6-22-2018

A New One Equation Turbulence Model Based on k-ε Closure with Elliptic Blending

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MEMS 500-09: Independent Study

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A new one equation turbulence model is developed based on the two equation $k-\epsilon$ model. In order to improve the accuracy of the model in the near wall region, especially for wall bounded flow with small regions of separation, the model is blended with an elliptic relaxation function. The results from the new model are compared to experimental data or DNS data for standard benchmark test cases. The open source CFD software OpenFOAM is used for the flow field calculations. It is shown by computing a number of benchmark cases that the results from the new one-equation turbulence model with elliptic blending improve the accuracy compared to the original one-equation model.

Nomenclature

C_f = skin friction coefficient
Re = Reynolds number
S = strain rate magnitude
$\widetilde{\nu}_t$ = eddy viscosity
ν = kinematic viscosity
κ = von Karman constant
k = turbulent kinetic energy
ϵ = turbulent dissipation
σ = Prandtl number
σ_k = turbulent Prandtl number for kinetic energy

I. Introduction

In recent years, a number of one-equation turbulence models have been proposed for the solution of Reynolds-Averaged Navier-Stokes (RANS) equations. Some of these are Menter's one-equation eddy viscosity model based on k- ε model [1], Wray-Agarwal model [2], Rahman-Agarwal-Siikonen (RAS) model [3], and one-equation model based on two-equation k-kL model [4]. One of the most well-known and widely used one-equation model is the Spalart-Allmaras (SA) model [5]. In the category of two-equation models, most well-known models and widely used models are k-ε model [6], Wilcox k-ω model [7] and SST $k-\omega$ model [8]. To improve the accuracy, SA model and one- and two-equation models have been revised several times over the years as noted in NASA TMR [9].

Menter [1] considered the standard $k-\epsilon$ model and proposed a one-equation eddy-viscosity model employing some assumptions. Because of close connection with the standard $k-\epsilon$ model, his model has some deficiencies compared to the k-ϵ model. As a result, compared to the one-equation SA model or the two-equation SST $k-\omega$ model, the one-equation eddy viscosity model based on $k-\epsilon$ modeldoes not perform that well.

In this paper, the original one $-$ equation model of Menter is modified and combined with an elliptic relaxation. The new model is tested on two incompressible flow cases: subsonic flow past a flat plate in zero pressure gradient and flow in a 2D channel at high Reynolds number. The results from the new one-equation model with elliptic blending show better agreement with DNS data in capturing the velocity profile in the entire boundary layer including the sub-layer, buffer layer and log layer compared to the model without elliptic blending.

II. Description of New One- Equation Model

One-equation eddy viscosity model based on the two-equation $k-\varepsilon$ model is described in this section. The one equation model is derived from the standard $k-\epsilon$ closure using the definition of the eddy viscosity $\widetilde{\nu_t} = C_\mu \frac{k^2}{\varepsilon}$ [1]. The final form of the one-equation model can be written as:

$$\frac{D\widetilde{\nu_t}}{Dt} = c_1 D_1 \widetilde{\nu_t} S - c_2 E_{1e} + \frac{\partial}{\partial x_j}\left(\left(\nu + \frac{\widetilde{\nu_t}}{\sigma}\right)\frac{\partial}{\partial x_j}(\widetilde{\nu_t})\right) \tag{1}$$

where $$\nu_t = D_2 \widetilde{\nu_t} \tag{2}$$

In Eq. (1) and Eq. (2), two damping functions D_1 and D_2 are given by:

$$D_1 = \frac{\nu_t + \nu}{\widetilde{\nu_t} + \nu} \tag{3}$$

$$D_2 = 1 - e^{-\left(\frac{\widetilde{\nu_t}}{\kappa \nu A^+}\right)^2} \tag{4}$$

The strain rate magnitude is defined as:

$$S = \sqrt{U_{i,j}\left(U_{i,j} + U_{j,i}\right)} \tag{5}$$

The term involving the inverse of von Karman length-scale is given as:

$$E_{k-\epsilon} = \widetilde{\nu_t}^2 \left(\frac{1}{L_{VK}}\right)^2 = \widetilde{\nu_t}^2 \left(\frac{\frac{\partial s}{\partial x_j}\frac{\partial s}{\partial x_j}}{s^2}\right) \tag{6}$$

In Eq. (1), $$E_{1e} = c_3 E_{BB} tanh\left(\frac{E_{k-\epsilon}}{c_3 E_{BB}}\right) \tag{7}$$

where the Baldwin-Barth destruction term is defined as:

$$E_{BB} = \frac{\partial \widetilde{\nu_t}}{\partial x_j} \tag{8}$$

The coefficients used in one-equation eddy viscosity model in Eq. (1) are listed in Table 1.

Table 1 Constants used in the one-equation model given by Eq. (1)

c_1	σ	c_2	κ	A^+	c_3
0.144	1	1.86	0.41	13	7

III. Development of One-Equation Model with Elliptic Blending

Based on the work of Han et al. [10], it can be concluded that an elliptic relaxation applied to a turbulence model can capture the anisotropic low Reynolds number near wall effects more accurately. Following the form of the elliptic blending in WA2018EB model [10], elliptic blending is added in the present model considered in this paper. The equation for elliptic blending can be expressed as:

$$-L_R^2 \nabla^2 P_R + P_R = \widetilde{\nu_t} S \tag{9}$$

where P_R is a production term. The coupled one-equation model with elliptic blending can be expressed as:

$$\frac{D\widetilde{\nu_t}}{Dt} = c_1 D_1 \widetilde{\nu_t} S - c_2 E_{1e} + \frac{\partial}{\partial x_j}\left(\left(\nu + \frac{\widetilde{\nu_t}}{\sigma}\right)\frac{\partial}{\partial x_j}(\widetilde{\nu_t})\right) + P_R - \widetilde{\nu_t} S \tag{10}$$

$$\nu_t = D_2 \widetilde{\nu_t} \tag{11}$$

$$-L_R^2 \nabla^2 P_R + P_R = \widetilde{\nu_t} S \tag{12}$$

$$L_R^2 = \frac{max(C_{3k\omega}\widetilde{\nu_t}, C_l \nu)}{S + \frac{C_l \nu}{{L_{ref}}^2}} \tag{13}$$

$$C_l = 4.0 + \sqrt{\chi} \tag{14}$$

In Eq. (13) for L_R, C_l is used to correct for the free stream behavior of L_R and $L_{ref} = 1$ is the reference length scale.

