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1 Introduction

Carlos Alchourrón was a scholar in the old tradition, with a vast culture and a passion for knowledge. His initial research, with Eugenio Bulygin on *Normative Systems* ([Alchourrón-Bulygin 71]), led him to the realization that legal reasoning is actually representative of a more general kind of reasoning. He subsequently concluded that classical mathematical logic was not appropiate for formalizing this *ampliative* and *non-deterministic* kind of reasoning.

His line of attack shows clearly in the characteristics of the AGM system of belief revision [AGM 85]. The language of mathematical logic was preserved and the only big departure from that tradition is the addition of a formalism to represent changes of a theory. The key element is a non-constructive *choice function* that provides for a selection of "worlds" (maximally consistent extensions of a theory), which allows a consistent view of the revision to be applied (*meet contraction*).

AGM thus provides a definition of a revised theory in terms of a fix point of the consequence operator. The real advantage of this system is not so much that of being a depiction of ampliative reasoning, but that of providing a formalization of a non-bayesian change of beliefs.

Several authors tailored AGM in order to implement computational procedures of ampliative reasoning ([Williams 96],[Boutillier 93]). All of these attempts raise the issue of defining the choice function. There are two hidden assumptions in Alchourrón's view. The first is that the final decision on the

*We aknowledge the warm hospitality of Antonio Martino and all the people we met in Pisa in October 1996, during the Workshop on Logic, Computation and Law in Honor of Carlos Alchourrón.

relevance of worlds rests entirely with an agent, individual or collective. The second one is that this agent delegates the routine tasks of computing all the theorems to the revision operator. The real procedural aspects of the reasoning are lost in the assumption that the agent can do the computation.

Almost in parallel with developments in belief revision, the Artificial Intelligence community advanced several formal representations of "common-sense" reasoning. Although much of that reasearch can hardly be distinguished from research on non-standard logics, the concern with the possibility of implementation gave it a special flavor. After a decade or so, in which all this research was collectively described as *non-monotonic logics*, a consensus began to arise that, in order to be implementable, systems should depart drastically from the classical logic tradition. The computation of consequence operators (the key elements in logical systems, according to Tarski) is equivalent to the *entscheidung* or decision problems in the respective logics. It is well known that decision problems are only solvable for particulary "well behaved" formal systems. So in general the dream of implementing non-monotonic systems in terms of consequence operators was abandoned. Instead of that, the ensuing consensus is that what really matters is the *justification* of an assertion, and that this justification is a result of a partial computation [Loui 91] [Pollock 87]. Moreover, a justification can be *defeated* by another one, if the last one involved more computation. Several systems were proposed, in which sentences are only accepted after a comparisson among the justifications for and against it. These different systems perform what is called *defeasible reasoning* (most recent examples can be found in [Vreeswijk 93],[Verheij 96]).

Alchourrón devoted his last years to the analysis of the notion of defeasibility. His idea was to develop a formal system capturing the essentials of defeasibility. Following the tradition in non-standard logics he proposed a new connective for the implication, with new properties. A *defeasible conditionalization* replaced material implication, in such a way that the consequence operator of the logic no longer has the properties of *modus ponens* and *strengthening the antecedent*. These properties of logical inference seemed to Alchourrón incompatible with the possibility of changing the status of a concluded proposition from "justified" to "non-justified".

The definition of the defeasible conditional is given in terms of material implication and a syntactic operator. Given a defeasible implication, this operator is the syntactic version of a choice function on worlds. The model theory provided by Alchourrón shows clearly how the idea of a choice function on worlds was his preferred device to make logical systems manageable. The idea is simply that in any interpretation the syntactic operator must be translated into a choice function, identifying chosen worlds, the worlds in which all the preconditions for a defeasible rule are met.

Again, an unstated assumption is that the "choice" is made by an agent able to evaluate all the possible consistent extensions of a theory. Clearly, Alchourrón had not in mind the possibility of computational implementation.

