<table>
<thead>
<tr>
<th>Mechanical Engineering and Materials Science Independent Study</th>
<th>Mechanical Engineering &amp; Materials Science</th>
</tr>
</thead>
</table>

12-12-2018

**Donahue Independent Study; Collection of Aerodynamic Stability and Control Lecture Notes**

Daniel Donahue  
*Washington University in St. Louis*

Dave Peters  
*Washington University in St. Louis*

Follow this and additional works at: https://openscholarship.wustl.edu/mems500

**Recommended Citation**

https://openscholarship.wustl.edu/mems500/80

This Final Report is brought to you for free and open access by the Mechanical Engineering & Materials Science at Washington University Open Scholarship. It has been accepted for inclusion in Mechanical Engineering and Materials Science Independent Study by an authorized administrator of Washington University Open Scholarship. For more information, please contact digital@wumail.wustl.edu.
The learning objective of this independent study was to further my own knowledge in the field of Flight Mechanics/Dynamics as well as attaining experience in creating coursework. This was done by creating 14 lectures worth of hand-written notes on the subject of Aircraft Stability and Control. I primarily used the book by Pamadi, “Performance, Stability, Dynamics, and Control of Airplanes 2\textsuperscript{nd} ed” while also pulling from my experiences as an aerodynamic stability and control engineer.

During the course of the semester I generated 42 pages of hand written notes that correspond to 14 lectures worth of material. It was agreed upon with my advisor, David Peters, that 3 pages of hand written notes would equate to a lecture. This number was based off of his experience with lecture materials where 3 pages of hand written notes, accompanied by in-person elaboration of the material, equates to one lecture worth of material.

Below is a syllabus of the 14 covered lectures with corresponding pages in the hand-written notes attached.

<table>
<thead>
<tr>
<th>Lecture</th>
<th>Topic</th>
<th>Note Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Syllabus and Aircraft Geometry</td>
<td>Syllabus and 0</td>
</tr>
<tr>
<td>2-3</td>
<td>Aircraft Stability and Control Definitions; Coordinate Axis</td>
<td>1-7</td>
</tr>
<tr>
<td>3-6</td>
<td>Derivation of Aircraft Equations of Motion</td>
<td>8-15</td>
</tr>
<tr>
<td>7-9</td>
<td>Longitudinal Characteristics</td>
<td>16-27</td>
</tr>
<tr>
<td>10</td>
<td>Maneuvering Performance</td>
<td>28-30</td>
</tr>
<tr>
<td>11-12</td>
<td>Lateral-Directional Characteristics</td>
<td>31-36</td>
</tr>
<tr>
<td>13-14</td>
<td>Dynamic Derivatives; Hinge Moments</td>
<td>37-41</td>
</tr>
</tbody>
</table>
1) SYLLABUS & AIRCRAFT GEOMETRY
2) AIRCRAFT STABILITY & CONTROL DEFINITIONS
   - FLIGHT VARIABLE DEFINITIONS OF INTEREST
     - COORDINATE SYSTEMS
       - BODY
       - STABILITY
       - WIND
       - INERTIAL
     - AIRCRAFT ACRO FORCES & MOMENTS NON-DIM
     - STANDARD ATM EQUATIONS

2) SIX DEGREES OF FREEDOM EQUATIONS OF MOTION

3) LATERAL-DIRECTIONAL CHARACTERISTICS
   - DIRECTIONAL STABILITY & CONTROL
   - LATERAL STABILITY & CONTROL
   - DEPARTURE RESISTANCE

4) PERFORMANCE PARAMETERS
   - TURN RATE
   - TURN RADIUS
   - RATE OF CLIMB

5) DYNAMIC DERIVATIVES
   - ROLL, YAW, PITCH & PLUNGING DAMPING
   - CROSS-AXIS DAMPING
   - TRADITIONAL METHODS

6) HINGE MOMENTS
AIRCRAFT TYPES, GEOMETRY, CONTROL SURFACES

TYPES OF AIRCRAFT
- COMMERCIAL
  • TRANSPORT (7-SEAT, A-SERIES, FREIGHTER, 380MP!
  • PERSONAL (CESSNA SMALL AIRCRAFT, RED-BOUL FLYERS)

- DEFENSE
  • FIGHTER/ATTACK (F-15, F/A-18, F-22, F-25, TOMCAT, ETC.)
  • BOMBER (STRAIGHTFORWARD, B-1, FLYING WING)
  • TRAINER (T-45, ETC.)
  • UAV [DRONE] (SCAN EAGLE, ETC.)
  • SURVEYANCE [STEALTH] (SR-71, A-12, SATELLITE)

GEOMETRY
- FUSELAGE
  • TUBULAR
  • SEARS-HAAK
  • RASTOR

- WING
  • CANARD (MISC.
  • LEX
  • HERSCHER
  • SWEPT
  • DELTA

- TAIL
  • V-TAIL
  • T-TAIL
  • H-TAIL

- MISC
  • THRUST-VECTORING
  • WINGLETS
  • VORTEX GENERATORS

- CONTROL SURFACES
  • AILERON
  • FLAPS (TEF/LEF) FIG1.24
  • FLAPERON
  • SLATS STRAKE
  • TRIM TABS
  • ELEVON
  • STABILATOR
  • SPEED BREAK
  • RUDDER

- AIRCRAFT BODY AXIS
  • X, Y, Z
  • φ, θ, ψ
  • LIFT, DRAG, SIDEFORCE

HOMEWORK
IDENTIFY WHICH C'S
DO WITH EACH AXIS
(1) Flight Dynamics: A Study of the Transient Motion of an Aerospace Vehicle Following a Perturbation from an Equilibrium State

- Equilibrium State: Vehicle state in which \( F(e) = M(e) = 0 \) where \( F \) and \( M \) represent external forces and moments applied.

- States of Interest: Steady-level flight, etc.

- Perturbation: Change in \( F \) and \( M \) caused by control surface deflections, turbulence, internal shifts, bare airframe stability, etc.

Flight Dynamics deals with steady and unsteady motion of a vehicle caused by disturbance, intentional or unintentional, applied to an initially trimmed motion state.

(2) Stability: The Quality of an Equilibrium State

- Equilibrium states of aircraft are generally designed to be hands-off conditions.

- Aircraft with inferior stability characteristics can potentially be dangerous, unacceptable.

- Static Stability:
  - Stable
  - Neutral
  - Unstable

- 3D Stability:
  - Static stability: If it returns to a state when perturbed.
DYNAMIC STABILITY → VEHICLE IS DYNAMICALLY STABLE IF, WHEN PERTURBED, IT EVENTUALLY RETURNS TO AN EQUILIBRIUM STATE.

STABLE → UNSTABLE → NEUTRAL

CONTROL → ABILITY, OR LACK THEREOF, TO STEER AN AEROSPACE VEHICLE FROM POINT A TO POINT B ALONG AN ACCEPTABLE FLIGHT PATH

STATIC CONTROL → ABILITY TO MAINTAIN A PRESCRIBED EQUILIBRIUM FLIGHT CONDITION

DYNAMIC CONTROL → ABILITY TO MAINTAIN AN ACCEPTABLE TRANSIENT MOTION FOLLOWING A PERTURBATION, THE ABILITY TO CHANGE FROM ONE TRIM CONDITION TO ANOTHER.

