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Vinay Chandrasekaran
Washington University in St. Louis

Guy Genin
Washington University in St. Louis

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Image Analysis to Quantitatively Estimate Catheter Bending Stiffness
Vinay D. Chandrasekaran

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Abstract
The performance of endovascular surgery is highly dependent on catheter stiffness, especially when navigating past tortuous anatomy. Stiff catheters lack the maneuverability of compliant catheters, but they provide necessary support for medical device delivery and intervention. Conversely, compliant catheters can easily navigate through the vasculature, but they are not stable enough for device delivery and intervention. Catheters vary in length, diameter, shape, and stiffness to accommodate various surgical situations. Although stiffness is of critical importance when considering catheter properties, biomedical companies offer no quantitative measure of a catheter’s bending rigidity. A method for determining bending rigidity would therefore allow surgeons to compare products and make informed decisions on which catheters to use in surgery.

Linear beam theory, also known as small deformation beam theory, is often used to determine beam stiffness. However, since catheters are made from soft, composite materials, they undergo large deformations when subject to relatively small loads, and the linear beam theory loses accuracy. A nonlinear formulation of Euler-Bernoulli beam theory was used to derive a nonlinear flexural rigidity equation. The nonlinear flexural rigidity equation was validated using finite element analysis. The analysis showed that nonlinear beam theory could accurately predict beam stiffness, even at very large strains, compared to linear beam theory which loses accuracy as strain increases.

A simple image analysis experiment was devised to obtain the necessary parameters to calculate catheter stiffness. Expired and used catheters, donated by Drs. Joshua Osbun and Mohammed Zayed, were used for experimentation. Each catheter sample was subject to two experimental treatments: one smaller applied external moment and one larger applied external moment. For each treatment, flexural rigidity was calculated from the linear and nonlinear theories. The consistency of both theories was measured as the percent difference between experimental treatments. Statistical testing was performed using a paired data T-test for difference between means.

For catheter samples with high variation in angular deflection between treatments (> 30°), the nonlinear beam theory was much more consistent than the linear beam theory for quantifying catheter stiffness (p < 0.01). Across all experimental data, there was not significant evidence to indicate that the nonlinear theory was more consistent at measuring flexural rigidity than the linear theory (p = 0.073 > 0.05). Finite element analysis shows that nonlinear beam theory predicts beam stiffness more accurately than linear beam theory across varying angular deflections. Since the data does not suggest statistical significance, the experimental procedure must be further refined before using nonlinear beam theory to quantify catheter stiffness. Possible sources of error and suggestions for future experimentation are discussed at the end of this paper.
Introduction
In transradial neurovascular intervention, surgeons must navigate past tortuous anatomy to reach the brain. Of notable difficulty is the transition from the subclavian artery to the carotid artery via the aortic arch. Compliant catheters, such as guide catheters, exceed when it comes to maneuverability and navigability, but they are too floppy to provide necessary stability during interventions. Stiffer catheters such as sheaths provide the necessary stability to perform interventions. However, they are too stiff to navigate around tight twists and turns. Coaxial systems of stiff sheaths and compliant guide catheters are often used to properly position sheaths within the patient. A vast selection of catheters are available to surgeons which vary in diameter, length, shape, functionality, and stiffness. Of key importance is catheter stiffness - surgeons often develop nuanced preferences for surgical equipment that offers optimal stiffness levels for particular applications. Despite its importance, biomedical companies offer no quantitative measure of catheter stiffness, and surgeons must rely on the word of drug reps, marketing schemes, experience, and anecdotal evidence from other surgeons to appraise a catheter’s bending stiffness.

A quantitative measure of a catheter’s resistance to bending, known as flexural rigidity or bending rigidity, would be of great help to endovascular surgeons. Deflection analysis employing beam theory is suitable for measuring the flexural rigidity of beams. It is possible to determine the bending rigidity of a cantilever beam under stress from a single point load knowing only two parameters: applied external moment \( M(x) \) and angular deflection \( \theta \) (Fig. (1)).

