Shape Optimization of Busemann-Type Biplane Airfoil for Drag Reduction under Nonlifting and Lifting Conditions Using Genetic Algorithms

Yi Tian
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SHAPE OPTIMIZATION OF BUSEMANN-TYPE BIPLANE AIRFOIL FOR DRAG REDUCTION UNDER NONLIFTING AND LIFTING CONDITIONS USING GENETIC ALGORITHMS

by

Yi Tian

A thesis presented to the School of Engineering and Applied Science of Washington University in St. Louis in partial fulfillment of the requirements for the degree of

Master of Science

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Yi Tian

Washington University in St. Louis
August 2015
Dedicated to my parents

I would like to dedicate this thesis to my beloved parents (Haiying Yu and Zuowei Tian) for their continuous unconditional support of my studies in the United States.

I love you forever!
ABSTRACT

SHAPE OPTIMIZATION OF BUSEMANN-TYPE BIPLANE AIRFOIL FOR DRAG REDUCTION UNDER NONLIFTING AND LIFTING CONDITIONS USING GENETIC ALGORITHMS

by

Yi Tian

Master of Science in Mechanical Engineering

Washington University in St. Louis, 2015

Research Advisor: Professor Ramesh K. Agarwal

The focus of this thesis is on the shape optimization of the Busemann-type biplane airfoil for drag reduction under both nonlifting and lifting conditions using genetic algorithms. The concept of the Busemann-type biplane airfoil was first introduced by Adolf Busemann in 1935. Under its design condition at a specific supersonic flow speed, the Busemann biplane airfoil eliminates all wave drag due to its symmetrical biplane configuration; however it produces zero lift. Previous research has shown that the original Busemann biplane airfoil design has a poor performance under off-design conditions as well. In order to solve this problem of zero lift and to improve the off-design-condition performance of the biplane airfoil, shape optimization of the asymmetric biplane airfoil is performed to minimize the drag while maximizing the lift. In this thesis, the commercially available CFD solver ANSYS FLUENT is employed for computing the inviscid flow past the biplane airfoil. An unstructured mesh is generated using ICEM software. A second-order accurate steady density-based solver is employed to compute the supersonic flow field. A single-objective genetic algorithm (SOGA) is employed to optimize the Busemann biplane airfoil shape under nonlifting condition to
minimize the drag coefficient and a multi-objective genetic algorithm (MOGA) is employed to
optimize the Busemann biplane airfoil shape under lifting condition to maximize both the lift
coefficient and the lift to drag ratio simultaneously. Both results obtained by using SOGA and
MOGA show significant improvement in the design and off-design-condition performance of the
optimized Busemann biplane airfoil compared to the original one.
Chapter 1

Introduction

This chapter provides the background of the supersonic airfoils of commercial airplanes and the motivation behind the study of the optimization of the shape of such airfoils. The scope of this thesis is also included.

1.1 Motivation

For decades, the speed of commercial aircrafts has been bounded by the sound barrier. Even the most successful supersonic transport (SST) plane, the Concorde, could only be deployed in very few routes due to government regulation, low efficiency and excessive noise generation. Since the retirement of Concorde in 2003, the desire of developing a replacement for Concorde still remains. In order to accomplish that, one of the biggest design challenges is to eliminate, or at least greatly reduce, the strong bow shock wave generated during supersonic flight, which can cause high wave drag and substantial noise. This research employs a Single-Objective Genetic Algorithm (SOGA) and a Multi-Objective Genetic Algorithm (MOGA) to optimize the airfoil shape to minimize the wave drag of supersonic commercial aircrafts.


1.2 Background

Computational Fluid Dynamics (CFD) is routinely used in a variety of industrial applications as an analysis tool to simulate and validate new concepts and designs of systems and components that involve fluid flow including both liquid and gas. Components such as wings, fuselage, tails and engines are routinely analyzed for both commercial and fighter aircrafts using CFD.

At supersonic speed, a bow shock is generated ahead of the airplane. This shock wave generates a substantially high wave drag which needs to be overcome by the engine by providing a much higher thrust compared to that for a conventional subsonic/transonic airplane, which results in higher fuel consumption and low propulsive efficiency. In order to address the high-wave-drag problem at supersonic speed, a biplane concept was proposed by Adolf Busemann in 1935 [1] which can potentially avoid the formation of the bow shock and thus does not create sonic boom. During the period from 1935 to 1960, substantial research was conducted on the Busemann biplane concept. Moeckel [2] and Licher [3] performed the theoretical analysis of the optimized lifting supersonic biplanes. Tan [4] took a further step and derived analytical expressions for the drag and lift of a three dimensional supersonic biplane with a finite span of rectangular plan form. Some experimental results were obtained by Ferri [5] using a wind tunnel and comparisons were made between the experimental and analytical results. During the past ten years, considerable interest has again generated in supersonic biplane airfoils. Igra and Arad [6] tested and analyzed the effects of different parameters on the drag coefficient of the Busemann airfoil at various flow conditions. Kusunose et al. [7] proposed a concept for the next generation supersonic transport using the Busemann biplane design. A series of studies using both computational fluid dynamics (CFD) methods and wind-tunnel experimental methods have been completed by Kusunose’s research group [7-16]. Recently,
Hu et al. [17] employed a multi-point adjoint-based aerodynamic design and optimization method to improve the baseline Busemann biplane airfoil’s off-design performance and alleviate the flow hysteresis problem. They also addressed the problem of minimizing its drag by shape optimization.

1.3 Scope of the Thesis

This thesis addresses the aerodynamic shape optimization of the two-dimensional symmetric Busemann biplane airfoil under both design and off-design conditions for reducing its wave drag and the alleviation of the flow-hysteresis and choked-flow effects using a single-objective genetic algorithm. The symmetric Busemann biplane airfoil generates zero lift; this problem is addressed by introducing asymmetry in the shape of the two profiles of the biplane. The asymmetric configuration is shape optimized for maximum lift and minimum drag by employing a multi-objective genetic algorithm. The flow field is computed using the commercial CFD software ANSYS FLUENT. Body-fitted H-grids around the airfoils are generated using the ICEM software. Random airfoil shapes with constraints in a given generation of the genetic algorithm are generated by employing the Bezier Curves (third-order polynomials).
Chapter 2

Flow Field Simulation of Busemann Biplane Airfoil

This chapter presents the physical characteristics of the Busemann-type biplane airfoil and the simulation of supersonic flow past the Busemann biplane airfoil under both design and off-design conditions. A total of three types of flows are simulated: an impulsive uniform flow; a flow during acceleration; and a flow during deceleration.

2.1 Wave Drag of Supersonic Thin Airfoil

When measuring the total drag of an airplane wing, several types of drag are taken into consideration: skin friction drag, wave drag, pressure drag and vortex drag. However in supersonic cruise flight, the wave drag, which is the drag caused by the existence of strong shock waves, is the dominant component among all others [17].

In order to solve for the lift and drag of an airfoil in supersonic flow, the shock expansion theory is generally employed. Under the conditions that the airfoil is thin and the angle of attack is small, the lift and drag of an airfoil in supersonic flow can be approximately determined by the analytical expressions using the thin airfoil theory [19]. The lift and drag coefficients of an airfoil are defined in Eq. (2.1) and Eq. (2.2).
where \( L \) is the lift, \( D \) is the wave drag of the airfoil, \( c \) is the chord length and \( q \) is the dynamic pressure defined as

\[
q = \frac{1}{2} \rho_{\infty} U_{\infty}^2
\]  

It should be noted that in Eq. (2.1) and Eq. (2.2), a wingspan of unit length is assumed. According to thin airfoil theory, Eq. (2.4) and Eq. (2.5) provide the expressions for \( c_l \) and \( c_d \) of an arbitrary two-dimensional thin airfoil at small angle of attack shown in Figure 2.1.

\[
c_l = \frac{4\alpha_0}{\sqrt{M_{\infty}^2 - 1}}
\]  

\[
c_d = \frac{4}{\sqrt{M_{\infty}^2 - 1}} \left[ \left( \frac{dh}{dx} \right)^2 + \alpha_0^2 + \alpha_c^2(x) \right]
\]

It is clear from Eq. (2.4) that the lift of the thin airfoil in supersonic flow simply depends on the mean angle of attack, while the wave drag consists of three parts: drag caused by the angle of attack; drag caused by the camber of the airfoil and drag caused by the thickness of the airfoil.
2.2 Advantageous Effects of Busemann Biplane Airfoil

The biplane airfoil concept proposed by Adolf Busemann [1] is simply to divide a standard diamond-shaped airfoil into two identical components and place the triangular surfaces facing each other at a certain distance apart, as shown in Figure 2.2. The Busemann biplane airfoil design works excellently to reduce the drag due to angle of attack and the drag due to thickness, referred to as the wave reduction effect and the wave cancellation effect respectively.
2.2.1 Wave Reduction Effect

Although the drag due to angle of attack cannot be completely eliminated, it can be significantly reduced by employing the multiple-airfoil concept [7]. To better show this, the general airfoil is simplified to a flat plate airfoil. Thus, $c_l$ is the same as in Eq. 2.4 while $c_d$ can be simplified to

$$c_d = \frac{4\alpha_0^2}{\sqrt{M_\infty^2 - 1}} \quad (2.6)$$

This indicates that the lift coefficient $c_l$ of a general airfoil is proportional to the angle of attack while the drag coefficient $c_d$ of a general airfoil is proportional to the square of the angle of attack.

![Wave reduction effect of the biplane airfoil](image)

Figure 2.3 Wave reduction effect of the biplane airfoil [17]

In the case of cascade of airfoils which consists of $n$ parallel flat plates with the same chord length $c$ as the single flat plate airfoil shown in Figure 2.3, the angle of attack $\alpha_n$ is set at $\alpha_n = \alpha_0/n$ since the total lift of this $n$-plate cascade of airfoils $c_l_n$ is set to be the same as that of the single plate airfoil $c_{ls}$ as described by Eq. (2.7).
The wave drag of this $n$-plate cascade of airfoils $c_{dn}$ equals to the summation of the wave drag of all of the plates:

$$c_{dn} = n \cdot \frac{4 \alpha_n}{\sqrt{M_{\infty}^2 - 1}} = n \cdot \frac{4 \alpha_0}{n \sqrt{M_{\infty}^2 - 1}} = c_{ds}$$  \hspace{1cm} (2.7)$$

It can been seen from Eq. (2.8) that the $n$-plate cascade of airfoils produces $\frac{1}{n}$ of the wave drag compared to the single flat plate airfoil while having the same amount of lift (Eq. (2.7)).