FLIGHT VARIABLE DEFINITIONS OF INTEREST

BODY AXIS COORDINATE SYSTEM

COORDINATE

VELOCITY

ROTATIONAL VELOCITY

AERO FORCE

AERO MOMENT

AXIAL FORCE

NORMAL FORCE
\[ \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{body}} = \begin{bmatrix} \cos \phi & 0 & -\sin \phi \\ 0 & 1 & 0 \\ \sin \phi & 0 & \cos \phi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{stability}} \]

\[ \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{stability}} = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{body}} \]

Check signs

\[
\begin{align*}
L &= \text{Lift} = -z_B \cos \alpha + x_B \sin \alpha = N \cos \theta \phi A \sin \phi \\
D &= \text{Drag} = -x_B = -z_B \sin \alpha - x_B \cos \alpha = N \sin \phi \phi A \cos \phi
\end{align*}
\]

W = \text{Weight}

T = \text{Thrust}
\[ \alpha = \tan^{-1}\left( \frac{w}{u} \right) = \cos^{-1}\left( \frac{u}{\sqrt{u^2+w^2}} \right) = \sin^{-1}\left( \frac{w}{\sqrt{u^2+w^2}} \right) \]

\[ \beta = \sin^{-1}\left( \frac{v}{v_T} \right) = \tan^{-1}\left( \frac{v}{\sqrt{v^2+w^2}} \right) = \cos^{-1}\left( \frac{\sqrt{v^2+w^2}}{v_T} \right) \]

\[ \alpha_T = \cos^{-1}\left( \frac{u}{v_T} \right) = \sin^{-1}\left( \frac{\sqrt{v^2+w^2}}{v_T} \right) = \tan^{-1}\left( \frac{\sqrt{v^2+w^2}}{u} \right) \]

\[ \beta_B = \tan^{-1}\left( \frac{v}{u} \right) = \cos^{-1}\left( \frac{u}{\sqrt{u^2+v^2}} \right) = \sin^{-1}\left( \frac{v}{\sqrt{u^2+v^2}} \right) \]

\[ \beta = \tan^{-1}(\tan \beta_B \cos \alpha) \]

\[ \alpha_T = \cos^{-1}(\cos \alpha \cos \beta) \]

\[ v_T = \sqrt{u^2+v^2+\omega^2} \]

\[ u = v_T \cos \alpha \cos \beta \]

\[ v = v_T \sin \beta \]

\[ \omega = \sin \omega \cos \beta \]
STABILITY TO WIND

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_W =
\begin{bmatrix}
\cos \beta & \sin \beta & 0 \\
-\sin \beta & \cos \beta & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_S
\]

BODY TO WIND

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_W =
\begin{bmatrix}
\cos \alpha \cos \beta & \sin \alpha & \sin \alpha \cos \beta \\
-\cos \alpha \sin \beta & \cos \alpha & -\sin \alpha \sin \beta \\
-\sin \alpha & 0 & \cos \alpha
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_B
\]

INERTIAL AXIS SYSTEM

ORDER MATTERS
INERTIAL TO BODY AXIS SYSTEM

\[
\begin{pmatrix}
X_N \\
Y_E \\
Z_D
\end{pmatrix} =
\begin{pmatrix}
\cos \phi \cos \theta & \cos \theta \sin \phi - \sin \theta \cos \phi & \cos \theta \sin \phi + \sin \theta \cos \phi \\
\sin \theta \cos \phi & \sin \theta \sin \phi + \cos \phi \cos \phi & \cos \theta \sin \phi - \sin \theta \cos \phi \\
-\sin \phi & \cos \phi & \cos \phi \\
\end{pmatrix}
\begin{pmatrix}
X_B \\
Y_B \\
Z_B
\end{pmatrix}
\]

\[
C =
\begin{pmatrix}
\cos \phi & \sin \phi & 0 \\
-\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

\[
B =
\begin{pmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{pmatrix}
\]

\[
A =
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \phi & \sin \phi \\
0 & -\sin \phi & \cos \phi
\end{pmatrix}
\]

\[
\begin{pmatrix}
X_N \\
Y_E \\
Z_D
\end{pmatrix} = (ABC)^{-1}
\begin{pmatrix}
X_B \\
Y_B \\
Z_B
\end{pmatrix}
\]

\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix}_B = ABC
\begin{pmatrix}
X_E \\
Y_E \\
Z_E
\end{pmatrix}
\]
AERODYNAMIC

\[ F_X = -A \quad \rightarrow \quad \text{OPPOSITE OF AXIAL FORCE} \]
\[ F_Y = SF \quad \rightarrow \quad \text{SIDE FORCE} \]
\[ F_z = -N \quad \rightarrow \quad \text{OPPOSITE OF NORMAL FORCE} \]

\[ \bar{F} = \frac{1}{2} \rho V^2 = 0.7 \rho M^2 \]
\[ b = \text{span} \]
\[ c = \text{chord} \] (MAC)

\[ F_X = \bar{F} S C_X \]
\[ F_Y = \bar{F} S C_Y \]
\[ F_Z = \bar{F} S C_Z \]
\[ L = \bar{F} S b C_L \]
\[ M = \bar{F} S b C_M \]
\[ N = \bar{F} S b C_N \]

STANDARD ATMOSPHERE MODEL EQUATIONS

IDEAL GAS LAW: \[ \rho = \frac{p}{\sqrt{RT}} \]

HYDROSTATIC EQUILIBRIUM: \[ \frac{dp}{dh} = -\rho \]

TEMPERATURE GRADIENTS: \[ T = T_{BASE} + \Delta T \]

RATIOS: \[ \Theta = \frac{T}{T_0} \quad \sigma = \frac{p}{p_0} \quad \gamma = \frac{T}{T_0} = \frac{\rho}{\rho_0} \]

SPEED OF SOUND: \[ a = \sqrt{\gamma RT} \]

DYNAMIC PRESSURE: \[ \bar{F} = \frac{1}{2} \rho V^2 = 0.7 \rho M^2 \]

TRUE AIRSPEED: \[ V_T = M_0 \]

EQUIVALENT AIRSPEED: \[ V_E = V_T \sqrt{\Theta} \]

TEMPERATURE RATIO: \[ \frac{\Theta}{\Theta_0} = 1 + \frac{V - 1}{2} M^2 = 1 + 0.2 M^2 \]
DERIVATION OF AIRCRAFT EQUATIONS OF MOTION

FORCES

STARTING IN THE INERTIAL REFERENCE FRAME

→ NEWTON'S 2nd LAW ONLY APPLICABLE IN THIS FRAME

\[ F = \frac{\partial}{\partial t} (m \cdot \dot{v}) \]
\[ = \dot{m} \frac{\partial}{\partial t} (m) + m \frac{\partial}{\partial t} (\dot{v}) \]

ASSUMPTION #1: CONSERVATION OF MASS, \( \frac{\partial m}{\partial t} = 0 \)

→ \( \ddot{v} = \dot{m} \frac{\partial}{\partial t} (\dot{v}) \)

ASIDE: NOT TRUE FOR AIRCRAFT, BUT REASONABLE FOR TIME SCALES WE ARE LOOKING AT. FUEL BURN.

WANT TO SWITCH TO A BODY AXIS REFERENCE FRAME

→ BODY AXIS IS EASILY MEASURED ONBOARD AN AIRCRAFT

→ EASY TO REFERENCE ABOUT A/C CG

RECALL: \( \frac{\partial I}{\partial t} \dot{\Delta} = \frac{\partial^3}{\partial t^3} \dot{\Delta} + \nabla \times \dot{\Delta} \) \( I \rightarrow \) INERTIAL

\( \dot{\Delta} \rightarrow \) BODY

→ VECTOR DIFFERENTIATION REQUIRES ATTENTION TO ROTATING AXIS SYSTEMS.

\[ \frac{\partial^3}{\partial t^3} \Delta \dot{v} + \dot{\nabla} \times \dot{v} = \dot{v} + \nabla \times \dot{v} \Rightarrow \dot{v}_{CG} + \Omega \times \dot{v}_{CG} \]

→ \( \ddot{v} = \dot{m} \left( \dot{v}_{CG} + \Omega \times \dot{v}_{CG} \right) \)

\[ \Omega \rightarrow \] REPRESENTS THE ROTATION OF THE BODY-FIXED FRAME WITH RESPECT TO IRF

→ ABOVE EQN IS IN BODY REFERENCE FRAME
MOMENTS

NEWTON'S SECOND LAW FOR ROTATIONAL MOTIONS

\[ \dot{M} = \frac{d}{dt}(\dot{\tau}) \]