Moreover, in his 1993 paper he rejected this approach in favor of one based on AGM [Alchourrón 93]. But in his posthumous paper of 1996, the epistemological reasons that led to that rejection are no longer present (maybe because he realized the deep connection between both approaches) [Alchourrón 96].

The rest of this paper will be devoted to showing that Alchourrón's defeasible conditionals are a cumbersome way to handle defeasibility: these conditionals hide defeasibility's essential procedural aspects, and relegate them to a mere clerical task of explicitating the consequences of the choices made by an external agent. We aver that defeasible reasoning is more than that: it is a process of deliberation that provides constructive justifications and avoids the struggle with complexity, simply because every partial justification counts. More computation may lead to the defeat of a previously justified conclusion. Defeasible reasoning is an open process, where the notions of *entscheidung* or *completeness* are no longer relevant. Their importance is overthrown by the relevance of criteria for *efficient* adjudication.

So it seems paradoxical that, the representation of legal reasoning being his main goal, Alchourrón chose the logical approach instead of defeasible reasoning. In the following sections we will briefly introduce both approaches, in order to compare them and to show why the *Zeitgeist* favors defeasible reasoning.

2 A Logic for Defeasible Conditionals

The vocabulary of the logical system presented by Alchourrón is typical of a modal sentential logic with the addition of a *strict implication* $\Rightarrow$. The syntax is the same as that of system T, but not allowing nested strict implications.

To this modal framework, a monadic operator f is added such that ([Alchourrón 93],[Alchourrón 96]):

- if A is a sentence then fA is a sentence
- *(expansion)* $\vdash fA \rightarrow A$

This operator allows the definition of the *defeasible conditional connective* $>$:

Definition 1

$$A > B \;=\; fA \Rightarrow B$$

The following are properties of the defeasible conditionals:

- *(extensionality)* $\vdash (A \Leftrightarrow B) \rightarrow (fA \Leftrightarrow fB)$
- *(limit expansion)* $\vdash \Diamond A \rightarrow \Diamond fA$ (where $\Diamond$ is the modal operator *possible*)
- *(hierarchical ordering)*

$$\vdash (f(A \vee B) \Leftrightarrow fA) \vee (f(A \vee B) \Leftrightarrow fB) \vee (f(A \vee B) \Leftrightarrow (fA \vee fB)$$

And properties lacking in the behavior of these conditionals are (in order to avoid a colapse of defeasible conditionals into material implication):

- *(defeasible modus ponens)* $\vdash (A > B) \to (A \to B)$
- *(strengthening the antecedent)* $\vdash (A > B) \to ((A \wedge C) > B)$

All these conditions define the system *DFT*. Its semantics is given in the following:

Definition 2 *A model for* DFT *is:*

$$\langle W, [|\ |], Ch^{\alpha} \rangle$$

where W is the set of worlds, $[|\ |]$ is such that for any sentence A, $[|A|] \in 2^{W}$ (that is, to each sentence it assigns a subset of W). Ch^{α} is a choice function *that, for any sentence A gives a subset of W, the worlds in which f is true,* from the point of view of an agent α

The following are the semantic versions of the axioms for f and $>$:

- *(expansion)* $Ch^{\alpha}(A) \subseteq [|A|]$
- *(extensionality)* If $[|A|] = [|B|]$ then $Ch^{\alpha}(A) = Ch^{\alpha}(B)$
- *(limit expansion)* If $[|A|] \neq \emptyset$ then $Ch^{\alpha}(A) \neq \emptyset$
- *(hierarchical ordering)*

$$Ch^{\alpha}(A \vee B) = \begin{cases} Ch^{\alpha}(A) & \text{or} \\ Ch^{\alpha}(B) & \text{or} \\ Ch^{\alpha}(A) \cup Ch^{\alpha}(B) & \end{cases}$$

The interpretations of f, $\Rightarrow$ and $>$ are:

