in |z_k^1 - z_k^j \neq 0\}$, I can simplify (3.3) to (3.4), and further to (3.5) with a change of variables.

$$0 \leq p_j - p_1 + t \sum_{i \in I_j} (1 - 2z_j^1) - 2(1 - 2z_j^1) x_j \tag{3.4}$$

$$0 \leq p_j - p_1 + t \sum_{i \in I_j} \delta_j - 2\delta_j x_j \tag{3.5}$$

In order to find the volume of $S_1(\mathbf{p})$ I must to find out which of those constraints are redundant and which bind. Define a type j constraint to be a constraint imposed by a firm k such that $|I_k| = j$. Geometrically speaking, type j constraints are imposed by type j firms, which differ from z^1 in j coordinates.

Lemma 2. *In equilibrium, only type one constraints bind.*

Proof. See Appendix B. □

This means that firms only compete directly with their neighbors in equilibrium, so the region that a firm captures is just a hyperrectangle. Thus, for $p_1 \geq p$, the profit of firm 1 is given by equation (3.6).

$$\pi_1(\mathbf{p}) = p_1 (\prod_{i=1}^{n} \frac{\delta_i t + p - p_1}{2\delta_i t}) \tag{3.6}$$

For a set of number, A, let $hm(A)$ be the harmonic mean of A.

Theorem 1. *If 2^n firms locate in a reflectively symmetric positioning on an n-cube and compete in prices, there exists a unique symmetric equilibrium in which firms charge $p^* = \frac{t\ hm(\{\delta_i\}_{i=1}^n)}{n}$ and earn profits equal to $\pi^* = \frac{t\ hm(\{\delta_i\}_{i=1}^n)}{2^n n}$.*

Proof. See Appendix B. □

In general, both prices and profits decrease with dimension. Profits obviously decline in n, since the same volume must be split among more and more firms. In an n-cube, each firm has n neighbors with which it competes directly. More neighbors lead to a decrease in prices because the measure of consumers that a firm captures from one of its neighbors interacts positively with the measure of consumers that it captures from each of its other neighbors. This gives firms a strong incentive to cut prices to expand their market share, leading to more savage competition and a lower equilibrium price. This effect scales with each additional direct competitor.

A cursory examination of other multi-firm Hotelling models such as the circular road or Von Ungern-Sternberg's pyramid shows that the number of firms has no impact on pricing behavior as long as the distance between any two firms is preserved. In those models, in which consumers only live on one dimensional edges between firms, firms compete independently with each of their neighbors. More firms are either irrelevant, as in Salop, or enter the profit function as a linear multiplier, as in Von Ungern-Sternberg.

Note that both prices and profits increase in each δ_j, the distance between neighbors. As is common in Hotelling models, firms can charge higher prices whenever they are located further apart. The δ_j's cannot be greater than 1, so the reflectively symmetric arrangement that maximizes firm profits is given by $z_1 = 0$, or a firm located on each vertex. Call this positioning of firms the *basic positioning*. At first glance, the basic positioning appears to follow the principle of maximal differentiation, as it represents the arrangement of 2^n firms on an n-cube that maximizes the distance between firms. However, the principle of maximal differentiation is a statement about equilibrium behavior in a location choice game and, thus far, I have only worked with an exogenous location game.

3.4 Offsets

In this section I will introduce an alternate positioning so that I can partially divorce the number of dimensions from the number of firms. I show, using this alternate positioning, that there exists an arrangement of 2^n firms on an m-cube such that each firm earns more profits than any firm in any reflectively symmetric positioning of 2^n firms on an n-cube whenever $m > n$. While increasing the dimensions of the hypercube generally exposes each firm to more axes of competition which drive prices down, it also allows firms to locate further from one another, driving prices up. Given the results of this section, the latter effect dominates.

Consider a collection of m firms on an n-cube in some symmetric positioning, A. In a symmetric equilibrium, each firm will earn the same profit, and that profit will be determined by m, n, and the location of the firms. Let the function $\tilde{\pi}(m, n, A)$ give such profits.

Proposition 1. *There exists an arrangement of firms B such that* $\tilde{\pi}(m, n+1, B) \geq \tilde{\pi}(m, n, A)$.

Proof. To get the result, simply replicate the positioning given by A on one of the n dimensional sides of the (n+1)-cube. The regions captured by the firms will be the same as on the n-cube, but prismed with length 1. Then, firms will face the same profit functions, make the same decisions and earn the same profit. □

With this proposition, I need only show that 2^n firms can be made to be strictly better off in an (n+1)-cube than they can be in an n-cube. Recall that the basic positioning will generate the most profit of any reflectively symmetric arrangement. In the basic positioning, $\delta_i = 1$ for every i, and so the profit of each firm is given by $\frac{t}{n2^n}$. Therefore, it suffices to construct a positioning of 2^n firms on an (n+1)-cube that dominates the basic positioning on an n-cube.

The positioning I would like to examine is such that there is a distance of at least $\sqrt{2}$ between any two firms, or that no firms are neighbors in the sense of the previous section. In order to do this, position a firm at every vertex z^i such that $< z^i, z^i >= 2k$ for some integer k. It is easy to verify that this takes exactly 2^n firms in an (n+1)-cube. To verify that

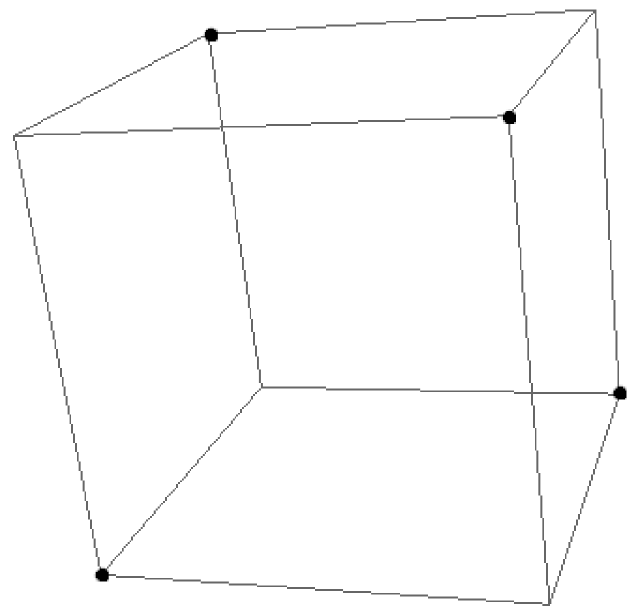


Figure 3.2: The offset positioning on a 3 cube

there are no adjacent firms, note that it's a collection of type 2 k firms for some k. Obviously a type 2 firm cannot be adjacent to a type 4 firm. If two type 2 firms are distinct, one of them must have a zero where the other has a 1 and, to compensate, must have a 1 somewhere that the other has a zero. Therefore, we have a positioning of exactly 2^n firms such that no two firms are adjacent. Refer to this arrangement as the *offset positioning*. Figure 3.2 shows the offset positioning in three dimensions, with the black dots representing the location of firms.

Once again, I can appeal to symmetry and consider only the problem of firm 1, located at the origin, and assume that all other firms charge a uniform price. Suppose that firm 1's price, p_1 is at least as large as the competitors price, p. When $p_1 \geq p$ in the basic case last section, only type 1 constraints bound. Here, there are no type 1 constraints, and so only type 2 constraints bind. The argument for this is extremely similar to Proposition 2 from the last section and is omitted. Type 2 constraints are of the form

$$x_i \leq \frac{2t - p_1 + p}{2t} - x_j$$

Because every type 2 vertex is occupied, there is a constraint of this form for every i,j.

Theorem 2. *There exists a symmetric arrangement of 2^n firms on an (n+1) cube such that each firm earns more profit in equilibrium than it is possible for any firm to earn in a reflectively symmetric equilibrium of 2^n firms on an n-cube.*

Proof. See Appendix B. □

Theorem 2 allows me to compare two markets that differ only in their Preference complexity. Suppose that there are n characteristics of good A and some m > n characteristics of good B. If the markets have the same number of consumers and the same number of firms competing, theorem 2 says that each firm in the market for good B ought to make more profit. Suppose, instead, that there is a uniform fixed cost, F, of entering either market A or market B. Then, each firm must earn a profit of at least F in equilibrium in order to survive. Since firms in market B earn more than those in market A, if entry is allowed, there should be more firms in market B than in market A.

Here, I should clarify what is meant by a firm in this model. Each firm is located at a single bundle of characteristics, which means that they sell a product. What the model actually suggests is that the more complicated a product space is, the more individual products should be observed in equilibrium. This claim can be confirmed by a variety of observations like the milk/cheese comparison that was made in the introduction. While it is intuitive to claim that the more kinds of products that are possible, the more should exist, Theorem 2 provides concrete theoretical grounding for why this should be true even in the case of fixed demand for the overall good.

3.5 Endogenous Location

While the dominance of the offset positioning over the basic positioning is indicative that firms ought to be better off in higher dimensions, it is not a concrete proof. Firms can position themselves to make higher profits in higher dimensions but it remains to be seen if they would. Suppose that 2^n firms can choose both their price and their location on an n-cube and on an (n+1)-cube. The natural question is which of the two set ups will yield a higher profit for firms. Unfortunately, as I will show in this section, the question is generally unanswerable. There is no symmetric pure strategy equilibrium for four firms on a square, and no reason to believe that there is such an equilibrium in higher dimensions.

To show that there is no pure strategy symmetric equilibrium, let Z be the square $[0,1]x[0,1]$ and fix t=1. As before, a mass 1 of consumers are uniformly distributed on Z. The utility of a consumer located at x and purchasing a product from firm located at y that charges price p is given by $u_x(y,p) = -d(x,y)^2 - p$, where d is the Euclidean distance. Each consumer will maximize their utility by purchasing one unit of the good from the firm which gives them the least disutility.

There are four firms, labeled firm 1 through firm 4. Each firm may choose its location in Z, $z_i = (a_i, b_i)$, and its price, $p_i \in \mathbb{R}_+$. In the fixed location case, symmetry simply meant that all firms charged the same price. With endogenous location, I need to extend symmetry to location decisions as well. There are two basic ways for four points to exhibit symmetry on a square. They may be rotationally symmetric, invariant to a rotation of the square, or reflectively symmetric, invariant to reflections of the square. Given a location (a,b), there exists a unique rotationally symmetric positioning for the other three firms, given by $h(a,b) = ((a,b),(1-b,a),(1-a,1-b),(b,1-a))$. Similarly, for a given (a,b), $g(a,b) = ((a,b),(1-a,b),(1-a,1-b),(a,1-b))$ provides the unique reflectively symmetric positioning. Given the symmetry of the square, I can assume without loss of generality that $a \in [0, \frac{1}{2}]$, and $b \in [0, a]$.

As in the fixed location model, define T_i and π_i^e such that

$$T_i(\mathbf{z}, \mathbf{p}) := \{y \in Z | \ p_i + d(y, z^i) \leq p_j + d(y, z^j) \ \forall_j\}$$

$$\pi_i^e(\mathbf{z}, \mathbf{p}) := p_i V(T_i(\mathbf{z}, \mathbf{p}))$$

In this model, a symmetric pure strategy equilibrium is defined as follows.

Definition 2. *A* symmetric Nash equilibrium *is a vector of prices,* (p_1, p_2, p_3, p_4)*, a vector of locations* (z^1, z^2, z^3, z^4) *and a map* $F : [0,1]^2 \to \{1, 2, 3, 4\}$ *such that:*

1. $(z^1, z^2, z^3, z^4) = g(z^1)$ *or* $(z^1, z^2, z^3, z^4) = h(z^1)$.

2. *F(x)=i if* $\forall_j, p_i + td(x, z^i)^2 < p_j + td(x, z^j)^2$ *and only if* $\forall_j, p_i + td(x, z^i)^2 \leq p_j + td(x, z^j)^2$.

3. $T_i = \{y \in Z | F(y) = i\}$.

4. *For each i,* p_i *maximizes* $p_i V(T_i)$.

5. $p_i = p_j \ \forall_{i,j}$.

At first, consider only rotationally symmetric equilibria. Any candidate equilibrium can be summarized by the actions of one firm, since prices are uniform and the location can be fed through $h(.)$. To check if any candidate equilibrium is actually an equilibrium, I must find the profit function of a potential deviant in terms of his deviation and the actions of his competitors according to the candidate. Let the potential equilibrium be (a,b,p) and the deviation be $(\hat{a}, \hat{b}, \hat{p})$. If a potential deviant, firm 1 for simplicity, maximizes his profit at (a,b,p), then (a,b,p) is an equilibrium. In the case of a symmetric allocation, each firm charges the same price and is located symmetrically about the square, inducing them to

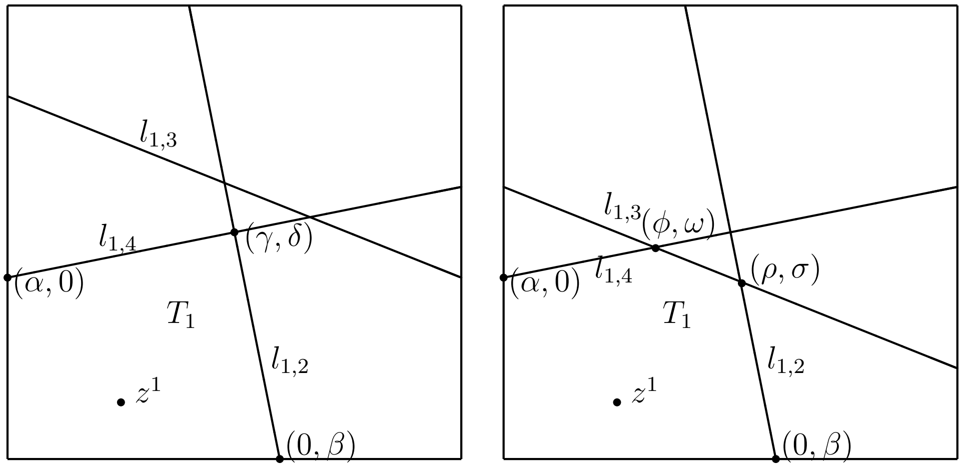


Figure 3.3: (a) T_1 when $l_{1,3}$ does not bind, (b) T_1 when $l_{1,3}$ binds

split the consumers evenly. Then the profit from any symmetric allocation summarized by (a,b,p) will simply be $\frac{p}{4}$.

T_1 is made up of three constraints, one for each of the deviant's competitors. The sets below are the consumers that prefer firm 1 to firms 2, 3, and 4 respectively.

$$\{(x,y) \in Z | x \leq \frac{1 + a^2 - \hat{a}^2 - 2b + b^2 - \hat{b}^2 + p - \hat{p} - 2ay + 2\hat{b}y}{2(1 - \hat{a} - b)}\} \tag{3.7}$$

$$\{(x,y) \in Z | y \leq \frac{2 + a^2 - \hat{a}^2 - 2b + b^2 - \hat{b}^2 + p - \hat{p} - 2a(1-x) - 2x(1-\hat{a})}{2(1 - b - \hat{b})}\} \tag{3.8}$$

$$\{(x,y) \in Z | y \leq \frac{1 + a^2 - \hat{a}^2 - 2a + b^2 - \hat{b}^2 + p - \hat{p} - 2\hat{a}x + 2bx}{2(1 - a - \hat{b})}\} \tag{3.9}$$

Call the line of consumers that are indifferent between purchasing from firm 1 and firm 2 (constraint (3.7), but holding with equality) $l_{1,2}$, and likewise for the other lines of indifference. Figure 3.3 shows the $[0,1]x[0,1]$ square from the perspective of the first firm: each of the constraints represented by $l_{1,j}$ is plotted, bounding the region that firm 1 captures, T_1. Figure 3.3(a) shows the case in which $l_{1,3}$ is non-binding, while Figure 3 (b) shows the binding case.

As indicated by the figure, define the point $(\alpha, 0)$ to be the intersection of $l_{1,4}$ and the y-axis. Let the other variables, $(\beta, \gamma, \delta, \phi, \omega, \rho, \sigma)$ be defined similarly in keeping with the figures[1]. $V(T_1)$ can be derived from basic geometry in each of the cases. In the case in picture 1, $\pi^1 = \hat{p}V(T_1) = \hat{p}\frac{\alpha\gamma+\beta\delta}{2}$, while in picture 2, $\pi^2 = \hat{p}V(T_1) = \hat{p}\frac{\alpha\phi+\omega\rho-\sigma\phi+\beta\sigma}{2}$, where the superscripts refer to forms of the profit function rather than the profit function for different firms.

Clearly the crossover point from the first expression to the second occurs when $\phi = \rho$. Thus, the full profit function is given by the following:

$$\pi^e(a, b, p, \hat{a}, \hat{b}, \hat{p}) = \begin{cases} \hat{p}\frac{\alpha\gamma+\beta\delta}{2} & \text{if } \rho \geq \phi, \\ \hat{p}\frac{\alpha\phi+\omega\rho-\sigma\phi+\beta\sigma}{2} & \text{if } \rho < \phi. \end{cases} \tag{3.10}$$

Let a triple (a^*, b^*, p^*) be said to be a local (global) maximum at itself with respect to π^e if $\pi^e(a^*, b^*, p^*, \hat{a}, \hat{b}, \hat{p})$ is maximized locally (globally) over $(\hat{a}, \hat{b}, \hat{p})$ at (a^*, b^*, p^*).

Then, in this model, a rotationally symmetric pure strategy equilibrium will be a triple, (a^*, b^*, p^*) that is a global maximum at itself with respect to π^e. The process for investigating reflectively symmetric equilibria is similar, but using $g(.)$ to determine the location of the deviant's competitors rather than $h(.)$. All of the methodology remains the same in the reflective case.

Theorem 3. *There is no symmetric pure strategy equilibrium of this game.*

Proof. To show that there are no pure strategy symmetric equilibria in this game, I must show that there are no pure strategy reflectively symmetric equilibria and also no pure strategy rotationally symmetric equilibria. In appendix B, I formally prove that there are no pure strategy rotationally symmetric equilibria. The proof for the reflective case is similar. □

[1]Full expressions for $(\alpha, \beta, \gamma, \delta, \rho, \sigma, \phi, \omega)$ as well as the full profit function and calculations are omitted for clarity, but are available upon request.