\( \dot{M} \) is the sum of externally applied moments

\[ \frac{d}{dt}(\dot{\tau}) \] is rate of change of angular momentum

ADDS: ANGULAR MOMENTUM

ANGULAR MOMENTUM CAN BE THOUGHT OF AS LINEAR MOM. WITH A MOMENT ARM

Lin. mom. = \( m \ddot{r} \), \( \ddot{r} = \mathbf{v} \)

Ang. mom. = \( \mathbf{r} \times m \mathbf{\dot{r}} = \mathbf{L} = \mathbf{r} \mathbf{\dot{v}} \)

IN THIS CASE, INDIVIDUAL MASSES NEED TO BE CONSIDERED:

\[ \dot{H}_m = \mathbf{r}_m \times d\mathbf{m} \ddot{r} = d\mathbf{m} (\mathbf{r}_m \times \mathbf{\dot{r}}) \]

SUMMING OVER ALL MASS:

\[ \dot{H} = \int \dot{H}_m = \int \mathbf{r}_m \times \mathbf{\dot{r}} d\mathbf{m} \]

\[ \int \] DIFFERENTIATING TO GET

\[ \dot{H} = \int (\mathbf{r}_m \times \mathbf{\dot{r}}) d\mathbf{m} = \dot{M} \]

WANT TO GET TO BODY REFERENCE FRAME

\[ \mathbf{r}_x = \mathbf{r}_b \]

\[ \mathbf{r}_z = \mathbf{r}_b + \mathbf{\omega} \times \mathbf{r}_b \]

\[ \mathbf{r}_z = \mathbf{r}_b + \mathbf{\omega} \times \mathbf{r}_b + \mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{r}_b) + \mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{r}_b) \]

\[ = \mathbf{r}_b + 2 \mathbf{\omega} \times \mathbf{r}_b + \mathbf{\omega} \times \mathbf{r}_b + \mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{r}_b) \]

\[ \mathbf{\dot{r}}_m = \int (\mathbf{\dot{r}}_b + 2 \mathbf{\omega} \times \mathbf{r}_b + \mathbf{\omega} \times \mathbf{r}_b + \mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{r}_b)) \]
Assumption #2: Airplanes are rigid bodies
\[ \mathbf{\dot{\omega}} = 0 \]

Essentially saying that elements of mass in an airplane do not change position with one another time.

Airplanes are not rigid bodies; structure flexes, fuel sloshes, propellers or compressors or turbines rotate, reasonably assumption for flying qualities.

\[ \mathbf{\dot{M}} = \int \mathbf{r} \times (\mathbf{\dot{r}} \times \mathbf{\tau} + \dot{\mathbf{r}} \times (\mathbf{\tau} \times \mathbf{\tau})) \, dt \]

Assumption #3: Our ERF will be a flat, non-rotating earth reference frame.

Valid if under 30/30s, or less than 1 min.

Assumption #4: Allows us to define \( \mathbf{\dot{\omega}} \) as the aircraft angular rotation relative to earth.

Assumption #4: Atmosphere is at rest relative to flat-earth

\[ \Rightarrow \text{No winds.} \]

Assign the vectors in notation:

\[ \mathbf{\dot{r}} = F_x \hat{\mathbf{e}}_x + F_y \hat{\mathbf{e}}_y + F_z \hat{\mathbf{e}}_z \]

\[ \mathbf{\dot{V}} = U \hat{\mathbf{e}}_x + V \hat{\mathbf{e}}_y + W \hat{\mathbf{e}}_z \]

\[ \mathbf{\dot{\omega}} = \omega_\theta \hat{\mathbf{e}}_\theta + \omega_\psi \hat{\mathbf{e}}_\psi + \hat{\mathbf{e}}_\phi \]

\[ \mathbf{\dot{M}} = L \hat{\mathbf{e}}_L + M \hat{\mathbf{e}}_M + N \hat{\mathbf{e}}_N \]

\[ \mathbf{\dot{\omega}} = \dot{\omega}_x \hat{\mathbf{e}}_x + \dot{\omega}_y \hat{\mathbf{e}}_y + \dot{\omega}_z \hat{\mathbf{e}}_z + \left[ U \dot{\mathbf{e}}_x + V \dot{\mathbf{e}}_y + W \dot{\mathbf{e}}_z \right] \]

\[ [J] = 0 \text{ in body fixed frame} \]
\[ \mathbf{F} = m \left( \mathbf{\dot{V}}_{\text{co}} + \mathbf{\dot{r}} \times \mathbf{\dot{V}}_{\text{co}} \right) \]

\[ \mathbf{\dot{r}} \times \mathbf{\dot{V}}_{\text{co}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \mathbf{p} & \mathbf{q} & \mathbf{r} \\ \mathbf{u} & \mathbf{v} & \mathbf{w} \end{vmatrix} = (\mathbf{q} \mathbf{w} - \mathbf{r} \mathbf{v}) \mathbf{k} + (\mathbf{r} \mathbf{u} - \mathbf{p} \mathbf{w}) \mathbf{j} + (\mathbf{p} \mathbf{v} - \mathbf{q} \mathbf{u}) \mathbf{i} \]

\[ \rightarrow \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} m \left( \mathbf{\dot{u}} + \mathbf{a} \mathbf{w} - \mathbf{R} \mathbf{v} \right) \\ m \left( \mathbf{\dot{v}} + \mathbf{R} \mathbf{u} - \mathbf{P} \mathbf{w} \right) \\ m \left( \mathbf{\dot{w}} + \mathbf{P} \mathbf{v} - \mathbf{Q} \mathbf{u} \right) \end{bmatrix} \]

**Back to Moments**

\[ \mathbf{M} = \int_0^\mathbf{r} \mathbf{\dot{r}} \times \left( \mathbf{\dot{r}} \times \mathbf{\dot{r}}_\mathbf{B} + \mathbf{\dot{r}} \times \left( \mathbf{\dot{r}} \times \mathbf{\dot{r}}_\mathbf{B} \right) \right) \mathbf{d}m \]

\[ \mathbf{\dot{r}} \times \mathbf{\dot{r}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \mathbf{p} & \mathbf{q} & \mathbf{r} \\ \mathbf{u} & \mathbf{v} & \mathbf{w} \end{vmatrix} = (\mathbf{q} \mathbf{w} - \mathbf{r} \mathbf{v}) \mathbf{k} + (\mathbf{r} \mathbf{u} - \mathbf{p} \mathbf{w}) \mathbf{j} + (\mathbf{p} \mathbf{v} - \mathbf{q} \mathbf{u}) \mathbf{i} \]

\[ \mathbf{\dot{r}} \times (\mathbf{\dot{r}} \times \mathbf{\dot{r}}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \mathbf{p} & \mathbf{q} & \mathbf{r} \\ \mathbf{u} & \mathbf{v} & \mathbf{w} \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \mathbf{p} & \mathbf{q} & \mathbf{r} \\ \mathbf{u} & \mathbf{v} & \mathbf{w} \end{vmatrix} \]

\[ = \left( \mathbf{q} \mathbf{w} - \mathbf{r} \mathbf{v} \right) \mathbf{k} + \left( \mathbf{r} \mathbf{u} - \mathbf{p} \mathbf{w} \right) \mathbf{j} + \left( \mathbf{p} \mathbf{v} - \mathbf{q} \mathbf{u} \right) \mathbf{i} \]

\[ \mathbf{A} = \mathbf{\dot{r}} \times \mathbf{\dot{r}} + \mathbf{\dot{r}} \times (\mathbf{\dot{r}} \times \mathbf{\dot{r}}) = \left( \mathbf{q} \mathbf{w} - \mathbf{r} \mathbf{v} \right) \mathbf{k} + \left( \mathbf{r} \mathbf{u} - \mathbf{p} \mathbf{w} \right) \mathbf{j} + \left( \mathbf{p} \mathbf{v} - \mathbf{q} \mathbf{u} \right) \mathbf{i} \]
\[ T \mathbf{A} = \ldots \text{ a lot} \]

\[ T \mathbf{A} + (g(x^2 - zr^2) \mathbf{c} + (pr - g) \mathbf{e} + (pq z^2 - pr e^2) \mathbf{F} \]

