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Beurling-Lax representation for weighted Bergman-shift invariant subspaces

Abstract

The Beurling-Lax-Halmos theorem tells us that any invariant subspace $\mathcal{M}$ for the shift operator $S\colon f(z) \mapsto zf(z)$ on the vectorial Hardy space over the unit disk $H^2_{\mathcal{Y}} = \{f(z) = \sum_{j=0}^{\infty} f_j z^j \colon \|f\|^2 = \sum_{j\geq 0} \|f_j\|^2 < \infty\}$ (the Reproducing Kernel Hilbert Space with reproducing kernel $K(z,w) = (1 - z\overline{w})^{-1} I_{\mathcal{Y}}$) can be represented as $\mathcal{M} = M_\Theta H^2_{\mathcal{U}}$ where $M_\Theta \colon H^2_{\mathcal{U}} \to H^2_{\mathcal{Y}}$ is an isometric multiplication operator $M_\Theta \colon u(z) \mapsto \Theta(z)u(z)$. We focus on three constructions of $\Theta(z)$:

(1) **the wandering subspace construction of Halmos**: $\mathcal{M} = \bigoplus_{j\geq 0} S^j(\mathcal{M} \ominus S\mathcal{M}) = M_\Theta H^2_{\mathcal{U}}$ where M_Θ is constructed so that $M_\Theta \colon H^2_{\mathcal{U}} \to \mathcal{M} \ominus S\mathcal{M}$ is unitary;

(2) as the **Sz.-Nagy–Foias characteristic function** $\Theta_T(z)$ of the $C_{\cdot 0}$ contraction operator $T = P_{\mathcal{M}^\perp} S|_{\mathcal{M}^\perp}$, and

(3) via the **functional-model realization formula** $\Theta(z) = D + zC(I - zA)^{-1}B$ where $\left[\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right] = \left[\begin{smallmatrix} S^* & S^* M_\Theta \\ \mathbf{ev}_0 & \Theta(0) \end{smallmatrix}\right] \colon \left[\begin{smallmatrix} \mathcal{M}^\perp \\ \mathcal{U} \end{smallmatrix}\right] \to \left[\begin{smallmatrix} \mathcal{M}^\perp \\ \mathcal{Y} \end{smallmatrix}\right]$ with $\mathbf{ev}_0 \colon f \mapsto f(0)$ is unitary.

We discuss analogues of these results for the weighted Bergman-shift operator $S_n \colon f(z) \mapsto zf(z)$ acting on the weighted Bergman space $\mathcal{A}_{n,\mathcal{Y}} = \{f(z) = \sum_{j\geq 0} f_j z^j : \|f\|^2 = \sum_{j\geq 0} \mu_{n,j} \|f_j\|^2_{\mathcal{Y}} < \infty\}$ (where $\mu_{n,j} = 1/\binom{j+n-1}{j}$ are reciprocal binomial coefficients), or $\mathcal{A}_{n,\mathcal{Y}}$ is the Reproducing Kernel Hilbert Space with reproducing kernel equal to $K_{n,\mathcal{Y}}(z,w) = (1 - z\overline{w})^{-n} I_{\mathcal{Y}}$, as well as for the freely noncommutative weighted Bergman shift-tuple $\mathbf{S}_n = (S_{n,1}, \ldots, S_{n,d})$ where $S_{n,j} \colon f(z) \mapsto f(z)z_j$ $(j = 1, \ldots, d)$ on the weighted Bergman-Fock space $\mathcal{A}_{n,\mathcal{Y}}(\mathbb{F}_d)$ consisting of formal power series $f(z) = \sum_{\mathfrak{a}\in\mathbb{F}_d} f_{\mathfrak{a}} z^{\mathfrak{a}}$ for which $\|f\|^2 = \sum_{\mathfrak{a}\in\mathbb{F}_d} \mu_{n,|\mathfrak{a}|} \|f_{\mathfrak{a}}\|^2 < \infty$ ($\mathbb{F}_d$ is the free semigroup (also called the monoid) on d generators $1, \ldots, d$, $z = (z_1, \ldots, z_d)$ is a d-tuple of freely noncommuting indeterminates, $z^{\mathfrak{a}}$ is the noncommutative monomial $z^{\mathfrak{a}} = z_{i_1} \cdots z_{i_N}$ if $\mathfrak{a} = i_1 \cdots i_N$, $|\mathfrak{a}|$ is the **length** of $\mathfrak{a}$ (equal to N if $\mathfrak{a} = i_1 \cdots i_N$). This talk reports on joint work with Vladimir Bolotnikov of the College of William and Mary.

Talk time: 07/21/2016 6:00PM— 07/21/2016 6:20PM
Talk location: Crow 206

Special Session: Non-commutative inequalities. Organized by J.W. Helton and I. Klep.