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Rational Schur-Agler functions on polynomially-defined domains

Abstract

When $p(z_1, \ldots, z_d)$ is a polynomial in d variables that can be represented as

$$p(z_1, \ldots, z_d) = p_0 \det\left(I - K(\oplus_{j=1}^{d} z_j I_{n_j})\right),$$

where $p_0 \neq 0$ and K is a $(\sum_{j=1}^{d} n_j) \times (\sum_{j=1}^{d} n_j)$ contraction, then the rational inner function

$$f(z_1, \ldots, z_d) = \frac{\left(\prod_{j=1}^{d} z_j^{n_j}\right) \overline{p(1/\bar{z}_1, \ldots, 1/\bar{z}_d)}}{p(z_1, \ldots, z_d)}$$

is in the Schur-Agler class of the polydisk; that is, if $(T_1, \ldots, T_d)$ are commuting strict contractions then $\|f(T_1, \ldots, T_d)\| \leq 1$.

The converse question,"is every rational inner function in the Schur-Agler class of the polydisk necessarily of the above form?" led to questions regarding finite dimensional realizations of rational Schur-Agler functions, determinantal representations of stable polynomials, rational inner functions that are not Schur-Agler, and so forth. In this work we study these questions in Schur-Agler classes defined via a matrix-valued polynomial $\mathbf{P}$, leading to domains of the type

$$\mathcal{D}_{\mathbf{P}} := \{z = (z_1, \ldots, z_d) \in \mathbb{C}^d \ : \ \mathbf{P}(z)^* \mathbf{P}(z) < I\}.$$

Aside from the polydisk this general setting also includes the unit ball $\mathbb{B}^d$, and more generally, Cartan's classical domains. Using methods of Free Noncommutative Analysis, Systems Theory, and Algebraic Geometry, several new results were obtained. This talk is based on joint work with A. Grinshpan, D. S. Kaliuzhnyi-Verbovetskyi, and V. Vinnikov.

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