![Diagram of a cantilever beam deforming under external load](image)

**Fig. 1** A cantilever beam deforms in response to applied external forces. The tangent to the beam at the point load \( P \) deflects some angle \( \theta \) in response to an applied external moment.
Similarly, by fixing a catheter at one end and hanging a mass from the free end, $M(x)$ and $\theta$ may be determined through a simple image analysis experiment. Using Euler-Bernoulli beam theory and a linear approximation for beam curvature (Eq. (A11)), the following “linear” stiffness equation may be derived for calculating flexural rigidity (Eq.(1)) [1].

Linear flexural rigidity:

$$EI = \frac{PL^2}{2\tan(\theta)}$$  \hspace{1cm} (1)

Where $E$ is the elastic modulus $N/m^2$, $I$ is the second area of inertia of the beam’s cross section about the neutral axis (m$^4$), $P$ is applied external force (N), $L$ is the length of the moment arm (m), $\theta$ is the angular deflection at the point load (radians), and the quantity $EI$ is the flexural rigidity of the beam ($Nm^2$).

However, such a formulation is valid only for small angular deflections where beam rotation is negligible compared to unity (angles for which the square slope is much less than 1 (Eq. (A15))). Catheters, which are made from soft, composite materials, undergo large deformation when subject to relatively small loads. Thus, a nonlinear or large deformation beam theory is necessary to accurately quantify catheter stiffness. Using instead the exact, nonlinear definition of curvature in rectangular space, a “nonlinear” stiffness equation may be derived (Eq. (2)). Full derivations of Eq. (1) and Eq. (2) may be found in section A1 of the appendix.

Nonlinear flexural rigidity:

$$EI = \frac{PL^2 \sqrt{1 + \tan^2(\theta)}}{2\tan(\theta)}$$  \hspace{1cm} (2)

The variables in Eq. (2) are the same as in Eq. (1).
Validation of the Nonlinear Flexural Rigidity Equation Using Finite Element Analysis
Extensive validation of Eq. (2) was performed using finite element analysis. The nonlinear equation accurately predicts bending rigidity at large angles where the linear equation loses accuracy. However, at small angular deflections, both theories work reasonably well for quantifying beam stiffness. At $\theta = 45.4$ degrees, percent error for the nonlinear and linear equations is 0.0452% and 29.8% respectively. At $\theta = 6.82$ degrees, percent error for the nonlinear and linear equations was 0.0452% and 0.662% respectively (Fig. (2)).

![Graph showing linear and nonlinear flexural rigidity values](image)

**Fig. 2** Linear and nonlinear flexural rigidity values calculated from COMSOL displacements. Flexural rigidity values were calculated using finite element analysis displacement fields for various forces applied 420 mm from the fixed end of a 450 mm cantilever beam. The beam was a hollow cylinder with inner diameter of 1.778 mm, an outer diameter of 2.1 mm, and a known flexural rigidity of 1.856 Ncm$^2$. 
Obtaining Images of Strained Catheters in Static Equilibrium

Used and expired catheters were cut into smaller sections and clamped in a fixed-free configuration forming a cantilever beam. Each catheter sample was subject to two experimental treatments - one smaller and one larger applied external moment - hereby called treatment 1 and treatment 2 respectively. The mass of the applied point load was the same for both treatments. An increase in applied external moment was achieved by sliding the point load further along the length of the catheter (Fig. (3)).

![Experimental setup for image analysis experiments](image)

A) Treatment 1 for a 6 French Microvention SOFIA catheter sample. B) Treatment 2 for a 6 French Microvention SOFIA catheter sample.

