6-1-2018

An advanced course on finite element analysis, with application to the stress distribution in teeth

Eric Yoon
Washington University in St. Louis

Guy M. Genin
Washington University in St. Louis

An advanced course on finite element analysis, with application to the stress distribution in teeth

DongHwan Yoon
Department of Mechanical Engineering and Materials Science
Washington University, St. Louis, MO 63130

yoondh91@gmail.com

May 08, 2018

Abstract

The overall goal of my work is to gain insight into how tooth shape relates to its function. As a step towards this, I undertook an independent study project to further improve my skills on finite element analysis (FEA) this semester, and to combine this into my Master's thesis project work. Continuing from the previous independent study course, the tooth model was improved to eliminate singularities and a contact surface model was included to simulate contact stress problems. I believe that these series of problems will be useful to my research. This report contains an overview of some literature that I studied, and a summary of several finite element output plots that I found to be particularly instructive.

1. Introduction

The context in which this study was undertaken is the attachment of tendon to bone, which is a major challenge from the surgical, mechanical engineering, and tissue engineering perspectives [1-3]. For surgery, up to 94% of rotator cuff reattachments fail [4]. From the mechanical engineering perspective, the mechanisms of resilience at the insertion site are an area of ongoing research [5-11], and must overcome the free edge singularity problem [12-29]. From the tissue engineering perspective, the natural tendon to bone attachment does not grow back [4], and it is important to find ways to stabilize tissue without this attachment [20-23] and to guide regrowth of the transitional tissue [24-28]. Stabilization of tissue during healing is a topic that I am focusing on and have contributed to a conference paper on [29]. The question of resilience of tissues motivated my ongoing study of how carnivores capture and tear through flesh.

As a step towards this, I studied some basic solid mechanics, including some specialized problems from the textbook by Budynas [30], and studied an introduction to finite element analysis [31].

In this report, I present a few finite element results that overcome the issues from the previous independent study and demonstrate the results that could be useful for my Masters research.

2. Methods

The numerical portion of the study was conducted using the finite element method, and using commercial software (Abaqus/CAE) for the analysis. The steps involved in a finite element analysis are coming up with an idealized geometry, assigning idealized material properties, choosing boundary conditions, making a mesh, implementing the boundary conditions, solving the equations (equilibrium, strain displacement, and constitutive equations) by a matrix-based energy minimization method, and then validating results by mesh refinement [31].

Numerical simulations were performed to assess how teeth might be optimized to switch from cutting teeth that induce high principal stresses on an isotropic continuum to trapping teeth that induce compression of an isotropic continuum against a rigid simulated gumline. The first step in this study was a review of basic solid mechanics solutions for curved beams [30]. Thereafter, I evaluated how teeth both stress and constrain soft tissues. As mentioned above, the goal was to determine what shapes lead to high stresses at tooth tip, and what shapes lead to constriction of the soft tissue against a rigid gumline.

Teeth were modeled parametrically to shift from a nearly pyramidal canine to a hooked python-like tooth. Analyses were performed under plane strain conditions. Each tooth was treated as a pair of splines that intersected at a curved top. The teeth were each of a base *w* and a height of 2*w*. The curvature of the tooth was determined by moving the tip (sharp end) to the right in the Abaqus/CAE sketch interface, thereby increasing the distance *w'* (Figure 2). The tooth has a dimension of 2 x 4 x 1 (length, height, width) in arbitrary units (in this study mm). The top region of the tooth from the left end to the tip is referred to *w'* and the base of the tooth is referred to *w*. The parameter that determines the degree of curvature can be expressed as *w'*/*w* where in this study ranges from 0.5 to 2.25