3.6 Conclusions and Extensions

This paper introduces a model of product differentiation that considers the complexity of a product from a consumer's perspective rather than a firm's. This is the first model to explicitly model preference complexity and investigate its effect on market structure. My main result is that in an exogenous location setting, more individual products can be supported in a more complex product space. In other words, the more potential kinds of products that can exist in a market, the more products ought to exist in equilibrium. This result meshes well with stylized facts from many markets. While it makes sense that products with more potential variance ought to support more realized variance, I provide clear theoretical grounding for this idea.

Further, I find that allowing firms to compete with each other over many axes of product differentiation radically alters the nature of competition. Unlike previous Hotelling models, I find that firms' pricing depends on the number of direct competitors because capturing consumers from one firm has a positive effect on the measure captured from others. I also find a weak version of the principle of maximum differentiation: that firms would be better off locating as far from each other as possible, but that firms would not realize that allocation. Investigating a proper, endogenous location version of the model, I find that there need not be a symmetric, pure strategy equilibrium for more than two firms in more than one dimension. All of these results combine to call into question the practice of using one dimensional Hotelling models to characterize competition between many firms in a potentially complex product space.

To extend my work further, I would like to do more with the endogenous location game. If there is a way to perturb the game so that an equilibrium exists, I could investigate several interesting lines of research. Most immediately, I could address whether or not four firms would be better off competing in two dimensions or one, to generate or negate a much more powerful version of the main result of this paper. I suspect that they would be, as firms in any of the local equilibria of the game that I solved are, in fact, better off in two dimensions. Further, I could investigate how the equilibrium changes in response to changing travel costs, or if it's invariant. Lastly it would be interesting to compare rotationally and reflectively symmetric equilibria, to see which is easier to achieve or more profitable.

To develop the contributions of my framework further, a natural next step is to examine the effects of coalitions on the pricing behavior and profits of firms in a hypercube set up. A coalition in this setting would represent either a merger between two firms or a single firm selling multiple products. If individual firms behave in a qualitatively different way on a hypercube than they do on a circular road, then it makes sense that coalitions would as well. Additionally, unlike the circular road, a hypercube framework allows for large coalitions that treat every member firm symmetrically.

Chapter 4

Grandfather Clauses and Consumer Complacency

4.1 Introduction

A grandfather clause allows a certain group of people to be exempt from a wider change in circumstances. They are mostly found in legislation, these days often included to reduce opposition to the changes that a bill imposes from special interests. However, firms have been known to use grandfather clauses to reward their loyal customers. In economics, this practice typically takes the form of exempting existing subscribers of a service from a price hike, or allowing them to keep consuming the service but no longer offering it to new customers. These practices, while hardly widespread, have been a feature of telecom markets for decades. More recently, in May of 2014, the popular media streaming service Netflix offered a grandfather clause to all of its current customers shortly after announcing a general price hike. Despite this, there is no literature on either firms' motivations for offering grandfather clauses, or their effects on welfare. This is likely the case because it is difficult to imagine a world in which grandfather clauses make economic sense.

When a firm raises its price for new customers only, it must be responding to a spike in demand. Otherwise, it would not expect to attract any new customers, especially not at a higher price. Even assuming a demand spike, there must be some other force which drives firms to offer a grandfather clause instead of raising the price for everyone. Firms could do so as a tool for price discriminating between long term and short term customers, but there are a number of reasons that this is unlikely. Typically when price discriminating, a firm will want to charge a higher price for customers that value its products more. In this case, that means that firms would expect its preexisting customers to be less likely to enjoy their product than a newcomer. Alternatively, if there is an intrinsic value to the firm of having longstanding customers, such as word-of-mouth, then using longstanding contracts would be a more sensible approach. Even supposing that a price discrimination makes sense, a grandfather clause would generically be non-optimal. This is because the optimal discount would have to precisely balance out the increase in price generated by the demand shock.

In order for grandfather clauses to be optimal, there must be some qualitative difference between *keeping* a price and naming the same price two periods in a row. Consider two competing firms that offer a subscription service, such as Netflix and Amazon Prime. Since these two services are constantly adding new shows and services, as well as losing them, people's idiosyncratic tastes are changing each period. However, because their customers are engaged in hundreds if not thousands of markets, they do not typically search to discover their preferences each time they change. It is a rare Netflix subscriber that reconsiders whether or not they want to change services every month. However, consumers can be induced to search if something draws their attention to the market. There are two changes that can grab consumers' attention: 1) the price of their service increases, or 2) the non-idiosyncratic quality of the competition increases. In this way, when a firm develops an average improvement to the quality of its service, it will induce its competitor's consumers to search. Ordinarily, a firm that has markedly improved its quality would also raise its price. Here, however, they may wish to offer a grandfather clause in order to prevent their consumers from searching and finding out that their idiosyncratic tastes have shifted far enough such that they prefer the competition, despite the upswing in quality.

4.2 Model

4.2.1 A One Period Game

There are two firms that offer differentiated but competing subscription services. There is a mass one of consumers that have idiosyncratic tastes over these services. That is, each consumer values consuming product i at some average quality μ_i plus an iid preference shock ϵ_i with mean zero. Each consumer also suffers disutility equal to the price she paid, or:

$$u_i(p_i) = \mu_i + \epsilon_i - p_i \tag{4.1}$$

Then, a consumer will buy from firm 1 only if

$$\mu_1 + \epsilon_1 - p_1 \geq \mu_2 + \epsilon_2 - p_2 \tag{4.2}$$

Define $\epsilon = \epsilon_2 - \epsilon_1$ and let $\epsilon \sim F(x)$. To avoid having a positive measure of indifferent consumers, let $F(x)$ have no point masses. Then, the quantity of consumers that prefer firm 1 is equal to $F(\mu_1 - \mu_2 + p_2 - p_1)$. Assuming that the μ_i are high enough to ensure market coverage, only the difference, $\mu_1 - \mu_2$, has an impact on the model. Call this difference Δ. Lastly, assume that firms produce their services at a constant marginal cost, which can be safely normalized to zero. Then, firm profit functions are given by

$$\pi_1(p_1; p_2, \Delta) = p_1 F(\Delta + p_2 - p_1) \tag{4.3}$$

$$\pi_2(p_2; p_1, \Delta) = p_2(1 - F(\Delta + p_2 - p_1)) \tag{4.4}$$

Then there will be an equilibrium whenever the first order conditions are satisfied. For a simple example, suppose that $\epsilon \sim U[-a, a]$. Then trivial calculations will show that

$$p_1 = a + \frac{\Delta}{3} \tag{4.5}$$

$$p_2 = a - \frac{\Delta}{3} \tag{4.6}$$

$$q_1 = \frac{1}{2a}(a + \frac{\Delta}{3}) \tag{4.7}$$

$$q_2 = \frac{1}{2a}(a - \frac{\Delta}{3}) \tag{4.8}$$

$$\pi_1 = \frac{1}{2a}(a + \frac{\Delta}{3})^2 \tag{4.9}$$

$$\pi_2 = \frac{1}{2a}(a - \frac{\Delta}{3})^2 \tag{4.10}$$

4.2.2 A Two Period Game

For the two period game, let a superscript denote a time period, either 1 or 2. For this section, bear in mind that consumer search decisions are exogenous. Suppose that initially both firms have the same average quality so that $\Delta^1 = 0$. In the first period of the game, both firms enter the market and set their prices. Then, consumers search each firm and choose their favorite and the game plays out in the same manner as the previous subsection. Each month, the firms lose contracts and add content and generally change their particular product. Consumers tastes, then, migrate with new ϵ's being drawn. However, in each of these periods, nothing draws consumers' attention to the market and they do not search. After the first period, there are customers that would benefit from switching services, but aren't paying enough attention to realize this.

After a long period of time has passed, consumers original preferences and hence the service that they subscribe to no longer carries any meaningful information about their current tastes. That is, their second period ϵ's are uncorrelated with the first period's. Suppose that in this second period firm 1 announces a general improvement in the quality of its product, either through higher definition streaming, producing its own shows, or a particularly large addition of new content. Let the value of this improvement to the average customer be called ν. Then, in the second period, $\Delta^2 = \nu$. For the purposes of this paper, suppose that ν is exogenous and costless.

Because firm 1 has a very noticeable upswing in average quality and, presumably, advertised this, some of the customers from firm 2 will search both firms and possibly switch to firm 1. If firm 1 does not change its price, its consumers from the previous period will see no reason to search. Thus, firm 1 could attempt to hold onto its current customer base while attracting new customers by not increasing its price. Alternatively, it could name a higher price to reflect its competitive edge, and count on its improved quality to keep most, if not all, of its previous customers. Finally, it could try to get the best of both worlds by maintaining its previous price for loyal customers so that they do not search, while naming an optimal price for any newcomers. Obviously, the third option dominates the first, but in some cases the second may be best.

If the firm raises its price globally, all consumers will search. Then the situation is the same as in the previous one period case, but using $\Delta = \nu$ instead. If the firm offers a grandfather clause, then its loyal customers are effectively out of the market and the firms are competing only over the remaining mass $\frac{1}{2}$ of consumers that search. Once again, this competition will take the same form as in the single period model. Let superscripts denote the period and recall that $\Delta^1 = 0$. Let Π be the maximum profit a firm can achieve by naming a single price, and Π^{gfc} be the similar maximum if the firm uses a grandfather clause. Putting this all together, firm 1 faces the choice between the following profits:

$$\Pi^{gfc} = p_1^1 q_1^1 + q_2^1(p_1^2 q_1^2) \tag{4.11}$$

$$= \frac{a}{2} + \frac{1}{2}(\frac{1}{2a}(a + \frac{\nu}{3})^2) \tag{4.12}$$

$$\Pi = p_1^2 q_1^2 \tag{4.13}$$

$$= \frac{1}{2a}(a + \frac{\nu}{3})^2 \tag{4.14}$$

4.2.3 Equilibrium Behavior

Firm 1 will want to offer a grandfather clause when $\Pi^{gfc} > \Pi$, which simplifies to

$$\nu < 3a(\sqrt{2} - 1) \tag{4.15}$$

When the increase in quality is large enough to completely drive a competing firm out of the market $\nu \geq 3a$, then a firm has no motivation at all to offer a grandfather clause. There's no need to prevent its consumers from searching, since they'll stay with the firm anyway. When the increase in quality is tiny, firms have a large incentive to try to prevent their consumers from searching, since they can expect to lose approximately half of the ones that do. Then, firms will want to offer grandfather clauses when their average quality improvements are relatively small. This result does not depend on the unrealistic assumption that all of firm 2's customers search regardless of the size of firm 1's quality improvement. If there were some scaling function, $\beta(\nu)$, that determined the proportion of firm 2's customers that searched in terms of the size of firm 1's improvement, it would have no effect on firm 1's incentives to offer a grandfather clause or not, it would only reduce the overall profitability of making the improvement in the first place.

4.2.4 Welfare

In this model, grandfather clauses are unambiguously bad for total welfare. This can be seen without any explicit calculations, and applies beyond the simple example that I have worked through thus far. First, since firm and consumer welfare are both linear in transfer payments, money is a zero sum affair which can be discounted in total welfare calculations. Then, the only thing that matters is the matching values for consumers to firms. Grandfather clauses act to distort these by preventing some of firm 1's customers from searching and finding out that they would be better matched with firm 2. Furthermore, it is plain to see that grandfather clauses are uniformly bad for firm 2 by effectively reducing the size of its market. Grandfather clauses will be good for firm 1's welfare in the cases discussed in the previous subsection.

The most interesting question is whether or not grandfather clauses are good for consumer welfare. On the one hand, they reduce the average price paid by consumers. On the other hand, they cause some consumers to be matched poorly and receive a service that they do not enjoy very much. It bears asking then, is there a range of parameters in which firm 1 has an incentive to offer a grandfather clause that hurts consumers?

Returning to the simple, uniform example, the answer to this question can be calculated. In a case without grandfather clauses, we know that the market will behave as outlined in equations (4.5)-(4.10). Then, using equation (4.1), the total consumer welfare in such a world is given by

$$W = \int_{-a}^{\frac{\nu}{3}} (\mu + \nu - (a + \frac{\nu}{3}))\frac{1}{2a}\,\mathrm{d}x + \int_{\frac{\nu}{3}}^{a} (\mu + x - (a - \frac{\nu}{3}))\frac{1}{2a}\,\mathrm{d}x \tag{4.16}$$

where the first integral is for firm 1's consumers and the second is for firm 2's. This expression simplifies to

$$W = \mu - a + \frac{a}{4} + \frac{\nu}{2} + \frac{\nu^2}{36a} \tag{4.17}$$

Now, in the case with a grandfather clause, half of the market is content to remain with firm 1, paying a price of a, while the other half of the market behaves exactly as above. Then,

$$W^{gfc} = \frac{1}{2}(\mu + \nu - a) + \frac{1}{2}W \tag{4.18}$$

Clearly, $W^{gfc} > W$ if and only if $\mu + \nu - a > W$, which occurs when $\nu > 3a(3 - 2\sqrt{2})$. When ν is sufficiently small relative to a, the price savings (proportional to ν) are small relative to the forgone gains through search (proportional to a). Recall that firm 1 will only wish to offer a grandfather clause when $\nu < 3a(\sqrt{2} - 1)$. In this range, there are some ν's where a grandfather clause is detrimental to consumer welfare, and others in which it would enhance consumer welfare.

4.2.5 Investment

Hitherto, I have treated firm 1's improvement in overall quality, ν as exogenous. However, it is more realistic to suppose that firm 1 is able to invest in some research and development process in order to determine the magnitude of ν. Then, the natural question to investigae is whether a firm's ability to offer a grandfather clause would encourage or discourage investment.

Suppose that there exists a convex, increasing, and differentiable function $c(\nu)$ that gives the investment cost required to improve the average quality of the product by ν. Then, the firm will choose ν such that $\frac{\partial \pi}{\partial \nu} = \frac{\partial c}{\partial \nu}$. Let ν^{GFC} be the optimal level of improvement when the firm institutes a grandfather clause, and let ν^* be the optimal level of investment when a firm does not offer a grandfather clause.

Recall from equation (4.11) that in the case of a grandfather clause, the firm's profits are given by $p_1^1 q_1^1 + q_2^1(p_1^2 q_1^2)$. In the case without grandfather clauses, equation (4.13) provides the profits as $p_1^2 q_1^2$. Since the first period quantities do not depend on ν in any way, $\frac{\partial \Pi^{GFC}}{\partial \nu} = q_2^1 \frac{\partial \Pi}{\partial \nu}$. Since $q_2^1 < 1$ and $c(\nu)$ is convex, then $\nu^{GFC} < \nu^*$.

So far in this section, the firm's actual decision to offer a grandfather clause has been treated exogenously for clarity of exposition. Recall that, generally speaking, there exists some $\hat{\nu}$ such that a firm will want to offer a grandfather clause if and only if $\nu < \hat{\nu}$. Then, if $\nu^* \geq \hat{\nu}$, the option to offer a grandfather clause will have no effect on the equilibrium level of investment. However, if $\nu^* < \hat{\nu}$ then the firm will want to offer a grandfather clause and will instead only invest to the level of ν^{GFC}. That is, depending on the functions c and F, the ability to offer a grandfather clause will either reduce investment or have no effect.

4.3 Extensions

This paper is a very preliminary exploration of a hitherto unstudied phenomenon. A proper treatment of this question would need to do away with the exogenous consumer search behavior that I imposed in this model. If consumers' taste shocks were correlated period to period and search were costly, this model could use the tools developed by the rational inattention literature (Sims, 2003). Remember that consumers are perfectly informed of prices and average quality, but remain uncertain about their own, specific tastes. Consumers would optimally search infrequently, but would take into account changes to price and non-idiosyncratic quality. Then the measure of consumers who search would be a function that depended on the size of the change in average quality as well as the size of the price differential between competitors. The basic forces of the model, which encourage a dominant firm to offer a grandfather clause in order to discourage its consumers from searching, would still have traction without the draconian assumptions imposed in this paper.

An alternative, and less ambitious, reform to this paper would be to assume that there was some exogenous, but continuous and increasing, function $\alpha(p_1^2 - p_1^1)$ which determined the measure of consumers that searched as a function of the increase and prices. It is likely that in such a model a grandfather clause would not generally be optimal. Instead, firm 1 would probably like to name a cheaper price for its loyal customers than new customers, but still higher than the initial price. In this case, the behavior would less resemble a grandfather clause and more a rewards program, like those offered by airlines and credit cards. The strength of this effect would depend on the exact shape of the function α.

Last, by adding an element of research and development to the model and allowing both firms to innovate, I could examine what effects the potential for grandfather clauses has on a firms desire to innovate. They may make firms want to innovate more by yielding a higher potential reward. However, given the rival's ability to do the same, it's possible that grandfather clauses would lead to a prisoners' dilemma situation and actually reduce firm profits from innovation.

4.4 Conclusions

In this paper, I have given a recent, high profile example of a firm instituting a grandfather clause. I have shown the theoretical difficulties in explaining this behavior using obvious, existing models. To rectify this gap in our understanding, I propose a model in which grandfather clauses can be used to manipulate consumer search behavior to the benefit of a firm o the detriment of its competitors. Using a simple example, I explore the consequences of this behavior and find several interesting outcomes.

I find that even in an environment explicitly designed to promote them, grandfather clauses are not universally profitable. This may help explain why, while unquestionably present in market behavior, grandfather clauses seem rare. I find that grandfather clauses are most likely to be offered when the increase in quality that drives a spike in demand is modest. I also find that, while grandfather clauses are unambiguously harmful to total welfare, they have an ambiguous effect on consumer welfare. They help consumers when the increase in quality is relatively large, although there exists a range of parameters in which a firm would want to offer a grandfather clause and consumers would benefit on net from such an offer.

Bibliography

[1] Arbatskaya, Maria, Morten Hviid, and Greg Shaffer. 2006. On the use of low-price guarantees to discourage price cutting. *International Journal of Industrial Organization* 24:1139-1156.

