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Donahue Independent Study; Collection of Aerodynamic Stability and Control Lecture Notes

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Aerodynamic Stability & Control

*MEMS 500: Independent Study*

Donahue, Daniel
Daniel Donahue’s Independent Study

The learning objective of this independent study was to further my own knowledge in the field of Flight Mechanics/Dynamics as well as attaining experience in creating coursework. This was done by creating 14 lectures worth of hand-written notes on the subject of Aircraft Stability and Control. I primarily used the book by Pamadi, “Performance, Stability, Dynamics, and Control of Airplanes 2nd ed” while also pulling from my experiences as an aerodynamic stability and control engineer.

During the course of the semester I generated 42 pages of hand written notes that correspond to 14 lectures worth of material. It was agreed upon with my advisor, David Peters, that 3 pages of hand written notes would equate to a lecture. This number was based off of his experience with lecture materials where 3 pages of hand written notes, accompanied by in-person elaboration of the material, equates to one lecture worth of material.

Below is a syllabus of the 14 covered lectures with corresponding pages in the hand-written notes attached.

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0) SYLLABUS & AIRCRAFT GEOMETRY

1) AIRCRAFT STABILITY & CONTROL DEFINITIONS
   - FLIGHT VARIABLE DEFINITIONS OF INTEREST
     - COORDINATE SYSTEMS
       - BODY
       - STABILITY
       - WIND
       - INERTIAL
     - AIRCRAFT AGRO FORCES & MOMENTS NON-DIM
     - STANDARD ATM EQUATIONS

2) SIX DEGREES OF FREEDOM EQUATIONS OF MOTION

3) LATERAL-DIRECTIONAL CHARACTERISTICS
   - LATERAL-STABILITY & CONTROL
   - LATERAL STABILITY & CONTROL
   - DEPARTURE RESISTANCE

4) PERFORMANCE PARAMETERS
   - TURN RATE
   - TURN RADIUS
   - RATE OF CLimb

5) DYNAMIC DERIVATIVES
   - ROLL, YAW, PITCH & PLUNGING DAMPING
   - CROSS-AXIS DAMPING
   - TRADITIONAL METHODS

6) HINGE MOMENTS
AIRCRAFT TYPES, GEOMETRY, CONTROL SURFACES

TYPES OF AIRCRAFT
- COMMERCIAL
  - TRANSPORT (7-SERIES, A-SERIES, FREIGHTER, Z-BLIMP?)
  - PERSONAL (CESSNA SMALL AIRCRAFT, RED BULL FLYERS)
- DEFENSE
  - FIGHTER/ATTACK (F-15, F/A-18, F-22, F-35, TOMCAT, ETC.)
  - BOMBER (STRATOFORTRESS, B-1, FLYING WING)
  - TRAINER (T-45, ETC.)
  - UAV [DRONE] (SLOW EAGLE, ETC.)
  - SURVEILLANCE [STEALTH] (SR-71, A-12, SATELLITE)

GEOMETRY
- FUSELAGE
  - TUBULAR
  - SEARS-HAAK
  - ROTTING
- WING
  - CANARD (MISC.)
  - LEX
  - HERSHEY
  - SWEPT
  - DELTA
- TAIL
  - V-TAIL
  - T-TAIL
  - H-TAIL
- MISC
  - THRUST-VECTORIZATION
  - WINGLETS
  - VORTEX GENERATORS

CONTROL SURFACES
- AILEEON
- FLAPS (TEF/LEF) FIG. 24
- FLAPERON
- SLATS STABILATOR
- TRIM TABS
- ELEVON
- SPEED BREAK
- RUDDER

AIRCRAFT BODY AXIS
- X, Y, Z
- θ, ψ, γ
- LIFT, DRAG, SIDEFORCE

HOMEWORK
- IDENTIFY WHICH C65
- 65 WITH EACH AXIS
1. **FLIGHT DYNAMICS** ⇒ **A STUDY OF THE TRANSIENT MOTION OF AN AEROSPACE VEHICLE FOLLOWING A PERTURBATION FROM AN EQUILIBRIUM STATE**

   - **EQUILIBRIUM STATE** ⇒ **VEHICLE STATE** IN WHICH \( F(e) = M(e) = 0 \) WHERE \( F \) & \( M \) REPRESENT EXTERNAL FORCES & MOMENTS APPLIED

   - **STATES OF INTEREST:** STEADY-LEVEL FLIGHT, ETC.

   - **PERTURBATION** ⇒ **CHANGE IN F & M CAUSED BY CONTROL SURFACE DEVIATIONS, TURBULENCE, INTERNAL SHIFTS, **

   - **FLIGHT DYNAMICS DEALS WITH** STEADY & UNSTEADY MOTION OF A VEHICLE CAUSED BY A DISTURBANCE, UNINTENTIONAL OR UNINTENTIONAL, APPLIED TO AN INITIALLY TRIMMED MOTION STATE.

2. **STABILITY** ⇒ **THE QUALITY OF AN EQUILIBRIUM STATE**

   - **EQUILIBRIUM STATES OF AIRCRAFT ARE GENERALLY DESIGNED TO BE HANDS-OFF CONDITIONS**

   - **AIRCRAFT WITH INFERIOR STABILITY CHARACTERISTICS CAN POTENTIALLY BE DANGEROUS, UNACCEPTABLE**

   - **STATIC STABILITY:**

     - **STABLE:**
     - **NEUTRAL:**
     - **UNSTABLE:**

   - **3D STABILITY**

     - **STATIONARILY STABLE** IF IT RETURNS TO A STATE WHEN PERTURBED
Dynamic stability is the ability of a vehicle to return to an equilibrium state when disturbed. A vehicle is dynamically stable if, when perturbed, it eventually returns to an equilibrium state.

(3) Control means ability, or lack thereof, to steer an aerospace vehicle from point A to point B along an acceptable flight path.

Static control means ability to maintain a prescribed equilibrium flight condition.

Dynamic control means ability to maintain an acceptable transient motion following a perturbation, the ability to change from one trim condition to another.

Flight variable definitions of interest:

**Body Axis Coordinate System**

<table>
<thead>
<tr>
<th>Coordinate</th>
<th>Nose Wing Down</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
</tr>
</tbody>
</table>

| Velocity | $u$ | $v$ | $w$ |
| Rotational Velocity | $p$ | $q$ | $r$ |

| Aerodynamic Force | $X$ | $Y$ | $Z$ |
| Aerodynamic Moment | $L$ | $M$ | $N$ |

Roll Pitch Yaw

$X_B \rightarrow$ Axial Force = $-A$

$Z_B \rightarrow$ Normal Force = $-N$
Stability Axis

$V_T \rightarrow$ Flight Path

$\alpha \rightarrow$ Angle of Attack (Flight Path/Velocity to Body x)

$\gamma \rightarrow$ Flight Path Angle (Flight Path to Horizon)

$\Theta \rightarrow$ Pitch Angle (Horizon to X-Axis Body)

\[
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
_{BODY} =
\begin{bmatrix}
  \cos \Theta & 0 & -\sin \Theta \\
  0 & 1 & 0 \\
  \sin \Theta & 0 & \cos \Theta
\end{bmatrix}
_{STABILITY}
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
_{STABILITY} =
\begin{bmatrix}
  \cos \Theta & 0 & \sin \Theta \\
  0 & 1 & 0 \\
  -\sin \Theta & 0 & \cos \Theta
\end{bmatrix}
_{BODY}
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
_{BODY}

