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Contagion of a Crisis, Corporate Governance, and Credit Rating

Dong Chuhl Oh

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CONTAGION OF A CRISIS, CORPORATE GOVERNANCE, AND CREDIT RATING

by

Dong Chuhl Oh

A dissertation presented to the
Graduate School of Arts and Sciences
of Washington University in
partial fulfillment of the
requirements for the degree
of Doctor of Philosophy

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Abstract

My dissertation aims at understanding three critical issues confronting the financial world: contagion of a crisis, corporate governance, and credit rating. It contains four chapters.¹

Through examining the contagion of a crisis, Chapter 1 presents a model in which the contagion of a liquidity crisis between two non-financial institutions occurs due to the learning within a common creditor pool. After creditors observe what occurs in a firm’s rollover game, they conjecture each other’s "type," or the attitude toward the risk of a firm’s investment project. Creditors’ inference of others’ types then affects their own decisions with regard to the next firm that they lend to. Through the analysis of each firm’s "incidence of failure" – the threshold for a liquidity crisis – I demonstrate that the risk of contagion rises in an important way if originating from a firm that ex-ante faces a small probability of failure. I also offer policy proposals to mitigate the severity of contagion in such liquidity crises.

Then, Chapter 2 extends this contagion idea to delve into the effect of enhanced distribution of public information by the central bank on the contagion of a currency crisis between two countries. In the speculators’ learning process about each other’s aggressiveness in regard to the speculation as the contagion mechanism of a currency crisis, the impact of the contagion can be either negative or positive. Through the analysis of each country’s threshold for a currency crisis, I show that the public signal distributed by the central bank promotes the positive effect of the contagion and reduces the negative effect of the contagion on the currency crisis from the other country. I also demonstrate that the effectiveness of the high precision of the public signal depends on the ex-ante expected state of the economic fundamentals of the country.

¹ Chapter 3 and Chapter 4 are written jointly with Kyung Suh Park and Sung-Tae Kim, respectively.
Corporate governance is a major factor in determining the value of a firm. Hence, Chapter 3 examines how shareholders use the corporate governance structure – managerial incentive scheme – to maximize their utility in the product market competition. That is, we assess the effects of the competitive structure of a product market on a firm’s corporate governance structure. We show that shareholders determine the corporate governance structure, including the manager’s stock ownership and his governance power over the firm, in order to maximize their utility in the product market competition. We find that the manager’s stock ownership would be lower and his governance power over the firm would be higher in cases in which the firm’s product is more profitable or when competition in the product market is more severe. We also determine that the manager’s stock ownership and his governance power would tend to be higher in cases in which the manager’s private benefit of control tends to overly hurt the firm’s value.

Because a credit rating system is directly connected to the soundness of the whole corporate system, it is important to foster the competitive condition in the credit rating industry. Investigating the market structure of Korea’s credit rating industry during 1995 – 2000, Chapter 4 utilizes the Rosse-Panzar methodology to evaluate the Korean government’s financial restructuring policy for fostering the competitive condition in the credit rating industry after the 1997 financial crisis. We find that the degree of market competition in the credit rating industry increased after the implementation of the Korean government’s financial restructuring policy. Our analysis indicates that after the financial restructuring process the market structure of Korea’s credit rating industry became an oligopoly in a contestable market, which is economically equivalent to the structure of perfect competition.
Acknowledgements

Although I still have a long way to go as a scholar, I have finally produced my Ph.D. thesis, the first academic work in my doctoral studies. This achievement would not have been possible without the support from many people around me.

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1.1 Introduction

Contagion is a propagation of the solvency problems of a single institution to other institutions. It is one of the most striking features of a financial crisis, in that it causes the crisis to spread across countries and institutions. In the late 1990’s, most East Asian countries suffered severe financial crises via contagion across countries, the so-called "Asian Flu." When South Korea suffered the Asian Flu, the liquidity crisis spread from one firm to other firms even though their businesses were not closely related. For example, in January 1997, Hanbo Steel Group – the country’s fourteenth-largest conglomerate – went bankrupt, and within a few months, Jinro – the largest liquor group in Korea – failed. They had connections with each other only via common bank creditors. How do we explain these kinds of serial (contagious) failures of non-financial firms whose businesses are not related to each other?

In this paper, I present a model in which the contagion of a liquidity crisis between two unrelated non-financial institutions occurs due to the co-creditors’ learning about each other’s "type," or the attitude toward the risk of a firm’s investment project. Some fairly extensive studies deal with a contagion of the financial crisis among financial institutions and/or international financial markets based on their interlinkages and changes in asset prices (Allen and Gale (2000) and Cifuentes, Ferrucci, and Shin (2005) among others). However, studies on the contagion of the liquidity crisis among non-financial businesses have received only scant attention. My aim is to contribute to the understanding of the contagion phenomenon among non-financial institutions.

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2 This 1997 Korean financial crisis will be explained more specifically in section 1.5.
based on the idea that the contagion’s triggering mechanism is the learning within a common creditor pool. Specifically, I suggest that when co-creditors learn about each other’s "type," contagion is triggered.

I focus on the "self-fulfilling crisis" – a crisis that occurs just because creditors believe it is going to occur. The self-fulfilling nature of the crisis is important because a liquidity crisis in a firm is often viewed as resulting from a coordination failure among creditors. However, to approach the nature of the crisis as self-fulfilling tends to produce multiple equilibrium outcomes, and thus it is hard to demonstrate the contagion effect.\(^4\) Therefore, to obtain the unique equilibrium (the threshold for a liquidity crisis), I employ the global games method introduced by Carlsson and van Damme (1993). This method allows me to get the unique equilibrium in each firm and therefore capture the contagion effect in which a liquidity crisis in one firm affects the likelihood of a crisis in another.

Specifically, the global games setting of the firm and of the creditors is similar to the one used by Morris and Shin [M-S] (2004). M-S (2004) analyze the coordination game in the debt market by using tools of global games. They show that the creditors of a distressed borrower face a coordination problem (a rollover game among creditors). They further demonstrate that, without common knowledge on the fundamentals of the distressed borrower, the incidence of failure is uniquely determined, given that the creditors’ private information on the fundamentals is precise enough.\(^5\) However, they just tackle one firm’s rollover game among one type of creditors and do not cover the contagion of the liquidity crisis among firms, which is the central issue of my paper.

\(^4\) As Goldstein and Pauzner (2004) mention in their paper, models with multiple equilibria cannot capture the contagion effect in which a liquidity crisis in one firm affects the likelihood of a liquidity crisis in the other firm because they do not predict the likelihood of each particular equilibrium.

I extend M-S (2004)’s model to the case of two firms with two different types of creditors. By doing so, I explain the contagion phenomenon between two firms.

For the contagion setting between two firms, I generally refer to Goldstein and Pauzner [G-P] (2004). G-P (2004) use the global games method to explain the contagion phenomenon between two countries. They look at two countries that have independent fundamentals but share the same group of investors. In their model, a crisis in one country reduces agents’ wealth, which makes them more averse to the strategic risk associated with the unknown behaviors of other agents in the second country. This increases agents’ incentive to withdraw their investments in the second country. That is, the mechanism that generates the contagion in their model originates in a wealth effect. However, in my paper, I focus on the creditors’ learning about each other’s type as the contagion mechanism. In a coordination game setting, the learning process is very important because it can directly explain the creditors’ strategic behaviors, which in turn affect the probability of the liquidity crisis in the firm.

I examine a sequential framework in which the rollover game among creditors in firm $A$ takes place before it occurs in firm $B$. Creditors in my model hold loans for two firms’ investment

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6 Kyle and Xiong (2001) also explain the contagion of the financial crisis between two countries based on the wealth effect, even though they do not use the global games approach.

7 Angeletos, Hellwig, and Pavan (2007) study how learning about the underlying fundamentals influences the dynamics of coordination in a global game of regime change. Similarly, Manz (2002) shows that the failure of a single firm can trigger a chain of failures when investors learn about a common state influencing all firms within an industry, such as a proxy variable for the demand facing the products of all firms. Empirically, Lando and Nielsen (2010) conduct tests for default contagion effects among firms, based on ratings covariates. In my paper, however, I examine how learning about the types of other co-creditors plays a role as the contagion mechanism of the liquidity crisis between two firms. Taketa (2004a) analyzes the contagion phenomenon of currency crises between two countries using the global games method with the learning process of speculators. However, he does not numerically analyze the contagion effect and its severity. Focusing on non-financial institutions, I specifically analyze the contagion effects and suggest policy proposals to reduce the severity of contagion on the liquidity crisis from one firm to the other.

8 Chen (1999) shows that the systemic risk may occur in the absence of any interbank relations due to the first-come, first-served rule and to information externalities on the negative payoffs. That is, he models banking panics as the outcome of "information-based herding behavior" by depositors. However, the global games approach that I use in this paper has a quite different mechanism from the herding model. Morris and Shin (2003) differentiate the two as follows: "The global games analysis is driven by strategic complementarities and the highly correlated signals generated by the noisy observations technology. However, the sensitivity to the information structure arises in a purely static setting. The herding stories have no payoff complementarities and simple information structures, but
In each firm, they can either roll over their loans until maturity, in which case they can get full repayment from the firm if the investment project succeeds, or they can recall their loans in the interim stage, in which case they can get some premature liquidation value (collateral debt) but less than the full repayment amount. The success of the investment project depends on the fundamentals of the firm and on the number of creditors in that firm who keep rolling over their loans until maturity. That is, creditors’ coordination on whether or not to roll over the loans determines the likelihood of a liquidity crisis in the firm.

There are two types of creditors: one is "pessimistic" and the other is "optimistic." "Pessimistic" creditors worry about the failure of the firm’s investment project more than "optimistic" creditors do. In practice, the different types reflect both the strength of the balance sheet (the financial status) of each creditor and any information advantage on firm-related issues, including the economic situation. That is, a creditor with a weak balance sheet and/or with an information disadvantage on firm-related issues holds a more "pessimistic" attitude toward the risk he takes than one who has a strong balance sheet and/or an information advantage on firm-related issues.

Following the global games method, I assume that creditors do not have common knowledge on the fundamentals of firm $A$ and firm $B$. Rather, creditors get noisy signals about the firm’s fundamentals after they are realized. In this setting, based on the private signals about the firm’s fundamentals, different types of creditors uniquely determine both their own beliefs on the fundamentals of each firm and their own actions on whether or not to roll over the loans until maturity.

---

9 Co-creditors, for example, can be thought of as common bank creditors of different firms.

10 Izmalkov and Yildiz (2010) show that in strategic environments the relevant measure of sentiments (i.e., pessimistic / optimistic outlook) can vary arbitrarily and have a large impact on the strategic behavior even when there is little uncertainty.
maturity in that firm. After the rollover game in firm \( A \) ends, creditors observe the aggregate outcomes of firm \( A \), which depend on firm \( A \)'s fundamentals and on creditors’ actions in firm \( A \).

Observing what occurred in firm \( A \), creditors can conjecture other creditors’ types since the outcome of the rollover game in firm \( A \) depends on the different actions of different type creditors. Hence, before the rollover game in firm \( B \) occurs, creditors can revise their beliefs about other creditors’ types. After learning about other creditors’ types from the outcome of firm \( A \), creditors uniquely determine their beliefs on the fundamentals of firm \( B \) and their actions in firm \( B \). If there is a liquidity crisis in firm \( A \), and if firm \( B \) also suffers the liquidity crisis due to the creditors’ learning process, then there is a "contagion" of the liquidity crisis from firm \( A \) to firm \( B \). Moreover, I refer to the increased probability of the liquidity crisis in firm \( B \) due to the contagion as a "severity of contagion" on the liquidity crisis.

Having shown the severity of contagion on the liquidity crisis from firm \( A \) to firm \( B \), I demonstrate that the severity of contagion is greater when the originating firm’s "failure point" – the probability that the firm’s investment project will fail – has decreased. In other words, the liquidity crisis in a firm that has a small possibility of failing is more contagious than otherwise. This is a striking result compared with other contagion-related papers, which deal with contagion among international financial markets and/or financial institutions through capital linkages and asset price changes. In these papers, the larger the negative impact originating from worse fundamentals, the more severely other financial institutions or countries are affected through their linkages.

Also, I analyze the policy implications of reducing the severity of a liquidity crisis contagion from firm \( A \) to firm \( B \). Firm \( B \) can minimize the severity of a contagion from firm \( A \) to itself by setting the value of its collateral small, since the decreased value of the collateral is the increased
cost of not rolling over the loans from the creditors’ standpoint. The government can also play a role to reduce the severe contagion damage of the liquidity crisis by making the pessimistic creditors more optimistic about the success of the firm’s investment project (e.g., by providing bailouts to the firm that suffers a transitory liquidity problem) and by reducing the degree of incomplete information on the creditors’ types in the market (e.g., by implementing the financial disclosure policy that discloses the types of creditors).

Regarding creditors’ information structure, I find that increasing the accuracy of creditors’ information on the firm’s fundamentals lowers the failure point of the individual firm. However, in the same way that the severity of contagion is more serious when the originating firm’s failure point is lower, the severity of contagion is also more serious when creditors have more accurate information. That is, if the liquidity crisis occurs in the firm considered less likely to fail (i.e., the firm with a small failure point because creditors have precise information structure on the fundamentals), then it leads to a big surprise in the market, and thus the liquidity crisis can be more contagious. Based on this phenomenon, I argue that policies promoting transparency and precise information on the firm’s fundamentals are not a panacea in a crisis episode. Even though transparency reduces the probability of a crisis in one economy’s case, it worsens the severity of contagion among more than one economy.

The remainder of this paper is as follows. I present the model in section 1.2. In section 1.3, I solve for firm A’s equilibrium and firm B’s equilibrium, and show how the contagion of the liquidity crisis from firm A to firm B occurs through the creditors’ learning process. In section 1.4, I define "severity of contagion" on the liquidity crisis and discuss some policy implications to reduce this severity. In section 1.5, I discuss the applicability of my model to real-world phenomena, focusing on Korea’s 1997 financial crisis. Section 1.6 concludes.
1.2 Model

There are two firms: firm A and firm B. Both firms own no capital, and their investment projects are only financed by loans from creditors. There are two groups of creditors: group 1 and group 2. The order of events (see figure 1.1) is as follows.\(^{11}\)

![Timeline](image)

**Figure 1.1: Timeline**

First, nature determines what the creditors are like. Second, creditors lend their money to both firms A and B. Third, the states of each firm’s fundamentals \((\theta_A\) and \(\theta_B)\) are realized. Fourth, each creditor in each group \((j = 1, 2)\) receives a private signal \((x_{Aj})\) on the fundamentals of firm A. Fifth, each creditor decides whether or not to roll over the loan in firm A. Sixth, the exact realization of the fundamentals of firm A and the result of the creditors’ actions (i.e., firm A’s project failure or success) are known to all creditors after the rollover game in firm A ends.\(^{12}\) Seventh, each creditor in each group \((j = 1, 2)\) gets a private signal \((x_{Bj})\) on the fundamentals of firm B. Eighth, each creditor decides his action in firm B. Ninth, the exact realization of the

---

\(^{11}\) I generally follow G-P (2004)’s sequence. Note that the model is sequential: the activity takes place in firm A and then in firm B.

\(^{12}\) That is, before creditors decide on their actions, they did not know the exact value of the firm’s fundamentals. However, I assume that after the rollover game ends, creditors get to know the true value of the firm’s fundamentals. As G-P (2004) mention in their paper, in equilibrium, it is sufficient that creditors receive information regarding either the fundamentals or the aggregate behaviors of creditors since one can be inferred from the other.
fundamentals of firm $B$ as well as the aggregate behaviors in firm $B$ are known to all creditors.

Creditors are financing both investment projects of two firms: firm $A$ and firm $B$. In other words, two firms share the same creditors. There are two groups of creditors: group 1 and group 2, both consisting of a continuum of small creditors, so that any individual creditor’s stake is negligible as a proportion of the whole.  

The size of group 1 is $\lambda$ and that of group 2 is $(1 - \lambda)$, where $0 < \lambda < 1$. There exists uncertainty about the creditors’ type, that is, about the creditors’ attitudes toward the risk (bullishness) of a firm’s investment project. Thus, group 1’s type is its own private information. There are two possible types of group 1 creditors: "pessimistic," with probability $q$, and "optimistic" with probability $(1 - q)$.  

"Pessimistic" creditors worry about the failure of the firm’s investment project more than "optimistic" creditors do. The different types reflect both the strength of the balance sheet (the financial status) of each creditor and any information advantage on firm-related issues, including the economic situation.  

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13 Corsetti, Dasgupta, Morris, and Shin [C-D-M-S] (2004) use the global games approach to consider the implication of the existence of a large speculator like George Soros in a currency crisis in the dynamic setting. But they do not cover the contagion effect there. Based on C-D-M-S (2004), Taketa (2004b) analyzes the implication of the presence of a large speculator in contagious currency crises: making countries more vulnerable to crises but mitigating the contagion of crises across countries. In my paper, for simplicity, I focus purely on all small players in the static/simultaneous game setting.

14 That is, nature randomly chooses the type of group 1 creditors: "pessimistic" or "optimistic." Group 1 creditors know their own type, but group 2 creditors do not know the type of group 1 creditors. Group 2 creditors can just expect that the type of all group 1 creditors is "pessimistic" with probability $q$ or "optimistic" with probability $(1 - q)$. However, group 1’s type can be revealed to group 2 creditors after the rollover game in firm $A$ ends, which I tackle in section 1.3.2.

15 Alternatively, I can set the type of group 2 creditors as being "optimistic." It does not affect the contagion result of my model because the type of group 2 creditors is public information in the market. Of course, the type of group 2 creditors affects the probability of the liquidity crisis in each individual firm.

16 In practice, a creditor’s financial status can change over time and his informativeness is different for firm $A$ and firm $B$. In my work, for simplicity, I assume that a creditor’s financial status does not change in the course of the model’s
disadvantage on firm-related issues holds a more "pessimistic" attitude toward the risk he takes than one who has a strong balance sheet and/or an information advantage on firm-related issues. I assume that "pessimistic" creditors use $\delta_P$ as their discount factor, which is less than $\delta_O$ – the discount factor of "optimistic" creditors (i.e., $0 < \delta_P < \delta_O < 1$). That is, "pessimistic" (bearish) creditors put less present value on the firm’s investment project than "optimistic" (bullish) creditors do.

The state of firm $i$’s fundamentals is $\theta_i$, where $i = A, B$. $\theta_i$ can be interpreted as a measure of the ability of firm $i$ to meet short-term claims from creditors. The higher value of $\theta_i$ refers to the better fundamentals. $\theta_i$ is randomly drawn from the real line after both firms raise funds from creditors and invest the funds in their projects. I assume that $\theta_A$ and $\theta_B$ are independent, which means that there is no linkage of fundamentals between firm $A$ and firm $B$.

After $\theta_i$ ($i = A, B$) is realized, the rollover game among creditors takes place in sequences: firm $A$ first and then firm $B$. In each firm’s rollover game, there are two periods: period 1 (interim stage) and period 2 (maturity), in which creditors lend for a firm’s investment project. The investment project of each firm is completed in period 2 and yields the return $v_i$ ($i = A, B$), which is uncertain initially because it depends on the creditors’ actions in period 1. Financing of firm $A$ and firm $B$ is undertaken by a standard debt contract. For simplicity, I assume that both firms have the same debt contract. That is, the face value of the repayment is $L$, and each creditor receives this full amount in period 2 if the realized value of $v_i$ is large enough to cover the repayment of debt.

---

17 This two-period rollover game among creditors is directly based on M-S (2004)’s model.
18 In general, firms use various debt contracts and they can screen the types of creditors. However, in my model, I explain creditors’ learning process about each other’s type by simply focusing on the standard debt contract. Analyzing creditors’ learning process, I define the contagion of the liquidity crisis from firm $A$ to firm $B$ in section 1.3.3.
At period 1, before the final realization of \( v_i \), the creditors have an opportunity to review their investment. Hence, in this period, creditors have to decide whether or not to roll over their loans until period 2. The loans are collateralized, and if creditors collect and liquidate the collateral after they do not roll over the loans (period 1), the liquidation value of the seized collateral is \( K^* \in (0, L) \). However, if the creditors collect and liquidate the collateral because they cannot get the full repayment after they roll over the loans (period 2), the liquidation value of the seized collateral is \( K_* \), which is less than \( K^* \) (i.e., \( K_* < K^* < L \)). That is, if I denote the proportion of creditors who do not roll over the loans of firm \( i \) at period 1 by \( l_i \) (\( i = A, B \)), then the firm’s investment project fails if and only if \( l_i > \theta_i \) and creditors get \( K_* \) at period 2.\(^{19}\)

As M-S (2004) do, for the simplicity of my discussion, I normalize the payoffs so that \( L = 1 \) and \( K_* = 0 \). Then, creditors who do not roll over the loans at period 1 get \( K \), which is in \((0,1)\).\(^{20}\)

In summary, the present values of the payoffs at period 1 to a creditor are given by the following matrix:

<table>
<thead>
<tr>
<th>Rollover</th>
<th>Project succeeds</th>
<th>Project fails</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not rollover</td>
<td>( \delta_m \cdot 1 = \delta_m )</td>
<td>( K )</td>
</tr>
<tr>
<td>Rollover</td>
<td>( 0 )</td>
<td>( K )</td>
</tr>
</tbody>
</table>

where \( m \) is \( P \) for a "pessimistic" creditor or \( O \) for an "optimistic" creditor. I assume

\[
0 < K < \delta_P < \delta_O < 1.
\]

If the creditors know the value of \( \theta_i \) perfectly before deciding whether or not to roll over the loans (period 1), their optimal strategies are like Obstfeld (1996)’s self-fulfilling story, as follows. If \( \theta_i > 1 \), then creditors will roll over their loans irrespective of other creditors’ actions because the project survives even if every other creditor recalls. Conversely, if \( \theta_i \leq 0 \), then it is optimal for creditors not to roll over the loans since the state of the fundamentals of the firm is so bad that the

\(^{19}\) The firm remains in operation given that \( \theta_i \) is large enough to meet the claims from creditors. Otherwise, it is pushed into default. Specifically, if \( \theta_i \geq l_i \), then the firm’s investment project succeeds and the realized value of \( v_i \) is equal to \( V \) which is a constant greater than \( L \). Meanwhile, if \( l_i > \theta_i \), then the project fails and \( v_i = K_* \).

\(^{20}\) The exact value of \( K \) is \( \frac{K^* - K_*}{L - K_*} \) by normalizing the payoffs, and it is in \((0,1)\) since \( K_* < K^* < L \).
project will fail even if all other creditors roll over their loans. When \( \theta_i \in (0, 1] \), the coordination problem among the creditors occurs. If all other creditors roll over their loans, then the payoff to rolling over the loan is 1 at maturity (period 2)\(^21\), so that rolling over the loan yields more than the premature liquidation value \( K \). Meanwhile, if everyone else recalls the loan, then the payoff is 0, which is less than \( K \), so that early liquidation is optimal. Hence, the common knowledge assumption of creditors on \( \theta_i \) leads to multiple equilibria.\(^22\)

To get the unique equilibrium, I apply the global games method here: \( \theta_i \) is not the common knowledge. Rather, at period 1 when creditors decide whether or not to roll over the loans, they receive private information concerning \( \theta_i \), but it is not perfect. In other words, each creditor in group \( j \) (\( j = 1, 2 \)) gets the private signal: \( x_{ij} = \theta_i + \varepsilon_{ij} \), where \( \varepsilon_{ij} \) is uniformly distributed over the interval \([ -\varepsilon, \varepsilon ]\).\(^23\) Note that the creditor’s present value (at period 1) of the expected utility of rolling over the loan based on his private signal is \( U = \delta_m \cdot \Pr [\theta_i \geq l_i \mid x_{ij}] \), where \( m = P \) or \( O \), and that of recalling the loan is \( K \). A strategy for the creditor is a decision rule which maps each realization of \( x_{ij} \) to an action – rolling over the loan or not rolling over the loan. An equilibrium strategy consists of (1) a firm’s switching fundamentals (\( \bar{\theta}_i \)) below which the project fails (i.e., a liquidity crisis occurs in the firm) and (2) the creditors’ switching private signal (\( \bar{x}_{ij} \)) such that every creditor who receives a signal lower than \( \bar{x}_{ij} \) does not roll over the loan.\(^24\)

In the following section, I solve for the equilibrium strategy of firm \( A (\bar{\theta}_A \) and \( \bar{x}_{Aj} \), where

---

21 At period 1, the present value of 1 is \( \delta_P \) for "pessimistic" creditors or \( \delta_O \) for "optimistic" creditors.

22 As M-S (2004) mention in their paper, this type of coordination problem among creditors is analogous to the bank run problem of Diamond and Dybvig [D-D] (1983). However, D-D (1983) do not cover contagion issues. They just focus on analyzing the coordination failure among patient depositors in one bank and show the result of multiple equilibria. M-S (2004) consider both private and public signals on the firm’s fundamentals. In my paper, for simplicity, I just assume that creditors get the private signals on the firm’s fundamentals.

23 As M-S (1998, 2003, 2004) discuss in the literatures, even if \( \varepsilon \) becomes very small, the realization of \( \theta_i \) will not be common knowledge among the creditors. Moreover, in this case, M-S (1998, 2003, 2004) and C-D-M-S (2004) show that the equilibrium strategy consists of a unique value of a firm’s switching fundamentals and a unique value of the creditors’ switching private signal.
After the rollover game in firm A ends, every creditor observes what occurred in firm A, including the exact value of $\theta_A$. Here, group 2 creditors can conjecture or learn the "type" of group 1 creditors based on the outcome in firm A (i.e., whether a liquidity crisis in firm A occurred or not) and on firm A’s switching fundamentals. Next, I solve for the equilibrium strategy of firm B ($\bar{\theta}_B$ and $\bar{x}_{Bj}$, where $j = 1, 2$), which is affected by creditors’ revised beliefs – which are formed after the rollover game in firm A ends – about other creditors’ types. This explains how and why a liquidity crisis in firm A can trigger a liquidity crisis in firm B (i.e., it explains a contagion of the liquidity crisis from firm A to firm B).

### 1.3 Solving the Model

#### 1.3.1 Equilibrium in Firm A

Firm A’s equilibrium strategy consists of (1) a firm’s switching fundamentals ($\bar{\theta}_A$) below which the project fails (i.e., a liquidity crisis occurs in firm A) and (2) the creditors’ switching private signal ($\bar{x}_{Aj}$) such that every creditor who receives a signal lower than $\bar{x}_{Aj}$ does not roll over the loan. Here, the equilibrium values $\bar{\theta}_A$ and $\bar{x}_{Aj}$ are as follows:

\[
\bar{\theta}_A = \begin{cases} 
\theta^*_{AP} & \text{if the type of group 1 creditors is "pessimistic";} \\
\theta^*_{AO} & \text{if the type of group 1 creditors is "optimistic"}
\end{cases}
\]

\[
\bar{x}_{A1} = \begin{cases} 
x^*_{A1P} & \text{if group 1 creditors are "pessimistic";} \\
x^*_{A1O} & \text{if group 1 creditors are "optimistic"}
\end{cases}
\]

\[
\bar{x}_{A2} = \bar{x}_{A2}.
\]

After getting a private signal in period 1, each creditor has to decide whether or not to roll over the loan. The indifference condition gives the following equation:

\[
\text{payoff from recalling} = \frac{K}{(1.1)} = \delta_m \cdot \text{Pr [rollover is successful | } \bar{x}_{Aj}] \cdot \text{PV of the payoff from successful rollover}
\]

Also, note that the critical threshold value of firm A’s fundamentals (i.e., switching fundamentals) is determined when the proportion of creditors who do not roll over the loans ($l_A$) is equal to $\theta_A$. 

