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Donahue Independent Study; Collection of Aerodynamic Stability and Control Lecture Notes

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Aerodynamic Stability & Control

MEMS 500: Independent Study

Donahue, Daniel

Daniel Donahue's Independent Study

The learning objective of this independent study was to further my own knowledge in the field of Flight Mechanics/Dynamics as well as attaining experience in creating coursework. This was done by creating 14 lectures worth of hand-written notes on the subject of Aircraft Stability and Control. I primarily used the book by Pamadi, "Performance, Stability, Dynamics, and Control of Airplanes 2nd ed" while also pulling from my experiences as an aerodynamic stability and control engineer.

During the course of the semester I generated 42 pages of hand written notes that correspond to 14 lectures worth of material. It was agreed upon with my advisor, David Peters, that 3 pages of hand written notes would equate to a lecture. This number was based off of his experience with lecture materials where 3 pages of hand written notes, accompanied by in-person elaboration of the material, equates to one lecture worth of material.

Below is a syllabus of the 14 covered lectures with corresponding pages in the hand-written notes attached.

Lecture	**Topic**	**Note Pages**
1	Syllabus and Aircraft Geometry	Syllabus and 0
2-3	Aircraft Stability and Control Definitions; Coordinate Axis	1-7
3-6	Derivation of Aircraft Equations of Motion	8-15
7-9	Longitudinal Characteristics	16-27
10	Maneuvering Performance	28-30
11-12	Lateral-Directional Characteristics	31-36
13-14	Dynamic Derivatives; Hinge Moments	37-41

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0) SYLLABUS & AIRCRAFT GEOMETRY

1) AIRCRAFT STABILITY & CONTROL DEFINITIONS

FLIGHT VARIABLE DEFINITIONS OF INTEREST

- COORDINATE SYSTEMS
    - BODY ← AIRCRAFT STATION AXES
    - STABILITY
    - WIND
    - INERTIAL
- ~~AIRCRAFT~~ AERO FORCES & MOMENTS NON-DIM
- STANDARD ATM EQUATIONS

2) SIX DEGREES OF FREEDOM EQUATIONS OF MOTION

3) LONGITUDINAL CHARACTERISTICS
- LIFT & WINGS
- PITCHING MOMENT & TRIM
- DRAG

4) PERFORMANCE PARAMETERS
- TURN RATE
- TURN RADIUS
- RATE OF CLIMB

5) LATERAL-DIRECTIONAL CHARACTERISTICS
- DIRECTIONAL STABILITY & CONTROL
- LATERAL STABILITY & CONTROL
- DEPARTURE RESISTANCE

6) DYNAMIC DERIVATIVES
- ROLL, YAW, PITCH & PLUNGING ~~DAMPING~~ DAMPING
- CROSS-AXIS DAMPING
- TRADITIONAL METHODS

7) HINGE MOMENTS

AIRCRAFT TYPES, GEOMETRY, CONTROL SURFACES

TYPES OF AIRCRAFT

- COMMERCIAL
  - TRANSPORT (7-SERIES, A-SERIES, FREIGHTER, ?BLIMP?)
  - PERSONAL (CESSNA SMALL AIRCRAFT, RED-BULL FLYERS)

- DEFENSE
  - FIGHTER/ATTACK (F-15, F/A-18, AV-8B F-22, F-35, TOMCAT, A-10 ETC.)
  - BOMBER (STRATOFORTRESS, B-1, FLYING WING)
  - TRAINER (T-45, ETC.)
  - UAV [DRONE] (SCAN EAGLE, ETC.)
  - SURVELANCE [STEALTH] (SR-71, A-12, SATELLITE)

GEOMETRY

- FUSELAGE
  - TUBULAR
  - SEARS-HAAK
  - RADOME
- WING
  - CANARD (MISC.)
  - LEX
  - HERSHEY
  - SWEPT
  - DELTA
- TAIL
  - V-TAIL
  - T-TAIL
  - H-TAIL
- MISC
  - THRUST-VECTORING
  - WINGLETS
  - VORTEX GENERATORS

- CONTROL SURFACES
  - AILERON
  - FLAPS (TEF/LEF) FIG 1.24
  - FLAPERON
  - SLATS STRAKE
  - TRIM TABS
  - ELEVON
  - STABILATOR
  - SPEED BREAK
  - RUDDER

- AIRCRAFT BODY AXIS
  - x, y, z
  - p, q, r
  - LIFT, DRAG, SIDEFORCE

~~HOMEWORK~~

~~IDENTIFY WHICH C.S.'s GO WITH EACH AXIS~~

10/22/18 DGD

~~LONGITUDINAL AXIS~~

AIRCRAFT FLIGHT DYNAMICS, STABILITY & CONTROL

(1) (2) (3)

(1) FLIGHT DYNAMICS → A STUDY OF THE ~~TRANSITION~~ MOTION OF AN ~~AIRPLANE~~ AEROSPACE VEHICLE FOLLOWING A PERTURBATION FROM AN EQUILIBRIUM STATE

EQUILIBRIUM STATE → VEHICLE ~~MOTION~~ STATE IN WHICH $F(t) = M(t) = 0$ WHERE F & M REPRESENT EXTERNAL FORCES & MOMENTS APPLIED

STATES OF INTEREST: ~~~~ STEADY-LEVEL FLIGHT, ETC.

PERTURBATION → CHANGE IN F & M CAUSED BY CONTROL SURFACE DEFLECTIONS, TURBULENCE, INTERNAL SHIFTS, ~~&~~ BARE AIRFRAME STABILITY, ETC.

FLIGHT DYNAMICS DEALS WITH STEADY & UNSTEADY MOTION OF A VEHICLE CAUSED BY A DISTURBANCE, ~~INTERNAL OR UN~~ INTENTIONAL OR UNINTENTIONAL, APPLIED TO AN INITIALLY TRIMMED MOTION STATE.

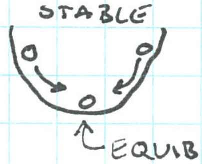
(2) STABILITY → THE QUALITY OF AN EQUILIBRIUM STATE

EQUILIBRIUM STATES OF AIRCRAFT ARE GENERALLY DESIGNED TO BE HANDS-OFF CONDITIONS

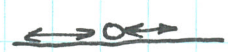
→ AIRCRAFT WITH INFERIOR STABILITY CHARACTERISTICS CAN POTENTIALLY BE DANGEROUS, UNACCEPTABLE

STATIC STABILITY:

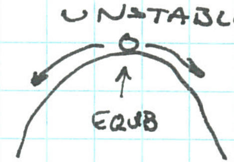
STABLE



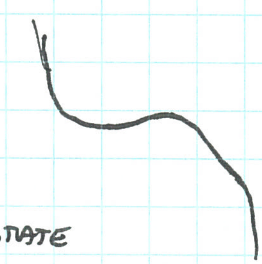
NEUTRAL



UNSTABLE



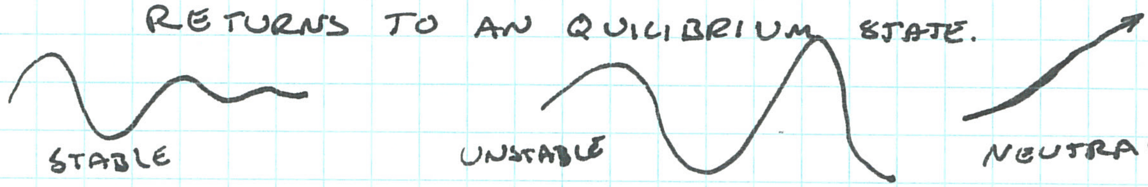
→ 3D STABILITY



→ X_0 DETERMINATE

STATICALLY STABLE IF IT RETURNS TO A STATE WHEN PERTURBED

DYNAMIC STABILITY → VEHICLE IS ~~EL~~ A DYNAMICALLY STABLE IF, WHEN PERTURBED, IT EVENTUALLY RETURNS TO AN QUILIBRIUM STATE.



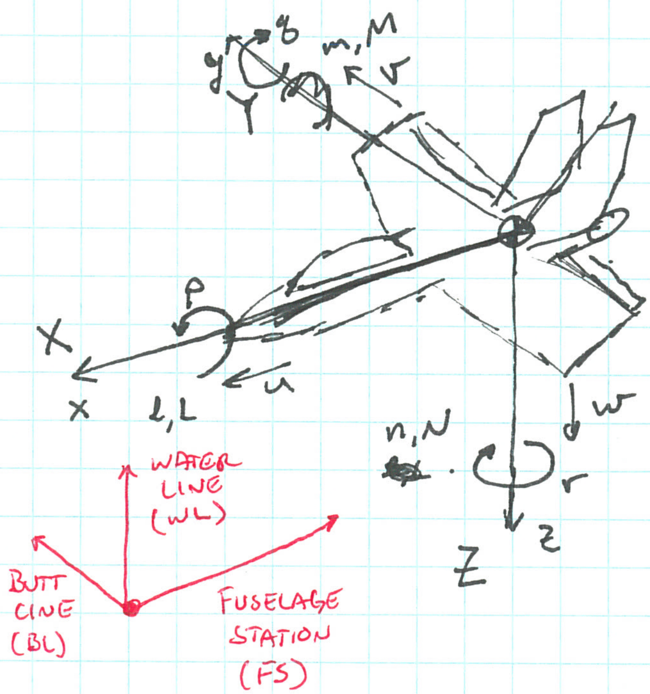
(3) CONTROL → ABILITY, OR LACK THEREOF, TO STEER AN AEROSPACE VEHICLE FROM POINT A TO POINT B ALONG AN ACCEPTABLE FLIGHT PATH

STATIC CONTROL → ABILITY TO MAINTAIN A PRESCRIBED EQUILIBRIUM FLIGHT CONDITION

DYNAMIC CONTROL → ABILITY TO MAINTAIN AN ACCEPTABLE TRANSIENT MOTION FOLLOWING A PERTURBATION. THE ABILITY TO CHANGE FROM ONE TRIM CONDITION TO ANOTHER.

FLIGHT VARIABLE DEFINITIONS OF INTEREST

BODY AXIS COORDINATE SYSTEM



	NOSE	WING	DOWN
COORDINATE	x	y	z
VELOCITY	u	v	w
ROTATIONAL VELOCITY	p	q	r
AERO FORCE	X	Y	Z
AERO MOM.	ℓ, L	m, M	n, N
	ROLL	PITCH	YAW

OPP

X_B → AXIAL FORCE $= -A$

Z_B → NORMAL FORCE $= -N$

OPP

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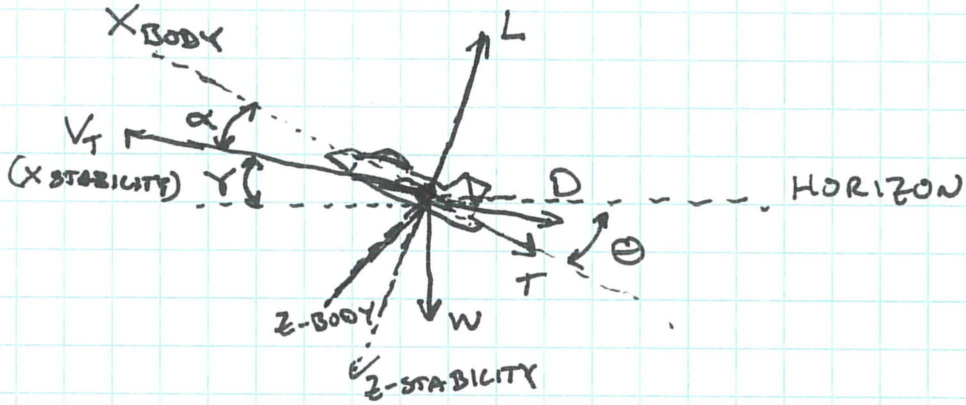
STABILITY AXIS



$V_T \rightarrow$ FLIGHT PATH

$\alpha \rightarrow$ ANGLE OF ATTACK (FLIGHT PATH/VELOCITY TO BODY X)
$\gamma \rightarrow$ FLIGHT PATH ANGLE (FLIGHT PATH TO HORIZON)
$\Theta \rightarrow$ PITCH ANGLE (HORIZION TO X-AXIS BODY)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{BODY} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{STABILITY}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{STABILITY} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{BODY}$$

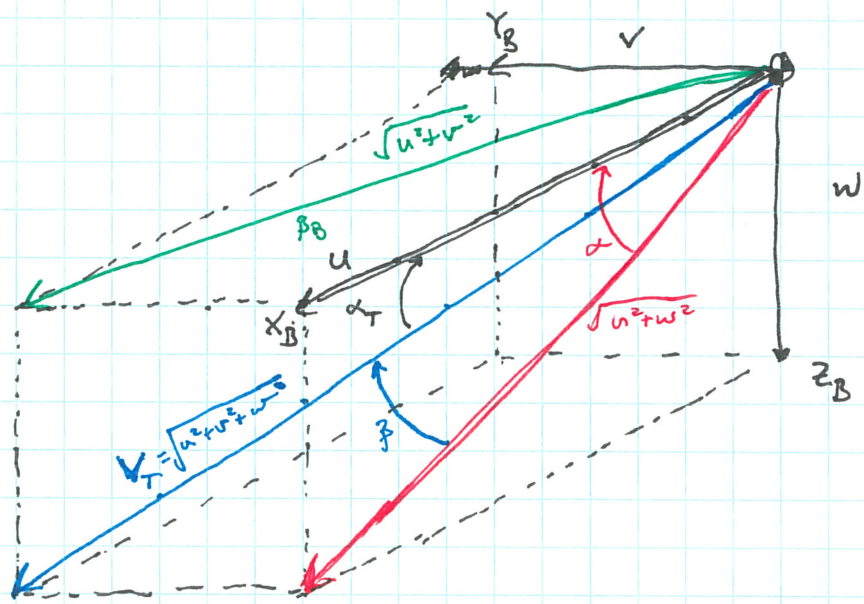
CHECK SIGNS

$L = \text{LIFT} = -Z_S = -Z_B\cos\alpha + X_B\sin\alpha = +N\cos\alpha \pm A\sin\alpha$

$D = \text{DRAG} = -X_S = -Z_B\sin\alpha - X_B\cos\alpha = \pm N\sin\alpha \pm A\cos\alpha$