Image Analysis to Quantitatively Determine Catheter Stiffness

Images of catheter samples were analyzed using ImageJ [2] to obtain the necessary parameters. There are a few discrepancies between catheter samples and the idealized cantilever beam that was used to derive the flexural rigidity equations (Appendix A1). For instance, the beam axis of catheter samples is not perfectly straight, and it is practically impossible to have the initial beam axis aligned perpendicular to the force of gravity. These discrepancies will be further discussed in later sections. The parameters of interest, $\theta$ and $M(x)$, were obtained using a specific protocol to ensure consistency between measurements.
**Measuring Angular Deflection (θ)**
The angular deflection is the difference between the initial and final angles of the tangent at the point load (Eq. (3)). Graph paper was used as a background to help ensure consistent angle measurements. All angle measurements were made between the tangent line of the catheter at the point load and the most convenient horizontal grid line.

Angular deflection: \[ θ = θ_f - θ_i \]  

Where \( θ_i \) and \( θ_f \) are the initial and final angles from horizontal. Here, angles above horizontal were considered negative and angles below horizontal were considered positive. Often times, catheter samples were angled slightly upward before the point load was applied giving a negative initial angle (\( θ_i \)). However, the final angle (\( θ_f \)) and total angular deflection (\( θ \)) were always positive.

**Protocol for obtaining angular deflection (θ):**

1. Images of catheter samples before and after the point load was applied were uploaded to ImageJ.

2. The segmented line tool was used to approximate the arc length of the catheter from the clamped end to the point load. In this case, the arc length is approximately 1889.806 pixels.

3. Using the previous measurement as a reference, the segmented line tool was used to approximate the position where the point load will be applied on the unstrained “before” image.
4. Using the segmented line tool as a guide, the image frame was fit around the local area where the point load would be applied.

5. The image was made binary to easily distinguish catheter edges.

6. Using the binarized image, the tangent line to the point load was approximated using the angle tool.
7. After approximating the tangent line, the initial angle from the horizontal \( \theta_i \) was found relative to the horizontal graph lines.

8. \( \theta_f \) was found in a similar manner.

9. Angular deflection \( \theta \) was found using Eq. (3).
Measuring Applied External Moment ($M(x)$)

The applied external moment, $M(x)$, is the resulting moment from the point load. Catheter mass was small compared to the applied point load, so bending caused by the catheter’s own weight was ignored. The mass of the applied point load ($P$) was measured using an analytical balance. The length of the moment arm ($L$) was obtained from image analysis.

Protocol for obtaining length of the moment arm ($L$):

1. Images of strained catheter samples were uploaded to ImageJ. The ruler in the background was used to obtain a pixel to cm conversion factor.

2. The end of the moment arm was centered directly above the point load using the horizontal and vertical gridlines as an aid. The horizontal gridlines were assumed to be perpendicular to gravity. The segmented line tool was used as an aid to help position the end of the moment arm directly above the point load.
3. The length of the moment arm was measured in pixels. Using the conversion factor obtained in step 1, the length of the moment arm was converted to metric units.

Results

After obtaining $M(x)$ and $\theta$, flexural rigidity was calculated using the nonlinear and linear beam theories (Eq. (1) and Eq. (2)). For each catheter sample, four calculations were performed - linear and nonlinear flexural rigidity calculations for experimental treatments 1 and 2. The consistency of the linear and nonlinear theories was measured as the percent difference ($X_l$ and $X_{nl}$) between treatments 1 and 2 (Eq. (4a) and Eq. (4b)).

$$X_l = \frac{|EI_{2,l} - EI_{1,l}|}{(EI_{2,l} + EI_{1,l})/2}$$  \hspace{1cm} (4a)

Percent difference, linear:

$$X_{nl} = \frac{|EI_{2,nl} - EI_{1,nl}|}{(EI_{2,nl} + EI_{1,nl})/2}$$  \hspace{1cm} (4b)

Percent difference, nonlinear:

Above, the subscripts $l$ and $nl$ stand for linear and nonlinear, and the subscripts 1 and 2 stand for experimental treatments 1 and 2. For example, $EI_{2,nl}$ refers to the nonlinear flexural rigidity calculation for experimental treatment 2.