12
Using equation (1.1) for each creditor and the condition of the critical threshold value of firm A’s fundamentals, I calculate the unique equilibrium values: the switching fundamentals of firm A ($\theta^*_{AP}$ and $\theta^*_{AO}$) and the switching private signals ($x^*_{A1P}$, $x^*_{A1O}$, and $x^*_{A2}$). Firm A’s equilibrium is summarized in the following proposition.

**Proposition 1** There exists a unique equilibrium strategy in firm A that consists of (1) a firm’s switching fundamentals ($\theta_A$) below which the project fails (i.e., a liquidity crisis occurs in firm A) and (2) the creditors’ switching private signal ($\bar{x}_{Aj}$, $j = 1, 2$) such that every creditor who receives a signal lower than $\bar{x}_{Aj}$ does not roll over the loan. Specifically, firm A’s switching fundamentals are

$$
\theta^*_{AP} = \frac{K}{\delta_P} (1 - \Sigma_1),
$$

$$
\theta^*_{AO} = \frac{K}{\delta_P} (1 - \Sigma_1 - \Sigma_2);
$$

and the creditors’ switching private signals are

$$
x^*_{A1P} = \frac{K}{\delta_P} (1 - \Sigma_1 + \Sigma_3),
$$

$$
x^*_{A1O} = \frac{K}{\delta_P} \left( 1 - \Sigma_1 - \Sigma_2 + \frac{\delta_P}{\delta_O} \Sigma_3 \right),
$$

$$
x^*_{A2} = \frac{K}{\delta_P} (1 - \Sigma_1 - (1 - q) \Sigma_2 + \Sigma_3),
$$

where

$$
\Sigma_1 = \frac{\lambda (1 - \lambda) (1 - q) (\delta_O - \delta_P)}{\delta_O (1 + 2\varepsilon - \lambda)},
$$

$$
\Sigma_2 = \frac{2\lambda \varepsilon (\delta_O - \delta_P)}{\delta_O (1 + 2\varepsilon - \lambda)},
$$

and

$$
\Sigma_3 = \left( \frac{2K - \delta_P}{K} \right) \varepsilon.
$$
Creditors in group 1 ("pessimistic type") do not roll over the loans in firm A
Creditors in group 2 do not roll over the loans in firm A

Figure 1.3: Creditors’ Switching Private Signals in Firm A

Note that $\theta_{AP}^* > \theta_{AO}^*$ and $x_{A1P}^* > x_{A2}^* > x_{A1O}^*$ hold since $\lambda$, $q$, and $\varepsilon$ are in $(0, 1)$, and $0 < \delta_P < \delta_O < 1$ (see figure 1.2 and figure 1.3). The intuition of the inequalities is the following. $x_{A1P}^*$ is greater than $x_{A1O}^*$ because the pessimistic creditors are more likely not to roll over the loans than optimistic creditors. By the same logic, $\theta_{AP}^*$ is greater than $\theta_{AO}^*$ because firm A’s project will be more likely to fail (i.e., will be liquidated early) if group 1 creditors are pessimistic.

1.3.2 Equilibrium in Firm B

Now every creditor observes what occurred in firm A, including the exact value of $\theta_A$. This conveys information about the type of group 1 creditors to the market because different types use different switching signals, resulting in different outcomes in firm A under certain conditions.

There are two possible scenarios. First, if $\theta_A \notin [\theta_{AO}^*, \theta_{AP}^*]$, then the type of group 1 creditors is not revealed. Why? If $\theta_A \leq \theta_{AO}^*$, then the liquidity crisis certainly occurs in firm A regardless of the type of group 1 creditors. Meanwhile, if $\theta_A \geq \theta_{AP}^*$, then the liquidity crisis never occurs in firm A regardless of the type of group 1 creditors. Hence, if $\theta_A \notin [\theta_{AO}^*, \theta_{AP}^*]$, group 2 creditors do not get to know the type of group 1 creditors and face the same rollover game, which was played.
in firm A, in determining whether or not to roll over the loans in firm B.\footnote{Note that in this case ($\theta_A \notin [\theta_{AO}^*, \theta_{AP}^*]$), even though the number of creditors who did not roll over their loans is known, the type of group 1 creditors is not revealed since $x_{A1}$ is in the $\varepsilon$-neighborhood of $\theta_A$; and $x_{A1P}$ and $x_{A1O}$ are very closely located around $\theta_{AP}^*$ and $\theta_{AO}^*$, respectively.}

Next, however, if $\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$, then the type of group 1 creditors is revealed to the market. Conditional on such $\theta_A$, the liquidity crisis occurs in firm A if and only if group 1 creditors are pessimistic. Likewise, conditional on such $\theta_A$, which is between $\theta_{AO}^*$ and $\theta_{AP}^*$, the liquidity crisis does not occur in firm A if and only if group 1 creditors are optimistic. Hence, if $\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$, then the new rollover game is played by creditors determining whether or not to roll over the loans in firm B.

In the following, I explain the two scenarios: \(\theta_A \notin [\theta_{AO}^*, \theta_{AP}^*]\) and \(\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]\). In each scenario, I derive the equilibrium strategy (i.e., $\tilde{\theta}_B$ and $\bar{x}_{Bj}$, $j = 1, 2$).

**1.3.2.1 Scenario 1: $\theta_A \notin [\theta_{AO}^*, \theta_{AP}^*]$**

In this scenario, the type of group 1 creditors is not revealed. Hence, the equilibrium values of the switching fundamentals of firm B and the switching private signals are exactly the same as those of firm A. This is the benchmark case of firm B, and particularly, the benchmark switching fundamentals of firm B are (1) $\theta_{AO}^*$ if group 1 creditors are optimistic, and (2) $\theta_{AP}^*$ if group 1 creditors are pessimistic.

**1.3.2.2 Scenario 2 – 1: Liquidity crisis in firm A when $\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$**

This scenario implies that the type of group 1 is "pessimistic." In this case, creditors in both group 1 and group 2 have the same switching strategy signal (say $x_B^*$). Hence, the equilibrium strategy consists of (1) a firm’s switching fundamentals ($\theta_{BP}^*$) below which the project fails (i.e., a liquidity crisis occurs in firm B) and (2) the creditors’ switching private signal ($x_B^*$) such that every creditor who receives a signal lower than $x_B^*$ does not roll over the loan. Here, I get the
following equilibrium strategy:

\[ \theta_{BP}^* = \frac{K}{\delta_P}, \]
\[ x_B^* = \frac{K}{\delta_P} (2\varepsilon + 1) - \varepsilon. \]

1.3.2.3 Scenario 2-2: No liquidity crisis in firm A, when \( \theta_A \in [\theta_{AO}, \theta_{AP}] \)

This scenario implies that the type of group 1 is "optimistic." In this case, creditors in both group 1 and group 2 have different switching strategy signals (say \( x_{B1}^* \) for group 1 and \( x_{B2}^* \) for group 2). Hence, the equilibrium strategy consists of (1) a firm’s switching fundamentals (\( \theta_{BO}^* \)) below which the project fails (i.e., firm B suffers a liquidity crisis) and (2) the creditors’ switching private signals (\( x_{B1}^* \) for group 1 and \( x_{B2}^* \) for group 2) such that every creditor in group 1 who receives a signal lower than \( x_{B1}^* \) does not roll over the loan and that every creditor in group 2 who receives a signal lower than \( x_{B2}^* \) does not roll over the loan. Here, I get the following equilibrium strategy:

\[ \theta_{BO}^* = \frac{\lambda K}{\delta_O} + \frac{(1 - \lambda) K}{\delta_P}, \]
\[ x_{B1}^* = \frac{K (\lambda + 2\varepsilon)}{\delta_O} + \frac{(1 - \lambda) K}{\delta_P} - \varepsilon, \]
\[ x_{B2}^* = \frac{\lambda K}{\delta_O} + \frac{K (1 - \lambda + 2\varepsilon)}{\delta_P} - \varepsilon. \]

Note that \( \theta_{BO}^* < \theta_{BP}^* \) and \( x_{B1}^* < x_{B2}^* < x_B^* \) hold since \( \lambda \) and \( \varepsilon \) are in \((0, 1)\), and \( 0 < \delta_P < \delta_O < 1 \). The intuition of the inequalities is the following. \( x_B^* \) is greater than \( x_{B1}^* \) and \( x_{B2}^* \) because when all creditors are pessimistic, they are more likely not to roll over the loans than when there exist optimistic creditors. By the same logic, \( \theta_{BP}^* \) is greater than \( \theta_{BO}^* \) because firm B’s project will be more likely to fail (i.e., will be liquidated early) if group 1 creditors are pessimistic.

Now, firm B’s equilibrium is summarized in the following proposition.26

---

26 The proof of this result directly follows from the derivation of firm B’s equilibrium strategy in each scenario and the same logic as the proof of Proposition 1 given in the Appendix.
**Proposition 2**  Conditional on the realized underlying state of the fundamentals of firm A ($\theta_A$) and whether a liquidity crisis occurs in firm A or not, there exists a unique equilibrium in firm B.

1. When the realized underlying state of the fundamentals of firm A ($\theta_A$) is not in the interval $[\theta_{AO}^*, \theta_{AP}^*]$, the same equilibrium values (switching fundamentals and switching private signals) as those of firm A are obtained irrespective of whether a liquidity crisis occurs in firm A or not;

2. When $\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$ and there is a liquidity crisis in firm A, every creditor in any group does not roll over the loan if his signal $x_{Bj}$ ($j = 1, 2$) is below $x_{B1}$ and does roll over the loan if the signal is above;

3. When $\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$ but there is no liquidity crisis in firm A, each creditor in group 1 does not roll over the loan if his signal $x_{B2}$ is below $x_{B1}$ and each creditor in group 2 does not roll over the loan if his signal $x_{B2}$ is below $x_{B2}^*$.

**1.3.3 Contagion of the Liquidity Crisis from Firm A to Firm B**

**1.3.3.1 What is contagion?**

In this paper, contagion is defined as a propagation of the solvency problems between two firms, and the contagion of the liquidity crisis from firm A to firm B is propagated by creditors who determine whether or not to roll over the loans. After observing what happened in firm A, creditors update their beliefs about other creditors’ types and reflect this information in their optimal decisions in firm B.

If the realized value of the fundamentals of firm A ($\theta_A$) is quite bad, which means $\theta_A \leq \theta_{AO}^*$, then firm A suffers a liquidity crisis regardless of the type of group 1 creditors. In this case, the type of group 1 creditors is not revealed. So if $\theta_A \leq \theta_{AO}^*$, it does not cause the contagion of the liquidity crisis from firm A to firm B because group 2 creditors’ decisions in firm B are not affected by the situation in firm A. Only when $\theta_A$ is between $\theta_{AO}^*$ and $\theta_{AP}^*$ and when there is a liquidity crisis in firm A, can I discuss whether there is a contagion of the liquidity crisis from firm A to firm B.

As I discussed in section 1.3.2, if $\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$ and there is no liquidity crisis in firm A, this implies that the type of group 1 creditors is "optimistic." This information is reflected in
group 2 creditors’ decisions, and \( \theta^*_B \) is determined. Likewise, if \( \theta_A \in [\theta^*_{AO}, \theta^*_{AP}] \) and there is a liquidity crisis in firm \( A \), this implies that the type of group 1 creditors is "pessimistic." This information is reflected in group 2 creditors’ decisions, and \( \theta^*_B \) is determined. That is, only when \( \theta_A \in [\theta^*_{AO}, \theta^*_{AP}] \), does the behavior of creditors in firm \( A \) affect the behavior of creditors in firm \( B \).

Now if the realized value of the fundamentals of firm \( B (\theta_B) \) is quite bad, which means \( \theta_B \leq \theta^*_B \), then firm \( B \) suffers a liquidity crisis regardless of the occurrence of the liquidity crisis in firm \( A \). Hence in this case, even though there are liquidity crises in both firms, I cannot say that there is an actual contagion of the solvency problems from firm \( A \) to firm \( B \). Meanwhile, if \( \theta_B \) is between \( \theta^*_B \) and \( \theta^*_B \) and there is a liquidity crisis in firm \( B \), then this is the contagion of the liquidity crisis from firm \( A \) to firm \( B \) since there can be the liquidity crisis in firm \( B \) in \( \theta_B \in [\theta^*_B, \theta^*_B] \) only when there was the liquidity crisis in firm \( A \) in \( \theta_A \in [\theta^*_A, \theta^*_A] \) (see figure 1.4).

**Definition 1** Contagion of the liquidity crisis from firm \( A \) to firm \( B \) is that there is a liquidity crisis in firm \( B \) due to creditors’ learning when \( \theta_B \in [\theta^*_B, \theta^*_B] \); and there is a liquidity crisis in firm \( A \) when \( \theta_A \in [\theta^*_A, \theta^*_A] \).

![Figure 1.4: Contagion of Liquidity Crisis from Firm A to Firm B](image-url)
1.3.3.2 Scenario 1 versus scenario 2

Now, let’s compare scenario 1 ($\theta_A \notin [\theta_{AO}^*, \theta_{AP}^*]$) with scenario 2 ($\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$). Scenario 1 provides the benchmark switching fundamentals ($\theta_{AO}^*$ and $\theta_{AP}^*$) in firm $B$. Meanwhile, scenario 2 provides the new switching fundamentals ($\theta_{BO}^*$ and $\theta_{BP}^*$) in firm $B$. By comparing the values of these switching fundamentals, I get the following lemma (see figure 1.5).

**Lemma 3** $\theta_{BO}^* < \theta_{AO}^* < \theta_{AP}^* < \theta_{BP}^*$.

**Proof.** From the values of $\theta_{AO}^*$, $\theta_{AP}^*$, $\theta_{BO}^*$, and $\theta_{BP}^*$, I get

$$
\theta_{BP}^* - \theta_{AP}^* = \frac{\lambda K (\delta_O - \delta_P)}{\delta_O \delta_P} \left( \frac{(1 - \lambda)(1 - q)}{1 + 2\varepsilon - \lambda} \right) > 0,
$$

$$
\theta_{AO}^* - \theta_{BO}^* = \frac{\lambda K (\delta_O - \delta_P)}{\delta_O \delta_P} \left( \frac{q(1 - \lambda)}{1 + 2\varepsilon - \lambda} \right) > 0.
$$

From the fact that $\theta_{AP}^* - \theta_{AO}^* > 0$, $\theta_{BO}^* < \theta_{AO}^* < \theta_{AP}^* < \theta_{BP}^*$ hold.

![Figure 1.5: Values of Switching Fundamentals (Firm B)](image)

The intuition of the inequalities is as follows. If the type of group 1 creditors is revealed and is "optimistic," then the liquidity crisis is less likely to occur in firm $B$ compared to the case where the type is not revealed (i.e., $\theta_{BO}^* < \theta_{AO}^*$). Meanwhile, if the type of group 1 creditors is revealed and is "pessimistic," then the liquidity crisis more likely occurs in firm $B$ compared to the case where the type is not revealed (i.e., $\theta_{BP}^* > \theta_{AP}^*$). That is, if the type of group 1 creditors is revealed, the possibility of whether the liquidity crisis occurs or not in firm $B$ will become more clear than if the type is not revealed. From this result, I argue that the revelation of the type of group 1 creditors is not always good for firm $B$. It depends on the realized type (i.e., "pessimistic"
or "optimistic") of group 1 creditors.

1.4 Comparative Statics and Policy Implications

In this section, after defining the severity of contagion on the liquidity crisis, I show that the impact of the contagion originating from the firm considered less likely to fail is bigger than otherwise. Then, by doing comparative statics on the severity of contagion, I suggest some policy implications to reduce the severity of contagion.

1.4.1 Severity of Contagion on the Liquidity Crisis

In section 1.3.3, I showed that $\theta_{BO}^* < \theta_{AO}^* < \theta_{AP}^* < \theta_{BP}^*$ hold. What does this imply? This means that if the type of group 1 creditors is revealed as being "pessimistic," then the probability of a liquidity crisis in firm $B$ is increased by the difference between $\theta_{BP}^*$ and $\theta_{AP}^*$. This is a negative effect of the contagion on the liquidity crisis in firm $B$. If the type of group 1 creditors is revealed as being "optimistic," then the probability of a liquidity crisis in firm $B$ is decreased by the difference between $\theta_{AO}^*$ and $\theta_{BO}^*$. This can be interpreted as a positive effect of reducing the probability of a liquidity crisis in firm $B$ via the revelation of the optimistic type of group 1 creditors. Focusing on the negative effect of the contagion on the liquidity crisis in firm $B$, I define the severity of contagion as the difference between the new switching fundamentals for pessimistic-type creditors ($\theta_{BP}^*$) and the benchmark switching fundamentals for pessimistic-type creditors ($\theta_{AP}^*$).

**Definition 2** Severity of contagion on the liquidity crisis in firm $B$ is the increased probability of a liquidity crisis in firm $B$ due to the negative effect of the contagion: the difference between the new switching fundamentals $\theta_{BP}^*$ and the benchmark switching fundamentals $\theta_{AP}^*$. Specifically, it is expressed by

$$ SC := \theta_{BP}^* - \theta_{AP}^* = \frac{\lambda (1 - \lambda) (\delta_O - \delta_P) (1 - q) K}{\delta_O \delta_P (1 + 2 \varepsilon - \lambda)}, $$

which is greater than 0 since $\lambda, \varepsilon, q, K, \delta_O, \text{ and } \delta_P$ are in $(0, 1)$.28

---

27 That is, I define the severity of contagion on the liquidity crisis in firm $B$ as an increase in the probability of a liquidity crisis in firm $B$ due to the creditors’ learning from the liquidity crisis in firm $A$.

28 How can I express the positive effect of reducing the probability of a liquidity crisis in firm $B$ due to the revelation of
Now, I get the following proposition.

**Proposition 4** The liquidity crisis in a firm with a small possibility of failing is more contagious than otherwise.

**Proof.** I need to show that the severity of contagion \((\theta^*_BP - \theta^*_AP)\) decreases with \(\theta^*_AP\). This is trivial since the decrease of \(\theta^*_AP\) will increase the difference between \(\theta^*_BP\) and \(\theta^*_AP\). Specifically I can express \(\theta^*_AP\) as

\[
\theta^*_AP = \frac{K}{\delta_P} \left( 1 + \frac{\lambda (1 - \lambda) (\delta_O - \delta_P) (1 - q)}{\delta_O (-1 - 2\varepsilon + \lambda)} \right)
\]

By arranging the above equation, I get

\[
SC = -\theta^*_AP + \frac{K}{\delta_P}.
\]

which implies that the severity of contagion \(SC\) decreases with \(\theta^*_AP\).

This proposition illustrates that the severity of contagion decreases with the level of firm A’s failure point (i.e., firm A’s switching fundamentals). It implies that the occurrence of the liquidity crisis in the firm considered less likely to fail (i.e., the firm having a lower failure point) would lead to a huge surprise in the market, and hence the liquidity crisis is likely to become more contagious than otherwise. In summary, the probable contagion of the liquidity crisis in the firm considered less likely to fail is much bigger than in the firm considered to be not strong enough to

---

The optimistic type of group 1 creditors? It is \(\theta^*_AO - \theta^*_BO = \frac{\lambda (1 - \lambda) (\delta_O - \delta_P) qK}{\delta_O \delta_P (1 + 2\varepsilon - \lambda)}\), which is only different in the term of \(q\) from \((1 - q)\) of \(SC\). That is, the sign of the comparative statics of \(\theta^*_AO - \theta^*_BO\) with respect to the variables that comprise it is exactly the same as \(SC\)’s case except for \(q\). This brings the trade-off relation of the policy proposals for reducing the severity of contagion, which I tackle in latter sections (e.g., the initial policies regarding \(K\), \(\varepsilon\), and \(\lambda\)). In other words, if the government and/or the firm takes measures to reduce the severity of contagion initially, the positive effect of reducing the probability of a liquidity crisis in firm \(B\) due to the revelation of the optimistic type of group 1 creditors is also reduced by those measures. This implies that the effectiveness of the pre-determined policies by the government and/or the firm depends on the type of group 1 creditors.

By the definition of firms’ switching fundamentals, the low value of the switching fundamentals means that the firm fails with a small probability. That is, the value of firm’s switching fundamentals can be interpreted as its failure point.
endure the liquidity crisis.

This is a noticeable result since other contagion-related papers that deal with contagion among international financial markets and/or financial institutions through capital linkages and asset price changes insist that the larger the negative impact originating from worse fundamentals, the more severely other financial institutions or countries are affected through their linkages. In my work, however, I find that the severity of contagion is more serious when the originating firm’s failure point is lower. This finding is based on the following conditions: 1) I focus on the co-creditors’ learning process between two non-financial institutions whose businesses are not related to each other (i.e., independent fundamentals) as the contagion triggering mechanism; 2) I assume that the exact realization of the fundamentals of the originating firm and the result of creditors’ actions (i.e., the failure or success of the firm’s project) are known to creditors before they determine their actions in the other firm.

1.4.2 Changes in the Value of the Collateralized Debt ($K$)

As M-S (2003, 2004) mention in their papers, increasing the value of the collateral ($K$) has two contrasting effects: first, it increases the value of the debt (loan) in the event of default (i.e., the direct effect\(^{30}\)); second, it increases the range of $\theta$ at which default occurs (i.e., the strategic effect\(^{31}\)). In the contagion context, I find that the strategic effect outweighs the direct effect, which means that decreasing the value of the collateral ($K$) is helpful to reduce the severity of contagion on the side of firm $B$.

**Proposition 5**  
Severity of contagion on the liquidity crisis in firm $B$ is reduced by a decrease in the value of its collateral ($K$).

\(^{30}\) In a similar context, Besanko and Thakor (1987) and Greenbaum and Thakor (2007) argue the signaling issue of the collateral: low-risk borrowers choose contracts with high collateral requirements because their low-risk means that the chance of defaulting and losing the collateral to the creditors is lower; hence, offering the high collateral is less onerous.

\(^{31}\) In my model, I can verify this strategic effect result from $\theta_{A_P}^*$ and $\theta_{B_P}^*$. 

22
Proof.

\[
\frac{\partial \text{SC}}{\partial K} = \frac{\lambda (1 - \lambda) (\delta_O - \delta_P) (1 - q)}{\delta_O \delta_P (1 + 2\varepsilon - \lambda)} > 0,
\]

which implies that if firm B decreases the value of K, then the severity of contagion on the liquidity crisis in firm B (SC) will be reduced. ■

What is the intuition of this proposition? The decreased value of the collateral is the increased cost of not rolling over the loans from the creditors’ standpoint. In other words, creditors have more incentive than otherwise to roll over their loans until maturity when the value of the collateral is small. Hence, firm B can reduce the severity of contagion on the liquidity crisis from firm A to itself by setting the value of its collateral small.

1.4.3 Changes in the Gap of Discount Factors (\(\delta_O\) and \(\delta_P\))

Diamond and Dybvig (1983) argue in their paper that deposit insurance by the government can prevent bank runs even though it might generate a moral hazard problem. That is, patient agents know that withdrawal by others is not going to harm their long-term return, and they will not withdraw their deposits. Likewise, let’s think about the government bailouts to a firm that suffers a transitory liquidity problem.\(^{32}\) After observing a liquidity crisis in firm A and getting to know that the type of group 1 creditors is pessimistic, the government can expect the contagion of the liquidity crisis from firm A to firm B. If the government provides bailouts to firm B, which is thought of as suffering a transitory liquidity problem even though the state of its fundamentals is not too bad, then it is a good signal for the success of firm B’s investment project in the market. In this case, pessimistic creditors become more optimistic toward the success of firm B’s investment project (i.e., \(\delta_P \rightarrow \delta_O\)).\(^ {33}\) That is, the gap between \(\delta_O\) and \(\delta_P\) decreases, and hence it reduces

\(^{32}\)Note that the government’s provision of bailouts is not the full insurance on the success of firm’s investment project. Here, I just focus on creditors’ optimism toward a firm’s fundamentals. However, in the whole economy’s point of view, if the government implements fiscal and/or monetary expansion policies, then pessimistic creditors become

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the severity of contagion on the liquidity crisis in firm $B$.\footnote{In the extreme case where the government "fully" guarantees firm $B$'s investment project, there occurs no contagion of the liquidity crisis from firm $A$ to firm $B$.} I summarize this argument in the following proposition.

**Proposition 6** Severity of contagion on the liquidity crisis in firm $B$ is reduced by a decrease in $(\delta_O - \delta_P)$, which is obtained by the government’s provision of bailouts to firm $B$.

**Proof.**

\[
\frac{\partial \text{SC}}{\partial (\delta_O - \delta_P)} = \frac{\lambda (1 - \lambda) (1 - q) K}{\delta_O \delta_P (1 + 2\varepsilon - \lambda)} > 0,
\]
which implies that if the government decreases $(\delta_O - \delta_P)$ by providing bailouts to firm $B$, then the severity of contagion on the liquidity crisis in firm $B$ (SC) will be reduced. ■

However, the government’s bailout policy may have two main problems, as follows. First, the government cannot easily distinguish between the insolvency risk and the illiquidity risk of the firm (Morris and Shin (2009)). As Thakor (2008) argues, the government’s bailout should be primarily intended to stave off the bankruptcy / illiquidity problem and to recover the investors’ sapped confidence. It cannot fix a broken business model. Second, as Fischer (1999) points out, the existence of this "lender-of-last-resort" creates a moral hazard problem with respect to the actions of both creditors and firms. Hence, the government’s bailout policy should be implemented under systematic guidelines, with proper surveillance and regulations, to achieve an improvement in the whole economy’s welfare (Fischer (1999), Gai, Hayes, and Shin (2001), and Schneider and Tornell (2004) among others).

1.4.4 Changes in the Information Structure ($\varepsilon$)

As creditors’ information on the firm’s fundamentals becomes very precise (i.e., $\varepsilon \to 0$), the value of the firm’s switching fundamentals is decreased.\footnote{In my model, I can verify this result from $\theta^*_{AP}$ for instance. By the way, note that $\theta^*_{BP}$ and $\theta^*_{BO}$ do not have $\varepsilon$ since} Heinemann and Illing (2002) similarly more optimistic and thus $\delta_P \to \delta_O$.\footnote{In the extreme case where the government "fully" guarantees firm $B$’s investment project, there occurs no contagion of the liquidity crisis from firm $A$ to firm $B$.}
emphasize the role of transparent / precise information in a crisis episode. Now, what is the effect of small noise (i.e., precise information on the firm’s fundamentals) on the severity of contagion? Will precise information on the firm’s fundamentals reduce the severity of contagion? The result looks very surprising, for I find that it increases the severity of contagion.

**Proposition 7**  
Severity of contagion on the liquidity crisis in firm B increases with the accuracy of the information structure.

**Proof.**

\[
\frac{\partial SC}{\partial \varepsilon} = -\frac{2\lambda (1 - \lambda) (\delta_O - \delta_P) (1 - q) K}{\delta_O \delta_P (1 + 2\varepsilon - \lambda)^2} < 0,
\]

which implies that if creditors’ (private) information on the firm’s fundamentals becomes very precise (i.e., \(\varepsilon \rightarrow 0\)), then the severity of contagion on the liquidity crisis in firm B (SC) will be increased. 

