Muckenhoupt Hamiltonians, triangular factorization, and Krein orthogonal entire functions

Abstract

According to classical results by M. G. Krein and L. de Branges, for every positive measure $\mu$ on the real line $\mathbb{R}$ such that $\int_{\mathbb{R}} \frac{d\mu(t)}{1+t^2} < \infty$ there exists a Hamiltonian $H$ such that $\mu$ is the spectral measure for the corresponding canonical Hamiltonian system $JX' = zHX$. In the case where $\mu$ is an even measure from Steklov class on $\mathbb{R}$, we show that the Hamiltonian $H$ normalized by $\det H = 1$ belongs to the classical Muckenhoupt class $A_2$. Applications of this result to triangular factorizations of Wiener-Hopf operators and Krein orthogonal entire functions will be also discussed.