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Matrix convex sets: Inclusions, completely positive interpolation and minimality

Abstract

This talk is complementary to the plenary talk to be given by Orr Shalit in IWOTA 2016, and is a part of a joint work with Kenneth R. Davidson, Orr Moshe Shalit and Baruch Solel.

A matrix convex set is a stratified set of the form $\mathcal{S} = \cup_{n \geq 1} \mathcal{S}_n$, where each $\mathcal{S}_n$ is comprised of d-tuples of $n \times n$ matrices, and is closed under direct sums and application of unital completely positive maps from M_n to M_k. To each d-tuple A, we associate a matrix convex set $\mathcal{W}(A)$ called the *matrix range* of A.

For two d-tuples $A = (A_1, ..., A_d)$ and $B = (B_1, ..., B_d)$, we show that there exists a UCP map sending A_i to B_i if and only if $\mathcal{W}(B) \subseteq \mathcal{W}(A)$. From this we recover results of Helton, Klep and McCullough in the case of real symmetric matrices and of Li and Poon in the case of Hermitian commuting matrices.

We say that a d-tuple A of operators on a Hilbert space H is *minimal* if there is no subspace $\{0\} \neq H' \subsetneq H$ that reduces A and $\mathcal{W}(A|_{H'}) = \mathcal{W}(A)$. Generalizing work of Helton, Klep and McCullough in the case of real symmetric matrices, we show for two minimal compact d-tuples of operators A and B, that $\mathcal{W}(A) = \mathcal{W}(B)$ if and only if A and B are simultaneously unitarily conjugate. We give an example of a d-tuple of non-compact operators where the situation is more complicated, and provide a reasonable alternative to the above theorem in cases where the C*-algebras generated by each d-tuple contains no compact operator.

Talk time: 07/19/2016 6:00PM— 07/19/2016 6:20PM
Talk location: Crow 206

Special Session: Multivariable operator theory. Organized by H. Woerdeman.