If creditors’ information on the firm’s fundamentals is very accurate (i.e., if \(\varepsilon\) is very small), then the probability of firm A’s liquidity crisis is reduced (i.e., the failure point of firm A (\(\theta_{AP}\)) becomes lower). However, if there occurs the liquidity crisis in firm A even though the probability of the failure is low, then the contagion of the liquidity crisis to firm B is more severe. This can be interpreted as what Proposition 4 addressed. That is, if the liquidity crisis occurs in the firm considered less likely to fail (i.e., the firm having a small failure point (\(\theta_{AP}\) via small \(\varepsilon\)), then it leads to a big shock in the market and thus the liquidity crisis can be more contagious. Based on this result, I argue that the policy for agents’ transparent / precise information on the fundamentals is not a panacea in a crisis episode. Even though transparency reduces the probability of a crisis in the case of one economy, it worsens the severity of contagion in the crisis among more than one economy.

the type of group 1 creditors is the known fact when those switching fundamentals are determined.
1.4.5 Changes in the Size of Group 1 ($\lambda$)

The size of group 1 creditors, which is measured by $\lambda$, represents incomplete information in the market. That is, even though the type of group 2 creditors is "pessimistic," which is public information in the market, the type of group 1 creditors is not known in the market initially. What is the effect of this incomplete information on the severity of contagion? In other words, what is the impact of the degree of incomplete information on the contagion? I show the effect of $\lambda$ on the severity of contagion when $\varepsilon$ converges to zero in the following proposition.\(^{36}\)

**Proposition 8**  
Severity of contagion on the liquidity crisis in firm $B$ is reduced by a decrease in the size of group 1.

**Proof.**

$$\frac{\partial SC}{\partial \lambda} = \frac{(\delta_O - \delta_P) (1 - q)K}{\delta_O \delta_P} > 0 \text{ as } \varepsilon \rightarrow 0,$$

which implies that as the size of group 1 becomes smaller and as creditors’ (private) information on the firm’s fundamentals becomes very precise, the severity of contagion on the liquidity crisis in firm $B$ (SC) will be decreased. \(\blacksquare\)

What does this proposition imply? As I discussed above, the size of group 1 stands for incomplete information in the market initially. If the size of this incomplete information becomes small, then the contagion of the liquidity crisis becomes less severe. Hence, the government can mitigate the severity of contagion by regulating the size of this incomplete information. For example, the government reinforces creditors to reveal their types via its financial disclosure policy (i.e., to disclose their financial information in the market).\(^{37}\)

---

\(^{36}\) When $\varepsilon$ does not converge to zero, the effect of $\lambda$ on the severity of contagion depends on the relative sizes of $\lambda$ and $\varepsilon$. Hence, here I tackle the case where $\varepsilon$ converges to zero, which means that the information of creditors on the firm’s fundamentals is very precise.

\(^{37}\) Note that even though this revelation policy is helpful to reduce the severity of contagion, it is not always good for the individual firm, which I discussed in section 1.3.3.
financial disclosure perfectly reveals the type of group 1 creditors in the market, there occurs no learning process among creditors, and thus there is no contagion of a liquidity crisis from firm A to firm B.

Related to the issue of the revelation of the type of group 1 creditors via the financial disclosure policy, what is the effect of the type of group 1 creditors on the severity of contagion? Since the type of group 1 creditors is "pessimistic" with probability \( q \), I find that the severity of contagion decreases with \( q \).\(^{38}\) It implies that if group 2 creditors initially expect that group 1 creditors are more likely the same type as theirs, then the learning process of creditors’ type does not have as much impact on the contagion of the liquidity crisis as otherwise.\(^{39}\)

### 1.5 Discussion in Real-World Phenomena

#### 1.5.1 Korea’s 1997 Financial Crisis

In order to assess the applicability of my model to real-world phenomena, let us revisit 1997 Korean financial crisis in the middle of the Asian Flu. According to Akama, Noro, and Tada [A-N-T] (2003), Korean firms were highly leveraged by short-term loans from domestic and foreign banks. By the end of 1996, the corporate debt relative to the nominal GDP ratio was over 1.6, and the external debt to the GDP ratio reached approximately 25%, in which the share of short-term debt out of the total external debt peaked at 58%. This fact is parallel to the debt-financing assumption of my model. A-N-T (2003) also argue that Korea had a bank-centered financial system. As of the end of 1997, among 26 domestic commercial banks, 16 nationwide commercial banks\(^{40}\) were actually common bank creditors of the top 30 conglomerates in

\[^{38}\] \( \theta_{AP}^* \) is increasing in \( q \), but \( \theta_{BP}^* \) is independent of \( q \). That is, \( SC (:= \theta_{BP}^* - \theta_{AP}^*) \) is decreasing in \( q \). Specifically, I get \( \frac{\partial SC}{\partial q} = -\frac{\lambda (1-\lambda)(\delta_0 - \delta_1)K}{\delta_0 \delta_1 (1+2x-\lambda)} < 0 \).

\[^{39}\] I check that if \( q < \frac{1}{2} \), \( SC (:= \theta_{BP}^* - \theta_{AP}^*) \) is greater than the positive effect of reducing the probability of the liquidity crisis in firm \( B \) due to the revelation of the optimistic type of group 1 creditors: \( \theta_{AO}^* - \theta_{BO}^* \).

\[^{40}\] The others were local commercial banks.
Korea.\textsuperscript{41} This means that Korean firms have the same co-lending banks, which is parallel to the co-creditors’ assumption of my model. In summary, the overall business situation of Korean firms at 1997 shows debt-rollover coordinations among co-creditors.

According to Rhee (1998), the bankruptcy of the Hanbo Steel Group in January 1997 was a sobering experience for co-lending banks. They started to strictly reexamine the profitability of their loans on other companies and to call in most of short-term loans. This led to a "domino effect" as more and more companies suffered liquidity crises. For example, Kia Motors – Korea’s eighth-largest conglomerate – failed even though its reputation in the market was fairly good.\textsuperscript{42} The rush continued, and as I mentioned in the Introduction, Jinro – Korea’s nineteenth-largest conglomerate and also the largest liquor group – failed in September 1997. By the end of 1997, over 15,000 companies, large and small, went bankrupt.\textsuperscript{43} In the process of serial firms’ failures, we can observe the following phenomenon. Foreign banks (especially, Japanese and U.S. banks\textsuperscript{44}) pulled out their money en masse, and some Korean domestic banks (e.g., Korea First Bank (KFB)\textsuperscript{45}) dramatically stopped rolling over their loans first. Then, other co-lending banks followed to stop rolling over their loans in other firms.

The interpretation of Korea’s 1997 financial crisis is consistent with my model. Observing Hanbo Steel Group’s liquidity crisis, common bank creditors could conjecture or learn other creditors’ types. Here, foreign banks and KFB, for example, can be thought of as pessimistic creditors in my model due to an information disadvantage and a weak balance sheet, respectively.

\textsuperscript{41} That is, those commercial banks lent their money to multiple firms, including top 30 conglomerates.
\textsuperscript{42} In 1998, Kia was merged by Hyundai Motor.
\textsuperscript{43} As I mentioned above, these companies generally have common bank creditors.
\textsuperscript{45} KFB went bankrupt right after Jinro’s failure.
More specifically, foreign banks can be considered as group 2 creditors in my model because they had an information disadvantage on the overall business situations of Korea compared to Korean domestic banks, and this fact was known in the market. Of course, some Korean domestic commercial banks can be treated as group 2 creditors if their bad financial states were known among creditors. In the case of KFB, its financial status was unknown initially, and thus I can interpret KFB as being in group 1 in my model. Reflecting the new information of other banks’ types revealed after Hanbo Steel Group’s liquidity crisis, co-bank creditors decided their own actions – rolling over their loans or not – in other firms.

Noting that there were no fundamental linkages among many firms that went bankrupt; and noting the leading roles of foreign banks and KFB on serial rushes in the market, I clearly argue that Korea’s 1997 financial crisis provides empirical evidence that supports my model of the contagion triggering mechanism: co-creditors’ learning about each other’s type. That is, by learning about other creditors’ types from former firms’ debt-rollover coordinations, creditors determine their own actions toward next firms.\textsuperscript{46} Note that Korea’s 1997 financial crisis is different from simple herding stories, which rely solely on the sequential choices of players. It demonstrates the newly repeated static debt-rollover coordination games in other firms among co-creditors via their learning about other creditors’ types from the former firms’ debt-rollover games. Moreover, in the static coordination game setting of each firm, there exist payoff (strategic) complementarities among co-creditors, unlike in the simple herding model.

\subsection*{1.5.2 Experimental Analysis of the Model}

Data constraints on the full financial information of creditors and firms may make it quite

\textsuperscript{46} In my model of two firms, for simplicity of analysis, I assume that creditors’ types remain the same in the course of two debt-rollover games among creditors. Of course, in the general sequential case of more than two firms, like Korea’s 1997 financial crisis, I should consider the dynamic effects of changes in creditors’ wealth from former firms’ rollover games on changes in creditors’ types. In other words, I need to extend my analysis to include the more dynamic learning process of co-creditors on other creditors’ types with their wealth changes.
difficult to empirically estimate the model. Experimental analysis would then be a good potential work to test my theoretical predictions. In fact, Heinemann, Nagel, and Ockenfels [H-N-O] (2004) design an experiment to test the speculative-attack model of M-S (1998). They conclude that the switching strategy in the theory of global games is an important reference point, providing correct predictions for comparative statics with respect to parameters of the payoff function. Taketa, Suzuki-Loffelholz, and Arikawa [T-S-A] (2009) conduct an experiment designed to imitate the C-D-M-S (2004) model and to support the argument that the presence of a large speculator causes other speculators to be more aggressive in their attacks.

To properly test my model of contagion in an experiment, I need to investigate creditors’ learning behavior about the type of other creditors from the former firm’s debt-rollover coordination game among them on top of the experimental analysis settings of H-N-O (2004) and T-S-A (2009). Hence, I can utilize the "belief-learning" model experiment in which players do not learn about which strategies work best; they learn about what others are likely to do, then use those updated beliefs to change their attractions and hence to change what strategies they choose (Camerer, Ho, and Chong (2001) and Fudenberg and Levine (1998) among others). By doing so in the experiment, I will be able to observe how switching strategies change according to creditors’ learning processes and to test the predictions for comparative statics in my model.

1.6 Concluding Remarks

This paper, focusing on liquidity crises in non-financial institutions, explores contagion: the phenomenon that occurs when the states of two firms’ fundamentals are not closely related, but still, what happens in one firm affects the optimal behaviors of creditors and thus what happens in the other firm. The contagion mechanism between two non-financial firms is based on the co-creditors’ learning about each other’s type, which has received little attention from the existing
literature. Examining the creditors’ learning process is very important, because in a rollover coordination game, creditors’ beliefs about others’ types affect the probability of the occurrence of the liquidity crisis in the firm, i.e., the creditors’ learning process can be very useful in explaining the creditors’ strategic behaviors in a coordination game. Learning and revising beliefs about others’ types after observing what occurred in one firm, creditors determine their actions in a latter firm, which affects the probability of a liquidity crisis in that latter firm. I discussed the real-world example (i.e., Korea’s financial crisis in 1997), which supports my model.

By analyzing the contagion process with creditors’ learning, I found a noticeable feature of the contagion that is different from previous contagion-related literatures: the contagion impact of the liquidity crisis originating from the firm having a lower failure point is more severe than otherwise under the assumptions that the exact realization of the fundamentals of the firm and the result of creditors’ actions in that firm are known to creditors before they decide their actions in the other firm. Moreover, even though increasing the accuracy of creditors’ information on the firm’s fundamentals reduces the probability of the liquidity crisis in an individual firm, it increases the severity of contagion. I dealt with policy proposals addressing how to mitigate the severity of contagion, including the government’s provision of bailouts to the firm suffering a transitory liquidity problem and its financial disclosure policy. Also, the firm can initially reduce the severity of contagion by setting the value of its collateral small.
Appendix
Proof of Proposition 1

First, let’s think about the decisions of group 1 creditors. They privately know their own type ("pessimistic" or "optimistic") and also know group 2 creditors’ type ("pessimistic"). Hence, they know the value of $\bar{\theta}_A : \theta^*_A P$ or $\theta^*_A O$. Note that $\varepsilon_{A_j} := x_{A_j} - \theta_A$ is uniformly distributed over the interval $[-\varepsilon, \varepsilon]$. So, the equation (1.1) becomes:

$$K = \Pr \left[ \text{rollover is successful} \mid \bar{x}_{A_1}, \bar{\theta}_A \right] \cdot \delta_{m}$$

$$= \Pr \left[ \theta_A \geq \bar{\theta}_A \mid \bar{x}_{A_1}, \bar{\theta}_A \right] \cdot \delta_{m}$$

$$= \Pr \left[ \bar{x}_{A_1} - \varepsilon_{A_1} \geq \bar{\theta}_A \mid \bar{x}_{A_1}, \bar{\theta}_A \right] \cdot \delta_{m}$$

$$= \Pr \left[ \varepsilon_{A_1} \leq \bar{x}_{A_1} - \bar{\theta}_A \mid \bar{x}_{A_1}, \bar{\theta}_A \right] \cdot \delta_{m}$$

$$= \frac{\bar{x}_{A_1} - \bar{\theta}_A + \varepsilon}{2\varepsilon} \cdot \delta_{m}. \tag{A1}$$

From (A1), I get the following two equations:

$$K = \frac{x^*_{A_1 P} - \theta^*_A P + \varepsilon}{2\varepsilon} \cdot \delta_{P}, \tag{A2}$$

$$K = \frac{x^*_{A_1 O} - \theta^*_A O + \varepsilon}{2\varepsilon} \cdot \delta_{O}. \tag{A3}$$

Next, let’s think about the decisions of group 2 creditors. They know their own type ("pessimistic") but do not know the type of group 1 creditors. They can just conjecture the probability that the type of group 1 creditors is "pessimistic" as $q$. They do not know the value of $\bar{\theta}_A : \theta^*_A P$ or $\theta^*_A O$, either. Then the equation (1.1) becomes:

$$K = \Pr \left[ \text{rollover is successful} \mid x^*_{A_2} \right] \cdot \delta_{P}$$

$$= \left\{ \Pr \left[ \text{rollover is successful when 1’s are pessimistic} \mid x^*_{A_2} \right] + \Pr \left[ \text{rollover is successful when 1’s are optimistic} \mid x^*_{A_2} \right] \right\} \cdot \delta_{P}$$

$$= q \times \Pr \left[ \theta_A \geq \theta^*_{A P} \mid x^*_{A_2} \right] \cdot \delta_{P} + (1 - q) \times \Pr \left[ \theta_A \geq \theta^*_{A O} \mid x^*_{A_2} \right] \cdot \delta_{P}$$

$$= q \times \frac{x^*_{A_2} - \theta^*_A P + \varepsilon}{2\varepsilon} \cdot \delta_{P} + (1 - q) \times \frac{x^*_{A_2} - \theta^*_A O + \varepsilon}{2\varepsilon} \cdot \delta_{P}. \tag{A4}$$

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Lastly, let’s think about the critical threshold value of firm A’s fundamentals (i.e., switching fundamentals). The proportion of creditors who do not roll over the loans is expressed as follows:

\[ l_A (\theta_A) = \lambda \Pr [x_{A1} \leq \bar{x}_{A1} \mid \theta_A] + (1 - \lambda) \Pr [x_{A2} \leq x^*_{A2} \mid \theta_A] \]

\[ = \lambda \Pr [\theta_A + \varepsilon_{A1} \leq \bar{x}_{A1} \mid \theta_A] + (1 - \lambda) \Pr [\theta_A + \varepsilon_{A2} \leq x^*_{A2} \mid \theta_A] \]

\[ = \lambda \Pr [\varepsilon_{A1} \leq \bar{x}_{A1} - \theta_A \mid \theta_A] + (1 - \lambda) \Pr [\varepsilon_{A2} \leq x^*_{A2} - \theta_A \mid \theta_A] \]

\[ = \lambda \frac{\bar{x}_{A1} - \theta_A + \varepsilon}{2\varepsilon} + (1 - \lambda) \frac{x^*_{A2} - \bar{\theta}_A + \varepsilon}{2\varepsilon}. \]

The critical threshold value is determined by:

\[ \bar{\theta}_A = l_A (\bar{\theta}_A) = \lambda \frac{\bar{x}_{A1} - \bar{\theta}_A + \varepsilon}{2\varepsilon} + (1 - \lambda) \frac{x^*_{A2} - \bar{\theta}_A + \varepsilon}{2\varepsilon}. \] (A5)

From equation (A5), I get the following two equations:

\[ \theta^*_{AP} = \lambda \frac{x^*_{A1P} - \theta^*_{AP} + \varepsilon}{2\varepsilon} + (1 - \lambda) \frac{x^*_{A2} - \theta^*_{AP} + \varepsilon}{2\varepsilon}, \] (A6)

\[ \theta^*_{AO} = \lambda \frac{x^*_{A1O} - \theta^*_{AO} + \varepsilon}{2\varepsilon} + (1 - \lambda) \frac{x^*_{A2} - \theta^*_{AO} + \varepsilon}{2\varepsilon}. \] (A7)

Solving equations (A2), (A3), (A4), (A6), and (A7), I get \( x^*_{A1P}, x^*_{A1O}, x^*_{A2}, \theta^*_{AP}, \) and \( \theta^*_{AO}. \)

The unique equilibrium values of the switching fundamentals of firm A (\( \theta^*_{AP} \) and \( \theta^*_{AO} \)) and the creditors’ switching private signals \( (x^*_{A1P}, x^*_{A1O}, \) and \( x^*_{A2}) \) are as follows:

\[ \theta^*_{AP} = \frac{K}{\delta_P} (1 - \Sigma_1), \]

\[ \theta^*_{AO} = \frac{K}{\delta_P} (1 - \Sigma_1 - \Sigma_2), \]

\[ x^*_{A1P} = \frac{K}{\delta_P} (1 - \Sigma_1 + \Sigma_3), \]

\[ x^*_{A1O} = \frac{K}{\delta_P} \left( 1 - \Sigma_1 - \Sigma_2 + \frac{\delta_P}{\delta_O} \Sigma_3 \right), \]

\[ x^*_{A2} = \frac{K}{\delta_P} (1 - \Sigma_1 - (1 - q) \Sigma_2 + \Sigma_3), \]

where

\[ \Sigma_1 = \frac{\lambda (1 - \lambda) (1 - q) (\delta_O - \delta_P)}{\delta_O (1 + 2\varepsilon - \lambda)}, \quad \Sigma_2 = \frac{2\lambda \varepsilon (\delta_O - \delta_P)}{\delta_O (1 + 2\varepsilon - \lambda)}, \quad \text{and} \quad \Sigma_3 = \left( \frac{2K - \delta_P}{K} \right) \varepsilon. \]
I also need to show that every creditor in each group strictly prefers not to roll over the loan (prefers to roll over the loan) if his private signal is less than (greater than) the switching private signal conditional on \( \theta_{AP}^* \) and \( \theta_{AO}^* \). Suppose that every other creditor follows the switching strategy. Then, an individual creditor in each group takes \( \theta_{AP}^* \) and \( \theta_{AO}^* \) as given. From equations (A2), (A3), and (A4), the present value of the expected payoff of rolling over the loan is strictly increasing in the switching private signals \((x_{A1P}^*, x_{A1O}^*, \text{ and } x_{A2}^*)\) given \( \theta_{AP}^* \) and \( \theta_{AO}^* \). Therefore, for any private signal greater than the switching signal, the expected payoff of rolling over the loan is strictly greater than that of not rolling over. Thus, it is optimal for a creditor to follow the switching strategy, given that every other creditor follows the switching strategy.

**Derivation of \( \theta_{BP}^* \) and \( x_B^* \)**

The proportion of creditors who do not roll over the loans conditional on \( \theta_B \) is expressed as follows:

\[
l_B (\theta_B) = \Pr [x_B \leq x_B^* | \theta_B]
\]

\[
= \Pr [\theta_B + \varepsilon_B \leq x_B^* | \theta_B]
\]

\[
= \Pr [\varepsilon_B \leq x_B^* - \theta_B | \theta_B]
\]

\[
= \frac{x_B^* - \theta_B + \varepsilon}{2\varepsilon}.
\]

The critical threshold value of firm B’s fundamentals (i.e., switching fundamentals) is determined by:

\[
\theta_{BP}^* = l_B (\theta_{BP}^*) = \frac{x_B^* - \theta_{BP}^* + \varepsilon}{2\varepsilon}.
\]

(A8)

From the fact that creditors’ present value of the expected utility of rolling over the loans
should be equal to the payoff from recalling the loan in the indifference condition, I get:

\[
K = \Pr \{ \text{rollover is successful} \mid x_B^* \} \cdot \delta_P
\]

\[
= \Pr \{ \theta_B \geq \theta_{BP}^* \mid x_B^* \} \cdot \delta_P
\]

\[
= \Pr \{ x_B^* - \varepsilon_B \geq \theta_{BP}^* \mid x_B^* \} \cdot \delta_P
\]

\[
= \Pr \{ \varepsilon_B \leq x_B^* - \theta_{BP}^* \mid x_B^* \} \cdot \delta_P
\]

\[
= \frac{x_B^* - \theta_{BP}^* + \varepsilon}{2\varepsilon} \delta_P. \tag{A9}
\]

From equations (A8) and (A9), I get the equilibrium strategy:

\[
\theta_{BP}^* = \frac{K}{\delta_P},
\]

\[
x_B^* = \frac{K}{\delta_P} (2\varepsilon + 1) - \varepsilon.
\]

**Derivation of \( \theta_{BO}^*, x_{BO1}^*, \text{ and } x_{BO2}^* \)**

The proportion of creditors who do not roll over the loans conditional on \( \theta_B \) is expressed as follows:

\[
l_B(\theta_B) = \lambda \Pr \{ x_B^1 \leq x_B^1 \mid \theta_B \} + (1 - \lambda) \Pr \{ x_B^2 \leq x_B^2 \mid \theta_B \}
\]

\[
= \lambda \Pr \{ \theta_B + \varepsilon B1 \leq x_B^1 \mid \theta_B \} + (1 - \lambda) \Pr \{ \theta_B + \varepsilon B2 \leq x_B^2 \mid \theta_B \}
\]

\[
= \lambda \Pr \{ \varepsilon B1 \leq x_B^1 - \theta_B \mid \theta_B \} + (1 - \lambda) \Pr \{ \varepsilon B2 \leq x_B^2 - \theta_B \mid \theta_B \}
\]

\[
= \lambda \frac{x_B^1 - \theta_B + \varepsilon}{2\varepsilon} + (1 - \lambda) \frac{x_B^2 - \theta_B + \varepsilon}{2\varepsilon}.
\]

The critical threshold value of firm \( B \)'s fundamentals (i.e., switching fundamentals) is determined by:

\[
\theta_{BO}^* = l_B(\theta_{BO}^*) = \lambda \frac{x_B^1 - \theta_{BO}^* + \varepsilon}{2\varepsilon} + (1 - \lambda) \frac{x_B^2 - \theta_{BO}^* + \varepsilon}{2\varepsilon}. \tag{A10}
\]

From the fact that creditors’ present value of the expected utility of rolling over the loans should be equal to the payoff from recalling the loans in the indifference condition, I get the
following equations for "optimistic" group 1 creditors and "pessimistic" group 2 creditors:

\[
K = \Pr [\text{rollover is successful} \mid x^*_{B1}] \cdot \delta_O
\]

\[
= \Pr [\theta_B \geq \theta^*_BO \mid x^*_{B1}] \cdot \delta_O
\]

\[
= \Pr [x^*_{B1} - \varepsilon_{B1} \geq \theta^*_BO \mid x^*_{B1}] \cdot \delta_O
\]

\[
= \Pr [\varepsilon_{B1} \leq x^*_{B1} - \theta^*_BO \mid x^*_{B1}] \cdot \delta_O
\]

\[
= \frac{x^*_{B1} - \theta^*_BO + \varepsilon}{2\varepsilon} \delta_O,
\]  \hspace{1cm} (A11)

and

\[
K = \Pr [\text{rollover is successful} \mid x^*_{B2}] \cdot \delta_P
\]

\[
= \Pr [\theta_B \geq \theta^*_BO \mid x^*_{B2}] \cdot \delta_P
\]

\[
= \Pr [x^*_{B2} - \varepsilon_{B2} \geq \theta^*_BO \mid x^*_{B2}] \cdot \delta_P
\]

\[
= \Pr [\varepsilon_{B2} \leq x^*_{B2} - \theta^*_BO \mid x^*_{B2}] \cdot \delta_P
\]

\[
= \frac{x^*_{B2} - \theta^*_BO + \varepsilon}{2\varepsilon} \delta_P.
\]  \hspace{1cm} (A12)

From equations (A10), (A11), and (A12), I get the equilibrium strategy:

\[
\theta^*_{BO} = \frac{\lambda K}{\delta_O} + \frac{(1 - \lambda) K}{\delta_P},
\]

\[
x^*_{B1} = \frac{K (\lambda + 2\varepsilon)}{\delta_O} + \frac{(1 - \lambda) K}{\delta_P} - \varepsilon,
\]

\[
x^*_{B2} = \frac{\lambda K}{\delta_O} + \frac{K (1 - \lambda + 2\varepsilon)}{\delta_P} - \varepsilon.
\]
References


Chapter 2 Public Information in a Contagious Currency Crisis

2.1 Introduction

Welfare effects of public information are very important criteria for designing public policies in many economies. Morris and Shin (2002, 2005) analyze a class of economies characterized by strategic complementarities of agents’ actions in an asymmetric-information environment. Agents have private information about the fundamentals of the economy and can obtain public information provided by a social agent such as the central bank. They show that the transparency in communication – disclosure of information obtained by the central bank to the private agents – could imply reduction in social welfare. In a similar context, Metz (2002) analyzes the effects of private and public information of speculators on the probability of a currency crisis in a country. However, these papers do not cover the contagion issue among economies.

In this paper, I examine the effect of public information disseminated by the central bank on the contagion of a currency crisis between two countries. Based on Oh (2010) and Taketa (2004), I focus on the speculators’ learning behavior about each other’s "type" – the aggressiveness on the speculation – as the contagion triggering mechanism between two countries which have central banks that publish economic data and statistics to the international financial market. This publicly available information is the source for speculators to guess the economic fundamentals of each country. I analyze the influence of varying information structure including the precision of public information disseminated by the central bank on the contagion of a currency crisis between two countries.

The concept of the contagion in this paper is more extended than the traditional one. Previous
studies (e.g., Oh (2010)) typically define the contagion as the negative effect of the bad outcome in the former economy on the outcome in the latter economy. However, as Manz (2002) properly points out, there is also the positive effect of the good outcome in the former economy on the outcome in the latter one. That is, a good result in the former economy can rescue the latter, which is a positive aspect of the contagion. Hence, in this paper, I define the contagion with two directions. One is the "Bad Contagion" which is the negative effect of the contagious currency crisis from the former country to the latter country. That is, a currency crisis in the former country increases the probability of a currency crisis in the latter country. The other is the "Good Contagion" which is the positive effect of the contagion. That is, no currency crisis in the former country lowers the probability of a currency crisis in the latter country.

As Morris and Shin (2003, 2004) analyze, in my model, the uniqueness of each country’s equilibrium – the threshold for a currency crisis – is guaranteed if the precision of speculators’ private information about each country’s economic fundamentals is large enough relative to the precision of public information about the economic fundamentals distributed by each country’s central bank. Under this condition, I find that the higher the public signal about the economic fundamentals disseminated by the central bank, the lower the effect of the "Bad Contagion" and the higher the effect of the "Good Contagion" will be. Moreover, I demonstrate that the sheer increase in the amount of public information distributed by the central bank is not enough to prevent the bad contagious currency crisis from the other country. In particular, only when the state of economic fundamentals is ex-ante expected to be sound, can distributing the precise public information promote the effect of the "Good Contagion" and reduce the effect of the "Bad Contagion" on the currency crisis from the other country.

