Application of Wray-Agarwal Model to Turbulent Flow in a 2D Lid-Driven Cavity and a 3D Lid-Driven Box

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ABSTRACT

Application of Wray-Agarwal Model to Turbulent Flow in a 2D Lid-Driven Cavity and a 3D Lid-Driven Box

by

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In this thesis, various turbulence models are used for simulating internal vortical flow, both turbulent and laminar, with large recirculation by considering the flow in a 2-D lid-driven square cavity and a 3-D lid driven cubic box. The accuracy of the newly developed Wray-Agarwal (WA) one equation turbulence model is compared against two well-known industry standard turbulence models; the Spalart-Allmaras (SA) and the Shear-Stress-Transport (SST) $k-\omega$ models. The simulations are performed by numerically solving the Reynolds-Averaged Navier-Stokes (RANS) equations in conjunction with WA, SA and SST $k-\omega$ models and comparing the results with the available experimental data and Large Eddy Simulation (LES) results. 2-D numerical solutions are obtained at Reynolds numbers of 10,000, 20,000, 50,000, and 100,000. 3-D numerical solutions are obtained at Reynolds numbers of 3200 and 10,000. All numerical calculation are compared with other numerical results available in the literature. The open-source CFD code OpenFOAM is used to compute the flow field. Computational results clearly demonstrate that the Wray-Agarwal model outperforms in accuracy the Spalart-Allmaras and Shear-Stress-Transport $k-\omega$ models at all Reynolds numbers considered.
Chapter 1

1. Introduction

1.1 Motivation
Computations of incompressible viscous flow in a lid-driven square cavity, and a cubic box, are widely used benchmark cases for testing the accuracy and efficiency of numerical algorithms for the solution of Navier-Stokes equations. An excellent review of the widely solved problem of incompressible flow in a 2D cavity lid-driven has been given by Shankar and Deshpande [1], and for the 3D cubic box by Agarwal [2], Koseff and Street [3]. A vast majority of papers investigating this problem have solved it for laminar flow at Reynolds numbers Re ranging from 100 to 10,000 based on the cavity dimension and the lid velocity. Very few papers solve for flows for Re >10,000 employing the Reynolds-Averaged Navier-Stokes (RANS) in conjunction with a turbulence model, or Large-Eddy Simulation (LES) or Direct Numerical Simulation (DNS). The aim of this thesis is to compute numerical solutions for a lid-driven 2D square cavity and 3D cubic box using the RANS in conjunction with three turbulence models; a recently developed one-equation Wray-Agarwal (WA) [4, 5] model, the one-equation Spalart-Allmaras (SA) model [6], and the two-equation Shear-Stress-Transport (SST) k-ω [7] model. The goal is to evaluate the relative accuracy of the three turbulence models by computing the mesh independent solutions using the same grid and numerical algorithm for the three models and comparing the solutions with available LES and DNS solutions and experimental data.

1.2 Brief Review of Literature
The 2D lid-driven cavity flow has been used as a benchmark case to test the accuracy and efficiency of various numerical algorithms for incompressible Navier-Stokes equations for more than five decades. The earliest paper computing the solution at Re=10,000 demonstrating the presence of an eddy in the upper left corner in addition to eddies in both the bottom corners of the cavity is by Agarwal [8]. He employed a third-order accurate upwind scheme for the convection terms and a second-order central differencing for the viscous terms on a very fine grid in order to obtain an accurate solution using the stream-function/velocity formulation for the Navier-Stokes equations. The most cited papers for laminar flow in a square cavity for Re≤10,000 and Re=20,000 are due to Ghia et al. [9] and Erturk et al. [10] respectively. For 3D flow in a lid-driven cubic box most notable
results are due to Koseff and Street [3] who solved the RANS equations with SA model to obtain numerical compared with experimental data at Re=3200 and Re=10,000. Very few papers in the literature solve these two problems using turbulence modeling at very high Reynolds numbers.

1.3 Objective
The goal of research in this thesis is to benchmark the accuracy and efficiency of the one-equation WA turbulence model. Three turbulence models (WA, SA, and SST \(k-\omega\)) are employed in conjunction with the RANS equations to obtain the numerical solutions for flow in a lid-driven square cavity and a cubic box. Solutions are compared to available laminar LES and DNS computations, and experimental data to evaluate the accuracy and efficiency of the WA model compared to the SA and SST \(k-\omega\) models at high Reynolds numbers considered. The computations are performed using the open-source Computational Fluid Dynamics (CFD) software OpenFOAM [11].

1.4 Turbulence Models
The three models (WA, SA, and SST \(k-\omega\)) used in this study in conjunction with the Reynolds-Averaged Navier-Stokes (RANS) equations, are briefly described below.