**RECOGNIZE MOMENTS AND PRODUCTS OF INERTIA**

\[
\begin{align*}
I_{xx} &= \int (y^2 + z^2) \, dm \\
I_{yy} &= \int (x^2 + z^2) \, dm \\
I_{zz} &= \int (x^2 + y^2) \, dm \\
I_{xy} &= \int xy \, dm \\
I_{xz} &= \int xz \, dm \\
I_{yz} &= \int yz \, dm \\

\end{align*}
\]

**GET TO:**

\[
\mathbf{M} = \begin{bmatrix}
L \\
M \\
N
\end{bmatrix} = \begin{bmatrix}
\dot{r} I_{xx} + \dot{r} r I_{ee} - \dot{r} r I_{gy} + (e^2 - g^2) I_{xy} + (p - \dot{r} g) I_{xy} - (\dot{r} + pr) I_{xe} \\
\dot{g} I_{yy} + \dot{p} r I_{xx} - \dot{p} r I_{ez} + (p^2 - r^2) I_{xz} - (\dot{r} - pr) I_{xe} - (\dot{g} + pr) I_{ye} \\
\dot{r} I_{ee} + \dot{p} g I_{gy} - \dot{p} g I_{xy} + (e^2 - p^2) I_{yx} + (\dot{e} - \dot{g} r) I_{ye} - (\dot{e} + pr) I_{ye}
\end{bmatrix}
\]

**ASSUMPTION #5: Xyz is a plane of symmetry**

\[ \Rightarrow I_{xy} = I_{yx} = 0 \]

**Mostly True, Asymmetric Fuel and Stores can cause it to be invalid.**

\[
\Rightarrow \mathbf{M} = \begin{bmatrix}
L \\
M \\
N
\end{bmatrix} = \begin{bmatrix}
\dot{r} I_{xx} + \dot{r} r I_{ee} - \dot{r} r I_{gy} - (\dot{r} + pr) I_{xe} \\
\dot{g} I_{yy} + \dot{p} r I_{xx} - \dot{p} r I_{ez} + (p^2 - r^2) I_{xz} - (\dot{r} - pr) I_{xe} - (\dot{g} + pr) I_{ye} \\
\dot{r} I_{ee} + \dot{p} g I_{gy} - \dot{p} g I_{xy} + (e^2 - p^2) I_{yx} + (\dot{e} - \dot{g} r) I_{ye} - (\dot{e} + pr) I_{ye}
\end{bmatrix}
\]
LOOK AT LHS TERMS

\[ F_x = F_{x \text{ aero}} + F_{x \text{ thrust}} + F_{x \text{ gravity}} = \frac{g}{8} S C_x + F_{x \text{ eng}} + F_{x g} \]

\[ F_y = F_{y \text{ aero}} + F_{y \text{ thrust}} + F_{y \text{ gravity}} = \frac{g}{8} S C_y + F_{y \text{ eng}} + F_{y g} \]

\[ F_z = F_{z \text{ aero}} + F_{z \text{ thrust}} + F_{z \text{ gravity}} = \frac{g}{8} S C_z + F_{z \text{ eng}} + F_{z g} \]

\[ F_{\text{earth}} = m \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \]

\[ F_{\text{body}} = m \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} = m \begin{bmatrix} -g \sin \theta \\ g \cos \theta \sin \phi \\ g \cos \theta \cos \phi \end{bmatrix} \]

\[ L = L_{\text{aero}} + L_{\text{thrust}} = \frac{g}{8} S b C_C + L_{\text{eng}} \]

\[ M = M_{\text{aero}} + M_{\text{thrust}} = \frac{g}{8} S c C_m + M_{\text{eng}} - \Omega_{\text{eng}} I_{\text{eng}} \]

\[ N = N_{\text{aero}} + N_{\text{thrust}} = \frac{g}{8} S b C_m + N_{\text{eng}} + \Omega_{\text{eng}} I_{\text{eng}} \]

**NOTE:** NO GRAVITY TERMS DUE TO FORCES ACTING THROUGH THE CG.

\[ L_{\text{eng}} I_{\text{eng}} \text{ TERMS COME FROM ENGINE GYROSCOPIC EFFECTS OF SPINNING COMPONENTS.} \]
\[ \rightarrow \text{ NONE IN ROLL DUE TO AXIS OF ROTATION} \]
\[ \rightarrow \text{ ASSUME THRUST AXIS AS BODY AXIS (SMALL INSTALLATION ANGLES)} \]

**VECTORS FORM E.O.M.'S**

\[
\begin{bmatrix}
\frac{g}{8} S C_x + F_{x \text{ eng}} - m \frac{g}{8} \sin \theta \\
\frac{g}{8} S C_y + F_{y \text{ eng}} + mg \cos \theta \sin \phi \\
\frac{g}{8} S C_z + F_{z \text{ eng}} + mg \cos \theta \cos \phi
\end{bmatrix} =
\begin{bmatrix}
m (\dot{u} + g \dot{w} - \dot{r} v) \\
m (\dot{v} + \dot{r} u - \dot{p} w) \\
m (\dot{w} + \dot{p} v - \dot{q} u)
\end{bmatrix}
\]
\[
\mathbf{\dot{m}} = \begin{bmatrix} \mathbf{\dot{I}_{xx}} + \mathbf{\dot{I}_{xy}} = \mathbf{\dot{I}_{xx}} + \mathbf{\dot{I}_{xy}} \\ \mathbf{\dot{\omega}} = \mathbf{\dot{I}_{xx}} - \mathbf{\dot{I}_{xy}} = \mathbf{\dot{I}_{xx}} - \mathbf{\dot{I}_{xy}} \end{bmatrix} = \begin{bmatrix} \hat{r} \mathbf{I}_{xx} + \mathbf{\hat{r}} \mathbf{I}_{xy} - \mathbf{\hat{r}} \mathbf{I}_{xy} - (\mathbf{\hat{r}} \mathbf{I}_{xx} + \mathbf{\hat{r}} \mathbf{I}_{xy}) \mathbf{I}_{xx} \\ \mathbf{\hat{r}} \mathbf{I}_{xx} + \mathbf{\hat{r}} \mathbf{I}_{xy} - \mathbf{\hat{r}} \mathbf{I}_{xy} - (\mathbf{\hat{r}} \mathbf{I}_{xx} + \mathbf{\hat{r}} \mathbf{I}_{xy}) \mathbf{I}_{xx} \end{bmatrix}
\]

Six Degrees of Freedom Equations of Motion Body Axis

\[
\begin{align*}
\mathbf{\dot{u}} &= \mathbf{v} - \mathbf{\hat{g}} \mathbf{w} + \frac{\mathbf{S}}{m} \mathbf{c}_x + \frac{\mathbf{F}_{\text{ext}}}{m} - \mathbf{g} \mathbf{w} \\
\mathbf{\dot{v}} &= \mathbf{\hat{r}} \mathbf{w} - \mathbf{r} \mathbf{u} + \frac{\mathbf{S}}{m} \mathbf{c}_y + \frac{\mathbf{F}_{\text{ext}}}{m} + g \cos \theta \sin \phi \\
\mathbf{\dot{w}} &= \mathbf{\hat{g}} \mathbf{u} + \mathbf{\hat{r}} \mathbf{v} + \frac{\mathbf{S}}{m} \mathbf{c}_z + \frac{\mathbf{F}_{\text{ext}}}{m} + g \cos \theta \cos \phi \\
\mathbf{\dot{\phi}} &= \frac{\mathbf{I}_{xx} - \mathbf{I}_{xy}}{\mathbf{I}_{xx}} \mathbf{g} \mathbf{r} + \frac{\mathbf{I}_{xx}}{\mathbf{I}_{xx}} \mathbf{r} + \mathbf{S} \mathbf{c}_x + \frac{\mathbf{L}_{\text{ext}}}{\mathbf{I}_{xx}} \\
\mathbf{\dot{\theta}} &= \frac{\mathbf{I}_{yy} - \mathbf{I}_{yx}}{\mathbf{I}_{yy}} \mathbf{g} \mathbf{r} + \frac{\mathbf{I}_{yy}}{\mathbf{I}_{yy}} \mathbf{r} + \mathbf{S} \mathbf{c}_y + \frac{\mathbf{L}_{\text{ext}}}{\mathbf{I}_{yy}} \\
\mathbf{\dot{\psi}} &= \frac{\mathbf{I}_{zz} - \mathbf{I}_{xy}}{\mathbf{I}_{xx}} \mathbf{g} \mathbf{r} + \frac{\mathbf{I}_{zz}}{\mathbf{I}_{zz}} \mathbf{r} + \mathbf{S} \mathbf{c}_z + \frac{\mathbf{L}_{\text{ext}}}{\mathbf{I}_{zz}}
\end{align*}
\]