Check Signs

\[
L = \text{LIFT} = -Z_S = -Z_0 \cos \alpha + X_B \sin \alpha + N \cos \Theta \cdot A \sin \Theta
\]

\[
D = \text{DRAG} = -X_S = -Z_0 \sin \alpha - X_B \cos \alpha + N \sin \Theta \cdot A \cos \Theta
\]

\[
W = \text{WEIGHT}
\]

\[
T = \text{THRUST}
\]
\[ \alpha = \tan^{-1}\left(\frac{w}{u}\right) = \cos^{-1}\left(\frac{u}{\sqrt{u^2+w^2}}\right) = \sin^{-1}\left(\frac{w}{\sqrt{u^2+w^2}}\right) \]

\[ \beta = \sin^{-1}\left(\frac{v}{V_T}\right) = \tan^{-1}\left(\frac{w}{V_T}\right) = \cos^{-1}\left(\frac{\sqrt{V_T^2+w^2}}{V_T}\right) \]

\[ \alpha_T = \cos^{-1}\left(\frac{u}{V_T}\right) = \sin^{-1}\left(\frac{v^2+w^2}{V_T}\right) = \tan^{-1}\left(\frac{\sqrt{V_T^2+w^2}}{u}\right) \]

\[ \Rightarrow T = \text{TOTAL} \rightarrow \text{USED IN MISSILES} \]

\[ \beta_B = \tan^{-1}\left(\frac{v}{u}\right) = \cos^{-1}\left(\frac{u}{\sqrt{u^2+v^2}}\right) = \sin^{-1}\left(\frac{v}{\sqrt{u^2+v^2}}\right) \]

\[ \beta = \tan^{-1}\left(\tan \beta_B \cos \alpha\right) \]

\[ \alpha_T = \cos^{-1}\left(\cos \alpha \cos \beta\right) \]

\[ V_T = \sqrt{u^2+v^2+w^2} \]

\[ u = V_T \cos \alpha \cos \beta \]

\[ v = V_T \sin \beta \]

\[ w = \sin \omega \cos \beta \]
STABILITY TO WIND

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_W = \begin{bmatrix}
\cos \beta & \sin \beta & 0 \\
-\sin \beta & \cos \beta & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_S
\]

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_S = \begin{bmatrix}
\cos \beta & -\sin \beta & 0 \\
\sin \beta & \cos \beta & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_W
\]

BODY TO WIND

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_W = \begin{bmatrix}
\cos \alpha \cos \beta & \sin \beta & \sin \alpha \cos \beta \\
-\cos \alpha \sin \beta & \cos \beta & -\sin \alpha \sin \beta \\
-\sin \alpha & 0 & \cos \alpha
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_B
\]

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_B = \begin{bmatrix}
\cos \alpha \cos \beta & -\cos \alpha \sin \beta & -\sin \alpha \cos \beta \\
\sin \beta & \cos \beta & 0 \\
\sin \alpha \cos \beta & -\sin \alpha \sin \beta & \cos \alpha
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_W
\]

INERTIAL AXIS SYSTEM

\[
\begin{bmatrix}
X_B \\
Y_B \\
Z_B
\end{bmatrix}
\]

INERTIAL AXIS
BODY AXIS

ORDER MATTERS
INERTIAL TO BODY AXIS SYSTEM

\[
\begin{bmatrix}
X_N \\
Y_e \\
Z_d
\end{bmatrix} =
\begin{bmatrix}
\cos \phi \cos \psi & \cos \theta \sin \psi - \sin \theta \sin \phi & \cos \theta \cos \psi + \sin \theta \sin \phi \\
\sin \phi \cos \psi & \sin \theta \sin \psi + \cos \theta \cos \phi & \cos \theta \sin \psi - \sin \theta \sin \phi \\
-\sin \psi & \cos \psi \cos \phi & \cos \phi \sin \psi
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_B
\]

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_B =
\begin{bmatrix}
\cos \phi \cos \psi & \sin \theta \sin \psi & \cos \theta \sin \psi + \sin \theta \sin \phi \\
\cos \psi \sin \phi & \sin \theta \cos \psi + \cos \theta \cos \phi & \cos \theta \cos \psi - \sin \theta \sin \phi \\
-\sin \phi & \cos \phi \cos \psi & \cos \phi \sin \psi
\end{bmatrix}
\begin{bmatrix}
X_N \\
Y_e \\
Z_d
\end{bmatrix}
\]

\[
C =
\begin{bmatrix}
\cos \psi & \sin \psi & 0 \\
-\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
B =
\begin{bmatrix}
\cos \phi & 0 & -\sin \phi \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}
\]

\[
A =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & \sin \phi \\
0 & -\sin \phi & \cos \phi
\end{bmatrix}
\]

\[
\begin{bmatrix}
X_d \\
Y_e \\
Z_d
\end{bmatrix} = (ABC)^{-1}\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_B
\]

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_B = ABC \begin{bmatrix}
X_N \\
Y_e \\
Z_d
\end{bmatrix}
\]
AIRCRAFT FORCES AND MOMENTS

AERODYNAMIC

\[ F_{xb} = -A \rightarrow \text{OPPOSITE OF AXIAL FORCE} \]
\[ F_{yb} = Y = SF \rightarrow \text{SIDE FORCE} \]
\[ F_{zb} = -N \rightarrow \text{OPPOSITE OF NORMAL FORCE} \]
\[ \delta = \frac{1}{2} \rho V^2 = 0.7 \rho M^2 \]
\[ b = \text{SPAN} \]
\[ c = \text{CHORD (MAC)} \]
\[ F_{xb} = \delta S C_x \]
\[ F_{yb} = \delta S C_y \]
\[ F_{zb} = \delta S C_z \]
\[ L = \delta S b C_L \]
\[ M = \delta S c C_m \]
\[ N = \delta S b C_n \]

STANDARD ATMOSPHERE MODEL EQUATIONS

IDEAL GAS LAW: \[ p = \rho RT \]

HYDROSTATIC EQUILIBRIUM: \[ \frac{dp}{dH} = -\delta p \]

TEMPERATURE GRADIENTS: \[ T = T_{BASE} + L_n (H - H_{BASE}) \]

RATIOS: \[ \frac{T}{T_0} ; \frac{p}{p_0} ; \frac{\delta}{\delta_0} = \frac{\delta}{\delta_0} \]

SPEED OF SOUND: \[ a = \sqrt{\frac{\delta}{\rho}} \]

DYNAMIC PRESSURE: \[ \delta = \frac{1}{2} \rho V^2 = 0.7 \rho M^2 \]

TRUE AIRSPEED: \[ V_T = M a \]

EQUIVALENT AIRSPEED: \[ V_E = V_T \sqrt{\theta} \]

TEMPERATURE RATIO: \[ \frac{T}{T_0} = 1 + \frac{V-1}{2} M^2 = 1 + 0.2 M^2 \times \]
Derivation of Aircraft Equations of Motion

Forces

Starting in the Inertial Reference Frame

$\Rightarrow$ Newton's 2nd Law only applicable in this frame

\[
\dot{\mathbf{F}} = \frac{d}{dt}(m \dot{\mathbf{V}}) = \dot{\mathbf{V}} \frac{d}{dt}(m) + m \frac{d}{dt}(\dot{\mathbf{V}})
\]

Assumption #2: Conservation of Mass, $\frac{dm}{dt} = 0$

$\Rightarrow \dot{\mathbf{F}} = m \frac{d}{dt}(\dot{\mathbf{V}})$

Aside: Not true for aircraft, but reasonable for timescales we are looking at, fuel born.