In summary, for a combined cascade of multiple-airfoils configuration, the drag caused by the angle of attack or lift effect is reduced since the effective angle of attack or lift effect of each individual airfoil is reduced while the total lift of the combined cascade of multiple-airfoils remains the same. This reduction in wave drag is called the “Wave Reduction Effect” of a combined cascade of multiple-airfoils configuration [7].

\[2.2.2 \text{ Wave Cancellation Effect}\]

The Busemann-type biplane configuration significantly reduces the drag due to thickness as well. By adjusting the distance $\Delta z$ between the upper and lower components of the airfoil at a specific Mach
number, the strong shock wave generated at the leading edge of one of the components (upper or lower) of the airfoil reaches exactly at the mid-point of the opposite component and is cancelled by the expansion wave generated at that point. Theoretically speaking, the shock wave is completely cancelled, and thus no wave drag due to thickness of the airfoil is produced, as shown in Figure 2.4.

![Figure 2.4 Wave cancellation effect [17]](image)

However, because of the entropy increase caused by the shock wave generated between the upper and lower components of the airfoil and due to the nonlinear effects during the shock-expansion process, the wave drag cannot be completely eliminated, and therefore the theoretical zero-drag condition cannot be practically realized [19,20]. At other off-design Mach numbers, the shock wave is cancelled only partially, as shown in Figure 2.5. Thus, the wave drag of the Busemann biplane airfoil does not become zero under off-design conditions.
2.3 Flow Field Simulation of the Standard Diamond-Shaped Airfoil and the Baseline Busemann Biplane Airfoil under Design and Off-design Conditions

The flow fields of both the standard diamond-shaped airfoil and the Busemann biplane airfoil are computed at zero angle of attack (the zero-lift condition). For meaningful comparison, the total thickness of the two airfoils is set to be equal. For this specific case under consideration, the thickness to chord ratio of the diamond-shaped airfoil is $t/c = 0.1$, while the thickness to chord ratio of the Busemann airfoil is $t/c = 0.05$ for both its upper and lower components. The distance between the upper and lower components of the Busemann airfoil is set to be one half of the chord length to obtain the theoretical minimum drag under the design condition (Mach number $M = 1.7$). The angle of attack is set to be zero since we first consider the non-lifting design and off-design conditions by varying the Mach number below 1.7 and greater than 1.7.
2.3.1 **Mesh Generation**

The commercial meshing software ANSYS ICEM CFD is used to generate the mesh for computing the flow field. The far field boundary is set at 21-chord-length by 20.5-chord-length rectangle from the center of the airfoil as shown in Figure 2.6. Figure 2.7 and Figure 2.8 show the H mesh configuration generated for the simulations. There are 64 nodes around the two components both horizontally and vertically. In between the two components, the grid has a dimension of 32 nodes (vertically) × 64 nodes (horizontally). An ICEM replay script file is created to automatically generate the mesh for the flow past different airfoil shapes once it is called.

![Figure 2.6 Computational domain for the Busemann biplane airfoil configuration](image)

Figure 2.6 Computational domain for the Busemann biplane airfoil configuration
Figure 2.7 H-mesh generated in the computational domain around the Busemann biplane airfoil

Figure 2.8 Zoomed-in view of H-mesh generated in computational domain around the Busemann biplane airfoil
2.3.2 Steady-State Flow Field Simulation at $M < 1.7$

As has been discussed above, the baseline Busemann biplane airfoil has a much lower drag under its design condition with Mach number $M = 1.7$. However, it has a poor aerodynamic performance under off-design conditions, which may cause much higher drag compared to the standard diamond-shaped airfoil. The configurations of both the standard diamond-shaped airfoil and the baseline Busemann biplane airfoil are shown in Figure 2.9.

Figure 2.9 Configurations of standard diamond-shaped airfoil (left) and baseline Busemann biplane airfoil (right)

Figure 2.10 shows the drag coefficient $c_d$ of both the standard diamond-shaped airfoil and the baseline Busemann biplane airfoil under zero-lift condition over a range of Mach numbers ($0.3 \leq M \leq 3.3$). The simulations are performed using ANSYS FLUENT; the flow field is initialized with an impulsive uniform flow.
Figure 2.10 $c_d$ comparison of two different airfoils under non-lifting condition

As shown in Figure 2.10, the baseline Busemann biplane airfoil has higher drag compared to the standard diamond-shaped airfoil when the Mach number is low ($0.3 \leq M \leq 1.6$). In the range $1.7 \leq M \leq 2.7$ however, the drag generated by the baseline Busemann biplane airfoil is smaller than that of the standard diamond-shaped airfoil, especially at $M = 1.7$ which is the design condition for the Busemann airfoil. Table 2.1 shows a detailed comparison of the drag coefficient $c_d$ of these two airfoils for the range of Mach numbers $0.3 \leq M \leq 3.3$. 

![Cd Comparison Diagram](image)
Table 2.1 Drag coefficient $c_d$ for diamond-shaped airfoil and Busemann airfoil over a range of Mach numbers

<table>
<thead>
<tr>
<th>M</th>
<th>$c_d$ (Diamond)</th>
<th>$c_d$ (Busemann)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
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<td>4.45E-03</td>
</tr>
<tr>
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<tr>
<td>3.3</td>
<td>1.29E-02</td>
<td>1.28E-02</td>
</tr>
</tbody>
</table>

Both Figure 2.10 and Table 2.1 demonstrate the advantage of the Busemann biplane airfoil since it produces much lower drag near its design condition ($M = 1.7$) due to the wave reduction and wave cancellation effects. The simulations shown above demonstrate that the baseline Busemann biplane airfoil has a very good performance at its design condition ($M = 1.7$), while it has much higher drag at off-design conditions due to the choked-flow phenomenon at lower Mach numbers ($M < 1.7$).
Figure 2.1 through Figure 2.14 show the flow field around the baseline Busemann biplane airfoil under two different off-design conditions both of which result in high drag but due to different reasons. At $M = 0.8$, as shown in Figure 2.11, the flow between the two airfoil components reaches Mach one at the midchord of the airfoil and then increases further downstream to supersonic. After the trailing edge of the airfoil however, the flow speed again drops to subsonic and hence forms a vertical shock wave. The supersonic flow in the rear part of the airfoil (after the mid-point) creates the low-pressure region, as shown in Figure 2.12, which leads to higher wave drag ($c_d = 0.148$) at this off-design condition of $M = 0.8$. At $M = 1.6$ however, as shown in Figure 2.13, the flow field is very different from the $M = 0.8$ subsonic condition discussed above. A bow shock is formed in front of the leading edge of the airfoil. The flow speed drops to subsonic and a high-pressure region, as shown in Figure 2.14, is created which again generates a high drag ($c_d = 0.0926$).
Figure 2.12 Pressure contours around the Busemann airfoil at $M = 0.8$

Figure 2.13 Velocity contours around the Busemann airfoil at $M = 1.6$
2.3.3 Flow Field Simulation during Acceleration and Deceleration

Besides the high drag been generated under off-design conditions, there is an even worse problem caused by the flow-hysteresis phenomenon during acceleration and choked-flow phenomenon during deceleration that need to be addressed under off-design conditions. To demonstrate these phenomena, flow field simulations are conducted both under acceleration and deceleration using the previous simulation results shown in Figure 2.10 as the initial condition. Figure 2.15 shows two separated $c_d$ curves during acceleration and deceleration of the biplane airfoil. As been shown in Figure 2.15, the change in acceleration and deceleration $c_d$ curves occurs in the range $M = 1.6$ to $M = 2.1$, where the blue dash line represents the $c_d$ of the Busemann biplane airfoil during acceleration and the red solid line represents the $c_d$ of the Busemann biplane airfoil during deceleration. This separation between the two $c_d$ curves is caused by the flow-hysteresis phenomenon during acceleration and the choked-flow phenomenon during deceleration. Therefore,
in order to minimize the difference in the drag coefficients of the Busemann biplane airfoil during acceleration and deceleration as well as to significantly decrease the drag, shape optimization of the Busemann airfoil is conducted using SOGA to eliminate the flow-hysteresis phenomenon during acceleration and the choked-flow phenomenon during deceleration; it is discussed in Chapter 3. Table 2.2 compares the $c_d$ for diamond-shaped airfoil and Busemann airfoil under acceleration and deceleration at various Mach numbers.

Figure 2.15 $c_d$ plots for diamond-shaped airfoil and Busemann airfoil during acceleration and deceleration under non-lifting condition.
2.3.3.1 Flow Field of Busemann Biplane Airfoil during Acceleration

In this section, the flow-hysteresis phenomenon during acceleration is examined for the baseline Busemann biplane airfoil under non-lifting condition. The pressure coefficient contours for flow past the Busemann airfoil during acceleration are shown in Figure 2.16. To simulate the non-lifting condition, the angle of attack is set to zero. The resulting non-lifting flow field around the Busemann airfoil is shown in Figure 2.16 at various supersonic Mach numbers ranging from $M = 1.7$ to $M = 2.13$. As shown in Figure 2.16, the bow shock exists in front of the Busemann airfoil and does not disappear until the Mach number reaches 2.13. It can also be noticed that there is a subsonic region behind the bow shock between the upper and lower components of the airfoil where the pressure coefficients are high. The result of the presence of this bow shock in front of the airfoil is a substantial increase in drag compared to that at the design condition. However, when the Mach number increases from 2.12 to 2.13, the bow shock is swallowed into the upper and lower components of the Busemann airfoil and is replaced by two oblique shock waves, and the subsonic region between the two airfoil components finally disappears as shown in Figure 2.16(f). The drag coefficient of the airfoil also decreases dramatically and the flow past the airfoil develops into a state similar to that under the design condition. Figure 2.16 illustrates the poor performance of the baseline Busemann airfoil under off-design conditions. During acceleration, the design condition cannot be achieved at Mach number 1.7. And the drag coefficient at March number 1.7 is much higher ($c_d = 0.0923$) compared to the standard diamond-shaped airfoil ($c_d = 0.0292$). Furthermore due to the flow-hysteresis phenomenon, the drag coefficients $c_d$ during acceleration and deceleration are different as shown in Figure 2.15 and Table 2.2.
Table 2.2 Drag Coefficients $c_d$ for Busemann airfoil and standard diamond airfoil at various Mach numbers