[2] Astorne-Figari, Carmen, and Aleksandr Yankelevich. 2014. Consumer search with asymmetric price sampling. *Economics Letters* 122:331-333.

[3] Belton, Terrence M. 1987. A model of duopoly and meeting or beating competition. *International Journal of Industrial Organization* 5:399-417.

[4] Biswas, Abhijit, Chris Pullig, Mehmet I. Yagci, and Dwane H. Dean. 2002. Consumer evaluation of low price guarantees: The moderating role of reference price and store image. *Journal of Consumer Psychology* 12:107-118.

[5] Caplin, Andrew, and Barry Nalebuff. 1991. "Aggregation and Imperfect Competition: On the Existence of Equilibrium." Econometrica, 59(1): 25-59

[6] Chen, Yuxin, Chakravarthi Narasimhan, and Z. John Zhang. 2001. Consumer heterogeneity and competitive price-matching guarantees. *Marketing Science* 20:300-314.

[7] Corts, Kenneth S. 1996. On the competitive effects of price-matching policies. *International Journal of Industrial Organization* 15:283-299.

[8] C. d'Aspremont, J. J. Gabszewicz, and J.-F. Thisse, On Hotelling's "stability in competition", Econometrica 47 (1979), 1145-1150.

[9] De los Santos, Babur. 2012. Consumer search on the internet. Working Paper, Kelley School of Business, Indiana University.

[10] DelVecchio, Devon. 2005. Deal-prone consumers' response to promotion: The effects of relative and absolute promotion value. *Psychology & Marketing* 22:373-391.

[11] Doyle, Christopher. 1988. Different selling strategies in Bertrand oligopoly. *Economic Letters* 28:387-390.

[12] Dugar, Subhasish, and Todd Sorensen. 2006. Hassle costs, price-matching guarantees and price competition: an experiment. *Review of Industrial Organization* 28:359-378.

[13] Hay, George A. 1982. Oligopoly, shared monopoly, and antitrust law. *Cornell Law Review* 67:439-481.

[14] Hess, James D., and Eitan Gerstner. 1991. Price-matching policies: An empirical case. *Managerial and Decision Economics* 12:305-315.

[15] Hotelling, H., 1929, Stability in Competition, The Economic Journal, 153(39), 41-57.

[16] Hviid, Morten, and Greg Shaffer. 1999. Hassle costs: The Achilles heel of price-matching guarantees. *Journal of Economics and Management Strategy* 8:489-522.

[17] Hviid, Morten, and Greg Shaffer. 2010. Matching own prices, rivals' prices or both? *The Journal of Industrial Economics* 58:479-506.

[18] Irmen A. and J.-F. Thisse, 1998. Competition in multi-characteristics spaces: Hotelling was almost right. Journal of Economic Theory 78, 76-102.

[19] Jain, Sanjay, and Joydeep Srivastava. 2000. An experimental and theoretical analysis of price-matching refund policies. *Journal of Marketing Research* 37:351-362.

[20] Janssen, Maarten CW, Josè Luis Moraga-Gonzàlez, and Matthijs R. Wildenbeest. 2005. Truly costly sequential search and oligopolistic pricing. *International Journal of Industrial Organization* 23:451-466.

[21] Janssen, Maarten CW, and Marielle C. Non. 2008. Advertising and consumer search in a duopoly model. *International Journal of Industrial Organization* 26:354-371.

[22] Janssen, Maarten CW, and Alexei Parakhonyak. 2013. Price matching guarantees and consumer search. *International Journal of Industrial Organization* 31:1-11.

[23] Janssen, Maarten CW, and Alexei Parakhonyak. 2014. Consumer search markets with costly revisits. *Economic Theory* 55:481-514.

[24] Lancaster, K.J. (1966), A New Approach to Consumer Theory, Journal of Political Economy, 74: 132-157.

[25] Lichtenstein, Donald R., Richard G. Netemeyer, and Scot Burton. 1990. Distinguishing coupon proneness from value consciousness: An acquisition-transaction utility theory perspective. *The Journal of Marketing*, 54:54-67.

[26] Mago, Shakun Datta, and Jennifer G. Pate. 2009. An experimental examination of competitor-based price matching guarantees. *Journal of Economic Behavior & Organization* 70:342-360.

[27] Meurer, Michael, and Dale O. Stahl. 1994. Informative advertising and product match. *International Journal of Industrial Organization* 12:1-19.

[28] Moorthy, Sridhar, and Ralph A. Winter. 2006. Price-matching guarantees. *RAND Journal of Economics* 37:449-465.

[29] Moorthy, Sridhar, and Xubing Zhang. 2006. Price matching by vertically differentiated retailers: Theory and evidence. *Journal of Marketing Research* 43:156-167.

[30] Moraga-Gonzàlez, Josè Luis, and Matthijs R. Wildenbeest. 2008. Maximum likelihood estimation of search costs. *European Economic Review* 52:820-848.

[31] Narasimhan, Chakravarthi. 1988. Competitive promotional strategies. *Journal of Business* 61:427-449.

[32] Png, Ivan PL, and David Hirshleifer. 1987. Price discrimination through offers to match price. *Journal of Business* 60:365-383.

[33] Robert, Jacques, and Dale O. Stahl. 1993. Informative price advertising in a sequential search model. *Econometrica* 61:657-686.

[34] Salop, S.C. Monopolistic Competition with Outside Goods. Bell Journal of Economics, Vol. 10 (1979), pp. 141-156.

[35] Salop, Steven C. 1986. Practices that (credibly) facilitate oligopoly coordination. In *New Developments in the Analysis of Market Structure* edited by Joseph E. Stiglitz and G. Frank Mathewson. MIT Press, Cambridge, MA., p. 265.

[36] Srivastava, Joydeep, and Nicholas Lurie. 2001. A consumer perspective on price-matching refund policies: effect on price perceptions and search behavior. *Journal of Consumer Research* 28:296-307.

[37] Stahl, Dale O. 1989. Oligopolistic pricing with sequential consumer search. *American Economic Review* 79:700-712.

[38] Sims, C. A. (2003). Implications of rational inattention. Journal of Monetary Economics, 50(3), 665-690.

[39] Varian, Hal R. 1980. A model of sales. *American Economic Review* 70:651-659.

[40] Von Ungern-Sternberg, T., 1991, Monopolistic competition in the pyramid, The Journal of Industrial Economics, XXXIX (4), 355-368

[41] Weitzman, Martin L. 1979. Optimal search for the best alternative. *Econometrica* 47:641-654.

[42] Zhang, Z. John. 1995. Price-matching policy and the principle of minimum differentiation. *Journal of Industrial Economics* 43:287-299.

Appendices

A Proofs for Chapter 1

We first define some useful notation. As with shoppers, for non-shoppers who have searched both firms, we assume that $\theta_N \in [0,\ 1]$ will ignore price-matching guarantees and always purchase from the firm with the lower listed price. The remaining $1 - \theta_N$ will invoke a price-matching guarantee at the last firm they stopped in when one is available and necessary to obtain the lower price there and purchase from the firm with the lower listed price otherwise. Let $\alpha_{S(N)} \in \left[0,\ \theta_{S(N)}\right]$ be the proportion of shoppers (non-shoppers) who buy from the first firm they searched after having observed the same price listed in both firms.[1] Let γ be the proportion of non-shoppers who do not search after freely observing a price of r_j at firm i.

Definition 1. *We say that firms have a mutual mass point when each firm has a mass point at the same price. We say that firms have a mutual break when each firm's equilibrium support has a break over the same price interval.*

Proof of Proposition 1

The proposition follows directly from the proof of Claims 1A and 1B below and from Weitzman's (1979) Pandora's Rule, which implies that non-shoppers' reservation prices associated with each firm must be the same when consumers are indifferent regarding which firm's price to sample first. In particular, Pandora's Rule rules out support types 3 and 4 in Claims 1A and 1B. The proofs of Claims 1A and 1B follows in the same vein as the proofs of Propositions 2 through 5 in Narasimhan (1988). However, various complications

[1] The restriction $\alpha_{S(N)} \leq \theta_{S(N)}$ is used for mathematical tractability. It says that when a firm undercuts a tie, it cannot lose customers.

arise because consumers in our model follow an optimal search rule and firms have the ability to price-match. Therefore, in order to aid the reader, we first outline the intuition behind each of the five steps used to prove Claims 1A and 1B.

Step 1.1. $v \geq \min\{\bar{p}_1,\ \bar{p}_2\} \geq \underline{p}_1 = \underline{p}_2 = \underline{p} \geq 0$.

Proof intuition. A firm, i, that prices below its rival's lowest price, $\underline{p}_j$, captures the same number of consumers everywhere on $[\underline{p}_i, \underline{p}_j)$, such that its profit is increasing in price over this interval, a contradiction. Prices below zero result in negative profits. A firm i that prices above v can only profit from consumers who accept an offer to match a price no higher than v from firm j. Firm i cannot lose money from such consumers by shifting mass above v down to v, but now expects to make sales if firm j prices above v. □

Step 1.2. *There are no mutual mass points.*

Proof intuition. This claim follows via a standard mass point undercutting argument (in this case, borrowed from a draft version of Janssen and Non 2008). □

Step 1.3. *The only possible breaks in the equilibrium supports are:*

(i) If $\bar{p}_i < \bar{p}_j$, there is a break at $(\bar{p}_i,\ \bar{p}_j)$.

(ii) If $r = r_i = r_j < \bar{p}_i = \bar{p}_j$, there may be a mutual break with lower bound r.

(iii) If $r_i \neq r_j$ and firm i has a mass point at r_j, there may be a mutual break with lower bound r_j.

Proof intuition. Because a firm's price distribution function is constant over a break, in general, it's rival's profit will be higher at one end of the break than the other, or otherwise be increasing in price along the break, which cannot be the case in a mixed strategy equilibrium. The potential exceptions to this argument are items (i) to (iii) listed above. □

Step 1.4. *Firm i does not have a mass point in the lower bound or the interior of firm j's equilibrium support, except possibly at r_j.*

Proof intuition. The proof of this step follows in a similar fashion to that of Step 1.2, but relies on the convexity of firm supports away from r_j such that continuous mass above a rival mass point can be shifted below it for a gain in profit. □

Step 1.5. *If $\bar{p} = \bar{p}_1 = \bar{p}_2$ then either*

(i) $\bar{p} = \min\{v,\ r_1,\ r_2\}$*, the supports have no breaks, and at most one firm can have a mass point at $\bar{p}$, or*

(ii) $\bar{p} = \min\{v,\ max\{r_1,\ r_2\}\}$*, there is a mutual break above* $\min\{r_1,\ r_2\} < \bar{p}$*, firm i has a mass point at r_j, and firm j has a mass point at $\bar{p}$.*

Proof intuition. Using Steps 1.1 to 1.4, this step rules out item (ii) in Step 1.3 and places restrictions on item (iii). Additionally, when firms have the same convex support, this claim entails that $\bar{p} = \min\{v, r_1, r_2\}$. At $\bar{p}$, firm i only sells to those shoppers who invoke price-matching guarantees to attain firm j's price. When $\bar{p} < \min\{v, r_1, r_2\}$, firm i can increase profit by raising prices paid by captive non-shoppers without decreasing shopper profit. When $\bar{p} > \min\{v, r_1, r_2\}$, firm i can do better by lowering $\bar{p}$ to the point that it sells to non-shoppers. □

Step 1.5 allows us to narrow down the possible supports to item (i) in Step 1.3 and the two items in Step 1.5. By supposing that all consumers are indifferent regarding which firm to sample first, we can rely on Weitzman's (1979) Pandora's Rule to further narrow the supports to item (i) in Step 1.5. Finally, if firm i prices with a mass point at r_j in equilibrium, but some non-shoppers searched upon observing r_j, firm i would have an incentive to shift the mass point slightly below r_j, a contradiction. We next proceed to the complete proof.

Claim 1A. *Suppose that firms are exogenously required to offer price-matching guarantees and that $\theta_S \in (0,\ 1]$. In equilibrium, firms play mixed pricing strategies with $\underline{p}_1 = \underline{p}_2 = \underline{p} <$ $\min\{\bar{p}_1,\ \bar{p}_2\}$. The supports of the firm pricing distributions can only take one of the four following forms:*

1. *Completely symmetric, no breaks: $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v,\ r_1 = r_2\}$.*
2. *Single mass point, no breaks: firm i has a mass point at $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v,\ r_j\}$, $r_j \leq r_i$.*
3. *Two mass points, mutual break: firm j has a mass point at $r_i < \min\{v,\ r_j\}$; mutual break over $(r_i,\ p^u)$ for $p^u \in (r_i,\ \bar{p})$; $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v,\ r_j\}$, firm i has a mass point at $\bar{p}$.*
4. *Two mass points, single break: firm j has a mass point at $\bar{p}_j = r_i < \min\{v,\ r_j\}$; firm i has a break over $(r_i,\ \bar{p}_i)$ for $\bar{p}_i = \min\{v,\ r_j\}$ and a mass point at $\bar{p}_i$.*

The following steps complete the proof of Claim 1A.

Step 1A.1. $v \geq \min\{\bar{p}_1,\ \bar{p}_2\} \geq \underline{p}_1 = \underline{p}_2 = \underline{p} \geq 0$.

Proof. Suppose $\underline{p}_1 < \underline{p}_2 \leq v$. Then, for $p_1 \in \left[\underline{p}_1, \underline{p}_2\right)$, firm 1's expected profit is

$$\begin{aligned} &p_1 \{\mu [\theta_S + (1-\beta_S)(1-\theta_S)] \\ &+ (1-\mu)\{\beta_N + (1-\beta_N)\{[1-F_2(r_1)] + (1-\gamma)\Pr(p_2 = r_1)\}\}\} \end{aligned} \tag{19}$$

(because $\underline{p}_2 < r_2$ by definition), which is increasing in p_1, contradicting the equilibrium. If $\underline{p}_1 \leq v < \underline{p}_2$, for $p_1 \in \left[\underline{p}_1, v\right)$, firm 1's expected profit is given by Expression (19), which is increasing in p_1, so it must be the case that $\underline{p}_1 = v$. But if $v = \underline{p}_1 < \underline{p}_2$, then $F_1(v) = 1$ (because firm 1 does not make any profit at prices above v) and firm 2 expects profit of $\mu\beta_S(1-\theta_S)v$ everywhere on its support. For sufficiently small $\varepsilon > 0$, firm 2 benefits by shifting its mass to $v-\varepsilon$ for expected profit of $(v-\varepsilon)\{\mu[\theta_S + \beta_S(1-\theta_S)] + (1-\mu)(1-\beta_S)\}$.

Finally, if $v < \underline{p}_1 \leq \underline{p}_2$, then both firms make zero profits and either can increase profit by shifting mass to v, so $\underline{p}_2 \leq \underline{p}_1$. By a similar argument, $\underline{p}_2 \leq \underline{p}_1$ and $v \geq \underline{p}_1 = \underline{p}_2 = \underline{p}$. Because prices below zero result in negative profit, $\underline{p} \geq 0$.[2]

Suppose $v < \min\{\bar{p}_1,\ \bar{p}_2\}$. Then, for $p_i > v$, firm i expects no profit with probability $\Pr(p_j > v) > 0$ and because consumers never purchase at prices above v, firm i will only profit from consumers who accept its price-match offer after they had rejected a price no higher than v at firm j. Thus, firm i cannot lose money from such consumers by shifting mass above v down to v. However, by doing so, it now also expects to earn a positive profit with probability $\Pr(p_j > v)$, a contradiction. □

Step 1A.2. *There are no mutual mass points.*

Proof. Suppose that there is a mutual mass point at p. Firm 1's expected profit at p when firm 2 charges p as well is

$$\begin{aligned}
&p\,\{\mu\,[\beta_S\alpha_S + (1-\beta_S)(1-\alpha_S)] \\
&\quad + (1-\mu)\,\{\beta_N\,[\mathbb{I}_{p<r_2} + [\gamma + \alpha_N(1-\gamma)]\,\mathbb{I}_{p=r_2} + \alpha_N\mathbb{I}_{p>r_2}] \\
&\quad + (1-\beta_N)(1-\alpha_N)\,[(1-\gamma)\,\mathbb{I}_{p=r_1} + \mathbb{I}_{p>r_1}]\}\}
\end{aligned} \tag{20}$$

where $\mathbb{I}$ is an indicator function. Suppose instead that firm 1 deviates to $p-\varepsilon$ while firm 2 maintains its price at p. Firm 1's expected profit will be

$$\begin{aligned}
&(p-\varepsilon)\,\{\mu\,[\beta_S\theta_S + (1-\beta_S)] \\
&\quad + (1-\mu)\,\{\beta_N\,[\mathbb{I}_{p-\varepsilon<r_2} + [\gamma + \theta_N(1-\gamma)]\,\mathbb{I}_{p-\varepsilon=r_2} + \theta_N\mathbb{I}_{p-\varepsilon>r_2}] \\
&\quad + (1-\beta_N)\,[(1-\gamma)\,\mathbb{I}_{p=r_1} + \mathbb{I}_{p>r_1}]\}\}
\end{aligned} \tag{21}$$

Expression (20) is smaller than Expression (21) provided that ε is sufficiently small.

Suppose firm 2 chooses a price other than p. Lowering the price charged never reduces the number of sales so the loss to firm 1 from lowering the price by ε is at most ε. However, when p is charged with positive probability, lowering the price by ε will with positive prob-

[2] If $\underline{p} = 0$, then there must be zero density at $\underline{p} = 0$ because at $p_i = \varepsilon < \min\{r_j,\ v\}$, firm i will make money off its non-shoppers.

ability lead to a gain and with the complementary probability at worst lead to a loss of ε. Therefore, by shifting its mass point at p to $p-\varepsilon$ for sufficiently small ε firm 1 increases its expected profit, a contradiction. For the case $\alpha_S = \theta_S$, $\alpha_N = \theta_N$, and $\beta_N = \beta_S = 1$, firm 1 cannot profitably undercut the mutual mass point, but firm 2 can. □

Step 1A.3. *The only possible breaks in the equilibrium supports are:*

(i) If $\bar{p}_i < \bar{p}_j$, there is a break at $(\bar{p}_i,\ \bar{p}_j) \in S_j$.