Want to switch to a Body Axis Reference Frame

$\Rightarrow$ Body Axis is easily measured onboard an aircraft

$\Rightarrow$ Easy to reference about A/C CG

Recall:

\[
\frac{d}{dt} \mathbf{A} = \frac{d^2}{dt^2} \mathbf{A} + \overrightarrow{\mathbf{\Omega}} \times \mathbf{A}
\]

$I \Rightarrow$ Inertial

$b \Rightarrow$ Body

$\Rightarrow$ Vector differentiation requires attention to rotating axis systems.

\[
\frac{d}{dt}(\dot{\mathbf{V}}) = \frac{d^2}{dt^2} \mathbf{V} + \overrightarrow{\mathbf{\Omega}} \times \mathbf{V} = \dot{\mathbf{V}} + \mathbf{\Omega} \times \dot{\mathbf{V}} \Rightarrow \dot{\mathbf{V}}_{CG} + \mathbf{\Omega} \times \mathbf{V}_{CG}
\]

\[
\Rightarrow \dot{\mathbf{F}} = m \left( \dot{\mathbf{V}}_{CG} + \mathbf{\Omega} \times \mathbf{V}_{CG} \right)
\]

Again, static accelerates

$\overrightarrow{\mathbf{\Omega}}$ represents the rotation of the body-fixed frame with respect to IRF

$\Rightarrow$ Above eqn is in body reference frame
MOMENTS

NEWTON'S SECOND LAW FOR ROTATIONAL MOTIONS

\[ \vec{M} = \frac{d}{dt}(\vec{\omega}) \]

\[ \vec{M} \] is the sum of externally applied moments

\[ \frac{d}{dt}(\vec{\omega}) \] is rate of change of angular momentum

ADJOINT: ANGULAR MOMENTUM

ANGULAR MOMENTUM CAN BE THOUGHT OF AS LINEAR MOM. WITH A MOMENT ARM

LIN. MOM. = \( m \vec{v} \quad p = mv \)

ANG. MOM. = \( \vec{r} \times m \vec{v} = L = rmv \)

IN THIS CASE, INDIVIDUAL MassES NEED TO BE CONCERNED:

\[ \vec{H}_{\text{lin}} = \vec{r} \times dm \vec{v} = dm (\vec{r} \times \vec{v}) \]

SUMMING OVER ALL Mass,

\[ \vec{H} = \int \vec{H}_{\text{lin}} = \int \vec{r} \times \vec{v} \, dm \]

\[ \Rightarrow \text{DIFFERENTIATING TO GET} \]

\[ \dot{\vec{H}} = \int (\vec{\omega} \times \vec{r}) \, dm = \vec{M} \]

\[ \text{WANT TO GET TO BODY REFERENCE FRAME} \]

\[ \vec{r}_A = \vec{r}_B \]

\[ \vec{r}_A = \vec{r}_B + \vec{r}_B \times \vec{\omega}_B \]

\[ \vec{r}_A = \vec{r}_B + \vec{\omega}_B \times \vec{r}_B + \vec{\omega}_B \times (\vec{r}_B + \vec{\omega}_B \times \vec{r}_B) + \vec{\omega}_B \times \vec{\omega}_B \]

\[ = \vec{r}_B + 2 \vec{\omega}_B \times \vec{r}_B + \vec{\omega}_B \times \vec{r}_B + \vec{\omega}_B \times (\vec{\omega}_B \times \vec{r}_B) \]

\[ \Rightarrow \vec{M}_B = \int \vec{\omega}_B \times \left( \vec{r}_B + 2 \vec{\omega}_B \times \vec{r}_B + \vec{\omega}_B \times \vec{r}_B + \vec{\omega}_B \times (\vec{\omega}_B \times \vec{r}_B) \right) \]
ASSUMPTION #2: AIRPLANES ARE RIGID BODIES

\[ \hat{\mathbf{P}} = \hat{\mathbf{P}} = 0 \]

Essentially saying that elements of mass in an airplane do not change position with one another time.

\[ \Rightarrow \mathbf{M} = \int \rho \mathbf{r} \times (\hat{\mathbf{r}} \times \mathbf{r} + \hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \mathbf{r})) \, d\mathbf{m} \]

ASSUMPTION #3: OUR ERF WILL BE A FLAT, NON-ROTATING EARTH REFERENCE FRAME.

VALID IF UNDER 3KPS - 5KPS, OR LESS THAN 1 MIN.

\[ \Rightarrow \text{Assumption #3 allows us to define } \hat{\mathbf{r}} \text{ as the aircraft angular rotation relative to earth} \]

ASSUMPTION #4: ATMOSPHERE IS AT REST RELATIVE TO FLAT-EARTH

\[ \Rightarrow \text{No winds.} \]

ASSIGN THE VECTORS IN NOTATION:

\[ \mathbf{F} = F_x \hat{\mathbf{E}} + F_y \hat{\mathbf{S}} + F_z \hat{\mathbf{K}} \]

\[ \mathbf{V}_{0a} = U \hat{\mathbf{E}} + V \hat{\mathbf{S}} + W \hat{\mathbf{K}} \]

\[ \dot{\mathbf{r}} = \dot{X} \hat{\mathbf{E}} + \dot{Y} \hat{\mathbf{S}} + \dot{Z} \hat{\mathbf{K}} \]

\[ \mathbf{M}_{eq} = LE + MS + NK \]

\[ \mathbf{\omega} = X \hat{\mathbf{E}} + Y \hat{\mathbf{S}} + Z \hat{\mathbf{K}} \]

\[ \Rightarrow \mathbf{\dot{V}}_{0a} = \dot{V} \hat{\mathbf{K}} + V \hat{\mathbf{S}} + W \hat{\mathbf{K}} + [U \frac{\partial \mathbf{E}}{\partial E} + V \frac{\partial \mathbf{S}}{\partial S} + W \frac{\partial \mathbf{K}}{\partial K}] \]

\[ [J] = 0 \text{ in body fixed frame} \]
BACK TO FORCES

\[ \vec{F} = m \left( \dot{\vec{V}}_{co} + \vec{r} \times \dot{\vec{V}}_{co} \right) \]

\[ \vec{r} \times \dot{\vec{V}}_{co} = \begin{vmatrix}
\vec{e}_x & \vec{e}_y & \vec{e}_z \\
\vec{r} & \vec{q} & \vec{r} \\
u & w & w
\end{vmatrix} = (Q\omega - R\nu)\hat{c} + (RU - P\nu)\hat{s} + (P\nu - Q\omega)\hat{k} \]

\[ \rightarrow \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix}
m (\dot{u} + Q\omega - R\nu) \\
m (\dot{v} + RU - P\nu) \\
m (\dot{w} + P\nu - Q\omega)
\end{bmatrix} \]

BACK TO MOMENTS

\[ \vec{M} = \int_{B_1} \vec{r}_B \times \left( \vec{r}_B \times \vec{r}_B + \vec{r} \times (\vec{r} \times \vec{r}_B) \right) \, dm \]

\[ \vec{r} \times \vec{r}_B = \begin{vmatrix}
\vec{e}_x & \vec{e}_y & \vec{e}_z \\
\vec{r} & \vec{q} & \vec{r} \\
u & w & w
\end{vmatrix} = (Q\omega - Q\omega)\hat{c} + (R \omega - P\nu)\hat{s} + (P\nu - Q\omega)\hat{k} \]

\[ \vec{r} \times (\vec{r} \times \vec{r}_B) = \begin{vmatrix}
\vec{e}_x & \vec{e}_y & \vec{e}_z \\
\vec{r} & \vec{q} & \vec{r} \\
u & w & w
\end{vmatrix} = \begin{vmatrix}
\hat{c} & \hat{s} & \hat{k} \\
\vec{r} & \vec{q} & \vec{r} \\
u & w & w
\end{vmatrix} = (\hat{c} - \hat{s} - \hat{k}) \vec{c} + (\hat{s} - \hat{c} + \hat{k}) \vec{s} + (\hat{c} - \hat{s} + \hat{k}) \vec{k} \]

\[ \vec{A} = \vec{r} \times \vec{r}_B + \vec{r} \times (\vec{r} \times \vec{r}_B) = \begin{vmatrix}
\hat{c} & \hat{s} & \hat{k} \\
\vec{r} & \vec{q} & \vec{r} \\
u & w & w
\end{vmatrix} = \begin{vmatrix}
\hat{c} & \hat{s} & \hat{k} \\
\vec{r} & \vec{q} & \vec{r} \\
u & w & w
\end{vmatrix} = (\hat{c} - \hat{s} - \hat{k}) \vec{c} + (\hat{s} - \hat{c} + \hat{k}) \vec{s} + (\hat{c} - \hat{s} + \hat{k}) \vec{k} \]
\[
\frac{\hat{r}}{x \hat{A}} = \ldots \text{ A lot}
\]

\[
\frac{\hat{r}}{x \hat{A}} + (g(x^2 - q r x^2)) \hat{c} + (p r (y^2 - p r y^2)) \hat{c} + (p g (z^2 - p g z^2)) \hat{E}
\]