1.4.1 Spalart-Allmaras (SA) One-Equation model
The Spalart-Allmaras model defines eddy viscosity with the following equation:

\[
\mu_t = \rho \tilde{v} f_{v1}
\]  

where \(\tilde{v}\) is given by the equation:

\[
\frac{\partial \tilde{v}}{\partial t} + u_j \frac{\partial \tilde{v}}{\partial x_j} = c_{b1} (1 - f_{t2}) \tilde{S} \tilde{v} - \left[ c_{w1} f_w - \frac{c_{b1}}{\kappa^2} f_{t2} \right] \left( \frac{\tilde{v}}{d} \right)^2 + \frac{1}{\sigma} \left[ \frac{\partial}{\partial x_j} \left( (v + \tilde{v}) \frac{\partial \tilde{v}}{\partial x_j} \right) + c_{b2} \frac{\partial \tilde{v}}{\partial x_i} \frac{\partial \tilde{v}}{\partial x_i} \right]
\]  

This is an industry standard one-equation model which is efficient and accurate. \(c_{b1}, c_{w1}, \) and \(c_{b2}\) are calibrated constants from a flow calculation over a flat plate as is standard industry practice. \(f_w\) accounts for the wall blocking effect and \(d\) is the distance from the field point to the nearest wall. More information for the equations above is given in Refs. [6].
1.4.2 Menter’s SST $k$-$\omega$ Two-Equation model

This is the second most widely used model in the industry. The eddy viscosity is defined by the following equation

$$v_t = \frac{a_i k}{\text{max}(a_i, \omega; |S|F_2)}$$

(3)

where $k$ and $\omega$ are given by the equations:

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = P_k - \beta_k \omega + \frac{\partial}{\partial x_j} \left[ (v + \sigma_k v_T) \frac{\partial k}{\partial x_j} \right]$$

(4)

$$\frac{\partial \omega}{\partial t} + U_j \frac{\partial \omega}{\partial x_j} = \alpha S^2 - \beta \omega^2 + \frac{\partial}{\partial x_j} \left[ (v + \sigma_\omega v_T) \frac{\partial \omega}{\partial x_j} \right] + 2(1 - F_1) \sigma_\omega^2 \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}$$

(5)

This is a two-equation model which is generally more accurate than the SA model but requires more computational time to solve. More information for the equations above is given in Refs. [7].

1.4.3 Wray-Agarwal (WA) One-Equation model

The WA model is based on the $k$-$\omega$ closure. It has the characteristics of the $k$-$\omega$ model near the wall and that of $k$-$\epsilon$ model away from the wall. Being a one-equation model, it retains the efficiency of the SA model but has the accuracy competitive with the SST $k$-$\omega$ model. The model determines the eddy viscosity by the equations where:

$$R = \frac{k}{w}$$

(6)

$$\mu_T = f_\mu \rho R$$

(7)

The transport equation for $R$ is given by:

$$\frac{\partial \rho R}{\partial t} + \frac{\partial \rho u_j R}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_\mu \mu_T) \frac{\partial R}{\partial x_j} \right] + \rho C_{1s} R S + \rho f_1 C_{2k\omega} \frac{\partial R}{\partial x_j} \frac{\partial S}{\partial x_j} - (1 - f_1) \rho C_{2\epsilon} \left( \frac{R}{S} \right)^2$$

(8)

Where $f_1$ determines the $k$-$\epsilon$ or the $k$-$\omega$ behavior of the model away from the wall and near the wall respectively, and $f_\mu$ accounts for the wall blocking effect.

The model successfully takes the two equation $k$-$\omega$ closure for eddy viscosity and reduces it to one equation. This model has shown better accuracy and efficiency compared to SA and SST $k$-$\omega$ models for both wall-bounded internal and external flows with small regions of separation and free shear flows [4, 5]. The goal of this research aims to test the ability in computing vortical flows.
Chapter 2

2. Computational Approach for Flow in a 2D Lid Driven Square Cavity and a Cubic Box

This chapter briefly describes the features of the Computational Fluid Dynamics (CFD) software employed and the computational methodology in regard to mesh size and boundary conditions.

2.1 Computational Fluid Dynamics Software

Computational Fluid Dynamics (CFD) is a field in fluid mechanics which employs the numerical methods to discretize the governing equations of fluid dynamics on a set of points known as the mesh, around or inside a body. It then solves these discretized algebraic equations equations on a computer to determine the flow field.