(ii) If $r = r_i = r_j < \bar{p}_i = \bar{p}_j$, there may be a mutual break with lower bound r.

(iii) If $r_i \neq r_j$ and firm i has a mass point at r_j, there may be a mutual break with lower bound r_j.

Proof. Let S_1 and S_2 be respectively, the equilibrium supports for firms 1 and 2. Define $H = \left(p^d,\ p^u\right) \in \mathrm{int}(S_1 \cap S_2)$.

Suppose first, without loss of generality, that in equilibrium, firm 2 has no support over H, but that firm 1 does. Firm 1's expected profit at some $p_1 \in H$ is

$$\begin{aligned}
&\mu \left\{p_1 \left(\theta_S \beta_S + 1 - \beta_S\right)\left[1 - F_2\left(p_1\right)\right] + \left(1 - \theta_S\right)\left(1 - \beta_S\right) \mathrm{E}\left[p_2 | p_2 < p_1\right] F_2\left(p_1\right)\right\} \\
&+ (1-\mu)\left\{p_1 \beta_N \left\{\mathbb{I}_{p_1 < r_2} + \left\{\gamma + \theta_N (1-\gamma)\left[1 - F_2\left(p_1\right)\right]\right\} \mathbb{I}_{p_1 = r_2}\right.\right. \\
&\left. + \theta_N \left[1 - F_2\left(p_1\right)\right] \mathbb{I}_{p_1 > r_2}\right\} \\
&+ \left(1 - \beta_N\right)\left\{p_1 \left[1 - F_2\left(r_1\right) + (1-\gamma) \Pr\left(p_2 = r_1\right)\right] \mathbb{I}_{p_1 < r_1} + p_1 \left[1 - F_2\left(r_1\right)\right] \mathbb{I}_{p_1 = r_1}\right. \\
&+ \left\{p_1 \left[1 - F_2\left(p_1\right)\right] + r_1 \left(1 - \theta_N\right)(1-\gamma) \Pr\left(p_2 = r_1\right)\right. \\
&\left.\left.\left. + \left(1 - \theta_N\right) \mathrm{E}\left[p_2 | r_1 < p_2 < p_1\right]\left[F_2\left(p_1\right) - F_2\left(r_1\right)\right]\right\} \mathbb{I}_{p_1 > r_1}\right\}\right\}
\end{aligned} \tag{22}$$

As firm 1 raises p_1 along H, its expected profit is increasing because $F_2\left(p_1\right)$ is constant along H (and equal to $F_2\left(r_1\right)$ if $r_1 \in H$). Thus, if $r_2 \notin H$, firm 1 could increase expected profits by shifting all its mass in H slightly below p^u (to p^u if firm 2 does not have a mass point there), a contradiction. If $r_2 \in H$, firm 1 can increase expected profits by shifting all mass in $\left(p^d,\ r_2\right)$ slightly below r_2, and all mass in $\left(r_2,\ p^u\right)$ either slightly below r_2 or to p^u, again contradicting the equilibrium. A similar argument applies when firm 1 has no support over H, but firm 2 does. This tells us that any breaks in $S_1 \cap S_2$ are mutual.

Now suppose that neither firm randomizes over H in equilibrium. Suppose first that $p^d \neq r_1$, $p^d \neq r_2$ and that neither firm has a mass point at p^d. Then either firm 1 has a strictly higher expected profit at p^u (or slightly below r_2 if $r_2 \in H$) than at p^d, or firm 2 has a strictly higher expected profit at p^u (or slightly below r_1 if $r_1 \in H$) than at p^d, or possibly both, if neither firm has a mass point at p^u, contradicting the equilibrium.

Suppose that firm i has a mass point at $p^d \neq r_j$. Because there are no mutual mass points, firm i could increase profits by shifting its mass point to p^u (or slightly below p^u if firm j has a mass point there, or slightly below r_j if $r_j \in H$).

If $p^d = r_j \neq r_i$ and firm i has no mass point at p^d, firm j's expected profit will be strictly higher at p^u (or slightly below p^u if firm i has a mass point there, or slightly below r_i if $r_i \in H$) than at p^d. But if firm i does have a mass point at p^d, then it is possible that profits are the same at p^d and p^u for each firm. If $\gamma \neq 1$, firm i can profitably deviate by shifting its mass point slightly below p^d. In doing so, it retains $1-\gamma$ non-shoppers who search after observing a price of r_j and have a positive probability of purchasing from firm j. However, if $\gamma = 1$, neither firm may have a profitable deviation. This may also be the case if, $p^d = r_1 = r_2$.

From Step 1A.1, we know that both S_1 and S_2 have the same lower bound, $\underline{p}$, so $S_1 \Delta S_2 \in (\min\{\bar{p}_1,\ \bar{p}_2\},\ \max\{\bar{p}_1,\ \bar{p}_2\}]$. Suppose, without loss of generality, that $\bar{p}_1 > \bar{p}_2$. At $p_1 \in (\bar{p}_2,\ \bar{p}_1]$, firm 1's expected profit is

$$\begin{aligned} &\mu (1-\theta_S)(1-\beta_S)\,\mathrm{E}[p_2] + (1-\mu)\{p_1\beta_N (\mathbb{I}_{p_1<r_2} + \gamma \mathbb{I}_{p_1=r_2})\,\mathbb{I}_{p_1\le v} \\ &+ (1-\theta_N)(1-\beta_N)\{\mathrm{E}[p_2|r_1<p_2][1-F_2(r_1)] + r_1(1-\gamma)\Pr(p_2=r_1)\}\} \end{aligned} \tag{23}$$

If $\bar{p}_2 < r_2$, then for $\beta_N \neq 0$, firm 1's expected profit is increasing in p_1 along $(\bar{p}_2,\ \min\{v,\ r_2\})$ and is strictly greater anywhere in $(\bar{p}_2,\ \min\{v,\ r_2\})$ than at any price above $\min\{v,\ r_2\}$. As a result, for $\varepsilon > 0$ sufficiently small, firm 1 can increase profit by shifting mass in

$(\bar{p}_2,\ \bar{p}_1]$ to $\min\{v,\ r_2\}-\varepsilon$ (likewise if $\bar{p}_2 = \min\{v,\ r_2\}$). Therefore, when $\bar{p}_2 \leq r_2$, either $S_1\Delta S_2 = \{\bar{p}_1\} = \{\min\{v,\ r_2\}\}$, or $S_1\Delta S_2 = \varnothing$. Suppose $S_1\Delta S_2 = \{\bar{p}_1\}$. If firm 2 has no mass point at $\bar{p}_2$, this means that firm 1's expected profit at $\bar{p}_1$ is strictly higher than its expected profit at $\bar{p}_2$, a contradiction. If firm 2 has a mass point at $\bar{p}_2 \neq r_1$, Because there are no mutual mass points, firm 2 can profitably shift the mass point to slightly below $\bar{p}_1$ (or slightly below r_1 if $r_1 \in (\bar{p}_2,\ \bar{p}_1]$). However, if $\gamma = 1$, firm 2 has a mass point at $\bar{p}_2 = r_1$, and $F_1(r_1)$ is large enough, then neither firm may have a profitable deviation. Following the proof of Step 1A.5, we will discuss why an equilibrium where $r_2 < \bar{p}_2 < \bar{p}_1$ cannot exist. A similar argument applies when $\bar{p}_2 > \bar{p}_1$. $\square$

Corollary 1A.1. *The equilibrium supports are the same except if* $\bar{p}_i = r_j < \bar{p}_j = \min\{v,\ r_i\}$.

Step 1A.4. *Firm i does not have a mass point in the lower bound or the interior of firm j's equilibrium support, except possibly at r_j.*

Proof. Suppose, without loss of generality, that firm 2 has a mass point at $p \in S_1\backslash\{\bar{p}_1\}$, and suppose that $p \neq r_1$. Firm 1's expected profit at $p-\varepsilon$ when firm 2 charges p is given by Expression (21), whereas its expected profit at $p+\varepsilon$ is

$$\begin{aligned}&\mu p(1-\theta_S)(1-\beta_S)+(1-\mu)\{(p+\varepsilon)\beta_N(\mathbb{I}_{p+\varepsilon<r_2}+\gamma\mathbb{I}_{p+\varepsilon=r_2})\\&+p(1-\theta_N)(1-\beta_N)[(1-\gamma)\mathbb{I}_{p=r_1}+\mathbb{I}_{p>r_1}]\}\end{aligned} \tag{24}$$

Expression (24) is smaller than Expression (21) provided that ε is sufficiently small. Suppose firm 2 chooses a price other than p. Lowering the price charged never reduces the number of sales so the loss to firm 1 from lowering the price by 2ε or less is at most 2ε. However, when p is charged with positive probability, lowering the price by 2ε or less will with positive probability lead to a gain and with the complementary probability at worst lead to a loss of 2ε. Therefore, by shifting its mass between p and $p+\varepsilon$ to $p-\varepsilon$ for sufficiently small ε, firm 1 increases its expected profit, a contradiction. By a similar argument, firm 1 cannot have a mass point at $p \in S_2\backslash\{\bar{p}_2\}$, except possibly if $p = r_2$. $\square$

Step 1A.5. *If $\bar{p} = \bar{p}_1 = \bar{p}_2$ then either*

(i) $\bar{p} = \min\{v,\ r_1,\ r_2\}$*, the supports have no breaks, and at most one firm can have a mass point at* $\bar{p}$*, or*

(ii) $\bar{p} = \min\{v,\ \max\{r_1,\ r_2\}\}$*, there is a mutual break above* $\min\{r_1,\ r_2\} < \bar{p}$*, firm i has a mass point at r_j, and firm j has a mass point at* $\bar{p}$*.*

Proof. Suppose that $\bar{p} = \bar{p}_1 = \bar{p}_2$ and neither firm has a mass point at $\bar{p}$. From Steps 1A.1 and 1A.4 we know that $\underline{p} < \bar{p} \leq v$. Suppose, without loss of generality, that $\bar{p} < \min\{v,\ r_2\}$. At $\bar{p}$, firm 1's expected profit is given by Expression (23) (with $p_1 = \bar{p}$), which is increasing in p_1 along $(\bar{p},\ \min\{v,\ r_2\})$ when $\beta_N \neq 0$, a contradiction. Suppose instead that $\bar{p} > \min\{v,\ r_2\} = r_2$. For any $p_1 \in (r_2,\ \bar{p})$, in equilibrium, $\mathrm{E}\,\pi_1(\bar{p}) = \mathrm{E}\,\pi_1(p_1,\ F_2(p_1))$. $\mathrm{E}\,\pi_1(\bar{p})$ is given by Expression (23) (with $p_1 = \bar{p}$). If $r_2 \geq r_1$, for $p_1 \in (r_2,\ \bar{p})$, $\mathrm{E}\,\pi_1(p_1,\ F_2(p_1))$ equals

$$\begin{aligned}&\mu\{p_1(\theta_S\beta_S + 1 - \beta_S)[1 - F_2(p_1)] + (1-\theta_S)(1-\beta_S)\,\mathrm{E}[p_2|p_2 < p_1]\,F_2(p_1)\}\\&+(1-\mu)\{p_1\beta_N\theta_N[1-F_2(p_1)] + (1-\beta_N)\{r_1(1-\theta_N)(1-\gamma)\Pr(p_2 = r_1)\\&+p_1[1-F_2(p_1)] + (1-\theta_N)\,\mathrm{E}[p_2|r_1 < p_2 < p_1][F_2(p_1) - F_2(r_1)]\}\}\end{aligned} \tag{25}$$

Setting Expression (23) equal to Expression (25) and differentiating with respect to p_1 yields

$$\begin{aligned}&[\mu(\theta_S\beta_S + 1 - \beta_S) + (1-\mu)(\theta_N\beta_N + 1 - \beta_N)][1 - F_2(p_1)]\\&-[\mu\theta_S + (1-\mu)\theta_N]\,p_1F_2{}'(p_1) = 0\end{aligned} \tag{26}$$

Solving the differential equation given by Equation (26) using the initial value $F_2(\bar{p}) = 1$ gives us $F_2(p_1) = 1$ for all $p_1 \in (r_2,\ \bar{p}]$, a contradiction. Similarly, if $r_1 \in (r_2,\ \bar{p})$, then Expression (25) represents firm 1's expected profit at $(r_1,\ \bar{p})$ and $F_2(p_1) = 1$ for all $p_1 \in (r_1,\ \bar{p}]$, a contradiction. If on the other hand, $r_1 \geq \bar{p}$, $\mathrm{E}\,\pi_1(\bar{p})$ becomes $\mu(1-\theta_S)(1-\beta_S)\,\mathrm{E}[p_2]$ while $\mathrm{E}\,\pi_1(p_1,\ F_2(p_1))$ at $p_1 \in (r_2,\ \bar{p})$ becomes

$$\begin{aligned}&\mu\{p_1(\theta_S\beta_S + 1 - \beta_S)[1 - F_2(p_1)] + (1-\theta_S)(1-\beta_S)\,\mathrm{E}[p_2|p_2 < p_1]\,F_2(p_1)\}\\&+(1-\mu)\,\beta_N\theta_Np_1[1-F_2(p_1)]\end{aligned} \tag{27}$$

Setting $\mu(1-\theta_S)(1-\beta_S)\,\mathrm{E}[p_2]$ equal to Expression (27) and solving the resulting differential equation using the initial value $F_2(\bar{p}) = 1$ again gives us $F_2(p_1) = 1$ for all $p_1 \in (r_2,\ \bar{p}]$, a contradiction. Hence, for $\beta_N \neq 0$, $\bar{p} = \min\{v,\ r_2\}$. By a similar argument, for $\beta_N \neq 1$, $\bar{p} = \min\{v,\ r_1\}$, so when neither firm has a mass point at $\bar{p}$, $\bar{p} = \min\{v,\ r_1,\ r_2\}$. From Step 1A.2, we know that at most one firm can have a mass point at $\bar{p}$, say firm j. If $\gamma = 1$ or $v < r_i$, then following the argument in the paragraph above, $\bar{p} = \min\{v,\ r_i\}$. Otherwise, firm j cannot have a mass point at $\bar{p}$ (using reasoning similar to that in the proof of Step 1A.3). Moreover, if $r_j \geq r_i$, then $\bar{p} = \min\{v,\ r_1,\ r_2\}$ and from Step 1A.3, we know that the firm supports have no breaks. Conversely, suppose $r_j < r_i$ (and therefore, $r_j < v$). Without loss of generality, let $i = 1$. From Step 1A.4, we know that firm 2 cannot have a mass point at r_2. At r_2, firm 1 expects profit of

$$\begin{aligned}&\mu\{r_2(\theta_S\beta_S + 1 - \beta_S)[1 - F_2(r_2)] + \\ &(1-\theta_S)(1-\beta_S)\,\mathrm{E}[p_2|p_2 < r_2]\,F_2(r_2)\} + (1-\mu)\,\beta_N r_2\end{aligned} \tag{28}$$

whereas at $p_1 \in (r_2,\ \bar{p})$, $\mathrm{E}\,\pi_1(p_1,\ F_2(p_1))$ is given by Expression (27). By definition, for $p_1 \in (r_2,\ \bar{p})$, $0 < F_2(r_2) \leq F_2(p_1)$, so for p_1 close enough to r_2, Expression (28) is strictly greater than Expression (27). Therefore, r_2 must be the lower bound for a break in S_1 and we are in Case (iii) of Step 1A.3). $\square$

Notice that Step 1A.5 rules out Case (ii) in Step 1A.3. Moreover, following the same procedure used in Step 1A.5, it is easy to show that an equilibrium where $r_j < \bar{p}_j < \bar{p}_i$ cannot exist. In particular, by setting $\mathrm{E}\,\pi_i(\bar{p}_i) = \mathrm{E}\,\pi_i(p_i,\ F_j(p_i))$ for $p_i \in (r_j,\ \bar{p}_j]$ and solving for F_j, we see that $F_j(p_i) = 1$ for all $p_i \in (r_j,\ \bar{p}_j]$, a contradiction.

