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A Nagy-Foias model for commuting pairs of contractions

Abstract

The starting point for the Nagy-Foias model for a contractive operator T on Hilbert space is Sz.-Nagy's observation that T has a canonical minimal unitary dilation to a larger Hilbert space. For a *pair* $T = (T_1, T_2)$ of commuting contractions, Ando's theorem asserts that there exist commuting unitary dilations of T to larger Hilbert spaces, and one might aspire to extend the Nagy-Foias model to such operator pairs. However, the dilations provided by Ando's theorem are far from being canonical, and this fact appears to rule out a good model theory for T.

It has recently been shown that nevertheless there *is* such a model theory, though it requires a slight shift in perspective. One focuses on the commuting pair (S, P), where $S = T_1 + T_2$, $P = T_1 T_2$. The operator pair (S, P) is a Γ-*contraction*, which means that the set

$$\Gamma = \{(z + w, zw) : |z| \leq 1, \, |w| \leq 1\}$$

is a spectral set for (S, P). One constructs a canonical dilation, and thereafter a functional model, not for the individual operators T_1, T_2, but for the pair (S, P). The model parallels closely the original Nagy-Foias model.

This line of investigation was begun by Agler and Young, and successfully developed and brought to a conclusion by members of the Indian school of operator theory.

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