<table>
<thead>
<tr>
<th>Busemann Acceleration</th>
<th>Busemann Deceleration</th>
<th>Standard Diamond</th>
</tr>
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<tbody>
<tr>
<td>M</td>
<td>Cd</td>
<td>M</td>
</tr>
<tr>
<td>0.3</td>
<td>4.45E-03</td>
<td>0.3</td>
</tr>
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<td>3.3</td>
<td>1.28E-02</td>
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</tr>
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</table>
Figure 2.16 $c_p$-contours around the Busemann biplane airfoil with zero-lift during acceleration.
2.3.3.2 Flow Field of Busemann Biplane Airfoil during Deceleration

In this section, the choked-flow phenomenon of the baseline Busemann biplane airfoil during deceleration is examined. The contours of pressure coefficients for flow past the Busemann airfoil during deceleration are shown in Figure 2.17. To simulate the non-lifting condition, the angle of attack is set to zero as before. The resulting non-lifting flow field around the Busemann airfoil is shown in Figure 2.17 at various supersonic Mach numbers ranging from $M = 1.8$ to $M = 1.62$. As shown in Figure 2.17, a different flow field appears within the small range near the design Mach number ($M = 1.7$); a relatively high drag coefficient still occurs as the Mach number further decreases during deceleration. A strong bow shock is formed in front of the airfoil when the Mach number drops from 1.63 to 1.62, while the drag coefficient increases dramatically from 0.005594 to 0.0926, which is substantially higher than that of the standard diamond-shaped airfoil ($c_d = 0.03158$). The flow between the two components of the airfoil is choked at the location of the maximum thickness of the Busemann airfoil and a subsonic region is formed. This is also a clear indication of the poor performance of the baseline Busemann biplane airfoil at off-design conditions since the drag coefficient of the Busemann airfoil is much higher than that of the standard diamond-shaped airfoil for $M \leq 1.62$.

In conclusion, the baseline Busemann biplane airfoil produces a substantially higher drag in the low Mach number range (below the design Mach number of $M = 1.7$). Additionally, it is necessary to accelerate the Busemann biplane airfoil to a much higher Mach number ($M = 2.13$) to reach the shockwave-swallowing state, while producing dramatically higher drag and largely decreasing its efficiency, and then decelerate to a lower velocity to achieve the design condition at Mach number.
$M = 1.7$. As a result, the baseline Busemann biplane airfoil design needs to be modified and optimized to avoid or at least reduce the high drag coefficient caused by the flow-hysteresis phenomenon during acceleration and the choked-flow phenomenon during deceleration.

![Flow contours around the Busemann biplane airfoil with zero-lift during deceleration for different Mach numbers.](image)

- **a)** $M_\infty = 1.8, c_d = 0.003638$
- **b)** $M_\infty = 1.7, c_d = 0.002183$
- **c)** $M_\infty = 1.65, c_d = 0.004144$
- **d)** $M_\infty = 1.64, c_d = 0.004778$
- **e)** $M_\infty = 1.63, c_d = 0.005594$
- **f)** $M_\infty = 1.62, c_d = 0.009261$

Figure 2.17 $c_p$-contours around the Busemann biplane airfoil with zero-lift during deceleration
Chapter 3

Optimization of Busemann-Type Biplane Airfoil

In this chapter, the shape optimization procedure for the baseline Busemann biplane airfoil using the genetic algorithms (GA) and the results of the optimized Busemann-type airfoil under both non-lifting and lifting conditions are presented.

3.1 Genetic-Algorithm-Based Optimization Procedure

The optimization process is established by coupling a Single-Objective Genetic Algorithm (SOGA) or a Multi-Objective Genetic Algorithm (MOGA) based optimization method with the mesh generation software ANSYS-ICEM and the CFD solver ANSYS-FLUENT as shown in Fig. 3.1.

![Figure 3.1 Schematic of information flow in the optimization process](image_url)
Every individual (airfoil) in each generation of SOGA/MOGA is represented by a set of control points, which randomly generate the airfoil shape by using the Bezier Curves. The mesh generation software ICEM is used to generate a two-dimensional structured mesh around the airfoil as an input to the CFD solver FLUENT, which is then used to calculate the supersonic inviscid flow fields for specific flow conditions. Based on the fitness values of all airfoil shapes in a given generation, SOGA/MOGA is applied to create the next generation of airfoils and this whole process is repeated until the optimal fitness value is obtained. The airfoil shape that corresponds to the optimal fitness value is the final shape of the optimized airfoil [18].

3.1.1 Overview of Genetic Algorithm

Genetic algorithms (GAs) are a class of stochastic optimization algorithms inspired by the biological evolution [20]. They efficiently exploit historical information to speculate on new offspring with improved performance [21]. For the Busemann biplane airfoil in particular, genetic algorithm is used to generate new shapes that produce much lower drag by minimizing the flow-hysteresis phenomenon during acceleration and the choked-flow phenomenon during deceleration. Generally, the GA employs the following steps to complete the optimization process:

1. Initialization: Randomly generates a group of individuals.
2. Evaluation: Evaluates the fitness of each individual generated.
3. Natural selection: Individuals that have the lowest fitness are removed.
4. Reproduction: Pairs of the individuals are picked to produce the offspring, which is often done by roulette wheel sampling. A crossover function is then used to produce the offspring.
5. Mutation: Randomly modifies some small percentage of the population.
6. Check for convergence: If the current generation has converged, the individual having the best fitness will be returned. Otherwise, the process will be repeated starting from step 2 until the pre-defined tolerance criteria for acceptable change in fitness between generations is met.

3.2 Shape Optimization of Busemann Biplane Airfoil under Nonlifting Condition

3.2.1 Application of Single-Objective Genetic Algorithm (SOGA)

For the non-lifting condition, where the lift coefficient \( c_l = 0 \), a Single-Objective Genetic Algorithm (SOGA) is employed for the shape optimization of the Busemann biplane airfoil. The single objective to be achieved is to minimize the drag coefficient \( c_d \). Multiple design points (Mach numbers) are used during the optimization process due to the flow-hysteresis and choked-flow phenomena caused by the baseline Busemann biplane airfoil during acceleration and deceleration respectively. For the multi-point optimization, the fitness function employed is a weighted average of the drag coefficients \( c_d \), which can be written in the form:

\[
I = \frac{\sum_{i=1}^{n} w_i l_i}{\sum_{i=1}^{n} w_i} \tag{3.1}
\]

where \( I \) is the weighted average drag coefficient and “\( i \)” denotes a design point (related to Mach number). For the optimization process for nonlifting condition, an evenly weighted average of drag coefficients \( c_d \) at different Mach numbers is employed. Therefore Eq. (3.1) reduces to
Since the wave drag of the Busemann biplane airfoil is much lower when the flow is unchoked, it is highly desirable that the strong bow shock wave in front of the airfoil is swallowed into the area between the upper and lower components of the airfoil before the flow speed approaches the design Mach number \((M = 1.7)\). Although a higher weight can be assigned to the most important design Mach number \((M = 1.7)\) to produce a lower drag at that design point, previous research conducted by Hu et al. [17] has shown that a slightly higher drag coefficient \(c_d\) is obtained at lower Mach numbers during acceleration and deceleration, as given in Table 3.1 and Table 3.2 respectively if such an uneven weighting method is used.

Table 3.1 \(c_d\) comparison at zero lift condition during acceleration (1 count = 0.0001) [17]

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<td>906</td>
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<td>419</td>
<td>376</td>
<td>332</td>
<td>112</td>
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<tr>
<td>Optimized (uneven)</td>
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<td>486</td>
<td>428</td>
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<td>336</td>
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<td>101</td>
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</table>

Table 3.2 \(c_d\) comparison at zero lift condition during deceleration (1 count = 0.0001) [17]

<table>
<thead>
<tr>
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<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
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<tbody>
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<td>889</td>
<td>906</td>
<td>928</td>
<td>957</td>
<td>996</td>
<td>1050</td>
</tr>
<tr>
<td>Optimized (even)</td>
<td>106</td>
<td>112</td>
<td>127</td>
<td>152</td>
<td>419</td>
<td>473</td>
<td>527</td>
</tr>
<tr>
<td>Optimized (uneven)</td>
<td>101</td>
<td>107</td>
<td>122</td>
<td>146</td>
<td>428</td>
<td>486</td>
<td>539</td>
</tr>
</tbody>
</table>

Here, we have a total of seven design points in the Mach numbers range \(M = 1.1\) to \(M = 1.7\), for both acceleration and deceleration. The reason for picking these seven design points is that this
range of Mach numbers, \( M = 1.1 \) to \( M = 1.7 \), provides the critical region before the Mach number increases to the design condition \( M = 1.7 \); it is the region in which we want to keep the drag coefficient as low as possible. Both acceleration and deceleration scenarios are considered in order to reduce the flow-hysteresis phenomenon and the choked flow phenomenon respectively.

A code in MATLAB package is developed and utilized in the optimization process of the airfoil as well as for ICEM meshing and FLUENT flow field calculations. All SOGA parameters are defined based on the GA methodology, as shown in Table 3.3.