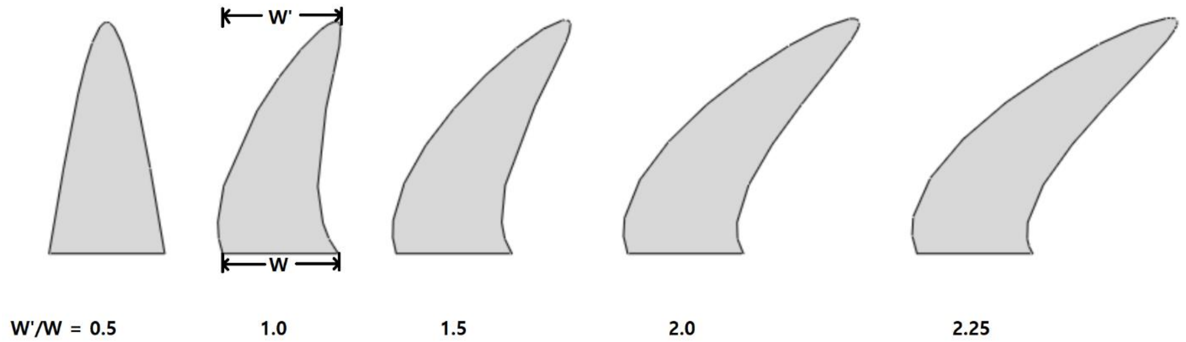

Figure 1. General view of tooth model with annotations of w and w'

A tissue was placed over the tooth (Figure 3). The right and left boundaries of the tissues, placed a distance 4*w* away from the middle of the tooth. The height of the tissue was 4*w* away from the base of the tooth. A gumline was placed at the bottom of the tooth and was assigned the same material properties as the tooth, described below. The tooth/gumline and the tissue were not allowed to interpenetrate.

The problems studied here were idealized teeth on an elastic foundation contacting with a softer material, which would ideally resemble a tendon. As a first approximation, the teeth and softer tissue were modeled as linear hyperelastic and as isotropic. The hyperelasticity was irrelevant for the tooth due to its high relative stiffness and strength. Also, the tooth was modeled as a solid rather than multilayered structure due to the stresses that were very small compared to its failure strength: the tooth was effectively rigid compared to the soft tissue. The Young's modulus and Poisson's ratio were set to 14 GPa and 0.3 for the tooth, respectively. These values correspond to human cortical bone which I have used as a reference material that would allow me to gain insight [32-36].

The models were two dimensional, and plane strain, linear interpolation quadrilateral and triangular elements were used.

Abaqus was used to refine the mesh until the strain energy and peak principal stress in a model did not change more than a few percent with additional refinement. The corresponding plots of the maximum principal stress, strain tensor energy and the strain energy density were studied.

The mesh size can be controlled through the graphical user interface in Abaqus/CAE. However for the purpose of this study the finer upper is the part which simulates the soft tendon, is the part of more interest, therefore used a finer mesh (Figure 3). The bottom part which resembles the tooth has a larger mesh. The graphic interface in Abaqus/CAE allows the user to change the mesh size and element type (quadratic or linear). For the purpose of this study, quadratic elements were used.

Abaqus was used to refine the mesh until the strain energy and peak stress did not change more than a few percent with additional refinement. The corresponding plots of the maximum principal stress, strain tensor energy and the strain energy density will be attached.

3. Results and Discussion

Results showed that stresses in the tendon were highly localized to the tip of the tooth, with stress concentrations well above 10 at the contact point (Figure 4). This is consistent with the sharp nature of the rounded tip of a tooth, and is expected for an appendage that must penetrate tissue. In subsequent analyses, the objective was to determine the degree to which changes to the tooth affected the degree of this stress concentration. The deformed shape of the tendon and tooth model implies that the model created acts as expected. (Figure 5).

The maximum tensile principal stress follows what would be expected in a cantilever beam with the boundary conditions used (Figure 2). For the curved tooth, the tensile stresses were in general higher on the loaded face, and the principal stress was zero on the back face, consistent with what is expected for flexure of a beam [30]. Two artifacts appear. The first is a stress concentration at the point that was fixed, in the lower left hand corner. This arose because of the choice made to have rollers on the bottom boundary and one fixed point. However, in other simulations where the bottom boundary was "encastre" [31], meaning that the displacement was fixed to zero, a stress concentration known as a Williams free-edge singularity appeared at that corner [44]. The stress concentrations or stress singularities can be suppressed by choosing different boundary conditions, such as a foundation that is elastic in shear or a cohesive zone model, which is used in fracture studies [31,46]. Although the understanding of these mechanisms falls under multi-scale modeling that is beyond the scope of what is needed for this study, phenomenological models can be used to account for how microstructure relates to continuum behavior [45-46]. The second is an hourglass effect [31]. Here, the oscillatory nature of the free edge singularity shows up as a series of errors in the estimation of displacements, which makes neighboring quadrilateral elements look like hourglasses [31]. The hourglass effect can be suppressed by choosing elements with "hourglass control" or by choosing triangular elements [31].