RECOGNIZE MOMENTS & PRODUCTS OF INERTIA

\[
I_{xx} = \int (y^2 + z^2) \, dm \\
I_{yy} = \int (x^2 + z^2) \, dm \\
I_{zz} = \int (x^2 + y^2) \, dm
\]

GET TO:

\[
\begin{bmatrix}
\hat{L} \\
\hat{M} \\
\hat{N}
\end{bmatrix} =
\begin{bmatrix}
\hat{r} I_{xx} + q r I_{zz} - q r I_{zy} + (\hat{c}^2 - \hat{g}^2) I_{yy} + (\hat{c} + \hat{g}) I_{xy} - (\hat{c} + \hat{g}) I_{zx} \\
\hat{r} I_{zy} + p r I_{xx} - p r I_{zz} + (p \hat{c}^2 - r \hat{g}^2) I_{xy} - p r I_{zx} - (\hat{c} + \hat{g} r) I_{y} \\
\hat{r} I_{zx} + p g I_{yy} - p g I_{xx} + (p^2 - r^2) I_{xy} - (p^2 - q r) I_{xz} - (\hat{c} + \hat{g} r) I_{y}
\end{bmatrix}
\]

ASSUMPTION #5: \(x, y, z\) IS A PLANE OF SYMMETRY

\[\Rightarrow I_{xy} = I_{yx} = 0\]

MOSTLY TRUE, ASYMMETRIC FUEL & STORES CAN CAUSE IT TO BE INVALID.

\[\Rightarrow \hat{M} \rightarrow \begin{bmatrix}
\hat{L} \\
\hat{M} \\
\hat{N}
\end{bmatrix} =
\begin{bmatrix}
\hat{r} I_{xx} + q r I_{zz} - q r I_{zy} - (\hat{c} + \hat{g}) I_{xx} \\
\hat{r} I_{zy} + p r I_{xx} - p r I_{zz} + (p^2 - r^2) I_{xy} \\
\hat{r} I_{zx} + p r I_{yy} - p r I_{xx} - (p^2 - r^2) I_{xz}
\end{bmatrix}\]
LOOK AT LHS TERMS

\[ F_x = F_{x\,\text{AERO}} + F_{x\,\text{THRESS}} + F_{x\,\text{GRAVITY}} = \dot{\theta} S C_x + F_{x\,\text{ENG}} + F_{x\,g} \]

\[ F_y = F_{y\,\text{AERO}} + F_{y\,\text{THRESS}} + F_{y\,\text{GRAVITY}} = \dot{\theta} S C_y + F_{y\,\text{ENG}} + F_{y\,g} \]

\[ F_z = F_{z\,\text{AERO}} + F_{z\,\text{THRESS}} + F_{z\,\text{GRAVITY}} = \dot{\theta} S C_z + F_{z\,\text{ENG}} + F_{z\,g} \]

\[ \dot{\omega}_\text{EARTH} = m \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} \]

\[ \dot{\omega}_\text{BODY} = m (ABC) \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} = m \begin{bmatrix} -\dot{\theta} \sin \phi \\ \dot{\theta} \cos \phi \sin \phi \\ \dot{\theta} \cos \phi \cos \phi \end{bmatrix} \]

\[ L = L_{\text{AERO}} + L_{\text{THRESS}} = \dot{\theta} S b C_e + L_{\text{ENG}} \]

\[ M = M_{\text{AERO}} + M_{\text{THRESS}} = \dot{\theta} S c C_m + M_{\text{ENG}} - \Omega_{\text{ENG}} I_{\text{ENG}} \]

\[ N = N_{\text{AERO}} + N_{\text{THRESS}} = \dot{\theta} S b C_m + N_{\text{ENG}} + \Omega_{\text{ENG}} I_{\text{ENG}} \]

**NOTE:** NO GRAVITY TERMS DUE TO FORCES ACTING THROUGH THE CG.

\[ \Omega_{\text{ENG}} I_{\text{ENG}} \] TERMS COME FROM ENGINE

GYROSCOPIC EFFECTS OF SPINNING COMPONENTS.

- NONE IN ROLL DUE TO AXIS OF ROTATION
- ASSUME THRUST AXIS AS BODY AXIS (SMALL INSTALLATION ANGLES)

VECTOR FORM E.O.M.'S

\[ \dot{\omega} = \begin{bmatrix} \dot{\theta} S C_x + F_{x\,\text{ENG}} - m \dot{\theta} \sin \phi \\ \dot{\theta} S C_y + F_{y\,\text{ENG}} + m g \cos \phi \cos \phi \\ \dot{\theta} S C_z + F_{z\,\text{ENG}} + m g \cos \phi \cos \phi \end{bmatrix} = \begin{bmatrix} m (u + g w - r v) \\ m (\dot{v} + r u - p w) \\ m (\dot{w} + p v - g u) \end{bmatrix} \]
\[
\begin{bmatrix}
\dot{\mathbf{q}}_{SE} \\
\dot{\mathbf{c}}_{SE} \\
\dot{\mathbf{q}}_{SE}
\end{bmatrix}
= \begin{bmatrix}
\mathbf{M} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{M} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{M}
\end{bmatrix}
\begin{bmatrix}
\mathbf{I}_{xx} \\
\mathbf{I}_{yy} \\
\mathbf{I}_{zz}
\end{bmatrix}
\begin{bmatrix}
\dot{\mathbf{r}} \\
\dot{\omega}_z \\
\dot{\omega}_y \\
\dot{\omega}_x
\end{bmatrix}
+ \mathbf{F}_{ext}
\]

\text{SIX DEGREES OF FREEDOM EQUATIONS OF MOTION BODY AXIS}

\[
\dot{\mathbf{u}} = \mathbf{v} - \mathbf{g} \sin \theta
\]

\[
\dot{\mathbf{v}} = \mathbf{u} + \mathbf{g} \cos \theta \sin \phi
\]

\[
\dot{\mathbf{w}} = \mathbf{g} \sin \phi
\]

\[
\dot{\phi} = \frac{\mathbf{I}_{yy} - \mathbf{I}_{zz}}{\mathbf{I}_{xx}} \mathbf{g} \mathbf{r} + \frac{\mathbf{I}_{zz}}{\mathbf{I}_{xx}} (\mathbf{r} + \mathbf{p}) + \frac{\mathbf{I}_{zz}}{\mathbf{I}_{xx}} \frac{\mathbf{r} \mathbf{S}_{c} + \mathbf{I}_{zz} \mathbf{c} + \mathbf{I}_{zz} \mathbf{m}}{\mathbf{I}_{xx}}
\]

\[
\dot{\theta} = \frac{\mathbf{I}_{xx} - \mathbf{I}_{yy}}{\mathbf{I}_{yy}} \mathbf{p} \mathbf{r} + \frac{\mathbf{I}_{xx}}{\mathbf{I}_{yy}} (\mathbf{r}^2 - \mathbf{p}^2) + \frac{\mathbf{I}_{xx}}{\mathbf{I}_{yy}} \frac{\mathbf{r} \mathbf{S}_{c} + \mathbf{I}_{xx} \mathbf{m}}{\mathbf{I}_{yy}}
\]

\[
\dot{\psi} = \frac{\mathbf{I}_{yy} - \mathbf{I}_{zz}}{\mathbf{I}_{zz}} \mathbf{p} \mathbf{b} + \frac{\mathbf{I}_{zz}}{\mathbf{I}_{zz}} (\mathbf{p} - \mathbf{g} \mathbf{r}) + \frac{\mathbf{I}_{zz}}{\mathbf{I}_{zz}} \frac{\mathbf{r} \mathbf{S}_{b} + \mathbf{I}_{zz} \mathbf{m}}{\mathbf{I}_{zz}} + \frac{\mathbf{I}_{zz} \mathbf{S}_{c}}{\mathbf{I}_{zz}} \beta + \frac{\mathbf{I}_{zz} \mathbf{m}}{\mathbf{I}_{zz}}
\]