The calibrated coefficients used in the one-equation model with elliptic blending are listed in Table 2.

Table 2 Constants used in the one-equation model with elliptic blending

c_1	σ	c_2	κ	A^+	C_3	C_{3kw}
0.308	1	3.097	0.41	8.36	7	0.539

IV. Tests Cases and Results

This section shows the comparison of results computed by one-equation model based on k-ε closure (designated as KE model) and the one-equation model with elliptic blending（designated as KEEB model). Two benchmark cases are tested to show the performance and accuracy of these two models KE and KEEB). These cases are the flow over a flat plate and flow in a 2D channel at different Reynolds numbers. The results are compared with the DNS data or experimental results for each case to compare the KE and KEEB models. All the results are computed by using the open source CFD software OpenFOAM.

A. Zero Pressure Gradient Boundary-Layer Flow past a Flat Plate

Flow past a flat plate is a basic case used to verify the accuracy of any turbulence model. Figure 1 shows the computational setup and boundary conditions from NASA TMR [9]. Figure 2 shows the computation result for wall skin-friction coefficient C_f vs. Reynolds number Re based on length in x direction (Re_x)and their comparison with experimental data. Re_x is defined as:

$$Re_x = \frac{\rho_\infty U_\infty x}{\mu_\infty} \tag{15}$$

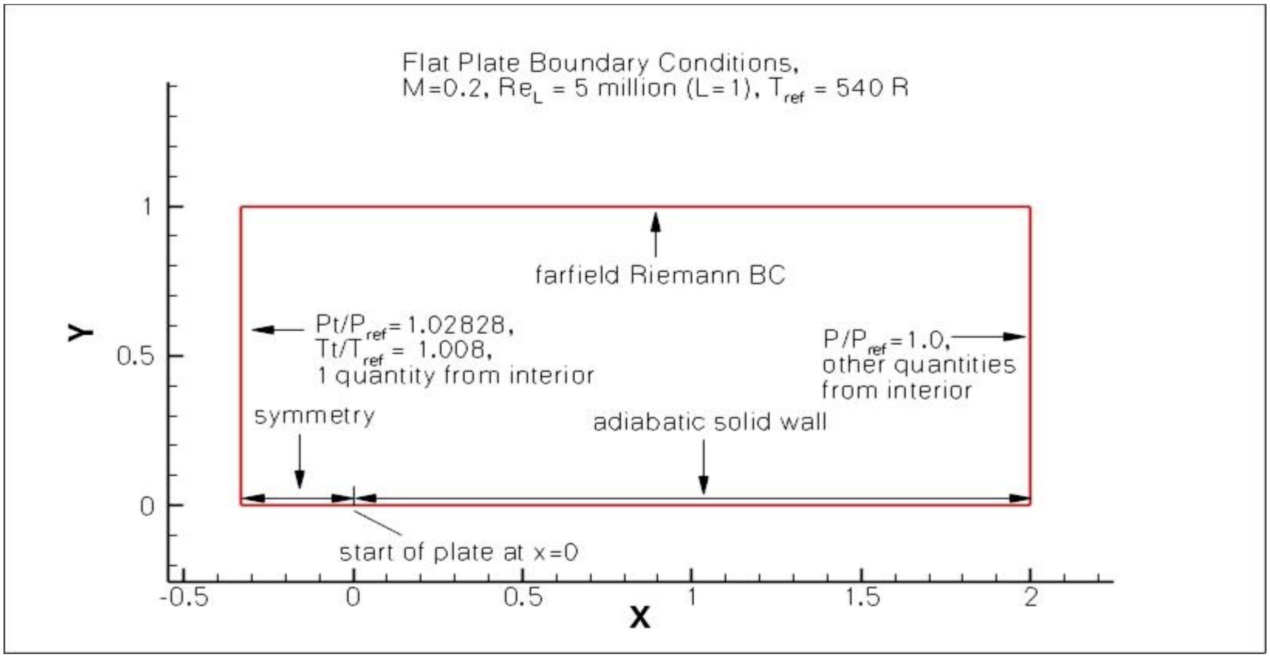

Fig.1 Flat plate geometry and boundary conditions [9].