- $[|fA|] = Ch^{\alpha}(A)$
- $w \in [|A \Rightarrow B|]$ iff $[|A|] \subseteq [|B|]$
- $w \in [|A > B|]$ iff $Ch^{\alpha}(A) \subseteq [|B|]$

f (as the syntactic counterpart of Ch^{α}), being defined in terms of an agent α, licenses arbitrariness in the definition of this system. The intuitive meaning of fA is $A \wedge A_1 \vee \ldots \wedge A_n$, where $A_1, \ldots A_n$ are the assumptions "associated" (according to α) with A. So, if $A > B$, A is not a sufficient condition for B. It is a part of a suficient condition (fA) or, in von Wright's terms, a *contributory condition.*

When particular knowledge is incorporated as a set of proper axioms $\mathcal{K}$, the system $DFT + \mathcal{K}$ provides theorems that differ according the agents that define f (therefore the system should be called $DFT^{\alpha} + \mathcal{K}$).

To see how this system works and how little it captures of the idea of defeasible reasoning we present the following example:

Example 1 *Consider the following sentences:*

- Birds fly
- Penguins do not fly
- Penguins are Birds

Their representation in the language of $DFT + \mathcal{K}$ *is, respectively:*

- $B(a) > F(a)$
- $P(a) > \neg F(a)$
- $P(a) \Rightarrow B(a)$

where we assume that each formula is a schema, where a *represents a constant provided by the finite knowledge base* $\mathcal{K}$. *Then we have the following:*

$$\mathcal{K} = \{B(Opus), P(Opus)\}$$

Given two external agents, α, β *suppose that their choices are:*

- $Ch^{\alpha}(B(Opus)) = [|B(Opus) \wedge \neg P(Opus)|]; Ch^{\alpha}(P(Opus)) = [|P(Opus)|]$
- $Ch^{\beta}(B(Opus)) = [|B(Opus) \wedge \neg P(Opus)|]; Ch^{\beta}(P(Opus)) = [|P(Opus) \wedge \neg D(Opus)|]$ *(where* $D(Opus)$ *represents the sentence "Opus is dead")*

Thus $f^{\alpha}(B(Opus)) = f^{\beta}(B(Opus)) = (B(Opus) \wedge \neg P(Opus))$ *but* $f^{\alpha}(P(Opus)) = P(Opus)$ *and* $f^{\beta}(P(Opus)) = (P(Opus) \wedge \neg D(Opus))$.

For α, *therefore the defeasible conditionals are equivalent to:*

$$B(Opus) \wedge \neg P(Opus) \Rightarrow F(Opus)$$

$$P(Opus) \Rightarrow \neg F(Opus)$$

and so (since the first statement has a non valid antecedent) $\neg F(Opus)$ *can be inferred.*

But for β *the defeasible conditionals are equivalent to:*

$$B(Opus) \wedge \neg P(Opus) \Rightarrow F(Opus)$$

$$P(Opus) \wedge \neg D(Opus) \Rightarrow \neg F(Opus)$$

and so, nothing can be inferred about the flying capabilities of Opus, because both rules have non-valid antecedents.

This example shows also that Alchourrón's notion of defeasibility is more conservative than the most conservative logical formalisms of non-monotonic reasoning in Artificial Intelligence. Even *Circumscription* (see [McCarthy 86]), solves in itself (without relaying on an "external agent") the problem of defining the relevant *frame* for the problem.

It is, simply put, that the defeasible conditionals in *DFT* represent incomplete information and the system in itself does not even attempt to give an answer without external interference.

3 Defeasible Reasoning

Defeasible reasoning is concerned with finding *warrants* for sentences. What distinguishes this type of reasoning from logic is that it is fundamentally procedural: the warrant of sentences is not defined in terms of the consistency of the warranted sentences with a set of axioms, but in terms of the procedure followed to support them [Simari-Loui 92], [Vreeswijk 97].[1]

The procedure of justification is called *argumentation*. The idea is that arguments or *defeasible proofs* are derived for and against a sentence. Arguments can be partially ordered in terms of their "conclusive strength". If given two arguments $\mathcal{H}_1$ and $\mathcal{H}_2$ support A and $\neg A$, respectively, and $\mathcal{H}_1 \lhd \mathcal{H}_2$ (where $\lhd$ represents the relation *less strong*) then we say that $\mathcal{H}_2$ *defeats* $\mathcal{H}_1$. It follows that $\neg A$ is justified (if no $\mathcal{H}_3$ defeats $\mathcal{H}_2$).