\text{OTHER EQUATIONS}

\[
\mathbf{V}_T = \sqrt{\mathbf{u}^2 + \mathbf{v}^2 + \mathbf{w}^2}
\]

\[
\mathbf{u} = \mathbf{V}_T \cos \alpha \cos \beta
\]

\[
\mathbf{v} = \mathbf{V}_T \cos \alpha \sin \beta
\]

\[
\mathbf{w} = \mathbf{V}_T \sin \beta
\]

\[
\mathbf{a}_x = \tan^{-1} \frac{\mathbf{w}}{\mathbf{u}}
\]

\[
\mathbf{b} = \sin^{-1} \frac{\mathbf{v}}{\mathbf{V}_T}
\]

\[
\dot{\alpha} = \frac{1}{\mathbf{V}_T \cos \beta} (\mathbf{v} \cos \alpha - \mathbf{u} \sin \alpha)
\]

\[
\dot{\beta} = \frac{1}{\mathbf{V}_T} (\mathbf{u} \cos \alpha \sin \beta - \mathbf{w} \sin \alpha \sin \beta)
\]

\[
\dot{\mathbf{V}}_T = \mathbf{V}_T \cos \alpha \cos \beta + \mathbf{V}_T \sin \beta + \mathbf{w} \sin \alpha \cos \beta
\]
BODY AKIS E.O.M FOR AIRCRAFT

\[
\dot{V}_T = \frac{1}{\cos^2 \alpha \cos \phi} \left[ RV_T \sin \beta - g \sin \phi + \frac{\bar{F}_S}{m} C_x + \frac{F_{ex}}{m} \right] + V_T (\dot{\alpha} - \dot{\beta}) \tan \beta + \dot{\beta} V_T \tan \beta
\]

\[\dot{\alpha} = \frac{1}{\cos \beta \cos \phi} \left[ -p \sin \beta + \dot{\beta} \sin \beta \sin \phi + \frac{\bar{F}_S}{V_T} \cos \phi \cos \beta \right] + \frac{\bar{F}_S}{m V_T} C_x + \frac{F_{ex}}{m V_T} \]

\[- \frac{\dot{V}_T \tan \alpha}{V_T} + g\]

\[\dot{\beta} = \frac{1}{V_T \cos \beta} \left[ -V_T \sin \beta + g \cos \theta \sin \phi + \frac{\bar{F}_S}{m} C_y + \frac{F_{ey}}{m} \right] + p \sin \alpha - \bar{F} \cos \alpha \]

INTERESTING TERMS IN EOM

\[\dot{\beta} \text{ TERM} = f \left( p \sin \alpha - \bar{F} \cos \alpha \right)\]

- TRANSFORMATION OF \( \alpha \) & \( \beta \) INTO EACH OTHER
- NEED TO COORDINATE \( \dot{p} \) & \( \dot{r} \) IN ORDER TO NOT DRIVE \( \beta \) TO DEPARTURE

\[\dot{\beta} = \dot{r} = \text{ const.}\]

- KINEMATIC & INERTIAL COUPLING
LONGITUDINAL CHARACTERISTICS

LONGITUDINAL AXIS RUNS ALONG X-AXIS & ITS CHARACTERISTICS ACT AS ROTATION ABOUT THE Y-AXIS

COEFFICIENTS & RATES OF INTEREST

- $C_L$ -> LIFT COEFFICIENT
- $C_{D_n}$ -> PITCHING MOMENT COEFFICIENT
- $C_D$ -> DRAG COEFFICIENT

$\dot{\theta}$ -> PITCH RATE

$\delta_H$ -> TAIL DEFLECTION (CONTROL SURFACE) $\uparrow$ = TE DEW

LIFT

THE LIFT CURVE

\[
\begin{align*}
C_L & \quad \text{vs} \quad \xi \\
C_{L_{\text{max}}} & \quad \uparrow \\
\xi & \quad \uparrow \\
\end{align*}
\]

SYMMETRIC AIRFOILS HAVE LINEAR SLOPE UP TO A MAX, THEN THE LINEARITY ENDS UP TO A MAXIMUM $C_L$ AND THEN DIMINISHES.

SLOPE & MAXIMUM LIFT ARE FUNCTIONS OF WING GEOMETRY & SPEED
WINGS

-> COLLECTION OF AIRFOIL SECTIONS
- AIRFOILS ARE 2D
- WINGS ARE 3D

\[ \text{SPAN} \quad b \]
\[ \text{CL} \quad \text{ROOT CHORD} \quad C_r \]

AIRFOIL

C-CHORD

LAMBERT LINE

LEADING EDGE

TRAILING EDGE

LOWER SURFACE

WING AREA: \( S = \frac{bC}{2} \Rightarrow \text{MEAN CHORD} \times \text{SPAN} \)

TAPER RATIO: \( t = \frac{C_{\text{tip}}}{C_{\text{root}}} \Rightarrow \text{TIP CHORD} / \text{ROOT CHORD} \)

ASPECT RATIO: \( AR = \frac{b^2}{S} \Rightarrow \text{SPAN}^2 / \text{WING AREA} \)

\[ = \frac{b}{c} \text{ FOR} \]

MEAN AERODYNAMIC CHORD, \( \bar{C} \) OR MAC

\( \Rightarrow \text{CHORD WHERE AEROdynamic FORCES ARE ASSUMED TO ACT THROUGH} \)

\( \Rightarrow \text{PASSES THROUGH CENTROID OF AREA OF THE SEMI-SPAN} \)

\[ \bar{C} = \frac{2}{3} C_r \frac{1+a+x^2}{1+a} \]

\[ C_L \]

[Diagram showing CL vs AR for different values of AR: 2.5, 5, 10, 15-20]

FIGHTERS \( \sim \text{AR} \ 2.5-3.5 \)
TRANSOFT COMMERCIAL \( \sim \text{AR} \ 6-8 \)
GLIDER \( \sim \text{AR} \ 15-20 \)
High aspect ratio wings have steeper lift curve slope, but separation occurs at lower AoA's.

Sweep reduces lift curve slope, but promotes a more gradual stall.

High sweep, ~60° and more, creates strong leading edge vortices that enhance LE suction and introduce nonlinear contribution to lift.

Camber effects on lift curve slope:

Note that unsymmetrical airfoil provides non-zero lift at zero angle of attack.
PITCHING MOMENT & TRIM

DEFINITIONS

CENTER OF PRESSURE (C.P., C.P.)

- Location on aircraft where distributed aerodynamic loads produce no pitching moment

\[ Z_{M_{C.P.}} = 0 \]  - Distributed lift & drag can be replaced with point loads

\[ C_m = 0 \]  - Moves with flight condition or configuration

  - Forces change with airspeed & AOA
  - Aircraft is controlled by reconfiguring to move C.P. (control surfaces)

AERODYNAMIC CENTER (A.C., A.C.)