Claim 1A: Support Type 4. Without loss of generality, suppose that β_N is such that $\bar{p}_1 = r_2 < \min\{v,\ r_1\} = \bar{p}_2$. Then a complete solution to an equilibrium with support type 4, if one exists, requires the following set of equations to hold.

$$\begin{aligned}&\mathrm{E}\,\pi_1\left(\underline{p}\right)=\mathrm{E}\,\pi_1\left(p_1,F_2\left(p_1\right)\right)\;\Leftrightarrow\underline{p}\left[\mu\left(\beta_S\theta_S+1-\beta_S\right)+\left(1-\mu\right)\beta_N\right]\\&=\;\mu\left\{p_1\left(\beta_S\theta_S+1-\beta_S\right)\left[1-F_2\left(p_1\right)\right]\right.\\&+\;\left.\left(1-\theta_S\right)\left(1-\beta_S\right)\mathrm{E}\left[p_2|p_2<p_1\right]F_2\left(p_1\right)\right\}+\left(1-\mu\right)\beta_Np_1\end{aligned}\tag{29}$$

$$\begin{aligned}&\mathrm{E}\,\pi_1\left(\underline{p}\right)=\mathrm{E}\,\pi_1\left(r_2,F_2\left(r_2\right)\right)\;\Leftrightarrow\underline{p}\left[\mu\left(\beta_S\theta_S+1-\beta_S\right)+\left(1-\mu\right)\beta_N\right]\\&=\;\mu\left\{r_2\left(\beta_S\theta_S+1-\beta_S\right)\Pr\left(p_2=\bar{p}_2\right)\right.\\&+\;\left.\left(1-\theta_S\right)\left(1-\beta_S\right)\mathrm{E}\left[p_2|p_2<r_2\right]F_2\left(r_2\right)\right\}+\left(1-\mu\right)\beta_Nr_2\end{aligned}\tag{30}$$

$$\begin{aligned}&\mathrm{E}\,\pi_2\left(\underline{p}\right)=\mathrm{E}\,\pi_2\left(p_2,F_1\left(p_2\right)\right)\\&\Leftrightarrow\underline{p}\left\{\mu\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]+\left(1-\mu\right)\left(1-\beta_N\right)\right\}=\left(1-\mu\right)\left(1-\beta_N\right)p_2\\&+\mu\left\{p_2\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]\left[1-F_1\left(p_2\right)\right]+\left(1-\theta_S\right)\beta_S\,\mathrm{E}\left[p_1|p_1<p_2\right]F_1\left(p_2\right)\right\}\end{aligned}\tag{31}$$

$$\begin{aligned}&\mathrm{E}\,\pi_2\left(\underline{p}\right)=\mathrm{E}\,\pi_2\left(\bar{p}_2\right)\;\Leftrightarrow\underline{p}\left\{\mu\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]+\left(1-\mu\right)\left(1-\beta_N\right)\right\}\\&=\mu\left(1-\theta_S\right)\beta_S\,\mathrm{E}\left[p_1\right]+\left(1-\mu\right)\left(1-\beta_N\right)\bar{p}_2\end{aligned}\tag{32}$$

$$\int_{\underline{p}}^{r_2}F_1(p)\mathrm{d}p+r_1-r_2=c\tag{33}$$

$$\int_{\underline{p}}^{r_2}F_2(p)\mathrm{d}p=c\tag{34}$$

$$\Pr\left(p_1=r_2\right)=1-\lim_{\varepsilon\to0-}F_1\left(r_2-\varepsilon\right)\in\left(0,\;1\right)\tag{35}$$

In addition to Equations (29) to (35), firm 2 must have an expected price which is strictly lower than that of firm 1. Moreover, the inequality, $\mathrm{E}\,\pi_1\left(r_2,\;F_2\left(r_2\right)\right)>\mathrm{E}\,\pi_1\left(\bar{p}_2-\varepsilon,\;F_2\left(\bar{p}_2-\varepsilon\right)\right)$ must hold for all $\varepsilon\in\left(0,\;\bar{p}_2-r_2\right)$. That is, firm 1 must not wish to deviate above r_2.

We can use the following procedure to look for equilibrium. First, we use Equation (29) and (31) to solve for F_2 and F_1 respectively, in terms of $\underline{p}$. Plugging F_2 into Equation (34) and using Equation (30) to solve for $\underline{p}$ we obtain r_2 in terms of $\Pr\left(p_2=\bar{p}_2\right)$. Plugging F_1 into Equation (35) yields $\Pr\left(p_1=r_2\right)$ in terms of $\Pr\left(p_2=\bar{p}_2\right)$. Rewriting F_1 in terms of $\Pr\left(p_2=\bar{p}_2\right)$ and plugging into Equation (33) yields r_1 in terms of $\Pr\left(p_2=\bar{p}_2\right)$. Finally, using Equation (32) to solve for $\underline{p}$ and setting this equal to the solution obtained from Equation (30) we can rewrite r_1 as an alternate function of $\Pr\left(p_2=\bar{p}_2\right)$. Setting the two expressions for r_1 equal to each other, we can now solve for $\Pr(p_2 = \bar{p}_2)$ in terms of the exogenous parameters. An equilibrium exists only if there is a solution to $\Pr(p_2 = \bar{p}_2)$ in the interval $[0,\ 1]$ such that non-shoppers strictly prefer to search firm 2 first and firm 1 does not wish to deviate above r_2.

Claim 1B. *Suppose that firm 1 is exogenously required to offer price-matching guarantees while firm 2 is required not to. In equilibrium, firms play mixed pricing strategies with* $\underline{p}_1 = \underline{p}_2 = \underline{p} < \min\{\bar{p}_1,\ \bar{p}_2\}$. *The supports of the firm pricing distributions can only take one of the four following forms:*

1. *Completely symmetric, no breaks:* $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v,\ r_1 = r_2\}$.

2. *Single mass point, no breaks: firm i has a mass point at* $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v,\ r_j\}$, $r_j \leq r_i$.

3. *Two mass points, mutual break: firm j has a mass point at* $r_i < \min\{v,\ r_j\}$*; mutual break over* $(r_i,\ p^u)$ *for* $p^u \in (r_i,\ \bar{p})$*;* $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v,\ r_j\}$*, firm i has a mass point at* $\bar{p}$.

4. *Two mass points, single break: firm j has a mass point at* $\bar{p}_j = r_i < \min\{v,\ r_j\}$*; firm i has a break over* $(r_i,\ \bar{p}_i)$ *for* $\bar{p}_i = \min\{v,\ r_j\}$ *and a mass point at* $\bar{p}_i$.

The following steps complete the proof of Claim 1B.

Step 1B.1. $v \geq \min\{\bar{p}_1,\ \bar{p}_2\} \geq \underline{p}_1 = \underline{p}_2 = \underline{p} \geq 0$.

Proof. Suppose $\underline{p}_1 < \underline{p}_2 \leq v$. Then, for $p_1 \in \left[\underline{p}_1, \underline{p}_2\right)$, firm 1's expected profit is

$$p_1 \left\{\mu + (1-\mu)\left\{\beta_N + (1-\beta_N)\left\{[1 - F_2(r_1)] + (1-\gamma)\Pr(p_2 = r_1)\right\}\right\}\right\} \tag{36}$$

which is increasing in p_1, contradicting the equilibrium. If $\underline{p}_1 \leq v < \underline{p}_2$, for $p_1 \in \left[\underline{p}_1, v\right)$, firm 1's expected profit is given by Expression (36), which is increasing in p_1, so it must be the case that $\underline{p}_1 = v$. But if $v = \underline{p}_1 < \underline{p}_2$, firm 2 makes no profit on its support and for sufficiently small $\varepsilon > 0$, it benefits by shifting its mass to $v - \varepsilon$. Finally, if $v < \underline{p}_1 \leq \underline{p}_2$, then both firms make zero profits and either can increase profit by shifting mass to v, so $\underline{p}_2 \leq \underline{p}_1$. Suppose $\underline{p}_2 < \underline{p}_1 \leq v$. Then, for $p_2 \in \left[\underline{p}_2, \underline{p}_1\right)$, firm 2's expected profit is

$$\begin{aligned} &p_2 \left\{\mu \left[\theta_S + \beta_S \left(1 - \theta_S\right)\right]\right. \\ &\quad + \left(1 - \mu\right) \left\{\left(1 - \beta_N\right) + \beta_N \left\{\left[1 - F_1\left(r_2\right)\right] + \left(1 - \gamma\right) \Pr\left(p_1 = r_2\right)\right\}\right\}\right\} \end{aligned} \tag{37}$$

(because $\underline{p}_1 < r_1$ by definition) which is increasing in p_2, again contradicting the equilibrium. If $\underline{p}_2 \leq v < \underline{p}_1$, for $p_2 \in \left[\underline{p}_2, v\right)$, firm 2's expected profit is given by Expression (37), which is increasing in p_2, so it must be the case that $\underline{p}_2 = v$. But if $v = \underline{p}_2 < \underline{p}_1$, then $F_1(v) = 1$ (because firm 2 does not make any profit at prices above v) and firm 1 expects profit of $\mu \left(1 - \beta_S\right) \left(1 - \theta_S\right) v$ everywhere on its support. For sufficiently small $\varepsilon > 0$, firm 1 benefits by shifting its mass to $v - \varepsilon$ for expected profit of $\left(v - \varepsilon\right) \left\{\mu \left[\theta_S + \left(1 - \beta_S\right) \left(1 - \theta_S\right)\right] + \left(1 - \mu\right) \beta_S\right\}$ Thus, $v \geq \underline{p}_1 = \underline{p}_2 = \underline{p}$. Because prices below zero result in negative profit, $\underline{p} \geq 0$.[3]
The proof that $v \geq \min\left\{\bar{p}_1,\ \bar{p}_2\right\}$ follows precisely that in Step 1A.1. □

Step 1B.2. *There are no mutual mass points.*

Proof. Suppose that there is a mutual mass point at p. Firm 1's expected profit at p when firm 2 charges p as well is given by Expression (20). Suppose instead that firm 1 deviates to $p - \varepsilon$ while firm 2 maintains its price at p. Firm 1's expected profit will be

$$\left(p - \varepsilon\right) \left\{\mu + \left(1 - \mu\right) \left\{\beta_N + \left(1 - \beta_N\right) \left[\left(1 - \gamma\right) \mathbb{I}_{p = r_1} + \mathbb{I}_{p > r_1}\right]\right\}\right\} \tag{38}$$

Expression (20) is smaller than Expression (38) provided that ε is sufficiently small.
Suppose firm 2 chooses a price other than p. Lowering the price charged never reduces the number of sales so the loss to firm 1 from lowering the price by ε is at most ε. However, when p is charged with positive probability, lowering the price by ε will with positive prob-

[3] If $\underline{p} = 0$, then there must be zero density at $\underline{p} = 0$ because at $p_i = \varepsilon < \min\left\{r_j,\ v\right\}$, firm i will make money off its non-shoppers.

ability lead to a gain and with the complementary probability at worst lead to a loss of ε. Therefore, by shifting its mass point at p to $p-\varepsilon$ for sufficiently small ε firm 1 increases its expected profit, a contradiction. $\square$

Step 1B.3. *The only possible breaks in the equilibrium supports are:*

(i) If $\bar{p}_i < \bar{p}_j$, there is a break at $(\bar{p}_i,\ \bar{p}_j) \in S_j$.

(ii) If $r = r_i = r_j < \bar{p}_i = \bar{p}_j$, there may be a mutual break with lower bound r.

(iii) If $r_i \neq r_j$ and firm i has a mass point at r_j, there may be a mutual break with lower bound r_j.

Proof. Let S_1 and S_2 be respectively, the equilibrium supports for firms 1 and 2. Define $H = \left(p^d,\ p^u\right) \in \operatorname{int}(S_1 \cap S_2)$.

Suppose first that in equilibrium, firm 2 has no support over H, but that firm 1 does. Firm 1's expected profit at some $p_1 \in H$ is

$$\begin{aligned}
&\mu \left\{p_1 \left[1 - F_2\left(p_1\right)\right] + \left(1-\theta_S\right)\left(1-\beta_S\right) \mathrm{E}\left[p_2 | p_2 < p_1\right] F_2\left(p_1\right)\right\} \\
&+ (1-\mu)\left\{p_1 \beta_N \left\{\mathbb{I}_{p_1<r_2} + \left\{\gamma + (1-\gamma)\left[1-F_2\left(p_1\right)\right]\right\} \mathbb{I}_{p_1=r_2} + \left[1-F_2\left(p_1\right)\right] \mathbb{I}_{p_1>r_2}\right\}\right. \\
&+ \left(1-\beta_N\right)\left\{p_1\left[1-F_2\left(r_1\right)+(1-\gamma)\Pr\left(p_2=r_1\right)\right] \mathbb{I}_{p_1<r_1} + p_1\left[1-F_2\left(r_1\right)\right] \mathbb{I}_{p_1=r_1}\right. \\
&+ \left\{p_1\left[1-F_2\left(p_1\right)\right] + r_1\left(1-\theta_N\right)(1-\gamma)\Pr\left(p_2=r_1\right)\right. \\
&\left.\left.\left.+\left(1-\theta_N\right)\mathrm{E}\left[p_2 | r_1 < p_2 < p_1\right]\left[F_2\left(p_1\right)-F_2\left(r_1\right)\right]\right\}\mathbb{I}_{p_1>r_1}\right\}\right\}
\end{aligned} \tag{39}$$

As firm 1 raises p_1 along H, its expected profit is increasing because $F_2\left(p_1\right)$ is constant along H (and equal to $F_2\left(r_1\right)$ if $r_1 \in H$). Thus, if $r_2 \notin H$, firm 1 could increase expected profits by shifting all its mass in H slightly below p^u (to p^u if firm 2 does not have a mass point there), a contradiction. If $r_2 \in H$, firm 1 can increase expected profits by shifting all mass in $\left(p^d,\ r_2\right)$ slightly below r_2, and all mass in $\left(r_2,\ p^d\right)$ either slightly below r_2 or to p^u, again contradicting the equilibrium.

Conversely, suppose that firm 1 has no support over H, but that firm 2 does. Firm 2's expected profit at some $p_2 \in H$ is

$$
\begin{aligned}
&\mu p_2\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]\left[1-F_1\left(p_2\right)\right]+\left(1-\mu\right)p_2\\
&\times\left\{\left(1-\beta_N\right)\left\{\mathbb{I}_{p_2<r_1}+\left\{\gamma+\theta_N\left(1-\gamma\right)\left[1-F_1\left(p_2\right)\right]\right\}\mathbb{I}_{p_2=r_1}\right.\right.\\
&\left.+\theta_N\left[1-F_1\left(p_2\right)\right]\mathbb{I}_{p_2>r_1}\right\}\\
&+\beta_N\left\{\left[1-F_1\left(r_2\right)+\left(1-\gamma\right)\Pr\left(p_1=r_2\right)\right]\mathbb{I}_{p_2<r_2}\right.\\
&\left.\left.+\left[1-F_1\left(r_2\right)\right]\mathbb{I}_{p_2=r_2}+\left[1-F_1\left(p_2\right)\right]\mathbb{I}_{p_2>r_2}\right\}\right\}
\end{aligned}
\tag{40}
$$

As firm 2 raises p_2 along H, its expected profit is increasing because $F_1(p_2)$ is constant along H (and equal to $F_1(r_2)$ if $r_2 \in H$). Thus, if $r_1 \notin H$, firm 2 could increase expected profits by shifting all its mass in H slightly below p^u (to p^u if firm 2 does not have a mass point there), a contradiction. If $r_1 \in H$, firm 2 can increase expected profits by shifting all mass in $\left(p^d,\ r_1\right)$ slightly below r_1, and all mass in $(r_1,\ p^u)$ either slightly below r_1 or to p^u, again contradicting the equilibrium. Thus, any breaks in $S_1 \cap S_2$ are mutual.

The remainder of this proof follows similarly to that in Step 1A.3. □

Corollary 1B.1. *The equilibrium supports are the same except if* $\bar{p}_i = r_j < \bar{p}_j = \min\{v,\ r_i\}$.

Step 1B.4. *Firm i does not have a mass point in the lower bound or the interior of firm j's equilibrium support, except possibly at r_j.*

Proof. The proof to show that firm 2 does not have a mass point at $p \in S_1 \backslash \{\bar{p}_1\}$ proceeds precisely as that in Step 1A.4. Suppose instead that firm 1 has a mass point at $p \in S_2 \backslash \{\bar{p}_2\}$ and that $p \neq r_2$. Firm 2's expected profit at $p-\varepsilon$ when firm 1 charges p is

$$
\begin{aligned}
&(p-\varepsilon)\left\{\mu\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]+\left(1-\mu\right)\right.\\
&\times\left\{\left(1-\beta_N\right)\left[\mathbb{I}_{p-\varepsilon<r_1}+\left[\gamma+\theta_N\left(1-\gamma\right)\right]\mathbb{I}_{p-\varepsilon=r_1}+\theta_N\mathbb{I}_{p-\varepsilon>r_1}\right]\right.\\
&\left.\left.+\beta_N\left[\left(1-\gamma\right)\mathbb{I}_{p=r_2}+\mathbb{I}_{p>r_2}\right]\right\}\right\}
\end{aligned}
\tag{41}
$$

whereas its expected profit at $p+\varepsilon$ is

$$
\left(1-\mu\right)\left(p+\varepsilon\right)\left(1-\beta_N\right)\left(\mathbb{I}_{p+\varepsilon<r_1}+\gamma\mathbb{I}_{p+\varepsilon=r_1}\right) \tag{42}
$$

Expression (42) is smaller than Expression (41) provided that ε is sufficiently small. Suppose firm 1 chooses a price other than p. Lowering the price charged never reduces the number of sales so the loss to firm 2 from lowering the price by 2ε or less is at most 2ε. However, when p is charged with positive probability, lowering the price by 2ε or less will with positive probability lead to a gain and with the complementary probability at worst lead to a loss of 2ε. Therefore, by shifting its mass between p and $p+\varepsilon$ to $p-\varepsilon$ for sufficiently small ε, firm 2 increases its expected profit, a contradiction. $\square$

Step 1B.5. *If $\bar{p}=\bar{p}_1=\bar{p}_2$ then either*

(i) $\bar{p}=\min\{v,\ r_1,\ r_2\}$*, the supports have no breaks, and at most one firm can have a mass point at* $\bar{p}$*, or*

(ii) $\bar{p}=\min\{v,\ \mathit{max}\{r_1,\ r_2\}\}$*, there is a mutual break above* $\min\{r_1,\ r_2\}<\bar{p}$*, firm i has a mass point at r_j, and firm j has a mass point at* $\bar{p}$*.*

Proof. Suppose that $\bar{p}=\bar{p}_1=\bar{p}_2$ and neither firm has a mass point at $\bar{p}$. From Steps 1B.1 and 1B.4 we know that $\underline{p}<\bar{p}\leq v$. Suppose that $\bar{p}<\min\{v,\ r_2\}$. At $p_1\in[\bar{p},\ \min\{v,\ r_2\})$, firm 1's expected profit is

$$\begin{aligned}&\mu\left(1-\theta_S\right)\left(1-\beta_S\right)\mathrm{E}\left[p_2\right]\\&+\ \left(1-\mu\right)\left\{\beta_N p_1+\left(1-\theta_N\right)\left(1-\beta_N\right)\left\{\mathrm{E}\left[p_2|r_1<p_2\right]\left[1-F_2\left(r_1\right)\right]\right.\right.\\&\left.\left.+\,r_1\left(1-\gamma\right)\Pr\left(p_2=r_1\right)\right\}\right\}\end{aligned}\tag{43}$$

which is increasing in p_1 when $\beta_N\neq 0$, a contradiction. Suppose instead that $\bar{p}>\min\{v,\ r_2\}=r_2$. For any $p_1\in(r_2,\ \bar{p})$, in equilibrium, $\mathrm{E}\,\pi_1(\bar{p})=\mathrm{E}\,\pi_1(p_1,\ F_2(p_1))$. $\mathrm{E}\,\pi_1(\bar{p})$ equals

$$\begin{aligned}&\mu\left(1-\theta_S\right)\left(1-\beta_S\right)\mathrm{E}\left[p_2\right]\\&+\ \left(1-\mu\right)\left(1-\theta_N\right)\left(1-\beta_N\right)\left\{\mathrm{E}\left[p_2|r_1<p_2\right]\left[1-F_2\left(r_1\right)\right]\right.\\&\left.+\,r_1\left(1-\gamma\right)\Pr\left(p_2=r_1\right)\right\}\end{aligned}\tag{44}$$