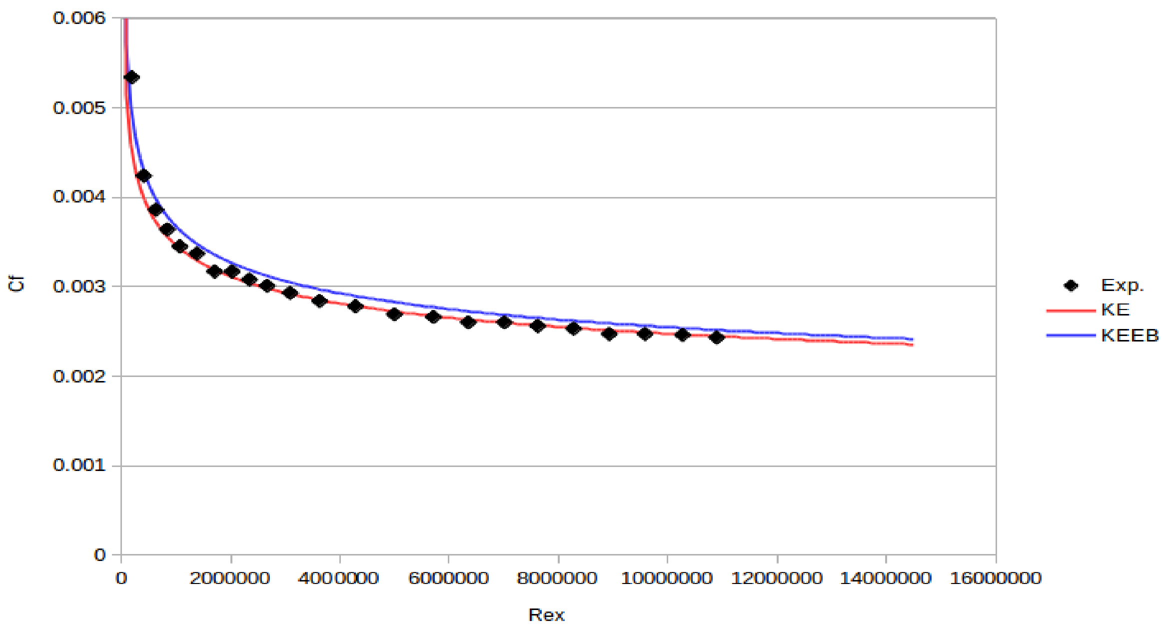

Fig.2 Comparison of computed C_f on the flat plate with the experimental data.

Figure2 shows that the one-equation KE model has slightly better accuracy compared to the KEEB model. The KE model completely matches the experimental data. However, the errors between the results computed by KEEB model and experimental data are quite acceptable when considering the significant improvement of the results of KEEB model in the 2D channel case as shown in the next section.

B. Flow in a 2D Channel at Different Reynolds Numbers

Fully developed turbulent flow in a channel is another basic test case frequently used to assess the accuracy of various turbulence models. Figures 3 -12 show results in a simple channel flow at several different friction Reynolds number ranging from Reτ = 182 to 5200, and are compared with DNS data by Lee and Moser [11].

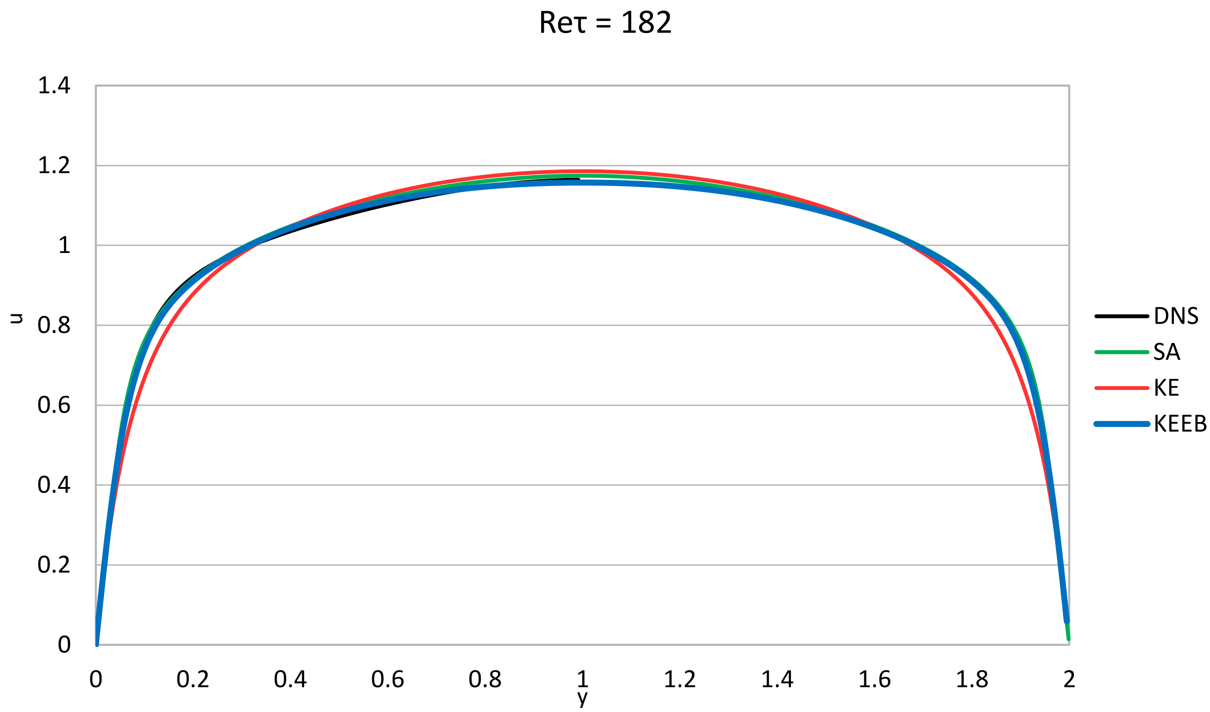

Fig.3 Comparison of velocity profile in turbulent channel flow at Reτ = 182.