The choice of the formal language in which to develop this approach implies a departure from Alchourrón's approach. While he incorporates the defeasible conditional in the object language, we lift all the conditionals to the meta-language, treating them as *rules.Rules* are easy to implement, and this is a very important consideration in the design of defeasible reasoning systems. Rules, with their "IF-THEN" nature, seem to be more adequate to represent the means by which a reasoner generates arguments. In contrast, in classical logic, the equivalence of the material implication $A \rightarrow B$ with $\neg A \vee B$ eliminates all the rule-like characteristics from the conditional.

We will consider a finite set $\mathcal{L}$ of instantiated sentences in a first order language (with the conventional connectives, except material implication).[2] $\mathcal{L}$ is constructed applying a finite set of rules of inference on a finite atomical knowledge base $\mathcal{K}$:

- *strict rules of inference* $\mathcal{A} \Rightarrow A$, where $\mathcal{A} \subseteq \mathcal{L}, A \in \mathcal{L}$ [3]
- *defeasible rules of inference* $\mathcal{A} > A$ (where $>$ is no longer a connective but a meta-linguistic symbol)

The recursive application of rules of inference defines what is called an *argument*:

Definition 3 *The pair $\langle \mathcal{H}, A \rangle$ is called an argument for A, where:*

- $A \in \mathcal{L}$

[1] We present a very simplified version of defeasible reasoning, in which important features are omitted. This is in order to not mislead the reader with technical details that are not relevant for the comparison with Alchourrón's system.

[2] So, $\mathcal{L}$ does not include universal or existential quantified formulas, as well as instantiated material implications.

[3] Every implication in the object language should be subsituted by a strict rule of inference, in order to treat the conditionals (defeasible or strict) in a similar way.

- $\mathcal{H}$ *is a finite tree, with nodes in* $\mathcal{L}$. *The root node is* A *and each node* B *has children* $\mathcal{B}$ *iff it exists a rule* $\mathcal{B} \Rightarrow B$ *or a rule* $\mathcal{B} > B$. *The leaves of* $\mathcal{H}$ *are sentences in* $\mathcal{K}$

- *for at least one node* B *in* $\mathcal{H}$, *and its children* $\mathcal{B}$, *it exists a defeasible rule of inference* $\mathcal{B} \ > \ B$

Probative strength can have several characterizations. We will consider here the following:

Definition 4 *Given two arguments* $\langle\mathcal{H}, A\rangle$ *and* $\langle\mathcal{H}', A'\rangle$, *we say the first one has less probative strenght* ($\langle\mathcal{H}, A\rangle \lhd \langle\mathcal{H}', A'\rangle$) *iff*

- *there exists a node* A'' *in* $\mathcal{H}$ *such that there exists a subset of nodes* $\mathcal{A}''$ *of the tree* $\mathcal{H}'$ *and a strict rule* $\mathcal{A}'' \Rightarrow A''$

- *there exists a node* A'' *in* $\mathcal{H}'$ *such that there is no subset of nodes* $\mathcal{A}''$ *of the tree* $\mathcal{H}$ *and a strict rule* $\mathcal{A}'' \Rightarrow A''$

This relation allows to define the stronger relationship of *defeat*:

Definition 5 *Given* $\langle\mathcal{H}, A\rangle$ *and* $\langle\mathcal{H}', A'\rangle$, *we say that* $\langle\mathcal{H}', A'\rangle$ *defeats* $\langle\mathcal{H}, A\rangle$ *(symbolized* $\langle\mathcal{H}, A\rangle \ll \langle\mathcal{H}', A'\rangle$*) if:*

- *there exists a subtree* $\mathcal{H}''$ *of* $\mathcal{H}$, *with root* $\neg A'$ *and* $\langle\mathcal{H}'', \neg A'\rangle \lhd \langle\mathcal{H}', A'\rangle$