If $r_2\geq r_1$, for $p_1\in(r_2,\ \bar{p})$, $\mathrm{E}\,\pi_1(p_1,\ F_2(p_1))$ equals

$$\begin{aligned}&\mu\left\{p_1\left[1-F_2\left(p_1\right)\right]+\left(1-\theta_S\right)\left(1-\beta_S\right)\mathrm{E}\left[p_2|p_2<p_1\right]F_2\left(p_1\right)\right\}\\&+\ \left(1-\mu\right)\left\{p_1\beta_N\left[1-F_2\left(p_1\right)\right]+\left(1-\beta_N\right)\left\{r_1\left(1-\theta_N\right)\left(1-\gamma\right)\Pr\left(p_2=r_1\right)\right.\right.\\&\left.\left.+\,p_1\left[1-F_2\left(p_1\right)\right]+\left(1-\theta_N\right)\mathrm{E}\left[p_2|r_1<p_2<p_1\right]\left[F_2\left(p_1\right)-F_2\left(r_1\right)\right]\right\}\right\}\end{aligned}\tag{45}$$

Setting Expression (44) equal to Expression (45) and differentiating with respect to p_1 yields

$$1-F_2\left(p_1\right)-\left[\mu\left(\theta_S+\beta_S-\theta_S\beta_S\right)+\left(1-\mu\right)\left(\theta_N+\beta_N-\theta_N\beta_N\right)\right]p_1F_2{}'\left(p_1\right)=0\tag{46}$$

Solving the differential equation given by Equation (46) using the initial value $F_2\left(\bar{p}\right)=1$ gives us $F_2\left(p_1\right)=1$ for all $p_1\in\left(r_2,\ \bar{p}\right]$, a contradiction. Similarly, if $r_1\in\left(r_2,\ \bar{p}\right)$, then Expression (45) represents firm 1's expected profit at $\left(r_1,\ \bar{p}\right)$ and $F_2\left(p_1\right)=1$ for all $p_1\in\left(r_1,\ \bar{p}\right]$, a contradiction. If on the other hand, $r_1\geq\bar{p}$, $\mathrm{E}\,\pi_1\left(\bar{p}\right)$ becomes $\mu\left(1-\theta_S\right)\left(1-\beta_S\right)\mathrm{E}\left[p_2\right]$ while $\mathrm{E}\,\pi_1\left(p_1,\ F_2\left(p_1\right)\right)$ at $p_1\in\left(r_2,\ \bar{p}\right)$ becomes

$$\begin{aligned}&\mu\left\{p_1\left[1-F_2\left(p_1\right)\right]+\left(1-\theta_S\right)\left(1-\beta_S\right)\mathrm{E}\left[p_2|p_2<p_1\right]F_2\left(p_1\right)\right\}\\&+\ \left(1-\mu\right)\beta_Np_1\left[1-F_2\left(p_1\right)\right]\end{aligned}\tag{47}$$

Setting $\mu\left(1-\theta_S\right)\left(1-\beta_S\right)\mathrm{E}\left[p_2\right]$ equal to Expression (47) and solving the resulting differential equation using the initial value $F_2\left(\bar{p}\right)=1$ again gives us $F_2\left(p_1\right)=1$ for all $p_1\in\left(r_2,\ \bar{p}\right]$, a contradiction. Hence, for $\beta_N\neq0$, $\bar{p}=\min\left\{v,\ r_2\right\}$. Now suppose that $\bar{p}<\min\left\{v,\ r_1\right\}$. At $p_2\in[\bar{p},\ \min\left\{v,\ r_1\right\})$, firm 2's expected profit is $\left(1-\mu\right)\left(1-\beta_N\right)p_2$, which is increasing in p_2 when $\beta_N\neq1$, a contradiction. Suppose instead, that $\bar{p}>\min\left\{v,\ r_1\right\}$. But then, at $\bar{p}$, firm 2 expects no profit, a contradiction. Thus, $\bar{p}=\min\left\{v,\ r_1\right\}$, so when neither firm has a mass point at $\bar{p}$, $\bar{p}=\min\left\{v,\ r_1,\ r_2\right\}$.

From Step 1B.2, we know that at most one firm can have a mass point at $\bar{p}$, say firm j. If $\gamma=1$ or $v<r_i$, then following the argument in the paragraph above, $\bar{p}=\min\left\{v,\ r_i\right\}$. Otherwise, firm j cannot have a mass point at $\bar{p}$ (using reasoning similar to that in the proof of Step 1B.3). Moreover, if $r_j\geq r_i$, then $\bar{p}=\min\left\{v,\ r_1,\ r_2\right\}$ and from Step 1B.3, we know that the firm supports have no breaks. Conversely, suppose $r_j<r_i$ (and therefore,

$r_j < v$). First, let $i = 1$. From Step 1B.4, we know that firm 2 cannot have a mass point at r_2. At r_2, firm 1 expects profit of

$$\mu \left\{ r_2 \left[1 - F_2 \left(r_2 \right) \right] + \left(1 - \theta_S \right) \left(1 - \beta_S \right) \mathrm{E} \left[p_2 | p_2 < r_2 \right] F_2 \left(r_2 \right) \right\} + \left(1 - \mu \right) \beta_N r_2 \tag{48}$$

whereas at $p_1 \in (r_2,\ \bar{p})$, $\mathrm{E}\,\pi_1 \left(p_1,\ F_2 \left(p_1 \right) \right)$ is given by Expression (47). By definition, for $p_1 \in (r_2,\ \bar{p})$, $0 < F_2 \left(r_2 \right) \leq F_2 \left(p_1 \right)$, so for p_1 close enough to r_2, Expression (48) is strictly greater than Expression (47). Therefore, r_2 must be the lower bound for a break in S_1 and we are in Case (iii) of Step 1B.3. Now let $j = 1$. From Step 1B.4, we know that firm 1 cannot have a mass point at r_1. At r_1, firm 2 expects profit of

$$r_1 \left\{ \mu \left[\left(1 - \beta_S \right) \theta_S + \beta_S \right] \left[1 - F_1 \left(r_1 \right) \right] + \left(1 - \mu \right) \left(1 - \beta_N \right) \right\} \tag{49}$$

whereas at $p_2 \in (r_1,\ \bar{p})$, $\mathrm{E}\,\pi_2 \left(p_2,\ F_1 \left(p_2 \right) \right)$ is given by

$$p_2 \left\{ \mu \left[\left(1 - \beta_S \right) \theta_S + \beta_S \right] + \left(1 - \mu \right) \left(1 - \beta_N \right) \theta_N \right\} \left[1 - F_1 \left(p_2 \right) \right] \tag{50}$$

By definition, for $p_2 \in (r_1,\ \bar{p})$, $0 < F_1 \left(r_1 \right) \leq F_1 \left(p_2 \right)$, so for p_2 close enough to r_1, Expression (49) is strictly greater than Expression (50). Therefore, r_1 must be the lower bound for a break in S_2 and we are again in Case (iii) of Step 1B.3. $\square$

Notice that Step 1B.5 rules out Case (ii) in Step 1B.3.

Proof of Proposition 2

Proof. In equilibrium, a firm must be indifferent between any price in its support. Therefore, for any p_i in the support of F_j, $\mathrm{E}\,\pi_i \left(\underline{p} \right) = \mathrm{E}\,\pi_i \left(p_i,\ F_j \left(p_i \right) \right)$. Then for firm 1, the profit equality condition is given by

$$\begin{aligned} &\mu \left\{ p_1 \left(\beta_S \theta_S + 1 - \beta_S \right) \left[1 - F_2 \left(p_1 \right) \right] + \left(1 - \theta_S \right) \left(1 - \beta_S \right) \mathrm{E} \left[p_2 | p_2 < p_1 \right] F_2 \left(p_1 \right) \right\} \\ &+ \left(1 - \mu \right) \beta_N p_1 = \underline{p} \left[\mu \left(\beta_S \theta_S + 1 - \beta_S \right) + \left(1 - \mu \right) \beta_N \right] \end{aligned} \tag{51}$$

Differentiating Equation (51) with respect to p_1 and rearranging gives

$$\mu\left\{\left(\beta_S\theta_S+1-\beta_S\right)F_2\left(p_1\right)+\theta_S p_1 F_2'\left(p_1\right)-\beta_S\theta_S-1+\beta_S\right\}-\left(1-\mu\right)\beta_N=0 \tag{52}$$

Solving the differential equation given by Equation (52) using the initial value $F_2\left(\underline{p}\right)=0$ gives

$$F_2\left(p\right)=\left[1+\frac{\left(1-\mu\right)\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)}\right]\left[1-\left(\frac{\underline{p}}{p}\right)^{\frac{\beta_S\theta_S+1-\beta_S}{\theta_S}}\right] \tag{53}$$

We can similarly solve for $F_1\left(p\right)$ to get

$$F_1\left(p\right)=\left\{1+\frac{\left(1-\mu\right)\left(1-\beta_N\right)}{\mu\left[\left(1-\beta_S\right)\theta_S+\beta_S\right]}\right\}\left[1-\left(\frac{\underline{p}}{p}\right)^{\frac{\left(1-\beta_S\right)\theta_S+\beta_S}{\theta_S}}\right] \tag{54}$$

Without loss of generality, suppose that firm 1 is the one with the mass point at $\bar{p}$. Setting $F_2\left(\bar{p}\right)=1$ to solve for $\underline{p}$ in terms of $\bar{p}$ gives

$$\underline{p}=\bar{p}\left[\frac{\left(1-\mu\right)\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)+\left(1-\mu\right)\beta_N}\right]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}} \tag{55}$$

Substituting Equation (55) into Equation (53) gives

$$\begin{aligned}F_2\left(p\right)=&\left[1+\frac{\left(1-\mu\right)\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)}\right]\\&\times\left[1-\frac{\left(1-\mu\right)\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)+\left(1-\mu\right)\beta_N}\left(\frac{\bar{p}}{p}\right)^{\frac{\beta_S\theta_S+1-\beta_S}{\theta_S}}\right]\end{aligned} \tag{56}$$

When $r_2\leq v$, $\bar{p}=r_2$. Optimal search requires that Equation (2.1) holds. Substituting Equation (56) into Equation (2.1) yields

$$\begin{aligned}&\left[1+\frac{\left(1-\mu\right)\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)}\right]\\&\times\int_{\underline{p}}^{r_2}\left[1-\frac{\left(1-\mu\right)\beta_N}{\mu\left(\beta_S\theta_S+1-\beta_S\right)+\left(1-\mu\right)\beta_N}\left(\frac{r_2}{p}\right)^{\frac{\beta_S\theta_S+1-\beta_S}{\theta_S}}\right]\mathrm{d}p=c\end{aligned} \tag{57}$$

Integrating to solve for r_2 in terms of μ, θ_S, c, β_S and β_N, we get

$$\begin{aligned}r_2\left(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N\right)=c&\left\{1-\frac{\left[\left(1-\mu\right)\beta_N\right]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}}}{\mu\left(1-\beta_S\right)\left(1-\theta_S\right)}\right.\\\times&\left.\left\{\left[\mu\left(\beta_S\theta_S+1-\beta_S\right)+\left(1-\mu\right)\beta_N\right]^{\frac{\left(1-\beta_S\right)\left(1-\theta_S\right)}{1-\beta_S\left(1-\theta_S\right)}}-\left[\left(1-\mu\right)\beta_N\right]^{\frac{\left(1-\beta_S\right)\left(1-\theta_S\right)}{1-\beta_S\left(1-\theta_S\right)}}\right\}\right\}^{-1}.\end{aligned} \tag{58}$$

By assumption, non-shoppers are indifferent between which firm to sample first. Weitzman's (1979) Pandora's Rule implies

$$\begin{aligned}&\int_{\underline{p}}^{r_2} F_2(p)\mathrm{d}p = \int_{\underline{p}}^{r_2} F_1(p)\mathrm{d}p \\ \Leftrightarrow &\int_{\underline{p}}^{r_2} p\mathrm{d}F_2(p) = \lim_{x\to r_2-}\int_{\underline{p}}^{x} p\mathrm{d}F_1(p) + \bar{p}\left[1-\lim_{x\to r_2-} F_1(x)\right]\end{aligned} \tag{59}$$

The first equation follows from Pandora's Rule and non-shoppers' indifference (so $r_2 = r_1 = r$); the second, which sets the expected price of the two firms equal to each other, follows from integration by parts together with the fact that $\bar{p}$ $=\min\{v,\ r\}$ in equilibrium. By setting the expected price of firm 1 equal to that of firm 2, we can solve for β_N in terms of β_S and the other parameters. To obtain an expression for the expected price of each firm we proceed as in Janssen, Moraga-Gonzàlez, and Wildenbeest (2005). For firm 2, we solve for p using Equation (56) to get

$$p = \bar{p}\left\{\frac{(1-\mu)\beta_N}{(1-\mu)\beta_N + \mu(\beta_S\theta_S + 1 - \beta_S)[1-F_2(p)]}\right\}^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}} \tag{60}$$

Using a change of variables with $u = F_2(p)$, we can write $\mathrm{E}_2[p] = \int_0^1 p\mathrm{d}u$. Substituting in p from Equation (60) gives

$$\mathrm{E}_2[p] = \bar{p}\int_0^1 \left[\frac{(1-\mu)\beta_N}{(1-\mu)\beta_N + \mu(\beta_S\theta_S + 1 - \beta_S)(1-u)}\right]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}} \mathrm{d}u \tag{61}$$

Integrating and rearranging yields

$$\begin{aligned}\mathrm{E}_2[p] = &\frac{\bar{p}[(1-\mu)\beta_N]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}}}{\mu(1-\beta_S)(1-\theta_S)} \\ &\times\left\{[\mu(\beta_S\theta_S+1-\beta_S)+(1-\mu)\beta_N]^{\frac{(1-\beta_S)(1-\theta_S)}{1-\beta_S(1-\theta_S)}} - [(1-\mu)\beta_N]^{\frac{(1-\beta_S)(1-\theta_S)}{1-\beta_S(1-\theta_S)}}\right\}\end{aligned} \tag{62}$$

Proceeding similarly for firm 1, we get

$$
\begin{aligned}
rcl\mathrm{E}_1[p] = \quad & \bar{p}\left[1-\lim_{x\to\bar{p}-}F_1(x)\right] \\
& +\frac{\bar{p}\{\mu[(1-\beta_S)\theta_S+\beta_S]+(1-\mu)(1-\beta_N)\}^{\frac{\theta_S}{(1-\beta_S)\theta_S+\beta_S}}}{\mu\beta_S(1-\theta_S)} \\
& \times\left[\frac{(1-\mu)\beta_N}{\mu(\beta_S\theta_S+1-\beta_S)+(1-\mu)\beta_N}\right]^{\frac{\theta_S}{\beta_S\theta_S+1-\beta_S}} \\
& \times\Bigg\{\{\mu[(1-\beta_S)\theta_S+\beta_S]+(1-\mu)(1-\beta_N)\}^{\frac{\beta_S(1-\theta_S)}{(1-\beta_S)\theta_S+\beta_S}} \\
& -\left\{\mu[(1-\beta_S)\theta_S+\beta_S]\left[1-\lim_{x\to\bar{p}-}F_1(x)\right]+(1-\mu)(1-\beta_N)\right\}^{\frac{\beta_S(1-\theta_S)}{(1-\beta_S)\theta_S+\beta_S}}\Bigg\}
\end{aligned}
\tag{63}
$$

where

$$
\begin{aligned}
\lim_{x\to\bar{p}-}F_1(x) = & \left\{1+\frac{(1-\mu)(1-\beta_N)}{\mu[(1-\beta_S)\theta_S+\beta_S]}\right\} \\
& \times\left\{1-\left[\frac{(1-\mu)\beta_N}{\mu(\beta_S\theta_S+1-\beta_S)+(1-\mu)\beta_N}\right]^{\frac{(1-\beta_S)\theta_S+\beta_S}{\beta_S\theta_S+1-\beta_S}}\right\}
\end{aligned}
\tag{64}
$$

We can now implicitly solve for β_N as a function of the remaining parameters using $\mathrm{E}_1[p]=\mathrm{E}_2[p]$.

Define r_1^* and r_2^* as the equilibrium reservation prices. If $r_2(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N)\le v$, r_2^* is defined by Equation (58). Because firms are not concerned with prices above v, if $r_2(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N)>v$, we define r_2^* as positive infinity. According to Equation (59), we can set $r_1^*=r_2^*$. $\square$

Proof of Proposition 3

Proof. We proceed to prove Part (2) of the proposition first:

2. Plugging $\beta_S=\beta_N=1/2$ into Equation (61) gives

$$
\mathrm{E}[p]=r^*\int_0^1\left[\frac{1-\mu}{1-\mu+\mu(1+\theta_S)(1-u)}\right]^{\frac{2\theta_S}{1+\theta_S}}\mathrm{d}u \tag{65}
$$

where $r^*=\bar{p}$ by assumption. To see that r^* is increasing in c, observe that the integrand in Equation (65) is less than 1 for all values of of $u \in [0,\ 1)$, $\mu \in (0,\ 1)$, and $\theta_S \in (0,\ 1]$. Because the limits of integration are 0 and 1, this implies that $\mathrm{E}[p] < r^*$ and $r^* > 0$. Thus, the derivative of r^* with respect to c is clearly positive as well. The derivatives of the right hand side integrand in Equation (65) with respect to μ and θ_S are both negative, implying that $\mathrm{E}[p]/r^*$ is decreasing in μ and θ_S and according to Equation (2.1) (rewritten as $r^* - \mathrm{E}[p] = c$ using $\bar{p} = \min\{v, r^*\}$), so is r^*.