The remainder of this paper is as follows. I present the model in section 2.2. In section 2.3, I
solve for each country’s equilibrium in sequence and show how the latter country’s equilibrium is affected by speculators’ learning about each other’s type from the former country’s speculation process. In section 2.4, I define the concept of the "Bad Contagion" and the "Good Contagion," and discuss the role of public information distributed by the central bank in the contagion of a currency crisis. Section 2.5 concludes. Proofs are in the Appendix.

2.2 Model

There are two countries: country A and country B. The central bank of each country pegs the currency at some level. The economy in each country is characterized by the state of its underlying economic fundamentals, $\theta_i$ ($i = A, B$). A high value of $\theta_i$ refers to good fundamentals while a low value refers to bad fundamentals. I assume $\theta_i$ is drawn from the real line. Also, there is no linkage of economic fundamentals between country A and country B, which means that $\theta_A$ and $\theta_B$ are independent.

There are two groups of speculators: group 1 and group 2 in the foreign exchange market. Both groups consist of a continuum of small speculators, and hence each individual speculator’s stake is negligible as a proportion of the whole. I index the set of speculators by the unit interval [0, 1]. There exists some uncertainty about their attitude toward risk. Thus, group 1’s type is privately known to group 1 speculators. There are two possible types of group 1 with respect to the aggressiveness: one type is "bullish" with probability $q$ while another type is "chicken" with probability $1 - q$. That is, all the speculators in group 1 are bullish (chicken) with probability $q (1 - q)$. For simplicity, group 2’s type is always "bullish" and is common knowledge to all speculators. The size of group 1 is $\lambda$ while that of group 2 is $1 - \lambda$, where $0 < \lambda < 1$.

Each speculator disposes of one unit of the currency and can decide whether to short-sell this unit (i.e., attack the currency peg) or not. If the attack is successful, he gets a fixed payoff $D$.
Taking a speculative position in the market, however, also leads to costs of \( t \) for "bullish" speculators and \( t + \delta \), where \( \delta > 0 \), for "chicken" speculators. That is, I assume that speculators who pay more costs to attack the currency are of chicken type. I assume that a successful attack is profitable for any speculator: \( D - t - \delta > 0 \).

Let the proportion of attacking speculators be denoted by \( l_i \) \((i = A, B)\). If \( \theta_i \) is sufficiently high, the central bank is able to always defend the peg, irrespective of the number of attacking speculators. Nevertheless, if \( \theta_i \) is sufficiently low, the central bank abandons the peg in favor of a devaluation even if none of the speculators sells the currency. That is,

- if \( l_i \leq \theta_i \), the central bank keeps the peg: an attack is unsuccessful (no currency crisis);
- if \( l_i > \theta_i \), the central bank devalues the peg: an attack leads to success (currency crisis).

If the fundamental index \( \theta_i \) becomes common knowledge to speculators, I get the typical tripartition of fundamentals of a complete information game as in the original multiple equilibria model by Obstfeld (1996):47

- For \( \theta_i > 1 \), the currency peg is stable, since the economy is sound enough so that the central bank is always able to defend the peg.
- For \( \theta_i \leq 0 \), the central bank always abandons the peg, irrespective of the speculators’ actions and the currency peg is unstable.
- For \( 0 < \theta_i \leq 1 \), the currency peg is said to be ripe for attack. In this interval, if all speculators attack, the central bank will be forced to devalue, whereas the peg will be kept if the speculators do not attack. However, since the agents will only attack the currency if they believe in success and will refrain from attacking otherwise, their actions vindicate the initial beliefs so that expectations are self-fulfilling for this range of fundamentals.

Based on Metz (2002) and Morris and Shin (1998, 2004), the game between speculators and the central bank is structured as follows to avoid the multiple equilibria. Nature chooses a type of group 1 speculators. And the value of the fundamental index \( \theta_i \) is drawn from the real line. The value of \( \theta_i \) can be observed by the central bank, but not by the speculators. That is, \( \theta_i \) is not common knowledge to speculators. After having observed \( \theta_i \), the central bank disseminates a

47 I directly refer to Metz (2002) for the explanation about this multiple equilibria case.
public signal $y_i = \theta_i + v_i$, where $v_i \sim \mathcal{N}(0, 1/\alpha_i)$, $\alpha_i > 0$, and $\mathbb{E}[v_i\theta_i] = 0$, so that the noise parameter is independent of the truly chosen fundamental state. This signal is public in the sense that it is common knowledge to all market participants. The precision $\alpha_i$ of the public signal is exogenous to the model, i.e., $\alpha_i$ is chosen before the central bank gets to know the true value of $\theta_i$ and stays constant throughout the course of the game. The distribution of the noise parameter $v_i$ is common knowledge as well.

In addition to the public signal, speculators individually receive informative private signals on $\theta_i$. Each group $j$ ($j = 1, 2$) speculator gets the private signal $x_{ij} = \theta_i + \varepsilon_{ij}$, where $\varepsilon_{ij} \sim \mathcal{N}(0, 1/\beta)$ and $\beta > 0$. The noise parameters of the private signals are assumed to be independent of each other, of the fundamental state, and of the noise parameter in the public signal: $\mathbb{E} [\varepsilon_{ij}\varepsilon_{ik}] = 0$ for $j \neq k$, $\mathbb{E} [\varepsilon_{ij}\theta_i] = 0$, and $\mathbb{E} [\varepsilon_{ij}v_i] = 0$. The distributional properties of the noise parameter in the private signal are again presumed to be common knowledge to all speculators. However, as long as the precision $\beta$ of the private signals is finite, private signals might differ from each other and speculators cannot accurately establish the signals of their opponents. Note that due to the assumption of normally distributed noise parameters, the distribution of $\theta_i$ conditional on private and public information is normal as well, and thus the expected value of the unknown fundamental value of the economy conditional on private and public information is given by

$$\mathbb{E}[\theta_i|x_{ij}, y_i] = \frac{1}{\alpha_i+\beta} (\alpha_i y_i + \beta x_{ij})$$

with variance $\text{Var}[\theta_i|x_{ij}, y_i] = \frac{1}{\alpha_i+\beta}$.

The model is sequential: the speculation game described above takes place first in country $A$ and then in country $B$. The order of events is depicted in figure 2.1.
2.3 Solving the Model

As shown in Carlsson and van Damme (1993), Corsetti, Dasgupta, Morris, and Shin (2004), and Morris and Shin (1998, 2003, 2004), the switching strategy is the only equilibrium strategy in the setting above. The equilibrium strategy consists of the following values conditional on group 1’s type and the information structure: a unique value of the switching economic fundamentals $\bar{\theta}_i (i = A, B)$ up to which the government always abandons the peg, and a unique value of the switching private signal $\bar{x}_{ij} (j = 1, 2)$ such that every speculator who receives a signal lower than $\bar{x}_{ij}$ attacks the peg. That is, the equilibrium values $\bar{\theta}_i (i = A, B)$ and $\bar{x}_{ij} (j = 1, 2)$ belong to two situations of indifference: for $\theta = \bar{\theta}_i$ the government is indifferent between defending the currency peg and abandoning it, whereas speculators with $\bar{x}_{ij}$ are indifferent between attacking the peg and refraining from doing so.

2.3.1 Equilibrium in Country A

Country A’s equilibrium strategy (i.e., $\bar{\theta}_A$ and $\bar{x}_{Aj} (j = 1, 2)$) can be expressed as follows:
\[ \bar{\theta}_A = \begin{cases} \theta_{AB}^* & \text{if the type of group 1 speculators is "bullish";} \\ \theta_{AC}^* & \text{if the type of group 1 speculators is "chicken"} \end{cases} \]

\[ \bar{x}_{A1} = \begin{cases} x_{A1B}^* & \text{if group 1 speculators are "bullish";} \\ x_{A1C}^* & \text{if group 1 speculators are "chicken"} \end{cases} \]

\[ \bar{x}_{A2} = x_{A2}^*. \]

After receiving the private and public signals, each speculator has to decide whether to attack the currency, which leads to an uncertain payoff \( D \) but to costs of \( t \) for "bullish" type and \( t + \delta \) for "chicken" type. If he does not sell the currency, then a net profit is zero with certainty. Indifference between these two possible actions is achieved if both lead to the same expected net payoff:

\[ 0 = D \cdot \Pr[\text{Attack is successful} | \bar{x}_{Aj}] - t; \quad \text{(bullish type)} \]
\[ 0 = D \cdot \Pr[\text{Attack is successful} | \bar{x}_{Aj}] - t - \delta. \quad \text{(chicken type)} \]

Also, note that the critical threshold value of country A’s fundamentals (i.e., switching fundamentals) is determined when the proportion of speculators who attack the currency peg (\( l_A \)) is equal to \( \theta_A \).

Using the indifference condition for each type speculator and the condition of the critical threshold value of country A’s fundamentals, I obtain the unique equilibrium values: the switching fundamentals of country A (\( \theta_{AB}^* \) and \( \theta_{AC}^* \)) and the switching private signals (\( x_{A1B}^*, x_{A1C}^*, \) and \( x_{A2}^* \)).

**Proposition 9** There exists a unique equilibrium strategy in country A that consists of the switching economic fundamentals of country A (\( \bar{\theta}_A \)) and the switching private signal (\( \bar{x}_{Aj}, j = 1, 2 \)) of speculators.

Note that the uniqueness condition of the equilibrium strategy in country A is \( \beta > \frac{\alpha_A^2}{2\pi} \). The intuition follows Morris and Shin (2003, 2004). Since \( \alpha_A \) is the precision of the ex-ante distribution of \( \theta_A \), the depicted equilibrium above is unique as long as the precision of the
private signal ($\beta$) is high enough relative to the underlying uncertainty. Based on this uniqueness condition ($\beta > \frac{\sigma^2}{2\pi}$) of the equilibrium strategy in country $A$, I can verify that $\theta^*_A > \theta^*_C$ and $x^*_A > x^*_2 > x^*_A > x^*_C$ hold. The intuition of the inequalities is the following. $x^*_A > x^*_C$ because the bullish-type speculators are more likely to attack the peg than chicken-type speculators. By the same logic, $\theta^*_A$ is greater than $\theta^*_C$ because country $A$ will be more likely to suffer a currency crisis if speculators in group 1 are bullish.

2.3.2 Equilibrium in Country $B$

Now every speculator observes what occurred in country $A$, including the exact value of $\theta_A$. This conveys information about the type of speculators in group 1 to the market because different types use different switching signals, resulting in different outcomes in country $A$ under certain conditions.

There are two possible scenarios. First, if $\theta_A \notin [\theta^*_A, \theta^*_B]$, then the type of speculators in group 1 is not revealed. Why? If $\theta_A \leq \theta^*_A$, then the currency crisis certainly occurs in country $A$ regardless of the type of speculators in group 1. Meanwhile, if $\theta_A \geq \theta^*_B$, then the currency crisis never occurs in country $A$ regardless of the type of speculators in group 1. Hence, if $\theta_A \notin [\theta^*_A, \theta^*_B]$, speculators in group 2 do not get to know the type of speculators in group 1 and face the same game, which was played in country $A$, in determining whether or not to attack the peg in country $B$.

Next, however, if $\theta_A \in [\theta^*_A, \theta^*_B]$, then the type of speculators in group 1 is revealed to the market. Conditional on such $\theta_A$, the currency crisis occurs in country $A$ if and only if speculators in group 1 are bullish. Likewise, conditional on such $\theta_A$ which is between $\theta^*_A$ and $\theta^*_B$, the currency crisis does not occur in country $A$ if and only if group 1 speculators are chicken. Hence, if $\theta_A \in [\theta^*_A, \theta^*_B]$, then the new game is played by speculators whether or not to attack the peg in
country $B$.

In the following, I explain the two scenarios: $\theta_A \notin [\theta_{AC}^*, \theta_{AB}^*]$ and $\theta_A \in [\theta_{AC}^*, \theta_{AB}^*]$. In each scenario, I derive the equilibrium strategy (i.e., $\bar{\theta}_B$ and $\bar{x}_{Bj}$, $j = 1, 2$).

2.3.2.1 Scenario 1: $\theta_A \notin [\theta_{AC}^*, \theta_{AB}^*]$

In this scenario, the type of group 1 speculators is not revealed. Hence, the equilibrium values of the switching fundamentals of country $B$ and the switching private signals are exactly the same as those of country $A$. This is the benchmark case of country $B$, and particularly, the benchmark switching fundamentals of country $B$ are (1) $\theta_{AC}^*$ if the type of group 1 speculators is chicken, and (2) $\theta_{AB}^*$ if the type of group 1 speculators is bullish.

2.3.2.2 Scenario 2 - 1: Currency crisis in country $A$ when $\theta_A \in [\theta_{AC}^*, \theta_{AB}^*]$

This scenario implies that the type of group 1 is "bullish." Then, both speculators in group 1 and 2 have the same switching strategy signal (say $x_B^*$). Hence, the equilibrium strategy consists of (1) a country’s switching fundamentals ($\theta_{BB}^*$) below which the government abandons the peg (i.e., a currency crisis occurs in country $B$) and (2) the speculators’ switching private signal ($x_B^*$) such that every speculator who receives a signal lower than $x_B^*$ attacks the peg. I obtain the following equilibrium strategy:

$$
\theta_{BB}^* = \Phi \left[ \frac{\alpha_B}{\sqrt{\beta}} \left( \theta_{BB}^* - y_B - \frac{\sqrt{\alpha_B + \beta}}{\alpha_B} \Phi^{-1} \left( \frac{t}{D} \right) \right) \right],
$$

$$
x_B^* = \frac{\alpha_B + \beta}{\beta} \theta_{BB}^* - \frac{\alpha_B}{\beta} y_B - \frac{\sqrt{\alpha_B + \beta}}{\beta} \Phi^{-1} \left( \frac{t}{D} \right).
$$

Here, I can easily check that $\theta_{BB}^*$ is unique if $\beta > \frac{\alpha_B^2}{2\pi}$. Also, if $\theta_{BB}^*$ is unique, then $x_B^*$ is unique.

2.3.2.3 Scenario 2 - 2: No currency crisis in country $A$ when $\theta_A \in [\theta_{AC}^*, \theta_{AB}^*]$

This scenario implies that the type of group 1 is "chicken." Then, speculators in group 1 and 2 have different switching strategy signals (say $x_{B1}^*$ for group 1 and $x_{B2}^*$ for group 2). Hence, the equilibrium strategy consists of (1) a country’s switching fundamentals ($\theta_{BC}^*$) below which the
government abandons the peg (i.e., country $B$ suffers a currency crisis) and (2) the speculators’ switching private signal ($x_{B1}^*$ for group 1 and $x_{B2}^*$ for group 2) such that every speculator in group 1 who receives a signal lower than $x_{B1}^*$ attacks the peg and that every speculator in group 2 who receives a signal lower than $x_{B2}^*$ attacks the peg. I get the following equilibrium strategy:

$$
\theta_{BC}^* = \lambda \Phi \left[ \frac{\alpha_B}{\sqrt{\beta}} \left( \theta_{BC}^* - y_B - \frac{\sqrt{\alpha_B + \beta}}{\alpha_B} \Phi^{-1} \left( \frac{t + \delta}{D} \right) \right) \right] \\
+ (1 - \lambda) \Phi \left[ \frac{\alpha_B}{\sqrt{\beta}} \left( \theta_{BC}^* - y_B - \frac{\sqrt{\alpha_B + \beta}}{\alpha_B} \Phi^{-1} \left( \frac{t}{D} \right) \right) \right],
$$

$$
x_{B1}^* = \frac{\alpha_B + \beta}{\beta} \theta_{BC}^* - \frac{\alpha_B}{\beta} y_B - \frac{\sqrt{\alpha_B + \beta}}{\beta} \Phi^{-1} \left( \frac{t + \delta}{D} \right),
$$

$$
x_{B2}^* = \frac{\alpha_B + \beta}{\beta} \theta_{BC}^* - \frac{\alpha_B}{\beta} y_B - \frac{\sqrt{\alpha_B + \beta}}{\beta} \Phi^{-1} \left( \frac{t}{D} \right).
$$

Like $\theta_{BB}$ and $x_B^*$, I can easily check that $\theta_{BC}^*$ is unique if $\beta > \frac{\alpha_B^2}{2\pi}$. Also, if $\theta_{BC}^*$ is unique, then $x_{B1}^*$ and $x_{B2}^*$ are unique, respectively.

Based on the uniqueness condition ($\beta > \frac{\alpha_B^2}{2\pi}$) of the equilibrium strategy in country $B$, I can verify that $\theta_{BB}^* > \theta_{BC}^*$ and $x_B^* > x_{B2}^* > x_{B1}^*$ hold. The intuition of the inequalities is the following. $x_B^*$ is greater than $x_{B1}^*$ and $x_{B2}^*$ because when all speculators are bullish, they are more likely to attack the peg than when there exist chicken-type speculators. By the same logic, $\theta_{BB}^*$ is greater than $\theta_{BC}^*$ because country $B$ will be more likely to suffer a currency crisis if speculators in group 1 are bullish.

### 2.4 Contagion

#### 2.4.1 What Is Contagion?

In this paper, I define the contagion of a currency crisis between two countries as the effect of what occurs in country $A$ on what occurs in country $B$ due to the speculators’ learning process. That is, the probability of a currency crisis in country $B$ is affected by the speculators’ revised beliefs about other speculators’ types from what occurs in country $A.
As I discussed in section 2.3.2, the probability of a currency crisis in country B is reduced when the type of group 1 speculators is revealed as being "chicken" after the speculation game in country A ends. This can be interpreted as a positive effect of the contagion from country A to country B. In this sense, like the analysis of Manz (2002), I define $\theta_{AC}^* - \theta_{BC}^*$ as the "Good Contagion (GC)" of the currency crisis from country A to country B. Meanwhile, if the type of group 1 speculators is revealed as being "bullish" after the speculation game in country A ends, then the probability of a currency crisis in country B is increased by $\theta_{BB}^* - \theta_{AB}^*$. This is a negative effect of the contagion from country A to country B and thus is defined as the "Bad Contagion (BC)" of the currency crisis from country A to country B. I can easily check that BC and GC are greater than zero if both countries’ public signals and the precisions of these public signals are the same (i.e., homogeneity condition: $y_A = y_B$ and $\alpha_A = \alpha_B$) (see figure 2.2).

![Contagion – Good Contagion (GC) / Bad Contagion (BC)](image)

**Figure 2.2: Contagion – Good Contagion (GC) / Bad Contagion (BC)**

### 2.4.2 Effects of Public Information on Contagion

Now, let’s discuss the effects of country B’s public information on the contagion. That is, in the following, I examine the influence that different parameters (public signal: $y_B$; and the precision of the public signal: $\alpha_B$) exert on the "Good Contagion (GC)" and the "Bad Contagion (BC)" given the fact that the uniqueness of the equilibrium is guaranteed, i.e., $\beta > \frac{\alpha_B^2}{2\pi}$.

**Proposition 10** The public signal distributed by the central bank of country B (i.e., $y_B$) promotes the effect of the "Good Contagion (GC)" and reduces the effect of the "Bad Contagion (BC)."

Proposition 10 implies that if the central bank distributes / announces more public information
about the economic fundamentals of its country in the international financial market, then it can increase the positive effect of the contagion and decrease the negative effect of the contagion on the currency crisis from the other country. That is, overall, the higher the public signal about the economic fundamental state of the country, the lower the bad contagious currency crisis from the other country will be. Meanwhile, I also find that the public signal distributed by the central bank of country \( A \) (i.e., \( y_A \)) reduces the effect of the "Good Contagion (GC)" and worsens the effect of the "Bad Contagion (BC)" on the currency crisis of country \( B \).\(^{48}\) I can interpret this result as follows. Even though the higher public signal about the fundamental state of the country’s economy lowers the bad contagious currency crisis from the former country as Proposition 10 addresses, it triggers a negative impact on the currency crisis of the latter country.

**Proposition 11**  
1. If the state of country \( B \)’s fundamentals is ex-ante expected to be bad, then the precision of the public signal distributed by the central bank of country \( B \) (i.e., \( \alpha_B \)) reduces the effect of the "Good Contagion (GC)" and worsens the effect of the "Bad Contagion (BC)."

2. If the state of country \( B \)’s fundamentals is ex-ante expected to be good, then the precision of the public signal distributed by the central bank of country \( B \) (i.e., \( \alpha_B \)) promotes the effect of the "Good Contagion (GC)" and reduces the effect of the "Bad Contagion (BC)."

Proposition 11 shows the effectiveness of the public signal’s precision on the contagion in currency crises. Interestingly, the high precision of the public signal does not always bring a good result to lowering the bad contagious currency crisis from the other country. Only when the country’s economic fundamental state is ex-ante expected to be good, can the precision of the public signal lower the negative effect of the contagion and raise the positive effect of the contagion on the currency crisis from the other country.

\(^{48}\) That is, I check \( \frac{\partial \sigma_{GC}}{\partial y_A} < 0 \) and \( \frac{\partial \sigma_{BC}}{\partial y_A} < 0 \) for all \( y_A \) by applying intermediate value theorem (contrapositive statement), which implies that \( \frac{\partial GC}{\partial y_A} < 0 \) and \( \frac{\partial BC}{\partial y_A} > 0 \) hold.
2.5 Concluding Remarks

In this paper, focusing on the speculators’ learning process about each other’s type – the aggressiveness on the speculation (i.e., "bullish" vs. "chicken") – as the contagion triggering mechanism between two countries (e.g., Oh (2010) and Taketa (2004)), I explored the role of public information disseminated by the central bank of the country in the contagion of a currency crisis. With a traditional concept of the contagion in which the bad outcome in the former country negatively affects the outcome in the latter country (i.e., "Bad Contagion"), I dealt with the "Good Contagion" in which a positive result in the former country rescues the latter country. In particular, a revelation of the type of speculators as being "chicken" in the first country’s speculation game among speculators would reduce the probability of a currency crisis in the second country.

Under the uniqueness condition for each country’s equilibrium – the threshold for a currency crisis – I found that the higher the public signal about the fundamental state of the economy distributed by the central bank, the lower the effect of the "Bad Contagion" and the higher the effect of the "Good Contagion" will be. In addition, I showed that only when the country’s economic fundamental state is ex-ante expected to be good, can the central bank’s distribution of the precise public signals be effective in preventing the "Bad Contagion" of the currency crisis and in promoting the "Good Contagion" of the currency crisis from the other country. Hence, the welfare effectiveness of enhanced dissemination of public information by the central bank on the contagion between two economies depends on the economic fundamental state of the country.
Appendix

Proof of Proposition 9

First, let’s think about the decisions of group 1 speculators. They privately know their type ("bullish" or "chicken") and also know group 2 speculators’ type ("bullish"). Hence, they know the value of \( \hat{\theta}_A^* : \theta_{AB}^* \) or \( \theta_{AC}^* \). If the type of group 1 speculators is "bullish," then I get the following indifference equation:

\[
t = D \cdot \Pr [\theta_A \leq \theta_{AB}^* | x_{A1B}^*] \]
\[
= D \cdot \Phi \left[ \sqrt{\alpha_A + \beta} \left( \theta_{AB}^* - \frac{\alpha_A}{\alpha_A + \beta} y_A - \frac{\beta}{\alpha_A + \beta} x_{A1B}^* \right) \right], \quad \text{(B1)}
\]

where \( \Phi \) denotes the cumulated normal density. In the same way, I get the following indifference equation for "chicken" type group 1 speculators:

\[
t + \delta = D \cdot \Phi \left[ \sqrt{\alpha_A + \beta} \left( \theta_{AC}^* - \frac{\alpha_A}{\alpha_A + \beta} y_A - \frac{\beta}{\alpha_A + \beta} x_{A1C}^* \right) \right]. \quad \text{(B2)}
\]

Next, let’s think about the decision of group 2 speculators. They know only their own type ("bullish") but not the type of group 1 speculators. They can just conjecture the probability that the type of group 1 speculators is "bullish" as \( q \). Hence, the indifference equation for group 2 speculators is as follows:

\[
t = D \cdot \left\{ q \cdot \Phi \left[ \sqrt{\alpha_A + \beta} \left( \theta_{AB}^* - \frac{\alpha_A}{\alpha_A + \beta} y_A - \frac{\beta}{\alpha_A + \beta} x_{A2B}^* \right) \right] \right. \]
\[
\left. + (1 - q) \cdot \Phi \left[ \sqrt{\alpha_A + \beta} \left( \theta_{AC}^* - \frac{\alpha_A}{\alpha_A + \beta} y_A - \frac{\beta}{\alpha_A + \beta} x_{A2C}^* \right) \right] \right\}. \quad \text{(B3)}
\]

Lastly, let’s think about the critical threshold value of country A’s fundamentals (i.e., switching fundamentals). The proportion of speculators who attack country A’s currency is expressed as follows:

\[
l_A (\theta_A) = \lambda \cdot \Pr [x_{A1} \leq \bar{x}_{A1} | \theta_A] + (1 - \lambda) \cdot \Pr [x_{A2} \leq \bar{x}_{A2} | \theta_A] \]
\[
= \lambda \cdot \Phi \left[ \sqrt{\beta} (\bar{x}_{A1} - \theta_A) \right] + (1 - \lambda) \cdot \Phi \left[ \sqrt{\beta} (x_{A2}^* - \theta_A) \right].
\]
The critical threshold value is determined by:
\[
\bar{\theta}_A = l_A(\bar{\theta}_A) = \lambda \cdot \Phi \left[ \sqrt{\beta} (\bar{x}_A - \bar{\theta}_A) \right] + (1 - \lambda) \cdot \Phi \left[ \sqrt{\beta} (x_{A2}^* - \bar{\theta}_A) \right]. \tag{B4}
\]

From equation (B4), I get the following two equations:
\[
\theta_{AB}^* = \lambda \cdot \Phi \left[ \sqrt{\beta} (x_{A1B}^* - \theta_{AB}^*) \right] + (1 - \lambda) \cdot \Phi \left[ \sqrt{\beta} (x_{A2}^* - \theta_{AB}^*) \right], \tag{B5}
\]
\[
\theta_{AC}^* = \lambda \cdot \Phi \left[ \sqrt{\beta} (x_{A1C}^* - \theta_{AC}^*) \right] + (1 - \lambda) \cdot \Phi \left[ \sqrt{\beta} (x_{A2}^* - \theta_{AC}^*) \right]. \tag{B6}
\]