W = WEIGHT
T = THRUST

WIND AXIS



$$\alpha = \tan^{-1}\left(\frac{w}{u}\right) = \cos^{-1}\left(\frac{u}{\sqrt{u^2+w^2}}\right) = \sin^{-1}\left(\frac{w}{\sqrt{u^2+w^2}}\right)$$

$$\beta = \sin^{-1}\left(\frac{v}{V_T}\right) = \tan^{-1}\left(\frac{v}{\sqrt{u^2+w^2}}\right) = \cos^{-1}\left(\frac{\sqrt{u^2+w^2}}{V_T}\right)$$

$$\alpha_T = \cos^{-1}\left(\frac{u}{V_T}\right) = \sin^{-1}\left(\frac{\sqrt{v^2+w^2}}{V_T}\right) = \tan^{-1}\left(\frac{\sqrt{v^2+w^2}}{u}\right)$$

↳ T = TOTAL → USED IN MISSILES

$$\beta_B = \tan^{-1}\left(\frac{v}{u}\right) = \cos^{-1}\left(\frac{u}{\sqrt{u^2+v^2}}\right) = \sin^{-1}\left(\frac{v}{\sqrt{u^2+v^2}}\right)$$

$$\beta = \tan^{-1}(\tan\beta_B \cos\alpha)$$

$$\alpha_T = \cos^{-1}(\cos\alpha \cos\beta)$$

$$V_T = \sqrt{u^2+v^2+w^2}$$

$$u = V_T \cos\alpha \cos\beta$$

$$v = V_T \sin\beta$$

$$w = \sin\alpha \cos\beta$$

STABILITY TO WIND

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_W = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_S$$

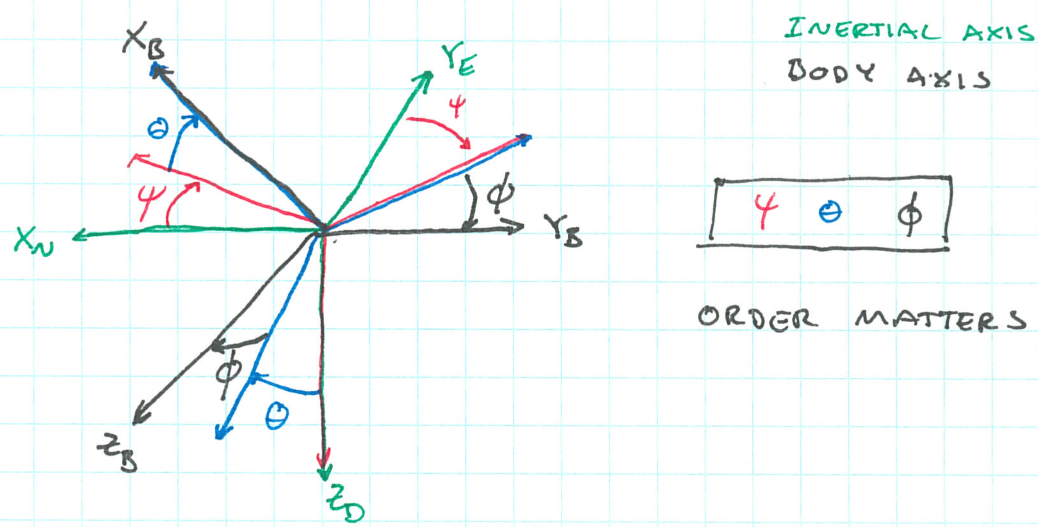
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_S = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_W$$

BODY TO WIND

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_W = \begin{bmatrix} \cos\alpha\cos\beta & \sin\beta & \sin\alpha\cos\beta \\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B = \begin{bmatrix} \cos\alpha\cos\beta & -\cos\alpha\sin\beta & -\sin\alpha \\ \sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & -\sin\alpha\sin\beta & \cos\alpha \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_W$$

INERTIAL AXIS SYSTEM



$\boxed{\psi \quad \theta \quad \phi}$

ORDER MATTERS

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INERTIAL TO BODY AXIS SYSTEM

$$\begin{bmatrix} X_N \\ Y_E \\ Z_D \end{bmatrix} = \begin{bmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi & \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi \\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B = \begin{bmatrix} \cos\psi\cos\theta & \sin\psi\cos\theta & -\sin\theta \\ \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \cos\theta\sin\phi \\ \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} X_N \\ Y_E \\ Z_D \end{bmatrix}$$

$$C = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

$$\begin{bmatrix} X_N \\ Y_E \\ Z_D \end{bmatrix} = (ABC)^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B \iff \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B = ABC \begin{bmatrix} X_N \\ Y_E \\ Z_D \end{bmatrix}$$

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AIRCRAFT AERODYNAMIC FORCES AND MOMENTS

$F_{X_B} = -A$ → OPPOSITE OF AXIAL FORCE

$F_{Y_B} = Y = SF$ → SIDE FORCE

$F_{Z_B} = -N$ → OPPOSITE OF NORMAL FORCE

$\bar{q} = \frac{1}{2}\rho V_T^2 = 0.7pM^2$

b = SPAN

c = CHORD (MAC)

$F_{X_B} = \bar{q} S C_X$ $\qquad L = \bar{q} S b C_l$

$F_{Y_B} = \bar{q} S C_Y$ $\qquad M = \bar{q} S c C_m$

$F_{Z_B} = \bar{q} S C_Z$ $\qquad N = \bar{q} S b C_n$

STANDARD ATMOSPHERE MODEL EQUATIONS

IDEAL GAS LAW : $p = \rho R T$

HYDROSTATIC EQUILIBRIUM: $\frac{dp}{dH} = -g\rho$

TEMPERATURE GRADIENTS $T = T_{BASE_n} + \lambda_n (H - H_{BASE_n})$

RATIOS: $\theta = \frac{T}{T_0}$; $\delta = p/p_0$; $\sigma = \frac{\rho}{\rho_0} = \frac{\delta}{\theta}$

SPEED OF SOUND: $a = \sqrt{\gamma R T}$

DYNAMIC PRESSURE: $\bar{q} = \frac{1}{2}\rho V_T^2 = 0.7pM^2$

TRUE AIRSPEED : $V_T = Ma$

EQUIVALENT AIRSPEED : $V_E = V_T\sqrt{\sigma}$

TEMPERATURE RATIO: $T_T/T_S = 1 + \frac{\gamma-1}{2}M^2 = 1 + 0.2M^2$ X

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DERIVATION OF AIRCRAFT EQUATIONS OF MOTION

FORCES

STARTING IN THE INERTIAL REFERENCE FRAME

→ NEWTON'S 2^{nd} LAW ONLY APPLICABLE IN THIS FRAME

~~FORCES~~

$$\vec{F} = \frac{d}{dt}(m\vec{V})$$
$$= \vec{V}\frac{d}{dt}(m) + m\frac{d}{dt}(\vec{V})$$

ASSUMPTION #1: CONSERVATION OF MASS, $\frac{dm}{dt} = 0$

ASIDE: NOT TRUE FOR AIRCRAFT, BUT REASONABLE FOR TIMESCALES WE ARE LOOKING AT. FUEL BURN.

$$\rightarrow \vec{F} = m\frac{d}{dt}(\vec{V})$$

WANT TO SWITCH TO A BODY AXIS REFERENCE FRAME

→ BODY AXIS IS EASILY MEASURED ONBOARD AN AIRCRAFT

→ EASY TO REFERENCE ABOUT A/C CG

RECALL: $\frac{d^I}{dt}\vec{A} = \frac{d^B}{dt}\vec{A} + \vec{\Omega}\times\vec{A}$

I → INERTIAL

B → BODY

→ VECTOR DIFFERENTATION REQUIRES ATTENTION TO ROTATING AXIS SYSTEMS.

$$\frac{d^I}{dt}(\vec{V}) = \frac{d^B}{dt}\vec{V} + \vec{\Omega}\times\vec{V} = \dot{\vec{V}} + \Omega\times\vec{V} \rightarrow \dot{\vec{V}}_{CG} + \Omega\times\vec{V}_{CG}$$

$$\rightarrow \boxed{\vec{F} = m\left(\dot{\vec{V}}_{CG} + \Omega\times\vec{V}_{CG}\right)}$$

~~MOMENTS~~

~~AGAIN, START IN IRF~~

$\vec{\Omega}$ REPRESENTS THE ROTATION OF THE BODY-FIXED FRAME WITH RESPECT TO IRF

→ ABOVE EQN IS IN BODY REFERENCE FRAME

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MOMENTS

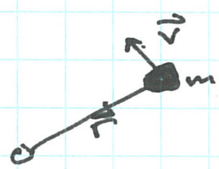
NEWTON'S SECOND LAW FOR ROTATIONAL MOTIONS

$$\vec{M} = \frac{d}{dt}(\vec{H})$$

$\vec{M}$ IS THE SUM OF EXTERNALLY APPLIED MOMENTS

$\frac{d}{dt}(\vec{H})$ IS RATE OF CHANGE OF ANGULAR MOMENTUM

ASIDE: ANGULAR MOMENTUM ~~ANGL~~



ANGULAR MOMENTUM CAN BE THOUGHT OF AS LINEAR MOM. WITH A MOMENT ARM

LIN. MOM. $= m\dot{\vec{r}}$ $\quad = p = mv$

ANG. MOM. $= \vec{r} \times m\dot{\vec{r}}$ $\quad = L = rmv$

IN THIS CASE, INDIVIDUAL SMALL MASSES NEED TO BE CONCERNED:

$$\vec{H}_{dm} = \vec{r} \times dm\,\dot{\vec{r}} = dm(\vec{r} \times \dot{\vec{r}})$$

SUMMING OVER ALL MASS,

$$\vec{H} = \int H_{dm} = \int \vec{r} \times \dot{\vec{r}}\, dm$$

~~ONLY APPLIES TO~~

$\rightarrow$ DIFFERENTIATING TO GET $\quad \dot{\vec{H}} = \int (\vec{r} \times \ddot{\vec{r}})\,dm = \vec{M}$

WANT TO GET TO BODY REFERENCE FRAME

$$\vec{r}_I = \vec{r}_B$$

$$\dot{\vec{r}}_I = \dot{\vec{r}}_B + \vec{\Omega} \times \vec{r}_B$$

$$\ddot{\vec{r}}_I = \ddot{\vec{r}}_B + \vec{\Omega} \times \dot{\vec{r}}_B + \vec{\Omega} \times (\dot{\vec{r}}_B + \vec{\Omega} \times \vec{r}_B) + \dot{\vec{\Omega}} \times \vec{r}_B$$

$$= \ddot{\vec{r}}_B + 2\vec{\Omega} \times \dot{\vec{r}}_B + \dot{\vec{\Omega}} \times \vec{r}_B + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_B)$$

$$\rightarrow \vec{M}_B = \int \vec{r}_B \times \left(\ddot{\vec{r}}_B + 2\vec{\Omega} \times \dot{\vec{r}}_B + \dot{\vec{\Omega}} \times \vec{r}_B + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_B)\right)$$

ASSUMPTION #2: AIRPLANES ARE RIGID BODIES

$\rightarrow \dot{\vec{r}} = \ddot{\vec{r}} = 0$

ESSENTIALLY SAYING THAT ELEMENTS OF MASS IN AN AIRPLANE DO NOT CHANGE POSITION WITH ONE ANOTHER TIME

→ AIRPLANES ARE NOT RIGID BODIES, STRUCTURE FLEXES, FUEL SLOSHES, PROPELLERS OR COMPRESSORS OR TURBINES ROTATE. REASONABLE ASSUMPTION FOR FLYING QUALITIES.

$$\rightarrow \boxed{\vec{M} = \int \vec{r} \times \left(\dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r})\right) dm}$$

ASSUMPTION #3: OUR IRF WILL BE A FLAT, NON-ROTATING EARTH REFERENCE FRAME.

VALID IF UNDER 3Kfps – 5Kfps, OR LESS THAN 1 min

→ ASSUMPTION #3 ALLOWS US TO DEFINE $\vec{\Omega}$ AS THE AIRCRAFT ANGULAR ROTATION RELATIVE TO EARTH

ASSUMPTION #4: ATMOSPHERE IS AT REST RELATIVE TO FLAT-EARTH → IE NO WINDS.

ASSIGN THE VECTORS IN NOTATION:

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$
$$\vec{V}_{cg} = U \hat{i} + V \hat{j} + W \hat{k}$$
$$\vec{\Omega} = P \hat{i} + Q \hat{j} + R \hat{k}$$
$$\vec{M}_{cg} = L \hat{i} + M \hat{j} + N \hat{k}$$
$$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$$

$$\rightarrow \dot{\vec{V}}_{cg} = \dot{U}\hat{k} + \dot{V}\hat{j} + \dot{W}\hat{k} + \left[U \frac{d}{dt}\hat{i} + V \frac{d}{dt}\hat{j} + W \frac{d}{dt}\hat{k} \right]$$

[ ] = 0 IN BODY FIXED FRAME

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BACK TO FORCES

$$\vec{F} = m\left(\dot{\vec{V}}_{cg} + \vec{\Omega}\times\vec{V}_{cg}\right)$$

$$\vec{\Omega}\times\vec{V}_{cg} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ P & Q & R \\ u & v & w \end{vmatrix} = (Qw - Rv)\hat{\imath} + (Ru - Pw)\hat{\jmath} + (Pv - Qu)\hat{k}$$

$$\rightarrow \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} m(\dot{u} + Qw - Rv) \\ m(\dot{v} + Ru - Pw) \\ m(\dot{w} + Pv - Qu) \end{bmatrix}$$

BACK TO MOMENTS

$$\vec{M} = \int \vec{r}_B \times \left(\dot{\vec{\Omega}}\times\vec{r}_B + \vec{\Omega}\times(\vec{\Omega}\times\vec{r}_B)\right) dm$$

$$\vec{\Omega}\times\vec{r} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ p & q & r \\ x & y & z \end{vmatrix} = (qz - ry)\hat{\imath} + (rx - pz)\hat{\jmath} + (py - qx)\hat{k}$$

$$\vec{\Omega}\times(\vec{\Omega}\times\vec{r}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ p & q & r \\ qz - ry & rx - pz & py - qx \end{vmatrix}$$

$$= (qpy - q^2x - r^2x + prz)\hat{\imath}$$

$$+ (rqz - r^2y - p^2y + pqx)\hat{\jmath}$$

$$+ (prx - p^2z - q^2z + rqy)\hat{k}$$

$$\vec{A} = \dot{\vec{\Omega}}\times\vec{r} + \vec{\Omega}\times(\vec{\Omega}\times\vec{r}) = (\dot{q}z - \dot{r}y + qpy - q^2x - r^2x + prz)\hat{\imath}$$

$$+ (\dot{r}x - \dot{p}z + rqz - r^2y - p^2y + pqx)\hat{\jmath}$$

$$+ (\dot{p}y - \dot{q}x + prx - p^2z - q^2z + rqy)\hat{k}$$

$\vec{r} \times \vec{A} = \quad \cdots \quad$ A LOT

$$\vec{r} \times \vec{A} + (qr\dot{x}^2 - qrx^2)\hat{i} + (pry^2 - pry^2)\hat{j} + (pqz^2 - pqz^2)\hat{k}$$