In this thesis, we employ the open-source CFD code OpenFOAM [11], which has an extensive library of numerical algorithms and turbulence models for the solutions of incompressible and compressible RANS equations, and for LES and DNS. In this research, incompressible RANS equations are solved using the well-known SIMPLE/PISO algorithm in conjunction with SA, SST $k$-$\omega$ and WA turbulence models. The SA and SST $k$-$\omega$ models are readily available in OpenFOAM. A new subroutine was written for the WA model described in Refs. [4, 5].

2.2 Computational Methodology

For 2D turbulent flow in a lid-driven square cavity shown in Figure 1, the flow field is computed at $Re = 10,000, 20,000, 50,000,$ and $100,000$. All sides of the cavity are considered to be of unit length and the viscosity of the fluid is varied to achieve the desired Reynolds number, since the velocity of the lid is also kept at a constant value of $U=1$. The Dirichlet boundary conditions are employed on the cavity walls as shown in Figure 1. In all cases, a fine mesh is chosen so as to obtain the mesh independent solutions.
For the lid-driven cubic box, shown in Figure 2, turbulent flow is computed at $Re = 3200$ and $10,000$. All sides of the box are considered to be of unit length. The velocity of the moving upper boundary is varied to achieve the required Reynolds number and the box is considered to be filled with air. Dirichlet boundary conditions at the box walls are with $u=v=w=0$ at all five walls of the cube except the top wall where the boundary condition is $u=U$, $v=0$ and $w=0$. Just like in the case of the 2D cavity flow, the mesh is chosen so as to obtain the mesh independent solutions.
Chapter 3

3. Results for Flow inside a 2D Lid-Driven Square Cavity

This Chapter shows the numerical results for turbulent flow in the 2D lid-driven cavity at Re = 10,000, 20,000, 50,000 and 100,000 using SA, SST $k$-$\omega$ and WA models. These numerical results are compared with the results of other investigators when available.

3.1 Velocity Profiles and Streamlines at Re=10,000

Mesh sizes ranging from 401x401 to 601x601 grid points were utilized by Erturk et al. [10]. Comparing with their results, a graded mesh of 200x200 with a 20:1 grid ratio from cavity interior to the walls provided accurate results in the present computations. This smaller mesh size was used to reduce the computational time while still obtaining mesh independent accurate steady state solutions. Mesh grading was utilized to increase the resolution near the cavity boundaries and to obtain accurate velocity and vorticity values inside the cavity. Figures 3(a) – 3(c) compare the streamline plots obtained with the WA, SA and SST $k$-$\omega$ models. The steady state solution for the WA model develops an extra eddy in the bottom right corner of the cavity as well as more pronounced and stronger eddies in the other corners. This is due to the Scale-Adaptive Simulation (SAS) character of the WA model in contrast SA and SST $k$-$\omega$ models which are not scale adaptive.

![Figure 3(a): Streamlines using the WA model at Re=10,000](image)
The turbulent flow streamlines from Figure 3(a) - 3(c) are compared with those from Erturk et al. [10] for laminar flow at Re=10,000 shown in Figure 4. These plots show that the WA model predicts the eddies in the three corners more accurately when compared to the very fine grid laminar solutions of Erturk et al. [10]. The comparison between Figure 3(a) and Figure 4 demonstrates that the SAS character of the WA model is also able to capture the laminar flow behavior of the flow accurately.
In Figures 5(a) and 5(b), velocity components $v$ and $u$ along the horizontal and vertical centerlines of the cavity respectively compared. The computations with WA, SA and SST $k-\omega$ models are compared with those of Erturk et al. [10] and Ghia et al. [9]. As can be seen from Figures 5(a) and 5(b), WA model outperforms both the SA and SST $k-\omega$ models with SA model having the worst performance. It should be noted that the computed turbulent flow results are compared with the laminar flow computations from Refs. [9, 10]. The WA models seems to be the only model able to reproduce the laminar results. Simulations in Refs. [9, 10] can be considered as DNS computations, but this point could be debatable unless extremely fine grid simulations are performed and compared.
3.2 Velocity Profiles and Streamlines at Re=20,000

For this case, a mesh size of 500x500 was chosen with mesh grading substantially increased to 1000:1 grid spacing ratio from cavity interior to walls in order to maximize the accuracy at the boundary walls without having to further increase the mesh size. Figures 6(a) – 6(d) show the
comparison of streamlines using WA, SA and SST $k$-$\omega$ models compared to the laminar flow numerical solution of Erturk et al [10]. Similar to what was observed at Reynolds number of 10,000, the WA model again has extra eddies in the three corners when compared to solution from the other turbulence models. WA model results again closely match the numerical solution from Erturk et al [10].

Figure 6(a): Streamlines using the WA model at Re=20,000

Figure 6(b): Streamlines using the SA model at Re=20,000
Figures 6(c) and 6(d) show the streamline using the SST $k$-$\omega$ model at $R=20,000$.