- Location on aircraft about which a change in lift does not change pitching moment

- Moves as airspeed changes, but insensitive to AOA, camber, or control deflections (low AOA)

- Moves with AOA as stall is approached

\[ \frac{dC_{max}}{dC_L} = 0 \]  - Aerodynamic center is the lift center

Note: Does not mean \[ C_m = 0 \]

PITCHING MOMENT RESULTS WHEN C.P. IS NOT LOCATED AT C.G.

- Weight & lift/drag not aligned

- Elimination of produced moment is called trimming by balancing the forces &

\[ \text{Net moment} = \text{lift} \times (C.P. \times \text{pos} - C.G. \times \text{pos}) + \text{drag} \times (C.P. \times \text{pos} - C.G. \times \text{pos}) \]
Center of pressure represents the CG location where the moment is zero.

Aerodynamic center represents CG where the moment slope is zero.

It is not convenient to express moments about C.P. as it changes with all states. Instead, move those forces to A.C. and express C_m about C.G.

Control deflections camber the aircraft, but do not change the aerodynamic center. Trias aircraft

\[
\frac{\mathrm{d}C_m}{\mathrm{d}C_l} \quad \text{unchanged}
\]

A.C. unchanged

Tail movement = camber & C
**Neutral Point**

- Center of gravity location which produces neutral aircraft stability.

- Neutral point is at the aerodynamic center.

\[
\Delta X_{N.P.} \quad \Delta Y_{N.P.} = \frac{dC_m}{dC_l}
\]

**Pitch Stability**

- Stable -> A perturbation in lift results in a pitching moment that restores the aircraft to its original, equilibrium state.

\[
\frac{dM_{CG}}{dL} < 0 \quad \text{or} \quad \frac{dC_{m_{CG}}}{dC_l} < 0
\]

- \( C_{m_{CG}} = C_l \frac{dC_{m_{CG}}}{dC_l} + C_{m_{AC}} = 0 \)

Results when aerodynamic center is aft of the CG.
Can express pitch stability as a function of angle of attack.

\[
\frac{dC_{m\alpha}}{dC_L} = \frac{dC_L}{d\alpha} = \frac{dC_{m\alpha}}{d\alpha} = C_{m\alpha}
\]

Pitch stability always lift curve slope (always positive).

\[ C_{m\alpha} \]

\[ \alpha \]

\[ C_{m\alpha} > 0 \text{ unstable} \]
\[ C_{m\alpha} < 0 \text{ stable} \]

\[ C_{m\alpha} \text{ is a more appropriate measure of pitch stability at high AoA when wings stall} \]

Caution should be made to look out for "deep stalls".

\[ C_{m\alpha} \]

\[ \text{Deep stall region} \]

\[ \text{Deep stall region} \rightarrow C_{m\alpha} \text{ drives to equilibrium point at a higher } \alpha \]

\[ \text{New equilibrium } \alpha \text{ at high AoA} \]

\[ \text{could augment w/ control surface, or "rock" out if small region.} \]
STATIC MARGIN

\[ \Delta x_{n.p.} = x_{n.p.} - x_{c.g.} \]

\[ \text{STATIC MARGIN} = \frac{\Delta x_{n.p.}}{c} = \frac{d C_m}{d C_L} \]

- NEGATIVE DISTANCE MEANS C.G. IS AHEAD OF N.P.
- LONGITUDINALLY STABLE
- NEGATIVE DISTANCE IS POSITIVE STATIC MARGIN

OFTEN TIMES DISTANCES ARE EXPRESSED AS % OF MAC

\[ \text{STATIC MARGIN} = \left[ \frac{x}{c} \right]_{n.p.} - \left[ \frac{x}{c} \right]_{c.g.} = - \left[ \frac{d C_m}{d C_L} \right] \]

\[ x_0 = x_{\text{MAC}} \quad \text{MEASURED POS. X IN -X DIRECTION} \]

NEW CONVERSION

- POSITIVE DIFFERENCE MEANS C.G. IS AHEAD OF N.P.
- AIRCRAFT LONGITUDINALLY STABLE
- POSITIVE DIFFERENCE IS A POSITIVE STATIC MARGIN

RULES OF THUMB FOR PITCH INSTABILITY LIMITS

- STATIC STABILITY RATIO REPRESENTS RATIO BETWEEN STATIC
  PITCH STABILITY & PRIMARY PITCH SURFACE CONTROL POWER

\[ -2 < \frac{M_0}{M_s} < 2 \]

- \( M_0/M_s < -2 \) : TOO UNSTABLE WITH INADEQUATE CONTROL POWER
- \( M_0/M_s > 2 \) : TOO STABLE WITH INADEQUATE CP
- \( \geq \text{SLUGGLISH RESPONSE} \)

- RATIO CAN BE CONSIDERED AS CHANGE IN TRIM CS RELATIVE POTRAWANG

\[ -2 < \left[ \frac{\delta h}{\alpha} \right]_{\text{TRIM}} < 2 \]

- TIME TO DOUBLE:

\[ T_2 \sim \sqrt{C_{m_{\alpha}} \cdot \frac{\delta h}{\alpha} \cdot \frac{85C}{98}} > 0.2 \text{s} \quad \text{PLمهندس/FCS CAN COUNTER} \]
DEFINITIONS SUMMARY

NEUTRAL POINT
- CG location that produces neutral aircraft pitch stability
- Aerodynamic center of aircraft
  - Change in lift does not change aero pitching moment
- Defined with controls fixed & no PCS feedbacks
- Typically defined at low AOA \( \Rightarrow f(M) \) only
- Typically a single neutral point is defined as an averaged sum across trim AOA for a range of gross weights & altitudes

\[
NP(\%MAC) = 100 \left( CG - \frac{dC_m}{dC_L} \right) = 100 \left( CG - \frac{C_m}{C_L} \right)
\]

STATIC MARGIN
- Distance between N.P. & C.G., positive when CG is fwd
- Pos SM comes from a negative \( \frac{dC_m}{dC_L} \) or negative \( C_m \)

\[
SM(\%MAC) = 100 \times \frac{\Delta K \omega}{2} = -100 \times \frac{dC_m}{dC_L} = -100 \times \frac{C_m}{C_L}
\]

CENTER OF PRESSURE
- Point at which aerodynamic loads produce no pitching moment
  - Lift & drag can be replaced with point loads
- Moves aft with increasing Mach number
- Seen in \( C_m \) vs. \( \alpha \)

\[ C_m \quad \text{more stable with increasing Mach} \]
TRIMMING SUMMARY

For an aircraft to fly straight and level, the forces must balance about the C.G.

\[ C_l \delta h = C_{l, aero-lift} + \frac{dC_m}{dC_l} C_l + \frac{dC_m}{d\delta h} \delta h = 0 \]

\[ C_m_{control} \]

TRIMMED LIFT BUILDUP

\[ \rightarrow \text{STABILITY OF A CONFIGURATION} \]

Collection of trimmed lift curves for all flight cond.

For given Mach and static margin:

\[ \alpha \quad \text{(UNTRIMMED LIFT CURVE)} \]

\[ \alpha \quad \text{(TRIMMED LIFT CURVE)} \]
A finite wing results in lift loss:

- Flow spills from high pressure side to low pressure side
- Reduced compression lift on lower surface
- Suction lift on upper surface

This limitation of span is a major source of drag.

Lift distribution is approximately elliptical.

Front view of wing.