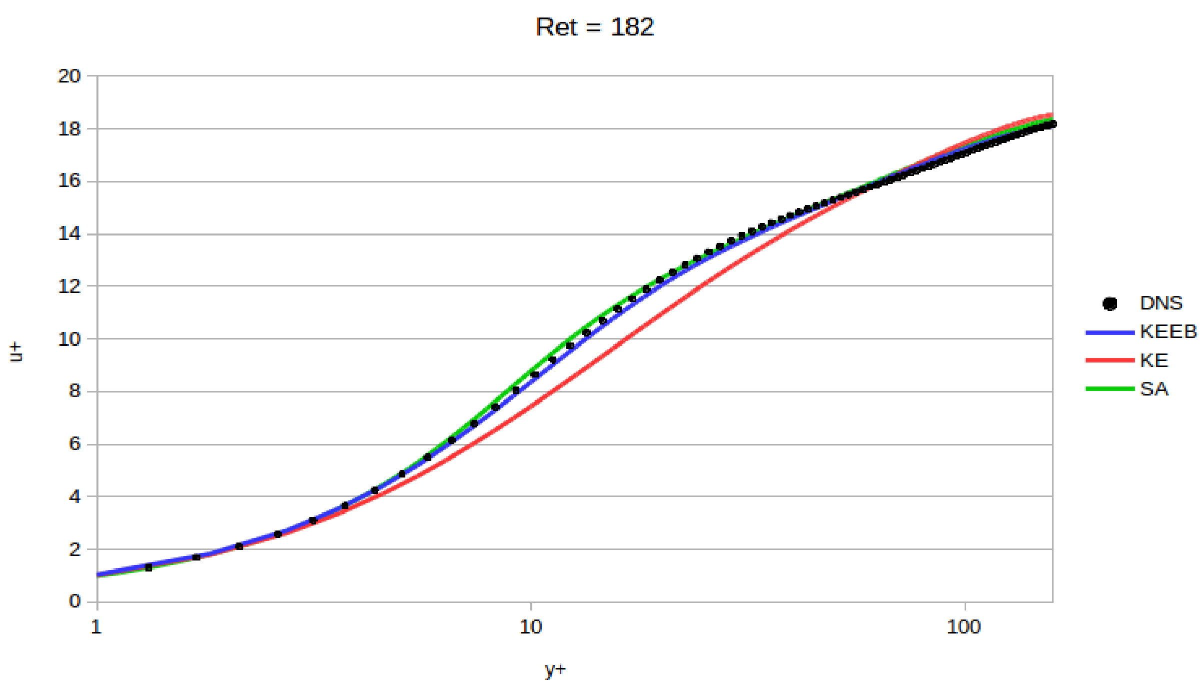

Fig.4 Comparison of velocity profile in log layer for turbulent flow in a channel at Reτ = 182.

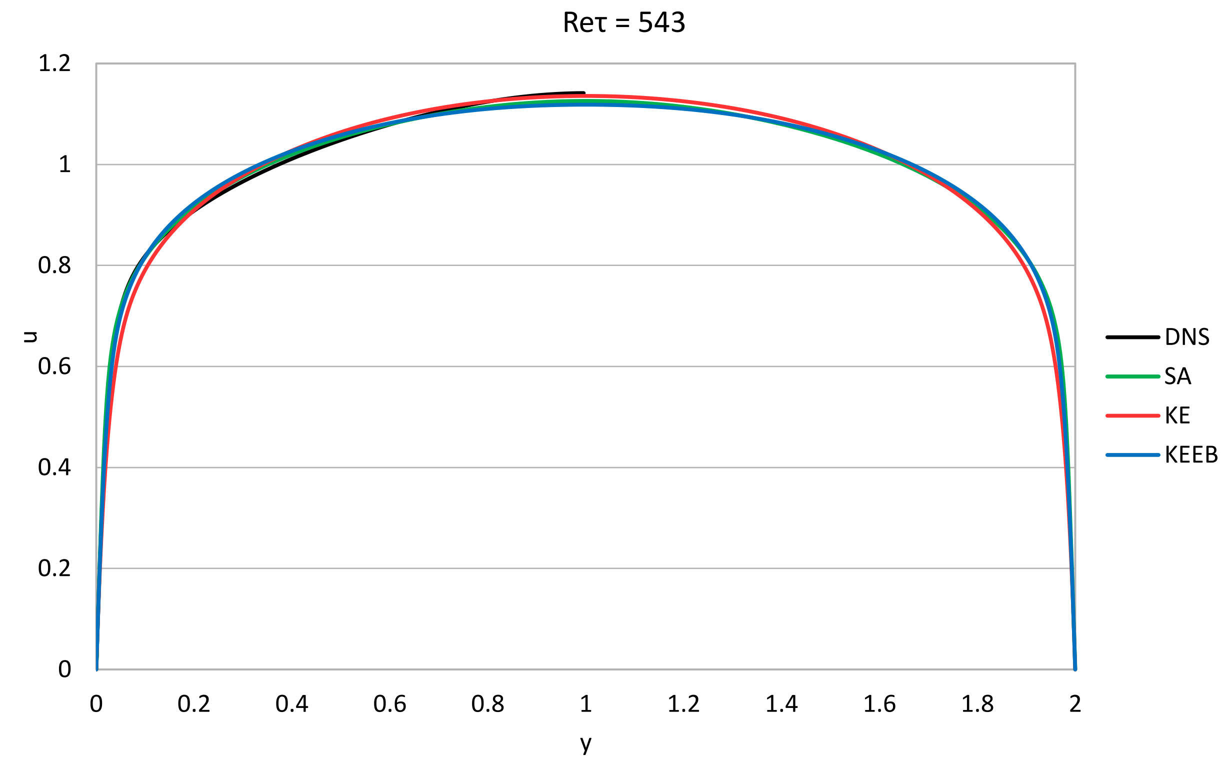

Fig.5 Comparison of velocity profile in turbulent channel flow at Reτ = 543.

