

Washington University in St. Louis  
**Washington University Open Scholarship**

---

Undergraduate Research Symposium Posters

Undergraduate Research

---

Fall 2016

# Application of the Dispersive Optical Model to $^{208}\text{Pb}$

Michael Keim

Follow this and additional works at: [https://openscholarship.wustl.edu/undergrad\\_research](https://openscholarship.wustl.edu/undergrad_research)



Part of the [Nuclear Commons](#)

---

## Recommended Citation

Keim, Michael, "Application of the Dispersive Optical Model to  $^{208}\text{Pb}$ " (2016). *Undergraduate Research Symposium Posters*. 108.  
[https://openscholarship.wustl.edu/undergrad\\_research/108](https://openscholarship.wustl.edu/undergrad_research/108)

This Unrestricted is brought to you for free and open access by the Undergraduate Research at Washington University Open Scholarship. It has been accepted for inclusion in Undergraduate Research Symposium Posters by an authorized administrator of Washington University Open Scholarship. For more information, please contact [digital@wumail.wustl.edu](mailto:digital@wumail.wustl.edu).



# Application of the Dispersive Optical Model to $^{208}\text{Pb}$

Michael A. Keim, Supervised by Professor Willem Dickhoff  
Department of Physics, Washington University, St. Louis, MO 63130, USA



## Introduction

Traditionally, elastic nucleon scattering experiments have been analyzed by parametrizing the interaction between a nucleon and the nucleus using optical potentials. Such analysis is unable to simultaneously describe both scattering and bound-state data, even though, in principle, scattering experiments also yield information about bound state data [1]. However, Mahaux and Sartor were able to use Kramers-Kronig dispersion relations with optical potentials in order to link nuclear reactions and nuclear structure [2]. The resulting dispersive optical model (DOM) is extensively reviewed in Ref. [3]. A extension of the DOM including a nonlocal, energy-independent Hartree-Fock (HF) term was implemented in Ref. [4], allowing for an interpretation of the DOM potential as a proper self-energy. Here we implement a self-energy similar to that employed in Ref. [5] to describe  $^{208}\text{Pb}$ . The self-energy includes real part connected to an imaginary through a dispersion relation.

$$\Re\Sigma(\mathbf{r}, \mathbf{r}'; E) = \Sigma^{HF}(\mathbf{r}, \mathbf{r}') + \Re\Sigma_d(\mathbf{r}, \mathbf{r}'; E)$$

The real part includes a nonlocal HF term, a Coulomb term, and a local spin-orbit term.

$$\Sigma^{HF}(\mathbf{r}, \mathbf{r}') = \Sigma_{nl}^{HF}(\mathbf{r}, \mathbf{r}') + \delta(\mathbf{r} - \mathbf{r}') (V^{so}(r) + V^C(r))$$

Nonlocality is represented as a Gaussian in the form proposed by Perey and Buck in Ref. [6]. The imaginary part has both a nonlocal contribution and a local spin-orbit term.

$$\Im\Sigma(\mathbf{r}, \mathbf{r}'; E) = \Im\Sigma_{nl}(\mathbf{r}, \mathbf{r}'; E) + \delta(\mathbf{r} - \mathbf{r}') W^{so}(r; E)$$

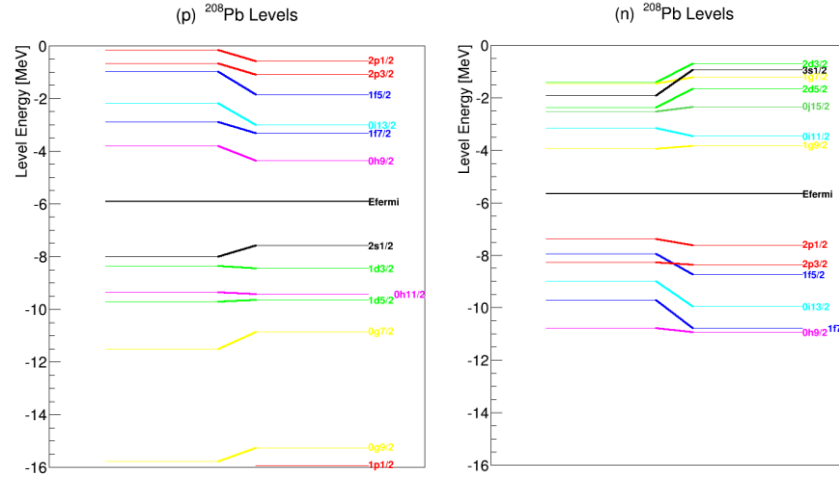
The dispersion integral, as in Ref. [7], can be expressed as

$$\begin{aligned} \Re\Sigma(\mathbf{r}, \mathbf{r}'; E) &= \Re\Sigma(\mathbf{r}, \mathbf{r}'; \varepsilon_F) \\ &+ \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\varepsilon_F} \Im\Sigma(\mathbf{r}, \mathbf{r}'; E') = \left( \frac{1}{E' - E} - \frac{1}{E' - \varepsilon_F} \right) dE' \\ &+ \frac{1}{\pi} \mathcal{P} \int_{\varepsilon_F}^{\infty} \Im\Sigma(\mathbf{r}, \mathbf{r}'; E') = \left( \frac{1}{E' - E} - \frac{1}{E' - \varepsilon_F} \right) dE' \end{aligned}$$