Solving equations (B1), (B2), (B3), (B5), and (B6), I obtain \( x_{A1B}^*, \ x_{A1C}^*, \ x_{A2}^*, \ \theta_{AB}^*, \) and \( \theta_{AC}^* \):
\[
x_{A1B}^* = \frac{\alpha_A + \beta}{\beta} \theta_{AB}^* - \frac{\alpha_A}{\beta} y_A - \frac{\sqrt{\alpha_A + \beta}}{\beta} \Phi^{-1} \left( \frac{t}{D} \right), \tag{S1}
\]
\[
x_{A1C}^* = \frac{\alpha_A + \beta}{\beta} \theta_{AC}^* - \frac{\alpha_A}{\beta} y_A - \frac{\sqrt{\alpha_A + \beta}}{\beta} \Phi^{-1} \left( \frac{t + \delta}{D} \right), \tag{S2}
\]
\[
x_{A2}^* = \theta_{AC}^* + \frac{1}{\sqrt{\beta}} \Phi^{-1} (K) = \theta_{AB}^* + \frac{1}{\sqrt{\beta}} \Phi^{-1} (G), \tag{S3}
\]
\[
\theta_{AB}^* = \frac{\alpha_A + \beta}{\beta} \left\{ \lambda \Phi (H) + (1 - \lambda) \Phi (F) \right\} - \frac{\alpha_A}{\beta} y_A - \frac{1}{\sqrt{\beta}} \Phi^{-1} (G)
- \frac{\sqrt{\alpha_A + \beta}}{\beta} \Phi^{-1} \left[ \frac{t}{D (1 - q)} - \frac{q}{1 - q} \Phi \left[ \frac{\alpha_A}{\sqrt{\alpha_A + \beta}} \left( \theta_{AB}^* - y_A + \frac{\sqrt{\beta}}{\alpha_A} \Phi^{-1} (G) \right) \right] \right], \tag{S4}
\]
\[
\theta_{AC}^* = \frac{\alpha_A + \beta}{\beta} \left\{ \lambda \Phi (L) + (1 - \lambda) \Phi (M) \right\} - \frac{\alpha_A}{\beta} y_A - \frac{1}{\sqrt{\beta}} \Phi^{-1} (K)
- \frac{\sqrt{\alpha_A + \beta}}{\beta} \Phi^{-1} \left[ \frac{t}{q D} - \frac{1 - q}{q} \Phi \left[ \frac{\alpha_A}{\sqrt{\alpha_A + \beta}} \left( \theta_{AC}^* - y_A + \frac{\sqrt{\beta}}{\alpha_A} \Phi^{-1} (K) \right) \right] \right], \tag{S5}
\]
where
\[
K := \frac{1}{1 - \lambda} \theta_{AC}^* - \frac{\lambda}{1 - \lambda} \Phi \left[ \frac{\alpha_A}{\sqrt{\beta}} \left( \theta_{AC}^* - y_A + \frac{\sqrt{\beta}}{\alpha_A} \Phi^{-1} \left( \frac{t + \delta}{D} \right) \right) \right],
\]
\[
G := \frac{1}{1 - \lambda} \theta_{AB}^* - \frac{\lambda}{1 - \lambda} \Phi \left[ \frac{\alpha_A}{\sqrt{\beta}} \left( \theta_{AB}^* - y_A + \frac{\sqrt{\beta}}{\alpha_A} \Phi^{-1} \left( \frac{t}{D} \right) \right) \right],
\]
and
\[
L := \sqrt{\beta} (x_{A1B}^* - \theta_{AB}^*)
= \frac{\alpha_A \sqrt{\beta}}{\alpha_A + \beta} \left( \theta_{AC}^* - y_A + \frac{1}{\sqrt{\beta}} \Phi^{-1} (K) \right) - \frac{\sqrt{\alpha_A + \beta}}{\sqrt{\beta}} \Phi^{-1} \left( \frac{t}{D} \right)
+ \frac{\alpha_A}{\sqrt{\beta} \sqrt{\alpha_A + \beta}} \Phi^{-1} \left[ \frac{t}{q D} - \frac{1 - q}{q} \Phi \left[ \frac{\alpha_A}{\sqrt{\alpha_A + \beta}} \left( \theta_{AC}^* - y_A + \frac{\sqrt{\beta}}{\alpha_A} \Phi^{-1} (K) \right) \right] \right],
\]
54
\[ M := \sqrt{\beta} (x_{A2}^* - \theta_{AB}^*) \]
\[ = \frac{\alpha_A \sqrt{\beta}}{\alpha_A + \beta} \left( \theta_{AC}^* - y_A + \frac{1}{\sqrt{\beta}} \Phi^{-1} (K) \right) \]
\[ - \frac{\sqrt{\beta}}{\sqrt{\alpha_A + \beta}} \Phi^{-1} \left[ \frac{t}{qD} - \frac{1 - q}{q} \Phi \left[ \frac{\alpha_A}{\sqrt{\alpha_A + \beta}} \left( \theta_{AC}^* - y_A + \frac{\sqrt{\beta}}{\alpha_A} \Phi^{-1} (K) \right) \right] \right], \]

\[ H := \sqrt{\beta} (x_{A1C}^* - \theta_{AC}^*) \]
\[ = \frac{\alpha_A \sqrt{\beta}}{\alpha_A + \beta} \left( \theta_{AB}^* - y_A + \frac{1}{\sqrt{\beta}} \Phi^{-1} (G) \right) - \frac{\sqrt{\alpha_A + \beta}}{\sqrt{\beta}} \Phi^{-1} \left( \frac{t + \delta}{D} \right) \]
\[ + \frac{\alpha_A}{\sqrt{\alpha_A + \beta}} \Phi^{-1} \left[ \frac{t}{D (1 - q)} - \frac{q}{1 - q} \Phi \left[ \frac{\alpha_A}{\sqrt{\alpha_A + \beta}} \left( \theta_{AB}^* - y_A - \frac{\sqrt{\beta}}{\alpha_A} \Phi^{-1} (G) \right) \right] \right], \]

\[ F := \sqrt{\beta} (x_{A2}^* - \theta_{AC}^*) \]
\[ = \frac{\alpha_A \sqrt{\beta}}{\alpha_A + \beta} \left( \theta_{AB}^* - y_A + \frac{1}{\sqrt{\beta}} \Phi^{-1} (G) \right) \]
\[ - \frac{\sqrt{\beta}}{\sqrt{\alpha_A + \beta}} \Phi^{-1} \left[ \frac{t}{D (1 - q)} - \frac{q}{1 - q} \Phi \left[ \frac{\alpha_A}{\sqrt{\alpha_A + \beta}} \left( \theta_{AB}^* - y_A - \frac{\sqrt{\beta}}{\alpha_A} \Phi^{-1} (G) \right) \right] \right]. \]

**Proof of the Uniqueness Condition:** \( \beta > \frac{\alpha_A^2}{2\pi} \)

The uniqueness is proved by the following three lemmas.

**Lemma 12** \( \frac{\partial}{\partial \theta_{AC}^*} \Phi \left[ \sqrt{\beta} (x_{A1C}^* - \theta_{AC}^*) \right] < 1 \) if \( \beta > \frac{\alpha_A^2}{2\pi} \).

**Proof.** From equation (B2), I obtain
\[ 0 = \phi \left[ \sqrt{\alpha_A + \beta} \left( \theta_{AC}^* - \frac{\alpha_A}{\alpha_A + \beta} y_A - \frac{\beta}{\alpha_A + \beta} x_{A1C}^* \right) \right] \sqrt{\alpha_A + \beta} \left( 1 - \frac{\beta}{\alpha_A + \beta} \frac{\partial x_{A1C}^*}{\partial \theta_{AC}^*} \right) \]
and thus,
\[ \frac{\partial x_{A1C}^*}{\partial \theta_{AC}^*} = \frac{\alpha_A + \beta}{\beta}. \]
Then, I get
\[
\frac{\partial}{\partial \theta^*_AC} \Phi \left[ \sqrt{\beta} \left( x^*_{A1C} - \theta^*_AC \right) \right] = \phi \left[ \sqrt{\beta} \left( x^*_{A1C} - \theta^*_AC \right) \right] \sqrt{\beta} \left( \frac{\partial x^*_{A1C}}{\partial \theta^*_AC} - 1 \right) = \phi \left[ \sqrt{\beta} \left( x^*_{A1C} - \theta^*_AC \right) \right] \sqrt{\beta} \left( \frac{\alpha_A + \beta}{\beta} - 1 \right) = \phi \left[ \sqrt{\beta} \left( x^*_{A1C} - \theta^*_AC \right) \right] \frac{\alpha_A}{\sqrt{\beta}} \leq \frac{1}{\sqrt{2\pi} \sqrt{\beta}} < 1.
\]

Lemma 13 \( \frac{\partial}{\partial \theta^*_AC} \Phi \left[ \sqrt{\beta} \left( x^*_{A2} - \theta^*_AC \right) \right] < 1 \) if \( \beta > \frac{\alpha_A^2}{2\pi} \).

Proof. From equation (B1), I can derive
\[
\frac{\partial x^*_{A1B}}{\partial \theta^*_AC} = \frac{\alpha_A + \beta}{\beta} \frac{\partial \theta^*_AB}{\partial \theta^*_AC}.
\]
From equation (B5), I get
\[
\frac{\partial \theta^*_AB}{\partial \theta^*_AC} = \lambda \phi (L) \sqrt{\beta} \left( \frac{\partial x^*_{A1B}}{\partial \theta^*_AC} - \frac{\partial \theta^*_AB}{\partial \theta^*_AC} \right) + (1 - \lambda) \phi (M) \sqrt{\beta} \left( \frac{\partial x^*_{A2}}{\partial \theta^*_AC} - \frac{\partial \theta^*_AB}{\partial \theta^*_AC} \right),
\]
where \( L := \sqrt{\beta} \left( x^*_{A1B} - \theta^*_AB \right) \) and \( M := \sqrt{\beta} \left( x^*_{A2} - \theta^*_AB \right) \). Rearranging the above equation,
\[
(1 - \lambda) \phi (M) \sqrt{\beta} \frac{\partial x^*_{A2}}{\partial \theta^*_AC} = \left[ 1 - \lambda \phi (L) \sqrt{\beta} \frac{\alpha_A}{\beta} + (1 - \lambda) \phi (M) \sqrt{\beta} \right] \frac{\partial \theta^*_AB}{\partial \theta^*_AC}
\]
and thus,
\[
\frac{\partial \theta^*_AB}{\partial \theta^*_AC} = \left[ 1 - \lambda \phi (L) \frac{\alpha_A}{\sqrt{\beta}} + (1 - \lambda) \phi (M) \sqrt{\beta} \right]^{-1} (1 - \lambda) \phi (M) \sqrt{\beta} \frac{\partial x^*_{A2}}{\partial \theta^*_AC}.
\]
Let
\[
P := \sqrt{\alpha_A + \beta} \left( \theta^*_AB - \frac{\alpha_A}{\alpha_A + \beta} y_A - \frac{\beta}{\alpha_A + \beta} x^*_{A2} \right),
\]
\[
Q := \sqrt{\alpha_A + \beta} \left( \theta^*_AC - \frac{\alpha_A}{\alpha_A + \beta} y_A - \frac{\beta}{\alpha_A + \beta} x^*_{A2} \right).
\]
Then, equation (B3) is rewritten as

\[ \frac{t}{D} = q\Phi (P) + (1 - q) \Phi (Q) \]

and from this equation, I get

\[ 0 = q\phi (P) \sqrt{\alpha_A + \beta} \left( \frac{\partial \theta_{AB}^*}{\partial \theta_{AC}} - \frac{\beta}{\alpha_A + \beta} \frac{\partial x_{A2}^*}{\partial \theta_{AC}} \right) + (1 - q) \phi (Q) \sqrt{\alpha_A + \beta} \left( 1 - \frac{\beta}{\alpha_A + \beta} \frac{\partial x_{A2}^*}{\partial \theta_{AC}} \right). \]

By rearranging the above equation, I get

\[ (1 - q) \phi (Q) = \left[ \frac{\{q\phi (P) + (1 - q) \phi (Q)\} \cdot \frac{\beta}{\alpha_A + \beta}}{1 - \lambda \phi (L)^{\frac{\alpha_A}{\sqrt{\beta}}} + (1 - \lambda) \phi (M)^{\sqrt{\beta}}} \right] \frac{\partial x_{A2}^*}{\partial \theta_{AC}}, \]

and thus,

\[ \frac{\partial x_{A2}^*}{\partial \theta_{AC}} = (1 - q) \phi (Q) \left[ \frac{\{q\phi (P) + (1 - q) \phi (Q)\} \cdot \frac{\beta}{\alpha_A + \beta}}{1 - \lambda \phi (L)^{\frac{\alpha_A}{\sqrt{\beta}}} + (1 - \lambda) \phi (M)^{\sqrt{\beta}}} \right]^{-1}. \]

Here, I observe

\[ \frac{(1 - \lambda) \phi (M)^{\sqrt{\beta}}}{1 - \lambda \phi (L)^{\frac{\alpha_A}{\sqrt{\beta}}} + (1 - \lambda) \phi (M)^{\sqrt{\beta}}} = \frac{(1 - \lambda) \phi (M) \beta}{\sqrt{\beta} - \lambda \phi (L) \alpha_A + (1 - \lambda) \phi (M) \beta} \]

\[ < \frac{\alpha_A}{\sqrt{2\pi}} - \lambda \frac{\alpha_A}{\sqrt{2\pi}} + (1 - \lambda) \phi (M) \beta \]

\[- \frac{\phi (M) \beta}{\sqrt{2\pi} \phi (M) + \beta} \]

\[ = \frac{\beta}{\alpha_A + \beta}. \]

Thus,

\[ \left[ \frac{\{q\phi (P) + (1 - q) \phi (Q)\} \cdot \frac{\beta}{\alpha_A + \beta}}{1 - \lambda \phi (L)^{\frac{\alpha_A}{\sqrt{\beta}}} + (1 - \lambda) \phi (M)^{\sqrt{\beta}}} \right]^{-1} > \left[ \frac{\{q\phi (P) + (1 - q) \phi (Q)\} \cdot \frac{\beta}{\alpha_A + \beta}}{1 - \lambda \phi (L)^{\frac{\alpha_A}{\sqrt{\beta}}} + (1 - \lambda) \phi (M)^{\sqrt{\beta}}} \right] \frac{\beta}{\alpha_A + \beta} - q\phi (P) \frac{\beta}{\alpha_A + \beta} \]

\[ = (1 - q) \phi (Q) \frac{\beta}{\alpha_A + \beta}. \]
Then, I get
\[
\frac{\partial x_{A2}^*}{\partial \theta_{AC}^*} = (1 - q) \phi(Q) \left[ \frac{\{q\phi(P) + (1 - q) \phi(Q)\} \beta}{(1 - \lambda)\phi(M)\sqrt{\beta}} \right]^{-1}
\]
\[
< (1 - q) \phi(Q) \frac{1}{(1 - q) \phi(Q) \frac{\beta}{\alpha + \beta}}
\]
\[
= \frac{\alpha + \beta}{\beta}.
\]

Finally, I obtain
\[
\frac{\partial}{\partial \theta_{AC}^*} \Phi \left[ \sqrt{\beta} (x_{A2}^* - \theta_{AC}^*) \right] = \phi \left[ \sqrt{\beta} (x_{A2}^* - \theta_{AC}^*) \right] \sqrt{\beta} \left( \frac{\partial x_{A2}^*}{\partial \theta_{AC}^*} - 1 \right)
\]
\[
< \phi \left[ \sqrt{\beta} (x_{A2}^* - \theta_{AC}^*) \right] \sqrt{\beta} \left( \frac{\alpha + \beta}{\beta} - 1 \right)
\]
\[
= \phi \left[ \sqrt{\beta} (x_{A2}^* - \theta_{AC}^*) \right] \frac{\alpha}{\sqrt{\beta}}
\]
\[
\leq \frac{1}{\sqrt{2\pi}} \frac{\alpha}{\sqrt{\beta}} < 1.
\]

Lemma 14  \( \theta_{AC}^* \) is unique if \( \beta > \frac{\alpha^2}{2\pi} \).

Proof. From equation (B6), I obtain
\[
\frac{\partial}{\partial \theta_{AC}^*} \left( \lambda \Phi \left[ \sqrt{\beta} (x_{A1C}^* - \theta_{AC}^*) \right] + (1 - \lambda) \Phi \left[ \sqrt{\beta} (x_{A2}^* - \theta_{AC}^*) \right] \right)
\]
\[
= \lambda \frac{\partial}{\partial \theta_{AC}^*} \Phi \left[ \sqrt{\beta} (x_{A1C}^* - \theta_{AC}^*) \right] + (1 - \lambda) \frac{\partial}{\partial \theta_{AC}^*} \Phi \left[ \sqrt{\beta} (x_{A2}^* - \theta_{AC}^*) \right]
\]
\[
< \lambda + (1 - \lambda) = 1,
\]
i.e., \( \theta_{AC}^* \) is unique if \( \beta > \frac{\alpha^2}{2\pi} \). ■

From these three lemmas, I can easily check that 5-tuple \((\theta_{AB}^*, \theta_{AC}^*, x_{A1B}^*, x_{A1C}^*, x_{A2}^*)\) is unique
if \( \beta > \frac{\alpha^2}{2\pi} \).
Proof of \( \theta_{AB}^* > \theta_{AC}^* \)

Let \( \widetilde{K} (\xi) \) and \( \widetilde{G} (\xi) \) be functions of \( \xi \) defined by

\[
\widetilde{K} := \xi + \frac{1}{\sqrt{\beta}} \Phi^{-1} (K (\xi)) ,
\]
\[
\widetilde{G} := \xi + \frac{1}{\sqrt{\beta}} \Phi^{-1} (G (\xi)) ,
\]

where

\[
K (\xi) := \frac{1}{1 - \lambda} \xi - \frac{\lambda}{1 - \lambda} \Phi \left[ \frac{\alpha_A}{\sqrt{\beta}} \left( \xi - y_A - \frac{\sqrt{\alpha_A + \beta}}{\alpha_A} \Phi^{-1} \left( \frac{t + \delta}{D} \right) \right) \right] ,
\]
\[
G (\xi) := \frac{1}{1 - \lambda} \xi - \frac{\lambda}{1 - \lambda} \Phi \left[ \frac{\alpha_A}{\sqrt{\beta}} \left( \xi - y_A - \frac{\sqrt{\alpha_A + \beta}}{\alpha_A} \Phi^{-1} \left( \frac{t}{D} \right) \right) \right] .
\]

Since \( \Phi \) and \( \Phi^{-1} \) are continuous and increasing functions, \( \widetilde{K} (\xi) \) and \( \widetilde{G} (\xi) \) are continuous and increasing functions if \( \beta > \frac{\alpha_A^2}{2\pi} \). Therefore, \( \widetilde{K} \) and \( \widetilde{G} \) are continuous and increasing, too. I can see that \( K (\xi) > G (\xi) \) for all \( \xi \), and thus \( \widetilde{K} (\xi) > \widetilde{G} (\xi) \).

\( \theta_{AC}^* \) is the solution of \( \widetilde{K} (\xi) = x_{A2}^* \) and \( \theta_{AB}^* \) is the solution of \( \widetilde{G} (\xi) = x_{A2}^* \). Since \( \widetilde{K} (\xi) > \widetilde{G} (\xi) \) and they are continuous and increasing, I conclude that \( \theta_{AB}^* \) is greater than \( \theta_{AC}^* \).

Proof of \( x_{A1B}^* > x_{A2}^* > x_{A1C}^* \)

Let \( f (\xi) \) and \( g (\xi) \) be functions of \( \xi \) defined by

\[
f (\xi) := \Phi \left[ \sqrt{\alpha_A + \beta} \left( \theta_{AB}^* - \frac{\alpha_A}{\sqrt{\alpha_A + \beta} + \beta} y_A - \frac{\beta}{\alpha_A + \beta} \xi \right) \right] ,
\]
\[
g (\xi) := \Phi \left[ \sqrt{\alpha_A + \beta} \left( \theta_{AC}^* - \frac{\alpha_A}{\sqrt{\alpha_A + \beta} + \beta} y_A - \frac{\beta}{\alpha_A + \beta} \xi \right) \right] .
\]

Both functions are continuous and decreasing. I can see that \( f (\xi) < \lambda \cdot f (\xi) + (1 - \lambda) \cdot g (\xi) < g (\xi) \) for \( 0 < \lambda < 1 \).

\( x_{A1B}^*, x_{A2}^*, \) and \( x_{A1C}^* \) are the solutions of \( f (\xi) = \frac{t}{D} \), \( \lambda \cdot f (\xi) + (1 - \lambda) \cdot g (\xi) = \frac{t}{D} \), and \( g (\xi) = \frac{t + \delta}{D} \), respectively. Since \( \frac{t}{D} = f (x_{A1B}^*) = \lambda \cdot f (x_{A2}^*) + (1 - \lambda) \cdot g (x_{A2}^*) < f (x_{A2}^*) \), \( x_{A1B}^* > x_{A2}^* \) holds. Also, since \( g (x_{A2}^*) < \lambda \cdot f (x_{A2}^*) + (1 - \lambda) \cdot g (x_{A2}^*) = \frac{t}{D} < \frac{t + \delta}{D} = g (x_{A1C}^*) \),
Derivation of $\theta_{BB}^*$ and $x_B^*$

The proportion of speculators who attack the peg conditional on $\theta_B$ is expressed as follows:

$$l_B(\theta_B) = \Pr [x_B \leq x_B^* | \theta_B] = \Phi \left[ \sqrt{\beta} (x_B^* - \theta_B) \right].$$

The critical threshold value of country $B$’s fundamentals (i.e., switching fundamentals) is determined by:

$$\theta_{BB}^* = l_B(\theta_{BB}^*) = \Phi \left[ \sqrt{\beta} (x_B^* - \theta_{BB}^*) \right]. \quad (B7)$$

From the speculators’ indifference condition of attacking, I get:

$$t = D \cdot \Phi \left[ \sqrt{\alpha_B + \beta} \left( \theta_{BB}^* - \frac{\alpha_B}{\alpha_B + \beta} y_B - \frac{\beta}{\alpha_B + \beta} x_B^* \right) \right]. \quad (B8)$$

From equations (B7) and (B8), I get the equilibrium strategy:

$$x_B^* = \frac{\alpha_B + \beta}{\beta} \theta_{BB}^* - \frac{\alpha_B}{\beta} y_B - \frac{\sqrt{\alpha_B + \beta} \Phi^{-1} \left( \frac{t}{D} \right)}{\beta}, \quad (S6)$$

$$\theta_{BB}^* = \Phi \left[ \frac{\alpha_B}{\sqrt{\beta}} \left( \theta_{BB}^* - y_B - \frac{\sqrt{\alpha_B + \beta}}{\alpha_B} \Phi^{-1} \left( \frac{t}{D} \right) \right) \right]. \quad (S7)$$

Derivation of $\theta_{BC}^*$, $x_{B1}^*$, and $x_{B2}^*$

The proportion of speculators who attack the peg conditional on $\theta_B$ is expressed as follows:

$$l_B(\theta_B) = \lambda \cdot \Pr [x_{B1} \leq x_{B1}^* | \theta_B] + (1 - \lambda) \cdot \Pr [x_{B2} \leq x_{B2}^* | \theta_B] = \lambda \cdot \Phi \left[ \sqrt{\beta} (x_{B1}^* - \theta_B) \right] + (1 - \lambda) \cdot \Phi \left[ \sqrt{\beta} (x_{B2}^* - \theta_B) \right].$$

The critical threshold value of country $B$’s fundamentals (i.e., switching fundamentals) is determined by:

$$\theta_{BC}^* = l_B(\theta_{BC}^*) = \lambda \cdot \Phi \left[ \sqrt{\beta} (x_{B1}^* - \theta_{BC}^*) \right] + (1 - \lambda) \cdot \Phi \left[ \sqrt{\beta} (x_{B2}^* - \theta_{BC}^*) \right]. \quad (B9)$$
From the speculators’ indifference condition of attacking, I get:

\[
t + \delta = D \cdot \Phi \left[ \sqrt{\frac{\alpha_B}{\alpha_B + \beta}} \left( \theta_{BC}^* - \frac{\alpha_B}{\alpha_B + \beta} y_B - \frac{\beta}{\alpha_B + \beta} x_{B1}^* \right) \right]
\]  
(B10)

for "chicken" group 1 speculators and

\[
t = D \cdot \Phi \left[ \sqrt{\frac{\alpha_B}{\alpha_B + \beta}} \left( \theta_{BC}^* - \frac{\alpha_B}{\alpha_B + \beta} y_B - \frac{\beta}{\alpha_B + \beta} x_{B2}^* \right) \right]
\]  
(B11)

for "bullish" group 2 speculators.

From equations (B9), (B10), and (B11), I get the equilibrium strategy:

\[
x_{B1}^* = \frac{\alpha_B + \beta}{\beta} \theta_{BC}^* - \frac{\alpha_B}{\beta} y_B - \frac{\sqrt{\alpha_B + \beta}}{\beta} \Phi^{-1} \left( \frac{t + \delta}{D} \right),
\]  
(S8)

\[
x_{B2}^* = \frac{\alpha_B + \beta}{\beta} \theta_{BC}^* - \frac{\alpha_B}{\beta} y_B - \frac{\sqrt{\alpha_B + \beta}}{\beta} \Phi^{-1} \left( \frac{t}{D} \right),
\]  
(S9)

\[
\theta_{BC}^* = \lambda \Phi \left[ \frac{\alpha_B}{\sqrt{\beta}} \left( \theta_{BC}^* - y_B - \frac{\sqrt{\alpha_B + \beta}}{\alpha_B} \Phi^{-1} \left( \frac{t + \delta}{D} \right) \right) \right] + (1 - \lambda) \Phi \left[ \frac{\alpha_B}{\sqrt{\beta}} \left( \theta_{BC}^* - y_B - \frac{\sqrt{\alpha_B + \beta}}{\alpha_B} \Phi^{-1} \left( \frac{t}{D} \right) \right) \right].
\]  
(S10)

**Proof of \( \theta_{BB}^* > \theta_{BC}^* \) and \( x_{B1}^* > x_{B2}^* > x_{B1}^* \)**

Let \( f_1 (\xi) = \Phi [a (\xi - b_1)] \) and \( f_2 (\xi) = \Phi [a (\xi - b_2)] \), where \( a > 0 \) and \( b_2 > b_1 \). Since \( \Phi (\xi) \) is an increasing function, \( f_1 (\xi) > f_2 (\xi) \) always holds. In this case, \( f_1 (\xi) > \lambda \cdot f_1 (\xi) + (1 - \lambda) \cdot f_2 (\xi) \) for \( 0 < \lambda < 1 \).

Put \( a = \frac{\alpha_B}{\sqrt{\beta}}, b_1 = y_B + \frac{\sqrt{\alpha_B + \beta}}{\alpha_B} \Phi^{-1} \left( \frac{t}{D} \right), \) and \( b_2 = y_B + \frac{\sqrt{\alpha_B + \beta}}{\alpha_B} \Phi^{-1} \left( \frac{t + \delta}{D} \right) \). Then, I observe that \( \theta_{BB}^* = f_1 (\theta_{BB}^*) \) and \( \theta_{BC}^* = \lambda \cdot f_2 (\theta_{BC}^*) + (1 - \lambda) \cdot f_1 (\theta_{BC}^*) \). I know that \( f_1 (0) \) and \( f_2 (0) \) are greater than 0. Since \( \beta > \frac{\sqrt{\alpha_B + \beta}}{2 \pi} \), both \( \frac{\partial f_1 (\xi)}{\partial \xi} \) and \( \frac{\partial f_2 (\xi)}{\partial \xi} \) are less than 1.

Hence, \( \theta_{BB}^* > \theta_{BC}^* \) for unique \( \theta_{BB}^* \) and \( \theta_{BC}^* \). From this result with equations (S6), (S8), and (S9), I can easily check that \( x_{B1}^* > x_{B2}^* > x_{B1}^* \) hold.

