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# Introduction to Finite Element Methods in Structural Analysis Final Report

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Introduction to Finite Element Methods in Structural Analysis Final Report

# Finite Element Analysis of a chair

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## Problem Description

A chair will be analyzed with FEM in this report. As shown in Figure 1, there is a force applied on the middle of the chair, it is assumed at the right middle and the value of it is the weight of an adult man, which is 75kg. The weight of the chair is assumed to be 1kg. The angle between the legs and the body is 60 degrees. The length of the leg is 40 centimeters, the cross section of the leg is 3cm*3cm, which is 9cm$^2$, and the width of the body is 34 centimeters.

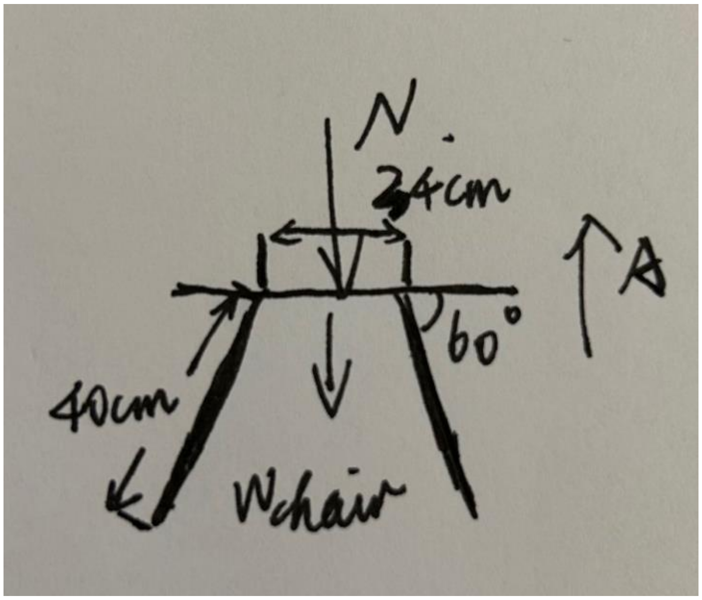

*Figure 1 chair FBD*

Since the failure will happen to the leg of the chair instead of the body of the chair, the only part needing to analyze is the leg. The material of the legs is steel. It is assumed that linear material property and static loading are applied in this case. The constraint is the connection between the body and the legs. It might fail if the weight of the man increases, or the chair gets used too much leading to excessive deflection.

## Analysis Method

The software used to analyze the chair is COMSOL. COMSOL helps find the max 1st principal strength and simulate the deformation of the chair with finite element method. 2D model is used in this simulation. In the real world, a chair is used to hold people when people sit on it. Therefore, there is pressure from the weight of the people, N, and the self-weight of the chair, $W_{chair}$ , unit in Newton. To simplify the problem, the total force F is assumed acted on the connection between the chair body and the chair leg vertically. As shown in Figure 2, total force F is decomposed to Fv, which is orthogonal to the chair leg, and Fa, which is axial to the chair leg.

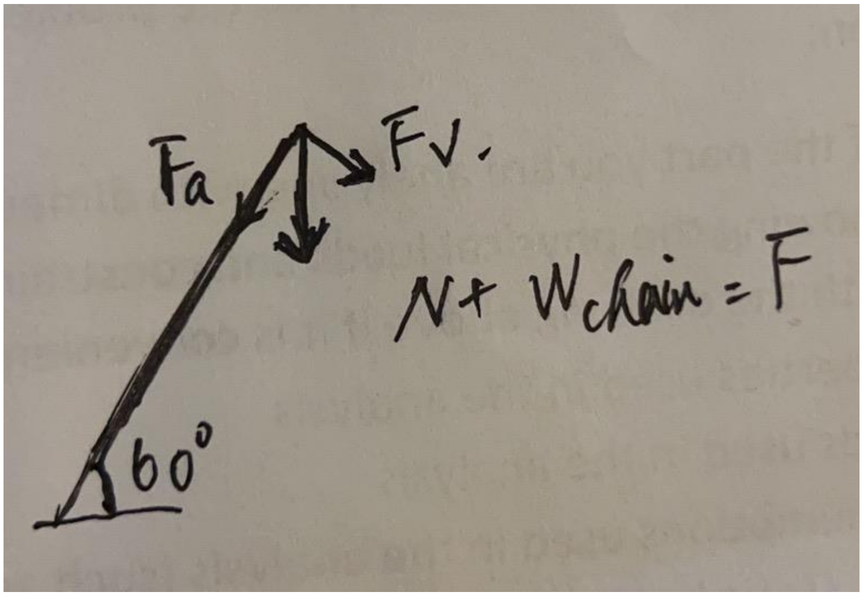

*Figure 2 Decomposition of the total force*

The constraint is at the connection between the chair leg and chair body, which point has no rotation and displacement. To model the real situation, it is simplified to analyze the leg only instead of the whole chair in COMSOL. To make it easier to see the deformation of the leg, the point of the connection is set at origin in COMSOL, where the constraint at.