The results up to this point showed that increasing the tooth length and curving the tooth more towards a python shape caused an increase in the contact area and improved the normal force. The most important aspects for an optimal shape were to have a firm grip on the tendon, while having low stress values. In order to relieve edge effects, the tooth model was set to penetrate the tendon model until half its total height. As expected the highest region of stress was found to be near the tip area.

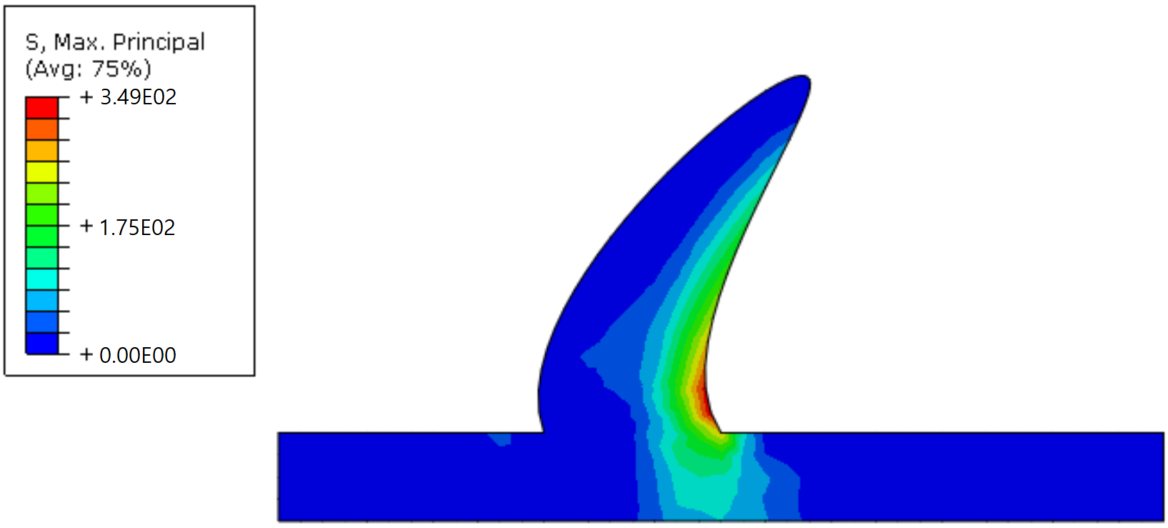

Figure 2. Maximum principal stress contour on tooth model of w'/w = 1.5