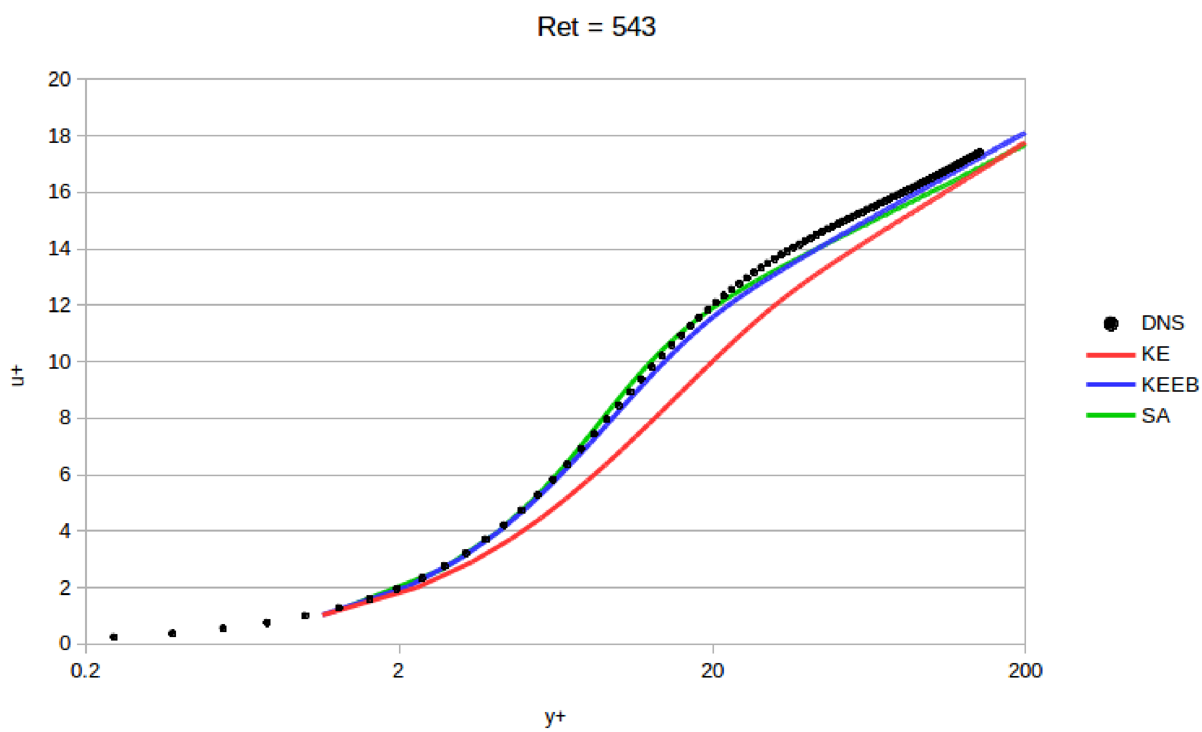

Fig.6 Comparison of velocity profile in log layer in turbulent channel at Reτ = 543.

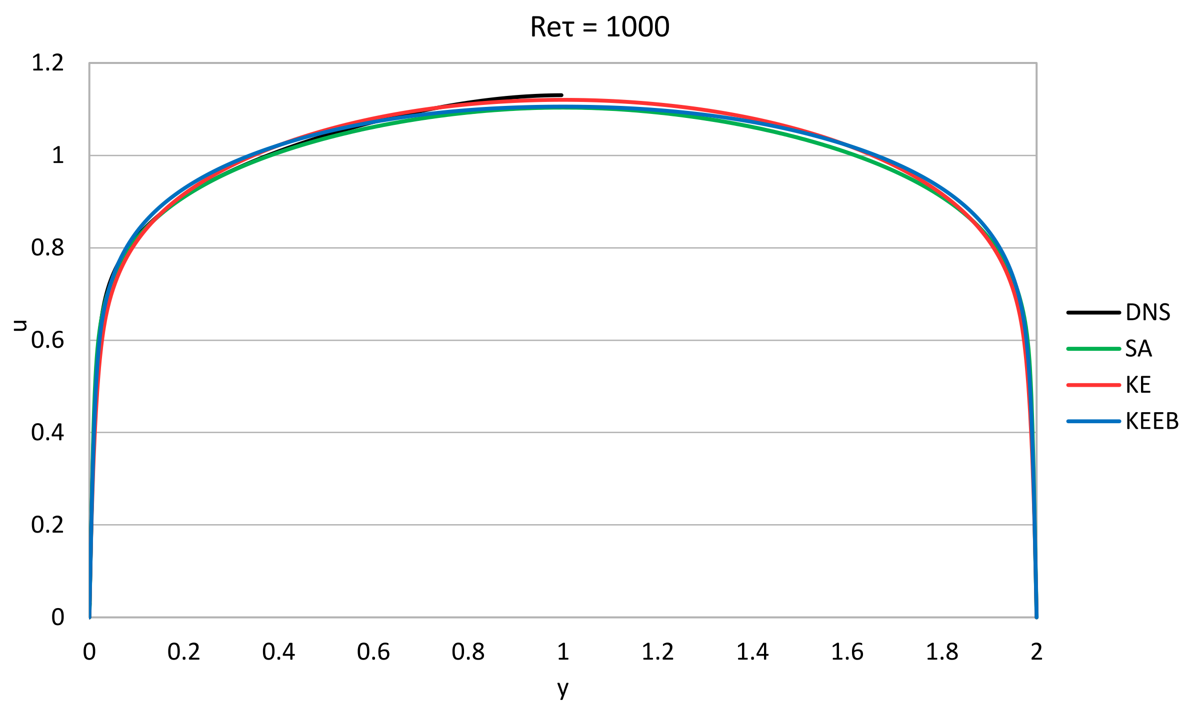

Fig.7 Comparison of velocity profile in turbulent channel flow at Reτ = 1000.