Figures 7(a) and 7(b) show the $v$ and $w$-velocity profiles along the horizontal and vertical centerlines of the cavity respectively obtained using the WA, SA and SST $k$-$\omega$ models. These results are compared to the LES results of Hashiguchi [12] and DNS results from Erturk et al. [10]. These
figures show that the WA model is in better agreement with the LES and DNS results and outperforms both the SA and SST $k-\omega$ models, with SA model again having the worst performance.

![v-velocity across the horizontal centerline of the cavity at Re=20,000](image)

**Figure 7(a):** Comparison of $v$-velocity profiles along the horizontal centerline of the cavity at Re=20,000

![u-velocity across the vertical centerline of the cavity at Re=20,000](image)

**Figure 7(b):** Comparison of $u$-velocity profiles along the vertical centerline of the cavity at Re=20,000
3.3 Velocity Profiles and Streamlines at Re=50,000

For this case, the mesh size is kept the same as before in section 3.2 for Re=20,000 with a highly graded mesh of 500x500 with a grid spacing ratio of 1000:1 from the cavity interior to the walls. Figure 8(a)-8(c) show the comparison of streamlines inside the cavity obtained using the WA, SA and SST \( k-\omega \) models, in case of SA and SST \( k-\omega \) models, the structure and number of corner vortices are largely unchanged in comparison to the Re=20,000 case, whereas the WA model shows the formation of extra secondary vortices in the three corners of the cavity. As noted by Erturk et al. [10], the number of secondary vortices should increase as the Reynolds number increases to Re=50,000.

![Figure 8(a): Streamlines using the WA model at Re=50,000](image)
There were no numerical solutions or experimental data readily available in the literature for comparison at Re=50,000; it can be observed that the WA model results follow the trends seen before at lower Reynolds number of Re=20,000. Figures 9(a) and 9(b) show the $v$ and $u$-velocity profiles along the horizontal and vertical centerlines of the cavity respectively obtained using the
three models. The trend in the compute velocity profiles is the same as in the case of Reynolds number of 20,000 and 10,000.

**Figure 9(a):** Comparison of $v$-velocity profiles along the horizontal centerline of the cavity at $Re=50,000$

**Figure 9(b):** Comparison of $u$-velocity profiles along the vertical centerline of the cavity at $Re=50,000$
3.4 Velocity Profiles and Streamlines at Re=100,000

For Re=100,000, a uniform mesh of size 2500x2500 grid points is employed. A highly graded mesh could not be employed since it resulted in unstable solutions. The numerical results obtained using the three turbulence models are compared to the LES of Hashiguchi [12]. Streamlines are plotted in Figure 10(a) - 10(c) using WA, SA, and SST $k$-$\omega$ models respectively; these compare to the vorticity field from LES shown in Figure 10(d). It should be noted that the streamline plots in figures 10(a) - 10(c) were obtained by projecting the numerical results from a fine grid onto a coarser mesh resolution of 500x500 size for better visualization. The streamline plots using the three models have the same general structure observed in the streamline plots at lower Reynolds numbers – a main primary eddy surrounded by three secondary eddies; two in the bottom corners and one in the top left corner. The streamline plot obtained from the WA model has a few additional small eddies. The vorticity plot obtained from LES solution of Hashiguchi [12] shown in Figure 10(d) has several smaller eddies in the three corners. Nonetheless, the WA model captures the eddy structures in the cavity corners closer to the LES simulations than that obtained by using the other two models.

Figure 10(a): Streamlines using the WA model at Re=100,000
Figure 10(b): Streamlines using the SA model at $Re=100,000$

Figure 10(c): Streamlines using the SST $k-\omega$ model at $Re=100,000$
Velocity components $v$ and $u$ obtained using WA, SA and SST $k$-$\omega$ models are compared to the only available LES solution of Hashiguchi [12]. Figures 11(a) and 11(b) show $v$-velocity and $u$-velocity profiles along the horizontal and vertical centerlines of the cavity respectively. Results from both the WA and SST $k$-$\omega$ models are in better agreement with the LES results than the numerical solution from SA. A graded finer mesh would most likely improve the results obtained from the WA model.

Figure 10(d): Vorticity Field using LES at Re=100,000, Hashiguchi [12]

Figure 11(a): Comparison of $v$-velocity profiles along the horizontal centerline of the cavity at Re=100,000
Figure 11(b): Comparison of $u$-velocity profiles along the vertical centerline of the cavity at $Re=100,000$
Chapter 4

4. Results for Flow inside a 3D Lid-Driven Cubic Box

This Chapter shows the results for turbulent flow in a 3D lid-driven cubic box at Re = 3,200 and 10,000 using the SA, SST \( k-\omega \) and WA models. These numerical results are compared with the results of other investigators. Each side of the box measures a unit length of 1 in the x, y and z directions.