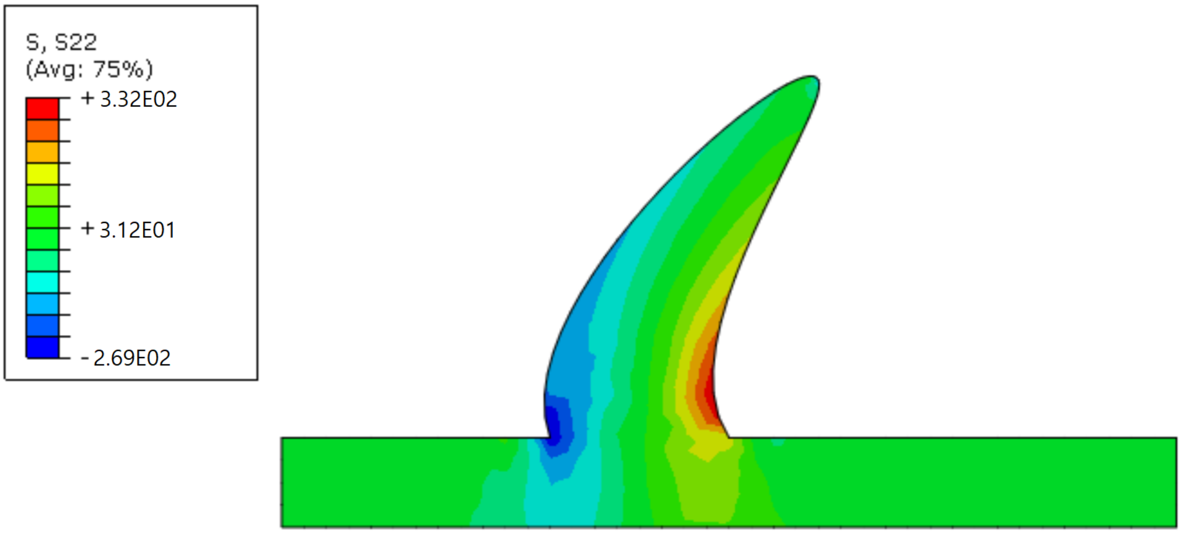

Figure 3. Normal strain in the vertical direction of tooth model of w'/w = 1.5

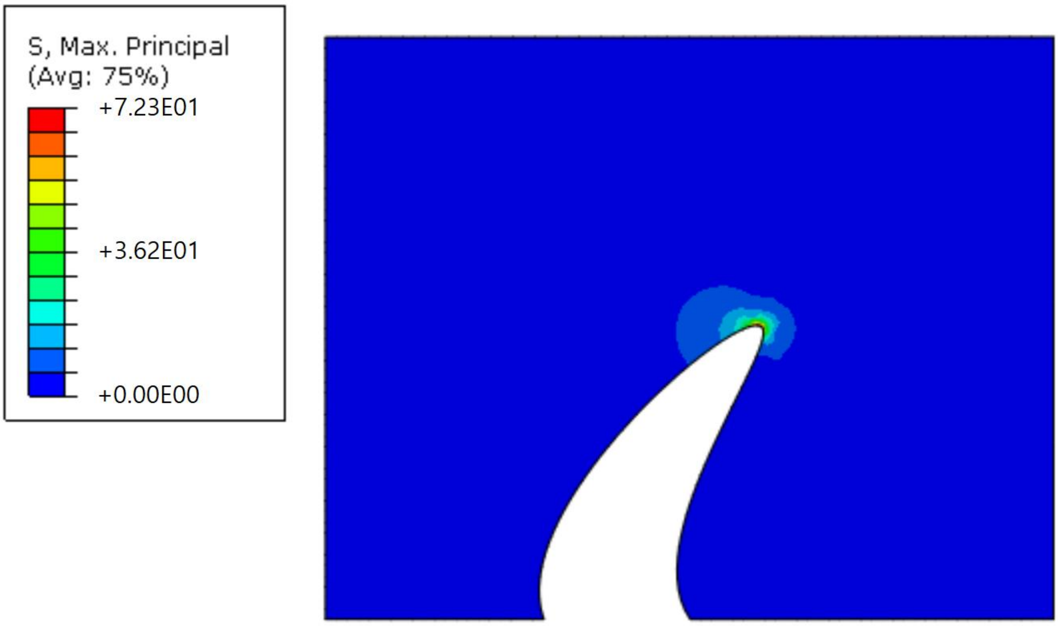

Figure 4. Stress contour of maximum normalized principal stress of w'/w = 1.5

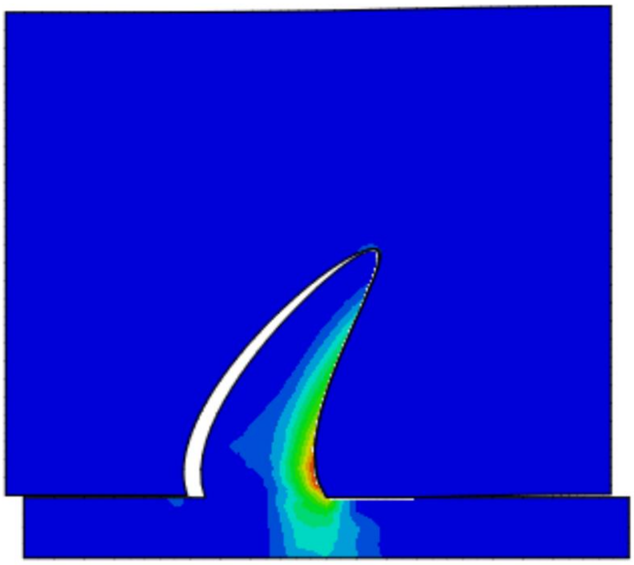

Figure 5. Deformed tendon and tooth model with scale factor of 30

4. Conclusions

I am confident that I have become proficient in using Abaqus in order to create and analyze multiple models in contact. This skill sets that I have acquired during the semester shall contribute to completing a Master's thesis project which is the learning objective of the study.