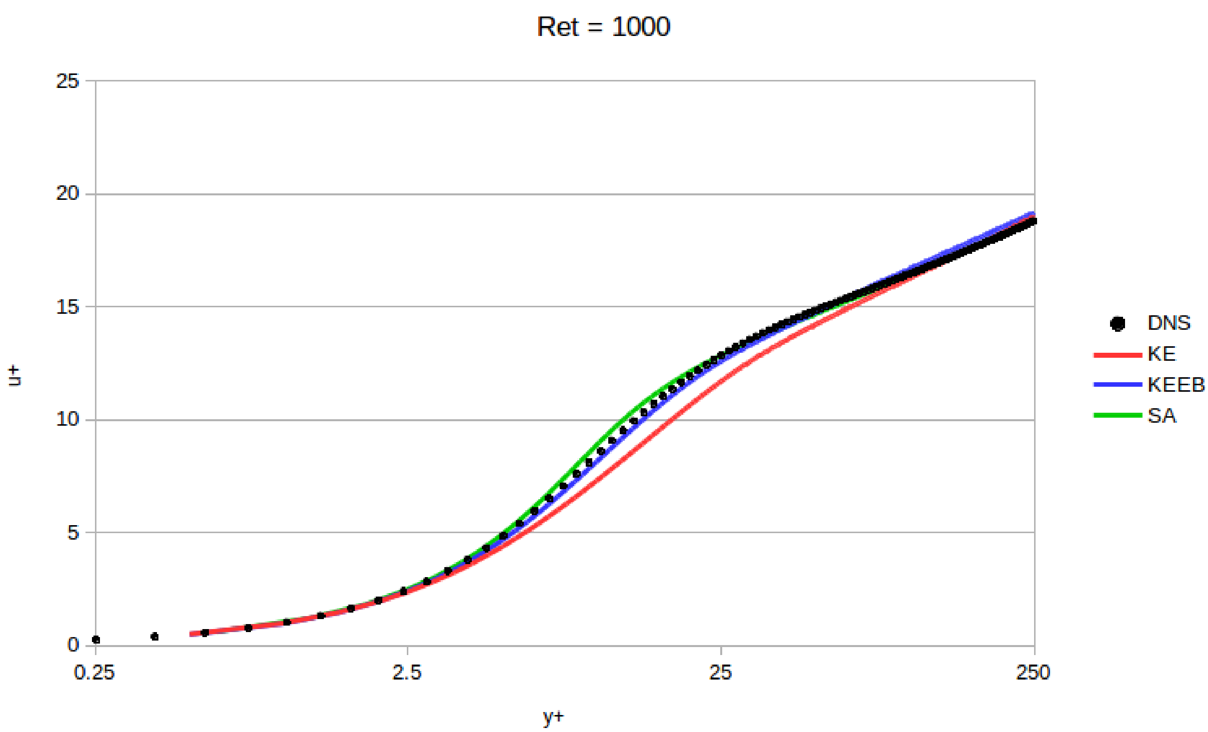

Fig.8 Comparison of velocity profile in turbulent channel at Reτ = 1000.

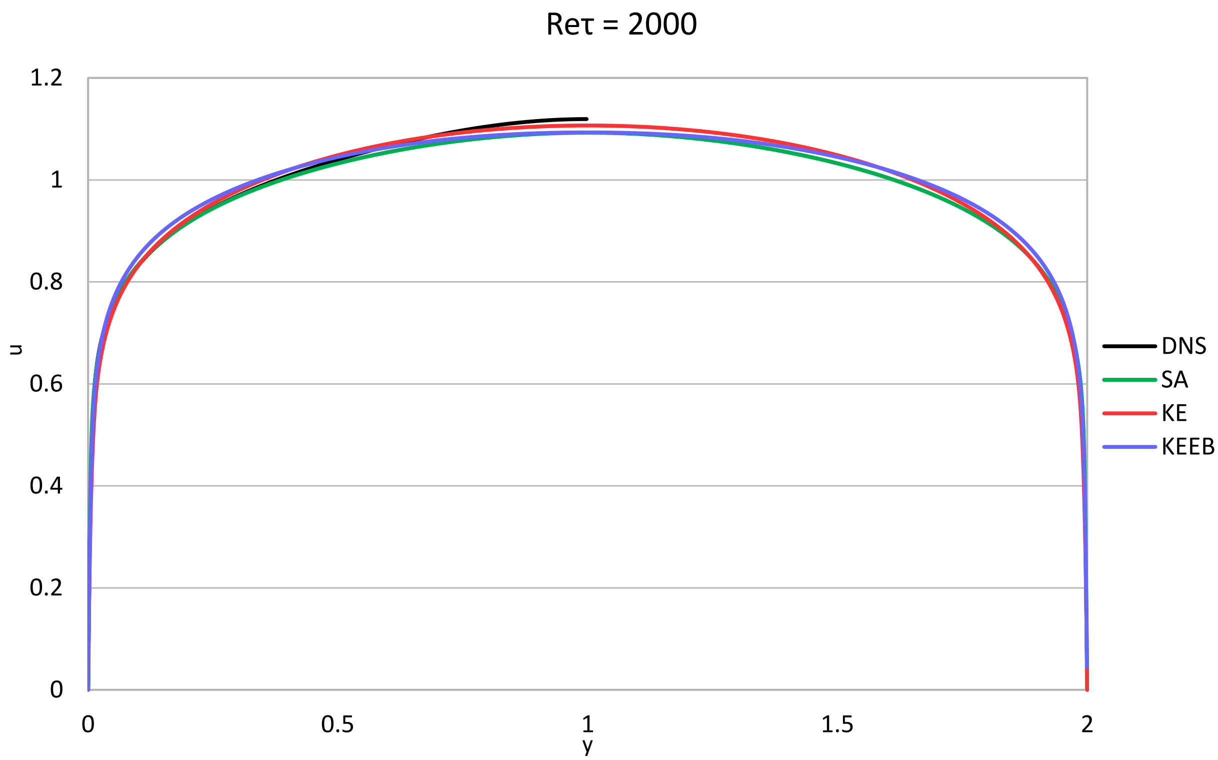

Fig.9 Comparison of velocity profile in turbulent channel flow at Reτ = 2000.