RECOGNIZE MOMENTS & PRODUCTS OF INERTIA

$$I_{xx} = \int (y^2+z^2)\,dm \qquad I_{xy} = \int xy\,dm$$

$$I_{yy} = \int (x^2+z^2)\,dm \qquad I_{xz} = \int xz\,dm$$

$$I_{zz} = \int (x^2+y^2)\,dm \qquad I_{yz} = \int yz\,dm$$

GET TO:

$$\vec{M} = \begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} \dot{p} I_{xx} + qrI_{zz} - qrI_{yy} + (r^2-q^2)I_{yz} + (pr-\dot{q})I_{xy} - (\dot{r}+pq)I_{xz} \\ \dot{q} I_{yy} + prI_{xx} - prI_{zz} + (p^2-r^2)I_{xz} - (\dot{r}-pq)I_{yz} - (\dot{p}+qr)I_{xy} \\ \dot{r} I_{zz} + pqI_{yy} - pqI_{xx} + (q^2-p^2)I_{xy} - (\dot{p}-qr)I_{xz} - (\dot{q}+pr)I_{yz} \end{bmatrix}$$

ASSUMPTION #5: $x_b z_b$ IS A PLANE OF SYMMETRY

$\rightarrow I_{xy} = I_{yx} = 0$

MOSTLY TRUE, ASYMMETRIC FUEL & STORES CAN CAUSE IT TO BE INVALID.

$$\rightarrow \vec{M} = \begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} \dot{p} I_{xx} + qrI_{zz} - qrI_{yy} - (\dot{r}+pq)I_{xz} \\ \dot{q} I_{yy} + prI_{xx} - prI_{zz} + (p^2-r^2)I_{xz} \\ \dot{r} I_{zz} + pqI_{yy} - pqI_{xx} - (\dot{p}-qr)I_{xz} \end{bmatrix}$$

11/02/18

LOOK AT LHS TERMS

$$F_x = F_{x_{AERO}} + F_{x_{THRUST}} + F_{x_{GRAVITY}} = \bar{q} S C_x + F_{x_{ENG}} + F_{x_g}$$

$$F_Y = F_{Y_{AERO}} + F_{Y_{THRUST}} + F_{Y_{GRAVITY}} = \bar{q} S C_Y + F_{Y_{ENG}} + F_{Y_g}$$

$$F_Z = F_{Z_{AERO}} + F_{Z_{THRUST}} + F_{Z_{GRAVITY}} = \bar{q} S C_Z + F_{Z_{ENG}} + F_{Z_g}$$

$$\vec{F}_{g_{EARTH}} = m \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$

$$\vec{F}_{g_{BODY}} = m\,(ABC) \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} = m \begin{bmatrix} -g \sin\theta \\ g\cos\theta \sin\phi \\ g\cos\theta\cos\phi \end{bmatrix}$$

$$L = L_{AERO} + L_{THRUST} = \bar{q} S b C_\ell + L_{ENG}$$

$$M = M_{AERO} + M_{THRUST} = \bar{q} S c C_m + M_{ENG} - \Omega_{ENG} I_{ENG}\, r$$

$$N = N_{AERO} + N_{THRUST} = \bar{q} S b C_n + N_{ENG} + \Omega_{ENG} I_{ENG}\, q$$

NOTE: NO GRAVITY TERMS DUE TO FORCES ACTING THROUGH THE CG.

$\Omega_{ENG} I_{ENG}$ TERMS COME FROM ENGINE GYROSCOPIC EFFECTS OF SPINNING COMPONENTS.
- → NONE IN ROLL DUE TO AXIS OF ROTATION
- → ASSUME THRUST AXIS ≈ BODY AXIS (SMALL INSTALLATION ANGLES)

VECTOR FORM E.O.M'S

$$\vec{F} = \begin{bmatrix} \bar{q} S C_x + F_{x_{ENG}} - mg\sin\theta \\ \bar{q} S C_Y + F_{Y_{ENG}} + mg\cos\theta\sin\phi \\ \bar{q} S C_Z + F_{Z_{ENG}} + mg\cos\theta\cos\phi \end{bmatrix} = \begin{bmatrix} m(\dot{u} + qw - rv) \\ m(\dot{v} + ru - pw) \\ m(\dot{w} + pv - qu) \end{bmatrix}$$

11/02/18

$$\vec{M} = \begin{bmatrix} \bar{q} S b C_\ell + L_{ENG} \\ \bar{q} S c C_m + M_{ENG} - \Omega_{ENG} I_{ENG}\, r \\ \bar{q} S b C_n + N_{ENG} + \Omega_{ENG} I_{ENG}\, q \end{bmatrix} = \begin{bmatrix} \dot{p} I_{xx} + q r I_{zz} - q r I_{yy} - (\dot{r} + p q) I_{xz} \\ \dot{q} I_{yy} + p r I_{xx} - p r I_{zz} + (p^2 - r^2) I_{xz} \\ \dot{r} I_{zz} + q p I_{yy} - q p I_{xx} - (\dot{p} - q r) I_{xz} \end{bmatrix}$$

SIX DEGREES OF FREEDOM EQUATIONS OF MOTION BODY AXIS

$$\dot{u} = r v - q w + \frac{\bar{q} S}{m} C_x + \frac{F_{X\,ENG}}{m} - g \sin\theta$$

$$\dot{v} = p w - r u + \frac{\bar{q} S}{m} C_Y + \frac{F_{Y\,ENG}}{m} + g \cos\theta \sin\phi$$

$$\dot{w} = q u - p v + \frac{\bar{q} S}{m} C_Z + \frac{F_{Z\,ENG}}{m} + g \cos\theta \cos\phi$$

$$\dot{p} = \frac{I_{yy} - I_{zz}}{I_{xx}} q r + \frac{I_{xz}}{I_{xx}} (\dot{r} + p q) + \frac{\bar{q} S b}{I_{xx}} C_\ell + \frac{L_{ENG}}{I_{xx}}$$

$$\dot{q} = \frac{I_{zz} - I_{xx}}{I_{yy}} p r + \frac{I_{xz}}{I_{yy}} (r^2 - p^2) + \frac{\bar{q} S c}{I_{yy}} C_m + \frac{M_{ENG}}{I_{yy}} - \frac{\Omega_E I_E}{I_{yy}} r$$

$$\dot{r} = \frac{I_{xx} - I_{yy}}{I_{zz}} p q + \frac{I_{xz}}{I_{zz}} (\dot{p} - q r) + \frac{\bar{q} S b}{I_{zz}} C_n + \frac{N_{ENG}}{I_{zz}} + \frac{\Omega_E I_E}{I_{zz}} q$$

OTHER EQUATIONS

$$V_T = \sqrt{u^2 + v^2 + w^2}$$

$$u = V_T \cos\alpha \cos\beta$$

$$v = V_T \sin\beta$$

$$w = V_T \sin\alpha \cos\beta$$

$$\dot{u} = \dot{V}_T \cos\alpha \cos\beta - \dot{\alpha} V_T \sin\alpha \cos\beta - \dot{\beta} V_T \cos\alpha \sin\beta$$

$$\dot{v} = \dot{V}_T \sin\beta + \dot{\beta} V_T \cos\beta$$

$$\dot{w} = \dot{V}_T \sin\alpha \cos\beta + \dot{\alpha} V_T \cos\alpha \cos\beta - \dot{\beta} V_T \sin\alpha \sin\beta$$

$$\alpha = \tan^{-1} \frac{w}{u}$$

$$\beta = \sin^{-1} \frac{v}{V_T}$$

$$\dot{\alpha} = \frac{1}{V_T \cos\beta} (\dot{w} \cos\alpha - \dot{u} \sin\alpha)$$

$$\dot{\beta} = \frac{1}{V_T} (\dot{v} \cos\beta - \dot{u} \cos\alpha \sin\beta - \dot{w} \sin\alpha \sin\beta)$$

$$\dot{V}_T = \dot{u} \cos\alpha \cos\beta + \dot{v} \sin\beta + \dot{w} \sin\alpha \cos\beta$$

11/02/18

BODY AXIS E.O.M FOR AIRCRAFT

$$\dot{V}_T = \frac{1}{\cos\beta\cos\alpha}\left[rV_T\sin\beta - g\sin\theta + \frac{\bar{q}S}{m}C_x + \frac{F_{XE}}{m}\right] + V_T(\dot{\alpha} - q)\tan\alpha + \dot{\beta}V_T\tan\beta$$

$$\dot{\alpha} = \frac{1}{\cos\beta\cos\alpha}\left[-p\sin\beta + \dot{\beta}\sin\beta\sin\alpha + \frac{g\cos\theta\cos\phi}{V_T} + \frac{\bar{q}S}{mV_T}C_z + \frac{F_E}{mV_T}\right] - \frac{\dot{V}_T\tan\alpha}{V_T} + q$$

$$\dot{\beta} = \frac{1}{V_T\cos\beta}\left[-\dot{V}_T\sin\beta + g\cos\theta\sin\phi + \frac{\bar{q}S}{m}C_Y + \frac{F_{YE}}{m}\right] + p\sin\alpha - r\cos\alpha$$

INTERESTING TERMS IN EOM

$\dot{\beta}$ TERM $= f(p\sin\alpha - r\cos\alpha)$

-> TRANSFORMATION OF α & β INTO EACH OTHER
-> NEED TO COORDINATE p & r IN ORDER TO NOT DRIVE β TO DEPARTURE

$\dot{p}, \dot{q}$ & $\dot{r}$ = ~~f(~~

-> KINEMATIC & INERTIAL COUPLING

12/04/2018

LONGITUDINAL CHARACTERISTICS

LONGITUDINAL AXIS RUNS ALONG X-AXIS & IT'S CHARACTERISTICS ACT ~~ABOUT~~ AS ROTATION ABOUT THE Y-AXIS

COEFFICIENTS & RATES OF INTEREST

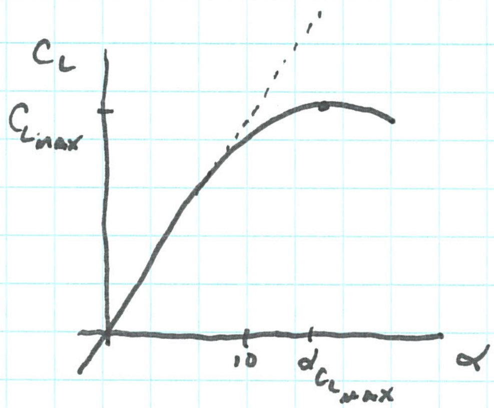
C_L -> LIFT COEFFICIENT
C_m -> PITCHING MOMENT COEFFICIENT
C_D -> DRAG COEFFICENT

q -> PITCH RATE

δ_H -> TAIL DEFLECTION (CONTROL SURFACE)
+ -> TE DOWN

LIFT

THE LIFT CURVE



SYMMETRIC AIRFOILS HAVE LINEAR SLOPE UP TO ~ 10α, BEYOND THAT THE LINEARITY ENDS UP TO A MAXIMUM C_L AND THEN DIMINISHES.

SLOPE & MAXIMUM LIFT ARE FUNCTIONS OF WING GEOMETRY & SPEED

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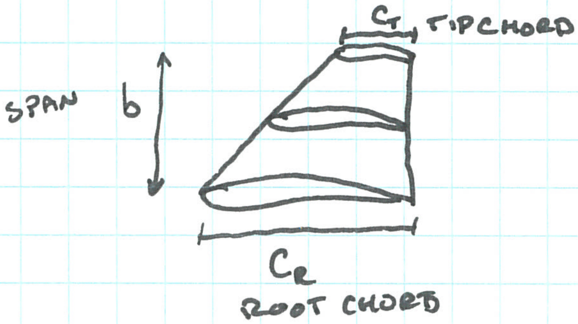
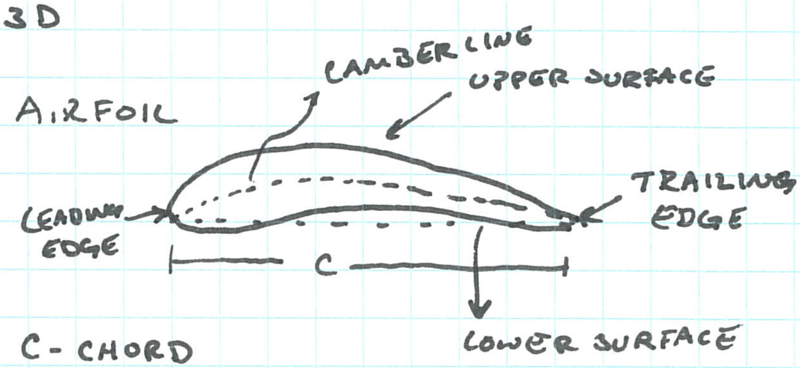
WINGS

-> COLLECTION OF AIRFOIL SECTIONS
  -> AIRFOILS ARE 2D
  -> WINGS ARE 3D





WING AREA : $S = b\bar{c}$ => MEAN CHORD × SPAN

TAPER RATIO : $\lambda = C_{TIP}/C_{ROOT}$ => TIP CHORD / ROOT CHORD

ASPECT RATIO: $AR = b^2/S$ => $(\text{SPAN})^2$/WING AREA

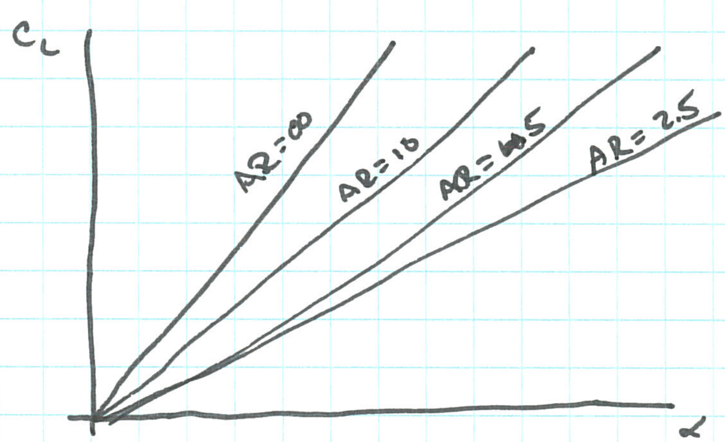
$= b/\bar{c}$ ~~FOR~~

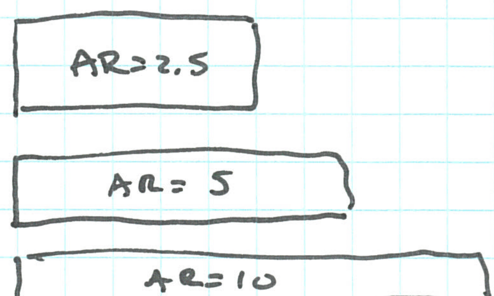
MEAN AERODYNAMIC CHORD, $\bar{c}$ OR MAC