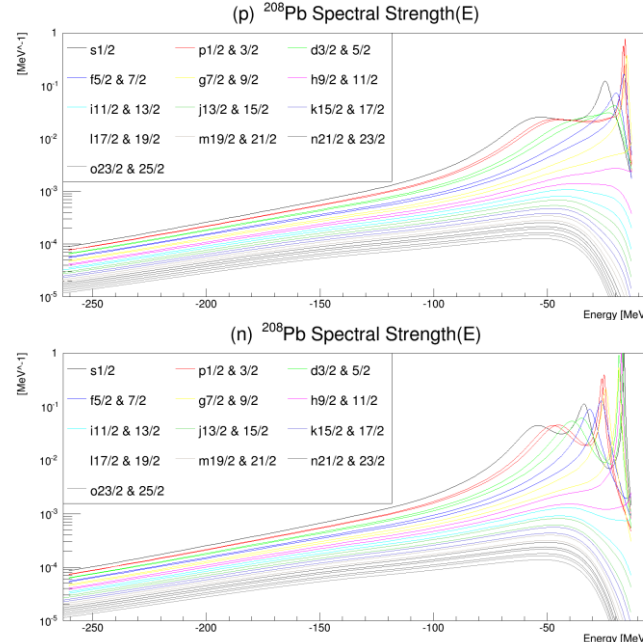
where  $\mathcal{P}$  represents the principal value and the Fermi energy  $\varepsilon_F$  is the average of the energy for adding and removing a nucleon. In this analysis, bound-state properties are given priority in constraining the self-energy since more positive energy data are available to fit. Presented are various properties derived from said self-energy and, specifically, the propagator, or Greens function as described in Ref. [5].

## Results

Overlap functions for nucleon removal or addition and corresponding quasihole or quasiparticle energies are obtained from a Schrödinger-like equation [8] with discrete solutions in the domain where the self-energy has no imaginary part. Below, for protons (left) and neutrons (right), experimental levels (left) are compared to DOM calculated levels (right).



Spectral functions provide information on the likelihood of a particle with a specific l-j at a specific position and at a specific energy. Below, by radially integrating over the hole spectral function the spectral strength is obtained for negative energies:



## Motivation

Properties of heavy nuclei such as  $^{208}\text{Pb}$  have implications for the physics of neutron stars:

- Schwarzschild stars, spherically-symmetric neutron stars in hydrostatic equilibrium, are sensitive to the equation of state of neutron-rich matter alone [9]
- The skin of a heavy nucleus is also composed of neutron-rich matter at a lower density [10]
- Ref. [10] proposes that the smaller the skin-thickness of  $^{208}\text{Pb}$ , the smaller the size of neutron stars
- Ref. [11] proposes an inverse correlation between the neutron-skin thickness and the density of a phase transition from nonuniform to uniform neutron-rich matter

## Conclusions

Provided are selected results from an initial fit to bound-state data of parameters describing a  $^{208}\text{Pb}$  nonlocal self-energy. This analysis necessitated several modifications from previous applications of the DOM for smaller nuclei. In order to achieve reasonable results for weakly bound energy levels, it was necessary to enlarge the radial grid used for self-energy calculations. Additionally, particles were found at very low energies, necessitating a lower integration minimum. In fact, non-negligible spectral strength is calculated beyond just l-j combinations with bound levels. The next step for the analysis is to fit to elastic-scattering data, which will provide greater constraints for the self-energy, allowing for the generation of a more accurate neutron skin useful to astrophysicists.

## References

- [1] Seth Waldecker. Improving the dispersive optical model toward a dispersive self-energy method. 2011 *Ph.D. thesis Washington University*
- [2] Mahaux C and Sartor R 1986 *Phys. Rev. Lett.* **57** 3015
- [3] Mahaux C and Sartor R 1991 *Adv. Nucl. Phys.* **20** 1
- [4] Dickhoff W H, Van Neck D, Waldecker S J, Charity R J and Sobotta L G 2010 *Phys. Rev. C* **82** 054306
- [5] Mohammadhossein Mahzoon. Implications of a Fully Nonlocal Implementation of the Dispersive Optical Model. 2015 *Ph.D. thesis Washington University*
- [6] Perey F and Buck B 1962 *Nucl. Phys.* **32** 353
- [7] Mahaux C and Sartor R 1989 *Nucl. Phys.* **A503** 525
- [8] Dickhoff W H and Van Neck D 2008 *Many-Body Theory Exposed!, 2nd edition* (New Jersey: World Scientific)
- [9] Piekarewicz J 2008 *Revista Mexicana de Fisica S* **54** 104
- [10] Horowitz C J and Piekarewicz J 2001 *Phys. Rev. C* **64** 062802
- [11] Horowitz C J and Piekarewicz J 2001 *Phys. Rev. Lett.* **86** 5647

## Acknowledgments

This work supported by the National Science Foundation under grants #1304242 and #1613362 and by the Washington University in St. Louis Office of Undergraduate Research. Enough thanks cannot be expressed for the guidance and supervision of Professor Willem Dickhoff. Mackenzie Atkinson and Professor Robert Charity also provided important advise and guidance.