**Proof of \( BC > 0 \) and \( GC > 0 \)**

- **BC > 0:**
I define \( L_A(\xi) \) by
\[
L_A(\xi) := \frac{\alpha_A}{\sqrt{\beta}} \xi - \frac{\alpha_A}{\sqrt{\beta}} y_A - \frac{\sqrt{\alpha_A + \beta}}{\sqrt{\beta}} \Phi^{-1}\left( \frac{t}{D} \right).
\]
Similarly, I can define \( M_A(\xi) \). From equation (B5), \( \theta_{AB}^* \) is the unique solution of
\[
\xi = \lambda \cdot \Phi(L_A(\xi)) + (1 - \lambda) \cdot \Phi(M_A(\xi)).
\]

I define \( L_B(\xi) \) by
\[
L_B(\xi) := \frac{\alpha_B}{\sqrt{\beta}} \xi - \frac{\alpha_B}{\sqrt{\beta}} y_B - \frac{\sqrt{\alpha_B + \beta}}{\sqrt{\beta}} \Phi^{-1}\left( \frac{t}{D} \right).
\]
From equation (B7), \( \theta_{BB}^* \) is the unique solution of \( \xi = \Phi(L_B(\xi)) \).

Assume \( y_A = y_B \) and \( \alpha_A = \alpha_B \). Then, \( L_A(\xi) = L_B(\xi) \). Since \( \Phi(L_B(\xi)) \) is increasing and \( \max \frac{d}{dx} \Phi(L_B(\xi)) < 1 \), I have the following property:
(a) If \( \xi < \theta_{BB}^* \), then \( \xi < \Phi(L_B(\xi)) \);
(b) If \( \xi > \theta_{BB}^* \), then \( \xi > \Phi(L_B(\xi)) \);
(c) If \( \xi = \theta_{BB}^* \), then \( \xi = \Phi(L_B(\xi)) \).

Since \( \theta_{AB}^* = \lambda \cdot \Phi(L_A(\theta_{AB}^*)) + (1 - \lambda) \cdot \Phi(M_A(\theta_{AB}^*)) < \Phi(L_A(\theta_{AB}^*)), \theta_{AB}^* < \theta_{BB}^* \) holds (i.e., \( BC > 0 \)).

**GC > 0:**
Since \( \theta_{AB}^* > \theta_{AC}^* \), in equation (B3), there exists some \( \varepsilon > 0 \) such that
\[
\frac{t}{D} = q \Phi \left[ \sqrt{\alpha_A + \beta} \left( \theta_{AB}^* - \frac{\alpha_A}{\alpha_A + \beta} y_A - \frac{\beta}{\alpha_A + \beta} x_{A2}^* \right) \right] + (1 - q) \Phi \left[ \sqrt{\alpha_B + \beta} \left( \theta_{AC}^* - \frac{\alpha_A}{\alpha_A + \beta} y_A - \frac{\beta}{\alpha_A + \beta} x_{A2}^* \right) \right]
\]
\[
= q \varepsilon + q \Phi \left[ \sqrt{\alpha_A + \beta} \left( \theta_{AC}^* - \frac{\alpha_A}{\alpha_A + \beta} y_A - \frac{\beta}{\alpha_A + \beta} x_{A2}^* \right) \right] + (1 - q) \Phi \left[ \sqrt{\alpha_B + \beta} \left( \theta_{AC}^* - \frac{\alpha_A}{\alpha_A + \beta} y_A - \frac{\beta}{\alpha_A + \beta} x_{A2}^* \right) \right]
\]
\[
= q \varepsilon + \Phi \left[ \sqrt{\alpha_A + \beta} \left( \theta_{AC}^* - \frac{\alpha_A}{\alpha_A + \beta} y_A - \frac{\beta}{\alpha_A + \beta} x_{A2}^* \right) \right] .
\]
From this, I obtain:
\[
x_{A2}^* = \frac{\alpha_A + \beta}{\beta} \theta_{AC}^* - \frac{\alpha_A}{\beta} y_A - \frac{\sqrt{\alpha_A + \beta}}{\beta} \Phi^{-1}\left( \frac{t}{D} - q \varepsilon \right) .
\]
From equation (B6), I obtain:
\[
\theta_{AC}^* = \lambda \Phi \left[ \frac{\alpha_A}{\sqrt{\beta}} \left( \theta_{AC}^* - y_A - \frac{\sqrt{\alpha_A + \beta}}{\alpha_A} \Phi^{-1}\left( \frac{t + \delta}{D} \right) \right) \right] + (1 - \lambda) \Phi \left[ \frac{\alpha_A}{\sqrt{\beta}} \left( \theta_{AC}^* - y_A - \frac{\sqrt{\alpha_A + \beta}}{\alpha_A} \Phi^{-1}\left( \frac{t}{D} - q \varepsilon \right) \right) \right] .
\]
Note that $\theta_{BC}^*$ was obtained as follows:

$$
\theta_{BC}^* = \lambda \Phi \left[ \frac{\alpha_B}{\sqrt{\beta}} \left( \theta_{BC}^* - y_B - \frac{\sqrt{\alpha_B + \beta}}{\alpha_B} \Phi^{-1} \left( \frac{t + \delta}{D} \right) \right) \right] \\
+ (1 - \lambda) \Phi \left[ \frac{\alpha_B}{\sqrt{\beta}} \left( \theta_{BC}^* - y_B - \frac{\sqrt{\alpha_B + \beta}}{\alpha_B} \Phi^{-1} \left( \frac{t}{D} \right) \right) \right].
$$

Let

$$
F_1 (\xi) := \lambda \Phi \left[ \frac{\alpha_A}{\sqrt{\beta}} \left( \xi - y_A - \frac{\sqrt{\alpha_A + \beta}}{\alpha_A} \Phi^{-1} \left( \frac{t + \delta}{D} \right) \right) \right] \\
+ (1 - \lambda) \Phi \left[ \frac{\alpha_A}{\sqrt{\beta}} \left( \xi - y_A - \frac{\sqrt{\alpha_A + \beta}}{\alpha_A} \Phi^{-1} \left( \frac{t}{D} - q \xi \right) \right) \right],
$$

$$
F_2 (\xi) := \lambda \Phi \left[ \frac{\alpha_B}{\sqrt{\beta}} \left( \xi - y_B - \frac{\sqrt{\alpha_B + \beta}}{\alpha_B} \Phi^{-1} \left( \frac{t + \delta}{D} \right) \right) \right] \\
+ (1 - \lambda) \Phi \left[ \frac{\alpha_B}{\sqrt{\beta}} \left( \xi - y_B - \frac{\sqrt{\alpha_B + \beta}}{\alpha_B} \Phi^{-1} \left( \frac{t}{D} \right) \right) \right].
$$

Then, $\theta_{AC}^*$ is the unique solution of $\xi = F_1 (\xi)$; and $\theta_{BC}^*$ is the unique solution of $\xi = F_2 (\xi)$.

Assume $y_A = y_B$ and $\alpha_A = \alpha_B$. Then, $F_1 (\xi) > F_2 (\xi)$ holds since $q > 0$. Under the uniqueness conditions,

$$
\left| \frac{d}{d\xi} F_1 (\xi) \right| < 1 \quad \text{and} \quad \left| \frac{d}{d\xi} F_2 (\xi) \right| < 1.
$$

Hence, $\theta_{BC}^* < \theta_{AC}^*$ holds (i.e., $GC > 0$).

**Proof of Proposition 10**

From equation (S10), I obtain:

$$
\frac{\partial \theta_{BC}^*}{\partial y_B} = \lambda \phi (\cdot) \left( \frac{\alpha_B}{\sqrt{\beta}} \frac{\partial \theta_{BC}^*}{\partial y_B} - \frac{\alpha_B}{\sqrt{\beta}} \right) + (1 - \lambda) \phi (\cdot) \left( \frac{\alpha_B}{\sqrt{\beta}} \frac{\partial \theta_{BC}^*}{\partial y_B} - \frac{\alpha_B}{\sqrt{\beta}} \right) \\
= \left( 1 - \lambda \phi (\cdot) \frac{\alpha_B}{\sqrt{\beta}} - (1 - \lambda) \phi (\cdot) \frac{\alpha_B}{\sqrt{\beta}} \right)^{-1} \left( -\lambda \frac{\alpha_B}{\sqrt{\beta}} \phi (\cdot) - (1 - \lambda) \frac{\alpha_B}{\sqrt{\beta}} \phi (\cdot) \right) < 0
$$

and thus $\frac{\partial GC}{\partial y_B} > 0$. That is, $GC$ increases with $y_B$.

From equation (S7), I obtain:

$$
\frac{\partial \theta_{BB}^*}{\partial y_B} = \phi (\cdot) \left( \frac{\alpha_B}{\sqrt{\beta}} \frac{\partial \theta_{BB}^*}{\partial y_B} - \frac{\alpha_B}{\sqrt{\beta}} \right) \\
= \left( 1 - \frac{\alpha_B}{\sqrt{\beta}} \phi (\cdot) \right)^{-1} \phi (\cdot) \left( -\frac{\alpha_B}{\sqrt{\beta}} \right) < 0
$$

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and thus $\frac{\partial BC}{\partial y_B} < 0$. That is, BC decreases with $y_B$.

**Proof of Proposition 11**

From equation (S10), I obtain:

$$
\frac{\partial \theta^*_B}{\partial \alpha_B} = \lambda \phi(\cdot) \left\{ \frac{1}{\sqrt{\beta}} \theta^*_B + \frac{\alpha_B}{\sqrt{\beta}} \frac{\partial \theta^*_B}{\partial \alpha_B} - \frac{y_B}{\sqrt{\beta}} - \frac{1}{\sqrt{\beta}} \frac{1}{2\sqrt{\alpha_B} + \beta} \Phi^{-1}\left( \frac{t + \delta}{D} \right) \right\} \\
+ (1 - \lambda) \phi(\cdot) \left\{ \frac{1}{\sqrt{\beta}} \theta^*_B + \frac{\alpha_B}{\sqrt{\beta}} \frac{\partial \theta^*_B}{\partial \alpha_B} - \frac{y_B}{\sqrt{\beta}} - \frac{1}{\sqrt{\beta}} \frac{1}{2\sqrt{\alpha_B} + \beta} \Phi^{-1}\left( \frac{t}{D} \right) \right\} \\
= \left( 1 - \lambda \phi(\cdot) \frac{\alpha_B}{\sqrt{\beta}} - (1 - \lambda) \phi(\cdot) \frac{\alpha_B}{\sqrt{\beta}} \right)^{-1} \left\{ \frac{1}{\sqrt{\beta}} \right\} \\
\times \left\{ \lambda \phi(\cdot) \left( \theta^*_B - y_B - \frac{1}{2\sqrt{\alpha_B} + \beta} \Phi^{-1}\left( \frac{t + \delta}{D} \right) \right) \\
+ (1 - \lambda) \phi(\cdot) \left( \theta^*_B - y_B - \frac{1}{2\sqrt{\alpha_B} + \beta} \Phi^{-1}\left( \frac{t}{D} \right) \right) \right\}.
$$

If $\theta^*_B > y_B + \frac{1}{2\sqrt{\alpha_B} + \beta} \Phi^{-1}\left( \frac{t + \delta}{D} \right)$, then $\frac{\partial \theta^*_B}{\partial \alpha_B} > 0$ and thus $\frac{\partial GC}{\partial \alpha_B} < 0$. That is, if the state of country B’s fundamentals is ex-ante expected to be bad, then GC decreases with $\alpha_B$. Meanwhile, if $\theta^*_B < y_B + \frac{1}{2\sqrt{\alpha_B} + \beta} \Phi^{-1}\left( \frac{t}{D} \right)$, then $\frac{\partial \theta^*_B}{\partial \alpha_B} < 0$ and thus $\frac{\partial GC}{\partial \alpha_B} > 0$. That is, if the state of country B’s fundamentals is ex-ante expected to be good, then GC increases with $\alpha_B$.

From equation (S7), I obtain:

$$
\frac{\partial \theta^*_B}{\partial \alpha_B} = \phi(\cdot) \left( \frac{1}{\sqrt{\beta}} \theta^*_B + \frac{\alpha_B}{\sqrt{\beta}} \frac{\partial \theta^*_B}{\partial \alpha_B} - \frac{y_B}{\sqrt{\beta}} - \frac{1}{\sqrt{\beta}} \frac{1}{2\sqrt{\alpha_B} + \beta} \Phi^{-1}\left( \frac{t}{D} \right) \right) \\
= \left( 1 - \frac{\alpha_B}{\sqrt{\beta}} \phi(\cdot) \right)^{-1} \phi(\cdot) \left( \frac{1}{\sqrt{\beta}} \left( \theta^*_B - y_B - \frac{1}{2\sqrt{\alpha_B} + \beta} \Phi^{-1}\left( \frac{t}{D} \right) \right) \right).
$$

If $\theta^*_B > y_B + \frac{1}{2\sqrt{\alpha_B} + \beta} \Phi^{-1}\left( \frac{t}{D} \right)$, then $\frac{\partial \theta^*_B}{\partial \alpha_B} > 0$ and thus $\frac{\partial BC}{\partial \alpha_B} > 0$. That is, if the state of country B’s fundamentals is ex-ante expected to be bad, then BC increases with $\alpha_B$. Meanwhile, if $\theta^*_B < y_B + \frac{1}{2\sqrt{\alpha_B} + \beta} \Phi^{-1}\left( \frac{t}{D} \right)$, then $\frac{\partial \theta^*_B}{\partial \alpha_B} < 0$ and thus $\frac{\partial BC}{\partial \alpha_B} < 0$. That is, if the state of country B’s fundamentals is ex-ante expected to be good, then BC decreases with $\alpha_B$.  

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References


3.1 Introduction

As Jensen and Meckling (1976) have accurately pointed out, a manager does not necessarily maximize his shareholders’ utility. The conflict of interests existing between shareholders and their manager is the result of the different objective functions of both sides, and the informational asymmetry between the manager and the shareholders allows the manager to pursue his own interests at the expense of his shareholders. Accordingly, the traditional solution to the agency problem involves strengthening the corporate governance of the firm, where shareholders monitor their manager and attempt to minimize the private benefit of control enjoyed by the manager. However, in reality, we still observe that managers continue to enjoy their private benefits of control in diverse internal corporate governances across firms and across countries. For example, we see that managers or controlling shareholders in the firms of emerging economies tend to enjoy greater governance power over their firms than would be allowed by their stock ownerships, relative to those in firms of developed economies (La Porta, Lopez-de-Silanes, and Shleifer (1999), Claessens, Djankov, and Lang (2000), and Lemmons and Lins (2003) among others).

This paper explains why we continue to observe the private benefits of control enjoyed by managers and why firms permit the different levels of internal governance that inevitably result in different levels of managerial perks. We surmise that shareholders’ decisions regarding the corporate governance of their firms might be associated with competition in the product market, because shareholders’ utility depends on different product market situations. We demonstrate herein that shareholders allow their firm’s manager to enjoy the private benefit of control in
order to maximize their utility, given the product market competition and its profitability. To maximize their utility in the firm’s competition in its product market, shareholders employ the corporate governance structure – the managerial incentive scheme – including the level of their manager’s governance / controlling power over the firm and the managerial ownership in stocks as a commitment device according to the prevailing market situation. Given this commitment, the manager enjoys his allowed private benefit of control with the shareholders’ consent, which affects his decisions as to the firm’s optimal production level in the product market competition.

Our paper is unique in that it models the positive side of the conflict of interests between shareholders and their firm’s manager as a commitment device, whereas existing papers have focused generally on the negative side of the private benefit of control as a value-reducing factor in corporate management (Walkling and Long (1984), Jensen (1986), and Ang, Cole, and Lin (2000) among others). In our paper, the private benefit of control functions as a component of managerial compensation, whereas the existing view of corporate governance purports to restrict the pursuit of the private benefit by the manager at the expense of his shareholders’ utility. We even claim that the stock ownership by the manager may work against the interests of other shareholders when we consider its negative effects on the firm’s competition in the product market.49

There are several extant papers that address the corporate structure as a commitment device. Brander and Lewis (1986) and Maksimovic (1988) stress that firms employ the financial structure as a commitment device to influence the product market equilibrium. Fershtman and Judd (1987) assert that a manager has an incentive to overinvest or overproduce when he maximizes the combination of the firm’s profits and sales in an oligopoly. Gertner, Gibbons, and Scharfstein

49 Existing papers, including Jensen and Meckling (1976) and Leland and Pyle (1977), assert that the relationship between the firm’s value and the ownership by the manager would be positive, as the managerial ownership functions as an incentive alignment device with other shareholders.
(1988) employ the financial structure as a signaling device for a firm’s cost function.\textsuperscript{50} Bolton and Sharfstein (1990) also rely on the strategic aspect of the conflict of interests between the manager and the investors, where the latter continue to provide new capital despite the moral hazard of the manager, only to attenuate the aggressive strategy of a competing firm. However, these papers do not consider the corporate governance structure to be a commitment device, which is the central issue of our paper.

Our paper contributes to the extant literature by considering the relationship between the corporate governance structure and the product market condition. An analysis of the linkage existing between corporate governance and production decisions is a particularly interesting issue, as corporate governance has been identified as a major determining factor in the value of a firm (Morck, Shleifer, and Vishny (1988), La Porta, Lopez-de-Silanes, Shleifer, and Vishny (2002), and Gompers, Ishii, and Metrick (2003) among others), and the production decision is a critical channel through which a firm’s corporate governance influences a firm’s performance in the product market.

Our paper is also related with the extant literature regarding the discrepancy between the control right and the security right of a manager (La Porta, Lopez-de-Silanes, Shleifer, and Vishny (2002), Mitton (2002), and Lemmon and Lins (2003) among others). The corporate governance structure we address in this paper includes the managerial stock ownership and the level of the manager’s controlling power over the firm. We can understand the former as being representative of his security right, and the latter as reflecting his control right. Existing papers addressing this issue have generally held that the discrepancy between the control right and the security right of

\textsuperscript{50} Likewise, Bagnoli and Watts (2009) assess the effects of the earnings management of a firm on product market competition with the other firm and how rivalry impacts both production decisions and disclosure (earnings management) decisions in an incomplete information Cournot duopoly model.
a manager is the source of the conflict of interests between the manager and his shareholders, and thus it hurts both the firm’s value and shareholders’ utility. However, our paper asserts that shareholders may allow and benefit from the discrepancy, and the manager enjoys more control power than his stock ownership would actually allows.

The theoretical model of our paper shows that the optimal level of the managerial stock ownership would be lower and the level of his controlling power over the firm would be higher if the product is more profitable, or if the competition in the product market is more severe. This is straightforward and intuitive. As the product is more profitable, it is better for the firm to increase its market share, and thus its production level, which is induced by the higher control right and the lower share ownership of the manager. If the product market competition is more severe, then the firm will lose less in terms of its product price, even though it increases its supply. Therefore, it would be better for the firm to exhort its manager to be more aggressive in his production decisions.

The remainder of this paper is organized as follows. In section 3.2, we establish a model wherein shareholders determine their firm’s corporate governance structure to influence their manager’s production decisions. In section 3.3, we solve the model and derive the propositions with empirical implications. Section 3.4 concludes. Proofs are provided in the Appendix.

3.2 Model

There are two competing firms in an industry: firm 1 and firm 2. We consider the following timeline. At $t = 0$, shareholders of firm $i$ ($i = 1, 2$) determine their firm’s corporate governance structure. In this paper, we focus on the managerial incentive scheme as the corporate governance structure of the firm. At $t = 1$, given his managerial incentive scheme, the manager determines the firm’s production level in the oligopoly, taking into consideration the consumers’ demand and
its own marginal costs.

Firm $i$’s manager receives monetary compensation consisting of the fixed pay ($F_i$) and shares ($\alpha_i$), and also enjoys the private benefit of control as non-monetary compensation by retaining governance / controlling power ($\beta_i$) over the firm.\footnote{Generally, we can even interpret the first part of the managerial compensation as reflecting the cashflow right of the manager and the second part as reflecting the control right of the manager, following the tradition of terminology described by La Porta, Lopez-de-Silanes, Shleifer, and Vishny (2002). The more profound the difference between these two components, the poorer the firm’s corporate governance appears to be.} The manager’s private benefit includes the utilities generated from all sorts of benefits by controlling the firm’s resources. In the model, we employ the investment size multiplied by the controlling power of the manager as a proxy for the level of his private benefit of control. This simplifying assumption follows the practical wisdom that the manager’s private benefit is related positively with the size of the firm: the larger the firm is, the larger are the corporate and human resources controlled by the manager, which tends to increase the manager’s private benefit. Additionally, the manager can enjoy a high degree of private benefit of control given the investment size, if he can exercise more controlling power over the firm without checks and balances by other monitors, such as the shareholders or the board of directors. Here, the manager’s governance / controlling power ($\beta_i$) represents the degree of discretion held by the manager. In summary, the manager’s private benefit is denoted by multiplying the firm’s investment size ($I_i$) – which is determined by the multiplication of the production level ($s_i$) and its cost ($c_i$) – and his controlling power ($\beta_i$).

We assume a linear demand structure between two firms in the industry, and hence the price of the product is:

$$p = D - \eta(s_i + s_{-i}),$$  \hspace{1cm} (3.1)

where a demand intercept $D$ can be interpreted as the level of market demand and the slope $\eta$
(> 0) can be interpreted as a proxy for the supply elasticity of price (i.e., \( \frac{\Delta \ln p}{\Delta \ln s} \)) in the industry.\(^{52}\)

We can then derive the gross profit of firm \( i \) by

\[
\Pi_i = (p - c) \times s_i = (D - \eta(s_i + s_{-i}) - c) \times s_i, \tag{3.2}
\]

where \( c \) is the per unit cost for the production level \( s_i \).

Now we set the following objective function of firm \( i \)'s manager:

\[
W_i = \alpha_i \{\Pi_i - F_i - k_i \beta_i I_i\} + F_i + \beta_i I_i, \tag{3.3}
\]

where

\[
F_i = \text{fixed pay for firm } i \text{'s manager},
\]

\[
\alpha_i = \text{manager's stock ownership } (0 < \alpha_i < 1),
\]

\[
\beta_i = \text{manager's control right / power over the firm } i \ (\beta_i \geq 0),
\]

\[
\Pi_i = \text{firm } i \text{'s gross profit before the fixed compensation to the manager}
\]
\[\text{and the loss due to the private benefit of control},\]

\[
I_i = \text{firm } i \text{'s investment size (i.e., } I_i = c_i \times s_i),\]

\[
k_i = \text{the degree that the manager’s private benefit decreases firm } i \text{'s profit.}
\]
\[\text{(i.e., wealth transfer from shareholders to the manager) } (k_i > 1)\]

Firm \( i \)'s manager determines the firm’s production level \( s_i \) to maximize his objective function, \( W_i \). The manager’s objective function shows the typical conflict of interests between shareholders and their manager. While the shareholders maximize the firm’s profit, the manager maximizes his utility, which consists of both the firm’s profit and the firm’s size. Here, we can interpret \( \alpha_i \) as the degree that aligns the manager’s preference with those of shareholders and \( \beta_i \) as the degree to which he pursues his own benefits. \( I_i \) is the firm’s investment size, which is a monetary proxy for the firm’s size. This is the required level of investment to produce \( s_i \), and we assume that the manager’s private benefit increases along with it. The manager’s private benefit negatively

---

\(^{52}\) As Fershtman and Judd (1987) have demonstrated, \( \eta \) denotes the effect of firms’ production level on the price of the product.
influences the firm’s gross profit, and thus we subtract $k_i \beta_i I_i$ from the firm’s gross profit where $k_i$ represents the degree to which the manager’s private benefit reduces the firm’s profit.\(^5\) $k_i$ would vary depending on the types of wealth transfer from shareholders to the manager. For example, $k_i$ would be lower in cases in which the manager misappropriates cash for his direct consumption, since even a minimal amount of cash can increase his utility. On the other hand, $k_i$ would be higher in cases in which the manager uses cash for an investment in a negative NPV project where he enjoys far less private benefits than the corporate resources consumed. In this case, huge quantities of corporate resources will be wasted away to marginally increase the manager’s utility.

Next, we establish the objective function of firm $i$’s shareholders, as follows:

$$\Psi_i = (1 - \alpha_i) \{\Pi_i - k_i \beta_i I_i - F_i\} - \frac{r_i}{2} (\bar{M}_i - \beta_i)^2. \tag{3.4}$$

Shareholders of firm $i$ determine their manager’s stock ownership ($\alpha_i$ where $0 < \alpha_i < 1$), control right ($\beta_i$ where $0 \leq \beta_i \leq \bar{M}_i$), and fixed pay ($F_i$) subject to the minimum compensation level, or the reservation utility ($R$) of the manager’s objective function.\(^5\) In the shareholders’ objective function $\Psi_i$, $\frac{r_i}{2} (\bar{M}_i - \beta_i)^2$ captures the monitoring cost of monitoring the manager. $r_i$ is the per unit monitoring cost and $\bar{M}_i$ is a constant that can be interpreted as the maximum $\beta_i$. That is, if $\beta_i = 0$, then the manager has no private benefit, but the shareholders pay a very high monitoring cost of $\frac{r_i}{2} \bar{M}_i^2$. In the other extreme case, shareholders could establish $\beta_i = \bar{M}_i$ and allow their manager the maximum private benefit. We can observe that a trade-off exists between the monitoring cost and the size of the manager’s private benefit of control. We also establish the monitoring cost as a convex function, such that the marginal cost of monitoring increases with smaller $\beta_i$.

\(^{53}\) We assume $k_i > 1$. Otherwise (i.e., $0 < k_i \leq 1$), the manager’s private benefit pursuing acts would be socially desirable, rendering any analysis on the agency problem uninteresting.

\(^{54}\) That is, $\alpha_i (\Pi_i - k_i \beta_i I_i - F_i) + F_i + \beta_i I_i \geq R$. 

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In the following section, we analyze the manager’s decision problem \((s_i \text{ at } t = 1)\) and shareholders’ decision problem \((\alpha_i, \beta_i, \text{ and } F_i \text{ at } t = 0)\) recursively.

### 3.3 Solving the Model and Its Empirical Implications

#### 3.3.1 Manager’s Decision Problem

##### 3.3.1.1 Optimal Production Level

At \(t = 1\), firm \(i\)’s manager determines the production level \((s_i)\) given \(\alpha_i, \beta_i, D, \text{ and } c_i\), to maximize his objective function, \(W_i\).

Firm \(i\)’s manager’s first-order optimal condition is as follows:

\[
FOC_i = \frac{\partial W_i}{\partial s_i} = G_i(s_i|s_{-i})
\]

\[
= \alpha_i \left(D - 2\eta s_i - \eta s_{-i} - c_i - s_i \frac{\partial c_i}{\partial s_i}\right) + (1 - k_i \alpha_i) \beta_i \frac{\partial I_i}{\partial s_i} = 0. \tag{3.5}
\]

For the existence of the unique solution, the second-order condition must be satisfied:

\[
SOC_i = \frac{\partial^2 W_i}{\partial s_i^2}
\]

\[
= -\alpha_i \left(2\eta + 2 \frac{\partial c_i}{\partial s_i} + s_i \frac{\partial^2 c_i}{\partial s_i^2}\right) + (1 - k_i \alpha_i) \beta_i \frac{\partial^2 I_i}{\partial s_i^2} < 0. \tag{3.6}
\]

We know that the second-order condition holds if the production function evidences constant returns to scale, for example.