Other Equations

\[
\begin{align*}
\mathbf{v}_t &= \sqrt{\mathbf{u}^2 + \mathbf{v}^2 + \mathbf{w}^2} \\
\mathbf{u} &= \mathbf{v}_t \cos \alpha \cos \beta \\
\mathbf{v} &= \mathbf{v}_t \sin \beta \\
\mathbf{w} &= \mathbf{v}_t \sin \alpha \cos \beta \\
\alpha &= \tan^{-1} \frac{\mathbf{w}}{\mathbf{u}} \\
\beta &= \sin^{-1} \frac{\mathbf{v}}{\mathbf{v}_t}
\end{align*}
\]

\[
\begin{align*}
\mathbf{\dot{\alpha}} &= \frac{1}{\mathbf{v}_t \cos \beta} (\mathbf{\dot{\mathbf{u}}} \cos \alpha - \mathbf{\dot{\mathbf{v}}} \sin \alpha) \\
\mathbf{\dot{\beta}} &= \frac{1}{\mathbf{v}_t} (\mathbf{\dot{\mathbf{v}}} \cos \beta - \mathbf{\dot{\mathbf{w}}} \cos \alpha \sin \beta - \mathbf{\dot{\mathbf{w}}} \sin \alpha \sin \beta) \\
\mathbf{\dot{\mathbf{v}}}_t &= \mathbf{\dot{\mathbf{u}}} \cos \alpha \cos \beta + \mathbf{\dot{\mathbf{v}}} \sin \beta + \mathbf{\dot{\mathbf{w}}} \sin \alpha \cos \beta
\end{align*}
\]
BODY AXIS E.O.M FOR AIRCRAFT

\[ \ddot{V}_T = \frac{1}{\cos \beta \cos \alpha} \left[ \gamma V_T \sin \beta - g \sin \theta + \frac{\bar{F}_S}{m} C_x + \frac{F_x e}{m} \right] + V_T (\ddot{\alpha} - g) \tan \beta + \dot{\beta} V_T \tan \beta \]

\[ \ddot{\alpha} = \frac{1}{\cos \beta \cos \phi} \left[ -p \sin \beta + \dot{\beta} \sin \beta \sin \phi - g \cos \theta \cos \phi + \frac{\bar{F}_S}{m V_T} C_x + \frac{F_x e}{m V_T} \right] \]

\[ -\frac{V_T \tan \alpha}{V_T} + g \]

\[ \dot{\beta} = \frac{1}{V_T \cos \beta} \left[ -\dot{V}_T \sin \beta + g \cos \theta \sin \phi + \frac{\bar{F}_S}{m} C_y + \frac{F_y e}{m} \right] + p \sin \alpha + \gamma \cos \alpha \]

INTERESTING TERMS IN EOM

\[ \ddot{\beta} \text{ TERM } = f (p \sin \alpha + \gamma \cos \alpha) \]

- TRANSFORMATION OF \( \alpha \) \& \( \beta \) INTO EACH OTHER
- NEED TO COORDINATE \( \dot{p} \) \& \( \dot{r} \) TO NOT DRIVE \( \beta \) TO DEPARTURE

\[ \dot{p}, \dot{r} = \dot{\gamma} \]

- KINEMATIC \& INERTIAL COUPLING
LONGITUDINAL CHARACTERISTICS

LONGITUDINAL AXIS runs along x-axis and its characteristics act as rotation about the y-axis.

COEFFICIENTS & RATES OF INTEREST

- $C_L$ → LIFT COEFFICIENT
- $C_{MA}$ → PITCHING MOMENT COEFFICIENT
- $C_D$ → DRAG COEFFICIENT

- $\beta$ → PITCH RATE
- $\delta_H$ → TAIL DEFLECTION (CONTROL SURFACE)

LIFT

THE LIFT CURVE

![Lift Curve Graph]

SYMMETRIC AIRFOILS have linear slope up to $C_L = 0$, beyond that the linearity ends up to a maximum $C_L$ and then diminishes.

SLOPE & MAXIMUM LIFT are functions of WING GEOMETRY & SPEED
WINGS

To collection of airfoil sections
- Airfoils are 2D
- Wings are 3D

Span b

Mean chord x span

Taper ratio: \( l = \frac{c_{tip}}{c_{root}} \) = Tip chord / root chord

Aspect ratio: \( AR = \frac{b^2}{S} = \frac{(\text{span})^2}{\text{wing area}} = \frac{b^2}{c} \) for

Mean aerodynamic chord, \( \bar{c} \) or MAC
- Chord where aerodynamic forces are assumed to act through
- Pass through centroid of area of the semi-span

\[ \bar{c} = \frac{2}{3} c_r \frac{1 + \lambda + \lambda^2}{1 + \lambda} \]

\[ CL \]

\[ \text{AR} = 2.5 \]

\[ \text{AR} = 5 \]

\[ \text{AR} = 10 \]

Fighters \( \sim \text{AR} 2.5 - 3.5 \)

Transport commercial \( \sim \text{AR} 6 - 8 \)

Glider \( \sim \text{AR} 15 - 20 \)
High aspect ratio wings have steeper lift curve slope, but separation occurs at lower AoA's.

Sweep reduces lift curve slope, but promotes a more gradual stall.

High sweep, ~60° and more, creates strong leading edge vortices that enhance LE suction and introduce non-linear contribution to lift.

Camber effects on lift curve slope:

Note that unsymmetrical airfoil provides non-zero lift at zero angle of attack.
PITCHING MOMENT & TRIM

DEFINITIONS

CENTER OF PRESSURE (C.P., C.P.)
- LOCATION ON AIRCRAFT WHERE DISTRIBUTED AERODYNAMIC LOADS PRODUCE NO PITCHING MOMENT
- \( Z_{M_{C.P.}} = 0 \)
- DISTRIBUTED LIFT & DRAG CAN BE REPLACED WITH POINT LOADS
- \( C_{m} = 0 \)
- MOVES W/ FLIGHT CONDITION OR CONFIGURATION
  - FORCES CHANGE W/ AIRSPEED & AOA
  - AIRCRAFT IS CONTROLLED BY RECONFIGURING TO MOVE C.P. (CONTROL SURFACES)

AERODYNAMIC CENTER (AC, A.C.)
- LOCATION ON AIRCRAFT ABOUT WHICH A CHANGE IN LIFT DOES NOT CHANGE PITCHING MOMENT
- MOVES AS AIRSPEED CHANGES, BUT INSENSITIVE TO AOA, CAMBER, OR CONTROL DEFLECTIONS (LOW AOA)
- MOVES WITH AOA AS STALL IS APPROACHED

\[ \frac{dC_{max}}{dC_{L}} = 0 \]
AERODYNAMIC CENTER IS THE LIFT CENTER
NOTE: DOES NOT MEAN \( C_{m} = 0 \)

PITCHING MOMENT RESULTS WHEN C.P. IS NOT LOCATED AT C.G.
- WEIGHT & LIFT/DRAG NOT ALIGNED
- ELIMINATION OF PRODUCED MOMENT IS CALLED "TRIMMING" BY BALANCING THE FORCES

NET MOMENT = LIFT \((C.P. \times \text{pos} - C.G. \times \text{pos})\) + DRAG \((C.P. \times \text{pos} - C.G. \times \text{pos})\)
Center of pressure represents the CG location where the moment is zero.