<table>
<thead>
<tr>
<th>GA Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generation Size</td>
<td>8 individuals per generation</td>
</tr>
<tr>
<td>Number of Generations</td>
<td>Maximum of 50 generations if convergence not obtained</td>
</tr>
<tr>
<td>Number of Design Variables</td>
<td>14 in total, 7 for acceleration and 7 for deceleration</td>
</tr>
<tr>
<td>Selection Type</td>
<td>Roulette Wheel Selection</td>
</tr>
<tr>
<td>Crossover Rate</td>
<td>0.7</td>
</tr>
<tr>
<td>Mutation Rate</td>
<td>0.1</td>
</tr>
<tr>
<td>Error of Mutation Constant</td>
<td>0.8, which determines how much mutation affects the curves as generations proceed</td>
</tr>
</tbody>
</table>

### 3.2.2 Airfoil Parameterization

The shape of the airfoil is generated by using the Bezier curves (third-order polynomials). Bezier curves are frequently used in computer graphics to produce curves which appear to be reasonably smooth at all scales. One of the main reasons that Bezier curves are used in computer graphics is that they can be efficiently constructed; each Bezier curve is simply defined by a set of control points [22].
For the non-lifting case in specific, with \( c_l = 0 \), since the upper and lower components of the airfoil are symmetric with respect to the horizontal axis, only the upper component of the airfoil needs to be defined; the lower component is the mirror image with respect to the horizontal axis. For the upper component of the airfoil, there are two lines that need to be drawn to define its shape. Since the thickness distribution for the entire airfoil remains the same as that of the baseline Busemann biplane airfoil, the \( y \)-coordinates of the lower line are defined by Eq. (3.3) and Eq. (3.4). Thus the upper line (which is a straight horizontal line for the baseline Busemann airfoil) is the only line that needs to be generated by the Bezier curves in order to define the shape of the whole biplane airfoil.

\[
y_{\text{low}} = y_{up} - 0.05 - 0.1 \times x_{up} \quad (-0.5 \leq x_{up} \leq 0) \tag{3.3}
\]
\[
y_{\text{low}} = y_{up} - 0.05 + 0.1 \times x_{up} \quad (0 \leq x_{up} \leq 0.5) \tag{3.4}
\]

In Eq. (3.3) and Eq. (3.4), the subscripts ‘low’ and ‘up’ correspond to the lower and upper line of the airfoil; the origin is at the center of the upper line. Two Bezier curves are used to generate the shape-defining upper line. Each Bezier curve is defined by a set of four control points. Each control point is constrained by a specified range of \( x \)- and \( y \)-coordinates as shown in Table 3.4. Figure 3.2 shows a randomly generated Busemann-type biplane airfoil shape using Bezier curves. Figure 3.3 and Figure 3.4 show the detailed geometry of Bezier curve #1 and Bezier curve #2 along with their control points.
Table 3.4 Coordinate range for each control point of the Bezier curves for the airfoil under nonlifting condition

<table>
<thead>
<tr>
<th>Bezier curve #1</th>
<th>Coordinate</th>
<th>Upper Limit</th>
<th>Lower Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>Fixed at x=-0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x2</td>
<td>-0.05</td>
<td>-0.5</td>
<td></td>
</tr>
<tr>
<td>x3</td>
<td>Fixed at x=-0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x4</td>
<td>Fixed at x=0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y1</td>
<td>0.25</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>y2</td>
<td>0.25</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>y3</td>
<td>Fixed at y=0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y4</td>
<td>Fixed at y=0.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bezier curve #2</th>
<th>Coordinate</th>
<th>Upper Limit</th>
<th>Lower Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>Fixed at x=0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x2</td>
<td>Fixed at x=0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x3</td>
<td>0.5</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>x4</td>
<td>Fixed at x=0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y1</td>
<td>Fixed at y=0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y2</td>
<td>Fixed at y=0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y3</td>
<td>0.25</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>y4</td>
<td>0.25</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.2 Randomly generated Busemann-type biplane airfoil shape using Bezier curves
Figure 3.3 Detailed geometry of Bezier curve #1 and its control points

Figure 3.4 Detailed geometry of Bezier curve #2 and its control points
### 3.2.3 Optimization Results

After implementing SOGA for 20 generations with 8 individuals in each generation, an optimal shape result for symmetric Busemann-type biplane airfoil under non-lifting condition with minimum drag is obtained. Figure 3.5 shows the geometry of the original Busemann biplane airfoil (red) and the optimized Busemann biplane airfoil (blue) under nonlifting condition. Figure 3.6 and Figure 3.7 show the two final Bezier curves used to generate the optimized Busemann biplane airfoil shape along with their control points.

![Geometry of both the original and optimized Busemann biplane airfoil under nonlifting condition](image-url)
Figure 3.6 Bezier curve #1 and its control points for the optimized Busemann biplane airfoil under nonlifting condition

Figure 3.7 Bezier curve #2 and its control points for the optimized Busemann biplane airfoil under nonlifting condition
The drag coefficients for the seven design points are compared in Table 3.5 and Table 3.6 for both the original and optimized Busemann biplane airfoil under nonlifting condition during acceleration and deceleration respectively. As shown in Table 3.5 and Table 3.6, the baseline Busemann biplane airfoil is choked at all Mach numbers within the optimization range; while the optimized Busemann biplane airfoil unchokes at $M = 1.5$ during acceleration and chokes at $M = 1.3$ during deceleration. Even under choked conditions during both acceleration and deceleration, the optimized Busemann biplane airfoil has significantly lower drag compared to the baseline Busemann biplane airfoil. The only point where the optimized Busemann biplane airfoil has a higher drag compared to the original airfoil is at $M = 1.7$ during deceleration.

Table 3.5 $c_d$ comparison for the original and optimized Busemann biplane airfoil at nonlifting condition during acceleration (1 count = 0.0001)

<table>
<thead>
<tr>
<th>Mach Number</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>1097</td>
<td>1040</td>
<td>999</td>
<td>967</td>
<td>943</td>
<td>926</td>
<td>923</td>
</tr>
<tr>
<td>Optimized (GA)</td>
<td>622</td>
<td>506</td>
<td>428</td>
<td>389</td>
<td>132</td>
<td>114</td>
<td>104</td>
</tr>
</tbody>
</table>

Table 3.6 $c_d$ comparison for the original and optimized Busemann biplane airfoil at nonlifting condition during deceleration (1 count = 0.0001)

<table>
<thead>
<tr>
<th>Mach Number</th>
<th>1.7</th>
<th>1.6</th>
<th>1.5</th>
<th>1.4</th>
<th>1.3</th>
<th>1.2</th>
<th>1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>32</td>
<td>926</td>
<td>940</td>
<td>967</td>
<td>999</td>
<td>1040</td>
<td>1098</td>
</tr>
<tr>
<td>Optimized (GA)</td>
<td>104</td>
<td>114</td>
<td>132</td>
<td>163</td>
<td>428</td>
<td>506</td>
<td>621</td>
</tr>
</tbody>
</table>

Figures 3.8 – 3.21 show the change in the pressure coefficient $c_p$ around the original Busemann biplane airfoil as the Mach number increases and decreases within the design-points range. The corresponding $c_p$ contours for the optimized Busemann biplane airfoil are shown in Figures 3.22 – 3.35 to illustrate the wave-cancelling effect.
Figure 3.8 $c_p$ contours around the original Busemann biplane airfoil at $M = 1.1$ during acceleration

Figure 3.9 $c_p$ contours around the original Busemann biplane airfoil at $M = 1.2$ during acceleration
Figure 3.10 $c_p$ contours around the original Busemann biplane airfoil at $M = 1.3$ during acceleration

Figure 3.11 $c_p$ contours around the original Busemann biplane airfoil at $M = 1.4$ during acceleration
Figure 3.12 $c_p$ contours around the original Busemann biplane airfoil at $M = 1.5$ during acceleration.

Figure 3.13 $c_p$ contours around the original Busemann biplane airfoil at $M = 1.6$ during acceleration.
Figure 3.14 $c_p$ contours around the original Busemann biplane airfoil at $M = 1.7$ during acceleration

Figure 3.15 $c_p$ contours around the original Busemann biplane airfoil at $M = 1.7$ during deceleration
Figure 3.16 $c_p$ contours around the original Busemann biplane airfoil at $M = 1.6$ during deceleration.

Figure 3.17 $c_p$ contours around the original Busemann biplane airfoil at $M = 1.5$ during deceleration.
Figure 3.18 $c_p$ contours around the original Busemann biplane airfoil at $M = 1.4$ during deceleration.

Figure 3.19 $c_p$ contours around the original Busemann biplane airfoil at $M = 1.3$ during deceleration.
Figure 3.20 $c_p$ contours around the original Busemann biplane airfoil at $M = 1.2$ during deceleration

Figure 3.21 $c_p$ contours around the original Busemann biplane airfoil at $M = 1.1$ during deceleration
Figure 3.22 $c_p$ contours around the optimized Busemann biplane airfoil at $M = 1.1$ during acceleration

Figure 3.23 $c_p$ contours around the optimized Busemann biplane airfoil at $M = 1.2$ during acceleration
Figure 3.24 $c_p$ contours around the optimized Busemann biplane airfoil at $M = 1.3$ during acceleration

Figure 3.25 $c_p$ contours around the optimized Busemann biplane airfoil at $M = 1.4$ during acceleration
Figure 3.26 $c_p$ contours around the optimized Busemann biplane airfoil at $M = 1.5$ during acceleration

Figure 3.27 $c_p$ contours around the optimized Busemann biplane airfoil at $M = 1.6$ during acceleration
Figure 3.28 $c_p$ contours around the optimized Busemann biplane airfoil at $M = 1.7$ during acceleration

Figure 3.29 $c_p$ contours around the optimized Busemann biplane airfoil at $M = 1.7$ during deceleration
Figure 3.30 $c_p$ contours around the optimized Busemann biplane airfoil at $M = 1.6$ during deceleration

Figure 3.31 $c_p$ contours around the optimized Busemann biplane airfoil at $M = 1.5$ during deceleration
Figure 3.32 $c_p$ contours around the optimized Busemann biplane airfoil at $M = 1.4$ during deceleration

Figure 3.33 $c_p$ contours around the optimized Busemann biplane airfoil at $M = 1.3$ during deceleration
Figure 3.34 $c_p$ contours around the optimized Busemann biplane airfoil at $M = 1.2$ during deceleration.

Figure 3.35 $c_p$ contours around the optimized Busemann biplane airfoil at $M = 1.1$ during deceleration.
Figure 3.36 shows the comparison of the drag coefficients for the standard diamond-shaped airfoil, the baseline Busemann biplane airfoil and the optimized Busemann biplane airfoil under nonlifting condition. As shown in the figure, the separation between the acceleration and deceleration lines for $c_d$ still exists for the optimized Busemann biplane airfoil, which means that the flow-hysteresis and the choked-flow effects are not totally eliminated. However, as clearly shown in Figure 3.36, the flow-hysteresis area has been significantly reduced and the drag increase during deceleration due to the choked-flow phenomenon is much smaller than that for the original Busemann biplane airfoil.