The nonlinear theory was expected to have a smaller percent difference between treatments 1 and 2 compared to the linear theory from finite element analysis ($\mu_{nl} < \mu_l$). This hypothesis was tested using a paired data T-test for difference between means against the null hypothesis $H_0$: $\mu_{nl} - \mu_l \geq 0$.

For experimental samples with very large variation in angular deflection between treatments (> 30°), the nonlinear beam theory was much more consistent than the linear beam theory (p < 0.01). However, over all data, there was not significant evidence to reject the null hypothesis (p = 0.073 > 0.05).
Discussion and Suggestions for Future Experimentation

The overall results ($p = 0.073$) juxtaposed with the large variation in angular deflection results ($p < 0.01$) suggest that, in many cases, flaws in experimentation account for greater error than the nonlinear theory corrects for. Thus, the experimental procedure should be refined until the measured stiffness values reflect the discriminatory power of the nonlinear theory as shown by finite element analysis. A few potential sources of error may account for the observed experimental error and be used to improve the procedure in future experiments.

To measure the length of the moment arm, a ruler was placed in the background of all images to obtain a conversion factor from pixels to metric units. However, the ruler was further away from the camera lens than the catheter samples. Length measurements are therefore larger than their actual values due to the perspective view of the camera. Since both moment arms for treatments 1 and 2 appear larger than they are, their apparent square differences are larger than their true square differences. That is, since an $L^2$ term exists in both rigidity equations, there exists a greater difference between the linear and nonlinear $L^2$ terms than there should be due to the increase in perceived $L$. Furthermore, since the linear beam theory tends to underestimate flexural rigidity [1], the disproportionate increase in square length acts as a sort of corrective factor for the linear theory. Future experiments should improve the means of length measurement through methods such as placing the ruler in the same plane as the catheter samples, or adjusting the camera’s position, lens, and settings to obtain a more isometric view of the sample rather than a perspective view [3-4].

Angular deflection measurements are susceptible to the precision error of image analysis. Due to image quality and pixelation error, the true angular deflection can only be approximated within some close range to the true angular deflection. Eq. (1) and Eq. (2) both use $\tan(\theta)$ representing $dw/dx$. The rate of change of $\tan(\theta)$ increases from $0^\circ$ to $90^\circ$, so the larger the $\theta$, the greater that any error in measured angular deflection affects $\tan(\theta)$. This increase in error due to pixelation error, which has an especially pronounced effect at larger angles [3], may have affected calculated differences between the linear and nonlinear theory.

Another issue is that the materials of the catheter might not be described perfectly by linear elasticity. All materials exhibit some degree of viscoelasticity, in which the mechanical response depends upon the rate and the time history of loading. To test for this, creep or relaxation tests should be performed, and the response spectra should be estimated using established techniques [5-7]. Techniques exist to find spectra for linear and nonlinear materials, and both should be checked [8-11].

Most significant may be the problem of torsion. Unlike the theoretical cantilever beam used to derive Eq. (1) and Eq. (2), the catheter samples were not perfectly straight, they often had an initial bending bias in multiple directions, and might have microstructures that tend to couple bending and torsion, as can arise in materials containing aligned fibers [12]. To correct for this, catheters were placed in the clamp such that they appeared as straight as possible from the perspective of the camera (any pronounced curvature would be in the $xy$ plane). One assumption used in the derivation of the flexural rigidity equations was that strain in the $y$ direction ($\nu$) is negligible since the beams are essentially two-dimensional (Appendix A1). Since, practically speaking, this was not the case, failure to take into account torsional strain may have contributed to experimental error. Future experiments may prevent this by careful selection of catheter samples which are as straight as possible.
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Appendix
Appendix for image analysis paper. Includes derivations of flexural rigidity equations used in this paper and specific information about how hypothesis testing was performed.