- *it does not exist a subtree* $\mathcal{H}'''$ *of* $\mathcal{H}'$, *with root* $\neg A$ *and* $\langle\mathcal{H}''', \neg A\rangle \lhd \langle\mathcal{H}, A\rangle$

The following property is straightforward:

Proposition 1 $\ll$ *is an* asymmetric *relationship*

Proof 1 *If* $\langle\mathcal{H}, A\rangle \ll \langle\mathcal{H}', A'\rangle$ *then it is immediate, from the definition of* $\ll$ *that it does not exist a subtree* $\mathcal{H}'''$ *of* $\mathcal{H}'$, *with root* $\neg A$ *such that* $\langle\mathcal{H}''', \neg A\rangle \lhd \langle\mathcal{H}, A\rangle$. *Therefore* $\langle\mathcal{H}', A'\rangle \not\ll \langle\mathcal{H}, A\rangle$.

Argumentation is the process by which arguments are generated for and against sentences. The goal is to decide if an initial sentence is warranted or not. A search procedure defines the notion of *warranted sentence*. Given the set of arguments on $\mathcal{L}$, $\mathcal{ARG}$ this procedure generates alternatively arguments supporting a sentence and arguments against that sentence or against sentences in their supports:

Procedure 1 *To decide if a sentence* $A_0 = A$ *is warranted or not the following* search procedure *must be applied:*

- PRO *generate a set of possible arguments supporting* A_0, $\mathcal{ARG}_0$, *that is, arguments of the form* $\langle\mathcal{H}_0, A_0\rangle$ [4]

[4] We assume here that there exists at least one argument supporting A_0, otherwise it would be trivially not warranted.

- CON *generate the set of arguments* $\mathcal{ARG}_1 \subseteq \mathcal{ARG} - \mathcal{ARG}_0$, *such that* $\langle \mathcal{H}_1, A_1 \rangle \in \mathcal{ARG}_1$ *iff there is a* $\langle \mathcal{H}_0, A_0 \rangle \in \mathcal{ARG}_0$ *such that* $\langle \mathcal{H}_0, A_0 \rangle \ll \langle \mathcal{H}_1, A_1 \rangle$

  if there is one of the supporting arguments, say $\langle \mathcal{H}_0^*, A_0 \rangle \in \mathcal{ARG}_0$, *for which there is no* $\langle \mathcal{H}_1, A_1 \rangle$, *such that* $\langle \mathcal{H}_0^*, A_0 \rangle \ll \langle \mathcal{H}_1, A_1 \rangle$, *we can say that* A_0 *is warranted. If not,*

- PRO *generate the set of arguments* $\mathcal{ARG}_2 \subseteq \mathcal{ARG} - \bigcup_{j<2} \mathcal{ARG}_j$ *such that* $\langle \mathcal{H}_2, A_2 \rangle \in \mathcal{ARG}_2$ *iff there exists* $\langle \mathcal{H}_1, A_1 \rangle \in \mathcal{ARG}_1$, *with* $\langle \mathcal{H}_1, A_1 \rangle \ll \langle \mathcal{H}_2, A_2 \rangle$

  if there is one of the arguments, say $\langle \mathcal{H}_1^*, A_1 \rangle \in \mathcal{ARG}_1$, *that cannot be defeated by any* $\langle \mathcal{H}_2, A_2 \rangle$, *we say that* A_0 *is not warranted. If not,*

- ...

- PRO *generate the set of arguments* $\mathcal{ARG}_{2k} \subseteq \mathcal{ARG} - \bigcup_{j<2k} \mathcal{ARG}_j$ *such that* $\langle \mathcal{H}_{2k}, A_{2k} \rangle \in \mathcal{ARG}_{2k}$ *iff there exists* $\langle \mathcal{H}_{2k-1}, A_{2k-1} \rangle \in \mathcal{ARG}_{2k-1}$, *with* $\langle \mathcal{H}_{2k-1}, A_{2k-1} \rangle \ll \langle \mathcal{H}_{2k}, A_{2k} \rangle$

  if there is one of the arguments, say $\langle \mathcal{H}_{2k-1}^*, A_{2k-1} \rangle \in \mathcal{ARG}_{2k-1}$, *that cannot be defeated by any* $\langle \mathcal{H}_{2k}, A_{2k} \rangle$, *we say that* A_0 *is not warranted. If not,*