Aerodynamic center represents CG where the moment slope is zero.

It is not convenient to express moments about C.P. as it changes with all states. Instead, move those forces to A.C. and express C_m about C.G.

Control deflections camber the aircraft, but do not change the aerodynamic center - triax aircraft.

\[ \frac{\Delta X_{C.P.}}{C} = \frac{C_{m_a}}{C_{L_a}} \]

\[ \frac{\Delta X_{A.C.}}{C} = \left( \frac{dC_m}{dC_L} \right)_A \]

\[ C_m = C_{m_{A.C.}} + \frac{dC_m}{dC_L} C_{L_x} \]

\[ \frac{dC_m}{dC_L} \text{ unchanged} = A.C. \text{ unchanged} \]

Tail movement = camber & C
NEUTRAL POINT

- Center of Gravity Location Which Produces Neutral Aircraft Stability

- Neutral Point is at the Aerodynamic Center

\[ \frac{\Delta X_{N.P.}}{C} = \frac{dC_m}{dC_L} \]

PITCH STABILITY

Stable \( \rightarrow \) A Perturbation in Lift Results in a Pitching Moment That Restores the Aircraft to its Original, Equilibrium State

Negative Derivative implies a Stable System

\[ \frac{dC_m_{cg}}{dL} < 0 \quad \text{or} \quad \frac{dC_m_{cg}}{dC_L} < 0 \]

\[ C_{m_{cg}} = C_L \frac{dcL}{dcL} + C_m_{ac} = 0 \]

Results When Aerodynamic Center is Aft of the CG
Can express pitch stability as a function of angle of attack.

\[ \frac{dC_m}{dC_l} = \frac{dC_l}{d\alpha} = \frac{dC_m}{d\alpha} = C_m \]

- Pitch stability
- Always lift curve slope (always positive)

\[ C_{m_i} \]

\[ \alpha \]

- \( C_{m_i} > 0 \) unstable
- \( C_{m_i} < 0 \) stable

\( C_m \) is a more appropriate measure of pitch stability at high AoA when wings stall.

Caution should be made to look out for "deep stalls".

Deep stall region \( \Rightarrow C_m \) drives to equilibrium point at a higher \( \alpha \)

\( \Rightarrow \) new equilibrium \( \alpha \) at high AoA

\( \Rightarrow \) could augment w/ control surface, or "rock" out of stall region.
STATIC MARGIN

\( \Delta x_{N.P.} = X_{N.P.} - X_{C.G.} \)

\( \Delta x_{N.P.} = \frac{\Delta x_{N.P.}}{\Delta x_{N.P.}} = \frac{dC_m}{dC_l} \)

- NEGATIVE DISTANCE MEANS C.G. IS AHEAD OF N.P.
- LONGITUDINALLY STABLE
- NEGATIVE DISTANCE IS POSITIVE STATIC MARGIN

OFTEN TIMES DISTANCES ARE EXPRESSED AS \( \% \) MAC

\[ \text{STATIC MARGIN} = \left( \frac{x}{c} \right)_{N.P.} - \left( \frac{x}{c} \right)_{C.G.} = -\left( \frac{dC_m}{dC_l} \right) \]

\( x_0 = x_{LEMAC} \) MEASURED POS. \( x \) IN -X DIRECTION

NEW CONVERSION

- POSITIVE DIFFERENCE MEANS C.G. IS AHEAD OF N.P.
- AIRCRAFT LONGITUDINALLY STABLE
- POSITIVE DIFFERENCE IS A POSITIVE STATIC MARGIN

RULES OF THUMB FOR PITCH INSTABILITY LIMITS

- STATIC STABILITY RATIO REPRESENTS RATIO BETWEEN STATIC PITCH STABILITY & PRIMARY PITCH SURFACE CONTROL POWER

\(-2 < \frac{M_{0}^{\infty}}{M_8} < 2\)  \( M_8/M_8 < -2 \): TOO UNSTABLE WITH INADEQUATE CONTROL POWER  -->  CAN STABILITY MARGINS

\( M_{0}^{\infty}/M_8 > 2 \): TOO STABLE WITH INADEQUATE CP  -->  SLEWISH RESPONSE

- RATIO CAN BE CONSIDERED AS CHANGE IN TRIM VS RELATIVE FTUREMAGA

\(-2 < \left[ \frac{\delta h}{\alpha} \right]_{\text{TREIN}} \leq 2\)

- TIME TO DOUBLE: \( T_2 \approx \frac{\ln(z)}{C_{m_{\alpha}} \cdot \frac{180}{\pi} \cdot \frac{85 C}{G_{\infty}}} > 0.2 \)  --> PILOT/FCS CAN COUNTER
DEFINITIONS SUMMARY

NEUTRAL POINT

- CG location that produces neutral aircraft pitch stability
- Aerodynamic center of aircraft
  - Change in lift does not change aero pitching moment
- Defined with controls fixed (no PCS feedback)
- Typically defined at low AoA \( f(M) \) only
- Typically a single neutral point is defined as a weighted sum across trim AoA for a range of gross weights and attitudes

\[
NP(\%OMAC) = 100 \left( CG - \frac{dCm}{dCL} \right) = 100 \left( CG - \frac{Cm0}{CL0} \right)
\]

STATIC MARGIN

- Distance between N.P. and C.G., positive when C.G. is fwd
- Pos SM comes from a negative \( \frac{dCm}{dCL} \) or negative \( Cm0 \)

\[
SM(\%OMAC) = 100 \times \frac{\Delta W_0}{2} = -100 \times \frac{dCm}{dCL} = -100 \times \frac{Cm0}{CL0}
\]

CENTER OF PRESSURE

- Point at which aerodynamic loads produce no pitching moment
  - Lift & drag can be replaced with point loads
- Moves aft with increasing Mach number
  - Seen in \( Cm \) vs. \( \alpha \)

More stable with increasing Mach
TRIMMING SUMMARY

For an aircraft to fly straight and level, the forces must balance about the C.G.

\[ C_L \delta_{H} = C_m_{LIFT} \frac{dC_m}{dC_L} \delta_{H} \]

\[ C_m_{LIFT} = C_{m_{LIFT}} \frac{dC_m}{dC_L} \]

\[ C_m_{CONTROL} \]

POSIITED FORCES ON TAIL

WEIGHT

\[ C_{m_{CONTROL}} \]

\[ C_L = C_{m_{C.G.}} \]

\[ \frac{dC_m}{dC_L} \]

\[ \frac{dC_m}{dC_L} \]

\[ \delta_{H} = 0 \]

\[ \delta_{H} \]

\[ \delta_{H} \]

TRIMMED LIFT BUILDUP

\[ \Rightarrow \] STATUS OF A CONFIGURATION

COLLECTION OF TRIMMED LIFT CURVES FOR ALL FLIGHT COND.

\[ C_L \]

\[ \text{MACH} \]

\[ \text{INCREASE} \]

\[ C_L \]

\[ \text{FOR GIVEN MACH & STATIC MARGIN} \]

\[ \frac{dC_m}{dC_L} \]

\[ \delta_{H} = + \]

\[ \delta_{H} = 0 \]

\[ \delta_{H} = - \]

\[ \text{UNTRIMMED LIFT CURVE} \]

\[ \text{TRIMMED LIFT CURVE} \]
A finite wing results in lift loss:
- Flow spills from high pressure side to low pressure side
- Reduced compression lift on lower surface
- Suction lift on upper surface

This limitation of span is a major source of drag.

Lift distribution is approximately elliptical.

Front view of wing.

