

Aljaž Zalar

Institute of Mathematics, Physics, and Mechanics, Ljubljana, Slovenia

Operator positivstellensätze for noncommutative polynomials positive on matrix convex sets

Abstract

In the talk we will study algebraic certificates of positivity for noncommutative (nc) operator-valued polynomials on matrix convex sets, such as the solution set D_L , called a free Hilbert spectrahedron, of the linear operator inequality (LOI) $L(X) = A_0 \otimes I + \sum_{j=1}^g A_j \otimes X_j \succeq 0$, where A_j are self-adjoint linear operators on a separable Hilbert space, X_j matrices and I is an identity matrix. If A_j are matrices, then $L(X) \succeq 0$ is called a linear matrix inequality (LMI) and D_L a free spectrahedron. For monic LMIs, i.e., $A_0 = I$, and nc matrix-valued polynomials the certificates of positivity were established by Helton, Klep and McCullough [1, 2]. We extend the characterization of the inclusion $D_{L_1} \subseteq D_{L_2}$ from monic LMIs to monic LOIs L_1 and L_2 . Using this characterization in a separation argument, we obtain a certificate for matrix-valued nc polynomials F positive semidefinite on a free Hilbert spectrahedron defined by a monic LOI. Finally, focusing on the algebraic description of the equality $D_{L_1} = D_{L_2}$, we remove the assumption of boundedness from the description in the LMIs case and present counterexamples for the extension to LOIs case.

References

- [1] J.W. Helton, I. Klep, S. McCullough: The convex Positivstellensatz in a free algebra, *Adv. Math.* **231** (2012) 516–534.
- [2] J.W. Helton, I. Klep, S. McCullough: The matricial relaxation of a linear matrix inequality, *Math. Program.* **138** (2013) 401–445.
- [3] A. Zalar, Operator Positivstellensätze for noncommutative polynomials positive on matrix convex sets. *Preprint* <http://arxiv.org/abs/1602.00765>.

Talk time: 07/21/2016 4:00PM— 07/21/2016 4:20PM

Talk location: Crow 206

Special Session: Non-commutative inequalities. Organized by J.W. Helton and I. Klep.