4.1 Velocity Profiles and Streamlines at Re=3,200

Two dimensional numerical solutions of a lid-driven cavity flow do not compare favorably with the data in the symmetry plane of a three-dimensional lid-driven cube, as observed by Koseff et al. [3]. For computing the flow at Reynolds number of 3,200, which is well within the laminar range, a mesh consisting of 200x200x100 grid points is used with uniform spacing. The numerical results obtained for the cubic box are compared to the experimental data from Koseff et al. [3]. Figures 12(a) – 12(d), Figures 13(a) – 13(d), and Figures 14(a) – 14(d) respectively display the streamline plots of the magnitude of velocity starting from the symmetry plane normal to the z axis at z=0.5 to z=0.8 in 0.1 increments for WA, SA and SST \( k-\omega \) models. All three models show very similar streamline patterns in the plane at z=0.5, 0.6, 0.7, and 0.8.

For each model, the vortex at the center of the plane is shifted slightly to the right at z=0.5 and begins to move toward the center of the plane at z=0.6, 0.7, and 0.8. There are primarily two secondary vortices in bottom the lower corners whose structure varies along the z-axis. It seems that a third eddy begins to appear at the upper left corner. In case of the WA and SST \( k-\omega \) models, the bottom eddy in the left corner seems to change its structure at z=0.8 and creates an upward flow on the left side wall. In case of the SA model, the vortices remain fairly similar along the z-axis with the bottom left corner eddy disappearing at z=0.6 and re-appearing between z=0.6 and z=0.7.
Figure 12(a): Streamlines using the WA model at $Re=3,200$ at $z=0.5$ (symmetry plane)

Figure 12(b): Streamlines using the WA model at $Re=3,200$ at $z=0.6$
Figure 12(c): Streamlines using the WA model at Re=3,200 at z=0.7

Figure 12(d): Streamlines using the WA model at Re=3,200 at z=0.8
Figure 13(a): Streamlines using the SA model at Re=3,200 at z=0.5 (symmetry plane)

Figure 13(b): Streamlines using the SA model at Re=3,200 at z=0.6
Figure 13(c): Streamlines using the SA model at $Re=3,200$ at $z=0.7$

Figure 13(d): Streamlines using the SA model at $Re=3,200$ at $z=0.8$
Figure 14(a): Streamlines using the SST $k-\omega$ model at $Re=3,200$ at $z=0.5$ (symmetry plane)

Figure 14(b): Streamlines using the SST $k-\omega$ model at $Re=3,200$ at $z=0.6$
Figure 14(c): Streamlines using the SST $k-\omega$ model at $Re=3,200$ at $z=0.7$

Figure 14(d): Streamlines using the SST $k-\omega$ model at $Re=3,200$ at $z=0.8$

Velocity profiles in the symmetry plane $z=0.5$ using the three models are compared to the experimental data from Koseff et al. [3]. Figures 15(a) and 15(b) show $v$-velocity and $u$-velocity profiles along the horizontal and vertical centerlines in the symmetry plane of the box respectively. Results from both the WA and SST $k-\omega$ models are in better agreement with the experimental data.
than the numerical solution from SA model. WA model more closely compares with the experimental data than the SA and SST $k$-$\omega$ models at Reynolds number of 3,200.

Figure 15(a): Comparison of $v$-velocity profiles across the horizontal centerline in the symmetry plane $z=0.5$ at $Re=3,200$

Figure 15(b): Comparison of $u$-velocity profiles across the vertical centerline in the symmetry plane $z=0.5$ at $Re=3,200$
4.2 Velocity Profiles and Streamlines at Re=10,000

For Reynolds number of 10,000, a mesh of 200x200x200 grid points with uniform spacing is employed. Figures 16(a) – 16(d), Figures 17(a) – 17(d), and Figures 18(a) – 18(d) respectively display the streamline plots of the magnitude of velocity beginning from the symmetry plane normal to the z axis at z=0.5 to z=0.8 in 0.1 increments using the WA, SA and SST k-ω models. At Re=10,000, the streamline patterns are not as smooth and consistent as they were at Re=3,200 in section 4.1. The result from the WA model exhibit a large central primary vortex with two very small secondary vorticities in the bottom left and right corners in the symmetry plane. As the z axis increases from z=0.5, the bottom left corner eddy develops another smaller eddy next to it at z=0.6, but it tends to become smaller at z=0.7 and disappear at z=0.8.