The drag of the optimized Busemann biplane airfoil in the subsonic region is also smaller than that of the original Busemann biplane airfoil, although it is slightly higher than that of the standard diamond-shaped airfoil for $0.6 < M < 0.85$. For both subsonic and supersonic conditions, the optimized Busemann biplane airfoil has been able to significantly reduce the wave drag compared to the original Busemann biplane airfoil. At the design condition Mach number of 1.7 however, the drag coefficient of the optimized Busemann biplane airfoil ($c_d = 0.01038$) is much higher than that of the original Busemann biplane airfoil ($c_d = 0.002182$). This is due to the fact that for the shape optimization, our focus has been reducing the flow-hysteresis and choked-flow effects and we chose to assign equal weights to all Mach numbers used as the multiple design points. To address this problem, we could have put more weight on the design condition ($M = 1.7$) during the optimization process.
Next we examine the details of the flow field during acceleration and deceleration for the optimized Busemann biplane airfoil and compare them with the original baseline Busemann biplane airfoil and the optimization results obtained by Hu et al. [17] using an adjoint-based optimization technique. Figure 3.37 and Figure 3.38 show the pressure coefficient contours of the optimized Busemann biplane airfoil under acceleration and deceleration respectively. Figure 3.39 and Figure 3.40 show the pressure coefficient contours of the optimized Busemann biplane airfoil using the adjoint-based technique [17] under acceleration and deceleration respectively. During acceleration, the flow-hysteresis effect still exists and a bow shock wave is formed in front of the airfoil. The swallowing of the bow shock wave happens when the Mach number increases from 1.49 to 1.50 in our GA optimization as shown in Figure 3.37(f); it happens when Mach number increases from 1.52 to 1.53 in the adjoint-based optimization as shown in Figure 3.39(e) and it happens when Mach number
increases from 2.12 to 2.13 for the original Busemann biplane airfoil as shown in Figure 2.16(f). The drag coefficient decreases from 0.03556 to 0.01316 in the present GA based optimization and from 0.03336 to 0.01221 in the adjoint-based optimization [17].

Figure 3.37 $c_p$ contours of the GA-optimized Busemann biplane airfoil with zero-lift during acceleration.
During deceleration, the choked-flow effect still exists, however, it is shifted to a lower Mach number of 1.36 in the GA optimization and 1.37 in the adjoint-based optimization; it occurs at Mach number of 1.6 in the original baseline Busemann airfoil.

Figure 3.38 $c_p$-contours of the GA-optimized Busemann biplane airfoil with zero-lift during deceleration
Figure 3.39 $c_p$-contours of the Adjoint-based-optimized Busemann biplane airfoil with zero-lift during acceleration [17]
Figure 3.40 $c_p$-contours of the Adjoint-based-optimized Busemann biplane airfoil with zero-lift during deceleration [17]
In conclusion, as shown in Figure 3.41, the drag coefficient of the GA-optimized Busemann biplane airfoil is significantly reduced comparing to the original Buseman biplane airfoil and it matches with the adjoint-based optimization result obtained by Hu et al. [17].
3.3 Shape Optimization of Busemann Biplane Airfoil under Lifting Condition

3.3.1 Application of Multi-objective Genetic Algorithm (MOGA)

For the lifting condition with the lift coefficient $c_l \neq 0$, a Multi-Objective Genetic Algorithm (MOGA) is employed for shape optimization of the Busemann biplane airfoil. The two objectives to be achieved are to minimize the drag coefficient $c_d$ while maximizing the lift coefficient $c_l$. Similar to that for the nonlifting condition described in section 3.2, a total of seven design points ranging from $M = 1.1$ to $M = 1.7$ are used during the optimization process. For the fitness functions for the lifting case, we use the sum of evenly-weighted average of both $c_d$ and $c_l$. The GA parameters used for the lifting case are listed in Table 3.7.

<table>
<thead>
<tr>
<th>GA Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generation Size</td>
<td>8 individuals per generation</td>
</tr>
<tr>
<td>Number of Generations</td>
<td>Maximum of 50 generations if convergence not obtained</td>
</tr>
<tr>
<td>Number of Design Variables</td>
<td>28 in total, 14 (7 for $c_d$ &amp; 7 for $c_l$) for acceleration and 14 (7 for $c_d$ &amp; 7 for $c_l$) for deceleration</td>
</tr>
<tr>
<td>Selection Type</td>
<td>Roulette Wheel Selection</td>
</tr>
<tr>
<td>Crossover Rate</td>
<td>0.7</td>
</tr>
<tr>
<td>Mutation Rate</td>
<td>0.1</td>
</tr>
<tr>
<td>Error of Mutation Constant</td>
<td>0.8, which determines how much mutation affects the curves as generations go on</td>
</tr>
</tbody>
</table>
3.3.2 Airfoil Parameterization

Similar to that for the nonlifting case, the random shapes of the airfoil are generated by using the Bezier curves with control points. For the lifting case with $c_l \neq 0$, since the upper and lower components of the biplane airfoil are not symmetric, both the upper and lower component need to be defined separately. The thickness distribution for both the upper and lower components is still kept the same as for the nonlifting case. Now, a total of four Bezier curves are needed to define the shape of the airfoil. The range of the $x$- and $y$-coordinates of the Bezier curves for the two components of the airfoil are listed in Table 3.8 and Table 3.9.

<table>
<thead>
<tr>
<th>Bezier curve #1</th>
<th>Coordinate</th>
<th>Upper Limit</th>
<th>Lower Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>Fixed at $x=-0.5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x2</td>
<td>-0.05</td>
<td>-0.5</td>
<td></td>
</tr>
<tr>
<td>x3</td>
<td>Fixed at $x=-0.05$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x4</td>
<td>Fixed at $x=0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y1</td>
<td>0.25</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>y2</td>
<td>0.25</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>y3</td>
<td>Fixed at $y=0.25$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y4</td>
<td>Fixed at $y=0.25$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bezier curve #2</th>
<th>Coordinate</th>
<th>Upper Limit</th>
<th>Lower Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>Fixed at $x=0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x2</td>
<td>Fixed at $x=0.05$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x3</td>
<td>0.5</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>x4</td>
<td>Fixed at $x=0.5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y1</td>
<td>Fixed at $y=0.25$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y2</td>
<td>Fixed at $y=0.25$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y3</td>
<td>0.25</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>y4</td>
<td>0.25</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.9 Coordinate range for each control point of the Bezier curves for the lower component of the airfoil under lifting condition

<table>
<thead>
<tr>
<th>Bezier curve #3</th>
<th>Coordinate</th>
<th>Upper Limit</th>
<th>Lower Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>Fixed at x=-0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x2</td>
<td>-0.05</td>
<td>-0.5</td>
<td></td>
</tr>
<tr>
<td>x3</td>
<td>Fixed at x=-0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x4</td>
<td>Fixed at x=0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y1</td>
<td>-0.2</td>
<td>-0.25</td>
<td></td>
</tr>
<tr>
<td>y2</td>
<td>-0.2</td>
<td>-0.25</td>
<td></td>
</tr>
<tr>
<td>y3</td>
<td>Fixed at y=-0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y4</td>
<td>Fixed at y=-0.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bezier curve #4</th>
<th>Coordinate</th>
<th>Upper Limit</th>
<th>Lower Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>Fixed at x=0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x2</td>
<td>Fixed at x=0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x3</td>
<td>0.5</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>x4</td>
<td>Fixed at x=0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y1</td>
<td>Fixed at y=-0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y2</td>
<td>Fixed at y=-0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y3</td>
<td>-0.2</td>
<td>-0.25</td>
<td></td>
</tr>
<tr>
<td>y4</td>
<td>-0.2</td>
<td>-0.25</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.42 shows a randomly generated Busemann-type biplane airfoil shape using Bezier curves for the lifting condition. Figures 3.43 - 3.46 respectively show the detailed geometry of Bezier curves #1 to #4 along with their control points.
Figure 3.42 Randomly generated Busemann-type biplane airfoil shape using Bezier curves for lifting condition

Figure 3.43 Detailed geometry of Bezier curve #1 and its control points
Figure 3.44 Detailed geometry of Bezier curve #2 and its control points

Figure 3.45 Detailed geometry of Bezier curve #3 and its control points
3.3.3 Optimization Results

After implementing MOGA for 20 generations with 8 individuals in each generation, an optimal shape for asymmetric Busemann-type airfoil under lifting condition with maximum $C_L$ and minimum $C_d$ is obtained. Figure 3.47 shows the geometry of the original Busemann biplane airfoil (red) and the optimized Busemann biplane airfoil (blue) under lifting condition. Figures 3.48 - 3.51 show the four Bezier curves used to generate the optimized Busemann biplane airfoil shape along with their control points.
Figure 3.47 Geometry of both the original and optimized Busemann biplane airfoil under lifting condition

Figure 3.48 Bezier curve #1 and its control points for the optimized Busemann biplane airfoil under lifting condition
Figure 3.49 Bezier curve #2 and its control points for the optimized Busemann biplane airfoil under lifting condition

Figure 3.50 Bezier curve #3 and its control points for the optimized Busemann biplane airfoil under lifting condition
The drag coefficients for the seven design points are compared in Table 3.10 and Table 3.11 for both the original and optimized Busemann biplane airfoil under lifting condition. As shown in Table 3.10 and Table 3.11, the baseline Busemann biplane airfoil is choked at all Mach numbers within the optimization range; while the optimized Busemann biplane airfoil unchokes at $M = 1.6$ during acceleration and chokes at $M = 1.3$ during deceleration. Even under choked conditions during both acceleration and deceleration, the optimized Busemann biplane airfoil has significantly lower drag compared to the baseline Busemann biplane airfoil. Similar to the nonlifting condition, the only point where the optimized Busemann biplane airfoil has a higher drag compare to the original airfoil is at $M = 1.7$ during deceleration.
Table 3.10 $c_d$ comparison for the original and optimized Busemann biplane airfoil at lifting condition during acceleration (1 count = 0.0001)

<table>
<thead>
<tr>
<th>Mach Number</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>1097</td>
<td>1040</td>
<td>999</td>
<td>967</td>
<td>943</td>
<td>926</td>
<td>923</td>
</tr>
<tr>
<td>Optimized</td>
<td>683</td>
<td>633</td>
<td>529</td>
<td>461</td>
<td>418</td>
<td>148</td>
<td>136</td>
</tr>
</tbody>
</table>

Table 3.11 $c_d$ comparison for the original and optimized Busemann biplane airfoil at lifting condition during deceleration (1 count = 0.0001)

<table>
<thead>
<tr>
<th>Mach Number</th>
<th>1.7</th>
<th>1.6</th>
<th>1.5</th>
<th>1.4</th>
<th>1.3</th>
<th>1.2</th>
<th>1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>32</td>
<td>926</td>
<td>940</td>
<td>967</td>
<td>999</td>
<td>1040</td>
<td>1098</td>
</tr>
<tr>
<td>Optimized</td>
<td>136</td>
<td>148</td>
<td>173</td>
<td>224</td>
<td>529</td>
<td>633</td>
<td>682</td>
</tr>
</tbody>
</table>

Figures 3.52 – 3.65 show the change in the pressure coefficient $c_p$ around the optimized Busemann biplane airfoil under lifting condition as the Mach number increases and decreases within the design-point range. As can be seen from these figures, the bow shock wave in front of the airfoil disappears at $M = 1.6$ during acceleration and is not generated until the flow speed drops down to $M = 1.3$ during deceleration.

