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A band formula for a Toeplitz commutant lifting problem

Abstract

The band method plays a fundamental role in solving a Toeplitz and Nehari interpolation problem; see [2]. The solution to the Nehari problem involves the inverses of $I - HH^*$ and $I - H^*H$ where H is the corresponding Hankel matrix. Here we will derive a similar result for a certain commutant lifing problem.

Let Θ be an inner function in $H^\infty(\mathcal{E}, \mathcal{Y})$ and $\mathcal{H}(\Theta)$ the subspace of $\ell^2_+(\mathcal{Y})$ defined by

$$\mathcal{H}(\Theta) = \ell^2_+(\mathcal{Y}) \ominus T_\Theta \ell^2_+(\mathcal{E})$$

where T_Θ is the Toeplitz operator determined by Θ. Clearly, $\mathcal{H}(\Theta)$ is an invariant subspace for the backward shift $S^*_{\mathcal{Y}}$. Consider the *data set* $\{A, T', S_{\mathcal{Y}}\}$ where A is a strict contraction mapping $\ell^2_+(\mathcal{U})$ into $\mathcal{H}(\Theta)$, the operator T' on $\mathcal{H}(\Theta)$ is the compression of $S_{\mathcal{Y}}$ to $\mathcal{H}(\Theta)$, that is,

$$T' = \Pi_{\mathcal{H}(\Theta)} S_{\mathcal{Y}} | \mathcal{H}(\Theta) \text{ on } \mathcal{H}(\Theta).$$

Here $\Pi_{\mathcal{H}(\Theta)}$ is the orthogonal projection from $\ell^2_+(\mathcal{Y})$ onto $\mathcal{H}(\Theta)$. Moreover, A intertwines $S_{\mathcal{U}}$ with T', that is, $T'A = AS_{\mathcal{U}}$. Given this data set the commutant lifting problem is to find all contractive Toeplitz operators T_Ψ such that

$$\Pi_{\mathcal{H}(\Theta)} T_\Psi = A. \tag{1}$$

This lifting problem includes the Nevanlinna-Pick and Leech interpolation problems. Using two different methods we will show that the set of all solutions are given by

$$\Psi = \big(\Upsilon_{12} + \Upsilon_{11} g\big)\big(\Upsilon_{22} + \Upsilon_{21} g\big)^{-1}.$$

Here g is a contactive analytic function acting between the appropriate spaces. Analogous to the band formulas in the Nehari interpolation problem, Υ_{jk} are determined by the inverses of $I - AA^*$ and $I - A^*A$. The proofs relay on different techniques. Finally, this is joint work with S. ter Horst and M.A. Kaashoek.

References

[1] C. Foias, A.E. Frazho, I. Gohberg, and M. A. Kaashoek, *Metric Constrained Interpolation, Commutant Lifting and Systems,* Operator Theory: Advances and Applications, **100**, Birkhäuser-Verlag, 1998.

[2] I. Gohberg, S. Goldberg, and M.A. Kaashoek, *Classes of Linear Operators*, Vol. II, Operator Theory: Advances and Applications, **63**, Birkhäuser-Verlag, Basel, 1993.

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