-> CHORD WHERE AERODYNAMIC FORCES ARE ASSUMED TO ACT THROUGH

-> PASSES THROUGH CENTROID OF AREA OF THE SEMI-SPAN

$$\bar{c} = \frac{2}{3} C_R \frac{1+\lambda+\lambda^2}{1+\lambda}$$





FIGHTERS ~ AR 2.5-3.5

TRANSPORT ~~COMMERCIAL~~ ~ AR 6-8

GLIDER ~ AR 15-20

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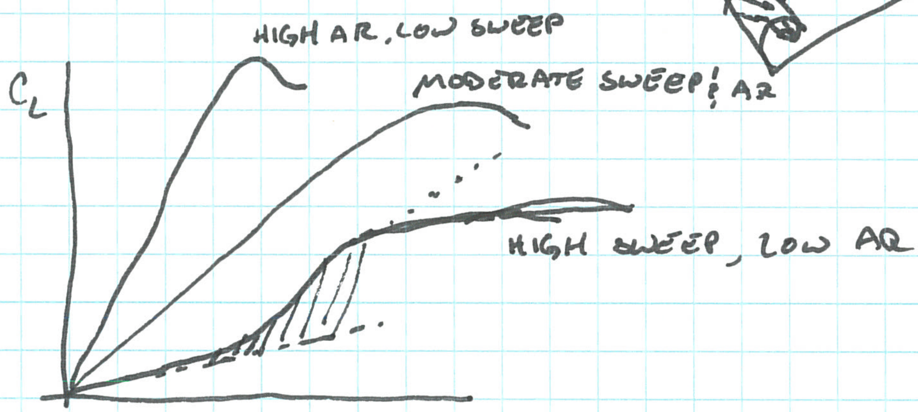
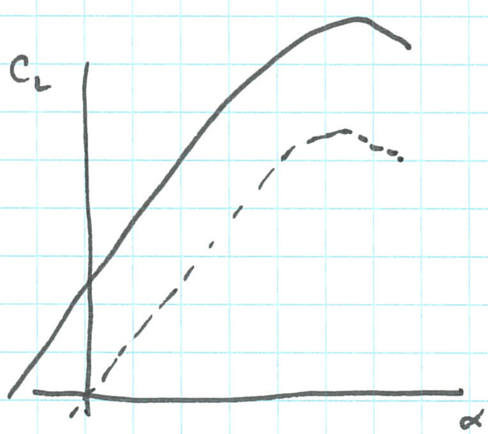
HIGH ASPECT RATIO WINGS HAVE STEEPER LIFT CURVE SLOPE, BUT SEPARATION OCCURS AT LOWER AOA'S

SWEEP REDUCES LIFT CURVE SLOPE, BUT PROMOTES A MORE GRADUAL STALL.

HIGH ~~SWEEP~~ SWEEP, ~60° & MORE, CREATES STRONG LEADING EDGE VORTICIES THAT ENHANCE LE SUCTION & INTRODUCE NONLINEAR CONTRIBUTION TO LIFT



CAMBER EFFECTS ON LIFT CURVE SLOPE



—— CAMBERED

- - - - UNCAMBERED

NOTE THAT UNSYMMETRIC AIRFOIL PROVIDES NON-ZERO LIFT AT ZERO ANGLE OF ATTACK

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PITCHING MOMENT & TRIM

DEFINITIONS

CENTER OF PRESSURE (CP, C.P.)

- LOCATION ON AIRCRAFT WHERE DISTRIBUTED ~~LOADS~~ AERODYNAMIC LOADS PRODUCE NO PITCHING MOMENT

$\Sigma M_{C.P.}=0$

- DISTRIBUTED LIFT & DRAG CAN BE REPLACED WITH POINT LOADS

$C_m=0$

- MOVES W/ FLIGHT CONDITION OR CONFIGURATION
  - FORCES CHANGE W/ AIRSPEED & AoA
  - AIRCRAFT IS CONTROLLED BY RECONFIGURING TO MOVE C.P. (CONTROL SURFACES)

AERODYNAMIC CENTER (AC, A.C.)

- LOCATION ON AIRCRAFT ABOUT WHICH A CHANGE IN LIFT DOES NOT CHANGE PITCHING MOMENT
- MOVES AS AIRSPEED CHANGES, BUT INSENSITIVE TO AoA, CAMBER, OR CONTROL DEFLECTIONS (LOW AoA)
- MOVES WITH AoA AS STALL IS APPROACHED

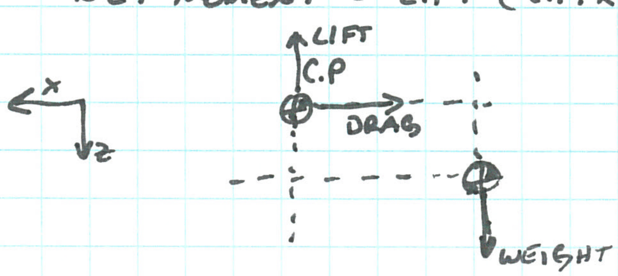
$\rightarrow \quad \frac{dC_{m_{a.c}}}{dC_L}=0$ AERODYNAMIC CENTER IS THE LIFT CENTER

NOTE: DOES NOT MEAN $C_m=0$

PITCHING MOMENT RESULTS WHEN C.P. IS NOT LOCATED AT C.G.

- WEIGHT & LIFT/DRAG NOT ALIGNED
- ELIMINATION OF PRODUCED MOMENT IS CALLED "TRIMMING" BY BALANCING THE FORCES ~~TO~~

NET MOMENT = LIFT (C.P. x-pos − C.G. x-pos) + DRAG × (C.P. z pos − C.G. z pos)



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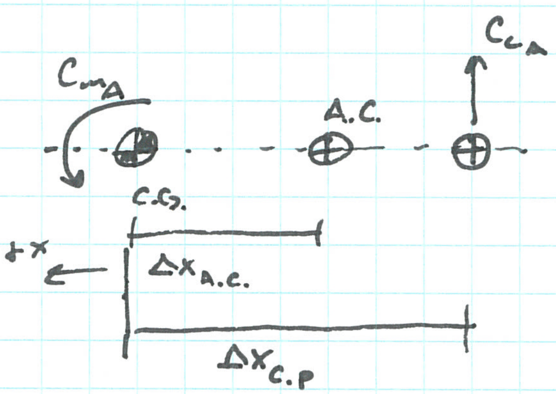
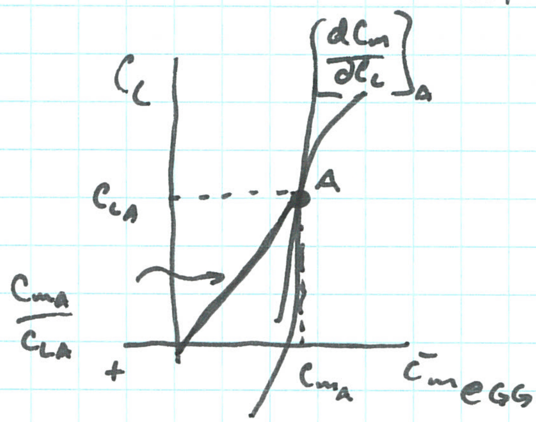




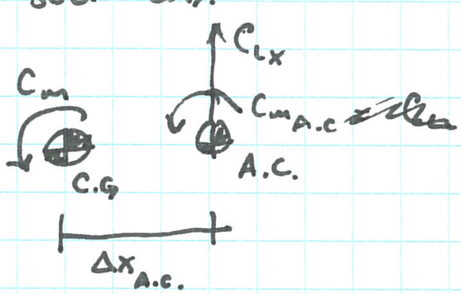
$$\frac{\Delta x_{C.P.}}{\bar{c}} = \frac{C_{m_A}}{C_{L_A}}$$

CENTER OF PRESSURE REPRESENTS THE C.G. LOCATION WHERE THE MOMENT IS ZERO

$$\frac{\Delta x_{A.C.}}{\bar{c}} = \left[\frac{dC_m}{dC_L}\right]_A$$

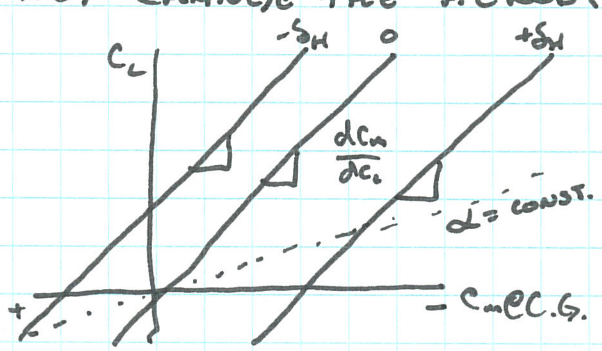
AERODYNAMIC CENTER REPRESENTS C.G. WHERE THE MOMENT SLOPE IS ZERO

IT IS NOT CONVIENIENT TO EXPRESS MOMENTS ABOUT C.P. AS IT CHANGES WITH ALL STATES.
INSTEAD, MOVE THOSE FORCES TO A.C. AND EXPRESS C_m ABOUT C.G.



$$C_m = C_{m_{A.C}} + \frac{dC_m}{dC_L} C_{L_X}$$

CONTROL DEFLECTIONS CAMBER THE AIRCRAFT, BUT DO NOT CHANGE THE AERODYNAMIC CENTER → TRIMS AIRCRAFT



$\frac{dC_m}{dC_L}$ UNCHANGED

=

A.C. UNCHANED

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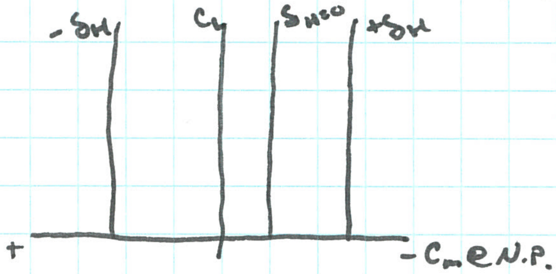
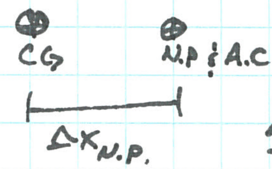
NEUTRAL POINT

→ CENTER OF GRAVITY LOCATION WHICH PRODUCES NEUTRAL AIRCRAFT STABILITY

→ NEUTRAL POINT IS AT THE AERODYNAMIC CENTER





$$\frac{\Delta x_{N.P.}}{\bar{c}} = \frac{dC_m}{dC_L}$$

PITCH STABILITY

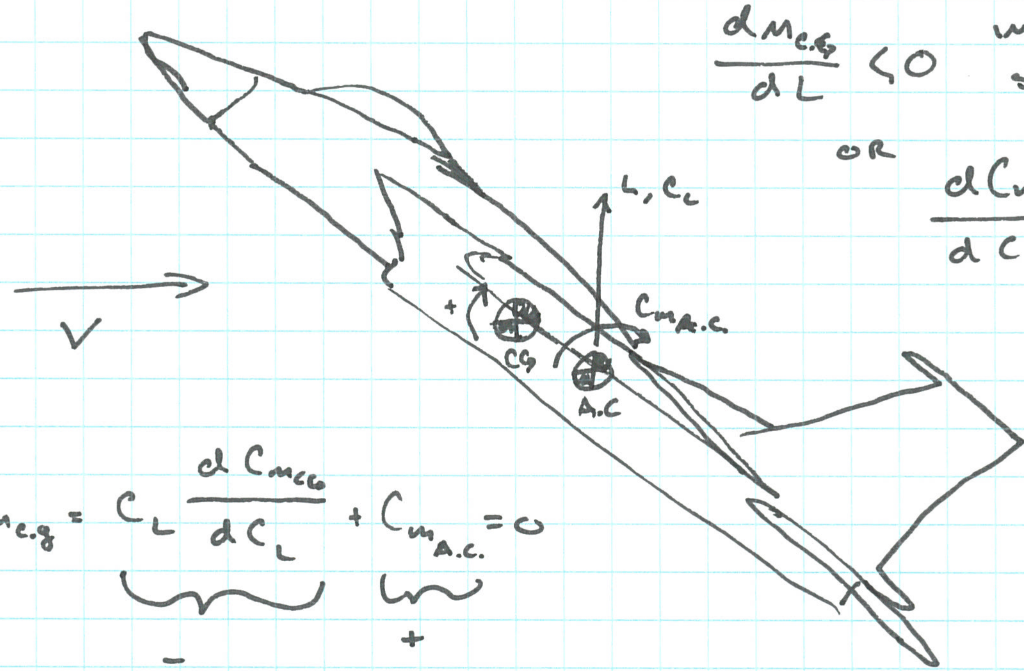
STABLE → A PERTURBATION IN LIFT RESULTS IN A PITCHING MOMENT THAT RESTORES THE AIRCRAFT TO IT'S ORIGINAL, EQUILIBRIUM STATE

$$\frac{dM_{c.g}}{dL} < 0$$

NEGATIVE DERIVATIVE IMPLIES A STABLE SYSTEM

OR

$$\frac{dC_{m_{CG}}}{dC_L} < 0$$



$$C_{m_{c.g}} = \underbrace{C_L \frac{dC_{m_{CG}}}{dC_L}}_{-} + \underbrace{C_{m_{A.C.}}}_{+} = 0$$

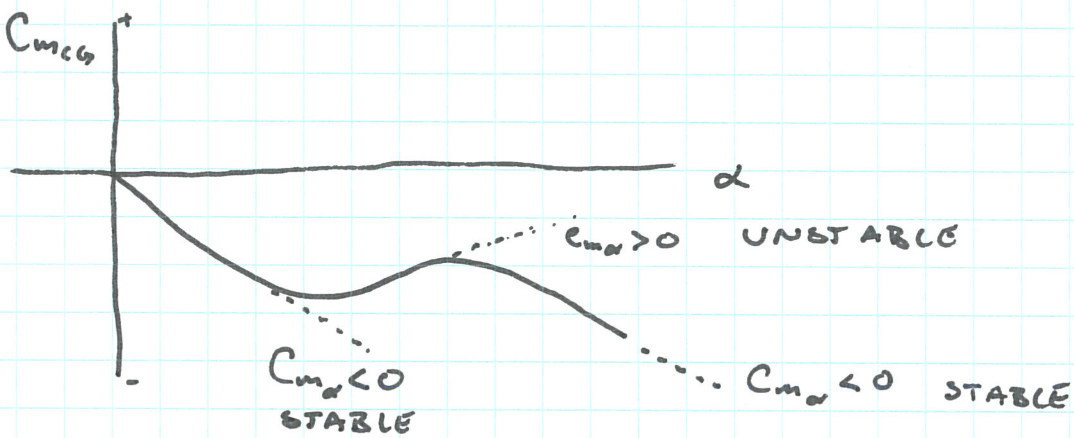
RESULTS WHEN AERODYNAMIC CENTER IS AFT OF THE CG

CAN EXPRESS PITCH STABILITY ~~AS~~ A FUNCTION OF ANGLE OF ATTACK

$$\frac{dC_{m_{CG}}}{dC_L} \cdot \frac{dC_L}{d\alpha} = \frac{dC_{m_{CG}}}{d\alpha} = C_{m_\alpha}$$