The results from the SA model show a large non-centered primary vortex with two secondary vorticities in the bottom left and right corners that are larger than those seen in the WA or SST k-ω models in the symmetry plane z=0.5. Similar to the situation with WA model, the lower left corner eddy develops another smaller eddy at z=0.6 which disappears as the flow moves from z=0.5 to z=0.8. The left corner eddy seems to disappear at z=0.8 and there appears to be an eddy forming in the upper left corner.

The results from the SST k-ω model also shows a non-centered primary vortex with one secondary eddy in the bottom right corner and an eddy forming in the upper left corner in the symmetry plane z=0.5. Moving from z=0.5 to z=0.8, the primary vortex tend to move toward the center. As in the case of the WA model, the bottom right eddy disappears at z=0.6 and then re-appears at z=0.7. A secondary eddy develops at z=0.6 and remains all the way to z=0.8.
Figure 16(a): Streamlines using the WA model at Re=10,000 at z=0.5 (symmetry plane)

Figure 16(b): Streamlines using the WA model at Re=10,000 at z=0.6
Figure 16(c): Streamlines using the WA model at $Re=10,000$ at $z=0.7$

Figure 16(d): Streamlines using the WA model at $Re=10,000$ at $z=0.8$
Figure 17(a): Streamlines using the SA model at Re=10,000 at z=0.5 (symmetry plane)

Figure 17(b): Streamlines using the SA model at Re=10,000 at z=0.6
Figure 17(c): Streamlines using the SA model at Re=10,000 at z=0.7

Figure 17(d): Streamlines using the SA model at Re=10,000 at z=0.8
Figure 18(a): Streamlines using the SST $k-\omega$ model at Re=10,000 at z=0.5 (symmetry plane)

Figure 18(b): Streamlines using the SST $k-\omega$ model at Re=10,000 at z=0.6
Numerical results are again compared to the experimental data obtained by Koseff et al. [3]. Figures 19(a) and 19(b) show $v$-velocity and $u$-velocity profiles along the horizontal and vertical centerlines in the symmetry plane ($z=0.5$) of the box respectively. The WA model accurately predicts the experimental data near the end walls and compares well with the data in both Figures 19(a) and 19(b), with a slight overshoot at $x=1$ in Figure 19(a). The results from the SA model and SST $k-\omega$
models do not seem to compare that well with the experimental data in Figures 19(a) and 19(b). The results from the SA model are similar to those from the WA model, however they do not compare well with the data near the boundary walls as shown in Figure 19(a) with a slight over-prediction at $x=0$ and under-prediction at $x=1$. As can be seen from Figure 19(b), the results from the SA model accurately predict the experimental data near boundary wall at $y=0$ and $y=1$ but show a poor performance in-between. The results from SST $k-\omega$ model slightly over-predict and under-predict the experimental data at $x=0$ and $x=1$ respectively as in the case of the SA model, however with slightly better accuracy. As can be seen from Figure 19(b), the SST $k-\omega$ model significantly outperforms the SA model and follows closely the results from WA model between the two boundary walls of the cubic box.

Figure 19(a): Comparisons of $v$-velocity profiles across the horizontal centerline in the symmetry plane at $Re=10,000$
Figure 19(a): Comparisons of $u$-velocity profiles across the vertical centerline in the symmetry plane at Re=10,000
Chapter 5

5. Conclusions

For two-dimensional turbulent flow simulations of flow in a lid-driven square cavity, the Wray-Agarwal (WA) turbulence model performed remarkably well compared to SA and SST $k$-$\omega$ turbulence models when matched the results against LES and DNS data. The WA model more accurately predicted the velocity profiles while the SA and SST $k$-$\omega$ models under-predicted the velocity peaks in the boundary layers near the walls of the cavity. This trend was noted in all simulations at Reynolds numbers of 10,000, 20,000, 50,000 and 100,000. In general, the SST $k$-$\omega$ results were closer to the LES and DNS data than the SA model results while the WA model results gave the best overall accuracy.

The three-dimensional simulations for a lid-driven cubic box however showed a very different trend using the three turbulence models. In the laminar flow regime at Reynolds number of 3,200, the WA model outperformed in accuracy compared to the other two models when predicting the experimental results while the SST $k$-$\omega$ model results followed closer to the experimental data than the SA model results. At Reynolds number of 10,000 all three models did not quite match the experimental data; however again the WA model performs very well near the boundary walls of the cube and overall gave much better accuracy than the other two models which also predicted the experimental data reasonable well but somewhat worse than the WA model.