We can solve the optimal production level \(s_i^* (i = 1, 2)\) from equations (3.5) and (3.6).\(^55\) For simplicity’s sake, we assume the constant returns to scale (c.r.t.s: i.e., \(\frac{\partial c_i}{\partial s_i} = 0, \frac{\partial^2 c_i}{\partial s_i^2} = 0, \frac{\partial I_i}{\partial s_i} = c_i, \frac{\partial^2 I_i}{\partial s_i^2} = 0 \text{ for } i = 1, 2\)) and the homogeneity between two firms (i.e., \(\alpha_1 = \alpha_2 = \alpha, \beta_1 = \beta_2 = \beta, k_1 = k_2 = k, \text{ and } c_1 = c_2 = c\)). Subsequently, we obtain the following equilibrium production

\(^{55}\) We deal with the condition for the product market equilibrium in the Appendix.
level and price of the product:

\begin{align*}
    s^* &= \frac{(1 - k\alpha)\beta c + \alpha(D - c)}{3\alpha \eta}, \\
    p^* &= D - 2\eta s^* = D - \frac{2((1 - k\alpha)\beta c + \alpha(D - c))}{3\alpha}.
\end{align*}

(3.7) \quad (3.8)

3.3.1.2 Effects of the Corporate Governance Structure on the Production Level

Now, we show the effects of \( \alpha, \beta, \) and \( k \) on the equilibrium production level \((s^*)\).\(^{56}\)

**Proposition 15** Under the assumptions of c.r.t.s and homogeneous firms, the equilibrium production level \((s^*)\) is decreasing in the level of the manager’s stock ownership \((\alpha)\); increasing in the level of his control right \((\beta)\); and decreasing in the inefficiency of the wealth transfer \((k)\).

The results are derived from the incentive of the manager who prefers to increase the production level to increase his private benefit of control, but only to the extent it does not overly harm his compensation from his stock ownership. The results also indicate that shareholders can influence their manager’s decisions regarding the production level by adjusting the managerial stock ownership and/or the manager’s control right.

The empirical implication of this proposition would be that we would perceive an overproduction problem when the industry is dominated by companies with managers who hold low share ownerships, but exercise monopolistic control power over their firms under dispersed ownership structures. We would also expect to observe a higher level of product supply and a lower price in industries wherein the wealth transfer from the shareholders to the manager is more in the form of the direct misappropriation of corporate cash for the manager’s private use, as such a misappropriation tends to consume fewer corporate resources.\(^{57}\)

Next, we can determine the opposite effects of \( \alpha, \beta, \) and \( k \) on the equilibrium price \((p^*)\).

\(^{56}\) We deal with the individual firm’s case in the Appendix.

\(^{57}\) Similarly, in industries where corporate resources are in the form of liquid assets such that they can be readily stolen away for the benefits of managers, we would expect to see higher production levels.
Corollary 16  The equilibrium price of the product \( (p^*) \) is increasing in the level of the manager’s stock ownership \( (\alpha) \); decreasing in the level of his control right \( (\beta) \); and increasing in the inefficiency of the wealth transfer \( (k) \).

Overall, Proposition 15 and Corollary 16 demonstrate the effects of the manager’s stock ownership \( (\alpha) \) and control right \( (\beta) \) on the production level \( (s^*) \) and the price of the product \( (p^*) \), given the competitive market structure in the product market. If \( \alpha \) is higher, then the manager places greater importance on the firm’s profits and will reduce the production level such that the overinvestment problem is mitigated. Meanwhile, if \( \beta \) is higher, then the manager will increase the production level because he places greater importance on his private benefit than on the firm’s profits. The inefficiency of the wealth transfer \( (k) \) from the shareholders to the manager also affects the manager’s decisions since higher \( k \) values impose higher costs on the manager as a shareholder and encourage him to reduce the production level. Of course, reductions in the production level also generally increase the equilibrium price.

3.3.2  Shareholders’ Decision Problem

3.3.2.1  Optimal Corporate Governance Structure

At \( t = 0 \), firm \( i \)'s shareholders determine the corporate governance structure \( (\alpha_i \) and \( \beta_i) \) considering their manager’s decision in the product market at \( t = 1 \) to maximize their objective function, \( \Psi_i \). Under the assumptions of c.r.t.s and homogeneity between the two firms, we solve the first-order conditions of the shareholders’ objective function, thus obtaining the following optimal \( \alpha^* \) and \( \beta^*\):\(^{58}\)

\[
\alpha^* = c \frac{4M\eta r - (k - 1)(D - c)}{\eta r [cM(k + 3) - (D - c)] - (k - 1)c^2(D - c)},
\]

\[
\beta^* = \frac{4M\eta r + (k - 1)c(D - c)}{4\eta r - (k - 1)^2c^2},
\]

\(^{58}\) The derivation is in the Appendix.
3.3.2.2 Effects of the Corporate Environment on the Corporate Governance Structure

Now, we show the effects of the competitive structure of the product market – such as the profitability of the product, the supply elasticity of price, and the degree to which the manager’s private benefit reduces the firm’s profits – on the governance structure (\(\alpha^*\) and \(\beta^*\)) of the firm in the equilibrium.

**Proposition 17** The manager’s stock ownership (\(\alpha^*\)) decreases and his control right (\(\beta^*\)) increases with the profitability of the product (i.e., \(\frac{\partial \alpha^*}{\partial D} < 0, \frac{\partial \beta^*}{\partial D} > 0, \frac{\partial \alpha^*}{\partial c} > 0, \text{ and } \frac{\partial \beta^*}{\partial c} < 0\)).

When a product generates a higher margin (i.e., \(D\) is higher and/or \(c\) is lower), it would be natural for the firm to increase its production level to the point at which the marginal revenue from the increase in the quantity to be sold equals the marginal cost from the lower price owing to the increased supply. Considering the conflict of interests deriving from the manager’s private benefit, a firm can credibly increase its production level, since the production decision is made by the manager, who maximizes his payoff including his benefit of control.

The results of this proposition are intuitive, since the shareholders can enjoy higher profit with a more aggressive manager who increases the production level given the higher control right and the lower stock ownership when the product is profitable in the market. The cost deriving from increases in the manager’s private benefit of control is more than offset by the benefit of higher production level and market share as the overall profit level increases. On the other hand, in cases in which the product is less profitable, lowering the level of supply would provide the firm with benefits, which are induced by the higher stock ownership and the manager’s lower control rights.

The theoretical results can explain why we continue to observe the high level of discrepancy between the control right and the stock ownership of managers in modern companies. As the
discrepancy increases, the production level of the firm rises in a given product market. Of course, such an ownership or governance structure would generate positive results only when the product market offers a profitable business, and the more profitable it is, the higher would be the discrepancy between the control right and the stock ownership.

Empirically, we would expect to observe the higher discrepancy in a profitable industry economy. We can anticipate that the discrepancy would be higher in an emerging economy in which profitable business opportunities abound than in an advanced economy. Such a conjecture has been confirmed by Claessens, Djankov, and Lang (2000) and Lemmon and Lins (2003), who demonstrate that firms in emerging economies, such as East Asian countries, tend to evidence higher levels of managerial control rights versus managerial security rights (stock ownership) of managers. On the other hand, we would also anticipate that the discrepancy would diminish as the economy grows into a developed one, as is also confirmed by La Porta, Lopez-de-Silanes, and Shleifer (1999), who review the ownership structures of large corporations in 27 wealthy economies.

**Proposition 18** The manager’s stock ownership \((\alpha^*)\) increases in the supply elasticity of price \((\eta)\) (i.e., \(\frac{\partial \alpha^*}{\partial \eta} > 0\)), whereas the control right \((\beta^*)\) decreases in the supply elasticity of price \((\eta)\) (i.e., \(\frac{\partial \beta^*}{\partial \eta} < 0\)).

Higher supply elasticity of price means that firms can achieve a larger price increase with a small reduction in the production level. As we can interpret this higher supply elasticity as less profound competition in the product market, we anticipate higher managerial ownership and lower control rights (i.e., smaller discrepancy) in more oligopolistic product markets.\(^{59}\) In

\(^{59}\) Generally, the products of less competitive markets tend to be more profitable than the products of competitive markets thus, we may expect to see higher control rights and lower stock ownership (i.e., higher discrepancy) in less competitive markets. However, if we consider the price effect of the product in a less competitive market, we obtain the opposite result; Proposition 18 addresses this issue.
oligopolistic or duopolistic markets, wherein higher prices can be obtained by lowering the equilibrium supply of a good, it would prove beneficial for firms to commit to smaller production levels by incentivizing their managers with higher stock ownerships, and inducing them to be more concerned with the performance of their firms than with the consumption of their private control benefits.

Given larger $\alpha$ and/or smaller $\beta$, the manager will place greater importance on the firm’s profit than on the firm’s size, and thus will reduce the production level. We also note that the monitoring cost affects optimal corporate governance, since smaller $\beta$ incurs higher monitoring costs. This proposition predicts that we would tend to observe less profound discrepancies between control rights and stock ownerships in a regulated industry, as such industries provide a more oligopolistic product market structure. Another empirical prediction would be that firms that manufacture necessity goods rather than luxury goods would also manifest a less profound discrepancy between the control rights and stock ownerships of their managers. Chhaochharia, Grinstein, Grullon, and Michaely (2009) find that U.S. corporations in industries with less competitions – as measured with the industry concentration data from the Economic Census Bureau – enjoyed significantly large efficiency gains from more efficient investment and production decisions after managerial incentives became more closely aligned with those of shareholders after the 2002 introduction of the Sarbanes-Oxley Act.

**Proposition 19** Levels of the manager’s stock ownership ($\alpha^*$) and control right ($\beta^*$) are both increasing in the inefficiency of the wealth transfer ($k$) (i.e., $\frac{\partial \alpha^*}{\partial k} > 0$ and $\frac{\partial \beta^*}{\partial k} > 0$).

Higher $k$ means that a firm experiences substantial costs from a marginal increase in their manager’s private benefits of control. One such case would be an investment in a pet project with
a negative NPV, such that the manager consumes a substantial quantity of corporate resources to achieve a relatively minimal increase in his private utility. One way to minimize the opportunistic behavior of the manager in such an environment would be to increase his stock ownership. However, the increased stock ownership encourages him to be less aggressive in the product market, which is counter-balanced by the higher control right ($\beta$).^60

The theoretical result can explain such structural changes in the ownership structure of firms as privatizations (delistings) or MBOs (Management Buyouts), where the ownership is concentrated in the hands of a few managers when firms suffer from certain types of managerial inefficiencies. The result predicts that privatizations or MBOs would occur when the private consumptions of corporate resources by managers tend to hurt overly the firms’ values. Smith (1990) investigates changes in operating performance after 58 MBOs of U.S. public companies executed during 1977 - 1986, and observes a significant increase in operating profits following a shift in corporate ownership structure under MBOs. Similar to our theoretical result, Smith (1990) argues that an increase in the equity holdings of corporate officers directly increases their private costs of shirking or consuming perks, thus bolstering managerial incentives to increase the firm’s operating efficiency.

3.4 Concluding Remarks

In this paper, we set up a theoretical model to demonstrate that the corporate governance structure, as represented by the manager’s stock ownership and his control right, could be employed as a commitment device and could affect the production level of the firm. We show that the manager’s stock ownership and his control right are affected by the profitability of the product, the supply elasticity of price, and the degree to which the manager’s private benefit reduces the

^60 We check the sign of $\frac{\partial s^*}{\partial k}$ to get $\frac{\partial s^*}{\partial k} = -\frac{c}{4\eta} \beta - \frac{(k-1)c}{4\eta} \frac{\partial \beta}{\partial k} < 0$ so that the manager will not increase the production level even though his $\beta$ is large. It shows that the effect of $\alpha$ on $s^*$ is greater than that of $\beta$. 79
firm’s profits. The private benefit of control is interpreted as a component of the managerial compensation and is allowed by shareholders who intend to affect their manager’s production decision. Our study is unique in that it models the positive side of the conflict of interests existing between shareholders and their manager as a commitment device in the firm’s product market competition.

The theoretical model provides a variety of empirical implications regarding the relationship between the product market structure and the firm’s corporate governance. Shareholders want their manager to be more aggressive in the profitable product market, since a higher market share can potentially lead to higher profits for their firm. They also want their manager to be more aggressive in a very competitive market, where the price level is given as an exogenous variable and the larger supply by an individual firm would not affect the price of the product. In summary, we can surmise that we would tend to observe lower managerial stock ownerships and higher control rights in industries or countries with higher profitability or with higher levels of competition. However, such a reactive optimization on the part of shareholders may alter the industry structure and the profitability of the product over a longer horizon, and may make the empirical testing of the model’s predictions quite difficult; this will be the subject of our future research.
Appendix

Condition for the Product Market Equilibrium

We obtain the total derivatives of (3.5) for each firm:

\[
\frac{\partial G_1(s_1)}{\partial s_1} ds_1 + \frac{\partial G_1(s_1)}{\partial s_2} ds_2 = 0, \tag{C1}
\]

\[
\frac{\partial G_2(s_2)}{\partial s_1} ds_1 + \frac{\partial G_2(s_2)}{\partial s_2} ds_2 = 0. \tag{C2}
\]

By rearranging equations (C1) and (C2), we obtain the following two equations:

\[
\frac{ds_2}{ds_1}(s_1) = \frac{-\alpha_1(2\eta + 2\frac{\partial c_1}{\partial s_1} + s_1\frac{\partial^2 c_1}{\partial s_1^2}) + (1 - k_1\alpha_1)\beta_1\frac{\partial^2 I_1}{\partial s_2^2}}{\eta\alpha_1} < 0, \tag{C3}
\]

\[
\frac{ds_2}{ds_1}(s_2) = \frac{-\alpha_2(2\eta + 2\frac{\partial c_2}{\partial s_2} + s_2\frac{\partial^2 c_2}{\partial s_2^2}) + (1 - k_2\alpha_2)\beta_2\frac{\partial^2 I_2}{\partial s_2^2}}{\eta\alpha_2} < 0. \tag{C4}
\]

Equations (C3) and (C4) represent the slopes of each firm’s reaction function (i.e., \(G_1(s_1|s_2)\) and \(G_2(s_2|s_1)\)).

The equilibrium condition in a Cournot model is \(\frac{ds_2}{ds_1}(s_1) < \frac{ds_2}{ds_1}(s_2)\) and hence we obtain the following condition:

\[
\alpha_1\alpha_2(2\eta + 2\frac{\partial c_1}{\partial s_1} + s_1\frac{\partial^2 c_1}{\partial s_1^2})(2\eta + 2\frac{\partial c_2}{\partial s_2} + s_2\frac{\partial^2 c_2}{\partial s_2^2})
\]

\[
- (1 - k_1\alpha_1)\beta_1\alpha_2(2\eta + 2\frac{\partial c_2}{\partial s_2} + s_2\frac{\partial^2 c_2}{\partial s_2^2})\frac{\partial^2 I_1}{\partial s_1^2}
\]

\[
- (1 - k_2\alpha_2)\beta_2\alpha_1(2\eta + 2\frac{\partial c_1}{\partial s_1} + s_1\frac{\partial^2 c_1}{\partial s_1^2})\frac{\partial^2 I_2}{\partial s_1^2}
\]

\[
+ (1 - k_1\alpha_1)(1 - k_2\alpha_2)\beta_1\beta_2\frac{\partial^2 I_1}{\partial s_1^2}\frac{\partial^2 I_2}{\partial s_2^2} - \eta^2\alpha_1\alpha_2 > 0.
\]

Under the assumption of constant returns to scale (c.r.t.s: i.e., \(\frac{\partial c_i}{\partial s_i} = 0, \frac{\partial^2 c_i}{\partial s_i^2} = 0, \frac{\partial I_i}{\partial s_i} = c_i, \frac{\partial^2 I_i}{\partial s_i^2} = 0\) for \(i = 1, 2\)), this condition always holds since \(3\eta^2\alpha_1\alpha_2 > 0\). In other cases, different results can follow depending on the relative sizes of \(\alpha_i\) and \(\beta_i\). To determine a more substantive empirical implication, let us assume that the firms have the same production function and \(k_i\). In this case, the firms will have the same managerial incentive scheme of \((\alpha_i, \beta_i)\). Then, we can simplify the
above condition as follows:

\[
\left\{ \alpha_i(3\eta + 2\frac{\partial c_i}{\partial s_i} + s_i \frac{\partial^2 c_i}{\partial s_i^2}) - (1 - k_i\alpha_i)\beta_i \frac{\partial^2 I_i}{\partial s_i^2} \right\} \\
\times \left\{ \alpha_i(\eta + 2\frac{\partial c_i}{\partial s_i} + s_i \frac{\partial^2 c_i}{\partial s_i^2}) - (1 - k_i\alpha_i)\beta_i \frac{\partial^2 I_i}{\partial s_i^2} \right\} > 0. \quad \text{(C5)}
\]

The value of the first curly bracket is always positive because of the second-order condition.\(^{61}\)

Meanwhile, we find that the value of the second curly bracket is positive only when the manager’s stock ownership is higher than some threshold level. Specifically, we can assert that, so long as managers enjoy the private benefits of control given by the control rights \((\beta_i)\), the product market can achieve equilibrium only if their stock ownerships \((\alpha_i)\) are larger than the threshold value \(l_i\), where \(l_i\) is defined as

\[
l_i = \frac{\beta_i \frac{\partial^2 I_i}{\partial s_i^2} + (\eta + 2\frac{\partial c_i}{\partial s_i} + s_i \frac{\partial^2 c_i}{\partial s_i^2})}{1 + k_i \beta_i \frac{\partial^2 I_i}{\partial s_i^2} + (\eta + 2\frac{\partial c_i}{\partial s_i} + s_i \frac{\partial^2 c_i}{\partial s_i^2})}.
\]

The intuition underlying this argument relies primarily on the manager’s incentive to overproduce given the relatively higher control right vis-a-vis his stock ownership. When the marginal benefit of increasing the level of production to obtain the higher private benefit of control dominates the marginal cost of the lower product price, managers will continue to expand their production levels, and no equilibrium will be achieved in the product market.

**Derivation of \(s^*\)**

Under c.r.t.s, the managers’ first-order conditions are arranged below.\(^{62}\)

\[
\begin{align*}
\frac{\partial W_1}{\partial s_1} &= \alpha_1(D - 2\eta s_1 - \eta s_2 - c_1) + (1 - k_1\alpha_1)\beta_1 c_1 = 0, \\
\frac{\partial W_2}{\partial s_2} &= \alpha_2(D - 2\eta s_2 - \eta s_1 - c_2) + (1 - k_2\alpha_2)\beta_2 c_2 = 0.
\end{align*}
\]

\(^{61}\) See SOC; in equation (3.6).

\(^{62}\) We assume \(0 < k_i\alpha_i < 1\).
We obtain the optimal production level for each rm as follows:

\[ s_1^* = \frac{1}{3\alpha_1\alpha_2\eta} \left[ 2\alpha_2 \{(1 - k_1\alpha_1)\beta_1c_1 + \alpha_1(D - c_1)\} - \alpha_1 \{(1 - k_2\alpha_2)\beta_2c_2 + \alpha_2(D - c_2)\} \right], \]

\[ s_2^* = \frac{1}{3\alpha_1\alpha_2\eta} \left[ 2\alpha_1 \{(1 - k_2\alpha_2)\beta_2c_2 + \alpha_2(D - c_2)\} - \alpha_2 \{(1 - k_1\alpha_1)\beta_1c_1 + \alpha_1(D - c_1)\} \right]. \]

By the assumption of homogeneity between two firms, let \( \alpha_1 = \alpha_2 = \alpha, \beta_1 = \beta_2 = \beta, \) \( k_1 = k_2 = k, \) and \( c_1 = c_2 = c. \) Then, we obtain:

\[ s^* = \frac{(1 - k\alpha)\beta c + \alpha(D - c)}{3\alpha\eta}. \]

**Individual Firm’s Case**

Here, we show the effects of the manager’s stock ownership \((\alpha_i)\) and his control right \((\beta_i)\) on the production level \((s_i)\) of firm \(i.\)

**Proposition 20** *Ceteris paribus, firm 1(2)’s production level is decreasing (increasing) in the level of manager 1’s stock ownership \((\alpha_1)\) and increasing (decreasing) in the level of his control right \((\beta_1)\). Moreover, firm 1(2)’s production level is decreasing (increasing) in the degree that manager 1’s private benefit reduces firm 1’s profit \((k_1)\). As the supply elasticity of price \((\eta)\) increases, both firms’ production levels are reduced.*

**Proof.** By total derivatives of the first-order condition with regard to \(s_1, s_2,\) and \(\alpha_1,\) we obtain the following equations:

\[
\frac{\partial G_1}{\partial s_1} \frac{ds_1}{d\alpha_1} + \frac{\partial G_1}{\partial s_2} \frac{ds_2}{d\alpha_1} + \frac{\partial G_1}{\partial \alpha_1} = 0,
\]

\[
\frac{\partial G_2}{\partial s_1} \frac{ds_1}{d\alpha_1} + \frac{\partial G_2}{\partial s_2} \frac{ds_2}{d\alpha_1} + \frac{\partial G_2}{\partial \alpha_1} = 0.
\]

Since \( \frac{\partial G_1}{\partial s_1} (= SOC_1) < 0, \frac{\partial G_2}{\partial s_2} (= SOC_2) < 0, \frac{\partial G_1}{\partial \alpha_2} = -\eta \alpha_1 < 0, \frac{\partial G_2}{\partial \alpha_1} = -\eta \alpha_2 < 0, \) \( \frac{\partial G_1}{\partial \alpha_1} = D - 2\eta s_1 - \eta s_2 - c_1 - s_1 \frac{\partial c_1}{\partial s_1} - k_1\beta_1 \frac{\partial \beta_1}{\partial \alpha_1} < 0 \) and \( \frac{\partial G_2}{\partial \alpha_1} = 0,\) we have the following results:

\[
\frac{ds_1}{d\alpha_1} = -\left(\frac{SOC_2}{SOC_1}(SOC_2 - \eta^2 \alpha_1 \alpha_2) \right) < 0,
\]

\[ (C6) \]
\[
\frac{ds_2}{d\alpha_1} = -\eta \alpha_2 \left\{ D - 2\eta s_1 - \eta s_2 - c_1 - s_1 \frac{\partial f_1}{\partial s_1} - k_1 \beta_1 \frac{\partial f_1}{\partial s_1} \right\} \frac{1}{(SOC_1)(SOC_2) - \eta^2 \alpha_1 \alpha_2} > 0. \tag{C7}
\]

Likewise, we get the following results for \( \beta_1, k_1, \) and \( \eta \):

\[
\begin{align*}
\frac{ds_1}{d\beta_1} &= -\frac{(SOC_2)(1 - k_1 \alpha_1) \frac{\partial f_1}{\partial s_1}}{(SOC_1)(SOC_2) - \eta^2 \alpha_1 \alpha_2} > 0, \tag{C8} \\
\frac{ds_2}{d\beta_1} &= -\frac{-\eta \alpha_2 (1 - k_1 \alpha_1) \frac{\partial f_1}{\partial s_1}}{(SOC_1)(SOC_2) - \eta^2 \alpha_1 \alpha_2} < 0, \\
\frac{ds_1}{dk_1} &= \frac{\alpha_1 \beta_1 (SOC_2) \frac{\partial f_1}{\partial s_1}}{(SOC_1)(SOC_2) - \eta^2 \alpha_1 \alpha_2} < 0, \\
\frac{ds_2}{dk_1} &= \frac{-\eta \alpha_1 \alpha_2 \beta_1 \frac{\partial f_1}{\partial s_1}}{(SOC_1)(SOC_2) - \eta^2 \alpha_1 \alpha_2} > 0, \tag{C9} \\
\frac{ds_1}{d\eta} &= \frac{\alpha_1 \{(2s_1 + s_2)(SOC_2) + \eta \alpha_2 (s_1 + 2s_2)\}}{(SOC_1)(SOC_2) - \eta^2 \alpha_1 \alpha_2} \text{ c.r.t.s} = \frac{-3\eta \alpha_1 \alpha_2 s_1}{(SOC_1)(SOC_2) - \eta^2 \alpha_1 \alpha_2} < 0, \tag{C10} \\
\frac{ds_2}{d\eta} &= \frac{\alpha_2 \{(s_1 + 2s_2)(SOC_1) + \eta \alpha_1 (2s_1 + s_2)\}}{(SOC_1)(SOC_2) - \eta^2 \alpha_1 \alpha_2} \text{ c.r.t.s} = \frac{-3\eta \alpha_1 \alpha_2 s_2}{(SOC_1)(SOC_2) - \eta^2 \alpha_1 \alpha_2} < 0. \tag{C11}
\end{align*}
\]

Equations (C6) and (C8) show that, in general, if the control right \( (\beta_i) \) is relatively stronger than the security right (i.e., stock ownership, \( \alpha_i \)), then the manager will tend to prefer an aggressive strategy and increase the production level. Next, equation (C10) implies that as the degree to which the manager’s private benefit decreases firm’s profit \( (k_i) \) increases, the firm’s production level is reduced. Because the manager has the share ratio of \( \alpha_i \), to that extent he will bear the negative effects from \( k_i \). We can confirm, from (C10), that as \( \alpha_i \) becomes larger, the effect of \( k_i \) is extended. Finally, equations (C12) and (C13) demonstrate that as the supply elasticity of price \( (\eta) \) increases, the firm’s production level decreases. The sign of \( \frac{ds_1}{d\eta} \) depends on the size of \( s_1 \) and \( s_2 \).

