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On singular integral operators with linear-fractional involutions

Abstract

We denote the Cauchy singular integral operator along a contour $\mathcal{L}$ by $(S_{\mathcal{L}}\varphi)(t) = \frac{1}{\pi i}\int\limits_{\mathcal{L}} \frac{\varphi(\tau)}{\tau - t} d\tau$ and the identity operator by $(I_{\mathcal{L}}\varphi)(t) = \varphi(t)$.

In the paper [1,2] we constructed a similarity transformation $F^{-1}AF = D$, between the singular integral operators A with the rotation operator $W_{\mathbb{T}}$ through the angle $2\pi/m$ on the unit circle $\mathbb{T}$, acting on the space $L_2(\mathbb{T})$, and a certain matrix characteristic singular integral operator without shifts acting on the space $L_2^m(\mathbb{T})$. For $m = 2$, we have $(W_{\mathbb{T}}\varphi)(t) = \varphi(-t)$,

$$A = a_0 I_{\mathbb{T}} + b_0 S_{\mathbb{T}} + a_1 W_{\mathbb{T}} + b_1 S_{\mathbb{T}} W_{\mathbb{T}}, \quad A \in [L_2(\mathbb{T})], D = uI_{\mathbb{T}} + vS_{\mathbb{T}}, \quad D \in [L_2^2(\mathbb{T})].$$

the right hand and left-hand side we reduced

$$B_{\mathbb{R}} = aI_{\mathbb{R}} + bQ_{\mathbb{R}} + cS_{\mathbb{R}} + dQ_{\mathbb{R}}S_{\mathbb{R}}, \ B_{\mathbb{R}} \in [L_2(\mathbb{R})], \ \mathbb{R} = (+\infty, -\infty),$$

where involution $\left(Q_{\mathbb{R}}\varphi\right)(x) = \frac{\sqrt{\delta^2+\beta}}{x-\delta}\varphi[\alpha(x)], \ \alpha(x) = \frac{\delta x+\beta}{x-\delta}, \ \delta^2 + \beta > 0,$
to a matrix characteristic singular integral operator without shift:

$$\mathcal{H}B\mathcal{F} = D_{\mathbb{R}_+}, \quad D_{\mathbb{R}_+} = u\mathbb{R}_+ I_{\mathbb{R}_+} + v\mathbb{R}_+ S_{\mathbb{R}_+},$$

acting on the space $L_2^2(\mathbb{R}_+, x^{-\frac{1}{4}})$, $\mathbb{R}_+ = (0, +\infty)$. We will refer to the formulas as operator equalities. Different applications of operator equalities to singular integral operators and to boundary value problems are considered.

[1] A. A. Karelin, On a relation between singular integral operators with a Carleman linear-fractional shift and matrix characteristic operators without shift, Boletin Soc. Mat. Mexicana Vol. 7 No. 12 (2001), pp. 235–246.

[2] A. Karelin, Aplications of operator equalities to singular integral operators and to Riemann boundary value problems, Math. Nachr. Vol. 280 No. 9-10 (2007), pp. 1108–1117.

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