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Hecke Algebra Characters Evaluatedat Kazhdan-Lusztig Basis Elements Give the Betti Numbers of Hessenberg Varieties

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A finite Coxeter group is a reflection group defined by a set of generating elements and a set of relations specifying the order of any product of two generators. These groups are related to a number of algebraic and geometric objects. Algebraically, Coxeter groups are associated to Hecke algebras, where each group element naturally corresponds to a basis element of the algebra. Geometrically, a Coxeter group is associated to both a root system—a set of vectors in a Euclidean space on which the group acts by reflection—and a Hessenberg variety.

In this thesis, we study the representations of Coxeter group Hecke algebras and the Betti numbers of Hessenberg varieties. Each Hecke algebra representation associates elements of the algebra to a matrix; the corresponding trace character of the representation maps an element of the algebra to the trace of its matrix under the representation. On a certain set of distinguished elements of the Hecke algebra—Kazhdan-Lusztig basis elements—these trace characters give polynomials. We prove for certain Hessenberg varieties and conjecture for all types that the Betti numbers of the variety are recorded as coefficients in these polynomials. Since the Betti numbers give information about the topology of the variety, as well as the geometry of the corresponding root system, our work illustrates a connection between the geometric and algebraic interpretations of finite Coxeter groups.