- CON *generate the set of arguments* $\mathcal{ARG}_{2k+1} \subseteq \mathcal{ARG} - \bigcup_{j<2k+1} \mathcal{ARG}_j$, *such that* $\langle \mathcal{H}_{2k+1}, A_{2k+1} \rangle \in \mathcal{ARG}_{2k+1}$ *iff there is a* $\langle \mathcal{H}_{2k}, A_{2k} \rangle \in \mathcal{ARG}_{2k}$ *such that* $\langle \mathcal{H}_{2k}, A_{2k} \rangle \ll \langle \mathcal{H}_{2k+1}, A_{2k+1} \rangle$

  if there is one of the supporting arguments, say $\langle \mathcal{H}_{2k}^*, A_{2k} \rangle \in \mathcal{ARG}_{2k}$, *for which there is no* $\langle \mathcal{H}_{2k+1}, A_{2k+1} \rangle$, *such that* $\langle \mathcal{H}_{2k}^*, A_{2k} \rangle \ll \langle \mathcal{H}_{2k+1}, A_{2k+1} \rangle$, *we can say that* A_0 *is warranted. If not,*

- ...

One of the properties of arguments generated in the search is that there are no cycles. That means that if an argument $\langle \mathcal{H}_m, A_m \rangle$ is in $\mathcal{ARG}_m$ it cannot be in $\mathcal{ARG}_k$, for $k > m$. This requirement, togheter with the fact that $\mathcal{ARG}$ is finite [5] ensure the following result:

Lemma 1 *The search procedure applied to a sentence* $A \in \mathcal{L}$ *terminates in finite time, indicating that* A *is warranted or not warranted*

[5] Because $\mathcal{L}$ and the set of rules of inference are both finite.

Proof 1 *Suppose that the procedure does not terminate. Then, for each argument in support of A, $\langle \mathcal{H}_0, A_0\rangle$, it can be generated a sequence $\{\langle \mathcal{H}_k, A_k\rangle\}_{k=0}^{\infty}$, where $\langle \mathcal{H}_k, A_k\rangle \ll \langle \mathcal{H}_{k+1}, A_{k+1}\rangle$. As cyclicity is not allowed, there does not exist a repeated argument in the sequence. Absurd, because $\mathcal{ARG}$ is finite.*

Therefore, there exists an argument $\langle \mathcal{H}_k, A_k\rangle$, such that there is no $\langle \mathcal{H}_{k+1}, A_{k+1}\rangle$, $\langle \mathcal{H}_k, A_k\rangle \ll \langle \mathcal{H}_{k+1}, A_{k+1}\rangle$. If k is an even number, $\langle \mathcal{H}_k, A_k\rangle$ was generated in a pro *stage, and it follows that A_0 is warranted. If k is odd, it follows that A_0 is not warranted.*

The following version of Example 1, shows how the search procedure can be applied: [6]

Example 2 *Consider again the case of Opus. Now, we represent the conditional expressions by means of rules of inference:*

- $P(a) \Rightarrow B(a)$
- $B(a) > F(a)$
- $P(a) > \neg F(a)$

[6] There exist another inductive characterization of *argumentation in stages*, by a *labeling* procedure:

Procedure 2 *The generation of arguments and the comparison among them is done in several stages:*

- *every $\langle \mathcal{H}, A\rangle$, defines a level-0 argument*
- *an argument $\langle \mathcal{H}', A'\rangle$ is an level-(n+1) argument if there is no level-k argument $\langle \mathcal{H}'', A''\rangle$, for $k \leq n$, such that $\langle \mathcal{H}', A'\rangle \ll \langle \mathcal{H}'', A''\rangle$*

This process has also as a goal to *warrant* conclusions. Formally:

Definition 6 *A sentence $A \in \mathcal{L}$ is* warranted *if it exist an argument $\langle \mathcal{H}, A\rangle$ and an m such that for each $n \geq m$, $\langle \mathcal{H}, A\rangle$ is a level-n argument*

The set of arguments $\mathcal{ARG}$ with the relation $\ll$ verifies the following:

Theorem 1 *There exists a warranted sentence A in $\mathcal{L}$*

Proof 1 *Suppose not. So, for any $A^0 \in \mathcal{L}$ and any $\langle \mathcal{H}^0, A^0\rangle \in \bar{\mathcal{L}}$, if $\langle \mathcal{H}^0, A^0\rangle$ is a level-n argument, there is a $m \geq n$ such that $\langle \mathcal{H}^0, A^0\rangle$ is not a level-m argument. Therefore it exists $\langle \mathcal{H}^1, A^1\rangle \in \bar{\mathcal{L}}$, an level-(m-1) argument such that $\langle \mathcal{H}^0, A^0\rangle \ll \langle \mathcal{H}^1, A^1\rangle$. This procedure can be repeated indefinitely: for each argument $\langle \mathcal{H}^k, A^k\rangle$ exists an argument $\langle \mathcal{H}^{k+1}, A^{k+1}\rangle$, such that $\langle \mathcal{H}^k, A^k\rangle \ll \langle \mathcal{H}^{k+1}, A^{k+1}\rangle$.*

The sequence $\{\langle \mathcal{H}^k, A^k\rangle\}_{k=0}^{\infty}$ has, by definition, no repeated elements. Absurd, because $\mathcal{ARG}$ is finite.

This result shows that when working with a finite knowledge base (that is the meaning of a finite $\mathcal{L}$), there is a procedure that without external intervention finds the sentences that the knowledge base justifies.

And again, we have that $P(Opus)$.

We want to see if $F(Opus)$ is a warranted sentence. We can generate an argument for it:

- $arg_0 = \langle \{P(Opus) \Rightarrow B(Opus), B(Opus) > F(Opus)\}, F(Opus) \rangle$

Now we can generate an argument against $F(Opus)$:

- $arg_1 = \langle \{P(Opus) > \neg F(Opus)\}, \neg F(Opus) \rangle$

It is immediate to see that $arg_0 \lhd arg_1$, because from $P(Opus)$ in arg_1 can be derived $B(Opus)$ (via $P(Opus) \Rightarrow B(OPus)$) in arg_0, while from $P(Opus), B(Opus)$ no node in the tree in arg_1 can be inferred. As the conclusions of both arguments are contradictory it follows that $arg_0 \ll arg_1$.

Moreover, no argument arg_2 can be generated against arg_1, so $F(Opus)$ is not warranted.

To apply the procedure to $\neg F(Opus)$, we can say that:

- $arg_0' = \langle \{P(Opus) > \neg F(Opus)\}, \neg F(Opus) \rangle$

and the argument against is:

- $arg_1' = \langle \{P(Opus) \Rightarrow B(Opus), B(Opus) > F(Opus)\}, F(Opus) \rangle$

But is clear that (as $arg_0' = arg_1$ and $arg_1' = arg_0$) $arg_0' \not\ll arg_1'$, and therefore $\neg F(Opus)$ is warranted.

An interesting feature of the search procedure is what happens in the case of *bounded resources* (time, computational power, etc.). The procedure can stop in any stage and provide a *tentative* answer. This aspect of defeasible reasoning makes it appropiate for representing ampliative inference where both computation and corrigibility can force revision. An outstanding example is legal reasoning with its *open texture*, which provides for robustness in both procedure and specification [Hart 61].