Figure 3.52 $c_p$ contours for the optimized Busemann biplane airfoil under lifting condition at $M = 1.1$ during acceleration.
Figure 3.53 $c_p$ contours for the optimized Busemann biplane airfoil under lifting condition at $M = 1.2$ during acceleration

Figure 3.54 $c_p$ contours for the optimized Busemann biplane airfoil under lifting condition at $M = 1.3$ during acceleration
Figure 3.5 $c_p$ contours for the optimized Busemann biplane airfoil under lifting condition at $M = 1.4$ during acceleration

Figure 3.6 $c_p$ contours for the optimized Busemann biplane airfoil under lifting condition at $M = 1.5$ during acceleration
Figure 3.5 $c_p$ contours for the optimized Busemann biplane airfoil under lifting condition at $M = 1.6$ during acceleration.

Figure 3.58 $c_p$ contours for the optimized Busemann biplane airfoil under lifting condition at $M = 1.7$ during acceleration.
Figure 3.5 $c_p$ contours for the optimized Busemann biplane airfoil under lifting condition at $M = 1.7$ during deceleration.

Figure 3.6 $c_p$ contours for the optimized Busemann biplane airfoil under lifting condition at $M = 1.6$ during deceleration.
Figure 3.61 $c_p$ contours for the optimized Busemann biplane airfoil under lifting condition at $M = 1.5$ during deceleration

Figure 3.62 $c_p$ contours for the optimized Busemann biplane airfoil under lifting condition at $M = 1.4$ during deceleration
Figure 3.63 $c_p$ contours for the optimized Busemann biplane airfoil under lifting condition at $M = 1.3$ during deceleration

Figure 3.64 $c_p$ contours for the optimized Busemann biplane airfoil under lifting condition at $M = 1.2$ during deceleration
Figure 3.6 shows the comparison of the drag coefficients for the standard diamond-shaped airfoil, the baseline Busemann biplane airfoil and the optimized Busemann biplane airfoil under lifting condition. As shown in the figure, the separation between the acceleration and deceleration $c_d$ lines still exists for the optimized Busemann biplane airfoil, which means that the flow-hysteresis and the choked-flow effects are not totally eliminated. However, as clearly shown in Figure 3.66, the flow-hysteresis area has been significantly reduced and the drag increase during deceleration due to the choked-flow phenomenon is much smaller than that for the original Busemann biplane airfoil. The drag of the optimized Busemann biplane airfoil in the subsonic region is smaller than that of the original Busemann biplane airfoil, although it is slightly higher than that of the standard diamond-shaped airfoil for $0.3 < M < 0.85$. For both subsonic and supersonic conditions, the optimized Buseman biplane airfoil has significantly reduced wave drag compared to the original Busemann biplane airfoil. Similar to the nonlifting condition, the drag coefficient of the optimized Busemann biplane airfoil at $M = 1.7$ ($c_d = 0.01362$) is higher than that of the original Busemann biplane airfoil ($c_d = 0.002182$).
Next we examine the details of the flow field in acceleration and deceleration conditions for the optimized Busemann biplane airfoil under lifting condition and compare them with the flow field of the original Busemann biplane airfoil. Figure 3.67 and Figure 3.68 show the pressure coefficient contours of the optimized Busemann biplane airfoil using GA under acceleration and deceleration respectively. During acceleration, the flow-hysteresis effect still exists and a bow shock wave is formed in front of the airfoil. The swallowing of the bow shock wave happens when the Mach number increases from 1.51 to 1.52 as shown in Figure 3.37(f) and from 2.12 to 2.13 for the original Busemann biplane airfoil as shown in Figure 2.16(f). The corresponding drag coefficient decreases from 0.04116 to 0.01665 for the optimized Busemann biplane airfoil.
Figure 3.6 $c_p$-contours of the GA-optimized Busemann biplane airfoil with lift during acceleration.
During deceleration, the choked-flow effect still exists; however, it is shifted to a lower Mach number of 1.38 compared to 1.6 for the original Busemann biplane airfoil.

Figure 3.68 $c_p$-contours of the GA-optimized Busemann biplane airfoil with lift during deceleration
In conclusion, the drag coefficient of the optimized Busemann biplane airfoil under lifting condition is significantly reduced compared to the original Buseman biplane airfoil as shown in Table 3.12.

Table 3.12 $c_l$ for the optimized Busemann biplane airfoil under lifting condition at different design points

<table>
<thead>
<tr>
<th>Mach Number</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_l$ (acceleration)</td>
<td>0.3319</td>
<td>0.3553</td>
<td>0.3061</td>
<td>0.2842</td>
<td>0.2858</td>
<td>0.2525</td>
<td>0.2713</td>
</tr>
<tr>
<td>$c_l$ (deceleration)</td>
<td>0.3306</td>
<td>0.3551</td>
<td>0.3064</td>
<td>0.3032</td>
<td>0.2678</td>
<td>0.2525</td>
<td>0.2712</td>
</tr>
</tbody>
</table>
Chapter 4

Conclusions

In this thesis, numerical simulations of the flow past the standard diamond-shaped airfoil and the baseline Busemann-type biplane airfoil have been conducted. An impulsive uniform flow, a flow during acceleration and a flow during deceleration are simulated. The original Busemann biplane airfoil has a poor performance at off-design conditions due to the flow-hysteresis phenomenon during acceleration and the choked-flow phenomenon during deceleration. For shape optimization, a Single-Objective Genetic Algorithm (SOGA) and a Multi-Objective Genetic Algorithm (MOGA) are employed to optimize the shape of the Busemann-type biplane airfoil under nonlifting and lifting condition respectively to improve its performance at off-design conditions. The commercially available CFD solver FLUENT is employed to calculate the flow field on an unstructured mesh generated using the ICEM software. A second-order accurate steady density-based solver in FLUENT is employed to compute the supersonic flow field. The optimization results for the nonlifting case show significant improvement in reducing the drag coefficient at off-design conditions for the optimized Buseman-type biplane airfoil shape compared to the original shape. The flow-hysteresis phenomenon during acceleration and the choked-flow phenomenon during deceleration are both alleviated significantly for the optimized shape. For the lifting case, the optimized Busemann biplane airfoil is able to significantly reduce the drag coefficient at off-design conditions while generating lift at the same time.
Appendix A

Mother Function for Optimization Code

% This is a simple multi-objective optimization code that can be used for
% CFD projects here in the lab. It can be adjusted for single or
dual-objective optimization, and is fully automated. Make sure you know
% the basics of a GA and have some familiarity with matlab before using
% this program. I'm going to try and comment it the best I can. THIS IS A
% GENERAL CODE AND HAS MULTIPLE COMPONENTS THAT MUST BE SPECIALIZED FOR YOUR
% PARTICULAR CASE. I've included "FilesToBeIndividualized.txt" that lists
% all those files. All functions and scripts are commented to help you
% follow what they are doing, but you should only have to change the
% particular files listed.

% This specific matlab file, "MOTHER", is the parent function from which
% everything else is called.

% -------------- Begin Code -------------- %

% Step 1: Defining parameters for use throughout simulation
%-------------------------------------------------------------
% The following functions start your program by defining key parameters for
% use throughout the code. Of these, GAparams and GeometryConstraints need
% to be completely individualized for your code. Instructions on how to
% individualize them can be found in their respective .m files.

% 1a) Run "StartOrRestart" to define initialization info
StartOrRestart;

% 1b) Run "GAparams" to define parameters for the GA
GAparams

% Run "GeometryConstraints" to define the specific geometry constraints and
% information this individual case

% Run "GlobalData" to define our shape and design variable cell arrays, but
% only if it is the initial run.
if strcmp(initRun,'Y') == 1
    [BEZIER_POINTS,DAUGHTER_CURVES,DES_VARS,OLD_DES_VARS] = GlobalData(genSize,nBC,Ncurves,Mcurves,nDV);
end

% Run "SimTimes" to define how long to pause for meshing and simulation
SimTimes

%---------------------------------------------------------------
% Step 2: Randomized Shapes Loop
%---------------------------------------------------------------
% This is the first step for the GA: creating, simulating, and evaluating a
% generation of random individuals. Once a set of random individuals has
% been created, it will be saved separately from the others for future use.
% If you are restarting the GA, as noted in the prompt for "initial run,"
% this entire step will be skipped and the code will begin at step 4.

if strcmp(initRun,'Y') == 1 || (strcmp(initRun,'Y') == 0 && generation == 1
&& numTrial <= genSize)
    RandomShapesLoop
end

%---------------------------------------------------------------
% Step 3: Genetic algorithm
%---------------------------------------------------------------
% Now that we have our generation of random curves and their data stored in
% our
% global matrices, we can run the genetic algorithm (GA) and then repeat for
% a loop using daughter curves. The genetic algorithm is simple once you
% understand it, but calls multiple functions. Read carefully.

if strcmp(initRun,'Y') == 1 || (strcmp(initRun,'Y') == 0 && generation == 1
&& numTrial == genSize+1)
    MOGAgeneral
    ResetVars
end

%---------------------------------------------------------------
% Step 4: Loop until converged
%---------------------------------------------------------------
% After the above 3 steps we now have our cell array "BEZIER_CURVES"
% containing all of the CP information for the newly generated curves. We
% also have saved the best curve's information. Now we cycle through a very
% similar loop to "RandomShapesLoop", except our "RandomIndividuals"
% function is replaced with something that is no longer random. Everything
% else is, for the most part, the same.

if maxGen ~= 1;
    while ConvergenceCheck(bestCurveVals,nDV,maxGen) == 0 % while the
        OptimizationLoop % solution is not converged
            MOGAgeneral
            ResetVars
        end
end

totalRuns = generation*genSize %#ok<NOPTS>

%---------------------------------------------------------------
% end of script
Appendix B

GAparams.m

% GAparams is a script to take in your parameters for the GA. It needs to % be individualized for your case.