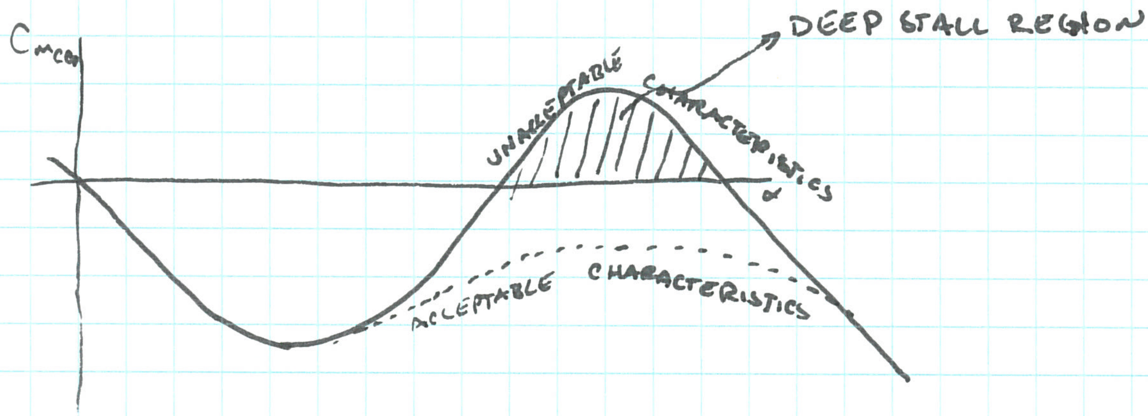
PITCH STABILITY

~~ALWAYS~~ LIFT CURVE SLOPE (ALWAYS POS ~~BEFORE STALL~~)



C_{m_α} IS A MORE APPROPRIATE MEASURE OF PITCH STABILITY AT HIGH AoA WHEN WINGS STALL

CAUTION SHOULD BE MADE TO LOOK OUT FOR "DEEP STALLS"



DEEP STALL REGION → C_{m_α} DRIVES TO EQUILIBRIUM POINT AT A HIGHER α

→ NEW EQUILIBRIUM α AT HIGH AoA

→ COULD AUGMENT W/ CONTROL SURFACES, OR "ROCK" OUT IF SMALL REGION.

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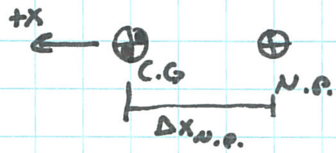
STATIC MARGIN

→ DISTANCE BETWEEN NEUTRAL POINT AND C.G.



$$\Delta x_{N.P.} = x_{N.P.} - x_{C.G.}$$

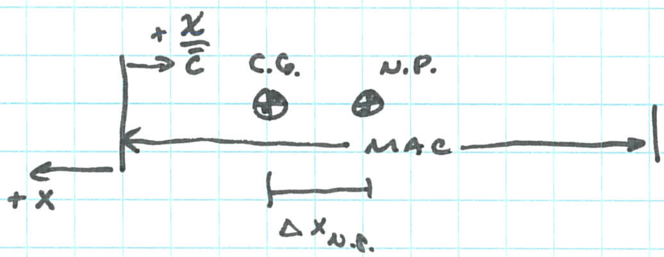
$$\text{STATIC MARGIN} = \frac{\Delta x_{N.P.}}{\bar{c}} = \frac{dC_m}{dC_L}$$

- NEGATIVE DISTANCE MEANS C.G. IS AHEAD OF N.P.
  - → LONGITUDINALLY STABLE
  - → NEGATIVE DISTANCE IS POSITIVE STATIC MARGIN

OFTEN TIMES DISTANCES ARE EXPRESSED AS %MAC

$$\text{STATIC MARGIN} = \left[\frac{x}{\bar{c}}\right]_{N.P.} - \left[\frac{x}{\bar{c}}\right]_{C.G.} = -\left[\frac{dC_m}{dC_L}\right]$$

$x_0 = x_{LEMAC}$ MEASURED POS. x IN $-x$ DIRECTION



NEW CONVENSION

- POSITIVE DIFFERENCE MEANS C.G. IS AHEAD OF N.P.
  - → AIRCRAFT LONGITUDINALLY STABLE
  - → POSITIVE DIFFERENCE IS A POSITIVE STATIC MARGIN

RULES OF THUMB FOR PITCH INSTABILITY LIMITS

- STATIC STABILITY RATIO REPRESENTS RATIO BETWEEN STATIC PITCH STABILITY & PRIMARY PITCH SURFACE CONTROL POWER

$$-2 < \frac{M_\alpha}{M_\delta} < 2$$

$M_\alpha / M_\delta < -2$: TOO UNSTABLE WITH INADEQUATE CONTROL POWER → LOW STABILITY MARGINS

$M_\alpha / M_\delta > 2$: TOO STABLE WITH INADEQUATE CP → SLUGGISH RESPONSE

- RATIO CAN BE CONSIDERED AS CHANGE IN TRIM CS RELATIVE TO TRIM AOA

$$-2 < \left[\delta_H / \alpha\right]_{TRIM} < 2$$

- TIME TO DOUBLE : $T_2 \approx \dfrac{\ln(2)}{\sqrt{C_{m_\alpha} \cdot \frac{180}{\pi} \cdot \frac{\bar{q} S c}{I_{yy}}}} > 0.2\,s$ → PILOT/FCS CAN COUNTER

12/05/2018

DEFINITIONS SUMMARY

NEUTRAL POINT

- CG LOCATION THAT PRODUCES NEUTRAL AIRCRAFT PITCH STABILITY
- AERODYNAMIC CENTER OF AIRCRAFT
    - CHANGE IN LIFT DOES NOT CHANGE AERO PITCHING MOMENT
- DEFINED WITH CONTROLS FIXED & NO FCS FEEDBACKS
- TYPICALLY DEFINED AT LOW AOA → f(M) ONLY
- TYPICALLY A SINGLE NEUTRAL POINT IS DEFINED AS A WEIGHTED SUM ACROSS 1g TRIM AOA FOR A RANGE OF GROSS WEIGHTS & ALTITUDES

$$NP(\%MAC) = 100\left(CG - \frac{dC_m}{dC_L}\right)$$
$$= 100\left(CG - \frac{C_{m\alpha}}{C_{L\alpha}}\right)$$

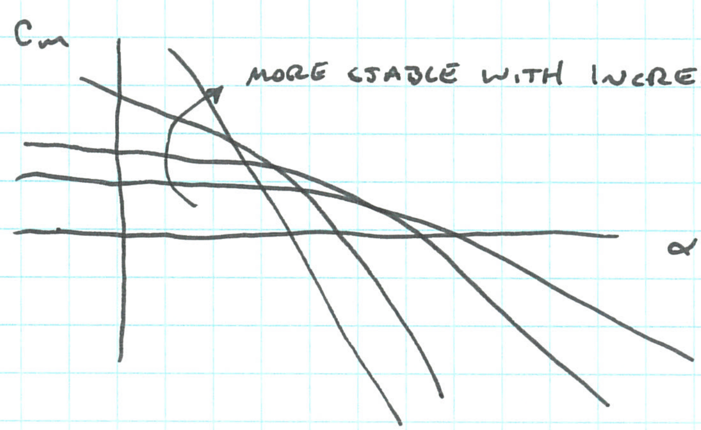
STATIC MARGIN

- DISTANCE BETWEEN N.P. & C.G., POSITIVE WHEN CG IS FWD
- POS SM COMES FROM A NEGATIVE dC_m/dC_L OR NEGATIVE $C_{m\alpha}$

$$SM(\%MAC) = 100 \times \frac{\Delta x_{NP}}{\bar{c}} = -100 \times \frac{dC_m}{dC_L} = -100 \times \frac{C_{m\alpha}}{C_{L\alpha}}$$

CENTER OF PRESSURE

- POINT AT WHICH AERODYNAMIC LOADS PRODUCE NO PITCHING MOMENT
    - LIFT & DRAG CAN BE REPLACED WITH POINT LOADS
- MOVES AFT WITH INCREASING MACH NUMBER
    - SEEN IN C_m vs. α



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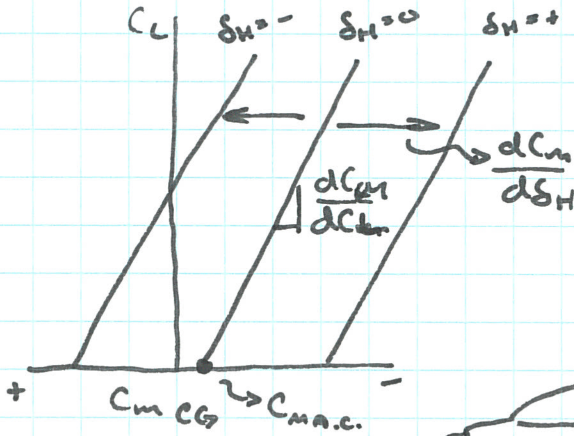
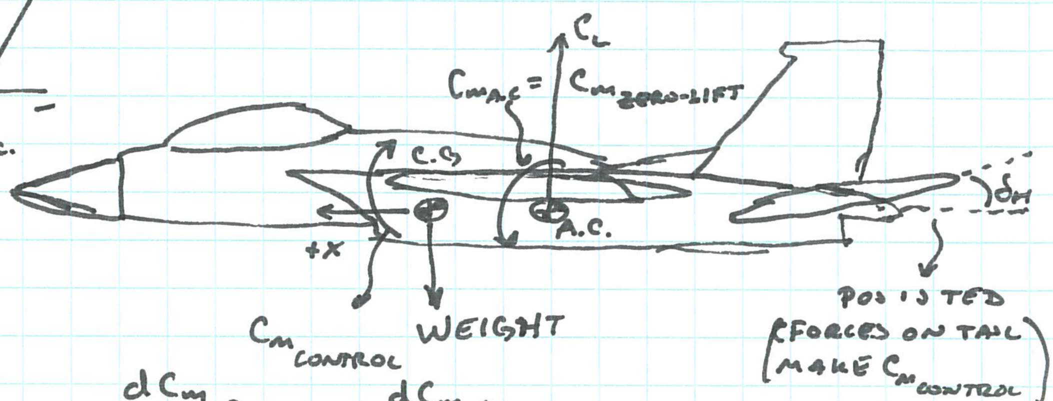
TRIMMING SUMMARY



FOR AN AIRCRAFT TO FLY STRAIGHT AND LEVEL, THE FORCES MUST BALANCE ABOUT THE C.G.



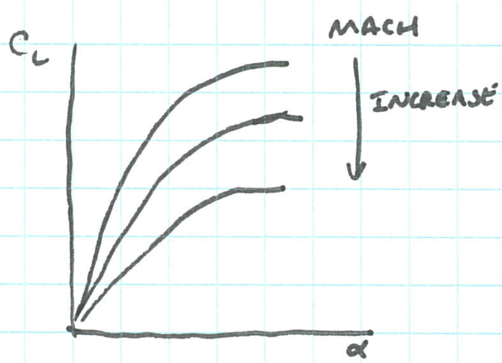
$$\Sigma C_{m_{C.G.}} = C_{m_{A.C}} + \underbrace{\frac{dC_m}{dC_L} C_L}_{C_{m_{LIFT}}} + \underbrace{\frac{dC_m}{d\delta_H} \delta_H}_{C_{m_{CONTROL}}} = 0$$

$\hookrightarrow$ FOR TRIM

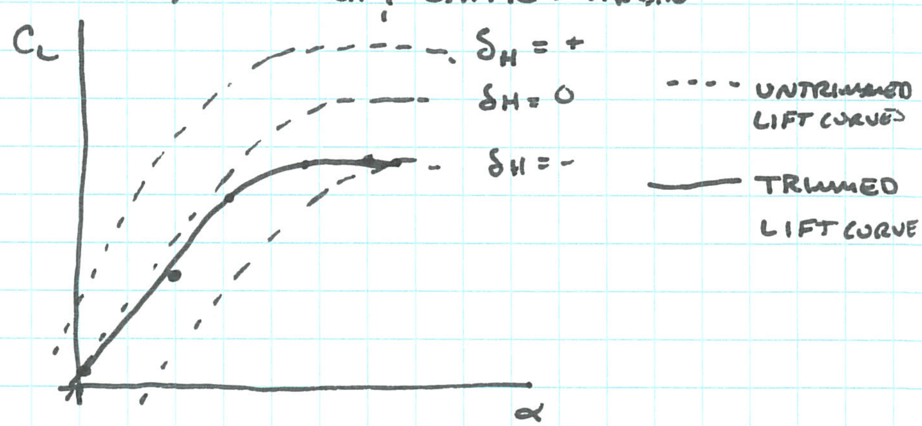
TRIMMED LIFT BUILDUP

→ ~~STATUS OF A CONFIGURATION~~

COLLECTION OF TRIMMED LIFT CURVES FOR ALL FLIGHT COND.