5. Acknowledgments

I thank my mentors Guy Genin, Victor Birman, and Stavros Thomopoulos, and also my labmate Steve Linderman, for their guidance on this study.

References

[1] Lu, H.H. and Thomopoulos, S., 2013. Functional attachment of soft tissues to bone: development, healing, and tissue engineering. Annual review of biomedical engineering, 15, pp.201-226.

[2] Birman, V., S. Thomopoulos, and G. M. Genin. "Challenges in attaching dissimilar materials." In Structural Interfaces and Attachments in Biology, S. Thomopoulos, V. Birman, and G. M. Genin, eds. New York: Springer, 2012.

[3] Genin, G. M. and Y. Liu. "Models for the mechanics of joining dissimilar materials." In Structural Interfaces and Attachments in Biology, S. Thomopoulos, V. Birman, and G. M. Genin, eds. New York: Springer, 2012.

[4] Galatz, L.M., Ball, C.M., Teefey, S.A., Middleton, W.D. and Yamaguchi, K., 2004. The outcome and repair integrity of completely arthroscopically repaired large and massive rotator cuff tears. J Bone Joint Surg Am, 86(2), pp.219-224.

[5] Birman, V., S. Thomopoulos, G.M. Genin. Multifunctional and Multiscale Natural Optimization of the Tendon-to-Bone Insertion Site: Composite Mechanics Lessons from Biology. Paper IMECE2015-50625. ASME International Mechanical Engineering Congress and Exposition (IMECE 2015). November 13-19, 2015, Houston, TX.

[6] Liu, Y., Birman, V., Chen, C., Thomopoulos, S. and Genin, G.M., 2011. Mechanisms of bimaterial attachment at the interface of tendon to bone. ASME Journal of Engineering Materials and Technology, 133(1), p.011006.

[7] Birman, V., Yanxin Liu, Stavros Thomopoulos, Guy M. Genin. Multiscale Optimization of Joints of Dissimilar Materials in Nature and Lessons for Engineering Applications. In Advanced Structured Materials. New York: Springer, 2012.

[8] Liu, Y.X., Thomopoulos, S., Birman, V., Li, J.S. and Genin, G.M., 2012. Bi-material attachment through a compliant interfacial system at the tendon-to-bone insertion site. Mechanics of Materials, 44, pp.83-92

[9] Liu, Y., A. G. Schwartz, V. Birman, S. Thomopoulos, and G. M. Genin. Adaptation of Developing Tendon-to-Bone Insertion Site to Optimize Stress Environment During Development. In Composite Materials: The Great Advance, pp. 7727-7735. Byte Press Publishers, 2013.

[10] Victor Birman, Guy M. Genin, and Stavros Thomopoulos. Multiscale Enthesis Mechanics. In Proceedings of the 20th International Conference on Composite Materials, Copenhagen, July 19-24, 2015.

[11] Deymier-Black, A.C., Pasteris, J.D., Genin, G.M. and Thomopoulos, S., 2015. Allometry of the tendon enthesis: mechanisms of load transfer between tendon and bone. ASME Journal of Biomechanical Engineering, 137(11), p.111005.

[12] Saadat, F., Deymier, A.C., Birman, V., Thomopoulos, S. and Genin, G.M., 2016. The concentration of stress at the rotator cuff tendon-to-bone attachment site is conserved across species. Journal of the Mechanical Behavior of Biomedical Materials, 62, pp.24-32.

[13] Hu, Y., Birman, V., Deymier-Black, A., Schwartz, A.G., Thomopoulos, S. and Genin, G.M., 2015. Stochastic interdigitation as a toughening mechanism at the interface between tendon and bone. Biophysical Journal, 108(2), pp.431-437.