Induced drag:

\[ D_i = \frac{L^2}{\pi \theta b^2} \] (also called drag due to lift)

Generating the same lift on a wing half the size quadruples the drag.

\[ C_{D_i} = \frac{C_l^2}{\pi AR} = \frac{D_i}{\theta S} \]

Camber shifts the induced drag curve.

\[ C_{D_i} = \frac{(C_l - C_{l_{\text{min}}})^2}{\pi AR} + \Delta C_{D_{\text{camber}}} \]

Reduced drag at higher AoA is in exchange for penalty at lower AoA.
DRAG TYPICALLY MODELED AS SUM OF
INDUCED DRAG & PARASITIC DRAG

\[ C_D = C_{D_0} + C_{D_i} \]

\[ C_{D_i} = \text{INDUCED DRAG} = \frac{C_L^2}{\pi AR} \]

\[ C_{D_0} = \text{PARASITIC DRAG (SUM OF OTHER DRAG)} \]

PARASITIC DRAG INCLUDES
- DRAG DUE TO CAMBER
- SKIN FRICTION DRAG
  - PRODUCED DUE TO BOUNDARY LAYER
  - EMPIRICAL EQUATIONS
- SHOCK WAVE DRAG
  - PRODUCED BY SHOCKS FORMING
- ZERO-LIFT DRAG

\[ C_D \]

\[ C_{D_i} \]

\[ C_{D_0} \]

\[ C_L \]

NOTE THAT TRIMMING THE AIRCRAFT PRODUCES TRIM DRAG
THAT IS DUE TO CONTROL SURFACE DEFLECTIONS &
STATIC MARGIN. CAN BE SUMMED INTO \( C_{D_0} \)

\[ C_D \to C_{D_{\text{zero-lift}}} + C_{D_{\text{lift}}} + C_{D_{\text{trim}}} \]
**PERFORMANCE**

**Rate of Climb** (Specific Excess Power, $P_s$)

\[ \text{Rate of Climb} = V \sin \gamma \]

\[ W \sin \gamma = T \cos \alpha - D \]

\[ \sin \gamma = \frac{T \cos \alpha - D}{W} \]

\[ P_s = V \sin \gamma = \frac{V}{W} (T \cos \alpha - D) \text{ ft/s or ft/\text{sec}} \]

**Specific Excess Power is a measure of instantaneous Rate of Climb**

"Think of it as usable energy."

**Load Factor**

- To any load, non-dimensionalized by weight

\[ N_L = \frac{L}{W} = \frac{C_L \bar{F} S}{W} = \frac{C_L \bar{F} S}{W} \]

\[ N_x = -\frac{x \text{ force}}{W} = -\frac{C_{x \bar{F}} S}{W} \quad \text{most common} \]

\[ N_y = \frac{Y}{W} = \frac{C_Y \bar{F} S}{W} \]

\[ N_{\text{limit}} \quad \text{Structural Design Limit Load} \]

\[ \text{Flight Design Gross Weight} \]

**Load Factor is also non-dimensional measurement of the aircraft acceleration as a multiple of "g's" (gravity)**

\[ \Rightarrow N_L = \frac{L}{W} = \frac{m \alpha}{mg} = \frac{a}{g} \]
TURN CAPABILITY

TURNING RADIUS: \( R = \frac{V^2}{g \sqrt{N_c^2 - 1}} \)

TURN RATE: \( \dot{\psi} = \frac{57.3 \cdot V}{\text{TURN RADIUS}} = 57.3 \cdot \frac{1}{V} \sqrt{N_c^2 - 1} \)

- **R = Radius in ft**
- **\dot{\psi} = Rate of turn deg/s**
- **V = Aircraft velocity ft/s**
- **g = Acceleration due to gravity - 32.174 ft/s^2**
- **N_c = Load factor normal to flight path**

-> **Higher speeds mean larger turn radius and lower turn rates**

-> **Maximum turn rate at a given speed is limited by load factor**

"Doughouse plots" -> Relationship between \( R, \dot{\psi}, P, \alpha, N_c \)
Ceiling Definitions

**Absolute Ceiling** - Altitude at a given airspeed at which the rate of climb is 0 fps using mil or max power

**Service Ceiling** - Altitude at a given airspeed at which the rate of climb is 100 fps using mil or max power

**Cruise Ceiling** - Flies at 300 fps using maximum continuous power (non-afterburning or reduced power)

**Combat Ceiling** - FL 118, 500 fps using mil or max power
LATERAL-DIRECTIONAL STABILITY

DIRECTIONAL STABILITY

VANT A RETURNING YAWING MOMENT TO BE GENERATED WHEN A DIRECTIONAL DISTURBANCE OCCURS:

→ DIRECTIONAL DISTURBANCE
   • INCREASE IN SIDESLIP (+β)

→ WANT A POSITIVE YAWING MOMENT TO FORM TO RETURN TO ZERO SIDESLIP

→ \( \frac{\delta n}{\delta \beta} > 0 \) OR \( \frac{SC_n}{\delta \beta} > 0 \)

→ \( C_{n \beta} > 0 \)

PREVAILING SIDESFORCE IS COMING FROM THE VERTICAL STABILIZERS
- RUDDERS (CONTROL SURFACE)
- VERTICAL TAILS (FIXED)

PREVAILING SIDESFORCE MAGNITUDE & LOCATION AFT OF CG CREATE YAWING MOMENT
- CAN FLY AT CONSTANT SIDESLIP (STEADY-HEADING SIDESLIP), REQUIRES RUDDER INPUT

CENTER & MAGNITUDE OF SF VARIES WITH FLIGHT CONDITIONS AND ALTITUDE
- MACH COMPRESSION EFFECTS
- AOA VARIES FLOW TO VERTICAL STABILIZERS \( \rightarrow \) CAN CHANGE \( C_{n \beta} = f(\alpha) \)
Directional Control from Vertical Tails

- Positive rudder deflection $\delta_r$ measured as to left

- Positive rudder deflection provides negative yawing moment

Looking at individual rudder:

- Rudder movement is applying camber to airfoil

- Side force generated instead of lift due to coordinate axis location.

Directional control is used to yaw the airplane and control sideslip. It also has input to rolling moment and side force.

- Note that canted vertical tails will provide a larger rolling moment when rudders are deflected

- Rudders generate rolling moment because pressure location is above CG.
LATERAL STABILITY

Positive Stability

\( - \) A disturbance generates a differential lift and rolling moment returning aircraft to zero sideslip

\[ \alpha > 0 \]

\[ \alpha > 0 \rightarrow \text{Negative rolling moment} \]

\[ \alpha < 0 \rightarrow \text{Positive rolling moment} \]

\( \alpha \) develops with a positive sideslip

\[ \alpha > 0 \rightarrow \text{Stable roll response reduces sideslip} \]

\[ \frac{S \ell}{\delta \beta} < 0 \quad \text{or} \quad \frac{S C_e}{\delta \beta} < 0 \quad \text{or} \quad C_{2 \beta} < 0 \quad \text{for} \ \alpha > 0 \]

\[ \frac{S \ell}{\delta \beta} > 0 \quad \text{or} \quad \frac{S C_e}{\delta \beta} > 0 \quad \text{or} \quad C_{2 \beta} > 0 \quad \text{for} \ \alpha < 0 \]

\( \beta = 0^\circ \)

\[ \frac{S \ell}{\delta \beta} < 0 \quad \text{or} \quad C_{2 \beta} < 0 \]

\( \rightarrow \) sideslip returns to zero
LATERAL CONTROL

- DIFFERENTIAL AILERONS/FAPS
- DIFFERENTIAL TAILS
- RUDDERS (SMALLER EFFECTS)
- SPOILERS

AILERON EXAMPLE

AILERON DEFLECTION
POS. TEO

\[
\Delta A = \frac{(s_{al} - s_{ar})}{2} = \Delta A
\]

\[
\Delta A_{net} = \frac{(s_{al} + s_{ar})}{2} = \Delta A_{net}
\]

* PROVIDES NEGATIVE ROLLING MOMENT

NOTE THAT LAT-DIR CONTROL SURFACES PROVIDE TERMS TO BOTH, HENCE LAT-DIR IS THOUGHT OF TOGETHER, VERY TOUGH TO GET ONE RESPONSE WITHOUT THE OTHER.