The results presented in this thesis demonstrated that the Wray-Agarwal model performs efficiently and accurately as a new one-equation turbulence model compared to the current two well-known industry standard models, namely the SA model and the SST $k$-$\omega$ model.
References


[8] OpenFoam®, v 2.4.0, openfoam.org, 2013.


Appendix A

OpenFOAM blockMeshDict files for 2D Square Cavity
A.1 Mesh for Re=10,000

FoamFile
{
    version 2.0;
    format ascii;
    class dictionary;
    object blockMeshDict;
}

convertToMeters 1;

vertices
(
    (0 0 0)
    (0.5 0 0)
    (1 0 0)
    (0 0.5 0)
    (0.5 0.5 0)
    (1 0.5 0)
    (0 1 0)
    (0.5 1 0)
    (1 1 0)
    (0 0 0.1)
    (0.5 0 0.1)
    (1 0 0.1)
    (0 0.5 0.1)
    (0.5 0.5 0.1)
    (1 0.5 0.1)
    (0 1 0.1)
    (0.5 1 0.1)
    (1 1 0.1)
);
blocks
(  
  hex (0 1 4 3 9 10 13 12) (100 100 1) simpleGrading (20 20 1)
  hex (1 2 5 4 10 11 14 13) (100 100 1) simpleGrading (0.05 20 1)
  hex (3 4 7 6 12 13 16 15) (100 100 1) simpleGrading (20 0.05 1)
  hex (4 5 8 7 13 14 17 16) (100 100 1) simpleGrading (0.05 0.05 1)
);

edges
(  
);

boundary
(  
  movingWall
  {  
    type wall;
    faces
    (  
      (6 15 16 7)  
      (7 16 17 8)  
    );  
  }  
  fixedWalls
  {  
    type wall;
    faces
    (  
      (3 12 15 6)  
      (0 9 12 3)  
      (0 1 10 9)  
      (1 2 11 10)  
      (2 5 14 11)  
      (5 8 17 14)  
    );  
  }  
  frontAndBack
  {  
    type empty;
    faces
    (  
      (0 3 4 1)  
      (1 4 5 2)  
      (3 6 7 4)  
      (4 7 8 5)  
    );  
  }  
);
mergePatchPairs
(
);
A.2 Mesh for Re=20,000

 foamFile
 {
  version 2.0;
  format ascii;
  class dictionary;
  object blockMeshDict;
 }

 convertToMeters 1;

 vertices
 (  
  (0 0 0)
  (0.5 0 0)
  (1 0 0)
  (0 0.5 0)
  (0.5 0.5 0)
  (1 0.5 0)
  (0 1 0)
  (0.5 1 0)
  (1 1 0)
  (0 0 0.1)
  (0.5 0 0.1)
  (1 0 0.1)
  (0 0.5 0.1)
  (0.5 0.5 0.1)
  (1 0.5 0.1)
  (0 1 0.1)
  (0.5 1 0.1)
  (1 1 0.1)
  );

 blocks
 (  
  hex (0 1 4 3 9 10 13 12) (250 250 1) simpleGrading (1000 1000 1)
  hex (1 2 5 4 10 11 14 13) (250 250 1) simpleGrading (0.001 1000 1)
hex (3 4 7 6 12 13 16 15) (250 250 1) simpleGrading (1000 0.001 1)
hex (4 5 8 7 13 14 17 16) (250 250 1) simpleGrading (0.001 0.001 1)
);

edges
(
);

boundary
(

    movingWall
    {
      type wall;
      faces
      (
        (6 15 16 7)
        (7 16 17 8)
      );
    }

    fixedWalls
    {
      type wall;
      faces
      (
        (3 12 15 6)
        (0 9 12 3)
        (0 1 10 9)
        (1 2 11 10)
        (2 5 14 11)
        (5 8 17 14)
      );
    }

    frontAndBack
    {
      type empty;
      faces
      (
        (0 3 4 1)
        (1 4 5 2)
        (3 6 7 4)
        (4 7 8 5)
        (9 10 13 12)
        (10 11 14 13)
        (12 13 16 15)
        (13 14 17 16)
      );
    }

);
} 
);

mergePatchPairs 
(
);

// ************************************************************************* //
A.3 Mesh for Re=50,000