[14] Liu, Y., Thomopoulos, S., Chen, C., Birman, V., Buehler, M.J. and Genin, G.M., 2014. Modelling the mechanics of partially mineralized collagen fibrils, fibres and tissue. Journal of The Royal Society Interface, 11(92), p.20130835.

[15] Thomopoulos, S., Marquez, J.P., Weinberger, B., Birman, V. and Genin, G.M., 2006. Collagen fiber orientation at the tendon to bone insertion and its influence on stress concentrations. Journal of Biomechanics, 39(10), pp.1842-1851.

[16] Genin, G.M., Kent, A., Birman, V., Wopenka, B., Pasteris, J.D., Marquez, P.J. and Thomopoulos, S., 2009. Functional grading of mineral and collagen in the attachment of tendon to bone. Biophysical Journal, 97(4), pp.976-985.

[17] Birman, V., Stavros Thomopoulos, Jenny Y. Hu, Guy M. Genin. Interdigitation of Materials and Its Implications for Engineering and Biological Attachments. Paper IMECE2013-62618. 2013 ASME International Mechanical Engineering Conference and Exhibition.

[18] Liu, Y., Victor Birman, Changqing Chen, Stavros Thomopoulos, Guy M. Genin. Tailoring the gross morphology of the tendon-to-bone insertion for the reduction of stress concentrations. Paper SBC2011-53636. Proceedings of the ASME 2011 Summer Bioengineering Conference (SBC2011).

[19] Liu, Y., Victor Birman, Changqing Chen, Stavros Thomopoulos, Guy M. Genin. Elastic stress singularities: Implications for the attachment of tendon to bone. Paper SBC2011-53724. Proceedings of the ASME 2011 Summer Bioengineering Conference (SBC2011).

[20] Linderman, S.W., Kormpakis, I., Gelberman, R.H., Birman, V., Wegst, U.G.K., Genin, G.M. and Thomopoulos, S., 2015. Shear lag sutures: Improved suture repair through the use of adhesives. Acta Biomaterialia, 23, pp.229-239.

[21] Smith, L.J., Deymier, A.C., Boyle, J.J., Li, Z., Linderman, S.W., Pasteris, J.D., Xia, Y., Genin, G.M. and Thomopoulos, S., 2016. Tunability of collagen matrix mechanical properties via multiple modes of mineralization. Interface Focus, 6(1), p.20150070.

[22] Smith, L., Xia, Y., Galatz, L.M., Genin, G.M. and Thomopoulos, S., 2012. Tissue-engineering strategies for the tendon/ligament-to-bone insertion. Connective Tissue Research, 53(2), pp.95-105.

[23] Lipner, J., Liu, W., Liu, Y., Boyle, J., Genin, G.M., Xia, Y. and Thomopoulos, S., 2014. The mechanics of PLGA nanofiber scaffolds with biomimetic gradients in mineral for tendon-to-bone repair. Journal of the Mechanical Behavior of Biomedical Materials, 40, pp.59-68.

[24] Kolluru, P.V., Lipner, J., Liu, W., Xia, Y., Thomopoulos, S., Genin, G.M. and Chasiotis, I., 2013. Strong and tough mineralized PLGA nanofibers for tendon-to-bone scaffolds. Acta Biomaterialia, 9(12), pp.9442-9450.

[25] Schwartz, A.G., Lipner, J.H., Pasteris, J.D., Genin, G.M. and Thomopoulos, S., 2013. Muscle loading is necessary for the formation of a functional tendon enthesis. Bone, 55(1), pp.44-51.

[26] Schwartz, A.G., Pasteris, J.D., Genin, G.M., Daulton, T.L. and Thomopoulos, S., 2012. Mineral distributions at the developing tendon enthesis. PLoS One, 7(11), p.e48630.

[27] Thomopoulos, S., Das, R., Birman, V., Smith, L., Ku, K., Elson, E.L., Pryse, K.M., Marquez, J.P. and Genin, G.M., 2011. Fibrocartilage tissue engineering: the role of the stress environment on cell morphology and matrix expression. Tissue Engineering Part A, 17(7-8), pp.1039-1053.