However, if we assume c.r.t.s, then the desired result \( \frac{ds_1}{d\eta} < 0 \) is derived.
Proof of Proposition 15

\[
\frac{\partial s^*}{\partial \alpha} = -\beta c < 0, \quad \frac{\partial s^*}{\partial \beta} = \frac{(1 - k\alpha)c}{3\eta\alpha} > 0, \quad \frac{\partial s^*}{\partial k} = -\alpha\beta c < 0.
\]

Proof of Corollary 16

\[
\frac{\partial p^*}{\partial \alpha} = \frac{2\beta c}{3\alpha^2} > 0, \quad \frac{\partial p^*}{\partial \beta} = -\frac{2(1 - k\alpha)c}{3\alpha} < 0, \quad \frac{\partial p^*}{\partial k} = \frac{2\beta c}{3} > 0.
\]

Derivation of \(\alpha^*\) and \(\beta^*\)

Under the assumptions of c.r.t.s and homogeneity between two firms, a firm’s equilibrium profit (\(\Pi^*\)) is determined as follows:

\[
\Pi^* = (p^* - c)s^*
= \left( D - c - \frac{2\{(1 - k\alpha)\beta c + \alpha(D - c)\}}{3\alpha} \right) \left( \frac{(1 - k\alpha)\beta c + \alpha(D - c)}{3\alpha\eta} \right)
= \frac{(1 - k\alpha)\beta c + \alpha(D - c)}{9\eta\alpha^2} \left[ \alpha(D - c) - 2(1 - k\alpha)\beta c \right].
\]

Meanwhile, we know \(F = \frac{R - \beta cs^* - \alpha(\Pi^* - k\beta cs^*)}{1 - \alpha}\).\(^{63}\)

Then, shareholders’ objective function is:

\[
\Psi = (1 - \alpha)(\Pi^* - k\beta I - F) - \frac{r}{2}(\bar{M} - \beta)^2
= (1 - \alpha) \left\{ \Pi^* - k\beta cs^* - \frac{R - \beta cs^* - \alpha(\Pi^* - k\beta cs^*)}{1 - \alpha} \right\} - \frac{r}{2}(\bar{M} - \beta)^2
= \Pi^* + (1 - k)\beta cs^* - R - \frac{r}{2}(\bar{M} - \beta)^2.
\]

Now we solve the first-order conditions of \(\Psi\) with respect to \(\alpha\) and \(\beta\), and thus we obtain the

\(^{63}\) If the manager’s shares (\(\alpha\)) increase above some threshold level, his fixed pay (\(F\)) can have a negative value. In this case, we can interpret that the manager buys shares by giving money (\(F\)) to the firm.
optimal \( \alpha^* \) and \( \beta^* \):

\[
\alpha^* = c \frac{4 \bar{M} \eta r - (k - 1)(D - c)}{\eta r [cM + 3 - (D - c)] - (k - 1)c^2(D - c)},
\]

\[
\beta^* = \frac{4 \bar{M} \eta r + (k - 1)c(D - c)}{4 \eta r - (k - 1)^2c^2}.
\]

For the uniqueness of \( \alpha^* \) and \( \beta^* \), the following second-order condition needs to be satisfied:

\[
4 \eta r - (k - 1)^2c^2 > 0.
\]  \hfill (C14)

Since \( 0 < \alpha < 1 \) and \( \beta > 0 \), the following conditions are also obtained:

\[
\bar{M}(k - 1)c - (D - c) > 0, \quad (C15)
\]

\[
\eta r - \frac{(k - 1)c(D - c)}{\bar{M}(k - 1)c - (D - c)} > 0. \quad (C16)
\]

**Proof of Proposition 17**

\[
\frac{\partial \alpha^*}{\partial D} = c \eta r \bar{M} \left\{ (k - 1)^2c^2 - 4 \eta r \right\} / (\cdot)^2 < 0, \quad (C14)
\]

\[
\frac{\partial \beta^*}{\partial D} = \frac{(k - 1)c}{4 \eta - (k - 1)^2c^2} > 0, \quad (C14)
\]

Meanwhile,

\[
\frac{\partial \alpha^*}{\partial c} = r \eta(D - c) \left\{ \bar{M}c^2(k - 1)^2 - 2(D - c)c(k - 1) + 4 \bar{M} \eta r \right\} / (\cdot)^2, \quad (I)
\]

where

\[
I_{(C14), (C15)} > (D - c)c(k - 1) - 2(D - c)c(k - 1) + 4 \bar{M} \frac{(k - 1)^2c^2}{4}
\]

\[
> (C15) -(D - c)c(k - 1) + (D - c)c(k - 1) = 0
\]

and thus \( \frac{\partial \alpha^*}{\partial c} < 0. \)

\[
\frac{\partial \beta^*}{\partial c} = -(k - 1) \left\{ -c^2(D - c)(k - 1)^2 + 8 \bar{M} \eta rc(k - 1) - 4 \eta r(D - c) \right\} / (\cdot)^2
\]

\[
< (C15) -(k - 1)(D - c) \left\{ -(k - 1)^2c^2 + 4 \eta r \right\} / (\cdot)^2 < 0. \quad (C14)
\]
Proof of Proposition 18

\[
\frac{\partial \alpha^*}{\partial \eta} = (k - 1)c r (D - c) \{ (k - 1)c \bar{M} - (D - c) \} \left/ \left(\cdot \right)^2 \right. \geq 0,
\]

\[
\frac{\partial \beta^*}{\partial \eta} = -4(k - 1)c r \{ (D - c) + \bar{M}(k - 1)c \} \left/ \left(\cdot \right)^2 \right. < 0.
\]

Proof of Proposition 19

\[
\frac{\partial \alpha^*}{\partial k} = c^2 \eta r \{ (D - c)^2 + 4 \eta r \bar{M}^2 \} \left/ \left(\cdot \right)^2 \right. > 0,
\]

\[
\frac{\partial \beta^*}{\partial k} = \{ c^3(D - c)(k - 1)^2 + 8c^2 \bar{M} \eta r(k - 1) - 4 \eta r c(D - c) \} \left/ \left(\cdot \right)^2 \right. > \{ c^3(D - c)(k - 1)^2 + (4 + 4) \eta r c(D - c) - 4 \eta r c(D - c) \} \left/ \left(\cdot \right)^2 \right.
\]

\[
= c(D - c) \{ (k - 1)^2 c^2 + 4 \eta r \} \left/ \left(\cdot \right)^2 \right. > 0.
\]
References


Chapter 4 Competition in Korea’s Credit Rating Industry: Effect of Financial Restructuring after the 1997 Financial Crisis

4.1 Introduction

In 1997, a series of bankruptcies of the chaebols destabilized the financial system of South Korea and triggered a currency crisis. To recover from this unprecedented financial crisis, the Korean government launched critical reforms of corporate and financial systems. In the financial restructuring process, the Korean government took measures for increasing the level of competition in the credit rating industry. Fostering the competitive condition in the credit rating industry is very important because the credit rating system is directly connected to the soundness of the whole corporate system.\textsuperscript{64} Hence, the Korean government legislated the "Enforcement Rule of the Use and Protection of Credit Information Act," which lowered the entry level to the credit rating industry and allowed foreign companies to hold shares in credit rating firms in Korea. News media and all the parties related with credit rating firms have argued that the level of competition in the credit rating industry has increased after the implementation of this legislation. However, there have been no studies that have directly addressed the problem of estimating competitive conditions in the credit rating industry.\textsuperscript{65}

The theory of contestability (Baumol (1982) and Baumol, Panzar, and Willig (1982)) argues that the threat of entry alone can lead to competitive conduct independent of the number of firms actually acting in the market given the fact that market entry and exit are free. If the market

\textsuperscript{64} Paul Schott Stevens, the President of the Investment Company Institute, stated in his testimony for the U.S. Senate Committee on Banking, Housing, and Urban Affairs, "I firmly believe that robust competition for the credit rating industry is the best way to promote the continued integrity and reliability of their ratings" (see http://conferences.ici.org/policy/ici_testimony/05_house_nrsro_tmny).

\textsuperscript{65} Regarding the banking industry, Lee (2003) investigates the effect of the Korean government’s financial restructuring policy on the degree of banking market competition after the 1997 financial crisis.
is contestable, the threat of market entry with price cutting by potential competitors enforces marginal cost pricing by incumbents. In equilibrium, potential competitors will not earn excess profits and thus no entry is likely to occur. In Korea’s credit rating industry, we can observe this contestable market structure after the 1997 financial crisis.\footnote{There are four competing credit rating firms in Korea: Korea Investors Service (KIS), Korea Ratings (KR), NICE Investors Service (NICE), and Seoul Credit Rating & Information.}

Because most of the sales of a credit rating firm come from the evaluation of the credit worthiness of the companies, it is reasonable for us to consider the amount of sales per company as a proxy for the rating fee (price) of a credit rating firm. According to NICE Investors Service (NICE) – one of the biggest credit rating firms in Korea – the amount of sales per company which was evaluated by NICE in the pre-crisis period (1995 - 1997) was $200,892 and that in the post-crisis period (1998 - 2000) was $183,323.\footnote{See http://www.nicerating.com.} Comparing these amounts of sales per company between two periods, we argue that the price was cut by the incumbent player (credit rating firm) to prevent the entry of potential competitor players in Korea’s credit rating industry during the financial crisis period.

In addition, the trends of the Herfindahl-Hirschman Index (HHI) from 1995 to 2000 show that the concentration ratio of Korea’s credit rating industry has declined (see Table 4.1)\footnote{The Herfindahl-Hirschman Index (HHI) is defined as the sum of the squared market share of each firm competing in a market (i.e., $HHI = \sum_{i=1}^{n} s_i^2$, where $s_i$ is the market share of the $i$-th firm). Each credit rating firm’s sales have been taken as the measure of the firm’s market share.}. We can observe that the HHI value dramatically dropped after the financial crisis period. In particular, the average HHI value of the pre-crisis period (1995 - 1997) is 4700 and that of the post-crisis period (1998 - 2000) is 4105.
Table 4.1: HHI for Korea’s Credit Rating Industry

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>HHI</td>
<td>4366</td>
<td>4891</td>
<td>4842</td>
<td>4552</td>
<td>3886</td>
<td>3878</td>
</tr>
</tbody>
</table>

In this paper, we examine the competitive condition (i.e., the market structure) of Korea’s credit rating industry during 1995 - 2000. We estimate reduced-form revenue equations on Korea’s credit rating firms over the period 1995 - 2000 and utilize the Rosse-Panzar methodology (Rosse and Panzar (1977, 1987)) to assess competitive conditions in the credit rating industry. To evaluate the effectiveness of the Korean government’s financial restructuring policy on the credit rating industry in 1997, we compare the competitive conditions of the pre-crisis period (1995 - 1997) and the post-crisis period (1998 - 2000).

The remainder of this paper is as follows. In section 4.2, we present the framework of the Rosse-Panzar test with a brief review of previous studies. Then, we discuss the data and empirical model of our studies. The estimation results are reported in section 4.3. Section 4.4 concludes.

4.2 Methodology and Data

4.2.1 What Is the Rosse-Panzar Test?

Rosse and Panzar [R-P] (1977, 1987) develop an empirical method to assess the competitive conditions in a market. It estimates the reduced-form revenue equations of the market participants derived from marginal revenue and cost functions with the zero profit constraint in equilibrium. With this method, we can discriminate the market structure as being oligopolistic, monopolistically competitive, and perfectly competitive. The methodology of R-P stems from a general equilibrium market model. It relies on the premise that firms, depending on the competitive behavior of market participants, will employ different pricing strategies in response to changes in factor input prices. That is, the degree of competition is measured by the extent to which changes in input prices are
reflected in firms’ equilibrium revenues.

Following Gutierrez de Rozas (2007), let’s consider a representative firm $i$. The twofold profit optimization condition applies at the industry and firm levels. At the former level, the zero profit constraint must hold:

$$R_i(y_i^*, Z^R_i) = C_i(y_i^*, W_i, Z^C_i),$$

(4.1)

where $R_i(\cdot)$ and $C_i(\cdot)$ refer to the revenue and cost functions of firm $i$; $y_i$ is the output of the firm; $W_i$ is a $K$-dimensional vector of factor input prices of firm $i$, $W_i = (w_{i1}, \ldots, w_{Ki})$; $Z^R_i$ is a vector of $J$ exogenous variables affecting the revenue function $Z^R_i = (z^R_{i1}, \ldots, z^R_{iJ})$ and $Z^C_i$ is a vector of $L$ exogenous variables that shift the cost function $Z^C_i = (z^C_{i1}, \ldots, z^C_{iL})$. At the individual firm level, marginal revenues must equal marginal costs:

$$R'_i(y_i^*, Z^R_i) = C'_i(y_i^*, W_i, Z^C_i).$$

(4.2)

From the above two conditions ((4.1) and (4.2)), the $H$-statistic is formulated as follows:

$$H = \sum_{k=1}^{K} \left( \frac{\partial R^*_i}{\partial w_{ki}} \cdot \frac{w_{ki}}{R^*_i} \right).$$

(4.3)

This formula evaluates the elasticity of total revenues with respect to changes in factor input prices. That is, the $H$-statistic denotes a single figure of the overall level of competition prevailing in the market under consideration. According to R-P (1977, 1987), the $H$-statistic ranges from minus infinity to unity. A negative $H$ arises when the competitive structure is a monopoly or a perfect colluding oligopoly. In both cases, an increase in input prices will translate into higher marginal costs, a reduction of equilibrium output and, subsequently, a fall in total revenues. If $H$ lies between zero and unity, the market structure is characterized by monopolistic competition. Under perfect competition, the $H$-statistic equals to unity. In this particular situation, a proportional increase in factor input prices induces an equiproportional change in revenues without distorting
the optimal output of any individual firm.

Contestable markets would also generate an $H$-statistic equal to unity. The contestability theory, first stated by Baumol (1982) and Baumol, Panzar, and Willig (1982), enables the existence of competition in highly concentrated scenarios under very restrictive circumstances, basically free entry and exit of market participants, i.e., neither economic nor legal entry barriers, completely costless exit, and highly price-elastic demands for industry’s output. On account of these features, the threat of potential new market participants forces firms to price their output in a competitive manner. Importantly, Shaffer (1983) derives the Lerner index by a function of the $H$-statistic. Hence, we can compare the degree of competition with the relative level of the $H$-statistic. That is, we can interpret that the market is more competitive as the $H$-statistic increases. Interpretations of the $H$-statistic are summarized in Table 4.2.

<table>
<thead>
<tr>
<th>$H$-statistic</th>
<th>Competitive Environment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H \leq 0$</td>
<td><em>Monopoly equilibrium</em></td>
</tr>
<tr>
<td></td>
<td><em>Perfect colluding oligopoly</em></td>
</tr>
<tr>
<td>$0 &lt; H &lt; 1$</td>
<td><em>Monopolistic competition</em></td>
</tr>
<tr>
<td>$H = 1$</td>
<td><em>Perfect competition</em></td>
</tr>
<tr>
<td></td>
<td><em>Oligopoly in a contestable market</em></td>
</tr>
</tbody>
</table>

4.2.2 Previous Studies on the R-P Test

A great number of empirical studies have assessed competitive conditions in banking markets. Based on the studies which are reviewed below, we will apply the R-P test to assess the competitive condition in Korea’s credit rating industry and examine the effectiveness of the Korean government’s financial restructuring policy to increase the competition level of the credit rating industry in 1997 when Korea suffered the financial crisis.
Studying data for Canadian banks, trust companies, and mortgage companies between 1982 and 1984, Nathan and Neave (1989) find for commercial banks a 1982 value of $H = 1.058$ which does not differ significantly from unity; however, for 1983 and 1984 they find $H = 0.680$ and $H = 0.729$, respectively, both significantly different from zero and unity. Molyneux, Lloyd-Williamson, and Thornton [M-L-T] (1994) utilize R-P methodology on a sample of French, German, Italian, Spanish, and UK banks for the period 1986 - 1989, and find that there is no change in market conduct of banks even though EC banking legislation has established relatively free access to member country banking systems. Like M-L-T (1994), De Bandt and Davis (2000), Bikker and Haaf (2002), and Claessens and Laeven (2004) test the changes in the degree of banking competition in EU, OECD countries, and the top fifty developed countries, respectively. Beyond banking markets, Fischer and Kamerschen (2003) employ the R-P test to calculate price-cost margins in selected airport-pair markets originating from Atlanta, and find the statistics generally positive and quite large, indicating that carriers are neither in perfect competition nor perfectly colluding.

### 4.2.3 Korea’s Credit Rating Industry: Data and Empirical Test Model

There are four competing credit rating firms in Korea: Korea Investors Service (KIS), Korea Ratings (KR), NICE Investors Service (NICE), and Seoul Credit Rating & Information. However, the big three players (KIS, KR, and NICE) occupy almost 98% of total sales in the credit rating industry, and thus we tackle those three firms in our analysis.

The R-P test is performed cross-sectionally on the data for individual firms for the years from 1995 to 2000.\footnote{While the sample size is small, as Shaffer (1993) points out in his paper, this sample is comparable to those of other studies of industry competition.} The accounting data are obtained from Financial Supervisory Service and NICE. To test the effectiveness of the Korean government’s financial restructuring policy, which

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\footnote{While the sample size is small, as Shaffer (1993) points out in his paper, this sample is comparable to those of other studies of industry competition.}
fostered the competitive condition in the credit rating industry during the financial crisis period, we decompose the duration as follows: 1995 - 1997 (pre-crisis period) vs. 1998 - 2000 (post-crisis period).

The forms of the revenue equations used are as follows:

\[
\ln (REV)_{i,t} = \alpha_i + \beta_1 \ln (W_L)_{i,t} + \beta_2 \ln (W_K)_{i,t} + \beta_3 \ln (W_F)_{i,t} + \beta_4 \ln (SL)_{i,t},
\]  
(4.4)

and

\[
\ln (REV)_{i,t} = \alpha_i + \beta_1 \left[ \ln (W_L)_{i,t} - \ln (W_F)_{i,t} \right] + \beta_2 \left[ \ln (W_K)_{i,t} - \ln (W_F)_{i,t} \right] + (\beta_1 + \beta_2 + \beta_3) \ln (W_F)_{i,t} + \beta_4 \ln (SL)_{i,t},
\]  
(4.5)

where \( i \) = individual credit rating firm; \( t \) = year. \( REV \) is the ratio of total revenue to total assets. \( W_L \) is the unit price of labor; \( W_K \) is the unit price of capital; and \( W_F \) is the unit price of funds. That is, \( W_L, W_K, \) and \( W_F \) are three factor prices. To take account of scale economies (i.e., size effect), we include sales of the firm (\( SL \)) as one of the control variables. The measures for the variables in equations (4.4) and (4.5) are summarized in Table 4.3; and Table 4.4 reports the summary statistics of the variables during the pre-crisis period (1995 - 1997) and the post-crisis period (1998 - 2000).\(^{71}\)

Note that equations (4.4) and (4.5) are exactly the same. The \( H \)-statistic is calculated by \( \beta_1 + \beta_2 + \beta_3.\(^{72}\) We examine the hypothesis whether \( H \) is equal to 0 by \( t \)-test of the co-efficient \( (\beta_1 + \beta_2 + \beta_3) \) from equation (4.5). Meanwhile, we examine the hypothesis whether \( H \) is equal to 1 by \( F \)-test of the sum of co-efficients \( (\beta_1 + \beta_2 + \beta_3) \) from equation (4.4). Lastly, to compare the difference between the \( H \)-statistics of 1995 - 1997 and 1998 - 2000 (i.e., the effectiveness of

\(^{70}\) In Table 4.3, note that "Total Funds" means "Net Worth."

\(^{71}\) In Table 4.4, we can briefly observe that after the 1997 financial crisis, a) the ratio of total revenue to total assets of a credit rating firm increased; b) the unit price of labor decreased; c) the unit price of capital and fund increased; d) the sales of a credit rating firm increased.

\(^{72}\) This is the sum of the factor price elasticities, which indicates how responsive revenue is to percentage change in factor prices.
the Korean government’s financial restructuring policy on the credit rating industry during the financial crisis period), we utilize the Chow breakpoint test (Chow (1960) and Greene (2003)) for checking the change in coefficient \((\beta_1 + \beta_2 + \beta_3)\) of each period from equation (4.5).\(^{73}\)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\ln (REV))</td>
<td>(\ln (\text{Gross Profit} / \text{Total Assets}))</td>
</tr>
<tr>
<td>(&lt;\text{Factor Prices}&gt;)</td>
<td></td>
</tr>
<tr>
<td>(\ln (W_L))</td>
<td>(\ln (\text{Personnel Expenses} / \text{Total Assets}))</td>
</tr>
<tr>
<td>(\ln (W_K))</td>
<td>(\ln (\text{Depreciation} / \text{Fixed Assets}))</td>
</tr>
<tr>
<td>(\ln (W_F))</td>
<td>(\ln (\text{Other Operating Expenses} / \text{Total Funds}))</td>
</tr>
<tr>
<td>(&lt;\text{Other Control Variable}&gt;)</td>
<td></td>
</tr>
<tr>
<td>(\ln (SL))</td>
<td>(\ln (\text{Sales}))</td>
</tr>
</tbody>
</table>

To accurately measure the effects of a certain policy in a given period, "difference in differences" (DID) is often used in economics or finance research (Rajan and Zingales (1998) and Angrist and Krueger (1999) among others). The basic premise of DID is to examine the effect of some sort of treatment by comparing the treatment group after treatment both to the treatment group before treatment and to some other control groups. Therefore, we should have both a treatment group and a control group in order to test the effect of the Korean government’s financial restructuring policy on the credit rating industry after the 1997 financial crisis. However, we cannot easily get control groups in this study, and thus it is hard for us to directly use the DID in our research. Due to this problem, instead of DID, in our paper we simply use the Chow breakpoint test for checking the presence of a structural break (i.e., the change in the competitive condition of the credit rating industry) between the pre-crisis period (1995 - 1997) and the post-crisis period (1998 - 2000). Hempell (2002) and Lee (2003) also test the hypothesis of no difference in competition level among German / Korean banks (respectively) between two periods by using the Chow breakpoint test.

\(^{73}\)
### Table 4.4: Summary Statistics of Variables

<table>
<thead>
<tr>
<th>Variables</th>
<th># of Obs.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln (REV)</td>
<td>9</td>
<td>-0.6311</td>
<td>0.3331</td>
<td>-0.1753</td>
<td>-1.2555</td>
</tr>
<tr>
<td>ln (WL)</td>
<td>9</td>
<td>-1.3304</td>
<td>0.1892</td>
<td>-1.0328</td>
<td>-1.6897</td>
</tr>
<tr>
<td>ln (WK)</td>
<td>9</td>
<td>-0.7643</td>
<td>0.7880</td>
<td>0.3905</td>
<td>-1.8010</td>
</tr>
<tr>
<td>ln (WF)</td>
<td>9</td>
<td>-3.3091</td>
<td>0.5263</td>
<td>-2.5342</td>
<td>-3.9944</td>
</tr>
<tr>
<td>ln (SL)</td>
<td>9</td>
<td>16.4397</td>
<td>0.7155</td>
<td>17.4975</td>
<td>15.5809</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th># of Obs.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln (REV)</td>
<td>9</td>
<td>-0.5423</td>
<td>0.4013</td>
<td>-0.0687</td>
<td>-1.2472</td>
</tr>
<tr>
<td>ln (WL)</td>
<td>9</td>
<td>-1.4188</td>
<td>0.3754</td>
<td>-0.9364</td>
<td>-2.2315</td>
</tr>
<tr>
<td>ln (WK)</td>
<td>9</td>
<td>-0.3281</td>
<td>0.7748</td>
<td>0.7384</td>
<td>-1.4048</td>
</tr>
<tr>
<td>ln (WF)</td>
<td>9</td>
<td>-3.0749</td>
<td>0.7152</td>
<td>-2.4540</td>
<td>-4.1730</td>
</tr>
<tr>
<td>ln (SL)</td>
<td>9</td>
<td>16.9543</td>
<td>0.6332</td>
<td>17.8998</td>
<td>15.8684</td>
</tr>
</tbody>
</table>

#### 4.3 Empirical Results

Table 4.5 gives the estimation results for the competitive condition for the sample periods of 1995 to 2000.\(^4\) \(H\)-statistics are calculated for each sub-period: pre-crisis period (1995 - 1997) and post-crisis period (1998 - 2000). In regressions [1] and [2], we just use factor prices (i.e., ln (WL), ln (WK), and ln (WF)) as independent variables. We add a control variable (i.e., ln (SL)) in regressions [3] and [4]. All tests confirm the good fit of our model. In particular, in each regression, the adjusted \(R^2\) is over 97%.

---

\(^4\) In Table 4.5, note that a) the values in parentheses are \(t\)-values; and in brackets are \(p\)-values; b) \(W\) is the Wald statistic (\(\sim F\)) to test the difference between \(H_{1995-1997}\) and \(H_{1998-2000}\); c) \(**\) and \(*\) indicate the significance at 1% and 5% levels.
Table 4.5: Regression Results of Competitive Condition

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.0070 (0.04)</td>
<td>1.0141** (9.01)</td>
<td>−1.5568 (−1.29)</td>
<td>1.8345 (1.05)</td>
</tr>
<tr>
<td>$\ln (W_L)$</td>
<td>1.1779** (14.25)</td>
<td>1.1225** (20.59)</td>
<td>1.0117** (6.79)</td>
<td>1.1901** (7.65)</td>
</tr>
<tr>
<td>$\ln (W_K)$</td>
<td>−0.1800** (−8.70)</td>
<td>−0.2298** (−8.35)</td>
<td>−0.1607** (−6.60)</td>
<td>−0.2445** (−5.64)</td>
</tr>
<tr>
<td>$\ln (W_F)$</td>
<td>−0.2391** (−7.63)</td>
<td>0.0128 (0.45)</td>
<td>−0.1405 (−1.73)</td>
<td>0.0196 (0.57)</td>
</tr>
<tr>
<td>$\ln (SL)$</td>
<td>—</td>
<td>—</td>
<td>0.1024 (1.31)</td>
<td>−0.0418 (−0.47)</td>
</tr>
<tr>
<td># of Obs.</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>adj-$R^2$</td>
<td>0.9828</td>
<td>0.9815</td>
<td>0.9850</td>
<td>0.9781</td>
</tr>
<tr>
<td>$H$-stat</td>
<td>0.7588</td>
<td>0.9055</td>
<td>0.7105</td>
<td>0.9652</td>
</tr>
<tr>
<td>$[H = 0]$</td>
<td>[0.0004]</td>
<td>[0.0001]</td>
<td>[0.0017]</td>
<td>[0.0013]</td>
</tr>
<tr>
<td>$[H = 1]$</td>
<td>[0.0486]</td>
<td>[0.1481]</td>
<td>[0.0376]</td>
<td>[0.8160]</td>
</tr>
<tr>
<td>$W$</td>
<td>—</td>
<td>45.1583**</td>
<td>—</td>
<td>5.8058*</td>
</tr>
</tbody>
</table>

From regressions [1] and [2], we see that the value of the $H$-statistic has increased from 0.7588 at the pre-crisis period to 0.9055 at the post-crisis period. The $H$ value of the pre-crisis period is significantly different from zero and also different from unity at the 5% significance level. That is, the market structure of the credit rating industry in the pre-crisis period shows monopolistic competition. The $H$ value of the post-crisis period is significantly different from zero but is not different from unity, which means that the competitive environment is an oligopoly in a contestable market. The Chow breakpoint test shows that these two $H$ values are significantly
different from each other, which implies that the competitive condition was dramatically improved after the implementation of the Korean government’s financial restructuring policy on the credit rating industry during the financial crisis period.

The result is also consistent with the case where we add a control variable in regressions [3] and [4]. That is, the $H$ value of the post-crisis period is 0.9652, which is higher than that of the pre-crisis period: 0.7105. Like the results of regressions [1] and [2], the Chow breakpoint test shows that these two values are significantly different from each other. Overall, regression results in Table 4.5 lead us to conclude that the Korean government’s financial restructuring policy for fostering the competitive condition in the credit rating industry after the 1997 financial crisis was fairly effective, and thus the credit rating firms began behaving in a more competitive manner.

4.4 Concluding Remarks

In this paper, we used Rosse-Panzar methodology to examine the competitive condition of Korea’s credit rating industry for the period 1995 to 2000. After the implementation of the Korean government’s financial restructuring policy for increasing the competition level of the credit rating industry in 1997, the value of the $H$-statistic has significantly increased, and the market structure has become an oligopoly in a contestable market, which can be economically interpreted as perfect competition. Hence, the empirical results lead us to conclude that the Korean government’s financial restructuring process in the credit rating industry was successful.

However, as one of our future potential works, we need to delve into the real effectiveness of improving the competitive condition in the credit rating industry on the quality of the credit rating itself. Examining the U.S. credit rating industry, Becker and Milbourn [B-M] (2010) find that increased competition brings a result of lowering quality ratings: rating levels go up; the correlation between ratings and market-implied yields falls; and the ability of ratings to predict
default worsens. In this sense, we do need to re-consider the current political issue in the Korea-U.S. Free Trade Agreement which allows foreign credit rating firms, including S&P and Moody’s, to directly enter Korea’s domestic credit rating industry only for the purpose of raising the degree of competition in the market.

However, B-M (2010) do not measure the degree of competition as an $H$-statistic. They assume the growth of Fitch’s market share as the measure of competition faced by other rating firms (S&P and Moody’s). In this sense, B-M (2010) do not capture the exact degree of competition / contestability in the U.S. credit rating industry.
References


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  • Citation for Best Public Interest Service Personnel, Jongno-Gu Office.

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  • 2005 Fulbright Graduate Study Award [Alternate Candidate], Korean-American Educational Commission.

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