**Induced Drag**

\[ D_i = \frac{L^2}{\pi b^2} \text{ (also called drag due to lift)} \]

\( \text{Generating the same lift on a wing half the size quadruples the drag} \)

\[ C_{DL} = \frac{C_l^2}{\pi AR} = \frac{D_i}{8 S} \]

Camber shifts induced drag curve.

\[ C_{DC} = \frac{(C_l - C_{l_{min}})^2}{\pi AR} + \Delta C_{DC_{camber}} \]

Reduced drag at higher AoA is exchanged for penalty at lower AoA.
DRAG TYPICALLY MODELED AS SUM OF INDUCED DRAG \& PARA\-SITIC DRAG

\[ C_D = C_{D_0} + C_{D_i} \]

\[ C_{D_i} = \text{INDUCED DRAG} = \frac{C_L^2}{\pi AR} \]

\[ C_{D_0} = \text{PARA\-SITIC DRAG (SUM OF OTHER DRAG)} \]

**PARA\-SITIC DRAG INCLUDES**
- drag due to camber
- skin friction drag
  - produced due to boundary layer
  - empirical equations
- shock wave drag
  - produced by shocks forming
- zero-lift drag

\[ C_L \]
\[ C_{D_0} \]
\[ C_{D_i} \]
\[ C_D \]

**NOTE THAT TRIMMING THE AIRCRAFT PRODUCES TRIM DRAG THAT IS DUE TO CONTROL SURFACE DEFORMATIONS \& STATIC MARGIN. CAN BE SUMMED INTO \( C_{D_0} \)**

\[ \rightarrow C_D = C_{D_{Z\text{ERO-LIFT}}} + C_{D_{L\text{IFT}}} + C_{D_{\text{TEIM}}} \]
PERFORMANCE

**Rate of Climb** (Specific Excess Power, $P_s$)

\[
\text{Rate of Climb} = V \sin \gamma \\
W \sin \gamma = T \cos \alpha - D \\
\sin \gamma = \frac{T \cos \alpha - D}{W}
\]

\[
P_s = V \sin \gamma = \frac{V}{W} (T \cos \alpha - D) \quad \text{ft/s or ft/mw}
\]

Specific Excess Power is a measure of instantaneous rate of climb.

Think of it as "usable energy".

**Load Factor**

\[
\begin{align*}
N_L &= \frac{L}{W} = \frac{C_L \overline{b} \overline{S}}{W} = \frac{C_L \overline{b} \overline{S}}{W} \\
N_x &= -\frac{\text{force}}{W} = -\frac{C_x \overline{b} \overline{S}}{W} \\
N_y &= \frac{Y}{W} = \frac{C_y \overline{b} \overline{S}}{W}
\end{align*}
\]

\[
N_{\text{limit}} = \frac{\text{Structural Design Limit Load}}{\text{Flight Design Gross Weight}}
\]

**Load Factor** is also a non-dimensional measure of the aircraft acceleration as a multiple of "g's" (gravity).

\[
\Rightarrow N_L = \frac{L}{W} = \frac{m a}{m g} = \frac{a}{g}
\]
**TURN CAPABILITY**

**TURNING RADIUS:** \( R = \frac{V^2}{g \sqrt{N_L^2 - 1}} \)

**TURN RATE:** \( \dot{\psi} = \frac{57.3 V}{\text{TURN RADIUS}} = 57.3 \frac{V}{V} \sqrt{N_L^2 - 1} \)

- \( R \): Radius in ft
- \( \dot{\psi} \): Rate of turn deg/s
- \( V \): Aircraft velocity ft/s
- \( g \): Acceleration due to gravity - 32.174 ft/s²
- \( N_L \): Load factor normal to flight path

→ Higher speeds mean larger turn radius; lower turn rates.

→ Maximum turn rate at a given speed is limited by load factor.

"Doughhouse plots" -> Relationship between \( R, \dot{\psi}, P_s, N_L \)

[Diagram showing turn rate, turn radius, Mach, corner speed, maximum turn rate, structural limit (load factor), increasing load factor, and intersection of maximum aerodynamic turning, maximum airframe strength.]
**Ceiling Definitions**

**Absolute Ceiling** - Altitude at a given airspeed at which the rate of climb is 0 fpm using MIL or MAX power.

**Service Ceiling** - Altitude at a given airspeed at which the rate of climb is 100 fpm using MIL or MAX power.

**Cruise Ceiling** - 11/11 300 fpm using maximum continuous power (non-afterburning, not reduced power).

**Combat Ceiling** - 11/11 500 fpm using MIL or MAX power.
**Lateral-Directional Stability**

**Directional Stability**

Want a returning yawing moment to be generated when a directional disturbance occurs:

- Directional disturbance
  - Increase in sideslip (+β)

- Want a positive yawing moment to form to return to zero sideslip

\[ \frac{\delta n}{\delta \beta} > 0 \quad \text{or} \quad \frac{SC_d}{\delta \beta} > 0 \]

\[ C_{nB} > 0 \]

Prevailing sideforce is coming from the vertical stabilizers:
- Rudders (control surface)
- Vertical tails (fixed)

Prevailing sideforce magnitude & location aft of CG create yawing moment:
- Can fly at constant sideslip (steady-maneuvering sideslip), requires rudder input

Center & magnitude of SF varies with flight conditions and altitude:
- Mach compressibility effects
- AOA varies flow to vertical stabilizers -> can change \( C_{nB} = f(\alpha) \)
DIRECTIONAL CONTROL FROM VERTICAL TAILS

- Positive rudder deflection \( \delta_r \) measured as to left

- Positive rudder deflection provides negative yawing moment

\[ \delta_R = \frac{(\delta_{RL} + \delta_{RR})}{2} \rightarrow \text{Symmetric rudder} \]

Looking at individual rudder:

- SFUL → Rudder movement is applying camber to airfoil

  • Side force generated instead of lift due to coordinate axis location.

Directional control is used to yaw the airplane and control sideslip. It also has input to rolling moment and side force.

→ Note that canted vertical tails will provide a larger rolling moment when rudders are deflected.

  • Rudders generate rolling moment because pressure location is above CG.
LATERAL STABILITY

POSITIVE STABILITY

→ A DISTURBANCE GENERATES A DIFFERENTIAL LIFT AND ROLLING MOMENT RETURNING AIRCRAFT TO ZERO SIDESLIP

→ \( \alpha > 0 \)

\( \alpha > 0 \) → NEGATIVE ROLLING MOMENT DEVELOPS WITH A POSITIVE SIDESLIP

\( \alpha < 0 \) → POSITIVE ROLLING MOMENT DEVELOPS WITH A POSITIVE SIDESLIP

→ STABLE ROLL RESPONSE REDUCES SIDESLIP

\[ \frac{S_l}{S_B} < 0 \text{ or } \frac{S c_e}{S_B} < 0 \text{ or } C_{l_B} < 0 \text{ for } \alpha > 0 \]

\[ \frac{S_l}{S_B} > 0 \text{ or } \frac{S c_e}{S_B} > 0 \text{ or } C_{l_B} > 0 \text{ for } \alpha < 0 \]

\( \Rightarrow \) SIDESLIP RETURNS TO ZERO
LATERAL CONTROL

LATERAL CONTROL

→ DIFFERENTIAL AILERONS/FLAPS
→ DIFFERENTIAL TAILS
→ RUDDERS (SMALLER EFFECTS)
→ SPOILERS

AILERON EXAMPLE

AILERON DEFLECTION

POS. TED

→ NEGATIVE DIFFERENTIAL AILERON SHOWN

* PROVIDES NEGATIVE ROLLING MOMENT

\[ \delta_A = \frac{(\delta_{AI} - \delta_{AR})}{2} = \delta_A \]

\[ \delta_{A_{AVG}} = \frac{(\delta_{AI} + \delta_{AR})}{2} = \delta_{A_{AVG}} \]

NOTE THAT LAT-DIR CONTROL SURFACES PROVIDE TERMS TO BOTH, HENCE LAT-DIR IS THOUGHT OF TOGETHER. VERY TOUGH TO GET ONE RESPONSE WITHOUT THE OTHER.

→ DIFFERENTIAL TAIL PROVIDES BOTH ROLLING MOMENT AND YAWING MOMENT, ETC.
DEPARTURE & DEPARTURE RESISTANCE

DEFINITION

-> A DEPARTURE IS A LOSS OF CONTROLLED FLIGHT AND HAPPENS WHEN AN AIRCRAFT IS MOVING IN AN UNCONTROLLED MANNER.