Triangular element and quadratic shape function are used to analyze the chair. As the “log” part in COMSOL shown, the number of degrees of freedom solved for is 13146. Figure 3 shows the mesh of the chair leg and what the model looks like in COMSOL.

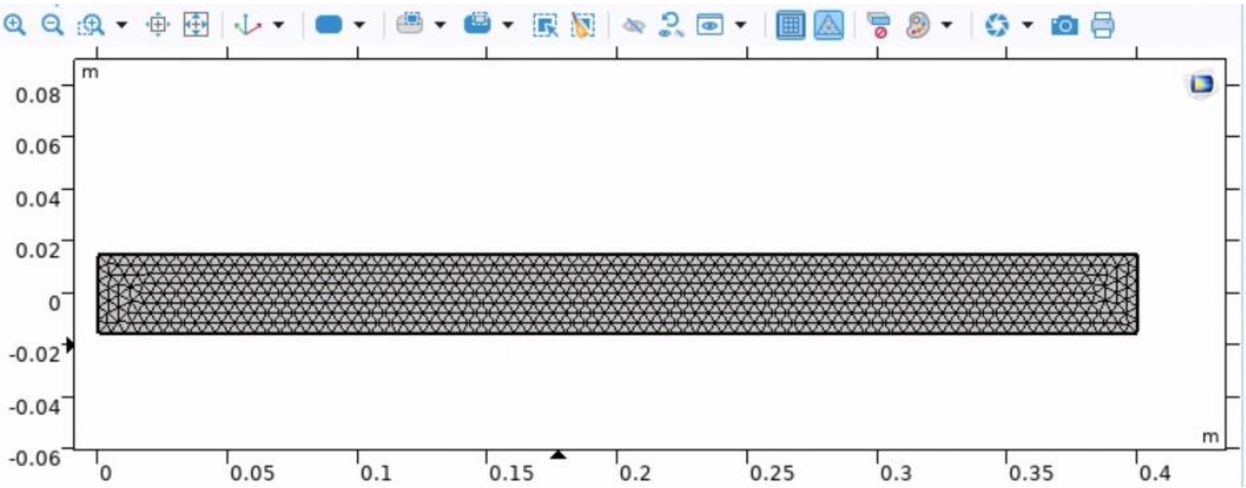

*Figure 3 the mesh of the chair leg*

## Analysis results

The result is shown in figure 4, choosing element size as 0.5cm. It shows that the maximum 1st principal stress is $3.83193 \times 10^7 N/m^2$ and the maximum displacement at the end of the leg in vertical is around 0.01cm.

To prove if this result is reasonable, Assuming the steel bar is made of ASTM A36 steel, which has a typical ultimate tensile strength of 400 to 550 MPa, we can estimate the maximum stress that the bar can withstand before it fails.

The maximum stress that the bar can withstand is given by:

maximum stress = maximum force / cross-sectional area

Assuming that the bar is loaded until it fails, the maximum stress would be equal to the ultimate tensile strength of the steel. Therefore, we can calculate the maximum force that the bar can withstand before failure as:

maximum force = ultimate tensile strength x cross-sectional area

For a 9 $cm^2$ cross-sectional area, the maximum force that the bar can withstand would be:

maximum force = 400MPa x 9 $cm^2$ = $2.32x10^{6}$ N

maximum stress = $2.32x10^{6}$ N / 9$cm^2$ = $2.58x10^{11}$ $N/m^2$

the deformation can be calculated using Hooke's law as:

deformation = maximum force x length / (cross-sectional area x modulus of elasticity)

where the modulus of elasticity for ASTM A36 steel is approximately 200 GPa.