% ---------------------------------------------------------------
% Step 1: Individuals per generation and maximum generations
% ---------------------------------------------------------------
% 1a) Define generation size (number of individuals per generation)
genSize = 8; %ind % must be greater than 1 and an even integer.

% Making sure genSize is a positive and even integer.
if genSize < 2 || rem(genSize,2) ~= 0 || fix(genSize) ~= genSize
    error('genSize must be a positive and even integer')
end

% 1b) Define maximum generations for the optimization
maxGen = 50; %ind % must be greater than 1 and an integer

% Making sure maxGen is a positive integer
if maxGen < 1 || fix(maxGen) ~= maxGen
    error('Maximum generations must be a positive integer')
end

% ---------------------------------------------------------------
% Step 2: Number of design variables (dVs)
% ---------------------------------------------------------------
% 2a) Define the number of design variables
% Number of design variables (nDV) is how many variables you need to keep % track of to use in your fitness function.
nDV = 28; %ind

% Making sure nDV is a positive integer
if nDV < 1 || fix(nDV) ~= nDV
    error('nDV must be a positive integer')
end

% 2b) Specify dV objectives
% List the design objective for each dv. If you are trying to minimize % them, input "'min'", if you are trying to maximize them, input "'max'" % (these inputs are without the " " but do include the ' ')
dvObj = cell(1,nDV); % allocates space

for i = 1:14
dvObj{i} = 'min';
end
for i = 15:28
dvObj{i} = 'max';
end

% DErrorCheck ensures you input the dV objectives correctly
DErrorCheck(dvObj,nDV)

% ------------------------------
% Step 3: Selection Type
% ------------------------------
% I have coded in roulette wheel selection and tournament selection.
% You define which type and its corresponding probability values here.
% Use 'rw' for roulette wheel and 't' for tournament (you need to
% apostrophes for these definitions)

% selectionType determines which type of selection will happen
selectionType = 'rw';
if strcmp(selectionType,'rw') == 1
    fprintf('Roulette Wheel Selection\n')
    k = 'not used';
elseif strcmp(selectionType,'t') == 1
    fprintf('Tournament Selection\n')
end

% Define tournament size (weaker individuals have a higher probability
% of being selected as the size becomes larger. When k = 1 then this
% behaves as random selection, which is bad so don't do it.
k = 2;
if k < 1 || k > genSize || fix(k) ~= k
    error('"k" value for tournament selection must be a positive integer
less than genSize')
end

% Make sure selection type is one of the options
error("selectionType" is not a viable option. (must be 't' or 'rw')
end

% ------------------------------
% Step 4: Crossover and mutation parameters
% ------------------------------
% 4a) Crossover parameter
% This is the probability that crossover occurs for any given variable
% point. It must be between 0 and 1.
crossRate = .7;

% makes sure the rate is an acceptable number
if crossRate >= 1 || crossRate <= 0
    error('Crossover rate is outside allowable bounds')
end

% 4b) Mutation rate
% This is the probability that any given variable control point is mutated.
% It helps increase genetic diversity to avoid local optimums. It must be
% between 0 and 1.

mutRate = .1; %ind
if mutRate >= 1 || mutRate <= 0
    error('Mutation rate is outside allowable bounds')
end

% 4c) Error of mutation constant (eomC)
% eomC determines how much mutation affects the curves as generations go
% on. Set to 0 for a ton of mutation variance (not recommended) and to 1
% for very little mutation (not recommended). Basically, as "generation"
% approaches
% "maxGens" the greatest change a mutation can make is 1-eomC*(distance to
% geometric limit).

eomC = .8; %ind

% makes sure eomC is between 0 and 1
if eomC >= 1 || eomC <= 0
    error('Error of mutation constant is outside allowable bounds')
end

%---------------------------------------------------------------
% end of script
Appendix C

MOGAgeneral.m

% This is a generic framework for an adaptable single objective or multi
% objective genetic algorithm. It is based on the inputs from "GAparams"
% and "GeometryConstraints"

% Step 1: Use your fitness function to evaluate each shape's fitness

fitness = FitnessFunction(genSize,DES_VARS);

% Step 2: Natural selection sorting

[BEZIER_POINTS,DES_VARS,sortedBPinfo,sortedTrialData,sortedFitness] = 
Sorter(generation,genSize,fitness,BEZIER_POINTS,DAUGHTER_CURVES,DES_VARS,OLD_DES_VARS);

% Step 3: Save data used for convergence analysis

% Save best curve as a vector for reference
for i = 1:nBC
    bestCurve{i}(2*generation-1:2*generation,:) = sortedBPinfo{i}(1:2,:);
end
filename = sprintf('%s_%d','bestCurve',i);
dlmwrite(filename,bestCurve{i},'-append','newline','pc')
end

% Save the best curve dVs as a vector for reference
for i = 1:nDV
    bestCurveVals(generation,i) = sortedTrialData(1,i+1);
end
dlmwrite('bestCurveVals.txt',bestCurveVals,'-append','newline','pc');

% Step 4: Create new generation

% Backup before
SaveCurves(BEZIER_POINTS,DAUGHTER_CURVES)

% Create new generation
DAUGHTER_CURVES =
DaughterCurveGenerator(sortedBPinfo,limCPinfo,vCPinfo,sortedFitness,genSize,
selectionType,crossRate,mutRate,DAUGHTER_CURVES,generation,maxGen,eomC,k);

% Define "old" matrices (sorted already)
BEZIER_POINTS = sortedBPinfo;

OLD_DES_VARS = DES_VARS;

% Backup new generation
SaveCurves(BEZIER_POINTS,DAUGHTER_CURVES)
SaveDVs(DES_VARS,OLD_DES_VARS)

% ----------------------------------------------------------------------
% end of function.
Appendix D

FitnessFunction.m

function [fitness] = FitnessFunction(genSize,DES_VARS)
% FitnessFunction needs to be individualized for you fitness function. It has
% access to DES_VARS and therefore can find the fitness. Save the fitness
% as a matrix of trial number in column 1 and fitness in column 2, and
% appropriately title it "fitness"

% My fitness function for the example case of minimizing curve length

% Normalized based on a bad (max) value
fitness = zeros(genSize,2);
fitness(:,1) = 1:genSize;
nDV = length(DES_VARS);
localDragHolder = zeros(1,nDV/2);
localLiftHolder = localDragHolder;
averageVals = zeros(genSize,3);
averageVals(:,1) = 1:genSize;

% Felix Code ---------------------------------------------------------------

% Drag
for j = 1:genSize
    for i = 1:nDV/2
        localDragHolder(i) = DES_VARS{i}(j,2);
    end
    % Finding any infinity values
    infIndex = find(localDragHolder == inf); %#ok<EPIND>
    if isempty(infIndex) == 1
        averageVals(j,2) = sum(localDragHolder)/(nDV/2);
    else
        averageVals(j,2) = nan;
    end
end

% Lift
for j = 1:genSize
    for i = (nDV/2+1):nDV
        localLiftHolder = abs(DS_VARS{i}(j,2));
    end
% Finding any negative infinity values
infIndex = find(localLiftHolder == -inf); %#ok<EFIND>
if isempty(infIndex) == 1
    averageVals(j,3) = sum(localLiftHolder)/(nDV/2);
else
    averageVals(j,3) = nan;
end

% Drag
minDragVal = min(averageVals(:,2));
maxDragVal = max(averageVals(:,2));
dragValues = averageVals(:,2);
infIndex = find(isnan(dragValues) == 1);
dragValues(infIndex) = maxDragVal; %#ok<FNDSB>
averageVals(:,2) = dragValues;
normalizedDrag = minDragVal./dragValues*100;

% Lift
maxLiftVal = max(averageVals(:,3));
minLiftVal = min(averageVals(:,3));
liftValues = averageVals(:,3);
infIndex = find(isnan(liftValues) == 1);
liftValues(infIndex) = minLiftVal; %#ok<FNDSB>
averageVals(:,3) = liftValues;
normalizedLift = liftValues./maxLiftVal*100;

% Pareto Optimal front weight (increases/decreases weight of design vars)
weight = .5;
if weight < 0 || weight > 1
    error('Weight is outside acceptable bounds');
end
fitness(:,2) = weight*normalizedLift + (1-weight)*normalizedDrag;
end

% end of function
Appendix E

GenerateIndividuals.m

function [BEZIER_POINTS] = GenerateIndividuals(numTrial,Mcurves,BEZIER_POINTS)

% GenerateIndividuals behaves largely the same as RandomIndividuals did,
% however, instead of creating randomized shapes it takes in curve data
% (BEZIER_POINTS) and generates the required curve acording to numTrial.
% This curve's point data is saved for being passed to ICEM.