1. Define $F(\mu, \theta_S, c; p)$ as the equilibrium distribution function when $\bar{p} = r^* < v$ and $F(\mu, \theta_S, v; p)$ as the equilibrium distribution function when $\bar{p} = v < r^*$. From Equation (56) and Part (2), we have

$$\begin{aligned}\frac{\partial F(\mu, \theta_S, c; p)}{\partial \mu} = \frac{1}{1+\theta_S} \Bigg\{ & \frac{1}{\mu^2}\left[\left(\frac{r^*}{p}\right)^{\frac{1+\theta_S}{2\theta_S}} - 1\right] \\ & - \frac{(1+\theta_S)(1-\mu)}{2\mu\theta_S p}\left(\frac{r^*}{p}\right)^{\frac{1-\theta_S}{2\theta_S}} \frac{\partial r^*}{\partial \mu} \Bigg\} > 0,\end{aligned} \tag{66}$$

$$\frac{\partial F(\mu, \theta_S, v; p)}{\partial \mu} = \frac{1}{\mu^2(1+\theta_S)}\left[\left(\frac{v}{p}\right)^{\frac{1+\theta_S}{2\theta_S}} - 1\right] \geq 0. \tag{67}$$

$$\begin{aligned}\frac{\partial F(\mu, \theta_S, c; p)}{\partial \theta_S} = \frac{1-\mu}{\mu(1+\theta_S)^2} \Bigg\{ & -1 + \left(\frac{r^*}{p}\right)^{\frac{1+\theta_S}{2\theta_S}} \Bigg\{ 1 + \\ & \frac{1+\theta_S}{2{\theta_S}^2}\left[\ln\left(\frac{r^*}{p}\right) - \frac{(1+\theta_S)\theta_S}{r^*}\frac{\partial r^*}{\partial \theta_S}\right]\Bigg\}\Bigg\} > 0,\end{aligned} \tag{68}$$

$$\begin{aligned}\frac{\partial F(\mu, \theta_S, v; p)}{\partial \theta_S} = & \frac{1-\mu}{\mu(1+\theta_S)^2} \\ & \times \left\{\left(\frac{v}{p}\right)^{\frac{1+\theta_S}{2\theta_S}}\left[1 + \ln\left(\frac{v}{p}\right)\frac{1+\theta_S}{2{\theta_S}^2}\right] - 1\right\} \geq 0.\end{aligned} \tag{69}$$

The inequalities in Equations (67) and (69) are strict for all $p \in [0,\ v)$. □

Proof of Proposition 4

Proof. We begin by showing that $\underline{p} \geq \min\{v, r_1, r_2\}$. Suppose conversely that $\underline{p} < \min\{v, r_1, r_2\}$. At any $p_1 \in (\underline{p}, \min\{v, r_1, r_2\})$, firm 1 captures $1-\beta_S$ shoppers, who pay $\min\{p_1,\ p_2\}$, and β_N, $\beta_N + (1-\gamma)(1-\beta_N)$, or all non-shoppers, depending on whether firm 2 prices below, at, or above r_1, where γ is the proportion of non-shoppers who do not search after freely observing a price of r_1 at firm 2. Non-shoppers who buy from firm 1 end up paying p_1. Thus, firm 1's profit is increasing in p_1, a contradiction.

It is straightforward to show that $v \geq \min\{\bar{p}_1,\ \bar{p}_2\} \geq \underline{p} \geq \min\{v,\ r_1,\ r_2\}$.[4] Moreover, from Equation (2.1) we know that $\min\{r_1,\ r_2\} > \underline{p}$, so $\min\{\bar{p}_1,\ \bar{p}_2\} = v = \underline{p}$. Suppose, without loss of generality, that $\bar{p}_2 > \bar{p}_1$. At any $p_2 \in (\bar{p}_1,\ \bar{p}_2]$, firm 2 expects profit of $\mu\beta_S v$. By shifting its mass in $(\bar{p}_1,\ \bar{p}_2]$ to v, firm 2 expects an additional profit of $(1-\mu)(1-\beta_N)v$, a contradiction. Using a similar argument we can rule out $\bar{p}_1 > \bar{p}_2$. Thus, $v = \bar{p}_1 = \bar{p}_2 = \bar{p} = \underline{p}$. Because the unique equilibrium is symmetric and employs pure strategies, we can define $F_1(p) = F_2(p) = F(p)$ as 0 for $p < v$ and as 1 for $p \geq v$. But then, using Equation (2.1) we get $r_1^* = r_2^* = v + c$. $\square$

Proof of Proposition 5

Proof. This proof follows very similarly to that of Proposition 2. In equilibrium, a firm must be indifferent between any price in its support. Therefore, for any p_i in the support of F_j, $\mathrm{E}\,\pi_i(\underline{p}) = \mathrm{E}\,\pi_i(p_i,\ F_j(p_i))$. Differentiating this profit equality with respect to p_i for $i = 1,\ 2$, rearranging, and solving the ensuing differential equation gives us F_1 and F_2 in the statement of the proposition.

A comparison of F_1 and F_2 will reveal that when $\beta_N < 1/2$, it must be that $\lim_{x \to \bar{p}-} F_2(x) < 0$, such that firm 2 is the one with the mass point at $\bar{p}$. As discussed in the body of the

[4] See Claim 1A, Step 1A.1, which holds for all θ_S.

article, when $\beta_S = 1/2$, consumer indifference regarding the first sample requires $\beta_N < 1/2$. Therefore, we may set $F_1(\bar{p}) = 1$ to solve for $\underline{p}$ in terms of $\bar{p}$. Substituting into F_1 gives

$$F_1(p) = \left\{1 + \frac{(1-\mu)(1-\beta_N)}{\mu\left[(1-\beta_S)\theta_S + \beta_S\right]}\right\} \left\{1 - \frac{(1-\mu)(1-\beta_N)}{(1-\mu)(1-\beta_N) + \mu\left[(1-\beta_S)\theta_S + \beta_S\right]} \frac{\bar{p}}{p}\right\} \tag{70}$$

When $r_1 \leq v$, $\bar{p} = r_1$. Optimal search requires that Equation (2.1) holds. Substituting Equation (70) into Equation (2.1) and integrating in order to solve for r_1 in terms of μ, θ_S, c, β_S and β_N, we get $r(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N)$ in the statement of the proposition.

By assumption, non-shoppers are indifferent between which firm to sample first. As per the proof of Proposition 2, Weitzman's (1979) Pandora's Rule implies that $r_1 = r_2 = r$ and $\mathrm{E}_1[p] = \mathrm{E}_2[p]$ in equilibrium. Using the expected price equality, we can now implicitly solve for β_N as a function of the remaining parameters. Following the same procedure as in Proposition 2, we obtain the following expressions for firms' expected prices:

$$\mathrm{E}_1[p] = \frac{\bar{p}(1-\mu)(1-\beta_N)}{\mu\left[(1-\beta_S)\theta_S + \beta_S\right]} \ln\left\{1 + \frac{\mu\left[(1-\beta_S)\theta_S + \beta_S\right]}{(1-\mu)(1-\beta_N)}\right\} \tag{71}$$

and

$$\begin{aligned}
\mathrm{E}_2[p] &= \bar{p}\left[1 - \lim_{x \to \bar{p}-} F_2(x)\right] \\
&+ \bar{p}\left\{\frac{(1-\mu)(1-\beta_N)}{\mu\left[(1-\beta_S)\theta_S + \beta_S\right] + (1-\mu)(1-\beta_N)}\right\} \frac{\left[\mu + (1-\mu)\beta_N\right]^{\beta_S + \theta_S - \beta_S\theta_S}}{\mu\left(1 + \beta_S\theta_S - \theta_S - \beta_S\right)} \\
&\times \left\{\left[\mu + (1-\mu)\beta_N\right]^{1+\beta_S\theta_S - \theta_S - \beta_S}\right. \\
&\left. - \left\{\mu\left[1 - \lim_{x \to \bar{p}-} F_2(x)\right] + (1-\mu)\beta_N\right\}^{1+\beta_S\theta_S - \theta_S - \beta_S}\right\}
\end{aligned} \tag{72}$$

where

$$\begin{aligned}
\lim_{x \to \bar{p}-} F_2(x) &= \left[1 + \frac{(1-\mu)\beta_N}{\mu}\right] \\
&\times \left\{1 - \left\{\frac{(1-\mu)(1-\beta_N)}{\mu\left[(1-\beta_S)\theta_S + \beta_S\right] + (1-\mu)(1-\beta_N)}\right\}^{\frac{1}{\beta_S + \theta_S - \beta_S\theta_S}}\right\}
\end{aligned} \tag{73}$$

Define r_1^* and r_2^* as the equilibrium reservation prices. If $r_1(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N)\ \leq\ v$, r_1^* is defined by $r(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N)$ in the statement of the proposition. Because firms are not concerned with prices above v, if $r_1(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N)\ >\ v$, we define r_1^* as positive infinity. According to Equation (59), we can set $r_1^* = r_2^*$. □

Proof of Proposition 6

Proof. The crux of this proof relies on the fact that for all $A\ \in\ (0,\ \infty)$, the function $A\ln(1+1/A)$, is strictly increasing in A. Moreover, $\lim_{A\to 0} A\ln(1+1/A) = 0$ and $\lim_{A\to\infty} A\ln(1+1/A) = 1$. We again prove Part (2) of the proposition first.

2. Let $A = [2(1-\mu)(1-\beta_N)]/[\mu(1+\theta_S)]$. Substituting $\beta_S = 1/2$ into $r(\mu,\ \theta_S,\ c,\ \beta_S,\ \beta_N)$ in Proposition 5 yields $r(\cdot) = c/[1 - A\ln(1+1/A)]$. This expression is clearly positive and we can see that it is increasing in A. Moreover, it can be seen that $r(\cdot)$ is decreasing (increasing) in μ or θ_S as A decreases (increases) in μ or θ_S. Some straightforward algebraic manipulation of $\partial A/\partial\mu$ and $\partial A/\partial\theta_S$ completes the proof.

1. Define $F_1(\mu,\theta_S,c;p)$ as the equilibrium distribution function when $\bar{p} = r^* < v$ and $F_1(\mu,\theta_S,v;p)$ as the equilibrium distribution function when $\bar{p} = v < r^*$. Additionally, substituting $\beta_S = 1/2$ and $\bar{p}$ into the expression for $F_1(p)$ in Proposition 5 and manipulating algebraically gives us $F_1(p) = 1 + A(1-\bar{p}/p)$.

   Suppose that $\bar{p} = v < r^*$. Because $1 - \bar{p}/p \leq 0$ for all p, $F_1(\mu,\theta_S,v;p)$ is decreasing in A (and strictly so for all $p\ \in\ [0,\ v)$). From the proof of Part (2), it follows that $F_1(\mu,\theta_S,v;p)$ is increasing in μ if and only if $\frac{\partial\beta_N}{\partial\mu}\ >\ -\frac{1-\beta_N}{\mu(1-\mu)}$ and increasing in θ_S if and only if $\frac{\partial\beta_N}{\partial\theta_S} > -\frac{1-\beta_N}{1+\theta_S}$.

   Suppose instead that $\bar{p}\ =\ r^*\ <\ v$. Because r^* is increasing in A, for any $p\ \in\ [0,\ r^*)$, $1\ -\ \bar{p}/p$ strictly falls in A, becoming more negative. As in the paragraph above, the remainder of the proof is now a direct consequence of the proof of Part (2).

Weitzman's (1979) Pandora's Rule (see in particular, Equation (59)) then implies the relationships regarding expected prices. □

Proof of Proposition 7

Proof. This proof follows very similarly to that of Proposition 5, but with β_N set to one or zero as appropriate.

1. Solving the usual profit equality condition, $\mathrm{E}\,\pi_i\left(\underline{p}\right) = \mathrm{E}\,\pi_i\left(p_i,\ F_j\left(p_i\right)\right)$ for F_1 and F_2 yields

$$F_1\left(p\right) = 1 - \frac{\underline{p}}{p} < \frac{1}{\mu}\left[1 - \left(\frac{\underline{p}}{p}\right)^{\frac{2}{1+\theta_S}}\right] = F_2\left(p\right) \tag{74}$$

for all $p \in \left(\underline{p}, \bar{p}\right)$. This implies that firm 1 has a higher reservation price and expected price, contradicting the assumption that non-shoppers prefer to sample it first.

2. The solution to the profit equality condition now gives us F_1 and F_2 in the second part of the statement of the proposition. Comparison of F_1 and F_2 will reveal that when $\beta_N = 0$, it must be that $\lim\limits_{x \to \bar{p}-} F_2\left(x\right) < 0$, such that firm 2 is the one with the mass point at $\bar{p}$. Therefore, we may set $F_1(\bar{p}) = 1$ to solve for $\underline{p}$ in terms of $\bar{p}$. Substituting into F_2 gives

$$F_2\left(p\right) = 1 - \left\{\frac{2(1-\mu)\bar{p}}{[2-\mu(1-\theta_S)]p}\right\}^{\frac{2}{1+\theta_S}} \tag{75}$$

When $r_2 \leq v$, $\bar{p} = r_2$. Optimal search requires that Equation (2.1) holds. Substituting Equation (75) into Equation (2.1) and integrating in order to solve for r_2 in terms of μ, θ_S, and c, we get $r_2\left(\mu,\ \theta_S,\ c\right)$ in the statement of the proposition. Define r_2^* as the equilibrium reservation price. If $r_2(\mu,\ \theta_S,\ c) \le v$, r_2^* is defined by $r_2(\mu,\ \theta_S,\ c)$. Because firms are not concerned with prices above v, if $r_2(\mu,\ \theta_S,\ c) > v$, we define r_2^* as positive infinity. □

B Proofs for Chapter2

Proof of Lemma 1

Lemma1: $\pi^i(\mathbf{p})$ is differentiable at $\forall_i, p_i = p$.

Proof. I want to show that, if a firm expects all other firms to charge a uniform price, that firm's profit function is everywhere differentiable in its own price. First note that $\pi(\mathbf{p}) = (p_1)V(S_1(\mathbf{p}))$. Since the product of two differentiable functions is differentiable, then it suffices to show that $V(S_1(\mathbf{p}))$ is differentiable in p_1. $S_1(\mathbf{p})$ is equal to the intersection of the sets of consumers that prefer firm 1 to firm j, for each j. Suppose that all other firms charge a uniform price p. The region $S_1(\mathbf{p})$ is given by the set of x satisfying constraints (76) and (77).

$$\forall_i, 0 \le x_i \tag{76}$$

$$\forall_{j\ge 1}, 0 \le p - p_1 + t\sum_{k=1}^{n}(z_k^j - x_k)^2 - t\sum_{k=1}^{n}(z_k^1 - x_k)^2 \tag{77}$$

Define $\alpha = p - p_1$. Suppose that $p_1 > p$ and $\alpha < 0$. From Lemma 2, only type one constraints bind. Then, $S_1(\mathbf{p})$ is merely a hyperrectangle with side length $\frac{\delta_i t+\alpha}{2t}$.

$$\text{If } \alpha < 0, \quad V(S_1(\mathbf{p})) = \prod_{i=1}^{n} \frac{\delta_i t + \alpha}{2t} := F(\alpha)$$

$F(\alpha)$ is clearly differentiable in both alpha and p_1. For the other case, suppose that $p_1 < p$. Every type of constraint will bind in this case. Once more consider the hypercube defined by the type 1 constraints. Let

$$A^j := \{x \in \mathbb{R}^n | x_i \leq \frac{\delta_i t + \alpha}{2t} \forall_i \text{ and } 0 \geq \alpha + t \sum_{i \in I_j} \delta_j - 2\delta_j x_j\}$$

This is the set of points that constrant j excludes from the region defined by the type 1 constraints. Consider a constraint from firm j of type k, $0 \leq \alpha + t \sum_{i=1}^{k} \delta_j - 2\delta_j x_j$. To find the measure of A^j, it will be helpful to define some notation. Let $\bar{x}_i = \frac{\delta_i t + \alpha}{2\delta_i t}$, so type one constraints are merely of the form $0 \leq \bar{x}_i$. Let $\sigma_i = \sum_{j=i+1}^{k} (1 - 2x_j)\delta_j$ if $i < k$ and zero else. Then the measure of A^j is given by the following integral:

$$\int_0^{\bar{x}_n} \ldots \int_{\frac{1}{2}}^{\bar{x}_k} \ldots \int_{\frac{\delta_i + \sigma_i}{2\delta_i}}^{\bar{x}_i} \ldots \int_{\frac{\alpha + \delta_1 t + t\sigma_1}{2\delta_1 t}}^{\bar{x}_1} 1 \, \mathrm{d}x_1 \, \mathrm{d}x_2 \ldots \mathrm{d}x_n$$

Since neither the constant function being integrated nor any of the bounds of integration depend on x_{k+1} through x_n, their only contribution to the integral will be to multiply by a constant. So the above reduces to

$$(\prod_{i=k+1}^{n} \bar{x}_i) \int_{\frac{1}{2}}^{\bar{x}_k} \ldots \int_{\frac{\delta_i + \sigma_i}{2\delta_i}}^{\bar{x}_i} \ldots \int_{\frac{\alpha + \delta_1 t + t\sigma_1}{2\delta_1 t}}^{\bar{x}_1} 1 \, \mathrm{d}x_1 \, \mathrm{d}x_2 \ldots \mathrm{d}x_k$$

The region A^j is, geometrically speaking, a hyperpyramidal hyperprism. That is, it is a k dimensional hyperpyramid that has been prismed into n-k other dimensions. Picturing the regions A^j for a three dimensional cube may make it clearer why this must be the case, and why such a shape would lead to the above integral. Evaluating the integrals yields a much simpler expression:

$$\frac{(k-1)^{k-1} \alpha^k}{2^k k! t^k \prod_{i=1}^{k} \delta_i} \prod_{i=k+1}^{n} (\frac{\alpha + t\delta_i}{2t\delta_i})$$