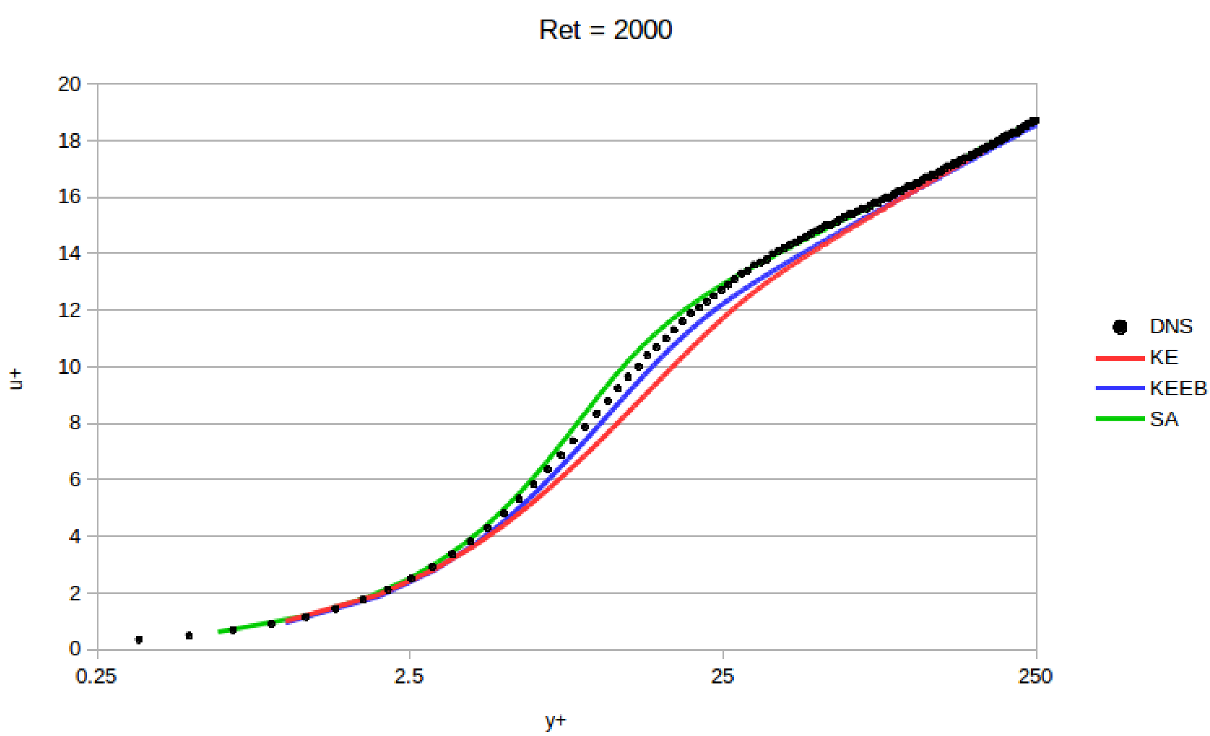

Fig.10 Comparison of velocity profile in log layer in turbulent channel at Reτ = 2000.