DIFFERENTIAL TAIL PROVIDES BOTH ROLLING MOMENT AND YAWING MOMENT, ETC.
**Départure Départure Resistance**

**Definition**

→ A DEPARTURE IS A LOSS OF CONTROLLED FLIGHT AND HAPPENS WHEN AN AIRCRAFT IS MOVING IN AN UNCONTROLLED MANNER.

- Can happen when operating envelope is exceeded
- Some departures can be recovered from

**Départure Resistance Parameter**

→ Helps determine likelihood of departures
→ Looks at directional stability & lateral stability together

\[ C_{\text{Foyn}} = C_{\beta} \cos \alpha - \frac{I_{xx}}{I_{xx}} C_{\beta} \sin \alpha \]

**Derivation from \( \dot{\beta} \) Equation:**

\[ \dot{\beta} = \dot{p} \sin \alpha - \dot{r} \cos \alpha \]

\[ \ddot{\beta} = \ddot{p} \sin \alpha + \dot{p} \cos \alpha - \ddot{r} \cos \alpha + \dot{r} \sin \alpha \]

→ \( \dot{\alpha} = 0 \) assumed

→ \( \ddot{\beta} = \ddot{p} \sin \alpha - \ddot{r} \cos \alpha \)

→ Assume no inertial coupling

\[ \ddot{p} = \ddot{\beta} \frac{C_{l}}{I_{xx}} \]

\[ \ddot{r} = \ddot{\beta} \frac{C_{l}}{I_{zz}} \]

\[ \ddot{\beta} = \ddot{\beta} \left[ \frac{C_{l}}{I_{xx}} \sin \alpha - \frac{C_{l}}{I_{zz}} \cos \alpha \right] \]
\[- \frac{\beta \frac{I_{\theta \theta}}{Q_{\phi \phi}}}{\dot{\beta}} = C_n \cos \alpha - \frac{I_{\theta \theta}}{I_{xx}} C_L \sin \alpha \]

Differentiate wrt \( \beta \)

\[\rightarrow C_{n\beta}^{\text{dynamic}} = C_{n\beta} \cos \alpha - \frac{I_{\theta \theta}}{I_{xx}} C_L \sin \alpha \]

Have resistance to departure when \( C_{n\beta}^{\text{dynamic}} \geq 0 \)

Remember:

For positive \( \alpha \)

Stable if: \( C_{n\beta} > 0 \)

\( C_L \beta < 0 \)

However, in practice...

\[\rightarrow C_{n\beta} \text{ and } C_L \beta \text{ reduce at higher AoAs due to flow separation on wing impinging on the roll & yaw authority devices} \]

\[\rightarrow \text{Notice } C_{n\beta} \text{ goes unstable!!} \]
DAMPING TERMS

EASIEST TO LOOK AT ROLL DAMPING FIRST

RETARDING LIFT

\[ \Delta V = \frac{p_b}{2} \]

in rad/s

VIEW OF WING TIP (RIGHT)

\[ \frac{p_b}{2V} = \tan \alpha \]

\( \Delta \alpha \) IS THE TIP HELIX ANGLE

ROTATION INCREASES THE ANGLE OF ATTACK, AND THEREFORE LIFT, ON THE DOWNWARD MOVING WING. THIS ROTATION DECREASES ANGLE OF ATTACK, AND LIFT, ON THE UPWARD MOVING WING. THIS COMBINATION GENERATES A RETARDING ROLLING MOMENT \( \Rightarrow \) ROLL DAMPING

ROLL DAMPING COEFFICIENT

\[ C_{LP} = \frac{2C_L}{\left( \frac{p_b}{2V} \right)} \]

TYPICALLY A NEGATIVE VALUE FOR POSITIVE STABILITY

OPPOSITE OF DAMPING IS PROPELLING. THIS IS A MOMENT DRIVING FORCE.
Comparable terms exist in other axis

**Yaw Damping Coefficient**

\[ C_{nx} = \frac{2C_m}{\frac{d}{2V}} \]

→ Positive yawing moment rotation induces velocity on vertical stabilizers and produces side force aft of CG. This generates restoring yawing moment.

**Pitch Damping Coefficient**

\[ C_{nb} = \frac{2C_n}{\frac{d}{2V}} \]

→ Positive pitching moment induces velocity on horizontal stabilizers, increasing lift at aft end, generating a restoring pitching moment.

**Plunging Derivative**

\[ C_{n\alpha} = \frac{2C_m}{\frac{d}{2V}} \]

Pure pitching motion \( \dot{\alpha} = 0 \) \( \theta = \sin \omega t \)

Pure plunging motion \( \dot{\alpha} = \sin \omega t \) \( \theta = 0 \)
CROSS-AXIS DAMPING COEFFICIENTS

YAW DAMPING DUE TO ROLL RATE COEFFICIENT

\[ C_{mp} = \frac{2C_m}{\frac{\delta}{2V}} \]

→ WHEN ROLLING, VELOCITY IS INDUCED ONTO THE VERTICAL STABILIZERS. THIS INDUCES A SIDE FORCE ON THE VERT STABILIZERS BEHIND THE CG, THUS PRODUCING A YAWING MOMENT.

ROLL DAMPING DUE TO YAW RATE COEFFICIENT

\[ C_{LR} = \frac{2C_r}{2\left(\frac{\delta}{2V}\right)} \]

→ WHEN YAWING, VELOCITY IS INDUCED ONTO THE VERTICAL STABILIZERS. THIS INDUCES A SIDE FORCE ABOVE THE CG, THUS GENERATING A ROLLING MOMENT.
**TRADITIONAL NON-DIMENSIONAL ROTATION RATE COMPONENTS**

Also known as the "direct method"
- Other methods exist
  - Kalviste
  - Excess Roll Rate
  - Forced Oscillation
- Simplest Method
- Non-Discontinuous

**TRADITIONAL NON-DIM ROTATIONAL RATES ARE DERIVED BY BREAKING THE TOTAL AIRCRAFT ROTATIONAL RATES INTO COMPONENTS**
- One "steady-state" component aligned with the velocity vector
- Three residual or oscillatory components aligned with body x, y, z axes.

\[
\Omega = \sqrt{p^2 + q^2 + r^2}
\]

\[
\Omega_v = p \cos \alpha \cos \beta + q \sin \beta + r \sin \alpha \cos \beta
\]

\[
\omega_{osc} = \Omega - \Omega_v
\]

\[
P_{osc} = p - \Omega_v \cos \alpha \cos \beta
\]

\[
\theta_{osc} = q - \Omega_v \sin \beta
\]

\[
\phi_{osc} = r - \Omega_v \sin \alpha \cos \beta
\]

\[
C_{D02V} = \frac{\Omega_v b}{2 V_T}
\]

\[
P_B = \frac{P_{osc} b}{2 V_T}
\]

\[
C_{L2B} = \frac{C_{D02V}}{2 V_T}
\]

\[
R_B = \frac{\phi_{osc} b}{2 V_T}
\]

\[
\text{Used in aerodynamic dynamic derivative terms}
\]
**Hinge Moments**

In order to rotate control surfaces about its hinge, necessary force is required to overcome the aerodynamic pressure that resists the motion.

If the aero forces on the surface overcome the applied actuator hinge moment, the control surface will move to a new, uncommanded position.

Hinge moment $H_e$ acts in the opposite direction of aero forces to counter the pressure.

Coefficient form:

$$C_H = \frac{H}{\frac{1}{2} \rho V^2 S_{\text{ref}} C_{\text{ref}}} = \frac{H}{\frac{\rho}{\hat{C}} S_{\text{ref}} C_{\text{ref}}}$$

Where $S_{\text{ref}}$ and $C_{\text{ref}}$ are surface dependent.