* CAN HAPPEN WHEN OPERATING ENVELOPE IS EXCEEDED

* SOME DEPARTURES CAN BE RECOVERED FROM

DEPARTURE RESISTANCE PARAMETER

-> HELPS DETERMINE LIKELIHOOD OF DEPARTURES

-> LOOKS AT DIRECTIONAL STABILITY & LATERAL STABILITY TOGETHER

\[ C_{\alpha \delta} \gamma = C_{\alpha} \beta \cos \alpha - \frac{I_{xx}}{I_{xx}} C_{\beta} \frac{\gamma}{\sin \alpha} \]

DERIVATION FROM \( \dot{\beta} \) EQUATION:

\[ \dot{\beta} = p \sin \alpha - r \cos \alpha \]

\[ \ddot{\beta} = \dot{p} \sin \alpha + p \cos \alpha \dot{\alpha} - r \cos \alpha + r \sin \alpha \dot{\alpha} \]

\[ \dot{\alpha} = 0 \hspace{1cm} \text{ASSUMED} \]

\[ \ddot{\beta} = \dot{p} \sin \alpha - r \cos \alpha \]

-> ASSUME NO INERTIAL COUPLING

\[ \dot{p} = \frac{\dot{\beta} \delta b}{C_{l}/I_{xx}} \]

\[ \dot{r} = \frac{\dot{\beta} \delta b}{C_{n}/I_{zz}} \]

\[ \ddot{\beta} = \frac{\dot{\beta} \delta b}{C_{l}/I_{xx}} \sin \alpha - \frac{C_{n}}{I_{zz}} \cos \alpha \]
\[-\frac{I_{yz}}{I_{zx}} = C_n \cos \alpha - \frac{I_{xz}}{I_{xx}} C_l \sin \alpha\]

Differentiate wrt \( \beta \)

\[C_{n,\beta,\text{dynamic}} = C_{n,\beta} \cos \alpha - \frac{I_{xz}}{I_{xx}} C_l \beta \sin \alpha\]

Have resistance to departures when \( C_{n,\beta,\text{dynamic}} \to 0 \)

Remember:

For positive \( \alpha \)

Stable is: \( C_{n,\beta} > 0 \)

\( C_{\beta} < 0 \)

However, in practice...

\[\rightarrow C_{n,\beta} < C_{\beta}\]

Reduce at higher AOA due to flow separation on wing impinging on the roll and yaw authority devices

\[\rightarrow \text{Notice } C_{n,\beta} \text{ goes unstable}!!\]
DAMPING TERMS

EASIEST TO LOOK AT ROLL DAMPING FIRST

$\Delta V = \frac{Pb}{2}$

$P$ in rad/s

VIEW OF WINGTIP (RIGHT)

$\Delta \alpha$ is the tip helix angle

$\frac{Pb}{2V} = \tan \Delta \alpha$

$\Delta \alpha$ is the tip helix angle

ROTATION INCREASES THE ANGLE OF ATTACK, AND THEREFORE LIFT, ON THE DOWNWARD MOVING WING. THIS ROTATION DECREASES ANGLE OF ATTACK, AND LIFT, ON THE UPWARD MOVING WING. THIS COMBINATION GENERATES A RETARDING ROLLING MOMENT -> ROLL DAMPING

ROLL DAMPING COEFFICIENT

$C_{LP} = \frac{2V}{Pb}$

TYPICALLY A NEGATIVE VALUE FOR POSITIVE STABILITY

OPPOSITE OF DAMPING IS PROPELLING. THIS IS A MOMENT DRIVING FORCE.
COMPARABLE TERMS EXIST IN OTHER AXES

**YAW DAMPING COEFFICIENT**

\[
C_{\text{mr}} = \frac{2 C_m}{\frac{d (\frac{C_l}{2 V})}{d t}}
\]

→ POSITIVE YAWING MOMENT ROTATION INDUCES VELOCITY ON VERTICAL STABILIZERS AND PRODUCES SIDE FORCE AFT OF C.G. THIS GENERATES RESTORING YAWING MOMENT.

**PITCH DAMPING COEFFICIENT**

\[
C_{\text{mb}} = \frac{2 C_m}{\frac{d (\frac{C_l}{2 V})}{d t}}
\]

→ POSITIVE PITCHING MOMENT INDUCES VELOCITY ON HORIZONTAL STABILIZERS, INCREASING LIFT AT AFT END, GENERATING A RESTORING PITCHING MOMENT.

**PLUNGING DERIVATIVE**

\[
C_{\text{m}} = \frac{2 C_m}{\frac{d (\frac{C_l}{2 V})}{d t}}
\]

**PURE PITCHING MOTION**
\[
\dot{\alpha} = 0 \quad \delta = \sin \omega t
\]

**PURE PLUNGING MOTION**
\[
\dot{\alpha} = \sin \omega t \quad \delta = 0
\]
**Cross-Axis Damping Coefficients**

**Yaw Damping Due to Roll Rate Coefficient**

\[ C_{\text{mp}} = \frac{2 \, \text{Cm}}{\text{d}(\text{Ps} / 2V)} \]

→ When rolling, velocity is induced onto the vertical stabilizers. This induces a side force on the vertical stabilizers behind the CG, thus producing a yawing moment.

**Roll Damping Due to Yaw Rate Coefficient**

\[ C_{\text{lr}} = \frac{2 \, \text{Cl}}{2(\text{la} / 2V)} \]

→ When yawing, velocity is induced onto the vertical stabilizers. This induces a side force above the CG, thus generating a rolling moment.
TRADITIONAL NON-DIMENSIONAL ROTATION RATE COMPONENTS

Also known as the "DIRECT METHOD"
- Other methods exist
  - Kalviste
  - Excess Roll Rate
  - Forced Oscillation
- Simplest method
- Non-Discontinuous

TRADITIONAL NON-DIM ROTATIONAL RATES ARE DERIVED BY BREAKING THE TOTAL AIRCRAFT ROTATIONAL RATES INTO COMPONENTS
- One "steady-state" component aligned with the velocity vector
- Three residual or oscillatory components aligned with body X, Y, Z axes.

\[ \Omega = \sqrt{\dot{p}^2 + \dot{q}^2 + \dot{r}^2} \]

\[ \Omega_v = p \cos \alpha \cos \beta + g \sin \beta + r \sin \alpha \cos \beta \]

\[ \Omega_{osc} = \Omega - \Omega_v \]

\[ P_{osc} = p - \Omega_v \cos \alpha \cos \beta \]

\[ \phi_{osc} = \beta - \Omega_v \sin \beta \]

\[ \rho_{osc} = r - \Omega_v \sin \alpha \cos \beta \]

\[ \Omega_{0} \Omega_v = \frac{\Omega_v b}{2V_T} \]

\[ P_B = \frac{P_{osc} b}{2V_T} \]

\[ C_{QB} = \frac{\phi_{osc} b}{2V_T} \]

\[ R_{B} = \frac{\rho_{osc} b}{2V_T} \]

\[ \left\{ \begin{array}{l}
\text{Used in aerodynamic dynamic derivative terms} \\
\end{array} \right. \]
Hinge moments

In order to rotate control surfaces about its hinge, necessary force is required to overcome the aerodynamic pressure that resists the motion.

If the aero forces on the surface overcome the applied actuator hinge moment, the control surface will move to a new, uncommanded position.

Hinge moment \( H_c \) acts in the opposite direction of aero forces to counter the pressure.

Coefficient form

\[
C_h = \frac{H}{\frac{1}{2} \rho V^2 S_{ref} \bar{c}_{ref}} = \frac{H}{\frac{\bar{c}}{S_{ref} \bar{c}_{ref}}}
\]

Where \( S_{ref} \) and \( \bar{c}_{ref} \) are surface dependent.