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Completely positive kernels: the noncommutative correspondence setting

Abstract

It is well known that a function $K : \Omega \times \Omega \to \mathcal{L}(\mathcal{Y})$ (where $\mathcal{L}(\mathcal{Y})$ is the set of all bounded linear operators on a Hilbert space $\mathcal{Y}$) being (1) a positive kernel in the sense of Aronszajn (i.e. $\sum_{i,j=1}^{N} \langle K(\omega_i, \omega_j) y_j, y_i \rangle \geq 0$ for all $\omega_1, \ldots, \omega_N \in \Omega$, $y_1, \ldots, y_N \in \mathcal{Y}$, and $N = 1, 2, \ldots$) is equivalent to (2) K being the reproducing kernel for a reproducing kernel Hilbert space $\mathcal{H}(K)$, and (3) K having a Kolmogorov decomposition $K(\omega, \zeta) = H(\omega)H(\zeta)^*$ for an operator-valued function $H : \Omega \to \mathcal{L}(\mathcal{X}, \mathcal{Y})$ where $\mathcal{X}$ is an auxiliary Hilbert space.

Recent work of the authors extended this result to the setting of free noncommutative functions (i.e. functions defined on matrices over a point set which respects direct sums and similarities) with the target set $\mathcal{L}(\mathcal{Y})$ of K replaced by $\mathcal{L}(\mathcal{A}, \mathcal{L}(\mathcal{Y}))$ where $\mathcal{A}$ is a C^*-algebra. In this talk, we discuss the next extension where the target set of K is replaced by $\mathcal{L}(\mathcal{A}, \mathcal{L}_a(\mathcal{E}))$ where $\mathcal{A}$ is a W^*-algebra and $\mathcal{L}_a(\mathcal{E})$ is the set of adjointable operators on a Hilbert W^*-module over a W^*-algebra $\mathcal{B}$. Various special cases of this result correspond to results of Kasparov, Murphy, and Szafraniec in the Hilbert C^*-module literature.

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