Therefore, the maximum deformation before failure for a 40 cm long and 9 $cm^2$ in cross-section steel bar made of ASTM A36 steel would be:

deformation = ($2.32x10^{6}$ N x 40 cm) / (9 $cm^2$ x 200 GPa) = 0.077 cm

As the calculation shown above, the maximum force can be applied on the chair is $2.32x10^{6}$ N, which is way higher than the total force applied on the chair in this situation. The maximum deformation before failure is calculated as 0.077cm, which is also way higher than 0.01cm shown on figure 4. Maximum stress from calculation is $2.58x10^{11}$ $N/m^2$ higher than the max 1st principal stress in the analysis. Hence, the result is reasonable.

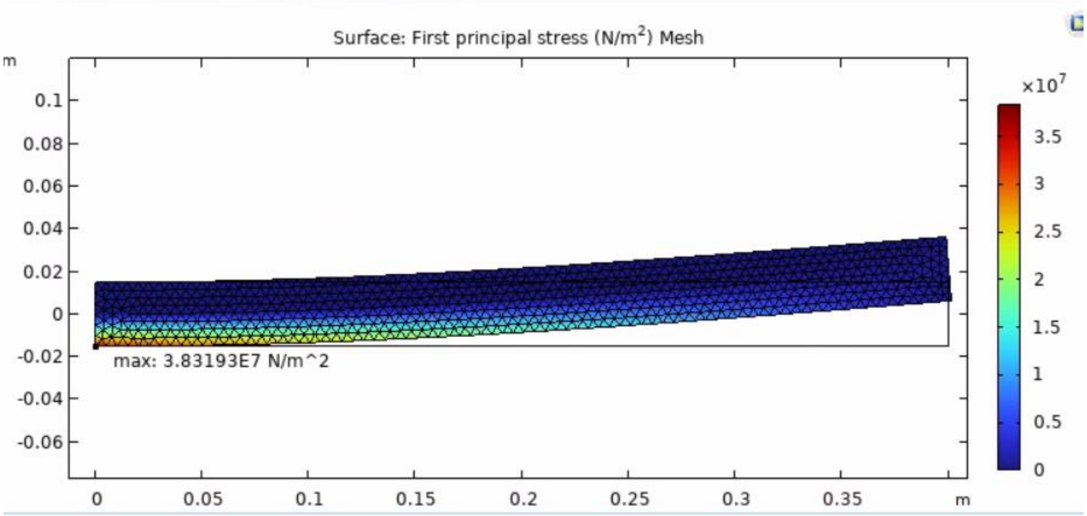

*Figure 4 analysis result*

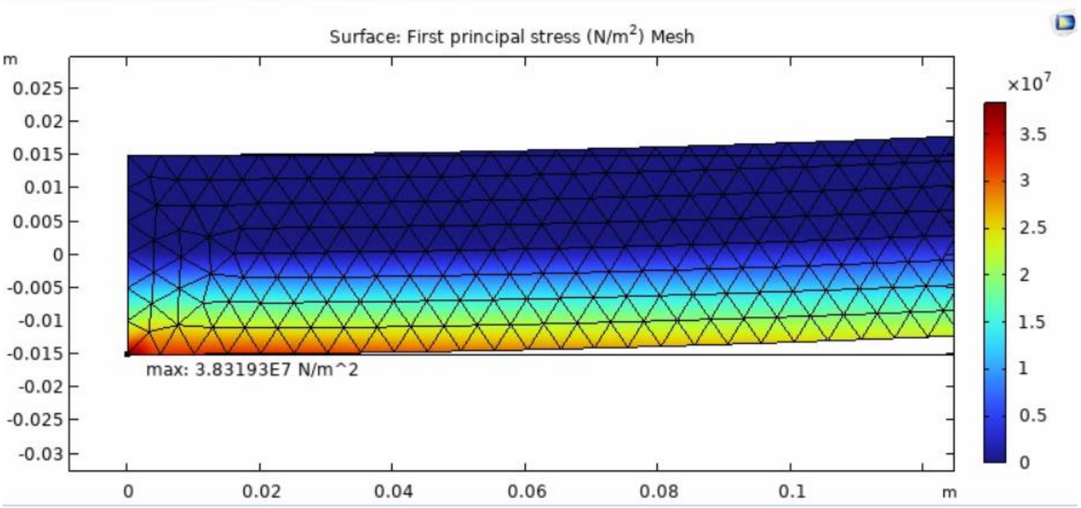

*Figure 5 Analysis result (Zoom in)*

To check if the mesh is sufficiently resolved, the result with element size 0.25cm, 0.125cm, 0.0625cm is shown in Figure 6, 7 and 8.