FOR GIVEN MACH & STATIC MARGIN



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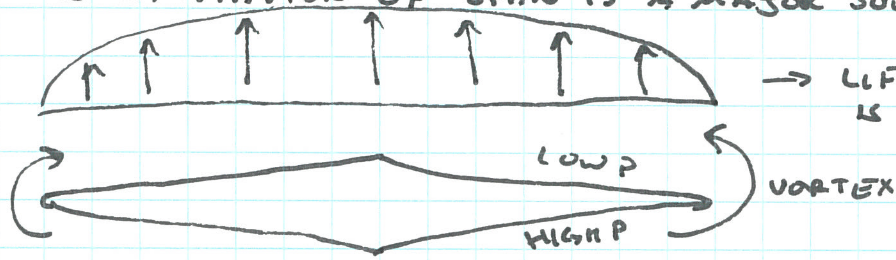
DRAG

A FINITE WING RESULTS IN LIFT LOSS

→ FLOW SPILLS FROM HIGH PRESSURE SIDE TO LOW PRESSURE SIDE

→ REDUCES COMPRESSION LIFT ON LOWER SURFACE & SUCTION LIFT ON UPPER SURFACE

THIS LIMITATION OF SPAN IS A MAJOR SOURCE OF DRAG

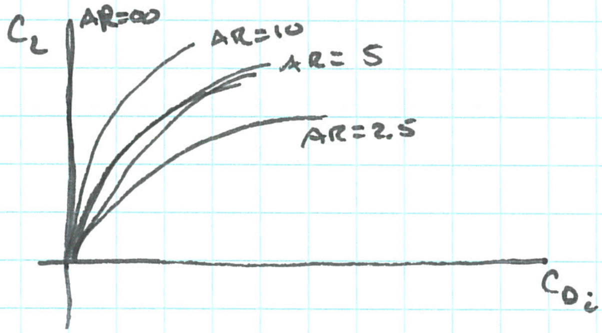


FRONT VIEW OF WING

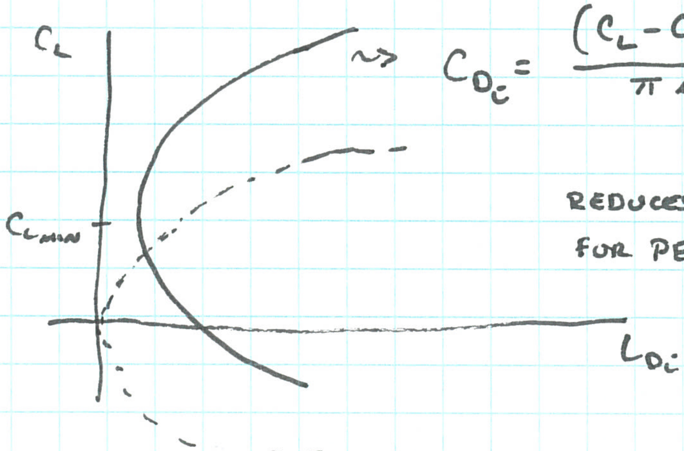
$$\text{INDUCED DRAG} = D_i = \frac{\text{LIFT}^2}{\pi q b^2}$$ (ALSO CALLED DRAG DUE TO LIFT)

↳ GENERATING THE SAME LIFT ON A WING HALF THE SIZE QUADRUPLES THE DRAG

$$C_{D_i} = \frac{C_L^2}{\pi AR} = \frac{D_i}{q S}$$



CAMBER SHIFTS INDUCED DRAG CURVE



$$C_{D_i} = \frac{(C_L - C_{L_{min}})^2}{\pi AR} + \Delta C_{D_{CAMBER}}$$

REDUCES DRAG AT HIGHER ~~ETC~~ AOA IN EXCHANGE FOR PENALTY AT LOWER AOA.

12/05/2018

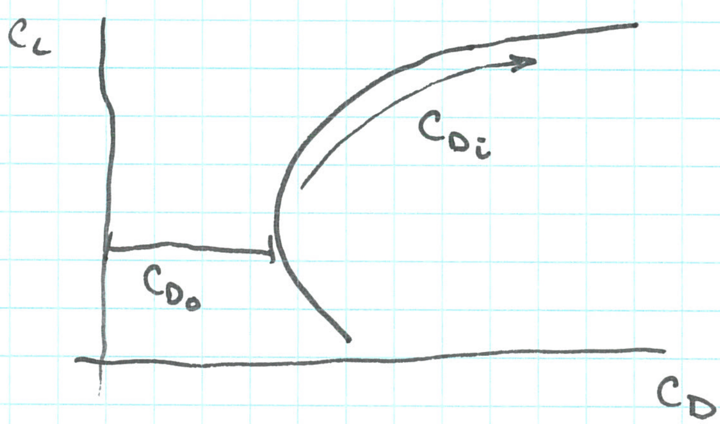
DRAG TYPICALLY MODELED AS SUM OF INDUCED DRAG & PARASITIC DRAG

$$C_D = C_{D_0} + C_{D_i}$$

C_{D_i} = INDUCED DRAG = $\dfrac{C_L^2}{\pi AR}$

C_{D_0} = PARASITIC DRAG (SUM OF OTHER DRAG)

PARASITIC DRAG INCLUDES
- DRAG DUE TO CAMBER
- SKIN FRICTION DRAG
  - PRODUCED DUE TO BOUNDARY LAYER
  - EMPIRICAL EQUATIONS
- SHOCK WAVE DRAG
  - PRODUCED BY SHOCKS FORMING
- ZERO-LIFT DRAG



NOTE THAT TRIMMING THE AIRCRAFT PRODUCES TRIM DRAG THAT IS DUE TO CONTROL SURFACE DEFLECTIONS & STATIC MARGIN. CAN BE SUMMED INTO C_{D_0}

$$\rightarrow \quad C_D = C_{D_{ZERO-LIFT}} + C_{D_{LIFT}} + C_{D_{TRIM}}$$

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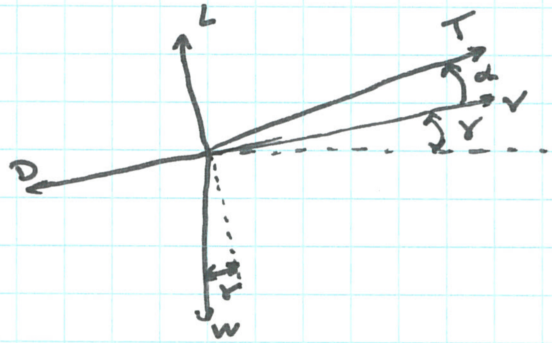
PERFORMANCE

RATE OF CLIMB (SPECIFIC EXCESS POWER, P_s)



RATE OF CLIMB = $V \sin\gamma$

$$W \sin\gamma = T\cos\alpha - D$$

$$\sin\gamma = \frac{T\cos\alpha - D}{W}$$

$$P_s = V\sin\gamma = \frac{V}{W}\left(T\cos\alpha - D\right) \quad ft/s \text{ or } ft/min$$

SPECIFIC EXCESS POWER IS A MEASURE OF INSTANTANEOUS RATE OF CLIMB

THINK OF IT AS "USABLE ENERGY"

LOAD FACTOR

→ ANY LOAD NON-DIMENSIONALIZED BY WEIGHT

$$N_L = \frac{L}{W} = \frac{C_L \bar{q} S}{W} = \frac{C_{L_\alpha} \bar{q} S}{W}$$

$$N_z = \frac{-Z_{force}}{W} = \frac{-C_z \bar{q} S}{W}$$

$$N_Y = \frac{Y}{W} = \frac{C_Y \bar{q} S}{W}$$

} MOST COMMON

$$N_{z_{LIMIT}} = \frac{\text{STRUCTURAL DESIGN LIMIT LOAD}}{\text{FLIGHT DESIGN GROSS WEIGHT}}$$

LOAD FACTOR IS ALSO NON-DIMENSIONAL MEASURE OF THE ACRAFT ACCELERATION AS A MULTIPLE OF "g's" (GRAVITY)

$$\rightarrow N_L = \frac{L}{W} = \frac{ma}{mg} = \frac{a}{g}$$

12/05/2019

TURN CAPABILITY

TURNING RADIUS : $R = \dfrac{V^2}{g\sqrt{N_L^2 - 1}}$

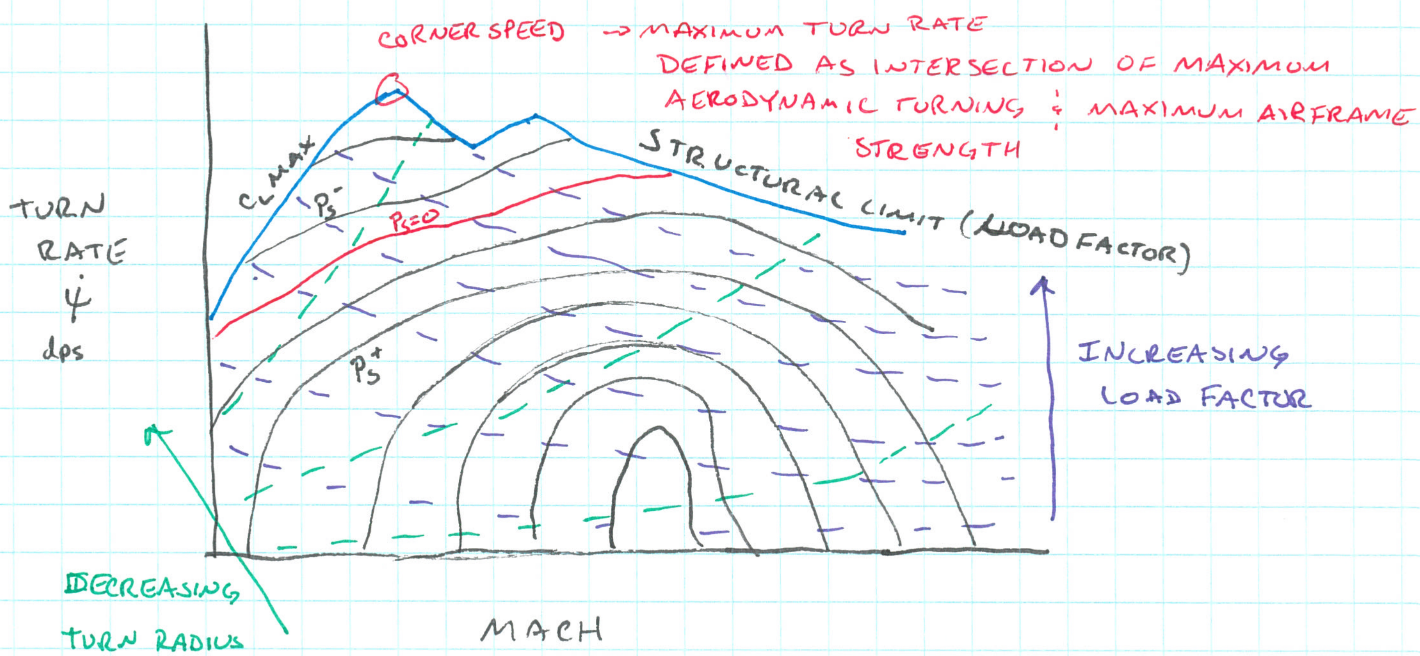
TURN RATE $= \dot{\psi} = \dfrac{57.3\,V}{\text{TURN RADIUS}} = 57.3\,\dfrac{g}{V}\sqrt{N_L^2-1}$

R = RADIUS IN FT
$\dot{\psi}$ = RATE OF TURN deg/s
V = AIRCRAFT VELOCITY ft/s
g = ACCELERATION DUE TO GRAVITY - 32.174 ft/s²
N_L = LOAD FACTOR NORMAL TO FLIGHT PATH

→ HIGHER SPEEDS MEAN LARGER TURN RADIUS & LOWER TURN RATES

→ MAXIMUM TURN RATE AT A GIVEN SPEED IS LIMITED BY LOAD FACTOR

"DOGHOUSE PLOTS" → RELATIONSHIP BETWEEN $R, \dot{\psi}, P_s, \& N_L$



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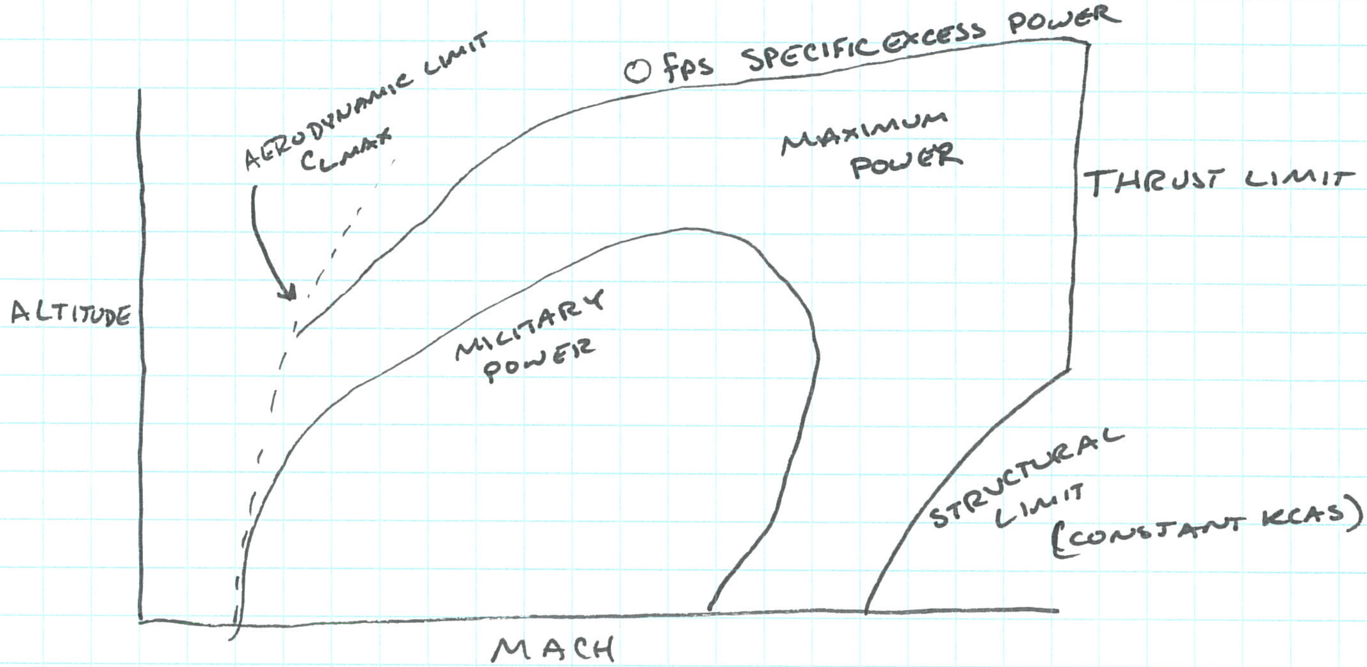
FLIGHT ENVELOPES



CEILING DEFINITIONS

ABSOLUTE CEILING - ALTITUDE AT A GIVEN AIRSPEED AT WHICH THE RATE OF CLIMB IS 0 fpm USING MIL OR MAX POWER

SERVICE CEILING - ALTITUDE AT A GIVEN AIRSPEED AT WHICH THE RATE OF CLIMB IS 100 fpm USING MIL OR MAX POWER

CRUISE CEILING - " " 300 fpm USING MAXIMUM CONTINUOUS POWER (NON-AFTER BURNING REDUCED POWER)