[28] Thomopoulos, S., Genin, G.M. and Galatz, L.M., 2010. The development and morphogenesis of the tendon-to-bone insertion What development can teach us about healing. Journal of Musculoskeletal & Neuronal Interactions, 10(1), pp.35-45.

[29] Linderman, S.W., Mikhail Golman, Donghwan Yoon, Victor Birman, Guy M. Genin, Stavros Thomopoulos. Strengthening tendon-to-bone repair with mechanically-optimized adhesives. 2017 Annual Meeting of the Orthopedic Research Society. San Diego, California March 19-22, 2017.

[30] Budynas, R., Advanced Strength and Applied Stress Analysis. New York: McGraw-Hill Science/Engineering/Math, 1998

[31] Hibbitt, D., Karlsson, B. and Sorensen, P., 2013. Abaqus/CAE user's guide.

[32] Thomopoulos, S., and G.M. Genin. "Tendon and Ligament Biomechanics." In Orthopedic Biomechanics. B. Winkelstein, ed. New York: Taylor & Francis, 2012.

[33] Liu, Y., Victor Birman, Changqing Chen, Stavros Thomopoulos, Guy M. Genin. On the mechanics of partially mineralized tissues and their implications for the attachment of tendon to bone. Paper SBC2011-53991. Proceedings of the ASME 2011 Summer Bioengineering Conference (SBC2011).

[34] Liu, Y., Schwartz, A.G., Birman, V., Thomopoulos, S. and Genin, G.M., 2014. Stress amplification during development of the tendon-to-bone attachment. Biomechanics and Modeling in Mechanobiology. 13(5), pp.973-983.

[35] Deymier-Black, A.C., Yiran An, Andrea G. Schwartz, Guy M. Genin, Stavros Thomopoulos, Asa H. Barber. Micrometer-scale Mechanical Properties Of The Tendon-to-bone Attachment. Paper SB3C2015-594. Summer Biomechanics, Bioengineering, and Biotransport Conference, Snowbird, Utah, June 17-20, 2015.

[36] Saadat, F., Birman, V., Thomopoulos, S. and Genin, G.M., 2015. Effective elastic properties of a composite containing multiple types of anisotropic ellipsoidal inclusions, with application to the attachment of tendon to bone. Journal of the Mechanics and Physics of Solids, 82, pp.367-377.

[37] Spencer, P., Ye, Q., Park, J., Parthasarathy, R., Marangos, O., Misra, A., Bohaty, B.S., Singh, V. and Laurence, J.S., 2013. Dentin/adhesive interface in teeth. In Structural Interfaces and Attachments in Biology (pp. 133-151). Springer New York.

[38] Lin, M., ShaoBao Liu, Feng Xu, TianJian Lu, BoFeng Bai, and Guy M. Genin. "Thermal Pain in Teeth: Heat Transfer, Thermomechanics and Ion Transport." In Transport in Biological Media, Sid M. Becker and Andrey V. Kuznetsov, eds, Amsterdam: Elsevier, 2013.

[39] Structural Interfaces and Attachments in Biology, S. Thomopoulos, V. Birman, and G. M. Genin, eds. New York: Springer, 2012.

[40] Babaei, B., Davarian, A., Lee, S.L., Pryse, K.M., McConnaughey, W.B., Elson, E.L. and Genin, G.M., 2016. Remodeling by fibroblasts alters the rate-dependent mechanical properties of collagen. Acta Biomaterialia, 37, pp.28-37.

[41] Babaei, B., Davarian, A., Pryse, K.M., Elson, E.L. and Genin, G.M., 2015. Efficient and optimized identification of generalized Maxwell viscoelastic relaxation spectra. Journal of the mechanical behavior of biomedical materials, 55, pp.32-41.

[42] Babaei, B., Abramowitch, S.D., Elson, E.L., Thomopoulos, S. and Genin, G.M., 2015. A discrete spectral analysis for determining quasi-linear viscoelastic properties of biological materials. Journal of The Royal Society Interface, 12(113), p.20150707.

[43] Lin, M., Genin, G.M., Xu, F. and Lu, T., 2014. Thermal Pain in Teeth: Electrophysiology Governed by Thermomechanics. Applied mechanics reviews, 66(3), p.030801.