/FoamFile
{
version 2.0;
format ascii;
class dictionary;
object blockMeshDict;
}

convertToMeters 1;

vertices
(
  (0 0 0)
  (0.5 0 0)
  (1 0 0)
  (0 0.5 0)
  (0.5 0.5 0)
  (1 0.5 0)
  (0 1 0)
  (0.5 1 0)
  (1 1 0)
  (0 0 0.1)
  (0.5 0 0.1)
  (1 0 0.1)
  (0 0.5 0.1)
  (0.5 0.5 0.1)
  (1 0.5 0.1)
  (0 1 0.1)
  (0.5 1 0.1)
  (1 1 0.1)
);

blocks
(
  hex (0 1 4 3 9 10 13 12) (250 250 1) simpleGrading (1000 1000 1)
  hex (1 2 5 4 10 11 14 13) (250 250 1) simpleGrading (0.001 1000 1)
hex (3 4 7 6 12 13 16 15) (250 250 1) simpleGrading (1000 0.001 1)
hex (4 5 8 7 13 14 17 16) (250 250 1) simpleGrading (0.001 0.001 1)
);

edges
(
);

boundary
(
    movingWall
    {
        type wall;
        faces
        (
            (6 15 16 7)
            (7 16 17 8)
        );
    }
    fixedWalls
    {
        type wall;
        faces
        (
            (3 12 15 6)
            (0 9 12 3)
            (0 1 10 9)
            (1 2 11 10)
            (2 5 14 11)
            (5 8 17 14)
        );
    }
    frontAndBack
    {
        type empty;
        faces
        (0 3 4 1)
        (1 4 5 2)
        (3 6 7 4)
        (4 7 8 5)
        (9 10 13 12)
        (10 11 14 13)
        (12 13 16 15)
        (13 14 17 16)
    );
```cpp
} 
);

mergePatchPairs
( 
);

// ************************************************************************* //
A.4 Mesh for Re=100,000

/*--------------------------------*- C++ -*----------------------------------*
 | =========                 |                                                 |
 | \      /  F ield         | OpenFOAM: The Open Source CFD Toolbox           |
 | \    /   O peration     | Version: 2.3.0                                 |
 |   \  /    A nd           | Web:      www.OpenFOAM.org                      |
 |    \/     M anipulation  |                                                 |
 \*---------------------------------------------------------------------------*/

FoamFile
{
    version 2.0;
    format ascii;
    class dictionary;
    object blockMeshDict;
}

// ******************************************************************************

convertToMeters 1;

vertices
(
    (0 0 0)
    (1 0 0)
    (1 1 0)
    (0 1 0)
    (0 0 0.1)
    (1 0 0.1)
    (1 1 0.1)
    (0 1 0.1)
);

blocks
(
    hex (0 1 2 3 4 5 6 7) (2500 2500 1) simpleGrading (1 1 1)
);

edges
(
);

boundary
(
    movingWall
    {
        type wall;
    }
);
faces
{
  (3 7 6 2);
}
fixedWalls
{
  type wall;
  faces
  {
    (0 4 7 3)
    (2 6 5 1)
    (1 5 4 0);
  }
}
frontAndBack
{
  type empty;
  faces
  {
    (0 3 2 1)
    (4 5 6 7);
  }
};
mergePatchPairs
(
);

//****************************************************************************/
Appendix B

OpenFOAM blockMeshDict files for 3D Cubic Box
B.1 Mesh for Re=3,200

/*--------------------------------*- C++ -*----------------------------------*\
 | =========                 |                                                 |
 | \      /  F ield         | OpenFOAM: The Open Source CFD Toolbox           |
 |  \    /   O peration     | Version:  2.3.0                                 |
 |   \  /    A nd           | Web:      www.OpenFOAM.org                      |
 |    \/     M anipulation  |                                                 |
\*---------------------------------------------------------------------------*/
FoamFile
{
    version 2.0;
    format ascii;
    class dictionary;
    object blockMeshDict;
}

convertToMeters 1;

vertices
(
    (0 0 0)
    (1 0 0)
    (1 1 0)
    (0 1 0)
    (0 0 1)
    (1 0 1)
    (1 1 1)
    (0 1 1)
);

blocks
(
    hex (0 1 2 3 4 5 6 7) (200 200 100) simpleGrading (1 1 1)
);

edges
(
);
boundary
(
  movingWall
  {
    type wall;
    faces
    (3 7 6 2);
  }
  fixedWalls
  {
    type wall;
    faces
    (0 4 7 3)
    (2 6 5 1)
    (1 5 4 0);
  }
  frontAndBack
  {
    type wall;
    faces
    (0 3 2 1)
    (4 5 6 7);
  }
);

mergePatchPairs
(
);

// ************************************************************************* //
faces
  (  
    (3 7 6 2)  
  );
}
fixedWalls
{
    type wall;
    faces
    (    
        (0 4 7 3)
        (2 6 5 1)
        (1 5 4 0)
    );
}
frontAndBack
{
    type wall;
    faces
    (    
        (0 3 2 1)
        (4 5 6 7)
    );
}
);

mergePatchPairs
(    
);

//************************************************************************* //
Vita

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August 2015