COMBAT CEILING - " " 500 fpm USING MIL OR MAX POWER

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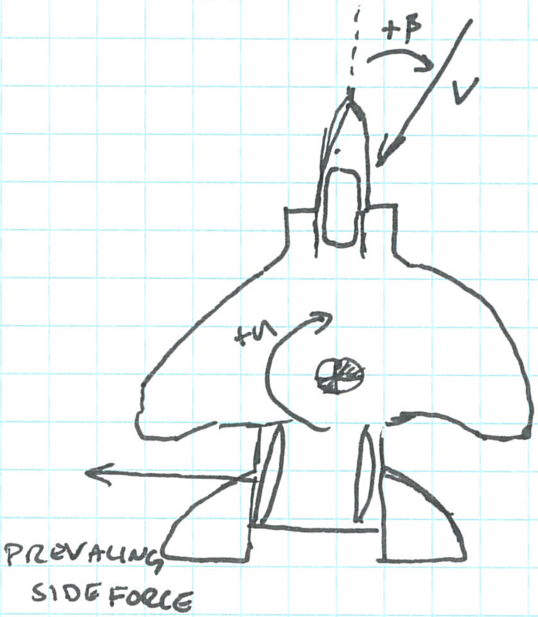
LATERAL-DIRECTIONAL ~~STABILITY~~ CHARACTERISTICS

DIRECTIONAL STABILITY



WANT A RETURNING YAWING MOMENT TO BE GENERATED WHEN A DIRECTIONAL DISTURBANCE OCCURS:

-> DIRECTIONAL DISTURBANCE
- INCREASE IN SIDESLIP (+β)

-> WANT A POSITIVE ~~SIDESLIP~~ YAWING MOMENT TO FORM TO RETURN TO ZERO SIDESLIP

-> $\frac{\delta n}{\delta \beta} > 0$ OR $\frac{\delta C_n}{\delta \beta} > 0$

-> $C_{n_\beta} > 0$

PREVAILING SIDEFORCE IS COMEING FROM THE VERTICAL STABILIZERS
- RUDDERS (CONTROL SURFACE)
- VERTICAL TAILS (FIXED)

PREVAILING SIDEFORCE MAGNITUDE & LOCATION AFT OF CG CREATE YAWING MOMENT
- CAN FLY AT CONSTANT SIDESLIP (STEADY-HEADING SIDESLIP), REQUIRES RUDDER INPUT

CENTER & MAGNITUDE OF SF VARIES WITH FLIGHT CONDITIONS AND ALTITUDE
- MACH COMPRESSIBILITY EFFECTS
- AOA VARRIES FLOW TO VERTICAL STABILIZERS -> CAN CHANGE $C_{n_\beta} = f(\alpha)$

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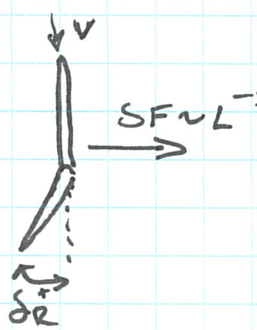
DIRECTIONAL CONTROL FROM VERTICAL TAILS

- POSITIVE RUDDER DEFLECTION δ_R MEASURED AS TE LEFT

→ POSITIVE RUDDER DEFLECTION PROVIDES NEGATIVE YAWING MOMENT

δ_{R_L} δ_{R_R} → $\delta_R = \dfrac{(\delta_{R_L} + \delta_{R_R})}{2}$ → SYMMETRIC RUDDER

LOOKING AT INDIVIDUAL RUDDER:



→ RUDDER MOVEMENT IS APPLYING CAMBER TO AIRFOIL

- SIDE FORCE GENERATED INSTEAD OF LIFT DUE TO COORDINATE AXIS LOCATION.

DIRECTIONAL CONTROL IS USED TO YAW THE AIRPLANE AND CONTROL SIDESLIP.
IT ALSO HAS INPUT TO ROLLING MOMENT AND SIDE FORCE

→ NOTE THAT CANTED VERTICAL TAILS WILL PROVIDE A LARGER ROLLING MOMENT WHEN RUDDERS ARE DEFLECTED

- RUDDERS GENERATE ROLLING MOMENT BECAUSE PRESSURE LOCATION IS ABOVE CG.

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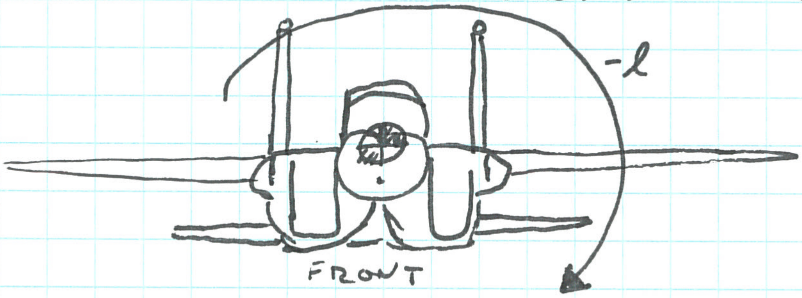
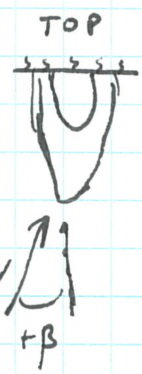
LATERAL STABILITY

POSITIVE STABILITY

→ A DISTURBANCE GENERATES A DIFFERENTIAL LIFT AND ROLLING MOMENT RETURNING AIRCRAFT TO ZERO SIDESLIP



→ $\alpha > 0$



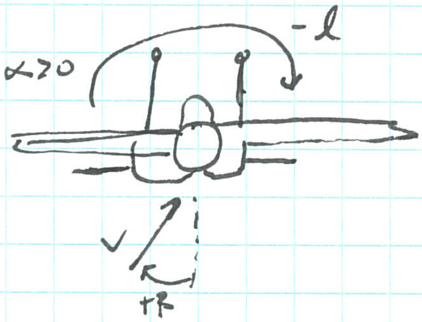
$\alpha > 0$ → NEGATIVE ROLLING MOMENT DEVELOPES WITH A POSITIVE SIDESLIP

$\alpha < 0$ → POSITIVE ROLLING MOMENT DEVELOPS WITH A POSITIVE SIDESLIP

→ STABLE ROLL RESPONSE REDUCES SIDESLIP

→ $\frac{\delta \ell}{\delta \beta} < 0$ OR $\frac{\delta C_\ell}{\delta \beta} < 0$ OR $C_{\ell_\beta} < 0$ FOR $\alpha > 0$

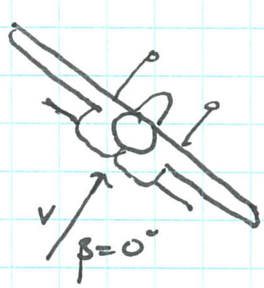
$\frac{\delta \ell}{\delta \beta} > 0$ OR $\frac{\delta C_\ell}{\delta \beta} > 0$ OR $C_{\ell_\beta} > 0$ FOR $\alpha < 0$



→

$\frac{\delta \ell}{\delta \beta} < 0$

$C_{\ell_\beta} < 0$



→ SIDESLIP RETURNS TO ZERO

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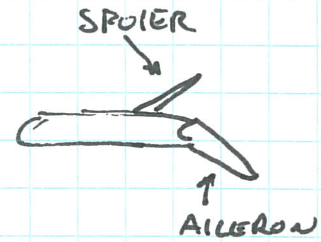
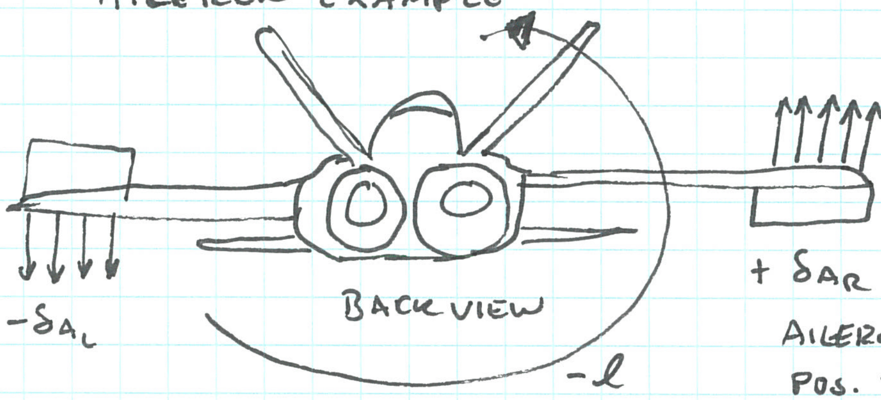
LATERAL CONTROL

LATERAL CONTROL
- -> DIFFERENTIAL AILERONS/FLAPS
- -> DIFFERENTIAL TAILS
- -> RUDDERS (SMALLER EFFECTS)
- -> SPOILERS



AILERON EXAMPLE



AILERON DEFLECTION
POS. TED

-> NEGATIVE DIFFERENTIAL AILERON SHOWN

$$\delta_{A_{DIFF}} = \frac{(\delta_{A_L} - \delta_{A_R})}{2} = \delta_A$$

- PROVIDES NEGATIVE ROLLING MOMENT

$$\delta_{A_{AVG}} = \frac{(\delta_{A_L} + \delta_{A_R})}{2} = \delta_{A_A}$$

. NOTE THAT LAT-DIR CONTROL SURFACES PROVIDE TERMS TO BOTH, HENCE LAT-DIR IS THOUGHT OF TOGETHER, VERY TOUGH TO GET ONE RESPONSE WITHOUT THE OTHER.

-> DIFFERENTIAL ~~TAIL~~ PROVIDES BOTH ROLLING MOMENT AND YAWING MOMENT. ETC.

12/06/2018

DEPARTURE & DEPARTURE RESISTANCE

DEFINITION

-> A DEPARTURE IS A LOSS OF CONTROLLED FLIGHT AND HAPPENS WHEN AN AIRCRAFT IS MOVING IN AN UNCONTROLLED MANNER.

- CAN HAPPEN WHEN OPERATING ENVELOPE IS EXCEEDED
- SOME DEPARTURES CAN BE RECOVERED FROM

DEPARTURE RESISTANCE PARAMETER

-> HELPS DETERMINE LIKELYHOOD OF DEPARTURES

-> LOOKS AT DIRECTIONAL STABILITY & LATERAL STABILITY TOGETHER

$$C_{n_{\beta_{DYN}}} = C_{n_\beta} \cos\alpha - I_{zz}/I_{xx}\, C_{l_\beta} \sin\alpha$$

DERIVATION FROM $\dot{\beta}$ EQUATION:

$$\dot{\beta} = p\sin\alpha - r\cos\alpha$$

$$\ddot{\beta} = \dot{p}\sin\alpha + p\cos\alpha\,\dot{\alpha} - \dot{r}\cos\alpha + r\sin\alpha\,\dot{\alpha}$$

-> $\dot{\alpha} = 0$ ASSUMED

-> $\ddot{\beta} = \dot{p}\sin\alpha - \dot{r}\cos\alpha$

-> ASSUME NO INERTIAL COUPLING

$$\dot{p} = \bar{q}Sb\, C_l/I_{xx}$$

$$\dot{r} = \bar{q}Sb\, C_n/I_{zz}$$

$$\ddot{\beta} = \bar{q}Sb\left[\frac{C_l}{I_{xx}}\sin\alpha - \frac{C_n}{I_{zz}}\cos\alpha\right]$$

12/06/2018

$$-\ddot{\beta}\frac{I_{zz}}{\bar{q}Sb} \simeq C_n \cos\alpha - I_{zz}/I_{xx}\, C_l \sin\alpha$$

DIFFERENTIATE WRT β

$$\rightarrow \quad C_{n_{\beta_{DYNAMIC}}} = C_{n_\beta}\cos\alpha - I_{zz}/I_{xx}\, C_{l_\beta}\sin\alpha$$

HAVE RESISTANCE TO DEPARTURES WHEN $C_{n_{\beta_{DYN}}} > 0$

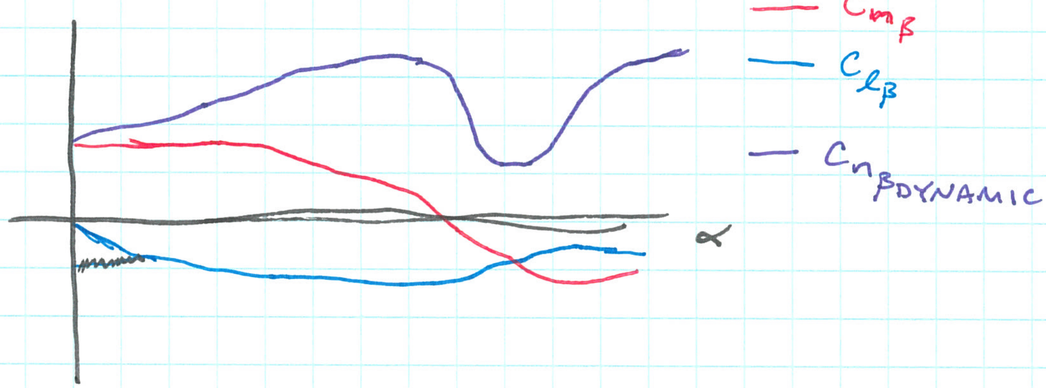
REMEMBER:

FOR POSITIVE α

STABLE IS: $C_{n_\beta} > 0$

$C_{l_\beta} < 0$

HOWEVER, IN PRACTICE...



$\rightarrow$ C_{n_β} & C_{l_β} REDUCE AT HIGHER AoAs DUE TO FLOW SEPARATION ON WING IMPINGING ON THE ROLL & YAW AUTHORITY DEVICES

$\rightarrow$ NOTICE C_{n_β} GOES UNSTABLE!!