(A1) - Formulation of Nonlinear and Linear Flexural Rigidity Equations
Timoshenko beam theory is typically used for beams with a small length to thickness (l/h) ratio, where shear strain has considerable effect on total strain. Since catheters have a large length to thickness ratio (l/h > 20), the simpler Euler-Bernoulli beam theory [1] was chosen as a starting point in the search for a suitable beam theory. Here a cantilever beam is considered with a single, freely moving, point load attached to the beam’s free end. The Euler-Bernoulli hypothesis along with assumptions appropriate for our application are what lead to the ultimate derivation of the linear and nonlinear flexural rigidity formulas presented in this paper.

The Euler-Bernoulli Hypothesis
1. Cross-sectional planes remain plane
2. Cross-sectional planes remain normal to the neutral axis
3. Isometric cross sections before and after bending

Assumptions
1. Isotropic material
2. Linear elastic modulus
3. Negligible membrane strain
4. Negligible torsional strain
5. Mass of catheter small compared to mass of point load

The Strain-Displacement Relationship
Considering an infinitesimal beam section, total strain can be modelled as a function of displacement \( \epsilon(x,y,z) = (u,v,w) \). Using assumptions 3 and 4, we can simplify the total strain as a function of \( z \) alone, defined as engineering strain [1].

\[
\epsilon(z) = \frac{l - l_o}{l_o}
\]

(A1)
Fig. A1  An infinitesimal beam section bending with radius of curvature $\rho$. The length of the neutral axis ($l_o$) remains unchanged after bending. The strain of any given beam segment parallel to the neutral axis may be calculated using the definition of arc length.

Using the definition of arc length, $ds = \rho d\theta$, where $\rho$ is radius of curvature, $\theta$ is angle, and $s$ is arc length, we can redefine strain in terms of curvature (Fig. (A1)).

$$
\epsilon(z) = \frac{(\rho + z)d\theta - (\rho)d\theta}{(\rho)d\theta} \quad (A2)
$$

Simplifying we have our strain displacement relationship where curvature ($\kappa$) is defined as $1/\rho$.

$$
\epsilon(z) = \frac{z}{\rho} = z\kappa \quad (A3)
$$

**Moment Curvature Relationship**

Looking at the beam’s cross section at the free end, where the point load is applied, we can see an internal moment resulting from tension and compression of the beam material about the neutral axis.

Fig. A2  Beam cross section showing internal reaction moment ($M(z)$) resulting from tension and compression of beam material about the neutral axis.
Since the beam is in static equilibrium, we know that the sum of all moments must be zero.

\[
\sum M = 0 \tag{A4}
\]

There are only two moments to consider. The applied external moment \( M(x) \) and the internal reaction moment \(-M(z)\).

\[
M(z) = M(x) \tag{A5}
\]

By definition, the internal moment has a fulcrum about the neutral axis. The applied external moment results only from the point load.

\[
\sum (r_z \times F) = M(x) \tag{A6}
\]

The internal reaction moment can be thought of as the sum of moments resulting from the compressive and tensile stresses of the material (Fig. (A2)).

\[
\int z \sigma dA = M(x) \tag{A7}
\]

The left hand side can be re-written in terms of strain with Young’s modulus.

\[
\int z E \epsilon dA = M(x) \tag{A8}
\]

Substitute strain with Eq. (A3) obtained from the strain displacement relationship.

\[
E \int z^2 \kappa dA = M(x) \tag{A9}
\]

Integrating, we obtain our final moment-curvature relationship.

\[
EI \kappa = M(x) \tag{A10}
\]