[44] Williams, M.L., 1952. Stress singularities resulting from various boundary conditions. Journal of applied mechanics, 19(4), pp.526-528.

[45] Buehler, M.J. and Genin, G.M., 2016. Integrated multiscale biomaterials experiment and modelling: a perspective. Interface Focus, 6(1), p.20150098.

[46] de Borst, R., Remmers, J.J. and Needleman, A., 2006. Mesh-independent discrete numerical representations of cohesive-zone models. Engineering fracture mechanics, 73(2), pp.160-177.

[47] "Pythonidae" Wikipedia: The Free Encyclopedia. Wikimedia Foundation, Inc. 22 July 2004. Web. 10 Aug. 2004, en.wikipedia.org/wiki/Pythonidae

[48] Hillson, Simon. Mammal bones and teeth: an introductory guide to methods of identification. Routledge, 2016.

[49] A.E. Anderson, Complete Tyrannosaurus rex skull, AMNH 5027, American Museum of Natural History, 1912.

[50] Seifert, Ashley W., Stephen G. Kiama, Megan G. Seifert, Jacob R. Goheen, Todd M. Palmer, and Malcolm Maden. "Skin shedding and tissue regeneration in African spiny mice (*Acomys*)." Nature 489, no. 7417 (2012): 561.

[51] Huang, G., Li, F., Zhao, X., Ma, Y., Li, Y., Lin, M., Jin, G., Lu, T.J., Genin, G.M. and Xu, F., 2017. Functional and Biomimetic Materials for Engineering of the Three-Dimensional Cell Microenvironment. *Chemical reviews*, *117*(20), pp.12764-12850.

[52] Lipner, J., Boyle, J.J., Xia, Y., Birman, V., Genin, G.M. and Thomopoulos, S., 2017. Toughening of fibrous scaffolds by mobile mineral deposits. *Acta biomaterialia*, *58*, pp.492-501.

[53] Genin, Guy M., and Stavros Thomopoulos. "The tendon-to-bone attachment: Unification through disarray." *Nature materials* 16, no. 6 (2017): 607.

[54] Shakiba, D., Babaei, B., Saadat, F., Thomopoulos, S. and Genin, G.M., 2017. The fibrous cellular microenvironment, and how cells make sense of a tangled web. *Proceedings of the National Academy of Sciences*, *114*(23), pp.5772-5774.

[55] Linderman, S., Genin, G., Thomopoulos, S., Ahn, K. and Birman, V.M., Washington University, 2017. *COMPOSTIONS AND METHODS FOR TISSUE REPAIR*. U.S. Patent Application 15/455,792.

[56] Babaei, B., Velasquez-Mao, A.J., Thomopoulos, S., Elson, E.L., Abramowitch, S.D. and Genin, G.M., 2017. Discrete quasi-linear viscoelastic damping analysis of connective tissues, and the biomechanics of stretching. *Journal of the mechanical behavior of biomedical materials*, *69*, pp.193-202.

[57] Linderman, S.W., Golman, M., Gardner, T.R., Birman, V., Levine, W.N., Genin, G.M. and Thomopoulos, S., 2018. Enhanced tendon-to-bone repair through adhesive films. *Acta biomaterialia*, *70*, pp.165-176.

[58] Deymier, A.C., An, Y., Boyle, J.J., Schwartz, A.G., Birman, V., Genin, G.M., Thomopoulos, S. and Barber, A.H., 2017. Micro-mechanical properties of the tendon-to-bone attachment. *Acta biomaterialia*, *56*, pp.25-35.

[59] Rossetti, L., Kuntz, L.A., Kunold, E., Schock, J., Müller, K.W., Grabmayr, H., Stolberg-Stolberg, J., Pfeiffer, F., Sieber, S.A., Burgkart, R. and Bausch, A.R., 2017. The microstructure and micromechanics of the tendon–bone insertion. Nature materials, 16(6), p.664.

[60] Tang, S.Y., 2017. Natural composites: The structure-function relationships of bone, cartilage, tendon/ligament, and the intervertebral disc. In *Biomedical Composites (Second Edition)*(pp. 1-16). New York: Elsevier.