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Free loci of matrix pencils and domains of noncommutative rational functions

Abstract

Consider a monic linear pencil $L(x) = I - A_1x_1 - \cdots - A_gx_g$ whose coefficients A_j are $d \times d$ matrices. It is naturally evaluated at g-tuples of matrices X using the Kronecker tensor product, which gives rise to its free locus $Z(L) = \{X : \det L(X) = 0\}$. Our main result is the following: $Z(L) \subseteq Z(L')$ if and only if the natural map sending the coefficients of L' to the coefficients of L induces a homomorphism $A'/\operatorname{rad} A' \to A/\operatorname{rad} A$. Since linear pencils are a key ingredient in studying noncommutative rational functions via linear systems realization theory, the above result leads to a characterization of all noncommutative rational functions with a given domain. Finally, an answer to a quantum version of Kippenhahn's conjecture on linear pencils will be given: if hermitian matrices $A_1, \ldots, A_g$ generate $M_d(\mathbb{C})$ as an algebra, then there exist hermitian matrices $X_1, \ldots, X_g$ such that $\sum_i A_i \otimes X_i$ has a simple eigenvalue.

The talk is based on the joint work with I. Klep.

Talk time: 07/19/2016 3:30PM— 07/19/2016 3:50PM
Talk location: Cupples I Room 113

Special Session: State space methods in operator and function theory. Organized by J. Ball and S. ter Horst.