**Derivation of Linear and Nonlinear Beam Theories**

Geometric nonlinearities are accounted for using the exact definition of curvature in rectangular space (Eq. (A15)). For linear beam theory, curvature is approximated as the rate of change of slope \( (d^2w/dx^2) \) since the square slope (beam rotation) is small compared to unity for small angular deflections. In engineering, the linear beam theory is often preferred since the integral is easier to do, and many structural problems do not deal with large angular deflections.
To solve for linear flexural rigidity ($EI$) in terms of $L$ and $\theta$, first substitute the linear approximation for curvature into the moment-curvature relationship.

$$EI \int_0^L \frac{d^2w}{dx^2} dx = \int_0^L M(x) dx \tag{A11}$$

From assumption 5, the only external moment to consider is the applied external moment from the point load.

$$EI \int_0^L \frac{d^2w}{dx^2} dx = \int_0^L P \cdot x dx \tag{A12}$$

$$EI \frac{dw}{dx} = \frac{PL^2}{2} \tag{A13}$$

The quantity $dw/dx$ is interpreted as $\tan(\theta)$.

$$EI = \frac{PL^2}{2 \tan(\theta)} \tag{A14}$$

The nonlinear solution requires a bit more work. Again, start off by substituting the exact definition of curvature into the moment-curvature relationship.

$$EI \frac{d^2w}{dx^2} [1 + (\frac{dw}{dx})^2]^{1.5} = M(x) \tag{A15}$$

Next, create a substitution relationship.

$$[1 + (dw/dx)^2] = u \tag{A16}$$

$$2 \frac{dw}{dx} \frac{d^2w}{dx^2} dx = du \tag{A17}$$

Plugging the above relationship into Eq.(A15) we get the following.

$$EI \int \frac{1}{2 \frac{dw}{dx} u^{1.5}} du = \int_0^L P \cdot x dx \tag{A18}$$

Rewrite slope in terms of $u$.

$$EI \int \frac{1}{2\sqrt{u - 1}u^{1.5}} du = \int_0^L P \cdot x dx \tag{A19}$$
Solving the integral gives the following equation.

$$EI \sqrt{\frac{u}{u-1}} = \frac{PL^2}{2}$$  \hfill (A20)

Backsubstitution give the following.

$$EI \frac{dw/dx}{\sqrt{1 + (dw/dx)^2}} = \frac{PL^2}{2}$$  \hfill (A21)

Substitute $dw/dx$ with $\tan(\theta)$.

$$EI = \frac{PL^2 \sqrt{1 + \tan^2(\theta)}}{2\tan(\theta)}$$  \hfill (A22)

(A2) - Hypothesis Testing

A paired data T-test for difference between means was performed on the data generated from the 32 catheter samples used in experimentation [13]. For each catheter sample, linear and nonlinear flexural rigidity was calculated for both experimental treatments. The consistency of linear and nonlinear calculations was measured as percent difference between experimental treatments. Each catheter sample had two statistical testing quantities, $X_l$ and $X_n$, representing the percent difference between experimental treatments for linear and nonlinear calculated values. The null hypothesis $H_0: \mu_l - \mu_n \leq 0$ was tested on two data sets. One data set including every catheter sample ($p = 0.073$) and another data set including catheter samples that had very large variation in angular deflection between treatments of greater than 30 degrees ($p < 0.01$). Statistical testing was performed using the following procedure.

1. Quantities $X_l$ and $X_n$ were calculated for each catheter sample.

2. The difference $D$ was calculated for each catheter sample using the following formula.

$$D = X_l - X_n$$  \hfill (A23)

3. The test statistic $T_{H_0}$ was calculated using the following equation.

$$T_{H_0} = \frac{\bar{D}}{\sqrt{S_D^2/n}}$$  \hfill (A24)

Where $S_D$ is the sample standard deviation, $\bar{D}$ bar is the average of all differences, and $n$ is the sample size of 32.

4. P-values were calculated using the cumulative $T_{n-1}$ distribution $G_{n-1}$ using the following formula.

$$p = 1 - G_{n-1}(T_{H_0})$$  \hfill (A25)
References