4 Comparison between systems

Alchourrón's DFT and the defeasible reasoning system presented in the previous section differ in several aspects. One of the differences is that defeasible reasoning is not concerned with semantical versions of the procedures. Anyway, it is possible to give one in order to make the differences more clear:

Definition 7 *A model for defeasible reasoning on L is:*

$$\langle W, [\| \ \|], Ch \rangle$$

where $w \in W$ is a set $\mathcal{A} \subseteq \mathcal{L}$, where there is no sentence A, such that both A and $\neg A$ are in w. A particular world w^* is distinguished, $\mathcal{K} \subseteq w^*$, containing *all the sentences of the knowledge base, and such that for every A such that $\mathcal{K} \vdash A$, $A \in w^*$.*

The interpretation correspondence $[|\ |]$ is such that for any sentence $A \in \mathcal{L}, A \notin w^*$, $[|A|] = \{w : w \vdash_{\mathcal{L}} A\}$, while for $A \in w^*$, $[|A|] = w^*$. Ch, the choice function, gives, for each sentence A, all the worlds that warrant A. That is, $Ch(A) = \{w$:such that $\langle w, A\rangle$ and A is warranted $\}$. For the case in which $A \in w^*$, we specify that $Ch(A) = w^*$

These definitions show immediatly that the following properties are valid:

- *(expansion)* $Ch(A) \subseteq [|A|]$
- *(extensionality)* If $[|A|] = [|B|]$ then $Ch(A) = Ch(B)$

and the following ones are not valid in general:

- *(limit expansion)* If $[|A|] \neq \emptyset$ then $Ch(A) \neq \emptyset$
- *(hierarchical ordering)*

$$Ch(A \vee B) = \begin{cases} Ch(A) & or \\ Ch(B) & or \\ Ch(A) \cup Ch(B) & \end{cases}$$

The non-validity of the property of *limit expansion* is a natural consequence of the fact that a *prima facie* derivation of a sentence (the possibility of finding an argument for a conclusion) does not imply that in the end the conclusion will be accepted (warranted). On the other hand, the property of *hierarchical ordering* seems to be also excessive for a defeasible system: the fact that a disjunction is warranted, does not mean that any one component sentence is warranted, nor that the support for the disjunction is equal to the union of the supports of the components.

It is obvious that *DFT*'s semantics differs from this one, but the most important feature to note is that the key difference resides in Alchourrón's f. In his system:

- $[|fA|]_{DFT} = Ch^{\alpha}(A)$ [7]

For defeasible reasoning we can extend the analogy, saying that there must exist a "function" f' such that:

- $[|f'A|]_{\mathcal{L}} = Ch_{\mathcal{L}}(A)$

[7] We use the subscript to distinguish, when necessary, the system in which the property holds.

Of course, f' is nothing other than the result of the *search procedure*. Not only the semantics is superfluous, but also the possibility of violating limit expansion and hierarchical ordering permit a more honest depiction of defeasibility.

The intuitive notion of defeasibility that Alchourrón intended is not fully captured in his system. But what eluded completely his analysis was the fact that what he needed was not a function giving the beliefs of an agent, but a *procedure*. That is, he lost sight of the fact that reasoning with defeasibility is a process of deliberation.

5 Conclusions

Logic was always considered as a discipline studying the process of *inference*. This notion encompassed deductive as well as non-deductive (or *ampliative*) inference. But in a short period in historical terms, from 1840 to 1910, logic was reduced only to the study of deductive inference, a process that can be explained by the heavy demand of solid foundations required by mathematics.

This trend is still very powerful, and well into the 1980s was even very influential in Artificial Intelligence. But the slow realization that deductive reasoning is the exception rather than the rule when the reasoner faces uncertainty, incomplete knowledge or limited deductive power, led the community to change the focus from axiomatic representations to procedural characterizations. Defeasible reasoning seems to be the paradigmatic case of this trend. Moreover, it can be seen as the appropiate formalism to represent ampliative inference.

The trend towards processes instead of axiomatic characterizations is certainly gaining momentum. New generations are learning (using computers) to think in terms of processes. Even mathematics is shaken by this trend, so it seems that the very core of logics is on the verge of great changes, maybe equal in magnitude (but in opposite sense) to those of the beginning of this century.

Therefore it is surprising, in a sense, that such a distinguished scholar as Carlos Alchourrón, while trying to establish the appropiate logic for legal reasoning (one of the most patent examples of ampliative reasoning) decided to remain in the waters of deductive logic.

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