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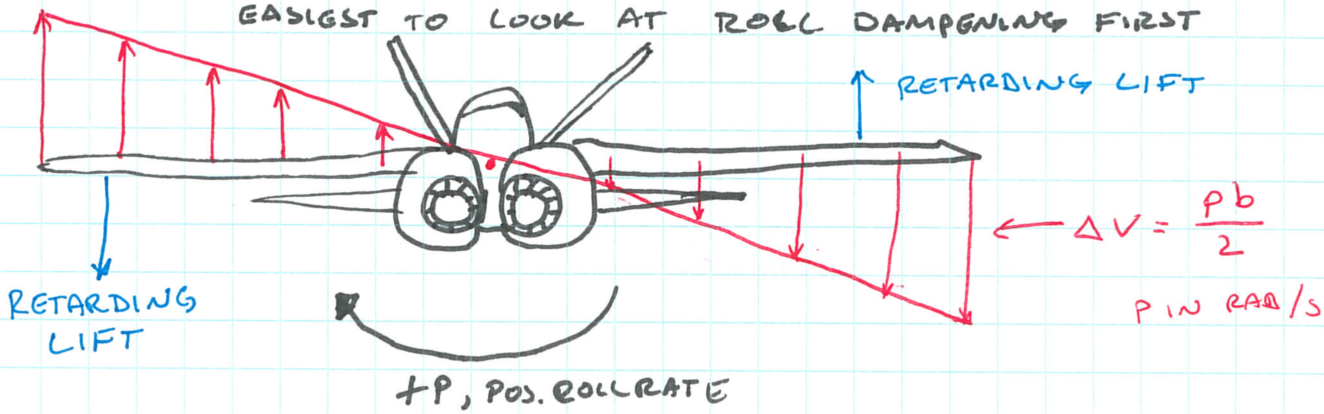
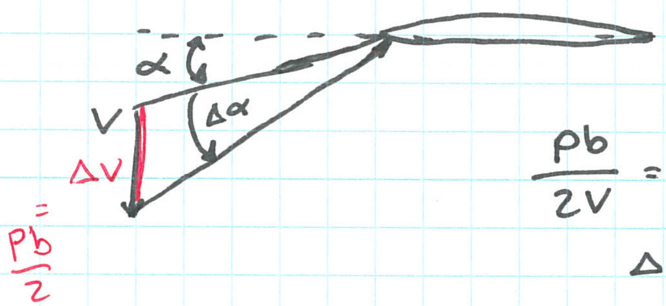
DAMPING TERMS

EASIEST TO LOOK AT ROLL DAMPENING FIRST



VIEW OF WINGTIP (RIGHT)



$$\frac{pb}{2V} = \tan \Delta\alpha$$

Δα IS THE TIP HELIX ANGLE

ROTATION INCREASES THE ANGLE OF ATTACK, AND THEREFORE LIFT, ON THE DOWNWARD MOVING WING. THIS ROTATION DECREASES ANGLE OF ATTACK, AND LIFT, ON THE UPWARD MOVING WING. THIS COMBINATION GENERATES A RETARDING ROLLING MOMENT → ROLL DAMPING

ROLL DAMPING COEFFICIENT

$$\rightarrow \quad C_{\ell_p} = \frac{\partial C_\ell}{\partial\left(\frac{pb}{2V}\right)}$$

TYPICALLY A NEGATIVE VALUE FOR POSITIVE STABILITY

OPPOSITE OF DAMPENING IS PROPELLING. THIS IS A MOMENT DRIVING FORCE.

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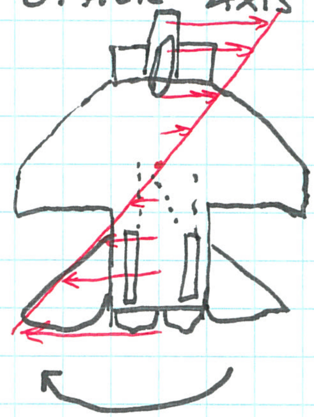
COMPARABLE TERMS EXIST IN OTHER AXIS

YAW DAMPING COEFFICIENT

$$C_{n_r} = \frac{\partial C_n}{\partial\left(\frac{rb}{2V}\right)}$$

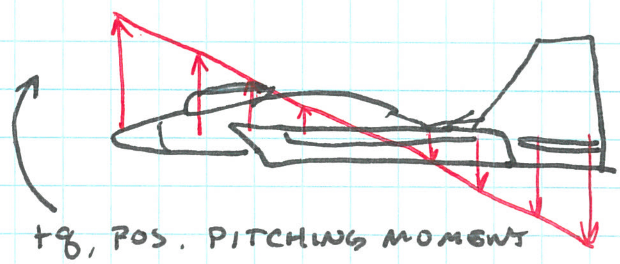


+r, POS. YAWING MOMENT

→ POSITIVE YAWING MOMENT ROTATION INDUCES VELOCITY ON VERTICAL STABILIZERS & PRODUCES SIDE FORCE AFT OF CG. THIS GENERATES RESTORING YAWING MOMENT.

PITCH DAMPING COEFFICIENT

$$C_{m_q} = \frac{\partial C_n}{\partial\left(\frac{q\bar{c}}{2V}\right)}$$

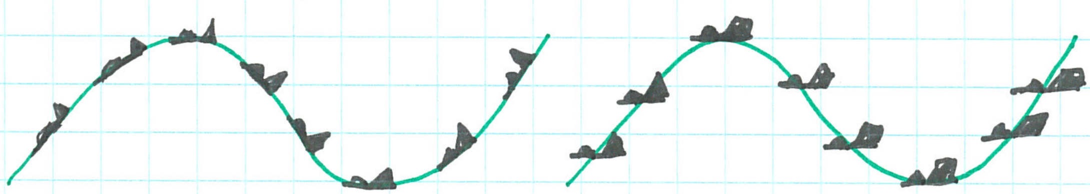


+q, POS. PITCHING MOMENT

→ POSITIVE PITCHING MOMENT INDUCES VELOCITY ON HORIZONTAL STABILIZERS, INCREASING LIFT AT AFT END, GENERATING A RESTORING PITCHING MOMENT.

PLUNGING DERIVATIVE

$$C_{m_{\dot\alpha}} = \frac{\partial C_m}{\partial\left(\frac{\dot\alpha\bar{c}}{2V}\right)}$$



PURE PITCHING MOTION

$\dot\alpha = 0 \quad q = \sin\omega t$

PURE PLUNGING MOTION

$\dot\alpha = \sin\omega t \quad q = 0$

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CROSS-AXIS DAMPING COEFFICIENTS

YAW DAMPING DUE TO ROLL RATE COEFFICIENT

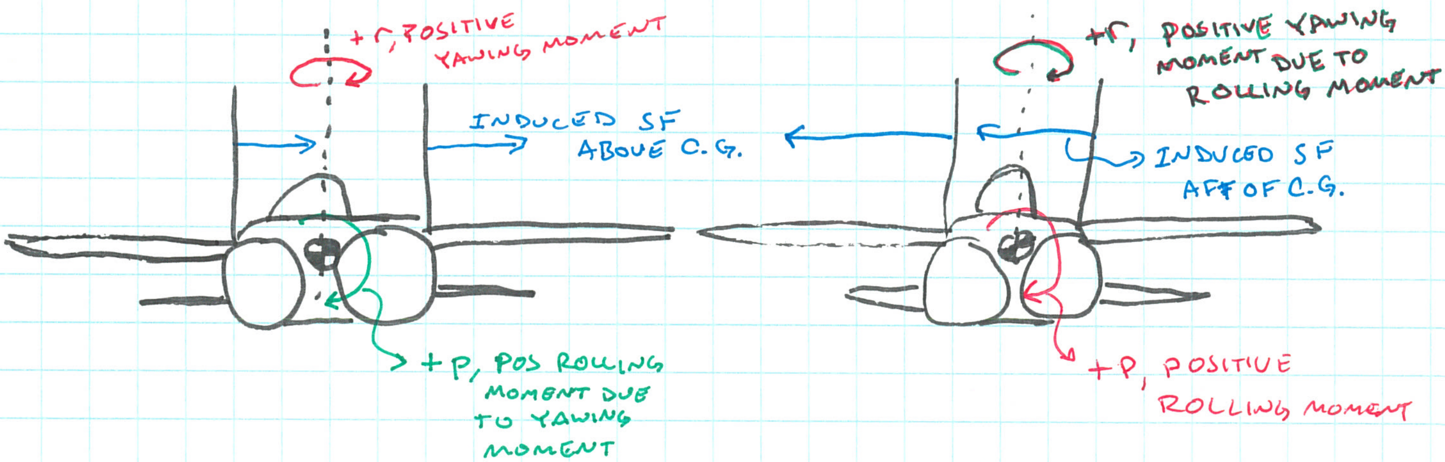
$$C_{n_p} = \frac{\partial C_n}{\partial\left(\frac{pb}{2V}\right)}$$

→ WHEN ROLLING, VELOCITY IS INDUCED ONTO THE VERTICAL STABILIZERS. THIS INDUCES A SIDE FORCE ON THE VERT STABILIZERS BEHIND THE CG, THUS PRODUCING A YAWING MOMENT.

ROLL DAMPING DUE TO YAW RATE COEFFICIENT

$$C_{l_r} = \frac{\partial C_l}{\partial\left(\frac{rb}{2V}\right)}$$

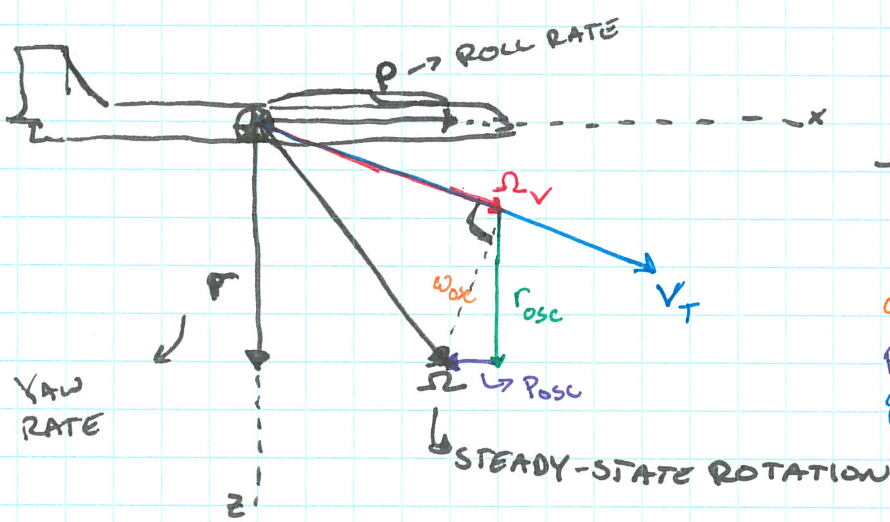
→ WHEN YAWING, VELOCITY IS INDUCED ONTO THE VERTICAL STABILIZERS. THIS INDUCES A SIDE FORCE ABOVE THE CG, THUS GENERATING A ROLLING MOMENT.



TRADITIONAL NON-DIMENSIONAL ROTATION RATE COMPONENTS

ALSO KNOWN AS THE "DIRECT METHOD"
- OTHER METHODS EXIST
  - KALVISTE
  - EXCESS ROLL RATE
  - FORCED OSCILLATION
- SIMPLEST METHOD
- NON-DISCONTINUOUS

TRADITIONAL NON-DIM ROTATIONAL RATES ARE DERIVED BY BREAKING THE TOTAL AIRCRAFT ROTATIONAL RATES INTO COMPONENTS
- ONE "STEADY-STATE" COMPONENT ALIGNED WITH THE VELOCITY VECTOR
- THREE RESIDUAL OR "OSCILLATORY" COMPONENTS ALIGNED WITH BODY X, Y, Z AXES.



$$\Omega = \sqrt{p^2 + q^2 + r^2}$$

$$\Omega_V = p\cos\alpha\cos\beta + q\sin\beta + r\sin\alpha\cos\beta$$

$$\omega_{osc} = \Omega - \Omega_V$$

$$p_{osc} = p - \Omega_V\cos\alpha\cos\beta$$

$$q_{osc} = q - \Omega_V\sin\beta$$

$$r_{osc} = r - \Omega_V\sin\alpha\cos\beta$$

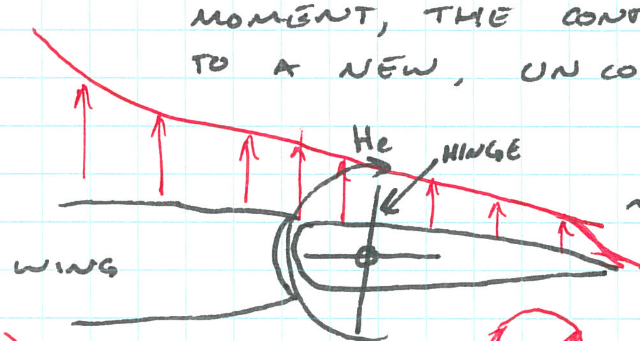
$$\left.\begin{aligned} OBO2V &= \frac{\Omega_V b}{2V_T} \\ PB &= \frac{p_{osc} b}{2V_T} \\ QB &= \frac{q_{osc}\bar{c}}{2V_T} \\ RB &= \frac{r_{osc} b}{2V_T} \end{aligned}\right\} \text{USED IN AERODYNAMIC DYNAMIC DERIVATIVE TERMS}$$

12/06/2018

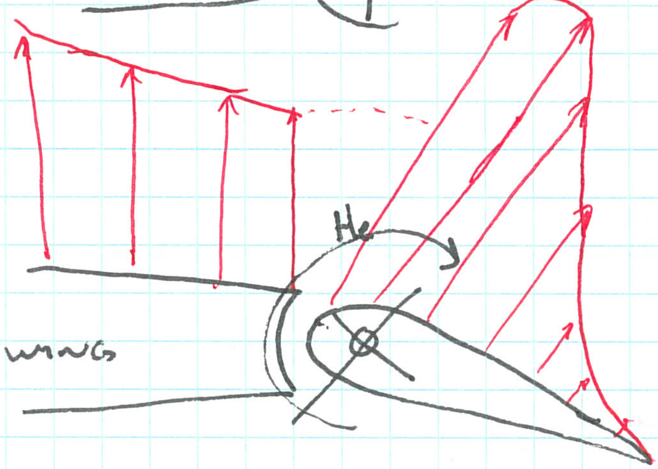
HINGE MOMENTS

IN ORDER TO ROTATE CONTROL SURFACES ABOUT IT'S HINGE, NECESSARY FORCE IS REQUIRED TO OVERCOME THE AERODYNAMIC PRESSURE THAT RESISTS THE MOTION

- IF THE AERO FORCES ON THE SURFACE OVERCOME THE APPLIED ACTUATOR HINGE MOMENT, THE CONTROL SURFACE WILL MOVE TO A NEW, UNCOMMANDED POSITION.



→ EVEN DISTRIBUTION OF FORCE FROM WING TO SURFACE



UNEVEN DISTRIBUTION OF FORCE

HINGE MOMENT H_e ACTS IN THE OPPOSITE DIRECTION ~~OF~~ AERO FORCES TO COUNTER THE PRESSURE

COEFFICIENT FORM

$$C_h = \frac{H}{\frac{1}{2}\rho V^2 S_{REF}\,\bar{c}_{REF}} = \frac{H}{\bar{q}\,S_{REF}\,\bar{c}_{REF}}$$

WHERE S_{REF} & $\bar{c}_{REF}$ ARE SURFACE DEPENDENT.