It should be noted that $V(A^j) = V(A^i)$ if firms i and j are both type k. This is true because the hypercube may be rotated to move firm i into the position of firm j while preserving firm zero at the origin. Lastly, note that there are $\binom{n}{k}$ type k firms. Consider the function

$$H(\alpha) := \sum_{j=2}^{2^n} V(A^j) = \sum_{k=2}^{n} \binom{n}{k} \frac{(k-1)^{k-1}\alpha^k}{2^k k! t^k \prod_{i=1}^{k} \delta_i} \prod_{i=k+1}^{n} (\frac{\alpha + t\delta_i}{2t\delta_i})$$

$H'(\alpha) \geq 0$ by inspection. If $\alpha > 0$, then $V(S_1(\mathbf{p})) = F(\alpha) - V(\bigcup_{j=1}^{2^n-1} A^j)$. If all of the A^j were pairwise disjoint, then the above would be equal to $F(\mathbf{p}) - H(\alpha)$. However, since the sets are not disjoint, construct the following sets.

$$E_2 := \{x \in \mathbb{R}^n | \exists_{i,j} \text{ such that } x \in A^i \cap A^j\}$$
$$E_3 := \{x \in \mathbb{R}^n | \exists_{i,j,k} \text{ such that } x \in A^i \cap A^j \cap A^k\}$$
$$\vdots$$

In general, let E_i be the set of x such that x is in the intersection of at least i A^j. Define $G(\alpha) := \sum_{k=2}^{2^n-1} V(E_k)$. Then:

$$V(S_0(\mathbf{p})) = F(\alpha) - H(\alpha) + G(\alpha)$$

$G'(\alpha) \geq 0$, since each A^j is strictly increasing in α, it must also the case that each E_i is also strictly increasing in α. Looking at the functional form of $H(\alpha)$, it is obvious that $H'(\alpha) \geq 0$. Since $H(\alpha)$ is a polynomial in α where each term is of degree at least two, $H'(\alpha)|_{\alpha=0} = 0$. Now, note that $G(\alpha)$ would grow fastest relative to $H(\alpha)$ in α if all of the new volume were in the intersection of every A^j. Suppose, for a moment, that that is the case. $H'(\alpha) = (2^n - 1)\frac{\partial}{\partial \alpha}A^j$, while $G(\alpha) = (2^n - 2)\frac{\partial}{\partial \alpha}A^j$. So even in this most ideal case, $G'(\alpha) \leq H'(\alpha)$. Finally, the chain rule tells us that $\frac{\partial}{\partial p_1}F(\alpha) = F'(\alpha)$, and likewise for $H(\alpha)$. From the above, $\frac{\partial}{\partial p_1}V(S_1(\mathbf{p}))$ is bounded between $\frac{\partial}{\partial p_1}F(\alpha)$ and $\frac{\partial}{\partial p_1}(F(\alpha) - H(\alpha))$.

Thus the left derivative of the demand function is bounded above and below by $\frac{\partial}{\partial p_1}F(\alpha)$. This is also the right hand derivative of $V(S_1(\mathbf{p}))$ at $p_1 = p$, and so $V(S_1(\mathbf{p}))$ is differentiable at $p_1 = p$. It is differentiable everywhere else by inspection. □

Proof of Lemma 2

Lemma 2: If 2^n firms are positioned in a reflectively symmetric arrangement on an n-cube, only type one constraints bind.

Proof. We may assume that the price charged by the firm we are considering, p, is weakly larger than that it expects its competitors to charge, $\hat{p}$. This is safe due to the differentiability of the profit function and the fact that, in equilibrium, they will all charge the same price. I wish to show that it is impossible for a vector to satisfy $2t\delta_j x_j \leq \delta_j t - p_1 + p$ for all j, but violate a constraint of the form $\sum_{i=1}^{k} 2t\delta_i x_i \leq p - p_1 + t\sum_{i=1}^{k}\delta_i$. It suffices to show that, for x_2 through x_k as large as they can be, the latter constraint is still looser than the type one constraint for x_1. Plugging these in, the type k constraint reduces to $x_1 \leq \frac{\delta_1 t + (k-2)(p_1 - p)}{2\delta_1 t}$, whereas the type one is given by $x_1 \leq \frac{\delta_1 t - p_1 + p}{2\delta_1 t}$. Since $p_1 \geq p$ by assumption and in equilibrium, this holds and the type k constraint is, in fact, redundant. □

Proof of Theorem 1

If 2^n firms locate in a reflectively symmetric positioning on an n-cube and compete in prices, there exists a unique symmetric equilibrium in which firms charge $p^* = \frac{t\ hm(\{\delta_i\}_{i=1}^n)}{n}$ and earn profits equal to $\pi^* = \frac{t\ hm(\{\delta_i\}_{i=1}^n)}{2^n n}$.

Proof. Recall that there are two forms of the profit function for a potential deviant, π^1 for when $p_1 < p$ and π^2 when $p_1 \geq p$. The difference stems from the fact that more constraints on the set of consumers captured may bind when $p_1 < p$, so that D_1 is weakly smaller in this case. Thus $\pi^1(\mathbf{p}) \leq \pi^2(\mathbf{p})\forall_{\mathbf{p}}$. In a candidate symmetric equilibrium, $p_1 = p$, so the profit function follows $\pi^2(\mathbf{p})$. Then, since π^2 is more optimistic, if firm 1 does not wish to deviate if it assumes that the profit function always follows π^2, it will never wish to deviate taking π^1 into account. For the rest of the proof, then, I will focus on $\pi^2(\mathbf{p}) = p_1(\prod_{i=1}^{n} \frac{\delta_i t + p - p_1}{2\delta_i t})$.

Any best response to an opposing strategy of p must either be on the boundary of the feasible set or satisfy a first order condition. The boundaries are easily ruled out because setting $p_1 = 0$ earns zero profit in equilibrium and cannot be optimal, and an extremely large p_1 ensures that $D_1 = 0 = \pi$. Therefore, the best response must satisfy the first order condition (78).

$$\prod_{i=1}^{n} \frac{\delta_i t + p - p_1}{2\delta_i t} + p_1 \sum_{i=1}^{n} \frac{-1}{2\delta_i t} \prod_{j \neq i} \frac{\delta_j t + p - p_1}{2\delta_j t} = 0 \tag{78}$$

It is safe to assume that $\delta_i + p - p_1 \neq 0 \forall_i$, otherwise $\pi = 0$. Then, dividing (78) through by $\prod_{i=1}^{n} \frac{\delta_i t + p - p_1}{2\delta_i t}$ and rearranging, the first order condition simplifies to (79).

$$p_1 = \frac{1}{\sum_{i=1}^{n} \frac{1}{\delta_i t - p_1 + p}} \tag{79}$$

Then the unique candidate symmetric equilibrium obeys $p_1 = p$ and yields

$$p^* = \frac{t}{\sum_{i=1}^{n} \frac{1}{\delta_i}}$$

□

To ensure that this candidate equilibrium is actually an equilibrium, note that proposition 3 of Caplin and Nalebuff (Caplin and Nalebuff, 1991) guarantees that π is quasiconcave in p_1. As long as p^* represents a local maximum, quasiconcavity guarantees that it is the global maximum. Brute computation shows that

$$\frac{\partial^2}{\partial^2 p_1}\pi^2(\mathbf{p})|_{\mathbf{p}=p^*} = -\sum_{i=1}^{n}\frac{1}{\delta_i} - \sum_{i=1}^{n}\frac{1}{\delta_i^2} < 0$$

Since the candidate equilibrium satisfies the first order condition at a locally concave point, it is a local and hence global maximum. It is the unique symmetric equilibrium because no other symmetric pairing can satisfy the first order condition for π^2, a necessary condition due to the differentiability of π.

Proof of Theorem 2

Proof. Since the basic positioning dominates any reflectively symmetric positioning of 2^n firms on an n-cube, it suffices to show that the offset positioning dominates the basic positioning. Recall that each firm in the basic positioning on an n-cube earns $\pi^* = \frac{t}{2^n n}$. Then this proof is simply a matter of showing that in any pricing equilibrium of the offset position on an (n+1)-cube firms earn profits greater than π^*.

First I must find the profit function of a potential deviant when firms are located in the offset positioning. Define $\tau = \frac{2t-p_1+p}{2t}$ and consider an (n+1)-cube. In any symmetric equilibrium, only type 2 constraints bind. The argument for this is similar to the proof of Lemma 1, and is thus omitted. I must find the volume of a region bounded by the following constraints

$$x_i \leq \tau - x_j \quad \forall_{i,j}$$

$$x_i \geq 0 \quad \forall_i$$

To clean up notation a bit, define $M_{i,j,k}$ as $Max\{x_i, x_j, x_k\}$. Then, the volume of this region can be given by

$$\int_0^{\tau} \int_0^{\tau - x_n} \int_0^{\tau - M_{n-1,n}} ... \int_0^{\tau - M_{1,2,...n}} 1 dx_1 dx_2 ... dx_{n+1} \tag{80}$$

Due to all of the maxima in the bounds of this integral, it is tiresome to evaluate directly. Instead, consider the volume of the subregion in which $x_{n+1} = M_{1,2,...n+1}$. This volume is given by (81) which simplifies to (82).

$$\int_0^{\tau} \int_0^{Min[x_{n+1}, \tau - x_{n+1}]} \int_0^{Min[x_{n+1}, \tau - x_{n+1}]} ... \int_0^{Min[x_{n+1}, \tau - x_{n+1}]} 1 dx_1 dx_2 ... dx_{n+1} \tag{81}$$

$$\int_{\frac{\tau}{2}}^{\tau} \int_0^{\tau - x_{n+1}} ... \int_0^{\tau - x_{n+1}} 1 dx_1 dx_2 ... dx_{n+1} + \int_0^{\frac{\tau}{2}} \int_0^{x_{n+1}} ... \int_0^{x_{n+1}} 1 dx_1 dx_2 ... dx_{n+1} \tag{82}$$

If x_{n+1} is the largest coordinate, any other coordinate that satisfies $x_j \leq \tau - x_{n+1}$ will satisfy all constraints. However, for x_{n+1} to be the largest coordinates, all others must be smaller than it, meaning they can be no larger than $Min[x_{n+1}, \tau - x_{n+1}]$. Then, after splitting the integral to get rid of the minima in the bounds, it is trivial to evaluate the integrals and get

$$\frac{\tau^{n+1}}{(n+1)2^{n+1}} + \frac{\tau^{n+1}}{(n+1)2^{n+1}} = \frac{\tau^{n+1}}{(n+1)2^n} \tag{83}$$

The labeling of coordinates is arbitrary, here. The overall region can be partitioned into n equal pieces, where each coordinate, in turn, is the largest. The total volume, therefore, must be n times the volume I just found, or $D_1 = \frac{\tau^{n+1}}{2^n}$. Expanding τ, $D_1 = \frac{(2t - p_1 + p)^{n+1}}{2^{n+1} t^{n+1} 2^n}$. Multiply D_1 by p_1 to get

$$\pi^0(\mathbf{p}) = p_1 \frac{(2t - p_1 + p)^{n+1}}{2^{n+1} t^{n+1} 2^n}$$

Consider the first order condition (84).

$$\frac{(2t - p_1 + p)^{n+1}}{2^{n+1}t^{n+1}2^n} - (n+1)p_1 \frac{(2t - p_1 + p)^n}{2^{n+1}t^{n+1}2^n} = 0 \tag{84}$$

With the symmetry condition, (84) easily reduces to (85).

$$2t = (n+1)p_1 \tag{85}$$

Then our candidate equilibrium for the offset positioning is $p = \frac{2t}{n+1}$. The proof that this is actually equilibrium follows the argument from the proof of Theorem 1 almost exactly. There can be no symmetric equilibria with $p < \frac{2t}{n+1}$ because every firm would have a profitable deviation to an incrementally higher price. Then each firm will earn at least $\frac{2t}{2^n(n+1)}$ in equilibrium which is equal to π^* when n=1 and strictly greater for $n \geq 2$. □

Proof of Theorem 3

Proof. In order for an allocation (a,b,p) to be an equilibrium, it must be the case either that it is a local maximum at itself with respect to π or that it lies on the boundary of the feasible set. Otherwise, the deviant would have a local profitable deviation.

First, consider the cases in which (a,b,p) is a local maximum at itself with respect to π.

In order to use calculus, the derivatives of $\pi(a, b, p, a, b, p)$ with respect to the fourth, fifth, and sixth variables must exist at any feasible, interior (a,b,p). At any point (a,b,p,a,b,p), $\rho = \phi$, which means that the derivatives of the two halves of the profit function, $\pi^1$and π^2, must agree at such points. Taking the derivatives and simplifying, we obtain that:

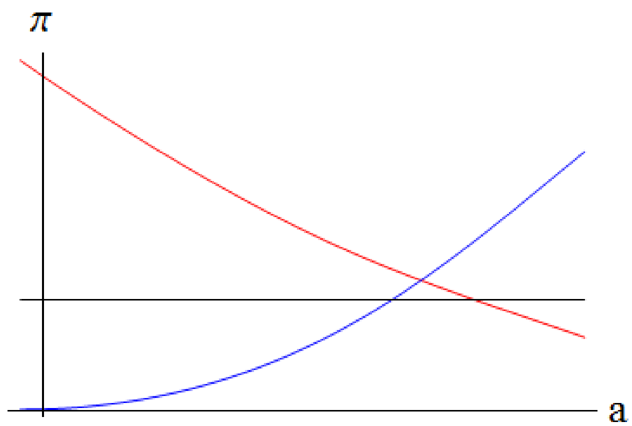


Figure 1: Profits from various location strategies

$$\frac{\partial\pi^1(a,b,p,a,b,p)}{\partial\hat{a}} = \frac{(1-2a)(3-4a-4b)p}{8(1-a-b)^2} = \frac{\partial\pi^2(a,b,p,a,b,p)}{\partial\hat{a}} \tag{86}$$

$$\frac{\partial\pi^1(a,b,p,a,b,p)}{\partial\hat{b}} = \frac{(1-2b)(3-4a-4b)p}{8(1-a-b)^2} = \frac{\partial\pi^2(a,b,p,a,b,p)}{\partial\hat{b}} \tag{87}$$

$$\frac{\partial\pi^1(a,b,p,a,b,p)}{\partial\hat{p}} = \frac{1-a-b-2p}{4(1-a-b)} = \frac{\partial\pi^2(a,b,p,a,b,p)}{\partial\hat{p}} \tag{88}$$

The derivatives clearly exist, so by taking the above expressions, setting them equal to zero and solving, I find that an interior (a,b,p) is a local maximum at itelf with respect to π only if:

$$(a,b,p) = (a, \frac{3}{4} - a, \frac{1}{8}) \tag{89}$$

Recall that it is assumed that $a \geq b$, so I restrict scrutiny on the above expression to $a \in (\frac{3}{8}, \frac{1}{2})$. Below is a picture with three graphs. The downward sloping graph is $\pi(a, \frac{3}{4} - a, \frac{1}{8}, \frac{1}{2}, .278, .12)$, the upward sloping graph is $\pi(a, \frac{3}{4} - a, \frac{1}{8}, .357, .357, .117)$, and the flat graph is $\pi(a, \frac{3}{4} - a, \frac{1}{8}, a, \frac{3}{4} - a, \frac{1}{8})$. All graphs are over the range $a \in [\frac{3}{8}, \frac{3}{4}]$.

It is clear that at least one of the sloped graphs is above the flat graph at every point in the continuum defined by (24). This means that at every point, a firm may make higher profit by deviating to one of (.357,.357,.117) or (.5,.278,.12) rather than staying in equilibrium. Then no point on the continuum is a global maximum at itself with respect to π and no such point is an equilibrium. It suffices to show that there are no boundary points with global maxima at themselves with respect to π.

First, consider the boundary in which p=0. If p=0, then firms cannot make any profit. Unless all firms are piled on top of each other, a firm may make positive profit by charging a extremely small but positive amount. If all firms are on top of each other, any firms will have incentive to move away so that they may make positive profit by charging a positive price. Therefore, there can be no equilibria with p=0.

Consider the boundary defined by b=0. The interior of this boundary is $a \in (0, \frac{1}{2})$ and $p \in \mathbb{R}_+$. Suppose that firms have their choices restricted to this boundary and define $\hat{\pi}(a, p, \hat{a}, \hat{p}) = \pi(a, 0, p, \hat{a}, 0, \hat{p})$. For there to be an equilibrium on the interior of this boundary, there must be an (a^*, p^*) that is a local maximum at itself with respect to $\hat{\pi}$. Taking derivatives and setting them equal to zero, it is easy to see that no such (a^*, p^*) exists on the interior of this boundary.

Using a similar technique for the a=$\frac{1}{2}$ boundary, we can find that such a point does exist. The candidate equilibrium is $(\frac{1}{2}, \frac{1}{4}, \frac{1}{8})$. Note, however, that this point is on the continuum scrutinized previously and cannot be an equilibrium.

Our last boundary is characterized by a=b. The local self optimum of this boundary is given by $(\frac{3}{8}, \frac{3}{8}, \frac{1}{8})$, which is also on the continuum and not an equilibrium.

We have now ruled out equilibria at all points except those of the form $(0, 0, p)$, $(\frac{1}{2}, 0, p)$, and $(\frac{1}{2}, \frac{1}{2}, p)$. For the reasoning explained earlier, all firms at $(\frac{1}{2}, \frac{1}{2})$ cannot be an equilibrium, as it would be impossible to make a positive profit. Again, by ruling out locally profitable deviations along the boundary, we need only examine the points $(0, 0, \frac{1}{2})$ and $(\frac{1}{2}, 0, \frac{1}{4})$. It is profitable to deviate from $(0, 0, \frac{1}{2})$ to $(\frac{1}{2}, \frac{1}{2}, \frac{9}{20})$, and from $(\frac{1}{2}, 0, \frac{1}{4})$ to $(.373, .373, .221)$.

□