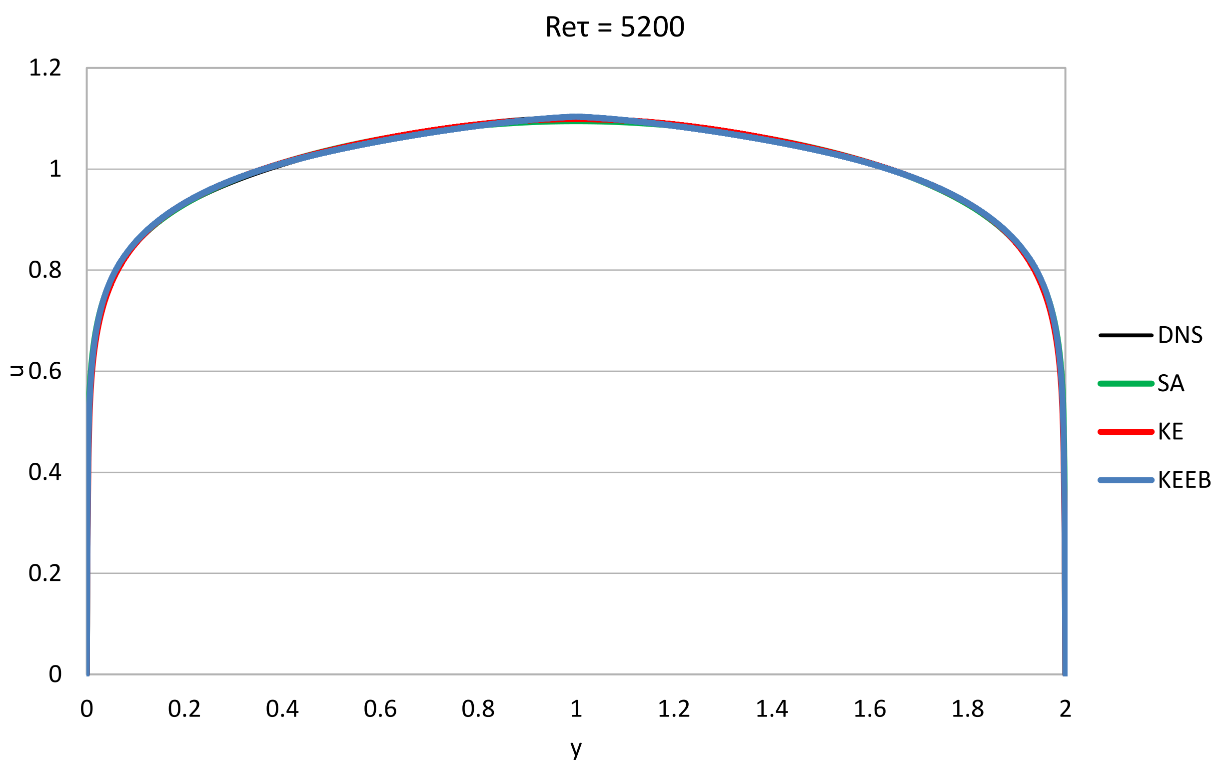

Fig.11 Comparison of velocity profile in turbulent channel flow at Reτ = 5200.

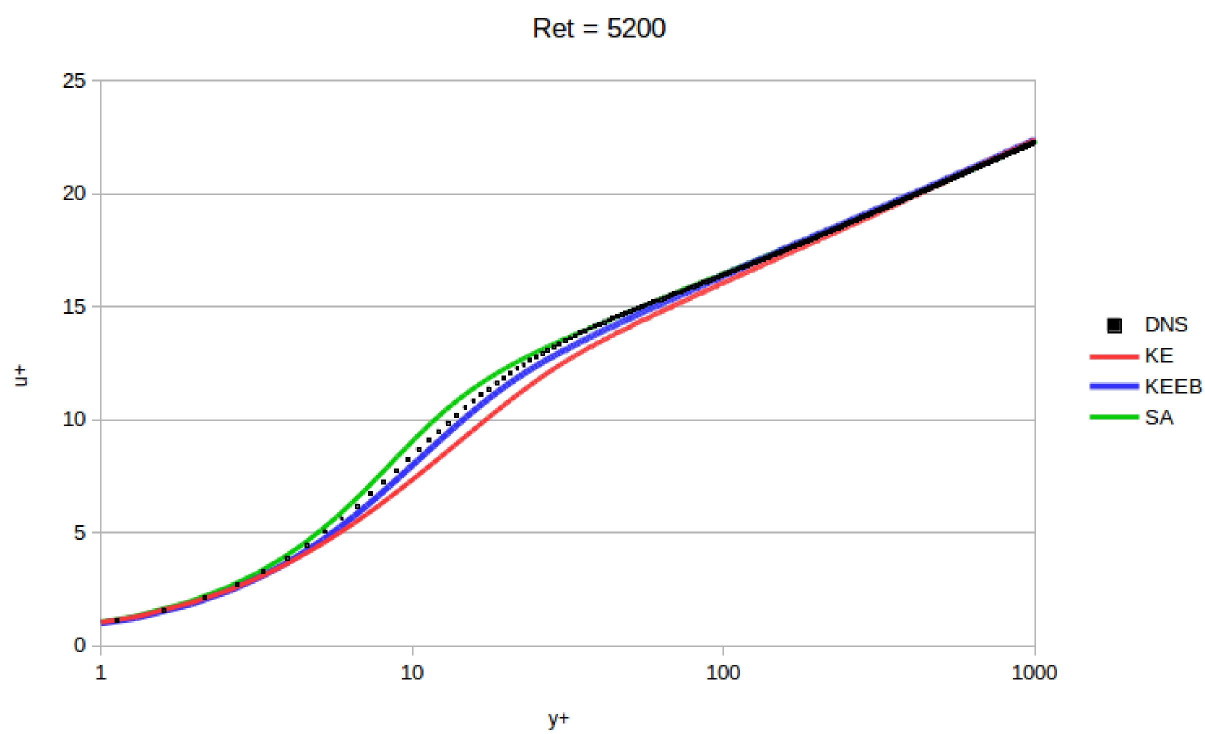

Fig.12 Comparison of velocity profile in log layer in turbulent channel flow at Reτ = 5200.

From Figures 3-12, it can be seen that both KE model and KEEB model can predict the fully developed turbulent channel flow velocity profile quite well. However, when Re_τ = 182, the results from KEEB model are in better agreement with DNS data compared to the KE model.

For the velocity profiles in the sublayer region, both models agree with the DNS data very well. However, in the buffer layer and log layer region, there is a large mismatch between the results from KE model computations and the DNS data. KEEB model shows significant improvement compared to the KE model in this region close to the channel wall, which demonstrates that the elliptic blending is beneficial in improving the performance of a turbulence model in buffer layer and log layer region. Overall, it can be concluded that KEEB model improves the results in computing wall bounded turbulent flows. .

V. Conclusions

It is shown that the proposed elliptic blending version of one-equation model based on $k - \epsilon$ closure (KEEB model) has better accuracy compared to the one-equation model based on $k - \epsilon$ closure (KE model). Both the models can predict the turbulent flow past a flat plate very well. However, for fully developed turbulent flow in a channel, KE model is not able to compute the buffer layer and log layer near the channel wall accurately. In contrast, the KEEB model shows significant improvement in calculation of the log layer for flow in the channel. In addition, KEEB model also shows its advantages when calculating the channel flow at low friction Reynolds number; the results from KEEB model are in better agreement with the DNS data compared to the KE model computations. Based on the results presented in this paper, it can be concluded that the KEEB model provides an improved approach over the KE model by incorporation of elliptic blending in KE model.

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