% If you want to graph each individual BC, set plotter = 1;
% If you want to graph all the BCs together, set plotterAll = 1;
% Be sure to set both to 0 before running the full GA. You don't want to
% come back to 400 graphs and your computer crashing.

plotter = 0; %ind
plotterAll = 0; %ind

%-------------------------------------------------------------------------%
% Step 1: Start of the loop and limit the geometry randomized inputs
%-------------------------------------------------------------------------%

nBC = length(Mcurves); % number of Bezier Curves (BCs) to iterate over
BPres = cell(1,nBC); % save points for later

for cNum = 1:nBC % for each BC

    j = 0; % counter variable
    n = Mcurves(cNum)-1; % used for Bezier Point (BP) function

    % Placehold the BCs as zeros to be filled in later
    BP = BEZIER_POINTS{cNum}(2*numTrial-1:2*numTrial,:); % gives our current curve

%-------------------------------------------------------------------------%
% Step 2: Bezier Function Generator
%-------------------------------------------------------------------------%

% This is the Bezier Function generator. Go to wikipedia Bezier Curves and
% you'll find this function under the equations section. THIS FUNCTION DOES
% NOT CHANGE EVER AND IS SOLELY DEPENDENT ON BPx, BPy, AND THE COUNTING
% VARIABLES. IT WILL WORK FOR ANY INPUT CURVES. DON'T MESS WITH IT UNLESS
% YOU KNOW WHAT YOU'RE DOING.

%---------------------------------------------------KEEP THIS AS IS---------------------------------------------------
nPoints = 100;
ux = zeros(1,nPoints);
uy = zeros(1,nPoints); % these are going to be the point files for ICEM
uylow = uy;
for t=0:1/(nPoints-1):1
    j=j+1;
    for i=0:n
        if i==0
            a=(1-t)^n;
        elseif i==n
            a=t^n;
        else
            a=(factorial(n)/(factorial(i)*factorial(n-i)))*(1-t)^(n-i);
        end
        ux(j)=ux(j)+BP(1,(i+1))*a;
        uy(j)=uy(j)+BP(2,(i+1))*a;
    end
    if cNum == 1
        uyLow(j)=uy(j)-0.05-0.1*ux(j);
    elseif cNum == 2
        uyLow(j)=uy(j)-0.05+.1*ux(j);
    elseif cNum == 3
        uyLow(j) = uy(j) +0.05+.1*ux(j);
    elseif cNum == 4
        uyLow(j) = uy(j) +0.05-.1*ux(j);
    end
end

% Step 3: Plotting
% Plotting the graphs is useful for checking the results in the beginning,
% but be sure to make plotter = 0 at the start of the function before
% running the optimization program.
if plotter == 1
    figure
    plot(ux,uy,'-r','LineWidth',4) % this plots the resulting curve
    hold on
    plot(BP(1,:),BP(2,:),'*g') % this plots the Bezier Points
    hold on
    axis equal
    grid on
end

% Step 4: Saving Shape Data to External Files and Global Matrices
% Saving the points to a file, Points_.txt, lets us easily load them into
% ICEM.
% See "HelpIcemAutoScript.txt" for some tips on how to most easily do so.
% Saving the CPs to a global matrix lets us call them later. We will also be
% saving the CPs to an external file for data back up: if for any reason the
% simulation shuts down, the most recent curve information will be
% available in a text file.

% Felix stuff -----------------------------------------------

uxlow = ux;

points = [ux;uy;zeros(1,length(ux))];
filename = sprintf('%s_%d','PointsBC',cNum);
fid = fopen(filename,'w');
fprintf(fid,'%5.0f %1.0f
', length(ux), 1);
fprintf(fid,'	%f	%f	%f
', points);
close(fid);

points = [uxlow;uylow;zeros(1,length(uxlow))];
filename = sprintf('%s_%d','PointsLow',cNum);
fid = fopen(filename,'w');
fprintf(fid,'%5.0f %1.0f
', length(ux), 1);
fprintf(fid,'	%f	%f	%f
', points);
close(fid);

% This saves the BC CPs to the global BEZIER_POINTS cell array
MatBeingUsed = BEZIER_POINTS{cNum};
MatBeingUsed(2*numTrial-1:2*numTrial,:) = BP;
BEZIER_POINTS{cNum} = MatBeingUsed;

% The formatting means that the CPs for our BC will exist in two rows, one
% for x values and one for y. If it's our first curve they will exist in rows
% 1 (x) and 2 (y). For our 12th curve they will bein rows 23 (x) and 24 (y).
% The size of BEZIER_POINTS was defined in GlobalData using our generation
% size
% and number of control points, so everything should fit perfectly. IF YOU
% HAVE ANY CURVES THAT EXIST IN THREE DIMENSIONS THE ABOVE CODE WILL NOT
% WORK. YOU'LL HAVE TO FIGURE THAT ONE OUT ON YOUR OWN.

% Save for later
BPres{cNum} = BP;
end

% Step 5: Saving results for easy viewing and global plotting.
% Results.txt is a file that will save your Bezier curve information and
% desVars information for each trial in each generation. We won't access
% any information from it as there are the global matrices and their
% backups, but it does bring all of the important information together into
% one text file that is easy to read.

% Saving a summary of things for easy viewing
fid = fopen('Results.txt','a');
fprintf(fid,'\nShape %i\n',numTrial);
for i = 1:cNum
    dlmwrite('Results.txt',BPres{i},'append')
end
fclose(fid);

% These final lines are used to plot all the BCs together on the same plot.
% To turn off, simply set plotterAll = 0 at the beginning of the code;

PointsBC = cell(1,nBC);
PointsLow = PointsBC;
PointsBCMirror = PointsBC;
PointsLowMirror = PointsBC;
if plotterAll == 1
    figure
    for i = 1:nBC
        % Plotting BCs
        filename = sprintf('%s_%d','PointsBC',i);
        PointsBC{i} = dlmread(filename);
        PointsBC{i} = PointsBC{i}(2:end,:); % gets rid of icem point count

        plot(PointsBC{i}(:,1),PointsBC{i}(:,2))
        hold on
        axis equal

        % Plotting low curves
        filename = sprintf('%s_%d','PointsLow',i);
        PointsLow{i} = dlmread(filename);
        PointsLow{i} = PointsLow{i}(2:end,:);

        plot(PointsLow{i}(:,1),PointsLow{i}(:,2))
        hold on
    end
end

end

%-----------------------------------------------
%end of function
Appendix F

DaughterCurveGenerator.m

function [DAUGHTER_CURVES] = DaughterCurveGenerator(sortedBPinfo,limCPinfo,vCPinfo,sortedFitness,genSize,selectionType,crossRate,mutRate,DAUGHTER_CURVES,generation,maxGen,eomC,k)
% will produce genSize of offspring curves

nBC = length(sortedBPinfo);
delete DaughterCurves.txt %-------------------------------------------------------------------------% Step 1: Selection function
%-------------------------------------------------------------------------
if strcmp(selectionType,'rw') == 1
    parNums = RouletteSampling(sortedFitness,genSize);
else
    parNums = TournamentSampling(genSize,k);
end
%-------------------------------------------------------------------------% Step 2: Crossover of selected curves
%-------------------------------------------------------------------------
maleCurve = cell(1,nBC);
femaCurve = maleCurve;
for i = 1:genSize % each couple produces 2 kids but we only save 1
    maleNum = parNums(i,1);
femaNum = parNums(i,2);
    for j = 1:nBC
        maleCurve{j} = sortedBPinfo{j}(2*maleNum-1:2*maleNum,:);
femaCurve{j} = sortedBPinfo{j}(2*femaNum-1:2*femaNum,:);
    end
% now we have the male and female curve information and can crossover
[newCurve] = Crossover(maleCurve,femaCurve,vCPinfo,crossRate);
    for j = 1:nBC
        DAUGHTER_CURVES{j}(2*i-1:2*i,:) = newCurve{j}; % store the info to DAUGHTER_CURVES
    end
end
%-------------------------------------------------------------------------% Step 3: Mutation of randomly selected curves
%-------------------------------------------------------------------------
%random mutation function
DAUGHTER_CURVES = Mutator(vCPinfo,limCPinfo,mutRate,genSize,DAUGHTER_CURVES,generation,maxGen,eomC);
end
Appendix G

Crossover.m

function [newCurve] = Crossover(male,fema,vCPinfo,crossRate)
% Exchange CP info between the curves

nBC = length(male);
new1 = male;
new2 = fema;

for i = 1:nBC % each Bezier curve
    for j = 1:length(vCPinfo{i}) % only the variable CPs
        doCrossover = (rand(1) <= crossRate); % 1 if we will, 0 if we won't
        if doCrossover == 1
            sM = male{i}(vCPinfo{i}(j)); % male value of index j
            sF = fema{i}(vCPinfo{i}(j)); % fema value of index j
            new1{i}(vCPinfo{i}(j)) = sF; % male now has fema value
            new2{i}(vCPinfo{i}(j)) = sM; % fema now has male value
        end
    end
end

% Randomly choose which curve
if rand(1) <= .5
    newCurve = new1;
else
    newCurve = new2;
end

end

% _________________________________________________________________
% end of function
Appendix H

Mutator.m

function [DAUGHTER_CURVES] =
Mutator(vCPinfo,limCPinfo,mutRate,genSize,DAUGHTER_CURVES,generation,maxGen,eomC)
% Mutator takes in the new daughter curves and iterates over all of their
% variable CPs. mutRate % of those vCPs will be mutated, or randomly
% altered.

% ------------------------------------------------------------------------
% Step 1: Define the effective mutation efficiency (eom)
% ------------------------------------------------------------------------
% We want eom to start at 1 and decrease to eomC by maxGen

eom = 1 - eomC*generation/maxGen;

% ------------------------------------------------------------------------
% Step 2: Mutate the curves
% ------------------------------------------------------------------------

nBC = length(DAUGHTER_CURVES);

for i = 1:genSize % iterate over all individuals
    for j = 1:nBC % iterate over each bezier curve
        curve2Mut = DAUGHTER_CURVES{j}(2*i-1:2*i,:); % define curve CPs
        for k = 1:length(vCPinfo{j}) % iterate over all vCPs
            doMutation = (rand(1) <= mutRate); % mutation probability is
            if doMutation == 1
                val2Mut = curve2Mut(vCPinfo{j}(k)); % value of point being
                mutated
                    if rand(1)<.5
                        edgeVal = limCPinfo{j}(k,1); % select one edge value
                    else
                        edgeVal = limCPinfo{j}(k,2); % otherwise select other
                    end
                curve2Mut(vCPinfo{j}(k)) = val2Mut + eom*rand(1)*(edgeVal-
                val2Mut); %curve moves some random distance towards that edge
            end
        end
        DAUGHTER_CURVES{j}(2*i-1:2*i,:) = curve2Mut; % replacing in DC matrix
    end
end
References


[22] https://www.math.ubc.ca/~cass/gfx/Bezier.html
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