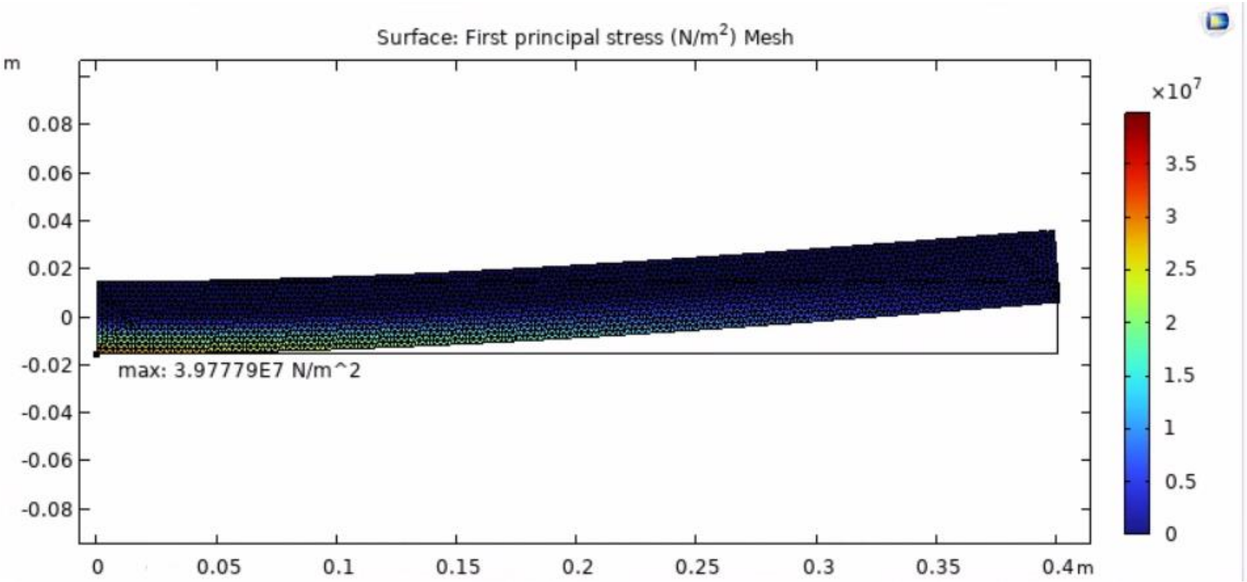

*Figure 6 analysis result (elSize=0.25cm)*

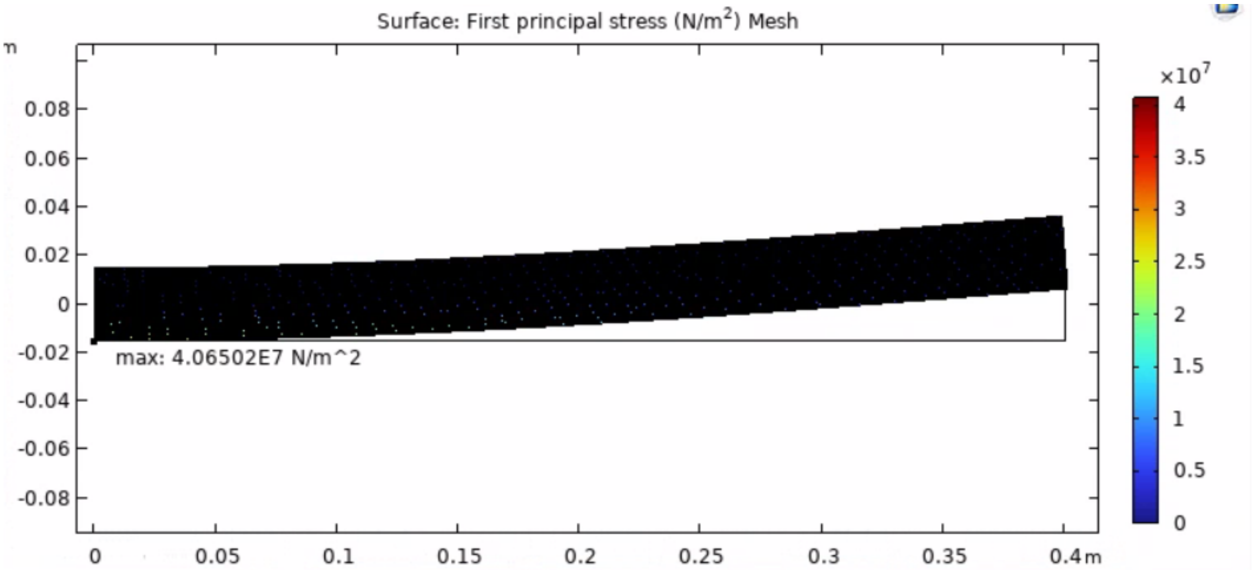

*Figure 7 analysis result (elSize=0.125cm)*

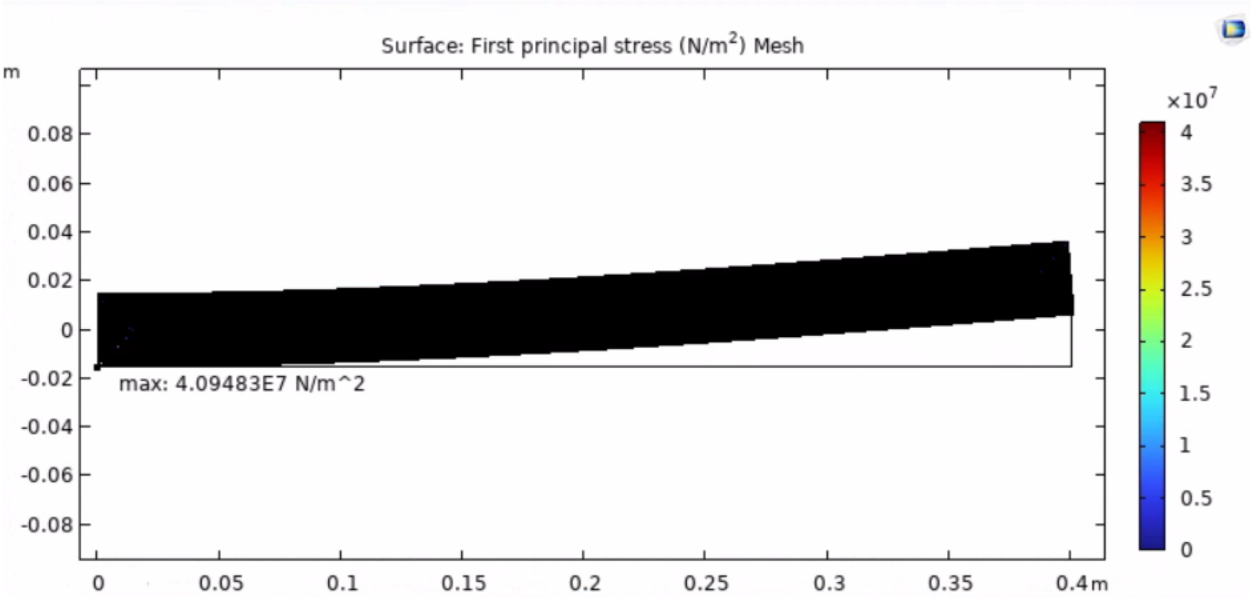

*Figure 8 analysis result (elSize=0.0625cm)*

As seen in the figures, the deformation and 1st maximum principal stress does not change dramatically.

## Discussion

The structure is not predicted to fail under the loading or constrain conditions in the scenario analyzed. As the result shown in previous part, the condition for the failure of the chair leg is harsh, which is not going to happen in real world in most cases. Fatigue failure will be the only way for the chair leg to fail since normal load will not make the chair leg crack right away. It can occur over time as the chair leg is repeatedly loaded and unloaded, leading to microscopic cracks that eventually grow and cause the leg to fail. It is a perfect structure, which has worked over centuries. There is plenty of room for improvement for the model. The model is too ideal and neglects a lot of factors that can affect the result analysis. Failure can also happen to the body of the chair. The decomposition of force can be massively different without the simplification in precious part. I am not too confident about my prediction since there are too many factors being neglected to simplify the problem. Fatigue tests will be recommended in the real world. This test involves subjecting the material to repeated loading and unloading cycles to determine its fatigue strength and behavior. The results can be compared to the model predictions to validate or calibrate the model. From personal consideration, Solidworks might work better than COMSOL. Solidworks can model the whole chair and do the analysis for the whole chair. Although the analysis data from Solidworks might not be as precise as that from COMSOL, the model is closer to the chair in the real world.